Math 402 **Practice 2** Spring 2021

(Due: Wed, Feb. 23)

Problem 1. Find all idempotents elements of the rings $\mathbb{Z}_9, \mathbb{Z}_{10}$ and $\mathbb{Z} \times \mathbb{Z}$.

Problem 2. Find all units of the rings $\mathbb{Z}_4 \times \mathbb{Z}_9$ and $\mathbb{Z} \times \mathbb{Z}$.

Problem 3. Show that there is no integral domain with exactly 6 elements.

Problem 4. Let R be a ring that contains at least 2 elements. Suppose that for each nonzero $a \in R$, there exists a unique $b \in R$ such that $aba = a$.

(a) Show that R has no zero divisor.

(b) Show that $bab = b$.

(c) Show that R has an identity.

(d) Show that R is a division ring.

Problem 5. Let $R = \mathbb{Z}/36\mathbb{Z}$ and let I be the ideal generated by 3 in R. Show that as a group, $(I, +)$ is isomorphic to $(\mathbb{Z}/12\mathbb{Z}, +)$. However, the ring $(I, +, \cdot)$ is not isomorphic to the ring $(\mathbb{Z}/12\mathbb{Z}, +, \cdot)$.