Math 402 | **Practice 1** | Spring 2021

(Due: Wed, Feb. 16)

Problem 1. Let $S = M_2(\mathbb{R})$ be the matrix ring over the real numbers. Which of the following subsets of S are rings under the same matrix addition and matrix multiplication as in S.

(1) $S_1 = \{A \in S : A^T = A\}$, the set of all symmetric matrices in S.
(2) $S_2 = \{A \in S : A^T = -A\}$, the set of all skew-symmetric matrices in S.
(3) The set S_3 of all upper triangular matrices in S.
(4) The set S_4 of all strict upper triangular matrices in S.

Problem 2. Let R be a ring such that $x^2 = x$ for all $x \in R$. Show that

(1) Let $x \in R$. Evaluate $(x + x)^2$. Show that $x + x = 0$.
(2) Evaluate $(x + y)^2$ for $x, y \in R$. Show that R is commutative.

Problem 3. Let R be a commutative ring with identity. Suppose that there exists an integer $n > 1$ such that $x^n = x$ for all $x \in R$. If m is a positive integer such that $a^m = 0$ for some $a \in R$, show that $a = 0$.

Problem 4. Let $R = \{a + bi : a, b \in \mathbb{Z}_3\}$, where $i^2 = -1$. Is R a commutative ring with identity? Is R a field?