Math 402 **Practice 5** Spring 2021

(Due: Tuesday, March. 23)

Problem 1. Let $a, b \in \mathbb{Z}$ and $f(x) = x^3 + ax^2 + bx + 1 \in \mathbb{Z}[x]$. For which values of a and b is $f(x)$ reducible in $\mathbb{Z}[x]$? What about in $\mathbb{Q}[x]$?

Problem 2. Find a polynomial in $\mathbb{Z}[x]$ that has $\sqrt{2} + \sqrt{3}$ as a root. Use it to prove that $\sqrt{2} + \sqrt{3}$ is irrational.

Problem 3. Let R be an integral domain. Show that $f(x) \in R[x]$ is irreducible if and only if $f(x + 1)$ is irreducible.

Problem 4. Determine whether the following polynomials are irreducible in the given ring or not.

(1) $f(x) = x^4 + x^3 + x^2 + x + 1$ in $\mathbb{Q}[x]$
(2) $f(x) = x^6 + 4x^3 + 1$ in $\mathbb{Q}[x]$