# Quiz $\sharp 4$

**Name (Print):** ______________________

**Problem 1** Let $A$ be a $3 \times 4$ matrix and $T$ be the linear transformation with standard matrix $A$. Determine whether the following statements are True or False:

1. ________ Let $b \in \mathbb{R}^3$. Then $b$ is in the range of $T$ if and only if the equation $Ax = b$ has at least one solution.
2. ________ The equation $Ax = \mathbb{O}$ has infinitely many solutions as $A$ has more columns than rows.
3. ________ $T$ is a one to one function as the columns of $A$ are linearly dependent.
4. ________ If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are vectors in $\mathbb{R}^4$ and the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent, then the set $\{T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3)\}$ is linearly dependent.
5. ________ A map $S : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is a linear transformation if and only if

$$S(c_1\mathbf{v}_1 + c_2\mathbf{v}_2) = c_1 S(\mathbf{v}_1) + c_2 S(\mathbf{v}_2)$$

for all vectors $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^n$ and scalars $c_1, c_2$.

**Problem 2** Let $T$ be a linear transformation with standard matrix

$$A = \begin{pmatrix} -1 & 2 & 1 & 0 & 3 \\ 0 & 2 & -4 & 6 & 2 \\ 0 & 0 & 0 & 2 & -2 \end{pmatrix},$$

that is, $T(x) = Ax$. Answer the following questions:

1. Determine the domain and codomain of $T$.
2. Do the columns of $A$ span $\mathbb{R}^3$? Is $T$ onto?
3. Find all vectors $x$ such that $T(x) = \mathbb{O}$. Write the answers in parametric vector form. Is $T$ one to one?