Homework 3- Due Sept. 24th.

Do problems 2.1.8, 2.1.14, 2.1.15, 2.2.1, 2.2.4, 2.2.8 from Durrett

and the following:

1. Let X be an exponential r.v. with parameter λ, i.e., $F_X(x) = (1 - e^{-\lambda x})1_{[0,\infty)}(x)$. Define the random variables

$$Y = \lfloor X \rfloor := \sup\{n \in \mathbb{Z} : n \leq x\} \text{ (the integer part of X)},$$

$$Z = X - \lfloor X \rfloor \text{ (the fractional part of X)}.$$

   (a) Compute the distributions of Y and of Z (for Y, its more natural to give the pmf: $\mathbb{P}(Y = n), n = 0, 1, 2, ...$; for Z compute either the distribution function or density function).

   (b) Show that Y and Z are independent.

2. Prove that if X is a random variable that is independent of itself, then X is a.s. constant.