Math 401-02 | **Practice 3** | Fall 2021

(Due: **Tuesday, Sep. 21**)

Instruction: Work in small groups

Problem 1. Let G be a group and let $a \in G$. The centralizer of a in G, denoted by $\mathbf{C}_G(a)$, is the set of all elements of G that commute with a, that is,

$$\mathbf{C}_G(a) = \{x \in G : xa = ax\}.$$

(1) Show that $\mathbf{C}_G(a)$ is a subgroup of G.
(2) Show that $\mathbf{C}_G(a) = \mathbf{C}_G(a^{-1})$.
(3) Let $G = \mathrm{S}_3$. Find $\mathbf{C}_G(g)$ for each $g \in G$.

Problem 2. Find the order of each element in $G = (\mathbb{Z}_{18}^*, \cdot)$. Is G a cyclic group? List all distinct cyclic subgroups of G.

Problem 3. Let H be a subgroup of a group G. Let $x \in G$ and define

$$xHx^{-1} = \{xhx^{-1} : h \in H\}.$$

(1) Show that xHx^{-1} is a subgroup of G. This subgroup is called a conjugate of H.
(2) Show that $|xHx^{-1}| = |H|$ if $|H|$ is finite.

Problem 4. Let p be a prime and let a, b be integers. Assume that p divides ab. Prove that p divides a or p divides b. (Do not use the Fundamental Theorem of Arithmetic).