Math 401-02 **Homework 2** Fall 2021

(Due: **Wednesday, September 15**)

Instruction: Turn in All problems. Use Tex/Latex if possible.

Problem 1. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 1 & 8 & 6 & 4 & 5 & 3 & 7 \end{pmatrix} \in \mathbf{S}_8$.

(1) Write σ as a product of disjoint cycles.
(2) Compute $\sigma^{-1}, \rho^{-1}, \sigma\rho, \sigma^{-1}\rho^{-1}, (\sigma\rho)^{-1}$, $\rho\sigma\rho^{-1}$ and σ^6, where $\rho = (136)(487)$.

Problem 2.

(1) Find $\gcd(27, 186)$ and integers u, v such that $\gcd(27, 186) = 27u + 186v$.
(2) Find the multiplicative inverse of $[37]$ in $\mathbb{Z}_{2021}$ if it exists.
(3) Let a, b be positive integers and let $d = \gcd(a, b)$. Prove that $\gcd(a/d, b/d) = 1$.
(4) Let a, b, n be integers with $n > 1$. Show that $\gcd(ab, n) = 1$ if and only if $\gcd(a, n) = 1$ and $\gcd(b, n) = 1$.

Problem 3. Construct the Cayley tables of the following groups. Use the tables to compute the inverse of each element in the group.

(1) $(\mathbb{Z}_8, +)$.
(2) $(\mathbb{Z}_{15}^*, \cdot)$.
(3) The symmetric group S_3 under the composition of functions.

Problem 4. Let G be a group. Show that G is abelian if one of the following conditions holds.

(1) $(ab)^{-1} = a^{-1}b^{-1}$ for all $a, b \in G$.
(2) $ab = ca$ implies $b = c$ for all $a, b, c \in G$.
(3) $(ab)^i = a^ib^i$ for three consecutive positive integers i, for all $a, b \in G$.