Other Matroids on Graphs and Signed, Gain, and Biased Graphs

Selections from
A Mathematical Bibliography of
Signed and Gain Graphs and Allied Areas
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Subject Classification Codes

A code in *lower case* means the topic appears implicitly but not explicitly. A code may be refined through being suffixed by a parenthesised code, as **S**(**M**) denoting signed matroids (while **S**: **M** would indicate matroids of signed objects; thus **S**(**M**): **M** means matroids of signed matroids).

A Adjacency matrix, eigenvalues.
Alg Algorithms.
Aut Automorphisms, symmetries, group actions.
Bic Bicircular matroids.
Col Vertex coloring.
D Duality (graphs, matroids, or matrices).
E Enumeration of types of signed graphs, etc.
EC Even-circle matroids.
Exp Expository.
Exr Interesting exercises (in an expository work).
G Connections with geometry, including linear programming, toric varieties, complex complement, etc.
Gen Generalization.
GG Gain graphs, voltage graphs, biased graphs; includes Dowling lattices.
GN Generalized or gain networks. (Multiplicative real gains.)
I Incidence matrix, Kirchhoff or Laplacian matrix.
M Matroids and geometric lattices, chain-groups, flows.
MF Matroidal families.
N Numerical and algebraic invariants (e.g., counts, polynomials, degrees) of signed, gain, biased graphs: number of bases, etc.
O Orientations, bidirected graphs.
P All-negative or antibalanced signed graphs; parity-biased graphs.
Ref Many references.
SG Signed graphs: mathematical properties.
SM Signed matroids.
Str Structure theory.
T Topology applied to graphs; surface embeddings. (Not applications to topology.)
X Extremal problems.

Standard symbols and terminology:

Γ is a graph (V, E) of order $n = |V|$, undirected, possibly allowing loops and multiple edges. It is normally finite unless otherwise indicated.
Σ is a signed graph (V, E, σ).
Φ is a gain graph (V, E, φ). $\|\Phi\|$ is its underlying graph.
Ω is a biased graph. $\|\Omega\|$ is its underlying graph.
$G(\ )$ is the frame (bias) matroid of a signed, gain, or biased graph.

$L(\ )$, $L_0(\ )$ are the lift and extended lift matroids.
polygon, circle: The graph of a simple closed path, or its edge set.
cycle: In an oriented graph or signed, gain, or biased graph, a frame matroid circuit oriented to have no source or sink.

A Mathematical Bibliography of
Signed and Gain Graphs and Allied Areas

Ravindra K. Ahuja, Thomas L. Magnanti, and James B. Orlin

1993a *Network Flows: Theory, Algorithms, and Applications.* Prentice Hall, Englewood Cliffs, N.J., 1993. MR 94e:90035.

Ch. 16: "Generalized flows". Sect. 15.5: "Good augmented forests and linear programming bases", Thm. 15.8, makes clear the connection between flows with gains and the frame matroid of the underlying gain graph. Some terminology: "breakeven cycle" = balanced polygon; "good augmented forest" = basis of the frame matroid, assuming the gain graph is connected and unbalanced. (**GN: M(Bases), Alg: Exp, Ref**)

Martin Aigner

1979a *Combinatorial Theory.* Grundl. math. Wiss., Vol. 234. Springer-Verlag, Berlin, 1979. Reprint: Classics in Mathematics. Springer-Verlag, Berlin, 1997. MR 80h:05002. Zbl. 415.05001, 858.05001 (reprint).

In §VII.1, pp. 333–334 and Exerc. 13–15 treat the Dowling lattices of $\mathrm{GF}(q)^\times$ and higher-weight analogs. (**GG: Gen: M: N, Str**)

Gautam Appa, Balázs Kotnyek, Konstantinos Papalamprou, and Leonidas Pitsoulis

2007a Optimization with binet matrices. *Operations Res. Letters* 35 (2007), 345–352. MR 2008a:90052. (**O: I(Gen), m**)

Thomas Andreae

1978a Matroidal families of finite connected nonhomeomorphic graphs exist. *J. Graph Theory* 2 (1978), 149–153. MR 80a:05160. Zbl. 401.05070.

Partially anticipates the "count" matroids of graphs (see Whiteley (1996a)). (**MF, Bic, EC**)

Christos A. Athanasiadis

†1996a Characteristic polynomials of subspace arrangements and finite fields. *Adv. Math.* 122 (1996), 193–233. MR 97k:52012. Zbl. 872.52006.

Treats the canonical lift representations (as affine hyperplane arrangements) of various gain graphs and signed gain graphs with additive gain group $\mathbb{Z}^+$. The article is largely a series of (sometimes brilliant) calculations of chromatic polynomials (*mutatis mutandis*, the characteristic polynomials of the representing arrangements) modulo a large integer q using gain graph coloring, though disguised as applications of Crapo–Rota's Critical Theorem. The fundamental principle is that, if q is larger than the largest gain of a circle, then $\mathbb{Z}^+$ can be replaced as gain group by $\mathbb{Z}_q^+$ without changing the chromatic polynomial (a consequence of Zaslavsky (1995b), Thm. 4.2)—and the analog for signed gain graphs, whose theory needs to be developed. A non-graphical result of the general method is a unified proof (Thm. 2.4) of the theorem of Blass and Sagan (1998a).

§3: "The Shi arrangements": these represent $\mathrm{Lat}^{\mathrm{b}}\{0,1\}\vec{K}_n$ and signed-graph analogs. §4: "The Linial arrangement": this represents $\mathrm{Lat}^{\mathrm{b}}\{1\}\vec{K}_n$. §5: "Other interesting hyperplane arrangements", treats: the arrangement representing $\mathrm{Lat}^{\mathrm{b}}\, AK_n$ where $A = \{-m, \dots, m{-}1, m\}$ [which is the semilattice of m-composed partitions; see Zaslavsky (2002a), Ex. 10.5, also Edelman and Reiner (1996a)], and several generalizations, including to arbitrary sign-symmetric gain sets L and to Weyl analogs; also, an antibalanced analog of the A_n Shi arrangement (Thm. 5.4); and more. Most impressive result: Thm. 5.2: Let A be a finite set of integers such that $0 \notin A = -A$ and let $A^0 = A \cup \{0\}$. For $\Phi = A^0 K_n$ and large integral λ, $\chi^*_\Phi(\lambda)/\lambda$ is the coefficient of $x^{\lambda-n}$ in $(1-x)^{-1} - f_A(x)/x$ where f_A is the ordinary generating function for A. From this $\chi^*_{AK_n}(\lambda)/\lambda$ is derived.

[The signed affinographic arrangements represent a kind of signed gain graph whose exact nature has not yet been penetrated by gain graph theory.] **(sg, gg: G, M, N)**

1997a A class of labeled posets and the Shi arrangement of hyperplanes. *J. Combin. Theory Ser. A* 80 (1997), 158–162. MR 98d:05008. Zbl. 970.66662.

The arrangement represents $\mathrm{Lat}^{\mathrm{b}}\{0,1\}\vec{K}_n$. **(gg: G, M, N)**

1998a On free deformations of the braid arrangement. *European J. Combin.* 19 (1998), 7–18. MR 99d:52008. Zbl. 898.52008.

The arrangements considered are the subarrangements of the projectivized Shi arrangements of type A_{n-1} that contain A_{n-1}. Thms. 4.1 and 4.2 characterize those that are free or supersolvable. The extended Shi arrangements, representing $L_0([1-a,a]\vec{K}_n)$ where $a \geq 1$, and a mild generalization, are of use in the proof. **(gg: G, M, N)**

1998b On noncrossing and nonnesting partitions for classical reflection groups. *Electronic J. Combin.* 5 (1998), Research Paper R42, 16 pp. (electronic).

§5, "Nonnesting partitions of fixed type", has calculations like those in (1996a) for affinographic arrangements representing additional types of gain graphs [of a kind that is not yet fully understood]. **(gg: G, m, N)**

1999a Extended Linial hyperplane arrangements for root systems and a conjecture of Postnikov and Stanley. *J. Algebraic Combin.* 10 (1999), 207–225. MR 2000i:52039. Zbl. 948.52012. **(gg: G, m, N)**

1999b Piles of cubes, monotone path polytopes, and hyperplane arrangements. *Discrete Comput. Geom.* 21 (1999), no. 1, 117–130. MR 99j:52015. Zbl. 979.52002.

The proof of Proposition 4.2 is essentially gain-graphic. **(gg: m: G: N)**

2000a Deformations of Coxeter hyperplane arrangements and their characteristic polynomials. In: *Singularities and Arrangements* (Proc., Arrangements – Tokyo, 1998), pp. 1–26. Adv. Studies Pure Math., 27. 2000. **(gg: G, m, N)**

G. David Bailey

20xxa Inductively factored signed-graphic arrangements of hyperplanes. Submitted.

Continues Edelman and Reiner (1994a). **(SG: G, M)**

V. Balachandran

1976a An integer generalized transportation model for optimal job assignment in computer networks. *Oper. Res.* 24 (1976), 742–759. MR 55 #12068. Zbl. 356.90028. **(GN: M(bases))**

V. Balachandran and G.L. Thompson

1975a An operator theory of parametric programming for the generalized transportation problem: I. Basic theory. II. Rim, cost and bound operators. III. Weight operators. IV. Global operators. *Naval Res. Logistics Quart.* 22 (1975), 79–100, 101–125, 297–315, 317–339. MR 52 ##2595, 2596, 2597, 2598. Zbl. 331.90048, 90049, 90050, 90051. **(GN: M)**

Egon Balas

1966a The dual method for the generalized transportation problem. *Management Sci.* 12 (1966), no. 7 (March, 1966), 555–568. MR 32 #7232. Zbl. 142, 166 (e: 142.16601). **(GN: M(bases))**

E. Balas and P.L. Ivanescu [P.L. Hammer]

1965a On the generalized transportation problem. *Management Sci.* 11 (1965), no. 1 (Sept., 1964), 188–202. MR 30 #4599. Zbl. 133, 425 (e: 133.42505). **(GN: M)**

Matthias Beck and Thomas Zaslavsky

2006a Inside-out polytopes. *Advances in Math.* 205 (2006), no. 1, 134–162. MR 2007e:-52017. Zbl. 1107.52009.

§5: "In which we color graphs and signed graphs." A geometric interpretation of signed graph coloring by lattice points and hyperplane arrangements unifies the chromatic and zero-free chromatic polynomials and gives immediate proofs of theorems on the chromatic polynomials and acyclic orientations. **(SG: Col: G, M: N)**

2006b The number of nowhere-zero flows in graphs and signed graphs. *J. Combin. Theory Ser. B* 96 (2006), no. 6, 901–918. MR 2007k:05084. Zbl. 1119.05105.

The nowhere-zero flow polynomial of a signed graph, for flows in an odd abelian group, and the integral nowhere-zero flow quasipolynomial with period 2. **(SG: Flows: G: M: N)**

Curtis Bennett and Bruce E. Sagan

1995a A generalization of semimodular supersolvable lattices. *J. Combin. Theory Ser. A* 72 (1995), 209–231. MR 96i:05180. Zbl. 831.06003.

To illustrate the generalization, most of the article calculates the chromatic polynomial of $\pm K_n^{(k)}$ (called $\mathcal{DB}_{n,k}$; this has half edges at k vertices), builds an "atom decision tree" for $k = 0$, and describes and counts the bases of $G(\pm K_n^{(k)})$ (called $\mathcal{D}_n$) that contain no broken circuits. **(SG: M, N, col)**

M.K. Bennett, Kenneth P. Bogart, and Joseph E. Bonin

1994a The geometry of Dowling lattices. *Adv. Math.* 103 (1994), 131–161. MR 95b:05050. Zbl. 814.51003.

Drawing an analogy between Desargues' and Pappus' theorems in projective spaces and similar incidence theorems in Dowling geometries. [The rigorous avoidance of gain graphs makes the results less obvious than they could be.] **(gg: M, G)**

Moussa Benoumhani

1996a On Whitney numbers of Dowling lattices. *Discrete Math.* 159 (1996), 13–33. MR 98a:06005. Zbl. 861.05004. **(gg: M: N)**

1997a On some numbers related to Whitney numbers of Dowling lattices. *Adv. Appl. Math.* 19 (1997), 106–116. MR 98f:05004. Zbl. 876.05001.

Generating polynomials and infinite generating series for multiples of Whitney numbers of the second kind, analogous to usual treatments of Stirling numbers. **(gg: M: N)**

1999a Log-concavity of Whitney numbers of Dowling lattices. *Adv. Appl. Math.* 22 (1999), 186–189. MR 2000i:05008. Zbl. 918.05003.

Logarithmic concavity of Whitney numbers of the second kind is deduced by proving that their generating polynomial has only real zeros. [Cf. Dur (1986a).] **(gg: M: N)**

Anders Björner and Bruce E. Sagan

1996a Subspace arrangements of type B_n and D_n. *J. Algebraic Combin.* 5 (1996), 291–314. MR 97g:52028. Zbl. 864.57031.

They study lattices $\Pi_{n,k,h}$ (for $0 < h \leq k \leq n$) consisting of all spanning subgraphs of $\pm K_n^\circ$ that have at most one nontrivial component K, for which K is complete and $|V(K)| \geq k$ if K is balanced, K is induced and $|V(K)| \geq h$ if K is unbalanced (also a generalization). Characteristic polynomial, homotopy and homology of the order complex, cohomology of the real complement. **(SG: G, M(Gen): N, col)**

Anders Björner, Michel Las Vergnas, Bernd Sturmfels, Neil White, and Günter M. Ziegler

1993a *Oriented Matroids.* Encyclop. Math. Appl., Vol. 46. Cambridge University Press, Cambridge, Eng., 1993. MR 95e:52023. Zbl. 773.52001.

The adjacency graph of bases of an oriented matroid is signed, using circuit signatures, to make the "signed basis graph". See §3.5, "Basis orientations and chirotopes", pp. 132–3. **(M: SG)**

Andreas Blass

1995a Quasi-varieties, congruences, and generalized Dowling lattices. *J. Algebraic Combin.* 4 (1995), 277–294. MR 96i:06012. Zbl. 857.08002. Errata. *Ibid.* 5 (1996), 167. MR 96i:06012, 1382046. Zbl. 857.08002.

Treats the generalized Dowling lattices of Hanlon (1991a) as congruence lattices of certain quasi-varieties, in order to calculate characteristic polynomials and generalizations. **(M(gg): Gen: N)**

Andreas Blass and Bruce Sagan

1997a Möbius functions of lattices. *Adv. Math.* 127 (1997), 94–123. MR 98c:06001. Zbl. 970.32977.

§3: "Non-crossing B_n and D_n". Lattices of noncrossing signed partial partitions. Atoms of the lattices are defined as edge fibers of the signed covering graph of $\pm K_n^\circ$, thus corresponding to edges of $\pm K_n^\circ$. [The "half edges" are perhaps best regarded as negative loops.] The lattices studied, called $NCB_n, NCD_n, NCBD_n(S)$, consist of the noncrossing members of the Dowling and near-Dowling lattices of the sign group, i.e., $\operatorname{Lat} G(\pm K_n^{(T)})$ for $T = [n], \varnothing, [n]\backslash S$, respectively. **(SG: G, N, cov)**

1998a Characteristic and Ehrhart polynomials. *J. Algebraic Combin.* 7 (1998), 115–126. MR 99c:05204. Zbl. 899.05003.

Signed-graph chromatic polynomials are recast geometrically by observing that the number of k-colorings equals the number of points of $\{-k, -k+1, \ldots, k-1, k\}^n$ that lie in none of the edge hyperplanes of the signed graph. The interesting part is that this generalizes to subspace arrangements of signed graphs and, somewhat *ad hoc*, to the hyperplane arrangements of the

exceptional root systems. [See also Athanisiadis (1996a), Zaslavsky (20xxi). For applications see articles of Sagan and Zhang.]
(SG, Gen: M(Gen), G: col, N)

T.B. Boffey

1982a *Graph theory in Oper. Research.* Macmillan, London, 1982. Zbl. 509.90053.

Ch. 10: "Network flow: extensions." 10.1(g): "Flows with gains," pp. 224–226. 10.3: "The simplex method applied to network problems," subsection "Generalised networks," pp. 246–250. **(GN: m(bases): Exp)**

Ethan D. Bolker

1977a Bracing grids of cubes. *Environment and Planning B* 4 (1977), 157–172.

The elementary 1-cycles associated with circuits of $G(-\Gamma)$ ("bicycles") are crucial. [Their first publication, I believe.] **(EC)**

1979a Bracing rectangular frameworks. II. *SIAM J. Appl. Math.* 36 (1979), 491–503. MR 81j:73066b. Zbl. 416.70010.

The elementary 1-cycles associated with circuits of $G(\Sigma)$ ("bicycles"), mostly for $\Sigma = -\Gamma$. General signed graphs appear at Thm. 7, p. 505. Dictionary: "Signed bicycle" = elementary 1-cycle (circulation) associated with a circuit. **(EC, SG: M, i)**

Joseph E. Bonin

See also M.K. Bennett.

1993a Automorphism groups of higher-weight Dowling geometries. *J. Combin. Theory Ser. B* 58 (1993), 161–173. MR 94k:51005. Zbl. 733.05027, (789.05017).

A weight-k higher Dowling geometry of rank n, $Q_{n,k}(\mathrm{GF}(q)^\times)$, is the union of all coordinate k-flats of $\mathrm{PG}(n-1,q)$: i.e., all flats spanned by k elements of a fixed basis. If $k>2$, the automorphism groups are those of $\mathrm{PG}(n-1,q)$ for $q>2$ and are symmetric groups if $q=2$. **(gg: Gen: M, Aut)**

1993b Modular elements of higher-weight Dowling lattices. *Discrete Math.* 119 (1993), 3–11. MR 94h:05018. Zbl. 808.06012.

See definition in (1993a). For $k>2$ the only nontrivial modular flats are the projective coordinate k-flats and their subflats. This gives some information about the characteristic polynomials [which, however, are still only partially known]. [Kung (1996a), §6, has further results.] **(gg: Gen: M: N)**

1995a Automorphisms of Dowling lattices and related geometries. *Combin. Probab. Comput.* 4 (1995), 1–9. MR 96e:05039. Zbl. 950.37335.

The automorphisms of a Dowling geometry of a nontrivial group are the compositions of a coordinate permutation, switching, and a group automorphism. A similar result holds, with two exceptions, if some or all coordinate points are deleted. [A third exception is missed: $Q_3'(\mathbb{Z}_3)$.] **(gg: M: Aut)**

1996a Open problem 6. A problem on Dowling lattices. In: Joseph E. Bonin, James G. Oxley, and Brigitte Servatius, eds., *Matroid Theory* (Proc., Seattle, 1995), pp. 417–418. Contemp. Math., Vol. 197. Amer. Math. Soc., Providence, R.I., 1996.

Problem 6.1. If a finite matroid embeds in the Dowling geometry of a group, does it embed in the Dowling geometry of some finite group? [No; see Brooksbank, Qin, Robertson, and Seress (2004a).] **(gg: M)**

2006a Extending a matroid by a cocircuit. *Discrete Math.* 306 (2006), no. 8–9, 812–819. MR 2006m:05045. Zbl. 1090.05008.

§4 concerns Dowling lattices. **(GG: M)**

Joseph E. Bonin and Kenneth P. Bogart

1991a A geometric characterization of Dowling lattices. *J. Combin. Theory Ser. A* 56 (1991), 195–202. MR 92b:05019. Zbl. 723.05033. **(gg: M)**

Joseph E. Bonin and Joseph P.S. Kung

1994a Every group is the automorphism group of a rank-3 matroid. *Geom. Dedicata* 50 (1994), 243–246. MR 95m:20005. Zbl. 808.05029. **(gg: M: Aut)**

Joseph E. Bonin and William P. Miller

1999a Characterizing combinatorial geometries by numerical invariants. *European J. Combin.* 20 (1999), 713–724. MR 2001a:51007. Zbl. 946.05020.

Dowling geometries are characterized amongst all simple matroids by numerical properties of large flats of ranks ≤ 7 (Thm. 3.4); amongst all matroids by their Tutte polynomials. **(gg: M)**

Joseph E. Bonin and Hongxun Qin

2000a Size functions of subgeometry-closed classes of representable combinatorial geometries. *Discrete Math.* 224 (2000), 37–60. MR 2001g:05031. Zbl. 968.52009.

Extremal matroid theory. The Dowling geometry $Q_3(\mathrm{GF}(3)^\times)$ appears as an exceptional extremal matroid in Thm. 2.10. The extremal subset of $\mathrm{PG}(n-1, q)$ that does not contain the higher-weight Dowling geometry $Q_{m,m-1}(\mathrm{GF}(q)^\times)$ (see Bonin 1993a) is found in Thm. 2.14.

(GG, Gen: M: X, N)

André Bouchet

1983a Nowhere-zero integral flows on a bidirected graph. *J. Combin. Theory Ser. B* 34 (1983), 279–292. MR 85d:05109. Zbl. 518.05058.

Introduces nowhere-zero flows on signed graphs. [The bidirection is inessential; it is a device to keep track of the flow.] A connected, coloop-free signed graph has a nowhere-zero integral flow with maximum weight ≤ 216. The value 216 cannot be replaced by 5, but: *Conjecture*(Bouchet): it can be replaced by 6. [See Khelladi (1987a) for some progress on this. See Jensen and Toft (1995a) for other contributions.] A topological application is outlined.

(SG: M, O, Flows)

Peter Brooksbank, Hongxung Qin, Edmund Robertson, and Ákos Seress

2004a On Dowling geometries of infinite groups. *J. Combin. Theory Ser. A* 108 (2004), no. 1, 155–158. MR 2005e:51014. Zbl. 1056.51011.

Solution of Bonin (1996a). They produce a finite gain graph that has gains in no finite group. **(gg: M)**

Gerald G. Brown and Richard D. McBride

1984a Solving generalized networks. *Management Sci.* 30 (1984), 1497–1523. Zbl. 554.-90032. **(GN: M(bases))**

Richard A. Brualdi and Nancy Ann Neudauer

1997a The minimal presentations of a bicircular matroid. *Quart. J. Math. Oxford* (2) 48 (1997), 17–26. MR 97m:05065. Zbl. 938.05023.

Minimal transversal presentations of $G(\Gamma, \varnothing)$, given Γ. **(Bic)**

Seth Chaiken

1982a A combinatorial proof of the all minors matrix tree theorem. *SIAM J. Algebraic Discrete Methods* 3 (1982), 319–329. MR 83h:05062. Zbl. 495.05018.

§4: "Extenson to signed graphs". Generalizing Zaslavsky (1982a), an all-minors matrix-tree theorem for weighted signed digraphs and a corollary for weighted signed graphs. Given: a signed graph on vertex set $[n]$. For a Kirchoff (or "Laplace")-type $n \times n$ matrix K (A in the paper), $K(\bar{U}, \bar{W})$ is K with the rows indexed by U and the columns indexed by W deleted. Take $U, W \subseteq V$ with $|U| = |W| = k \leq n$. Then $\det K(\bar{U}, \bar{W})$ is a sum of terms, one for each independent set F of rank $n-k$ in $G(\Sigma)$ in which each tree component contains just one vertex from U and one from W. Each term has a sign depending partly on the number of negative paths by which F links U to W and partly on the linking pattern, and with magnitude $4^c \cdot$(weight product of F), where $c = \#$ of circles in F. [The credit to Zaslavsky is overly generous: only the case $U = W = \varnothing$ is his; the others are new.] The digraph version replaces 4 by 2 and imposes conditions on arc directions in the tree and nontree components of F.

A brief remark describes a gain-graphic ("voltage-graphic") generalization. (**SG, GG: A, I, m**)

Vijaya Chandru, Collette R. Coullard, and Donald K. Wagner

1985a On the complexity of recognizing a class of generalized networks. *Oper. Res. Letters* 4 (1985), 75–78. MR 87a:90144. Zbl. 565.90078.

Determining whether a gain graph with real multiplicative gains has a balanced polygon, i.e., is not contrabalanced, is NP-hard. So is determining whether a real matrix is projectively equivalent to the incidence matrix of a contrabalanced real gain graph. (**GN, Bic: I, Alg**)

Zhi-Hong Chen, Ying-Qiang Kuang, and Hong-Jian Lai

1999a Connectivity of cycle matroids and bicircular matroids. *Ars Combin.* 52 (1999), 239–250. MR 2001d:05032. Zbl. 977.05027.

The relationship between graph structure and the Tutte, verticial, and cyclic connectivities of the bicircular matroid. (**Bic: Str**)

Timothy Y. Chow

2003a Symplectic matroids, independent sets, and signed graphs. *Discrete Math.* 263 (2003), 35–45. MR 2004a:05033. Zbl. 1014.05017.

§4, "From graphs to symplectic matroids": The matroid union of $G(\Gamma, \sigma)$ over all signatures of a fixed graph yields a symplectic matroid. (**SG: M**)

Lane Clark

2004a Limit theorems for associated Whitney numbers of Dowling lattices. *J. Combin. Math. Combin. Comput.* 50 (2004), 105–113. MR 2005b:06007. Zbl. 1053.06003.

Aymptotics of numbers introduced by Benoumhani (1997a). (**gg: M: N**)

Gérard Cornuéjols

See also M. Conforti.

2001a *Combinatorial Optimization: Packing and Covering.* CBMS-NSF Reg. Conf. Ser. in Appl. Math., Vol. 74. Soc. Indust. Appl. Math., Philadelphia, 2001. MR 2002e:90004. Zbl. 972.90059.

The topic is linear optimization over a clutter, esp. a "binary clutter", which is the class of negative circuits of a signed binary matroid. The class $\mathcal{C}_-(\Sigma)$ is an important example (see Seymour 1977a), as is its blocker $b\mathcal{C}_-(\Sigma)$ [which is the class of minimal balancing edge sets; hence the frustration index = minimum size of a member of the blocker.

Ch. 5: "Graphs without odd-K_5 minors", i.e., signed graphs without $-K_5$ as a minor. Some esp. interesting results: Thm. 5.0.7 (special case of Seymour (1977a), Main Thm.): The clutter of negative polygons of Σ has the "Max-Flow Min-Cut Property" (Seymour's "Mengerian" property) iff Σ has no $-K_4$ minor. Conjecture 5.1.11 is Seymour's (1981a) beautiful conjecture (his "weak MFMC" is here called "ideal"). §5.2 reports the partial result of Guenin (2001a). (See also §8.4.)

Def. 6.2.6 defines a signed graph "$G(A)$" of a $0, \pm 1$-matrix A, whose transposed incidence matrix is a submatrix of A. §6.3.3: "Perfect $0, \pm 1$-matrices, bidirected graphs and conjectures of Johnson and Padberg" (1982a), associates a bidirected graph with a system of 2-variable pseudoboolean inequalities; reports on Sewell (1997a) (*q.v.*).

§8.4: "On ideal binary clutters", reports on Cornuéjols and Guenin (2002a), Guenin (1998a), and Novick and Sebö (1995a) (*qq.v.*).

(**SM, SG: M, G, I(Gen), O: Exp, Ref, Exr**)

Gérard Cornuéjols and Bertrand Guenin

2002a Ideal binary clutters, connectivity, and a conjecture of Seymour. *SIAM J. Discrete Math.* 15 (2002), no. 3, 329–352. MR 2003h:05057. Zbl. 1035.90045.

A partial proof of Seymour's (1981a) conjecture. Main Thm.: A binary clutter is ideal if it has as a minor none of the circuit clutter of F_7, $\mathcal{C}_-(-K_5)$ or its blocker, or $\mathcal{C}_-(-K_4)$ or its blocker. Important are the lift and extended lift matroids, $L(M, \sigma)$ and $L_0(M, \sigma)$, defined as in signed graph theory. [See Cornuéjols (2001a), §8.4.] (**SM, SG: M, G**)

Collette R. Coullard

See also V. Chandru.

Collette R. Coullard, John G. del Greco, and Donald K. Wagner

††1991a Representations of bicircular matroids. *Discrete Appl. Math.* 32 (1991), 223–240. MR 92i:05072. Zbl. 755.05025.

§4: §4.1 describes 4 fairly simple types of "legitimate" graph operation that preserve the bicircular matroid. Thm. 4.11 is a converse: if Γ_1 and Γ_2 have the same connected bicircular matroid, then either they are related by a sequence of legitimate operations, or they belong to a small class of exceptions, all having order ≤ 4, whose bicircular matroid isomorphisms are also described. This completes the isomorphism theorem of Wagner (1985a). §5: If finitely many graphs are related by a sequence of legitimate operations (so their bicircular matroids are isomorphic), then they have contrabalanced real gains whose incidence matrices are row equivalent. These results are also found by a different approach in Shull *et al.* (1989a, 20xxa).

(**Bic: Str, I**)

1993a Recognizing a class of bicircular matroids. *Discrete Appl. Math.* 43 (1993), 197–215. MR 94i:05021. Zbl. 777.05036. (**Bic: Alg**)

1993b Uncovering generalized-network structure in matrices. *Discrete Appl. Math.* 46 (1993), 191–220. MR 95c:68179. Zbl. 784.05044. (**GN: Bic: I, Alg**)

George B. Dantzig

1963a *Linear Programming and Extensions.* Princeton Univ. Press, Princeton, N.J., 1963. MR 34 #1073. Zbl. (e: 108.33103).

Chapter 21: "The weighted distribution problem." 21-2: "Linear graph structure of the basis." (**GN: M(Bases)**)

John G. del Greco

See also C.R. Coullard.

1992a Characterizing bias matroids. *Discrete Math.* 103 (1992), 153–159. MR 93m:05050. Zbl. 753.05021.

How to decide, given a matroid M and a biased graph Ω, whether $M = G(\Omega)$. (**GG: M**)

Michael Doob

See also D.M. Cvetković.

1973a An interrelation between line graphs, eigenvalues, and matroids. *J. Combin. Theory Ser. B* 15 (1973), 40–50. MR 55 #12573. Zbl. 245.05125, (257.05132).

Along with Simões-Pereira (1973a), introduces to the literature the even-circle matroid $G(-\Gamma)$ [previously invented by Tutte, unpublished]. The multiplicity of -2 as an eigenvalue (in characteristic 0) equals the number of independent even polygons $= n - \operatorname{rk} G(-\Gamma)$. In characteristic p there is a similar theorem, but the pertinent matroid is $G(\Gamma)$ if $p = 2$ and, when $p|n$, the matroid has rank 1 greater than otherwise [a fact that mystifies me]. (**EC: I**)

1974a Generalizations of magic graphs. *J. Combin. Theory Ser. B* 17 (1974), 205–217. MR 51 #274. Zbl. 271.05128, (287.05124).

Thm. 3.2 is the theorem of van Nuffelen (1973a), supplemented by the observation that it remains true in any characteristic except 2. (**EC: I**)

Peter Doubilet

1971a Dowling lattices and their multiplicative functions. In: *Möbius Algebras* (Proc. Conf., Waterloo, Ont., 1971), pp. 187–192. Univ. of Waterloo, Ont., 1971, reprinted 1975. MR 50 #9605. Zbl. 385.05008. (**GG: M**)

Peter Doubilet, Gian-Carlo Rota, and Richard Stanley

1972a On the foundations of combinatorial theory (VI): The idea of generating function. In: *Proceedings of the Sixth Berkeley Symposium on Mathematical Statistics and Probability* (Berkeley, Calif., 1970/71), Vol. II: *Probability Theory*, pp. 267–318. Univ. of California Press, Berkeley, Calif., 1972. MR 53 #7796. Zbl. 267.05002. Reprinted in: Gian-Carlo Rota, *Finite Operator Calculus*, pp. 83–134. Academic Press, New York, 1975. MR 52 #119. Zbl. 328.05007. Reprinted again in: Joseph P.S. Kung, ed., *Gian-Carlo Rota on Combinatorics: Introductory Papers and Commentaries*, pp. 148–199. Birkhäuser, Boston, 1995. MR 99b:01027. Zbl. 841.01031.

Section 5.3: Brief gain-graphic treatment of Dowling lattices. (**GG: M**)

T.A. Dowling

1971a Codes, packings, and the critical problem. In: *Atti del Convegno di Geometria Combinatoria e sue Applicazioni (Perugia, 1970)*, pp. 209–224. Ist. Mat., Univ. di Perugia, Perugia, Italy, 1971. MR 49 #2438. Zbl. 231.05029.

Pp. 221–223: The first intimations of Dowling lattices/geometries, as in (1973a, 1973b), and their higher-weight relatives (see Bonin 1993a).
(gg, Gen: M)

1973a A q-analog of the partition lattice. Ch. 11 in: J.N. Srivastava *et al.*, eds., *A Survey of Combinatorial Theory* (Proc. Internat. Sympos., Ft. Collins, Colo., 1971), pp. 101–115. North-Holland, Amsterdam, 1973. MR 51 #2954. Zbl. 259.05023.

Linear-algebraic progenitor of (1973b). Treats the Dowling lattice of group $\mathrm{GF}(q)^\times$ as naturally embedded in $\mathrm{PG}^{n-1}(q)$. Interesting is p. 105, Remark: One might generalize some results to any ambient (simple) matroid.
(gg: G, M: N)

††1973b A class of geometric lattices based on finite groups. *J. Combin. Theory Ser. B* 14 (1973), 61–86. MR 46 #7066. Zbl. 247.05019. Erratum. *Ibid.* 15 (1973), 211. MR 47 #8369. Zbl. 264.05022.

Introduces the Dowling lattices of a group, treated as lattices of group-labelled partial partitions. Equivalent to the frame matroid of complete $\mathfrak{G}$-gain graph $\mathfrak{G}K_n^\bullet$. [The gain-graphic approach was known to Dowling (1973a, p. 109) but first published in Doubilet, Rota, and Stanley (1972a).] Isomorphism, vector representation, Whitney numbers and characteristic polynomial. [The first and still fundamental paper.] **(gg: M: N)**

Thomas Dowling and Hongxun Qin

2005a Reconstructing ternary Dowling geometries. *Advances in Applied Mathematics* 34 (2005), no. 2, 358–365. MR 2005j:05017. Zbl. 1068.52017.

Thm. 1.5: The Dowling geometry $Q_r(\mathbb{Z}_2)$ is the only matroid of rank $r \geq 4$ such that every contraction by a point is $Q_{r-1}(\mathbb{Z}_2)$. **(sg: M)**

20xxa Excluded minors for classes of cographic matroids. Submitted. **(GG: M, T, SG)**

Arne Dür

1986a *Möbius Functions, Incidence Algebras and Power Series Representations.* Lecture Notes in Math., Vol. 1202. Springer-Verlag, Berlin, 1986. MR 88m:05005. Zbl. 592.05006.

Dowling lattices are an example of a categorial approach to incidence-algebra techniques in Ch. IV, §7. Computed are the characteristic polynomial and second kind of Whitney numbers. Binomial concavity, hence unimodality of the latter [*cf.* Stonesifer (1975a)] is proved by showing that a suitable generating polynomial has only distinct, negative roots [*cf.* Benoumhani (1999a)]. **(gg: M: N)**

Paul H. Edelman and Victor Reiner

1994a Free hyperplane arrangements between A_{n-1} and B_n. *Math. Z.* 215 (1994), 347–365. MR 95b:52021. Zbl. 793.05122.

Characterizes all $\Sigma \supseteq +K_n$ whose frame matroid $G(\Sigma)$ is supersolvable, free, or inductively free. Essentially, iff the negative links form a threshold graph. [Continued in Bailey (20xxa). Generalized in part to arbitrary gain groups in Zaslavsky (2001a).] **(sg: M, G, col)**

Richard Ehrenborg and Margaret A. Readdy

1998a On valuations, the characteristic polynomial, and complex subspace arrangements. *Adv. Math.* 134 (1998), 32–42. MR 98m:52018. Zbl. 906.52004.

An abstract additive approach to the characteristic polynomial, applied in particular to "divisor Dowling arrangements" of hyperplanes and certain interpolating arrangements. Let $\Phi = \mathfrak{G}_1 K_1 \cup \cdots \cup \mathfrak{G}_n K_n$, where $V(K_i) = \{v_1, \ldots, v_i\}$ and $\mathfrak{G}_1 \geq \cdots \geq \mathfrak{G}_n$ is a chain of subgroups of a gain group $\mathfrak{G} = \mathfrak{G}_1$. When $\mathfrak{G}$ is finite cyclic, the complex hyperplane representation of $\Phi^\bullet$ is a "divisor Dowling arrangement". [Its polynomial equals the chromatic polynomial of $\Phi^\bullet$, which is easily computed via gain-graph coloring without the restriction to cyclic gain group. The same appears to be true for the other arrangements treated herein.] **(gg: M: G, N)**

1999a On flag vectors, the Dowling lattice, and braid arrangements. *Discrete Computat. Geom.* 21 (1999), 389–403. MR 2000a:52037. Zbl. 941.52021.

Canonical complex hyperplane representation of the Dowling lattice of $\mathbb{Z}_k$. P. 395: an interesting EL-labelling of the Dowling lattice by a [disguised lexicographic] ordering of atoms. Thm. 4.9 is a recursive formula for its **ab**-index. Thm. 5.2: the **c**-2**d**-index of the face lattices in case $k = 1, 2$, i.e., those of the real root system arrangements A_n^* and B_n^*. §6 presents a combinatorial description of the face lattice of B_n^* [which it is interesting to compare with that in Zaslavsky (1991b)]. [Dictionary: very confusingly, "region" = face.] **(gg: M: G, N)**

2000a The Dowling transform of subspace arrangements. *J. Combin. Theory Ser. A* 91 (2000), 322–333. MR 2001k:52038. Zbl. 962.05005.

The group expansion of an ordinary graph is generalized to expansion of an $\mathbb{R}^*_{>0}$-gain graph by a finite cyclic subgroup of $\mathbb{C}^*$, with correspondingly generalized formulas for the chromatic polynomial. The computations are technically incorrect; they should be done by gain-graph coloring. [Dictionary: "directed cycle" = polygon (not directed).] **(GG: G, N)**

Joyce Elam, Fred Glover, and Darwin Klingman

1979a A strongly convergent primal simplex algorithm for generalized networks. *Math. Oper. Res.* 4 (1979), 39–59. MR 81g:90049. Zbl. 422.90081. **(GN: M(bases), I)**

Lori Fern [Lori Koban]

See also L. Koban.

Lori Fern, Gary Gordon, Jason Leasure, and Sharon Pronchik

2000a Matroid automorphisms and symmetry groups. *Combinatorics, Probability and Computing* 9 (2000), 105-123. MR 2001g:05034. Zbl. 960.05055.

Consider a subgroup W of the hyperoctahedral group Oc_n that is generated by reflections. Let $M(W)$ be the vector matroid of the vectors corresponding to reflections in W. The possible direct factors of any automorphism group of $M(W)$ are S_k, Oc_k, and Oc_k^+. The proof is stricly combinatorial, via signed graphs. **(SG: M: Aut, G)**

Rigoberto Flórez

2006a Lindström's conjecture on a class of algebraically non-representable matroids. *European J. Combin.* 27 (2006), no. 6, 896–905. MR 2006m:05048. Zbl. 1090.05010.

Lindström conjectured that a certain matroid $M(n)$ is algebraically non-representable if n is nonprime. Proved by showing that $M(n)$ extends by harmonic conjugation to $L_0(\mathbb{Z}_n K_3)$, which in turn extends to a contradiction if n is composite. **(gg: M)**

20xxb Harmonic conjugation in harmonic matroids. Submitted.

In a harmonic matroid H, harmonic conjugates exist and are unique. If $L_0(\mathfrak{G}K_3) \subseteq H$ and $\mathfrak{G} = \mathbb{Z}$ or $\mathbb{Z}_p$, then the closure of L_0 under harmonic conjugation is a projective plane over $\mathbb{Q}$ or $\mathrm{GF}(p)$, as appropriate. **(gg: M)**

Rigoberto Flórez and David Forge

2007a Minimal non-orientable matroids in a projective plane. *J. Combin. Theory Ser. A* 114 (2007), no. 1, 175–183. MR 2007h:05031. Zbl. 1120.52012.

The minimal matroids are contained in lift matroids of $\mathbb{Z}_n K_3$. **(gg: M)**

Rigoberto Flórez and Thomas Zaslavsky

20xxa Biased graphs. VI. Synthetic geometry. In preparation. **(GG: M, G)**

David Forge

See also P. Berthomé and R. Flórez.

David Forge and Thomas Zaslavsky

2007a Lattice point counts for the Shi arrangement and other affinographic hyperplane arrangements. *J. Combin. Theory Ser. A* 114 (2007), no. 1, 97–109. MR 2007i:52026. Zbl. 1105.52014.

The number of proper integral m-colorings of a rooted integral gain graph (root v_0 and a function $h : V \to \mathbb{Z}$ such that there are root edges ge_{0i} for all $g \in (-\infty, h_i]$; otherwise the gain graph is finite). **(GG: G, N, M)**

Toshio Fujisawa

1963a Maximal flow in a lossy network. In: J.B. Cruz, Jr., and John C. Hofer, eds., *Proceedings, First Annual Allerton Conference on Circuit and System Theory* (Monticello, Ill., 1963), pp. 385–393. Dept. of Electrical Eng. and Coordinated Sci. Lab., Univ. of Illinois, Urbana, Ill., [1963]. **(GN: M(bases))**

Gilles Gastou and Ellis L. Johnson

1986a Binary group and Chinese postman polyhedra. *Math. Programming* 34 (1986), 1–33. MR 88e:90060. Zbl. 589.52004.

§10 introduces the co-postman and "odd circuit" problems, treated more thoroughly in Johnson and Mosterts (1987a) (q.v). "Odd" edges and circuits are precisely negative edges and polygons in an edge signing. The "odd circuit matrix" represents $L(\Sigma)$ (p. 30). **(SG: I, M(Bases), Alg)**

James F. Geelen and A.M.H. Gerards

2005a Regular matroid decomposition via signed-graphs. *J. Graph Theory* 48 (2005), no. 1, 74–84. MR 2005h:05037. Zbl. 1055.05024.

The lift matroid. **(SG: M: Str)**

A.M.H. Gerards

See also M. Chudnovsky and J.F. Geelen.

††1990a *Graphs and polyhedra: Binary spaces and cutting planes.* CWI Tract, 73. Centrum voor Wiskunde en Informatica, Amsterdam, 1990. MR 92f:52027. Zbl. 727.90044.

(Very incomplete annotation.) Thm.: Given Σ, the set $\{x \in \mathbb{R}^n : d_1 \leq x \leq d_2, b_1 \leq I(\Sigma)^{\mathrm{T}} x \leq b_2\}$ has Chvatal rank ≤ 1 for all integral vectors d_1, d_2, b_1, b_2, iff Σ contains no subdivided $-K_4$. **(SG: I, G, Str)**

1994a An orientation theorem for graphs. *J. Combin. Theory Ser. B* 62 (1994), 199–212. MR 96d:05051. Zbl. 807.05020. **(p, sg: M, O)**

1995a On Tutte's characterization of graphic matroids—a graphic proof. *J. Graph Theory* 20 (1995), 351–359. MR 96h:05038. Zbl. 836.05017.

Signed graphs used to prove Tutte's theorem. The signed-graph matroid employed is the extended lift matroid $L_0(\Sigma)$ ("extended even cycle matroid"). The main theorem (Thm. 2): Let Σ be a signed graph with no $-K_4$, $\pm K_3$, $-Pr_3$, or Σ_4 link minor; then Σ can be converted by Whitney 2-isomorphism operations ("breaking" = splitting a component in two at a cut vertex, "glueing" = reverse, "switching" = twisting across a vertex 2-separation) to a signed graph that has a balancing vertex ("blocknode"). Here Σ_4 consists of $+K_4$ with a 2-edge matching doubled by negative edges and one other edge made negative.

More translation: His "Σ" is our E_-. "Even, odd" = positive, negative (for edges and polygons). "Bipartite" = balanced; "almost bipartite" = has a balancing vertex. **(SG: M, Str, I)**

A.M.H. Gerards, L. Lovász, A. Schrijver, P.D. Seymour, C.-S. Shi, and K. Truemper

†1990a Manuscript in preparation, 1990.

Extension of Gerards and Schrijver (1986b). [Same comments apply. The proliferating authorship may prevent this major contribution from ever being published—though one hopes not! See Seymour (1995a) for description of two main theorems.] **(SG: Str, M, T)**

A.M.H. Gerards and A. Schrijver

1986b Signed graph – regular matroids – grafts. Research Memorandum, Faculteit der Economische Wetenschappen, Tilburg Univ., 1986.

Essential, major theorems. The (extended) lift matroid of a signed graph is one of the objects studied. Some of this material is published in Gerards (1990a). This paper is in the process of becoming Gerards, Lovász, *et al.* (1990a). **(SG: Str, M)**

Robert Gill

1998a The number of elements in a generalized partition semilattice. *Discrete Math.* 186 (1998), 125–134. MR 99e:52014. Zbl. 956.52009.

The semilattice is the intersection semilattice of a affinographic hyperplane arrangement representing $[-k,k]K_n$ [and is therefore isomorphic to the geometric semilattice of all k-composed partitions of a set; see, e.g., Zaslavsky (2002a), Ex. 10.5]. The rank and the Whitney numbers of the first kind are calculated. See Kerr (1999a) for homology. **(gg: m: G, N)**

2000a The action of the symmetric group on a generalized partition semilattice. *Electronic J. Combin.* 7 (2000), Research Paper 23, 20 pp. (electronic). MR 2001g:05107. Zbl. 947.06001.

See (1998a). **(gg: m: G, N, Aut)**

Omer Giménez, Anna de Mier, and Marc Noy

2005a On the number of bases of bicircular matroids. *Ann. Combin.* 9 (2005), no. 1, 35–45. MR 2005m:05049. Zbl. 1059.05030.

The number of bases is bounded above by $C^n\cdot$(number of spanning trees) in a simple graph but not in a multigraph. More precise results for K_n and $K_{n,m}$. [See Neudauer, Meyers, and Stevens (2001a) and Neudauer and Stevens (2001a).] **(Bic: I)**

Omer Giménez and Marc Noy

2006a On the complexity of computing the Tutte polynomial of bicircular matroids. *Combin. Probab. Comput.* 15 (2006), no. 3, 385–395. MR 2007a:05029. Zbl. 1094.05013.

Known NP-hardness results for transversal matroids apply to their proper subclass, bicircular matroids, with a few possible exceptions. (**Bic: I: Alg**)

Fred Glover

See also J. Elam.

F. Glover, J. Hultz, D. Klingman, and J. Stutz

1978a Generalized networks: A fundamental computer-based planning tool. *Management Sci.* 24 (1978), 1209–1220. (**GN: Alg, M(bases): Exp, Ref**)

Fred Glover, Darwin Klingman, and Nancy V. Phillips

1992a *Network Models in Optimization and Their Applications in Practice.* Wiley-Interscience, New York, 1992.

Textbook. See especially Ch. 5: "Generalized networks." (**GN: Alg: Exp**)

Eric Gottlieb

2003a On the homology of the h,k-equal Dowling lattice. *SIAM J. Discrete Math.* 17 (2003), no. 1, 50–71. MR 2004k:05209. Zbl. 1033.05098.

The lattice is the subposet of $\mathrm{Lat}\, G(\mathfrak{G}K_n)$ consisting of the flats whose nontrivial balanced components have order $\geq k$ and whose unbalanced component, if any, has order $\geq h$. If $|\mathfrak{G}| = 2$ and $h \leq k$ we have the lattice of Björner and Sagan (1996a).**Emanuele Delucchi**

2007a Nested set complexes of Dowling lattices and complexes of Dowling trees. *J. Algebraic Combin.* 26 (2007), no. 4, 477–494.

Studies Dowling trees (cf. Hultman 2007a). (**gg: M: N**)

(**gg: M: N**)

Eric Gottlieb and Michelle L. Wachs

2000a Cohomology of Dowling lattices and Lie (super)algebras. *Adv. in Appl. Math.* 24 (2000), no. 4, 301–336. MR 2001i:05161. Zbl. 1026.05104.

Two monomorphisms of the cohomology of the order complex of the lattice of flats of $Q_n(\mathfrak{G})$, upon which $\mathfrak{S}_n \wr \mathfrak{G}$ acts as operators, into enveloping algebras of certain Lie algebras and Lie superalgebras. (**gg: M: N**)

Richard C. Grinold

1973a Calculating maximal flows in a network with positive gains. *Oper. Res.* 21 (1973), 528–541. MR 50 #3900. Zbl. 304.90043.

Objective: to find the maximum output for given input. Basic solutions correspond to bases of $G(\Phi')$, Φ' being the underlying gain graph Φ together with an unbalanced loop adjoined to the sink. Onaga (1967a) also treats this problem. (**GN: M(bases), Alg**)

Jerrold W. Grossman, Devadatta M. Kulkarni, and Irwin E. Schochetman

1994a Algebraic graph theory without orientation. *Linear Algebra Appl.* 212/213 (1994), 289–307. MR 96b:05111. Zbl. 817.05047.

Incidence matrix $D(-\Gamma)$ (unoriented incidence matrix of Γ; here called M), Kirchoff or "Laplacian" matrix of $-\Gamma$, the even-circle ("even circuit") matroid $G(-\Gamma)$, a partial all-minors matrix-tree theorem [completed in Bapat, Grossman, and Kulkarni (1999a)]. [This part is not new. See van

Nuffelen (1973a) for rank($D(-\Gamma)$); Zaslavsky (1982a), §8 for both matrices; Tutte (1981a), Doob (1973a), and Simões-Pereira (1973a) for the matroid; Chaiken (1982a) for the whole matrix-tree theorem.]

§§4, 5: Vector spaces associated with $G(-\Gamma)$ and its dual, expressed both combinatorially in terms of vectors associated with matroid circuits and cocircuits (of two kinds) and as null and row spaces of $D(-\Gamma)$ and $D(-\Gamma)^{\mathrm{T}}$. E.g., in §5 is the all-negative version of: A basis for $\mathrm{Nul}\, D(\Sigma)^{\mathrm{T}}$ consists of one switching function positivizing each balanced component of Σ. [The viewpoint, going from matroids to vector spaces over fields, usually with characteristic $\neq 2$, contrasts sharply with that of Tutte (1981a), who starts with integral chain groups ($\mathbb{Z}$-modules) and ends with chain-group properties and matroids. This is the only thorough development I know of vector spaces of a signed graph before Chen and Wang (20xxa), despite some aspects' having appeared e.g. in Bolker (1977a, 1979a) and Tutte (1981a). It will be still more valuable if it is extended to $\mathbb{R}^*$-gain graphs and to F^*-gain graphs for any field F.]

Dictionary: " k-reduced spanning substructure" $\cong$ independent set of rank $n-k$ in $G(-\Gamma)$; "quasi edge cut" = balancing set; "quasibond" = minimal balancing set; "even circuit" = positive closed walk; "bowtie" = contrabalanced handcuff; "marimba stick" = half edge. (**EC, p: I, D**)

1995a On the minors of an incidence matrix and its Smith normal form. *Linear Algebra Appl.* 218 (1995), 213–224. MR 95m:15020. Zbl. 819.05043.

Rank of $D(-\Gamma)$ (the unoriented incidence matrix of Γ) [as in van Nuffelen (1973a)]. Finds all possible values of determinants of minors of $D(-\Gamma)$ [repeating and refining Zaslavsky (1982a), §8A] and of maximal nonsingular minors. Consequences are the Smith normal form of $D(-\Gamma)$ (§3) and the total integrality of some integer programs with $D(-\Gamma)$ as coefficient matrix. (**p: I, ec, G**)

Phil Hanlon

1984a The characters of the wreath product group acting on the homology groups of the Dowling lattices. *J. Algebra* 91 (1984), 430–463. MR 86j:05046. Zbl. 557.20009. (**gg: M: Aut**)

1988a A combinatorial construction of posets that intertwine the independence matroids of B_n and D_n. Manuscript, 1988.

Computes the Möbius functions of posets obtained from $\mathrm{Lat}\, G(\pm K_n^\circ)$ by discarding those flats with unbalanced vertex set in a given lower-hereditary list. Examples include $\mathrm{Lat}\, G(\pm K_n^{(k)})$, the exponent denoting the addition of k negative loops. Generalized and superseded by Hanlon and Zaslavsky (1998a). (**sg: M: Gen: N**)

1991a The generalized Dowling lattices. *Trans. Amer. Math. Soc.* 325 (1991), 1–37. MR 91h:06011. Zbl. 748.05043.

The lattices are based on a rank, n, a group, and a meet sublattice of the lattice of subgroups of the group. The Dowling lattices are a special case. (**gg: M: Gen: N**)

Phil Hanlon and Thomas Zaslavsky

1998a Tractable partially ordered sets derived from root systems and biased graphs.

Order 14 (1997–98), 229–257. MR 2000a:06016. Zbl. 990.03811.

Computes the characteristic polynomials (Thm. 4.1) and hence the Möbius functions (Cor. 4.4) of posets obtained from $\mathrm{Lat}\, G(\Omega)$, Ω a biased graph, by discarding those flats with unbalanced vertex set in a given lower-hereditary list. Examples include $\mathrm{Lat}\, G(\mathfrak{G}K_n^{(k)})$ where $\mathfrak{G}$ is a finite group, the exponent denoting the addition of k unbalanced loops. The interval structure, existence of a rank function, covering pairs, and other properties of these posets are investigated. There are many open problems.
(GG: M, Gen: N, Str, Col)

Kurt Hässig

1979a *Graphentheoretische Methoden des Operations Research.* Leitfaden der angew. Math. und Mechanik, 42. B.G. Teubner, Stuttgart, 1979. MR 80f:90002. Zbl. 397.90061.

Ch. 5: "Verallgemeinerte Fluss- und Potentialdifferenzen-probleme." The lift matroid arises from a side condition, i.e., extra row, added to the incidence matrix of the graph. [The side condition is expressed graphically by additive gains.] **(GN: I, M: Exp, Ref)**

Patrick Headley

1997a On a family of hyperplane arrangements related to the affine Weyl groups. *J. Algebraic Combin.* 6 (1997), 331–338. MR 98e:52010. Zbl. 911.52009.

The characteristic polynomials of the Shi hyperplane arrangements $\mathcal{S}(W)$ of type W for each Weyl group W, evaluated computationally. $\mathcal{S}(W)$ is obtained by splitting the reflection hyperplanes of W in two in a certain way; thus $\mathcal{S}(A_{n-1})$ splits the arrangement representing $\mathrm{Lat}\, G(K_n)$—more precisely, it represents $\mathrm{Lat}^{\mathrm{b}}\{0,1\}\vec{K}_n$; that of type B_n splits the arrangement representing $\mathrm{Lat}\, G(\pm K_n^\bullet)$, and so on. [See also Athanasiadis (1996a).]
(gg: G, M, N)

Anthony Henderson

2006a Plethysm for wreath products and homology of sub-posets of Dowling lattices. *Electronic J. Combin.* 13 (2006), no. 1, Research article R87, 25 pp. (electronic).

The subposets are $Q_n^{1 \bmod d}(\mathfrak{G})$ where $d > 1$, whose elements are the flats $A \subseteq E(\mathfrak{G}K_n^\bullet)$ such that d divides the order of the unbalanced part and the number of vertices every balanced component is $\equiv 1 \bmod d$. **(gg: M: Aut)**

Yao Ping Hou and Li Juan Wei

1999a Whitney numbers of the second kind for Dowling lattices. (In Chinese. English and Chinese summaries.) *Acta Sci. Natur. Univ. Norm. Hunan.* 22 (1999), No. 3, 6–10. MR 2000k:05017. Zbl. 948.05004.

Combinatorial proof of an explicit formula for W_k [possibly the standard one?]. Studies "associated numbers" W_k^r. Proved: $W_{n-k} \leq W_k$ for $k \leq 3$ [this must mean $W_k \leq W_{n-k}$ and must have some restriction on n; well known for $k=1$]. **(gg: M: N)**

2007b Link complexes of subspace arrangements. *Europ. J. Combin.* 28 (2007), no. 3, 781–790. MR 2007m:52029. Zbl. 1113.52038.

Interprets chromatic polynomials of signed graphs in terms of Hilbert polynomials. **(SG: N)**

John Hultz and D. Klingman

1979a Solving singularly constrained generalized network problems. *Appl. Math. Optim.* 4 (1978), 103–119. MR 57 #15414. Zbl. 373.90075. (**GN: M(bases)**)

John J. Jarvis and Anthony M. Jezior

1972a Maximal flow with gains through a special network. *Oper. Res.* 20 (1972), 678–688. MR 47 #6286. Zbl. 241.90021. (**GN: M(bases)**)

Paul A. Jensen and J. Wesley Barnes

1980a *Network Flow Programming.* Wiley, New York, 1980. MR 82f:90096. Zbl. 502.-90057. Reprinted by: Robert E. Krieger, Melbourne, Fla., 1987. MR 89a:90152.

§1.4: "The network-with-gains model." §2.8: "Networks with gains—-example applications." Ch. 9: "Network manipulation algorithms for the generalized network." Ch. 10: "Generalized minimum cost flow problems." (**GN: M(bases)**)

Guangfeng Jiang and Jianming Yu

2004a Supersolvability of complementary signed-graphic hyperplane arrangements. *Australasian J. Combin.* 30 (2004), 261–276. MR 2005j:05042.

Characterizes supersolvability of $G(K_n, \sigma)$. A special case of Zaslavsky (2001a). (**SG: G: m**)

Ellis L. Johnson

See also J. Edmonds and G. Gastou.

1965a Programming in networks and graphs. Report ORC 65-1, Operations Research Center, Univ. of California, Berkeley, Calif., Jan. 1965.

§7: "Flows with gains." §8: "Linear programming in an undirected graph." §9: "Integer programming in an undirected graph." (**GN: I, M(bases)**)(**ec: I, M(bases), Alg**)

1966a Networks and basic solutions. *Oper. Res.* 14 (1966), 619–623. (**GN**)

Jeff Kahn and Joseph P.S. Kung

1980a Varieties and universal models in the theory of combinatorial geometries. *Bull. Amer. Math. Soc. (N.S.)* 3 (1980), 857–858. MR 81i:05051. Zbl. 473.05025.

Announcement of (1982a). (**gg: M**)

††1982a Varieties of combinatorial geometries. *Trans. Amer. Math. Soc.* 271 (1982), 485–499. MR 84j:05043. Zbl. 503.05010. Reprinted in: Joseph P.S. Kung, *A Source Book in Matroid Theory*, pp. 395–409, with commentary, pp. 335–338. Birkhäuser, Boston, 1986. MR 88e:05028. Zbl. 597.05019.

A "variety" is a class closed under deletion, contraction, and direct summation and having for each rank a "universal model", a single member containing all others. There are two nontrivial types of variety of finite matroids: matroids representable over GF(q), and gain-graphic matroids with gains in a finite group $\mathfrak{G}$. The universal models of the latter are the Dowling geometries $Q_n(\mathfrak{G})$.

It is incidentally proved (Section 7, pp. 490–492) that Dowling geometries of non-group quasigroups cannot exist in rank $n \geq 4$. (**gg: M**)

1986a A classification of modularly complemented geometric lattices. *European J. Combin.* 7 (1986), 243–248. MR 87i:06026. Zbl. 614.05018.

A geometric lattice of rank ≥ 4, if not a projective geometry with a few points deleted, is a Dowling lattice. (**gg: M**)

Jeff L. Kennington and Richard V. Helgason

1980a *Algorithms for Network Programming.* Wiley, New York, 1980. MR 82a:9013. Zbl. 502.90056.

Ch. 5: "The simplex method for the generalized network problem."
(GN: M(Bases): Exp)

Julie Kerr

1999a A basis for the top homology of a generalized partition lattice. *J. Algebraic Combin.* 9 (1999), 47–60. MR 2000k:05265. Zbl. 921.05063.

The lattice is isomorphic to the semilattice of k-composed partitions of a set with a top element adjoined. (See R. Gill (1998a).) **(gg: m: G, T)**

A. Khelladi

1987a Nowhere-zero integral chains and flows in bidirected graphs. *J. Combin. Theory Ser. B* 43 (1987), 95–115. MR 88h:05045. Zbl. 617.90026.

Improves the result of Bouchet (1983a) about nowhere-zero integral flows on a signed graph. Σ has such an 18-flow if 4-connected, a 30-flow if 3-connected and without a positive triangle, and in some cases a 6-flow (proving Bouchet's conjecture in those cases). **(SG: M: Flows)**

1999a Colorations généralisées, graphes biorientés et deux ou trois choses sur François. Symposium à la Mémoire de François Jaeger (Grenoble, 1998). *Ann. Inst. Fourier (Grenoble)* 49 (1999), 955–971. MR 2000h:05083. Zbl. 917.05026.

Comments on the results of Bouchet (1983a) and Khelladi (1987a).
(SG: M, Flows)

Victor Klee

1971a The greedy algorithm for finitary and cofinitary matroids. In: Theodore S. Motzkin, ed., *Combinatorics*, pp. 137–152. Proc. Symp. Pure Math., Vol. 19. Amer. Math. Soc., Providence, R.I., 1971. MR 48 #10865. Zbl. 229.05031.

Along with Simões-Pereira (1972a), invents the bicircular matroid (here, for infinite graphs). **(Bic)**

Lori Koban [Lori Fern]

See also L. Fern.

2004a Comments on "Supersolvable frame-matroid and graphic-lift lattices" by T. Zaslavsky. *European J. Combin.* 25 (2004), 141–144. MR 2004k:05054. Zbl. 1031.05032.

Correction to Thm. 2.1 and an improved (and corrected) proof of Thm. 2.2 of Zaslavsky (2001a). **(GG: M)**

2004b *Two Generalizations of Biased Graph Theory: Circuit Signatures and Modular Triples of Matroids, and Biased Expansions of Biased Graphs.* Doctoral dissertation, Binghamton University, 2004.

Chapter 1: "Circuit signatures and modular triples." When can gains be applied to matroids, as they are to graphs in Zaslavsky (1991a), to produce a linear class of circuits and hence a lift matroid? Theorem 1.4.1: When the group has exponent > 2, one needs a ternary circuit signature, thus a ternary matroid. Theorem 1.4.5: When the group has exponent 2 the matroid must be binary (no circuit signature is required). **(M: GG: Gen)**

Chapter 2: "Biased expansions of biased graphs." Generalizes group and biased expansions of a graph and the chromatic (and bias-matroid charac-

teristic) polynomial formulas (Zaslavsky 1995b, 20xxj) to expansions of a biased graph. Chapter 3: "When are biased expansions actually group expansions?" Partial results about characterizing biased expansions of biased graphs that are group expansions; counterexamples to several plausible conjectures. **(GG: M, N, G)**

2008a A modular triple characterization of circuit signatures. *Europ. J. Combin.* 29 (2008), no. 1, 159–170.

Several kinds of circuit signatures of a matroid can be characterized through modular triples of copoints or circuits. These include lift signatures. **(GG: Gen: M)**

Joseph P.S. Kung

See also J.E. Bonin and J. Kahn.

1986a Numerically regular hereditary classes of combinatorial geometries. *Geom. Dedicata* 21 (1986), 85–105. MR 87m:05056. Zbl. 591.05019.

Examples include Dowling geometries, Ex. (6.2), and the frame matroids of full group expansions of graphs in certain classes; see pp. 98–99. **(GG: M)**

1990a Combinatorial geometries representable over GF(3) and GF(q). I. The number of points. *Discrete Computat. Geom.* 5 (1990), 83–95. MR 90i:05028. Zbl. 697.51007.

The Dowling geometry over the sign group is the largest simple ternary matroid not containing the "Reid matroid". **(sg: M: X)**

1990b The long-line graph of a combinatorial geometry. II. Geometries representable over two fields of different characteristic. *J. Combin. Theory Ser. B* 50 (1990), 41–53. MR 91m:51007. Zbl. 645.05026.

Dowling geometries used in the proof of Prop. (1.2). **(gg: M)**

1993a Extremal matroid theory. In: Neil Robertson and Paul Seymour, eds., *Graph Structure Theory* (Proc., Seattle, 1991), pp. 21–61. Contemp. Math., Vol. 147. Amer. Math. Soc., Providence, R.I., 1993. MR 94i:05022. Zbl. 791.05018.

Survey and new results. See: §2.7: "Gain-graphic matroids." P. 30, fn. 9. §4.3: "Varieties." §4.5. "Framed gain-graphic matroids." §6.4: "Matroids representable over two different characteristics." §8: "Concluding remarks," on a possible ternary analog of Seymour's decomposition theorem. **(GG: M: X, Str, Exp, Ref)**

1993b The Radon transforms of a combinatorial geometry. II. Partition lattices. *Adv. Math.* 101 (1993), 114–132. MR 95b:05051. Zbl. 786.05018.

Dowling lattices are lower-half Sperner. The proof is given only for partition lattices. **(gg: M)**

1996a Matroids. In: M. Hazewinkel, ed., *Handbook of Algebra*, Vol. 1, pp. 157–184. North-Holland (Elsevier), Amsterdam, 1996. MR 98c:05040. Zbl. 856.05001.

§6.2: "Gain-graphic matroids." I.e., frame matroids of gain graphs. **(GG: M: Exp)**

1996b Critical problems. In: Joseph E. Bonin, James G. Oxley, and Brigitte Servatius, eds., *Matroid Theory* (Proc., Seattle, 1995), pp. 1–127. Contemp. Math., Vol. 197. Amer. Math. Soc., Providence, R.I., 1996. MR 97k:05049. Zbl. 862.05019.

A remarkable more-than-survey with numerous new results and open problems. §4.5: "Abstract linear functionals in Dowling group geometries". §6:

"Dowling geometries and linear codes", concentrates on higher-weight Dowling geometries, extending Bonin (1993b). §7.4: "Critical exponents of classes of gain-graphic geometries". §7.5: "Growth rates of classes of gain-graphic geometries". §8.5: "Jointless Dowling group geometries". Corollary 8.30. §8.11: "Tangential blocks in $\mathscr{Z}(A)$". Also see pp. 56, 61, 88, 92, 114. The matroids are the frame matroids of gain graphs. **(GG, Gen: M)**

1998a A geometric condition for a hyperplane arrangement to be free. *Adv. Math.* 135 (1998), 303–329. MR 2000f:05023. Zbl. 905.05017.

Delete from a Dowling geometry a subset S that contains no whole plane. Found: necessary and sufficient conditions for the characteristic polynomial to factor completely over the integers. When the geometry corresponds to a hyperplane arrangement, many more of the arrangements are not free than are free; however, if S contains no whole line, all are free (so the characteristic polynomial factors completely over $\mathbb{Z}$) while many are not supersolvable. **(gg: M: N)**

2000a Critical exponents, colines, and projective geometries. *Combin. Probab. Comput.* 9 (2000), 355–362. MR 2002f:05048. Zbl. 974.51008.

Higher-weight Dowling geometries yield counterexamples to a conjecture. **(gg: Gen: M: N)**

2001a Twelve views of matroid theory. In: Sungpyo Hong *et al.*, eds., *Combinatorial & Computational Mathematics* (Proc., Pohang, 2000), pp. 56–96. World Scientific, Singapore, 2001. MR 2002i:05028.

§5, "Graph theory and lean linear algebra": "lean" means at most 2 nonzero coordinates, hence gain graphs. **(GG: M)**

2002a Curious characterizations of projective and affine geometries. Special issue in memory of Rodica Simion. *Adv. Appl. Math.* 28 (2002), 523–543. MR 2003c:51008. Zbl. 1007.51001.

Dowling geometries $G(\mathfrak{G}K_n^\bullet)$ (if $|\mathfrak{G}| > 2$) and jointless Dowling geometries $G(\mathfrak{G}K_n)$ (if $|\mathfrak{G}| > 4$) exemplify Lemma 3.4, which says that 5 numbers characterize the line sizes in a simple matroid with all lines of size 2, 3, or l. **(gg: M: N)**

Joseph P.S. Kung and James G. Oxley

1988a Combinatorial geometries representable over GF(3) and GF(q). II. Dowling geometries. *Graphs Combin.* 4 (1988), 323–332. MR 90i:05029. Zbl. 702.51004.

For $n \geq 4$, the Dowling geometry of rank n over the sign group is the unique largest simple matroid of rank n that is representable over GF(3) and GF(q). **(sg: M: X)**

M. Loréa

1979a On matroidal families. *Discrete Math.* 28 (1979), 103–106. MR 81a:05029. Zbl. 409.05050.

Discovers the "count" matroids of graphs (see Whiteley (1996a)). **(MF, Bic(Gen))**

Janice R. Lourie

1964a Topology and computation of the generalized transportation problem. *Management Sci.* 11 (1965) (Sept., 1964), no. 1, 177–187. **(GN: M(bases))**

Dănuţ Marcu

1987a Note on the matroidal families. *Riv. Math. Univ. Parma* (4) 13 (1987), 407–412. MR 89k:05025.

Matroidal families of (multi)graphs (see Simões-Pereira (1973a)) correspond to functions on all isomorphism types of graphs that are similar to matroid rank functions, e.g., submodular. This provides insight into matroidal families, e.g., it immediately shows there are infinitely many.

[Many of Marcu's articles known to be plagiarized. See MR 97a:05095 and Zbl. 701.51004. Also see MR 92a:51002, 92b:51026, 92h:11026, 97k:05050.] (**MF, Bic, EC**)

Harry Markowitz

1955a Concepts and computing procedures for certain X_{ij} programming problems. In: H.A. Antosiewicz, ed., *Proceedings of the Second Symposium in Linear Programming* (Washington, D.C., 1955), Vol. II, pp. 509–565. Nat. Bur. Standards of U.S. Dept. of Commerce, and Directorate of Management Analysis, DCS Comptroller, HQ, U.S. Air Force, 1955. Sponsored by Office of Scientific Res., Air Res. and Develop. Command. MR 17, 789.

Also see RAND Corporation Paper P-602, 1954. (**GN: m(bases)**)

J.H. Mason

1977a Matroids as the study of geometrical configurations. In: *Higher Combinatorics* (Proc. NATO Adv. Study Inst., Berlin, 1976), pp. 133–176. NATO Adv. Study Inst. Ser., Ser. C: Math. Phys. Sci., Vol. 31. Reidel, Dordrecht, 1977. MR 80k:05037. Zbl. 358.05017.

§§2.5-2.6: "The lattice approach" and "Generalized coordinates", pp. 172–174, propose a purely matroidal and more general formulation of Dowling's construction of his lattices. (**gg(Gen): M**)

1981a Glueing matroids together: A study of Dilworth truncations and matroid analogues of exterior and symmetric powers. In: *Algebraic Methods in Graph Theory* (Proc., Szeged, 1978), Vol. II, pp. 519–561. Colloq. Math. Soc. János Bolyai, 25. North-Holland, Amsterdam, 1981. MR 84i:05041. Zbl. 477.05022.

Dowling matroids are an example in §1. (**gg: M**)

Laurence R. Matthews

1977a Bicircular matroids. *Quart. J. Math. Oxford* (2) 28 (1977), 213–227. MR 58 #21732. Zbl. 386.05022.

Thorough study of bicircular matroids, introduced by Klee (1971a) and Simões-Pereira (1972a). (**Bic**)

1978a Properties of bicircular matroids. In: *Problèmes Combinatoires et Théorie des Graphes* (Colloq. Internat., Orsay, 1976), pp. 289–290. Colloques Internat. du CNRS, 260. Editions du C.N.R.S., Paris, 1978. MR 81a:05030. Zbl. 427.05021. (**Bic**)

1978b Matroids on the edge sets of directed graphs. In: *Optimization and Operations Research* (Proc. Workshop, Bonn, 1977), pp. 193–199. Lecture Notes in Economics and Math. Systems, 157. Springer-Verlag, Berlin, 1978. MR 80a:05103. Zbl. 401.05031. (**gg: M**)

1978c Matroids from directed graphs. *Discrete Math.* 24 (1978), 47–61. MR 81e:05055. Zbl. 388.05005.

Invents poise, modular poise, and antidirection matroids of a digraph.

(gg: M)

1979a Infinite subgraphs as matroid circuits. *J. Combin. Theory Ser. B* 27 (1979), 260–273. MR 81e:05056. Zbl. 433.05018. **(Bic: Gen)**

Laurence R. Matthews and James G. Oxley

1977a Infinite graphs and bicircular matroids. *Discrete Math.* 19 (1977), 61–65. MR 58 #16348. Zbl. 386.05021. **(Bic)**

Jean François Maurras

1972a Optimization of the flow through networks with gains. *Math. Programming* 3 (1972), 135–144. MR 47 #2993. Zbl. 243.90048. **(GN: M)**

Dillon Mayhew

2005a Inequivalent representations of bias matroids. *Combin. Probab. Comput.* 14 (2005), 567–583. MR 2006j:05040. Zbl. 1081.05021.

The number of inequivalent representations of a frame matroid over a fixed finite field is bounded, if the matroid does not have a free swirl $G(2C_n, \varnothing)$ as a minor. **(GG: M)**

William P. Miller

See also J.E. Bonin.

1997a Techniques in matroid reconstruction. *Discrete Math.* 170 (1997), 173–183. MR 98f:05039. Zbl. 878.05020.

Dowling matroids are reconstructible from their hyperplanes, their deletions, and their contractions. **(gg: M)**

Edward Minieka

1972a Optimal flow in a network with gains. *INFOR* 10 (1972), 171–178. Zbl. 234.90012. **(GN: M(indep))**

1978a *Optimization Algorithms for Networks and Graphs.* Marcel Dekker, New York and Basel, 1978. MR 80a:90066. Zbl. 427.90058.

§4.6: "Flows with gains," pp. 151–174. Also see pp. 80–81. **(GN: m(indep): Exp)**

Nancy Ann Neudauer

See also R.A. Brualdi.

2002a Graph representations of a bicircular matroid. *Discrete Appl. Math.* 118 (2002), 249–262. MR 2003b:05047. Zbl. 990.05025.

Survey of parts of Brualdi and Neudauer (1997a), Wagner (1985a), and Coullard, del Greco, and Wagner (1991a), with supplementary results on nice graphs whose bicircular matroid, $G(\Gamma, \varnothing)$, equals M. **(Bic)**

Nancy Ann Neudauer, Andrew M. Meyers, and Brett Stevens

2001a Enumeration of the bases of the bicircular matroid on a complete graph. Proc. Thirty-second Southeastern Intern. Conf. Combinatorics, Graph Theory and Computing (Baton Rouge, La., 2001). *Congr. Numer.* 149 (2001), 109–127. MR 2002m:05054. Zbl. 1003.05031.

Counts bases and connected bases. Very complicated formulas. [The results count labelled simple 1-trees and 1-forests. A 1*-tree* is a tree with one extra edge forming a circle. A 1*-forest* is a disjoint union of 1-trees. A connected basis of the bicircular matroid $G(K_n, \varnothing)$ for $n \geq 3$ is a labelled simple 1-tree;

a basis is a labelled simple 1-forest. Riddell (1951a) has a less complicated formula for 1-trees.] **(Bic: N(Bases))**

Nancy Ann Neudauer and Brett Stevens

2001a Enumeration of the bases of the bicircular matroid on a complete bipartite graph. *Ars Combin.* 66 (2003), 165–178. MR 2004a:05034. Zbl. 1075.05510.

Bases are counted and their structure compared to the spanning trees of the graph. [A basis is a simple, labelled 1-forest (cf. Neudauer, Meyer, and Stevens 2001a) whose circles are even.] **(Bic: N(Bases))**

Beth Novick and András Sebö

1995a On combinatorial properties of binary spaces. In: Egon Balas and Jens Clausen, eds., *Integer Programming and Combinatorial Optimization* (4th Internat. IPCO Conf., Copenhagen, 1995, Proc.), pp. 212–227. Lecture Notes in Computer Sci., Vol. 920. Springer-Verlag, Berlin, 1995. MR 96h:0503.

The clutter of negative circuits of a signed binary matroid (M, σ). Important are the lift and extended lift matroids, $L(M, \sigma)$ and $L_0(M, \sigma)$, defined as in signed graph theory. An elementary result: the clutter is signed-graphic iff $L_0(M, \sigma)/e_0$ is graphic (which is obvious). There are also more substantial but complicated results. [See Cornuéjols (2001a), §8.4.] **(SM, SG: M)**

1996a On ideal clutters, metrics and multiflows. In: William H. Cunningham, S. Thomas McCormick, and Maurice Queyrann, eds., *Integer Programming and Combinatorial Optimization* (5th Internat. IPCO Conf., Vancouver, 1996, Proc.), pp. 275–287. Lecture Notes in Computer Sci., Vol. 1084. Springer-Verlag, Berlin, 1996. MR 98i:90075. **(SM: M)**

Peter Orlik and Louis Solomon

1980a Unitary reflection groups and cohomology. *Invent. Math.* 59 (1980), 77–94. MR 81f:32017. Zbl. 452.20050. **(gg: M, G)**

1982a Arrangements defined by unitary reflection groups. *Math. Ann.* 261 (1982), 339–357. MR 84h:14006. Zbl. 491.51018. **(gg: M, G)**

1983a Coxeter arrangements. In: Peter Orlik, ed., *Singularities* (Arcata, Calif., 1981), Part 2, pp. 269–291. Proc. Symp. Pure Math., Vol. 40. Amer. Math. Soc., Providence, R.I., 1983. MR 85b:32016. **(gg: M, G)**

James B. Orlin

See also R.K. Ahuja, M. Kodialam, and R. Shull.

1985a On the simplex algorithm for networks and generalized networks. *Math. Programming Study* 24 (1985), 166–178. MR 87k:90102. Zbl. 592.90031. **(GN: M(Bases): Alg)**

James G. Oxley

See also J.P.S. Kung and L.R. Matthews.

James Oxley, Dirk Vertigan, and Geoff Whittle

1996a On inequivalent representations of matroids over finite fields. *J. Combin. Theory Ser. B* 67 (1996), 325–343. MR 97d:05052. Zbl. 856.05021.

§5: Free swirls, $G(2C_n, \varnothing)$ $(n \geq 4)$, mentioning their relationship to Dowling lattices, and complete free spikes, $L_0(2C_n, \varnothing)$. **(GG: M)**

Steven R. Pagano

†1998a Separability and Representability of Bias Matroids of Signed Graphs. Doctoral thesis, Dept. of Mathematical Sciences, Binghamton University, 1998.

Ch. 1: "Separability". Graphical characterization of bias-matroid k-separations of a biased graph. Also, some results on the possibility of k-separations in which one or both sides are connected subgraphs. (**GG: M: Str**)

Ch. 2: "Representability". The frame matroid of every signed graph is representable over all fields with characteristic $\neq 2$. For which signed graphs is it representable in characteristic 2 (and therefore representable over GF(4), by the theorem of Geoff Whittle, A characterization of the matroids representable over GF(3) and the rationals. *J. Combin. Theory Ser. B* 65 (1995), 222–261. MR 96m:05046. Zbl. 835.05015.)? Solved (for 3-connected signed graphs having vertex-disjoint negative circles and hence nonregular matroid). There are two essentially different types: (i) two balanced graphs joined by three independent unbalanced digons; (ii) a cylindrical signed graph, possibly with balanced graphs adjoined by 3-sums. [See notes on Seymour (1995a) for definition of (ii) and for Lovász's structure theorem in the case without vertex-disjoint negative circles.]

Furthermore, the representations of these graphs in characteristic not 2 are all canonical signed-graphic, while any representations over GF(4) are canonical $\mathbb{Z}_3$-gain graphic. (**SG: M: I, Str, T**)

Ch. 3: "Miscellaneous results". (**SG: M: I, Str**)

20xxa Binary signed graphs. Submitted. (**SG: M: I, Str**)

20xxb Signed graphic GF(4) forbidden minors. Submitted. (**SG: M**)

20xxc GF(4)-representations of bias matroids of signed graphs: The 3-connected case. Submitted. (**SG: M: I, Str, T**)

Alexander Postnikov

1997a Intransitive trees. *J. Combin. Theory Ser. A* 79 (1997), 360–366. MR 98b:05036. Zbl. 876.05042.

§4.2 mentions the lift matroid of $\{1\}\vec{K}_n$, i.e., the integral poise gains of a transitively oriented complete graph, represented by the Linial arrangement. [See also Stanley (1996a).] (**GG: M, G**)

Alexander Postnikov and Richard P. Stanley

2000a Deformations of Coxeter hyperplane arrangements. *J. Combin. Theory Ser. A* 91 (2000), 544–597. MR 2002g:52032. Zbl. 962.05004.

The arrangements are the canonical affine-hyperplane lift representations of certain additive real gain graphs. Characteristic polynomials of the former, equalling zero-free chromatic polynomials of the latter, are calculated. And much more. (**gg: G, M, N**)

J. Scott Provan

1987a Substitutes and complements in constrained linear models. *SIAM J. Algebraic Discrete Methods* 8 (1987), 585–603. MR 89c:90072. Zbl. 645.90049.

§4: "Determinacy in a class of network models." [Fig. 1 and Thm. 4.7 hint at possible digraph version of signed-graph or gain-graph frame matroid.] (**?sg, gg: m(?bases): gen**)

Hongxun Qin

See also J.E. Bonin, P. Brooksbank, T.. Dowling, and D.C. Slilaty.

2004a Complete principal truncations of Dowling lattices. *Adv. Appl. Math.* 32 (2004), nos. 1–2, 364–379. MR 2005e:06003. Zbl. 1041.05019.

These matroids are determined by their Tutte polynomials, except that only the order of the group can be determined. (**gg: M: I**)

Hongxun Qin, Daniel C. Slilaty, and Xiangqian Zhou

20xxa The regular excluded minors for signed-graphic matroids. Submitted. (**SG: M: Str**)

Uriel G. Rothblum and Hans Schneider

1980a Characterizations of optimal scalings of matrices. *Math. Programming* 19 (1980), 121–136. MR 81j:65064. Zbl. 437.65038. (**gg: m**)

1982a Characterizations of extreme normalized circulations satisfying linear constraints. *Linear Algebra Appl.* 46 (1982), 61–72. MR 84d:90047. Zbl. 503.05032. (**gg: m**)

Bernard Roy

1970a *Algèbre moderne et théorie des graphes, orientées vers les sciences économiques et sociales. Tome II: Applications et problèmes spécifiques.* Dunod, Paris, 1970. MR 41 #5039. Zbl. 238.90073.

§IX.B.3.b: "Flots multiplicatifs et non conditionnels, ou k-flots." §IX.E.1.b: "Extension du problème central aux k-flots." §IX.E.2.c: "Quelques utilisations concretes des k-flots." (**GN: m(circuit): Exp**)

Irasema Sarmiento

1999a A characterisation of jointless Dowling geometries. 16th British Combinatorial Conference (London, 1997). *Discrete Math.* 197/198 (1999), 713–731. MR 99m:-51020. Zbl. 929.05016.

They are 4-closed (determined by their flats of rank 4). They are characterized, among all matroids, by the statistics of flats of rank ≤ 7 and therefore by their Tutte polynomials. There are exceptions in rank 3. (**GG: M: N**)

Rüdiger Schmidt

1979a On the existence of uncountably many matroidal families. *Discrete Math.* 27 (1979), 93–97. MR 80i:05029. Zbl. 427.05024.

The "count" matroids of graphs (see Whiteley (1996a)) and an extensive further generalization of bicircular matroids that includes frame matroids. His "partly closed set" is a linear class of circuits in an arbitrary "count" matroid. (**MF: GG, M, Bic, EC: Gen**)

Gary K. Schwartz

2002a On the automorphism groups of Dowling geometries. *Combin. Probab. Comput.* 11 (2002), no. 3, 311–321. MR 2004c:20005. Zbl. 1008.06007.

Aut $Q_n(\mathfrak{G})$ factors in a certain natural way if, but also only if, $\mathfrak{G}$ factors. (**gg: M: Aut**)

Charles Semple and Geoff Whittle

1996a Partial fields and matroid representation. *Adv. Appl. Math.* 17 (1996), 184–208. MR 97g:05046. Zbl. 859.05035.

§7: "Dowling group geometries". A Dowling geometry of a group $\mathfrak{G}$ has a partial-field representation iff G is abelian and has at most one involution. (**gg: M: I**)

P.D. Seymour

See also M. Chudnovsky; Gerards, Lovász, *et al.* (1990a); W. McCuaig; and N. Robertson.

1977a The matroids with the max-flow min-cut property. *J. Combin. Theory Ser. B* 23 (1977), 189–222. MR 57 #2960. Zbl. 375.05022.

The central example is $Q_6 = \mathcal{C}_-(-K_4)$, the clutter of (edge sets of) negative polygons in $-K_4$. P. 199: the extended lift matroid $L_0(-K_4) = F_7^*$, the dual Fano matroid. Result (3.4) readily generalizes (by the negative-subdivision trick) to: every $\mathcal{C}_-(\Sigma)$ is a binary clutter, that is, a port of a binary matroid. [This is also immediate from the construction of $L_0(\Sigma)$.]

P. 200, (i)–(iii): Amongst minor-minimal binary clutters without the "weak MFMC property" are the circuit clutter of F_7^* and $\mathcal{C}_-(-K_5)$ and its blocker.

Main Thm. (§5): A binary clutter is "Mengerian" (I omit the definition) iff it does not have $\mathcal{C}_-(-K_4)$ as a minor. (See p. 200 for the antecedent theorem of Gallai.)

[See Cornuéjols (2001a), Guenin (2001a) for more.] (**sg, P: M, G**)

1981a Matroids and multicommodity flows. *European J. Combin.* 2 (1981), 257–290. MR 82m:05030. Zbl. 479.05023.

Conjecture (based on (1977a)). A binary clutter has the weak MFMC property iff no minor is either the circuit clutter of F_7 or $\mathcal{C}_-(-K_5)$ or its blocker. (**sm, sg: M**)

Randy Shull, James B. Orlin, Alan Shuchat, and Marianne L. Gardner

1989a The structure of bases in bicircular matroids. *Discrete Appl. Math.* 23 (1989), 267–283. MR 90h:05040. Zbl. 698.05022.

[See Coullard, del Greco, and Wagner (1991a).] (**Bic(Bases)**)

Randy Shull, Alan Shuchat, James B. Orlin, and Marianne Lepp

1993a Recognizing hidden bicircular networks. *Discrete Appl. Math.* 41 (1993), 13–53. MR 94e:90122. Zbl. 781.90089. (**GN: Bic: I, Alg**)

1997a Arc weighting in hidden bicircular networks. Proc. Twenty-eighth Southeastern Internat. Conf. on Combinatorics, Graph Theory and Computing (Boca Raton, Fla., 1997). *Congressus Numer.* 125 (1997), 161–171. MR 98m:05181. Zbl. 902.90157. (**GN: Bic: I, Alg**)

Rodica Simion

2000a Combinatorial statistics on type-B analogues of noncrossing partitions and restricted permutations *Electronic J. Combin.* 7 (2000), Research Paper R9, 27 pp. (electronic). MR 2000k:05013. Zbl. 938.05003.

"Type-B noncrossing partitions" are certain signed partial partitions of the ground set; i.e., certain elements of the Dowling lattice of $\{\pm\}$. (**gg: M**)

J.M.S. Simões-Pereira

1972a On subgraphs as matroid cells. *Math. Z.* 127 (1972), 315–322. MR 47 #6522. Zbl. 226.05016, (243.05022).

"Cell" = circuit. Along with Klee (1971a), invents the bicircular matroid (here, for finite graphs) (Thm. 1). Suppose we have matroids on the edge sets of all [simple] graphs, such that the class of circuits is a [nonempty] union

of homeomorphism classes of connected graphs. Thm. 2: The polygon and bicircular matroids [and free matroids] are the only such matroids. (**Bic**)

1973a On matroids on edge sets of graphs with connected subgraphs as circuits. *Proc. Amer. Math. Soc.* 38 (1973), 503–506. MR 47 #3214. Zbl. 241.05114, 264.05126.

A family of (isomorphism types of) [simple] connected graphs is "matroidal" if for any Γ the class of subgraphs of Γ that are in the family constitute the circuits of a matroid on $E(\Gamma)$. Bicircular and even-circle matroids are the two nicest examples. A referee contributes the even-circle matroid [*cf.* Tutte (1981a), Doob (1973a)]. Thm.: The family cannot be finite [unless it is void or consists of K_2]. [See Marcu (1987a) for a valuable new viewpoint.] (**MF, Bic, EC**)

1975a On matroids on edge sets of graphs with connected subgraphs as circuits II. *Discrete Math.* 12 (1975), 55–78. MR 54 #7298. Zbl. 307.05129.

Partial results on describing matroidal families of simple, connected graphs. Five basic types: free [omitted in the paper], cofree, polygon, bicircular, and even-circle. If the family does not correspond to one of these, then every member has ≥ 3 independent polygons and minimum degree ≥ 3. (**MF, Bic, EC**)

1978a A comment on matroidal families. In: *Problèmes Combinatoires et Théorie des Graphes* (Colloq. Internat., Orsay, 1976), pp. 385–387. Colloques Internat. du CNRS, 260. Editions du C.N.R.S., Paris, 1978. MR 81b:05031. Zbl. 412.05023.

Two small additions to (1973a, 1975a); one is that a matroidal family not one of the five basic types must contain $K_{p,q(p)}$ for each $m \geq 3$, with $q(p) \geq p$. (**MF, Bic, EC**)

1992a Matroidal families of graphs. In: Neil White, ed., *Matroid Applications*, Ch. 4, pp. 91–105. Encycl. Math. Appl., Vol. 40. Cambridge Univ. Press, Cambridge, Eng., 1992. MR 93c:05036. Zbl. 768.05024.

"Count" matroids (see N. White (1996a)) in §4.3; Schmidt's (1979a) remarkable generalization in §4.4. (**MF, Bic, EC: Exp, Exr, Ref**)

Daniel Slilaty

2000a Orientations of Biased Graphs and Their Matroids. Doctoral dissertation, Dept. of Mathematical Sciences, Binghamton University, 2000.

Introducing orientation of biased graphs and biased signed graphs by means of proper circle orientations and their generalization, "graphical orientation schemes". The definition is chosen so as to produce orientations of the bias and complete lift matroids and (though not in the thesis) to model the orientation of the bias or complete lift matroid of, respectively, an $\mathbb{R}^*$- or $\mathbb{R}^+$-gain graph induced by its canonical bias or lift representation (Zaslavsky 2003b). Characterizations of equivalence of different orientation schemes. The completeness question: when do graphical orientation schemes yield all orientations of the frame matroid? Always, for additively biased (i.e., signed) graphs and for some other kinds of biased graphs. (**GG: O, M, SG**)

2002a Matroid duality from topological duality in surfaces of nonnegative euler characteristic. *Combin. Probab. Computing* 11 (2002), no. 5, 515–528. MR 2003i:05034. Zbl. 1009.05036.

Duality of matroids of biased graphs, obtained by defining gains through embedding in a surface and dualizing the graph in the surface.

(GG, SG: M, D, T)

2005a On cographic matroids and signed-graphic matroids. *Discrete Math.* 301 (2005), no. 12, 207–217. MR 2007c:05049. Zbl. 1078.05017. **(SG: M, T)**

2006a Bias matroids with unique graphical representations. *Discrete Math.* 306 (2006), no. 12, 1253–1256. **(GG: M: Str)**

2007a Projective-planar signed graphs and tangled signed graphs. *J. Combin. Theory Ser. B* 97 (2007), no. 5, 693–717. **(SG: T, Str)**

20xxa Connectivity in signed-graphic matroids. Submitted. **(SG: M:Str)**

Daniel C. Slilaty and Hongxun Qin

2007a Decompositions of signed-graphic matroids. *Discrete Math.* 307 (2007), nos. 17–18, 2187–2199. **(SG: M: Str)**

2008a The signed-graphic representations of wheels and whirls. *Discrete Math.* 308 (2008), no. 10, 1816–1825.

All frame matroids (of biased graphs) that are wheels and whirls, characterized topologically by embeddings in the projective plane (wheels) and the cylinder (whirls). **(GG: M: Str)**

2008b Connectivity in frame matroids. *Discrete Math.* 308 (2008), no. 10, 1994–2001.

Graphical biconnectivity of Ω vs. matroid connectivity of $G(\Omega)$, generalizing concepts developed by Wagner (1985a) for the bicircular matroid. **(GG: M: Str)**

Murali K. Srinivasan

1998a Boolean packings in Dowling geometries. *European J. Combin.* 19 (1998), 727–731. MR 99i:05059. Zbl. 990.10387.

Decomposes the Dowling lattice $Q_n(\mathfrak{G})$ into Boolean algebras, indexed in part by integer compositions, that are cover-preserving and centered above the middle rank. **(GG: M)**

Richard P. Stanley

See also P. Doubilet and A. Postnikov.

1996a Hyperplane arrangements, interval orders, and trees. *Proc. Nat. Acad. Sci. USA* 93 (1996), 2620–2625. MR 97i:52013. Zbl. 848.05005.

Deformed braid hyperplane arrangements, i.e., canonical affine hyperplanar lift representations of $\mathrm{Lat}^{\mathrm{b}}\,\Phi$ where $\|\Phi\| = K_n$ and edge ij has gain $l_i \in \mathbb{Z}$ when $i < j$. In particular (§4), all $l_i = 1$. Also (§5), the Shi arrangement, which represents $\mathrm{Lat}^{\mathrm{b}}\{0,1\}\vec{K}_n$. **(gg: G, M, N: Exp)**

1998a Hyperplane arrangements, parking functions and tree inversions. In: B.E. Sagan and R. Stanley, eds., *Mathematical Essays in Honor of Gian-Carlo Rota*, Progress in Math., Vol. 161, pp. 359–375. Birkhäuser, Boston, 1998. MR 99f:05006. Zbl. 980.39546. **(gg: G, M, N: Exp)**

1999a *Enumerative Combinatorics, Volume 2.* Cambridge Stud. Adv. Math., Vol. 62. Cambridge University Press, Cambridge, Eng., 1999. MR 2000k:05026. Zbl. 928.05001.

Exercise 5.50: The Shi arrangement [the affinographic hyperplane representation of $\{0,1\}\vec{K}_n$ with gain group $\mathbb{Z}^+$]. Exercise 5.41(h–i): The

Linial arrangement and its characteristic polynomial [= $\chi^*_{\{1\}\vec{K}_n}(\lambda)$]. Exercise 6.19(lll) conceals the Catalan arrangement [representing $\{0,\pm1\}\vec{K}_n$].
(gg: G, m, N: Exr, Exp)

J. Randolph Stonesifer

1975a Logarithmic concavity for a class of geometric lattices. *J. Combin. Theory Ser. A* 18 (1975), 216–218. MR 50 #9637. Zbl. 312.05019.

The second kind of Whitney numbers of a Dowling lattice are binomially concave, hence strongly logarithmically concave, hence unimodal. [*Famous Problem* (Rota). Generalize this.] **(gg: M: N)**

W.T. Tutte

†1981a On chain-groups and the factors of graphs. In: L. Lovász and Vera T. Sós, eds., *Algebraic Methods in Graph Theory* (Proc. Colloq., Szeged, 1978), Vol. 2, pp. 793–818. Colloq. Math. Soc. János Bolyai, 25. János Bolyai Math. Soc., Budapest, and North-Holland, Amsterdam, 1981. MR 83b:05104. Zbl. 473.05023.

The chain-group approach to the dual even-circle matroid, $G(-\Gamma)^*$. Developed entirely in terms of the group $\Delta(\Gamma)$ [topologically, $B^1(\Gamma,\mathbb{Z})$] of integral 1-coboundaries. Assuming Γ connected: "Dendroids of $\Delta(\Gamma)$" = bases of $G(-\Gamma)$; Thms. 8.6–7 give their structure in the bipartite and nonbipartite cases. Support of an elementary coboundary = circuit of $G(-\Gamma)^*$; this is a bond of Γ if Γ is bipartite (Thm. 7.5) and a minimal balancing set otherwise (Thm. 7.6). Thm. 7.8: Any coboundary times some power of 2 is a sum of primitive coboundaries. [*Problem*. Explain how this is related to total dyadicity of the incidence matrix.] "Rank of $\Delta(\Gamma)$" = $\operatorname{rk} G(-\Gamma)$; its value is given at the end of §8. §9 develops a relationship between "homomorphisms" of $\Delta(\Gamma)$ (linear functionals) and graph factors. §10: The dual chain group; characterization of circuits of $\operatorname{rk} G(-\Gamma)$. [It is amazing what can be done with nothing but integral 1-coboundaries. *Problem* 1. Extend Tutte's theory of integral chain groups to all signed graphs. Grossman, Kulkarni, and Schochetman (1994a) have a development over a field but this is very different, even aside from their opposite viewpoint that goes from matroids to vector spaces. *Problem* 2. Extend to signed hypergraphs, where each hyperedge has a function $\tau_e : V(e) \to \{+,-\}$, not distinguished from $-\tau_e$—as with bidirected graphs, choosing one of them corresponds to orienting e.]

[Tutte knew and lectured on $G(-\Gamma)^*$ and/or $G(-\Gamma)$ before anyone (Doob 1973a, Simões-Pereira 1973a) published it.—information from Neil Robertson.] **(sg: EC, D, i)**

Donald K. Wagner

See also V. Chandru and C.R. Coullard.

††1985a Connectivity in bicircular matroids. *J. Combin. Theory Ser. B* 39 (1985), 308–324. MR 87c:05041. Zbl. 584.05019.

Prop. 1 and Thm. 2 show that n-connectivity of the bicircular matroid $B(\Gamma)$ is equivalent to "n-biconnectivity" of Γ.

When do two 3-biconnected graphs have isomorphic bicircular matroids? §5 proves that 3-biconnected graphs with > 4 vertices have isomorphic bicircular matroids iff one is obtained from the other by a sequence of operations called "edge rolling" and "3-star rotation". This is the bicircular analog

of Whitney's polygon-matroid isomorphism theorem, but it is complicated. [An important theorem, generalized to all bicircular matroids in Coullard, del Greco, and Wagner (1991a). *Major Research Problems.* Generalize to frame matroids of biased graphs. Find the analog for lift matroids.]
(**Bic: Str**)

1988a Equivalent factor matroids of graphs. *Combinatorica* 8 (1988), 373–377. MR 90d:05071. Zbl. 717.05022.
"Factor matroid" = even-circle matroid $G(-\Gamma)$. Decides when $G(-\Gamma) \cong G(B)$ where B is a given bipartite, 4-connected graph. (**EC: Str**)

Neil L. White
See also A. Björner.
1986a A pruning theorem for linear count matroids. *Congressus Numerantium* 54 (1986), 259–264. MR 88c:05047. Zbl. 621.05009. (**Bic: Gen, MF**)

Neil White and Walter Whiteley
1983a A class of matroids defined on graphs and hypergraphs by counting properties. Unpublished manuscript, 1983.
See Whiteley (1996a) for an exposition and extension. (**Bic: Gen, MF**)

Walter Whiteley
1996a Some matroids from discrete applied geometry. In: Joseph E. Bonin, James G. Oxley, and Brigitte Servatius, eds., *Matroid Theory* (Proc., Seattle, 1995), pp. 171–311. Contemp. Math., Vol. 197. Amer. Math. Soc., Providence, R.I., 1996. MR 97h:05040. Zbl. 860.05018.
Appendix: "Matroids from counts on graphs and hypergraphs", which expounds and extends Loréa (1979a), Schmidt (1979a), and especially White and Whiteley (1983a), describes matroids on the edge sets of graphs (and hypergraphs) that generalize the bicircular matroid. The definition: given $m \geq 0$ and $k \in \mathbb{Z}$, S is independent iff $\varnothing \subset S' \subseteq S$ implies $|S'| \leq m|V(S')| + k$. (**Bic: Gen, MF**)(**Ref**)

Geoff Whittle
See also J. Oxley and C. Semple.
1989a Dowling group geometries and the critical problem. *J. Combin. Theory Ser. B* 47 (1989), 80–92. MR 90g:51008. Zbl. 628.05018.
A Dowling-lattice version of Crapo and Rota's critical problem is developed. Some minimal matroids whose critical exponent is k (i.e., tangential k-blocks) are given, one being $G(\pm K_n^\circ)$. (**gg: M: N**)

1989b A generalisation of the matroid lift construction. *Trans. Amer. Math. Soc.* 316 (1989), 141–159. MR 90b:05038. Zbl. 684.05014.
Examples include bicircular and frame matroids. (**GG: M, Bic**)

Zhaoyang Wu
2003a On the number of spikes over finite fields. *Discrete Math.* 265 (2003), 261–296. MR 2004b:05057.
A spike is $L_0(\Omega)$ where $\|\Omega\| = 2C_n$. (**gg: M: E**)

Young-Jin Yoon
1997a A characterization of supersolvable signed graphs. *Commun. Korean Math. Soc.* 12 (1997), 1069–1073. MR 99j:05165. Zbl. 945.05051.

Attempts to characterize supersolvability of $G(\Sigma)$ in terms of [bias-]simplicial vertices. [There are fundamental conceptual and technical errors, vitiating the entire paper; see Koban (2004a). For correct results see Zaslavsky (2001a) and Koban (2004a).] **(SG: M: Str)**

Thomas Zaslavsky

1977a Biased graphs. Unpublished manuscript, 1977.

Published, greatly expanded, as (1989a, 1991a, 1995b) and more; as well as (but restricted to signed graphs) (1982a, 1982b). **(GG: M)**

1980a Voltage-graphic geometry and the forest lattice. In: *Report on the XVth Denison-O.S.U. Math. Conf.* (Granville, Ohio, 1980), pp. 85–89. Dept. of Math., The Ohio State Univ., Columbus, Ohio, 1980. **(GG: M, Bic)**

1981a The geometry of root systems and signed graphs. *Amer. Math. Monthly* 88 (1981), 88–105. MR 82g:05012. Zbl. 466.05058.

Signed graphs correspond to arrangements of hyperplanes in $\mathbb{R}^n$ of the forms $x_i = x_j$, $x_i = -x_j$, and $x_i = 0$. Consequently, one can compute the number of regions of the arrangement from graph theory, esp. for arrangements corresponding to "sign-symmetric" graphs, i.e., having both or none of each pair $x_i = \pm x_j$. Simplified account of parts of (1982a, 1982b, 1982c), emphasizing geometry. **(SG: M, G, N)**

1981c Is there a theory of signed graph embedding? In: *Report on the XVIth Denison-O.S.U. Math. Conf.* (Granville, Ohio, 1981), pp. 79–82. Dept. of Math., The Ohio State Univ., Columbus, Ohio, 1981.

See (1997a). **(SG: T, M)**

††1982a Signed graphs. *Discrete Appl. Math.* 4 (1982), 47–74. MR 84e:05095a. Zbl. 476.05080. Erratum. *Ibid.* 5 (1983), 248. MR 84e:05095b. Zbl. 503.05060.

Basic results on: Switching (§3). Minors (§4). The frame matroid $G(\Sigma)$ in many cryptomorphisms (§5) (some erroneous: Thm. 5.1(f,g); partly corrected in the Erratum [and fully in (1991a)]), consistency of matroid with signed-graph minors; separators of $G(\Sigma)$. The signed covering graph $\tilde{\Sigma}$ (§6).

In §8A, the incidence and Kirchhoff matrices and matrix-tree theorem [different from that of Murasugi (1989a)] [generalized by Chaiken (1982a) to a weighted, all-minors version, both directed and undirected]. In §8B, vector representation of the matroid $G(\Sigma)$ by the incidence matrix [as multisubsets of root systems $B_n \cup C_n$].

Conjectures about the interrelation between representability in characteristic 2 and unique representability in characteristic 0 [since answered by Geoff Whittle (A characterisation of the matroids representable over GF(3) and the rationals. *J. Combin. Theory Ser. B* 65 (1995), 222–261. MR 96m:05046. Zbl. 835.05015) as developed by Pagano (1998a, 20xxc)].

Examples (§7) include: Sign-symmetric graphs and signed expansions $\pm\Gamma$. The all-negative graph $-\Gamma$, whose matroid (Cor. 7D.3; partly corrected in the Erratum) is the even-circle matroid (see Doob 1973a) and whose incidence matrices include the unoriented incidence matrix of Γ.

Generalizations to gain graphs (called "voltage graphs") mentioned in §9. **(SG, GG: M, I, G; EC)**

††1982b Signed graph coloring. *Discrete Math.* 39 (1982), 215–228. MR 84h:05050a. Zbl. 487.05027.

A "proper k-coloring" of Σ partitions V into a special "zero" part, possibly void, that induces a stable subgraph, and up to k other parts (labelled from a set of k colors), each of which induces an antibalanced subgraph. A "zero-free proper k-coloring" is similar but without the "zero" part. [The suggestion is that a signed analog of a stable vertex set is one that induces an antibalanced subgraph. *Problem.* Use this insight to develop generalizations of stable-set notions, such as cliques and perfection. *Example.* Let $\alpha(\Sigma)$, the "antibalanced vertex set number", be the largest size of an antibalance-inducing vertex set. Then $\alpha(\Gamma) = \alpha(+\Gamma \cup -K_n)$.] One gets two related chromatic polynomials. The chromatic polynomial, $\chi_\Sigma(2k+1)$, counts all proper k-colorings; it is essentially the characteristic polynomial of the frame matroid. It can often be most easily computed via the zero-free chromatic polynomial, $\chi^*_\Sigma(2k)$, which counts proper zero-free colorings: see (1982c).
(SG, GG: M, Col, N, O, G)

1982c Chromatic invariants of signed graphs. *Discrete Math.* 42 (1982), 287–312. MR 84h:05050b. Zbl. 498.05030.

Continuation of (1982b). The fundamental balanced expansion formulas, that express the chromatic polynomial in terms of the zero-free chromatic polynomial. Many special cases, treated in great detail: antibalanced graphs, signed graphs that contain $+K_n$ or $-K_n$, signed K_n's (a.k.a. two-graphs), etc. **(SG, GG: M, N, Col, O, G; EC)**

1982d Bicircular geometry and the lattice of forests of a graph. *Quart. J. Math. Oxford* (2) 33 (1982), 493–511. MR 84h:05050c. Zbl. 519.05020. **(GG: M, Bic, G, N)**

1982e Voltage-graphic matroids. In: Adriano Barlotti, ed., *Matroid Theory and Its Applications* (Proc. Session of C.I.M.E., Varenna, Italy, 1980), pp. 417–423. Liguore Editore, Naples, 1982. MR 87g:05003 (book). **(GG: M, EC, Bic, N)**

1987a The biased graphs whose matroids are binary. *J. Combin. Theory Ser. B* 42 (1987), 337–347. MR 88h:05082. Zbl. 667.05015.

For the frame (bias), lift, and extended lift matroids: forbidden-minor and structural characterizations. The latter for signed-graphiic frame matroids is superseded by a result of Pagano (1998a).

[Error in Cor. 4.3: In the last statement, omit "$G(\Omega) = L(\Omega)$." That is true when Ω has no loops, but may not be if Ω has a loop e (because Theorem 3(3) applies with unbalanced block e, but $(E \setminus e, e)$ is not a 2-separation).]
(GG: M: Str)

1989a Biased graphs. I. Bias, balance, and gains. *J. Combin. Theory Ser. B* 47 (1989), 32–52. MR 90k:05138. Zbl. 714.05057.

Fundamental concepts and lemmas of biased graphs. Bias from gains; switching of gains; characterization of balance [for which see also Harary, Lindstrom, and Zetterstrom (1982a)]. **(GG)**

1990a Biased graphs whose matroids are special binary matroids. *Graphs Combin.* 6 (1990), 77–93. MR 91f:05097. Zbl. 786.05020.

A complete list of the biased graphs Ω such that $G(\Omega)$, $L_0(\Omega)$, or $L(\Omega)$ is one of the traditional special binary matroids, $G(K_5), G(K_{33}), F_7$, their du-

als, and $G(K_m)$ (for $m \geq 4$) and R_{10}. [Unfortunately omitted are nonbinary matroids like the non-Fano plane and its dual.]

[There is an error. The graphs $\langle +K_n^\circ \rangle$ were overlooked in the last statement of Lemma 1H—due to an oversight in (1987a) Cor. 4.3—and thus in Props. 2A and 5A. A corrected last statement of Lemma 1H: "If Ω has no two vertex-disjoint negative circles, then $G(\Omega) = M \iff L(\Omega) = M$." In Prop. 2A, add $\Omega = \langle +K_3^\circ \rangle$ to the list for $G(K_4)$. In Prop. 5A, add $\Omega = \langle +K_{m-1}^\circ \rangle$ to the list for $G(K_m)$. Thanks to Stefan van Zwam (25 July 2007).] (**GG: M**)

††1991a Biased graphs. II. The three matroids. *J. Combin. Theory Ser. B* 51 (1991), 46–72. MR 91m:05056. Zbl. 763.05096.

Basic theory of the bias, lift, and complete lift matroids. Infinite graphs. Matroids that are intermediate between the bias and lift matroids. Several questions and conjectures. (**GG: M**)

1991b Orientation of signed graphs. *European J. Combin.* 12 (1991), 361–375. MR 93a:05065. Zbl. 761.05095.

Oriented signed graph = bidirected graph. The oriented matroid of an oriented signed graph. A "cycle" in a bidirected graph is a bias circuit (a balanced polygon, or a handcuff with both circles negative) oriented to have no source or sink. Cycles in Σ are compared with those in its signed (i.e., derived) covering graph $\tilde{\Sigma}$. The correspondences among acyclic orientations of Σ and regions of the hyperplane arrangements of Σ and $\tilde{\Sigma}$, and dually the faces of the acyclotope of Σ. Thm. 4.1: the net degree vector $d(\tau)$ of an orientation τ belongs to the face of the acyclotope that is determined by the union of all cycles. Cor. 5.3 (easy): a finite bidirected graph has a source or sink. (**SG: O, M, G**)(**SGw: N**)

1994a Frame matroids and biased graphs. *European J. Combin.* 15 (1994), 303–307. MR 95a:05021. Zbl. 797.05027.

A simple matroidal characterization of the frame matroids of biased graphs. (**GG: M**)

††1995b Biased graphs. III. Chromatic and dichromatic invariants. *J. Combin. Theory Ser. B* 64 (1995), 17–88. MR 96g:05139. Zbl. 857.05088.

Polynomials of gain and biased graphs: a four-variable polynomial specializes to the chromatic, dichromatic, and Whitney-number polynomials. The polynomials come in two flavors: unrestricted and balanced, depending on the edge sets that appear in their defining sums.

§4: "Gain-graph coloring". In Φ with gain group $\mathfrak{G}$, a "zero-free k-coloring" is a mapping $f : V \to [k] \times \mathfrak{G}$; it is "proper" if, when $e{:}vw$ is a link or loop and $f(v) = (i, g), f(w) = (i, h)$, then $h \neq g\varphi(e; v, w)$. A "k-coloring" is similar but the color set is enlarged by inclusion of a color 0; propriety requires the additional restriction that $f(v)$ and $f(w)$ are not both 0 (and $f(v) \neq 0$ if v supports a half edge). In particular, a "group-coloring" of Φ is a zero-free 1-coloring (ignoring the irrelevant numerical part of the color). A "partial group-coloring" is a group-coloring of an induced subgraph [which can only be proper if the uncolored vertices form a stable set]. The unrestricted and balanced chromatic polynomials count, respectively, unrestricted and zero-free proper k-colorings; the two

Whitney-number polynomials count all colorings, proper and improper, by their improper edge sets.

§5: "The matroid connection". The various polynomials are, in essence, frame matroid invariants and closely related to corresponding lift matroid and extended lift matroid invariants.

Almost infinitely many identities, some of them (esp., the balanced expansion formulas in §6) essential. Innumerable examples worked in detail. [The first half, to the middle of §6, is fundamental. The rest is more or less ornamental. Most of the results are, intentionally, generalizations of properties of ordinary graphs.] **(GG: N, M, Col)**

1997a Is there a matroid theory of signed graph embedding? *Ars Combinatoria* 45 (1997), 129–141. MR 97m:05084. Zbl. 933.05067. **(SG: M, T)**

2001a Supersolvable frame-matroid and graphic-lift lattices. *European J. Combin.* 22 (2001), 119–133. MR 2001k:05051. Zbl. 966.05013.

Biased graphs whose bias and lift matroids are supersolvable are characterized by a form of simplicial vertex ordering—with a few exceptions. As preliminary results, modular copoints are characterized [but incompletely in the bias-matroid case, as observed by Koban (2004a)]. §4: "Examples": 4a: "Group expansions and biased expansions"; 4b: "Near-Dowling and Dowling lift lattices"; 4c: "An extension of Edelman and Reiner's theorem" to general gain groups (see Edelman and Reiner (1994a)); 4d: "Bicircular matroids". [Written in 1992 and long delayed. Correction in Koban (2004a). Independently, Yoon (1997a) incorrectly attempted the case of $G(\Sigma)$. Jiang and Yu rediscovered the case of a signed K_n.] **(GG, SG: M, G)**

2002a Perpendicular dissections of space. *Discrete Comput. Geom.* 27 (2002), 303–351. MR 2003i:52026. Zbl. 1001.52011.

Given an additive real gain graph Φ on n vertices and n reference points Q_i in $\mathbb{E}^d$, use Φ to specify perpendicular hyperplanes to each of the lines Q_iQ_j by means of the "Pythagorean coordinate" along Q_iQ_j. For generic points, the number of regions is computable based on the fact that the generic hyperplane intersection lattice is $\mathrm{Lat}^{\mathrm{b}}\,\Phi$. Modifications of Pythagorean coordinates give intersection lattice $\mathrm{Lat}^{\mathrm{b}}(\|\Phi\|, \varnothing)$ or a slightly more complex variant, still for generic reference points. **(GG: G, M, N)**

2003a Faces of a hyperplane arrangement enumerated by ideal dimension, with application to plane, plaids, and Shi. *Geom. Dedicata* 98 (2003), 63–80. MR 2004f:52025. Zbl. 1041.52021.

§6, "Affinographic arrangements": hyperplane arrangements that represent the extended lift matroid $L_0(\Phi)$ where Φ is an additive real gain graph. Examples: the weakly-composed-partition, extended Shi, and extended Linial arrangements. The faces are counted in terms of dimension and dimension of the infinite part. **(GG: m, G, N)**

††2003b Biased graphs IV: Geometrical realizations. *J. Combin. Theory Ser. B* 89 (2003), no. 2, 231–297. MR 2005b:05057. Zbl. 1031.05034.

§§2–4: Various ways in which to represent the bias and lift matroids of a gain or biased graph over a skew field F. Bias matroid: canonical vector and hyperplanar representations (generalizing those of a graph) based on a gain group $\subseteq F^*$, Menelæan and Cevian representations (generalizations of theorems of Menelaus and Ceva), switching vs. change of ideal hyperplane, equational logic. Lift matroid: canonical vector and hyperplanar representations (the latter generalizing the Shi and Linial arrangements among others) based on a gain group $\subseteq F^+$, orthographic representation (an affine variation on canonical representation), Pythagorean representation (Zaslavsky 2002a). Both: effect of switching, nonunique gain-group embedding. §5: Effect of Whitney operations, separating vertex. §6: Matroids characterized by restricted general position. §7, "Thick graphs": A partial unique-representation theorem for biased graphs with sufficient edge multiplicity. §8: The 7 biased K_4's. **(GG: M, G, N)**

2007a Biased graphs. VII. Contrabalance and antivoltages. *J. Combin. Theory Ser. B* 97 (2007), no. 6, 1019–1040.

Contrabalanced graphs, whose gains are called antivoltages. Emphasis on the existence of antivoltages in $\mathbb{Z}_\mu$, $\mathbb{Z}$, and $\mathbb{Z}_p^k$ for application to canonical representation of the contrabalanced bias and lift matroids. The number of such antivoltages is a polynomial function of the group order or (for $\mathbb{Z}$) the bound on circle gains. **(GG: M, bic, G, N)**

20xxd Geometric lattices of structured partitions: I. Gain-graphic matroids and group-valued partitions. Manuscript, 1985 et seq. **(GG: M, N, col)**

20xxe Geometric lattices of structured partitions: II. Lattices of group-valued partitions based on graphs and sets. Manuscript, 1985 et seq. **(GG: M, N, col)**

20xxf Totally frustrated states in the chromatic theory of gain graphs. *European J. Combinatorics*, to appear.

Given a set Q of "spins", a state is $s: V \to Q$. The gain group $\mathfrak{G}$ acts on the spin set. In a permutation gain graph Φ with gain group $\mathfrak{G}$, edge $e{:}vw$ is "satisfied" if $s(w) = s(v)\varphi(e)$, otherwise "frustrated". A totally frustrated state (every edge is frustrated) generalizes a proper coloring. Enumerative theory, including deletion/contraction, a monodromy formula for the number of totally frustrated states, and a multivariate chromatic polynomial. An abstract partition function in the edge algebra. **(GG: Col: Gen: N, M)**

20xxi Big flats in a box. In preparation.

The naive approach to characteristic polynomials via lattice point counting (in characteristic 0) and Möbius inversion (as in Blass and Sagan (1998a)) can only work when one expects it to. [This is a theorem!] **(GG: G, M, N, col)**

20xxj Biased graphs. V. Group and biased expansions. In preparation. **(GG: M, G, N)**

20xxm Biased graphs. VIII. A cornucopia of examples. In preparation.

Numerous types of examples of biased graphs, many having particular theory of their own, e.g., Hamiltonian bias. **(GG: M, G)**

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1997a The characteristic polynomials of subarrangements of Coxeter arrangements. *Discrete Math.* 177 (1997), 245–248. MR 98i:52016. Zbl. 980.06614.

Blass and Sagan's (1998a) geometrical form of signed-graph coloring is used to calculate (I) characteristic polynomials of several versions of k-equal subspace arrangements (these are the main results) and (II) [also in Zhang (2000a)] the chromatic polynomials (in geometrical guise) of ordinary graphs extending K_n by one vertex, signed graphs extending $\pm K_n^\circ$ by one vertex, and $\pm K_n$ with any number of negative loops adjoined. (**sg: N, G, col**)

2000a The characteristic polynomials of interpolations between Coxeter arrangements. *J. Combin. Math. Combin. Comput.* 34 (2000), 109–117. MR 2001b:05220. Zbl. 968.32017.

Uses signed-graph coloring (in geometrical guise) to evaluate the chromatic polynomials (in geometrical guise) of all signed graphs interpolating between (1) $+K_n$ and $+K_{n+1}$ [i.e., ordinary graphs extending a complete graph by one vertex]; (2) $\pm K_{n-1}^\circ$ and $\pm K_n^\circ$; (3) $\pm K_n$ and $\pm K_n^\circ$ [known already by several methods, including this one]; (4a) $\pm K_{n-1}$ and $\pm K_{n-1} \cup +K_n$; (4b) $\pm K_{n-1} \cup +K_n$ and $\pm K_n$; and certain signed graphs interpolating (by adding negative edges one vertex at a time, or working down and removing them one vertex at a time) between (5) $+K_n$ and $\pm K_n^\circ$; (6) $+K_n$ and $\pm K_n$. In cases (1)–(3) the chromatic polynomial depends only on how many edges are added [which is obvious from the coloring procedure]. (**sg: N, col, G**)