Homework 11

Do the problems on webwork and turn the following problems in class on Dec. 10th.

Homework should be written neatly and clearly explained. If it requires more than one sheet, the sheets must be stapled. Include your name and id number in the top right corner of your homework.

Problem 1. Let $X_1, \dots, X_n$ be independent, identically distributed random variables with probability density function

$$f(x) = \begin{cases} 2\theta^2 x^{-3}, & \text{if } x > \theta, \\ 0, & \text{otherwise} \end{cases}$$

where θ is a constant greater than 0. Let $X_{(1)} = \min\{X_1, ..., X_n\}$. Compute the probability density function for $X_{(1)}$ and $\mathbb{E}[X_{(1)}]$.

Problem 2. One hundred athletes is trying to qualify for the finals. Each of them has to pass the preliminary stage and the advanced stage. Each athlete passes the preliminary stage with the probability 0.7 and the advanced stage with probability 0.2 independently. Each athlete passes stages independently of all other athletes.
Let N be the number of athletes that pass the preliminary stage and pass the advanced stage.
By the Central Limit Theorem we expect N to be approximately a normal random variable. Use the normal distribution to estimate the probability $P(N \geq 20)$.

Problem 3. Let $X_1, \dots, X_5$ be i.i.d. random variables uniformly disturbed on the interval $[0, 2]$. Let $X_{(1)} = \min\{X_1, \dots, X_5\}$. Simulate 1000 independent realization of $X_{(1)}$ and plot a histogram of the simulation. What is the empirical mean of your random sample?