Math 401-02 **Homework 1** Fall 2021

(Due: **Wednesday, September 1**)

Instruction: Turn in All problems. Use Tex/Latex.

Problem 1. Determine whether each of the following set with the indicated binary operation is a group or not.

(1) $G = \mathbb{R}$ and for $x, y \in G$, $x * y = x + y - 1$.
(2) $G = \mathbb{R}$ and for $x, y \in G$, $x * y = x + y + xy$.
(3) $G = \mathbb{R} - \{-1\}$ and for $x, y \in G$, $x * y = x + y + xy$.

Problem 2. For $a, b \in \mathbb{R}$ with $a \neq 0$, define $f_{a,b} : \mathbb{R} \to \mathbb{R}$ by $f_{a,b}(x) = ax + b$ for all $x \in \mathbb{R}$.

(1) Show that $f_{a,b} \circ f_{c,d} = f_{u,v}$ for some real numbers u, v. Give explicit values for u, v in terms of a, b, c, d.
(2) Show that $f_{a,b}^{-1}$ exists and find $c, d \in \mathbb{R}$ such that $f_{a,b}^{-1} = f_{c,d}$.
(3) Let $G = \{f_{a,b} : a, b \in \mathbb{R}, a \neq 0\}$. Show that G is a group under the composition of functions.
(4) Is G an abelian group?

Problem 3. Let G be an abelian group and let $f \in G$ be a fixed element in G. Define a new binary operation on G by $a \circ b := abf$ for all $a, b \in G$. Determine whether $(G, \circ)$ is a group or not.

Problem 4. Let

$$G = \left\{ \begin{pmatrix} a & b \\ 1-a & 1-b \end{pmatrix} : a, b \in \mathbb{R}, a \neq b \right\}.$$

Is G a group under the usual matrix multiplication? Explain.