Math 402 **Practice 6** Spring 2021

(Due: Tuesday, March. 30)

Problem 1. Determine whether the following polynomials are irreducible in the given ring or not.

(1) $f(x) = 3x^5 + 30x^4 - 20x^3 + 10x + 20$ in $\mathbb{Q}[x]$.
(2) $f(x) = x^4 + x + 1$ in $\mathbb{Q}[x]$

Problem 2. Let $\alpha = \sqrt{1+\sqrt{5}}$.

(1) Find a monic polynomial $f(x) \in \mathbb{Z}[x]$ of minimal degree such that $f(\alpha) = 0$.
(2) Verify that $f(x)$ is irreducible in $\mathbb{Q}[x]$.
(3) Show that the ring $\mathbb{Q}[\alpha] = \{g(\alpha) : g \in \mathbb{Q}[x]\}$ is isomorphic to $\mathbb{Q}[x]/\langle f(x)\rangle$. Deduce that $\mathbb{Q}[\alpha]$ is a field.

Problem 3. Let a, b be real numbers and let $f \in \mathbb{R}[x]$. Prove that if $a + bi$ is a root of f, then $a - bi$ is a root of f.