MATH 304-01 **Name (Print):** ______________________

Fall 2019

Test 1

September 25, 2019

Time Limit: 60 Minutes

This test contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page. Question 1 is a **True/False** question. Clearly PRINT your correct answer. Question 2 is multiple choice.

You are required to show your work on Questions 3 – 5 on this exam. Question 6 is a bonus question. Only do this question when you finish other questions. The following rules apply:

- **Turn off and put away your cell phone**.
- **No note nor book is allowed**.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**.

Do not write in the table to the right.

Question	Points	Score
1	30	
2	15	
3	20	
4	15	
5	20	
6	10	
Total:	110	

1. (30 points) For each statement below, indicate whether it is **True** or **False**:

(a) ________ If the matrix equation $A\mathbf{x} = \mathbf{b}$ is inconsistent, then $\mathbf{b}$ is not a linear combination of the columns of A.

(b) ________ The columns of a matrix A are linearly independent if and only if the equation $A\mathbf{x} = \mathbb{O}$ has the trivial solution.

(c) ________ If $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^3$ are linearly independent vectors and if v_3 is a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent.

(d) ________ If A is a 4×6 matrix, then the columns of A are linearly dependent.

(e) ________ If $\mathbf{x}$ is a nontrivial solution of the homogeneous equation $A\mathbf{x} = \mathbb{O}$, then every entry in $\mathbf{x}$ is nonzero.

(f) ________ Let A and B be matrices. If AB is defined, then $(AB)^T = A^T B^T$.

(g) ________ If A is a square $n \times n$ matrix, then the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is an onto map from $\mathbb{R}^n$ to $\mathbb{R}^n$.

(h) ________ Let A be a square $n \times n$ matrix. If the equation $A\mathbf{x} = \mathbf{b}$ is consistent for some vector $\mathbf{b} \in \mathbb{R}^n$, then A is invertible.

(i) ________ If the columns of a square $n \times n$ matrix A are linearly independent, then A^T is invertible.

(j) ________ If A and B are square $n \times n$ matrices and $AB = I_n$, then both A and B are invertible.

2. (15 points) Answer the following multiple choice questions.

(i) Consider a system of linear equations with **augmented** matrix

$$C = \left(\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 3 \\ 0 & 2 & 0 & -3 & 1 \\ 0 & 0 & -1 & 4 & -3 \end{array}\right).$$

Circle ALL the **correct** answers.

(a) This system has 4 variables and 3 equations.

(b) This system has two free variables.

(c) The system has a unique solution since C has a pivot in every row.

(d) The system has infinitely many solutions since it has a free variable.

(e) This system is inconsistent since it has a nonzero row.

(ii) Let T be a linear transformation with standard matrix

$$A = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 3 & 4 \end{pmatrix}.$$

Circle **ALL the correct** answers.

(a) The domain of T is $\mathbb{R}^3$.

(b) The codomain of T is $\mathbb{R}^3$.

(c) T is one to one.

(d) T is onto.

(e) T is invertible since A has a pivot position in every row.

(iii) Let A be a square 4×4 matrix. Assume that the equation $A\mathbf{x} = \mathbf{b}$ is **inconsistent** for some vector $\mathbf{b} \in \mathbb{R}^4$. Which of the following statements are **correct**?

(a) The equation $A\mathbf{x} = \mathbb{O}$ has only trivial solution.

(b) The equation $A\mathbf{x} = \mathbb{O}$ has infinitely many solutions.

(c) The columns of A do not span $\mathbb{R}^4$.

(d) The columns of A span $\mathbb{R}^4$.

(e) A is not invertible.

3. (20 points) Let $T : \mathbb{R}^4 \longrightarrow \mathbb{R}^3$ be a linear transformation with standard matrix

$$A = \begin{pmatrix} 1 & 0 & 1 & -1 \\ 2 & 1 & 3 & 3 \\ 1 & -1 & 1 & -4 \end{pmatrix}.$$

(a) Find the reduced row-echolon form (RREF) of A. List all the elementary row operations performed.

(b) Which columns of A are pivot columns?

(c) Find all vectors $\mathbf{x} \in \mathbb{R}^4$ such that $T(\mathbf{x}) = \mathbb{O}$. Write the answers in parametric vector form.

(d) Is T one to one ? onto? Justify your answer.

4. (15 points) Let $A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 4 \\ 1 & -1 & 4 \end{pmatrix}$.

1. Determine whether A is invertible or not. Find A^{-1} if it exists.
2. Is A^T invertible? Find $(A^T)^{-1}$ if it exists.

5. (20 points) Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 + x_3 \\ 2x_2 + x_3 \\ x_1 + 5x_2 + 4x_3 \end{bmatrix}.$$

(a) Determine the standard matrix A for T.

(b) Show that T is not onto. Describe the set of all $\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \in \mathbb{R}^3$ for which $\mathbf{y}$ is in the range of T.

6. (10 points) (Bonus) Let A be an $m \times n$ matrix with $1 \leq m < n$. Is it possible to find an $n \times m$ matrix B such that $BA = I_n$? Explain.