Homework 7- Due Oct. 29.

Do problems 3.3.13, 3.3.16, 3.4.4, 3.4,6, 3.4.12 from Durrett, and the following

1. Let $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ be the density function of the standard normal distribution, and let $\Phi(x) = \int_{\infty}^{x} f(u)du$ be its C.D.F. Prove the inequalities

$$\frac{1}{x + x^{-1}} f(x) \leq 1 - \Phi(x) \leq \frac{1}{x} f(x), \text{ for } (x > 0)$$

Hint: To prove the upper bound, use the fact that for $t > x$ we have $e^{-t^2/2} \leq (t/x)e^{-t^2/2}$. For the lower bound, use the identity

$$\frac{d}{dx}\left(\frac{e^{-x^2/2}}{x}\right) = -\left(1 + \frac{1}{x^2}\right) e^{-x^2/2}$$

to compute $\int_x^{\infty}(1 + u^{-2})e^{-u^2/2}du$, which can then be bounded by $(1 + x^{-2}) \int_x^{\infty} e^{-u^2/2}du$.

Optional- Can you find a better bound?