Math 401-02 **Practice 1** Fall 2021

(Due: **Tuesday, August 31**)

Instruction: Work in small groups

Problem 1. Determine whether each of the following set with the indicated binary operation is a group or not.

(1) $G = \mathbb{R} - \{0\}$ and for $x, y \in G$, $x * y = xy/3$.
(2) $G = \mathbb{R} - \{0\}$ and for $x, y \in G$, $x * y = x^2 y$.
(3) $G = \mathbb{R}^+$ and $x \circ y = x\sqrt{y}$ for all $x, y \in G$.

Problem 2. Let

$$G = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, c \in \mathbb{R} - \{0\}, b \in \mathbb{R} \right\}$$

and let H be a subset of G in which the entries on the main diagonal are the same, that is, $a = c$. Determine whether G and H are groups under matrix multiplication or not.

Problem 3. Let

$$G = \left\{ \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} : a \in \mathbb{Z} \right\}.$$

Show that G is a group with the usual matrix multiplication.