Math 402 **Practice 4** Spring 2021

(Due: Tuesday, March. 16)

Problem 1.

(1) Let $f(x) = x^3 + 3x + 2$. Is $f(x)$ irreducible in $\mathbb{F}_7[x]$? How's about $\mathbb{F}_{19}$?
(2) Factor $x^5 + x + 1$ into irreducible polynomials in $\mathbb{F}_2[x]$.

Problem 2. Find the greatest common divisor of $f(x) = 3x^4+x^3+2x^2+1$ and $g(x) = x^2+4x+2$ in $\mathbb{F}_5[x]$.

Problem 3. Let D be a Euclidean domain with degree function d. Assume $u \in D$. Show that u is a unit if and only if $d(u) = d(1)$.

Problem 4. Let R be a commutative ring with identity and let I be an ideal of R.

(1) Show that $I[x]$ is an ideal of $R[x]$.
(2) If I is a prime ideal of R, then $I[x]$ is a prime ideal of $R[x]$.