A Mathematical Bibliography of Signed and Gain Graphs and Allied Areas

Compiled and Annotated by

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Colleagues:
HELP!
If you have any suggestions whatever for items to include in this bibliography, or for other changes, please let me hear from you. Thank you.

Preface

> [I]t should be borne in mind that incompleteness is a necessary concomitant of every collection of whatever kind. Much less can completeness be expected in a first collection, made by a single individual, in his leisure hours, and in a field which is already boundless and is yet expanding day by day.
>
> —Robert Edouard Moritz, preface to *Memorabilia Mathematica: The Philomath's Quotation Book*, 1914.

A *signed graph* is a graph whose edges are labeled by signs. This is a bibliography of signed graphs and related mathematics.

Several kinds of labelled graph have been called "signed" yet are mathematically very different. I distinguish four types:

- *Group-signed graphs*: the edge labels are elements of a 2-element group and are multiplied around a circle (or along any walk). Among the natural generalizations are larger groups and vertex signs.
- *Sign-colored graphs*, in which the edges are labelled from a two-element set that is acted upon by the sign group: − interchanges labels, + leaves them unchanged. This is the kind of "signed graph" found in knot theory. The natural generalization is to more colors and more general groups—or no group.
- *Weighted graphs*, in which the edge labels are the elements $+1$ and -1 of the integers or another additive domain. Weights behave like numbers, not signs; thus I regard work on weighted graphs as outside the scope of the bibliography—except (to some extent) when the author calls the weights "signs".
- Labelled graphs where the labels have no structure or properties but are called "signs" for any or no reason.

Each of these categories has its own theory or theories, generally very different from the others, so in a logical sense the topic of this bibliography is an accident of terminology. However, narrow logic here leads us astray, for the study of true signed graphs, which arise in numerous areas of pure and applied mathematics, forms the great majority of the literature. Thus I regard as fundamental for the bibliography the notions of *balance* of a circle (sign product equals +, the sign group identity) and the vertex-edge incidence matrix (whose column for a negative edge has two $+1$'s or two -1's, for a positive edge one $+1$ and one -1, the rest being zero); this has led me to include work on *gain graphs* (where the edge labels are taken from any group) and "consistency" in *vertex-signed graphs*, and generalizable work on two-graphs (the set of unbalanced triangles of a signed complete graph) and on even and odd circles and paths in graphs and digraphs.

Nevertheless, it was not always easy to decide what belongs. I have employed the following principles:

Only works with mathematical content are entered, except for a few representative purely applied papers and surveys. I do try to include:

- Any (mathematical) work in which signed graphs are mentioned by name or signs are put on the edges of graphs, regardless of whether it makes essential use of signs. (However, due to lack of time and in order to maintain "balance" in the bibliography, I have included only a limited selection of items concerning binary clutters and postman theory, two-graphs, signed digraphs in qualitative matrix theory, and knot theory. For clutters, see Cornuéjols (2001a); for postman theory, A. Frank (1996a). For two-graphs, see any of the review articles by Seidel. For qualitative matrix theory, see e.g. Maybee and Quirk (1969a) and Brualdi and Shader (1995a). For knot theory there are uncountable books and surveys.)

- Any work in which the notion of balance of a circle plays a role. Example: gain graphs. (Exception: purely topological papers concerning ordinary graph embedding.)
- Any work in which ideas of signed graph theory are anticipated, or generalized, or transferred to other domains. Examples: vertex-signed graphs; signed posets and matroids.
- Any mathematical structure that is an example, however disguised, of a signed or gain graph or generalization, and is treated in a way that seems in the spirit of signed graph theory. Examples: even-cycle and bicircular matroids; bidirected graphs; binary clutters (which are equivalent to signed binary matroids); some of the literature on two-graphs and double covering graphs.
- And some works that have suggested ideas of value for signed graph theory or that have promise of doing so in the future.

As for applications, besides works with appropriate mathematical content I include a few (not very carefully) selected representatives of less mathematical papers and surveys, either for their historical importance (e.g., Heider (1946a)) or as entrances to the applied literature (e.g., Taylor (1970a) and Wasserman and Faust (1994a) for psychosociology and Trinajstić (1983a) for chemistry). Particular difficulty is presented by spin glass theory in statistical physics—that is, Ising models and generalizations. Here one usually averages random signs and weights over a probability distribution; the problems and methods are rarely graph-theoretic, the topic is very specialized and hard to annotate properly, but it clearly is related to signed (and gain) graphs and suggests some interesting lines of graph-theoretic research. See Mézard, Parisi, and Virasoro (1987a) and citations in its annotation.

Plainly, judgment is required to apply these criteria. I have employed mine freely, taking account of suggestions from my colleagues. Still I know that the bibliography is far from complete, due to the quantity and even more the enormous range and dispersion of work in the relevant areas. I will continue to add both new and old works to future editions and I heartily welcome further suggestions.

There are certainly many errors, some of them egregious. For these I hand over responsibility to Sloth, Pride, Ambition, Envy, and Confusion. (Corrections, however, will be gratefully accepted by me.) And as Diedrich Knickerbocker says:

> Should any reader find matter of offense in this [bibliography], I should heartily grieve, though I would on no acount question his penetration by telling him he was mistaken, his good nature by telling him he was captious, or his pure conscience by telling him he was startled at a shadow. Surely when so ingenious in finding offense where none was intended, it were a thousand pities he should not be suffered to enjoy the benefit of his discovery.

Acknowledgement

I cannot name all the people who have contributed advice and criticism, but many of the annotations have benefited from suggestions by the authors or others and a number of items have been brought to my notice by helpful correspondents. I am very grateful to you all. Thanks also to the people who maintain the invaluable MR and Zbl indices (and a special thank-you for creating our own MSC classification: 05C22). However, I insist on my total responsibility for the final form of all entries, including such things as my restatement of results in signed or gain graphic language and, of course, all the praise and criticism (but not errors; see above) that they contain.

Bibliographical Data

Authors' names are given usually in only one form, even should the name appear in different (but recognizably similar) forms on different publications. Journal abbreviations follow the style of *Mathematical Reviews* (MR) with minor 'improvements'. Reviews and abstracts are cited from MR and its electronic form MathSciNet, from *Zentralblatt für Mathematik* (Zbl) and its electronic version (For early volumes, "Zbl VVV, PPP" denotes printed volume and page; the electronic item number is "(e VVV.PPPNN)".), and occasionally from *Chemical Abstracts* (CA) or *Computing Reviews* (CR). A review marked (*q.v.*) has significance, possibly an insight, a criticism, or a viewpoint orthogonal to mine.

Some—not all—of the most fundamental works are marked with a ††; some almost as fundamental have a †. This is a personal selection.

Annotations

I try to describe the relevant content in a consistent terminology and notation, in the language of signed graphs despite occasional clumsiness (hoping that this will suggest generalizations), and sometimes with my [bracketed] editorial comments. I sometimes try also to explain idiosyncratic terminology, in order to make it easier to read the original item. Several of the annotations incorporate open problems (of widely varying degrees of importance and difficulty).

I use these standard symbols:

Graphs

- Γ is a graph (V, E) of order $n = \#V$, undirected, possibly allowing loops and multiple edges. It is normally finite unless otherwise indicated.
- Δ is often another graph.
- $\Gamma{:}X$ is the induced subgraph on $X \subseteq V$; also, $\Gamma{:}\pi := \bigcup_{X \in \pi} \Gamma{:}X$ and $\Gamma(\pi) := \Gamma \setminus E{:}\pi$, for π a partition of V.
- Γ^c is the complement of a simple graph.
- $c(\ )$ is the number of connected components of a graph.

Signed, Gain, and Biased Graphs

- Σ is a signed graph (V, E, σ) of order n. $|\Sigma|$ is its underlying graph.
- σ is the signature (sign function) of Σ.
- E^+, E^- are the sets of positive and negative edges of Σ.
- Σ^+, Σ^- are the corresponding spanning subgraphs (unsigned).
- K_Δ is (K_n, σ) where $E^- = E(\Delta)$. Same as $K_n(-\Delta)$.
- $\Gamma(-\Delta)$ is (Γ, σ) where $E^- = E(\Delta)$.
- $[\Sigma]$ is the switching class of Σ.
- $\widetilde{\Sigma}$ is the double covering graph of Σ.
- Φ is a gain graph (V, E, φ) with gain function φ. $\|\Phi\|$ is its underlying graph.
- φ is the gain function of a gain graph Φ.
- $[\Phi]$ is the switching class of Φ.
- $\sim$ means that two signed or gain graphs are switching equivalent (with the same underlying graph and no automorphism).
- $\simeq$ means that two signed or gain graphs are switching isomorphic (with isomorphic underlying graphs, which may be the same graph).
- $\cong$ denotes isomorphism.
- $\langle\Sigma\rangle, \langle\Phi\rangle$ are the biased graphs of Σ and Φ.
- Ω is a biased graph. $\|\Omega\|$ is its underlying graph.
- $l(\ )$ is the frustration index (line index of imbalance).
- $l_0(\ )$ is the frustration number (vertex frustration number, vertex elimination number, balanced circle transversal number).
- $\Lambda(\ )$ is a line graph. $\Lambda(\Gamma)$ is that of a graph. For a signed or gain graph, Λ_{BC} is that of Behzad–Chartrand (1969a); $\Lambda_\times$ is that of M. Acharya (2009a), $\Lambda_\bullet$ is that of M. Acharya (*cf.* B.D. Acharya (2010a)), Λ is that of Zaslavsky (1979a), (1984c), (2010b), (20xxa).

Matrices

- $A(\ )$ is the adjacency matrix.
- $\mathrm{H}(\ )$ (Eta) is the incidence matrix.
- $L(\ )$ is the Laplacian matrix, $= \mathrm{H}(\ )\mathrm{H}(\ )^{\mathrm{T}}$.

$\lambda_1 = \lambda_{\max}$ and $\lambda_n = \lambda_{\min}$, the largest and least eigenvalues of a matrix, normally the adjacency matrix.

Matroids and Hyperplanes

$\mathcal{H}[\ ]$ is the hyperplane arrangement of a graph (ordinary, signed, or gain).
$\mathcal{L}(\ )$ is the intersection (semi)lattice of a hyperplane arrangement.
$\mathbf{M}(\ )$ [formerly written $G(\ )$] is the graphic (cycle) matroid of a graph.
$\mathbf{F}(\ )$ [formerly written $G(\ )$] is the frame (formerly bias) matroid of a signed, gain, or biased graph.
$\mathbf{L}(\ )$ is the lift matroid. [For line graphs see Λ. For the Laplacian matrix see L.]
$\mathbf{L}_\infty(\ )$ [formerly and also written $\mathbf{L}_0(\ )$] is the extended lift matroid, $\mathbf{L}+e_\infty$ [or $\mathbf{L}+e_0$].

Some standard terminology—much more will be found in the *Glossary* (Zaslavsky (1998c)):

circle: The graph of a simple closed path, or its edge set. (Also, "circuit", "cycle", "polygon", "simple cycle", etc.)
cycle: In a digraph, a coherently directed circle, i.e., "dicycle". More generally: in an oriented signed, gain, or biased graph, a matroid circuit (usually, of the frame matroid) oriented to have no source or sink.

Subject Classification Codes

A code in *lower case* means the topic appears implicitly but not explicitly. A suffix **w** on **Sgnd**, **SG**, **SD**, **VS** denotes signs used as weights, i.e., treated as the numbers +1 and −1, added, and (usually) the sum compared to 0. A suffix **c** on **SG**, **SD**, **VS** denotes signs used as colors (often written as the numbers +1 and −1), usually permutable by the sign group. In a string of codes a colon precedes subtopics. A code may be refined through being suffixed in parentheses, as **Sgnd**(**Matrd**) denoting signed matroids while **Sgnd**: **Matrd** would indicate matroids of signed objects; thus **Sgnd**(**Matrd**): **Matrd** means matroids of signed matroids.

Adj Adjacency matrix.
Algor Algorithms.
Algeb Algebraic structures upon signed, gain, or biased graphs or digraphs.
Appl Applications other than (**Chem**), (**Phys**), (**Biol**), (**PsS**) (partial coverage).
Aut Automorphisms, symmetries, group actions.
Bal Balance (mathematical), cobalance; harmony, "consistency" of vertex signs.
Bic Bicircular matroids.
Biol Applications to biology (partial coverage).
Chem Applications to chemistry (partial coverage).
Circ Circles. (**Cyc**) for directed circles.
Clu Clusterability.
Col Vertex coloring.
Cov Covering graphs, double coverings.
Cyc Directed cycles. (**Circ**) for undirected.
Du Duality (graphs, matroids, or matrices).
Dyn Dynamics in (di)graphs. Predicting aspects, e.g., an edge sign.
Eig Eigenvalues, eigenvectors, characteristic polynomial, energy.
Enum Enumeration of types of signed graphs, etc.
ECyc Even-cycle matroids.
ECol Edge coloring.
Exp Expository.
Exr Interesting exercises (in an expository work).
Fr Frustration (imbalance), esp. frustration index (line index of imbalance), other measures; minimum balancing set.
Geom Connections with geometry, e.g., linear programming, complex complement.
GD Digraphs with gains (or voltages).
Gen Generalization.
GG Gain, voltage, and biased graphs; includes Dowling lattices (with (**Matrd**)).
GN Generalized or gain networks. (Multiplicative real gains.)
GH Hypergraphs with gains.
Hom Homomorphisms *et al.*
Incid Incidence matrix.
KG Signed complete graphs.
Knot Connections with knot theory (sparse coverage if signs are purely notational).
Lap Laplacian matrix $L(\ )$.
Lab Nonalgebraic labelling of signed, gain, or biased graphs, e.g., gracefulness.
LG Line graphs. (**Gen**): Jump graphs, total graphs, *et al.*
Matrd Matroids and geometric lattices, chain-groups (not signed matroids).
MtrdF Matroidal families.
Invar Numerical and algebraic invariants of signed, gain, biased graphs: polynomials, degree sequences, number of bases, etc.
Ori Orientations, bidirected graphs.

OG Ordered gains.
Par All-negative or antibalanced signed graphs, parity-biased graphs.
par Includes results on even or odd paths or circles (partial coverage) that may generalize from antibalanced to all signed graphs.
Phys Applications in physics (limited coverage).
PsS Psychological, sociological, and anthropological applications (partial coverage).
QM Qualitative (sign) matrices: sign patterns, sign stability, sign solvability, etc.: graphical methods.
qm Qualitative (sign) matrices are implicit or are motivation.
Rand Random signs or gains, signed or gain graphs.
Ref Many references.
Sgnd Signed objects other than graphs and hypergraphs: mathematical properties.
SD Signed digraphs: mathematical properties.
SG Signed graphs: mathematical properties. See (**Par**) for only all-negative, possibly implicit as with (**par**) for signless Laplacian.
SH Signed hypergraphs: mathematical properties.
QSol Sign solvability, sign nonsingularity (partial coverage).
QSta Sign stability (partial coverage).
State State space, state space landscape; ground state landscape (very partial coverage).
Str Structure theory.
Sw Switching of signs or gains.
Top Topology applied to graphs; surface embeddings. (Not applications to topology.)
TG Two-graphs, graph (Seidel) switching (partial coverage).
VS Vertex-signed graphs ("marked graphs"); signed vertices and edges.
WD Weighted digraphs.
WG Weighted graphs.
WH Weighted hypergraphs.
Xtreml Extremal problems.

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[Maria Abi Aad]

See M. Abi Aad (under 'Ab').

Ahmad Abdi & Bertrand Guenin

2014a The cycling property for the clutter of odd st-walks. In: *Integer Programming and Combinatorial Optimization* (17th Int. IPCO, Bonn, 2014), pp. 1–12. Lect. Notes in Computer Sci., Vol. 8494. Springer, Cham, 2014. MR 3252290. Zbl 1418.90207.

Extended abstract of (2018a). **(SG)**

2017a The two-point Fano and ideal binary clutters. In: Friedrich Eisenbrand and Jochen Koenemann, eds., *Integer Programming and Combinatorial Optimization* (19th Int. IPCO, Waterloo, Ont., 2013), pp. 1–12. Lect. Notes in Computer Sci., Vol. 10328. Springer, Cham, 2017. MR 3678769. Zbl 1418.90208. **(SG: Matrd)**

2018a Packing odd T-joins with at most two terminals. *J. Graph Theory* 87 (2018), no. 4, 587–652. MR 3767184. Zbl 1386.05147. arXiv:1410.7423.

Dictionary: "odd" = negative. **(SG)**

2019a The minimally non-ideal binary clutters with a triangle. *Combinatorica* 39 (2019), no. 4, 719–752. MR 4015349. Zbl 1438.90393. **(SG: Matrd)**

2019b The two-point Fano and ideal binary clutters. *Combinatorica* 39 (2019), no. 4, 753–777. MR 4015350. Zbl 1449.05041. **(SG: Matrd)**

Normalah S. Abdulcarim

See M.M. Mangontarum.

Takuro Abe

2009a The stability of the family of B_2-type arrangements. *Commun. Algebra* 37 (2009), no. 4, 1193–1215. MR 2510979 (2010d:32027). Zbl 1194.32014.

The arrangements are affino-signed-graphic arrangements. **(sg, gg: Geom)**

2012a On a conjecture of Athanasiadis related to freeness of a family of hyperplane arrangements. *Math. Res. Lett.* 19 (2012), no. 2, 469–474. MR 2955776. Zbl 272.32024. arXiv:1110.0303. **(SG: Geom)**

Takuro Abe, Koji Nuida, & Yasuhide Numata

2009a An edge-signed generalization of chordal graphs, free multiplicities on braid arrangements, and their characterizations. In: Christian Krattenthaler, Volker Strehl, and Manuel Kauers, eds., *21st International Conference on Formal Power Series and Algebraic Combinatorics* (FPSAC, Hagenburg, Austria, 2009), pp. 1–12. Discrete Math. Theor. Computer Sci., Nancy, France, 2009. MR 2721497 (2011j:05130). **(SG: Str, Geom)**

2009b Signed-eliminable graphs and free multiplicities on the braid arrangement. *J. London Math. Soc.* (2) 80 (2009), no. 1, 121–134. MR 2520381 (2010k:32039).

Zbl 1177.32017. arXiv:0712.4110 . **(SG: Str, Geom)**

Takuro Abe, Daisuke Suyama, & Shuhei Tsujie

2015a The freeness of Ish arrangements. In: *Proceedings of the 27th International Conference on Formal Power Series and Algebraic Combinatorics* (FPSAC, Daejon, South Korea, 2015), pp. 273–284. Discrete Math. Theor. Computer Sci., Nancy, France, 2015. MR 3470870. Zbl 1335.05189.

Partial publication of (2017a). **(gg: Geom: Algeb)**

2017a The freeness of Ish arrangements. *J. Combin. Theory Ser. A* 146 (2017), 169–183. MR 3574228. Zbl 1351.05224. arXiv:1410.2084.

Ish arrangements are supersolvable, and more. [Annot. 19 Dec 2020.] **(gg: Geom: Algeb)**

Toshiki Abe

See Wang, Qian, and Abe (2019a).

Peter Abell

See also H.Z. Deng, B. Kujawski, and M. Ludwig.

1968a Structural balance in dynamic structures. *Sociology* 2 (1968), no. 3, 333–352. **(SG, PsS: Bal, Fr)**

1969a Structural balance: a clarification. *Sociology* 3 (1969), no. 3, 421–422.

Technical correction to probability estimates in (1968a). [Annot. 29 Aug 2013.] **(SG: Fr)**

Peter Abell & Robin Jenkins

1967a Perception of the structural balance of part of the international system of nations. *J. Peace Res.* 4 (1967), no. 1, 76–82. **(PsS)(SG: Bal: Exp)**

Peter Abell & Mark Ludwig

2009a Structural balance: A dynamic perspective. *J. Math. Sociology* 33 (2009), no. 2, 129–155. Zbl 1169.91434.

Dynamics of signed graphs in a space of sign probabilities and tolerance of imbalance. There are three discernibly different domains of dynamical behavior. [Continued in Deng and Abell (2010a) and Kujawski, Ludwig, and Abell (2010a).] [Annot. 10 Sept, 9 Dec 2009.] **(SG, PsS: Bal, Fr, Dyn: Algor)**

Robert P. Abelson

See also M.J. Rosenberg.

1967a Mathematical models in social psychology. In: Leonard Berkowitz, ed., *Advances in Experimental Social Psychology*, Vol. 3, pp. 1–54. Academic Press, New York, 1967.

§ II: "Mathematical models of social structure." Part B: "The balance principle." Reviews basic notions of balance and clusterability in signed (di)graphs and measures of degree of balance or clustering. Notes that signed K_n is balanced iff $I + A = vv^{\mathrm{T}}$, $v = \pm 1$-vector. Proposes: degree of balance $= \lambda_{\max}(I + A(\Sigma))/n$. [*Cf.* Phillips (1967a).] Part C, 3: "Clusterability revisited." **(SG, SD: Bal, Clu, Fr, Adj)**

R.P. Abelson, E. Aronson, W.J. McGuire, T.M. Newcomb, M.J. Rosenberg, & P.H. Tannenbaum, eds.

1968a *Theories of Cognitive Consistency: A Sourcebook.* Rand-McNally, Chicago, Ill., 1968. **(PsS)**

Robert P. Abelson & Milton J. Rosenberg

† 1958a Symbolic psycho-logic: a model of attitudinal cognition. *Behavioral Sci.* 3 (1958), 1–13.

$R(\Sigma)$ They introduce a modified adjacency matrix R, called the "structure matrix" [I call it the Abelson–Rosenberg adjacency matrix], with entries o, p, n, a for, respectively, nonadjacency [0 in the usual adjacency matrix A], positive and negative adjacency [$+1$, -1 in A] and simultaneous positive and negative adjacency [0 or indeterminate in A]. They define an algebra (i.e., associative, commutative, and distributive addition and multiplication) of these symbols (p. 8): o acts as 0, p acts as 1, $pn = n$, $n^2 = p$, $a = p + n$, $x + x = x$ and $ax = a$ for $x \neq 0$. In the algebra one can decide balance of Σ via the permanent of $I + R$: Σ is balanced if $\mathrm{per}(I + R) = p$ and unbalanced if $\mathrm{per}(I + R) = a$. (The "straightforward but space-consuming" proof is omitted [and the theorem is not completely correct]. They state that the permanent cannot equal n or o [but that is an error].) [See Harary–Norman–Cartwright (1965a) for more on this matrix, and Zaslavsky (2010b), Thm. 2.1, for a matrix with more precise counting properties.] They introduce a clumsy form of switching in terms of the Hadamard product of R with a "passive T-matrix" [oversimplifying, that is a matrix obtained by switching the square all-p's matrix; the actual definition involves operators s and c and is more interesting]. Thm. 11: Switching preserves balance. [This was never followed up in signed graphs. Switching signed graphs became significant with Zaslavsky (1982a).]

They propose (p. 12) "complexity" [= frustration index $l(\Sigma)$] as a measure of imbalance. [*Cf.* Harary (1959b).] Thm. 12: Switching preserves frustration index. Thm. 14: $\max l(\Sigma)$, taken over all signed graphs Σ of order n, equals $\lfloor (n-1)^2/4 \rfloor$. (Proof omitted. [Proved by Petersdorf (1966a) and Tomescu (1973a) for signed K_n's and hence for all signed simple graphs of order n.]) **(PsS)(SG: Adj, Bal, sw, Fr)**

Maria Abi Aad

See A. El Sahili.

Aida Abiad, Francesco Belardo, & Antonina P. Khramova

20xxa A switching method for constructing cospectral gain graphs. Submitted. arXiv:2304.03555. **(GG: Adj: Eig, Sw(Gen))**

Aida Abiad, Raffaella Mulas, & Dong Zhang

2021a Coloring the normalized Laplacian for oriented hypergraphs. *Linear Algebra Appl.* 629 (2021), 192–207. MR 4295981. arXiv:2008.03269. **(SH: Lap: Eig)**

Pierre Aboulker, Pierre Charbit, Nicolas Trotignon, & Kristina Vušković

2015a Vertex elimination orderings for hereditary graph classes. *Discrete Math.* 338 (2015), 825–834. MR 3303861. Zbl 1306.05202. arXiv:1205.2535. **(sg: Str, Algor)**

Pierre Aboulker, Marko Radovanović, Nicolas Trotignon, Théophile Trunck, & Kristina Vušković

2014a Linear balanceable and subcubic balanceable graphs. *J. Graph Theory* 75 (2014), no. 2, 150–166. MR 3150570. Zbl 1280.05056. **(SG: Bal(Gen))**

Lowell Abrams

2017a Families of fixed-point cellular rotations. *European J. Combin.* 63 (2017), 197–215. MR 3645794. Zbl 1365.05201. **(Top: GG)**

Lowell Abrams & Joanna A. Ellis-Monaghan

2022a New dualities from old: Generating geometric, Petrie, and Wilson dualities and trialities of ribbon graphs. *Combin. Probab. Comput.* 31 (2022), no. 4, 574–597. MR 4439773. arXiv:1901.03739.
"Ribbon graphs" (Bollobás and Riordan (2002a)) are orientation-embedded signed graphs (*cf.* Zaslavsky (1992a)). **(sg: Top)**

Lowell Abrams & Daniel Slilaty

2015a The minimal $\mathbb{Z}_n$-symmetric graphs that are not $\mathbb{Z}_n$-spherical. *European J. Combin.* 46 (2015), 95–114. MR 3305348. Zbl 1307.05102. **(Top: GG, SG)**

2015b Cellular automorphisms and self-duality. *Trans. Amer. Math. Soc.* 367 (2015), no. 11, 7695–7773. MR 3391898. Zbl 6479446. **(Top: GG, SG)**

M. Abreu, M.J. Funk, D. Labbate, & V. Napolitano

2013a On the ubiquity and utility of cyclic schemes. *Australasian J. Combin.* 55 (2013), 95–120. MR 3058329. Zbl 1278.05246. arXiv:1111.3265. **(GG: Cov)**

Nair Maria Maia de Abreu [Nair Abreu]

See also M.A.A. de Freitas, L.S. de Lima, A. Oliveira, and C.S. Oliveira.

Nair Abreu, Domingos M. Cardoso, Ivan Gutman, Enide A. Martins, & María Robbiano

2011a Bounds for the signless Laplacian energy. *Linear Algebra Appl.* 435 (2011), no. 10, 2365–2374. MR 2811121 (2012f:05164). Zbl 1222.05143. **(sg: par: Eig)**

Nair M.M. Abreu, Domingos M. Cardoso, Enide A. Martins, Maria Robbiano, & B. San Martín

2012a On the Laplacian and signless Laplacian spectrum of a graph with k pairwise co-neighbor vertices. *Linear Algebra Appl.* 437 (2012), 2308–2316. MR 2954492. Zbl 1247.05132. **(par: Lap: Eig)**

Nair Maria Maia de Abreu & Vladimir Nikiforov

2012a Maxima of the Q-index: abstract graph properties. *Electronic J. Linear Algebra* 23 (2012), art. 55, 782–789. MR 2992393. Zbl 1252.05116. **(par: Lap: Eig)**

2013a Maxima of the Q-index: graphs with bounded clique number. *Electronic J. Linear Algebra* 26 (2013), art. 9, 121–130. MR 3065852. Zbl 1282.05165. arXiv:1308.1653.
The largest eigenvalue of $L(-\Gamma)$ is $\leq 2n(1-1/\omega)$ (ω = clique number), with equality if Γ is complete ω-partite and regular. [Annot. 20 Jan 2015.] **(par: Lap: Eig)**

Seyed Ebrahim Abtahi

See P. Esmailian.

B. Devadas Acharya [Belmannu Devadas Acharya]

See also M.K. Gill, S.B. Rao, D. Sinha, and H.B. Walikar.

1973a On the product of p-balanced and l-balanced graphs. *Graph Theory Newsletter* 2 (Jan., 1973), no. 3, Results Announced No. 1. (**SG, VS: Bal**)

1979a New directions in the mathematical theory of balance in cognitive organizations. MRI Tech. Rep. No. HCS/DST/409/76/BDA (Dec., 1979). Mehta Research Institute of Math. and Math. Physics, Allahabad, 1979.
(**SG, SD: Bal, Adj, Ref**)(**PsS: Exp, Ref**)

1979b A programme logic for listing of sigraphs, their characteristic polynomials, and their spectra. *Graph Theory Newsletter* 9 (1979), no. 2, 1.

Abstract of a plan for computation. (**SG: Adj: Algor**)

† 1980a Spectral criterion for cycle balance in networks. *J. Graph Theory* 4 (1980), 1–11. MR 0558448 (81e:05097) (*q.v.*). Zbl 445.05066.

A signed simple graph Σ with positive edge weights w is balanced iff $A(\Sigma, w)$ has the same spectrum as $A(|\Sigma|, w)$. A weighted, signed simple digraph $(\vec{\Gamma}, \sigma, w)$ is cycle balanced (every directed cycle is positive) iff $A(\vec{\Gamma}, \sigma, w)$ has the same spectrum as $A(\vec{\Gamma}, w)$. [Improved for connected, unweighted signed graphs in Stanić (2019c).]

Proposed measure of imbalance: the proportion of corresponding coefficients where the characteristic polynomials $p(A(\Sigma); \lambda)$ and $p(A(|\Sigma|); \lambda)$ differ. [See M.K. Gill (1981b).] [Annot. rev. 4 Apr 2012, 30 Nov 2014, 19 Dec 2020].] (**SD, SG: Bal, Adj**)

1980b An extension of the concept of clique graphs and the problem of K-convergence to signed graphs. *Nat. Acad. Sci. Lett. (India)* 3 (1980), 239–242. Zbl 491.05052.
(**SG: LG, Clique graph**)

1980c Applications of sigraphs in behavioural sciences. M.R.I. Tech. Rep. No. DST/-HCS/409/79 (June, 1980). Mehta Research Institute of Math. and Math. Physics, Allahabad, 1979.

[Annotation is very incomplete.] Let $\Sigma_1 \vee \Sigma_2$ be the join of underlying graphs, with edge signs $\{\pm 1\}$ as in $\Sigma_1 \cup \Sigma_2$ and with $\sigma(v_1 v_2) := \max(\mu_1(v_1), \mu_2(v_2))$, where $\mu(v) := \prod_{vw \in E} \sigma(vw)$. [Annot. 20 July 2009.]
(**SG**)

1981a On characterizing graphs switching equivalent to acyclic graphs. *Indian J. Pure Appl. Math.* 12 (1981), 1187–1191. MR 0634306 (82k:05089). Zbl 476.05069.

Begins an attack on the problem of characterizing by forbidden induced subgraphs the simple graphs that switch to forests. Among them are K_5 and C_n, $n \geq 7$. *Problem.* Find any others that may exist. [Solved by Hage and Harju (2004a). Forests that switch to forests were characterized by Hage and Harju (1998a).] (**TG: Sw**)

1982a Connected graphs switching equivalent to their iterated line graphs. *Discrete Math.* 41 (1982), 115–122. MR 0676870 (84b:05078). Zbl 497.05052. (**LG, TG**)

1982b Even edge colorings of a graph: II. A lower bound for maximum even edge-coloring index. *Nat. Acad. Sci. Lett. (India)* 5 (1982), no. 3, 97–99.
(**bal: Gen**)

1983a Even edge colorings of a graph. *J. Combin. Theory Ser. B* 35 (1983), 78–79. MR 0723571 (85a:05034). Zbl 505.05032, (Zbl 515.05030).

Find the fewest colors to color the edges so that in each circle the number of edges of some color is even. [Possibly, inspired by §2 of Acharya and Acharya (1983a).] **(bal: Gen)**

1983b A characterization of consistent marked graphs. *Nat. Acad. Sci. Lett. (India)* 6 (1983), no. 12, 433–440. MR 0884837 (no rev). Zbl 552.05052.

Converts a vertex-signed graph (Γ, μ) into a signed graph Σ such that (Γ, μ) is consistent (as in Beineke and Harary (1978b)) iff every circle in Σ is all negative or has an even number of all-negative components. [See Joglekar, Shah, and Diwan (2010a) for the definitive result on consistency.] **(VS, SG: bal)**

1984a Some further properties of consistent marked graphs. *Indian J. Pure Appl. Math.* 15 (1984), 837–842. MR 0757960 (86a:05101). Zbl 552.05053.

Notably: nicely characterizes consistent vertex-signed graphs in which the subgraph induced by negative vertices is connected. [Subsumed by S.B. Rao (1984a).] **(VS: bal)**

1984b Combinatorial aspects of a measure of rank correlation due to Kendall and its relation to social preference theory. In: B.D. Acharya, ed., *Proceedings of the National Symposium on Mathematical Modelling* (Allahabad, 1982). M.R.I. Lect. Notes Appl. Math., No. 1. Mehta Research Institute of Math. and Math. Physics, Allahabad, India, 1984.

Includes an exposition of Sampathkumar and Nanjundaswamy (1973a). **(SG: KG: Exp)**

1985a *Signed Graphs With Applications in Behavioural Sciences*. M.R.I. Lect. Notes Appl. Math., No. 3. Mehta Research Institute of Math. and Math. Physics, Allahabad, 1985. **(SG: PsS)**

1986a An extension of Katai-Iwai procedure to derive balancing and minimum balancing sets of a social system. *Indian J. Pure Appl. Math.* 17 (1986), 875–882. MR 0851878 (87k:92037). Zbl 612.92019.

Expounds the procedure of Katai and Iwai (1978a). Proposes a generalization to those Σ that have a certain kind of circle basis. Construct a "dual" graph whose vertex set is a circle basis supplemented by the sum of basic circles. A "dual" vertex has sign as in Σ. Let T = set of negative "dual" vertices. A T-join in the "dual", if one exists, yields a negation set for Σ. [A minimum T-join need not yield a minimum negation set. Indeed the procedure is unlikely to yield a minimum negation set (hence the frustration index $l(\Sigma)$) for all signed graphs, since it can be performed in polynomial time while $l(\Sigma)$ is NP-complete. *Questions*. To which signed graphs is the procedure applicable? For which ones does a minimum T-join yield a minimum negation set? Do the latter include all those that forbid an interesting subdivision or minor (*cf.* Gerards and Schrijver (1986a), Gerards (1988a), (1989a))?] **(SG: Fr: Algor)**

2009a Role of cognitive balance in some notions of graph labelings: Influence of Frank Harary, Fritz Heider, Gustav Kirchhoff and Leonhard Euler. *Bull. Allahabad Math. Soc.* 24 (2009), no. 2, 391–413. MR 2597634 (no rev). Zbl 1221.05278. **(SG, SD: Bal, sw)**

2010a Signed intersection graphs. *J. Discrete Math. Sci. Cryptography* 13 (2010), no. 6, 553–569. MR 2791608 (2011m:05129). Zbl 1217.05170.

Signed hypergraph: hypergraph $H = (X, E)$ with $\sigma_H : E \to \{+, -\})$. Canonical marking $\mu_{\sigma_H}(x) := \prod_{e \ni x} \sigma_H(e)$ $(x \in X)$. Intersection edge sign $\sigma_\Omega(ef) := \prod_{x \in e \cap f} \mu_{\sigma_H}(x)$. The signed intersection graph $\Omega(H, \sigma)$ is the intersection graph of H with signature σ_Ω. Main example: Maximal-clique hypergraph $\mathcal{K}(\Xi)$ of a signed graph Ξ with $X = \{$maximal cliques of $|\Xi|\}$, signature $\sigma_{\mathcal{K}}(Q) := \prod_{v \in Q} \mu_\sigma(Q)$ for a max clique Q. Which signed graphs are $\Omega(\mathcal{K}(\Xi))$? Thm. 3.3: Σ is a maxclique signed graph iff it has an edge clique cover with the Helly property, whose members induce homogeneously signed subgraphs, an even number of which are all-negative.

On orbits of the operator $\mathcal{K}$: Thm. 5.1: $\mathcal{K}^m(\Sigma) = \mathcal{K}^n(\Sigma)$ iff $\mathcal{K}^m(|\Sigma|) = \mathcal{K}^n(|\Sigma|)$, $\exists\ m < n$. However (§7), $m = 0$ (Σ is "$\mathcal{K}$-periodic) may hold for $|\Sigma|$ but not Σ. *Problem* 7.2. Characterize $\mathcal{K}$-periodic signed graphs. [Annot. 28 Aug 2010.] (**SH, SG: lg**)

§8, "Signed line graphs": Taking edges instead of max cliques defines a line graph $\Lambda_\bullet(\Sigma)$ with signature $\sigma_\bullet(ef) := \mu_\sigma(e \cap f)$ (due to M. Acharya [M.K. Gill] (1982a), Acharya and Acharya (2015a)). [Annot. 28 Aug 2010.] (**SG: LG**)

2010b Mathematical chemistry: Basic issues. In: *Graph Theory Applied to Chemistry* (Proc. Nat. Workshop, Pala, Kerala, India, 2010), Ch. 2.2, pp. 26–46.

§2.2.9, "Newer vistas": Signed hypergraphs, signed semigraphs. [Annot. 31 Aug 2010.] (**SG: Gen, SH: Exp**)

2011a On notions generalizing combinatorial graphs, with emphasis on linear symmetric dihypergraphs. *Bull. Allahabad Math. Soc.* 26 (2011), no. 2, 229–258. MR 2984886 (no rev). Zbl 1257.05054.

Many generalizations of graphs and digraphs. Mainly historical and expository. [Annot. 31 Jan 2012.]
(**SG, SD, Gen: Exp, Ref**)(**SG, SD, Gen**)

2012a Set-valuations of a signed digraph. Set Valuations, Signed Graphs and Geometry (IWSSG-2011, Int. Workshop, Mananthavady, Kerala, 2011). *J. Combin. Inform. System Sci.* 37 (2012), no. 2-4, 145–167. Zbl 1301.05155. (**SD, SG**)

2012b Minus domination in a signed graph. Set Valuations, Signed Graphs and Geometry (IWSSG-2011, Int. Workshop, Mananthavady, Kerala, 2011). *J. Combin. Inform. System Sci.* 37 (2012), no. 2-4, 333–343. Zbl 1300.05118.

Unlike with graphs, not every signed graph admits a minus domination function, i.e., $\mathbf{f} \in \{0, \pm 1\}^V$ such that $(I + A(\Sigma))\mathbf{f} \geq \mathbf{1}$. [Continued in Shreyas and Joseph (2020a). *Cf.* Walikar, Motammanavar, and Acharya (2015a) for signed domination.] [Annot. 18 May 2018.] thru (**SG: Dom: Lab**)

2013a Domination and absorbance in signed graphs and digraphs I. Foundations. *J. Combin. Math. Combin. Computing* 84 (2013), 5–20. MR 3076750. Zbl 1274.05205.

$D \subseteq V$ dominates Σ if D dominates $|\Sigma|$ and $\bigcup_{u \in D} E(u)$ is balanced. Domination and absorbance in a signed digraph are similar, using only

outgoing, resp. incoming, arcs at D. Special attention to independent dominating and absorbing sets. [Annot. 22 Apr 2023.]
(**SG, SD: Dom**)

B. Devadas Acharya & Mukti Acharya [M.K. Gill]

1983a A graph theoretical model for the analysis of intergroup stability in a social system. Manuscript, 1983.

The first half (most of §1) was improved and published as (1986a).

The second half (§§2–3) appears to be unpublished. Given: a graph Γ, a vertex signing μ, and a covering $\mathcal{F}$ of $E(\Gamma)$ by cliques of size ≤ 3. Define a signed graph S by $V(S) = \mathcal{F}$ and $QQ' \in E(S)$ when at least half the elements of Q or Q' lie in $Q \cap Q'$; sign QQ' negative iff there exist vertices $v \in Q \backslash Q'$, and $w \in Q' \backslash Q$ such that $\mu(v) \neq \mu(w)$. Suppose there is no edge QQ' in which $\#Q = 3$, $\#Q' = 2$, and the two members of $Q \backslash Q'$ have differing sign. [This seems a very restrictive supposition.] Main result (Thm. 7): S is balanced. The definitions, but not the theorem, are generalized to multiple vertex signs μ, general clique covers, and clique adjacency rules that differ slightly from that of the theorem.
(**GG, VS, SG: Bal**)

1986a New algebraic models of social systems. *Indian J. Pure Appl. Math.* 17 (1986), 150–168. MR 0830552 (87h:92087). Zbl 591.92029.a

Four criteria for balance in an arbitrary gain graph. [Also see Harary, Lindström, and Zetterström (1982a).] (**GG: Bal, sw**)

2015a Dot-line signed graphs. *Ann. Pure Appl. Math.* 10 (2015), no. 1, 21–27.
(**SH: VS, SG: LG(Gen)**)

20xxa Consistent marked hypergraphs. Submitted. (**SH(Gen): VS: Bal**)

Belmannu Devadas Acharya, Mukti Acharya, & Deepa Sinha

2008a Cycle-compatible signed line graphs. *Indian J. Math.* 50 (2008), no. 2, 407–414. MR 2517744 (2010h:05142). Zbl 1170.05032.

Characterizes when $\Lambda_{BC}(\Sigma)$, the Behzad–Chartrand (1969a) line graph, with vertex signs σ is harmonious. Dictionary: "cycle compatible" = harmonious (the product of all edge and vertex signs on each circle is positive). [Annot. 14 Oct 2009.] (**SG, VS: LG: Bal**)

2009a Characterization of a signed graph whose signed line graph is S-consistent. *Bull. Malaysian Math. Sci. Soc.* (2) 32 (2009), no. 3, 335–341. MR 2562172 (2010m:05135). Zbl 1176.05032.

Let Σ be a signed simple graph. Thm. 2.1: The line graph $\Lambda(|\Sigma|)$, with vertex signs σ, is consistent (as in Beineke–Harary (1978b)) iff Σ is balanced and, in Σ, a vertex of degree ≥ 4 has only positive edges, while a trivalent vertex v with negative edges has two such edges, which lie in every circle on v. [Slilaty and Zaslavsky (2015a) have a constructive approach. Sinha and Acharya (2016a) generalize to iterated line graphs.] [Cor. 1: A positive edge at a vertex with two negative edges is an isthmus. Cor. 2: Let Σ be 2-connected. $(\Lambda(|\Sigma|), \sigma)$ is consistent iff Σ is balanced and every negative edge has endpoints of degree ≤ 2. *Problem.* Find a structural characterization, by means of which all such

Σ can be constructed. Zaslavsky (2016a) has solutions.] [Annot. 2 Oct 2009, rev 15 Oct, 3 Nov 2013, 23 Jan 2014.] **(SG, VS: LG: Bal)**

B.D. Acharya, M.K. Gill, & G.A. Patwardhan

1984a Quasicospectral graphs and digraphs. In: B.D. Acharya, ed., *Proceedings of the National Symposium on Mathematical Modelling* (Allahabad, 1982), pp. 133–144. M.R.I. Lect. Notes Appl. Math., No. 1. Mehta Research Institute of Math. and Math. Physics, Allahabad, 1984. MR 0766920 (86c:05087). Zbl 556.05048.

Continues M.K. Gill (1981a). A signed graph, or digraph, is "cycle-balanced" if every circle, or every cycle, is positive. Graphs, or digraphs, are "quasicospectral" if they have cospectral signings, "strictly quasicospectral" if they are quasicospectral but not cospectral, "strongly cospectral" if they are cospectral and have cospectral cycle-unbalanced signings. There exist arbitrarily large sets of strictly quasicospectral digraphs, which moreover can be assumed strongly connected, weakly but not strongly connected, etc. There exist pairs of unbalanced, strictly quasicospectral graphs; existence of larger sets is unsolved. There exist arbitrarily large sets of nonisomorphic, strongly cospectral connected graphs; also, of weakly connected digraphs, which moreover can be taken to be strongly connected, unilaterally connected, etc. Proofs, based on ideas of A.J. Schwenk, are sketched. **(SD, SG: Eig)**

Belmannu Devadas Acharya & Shalini Joshi

2003a On the complement of an ambisidigraph. [Abstract.] Proc. R.C. Bose Centenary Sympos. Discrete Math. Appl. (Kolkata, 2002). *Electronic Notes Discrete Math.* 15 (2003), 5. MR 2159023 (no rev). Zbl 1184.05100.

The complement of a signed digraph D without loops or multiple signed arcs (a loopless, simply signed digraph, or "ambisidigraph") is defined in the obvious way. Observation: If D or D^c contains a directed cycle of length $2k+1$, then one of them contains a positive such cycle. **(SD)**

2004a Semibalance in signed digraphs. In: *Proceedings of the International Conference on Recent tRends and New Directions of REsearch in Cybernetics and Systems* (Inst. Adv. Study Sci. and Technology, Guwahati, India, 2004). [2004?]. **(sg: SD: Bal)**

2005a Mathematical modelling in social psychology–social networks. *Everyman's Science* 40 (2005), no. 2, 124–128.

Popular exposition including ambisidigraphs (*cf.* (2003a)). [Annot. 7 Apr 2012.] **(SD: Exp)**

2011a Some reflections on discrete mathematical models in behavioral, cognitive and social sciences. In: Johan van Benthem, Amitabha Gupta, and Rohit Parikh, eds., *Proof, Computation and Agency: Logic at the Crossroads*, Ch. 11, pp. 277–307. Synthèse Library, Vol. 352. Springer Netherlands, 2011. MR 3290119 (no rev). Zbl 1315.91015. **(PsS: SG)**

B.D. Acharya, S. Joshi, A.R. Rao, & S.B. Rao

2003a A Ramsey theorem for strongly connected ambisidigraphs. Manuscript, 2003.

Sequel to Acharya and Joshi (2003a). For which loopless, simply signed digraphs D do both D and D^c contain no positive 3-cycle? Thm.: If strongly connected, D has order < 6. An attempt to use this to describe all loopless, simply signed digraphs that contain no positive 3-cycle. **(SD: Str)**

Mukti Acharya [Mukhtiar Kaur Gill]

See also B.D. Acharya, D. Antoney, M.K. Gill, R. Jain, P.B. Joshi, A.J. Mathias, Pranjali, S.B. Rao, and D. Sinha.

1988a Switching invariant three-path signed graphs. In: M.N. Gopalan and G.A. Patwardhan, eds., *Optimization, Design of Experiments and Graph Theory* (Proc. Sympos. in Honour of Prof. M.N. Vartak, Bombay, 1986), pp. 342–345. Indian Inst. of Technology, Bombay, 1988. MR 0998809 (90b:05102). Zbl 744.05054.

See Gill and Patwardhan (1986a) for the k-path signed graph of Σ. The equation $\Sigma \simeq D_3(\Sigma)$ is solved. [Annot. 29 Apr 2009.] **(SG, Sw)**

2009a $\times$-line signed graphs. Int. Conf. Recent Developments Combin. Graph Theory (Krishnankoil, Tamil Nadu, India, 2007). *J. Combin. Math. Combin. Comput.* 69 (2009), 103–111. MR 2517311 (no rev). Zbl 1195.05031.

$\Lambda_\times$ $\Lambda_\times(\Sigma) := (\Lambda(|\Sigma|), \sigma_\times)$ where $\sigma_\times(ef) := \sigma(e)\sigma(f)$. (Contrast with line graphs of Behzad–Chartrand (1969a), or Zaslavsky (2010b), (2012c), (20xxa), or M. Acharya in B.D. Acharya (2010a).) [The definition originated in M.K. Gill (1982a). Publication of this article was delayed by many years.] [Annot. rev. 20 Dec 2010, 1 Sept 2012.] **(SG: LG)**

2010a Square-sum graphs: Some new perspectives. In: *International Conference on Recent Trends in Graph Theory and Combinatorics (ICRTGC-2010)* (Cochin, 2010) [Summaries], pp. 114–119. Dept. of Mathematics, Cochin Univ. of Science and Technology, 2010.

P. 119: Summary of k-square-sum signed graphs, where k edges classes are square-sum with the same vertex labels. $k = 2$ is signed graphs. [Annot. 30 Aug 2010.] **(SGc: Lab)**

2012a Quasicospectrality of graphs and digraphs: A creative review. Set Valuations, Signed Graphs and Geometry (IWSSG-2011, Int. Workshop, Mananthavady, Kerala, 2011). *J. Combin. Inform. System Sci.* 37 (2012), no. 2-4, 241–256. Zbl 1301.05207.

Graphs or digraphs are quasicospectral if they have cospectral signatures (signatures with the same adjacency spectrum). Properties and examples of quasicospectral graphs and digraphs that are not cospectral. Definitions and results from B.D. Acharya–Gill–Patwardhan (1984a) *et al.*, as well as new results. [Annot. 4 Apr 2012.]
(SG, SD: Eig: Exp)(SG, SD: Eig)

2017a C-cordial labeling in signed graphs. Int. Conf. Current Trends in Graph Theory and Computation (CTGTC-2016, New Delhi). *Electronic Notes Discrete Math.* 63 (2017), 11–22. MR 3754786. Zbl 1383.05276.
(VS: SG, SH: Lab)

2017b Total C-cordial labeling in signed graphs – I. *Indian J. Discrete Math.* 3 (2017), no. 2, 85–96. MR 4317606. **(VS: SG: Lab)**

Mukti Acharya, Ivy Chakrabarty, & Joseph Varghese Kureethara

20xxa Divisor cordial signed graphs. Submitted.

Divisor cordial labeling: $f : V \to [n]$ such that $v \sim w$ if $\sigma(vw) = +$

and $v \not\sim w$ if $\sigma(vw) = -$, where $v \sim w$ means $f(v)|f(w)$ or $f(w)|f(v)$. [Annot. 1 Jan 2024.] (**SG: Lab**)

Mukti Acharya, Rashmi Jain, & Sangita Kansal

2014a Some results on the splitting signed graphs $\mathfrak{S}(S)$. *J. Combin. Inform. System Sci.* 39 (2014), no. 1-4, 23–32. Zbl 1358.05131. (**SG, VS: Bal, Bal(Gen)**)

2015a Results on lict signed graphs $L_c(S)$. *J. Discrete Math. Sci. Cryptography* 18 (2015), no. 6, 727–742. MR 3435215 (no rev).

"Lict" = line-cutpoint, an extension of the line graph. $V(\Sigma)$ is signed by μ_σ, the canonical vertex signature. $L_c(S)$ or rather $\Lambda_c(\Sigma)$ has vertex set $E \cup C$ where $C = \{\text{cutpoints of } \Sigma\}$, edge uv iff u, v are adjacent or incident, and uv negative iff u and v are negative. $\Lambda_c{:}E = \Lambda_{BC}(\Sigma)$, the Behzad–Chartrand (1969a) line graph. Characterized: (1) The signed K_n, C_n, and $K_{r,s}$ that are $\Lambda_{BC}(\Sigma)$ or $\Lambda_c(\Sigma)$. (2) Σ such that $\Lambda_{BC}(\Sigma)$ or $\Lambda_c(\Sigma)$ is balanced. (3) Σ that are switching equivalent to $\Lambda_{BC}(\Sigma)$ or $\Lambda_c(\Sigma)$ or their negatives. [Annot. 8 Jan 2016.] (**SG: LG(Gen): Bal, KG**)

2016a On $\bullet$-lict signed graphs $L_{\bullet_c}(S)$ and $\bullet$-line signed graphs $L_\bullet(S)$. *Trans. Combin.* 5 (2016), no. 1, 37–48. MR 3462889. Zbl 1463.05234. (**SG: LG(Gen)**)

2017a Vertex equitable labeling of signed graphs. Int. Conf. Current Trends in Graph Theory and Computation (CTGTC-2016, New Delhi). *Electronic Notes Discrete Math.* 63 (2017), 461–468. MR 3754836. Zbl 1383.05139. (**SG: Lab**)

Mukti Acharya & Joseph Varghese Kureethara

2021a Parity labeling in signed graphs. *J. Prime Res. Math.* 17 (2021), no. 2, 1–7. MR 4307053. arXiv:2012.07737.

Parity labeling: $f : V \leftrightarrow [n]$ such that $\sigma(uv) = (-1)^{f(u)+f(v)}$. [I.e., $E^- =$ cut with nearly equal sides.] *rna* number := $\min |E^-|$ over all parity signatures. Minor results. *Cf.* Acharya–Kureethara–Zaslavsky (2021a), Sehrawat–Bhattacharjya (20xxb), and esp. Kang, Chen, and Jin (2022a). [Annot. 7 Oct 2019.] (**Lab: SG**)

Mukti Acharya, Joseph Varghese Kureethara, & Thomas Zaslavsky

2021a Characterizations of some parity signed graphs. *Australasian J. Combin.* 81 (2021), no. 1, 89–100. MR 4312564. Zbl 1483.05071. arXiv:2006.03584.

Continues Acharya and Kureethara (2021a). Characterizations of parity signed graphs and in more detail parity signed stars, bistars, cycles, paths and complete bipartite graphs. *rna* numbers for some parity signed graphs. More examples in Ranjith–Kureethara (2020a), Reshma–Gayathri–Rajendra (2021a). [Annot. 13 Dec 2020.] (**Lab: SG**)

Mukti Acharya, Pranjali, Atul Gaur, & Amit Kumar

2017a Line signed graph of a signed total graph. Int. Conf. Current Trends in Graph Theory and Computation (CTGTC-2016, New Delhi). *Electronic Notes Discrete Math.* 63 (2017), 389–397. MR 3754828. Zbl 1383.05140. (**SG: LG**)

Mukti Acharya & Pranjali Sharma

2016a Balanced signed total graphs of commutative rings. *Graphs & Combin.* 32

(2016), no. 4, 1585–1597. MR 3514985. Zbl 1342.05060. **(SG: Algeb: Bal)**

Mukti Acharya & Tarkeshwar Singh

2002a Characterization of sigraphs whose negations are switching equivalent to their iterated line sigraphs. Manuscript, 2002(?).

The signed simple graphs Σ (which necessarily are signed circles) such that $-\Sigma$ is switching isomorphic to any of its iterated Behzad–Chartrand (1969a) line graphs. [Annot. 20 July 2009.] **(SG: Sw, LG)**

2003a Graceful signed graphs: III, The case of signed cycles in which the negative sections form a maximum matching. *Graph Theory Notes N.Y.* 45 (2003), 11–15. MR 2040207 (no rev).

See (2004a). Here the graph is a circle and the second color class is a maximum matching. **(SGc: Lab)**

2003b Skolem graceful signed graphs. Proc. R.C. Bose Centenary Sympos. Discrete Math. Appl. (Kolkata, 2002). *Electronic Notes Discrete Math.* 15 (2003), 10–11. MR 2159025 (no rev). Zbl 1184.05108.

Announcement of (2010a). **(SGc: Lab: Exp)**

2003c Construction of certain infinite families of graceful sigraphs from a given graceful sigraph. Manuscript, 2003.

Let $\vee$ denote the join of graphs or (defined in B.D. Acharya (1980c)) signed graphs. Thms.: If Σ is gracefully numbered, so are $\Sigma \mathbin{\dot\cup} K_t^c$ and $(\Sigma \mathbin{\dot\cup} K^c_{\#E-\#V+1}) \vee K_t^c$. All $(K_2 \vee K_r^c, \sigma)$ are gracefully numbered. [Annot. 20 July 2009.] **(SGc: Lab)**

2004a Graceful signed graphs. *Czechoslovak Math. J.* 54(129) (2004), no. 2, 291–302. MR 2059251 (2005a:05193). Zbl 1080.05529.

[Generalizing the definition in the article: Given: a graph with r-colored edges; integers $k, d > 0$. Required: a (k, d)-*graceful labelling*, i.e., an injection $\lambda : V \to \{0, 1, \ldots, k + (\#E - 1)d\}$ so that, if $f(vw) := |\lambda(v) - \lambda(w)|$, then f restricted to each color class is injective with range $k, k + d, \ldots$.]

The article concerns the case $r = 2$ with "results of our preliminary investigation". *Conjecture.* Every 2-colored circle of length ≥ 3 is (k, d)-graceful. **(SGc: Lab)**

2004b Graceful signed graphs V. The case of union of signed cycles of the length three with one vertex in common. *Int. J. Management Systems* 20 (2004), no. 3, 245–254.

Σ is a signed windmill with $k > 1$ blades. Only rim edges may be negative. Thm.: Σ is graceful $\implies$ $k \equiv 0 \bmod 4$ and $\#E^-$ is even, or $k \equiv 1 \bmod 4$, or $k \equiv 2 \bmod 4$ and $\#E^-$ is odd. Thm.: If $k \equiv 0, 1 \bmod 4$ and all rim edges are negative, then Σ is graceful. Thm.: If $k \equiv 2 \bmod 4$ and all rim edges but one are negative, then Σ is graceful. Also see Singh (2009a). [Annot. 21 July 2010.] **(SGc: Lab)**

2004c A characterization of signed graphs whose negation is switching equivalent to its iterated line sigraphs. In: R.J. Wilson, R. Balakrishnan and G. Sethuraman, eds., *Proceedings of the Conference of Graph Theory and Applications* (CGTA-

2001), pp. 15–24. Narosa Publishing House, New Delhi, 2004. (**SG: Sw, LG**)

2005a Graceful signed graphs: II. The case of signed cycles with connected negative sections. *Czechoslovak Math. J.* 55(130) (2005), no. 1, 25–40. MR 2121654 (2005m:05192). Zbl 1081.05097.

Proof of the conjecture of (2004a) for a circle of length $\not\equiv 1 \pmod 4$ where the negative edge set is connected. (**SGc: Lab**)

2009a Skolem graceful signed stars. *J. Combin. Math. Combin. Comput.* 69 (2009), 113–124. MR 2517312 (2010e:05257). Zbl 1195.05065. (**SGc: Lab**)

2010a Skolem graceful signed graphs. *Utilitas Math.* 82 (2010), 97–109. MR 2663369 (2011h:05218). Zbl 1232.05198.

From Singh (2003a), Ch. III. See (2003b), Singh (2008a). "Skolem gracefulness" is the $(0,1)$-gracefulness of (2004a). Thm.: A signed k-edge matching is Skolem graceful iff $k \equiv 0 \pmod 4$ and $\#E^-$ is even, or $k \equiv 2 \pmod 4$ and $\#E^-$ is odd, or $k \equiv 1 \pmod 4$. Curiously complementary to the theorem of Singh (2008a). [Annot. 20 July 2009.] (**SGc: Lab**)

2013a Embedding of signed graphs in graceful signed graphs. *Ars Combin.* 108 (2013), 421–426. MR 3112764. Zbl 1313.05162.

See (2004a). Every signed graph whose vertices have distinct nonnegative integral labels is an induced subgraph of a signed graph with $(1,1)$-graceful labels. (**SGc: Lab**)

Mukti Acharya & Deepa Sinha

2002a A characterization of signed graphs that are switching equivalent to their jump signed graphs. *Graph Theory Notes N.Y.* 43 (2002), 7–8. MR 1960487 (no rev). (**SG: LG**)

2003a A characterization of sigraphs whose line sigraphs and jump sigraphs are switching equivalent. *Graph Theory Notes N.Y.* 44 (2003), 30–34. MR 2002894. (**SG: LG**)

2003b A characterization of line sigraphs. Proc. R.C. Bose Centenary Sympos. Discrete Math. Appl. (Kolkata, 2002). *Electronic Notes Discrete Math.* 15 (2003), 12. MR 2159026 (no rev).

Abstract of (2005a). (**SG: LG**)

2005a Characterizations of line sigraphs. *Nat. Acad. Sci. Lett. (India)* 28 (2005), no. 1-2, 31–34. MR 2127289 (no rev).

Thm.: A signed simple graph Σ is the Behzad–Chartrand (1969a) line graph of a signed graph iff the underlying graph is a line graph and Σ is "sign compatible" (Sinha (2005a)). [Annot. 27 Apr 2009, 12 Oct 2010.] (**SG: LG**)

2006a Common-edge sigraphs. *AKCE Int. J. Graphs Combin.* 3 (2006), no. 2, 115–130. MR 2285459 (2007k:05083). Zbl 1119.05053.

The common-edge signed graph $C_E(\Sigma)$ is the second line graph $\Lambda^2(|\Sigma|)$ with signs $\sigma_{C_E}\{ef, fg\} = \sigma(f)$. Characterized in whole or part: When this is balanced (rarely), or isomorphic to Σ (rarely), or switching isomorphic to the Behzad–Chartrand (1969a) line graph $\Lambda_{BC}(\Sigma)$ (rarely),

or switching equivalent to $\Lambda^2_{BC}(\Sigma)$. There are notions of consistency and compatibility of $C_E(\Sigma)$ with respect to a vertex signature of Σ, that seem ill defined. (**SG: LG: Gen**)

2013a Characterization of signed line digraphs. *Discrete Appl. Math.* 161 (2013), no. 9, 1170–1172. MR 3030606. Zbl 1277.05078. (**SD: LG**)

Sibel Adalı

See Y. Qian.

Gbemisola Adejumo, P. Robert Duimering, & Zhehui Zhong

2008a A balance theory approach to group problem solving. *Social Networks* 30 (2008), 83–99. (**PsS, SG: Fr**)

Bibhas Adhikari

See also R. Singh.

Bibhas Adhikari, Subhashish Banerjee, Satyabrata Adhikari, & Atul Kumar

2017a Laplacian matrices of weighted digraphs represented as quantum states. *Quantum Inform. Process.* 16 (2017), 1–22. arXiv:1205.2747. MR 3605850. Zbl 1373.81044. (**par: Adj**)(**gg(Gen): Adj**)

Bibhas Adhikari, Amrik Singh, & Sandeep Kumar Yadav

2023a Corona product of signed graphs and its application to modeling signed networks. *Discrete Math. Algor. Appl.* 15 (2023), no. 1, art. 2250062, 32 pp. MR 4543880. Zbl 1516.05080. arXiv:1908.10018.

Defines a corona product $\Sigma \circ \Sigma'$ via edge and vertex signs. Detailed study of repeated corona product $\Sigma^{(m)} := \Sigma \circ \Sigma \circ \cdots$ as model of real-world signed networks: eigenvalues, signed degrees, numbers of sign types of triangles, etc.; e.g. (§3.4), $\#E^+ \gg \#E^-$ and most triangles are all positive. [*Question.* How much of the excess of all-positive triangles is due only to the large excess of positive edges?]

Dictionary: "d" means $d^\pm$ (net degree); "$d^\pm$" means d (degree); "signed Laplacian" $= L(\Sigma)$; "signless Laplacian" $= L(-\Sigma)$. [Annot. 3 Nov 2023.] (**SG, VS: Bal, Eig: Adj, Lap**)

Satyabrata Adhikari

See B. Adhikari.

Chandrashekar Adiga, E. Sampathkumar, & M.A. Sriraj

2014a Color energy of a unitary Cayley graph. *Discuss. Math. Graph Theory* 34 (2014), 707–721. MR 3268686. Zbl 1303.05056.

Energy of L-matrix (*cf.* Sampathkumar and Sriraj (2013b)) of a colored unitary Cayley graph and a colored gcd-graph. [Annot. 14 Oct 2014.] (**sgw: Adj: Eig**)

Chandrashekar Adiga, E. Sampathkumar, M.A. Sriraj, & Shrikanth A.S.

2013a Color energy of a graph. *Proc. Jangjeon Math. Soc.* 16 (2013), no. 3, 335–351. MR 3100089. Zbl 1306.05140.

Definitions at Sampathkumar and Sriraj (2013b). "Color matrix" := $A(\Sigma)$; "color energy" := energy of $A(\Sigma)$. Elementary results on characteristic polynomial, eigenvalues of $A(\Sigma)$. Thm. 2.4: Energy $\leq \sqrt{2n\#E(\Sigma)}$. "Complement" formed by complementing Σ^+ in $(K_n{:}\pi)^c$. Examples:

K_n, $K_{n,m}$, C_n, CP(n), etc., and complements. [No mention of signed graphs.] [2.2, 2.4 for all signed (di)graphs are in Bhat (2017a).] Sequels: Adiga–Sampathkumar–Sriraj (2014a), Sampathkumar–Pushpalatha–Sriraj (2016a), *et al.*] [For a strange "signed-graphic" generalization: Joshi–Joseph–Acharya (20xxa).] [Annot. 21 Dec 2018.]
(sgw: Adj: Eig)

Chandrashekar Adiga, Shrikanth A.S., & Shivakumar Swamy C.S.

2012a A note on 1-edge balance index set. *Int. J. Math. Combin.* 2012 (2012), no. 3, 113–117. Zbl 1276.05102.
Like Adiga, Subbaraya, Shrikanth, and Sriraj (2011a) for wheels and certain graphs of Mycielski. [Annot. 29 Dec 2015.] **(sgw: vsw: Invar)**

Chandrashekar Adiga, C.K. Subbaraya, A.S. Shrikanth, & M.A. Sriraj

2011a On 1-edge balance index set of some graphs. *Proc. Jangjeon Math. Soc.* 14 (2011), no. 3, 319–331. MR 3183856 (no rev). Zbl 1226.05209.
Let $I(\sigma) := \sum_v |d^{\pm}(v)|$ ($d^{\pm}$ = net degree). The titular index set is $\{I(\sigma) : \#E^+(\sigma) = \lfloor \frac{1}{2}\#E \rfloor$ or $\lceil \frac{1}{2}\#E \rceil\}$. The set is determined for two kinds of graph. [Annot. 29 Dec 2015.]
[Simplified: For $S \subseteq E$, $I(S) := \sum_v |d_\Gamma(v) - 2d_S(v)|$. The set is $\{I(S) : |\#E - 2\#S| \leq 1\}$.] [Annot. 19 May 2019.] **(sgw: vsw: Invar)**

2013a On vertex balance index set of some graphs. *Bull. Iranian Math. Soc.* 39 (2013), no. 4, 627–634. MR 3108880. Zbl 1301.05283.
Given $\zeta : V \to \{+,-\}$, set $\sigma(uv) := \zeta(u)\zeta(v)$ and $J(\zeta) := \big|\#E^+(\sigma) - \#E^-(\sigma)\big|$. The titular set is $\{J(\zeta) : \#\zeta^{-1}(0) = \lfloor \frac{1}{2}\#V \rfloor$ or $\lceil \frac{1}{2}\#V \rceil\}$. The set is determined for four kinds of graph including K_n, $K_{r,s}$. [Annot. 29 Dec 2015.]
[Simplified: For $X \subseteq V$, $J(X) := |\#E - 2\#E(X, X^c)|$. The set is $\{J(X) : |\#V - 2\#X| \leq 1\}$.] [Annot. 19 May 2019.] **(vsw: sgw: Invar)**

L. Adler & S. Cosares

1991a A strongly polynomial algorithm for a special class of linear programs. *Operations Res.* 39 (1991), 955–960. MR 1139965 (92k:90042). Zbl 749.90048.
The class is that of the transshipment problem with gains. Along the way, a time bound on the uncapacitated, demands-only flows-with-gains problem. **(GN: Incid(Du), Algor)**

Florian AdriaensFlorian Adriaens & Simon Apers

2023a Testing cluster properties of signed graphs. In: *WWW '23: Proceedings of the ACM Web Conference 2023* (Austin, Tex., 2023), pp. 49–59. Assoc. Computing Machinery, New York, 2023. **(SG: Clu: Algor)**

20xxa Testing properties of signed graphs. Submitted. arXiv:2102.07587.
(SG: Bal, Clu: Algor)

S.N. Afriat

1963a The system of inequalities $a_{rs} > X_r - X_s$. *Proc. Cambridge Philos. Soc.* 59 (1963), 125–133. MR 0141674 (25 #5071). Zbl 118.14901 (118, p. 149a).
Cf. Roy (1959a). **(GG: OG, Sw, bal)**

1974a On sum-symmetric matrices. *Linear Algebra Appl.* 8 (1974), 129–140. MR 0332838 (48 #11163). Zbl 281.15017. (**GG: Sw, bal**)

Amit Agarwal

See Harary, Lim, *et al.* (2004a).

A.A. Ageev, A.V. Kostochka, & Z. Szigeti

1995a A characterization of Seymour graphs. In: Egon Balas and Jens Clausen, eds., *Integer Programming and Combinatorial Optimization* (4th Int. IPCO Conf., Copenhagen, 1995), pp. 364–372. Lect. Notes in Computer Sci., Vol. 920. Springer, Berlin, 1995. MR 1367995 (96h:05157).

A Seymour graph satisfies with equality a general inequality between T-join size and T-cut packing. Thm.: A graph is not a Seymour graph iff it has a conservative ±1-weighting such that there are two circles with total weight 0 whose union is an antibalanced subdivision of $-K_n$ or $-Pr_3$ (the triangular prism). (**SGw: Str, Bal, Par**)

1997a A characterization of Seymour graphs. *J. Graph Theory* 24 (1997), 357–364. MR 1437297 (97m:05217). Zbl 970.24507.

Virtually identical to (1995a). (**SGw: Str, Bal, Par**)

Charu Aggarwal

See J.-L. Tang.

J.K. Aggarwal

See M. Malek-Zavarei.

Kalin Agrawal & William H. Batchelder

2012a Cultural Consensus Theory: Aggregating signed graphs under a balance constraint. In: S.J. Yang, A.M. Greenberg, and M. Endsley, eds., *Social Computing, Behavioral-Cultural Modeling and Prediction* (Proc. 5th Int. Conf., SBP 2012, College Park, Md.), pp. 53–60. Lect. Notes in Computer Sci., Vol. 7227. Springer, Berlin, 2012. (**SG: KG, PsS, Rand**)

Priyanka Agrawal, Vikas K. Garg, & Ramasuri Narayanam

2013a Link label prediction in signed social networks. In: *Proceedings of the Twenty-Third International Joint Conference on Artificial Intelligence* (IJCAI '13), pp. 2591–2597. AAAI Press, 2013. (**SG: Pred, PsS**)

Boris Aguilar

See A. Veliz-Cuba.

F. Aguilera-Granja

See M.C. Salas-Solís.

Ron Aharoni, Rachel Manber, & Bronislaw Wajnryb

1990a Special parity of perfect matchings in bipartite graphs. *Discrete Math.* 79 (1990), 221–228. MR 1044222 (91b:05140). Zbl 744.05036.

When do all perfect matchings in a signed bipartite graph have the same sign product? Solved. (**sg: bal, Algor**)(**qm: QSol**)

R. Aharoni, R. Meshulam, & B. Wajnryb

1995a Group weighted matchings in bipartite graphs. *J. Algebraic Combin.* 4 (1995), 165–171. MR 1323746 (96a:05111). Zbl 950.25380.

Given an edge weighting $w : E \to \mathfrak{K}$ where $\mathfrak{K}$ is a finite abelian group. Main topic: perfect matchings M such that $\sum_{e \in M} w(e) = 0$ [I'll call them 0-weight matchings]. (Also, in §2, $= c$ where c is a constant.) Generalizes and extends Aharoni, Manber, and Wajnryb (1990a). [Continued by Kahn and Meshulam (1998a).] **(GGw)**

Prop. 4.1 concerns vertex-disjoint circles whose total sign product is + in certain signed digraphs. **(SD: Circ)**

Amnon Aharony

1978a Low-temperature phase diagram and critical properties of a dilute spin glass. *J. Phys. C* 11 (1978), L457–L463.

Physics of a random signed subgraph of Γ: p, q, r = probabilities of +, −, or no edge. $r = 0$ is a randomly signed Γ. $p = 0$ is a random subgraph $-\Gamma_1$. Edges may have weights but the signs are most significant (pp. L461–2). Bipartite graphs ("simple systems, with two sublattices") give easier results; e.g., switching exchanges p and q, and transforms all-negative to all-positive. Analysis by the replica method: replicate the graph randomly n times. For temperature $T \to 0$: The case $p = q$ has special properties. The limit $r \to 0$ gives all-positive (ferromagnetic) behavior because "only [constant states $\zeta : V \to \{+1, -1\}$] contribute to the partition function." $T > 0$: Special cases for equal weights, similarly to Houtappel (1950b), Newell (1950a). The replica method's limitations include failure at $T \to 0$ when the signed subgraph is unbalanced ("frustrated") (p. L463). [An interesting study. *Problem.* Interpret the replica method and results in terms of random signed graphs.] [Annot. 21 Jun 2012.] **(Phys, SG, WG: Rand, Fr, sw)**

Uzma Ahmad, Saieed Akbari, Saira Hameed, Mohammad Ali Nematollahi, & Faiza Saeed

2022a Addendum to "Spectral characterizations of signed cycles". *Linear Algebra Appl.* 651 (2022), 83–89. MR 4445166. Zbl 1493.05141.

Corrects oversights about negative even circles in Akbari, Belardo, Dodongeh, and Nematollahi (2018a). [Annot. 26 Dec 2022.] **(SG: Adj, Lap: Eig)**

Saba Ahmadi, Samir Khuller, & Barna Saha

2019a Min-max correlation clustering via multicut. In: Andrea Lodi, ed., *Integer Programming and Combinatorial Optimization* (20th Int. Conf., IPCO 2019, Ann Arbor, Mich.), pp. 13–26. Lect. Notes in Computer Sci., Vol. 11480. Springer, Cham, 2019. MR 3950874. Zbl 1436.90114. arXiv:907.00117.

Cluster Σ to minimize negative edges within each cluster. [Annot. 17 Sept 2022.] **(SG: Clu: Algor)**

Saeed Ahmadizadeh, Iman Shames, Samuel Martin, & Dragan Nešić

2017a On eigenvalues of Laplacian matrix for a class of directed signed graphs. *Linear Algebra Appl.* 523 (2017), 281–306. MR 3624677. Zbl 1369.05132. arXiv:-1705.04406. Corrigendum. *Linear Algebra Appl.* 530 (2017), 541–557. MR 3672976 (no rev). Zbl 1391.05155. **(SD: Lap: Eig)**

Luis von Ahn

2008a Science of the Web: 15-396. Networks II: Structural Balance. Course slides. URL `http://www.scienceoftheweb.org`. Dept. of Computer Science, Carnegie Mellon University.

Triangle ("triad") balance and balance. (**SG: Bal: Exp**)

Ravindra K. Ahuja, Thomas L. Magnanti, & James B. Orlin

1993a *Network Flows: Theory, Algorithms, and Applications*. Prentice Hall, Englewood Cliffs, N.J., 1993. MR 1205775 (94e:90035).

§12.6: "Nonbipartite cardinality matching problem". Nicely expounds theory of blossoms and flowers (Edmonds (1965a), etc.). Historical notes and references at end of chapter. (**par: ori, Algor: Exp, Ref**)

§5.5: "Detecting negative cycles" (i.e., sum < 0); §12.7, subsection "Shortest paths in directed networks". Weighted arcs with negative weights allowed. Techniques for detecting negative cycles and, if none exist, finding a shortest path. (**OG: Algor: Exp**)

Ch. 16: "Generalized flows". §15.5: "Good augmented forests and linear programming bases", Thm. 15.8, makes clear the connection between flows with gains and the frame matroid of the underlying gain graph. Some terminology: "breakeven cycle" = balanced circle; "good augmented forest" = basis of the frame matroid, assuming the gain graph is connected and unbalanced. (**GN: Matrd(Bases), Algor: Exp, Ref**)

Martin Aigner

1979a *Combinatorial Theory*. Grundl. math. Wiss., Vol. 234. Springer-Verlag, Berlin, 1979. Reprint: Classics in Mathematics. Springer-Verlag, Berlin, 1997. MR 0542445 (80h:05002). Zbl 415.05001, Zbl 858.05001 (reprint).

In § VII.1, pp. 333–334 and Exerc. 13–15 treat the Dowling lattices of GF$(q)^\times$ and higher-weight analogs. (**GG, GG(Gen): Matrd: Invar, Str**)

1982a (as M. Aĭgner) *Kombinatornaya teoriya*. "Mir", Moscow, 1982. MR 0694072 (84b:05002).

Russian translation of Aigner (1979a) by V.V. Ermakov and V.N. Lyamin. Ed. and preface by G.P. Gavrilov.
(**GG, GG(Gen): Matrd: Invar, Str**)

Nir Ailon, Moses Charikar, & Alantha Newman

2005a Aggregating inconsistent information: ranking and clustering. In: *STOC'05: Proceedings of the 37th Annual ACM Symposium on the Theory of Computing* (Boston, 2005), pp. 684–693. Assoc. for Computing Machinery, New York, 2005. MR 2181673. Zbl 1192.90252.

Conference version of (2008a). (**SG: WG: Clu: Algor**)

2008a Aggregating inconsistent information: ranking and clustering. *J. ACM* 55 (2008), no. 5, Art. 23, 27 pp. MR 2456548 (2009k:68280).
(**SG: WG: Clu: Algor**)

Saieed Akbari

See also U. Ahmad.

Saieed Akbari, Francesco Belardo, Ebrahim Dodongeh, & Mohammad Ali Nematollahi

2018a Spectral characterizations of signed cycles. *Linear Algebra Appl.* 553 (2018), 307–327. MR 3809382. Zbl 1391.05126.

(C_n, σ) is spectrally unique for $A(\Sigma)$ iff n is odd or $n = 4$; finds most cospectral signed graphs otherwise. It is spectrally nonunique for $L(\Sigma)$ iff even and balanced; finds all cospectral signed graphs otherwise. Corrected for negative even circles in Ahmad, Akbari, *et al.* (2022a). [Annot. 9 May 2018, rev 26 Dec 2022.] **(SG: Adj, Lap: Eig)**

Saieed Akbari, Francesco Belardo, Farideh Heydari, Mohammad Maghasedi, & Mona Souri

2019a On the largest eigenvalue of signed unicyclic graphs. *Linear Algebra Appl.* 581 (2019), 145–162. MR 3982012. Zbl 1420.05070.

§2, "Signed graphs perturbations": For any Σ, effect on $\lambda_{\max}$ of adding an edge. $\lambda_{\max}$ and $\lambda_{\min}$ for $\sigma \equiv +1, -1$ vs. arbitrary σ. §3, "Unbalanced unicyclic graphs with extremal index": Those with max and min $\lambda_{\max}$. [Annot. 23 Jan 2020.] **(SG: Adj: Eig)**

S. Akbari, S. Bolouki, P. Hatami, & M. Siami

2009a On the signed edge domination number of graphs. *Discrete Math.* 309 (2009), no. 3, 587–594. MR 2499011 (2010e:05213). Zbl 1186.05089. arXiv:1008.3217.

See B.-G. Xu (2001a)). Assume $s_e > 0$, $\forall e$. Then $s \geq -n^2/16$, and $s = -(1 + o(1))n^2/54$ is attainable. [What is interesting is that $s < 0$ is possible.] Improved in Cherkashin and Prozorov (2022a). [Annot. 15 Aug 2022.] **(SGw)**

S. Akbari, S.M. Cioabă, S. Goudarzi, A. Niaparast, & A. Tajdini

2021a On a question of Haemers regarding vectors in the nullspace of Seidel matrices. *Linear Algebra Appl.* 615 (2021), 194–206. MR 4200822. Zbl 1459.05154. arXiv:2011.06435.

Nonsingular $A(K_n, \sigma)$ need not have a ± 1-nullvector. [Annot. 6 Feb 2021.] **(SG: KG: Adj)**

S. Akbari, A. Daemi, O. Hatami, A. Javanmard, & A. Mehrabian

2015a Nowhere-zero unoriented flows in hamiltonian graphs. *Ars Combin.* 120 (2015), 51–63. MR 3363263. Zbl 1363.05104.

Every signed Hamiltonian graph without a coloop has a nowhere-zero 12-flow: an improved result towards Bouchet's (1983a) conjecture. The proofs are for unoriented flows on a graph (i.e., flows on an all-negative signed graph, which are equivalent to signed-graph flows). Better results if there is a negative Hamilton circle C. Thm. 3.2: An 8-flow if $\Sigma \setminus C$ is connected. Thm. 3.3: A 6-flow if $\Sigma \setminus C$ is unbalanced. [Annot. 5 Feb 2010.] **(SG: Flows, ori)**

S. Akbari, M. Dalirrooyfard, K. Ehsani, & R. Sherkati

2016a A note on signed k-matching in graphs. *Australasian J. Combin.* 64 (2016), no. 2, 341–346. MR 3442494. Zbl 1333.05233.

Follows C.-P. Wang (2013a). Thm. 2: $\beta_S^k \geq n - k - c(\Gamma)$. Thm. 3: If Γ is connected and not Eulerian, $\beta_S^k = \max$ # odd trails in a minimal trail decomposition. [Annot. 17 Dec 2020.] **(sg)**

Saieed Akbari, Soudabeh Dalvandi, Farideh Heydari, & Mohammad Maghasedi

2019a On the eigenvalues of signed complete graphs. *Linear Multilinear Algebra* 67 (2019), no. 3, 433–441. MR 3909000. Zbl 1407.05119.

Thms.: Multiplicity of eigenvalue $\pm 1 \geq$ number of homogeneously positive (negative) vertices. Spectrum of regular Γ is related to that of K_Γ. [Annot. 17 Dec 2020.] (**SG: KG: Adj: Eig**)

2020a Signed complete graphs with maximum index. *Discuss. Math. Graph Theory* 40 (2020), no. 2, 393–403. MR 4060991. Zbl 1433.05138.

Thm. 8: For $\Sigma^- =$ tree of k edges, $\lambda_{\max}$ is max iff $\Sigma^- = K_{1,k}$. Thm. 10 relates eigenvalues of $H \subseteq K_n$ to those of K_H. Thm. 16: For nonspanning $H \subseteq K_n$ and most eigenvalues λ of H, $-1-2\lambda$ is an eigenvalue of $K_n(-H)$. [*Cf.* Ebrahim Ghorbani & Arezoo Majidi (2021a).] [Annot. 17 Dec 2020.] (**SG: KG: Adj: Eig**)

2020b On the multiplicity of -1 and 1 in signed complete graphs. *Utilitas Math.* 116 (2020), 21–32. MR 4243163. Zbl 1469.05069.

The multiplicities of ± 1 in $\operatorname{Spec} A(K_n,\sigma)$. Also, $\operatorname{Spec} A$ if $(K_n,\sigma)^- \cong K_{p,q,r}$. [Annot. 26 Dec 2022.] (**SG: KG: Adj: Eig**)

S. Akbari, M. Einollahzadeh, M.M. Karkhaneei, & M.A. Nematollahi

2020a Proof of a conjecture on the Seidel energy of graphs. *European J. Combin.* 86 (2020), art. 103078. MR 4047519. Zbl 1437.05130. arXiv:1901.06692.

"Seidel" means signed complete graphs. Thm. 2: Proof of Haemers' (2012a) conjecture. Thm. 1: Lower bound on p-power energy. [Annot. 10 Dec 2020.] (**SG: KG: Adj: Eig**)

S. Akbari, K. Etemadi, P. Ezzati, & M. Ghadiri

2015a Even and odd cycles passing a given edge or a vertex. Manuscript, 2015. arXiv:-1512.02443. (**sg: Par: Circ**)

S. Akbari, F.A.M. França, E. Ghasemian, M. Javarsineh, & L.S. de Lima

2021a The main eigenvalues of signed graphs. *Linear Algebra Appl.* 614 (2021), 270–280. MR 4209003. Zbl 1459.05155. (**SG: Adj: Eig**)

S. Akbari, A. Ghafari, K. Kazemian, & M. Nahvi

2020a Some criteria for a signed graph to have full rank. *Discrete Math.* 343 (2020), no. 8, art. 111910. MR 4081468. Zbl 1441.05097. arXiv:1708.07118.

Given simple Γ, there is σ such that $\operatorname{rk} A(\Gamma,\sigma) = n$ iff Γ has a $\{1,2\}$-factor. [Annot. 10 Dec 2020.] (**SG: Adj**)

S. Akbari, A. Ghafari, M. Nahvi, & M.A. Nematollahi

2020a Mixed paths and cycles determined by their spectrum. *Linear Algebra Appl.* 586 (2020), 325–346. MR 4027760. Zbl 1429.05117. arXiv:1806.03634.

The graphs are $\{\pm 1, \pm \mathrm{i}\}$-gain graphs. (**gg: Adj: Eig**)

Saieed Akbari, Ebrahim Ghorbani, Jack [Jacobus] H. Koolen, & Mohammad Reza Oboudi

2010a A relation between the Laplacian and signless Laplacian eigenvalues of a graph. *J. Algebraic Combin.* 32 (2010), no. 3, 459–464. MR 2721061 (2011i:05125). Zbl 1230.05196.

The sign-corrected coefficients of the characteristic polynomial of $L(-\Gamma)$ dominate those of $L(+\Gamma)$. [*Problem* 1. Prove they dominate those of $L(\Gamma,\sigma)$ for any σ. *Problem* 2. Generalize to any pair of signatures of Γ.] [Annot. 22 Nov 2010.] (**Par: Eig, Incid**)

2010b On sum of powers of the Laplacian and signless Laplacian eigenvalues of graphs. *Electronic J. Combin.* 17 (2010), art. R115, 8 pp. MR 2679569 (2011j:05189). Zbl 1218.05086. **(par: Lap: Eig)**

S. Akbari, E. Ghorbani, & M.R. Oboudi

2009a A conjecture on square roots of Laplacian and signless Laplacian eigenvalues of graphs. Manuscript. arXiv:0905.2118.

Conjecture. The sum s of singular values is larger for $\mathrm{H}(-\Gamma)$ than for $\mathrm{H}(+\Gamma)$. Dictionary: "incidence matrix" = the unoriented incidence matrix $\mathrm{H}(-\Gamma)$; "directed incidence matrix" = oriented incidence matrix $\mathrm{H}(+\Gamma)$. [*Problem.* Prove that $\max_\sigma s(\Gamma,\sigma) = s(-\Gamma)$, achieved uniquely for $[-\Gamma]$.] [Annot. 8 Oct 2010, rev 10 Dec 2020.] **(sg: Par: Incid: Eig)**

Saieed Akbari, Willem H. Haemers, Hamid Reza Maimani, & Leila Parsaei Majd

2018a Signed graphs cospectral with the path. *Linear Algebra Appl.* 553 (2018), 104–116. MR 3809370. Zbl 1391.05156. arXiv:1709.09853. **(SG: Adj: Eig)**

S. Akbari, H.R. Maimani, & L. Parsaei Majd

2018a On the spectrum of some signed complete and complete bipartite graphs. *Filomat* 32 (2018), no. 17, 5817–5826. MR 3899319.

Spectra of $K_n(-M)$ and $K_{p,q}(-M)$ for a matching M. A family of (K_n,σ) with symmetric spectrum. [*Cf.* Etsuo Segawa & Yusuke Yoshie (2021a).] [Annot. 17 Dec 2020.] **(SG: Adj: Eig)**

J. Akiyama, D. Avis, V. Chvátal, & H. Era

†† 1981a Balancing signed graphs. *Discrete Appl. Math.* 3 (1981), 227–233. MR 0675687 (83k:05059). Zbl 468.05066.

Bounds for $D(\Gamma)$, the largest frustration index $l(\Gamma,\sigma)$ over all signings of a fixed graph Γ (not necessarily simple) of order n and size $m = \#E$. Main Thm.: $\frac{1}{2}m - \sqrt{mn} \leq D(\Gamma) \leq \frac{1}{2}m$. Thm. 4: $D(K_{t,t}) \leq \frac{1}{2}t^2 - c_0 t^{3/2}$, where c_0 can be taken $= \pi/480$. Probabilistic methods are used. Thus, Thm. 2: Given Γ, $\mathrm{Prob}(l(\Gamma,\sigma) > \frac{1}{2}m - \sqrt{mn}) \geq 1 - (\frac{2}{e})^n$. Moreover, let $n_{\mathrm{b}}(\Sigma)$ be the largest order of a balanced subgraph of Σ. Thm. 5: $\mathrm{Prob}(n_{\mathrm{b}}(K_n,\sigma) \geq k) \leq \binom{n}{k}/2^{\binom{k}{2}}$. (The problem of evaluating $n - n_{\mathrm{b}}$ was raised by Harary (1959b).) Finally, Thm. 1: If Σ has vertex-disjoint balanced induced subgraphs with m' edges, then $l(\Sigma) \leq \frac{1}{2}(m - m')$. [See Poljak and Turzík (1982a) for an upper bound on $D(\Gamma)$, Solé and Zaslavsky (1994a) for lower and (bipartite) upper bounds; Brown and Spencer (1971a), Gordon and Witsenhausen (1972a) for $D(K_{t,t})$; Harary, Lindström, and Zetterström (1982a) for a result similar to Thm. 1.]
(SG: Fr, Rand)

Tatsuya Akutsu, Sven Kosub, Avraham A. Melkman, & Takeyuki Tamura

2012a Finding a periodic attractor of a Boolean network. *IEEE/ACM Trans. Comput. Biol. Bioinformatics* 9 (2012), no. 5, 1420–1421. **(SG: Dyn, Biol)**

Tatsuya Akutsu, Avraham A. Melkman, & Takeyuki Tamura

2012a Singleton and 2-periodic attractors of sign-definite Boolean networks. *Inform. Processing Lett.* 112 (2012), 35–38.

MR 2895503 (2012k:68098). Zbl 1233.68176. **(SD: Dyn)**

Ghadeer Alabandi, Jelena Tešić, Lucas Rusnak, & Martin Burtscher

2021a Discovering and balancing fundamental cycles in large signed graphs. In: *SC '21: Proc. Int. Conf. for High Performance Computing, Networking, Storage and Analysis* (St. Louis, 2021), art. 68, 14 pp. **(SG: Bal: Algor)**

Ansam I. Al-Aqtash

2014a *The Minimum Semidefinite Rank of Signed Graphs.* Doctoral dissertation, Central Michigan University, 2014.

Cf. Matar, Mitchell, and Narayan (2022a). **(SG: Adj: QM)**

M.J. Alava, P.M. Duxbury, C.F. Moukarzel, & H. Rieger

2001a Exact combinatorial algorithms: Ground states of disordered systems. In: C. Domb and J.L. Lebowitz, eds., *Phase Transitions and Critical Phenomena*, Vol. 18. Academic Press, San Diego, 2001. MR 2014388 (2004k:82040).

§7.1, "Random Ising magnets", (iv), "Frustrated magnets and spin glasses", introduces §7.4, "Ising spin glasses and Euclidean matching". §7.4.1, "Introduction and overview": Frustration index, in terms of Hamiltonian $H(s) := -\sum J_{ij}s_is_j$ where, mainly, $J_{ij} = \pm 1$ randomly (random signed graphs). Frustrated (negative) plaquettes (girth circles) in a lattice. §7.4.2, "Mapping to optimization problems": (i) "Mapping to a matching problem": Planar solution by dual matching as in Katai and Iwai (1978a) [not cited], Bieche, Maynard, Rammal, and Uhry (1980a), Barahona (1982b), *et al.* (ii) "Mapping to a cut problem": Equivalence to max cut. **(Phys: SG: Fr: Rand: Exp, Ref)**

§7.4.3, "Ground-state calculation in two dimensions": Behavior of ground state (fewest frustrated edges) as function of negative-edge density. Remarks on external magnetic field, cubic grid graphs. [Annot. 29 Aug 2012.] **(Phys: SG: Fr, State(fr): Exp, Ref)**

Abdullah Alazemi, Milica Anđelić, Francesco Belardo, Maurizio Brunetti, & Carlos M. da Fonseca

2019a Line and subdivision graphs determined by $\mathbb{T}_4$-gain graphs. *Mathematics* 7 (2019), no. 10, art. 926, 12 pp.

The gain group is $\{\pm 1, \pm \mathrm{i}\}$. **(GG: LG, LG(Gen), Eig, Incid)**

Şahin Albayrak

See J. Kunegis.

István Albert

See A. Saadatpour.

J. James Albert

See Santhi. M.

Rêka Albert

See A. Saadatpour.

Siavash Alemzadeh

See M.H. de Badyn.

John C. Alessio

1990a A synthesis and formalization of Heiderian balance and social exchange theory. *Social Forces* 68 (1990), no. 4, 1267–1285.

Signed digraphs and graphs, with weights, used to describe a novel kind of "balance", different from normal signed-graph balance, based on exchange between persons. [Annot. 24 Jan 2016.]
(PsS: SD, SG, WG: Bal(Gen))

S. Alexander & P. Pincus

1980a Phase transitions of some fully frustrated models. *J. Phys. A: Math. Gen.* 13 (1980), no. 1, 263–273.

Certain all-negative signed graphs where every edge is in a triangle: $d = 2$-dimensional triangular lattice and $d \geq 3$-dimensional face-centered cubic lattice. Phase phenomena depend on the parity of d. Odd d implies interesting infinities of switchings with minimum $\#E^{-}$. [Annot. 12 Aug 2012.] **(Par: Phys)**

Leonidas G. Alexopoulos

See I.N. Melas.

Carlos A. Alfaro, Hugo Corrales, & Carlos E. Valencia

2017a Critical ideals of signed graphs with twin vertices. *Adv. Appl. Math.* 86 (2017), 99–131. MR 3614193. Zbl 1391.13036. **(SD: Algeb)**

Artiom Alhazov, Ion Petre, & Vladimir Rogojin

2009a The parallel complexity of signed graphs: Decidability results and an improved algorithm. *Theor. Comput. Sci.* 410 (2009), no. 24–25, 2308–2315. MR 2522435 (2011a:68045). Zbl 1167.68022. **(SG: Algor)**

Luiz Emilio Allem

See R.E. Mansano.

Noga Alon

1996a Bipartite subgraphs. *Combinatorica* 16 (1996), no. 3, 301–311. MR 1417340 (97h:05081). Zbl 860.05043.

Lower bound on the largest bipartite subgraph of a simple graph with m edges. [I.e., upper bound on $l(-\Gamma)$. *Problem.* Generalize to $l(\Sigma)$.] [Annot. 8 Mar 2011, 19 May 2012.] **(sg: par: Fr)**

Noga Alon & Yoshimi Egawa

1985a Even edge colorings of a graph. *J. Combin. Theory Ser. B* 38 (1985), no. 1, 93–94. MR 0782628 (86f:05059). Zbl 556.05026.

Proves and improves a conjecture of B.D. Acharya (1983a). Thm.: The minimum number of colors for an "even edge coloring" = minimum number of colors so each color class is bipartite = $\lceil \log_2 \chi(\Gamma) \rceil$. [Zaslavsky (1987b) generalizes the latter to Σ.] **(par: bal: Gen)**

Noga Alon, Gregory Gutin, Eun Jung Kim, Stefan Szeider, & Anders Yeo

2010a Solving MAX-r-SAT above a tight lower bound. In: Moses Charikar, ed., *Proceedings of the Twenty-First Annual ACM-SIAM Symposium on Discrete Algorithms* (SODA 2010, Austin, Tex.), pp. 511–517. Soc. for Industrial and Appl. Math., Philadelphia, and Assoc. for Computing Machinery, New York, 2010. MR 2809695 (2012h:68266).

Extended abstract of (2011a). **(SG: Algor)**

2011a Solving MAX-r-SAT above a tight lower bound. *Algorithmica* 61 (2011), no. 3, 638–655. MR 2824999 (2012g:68101). Zbl 1242.68118. arXiv:0907.4573. (**SG: Algor**)

Noga Alon, Konstantin Makarychev, Yury Makarychev, & Assaf Naor

2005a Quadratic forms on graphs. In: *STOC'05: Proceedings of the 37th Annual ACM Symposium on Theory of Computing* (Baltimore, 2005), pp. 486–493. ACM Press, New York, 2005. MR 2181652 (2006g:05207). Zbl 1192.05168.

Extended abstract of (2006a).

2006a Quadratic forms on graphs. *Invent. Math.* 163 (2006), no. 3, 499–522. MR 2207233 (2008a:05156). Zbl 1082.05051.

N. Alon & T.H. Marshall

1998a Homomorphisms of edge-colored graphs and Coxeter groups. *J. Algebraic Combin.* 8 (1998), no. 1, 5–13. MR 1635549 (99i:05074). Zbl 911.05034.

(E.g., sign-colored graphs.) Thm.: For edge-colored graphs $\exists$ universal planar target. Bounds on target. [Relevant for signed-graph homomorphisms.] [Annot. 28 Dec 2019.] (**sgc(Gen): Hom**)

Stefanu Elias Aloysius, ed.

2011a *Dowling Geometry.* AlphaScript Publishing, 2011.

"… primarily consists of articles available from Wikipedia or other free sources online." This seems to mean copying Wikipedia. *Cf.* Chris Rand, "The odd tale of Alphascript Publishing and Betascript Publishing", URL `http://www.chrisrand.com/blog/2010/02/odd-tale-alphascript-publishing-betascript-publishing/` [Annot. 22 Jun 2018.] (**gg: Matrd**)

Yu. A. Al'pin

† 2014a Harary's theorem on signed graphs and reversibility of Markov chains. *J. Math. Sci.* 199 (2014), no. 4, 375–380. Trans. from Russian in *Zap. Nauchn. Semin. POMI* 419 (2013), 5–15. MR 3477903. Zbl 1343.60104.

§2, "Harary's theorem for digraphs over groups": Thm. 3. A gain digraph $(D, \varphi, \mathfrak{G})$, where D is strongly connected, is cycle balanced (every cycle has gain 1) $\Leftrightarrow$ every closed walk has gain 1 $\Leftrightarrow$ for each u, v all uv-paths have the same gain $\Leftrightarrow$ $\exists$ potential for φ. This makes explicit properties implicit in previous work, e.g., Belitskiĭ and Lyubich (1984a). [This actually generalizes Harary–Norman–Cartwright (1965a), Thm. 13.11 for signed digraphs, not Harary (1953a) for undirected graphs.] Thm. 4. Two potentials θ, θ' for φ differ by $\theta' = h\theta$ for some $h \in \mathfrak{G}$. Application to Markov chains. Dictionary: "path" = directed walk, "weight" = gain, "graph over a group" = gain digraph.

[The important aspect: Strong connection suffices for gain-graph properties; the arcs need not be reversible as in a gain graph.] [Annot. 22 Feb 2021.] (**GG: Bal, Appl**)

Claudio Altafini

See also N. Ballber Torres, G. Facchetti, A. Fontan, G. Iacono, G.D. Shi, and N. Soranzo.

2012a Dynamics of opinion forming in structurally balanced social networks. *PLoS ONE* 7 (2012), no. 6, art. 38135, 8 pp. + 6 supplements. URL `https://doi.`

org/10.1371/journal.pone.0038135 **(SD, sg: Bal, Dyn, PsS)**

2012b Achieving consensus on networks with antagonistic interactions. In: *51st IEEE Conference on Decision and Control* (CDC 2012, Maui, Hawaii), pp. 1966–1971. IEEE, 2012. **(SG: Bal)**

2013a Consensus problems on networks with antagonistic interactions. *IEEE Trans. Automatic Control* 58 (2013), no. 4, 935–946. MR 3038795. Zbl 1369.93433. **(SG: Bal)**

2013b Stability analysis of diagonally equipotent matrices. *Automatica J. IFAC* 49 (2013), no. 9, 2780–2785. MR 3084465. Zbl 1364.93318. **(SD: QM: QSta)**

Claudio Altafini & Gabriele Lini

2015a Predictable dynamics of opinion forming for networks with antagonistic interactions. *IEEE Trans. Automatic Control* 60 (2015), no. 2, 342–357. MR 3310162. Zbl 1360.91114. **(SG: Bal, Dyn)**

Dora Altbir

See E.E. Vogel.

Randolf Altmeyer

See D. Feng.

[Susan S. D'Amato]

See S.S. D'Amato (under 'D').

C. Amoruso & A.K. Hartmann

2004a Domain-wall energies and magnetization of the two-dimensional random-bond Ising model. *Phys. Rev. B* 70 (2004), art. 134425, 7 pp. arXiv:cond-mat/0401464. **(Phys, SG: Fr: State: Algor)**

Meirav Amram, Robert Shwartz, & Mina Teicher

2010a Coxeter covers of the classical Coxeter groups. *Int. J. Algebra Comput.* 20 (2010), no. 8, 1041–1062. MR 2747415 (2012c:20104). Zbl 1237.20031. arXiv:-0803.3010.

The structure of a quotient of a generalized Coxeter group depends on the presence of loops in the associated signed graph. [Annot. 17 Dec 2011.] **(SG)**

Javeria Amreen & Sudev Naduvath

20xxa Order sum signed graph of a group. Submitted. **(SG: Algeb)**

20xxb Non-inverse signed graph of a group. Submitted.

For group $\mathfrak{G}$ let $\Gamma := (\mathfrak{G}, E)$ where $g_1 g_2 \in E$ iff $g_1 = g_2^{-1}$. "Non-inverse signed graph" $= K_\Gamma$. Minor results. [Annot. 13 Oct 2023.] **(SG: Algeb: Bal, Clu, VS, Fr)**

Xinhui An

See J.F. Wang.

Milica Anđelić

See also A. Alazemi and S.K. Simić.

Milica Anđelić, Carlos M. da Fonseca, Slobodan K. Simić, & Dejan V. Tošić

2012a Connected graphs of fixed order and size with maximal Q-index: Some spectral bounds. *Discrete Appl. Math.* 160 (2012), no. 4-5, 448–459. MR 2876327 (2012k:05219). Zbl 1239.05115.

Bounds on $\lambda_{\max}(L(-\Gamma))$ where Γ is a nested split graph. (*Cf.* Cvetković, Rowlinson, and Simić (2007b), which shows nested split graphs maximize $\lambda_{\max}(L(-\Gamma))$.) [Annot. 2 Feb 2012.] (**sg: par: Eig**)

2012b Some further bounds for the Q-index of nested split graphs. *J. Math. Sci. (N.Y.)* 182 (2012), no. 2, 193–199. Trans. from *Sovrem. Mat. Prilozh.* 71 (2011). MR 3141339. Zbl 1254.05095. (**sg: par: Eig**)

Milica Anđelić, Tamara Koledin, & Zoran Stanić

2019a A note on the eigenvalue free intervals of some classes of signed threshold graphs *Special Matrices* 7 (2019), 218–225. MR 4044596. Zbl 1431.05098.

[*Cf.* Xiong and Hou (2022a), Mandal and Mehatari (20xxa).] (**SG: Adj: Eig**)

2020a On regular signed graphs with three eigenvalues. *Discuss. Math. Graph Theory* 40 (2020), no. 2, 405–416. MR 4060992. Zbl 1433.05139.

Constructions for signed regular graphs with exactly three eigenvalues. Determines all of order at most 10. [Annot. 29 Apr 2022.] (**SG: Adj: Eig**)

2021a Inequalities for Laplacian eigenvalues of signed graphs with given frustration number. *Symmetry* 2021 (2021), no. 13, art. 1902, 8 pp. (**SG: Fr, Lap: Eig**)

2023a Signed graphs whose all Laplacian eigenvalues are main. *Linear Multilinear Algebra* 71 (2023), no. 15, 2409–2425. MR 4646565.

That is, up to switching equivalence. Order ≤ 7 is sufficient but 8 is not. Two problems: (1) Find all signed graphs with the property up to switching equivalence. (2) Find all switchings with this property; solved for threshold graphs and for some cographs. [Annot. 7 Aug 2022.] (**SG: Lap: Eig**)

Milica Anđelić & Slobodan K. Simić

2010a Some notes on the threshold graphs. *Discrete Math.* 310 (2010), no. 17-18, 2241–2248. MR 2659175 (2011h:05145). Zbl 1220.05035. (**sg: par: Lap**)

Lars Døvling Andersen & Douglas D. Grant

1981a Homopolar circuits in polar graphs. *Czechoslovak Math. J.* 31 (1981), no. 2, 218–228. MR 0611077 (82e:05086). Zbl 0472.05038.

Cf. Zelinka (1976b). If there are no coherent circles, no loops, and no parallel edges with the same orientation, then $\#E \leq 4n - 4$ (equality is characterized) and $\delta \leq 6$. Sufficient conditions for an antidirected Hamiltonian circle. Dictionary: "polar graph" = switching class of bidirected graphs, "homopolar circuit" = antidirected circle. [Later work, only on digraphs: e.g., *cf.* Diwan, Frye, Plantholt, and Tipnis (2011a).] [Annot. 27 Jul 2013.] (**gg: Circ: Str**)(**sg: Ori: Circ**)

Laura Anderson, Ting Su, & Thomas Zaslavsky

20xxa Matroids of gain signed graphs. *Discrete Comput. Geom.* (in press). arXiv:-2206.00237.

†† Combines frame matroid $\mathbf{F}(\Sigma)$ of signed graph with lift matroid $\mathbf{L}(\Phi)$ of abelian gain graph to form extended matroid $\mathbf{M}_\infty(\Gamma, \sigma, \varphi)$ of graph with gains on signed edges. Basic properties of gain signed graphs (e.g., gain of a walk) and the matroid (rank, circuits, flats, etc.). Examples: Dimension of edge polytope (*cf.* Ohsugi and Hibi (1998a)), symmetric edge polytope (*cf.* Matsui *et al.* (2011a)), and bidirected generalization. [Annot. 1, 4 Jun 2022, rev 21 Oct 2023.] (**GG, SG: Matrd, Sw, Bal**)

P.W. Anderson
See S.F. Edwards.

Ascensión Andina-Díaz
See A. Parravano.

Kazutoshi Ando & Satoru Fujishige

1994a $\sqcup, \sqcap$-closed families and signed posets. Report no. 93813, Forschungsinstitut für Diskrete Math., Universität Bonn, 1994. (**sg: Ori**)

1996a On structures of bisubmodular polyhedra. *Math. Programming* 74 (1996), 293–317. MR 1407690 (97g:90102). Zbl 855.68107. (**sg: Ori**)

Kazutoshi Ando, Satoru Fujishige, & Takeshi Naitoh

1997a Balanced bisubmodular systems and bidirected flows. *J. Operations Res. Soc. Japan* 40 (1997), 437–447. MR 1476834 (98k:05073). Zbl 970.61830.
A balanced bisubmodular system corresponds to a bidirected graph that is balanced. The "flows" are arbitrary capacity-constrained functions, not satisfying conservation at a vertex. (**sg: Ori, Bal**)

Kazutoshi Ando, Satoru Fujishige, & Toshio Nemoto

1996a Decomposition of a bidirected graph into strongly connected components and its signed poset structure. *Discrete Appl. Math.* 68 (1996), 237–248. MR 1398276 (97c:05096). Zbl 960.53208. (**sg: Ori**)

1996b The minimum-weight ideal problem for signed posets. *J. Operations Res. Soc. Japan* 39 (1996), 558–565. MR 1435185 (98j:90084). Zbl 874.90188. (**sg: Ori**)

Thomas Andreae

1978a Matroidal families of finite connected nonhomeomorphic graphs exist. *J. Graph Theory* 2 (1978), 149–153. MR 0493394 (80a:05160). Zbl 401.05070.
Partially anticipates the "count" matroids of graphs (see Whiteley (1996a)). (**Bic, ECyc: Gen**)

Eleonora Andreotti & Raffaella Mulas

20xxa Spectra of signless normalized Laplace operators for hypergraphs. Submitted. arXiv:2005.14484. (**sh: Par: Lap: Eig**)

D. Angeli, M. Banaji, & C. Pantea

2014a Combinatorial approaches to hopf bifurcations in systems of interacting elements. *Commun. Math. Sci.* 12 (2014), no. 6, 1101–1133. MR 3194372. Zbl 1315.15007. arXiv:1301.7076. (**SD: QM, Dyn**)

David Angeli, Patrick De Leenheer, & Eduardo Sontag

2010a Graph-theoretic characterizations of monotonicity of chemical networks in reaction coordinates. *J. Math. Biol.* 61 (2010), no. 4, 581–616. MR 2672536 (2011d:92054). Zbl 1204.92038.

Dictionary: "J-graph" = a signed graph of a Jacobian matrix. "Species-reaction graph" ("SR-graph") = signed bipartite graph $(V_S, V_R, E, \sigma) =: \Sigma$; "reaction graph" ("R-graph") = $-\Sigma^2{:}V_R$; "species graph" ("S-graph") = $-\Sigma^2{:}V_S$ [where Σ^2 is the distance-2 signed graph: $V(\Sigma^2) := V_R \cup V_S$, $T_iT_j \in E^\varepsilon(\Sigma^2) \iff \exists$ path $T_iU_kT_j$ with $\sigma(T_iU_kT_j) = \varepsilon$]. Dictionary: "Simple loop" $\approx$ circle; "positive-loop property" = balance. In a signed bipartite graph, "e-loop, o-loop" = circle with $(-1)^{\#C/2}\sigma(C) = +$ or $-$. Prop. 4.5: $\Sigma^2{:}V_R$ is antibalanced iff all circles in Σ are e-loops and max $\deg(\Sigma{:}V_S) \leq 2$. Thm. 1 (oversimplified): A certain differential system is monotone iff $\Sigma^2{:}V_R$ is antibalanced (the R-graph is balanced). [Annot. 19 Feb 2010.] (**SG: Bal, sw, Geom, Chem**)

[A bipartite multiplicative gain graph $\Phi := (V_S, V_R, E, \varphi)$ may be defined by $\varphi(S_iR_j) :=$(a value from the stoichiometry matrix Γ). Circle C is "unitary" if $(-1)^{\#C/2}\varphi(C) = +1$.] Φ is implicated in the proof of geometrical Lemma 6.1. [Annot. 19 Feb 2010.] (**gg: Bal, Geom**)

David Angeli & Eduardo Sontag

2003a A note on multistability and monotone I/O systems. In: *Decision and Control, 2003* (42nd IEEE Conf., Maui, Hawaii, 2003), Vol. 1, pp. 67–72.

§IV, "Graphical conditions for strong monotonicity": Mentions signed (di)graph balance and monotonicity. [Annot. 1 Jan 2012.] (**SD, SG: Bal, Chem: Exp**)

2004a Interconnections of monotone systems with steady-state characteristics. In: Marcio S. de Queiroz, Michael Malisoff, and Peter Wolenski, eds., *Optimal Control, Stabilization and Nonsmooth Analysis*, pp. 135–154. Lect. Notes in Control Inform. Sci., Vol. 301. Springer, Berlin, 2004. MR 2079681 (2005f:93132). Zbl 1259.93092. (**SD, SG: Bal, Chem: Exp**)

2004b Multi-stability in monotone input/output systems. *Systems Control Lett.* 51 (2004), 185–202. MR 2037252 (2004k:93128). Zbl 1157.93509. (**SD, SG: Bal, Chem**)

2008a Oscillations in I/O monotone systems under negative feedback. *IEEE Trans. Automatic Control* 53 (2008), Special Issue on Systems Biology, 166–176. MR 2492561 (2009k:92005), MR 2605139 (no rev).

§II, p. 167, mentions signed (di)graph balance and monotonicity. [Annot. 1 Jan 2012.] (**SD, SG: Bal, Chem: Exp**)

2012a Remarks on the invalidation of biological models using monotone systems theory. In: *51st IEEE Conference on Decision and Control* (Maui, Hawaii, 2012), pp. 2989–2994. (**Biol, SD**)

J.C. Angles d'Auriac & R. Maynard

1984a On the random antiphase state of the $\pm J$ spin glass model in two dimensions. *Solid State Commun.* 49 (1984), no. 8, 785–790.

Signed square lattice graph: frustration index and ground states (minimum $\#E^-$ of switched Σ) via matching [*cf.* Katai and Iwai (1978a), Barahona (1981a), (1982a)]. Observed: natural clusters with relatively fixed spins (vertex signs) if the density of negative edges is in $(0.1, 0.2)$.

[Annot. 18 Aug 2012.] (**Phys, SG: Fr, State(fr): Algor**)

Marcin Anholcer, Bartłomiej Bosek, & Jarosław Grytczuk

2019a Weight choosability of oriented hypergraphs. *Ars Math. Contemp.* 16 (2019), 111–117. MR 3904720. Zbl 1416.05243.

The "orientation" maps $I \to \mathbb{C}^\times$, where I is the set of vertex-hyperedge incidences. (**SH(Gen): gg**)

Achu Aniyan & Sudev Naduvath

2020a Induced signed graphs of some classes of graphs. *Proc. Jangjeon Math. Soc.* 23 (2020), no. 2, 283–291. MR 4099090.

Induced signed graph Σ_f of Γ: given $f : V \to \mathbb{Z}$, $\sigma(uv) := (-1)^{f(u)-f(v)}$. §2: $f(u) = d(u)$, degree. Thm. 2.6: Σ_d is balanced. Thm. 2.7: $-\Sigma_d$ is balanced iff Γ is bipartite. §3: $f(u) = \varepsilon(u)$, eccentricity. Similar results. [Elementary. Balance of all Σ_f by Harary (1953a). $-\Sigma_f$ property because Σ_f is balanced.] §3: $f(u) =$ eccentricity. Similar results. Dictionary: "switched signed graph" = negation $-\Sigma$, not a switching of Σ. [Annot. 26 Sep, 13 Oct 2020.] (**SG: Lab: Bal**)

2020b On degree product induced signed graphs of graphs. In: Samayan Narayanamoorthy, ed., *Advances in Applicable Mathematics* (ICAAM2020, Coimbatore, India), art. 030018, 5 pp. AIP Conf. Proc., Vol. 2261. AIP Publishing, 2020.

Cf. (2020a). $\sigma(uv) := (-1)^{d(u)d(v)}$. Simple results. [Thms. 2, 4, 5, 7 are wrong.] [In general, define $\sigma(uv) := (-1)^{f(u)f(v)}$. Then $E^- = E{:}V_{\text{odd}}$ where $V_{\text{odd}} := \{v : f(v) \text{ odd}\}$.] [Annot. 17 Oct 2020.] (**SG: Bal, Fr**)

Christopher E. Anson

See K.C. Mondal.

T. Antal, P.L. Krapivsky, & S. Redner

†† 2005a Dynamics of social balance on networks. *Phys. Rev. E* 72 (2005), art. 036121, 10 pp. MR 2179924 (2006e:91124).

Models for the evolution of a signed K_n towards balance, with conclusions about the probable long-term behavior. A "state" of the graph is a signature. The unit of time t is $\#E = \binom{n}{2}$ steps of the process. The density of edges is $\rho := \#E^+/\binom{n}{2}$. The number of triangles with k negative edges (type k) is N_k; their density is $n_k := N_k/\binom{n}{3}$. The average density of type k triangles on a positive edge is $n_k^+ = (3-k)N_k/(n-2)\#E^+ = (3-k)n_k/(3n_0 + 2n_1 + n_2)$. Similarly, $n_k^- = kn_k/(3n_0 + 2n_1 + n_2)$.

"Local triad dynamics": At each step a random triangle T is chosen. If it is all negative, a random edge in T is chosen and negated. If it has one negative edge, a random edge in T is chosen and negated with probability p if it is negative and $1 - p$ if positive. If it is balanced there is no change. The process is repeated *ad infinitum*. Finite [i.e., fixed] n: For $p > 1/2$ the graph reaches all-positivity ("paradise") in time $C \log t$ and for $p = 1/2$ in time $C/\sqrt{2t}$. For $p < 1/2$ the graph reaches a balanced state which is not all positive, in super-exponential time. (Time is in the units described.) "Infinite" n [i.e., $n \to \infty$]: For $p < 1/2$ the density of negative edges approaches the stationary value $(1 + \sqrt{(3(1-2p)})^{-1}$. For $p > 1/2$ the network approaches all-positivity.

Thus, at $p = 1/2$ there is a phase transition. Differential equations arise in the densities, with coefficients π^+, π^- where $\pi^\varepsilon :=$ the probability that, in one step, the sign change is from ε to $-\varepsilon$; thus $\pi^+ = (1-p)n_1$ and $\pi^- = pn_1 + n_3$. A stationary state has $\pi^+ = \pi^-$. For infinite n the stationary states are in § III.B and temporal evolution of $\rho = \rho(t)$ is treated in § III.C. Finite n is in § III.D.

"Constrained triad dynamics": An edge is chosen randomly and is negated with probability 1 if the number of positive triangles increases, 0 if the number decreases, and 1/2 if the number remains the same. This corresponds to an Ising model with Hamiltonian $-\sum \sigma_i\sigma_j\sigma_k$, summed over all edge triples that form a triangle. This model approaches balance in time $C \log t$ with high probability if n is large. The other alternatives are to reach an unbalanced absorbing state, where every edge is more positive than negative triangles (a "jammed state"), or a trajectory where every edge is in equally many triangles of each sign (a "blinker"). Blinkers were not observed in the simulations. The probability of a jammed state decreases quickly as $n \to \infty$. The "final" state, if balanced, has Harary bipartition $V = V_1 \cup V_2$. For $\rho(0) \lesssim .4$, $\#V_1/\#V_2 approx 1$. As $\rho(0) \to \beta \approx .65$, $\#V_1/\#V_2 \to \infty$, i.e., one set becomes dominant. When $\rho(0) > \beta$, $V_1 = V$ and all edges are positive. (§ IV.B.) A jammed state can occur only when $n = 9$ or $n \geq 11$ (§ IV.C), e.g., certain 3-cluster states as in Davis (1967a). The number of jammed signatures $> 3^n \gg 2^{n-1} =$ number of balanced ones, notwithstanding that the probable long-term state is balanced (§ IV.C). [See Marvel, Strogatz, and Kleinberg (2009a), Abell and Ludwig (2009a), Kujawski, Ludwig, and Abell (2010a), Deng and Abell (2010a).]

Proposed research: Allow type 3 triangles (i.e., clustering). Allow incomplete graphs.

Dictionary: "network" = complete graph. [Annot. 27 Apr 2009.]

(SG: KG: Dyn: Bal)

2006a Social balance on networks: the dynamics of friendship and enmity. *Physica D* 224 (2006), no. 1-2, 130–136. MR 2301516 (2007k:91210). Zbl 1130.91041.

Similar to (2005a), with some details omitted and some additional results. [Annot. 27 Apr 2009.] **(SG: KG: Dyn: Bal)**

St. Antohe & E. Olaru

1981a Singned graphs homomorphism [*sic*]. [Signed graph homomorphisms.] *Bul. Univ. Galati Fasc. II Mat. Fiz. Mec. Teoret.* 4 (1981), 35–43. MR 0668767 (83m:05057).

A "congruence" is an equivalence relation R on $V(\Sigma)$ such that no negative edge is within an equivalence class. The quotient Σ/R has the obvious simple underlying graph and signs $\bar\sigma(\bar x\bar y) = \sigma(xy)$ [which is ambiguous]. A signed-graph homomorphism is a function $f : V_1 \to V_2$ that is a sign-preserving homomorphism of underlying graphs. [This is inconsistent, since the sign of edge $f(x)f(y)$ can be ill defined. The defect might perhaps be remedied by allowing multiple edges with different signs or by passing entirely to multigraphs.] The canonical map $\Sigma \to \Sigma/R$ is such a homomorphism. Composition of homomorphisms is well defined and associative; hence one has a category Graph$^{\text{sign}}$. The categorial product is $\prod_{i\in I} \Sigma_i :=$ Cartesian product of the $|\Sigma_i|$ with the component-wise signature $\sigma((\ldots, u_i, \ldots)(\ldots, v_i, \ldots)) := \sigma_i(u_i v_i)$. Some further elementary properties of signed-graph homomorphisms and congruences are proved. [The paper is hard to interpret due to mathematical ambiguity and grammatical and typographical errors.] **(SGc: Hom)**

Divya Antoney, Tabitha Agnes Mangam, & Mukti Acharya

20xxa Signed graphs from proper coloring of graphs. Submitted.

Parity signs (*cf.* Acharya and Kureethara (2021a)) on Γ derived from minimal proper colorings. [Annot. 12 Mar 2021.] **(Lab: SG)**

20xxa S-cordial and total S-cordial labeling in signed graphs. Submitted.

When does $\Lambda_{BC}(\Sigma)$ have nearly equal numbers of $+$ and $-$ vertices, or edges, or both together? [Annot. 1 Oct 2021.] **(SG, VS: LG)**

M. Venkata Anusha, R. Lakshmi, M. Siva Parvathi, & G.S. Shanmuga Priya

2021a Seidel energy and Seidel matrix energy of Euler totient Cayley graph. *Int. J. Analytical Exper. Modal Anal.* 13 (2021), no. 7, 309–320. **(sg: KG: Adj: Eig)**

Katsuaki Aoki

See M. Iri.

Mustapha Aouchiche & Pierre Hansen

2010a A survey of automated conjectures in spectral graph theory. *Linear Algebra Appl.* 432 (2010), 2293–2322. MR 2599861 (2011b:05139). Zbl 1218.05087.

Computer-generated conjectures. §4, "Signless Laplacian": Several computer-generated conjectures about eigenvalues of $L(-\Gamma)$; some are proved (mainly in Cvetković, Rowlinson, and Simić (2007b)) or disproved; some are difficult. [*Question.* How many generalize to all Σ, with or without proofs?] [Annot. 22 Jan 2012.] **(par: Lap: Eig)**

2013a A survey of Nordhaus–Gaddum type relations. *Discrete Appl. Math.* 161 (2013), no. 4-5, 466–546. MR 3015299. Zbl 1259.05083.

§6, "Spectral invariants": §6.3, "The eigenvalues of the signless Laplacian matrix": Nordhaus–Gaddum-type relations imply theorems from Gutman, Kiani, Mirzakhah, and Zhou (2009a) about the eigenvalues, singular values, incidence energy of $L(-\Gamma)$. Conjecture 6.19, generated by a computer—*cf.* (2010a): $\lambda_{\max}(L(-\Gamma)) + \lambda_{\max}(L(-\Gamma^c)) \leq 3n-4$; $\lambda_{\max}(L(-\Gamma)) \cdot \lambda_{\max}(L(-\Gamma^c)) \leq 2n(n-2)$; $=$ iff Γ is a star. [Annot. 22 Jan 2012.] **(par: Lap: Eig)**

2013b Two Laplacians for the distance matrix of a graph. *Linear Algebra Appl.* 439 (2013), 23–33. MR 3045220. Zbl 1282.05086.

The "distance signless Laplacian" is $\mathcal{D}^L(-\Gamma) := D + \mathcal{D}(\Gamma)$, where $D =$,diagonal degree matrix, $\mathcal{D} =$ distance matrix. Contrasts to the "distance Laplacian" $\mathcal{D}^L(\Gamma) := D - \mathcal{D}(\Gamma)$, in analogy to the Laplacian matrix $L(\Gamma) = D - A$ vs. signless Laplacian $L(-\Gamma) = D + A$.) [*Question.* Is there a signed-graphic distance matrix $\mathcal{D}(\Sigma)$ generalizing $\mathcal{D}(\Gamma)$ and $-\mathcal{D}(\Gamma)$, analogously to $A(\Sigma)$? E.g., is distance algebraically additive?] [Annot. 20 Mar 2016.] **(sg: par: Eig)**

2016a On the distance signless Laplacian of a graph. *Linear Multilinear Algebra* 64 (2016), no. 6, 1113–1123. MR 3479404. Zbl 1381.05015.

3479404 Further development of (2013b). [Annot. 20 Mar 2016.] (**sg: par: Eig**)

Mustapha Aouchiche, Pierre Hansen, & Claire Lucas

2011a On the extremal values of the second largest Q-eigenvalue. *Linear Algebra Appl.* 435 (2011), no. 10, 2591–2606. MR 2811141 (2012h:05184). Zbl 1222.05146.
(**par: Lap: Eig**)

Simon Apers

See F. Adriaens and K.-C. Chen.

Gautam Appa

See also L.S. Pitsoulis.

Gautam Appa & Balázs Kotnyek

2004a Rational and integral k-regular matrices. *Discrete Math.* 275 (2004), 1–15. MR 2026273 (2004m:05005). Zbl 1043.15011.

2-regular matrices include binet matrices (2006a). A key property of k-regular matrices is that solutions of integral equations are $1/k$-integral.
(**sg: Incid: Ori**)

2006a A bidirected generalization of network matrices. *Networks* 47 (2006), no. 4, 185–198. MR 2229861 (2008a:05157). Zbl 1097.05025.

Binet matrices are the network matrices of bidirected (or signed) graphs. Basic theory of binet matrices, generalizing that of network matrices, notably half-integrality theorems. [For a slight simplification see Bolker and Zaslavsky (2006a).] (**sg: Incid: Ori**)

Gautam Appa, Balázs Kotnyek, Konstantinos Papalamprou, & Leonidas Pitsoulis

2007a Optimization with binet matrices. *Operations Res. Lett.* 35 (2007), 345–352. MR 2320128 (2008a:90052). Zbl 1169.90407. (**Ori: Incid(Gen), matrd**)

Julio Aracena

See also F. Benitez, J.-P. Comet, J. Demongeot, and M. Montalva.

2008a Maximum number of fixed points in regulatory Boolean networks. *Bull. Math. Biol.* 70 (2008), no. 5, 1398–1409. MR 2421503 (2009d:05088). Zbl 1144.92323.

A regulatory Boolean network N is built on a signed digraph D. Thm. 6: If all (directed) cycles are positive then N has at least 2 fixed points. Thm. 9: N has at most 2^p fixed points, where $p :=$ minimum number of vertices that cover all positive cycles [unusually, not negative cycles!], and this is best possible. [Annot. 9 July 2009.]
(**SD: Dyn: Fr(Gen), Biol**)

J. Aracena, S. Ben Lamine, M.A. Mermet, O. Cohen, & J. Demongeot

2003a Mathematical modeling in genetic networks: Relationships between the genetic expression and both chromosomic breakage and positive circuits. *IEEE Trans. Systems, Man, Cybernetics B* 33 (2003), no. 5, 825–834.
(**SD, Biol: Dyn: Fr(Gen)**)

Julio Aracena, Jacques Demongeot, & Eric Goles

2004a Positive and negative circuits in discrete neural networks. *IEEE Trans. Neural Networks* 15 (2004), no. 1, 77–83.

Existence and upper bound on the number of fixed points of a "discrete neural network" $\mathcal{N}$, which consists of a real $n \times n$ matrix W, the

associated signed digraph D of order n, and a real vector b. A state is $x \in \{-1,+1\}^n$. A transition is $x \mapsto f(x) := \mathrm{sgn}_+(Wx-b)$ where $\mathrm{sgn}_+(t) := \mathrm{sgn}(t)$ except $\mathrm{sgn}_+(0) := +1$. Assume: D is connected; no component of f is constant, hence a cycle exists. Lemma 1: A cycle is positive iff it has a satisfied state. Thm. 1: If all cycles are positive, f has a fixed point. Thm. 2: If all cycles are negative, f has no fixed point. Thm. 3: $\#\{\text{fixed points}\} \leq 2^p$ where $p := \min(\text{size of vertex cover of positive cycles})$, and this is sharp. Dictionary: "positive feedback vertex set" = vertex cover of positive cycles = vertex set that covers all positive cycles; "circuit" = (directed) cycle; $\mathbf{1} = \mathrm{sgn}_+$. [Annot. 20 July 2009.]
(**SD: Dyn: Bal, Fr(Gen)**)

J. Aracena, E. Fanchon, M. Montalva, & M. Noual

2011a Combinatorics on update digraphs in Boolean networks. *Discrete Math.* 159 (2011), no. 6, 401–409. MR 2765431 (2012b:05271). Zbl 1209.05103.
Dictionary: "labeled digraph" = signed digraph. (**SD: Dyn**)

J. Aracena, E. Goles, A. Moreira, & L. Salinas

2009a On the robustness of update schedules in Boolean networks. *BioSystems* 97 (2009), 1–8.
A "labelled digraph" is essentially a signed digraph.
(**SD: Dyn**)(**sd: Dyn, Biol**)

Julio Aracena, Mauricio González, Alejandro Zuñiga, Marco A. Mendez, & Verónica Cambiazo

2006a Regulatory network for cell shape changes during *Drosophila* ventral furrow formation. *J. Theoretical Biol.* 239 (2006), 49–62. MR 2224512 (no rev).
Figs. 2, 3 show particular proposed genetic regulatory networks based on signed digraphs. §3.2 describes how the mathematical model of Aracena, Ben Lamine, *et al.* (2003a) and Aracena, Demongeot, and Goles (2004a) applies to the situation of this paper. [Annot. 20 July 2009.]
(**SD: Dyn: Appl(Biol)**)

Julio Aracena, Adrien Richard, & Lilian Salinas

2017a Fixed points in conjunctive networks and maximal independent sets in graph contractions. *J. Computer System Sci.* 88 (2017), 145–163. MR 3659350. Zbl 1371.68204. arXiv:1507.06141. (**SD: Dyn**)

Julián Aráoz, William H. Cunningham, Jack Edmonds, & Jan Green-Krótki

1983a Reductions to 1-matching polyhedra. Proc. Sympos. on the Matching Problem: Theory, Algorithms, and Applications (Gaithersburg, Md., 1981). *Networks* 13 (1983), 455–473. MR 0723693 (85d:90059). Zbl 525.90068.
"Minimum-cost capacitated b-matching problem in a bidirected graph B": minimize $\sum_e c_e x_e$ subject to $0 \leq x \leq u \in \{0,1,\ldots,\infty\}^E$ and $\mathrm{H(B)}x = b \in \mathbb{Z}^V$. Proves, by reduction to the ordinary perfect matching problem, Edmonds and Johnson's (1970a) description of the convex hull of feasible solutions. Dictionary: "lobe" = half edge.
(**sg: Ori: Incid, Algor, Geom**)

Marina Arav, F. Scott Dahlgren, & Hein van der Holst

2021a Signed graphs with stable maximum nullity at most two. *Linear Algebra Appl.* 620 (2021), 124–146. MR 4227776.

Cf. Arav, Hall, Li, and Hein van der Holst (2021a). (**SG: Adj(Gen)**)

Marina Arav, Frank J. Hall, Zhongshan Li, & Hein van der Holst

2013a The inertia set of a signed graph. *Linear Algebra Appl.* 439 (2013), no. 5, 1506–1529. MR 3067819. Zbl 1282.05058. arXiv:1208.5285. (**SG: Eig**)

2021a Two-connected signed graphs with maximum nullity at most two. *Linear Algebra Appl.* 611 (2021), 82–93. MR 4185137. Zbl 1461.05102. arXiv:1407.2525.

Studies $\max \mathrm{nul}(A)$ for symmetric matrix A with sign pattern $-A(\Sigma)$, where $|\Sigma|$ is simple and 2-connected. Equivalently, A also has Strong Arnold Property. Thm. 15: $\max \mathrm{nul}(A) \leq 2$ iff Σ has no $+K_4$, $-K_4$, $\pm K_3$, or $K_{2,3}(-e)$ minor. Dictionary: "even, odd" = positive, negative; "re-signing" = switching. [Annot. 6 Feb 2021.] (**SG: Adj**)

Marina Arav, Hein van der Holst, & John Sinkovic

2015a On the inertia set of a signed graph with loops. *Linear Algebra Appl.* 471 (2015), 169–183. MR 3314332. Zbl 1307.05091. arXiv:1402.4326. (**SG: Eig**)

2016a On the inertia set of a signed tree with loops. *Linear Algebra Appl.* 510 (2016), 361–372. MR 3551638. Zbl 1352.05081. (**SG: Adj: Eig**)

† 2016b Signed graphs whose signed Colin de Verdière parameter is two. *J. Combin. Theory Ser. B* 116 (2016), 440–455. MR 3425251. Zbl 1327.05142. arXiv:-1209.4628. (**SG: Adj, Str**)

Dan Archdeacon

1992a The medial graph and voltage-current duality. *Discrete Math.* 104 (1992), no. 2, 111–141. MR 1172842 (93i:05051). Zbl 757.05045.

The medial graph of $\Gamma \subset S$, a graph embedded in a surface, is a 4-regular graph $M \subset S$ that encodes Γ and its surface dual. Gains ("voltages") on Γ transfer to gains ("voltages") on M. Graphs have both gains and signs; signs are determined by face orientations. [*Question.* Does this suggest a gain-graphic, surface-embedding theory of 4-regular gain graphs? It gives such a theory under certain conditions: faces are 2-colorable and black face boundaries must have balanced gains.] Dictionary: "straight, twisted edge" = positive, negative edge. [Annot. 16 Jan 2012, rev 20 Nov 2016.] (**Top: GG, SG, Du**)

1995a Problems in topological graph theory. Manuscript, 1995. URL (2/1998) `http://www.emba.uvm.edu/~archdeac/papers/papers.html`

A compilation from various sources and contributors, updated every so often. "The genus sequence of a signed graph", p. 10: A conjecture due to Širáň (?) on the demigenus range (here called "spectrum" [though unrelated to matrices]) for orientation embedding of Σ, namely, that the answer to Question 1 under Širáň (1991b) is affirmative. [The term "parity embedding" is mistakenly used for orientation embedding of any signed graph; parity embedding is of an unsigned graph.] (**SG: Top**)

1996a Topological graph theory: a survey. Surveys in Graph Theory (Proc., San Francisco, 1995). *Congressus Numer.* 115 (1996), 5–54. Updated version at URL (2/1998) `http://www.emba.uvm.edu/~archdeac/papers/papers.html` MR 1411236 (98g:05044). Zbl 897.05026.

§2.5 describes orientation embedding (called "signed embedding" [although there are other kinds of signed embedding]) and switching (called "sequence of local switches of sense") of signed graphs with rotation systems. §5.5, "Signed embeddings", briefly mentions Širáň (1991b), Širáň and Škoviera (1991a), and Zaslavsky (1993a), (1996a). (**SG: Top: Exp**)

2005a Variations on a theme of Kuratowski. *Discrete Math.* 302 (2005), 22–31. MR 2179233 (2006g:05055). Zbl 1076.05027.

Mentions [and conflates] the theorems of Zaslavsky (1993a). [Annot. 20 Jun 2011.] (**Top: SG: Exp**)

Dan Archdeacon & Marisa Debowsky

2005a A characterization of projective-planar signed graphs. *Discrete Math.* 290 (2005), no. 2–3, 109–332. MR 2123383 (2005j:05041). Zbl 1060.05039.

Similar to Archdeacon–Širáň (1998a), but for the projective plane. (**SG, Sw: Top**)

Dan Archdeacon, Joan Hutchinson, Atsuhiro Nakamoto, Seiya Negam [Seiya Negami], & Katsuhiro Ota

2001a Chromatic numbers of quadrangulations on closed surfaces. *J. Graph Theory* 37 (2001), no. 2, 100–114. MR 1829924 (2002j:05044). Zbl 0979.05034.

[*Cf.* Nakamoto, Negami, and Ota (2002a), (2004a).] (**sg: Top: sw**)

Dan Archdeacon & Jozef Širáň

1998a Characterizing planarity using theta graphs. *J. Graph Theory* 27 (1998), 17–20. MR 1487782 (98j:05055). Zbl 887.05016.

A "claw" consists of a vertex and three incident half edges. Let C be the set of claws in Γ and T the set of theta subgraphs. Fix a rotation of each claw. Call $t \in T$ an "edge" with endpoints c, c' if t contains c and c'; sign it $+$ or $-$ according as t can or cannot be embedded in the plane so the rotations of its trivalent vertices equal the ones chosen for c and c'. This defines, independently (up to switching) of the choice of rotations, the "signed triple graph" $T^{\pm}(\Gamma)$. Theorem: Γ is planar iff $T^{\pm}(\Gamma)$ is balanced. (**SG, Sw: Top**)

Federico Ardila

2002a The Tutte polynomial of a hyperplane arrangement (extended abstract).

Extended abstract of (2007a). [Annot. 4 Oct 2014.] (**gg: bal: Geom, Invar**)

2003a *Enumerative and Algebraic Aspects of Matroids and Hyperplane Arrangements.* Ph.D. thesis, Massachusetts Institute of Technology, Cambridge, Mass., 2003. MR2717078 (no rev).

Ch. 2 is published as (2007a). [Annot. 4 Oct 2014.] (**gg: bal: Geom, Invar**)

2007a Computing the Tutte polynomial of a hyperplane arragement [[*sic*]]. *Pacific J. Math.* 230 (2007), no. 2, 1–26. MR 2318445 (2008g:52034). Zbl 1152.52011. arXiv:math/0409211.

Applies the "finite field method" of Athansiadis (1996a) (in §5 this is actually the modular gains method for computing chromatic polynomials

of integral gain graphs), cleverly extended, to compute Tutte polynomials (in the equivalent form of coboundary polynomials) of various gain graphs (in the equivalent form of affine hyperplane arrangements).

Thms. 4.2, 4.3: Coboundary polynomial of complete signed graph $\pm K_n^\bullet$ ("$\mathcal{B}_n$ arrangement") and complete signed link graph $\pm K_n$ ("$\mathcal{D}_n$ arrangement") in terms of generating functions. Thm. 4.4: Balanced coboundary polynomial of a contrabalanced multigraph $(\Gamma, \varnothing)$ (coboundary polynomial of "$\mathcal{A}_n^\#$ arrangement" [equivalently, the bicircular lift matroid $\mathbf{L}(\Gamma, \varnothing)$]). [Also computed, slightly generalized, in Zaslavsky (1995b), Ex. 3.4, $q^{\mathrm{b}}_{(\Gamma,\varnothing)}$.] Thm. 4.5: Coboundary polynomial of $-K_n$ ("threshold arrangement" $\mathcal{T}_n = \mathcal{H}[-K_n]$).

§5, "Deformations of the braid arrangement": Balanced subgraphs of A-expansions $A{\cdot}K_n$, where A is a finite set of integers, are employed to compute the coboundary polynomials of integral deformations $\mathcal{H}[A{\cdot}K_n]$ of the complete-graph ("'braid") arrangement $\mathcal{H}[K_n]$. Prop. 5.8 treats the Catalan arrangement $\mathcal{H}[\{0, \pm 1\}{\cdot}K_n]$ and its subarrangements. Thm. 5.14 treats the Catalan arrangement. Prop. 5.9 and Thm. 5.11 treat the Lineal arrangement $\mathcal{H}[\{1\}{\cdot}K_n$. Thm. 5.12 treats the Shi arrangement $\mathcal{H}[\{0, 1\}{\cdot}K_n]$. Thm. 5.13 treats the semiorder arrangement $\mathcal{H}[\{\pm 1\}{\cdot}K_n]$.

Dictionary: "finite field method" usually = modular gains method (Zaslavsky (2002a), §11.4 after (11.3); or see Berthomé *et al.* (2009a), Lemma 6.3); "type" = gain of edge; "planted graded graph with height function" = balanced integral gain graph with potential function; "planted graded A-graph" = balanced subgraph of $A{\cdot}K_n$; $\mathcal{E}_n = \mathcal{H}[A{\cdot}K_n]$; "threshold arrangement" = all-negative complete graph arrangement. [Annot. 4 Oct 2014.] (**sg, gg: bal: Geom, Invar**)

Federico Ardila, Federico Castillo, & Michael Henley

2014a The arithmetic Tutte polynomials of the classical root systems. In: *26th International Conference on Formal Power Series and Algebraic Combinatorics* (FPSAC 2014, Chicago), pp. 851–862. Discrete Math. Theor. Computer Sci. Proc., AT. Assoc. Discrete Math. Theor. Computer Sci., Nancy, 2014. MR 3466427.

Extended abstract of (2015a).

(**SG: Matrd(Gen): Invar, Enum**)(**SG: Geom: Exp**)

2015a The arithmetic Tutte polynomials of the classical root systems. *Int. Math. Res. Notices* 2015 (2015), no. 12, 3830–3877. MR 3356740. Zbl 1317.05091. arXiv:1305.6621.

These invariants of signed graphs are obtained by a substitution in generating functions of the ordinary Tutte polynomials. G.f. computed for A_{n-1}, B_n, C_n, D_n with respect to integer, root, and weight lattices. §4.1.2, "Enumeration of signed graphs": Counted by several parameters, e.g., number of balanced components. §4.2, "From classical root systems to signed graphs": Expository. $A_{n-1}, B_n, C_n, D_n \leftrightarrow +K_n, \pm K'_n, \pm K'_n, \pm K_n$, $'$ denoting half edges [but C_n should $\leftrightarrow \pm K_n^\circ$, with negative loops]. §4.3, "Computing the Tutte polynomials by signed graph enumeration". §5, "Arithmetic characteristic polynomials": Explicit, for some cases. Dictionary: "loop" = half edge (Lemma 4.9 depends on that). [Lemma 4.9: special case of Zaslavsky (1982a), Lemma 8A.2.]

[*Question.* How to compute arithmetic Tutte polynomials and characteristic polynomials for other signed graphs? Preferably, directly. How does the lattice framework fit in?] [Annot. 26 May 2018.]
(**SG: Matrd(Gen): Invar, Enum**)(**SG: Geom: Exp**)

Federico Ardila & Alexander Postnikov

2010a Combinatorics and geometry of power ideals. *Trans. Amer. Math. Soc.* 362 (2010), no. 8, 4357–4384. MR 2608410 (2011g:05322). Zbl 1226.05019. arXiv:0809.2143.

The polynomials $\widetilde{Z}_{\mathcal{A}}$ (Sokal's (2005a) "multivariate Tutte polynomial"), slightly normalized, and $S_{\mathcal{A}}$ are specializations of Traldi's (1989a) "weighted dichromatic polynomial", hence of Zaslavsky's (1992b) "parametrized" dichromatic and corank-nullity polynomials. [Annot. 17 Oct 2017.]
(**SGw(Gen): Invar, Geom**)

2015a Correction to "Combinatorics and geometry of power ideals": two counterexamples for power ideals of hyperplane arrangements. *Trans. Amer. Math. Soc.* 367 (2015), no. 5, 3759–3762. MR 3314823. Zbl 1315.05009. arXiv:1211.1368.
(**Geom**)

Samin Aref

2019a *Signed Network Structural Analysis and Applications with a Focus on Balance Theory.* Doctoral thesis, Univ. of Auckland, 2019. arXiv:1901.06845.
(**SG: Bal**)

Samin Aref, Andrew J. Mason, & Mark C. Wilson

2018a Computing the line index of balance using integer programming optimisation. In: Boris Goldengorin, ed., *Optimization Problems in Graph Theory: In Honor of Gregory Z. Gutin's 60th Birthday*, pp. 65–84. Springer Optim. Appl., Vol. 139. Springer, Cham, 2018. MR 3838926. Zbl 1416.05128. arXiv:1710.09876.
(**SG: Fr: Algor**)

2020a A modeling and computational study of the frustration index in signed networks. *Networks* 75 (2020), no. 1, 95–110. MR 4045043. arXiv:1611.09030.
(**SG: Fr: Algor**)

Samin Aref & Zachary Neal

20xxa Legislative effectiveness hangs in the balance: Studying balance and polarization through partitioning signed networks. Submitted. arXiv:1906.01696.

Two-step algorithm for the NP-complete problem of frustration index $l(\Sigma)$: first find upper and lower bounds by fast methods (by examples and LP relaxation), then use that to control the amount of computation. It also finds an optimal bipartition. [Annot. 15 Jul 2019.] (**SG: Fr: Algor, PsS**)

Samin Aref & Mark C. Wilson

2018a Measuring partial balance in signed networks. *J. Complex Networks* 6 (2018), no. 4, 566–595. MR 3842390. Zbl 1460.90039. arXiv:1509.04037. (**SG: Fr: Algor**)

2019a Balance and frustration in signed networks. *J. Complex Networks* 7 (2019), no.

2,163–189. MR 3941371. arXiv:1712.04628. (**SG: Fr: Algor**)

Alex Arenas

See S. Gómez.

Srinivasa R. Arikati & Uri N. Peled

1993a A linear algorithm for the group path problem on chordal graphs. *Discrete Appl. Math.* 44 (1993), 185–190. MR 1227703 (94h:68084). Zbl 779.68067.

Given a graph with edges weighted from a group. The weight of a path is the product of its edge weights in order (not inverted, as with gains). Problem: to determined whether between two given vertices there is a chordless path of given weight. This is NP-complete in general but for chordal graphs there is a fast algorithm (linear in $(\#E + \#V) \cdot$ (group order)). [*Question.* What if the edges have gains rather than weights?] (**WG: par(Gen): Algor**)

1996a A polynomial algorithm for the parity path problem on perfectly orientable graphs. *Discrete Appl. Math.* 65 (1996), 5–20. MR 1380065 (96m:05120). Zbl 854.68069.

Problem: Does a given graph contain an induced path of specified parity between two prescribed vertices? A polynomial-time algorithm for certain graphs. (*Cf.* Bienstock (1991a).) [*Problem.* Generalize to paths of specified sign in a signed graph.] (**par: Algor**)(**Ref**)

Nejat Arınık, Rosa Figueiredo, & Vincent Labatut

2017a Signed graph analysis for the interpretation of voting behavior. In: *International Conference on Knowledge Technologies and Data-driven Business (i-KNOW)* (Graz, Austria, 2017). arXiv:1712.10157. HAL hal-01583133. (**SG**)

2020a Multiple partitioning of multiplex signed networks: Application to European Parliament votes. *Social Networks* 60 (2020), 83–102. arXiv:1811.10337. HAL hal-02082574. (**SG: Clu: Algor, Appl**)

2020b Multiplicity and diversity: Analyzing the optimal solution space of the correlation clustering problem on complete signed graphs. *J. Complex Networks* 8 (2020), no. 6, art. cnaa025. arXiv:2011.05196. HAL hal-02994011. (**SG: KG: Fr**)

20xxb 2-Edge connected balanced subgraphs for correlation clustering problem. Submitted. HAL hal-02428305. (**SG: Clu: Algor**)

Esther M. Arkin & Christos H. Papadimitriou

1985a On negative cycles in mixed graphs. *Operations Res. Lett.* 4 (1985), 113–116. MR 0821170 (87h:68061). Zbl 585.05017.

Reinterpreted: Treat a mixed graph as a bidirected graph B where a walk can only enter a vertex along an in-arrow (thus introverted edges are irrelevant). Given integral gains in $\mathbb{Z}^+$, finding negative cycles is NP-complete, though it is P if B is all positive or all negative. Also, "it is NP-complete to find shortest cycles . . . , *even when the graph is known not to have negative cycles.*" [Annot. 18 Sept 2018.] (**ori: OG: Cyc**)

1986a On the complexity of circulations. *J. Algorithms* 7 (1986), 134–145. MR 0834086 (88a:68033). Zbl 603.68039.

As in (1985a), finding a feasible circulation is NP-complete for a general bidirected B but is P for all-negative (and all-positive) B. [Annot. 18 Sept 2018.] **(ori: OG: Flows)**

Drew Armstrong

2011a Hyperplane arrangements and diagonal harmonics. In: *23rd International Conference on Formal Power Series and Algebraic Combinatorics* (FPSAC 2011, Reykjavik, 2011), pp. 39–50. Discrete Math. Theor. Comput. Sci. Proc., AO. Assoc. Discrete Math. Theor. Comput. Sci., Nancy, 2011. MR 2820696. Zbl 1355.52014.

Introduces the Ish arrangement; *cf.* Armstrong and Rhoades (2012b). Extended abstract of (2013a). [Annot. 14 Mar 2013.] **(gg: Geom, Invar)**

2013a Hyperplane arrangements and diagonal harmonics. *J. Combin.* 4 (2013), no. 2, 157–190. MR 3096132. Zbl 6222541. arXiv:1005.1949. **(gg: Geom, Invar)**

Drew Armstrong & Brendon Rhoades

2012a The Shi arrangement and the Ish arrangement. In: *23rd International Conference on Formal Power Series and Algebraic Combinatorics* (FPSAC 2011, Reykjavik, 2011), pp. 51–62. Discrete Math. Theor. Comput. Sci. Proc., AO. Assoc. Discrete Math. Theor. Comput. Sci., Nancy, 2011. Zbl 1355.52015.

Extended abstract of (2012b). [Annot. 14 Mar 2013.] **(gg: Geom, Invar)**

2012b The Shi arrangement and the Ish arrangement. *Trans. Amer. Math. Soc.* 364 (2012), no. 3, 1509–1528. MR 2869184. Zbl 1238.05271. arXiv:1009.1655.

The integral gain graphs $0K_n \cup 1\vec{\Gamma}$ (the Shi gain graph, if $\Gamma = K_n$) and $0K_n \cup \{ie_{1j} : i \in [j-1]\}$ (the Ish gain graph, if $\Gamma = K_n$) have the same chromatic polynomial (but not Tutte polynomial) and the same numbers of acyclic orientations with specified properties. [$\vec{\Gamma}$ has $V = [n]$ and edges oriented upwards.] Proofs are [in effect] by counting proper integral colorations modulo a large prime. The results and proofs are cast in terms of the Shi and Ish hyperplane arrangements; *cf.* Athanasiadis (1996a), whose method is used, called the "finite field method". [*Cf.* Leven, Rhoades, and Wilson (2014a).] [Annot. 14 Mar 2013.] **(gg: Geom, Invar)**

S. Arockiaraj

See P. Jeyalakshmi.

A. Aromsawa & J. Poulter

2007a Domain wall entropy of the bimodal two-dimensional Ising spin glass. *Phys. Rev. B* 76 (2007), art. 064427, 5 pp. **(SG: fr, Phys)**

S. Arumugam

See J. Pereira and V. Vasanthi.

Ashraf P K

See also K.A. Germina and N.K. Sudev.

P.K. Ashraf & K.A. Germina

2014a Neighbourhood balanced domination in signed graphs. *Int. J. Math. Sci. Engineering Appl.* 8 (2014), no. 3, 193–203. **(SG: Dom)**

2015a On minimal dominating sets for signed graphs. *Adv. Appl. Discrete Math.* 15 (2015), no. 2, 101–112. MR 3381741. Zbl 1328.05138.

Definition (from B.D. Acharya (2013a)): $D \subseteq V(\Sigma)$ is "dominating" if for some vertex signature μ, $u \notin D$ implies $\exists\ v \in N(u) \cap D$, and $\mu(v) = \sigma(uv)\mu(u), \forall\ v \in N(u) \cap D$. Characterizes minimal such sets D (dominating in $|\Sigma|$ and three side conditions), and more. [Annot. 24 Mar 2017.] **(SG: Dom: VS, Invar)**

2016a Double domination in signed graphs. *Cogent Math.* 3 (2016), art. 1186135, 9 pp. MR 3625344.

Double dominating set D of Σ: double dominating set for $|\Sigma|$ such that the cut $E(D, D^c)$ is balanced. $\gamma_{\times 2}(\Sigma) := \min |D|$ can be 2 or n. **(SG: Dom)**

P.K. Ashraf, K.A. Germina, & N.K. Sudev

2018a A study on set-valuations of signed graphs. *Int. J. Math. Combin.* 2018 (2018), no. 1, 34–40. arXiv:1610.00698.

Set-valuation: $f : V \to \mathcal{P}(X)$ (injective), X a set, $f^{\oplus}(vw) := f(v) \oplus f(w)$; define Σ by $\sigma(vw) := (-1)^{|f(vw|}$. Long proofs, results trivial or false except Thms. 2.9–10: Σ has a set-valuation iff balanced. [Quick proof: switch by $\zeta(v) = (-1)^{|f(v)|}$.] [Annot. 3 Mar 2019.] **(SG: Bal)**

2019a On certain associated graphs of set-valued signed graphs. *Malaya J. Mat.* 7 (2019), no. 1, 113–117. MR 3938245. HAL hal-02263292.

Continues (2018a). Long proofs of trivial balance properties. [Annot. 3 Mar 2019.] **(SG: Bal)**

Ali Reza Ashrafi

See Z. Yarahmadi.

Jalal Askari, Ali Iranmanesh, & Kinkar Ch Das

2016a Seidel-Estrada index. *J. Inequalities Appl.* 2016 (2016), art. 120, 9 pp. MR 3486633. Zbl 1335.05106.

Seidel–Estrada index $:= \sum e^{\lambda}$ over all eigenvalues of $A(K_\Gamma)$. (Estrada index applied to signed complete graphs.) Bounds, and comparison with Seidel energy $\sum |\lambda|$. [Annot. 9 Nov 2020.] **(sg: KG: Adj: Eig)**

Fatihcan M. Atay

2012a On delay-induced stability in diffusively coupled discrete-time systems. *Afrika Mat.* 23 (2012), no. 1, 109–119. MR 2897768. Zbl 1253.93078. **(SG, SD: WG: Dyn)**

Fatihcan M. Atay & Bobo Hua

2016a On the symmetry of the Laplacian spectra of signed graphs. *Linear Algebra Appl.* 495 (2016), 24–37. MR 3462985. Zbl 1331.05134. **(SG: Lap: Eig)**

Fatihcan M. Atay & Shiping Liu

2020a Cheeger constants, structural balance, and spectral clustering analysis for signed graphs. *Discrete Math.* 343 (2020), no. 1, art. 111616, 26 pp. MR 4039398. Zbl

1429.05120. arXiv:1411.3530. **(SG: Bal, Fr: Invar, Eig, Algor)**

Fatihcan M. Atay & Hande Tunçel

2014a On the spectrum of the normalized Laplacian for signed graphs: Interlacing, contraction, and replication. *Linear Algebra Appl.* 442 (2014), 165–177. MR 3134361. Zbl 1282.05088. **(SG: Lap: Eig)**

Christos A. Athanasiadis

† 1996a Characteristic polynomials of subspace arrangements and finite fields. *Adv. Math.* 122 (1996), 193–233. MR 1409420 (97k:52012). Zbl 872.52006.

Treats the canonical lift representations (as affine hyperplane arrangements) of various gain graphs and signed gain graphs with additive gain group $\mathbb{Z}^+$. The article is largely a series of (sometimes brilliant) calculations of chromatic polynomials (*mutatis mutandis*, the characteristic polynomials of the representing arrangements) modulo a large integer q using gain graph coloring, though disguised as applications of Crapo–Rota's Critical Theorem. The fundamental principle is that, if q is larger than the largest gain of a circle, then $\mathbb{Z}^+$ can be replaced as gain group by $\mathbb{Z}_q^+$ without changing the chromatic polynomial (a consequence of Zaslavsky (1995b), Thm. 4.2)—and the analog for signed gain graphs, whose theory needs to be developed. A non-graphical result of the general method is a unified proof (Thm. 2.4) of the theorem of Blass and Sagan (1998a).

§3, "The Shi arrangements": these represent $\mathrm{Lat}^{\mathrm{b}}\{0,1\}\vec{K}_n$ and signed-graph analogs. §4: "The Linial arrangement": it represents $\mathrm{Lat}^{\mathrm{b}}\{1\}\vec{K}_n$. §5, "Other interesting hyperplane arrangements", treats: the arrangement representing $\mathrm{Lat}^{\mathrm{b}}\,AK_n$ where $A = \{-m, \dots, m-1, m\}$ [which is the semilattice of m-composed partitions; see Zaslavsky (2002a), Ex. 10.5, also Edelman–Reiner (1996a)], and several generalizations, including to arbitrary sign-symmetric gain sets L and to Weyl analogs; also, an antibalanced analog of the A_n Shi arrangement (Thm. 5.4); and more. Most impressive result: Thm. 5.2: Let A be a finite set of integers such that $0 \notin A = -A$ and let $A^0 = A \cup \{0\}$. For $\Phi = A^0 K_n$ and large integral λ, $\chi^*_\Phi(\lambda)/\lambda$ is the coefficient of $x^{\lambda-n}$ in $(1-x)^{-1} - f_A(x)/x$ where f_A is the ordinary generating function for A. From this $\chi^*_{AK_n}(\lambda)/\lambda$ is derived.

[The signed affinographic arrangements represent a kind of signed gain graph whose exact nature has not yet been penetrated by gain graph theory.] [Annot. 5 Feb 2024: Update: Solved in Anderson, Su, and Zaslavsky (20xxa).] **(sg, gg: Geom, Matrd, Invar)**

1996b *Algebraic Combinatorics of Graph Spectra, Subspace Arrangements and Tutte Polynomials.* Doctoral dissertation, Massachusetts Institute of Technology, 1996. MR 2716640 (no rev). **(SG: Geom, Invar, Col: Exp)**

1997a A class of labeled posets and the Shi arrangement of hyperplanes. *J. Combin. Theory Ser. A* 80 (1997), 158–162. MR 1472110 (98d:05008). Zbl 970.66662.

The arrangement represents $\mathrm{Lat}^{\mathrm{b}}\{0,1\}\vec{K}_n$.

(gg: Geom, Matrd, Invar)

1998a On free deformations of the braid arrangement. *European J. Combin.* 19 (1998),

7–18. MR 1600259 (99d:52008). Zbl 898.52008.

The arrangements considered are the subarrangements of the projectivized Shi arrangements of type A_{n-1} that contain A_{n-1}. Thms. 4.1 and 4.2 characterize those that are free or supersolvable. The extended Shi arrangements, representing $\mathbf{L}_\infty([1-a,a]\vec{K}_n)$ where $a \geq 1$, and a mild generalization, are of use in the proof. **(gg: Geom, Matrd, Invar)**

1998b On noncrossing and nonnesting partitions for classical reflection groups. *Electronic J. Combin.* 5 (1998), Research Paper R42, 16 pp. MR 1644234 (99i:05204). Zbl 898.05004.

§5, "Nonnesting partitions of fixed type", has calculations like those in (1996a) for affinographic arrangements representing additional types of integral gain graph [of a kind that is not yet fully understood]. **(gg: Geom, matrd, Invar)**

1999a Extended Linial hyperplane arrangements for root systems and a conjecture of Postnikov and Stanley. *J. Algebraic Combin.* 10 (1999), 207–225. MR 1723184 (2000i:52039). Zbl 0948.52012. arXiv:math/9705223.

Brief description as at (1998b). Solves a conjecture of Postnikov and Stanley (2000a). [Annot. 27 May 2018.] **(gg: Geom, Invar)**

1999b Piles of cubes, monotone path polytopes, and hyperplane arrangements. *Discrete Comput. Geom.* 21 (1999), no. 1, 117–130. MR 1661295 (99j:52015). Zbl 979.52002.

The proof of Proposition 4.2 is essentially gain-graphic. **(gg: matrd: Geom: Invar)**

2000a Deformations of Coxeter hyperplane arrangements and their characteristic polynomials. In: Michael Falk and Hiroaki Terao, eds., *Arrangements—Tokyo, 1998*, pp. 1–26. Adv. Studies Pure Math., 27. Kinokuniya, for the Mathematical Soc. of Japan, Tokyo, 2000. MR 1796891 (2001i:52035). Zbl 976.32016. **(gg: Geom, matrd, Invar)**

2004a Generalized Catalan numbers, Weyl groups and arrangements of hyperplanes. *Bull. London Math. Soc.* 36 (2004), 294–302. MR 2038717 (2005b:52055). Zbl 1068.20038. **(gg: Geom: Gen: Invar)**

2004b On a refinement of the generalized Catalan numbers for Weyl groups. *Trans. Amer. Math. Soc.* 357 (2004), no. 1, 179–196. MR 2098091 (2005h:20091). Zbl 1079.20057. **(gg: Geom: Gen: Invar)**

Christos A. Athanasiadis & Svante Linusson

1999a A simple bijection for the regions of the Shi arrangement of hyperplanes. *Discrete Math.* 204 (1999), 27–39. MR 1691861 (2000f:52031). Zbl 959.52019. **(gg: Geom)**

David Avis

See J. Akiyama.

Remi C. Avohou, Joseph Ben Geloun, & Etera R. Livine

2014a On terminal forms for topological polynomials for ribbon graphs: The N-petal flower. *European J. Combin.* 36 (2014), 348–366. MR 3131901. Zbl 1284.05067.

arXiv:1212.5961. **(sg: Top: Invar)**

F. Ayoobi, G.R. Omidi, & B. Tayfeh-Rezaie

2011a A note on graphs whose signless Laplacian has three distinct eigenvalues. *Linear Multilinear Algebra* 59 (2011), no. 6, 701–706. MR 2801363 (2012i:05158). Zbl 1223.05169. **(par: Lap: Eig)**

Ghodratollah Azadi

See V. Ghorbani.

Habib Azanchiler

See V. Ghorbani.

L. Babai & P.J. Cameron

2000a Automorphisms and enumeration of switching classes of tournaments. *Electronic J. Combin.* 7 (2000), Research Paper R38, 25 pp. MR 1773295 (2001h:-05048). Zbl 956.05050.

Tournaments are treated as nowhere-zero GF$(3)^+$-gain graphs based on K_n; "switching" is by negation in GF$(3)^+$. [Cheng–Wells (1986a) treats all digraphs as K_n with GF$(3)^+$-gains. GF$(3)^+$-gain switching differs from Babai–Cameron's switching.] [Annot. rev. 7 Jan 2016, 4 Nov 2017.] **(gg: KG: Sw, Aut, Enum)**

Maxim A. Babenko

2006a Acyclic bidirected and skew-symmetric graphs: algorithms and structure. In: Dima Grigoriev, John Harrison and Edward A. Hirsch, eds., *Computer Science—Theory and Applications* (Proc. 1st Int. Symp. Computer Sci. in Russia, CSR 2006, St. Petersburg, 2006), pp. 23–34. Lect. Notes in Computer Sci., Vol. 3967. Springer, Berlin, 2006. MR 2260979 (2007f:05165). Zbl 1185.05133.

"Skew-symmetric graph" = double covering digraph of a bidirected $-\Gamma$. "Weak acyclicity": No positive dicycle. "Strong acyclicity": No positive closed diwalk. Algorithm to test for weak acyclicity. Construction of weakly acyclic graphs from strongly acyclic ones. [Annot. 9 Sept 2010.] **(sg: Ori: Str, Cov, Algor)**

2006b On flows in simple bidirected and skew-symmetric networks. (In Russian.) *Problemy Peredachi Informatsii* 42 (2006), no. 4, 104–120. English trans. *Probl. Inf. Transm.* 42 (2006), no. 4, 356–370. MR 2278815 (2008i:90013).

$O(mn^{2/3})$ algorithm for integral max flow, improving on Gabow (1983a), showing that max flow takes no longer on a bidirected graph than on a digraph. The time bound follows from an upper bound on the max flow value. Also, an acyclic flow of value v is zero on all but $O(nv^{1/2})$ arcs. The technique involves transferring the flow to the double covering digraph. [Annot. 9 Sept 2010.] **(sg: Ori: Flows, Algor, Cov)**

2007a On an application of the structural theory of acyclic skew-symmetric digraphs. (In Russian.) *Vestnik Moskov. Univ. Ser. I Mat. Mekh.* (2007), no. 2, 65–66, 80. English trans. *Moscow Univ. Math. Bull.* 62 (2007), no. 2, 85–86. MR 2357046 (2008i:05146). Zbl 1164.05056.

The double covering graph of suitably oriented $-\Gamma$ [matching edges are introverted; nonmatching edges are extraverted] yields a proof that, if Γ has a unique perfect matching M, then M contains an isthmus. [Annot.

9 Sept 2010.] (**par: Ori**)

Maxim A. Babenko & Alexander V. Karzanov

2007a Free multiflows in bidirected and skew-symmetric graphs. *Discrete Appl. Math.* 155 (2007), 1715–1730. MR 2348356 (2008j:90102). Zbl 1152.90574.

Optimization of integral odd-vertex flows on a bidirected graph, without or with capacities. [Annot. 9 Sept 2010.] (**sg: Ori: Flows: Algor**)

2009a Minimum mean cycle problem in bidirected and skew-symmetric graphs. *Discrete Optimization* 6 (2009), no. 1, 92–97. MR 2483322 (2010a:05105). Zbl 1161.05327.

Minimizing the average weight in a cycle, or a closed trail, of an edge-weighted bidirected graph, in time $O(n^2 \min\{n^2, m \log n\})$. [Annot. 9 Sept 2010.] (**sg: Ori: Algor**)

Jayapal Baskar Babujee & Shobana Loganathan

2011a On signed product cordial labeling. *Appl. Math. (Irvine)* 2 (2011), 1525–1530.

Does Γ admit a balanced edge signature $\sigma = (+)^\zeta$ (ζ = switching function), such that $\#\zeta^{-1}(+) \approx \#\zeta^{-1}(-)$ and $\#E^+ \approx \#E^-$? ($\approx$ means difference ≤ 1.) If so, ζ is a "signed product cordial labeling". Some constructions. [Equivalently, as it is an unsigned graph property: Does Γ have a cut $D = E(X, V \setminus X)$ such that $\#X \approx \frac{1}{2}\#V$ and $\#D \approx \frac{1}{2}\#E$?] [Successors: Santhi and Albert (2015a), Rozario Raj and Manoharan (2016a), Shobana and Vasuki (2017a), *et al.*] [Annot. 27 Jan 2013, 11 Mar 2017, rev 27 Dec 2020.]

[Delphy and Devaraj (2011a) define "signed cordiality" based on edge signs.] [Annot. rev 26 Sept 2022.] (**Lab: VS: SG, Bal**)

Goksen Bacak-Turan

See R. Sundareswaran.

Constantin P. Bachas

1984a Computer-intractibility of the frustration model of a spin glass. *J. Phys. A* 17 (1984), L709–L712. MR 0763604 (85j:82043).

The frustration index decision problem on signed (3-dimensional) cubic lattice graphs is NP-complete. [Proof is incomplete; completed and improved by Green (1987a). Better result in Barahona (1982a).] (**SG: Fr: Algor**)

F. Bachmann

See B. Fierro.

Mathias Hudoba de Badyn, Siavash Alemzadeh, & Mehran Mesbahi

2017a Controllability and data-driven identification of bipartite consensus on nonlinear signed networks. In: *2017 IEEE 56th Annual Conference on Decision and Control (CDC)* (CDC2017, Melbourne, 2017), pp. 3557–3562. IEEE, 2017. arXiv:1709.06679. (**SG**)

Y. Bagheri, A.R. Moghaddamfar, & F. Ramezani

2020a The number of switching isomorphism classes of signed graphs associated with particular graphs. *Discrete Math.* 279 (2020), 25–33. MR 4092631. Zbl 1439.05100. arXiv:1909.06819.

The number of switching isomorphism classes of signed Γ. Thm. 3.2 on generalized Petersens: For $\Gamma = \mathrm{GP}(n,k)$ with prime $n > 5$ and $k \not\equiv \pm 1 \pmod{n}$, either $\mathrm{Aut}[\Gamma,\sigma] = \mathrm{Aut}\,\mathrm{GP}(n,k)$ or $|\mathrm{Aut}[\Gamma,\sigma]| \leq 2$. [An interesting dichotomy.] All 36 classes are shown for GP(7, 2). [Annot. 29 Jul 2019, rev 27 Jul 2022.] (**SG: Sw, Aut**)

Chun-Hsiang Bai & Bang Ye Wu

2012a Finding the maximum balanced vertex set on complete graphs. In: *Proceedings of the 29th Workshop on Combinatorial Mathematics and Computation Theory* (Taipei, 2012), pp. 42–51. National Taipei College of Business, Taipei, Taiwan, 2012. URL http://par.cse.nsysu.edu.tw/~algo/paper/paper_list12.htm

Algorithm for frustration number $l_0(K_n,\sigma)$. [Annot. 5 Jun 2017.] (**SG: KG: Fr: Algor**)

G. David Bailey

20xxa Inductively factored signed-graphic arrangements of hyperplanes. Manuscript, ca. 1998?

Continues Edelman–Reiner (1994a). (**SG: Geom, Matrd**)

Keith Baker

See J.O. Morrissette.

V. Balachandran

1976a An integer generalized transportation model for optimal job assignment in computer networks. *Operations Res.* 24 (1976), 742–759. MR 0439170 (55 #12068). Zbl 356.90028. (**GN: Matrd(bases)**)

V. Balachandran & G.L. Thompson

1975a An operator theory of parametric programming for the generalized transportation problem: I. Basic theory. II. Rim, cost and bound operators. III. Weight operators. IV. Global operators. *Naval Res. Logistics Quart.* 22 (1975), 79–100, 101–125, 297–315, 317–339. MR 0381706–MR 0381709 (52 #2595–2598). Zbl 331.90048–Zbl 331.90051. (**GN: Matrd**)

R. Balakrishnan & K. Ranganathan

2000a *A Textbook of Graph Theory.* Springer, New York, 2000. MR 1729781 (2000j:05001). Zbl 938.05001.

§10.6, "Application to social psychology": Short introduction to balance in signed graphs. §10.7: Exercises on balance. (**SG: Bal: Exp**)

2012a *A Textbook of Graph Theory.* Second ed. Springer, New York, 2012. MR 2977757. Zbl 1254.05001.

§1.11, "Application to social psychology": Short introduction to balance in signed graphs. (**SG: Bal: Exp**)

R. Balakrishnan & N. Sudharsanam

1982a Cycle-vanishing edge valuations of a graph. *Indian J. Pure Appl. Math.* 13 (1982), no. 3, 313–316. MR 0657670 (84d:05145). Zbl 485.05057.

$f : E(\Gamma) \to \mathbb{R}$ is "cycle-vanishing" if $f(C) := \sum_{e\in C} f(e) = 0$ for every circle. Thm. 3: f is cycle-vanishing iff $f(S) = 0$ for every series class of non-isthmus edges. Thm. 4: $\dim\{\text{cycle-vanishing } f\} = \#E -$ number of series classes of non-isthmus edges. Thm. 5: Connected Γ is 3-connected iff only $f = 0$ is cycle vanishing. [Specialized to a sign-weighted graph Σ, "cycle-vanishing" means $\#E^+(C) = \#E^-(C)$ for every circle. Thm. 3: σ is cycle-vanishing iff every series class of non-isthmus edges has evenly many edges, half positive and half negative. Etc. [*Cf.* B.G. Xu (2009a), Vijayakumar (2011a) for generalization.] [Annot. 16 Oct 2011.]

(**sgw: Gen**)

P. Balamuralidhar

See also H.K. Rath.

P. Balamuralidhar & M.A. Rajan

2011a Signed graph based approach for on-line optimization in cognitive networks. In: *2011 Third International Conference on Communication Systems and Networks* (COMSNETS, Bangalore, 2011). IEEE, 2011. (**SD: Algor, Appl**)

Egon Balas

1966a The dual method for the generalized transportation problem. *Management Sci.* 12 (1966), no. 7 (March, 1966), 555–568. MR 0189812 (32 #7232). Zbl 142.16601 (142, p. 166a). (**GN: Matrd(bases)**)

1981a Integer and fractional matchings. In: P. Hansen, ed., *Studies on Graphs and Discrete Programming*, pp. 1–13. North-Holland Math. Stud., 59. Ann. Discrete Math., 11. North-Holland, Amsterdam, 1981. MR 0653814 (84h:90084).

Linear (thus "fractional", meaning half-integral) vs. integral programming solutions to maximum matching. The difference of their maxima $= \frac{1}{2}$(max number of matching-separable vertex-disjoint odd circles). Also noted (p. 12): (max) fractional matchings in Γ correspond to (max) matchings in the double covering graph of $-\Gamma$. [*Question.* Does this lead to a definition of maximum matchings in signed graphs?]

(**par, ori: Incid, Geom, Algor, cov, Circ**)

E. Balas & P.L. Ivanescu [P.L. Hammer]

1965a On the generalized transportation problem. *Management Sci.* 11 (1965), no. 1 (Sept., 1964), 188–202. MR 0174395 (30 #4599). Zbl 133.42505 (133, p. 425e). (**GN: Matrd, Bal**)

A.R. Balasubramanian

20xxa Generalized threshold arrangements. Submitted. arXiv:1904.08903.

The arrangements have hyperplanes of the form $x_i + x_j = -l, -l+1, \ldots, m-1, m$ (integers) for $i < j$. [Annot. 28 Jan 2020.]

(**gg: Geom, Invar**)

K. Balasubramanian

1988a Computer generation of characteristic polynomials of edge-weighted graphs, heterographs, and directed graphs. *J. Computational Chem.* 9 (1988), 204–211.

Here a "signed graph" means, in effect, an acyclically oriented graph D along with the antisymmetric adjacency matrix $A_\pm(D) = A(+D \cup -D^{-1})$, D^{-1} being the converse digraph. [That is, $A_\pm(D) = A(D) - A(D)^{\mathrm{T}}$. The "signed graphs" are just acyclic digraphs with an antisymmetric adjacency matrix and, correspondingly, what we may call the 'antisymmetric characteristic polynomial'.] Proposes an algorithm for the polynomial. Observes in some examples a relationship between the characteristic polynomial of Γ and the antisymmetric characteristic polynomial of an acyclic orientation.

(**SD, wg: Eig: Invar: Algor, Chem**)

1991a Comments on the characteristic polynomial of a graph. *J. Computational Chem.* 12 (1991), 248–253. MR 1093297 (92b:92057).

Argues (heuristically) that a certain algorithm is superior to another, in particular for the antisymmetric polynomial defined in (1988a).

(**SD: Eig: Invar: Algor**)

1992a Characteristic polynomials of fullerene cages. *Chem. Phys. Lett.* 198 (1992), 577–586.

Computed for graphs of six different cages of three different orders, in both ordinary and "signed" (see (1988a)) versions. Observes a property of the "signed graph" polynomials [which is due to antisymmetry, as explained by P.W. Fowler (Comment on "Characteristic polynomials of fullerene cages". *Chem. Phys. Lett.* 203 (1993), 611–612)].

(**SD: Eig: Invar: Chem**)

1994a Are there signed cospectral graphs? *J. Chem. Information Computer Sci.* 34 (1994), 1103–1104.

The "signed graphs" are as in (1988a). Simplified contents: It is shown by example that the antisymmetric characteristic polynomials of two nonisomorphic acyclic orientations of a graph (see (1988a)) may be equal or unequal. [Much smaller examples are provided by P.W. Fowler, Comment on "Characteristic polynomials of fullerene cages". *Chem. Phys. Lett.* 203 (1993), 611–612).] [*Question.* Are there examples for which the underlying (di)graphs are nonisomorphic?] [For cospectrality of other kinds of signed graphs, see Acharya, Gill, and Patwardhan (1984a) (signed K_n's).] (**SD: Eig: Invar**)

R. Balian, J.M. Drouffe, & C. Itzykson

1974a Gauge fields on a lattice. I. General model. *Phys. Rev. D* 10 (1974), no. 10, 3376–3395.

Gain group SO(n) on a toroidal lattice graph (§ C, "Local invariance, gauge field, and minimal coupling), where SO(1) $= \{+1, -1\}$ (developed in (1975a)). [Annot. 12 Aug 2012.] (**SG: Phys**)

1975a Gauge fields on a lattice. II. Gauge-invariant Ising model. *Phys. Rev. D* 11 (1975), no. 8, 2098–2103.

Dictionary: "Ising model" = signed hypercubical lattice, "gauge invariance" = switching invariance, "plaquette" = quadrilateral. The partition function depends on $p^+ - p^-$ where $p^\varepsilon = \#$ plaquettes with sign ε and sometimes also $\#E^+ - \#E^-$. [Annot. 12 Aug 2012.]

(**SG: Phys, Sw, Fr**)

M.L. Balinski

1970a On recent developments in integer programming. *Proceedings of the Princeton Symposium on Mathematical Programming* (Princeton Univ., 1967), pp. 267–302. Princeton Univ. Press, Princeton, N.J., 1970. MR 0437023 (55 #9957). Zbl 222.90036.

Pp. 277–278 discuss integer programming problems on bidirected graphs in terms of the incidence matrix. **(ori: incid: par, Algor, Ref)**

Igor Balla, Felix Dräxler, Peter Keevash, & Benny Sudakov

2018a Equiangular lines and spherical codes in Euclidean space. *Invent. math.* 211 (2018), 179–212. MR 3742757. Zbl 1383.51035. arXiv:1606.06620. **(SG: KG: Adj: Eig)**

Núria Ballber Torres & Claudio Altafini

2016a Drug combinatorics and side effect estimation on the signed human drug-target network. *BMC Systems Biol.* 10 (2016), art. 74, 12 pp. **(SG: Cyc, bal)**

Murad Banaji

See also D. Angeli and N. Radde.

2010a Graph-theoretic conditions for injectivity of functions on rectangular domains. *J. Math. Anal. Appl.* 370 (2010), 302–311. MR 2651147 (2011f:26012). Zbl 1227.26006. **(SD)**

Murad Banaji & Gheorghe Craciun

2009a Graph-theoretic approaches to injectivity and multiple equilibria in systems of interacting elements. *Commun. Math. Sci.* 7 (2009), no. 4, 867–900. MR 2604624 (2011i:05126). Zbl 1195.05038. arXiv:0903.1190. **(SG, Chem)**

2010a Graph-theoretic criteria for injectivity and unique equilibria in general chemical reaction systems. *Adv. Appl. Math.* 44 (2010), 168–184. MR 2576846 (2010m:80010). Zbl 1228.05204.

Generalization of (2009a) to more general systems. [Annot. 26 Oct 2011.] **(SG, Chem)**

Murad Banaji & Carrie Rutherford

2011a P-matrices and signed digraphs. *Discrete Math.* 311 (2011), no. 4, 295–301. MR 3537201. Zbl 1222.05080. arXiv:1006.0152. **(SD: QM)**

Afonso S. Bandeira, Amit Singer, & Daniel A. Spielman

2013a A Cheeger inequality for the graph connection Laplacian. *SIAM J. Matrix Anal. Appl.* 34 (2013), no. 4, 1611–1630. MR 3138103. Zbl 1287.05081. arXiv:1204.3873.

Connection graph: A real-weighted $O(\mathbb{R}, d)$-gain graph. **(GG, WG: Lap)**

Debanjan Banerjee & Anita Pal

2015a Application of signed graph in decision making. *Int. J. Computers Technol.* 14 (2015), no. 6, 5825–5833.

"Character points" := sum of edge signs, at a vertex or in Σ. Positive = "good". Tenuous applications. [Annot. 11 Aug 2021.] **(SG, SD, VS: KG, Appl)**

Subhashish Banerjee

See B. Adhikari.

Jørgen Bang-Jensen, Stéphane Bessy, Bill Jackson, & Matthias Kriesell

2017a Antistrong digraphs. *J. Combin. Theory Ser. B* 122 (2017), 68–90. MR 3575196. Zbl 1350.05048.

Antidirected trails, i.e., coherent in poise gains. Dictionary: "even bicircular matroid" = even-cycle matroid $\mathbf{F}(-\Gamma)$ (*cf.* Doob (1973a)).

[*Cf.* antidirection matroid in Matthews (1978c). *Question.* Does this generalize to bidirected graphs?] [Annot. 10 Nov 2017, 30 May 2018.] **(gg: Str, Matrd)(sg: par: Ori)**

Jørgen Bang-Jensen & Gregory Gutin

1997a Alternating cycles and paths in edge-coloured multigraphs: A survey. *Discrete Math.* 165/166 (1997), 39–60. MR 1439259 (98d:05080). Zbl 876.05057.

A rich source for problems on bidirected graphs. An edge 2-coloration of a graph becomes an all-negative bidirection by taking one color class to consist of introverted edges and the other to consist of extroverted edges. An alternating path becomes a coherent path; an alternating circle becomes a coherent circle. [*General Problem.* Generalize to bidirected graphs the results on edge 2-colored graphs mentioned in this paper. (See esp. §5.) *Question.* To what digraph properties do they specialize by taking the underlying signed graph to be all positive?] [See e.g. Bánkfalvi and Bánkfalvi (1968a) (*q.v.*), Bang-Jensen and Gutin (1998a), Das and Rao (1983a), Grossman and Häggqvist (1983a), Mahadev and Peled (1995a), Saad (1996a).] **(par: ori: Paths, Circ)**

1998a Alternating cycles and trails in 2-edge-colored complete multigraphs. *Discrete Math.* 188 (1998), 61–72. MR 1630418 (99g:05072). Zbl 956.05040.

The longest coherent trail, having degrees bounded by a specified degree vector, in a bidirected all-negative complete multigraph that satisfies an extra hypothesis. Generalization of Das and Rao (1983a) and Saad (1996a), thus ultimately of Thm. 1 of Bánkfalvi and Bánkfalvi (1968a) (*q.v.*). Also, a polynomial-time algorithm. **(par: ori: Paths, Algor)**

M. Bánkfalvi & Zs. Bánkfalvi

1968a Alternating Hamiltonian circuit in two-coloured complete graphs. In: P. Erdős and G. Katona, eds., *Theory of Graphs* (Proc. Colloq., Tihany, 1966), pp. 11–18. Academic Press, New York, 1968. MR 0233731 (38 #2052). Zbl 159.54202.

Let B be a bidirected $-K_{2n}$ which has a coherent 2-factor. ("Coherent" means that, at each vertex in the 2-factor, one edge is directed inward and the other outward.) Thm. 1: B has a coherent Hamiltonian circle iff, for every $k \in \{2, 3, \ldots, n-2\}$, $s_k > k^2$, where $s_k :=$ the sum of the k smallest indegrees and the k smallest outdegrees. Thm. 2: The number of k's for which $s_k = k^2$ equals the smallest number p of circles in any coherent 2-factor of B. Moreover, the p values of k for which equality holds imply a partition of V into p vertex sets, each inducing B_i consisting of a bipartite [i.e., balanced] subgraph with a coherent Hamiltonian circle and in one color class only introverted edges, while in the other only extroverted edges. [*Problem.* Generalize these remarkable results to an arbitrary bidirected complete graph. The all-negative case will be these theorems; the all-positive case will give the smallest number of cycles in a covering by vertex-disjoint cycles of a tournament that has any such covering.] [See Bang-Jensen and Gutin (1997a) for further developments on alternating walks; also Busch, Jacobson, *et al.* (2013a), Busch, Mutar, and Slilaty (2022a).] **(par: ori: Circ)**

Zs. Bánkfalvi

See M. Bánkfalvi.

C. Bankwitz

1930a Über die Torsionszahlen der alternierenden Knotes. *Math. Ann.* 103 (1930), 145–161.

Introduces the sign-colored graph of a link diagram. [Further work by numerous writers, e.g., S. Kinoshita *et al.* and esp. Kauffman (1989a) and successors.] (**Knot: SGc**)

Nikhil Bansal, Avrim Blum, & Shuchi Chawla

2002a Correlation clustering. In: *Proc. 43rd Ann. IEEE Sympos. Foundations of Computer Science (FOCS '02)*, pp. 238–247. Zbl 1089.68085.

Preliminary version of (2004a). (**SG: KG: Clu: Algor**)

2004a Correlation clustering. Theoretical Advances in Data Clustering. *Machine Learning* 56 (2004), no. 1-3, 89–113. Zbl 1089.68085.

Clusterability index Q [minimum number of inconsistent edges; see Doreian and Mrvar (1996a) for notation] in signed complete graphs is NP-hard. Polynomial-time algorithms for approximate optimal clustering: up to a constant factor from Q (§3); probably within $1-\varepsilon$ of $\#E-Q$ for any ε (i.e., maximizing consistent edges within $1-\varepsilon$) (§4). §3: A 2-clustering within $3Q_2$ (Thm. 2). A clustering within cQ where $c \approx 20000$ (Thm. 13). §4: A clustering within εn^2 of $\#E-Q$ with high probability but slow in terms of $1/\varepsilon$ (Thm. 15). Asymptotically faster in terms of $1/\varepsilon$ (Thm. 22). The $1-\varepsilon$ factor results from the fact that $\#E-Q = \binom{n}{2} - Q > \frac{1}{2}\binom{n}{2}$ [so is not strong]. §6: "Random noise". §7: "Extensions", considers edge weights in $[-1,1]$ (thus allowing incomplete graphs). Thm. 23: An unweighted approximation algorithm will also approximate this case, assuming "linear cost": e costs $(1-w(e))/2$ if within a cluster and $(1+w(e))/2$ if between clusters. Thm. 24: The problem for clustering that minimizes the total weight of $+$ edges outside clusters and $-$ edges within clusters ("minimizing disagreements") is APX-hard. [Improved in Charikar–Guruswami–Wirth (2003a), (2005a), Swamy (2004a). Generalized in Demaine *et al.* (2006a).] [Annot. 22 Sept 2009.] (**SG: KG: Clu: Algor**)

Bo Bao, Rong Chen, & Genghua Fan

2021a Circuit covers of signed Eulerian graphs. *Electronic J. Combin.* 28 (2021), no. 1, art. P1.14, 12 pp. MR 4245247. Zbl 1456.05095. arXiv:1910.09999.

Assuming $\mathbf{F}(\Sigma)$ has no coloop, E can be covered by frame circuits so each edge is in exactly 6 circuits. [Annot. 28 Jan 2021.] (**SG: flows**)

R.B. Bapat

See also R. Singh.

2010a *Graphs and Matrices*. Hindustan Book Agency, New Delhi, and Springer, London, 2010. MR 2797201 (2012f:05001). Zbl 1248.05002.

§2.6, "$0-1$ Incidence matrix": The rank and related properties of the the unoriented incidence matrix. [*Cf.* van Nuffelen (1973a).] [Annot. 25 Aug 2011.] (**Par: Incid: Exp**)

Ravindra B. Bapat, Jerrold W. Grossman, & Devadatta M. Kulkarni

1999a Generalized matrix tree theorem for mixed graphs. *Linear Multilinear Algebra* 46 (1999), 299–312. MR 1729196 (2001c:05091). Zbl 940.05042.

Their “mixed graph” is a signed graph Σ: positive edges are called “directed” and negative edges “undirected”. The matrix-tree theorem is the unweighted case of Chaiken’s (1982a) all-minors theorem for signed graphs. The technical formalism differs somewhat. They point out that in case $U \cup W = V$, the minor is the sum of signed $\bar{U}\bar{W}$ matchings. Dictionary: “k-reduced substructure” $\cong$ independent set of rank $n-k$ in $\mathbf{F}(\Sigma)$; “quasibipartite” = balanced. Successor to Grossman, Kulkarni, and Schochetman (1994a) [*q.v.* for more dictionary]. **(sg: Incid)**

2000a Edge version of the matrix tree theorem for trees. *Linear Multilinear Algebra* 47 (2000), 217–229. MR 1785029 (2001d:05112). Zbl 960.05067.

Successor to (1999a). Their “mixed tree” T is a signed tree as in (1999a). Thm. 9 (simplified): The minor of $\mathrm{H}^{\mathrm{T}}\mathrm{H}$ (H is the incidence matrix of Σ) obtained by deleting rows corresponding to $E \subseteq E(\Sigma)$ and columns corresponding to $F \subseteq E(\Sigma)$ has determinant equal, up to sign, to the number of common SDR’s of vertex sets of components of $T \setminus E$ and $T \setminus F$. [Interesting, but edge signs are irrelevant because any tree switches to all positive.] Dictionary: “substructure” = subgraph allowing retention of edges incident to deleted vertices [thus they become loose or half edges]. [See (1999a) for more dictionary.] **(sg: Incid)**

R.B. Bapat & E. Ghorbani

2014a Inverses of triangular matrices and bipartite graphs. *Linear Algebra Appl.* 447 (2014), 68–73. MR 3200207. Zbl 1288.05152. arXiv:1303.2177.

Generalizes to rings the inverses of $A(T)$ for a tree; *cf.* Godsil (1985a). [Annot. 7 Aug 2019.] **(sg(Gen): Adj)**

R.B. Bapat, D. Kalita, & S. Pati

2012a On weighted directed graphs. *Linear Algebra Appl.* 436 (2012), no. 1, 99–111. MR 2859913. Zbl 1229.05198.

They are complex unit gain graphs Φ with simple underlying graph. Laplacian $L(\Phi)$ is obtained in the usual way from $\mathrm{H}(\Phi)$. §2, “D-similarity and singularity in weighted directed graphs”: Thm. 8: $L(\Phi)$ is singular iff $\Phi \sim \|\Phi\|$ iff Φ (assumed connected) is balanced. [*Cf.* Zaslavsky (2003b), §2.1 esp. Thm. 2.1(a), noting that $\operatorname{rk} L(\Phi) = \operatorname{rk} \mathrm{H}(\Phi) = \operatorname{rk} \mathbf{F}(\Phi)$.] §3, “Edge singularity of weighted directed graphs”: Elementary results on frustration index, appearing less elementary because treated indirectly, through eigenvalues, rather than directly, through the graph. Generalizing Y.Y. Tan and Fan (2008a) on signed graphs. §4, “3-Colored digraphs and their singularity”: Gains restricted to $\pm 1, \mathrm{i}$. Elementary results.

Dictionary: “weighted directed graph” = complex unit gain graph (*cf.* Reff (2012a)); “mixed graph” = signed graph; D-similarity” [diagonal similarity] = switching equivalence, “edge singularity” = frustration index. [Annot. 28 Oct 2011.] **(gg: Eig, Incid, Bal)**

R.B. Bapat & Devadatta M. Kulkarni

2000a Minors of some matrices associated with a tree. In: *Algebra and Its Applications* (Athens, Ohio, 1999), pp. 45–66. Contemp. Math., Vol. 259. American Math. Soc., Providence, R.I., 2000. MR 1778494 (2001h:05065). Zbl 979.05075.

Concerns a “mixed tree”, really an oriented signed tree without extroverted edges (see Bapat, Grossman, and Kulkarni (1999a)). The matrices are the incidence matrix H, the Laplacian matrix $\mathrm{H}\,\mathrm{H}^{\mathrm{T}}$, and the "edge Laplacian" $\mathrm{H}^{\mathrm{T}}\mathrm{H}$. Partly expository. New results concern Moore–Penrose inverses and their minor determinants. [Since a "mixed tree" is switching equivalent to an ordinary unsigned tree, their results should be identical to those for ordinary trees except for multiplication by a $V \times V$ diagonal matrix with signs on the diagonal.] **(sg: Incid)**

Nadav S. Bar

See N. Radde.

Francisco Barahona

1981a Balancing signed toroidal graphs in polynomial-time. Unpublished manuscript, 1981.

Given a 2-connected Σ whose underlying graph is toroidal, polynomial-time algorithms are given for calculating the frustration index $l(\Sigma)$ and the generating function of switchings Σ^μ by $\#E^-(\Sigma^\mu)$. The technique is to solve a Chinese postman (T-join) problem in the toroidal dual graph, T corresponding to the frustrated face boundaries. Generalizes (1982a). [See (1990a), p. 4, for a partial description.] **(SG: Fr, Algor)**

1982a On the computational complexity of Ising spin glass models. *J. Phys. A: Math. Gen.* 15 (1982), 3241–3253. MR 0684591 (84c:82022).

The frustration-index problem, that is, minimization of $\#E^-(\Sigma^\zeta)$ over all switching functions $\zeta : V \to \{\pm 1\}$, for signed planar and toroidal graphs and subgraphs of 3-dimensional grids. Analyzed structurally, in terms of perfect matchings in a modified dual graph, and algorithmically. 3-dimensional is NP-hard, even when the grid has only 2 levels; the former are polynomial-time solvable even with weighted edges.

Also, the problem of minimizing $\#E^-(\Sigma^\zeta) + \sum_v \zeta(v)$ for planar grids ("2-dimensional problem with external magnetic field"), which is NP-hard. [This corresponds to adding an extra vertex, positively adjacent to every vertex.] [See infinite analog in Istrail (2000a).]
(SG: Phys, Fr, Fr(Gen): Du, Algor)

1982b Two theorems in planar graphs. Unpublished manuscript, 1982. **(SG: Fr)**

1983a The max-cut problem on graphs not contractible to K_5. *Operations Res. Lett.* 2 (1983), no. 3, 107–111. MR 0717742 (84k:90048). Zbl 0525.90094.

Thm. 3.2: The real-weighted maximum cut problem is polynomial-time solvable for graphs not contractible to K_5. [Frustration index $l(-\Sigma)$ is the special case of weight$(e) = -\sigma(e) = \pm 1$.] [Annot. 19 Dec 2014.]
(sg, WG: fr, Algor)

1983b On some weakly bipartite graphs. *Operations Res. Lett.* 2 (1983), no. 5, 239–242. MR 0733782 (85a:05072). Zbl 0549.90087.

If the frustration number $l_0(-\Gamma) \leq 2$ (i.e., $-\Gamma \setminus \{u, v\}$ is balanced for some $u, v \in V$), then Γ is weakly bipartite (*cf.* Guenin (2001a)) and $l(-\Gamma)$ is polynomial-time computable. [*Problem.* Characterize Σ with $l_0 \leq 2$. For $l_0 \leq 1$ see Zaslavsky (1987c) with $\Omega = \Sigma$.] [Annot. 19 Dec 2014.] **(Par, SG: fr, Sw)**

1990a On some applications of the Chinese Postman Problem. In: B. Korte, L. Lovász, H.J. Prömel, and A. Schrijver, eds., *Paths, Flows and VLSI-Layout*, pp. 1–16.

Algorithms and Combin., Vol. 9. Springer-Verlag, Berlin, 1990. MR 1083374 (92b:90139). Zbl 732.90086.

§2: "Spin glasses." **(SG: Phys, Fr: Exp)**

§5: "Max cut in graphs not contractible to K_5," pp. 12–13. **(sg: fr: Exp)**

1990b Planar multicommodity flows, max cut, and the Chinese Postman Problem. In: William Cook and Paul D. Seymour, eds., *Polyhedral Combinatorics* (Proc. Workshop, Morristown, N.J., 1989), pp. 189–202. DIMACS Ser. Discrete Math. Theor. Computer Sci., Vol. 1. Amer. Math. Soc. and Assoc. Comput. Mach., Providence, R.I., 1990. MR 1105127 (92g:05165). Zbl 747.05067.

Negative cutsets, where signs come from a network with real-valued capacities. Dual in the plane to negative circles. See §2. **(SG: Du: Bal, Algor)**

Francisco Barahona & Adolfo Casari

1988a On the magnetisation of the ground states in two-dimensional Ising spin glasses. *Comput. Phys. Commun.* 49 (1988), 417–421. MR 0945813 (89d:82004). Zbl 814.90132. **(SG: State(fr): Algor)**

Francisco Barahona & Michele Conforti

1987a A construction for binary matroids. *Discrete Math.* 66 (1987), 213–218. MR 0900044 (88g:05039). Zbl 0644.05017. **(SG: Matrd)**

Francisco Barahona, Martin Grötschel, Michael Jünger, & Gerhard Reinelt

1988a An application of combinatorial optimization to statistical physics and circuit layout design. *Oper. Res.* 36 (1988), no. 3, 493–513. Zbl 646.90084.

Frustration index of a weighted signed graph (Ising ground state; via minimum) is reduced to weighted max-cut. The algorithm uses cutting planes on the cut polytope of the underlying graph, specifically applied to toroidal grids with an extra vertex. Fractional solutions appear occasionally, especially for signed graphs. Possible use of negative-circle constraints is mentioned. [Annot. 18 Aug 2012.] **(sg: Fr: Algor)**

Francisco Barahona, Martin Grötschel, & Ali Ridha Mahjoub

1985a Facets of the bipartite subgraph polytope. *Math. Operations Res.* 10 (1985), 340–358. MR 0793888 (87a:05123a). Zbl 578.05056.

The polytope $P_B(\Gamma)$ is the convex hull in $\mathbb{R}^E$ of characteristic vectors of bipartite edge sets. Various types of and techniques for generating facet-defining inequalities, thus partially extending the description of $P_B(\Gamma)$ from the weakly bipartite case (Grötschel and Pulleyblank (1981a)) in which all facets are due to edge and odd-circle constraints. [Some can be described best via signed graphs; see Poljak and Turzík (1987a).] [A brief expository treatment of the polytope appears in Poljak and Tuza (1995a).] **(sg: par: fr: Geom)**

Francisco Barahona & Enzo Maccioni

1982a On the exact ground states of three-dimensional Ising spin glasses. *J. Phys. A: Math. Gen.* 15 (1982), L611–L615. MR 0679090 (83k:82044).

Discusses a 3-dimensional analog of Barahona, Maynard, Rammal, and Uhry (1982a). There may not always be a combinatorial LP optimum;

hence LP may not completely solve the problem. (**SG: Phys, Fr, Algor**)

Francisco Barahona & Ali Ridha Mahjoub

1986a On the cut polytope. *Math. Programming* 36 (1986), 157–173. MR 0866986 (88d:05049). Zbl 616.90058.

Call $P_{\text{BS}}(\Sigma)$ the convex hull in $\mathbb{R}^E$ of characteristic vectors of negation sets (or "balancing [edge] sets") in Σ. Finding a minimum-weight negation set in Σ corresponds to a maximum cut problem, whence $P_{\text{BS}}(\Sigma)$ is a linear transform of the cut polytope $P_{\text{C}}(|\Sigma|)$, the convex hull of cuts. Conclusions follow about facet-defining inequalities of $P_{\text{BS}}(\Sigma)$. See §5: "Signed graphs". (**SG: Fr: Geom**)

1989a Facets of the balanced (acyclic) induced subgraph polytope. *Math. Programming Ser. B* 45 (1989), 21–33. MR 1017209 (91c:05178). Zbl 675.90071.

The "balanced induced subgraph polytope" $P_{\text{BIS}}(\Sigma)$ is the convex hull in $\mathbb{R}^V$ of incidence vectors of vertex sets that induce balanced subgraphs. Conditions are studied under which certain inequalities of form $\sum_{i\in Y} x_i \leq f(Y)$ define facets of this polytope: in particular, $f(Y) =$ max. size of balance-inducing subets of Y, $f(Y) = 1$ or 2, $f(Y) = \#Y - 1$ when $Y = V(C)$ for a negative circle C, etc. (**SG: Fr: Geom, Algor**)

1994a Compositions of graphs and polyhedra. I: Balanced induced subgraphs and acyclic subgraphs. *SIAM J. Discrete Math.* 7 (1994), 344–358. MR 1285575 (95i:90056). Zbl 802.05067.

More on $P_{\text{BIS}}(\Sigma)$ (see (1989a)). A balance-inducing vertex set in $\pm\Gamma =$ a stable set in Γ. [See Zaslavsky (1982b) for a different correspondence.] Thm. 2.1 is an interesting preparatory result: If $\Sigma = \Sigma_1 \cup \Sigma_2$ where $\Sigma_1 \cap \Sigma_2 \cong \pm K_k$, then $P_{\text{BIS}}(\Sigma) = P_{\text{BIS}}(\Sigma_1) \cap P_{\text{BIS}}(\Sigma_2)$. The main result is Thm. 2.2: If Σ has a 2-separation into Σ_1 and Σ_2, the polytope is the projection of the intersection of polytopes associated with modifications of Σ_1 and Σ_2. §5: "Compositions of facets", derives the facets of $P_{\text{BIS}}(\Sigma)$. (**SG: Geom, WG, Algor**)

F. Barahona, R. Maynard, R. Rammal, & J.P. Uhry

† 1982a Morphology of ground states of two-dimensional frustration model. *J. Phys. A: Math. Gen.* 15 (1982), 673–699. MR 0642302 (83c:82045).

Treats many important aspects of the quantity $l := \min_\zeta \#E^-(\Sigma^\zeta)$ [which equals the frustration index], over all switching functions ζ ("spin configurations σ" in the paper) of a signed graph, mainly a signed planar graph. ($\#E^-(\Sigma^\zeta)$ is the paper's $\frac{1}{2}(\#E + H)$, $H :=$ Hamiltonian.) They maximize $-H = W^+ + W^- - W^{+-}$ where $W^+ + W^- := \#$ unswitched positive edges $-$ # unswitched negative edges and $W^{+-} := \#$ switched positive edges $-$ # switched negative edges. Thus, $-H = \#E^+ - \#E^- = \#E - 2\#E^-$ after switching. Maximizing it $\iff$ minimizing $\#E^-$ over all ζ.

§2: "The frustration model as the Chinese postman's problem", describes how to find l when $|\Sigma|$ is planar, by solving a Chinese postman (T-join) problem in the dual graph, T corresponding to the frustrated (i.e., negative) face boundaries. The postman problem is solved by linear programming. [Solved independently by Katai and Iwai (1978a).] [Barahona (1981a) generalizes to signed toroidal graphs.]

§3: "Solution of the frustration problem by duality: rigidity". An edge is "rigid" if it has the same sign in every Σ^ζ that minimizes $\#E$ (such an ζ is a "ground state"). The endpoints of a rigid edge are called "solidary". Rigid edges are found via the dual linear program. The boundary contours of connected sets of frustrated faces play an important role.

§§4–5: "Numerical experimentation" and "Results", for a randomly signed square lattice graph. The proportion x of negative edges strongly affects the properties; esp., there is significant long-range order below but not above $x \approx .15$. [See Deng and Abell (2010a) for numerical results on random signed graphs.]

More general problems discussed are (1) allowing positive edge weights (due to variable bond strengths); (2) minimizing $\#E^-(\Sigma^\zeta) + c\sum_V \zeta(v)$, with $c \neq 0$ because of an external magnetic field. Then one cannot expect the LP to have a combinatorial optimum. [Annot. 20 Jan 2010.]

(**SG: Phys, Fr, Fr(Gen), Algor**)

F. Barahona & J.P. Uhry

1981a An application of combinatorial optimization to physics. *Methods Operations Res.* 40 (1981), 221–224. Zbl 461.90080. (**SG: Phys, Fr: Exp**)

John S. Baras

See G.-D. Shi.

S. Barik, D. Kalita, S. Pati, & G. Sahoo

2018a Spectra of graphs resulting from various graph operations and products: a survey. *Special Matrices* 6 (2018), 323–342. MR 3856591. Zbl 1423.05097.

The signless Laplacian $L(-\Gamma)$ is one of the matrices surveyed. No mention that it is signed-graphic. [Annot. 29 Dec 2020.]

(**Par: Lap: Eig: Exp**)

J. Wesley Barnes

See P.A. Jensen.

Adriano Barra

2008a Fluctuations induce transitions in frustrated sparse networks. *Fluctuation Noise Lett.* 8 (2008), L341. arXiv:0911.5144.

Physical quantities of a random signed graph, with states ("configurations") $s \in \{\pm 1\}^n$, studied by replication. [Annot. 29 Dec 2012.]

(**Phys: SG: Rand: Adj**)

Arun Kumar Baruah & Manoshi Kotoky

2015a Signed graph and its balance theory in transportation problem. *Int. J. Computer Appl.* 115 (2015), no. 12, 9–12.

Confused; nothing of value. [Annot. 13 Oct 2019.] (**SG: Bal, Appl**)

Tamer Başar

See W. Chen and X.-D. Chen.

Nino Bašić, Patrick W. Fowler, Tomaž Pisanski, & Irene Sciriha

2022a On singular signed graphs with nullspace spanned by a full vector: Signed nut graphs. *Discuss. Math. Graph Theory* 42 (2022), no. 4, 1351–1382. MR 4445856. arXiv:2009.09018.
(SG: Adj: Eig)

Lowell Bassett, John Maybee, & James Quirk

1968a Qualitative economics and the scope of the correspondence principle. *Econometrica* 36 (1968), 544–563. MR 0237165 (38 #5456). Zbl 217.26802 (217, p. 268b).

Lemma 3: A square matrix with every diagonal entry negative is sign-nonsingular iff every cycle is negative in the associated signed digraph. Thm. 4: A square matrix with negative diagonal is sign-invertible iff all cycles are negative and the sign of any (open) path is determined by its endpoints. And more. **(QM: QSol, QSta: sd)**

Vladimir Batagelj

See also P. Doreian, N. Kejžar, and W. de Nooy.

1990a [Closure of the graph value matrix.] (In Slovenian. English summary.) *Obzornik Mat. Fiz.* 37 (1990), 97–104. MR 1074109 (91f:05058). Zbl 704.05035.
(SG: Adj, Bal, Clu)

1994a Semirings for social networks analysis. *J. Math. Sociology* 19 (1994), 53–68. Zbl 827.92029. **(SG: Adj, Bal, Clu)**

1997a Notes on blockmodeling. *Social Networks* 19 (1997), 143–155.

§3, p. 6: Predicates to use for searching out balanced or clusterable partitions. [Annot. 10 Mar 2011.] **(SG: PsS, Algor)**

V. Batagelj & T. Pisanski

1979a On partially directed Eulerian multigraphs. *Publ. Inst. Math. (Beograd) (N.S.)* 25(39) (1979), 16–24. MR 0542818 (81a:05054). Zbl 418.05038. **(sg: Ori)**

William H. Batchelder

See K. Agrawal.

Christian Bauckhage

See J. Kunegis.

Thierry-Pascal Baum

See J. Demongeot.

Jan Baumbach

See S. Böcker.

Andrei Băutu & Elena Băutu

2007a Searching ground states of Ising spin glasses with particle swarms. *Rom. J. Phys.* 52 (2007), no. 3-4, 337–342.

Experimental results for $l(\Sigma)$ compared with known results. [Annot. 19 Aug 2012.] **(SG, Phys: Fr: Algor)**

2007b Searching ground states of Ising spin glasses with genetic algorithms and binary particle swarm optimization. In: Natalio Krasnogor *et al.*, eds., *Nature Inspired Cooperative Strategies for Optimization* (NICSO 2007, Int. Workshop, Acireale, Italy), pp. 85–94. Stud. Comput. Intelligence, Vol. 129. Springer, Berlin, 2008.

Compares the two algorithms for $l(\Sigma)$. [Annot. 19 Aug 2012.]
(SG, Phys: Fr: Algor)

2009a Particle swarms in statistical physics. In: Aleksandar Lazinica, ed., *Particle Swarm Optimization*, Ch. 4, pp. 77–88. InTech, Rijeka, Croatia, and Shanghai, 2009.

§4, "Binary particle swarm optimization and Ising spin glasses": The signed graph; spins and states; satisfied and frustrated edges; some history. In particle swarm optimization, each vertex acts as a cell in a cellular automaton, learning probabilistically, seeking a most satisfied spin $\zeta(v)$ in order to minimize $\#E_v^-(\Sigma^\zeta)$. [It seems that this local minimization suffers from the same potential instability as Mitra's (1962a) deterministic local minimization, hence is not accurate.] [Annot. 19 Aug 2012.] **(SG, Phys: State, Fr: Algor: Exp)**

Andrei Băutu, Elena Băutu, & Henri Luchian

2007a Particle swarm optimization hybrids for searching ground states of Ising spin glasses. In: Viorel Negru *et al.*, eds., *SYNASC 2007: Ninth International Symposium on Symbolic and Numeric Algorithms for Scientific Computing, 2007* (Timisoara, Romania), pp. 415–418. IEEE Computer Soc., Los Alamitos, Calif., 2007.

Particle swarm optimization combined with hill-climbing to find $l(\Sigma)$ (ground state of Ising model); a hybrid method is said to be promising. [Annot. 19 Aug 2012.] **(SG, Phys: State(fr): Algor)**

2008a Searching ground states of Ising spin glasses with a tree bond-based representation. In: Viorel Negru *et al.*, eds., *SYNASC 2008: 10th International Symposium on Symbolic and Numeric Algorithms for Scientific Computing* (Timisoara, Romania), pp. 501–506. IEEE Computer Soc., Los Alamitos, Calif., 2008.

Bond-based representation means recording switched edge ("bond") signs instead of vertex spins; *cf.* Pelikan and Hartmann (2007a), (2007b). Here, a state $s : V \to \{+1, -1\}$ is recorded as the signs of a spanning tree switched by s. This has ambiguity [by a factor of 2, obviously]. Negating one tree edge implies a chain of spin changes; this "may be considered a feature" [and its implications could be interesting]. Computational experiments tested the implied algorithm. [Annot. 19 Aug 2012.]
(SG, Phys: State(fr): Algor)

Andrei Băutu & Henri Luchian

2010a Particle swarm optimization with spanning tree representation for Ising spin glasses. In: *2010 IEEE Congress on Evolutionary Computation* (CEC 2010, Barcelona), pp. 1–6. IEEE, 2010.

Applies Băutu, Băutu, and Luchian (2008a). Shallower trees may produce better results due to the lesser effect of negating one tree edge. Computational comparisons of this and other algorithms for ground state (i.e., frustration index). [Annot. 19 Aug 2012.] **(SG, Phys: Fr: Algor)**

Elena Băutu

See A. Băutu.

Sean R.B. Bearden

See H. Manukian.

Laurent Beaudou, Florent Foucaud, & Reza Naserasr

2019a Homomorphism bounds of signed bipartite K_4-minor-free graphs and edge-colorings of $2k$-regular K_4-minor-free multigraphs. *Discrete Appl. Math.* 261 (2019), 40–51. MR 3958228. Zbl 1410.05083. arXiv:1811.03807. HAL hal-01915269. (**SG: Hom, ECol**)

2020a Smallest C_{2l+1}-critical graphs of odd-girth $2k+1$. In: Manoj Changat and Sandip Das, eds., *Algorithms and Discrete Applied Mathematics* (CALDAM 2020, Hyderabad, 2020), pp. 184–196. Lect. Notes in Computer Sci., Vol. 12016. Springer, Cham, 2020. MR 4074908. Zbl 1456.05117. HAL hal-03041342.

Conference version of (2022a). [Annot. 28 Dec 2022.] (**sg: Par: Hom**)

2022a Smallest C_{2l+1}-critical graphs of odd-girth $2k+1$. *Discrete Appl. Math.* 319 (2022) 564–575. MR 4457824. Zbl 1494.05043. arXiv:1610.03685. HAL hal-03784719. (**sg: Par: Hom**)

Laurent Beaudou, Penny Haxell, Kathryn Nurse, Sagnik Sen, & Zhouningxin Wang

20xxa Density of 3-critical signed graphs. Submitted. arXiv:2309.04450.

Circular coloring as in Naserasr, Wang, and Zhu (2021a). Such a 3-critical Σ has $|E| \geq \frac{1}{2}(3n-1)$, which is asymptotically best possible. Cors.: If $|\Sigma|$ is planar, girth ≥ 6 implies circular 3-colorability. If projective planar, ditto and girth 6 is best possible. [Annot. 23 Sept 2023.] (**SG: Col(Gen)**)

Matthias Beck & Mela Hardin

2015a A bivariate chromatic polynomial for signed graphs. *Graphs Combin.* 31 (2015), no. 5, 1211–1221. MR 3386004. Zbl 1327.05099. arXiv:1204.2568. (**SG: Col, Geom**)

Matthias Beck, Erika Meza, Bryan Nevarez, Alana Shine, & Michael Young

2015a The chromatic polynomials of signed Petersen graphs. *Involve* 8 (2015), no. 5, 825–831. MR 3404660. Zbl 1322.05073. arXiv:1311.1760.

Chromatic and zero-free chromatic polynomials of all six switching isomorphism types of signed Petersens (*cf.* Zaslavsky (2012b)) and all signed K_n's for $n \leq 5$. Each switching isomorphism type has a different chromatic polynomial and each has a different zero-free polynomial. Computer code in SAGE. [Annot. 2 Nov 2013.] (**SG: Col: Invar**)

Matthias Beck & Thomas Zaslavsky

2006a Inside-out polytopes. *Adv. Math.* 205 (2006), no. 1, 134–162. MR 2254310 (2007e:52017). Zbl 1107.52009. arXiv:0309330.

§5: "In which we color graphs and signed graphs." A geometric interpretation of signed graph coloring by lattice points and hyperplane arrangements unifies the chromatic and zero-free chromatic polynomials and gives immediate proofs of theorems on the chromatic polynomials and acyclic orientations. (**SG: Col: Geom, Matrd, Invar, Bal**)

2006b The number of nowhere-zero flows in graphs and signed graphs. *J. Combin. Theory Ser. B* 96 (2006), no. 6, 901–918. MR 2274083 (2007k:05084). Zbl 1119.05105. arXiv:math/0309330.

The nowhere-zero flow polynomial of a signed graph, for flows in an odd abelian group, and the integral nowhere-zero flow quasipolynomial

with period 2. [For even abelian groups see DeVos, Rollová, and Šámal (2019a).] (**SG: Flows: Geom, Matrd, Invar, Bal**)

2006c An enumerative geometry for magic and magilatin labellings. *Ann. Combin.* 10 (2006), no. 4, 395–413. MR 2293647 (2007m:05010). Zbl 1116.05071. arXiv:-math/0506315.

In magic labellings of a bidirected graph, the labels are distinct positive integers; at each vertex the sum over entering edge ends equals that over departing edge ends. Thms. (implicit): The number of magic labellings is a quasipolynomial function of the magic sum, if the magic sum is prescribed. It is also a quasipolynomial function of the upper bound on the labels, if an upper bound is prescribed. (**ori: Geom, Enum**)

§5: "Generalized exclusions." Complementarity rules in magic squares, etc., can be expressed by signed-graphic hyperplanes. (**sg: Lab: Geom, Enum**)

2010a Six little squares and how their numbers grow. *J. Integer Sequences* 13 (2010), art. 10.6.2, 43 pp. MR 2659218 (2011j:05052). Zbl 1230.05062. arXiv:1004.0282.

§3: "Semimagic squares." Counts magic labellings of the extraverted $-K_{3,3}$ by an explicit geometrical solution. Counted either by upper bound on the values or by magic sum. (**par: Lab: incid, Geom**)

Richard Behr

2017a Edges and vertices in a unique signed circle in a signed graph. *AKCE Int. J. Graphs Combin.* 14 (2017), no. 3, 224–232. MR 3695979. Zbl 1375.05122. arXiv:1610.02107.

In Σ, does edge e, or vertex v, lie in exactly one negative circle? Exactly one positive circle? The structure of Σ and the corresponding edges or vertices are determined for each question. [Annot. 21 Dec 2017.] (**SG: Circ: Str**)

2018a *Edge Coloring and Special Edges of Signed Graphs.* Doctoral dissertation, Binghamton University, 2018. MR 3818916 (no rev).

Ch. 2, "Edge coloring signed graphs": Vizing's edge-coloring theorem generalized to signed simple graphs. Early version of (2020a). Ch. 3, "Special edges": Same as (2017a). [Annot. 27 Apr 2018.] (**SG: ECol: lg**)(**SG: Str**)

† 2020a Edge coloring signed graphs. *Discrete Math.* 343 (2020), no. 2, art. 111654, 15 pp. MR 4040037. Zbl 1429.05058. arXiv:1807.11465.

Introduces edge coloring of signed graphs. Vizing's Theorem generalizes to signed simple graphs: the number of proper edge colorings is $\chi'(\Sigma) \leq \Delta(\Sigma) + 1$. A proper edge coloring of Σ is the same as a proper vertex coloring (*cf.* Zaslavsky (1982b)) of $-\Lambda(\Sigma)$, defined via bidirected graphs (*cf.* Zaslavsky (20xxa), (2010b)). An opposite definition ("antiproper coloring") is equivalent to balanced decomposition of $-\Sigma$ (*cf.* Zaslavsky (1987b)). Remarks on total coloring. [Independent partial proof by Zhang–Lu–Luo–Ye–Zhang (2020a) with results for signed planar graphs.] [Sign-independence: Janczewski, Turowski, and Wróblewski (2023a).] [Annot. 26 Aug 2018, rev 2 Jul 2020.] (**SG: ECol**)(**SG: LG: Ori, ECol**)

M. Behzad & G. Chartrand

1969a Line-coloring of signed graphs. *Elem. Math.* 24 (1969), 49–52. MR 0244098 (39 #5415). Zbl 175.50302 (175, p. 503b).

Λ_{BC} Their line graph $\Lambda_{BC}(\Sigma)$ of a signed simple graph Σ (not defined explicitly) is the line graph $\Lambda(|\Sigma|)$ with an edge negative when its two endpoints are negative edges in Σ. They "color" as in Cartwright and Harary (1968a) [i.e., clustering]. Characterized: Σ with colorable line graphs. Found: the fewest colors for line graphs of signed trees, K_n, and $K_{r,s}$. [For a more sophisticated kind of line graph see Zaslavsky (1984c), (2010b), (20xxa). For another line graph, see M. Acharya (2009a).]
(**SG: lg: Clu**)

Ghazaleh Beigi, Jiliang Tang, & Huan Liu

2016a Signed link analysis in social media networks. In: *Proceedings of the Tenth International AAAI Conference on Web and Social Media* (ICWSM 2016, Cologne), pp. 539–542. AAAI Press [Assoc. Advancement of Artificial Intelligence], Palo Alto, 2016. arXiv:1603.06878. (**PsS: SG, Algor**)

Amos Beimel, Aner Ben-Efraim, Carles Padró, & Ilya Tyomkin

2014a Multi-linear secret-sharing schemes. In: Yehuda Lindell, ed., *Theory of Cryptography* (Proc. 11th Theory of Cryptography Conf., TCC 2014, San Diego, 2014), pp. 394–418. Lect. Notes in Computer Sci., Vol. 8349. Springer, Berlin, 2014. MR 3183548. Zbl 1326.94071.
[*Cf.* Ben-Efraim (2016a).] (**gg: Matrd**)

Lowell W. Beineke & Frank Harary

1966a [As "W. Beineke and F. Harary"] Binary matrices with equal determinant and permanent. *Studia Sci. Math. Hungar.* 1 (1966), 179–183. MR 0207582 (34 #7397). Zbl 145.01505 (145, p. 15e). (**SD**)

1978a Consistency in marked digraphs. *J. Math. Psychology* 18 (1978), 260–269. MR 0522390 (80d:05026). Zbl 398.05040.

A "marked digraph" is a digraph $\vec{D}$ with signed vertices, $\vec{S} = \vec{D}, \mu)$ where $\mu : V \to \{+, -\}$. It is "consistent" if all diwalks from v to w have the same sign $\mu(W)$. The sign of a walk is the vertex sign product. Thm. 1. Asssuming $\vec{D}$ is strongly connected, $\vec{S}$ is consistent iff every dicycle is positive. [An important difference from signed graphs, where no restriction is needed.] Thm. 2. $\vec{S}$ is consistent iff V has a bipartition such that every arc with a positive tail lies within a set but no arc with a negative tail does so. Define $\sigma(\vec{uv} := \mu(u)$. Thm. 3. Asssuming $\vec{D}$ is strongly connected, this signed graph is balanced iff $\vec{S}$ is consistent. Thm. 4. A vertex-signed tournament $\vec{S}$ is consistent iff: When strongly connected, [it is all positive or] it has exactly two negative vertices u, v and, deleting uv, u is a source and v is a sink. When not strongly connected, it is consistent iff it is all positive, or it has one negative vertex which is a source or sink, or it has two negative vertices, one a source and the other a sink. Thm. 5. $\vec{D}$ has $\mu \not\equiv +$ such that $(\vec{D}, \mu)$ is consistent ("markable") iff $\exists\ \varnothing \subset V_0 \subset V$ such that, $\forall\ v$, all out-arcs from v, or none, go to V_0. [Annot. 16 Sept 2010.] (**VS**)

† 1978b Consistent graphs with signed points. *Riv. Mat. Sci. Econom. Social.* 1 (1978), 81–88. MR 0573718 (81h:05108). Zbl 493.05053.

A graph (not necessarily simple) with signed vertices is "consistent" if every circle has positive sign product. Thm. 2.2: Γ with all negative vertices is consistent iff bipartite. Thm. 2.3: 3-connected vertices must have the same sign. Thm. 3.3: Contracting an edge with positive endpoints preserves consistency and inconsistency. Further partial results. Open problem: A full characterization of consistent vertex-signed graphs. [For a good solution see Hoede (1992a). For the best solution see Joglekar, Shah, and Diwan (2010a).] [Annot. rev. 11 Sept 2010.] (**VS: Bal**)

Lowell W. Beineke & Robin J. Wilson, eds.

2009a *Topics in Topological Graph Theory.* Encycl. Math. Appl., Vol. 128. Cambridge Univ. Press, Cambridge, Eng., 2009. MR 2581536 (2011f:05005). Zbl 1170.05003.

Several chapters discuss the use of gain graphs (as "voltage graphs") to construct surface embeddings and covering graphs. Some mention edge signs *in re* orientation. Ch. 1: Gross and Tucker (2009a). Ch. 3: Gross (2009a). Ch. 8: Pisanski and Žitnik (2009a). Ch. 9: Kwak and Lee (2009a). Ch. 10: Širáň and Tucker (2009a). Ch. 11: Tucker (2009a). Ch. 12: A.T. White (2009a). Ch. 13: Grannell and Griggs (2009a).

[Strangely for a book "covering the full range of topological graph theory" (p. xvii), orientation embedding of signed graphs is virtually ignored. *Cf.*, e.g., Lins (1985a), Širáň and Škoviera (1991a), and Zaslavsky (1992a), (1993a).] [Annot. 12 Jun 2013.]

(**Top: GG, SG, Cov, Enum: Exp**)

Mohamed-Ali Belabbas

See X.-D. Chen.

Jacques Bélair, Sue Ann Campbell, & P. van den Driessche

1996a Frustration, stability, and delay-induced oscillations in a neural network model. *SIAM J. Appl. Math.* 56 (1996), 245–255. MR 1372899 (96j:92003). Zbl 840.92003.

The signed digraph of a square matrix is "frustrated" if it has a negative cycle. Somewhat simplified: a negative cycle is necessary for there to be oscillation caused by intraneuronal processing delay. (**SD: QM, Ref**)

Francesco Belardo

See also A. Abiad, S. Akbari, A. Alazemi, and J.F. Wang.

2014a Balancedness and the least eigenvalue of Laplacian of signed graphs. *Linear Algebra Appl.* 446 (2014), 133–147. MR 3163133. Zbl 1285.05112. (**SG: Eig, Fr**)

Francesco Belardo & Maurizio Brunetti

2019a Connected signed graphs L-cospectral to signed ∞-graphs. *Linear Multilinear Algebra* 67 (2019), no. 12, 2410–2426. MR 4017722. Zbl 1425.05067.

All connected Laplacian-cospectral mates of unbalanced "∞-graphs" (tight handcuffs). They are certain bicyclic graphs. [Annot. 18 Jan 2019.] (**SG: Lap: Eig**)

2021a Line graphs of complex unit gain graphs with least eigenvalue -2. *Electronic J. Linear Algebra* 37 (2021), 14–30. MR 4246513. Zbl 1464.05170.

Eigenvectors of -2 for $A(-\Lambda(\Phi)) = \mathrm{H}(\Phi)^T\mathrm{H}(\Phi) - 2I$. Dictionary: "line graph" = spectral line graph $\Lambda_{\text{Spec}}(\Phi) = -\Lambda(\Phi)$. [Annot. 2, 11 Jan 2020.] (**GG: LG: Adj: Eig**)

2022a On eigenspaces of some compound complex unit gain graphs. *Trans. Combin.* 11 (2022), no. 3, 131–152. MR 4443275. Zbl 1513.05237.

Compares eigenspaces of complex unit gain graph matrices: $L(\Phi)$, $A(\Lambda(\Phi))$, and subdivision $A(\Phi)$. [Annot. 18 Oct 2023.] (**GG: Eig: LG**)

† 2024a Limit points for the spectral radii of signed graphs. *Discrete Math.* 347 (2024), art. 113745, 20 pp.

Following the Hoffman program for graphs, finds all limit points. *Cf.* Wang, Dong, Hou, and Li (2023a). (**SG: Adj: Eig**)

Francesco Belardo, Maurizio Brunetti, Matteo Cavaleri, & Alfredo Donno

2021a Constructing cospectral signed graphs. *Linear Multilinear Algebra* 69 (2021), no. 14, 2717–2732. MR 4306815. Zbl 1472.05064. arXiv:1908.02220. (**SG: Adj: Eig**)

Francesco Belardo, Maurizio Brunetti, & Adriana Ciampella

2018a Signed bicyclic graphs minimizing the least Laplacian eigenvalue. *Linear Algebra Appl.* 557 (2018), 201–233. MR 3848268. Zbl 1396.05066. (**SG: Lap: Eig**)

2019a On the multiplicity of α as an $A_\alpha(\Gamma)$-eigenvalue of signed graphs with pendant vertices. *Discrete Math.* 342 (2019), 2223–2233. MR 3952139. Zbl 1418.05070. (**SG: Adj(Gen): Eig**)

2019b Edge perturbation on signed graphs with clusters: Adjacency and Laplacian eigenvalues. *Discrete Appl. Math.* 269 (2019), 130–138. MR 4016591. Zbl 1421.05051. (**SG: Adj, Lap: Eig**)

2021a Godsil-McKay switching for mixed and gain graphs over the circle group. *Linear Algebra Appl.* 614 (2021), 256–269. MR 4209002. Zbl 1459.05110. (**sg, GG: Adj: Eig**)

2021b Unbalanced unicyclic and bicyclic graphs with extremal spectral radius. *Czechoslovak Math. J.* 71 (2021), no. 2, 417–433. MR 4263178. (**SG: Adj: Eig**)

Francesco Belardo, Maurizio Brunetti, Nolan J. Coble, Nathan Reff, & Howard Skogman

2022a Spectra of quaternion unit gain graphs. *Linear Algebra Appl.* 632 (2022), 15–49 [2021]. MR 4319411. Zbl 1478.05088. (**GG: Adj, Lap, Incid: Eig**)

Francesco Belardo, Maurizio Brunetti, & Nathan Reff

2020a Balancedness and the least Laplacian eigenvalue of some complex unit gain graphs. *Discuss. Math. Graph Theory* 40 (2020), no. 2, 417–433. MR 4060993. Zbl 1433.05185. (**GG: Bal, Lap: Eig**)

Francesco Belardo, Maurizio Brunetti, & Vilmar Trevisan

2021a Locating eigenvalues of unbalanced unicyclic signed graphs. *Appl. Math. Comput.* 400 (2021), art. 126082. MR 4218433. Zbl 1508.05102. (**SG: Adj: Eig: Algor**)

Francesco Belardo, Sebastian M. Cioabă, Jack Koolen, & Jianfeng Wang

2018a Open problems in the spectral theory of signed graphs. *Art Discrete Appl. Math.* 1 (2018), art. #P2.10, 23 pp. MR 3997096. Zbl 1421.05052. arXiv:-907.04349.

Eigenvalue questions, almost all for $A(\Sigma)$, with a review of relevant knowledge. [Annot. 13 Jun 2019.] (**SG: Eig: Adj, KG**)

Francesco Belardo, Enzo M. Li Marzi, & Slobodan K. Simić

2016a Signed line graphs with least eigenvalue -2: The star complement technique. *Discrete Appl. Math.* 207 (2016), 29–38. MR 3497981. Zbl 1337.05051.

Produces a nice basis, derived from Σ, for the eigenspace of $+2$ of $\Lambda(\Sigma)$. Dictionary: *cf.* Belardo and Simić (2015a). [Annot. 11 Jan 2020.] (**SG: LG: Adj: Eig**)

Francesco Belardo, Enzo M. Li Marzi, Slobodan K. Simić, & Jianfeng Wang

2010a On the index of necklaces. *Graphs Combin.* 26 (2010), no. 2, 163–172. MR 2606492 (2011g:05171). Zbl 1231.05165.

The largest eigenvalue of $A(G)$ for $G =$ chain or necklace of cliques, via $L(-\Gamma)$ where $G = \Lambda(\Gamma)$. [Annot. 16 Jan 2012.] (**par: LG: Adj: Eig**)

2011a Graphs whose signless Laplacian spectral radius does not exceed the Hoffman limit value. *Linear Algebra Appl.* 435 (2011), no. 11, 2913–2920. MR 2825291 (2012k:05221). Zbl 1221.05229. (**par: Lap: Eig**)

Francesco Belardo & Paweł Petecki

2015a Spectral characterizations of signed lollipop graphs. *Linear Algebra Appl.* 480 (2015), 144–167. MR 3348518. Zbl 1315.05083. (**SG: Adj, Lap: Eig**)

Francesco Belardo, Paweł Petecki, & Jianfeng Wang

2016a On signed graphs whose second largest Laplacian eigenvalue does not exceed 3. *Linear Multilinear Algebra* 64 (2016), no. 9, 1785–1799. MR 3509501. Zbl 1341.05148. (**SG: Lap: Eig**)

Francesco Belardo, Tomaž Pisanski, & Slobodan K. Simić

2016a On graphs whose least eigenvalue is greater than -2. *Linear Multilinear Algebra* 64 (2016), no. 8, 1570–1582. MR 3503369. Zbl 1341.05149. Erratum. *Ibid.* 64 (2016), no. 8, I. MR 3503376.

§2, "Line graphs and their generalizations": Adjacency eigenvalues of spectral line graph $\Lambda_{\text{Spec}}(\Sigma)$ compared to Laplacian eigenvalues of Σ by standard method. Conj. 4.4 on signed graphs. Dictionary: *cf.* Belardo and Simić (2015a). [Annot. 4, 11 Jan 2020.] (**SG: Ori: Adj, Lap, LG: Eig**)

Francesco Belardo, Irene Sciriha, & Slobodan K. Simić

2016a On eigenspaces of some compound signed graphs. *Linear Algebra Appl.* 509 (2016), 19–39. MR 3546399. Zbl 1346.05161. (**SG: Adj, Lap, LG**)

Francesco Belardo & Slobodan K. Simić

2015a On the Laplacian coefficients of signed graphs. *Linear Algebra Appl.* 475 (2015), 94–113. MR 3325220. Zbl 1312.05078.

Λ_{Spec} §2: Adjacency eigenvalues of spectral line graph $-\Lambda_S(\Sigma)$ compared to Laplacian eigenvalues of Σ by standard method. §3: Coefficients of Laplacian characteristic polynomial. Dedò (1981a)-style matrix-tree theorem. §4, "Laplacian coefficients of signed unicyclic graphs". Dictionary: "oriented edge" = positive edge, "unoriented edge" = negative edge, "line graph" $\mathcal{L}(\Sigma_\eta)$ = reduced negated line graph $-\overline{\Lambda}(\Sigma,\eta)$, "subdivision graph" $\mathcal{S}(\Sigma,\eta)$ = signed vertex-edge incidence graph. [Annot. 5, 11 Jan 2020.] (**SG: Lap, LG**)

Francesco Belardo, Zoran Stanić, & Thomas Zaslavsky

2023a Total graph of a signed graph. *Ars Math. Contemp.* 3 (2023) [2022], art. P1.02, 17 pp. MR 4518838. Zbl 1504.05119. arXiv:1908.02001.

Total graph defined to have matrix properties. Two versions: combinatorial $\mathrm{T}_C(\Sigma)$ (with line graph $\Lambda(\Sigma)$ by Zaslavsky (20xxa), (2010b), *et al.*) and spectral $\mathrm{T}_S(\Sigma)$ (with $-\Lambda(\Sigma)$ by Belardo and Simić (2015a)). Frustration of line and total graphs. Spectra of total graph and iterated and combined total graphs. (For a spectrally unsuitable definition *cf.* Sinha–Garg (2011b).) [Annot. 1 Apr 2022.] (**SG: LG, LG(Gen): Adj: Eig**)

Francesco Belardo & Yue Zhou

2016a Signed graphs with extremal least Laplacian eigenvalue. *Linear Algebra Appl.* 497 (2016), 167–180. MR 3466641. Zbl 1331.05135.

Thms: Min and max $\lambda_{\min}(L(\Sigma))$ over unbalanced [connected] Σ are attained by an unbalanced triangular lollipop and $-K_n$. [Annot. 20 Mar 2016.] (**SG: Adj: Eig**)

Hacéne Belbachir & Imad Eddine Bousbaa

2013a Translated Whitney and r-Whitney numbers: A combinatorial approach. *J. Integer Sequences* 16 (2013), art. 13.8.6, 7 pp. MR 3118323. Zbl 1292.05050.

Introduces $w_{m,0}(n,k)$ (first kind) and $W_{m,0}(n,k)$ (second kind) = # of ways to permute (or partition) $[n]$ into k cycles (or blocks); color all but the least element in each using m colors. Also "translated Whitney–Lah numbers" and "translated r-Whitney numbers". Formulas and identities.

[Improvement: (1) Let group $\mathfrak{G}$ have order m; use color set $\mathfrak{G}$, least element colored ε; then $W_m(n,k,0) = W_k(\mathrm{Lat^b}\, Q_n(\mathfrak{G}))$ = Whitney number of balanced-flat semilattice of frame matroid $\mathbf{F}(\mathfrak{G}K_n^\bullet)$. Proved by Dowling (1973b) as amplified in Zaslavsky (1991a). Also *cf.* Remmel and Wachs (2004a). The Belbachir–Bousbaa case is $\mathfrak{G} = \mathbb{Z}_m$. (2) *Conjecture.* $w_m(n,k,0) = w_k(\mathrm{Lat^b}\, Q_n(\mathfrak{G})) = \pm$(# of permutations labelled by $\mathfrak{G}$, the Belbachir–Bousbaa case being $\mathfrak{G} = \mathbb{Z}_m$. By the same proof.]

Dictionary: "translated Whitney numbers" $\left[{n \atop k}\right]^{(m)}$, $\left\{{n \atop k}\right\}^{(m)} = w_{m,0}(n,k)$, $W_{m,0}(n,k)$ [0-Whitney numbers; *cf.* Gyimesi and Nyul (2018a)]; "mutation" = m-coloring; "dominant" = (here, for convenience) least.

["Whitney" should be "Dowling"; Whitney numbers are more general.] [Annot. 28 May 2018.] (**gg: matrd: Invar**)

G.R. Belitskiĭ & Yu.I. Lyubich

1984a *Norms of Matrices and Their Applications.* (In Russian.) Naukova Dumka, Kiev, 1984. MR 0799924 (87e:15002). Zbl 568.15016.

See Al'pin (2014a). [Annot. 22 Feb 2021.] (**gg: Bal**)

G.R. Belitskii & Yu.I. Lyubich [G.R. Belitskiĭ & Yu.I. Lyubich]

1984b *Matrix Norms and Their Applications.* Transl. A. Iacob from (1984a). Operator Theory Adv. Appl., Vol. 36. Birkhäuser, Basel, 1988. MR 1015711 (90g:15003). Zbl 645.15019. **(gg: Bal)**

A. Bellacicco & V. Tulli

1996a Cluster identification in a signed graph by eigenvalue analysis. In: *Matrices and Graphs: Theory and Applications to Economics* (full title *Proceedings of the Conferences on Matrices and Graphs: Theory and Applications to Economics*) (Brescia, 1993, 1995), pp. 233–242. World Scientific, Singapore, 1996. MR 1670488 (no rev). Zbl 914.65146.

Signed (di)graphs ("spin graphs") are defined. The main concepts are "dissimilarity", "balance", and "cluster" are defined and propositions are stated. Eigenvalues are mentioned. [This may be an announcement. There are no proofs. It is hard to be sure what is being said.] **(SD: Eig)**

Joachim von Below

1994a The index of a periodic graph. *Results Math.* 25 (1994), 198–223. MR 1273111 (95e:05081). Zbl 802.05054.

Here a periodic graph [of dimension m] is defined as a connected graph $\Gamma = \tilde{\Psi}$ where Ψ is a finite $\mathbb{Z}^m$-gain graph with gains contained in $\{\mathbf{0}, \mathbf{b}_i, \mathbf{b}_i - \mathbf{b}_j\}$. ($\mathbf{b}_1, \ldots, \mathbf{b}_m$ are the unit basis vectors of $\mathbb{Z}^m$.) Let us call such a Ψ a small-gain base graph for Γ. Any $\tilde{\Phi}$, where Φ is a finite $\mathbb{Z}^m$-gain graph, has a small-gain base graph Ψ; thus this definition is equivalent to that of Collatz (1978a). The "index" $I(\Gamma)$, analogous to the largest eigenvalue of a finite graph, is the spectral radius of $A(\|\Psi\|)$ (here written $A(\Gamma, N)$) for any small-gain base graph of Γ. The paper contains basic theory and the lower bound $L_m = \inf\{I(\Gamma) : \Gamma \text{ is } m\text{-dimensional}\}$, where $1 = L_1$, $\sqrt{9/2} = L_2 \leq L_3 \leq \cdots$. **(GG(Cov): Eig)**

Jean Bénabou

1996a Some geometric aspects of the calculus of fractions. European Colloq. Category Theory (Tours, 1994). *Appl. Categ. Structures* 4 (1996), 139–165. MR 1406095 (97g:18007). Zbl 874.18007.

Morphisms of signed graphs are employed in category-theoretic constructions. **(SG)**

Radel Ben-Av

See D. Kandel.

Edward A. Bender & E. Rodney Canfield

1983a Enumeration of connected invariant graphs. *J. Combin. Theory Ser. B* 34 (1983), 268–278. MR 0714450 (85b:05099). Zbl 532.05036.

§3: "Self-dual signed graphs," gives the number of n-vertex graphs that are signed, vertex-signed, or both; connected or not; self-isomorphic by reversing edge and/or vertex signs or not, for all $n \leq 12$. Some of this appeared in Harary, Palmer, Robinson, and Schwenk (1977a). **(SG, VS: Enum)**

Riccardo Benedetti

1998a A combinatorial approach to combings and framings of 3-manifolds. In: A. Balog, G.O.H. Katona, A. Recski, and D. Sa'sz, eds., *European Congress of*

Mathematics (Budapest, 1996), Vol. I, pp. 52–63. Progress in Math., Vol. 168. Birkhäuser, Basel, 1998. MR 1645797 (2000e:57033). Zbl 905.57018.

§8, "Spin manifolds", hints at a use for decorated signed graphs in the structure theory of spin 3-manifolds. **(sg: Appl: Exp)**

Aner Ben-Efraim

See also A. Beimel.

2016a Secret-sharing matroids need not be algebraic. *Discrete Math.* 339 (2016), 2136–2145. MR 3500143. Zbl 1337.05016. arXiv:403.6363. **(gg: Matrd)**

Joseph Ben Geloun

See R.C. Avohou.

Felipe Benítez, Julio Aracena, & Christopher Thraves Caro

20xxa The sitting closer to friends than enemies problem in the circumference. Submitted. arXiv:1811.02699.

Cf. Kermarrec and Thraves (2011a), (2014a). **(SG: Geom, KG)**

Samia Ben Lamine

See J. Aracena and J. Demongeot.

Curtis Bennett & Bruce E. Sagan

1995a A generalization of semimodular supersolvable lattices. *J. Combin. Theory Ser. A* 72 (1995), 209–231. MR 1357770 (96i:05180). Zbl 831.06003.

To illustrate the generalization, most of the article calculates the chromatic polynomial of $\pm K_n^{(k)}$ (called $\mathcal{DB}_{n,k}$; this has half edges at k vertices), builds an "atom decision tree" for $k = 0$, and describes and counts the bases of $\mathbf{F}(\pm K_n^{(k)})$ (called $\mathcal{D}_n$) that contain no broken circuits. **(SG: Matrd, Invar, col)**

M.K. Bennett, Kenneth P. Bogart, & Joseph E. Bonin

1994a The geometry of Dowling lattices. *Adv. Math.* 103 (1994), 131–161. MR 1265790 (95b:05050). Zbl 814.51003.

Drawing an analogy between Desargues' and Pappus' theorems in projective spaces and similar incidence theorems in Dowling geometries. [The rigorous avoidance of gain graphs makes the results less obvious than they could be.] **(gg: Matrd, Geom)**

Moussa Benoumhani

1996a On Whitney numbers of Dowling lattices. *Discrete Math.* 159 (1996), 13–33. MR 1415279 (98a:06005). Zbl 861.05004.

Cf. Dowling (1973b). Generating functions and identities for Whitney numbers of the first and second kinds, analogous to usual treatments of Stirling numbers. §2, "Whitney numbers of the second kind": $W_m(n,k) := W_k(Q_n(\mathfrak{G})) = W_k(\mathbf{F}(\mathfrak{G}K_n^\bullet))$ where $m = \#\mathfrak{G}$. E.g., Thm. 1: $\sum_n W_m(n,k)z^n/n! = [(e^{mz}-1)/m]^k e^z/k!$. Thm. 5: $\sum_n W_m(n,k)u^{n-k} = m^{k+1}/([1-u]/mu)_{k+1}$. §3, "Whitney numbers of the first kind": $w_m(n,k) := w_k(\mathbf{F}(\mathfrak{G}K_n^\circ))$. E.g., Thm. 10: $\sum_n w_m(n,k)z^n/n! = (1+mz)^{-1/m} \cdot \ln^k(1+mz)/k!m^k$. Thm. 12 is a reciprocity relation between $w_m(n,k)$ and $s(n,k)$. §4, "The integers maximizing $W_m(n,k)$ and $w_m(n,k)$": Partial, complicated results. [Annot. 30 Apr 2012.] **(gg: Matrd: Invar)**

1997a On some numbers related to Whitney numbers of Dowling lattices. *Adv. Appl. Math.* 19 (1997), 106–116. MR 1453407 (98f:05004). Zbl 876.05001.

Continuation of (1996a). §2, "Dowling polynomials": $D_m(n,x) := \sum_k W_m(n,k)x^k$. Generating function, recurrence, infinite series expression. §3 similarly studies $F_{m,1}(x) := \sum_k k!m^k W_m(n,k)x^k$ and $F_{m,1}(x) := \sum_k k!W_m(n,k)x^k$. §4, "Log-concavity of $k!W_m(n,k)$". Deduced from real negativity of zeros. [Annot. 1 May 2012.] **(gg: Matrd: Invar)**

1999a Log-concavity of Whitney numbers of Dowling lattices. *Adv. Appl. Math.* 22 (1999), 186–189. MR 1659426 (2000i:05008). Zbl 918.05003.

Logarithmic concavity of Whitney numbers of the second kind is deduced by proving that their generating polynomial has only real zeros. [*Cf.* Stonesifer (1975a), Dür (1986a), and Damiani, D'Antona, and Regonati (1994a).] **(gg: Matrd: Invar)**

Julien Bensmail

2019a On the 2-edge-coloured chromatic number of grids. *Australasian J. Combin.* 75 (2019), no. 3, 365–384. MR 4024807. Zbl 1429.05059. HAL hal-02264958.

The max homomorphic chromatic number of grids $\in \{8, 9, 10, 11\}$, improving ≤ 12 from Nesetril–Raspaud (2000a). [Improved to ≤ 9 in Dybizbański (2020a).] Dictionary: "2-edge-coloured graph" = signed graph, "2-edge-coloured chromatic number" = min order of homomorphism target. [Annot. 3 Sept 2020.] **(SG: Hom)**

Julien Bensmail, Sandip Das, Soumen Nandi, Théo Pierron, Sagnik Sen, & Éric Sopena

2022a On the signed chromatic number of some classes of graphs. *Discrete Math.* 345 (2022), no. 2, art. 112664, 20 pp. MR 4339573. arXiv:2009.12059. HAL hal-02947399.

Dictionary: "signed chromatic number" = homomorphic chromatic number (not chromatic number). **(SG: Hom: Invar)**

Julien Bensmail, François Dross, Nacim Oijid, & Éric Sopena

2022a Generalising the achromatic number to Zaslavsky's colourings of signed graphs. *Theor. Computer Sci.* 923 (2022), 196–221. MR 4436569. arXiv:2109.13627. HAL hal-0335582. **(SG: Col)**

Julien Bensmail, Soumen Nandi, Mithun Roy, & Sagnik Sen

2020a Classification of edge-critical underlying absolute planar cliques for signed graphs. *Australasian J. Combin.* 77 (2020), no. 1, 117–135. MR 4100300. Zbl 1444.05062. HAL hal-01919007. **(SG: Hom)**

20xxa On homomorphisms of planar signed graphs and absolute cliques. Submitted. HAL hal-01919007. **(SG: Hom)**

Josh Bentley

See A.-M. Yang.

C. Benzaken

See also P.L. Hammer.

C. Benzaken, S.C. Boyd, P.L. Hammer, & B. Simeone

1983a Adjoints of pure bidirected graphs. Proc. Fourteenth Southeastern Conf. on Combinatorics, Graph Theory and Computing (Boca Raton, Fla., 1983). *Congressus Numer.* 39 (1983), 123–144. MR 0734537 (85e:05077). Zbl 537.05024. **(sg: Ori: LG)**

Cl. Benzaken, P.L. Hammer, & B. Simeone

1980a Some remarks on conflict graphs of quadratic pseudo-boolean functions. In: L. Collatz, G. Meinardus, and W. Wetterling, eds., *Konstruktive Methoden der finiten nichtlinearen Optimierung* (Tagung, Oberwolfach, 1980), pp. 9–30. Int. Ser. Num. Math., 55. Birkhäuser, Basel, 1980. MR 0626538 (83e:90096). Zbl 455.90063. **(par: fr)(sg: Ori: LG)**

C. Benzaken, P.L. Hammer, & D. de Werra

1981a Threshold signed graphs. Res. Rep. No. 237, IMAG, Université Grenoble, Grenoble, 1981.

See (1985a). **(SG: Bal)**

1985a Threshold characterization of graphs with Dilworth number two. *J. Graph Theory* 9 (1985), no. 2, 245–267. MR 0797513 (87d:05135). Zbl 583.05048.

[They are identical to "threshold signed graphs", Calamoneri–Monti–Petreschi (2018a).] Γ is a threshold signed graph if $\exists\ a : V \to \mathbb{R}$, $S, T \in \mathbb{R}$, such that $v_iv_j \in E$ iff $|a_i + a_j| \geq S$ or $|a_i - a_j| \geq T$. [Proposed signed graph Σ: $-v_iv_j \in E$ iff $|a_i + a_j| \geq S$, $+v_iv_j \in E$ iff $|a_i - a_j| \geq T$. Then Γ = simplification of $|\Sigma|$. *Question.* Is Σ interesting?] [Annot. 16 Jan 2012.] **(VS, sg)**

Michele Benzi

See E. Estrada.

C. Berge & J.-L. Fouquet

1997a On the optimal transversals of the odd cycles. *Discrete Math.* 169 (1997), 169–175. MR 1449714 (98c:05094). Zbl 883.05088.

All-negative signed graphs in which the vertex frustration number equals the negative-circle vertex-packing number. This is called the "König property" [since it is a vertex König-type property for negative circles]. Example: the line graphs of cubic bipartite graphs. [*Problems.* Investigate for arbitrary signed and biased graphs.] **(par: Fr, Circ)**

Claude Berge & A. Ghouila-Houri

1962a *Programmes, jeu et reseaux de transport.* Dunod, Paris, 1962. MR 0192912 (33 #1137). Zbl 111.17302.

2^e partie, Ch. IV, §2: "Les reseaux de transport avec multiplicateurs." Pp. 223–229. **(GN: incid)**

1965a *Programming, Games and Transportation Networks.* Methuen, London; Wiley, New York, 1965. MR 0198964 (33 #7114).

English edition of (1962a).

Part II, 10.2: "The transportation network with multipliers." Pp. 221–227. **(GN: incid)**

1967a *Programme, Spiele, Transportnetze.* B.G. Teubner Verlagsgesellschaft, Leipzig, 1967, 1969. MR 0218106 (36 #1195). Zbl 183.23905, Zbl 194.19803.

German edition(s) of (1962a). **(GN: incid)**

Claude Berge & Bruce Reed

1999a Edge-disjoint odd cycles in graphs with small chromatic number. Symposium à la Mémoire de François Jaeger (Grenoble, 1998). *Ann. Inst. Fourier (Grenoble)* 49 (1999), 783–786. MR 1703423 (2000f:05051). Zbl 923.05034.

If $-\Gamma$ is an all-negative signed graph in which the frustration index equals the negative-circle edge-packing number for every subgraph, then $\chi(\Gamma) \leq 3$. [*Problem* 1. Is it natural to state this bound in terms of the chromatic number of $-\Gamma$? *Problem* 2. Generalize to arbitrary signed graphs.] **(par: Fr: Circ)**

2000a Optimal packings of edge-disjoint odd cycles. *Discrete Math.* 211 (2000), 197–202. MR 1735345 (2000h:05161). Zbl 945.05048.

An upper bound on the frustration index in terms of the negative-circle edge-packing number. **(par: Fr: Circ)**

Joseph Berger, Bernard P. Cohen, J. Laurie Snell, & Morris Zelditch, Jr.

1962a *Types of Formalization in Small Group Research.* Houghton Mifflin, Boston, 1962.

See Ch. 2: "Explicational models." **(PsS)(SG: Bal)(Ref)**

Nantel Bergeron

1991a A hyperoctahedral analogue of the free Lie algebra. *J. Combin. Theory Ser. A* 58 (1991), 256–278. MR 1129117 (93g:20015). Zbl 0759.17003. **(sg: Algeb, Geom)**

A. Nihat Berker

See D. Blankschtein.

Abraham Berman & Miriam Farber

2011a A lower bound for the second largest Laplacian eigenvalue of weighted graphs. *Electronic J. Linear Algebra* 22 (2011), 1179–1184. MR 2889818. Zbl 1252.05111.

§4, "The signless Laplacian": Upper bounds on second largest eigenvalue of $L(-\Gamma, w)$ for positively edge-weighted (Γ, w). [Annot. 20 Jan 2012.] **(par: WG: Eig)**

Abraham Berman & B. David Saunders

1981a Matrices with zero line sums and maximal rank. *Linear Algebra Appl.* 40 (1981), 229–235. MR 0629620 (82i:15029). Zbl 478.15013. **(QM, sd: ori)**

Abraham Berman, Naomi Shaked-Monderer, Ranveer Singh, & Xiao-Dong Zhang

2019a Complete multipartite graphs that are determined, up to switching, by their Seidel spectrum. *Linear Algebra Appl.* 564 (2019), 58–71. MR 3880868. Zbl 1405.05098. arXiv:1902.02575.

Each is determined up to switching by $\operatorname{Spec} A(K_\Gamma)$. [Annot. 30 Sep 2023.] **(sg: kg: Adj: Eig)**

Olivier Bernardi & Guillaume Chapuy

2010a Counting unicellular maps on non-orientable surfaces. In: *22nd International Conference on Formal Power Series and Algebraic Combinatorics* (FPSAC 2010), pp. 155–166. Discrete Math. Theor. Computer Sci. Proc., AN. The Association. Discrete Mathematics & Theoretical Computer Science (DMTCS), Nancy, 2010. MR 2673832 (2012m:05168). Zbl 1373.05042. **(sg: Top)**

2011a Counting unicellular maps on non-orientable surfaces. *Adv. Appl. Math.* 47 (2011), 259–275. MR 2803802 (2012i:05012). Zbl 1234.05019.

Counting one-face cellular orientation embeddings of all graphs of order n in U_h (sphere with h crosscaps). Exact formulas if all degrees are 1 and 3 (Cors. 8, 9); asymptotic (as $n \to \infty$) in general (Thm. 11). Dictionary: "twist" = negative edge, "flip" of vertex = switching. [*Cf.* Širáň and Škoviera (1991a).] [Annot. 3 Nov 2017.] **(sg: Top)**

Gilles Bernot

See A. Richard.

Daniel Irving Bernstein

2017a Completion of tree metrics and rank 2 matrices. *Linear Algebra Appl.* 533 (2017), 1–13. MR 3695897. Zbl 06778046. arXiv:1612.06797. **(gg)**

Pascal Berthomé, Raul Cordovil, David Forge, Véronique Ventos, & Thomas Zaslavsky

2009a An elementary chromatic reduction for gain graphs and special hyperplane arrangements. *Electronic J. Combin.* 16 (2009), no. 1, art. R121, 31 pp. MR 2546324 (2010k:05253). Zbl 1188.05076. arXiv:1001.4216. HAL inria-00491020.

Calculating chromatic functions (which satisfy deletion-contraction for zero-gain edges and equal 0 if there is a balanced loop) by eliminating or adding identity-gain edges. Application to integral, modular, and zero-free chromatic polynomials of the Shi, Linial, Catalan, and intermediate hyperplane arrangements via their gain graphs [*cf.* Stanley (1999a)]. [See Ardila (2007a) for some of the corresponding coboundary and Tutte polynomials.] **(GG: Invar, Geom)**

E.A. Bespalov

2014a On switching nonseparable graphs with switching separable subgraphs. *Siberian Electronic Math. Rep.* 11 (2014), 988–998. MR 3488478. Zbl 1326.05118.

Cf. D.S. Krotov (2010a). Classifies Γ for which deleting single vertices is insufficient. [Annot. 31 Jul 2018.] **(tg: Sw: Str)**

E.A. Bespalov & D.S. Krotov

2016a On a test for the switching separability of graphs modulo q. (In Russian.) *Sibirsk. Mat. Zh.* 57 (2016), no. 1, 10–24.

2016b On one test for the switching separability of graphs modulo q. (English trans.) *Siberian Math. J.* 57 (2016), no. 1, 7–17. MR 3499848. Zbl 1338.05211. arXiv:-1412.2947.

For $g : E(K_n) \to \mathbb{F}_q^+$, let $E(\Gamma_g) := E(K_n) \setminus g^{-1}(0)$. For $f : V \to \mathbb{F}_q$, let $\delta f(vw) := f(v) + f(w)$, i.e., $\delta f := \mathrm{H}(-\Gamma)f$. A "switching" of Γ_g (a nonstandard switching) = any $\Gamma_{g+\delta f}$. "Separable" = nontrivially disconnected; *cf.* D.S. Krotov (2010a). "Switching separable" = switches to a separable graph. Thm. 1 classifies Γ that are switching inseparable but with all single-vertex deletions switching separable: ∃ iff q even and odd $n > 4$; all are determined in Prop. 3. [Annot. 31 Jul 2018.]

(par: Sw(Gen), incid: Str)

Ouahiba Bessouf

1999a *Théorème de Menger dans les graphes biorientés*. Thèse de Magister, Fac. des Mathématiques, Université des Sciences et de la Technologie Houari Boumediene, Alger, 1999. **(SG: Ori, Str)**

Ouahiba Bessouf & Abdelkader Khelladi

2018a New concept of connection in bidirected graphs. *RAIRO–Oper. Res.* 52 (2018), 351–357. MR 3817469. Zbl 1398.05119. **(SG: Ori: Str)**

Ouahiba Bessouf, Abdelkader Khelladi, Meltem Öztürk & Alexis Tsoukiàs

2023a Bi-oriented graphs and four valued logic for preference modelling. *Ann. Oper. Res.* 328 (2023), 1239–1262. MR 4628468. HAL hal-03845896. **(SG: Ori: Appl)**

Ouahiba Bessouf, Abdelkader Khelladi, & Thomas Zaslavsky

2019a Transitive closure and transitive reduction in bidirected graphs. *Czechoslovak Math. J.* 69 (2019), no. 2, 295–315. MR 3959945. arXiv:1610.00179.

Fundamentals, generalizing from digraphs. Complications arise. Matroidal aspects: quasibalance and closure. [Annot. 4 Jun 2019.] **(SG: Ori: Str)**

20xxb New concept of connection in signed graphs. Under revision. arXiv:1708.01689. **(SG: Str)**

Stéphane Bessy

See J. Bang-Jensen.

Kenchappa S. Betageri

See also V.S. Shigehalli.

2016a The reduced color energy of graphs. *J. Computer Math. Sci.* 7 (2016), 13–20.

Cf. Adiga, Sampathkumar, *et al.* (2013a). **(sg: Adj: Eig)**

K.S. Betageri & G.H. Mokashi

2016a A note on reduced color energy of graphs. *J. Computer Math. Sci.* 7 (2016), no. 4, 203–212.

Cf. Adiga, Sampathkumar, *et al.* (2013a). **(sg: Adj: Eig)**

Nadja Betzler

See F. Hüffner.

D. Bharathi

See S. Sajana.

Mushtaq A. Bhat

See also S. Pirzada and T. Shamsher.

2015a *Energy of Graphs and Digraphs.* Ph.D. thesis, Univ. of Kashmir, Srinagar, 2015. **(SD: Adj: Eig)**

2017a Energy of weighted digraphs. *Discrete Appl. Math.* 223 (2017), 1–14. MR 3627295. Zbl 1471.05039. **(SG, WG: Eig)**

Mushtaq A. Bhat & S. Pirzada

2015a On equienergetic signed graphs. *Discrete Appl. Math.* 189 (2015), 1–7. MR 3348023. Zbl 1316.05055. **(SG: Adj: Eig)**

2016a Spectra and energy of bipartite signed digraphs. *Linear Multilinear Algebra* 64 (2016), no. 9, 1863–1877. MR 3509506. Zbl 1341.05150. arXiv:1501.00572. **(SG: Eig)**

2017a Unicyclic signed graphs with minimal energy. *Discrete Appl. Math.* 226 (2017), 32–39. MR 3659378. Zbl 1365.05118. **(SG: Adj: Eig)**

Mushtaq A. Bhat, U. Samee, & S. Pirzada

2018a Bicyclic signed graphs with minimal and second minimal energy. *Linear Algebra Appl.* 551 (2018), 18–35. MR 3797933. Zbl 1416.05130. **(SG: Adj: Eig)**

Pradeep G. Bhat & Sabitha D'Souza

2013a Energy of binary labeled graphs. *Trans. Combin.* 2 (2013), no. 3, 53–67. MR 3509506. Zbl 1302.05048.

The authors introduce a "label [adjacency] matrix" $A_l(\Gamma, \zeta)$ (see below) where $\zeta : V \to \{0, 1\}$ and a, b, c are distinct real numbers. They investigate the characteristic polynomial $\varphi(\lambda)$ and energy $E_l := \sum |\lambda_i|$ where λ_i are the eigenvalues. Thms. 2.1, 2.2: Top four coefficients of φ. Thms. 2.3, 2.4: Properties of E_l. Thm. 2.5: Eigenvalue upper bound. §3, "Label energies of some families of graphs": K_n, $K_{1,n}$, $K_{r,s}$, double star. Thm. 3.4: If K_n has m 0-labelled vertices, the eigenvalues are $-a$ (multiplicity $m - 1$), $-b$ (multiplicity $n - m - 1$), and the roots of a quadratic.

[*Problem.* Define a "generic adjacency matrix" $A_l(\mathrm{B})$ of a bidirected simple graph B by $a_{l;ij} = a$ if v_iv_j is extraverted, b if v_iv_j is introverted, c if v_iv_j is positive, and 0 otherwise, where a, b, c are generic numbers, indeterminates, etc. (For a non-simple graph, sum the values of edges v_iv_j.) Given a vertex-signed graph (Γ, ζ), bidirect it by $\tau(v_i, v_iv_j) := \zeta(v_i)$; that is, every vertex is a source (if $\zeta(v_i) = -$) or sink $\zeta(v_i) = +$). Note that $\{0, 1\} \cong \{+, -\}$. Does this matrix of a bidirected graph have interesting properties?] [Annot. 3 Oct 2013.] **(VS: Eig)(sg: ori: Eig)**

2015a Color Laplacian energy of a graph. *Proc. Jangjeon Math. Soc.* 18 (2015), no. 3, 321—330. MR 3410554. Zbl 1337.05036.

Cf. Adiga, Sampathkumar, *et al.* (2013a). **(sg: Lap: Eig)**

2017a Color signless Laplacian energy of graphs. *AKCE Int. J. Graphs Combin.* 14 (2017), 142–148. MR 3701601. Zbl 1372.05126.

Laplacian energy of $-\Sigma$, *cf.* Adiga, Sampathkumar, *et al.* (2013a). [Annot. 22 Dec 2018.] **(sg: Lap: Eig)**

R.N. Bhatt & A.P. Young

1985a Search for a transition in the three-dimensional $\pm J$ Ising spin-glass. *Phys. Rev. Lett.* 54 (1985), no. 9, 924–927. **(Phys, SG: Fr, State)**

1988a Numerical studies of Ising spin glasses in two, three, and four dimensions. *Phys. Rev. Lett.* 37 (1988), no. 10, 5606–5614. **(Phys, SG: Fr, State)**

Bikash Bhattacharjya

See D. Sehrawat.

Amitava Bhattacharya, Uri N. Peled, & Murali K. Srinivasan

2007a Cones of closed alternating walks and trails. *Linear Algebra Appl.* 423 (2007), no. 2-3, 351–365. MR 2312413 (2008j:05132). Zbl 1115.05067.

The cone of Eulerian real-weighted subgraphs of a bidirected all-negative signed graph. **(sg: par: Geom)**

2009a The cone of balanced subgraphs. *Linear Algebra Appl.* 431 (2009), no. 1-2, 266–273. MR 2522574 (2010h:05226). Zbl 1169.05372.

A "balanced subgraph" is an edge 2-colored graph where the red and blue degrees are equal at each vertex. [I.e., a signed graph whose net degree $d^{\pm}(v) = 0, \forall v$. Equivalent to an all-negative signed graph, oriented so that every vertex has equal in- and out-degree, which is the all-negative case of an Eulerian bidirected graph. P.D. Seymour, Sums of circuits, in *Graph Theory and Related Topics*, pp. 341–355, Academic Press, New York, 1979, treated the all-positive case.] The problem is to describe the facets of the convex cone generated by Eulerian subgraphs of an all-negative bidirected graph. [*Problem.* Solve for an arbitrary bidirected graph.] **(sg: Par, Ori: Geom)**

Anindya Bhattacharya & Rajat K. De

2008a Divisive Correlation Clustering Algorithm (DCCA) for grouping of genes: detecting varying patterns in expression profiles. *Bioinformatics* 24 (2008), no. 11, 1359–1366. **(sg: Clu: Algor, Biol)**

2010a Average correlation clustering algorithm (ACCA) for grouping of co-regulated genes with similar pattern of variation in their expression values. *J. Biomedical Informatics* 43 (2010), 560–568. **(sg: Clu: Algor, Biol)**

Gora Bhaumik

See P.A. Jensen.

V.N. Bhave

See E. Sampathkumar.

Mani Bhushan & Raghunathan Rengaswamy

2000a Design of sensor network based on the signed directed graph of the process for efficient fault diagnosis. *Ind. Eng. Chem. Res.* 39 (2000), 999–1019.

Another application to fault diagnosis in chemical engineering, this one to location of sensors. **(SD: Appl)**

Ginestra Bianconi

See V. Ciotti.

Christin Bibby

2018a Representation stability for the cohomology of arrangements associated to root systems. *J. Algebraic Combin.* 48 (2018), 51–75. MR 3836246. Zbl 1401.52033. arXiv:1603.08131.

Precursor of the marked Dowling posets of Bibby and Gadish (2018a). [Annot. 1 Feb 2019.] **(gg: matrd(Gen))**

Christin Bibby & Nir Gadish

2018a Combinatorics of orbit configuration spaces. *Sém. Lotharingien Combin.* 80B (2018), art. 72, 11 pp. MR 3940647. Zbl 1444.55008. arXiv:1804.06863.

Introduces marked Dowling ("S-Dowling") posets. The posets can be viewed as consisting of decorated flats of the frame matroid $\mathbf{F}(\mathfrak{G}K_n^\bullet)$. Dowling's $Q_n(\mathfrak{G}) = \operatorname{Lat}\mathbf{F}(\mathfrak{G}K_n^\bullet)$ is viewed as $\mathscr{F}^{\mathrm{b}} := \{$closed, balanced subgraphs B of $\mathbf{F}(\mathfrak{G}K_n^\bullet)$, suitably ordered. A marking is $m : V(B)^c \to S$. The marked Dowling poset is $\{(N, m) : N \in \mathscr{F}^{\mathrm{b}}\}$, suitably ordered. [More theory of marked Dowling posets in Delucchi–Girard–Paolini (2019a), Paolini (2020a).] [Annot. 1 Feb 2019.] (**gg: Matrd(Gen)**)

I. Bieche, R. Maynard, R. Rammal, & J.P. Uhry

1980a On the ground states of the frustration model of a spin glass by a matching method of graph theory. *J. Phys. A: Math. Gen.* 13 (1980), 2553–2576. MR 0582907 (81g:82037).

The frustration index and ground states of a planar square grid graph can be found by matching in the dual graph. [Solved for all planar graphs by Katai and Iwai (1978a), Barahona (1982b).] [Annot. 29 Aug 2012.] (**SG: Phys, Fr, State(fr), Algor**)

Dan Bienstock

1991a On the complexity of testing for odd holes and induced odd paths. *Discrete Math.* 90 (1991), 85–92. MR 1115733 (92m:68040a). Zbl 753.05046. Corrigendum. *Ibid.* 102 (1992), 109. MR 1168141 (92m:68040b). Zbl 760.05080.

Given a graph. Problem 1: Is there an odd hole on a particular vertex? Problem 2: Is there an odd induced path joining two specified vertices? Problem 3: Is every pair of vertices joined by an odd-length induced path? All three problems are NP-complete. [Obviously, one can replace the graph by a signed graph and "odd length" by "negative" and the problems remain NP-complete.] (**sg: Par: Circ, Paths: Algor**)

Norman Biggs

1974a *Algebraic Graph Theory.* Cambridge Math. Tracts, No. 67. Cambridge Univ. Press, London, 1974. MR 0347649 (50 #151). Zbl 284.05101.

Ch. 19: "The covering graph construction." The covering graphs of gain graphs, with emphasis on automorphisms. Let $\Phi := (\Gamma, \varphi)$ with gain group $\mathbb{Z}_2 E$ and $\varphi(e) = e$. Thm. 19.5: If Γ is t-transitive ($t \geq 1$) [and connected], then $\tilde{\Phi}$ is vertex transitive [actually, t-transitive] and has $n - c(\Gamma)$ components (all isomorphic). [The number of components and the isomorphism of components of $\tilde{\Phi}$ require only connectedness of Φ, because $\operatorname{Aut}\tilde{\Phi}$ acts transitively on each vertex fiber.] 19A: "Double coverings." The signed covering graph of $-\Gamma$. 19B: "The Desargues graph." With $P :=$ Petersen graph, $\widetilde{-P}$ is the Desargues graph. [Annot. 11 July 2009.]

[Tutte (1967a) implicitly develops the double covering of an oriented Σ; it is a self-converse orientation of $\tilde{\Sigma}$.] (**SG, GG: Cov, Aut, bal**)

1993a *Algebraic Graph Theory.* Second ed. Cambridge Math. Library. Cambridge Univ. Press, Cambridge, Eng., 1993. MR 1271140 (95h:05105). Zbl 797.05032.

As in (1974a), but 19A, 19B have become Additional Results 19a, 19b. (**SG, GG: Cov, Aut, bal**)

1997a International finance. In: Lowell W. Beineke and Robin J. Wilson, eds., *Graph Connections: Relationships between Graph Theory and other Areas of Mathematics*, Ch. 17, pp. 261–279. The Clarendon Press, Oxford, 1997. MR 1634542 (99a:05001) (book). Zbl 876.90014.

A model of currency exchange rates in which no cyclic arbitrage is possible, hence the rates are given by a potential function. [That is, the exchange-rate gain graph is balanced, with the natural consequences.] Assuming cash exchange without accumulation in any currency, exchange rates are determined. [*Cf.* Ellerman (1984a).]

(**GG, gn: Bal: Exp**)

Biju K

See S. Hameed.

Yonatan Bilu & Nathan Linial

2004a Ramanujan signing of regular graphs. *Combin. Probab. Comput.* 13 (2004), no. 6, 911–912. Zbl 1060.05040.

Conjecture 2 (based on (2006a)). Every d-regular Ramanujan graph can be signed so it has spectral radius $\leq 2\sqrt{d-1}$. *Conjecture* 3. The same for every d-regular graph. Dictionary: "2-lift" = signed covering graph. [Annot. 2 Mar 2011.] (**SG: Eig, Cov**)

2006a Lifts, discrepancy and nearly optimal spectral gap. *Combinatorica* 26 (2006), no. 5, 495–519. MR 2279667 (2008a:05160). Zbl 1121.05054.

Reproves the eigenvalue theorem of Fowler (2002a). [Annot. 2 Mar 2011, 13 Jan 2015.] (**SG: Eig, Cov**)

K. Binder & A.P. Young

1986a Spin glasses: Experimental facts, theoretical concepts, and open questions. *Rev. Modern Phys.* 58 (1986), no. 4, 801–976.

§ III.F.2, "Frustration and gauge invariance": A valuable summary of the state of knowledge and speculation at the time. Signed graphs with spin set $\{+1, -1\}$ (Ising spins) and $U(1)$ ("XY spins" = complex units). Frustration is treated via girth circles ("plaquettes") in lattice graphs, where the girth is 3 or 4 (triangular or square planar lattice). Analytic solutions being too difficult, results are numerical, qualitative, or for "simpler limiting cases". XY spins show quantization (*cf.* Villain (1977b)). For 3-dimensional lattices, plaquette duality leads to vector gains in a dual lattice, thence to closed paths of frustrated plaquettes.

In Ch. IV, "Mean-field theory": Complete-graph ("infinite range") models. § IV.A, "Sherrington-Kirkpatrick model and replica-symmetric solutions": Ising models ($\mathfrak{G} = \{+1, -1\}$). § IV.H, "Non-Ising models": Weighted edge signs are random variables. Spins may be normalized vectors (§1, "Isotropic vector spin glasses in zero field") or other. §3, "Other models": "p-spin couplings" = p-uniform complete hypergraphs. Energy valleys and their shapes. Potts models (signed graphs, spins are multivalued).

Dictionary: "site" = vertex, "bond" = edge, "state" = function $s : V \to \mathfrak{G}$, "spin" = value $s(v)$, "ferromagnetic" = positive, "antiferromagnetic" = negative, "quenched variable" = constant (instead of random variable), "gauge group" = gain group, "gauge transformation" = switch-

ing, "ground state" = state minimizing $\#(E \setminus E^{1\mathfrak{G}}(\Phi^s))$. [Annot. 17 Aug 2012.] (**Phys: sg, gg: Fr, State(fr), Sw, Exp, Ref**)

B.D. Bingham, D.D. Olesky, P. van den Driessche

2007a Potentially nilpotent and spectrally arbitrary even cycle sign patterns. *Linear Algebra Appl.* 421 (2007), 24–44. MR 2290683 (2007i:05112). Zbl 1112.05062.

"Even cycle pattern" means the signed digraph has negative cycles of every even length (remark before Thm. 4.1). [Annot. 13 Dec 2020.] (**sd: QM**)

Robert E. Bixby

1981a Hidden structure in linear programs. In: Harvey J. Greenberg and John S. Maybee, eds., *Computer-Assisted Analysis and Model Simplification* (Proc. Sympos., Boulder, Col., 1980), pp. 327–360; discussion, pp. 397–404. Academic Press, New York, 1981. MR 0617930 (82g:00016) (book). Zbl 495.93001 (book). (**GN**)

Türker Bıyıkoğlu & Josef Leydold

2010a Semiregular trees with minimal Laplacian spectral radius. *Linear Algebra Appl.* 432 (2010), 2335–2341. MR 2599863 (2011b:05114). Zbl 1225.05139.

In a semiregular tree T, all internal vertices have the same degree d. Thm. 2: Given $n, d \geq 3$, a semiregular T minimizes $\lambda_{\max}(L(T))$ iff it is a caterpillar. The proof is via $\operatorname{Spec} L(-T)$, which $= \operatorname{Spec} L(T)$ since a signed tree is balanced. [Annot. 21 Jan 2012.] (**sg: par: Eig**)

Türker Biyikoğlu, Marc Hellmuth, & Josef Leydold

† 2009a Largest eigenvalues of the discrete p-Laplacian of trees with degree sequences. *Electronic J. Linear Algebra* 18 (2009), 202–210. MR 2491656 (2010d:05089). Zbl 1169.05335.

The p-Laplacian ($1 < p < \infty$) generalizes the Laplacian matrix acting on vertex functions. [Generalizing to signed graphs:] Define the p-Laplacian of Σ by $\Delta_p(\Sigma)f(u) := \sum_{uv \in E} \operatorname{sgn}[f(u) - \sigma(uv)f(v)] \cdot |f(u) - \sigma(uv)f(v)|^{p-1}$. Then $p = 2$ gives $L(\Sigma)$.] The p-Laplacian of Γ is $\Delta_p(+\Gamma)$ and its signless p-Laplacian is $\Delta_p(-\Gamma)$. Prop. 3.3 *et seq.* concern $\Delta_p(-\Gamma)$. [Unlike with the Laplacian L, switching does not preserve properties, so signs matter in a tree.] [*Problem.* Generalize to signed graphs.] [Annot. 21 Jan 2012.] (**sg: par: Eig: Gen**)

Anders Björner & Bruce E. Sagan

1996a Subspace arrangements of type B_n and D_n. *J. Algebraic Combin.* 5 (1996), 291–314. MR 1406454 (97g:52028). Zbl 864.57031.

Lattices $\Pi_{n,k,h}$ (for $0 < h \leq k \leq n$) consisting of all spanning subgraphs of $\pm K_n^\circ$ that have at most one nontrivial component K, for which either K is balanced and complete and $\#V(K) \geq k$, or K is induced and $\#V(K) \geq h$. (Also a generalization of this.) Characteristic polynomial, homotopy and homology of the order complex, cohomology of the real complement. (**SG: Geom, Matrd(Gen): Invar, col**)

Anders Björner, Michel Las Vergnas, Bernd Sturmfels, Neil White, & Günter M. Ziegler

1993a *Oriented Matroids.* Encycl. Math. Appl., Vol. 46. Cambridge Univ. Press, Cambridge, Eng., 1993. MR 1226888 (95e:52023). Zbl 773.52001.

The adjacency graph of bases of an oriented matroid is signed, using circuit signatures, to make the "signed basis graph". See §3.5, "Basis orientations and chirotopes", pp. 132–3. (**Matrd: SG**)

Anders Björner & Michelle L. Wachs

2004a Geometrically constructed bases for homology of partition lattices of types A, B and D. *Electronic J. Combin.* 11 (2004), no. 2, art. R3, 26 pp. MR 2120098 (2005m:52029). Zbl 1064.05151.

§9, "Interpolating partition lattices": Homology of $\mathcal{L}(\mathcal{H}[\pm K_n^{(T)}])$ where T is the set of vertices that have half edges. [Annot. 12 Aug 2014.] (**sg: Geom**)

J.A. Blackman

See also J.R. Gonçalves and J. Poulter..

1982a Two-dimensional frustrated Ising network as an eigenvalue problem. *Phys. Rev. B* 26 (1982), no. 9, 4987–4996.

The signed square lattice graph, including effect of density of negative edges ("antiferromagnetic bonds"). §III, "Frustration": "Local-mode" eigenvectors of a Hermitian modification D of a Pfaffian adjacency matrix correspond bijectively to negative squares ("frustrated plaquettes"). Weight > 3 of frustrated edge (compared to 1 of satisfied edges) alters the ground state [by switching; because the graph is 4-regular]. §IV, "Frustration at higher density": Numerical studies of an N^2 grid with $N = 50$ [small by today's standards] suggest a change of behavior at $p := l(\Sigma)/\#E \approx .045$ [a value that surely depends on 4-regularity]. Dictionary: "gauge transformation" = switching, "gauge invariance" = switching invariance, "wrong bond" = frustrated edge [in a ground state].

Continued in Blackman and Poulter (1991a) and in Gonçalves, Poulter, and Blackman (1997a) and (1998a). [Annot. 17 May 2013.] (**Phys, SG: State(fr), Eig, WG**)

J.A. Blackman, J.R. Gonçalves, & J. Poulter

1998a Properties of the two-dimensional random-bond $\pm J$ Ising spin glass. *Phys. Rev. E* 58 (1998), no. 2, 1502–1507.

Extension and refinement of Blackman and Poulter (1991a). [Annot. 17 May 2013.] (**Phys, SG: Fr, Adj**)

J.A. Blackman & J. Poulter

1991a Gauge-invariant method for the $\pm J$ spin-glass model. *Phys. Rev. B* 44 (1991), no. 9, 4374–4386.

Square lattice graph with a definite proportion of negative edges. *Cf.* Poulter and Blackman (2001a) for triangular lattice. [Annot. 16 Aug 2018.] (**Phys: sg: Fr**)

Matthew Blair, Rigoberto Flórez, & Antara Mukherjee

2021a Geometric patterns in the determinant Hosoya triangle. *Integers* 21 (2021), art. A90, 24 pp. MR 4321092. Zbl 1490.11019.

Signed quadrilaterals are a tool in §3.1. [Annot. 8 Nov 2022.] (**SG: Appl**)

Franco Blanchini, Elisa Franco, & Giulia Giordano

2014a A structural classification of candidate oscillatory and multistationary biochemical systems. *Bull. Math. Biol.* 76 (2014), 2542–2569. MR 3266818. Zbl 1329.92041. **(SD, Biol: Dyn)**

Daniel Blankschtein, M. Ma, & A. Nihat Berker

1984a Fully and partially frustrated simple-cubic Ising models: Landau-Ginzburg-Wilson theory. *Phys. Rev. B* 30 (1984), no. 3, 1362–1365. **(Phys, SG: Fr, sw)**

Daniel Blankschtein, M. Ma, A. Nihat Berker, Gary S. Grest, & C.M. Soukoulis

1984a Orderings of a stacked frustrated triangular system in three dimensions. *Phys. Rev. B* 29 (1984), no. 9, 5250–5252.

Physics of $(-L_3)\times(+P_m)$, consisting of m all-negative triangular lattice layers $-L_3$, stacked vertically with vertical positive edges forming paths P_m $(0 \ll m \leq \infty)$. The horizontal triangles are negative (the layers are "totally frustrated") while the vertical squares are positive. Ground states ($\zeta : V \to \{+1,-1\}$ that minmize $\#(E^\zeta)^-$) are ground states of $-L_3$ (*cf.* Wannier (1950a)) repeated in every layer. [Annot. 18 Jun 2012.] **(Phys, SG: State(fr), sw)**

Jarosław Błasiok

See C.E. Tsourakakis.

Andreas Blass

1995a Quasi-varieties, congruences, and generalized Dowling lattices. *J. Algebraic Combin.* 4 (1995), 277–294. MR 1346885 (96i:06012). Zbl 857.08002. Errata. *Ibid.* 5 (1996), 167. MR 1382046. Zbl 857.08002.

Treats the generalized Dowling lattices of Hanlon (1991a) as congruence lattices of certain quasi-varieties, in order to calculate characteristic polynomials and generalizations. **(Matrd(gg): Gen: Invar)**

Andreas Blass & Frank Harary

1982a Deletion versus alteration in finite structures. *J. Combin. Inform. System Sci.* 7 (1982), 139–142. MR 0685507 (84d:05087). Zbl 506.05038.

The theorem that deletion index = negation index of a signed graph (Harary (1959b)) is shown to be a special case of a very general phenomenon involving hereditary classes of "partial choice functions". Another special case: deletion index = alteration index of a gain graph [an immediate corollary of Harary–Lindström–Zetterström (1982a), Thm. 2]. **(SG, GG: Bal, Fr)**

Andreas Blass & Bruce Sagan

1997a Möbius functions of lattices. *Adv. Math.* 127 (1997), 94–123. MR 1445364 (98c:06001). Zbl 970.32977.

§3: "Non-crossing B_n and D_n". Lattices of noncrossing signed partial partitions. Atoms of the lattices are defined as edge fibers of the signed covering graph of $\pm K_n^\circ$, thus corresponding to edges of $\pm K_n^\circ$. [The "half edges" are perhaps best regarded as negative loops.] The lattices studied, called $NCB_n, NCD_n, NCBD_n(S)$, consist of the noncrossing members of the Dowling and near-Dowling lattices of the sign group, i.e.,

Lat $\mathbf{F}(\pm K_n^{(T)})$ for $T = [n], \varnothing, [n]\backslash S$, respectively.
(**SG: Geom, Matrd(Gen), Invar, cov**)

1998a Characteristic and Ehrhart polynomials. *J. Algebraic Combin.* 7 (1998), 115–126. MR 1609889 (99c:05204). Zbl 899.05003.

Signed-graph chromatic polynomials are recast geometrically by observing that the number of k-colorings equals the number of points of $\{-k, -k+1, \dots, k-1, k\}^n$ that lie in none of the edge hyperplanes of the signed graph. The interesting part is that this generalizes to subspace arrangements of signed graphs and, somewhat *ad hoc*, to the hyperplane arrangements of the exceptional root systems. [*Cf.* Athanisiadis (1996a), Zaslavsky (20xxi). For applications see articles of Sagan and P. Zhang.]
(**SG, Gen: Matrd(Gen), Geom: col, Invar**)

Matthew Bloss

2003a G-colored partition algebras as centralizer algebras of wreath products. *J. Algebra* 265 (2003), no. 2, 690–710. MR 1987025 (2004e:20020). Zbl 1028.20007.

Let $\mathfrak{G}$ denote any group. The algebra is $\mathbb{C}\,\mathrm{Lat}^{\mathrm{b}}\,\mathbf{F}(\mathfrak{G}K_{2k}(U,W))$ where $\mathrm{Lat}^{\mathrm{b}}\,\mathbf{F}(\mathfrak{G}K_{2k}(U,W))$ = the semilattice of balanced flats of the Dowling lattice $Q_{2k}(\mathfrak{G})$ on a set $V := U \mathbin{\dot\cup} W$ of $2k$ vertices, $U := \{u_1, \dots, u_k\}$, and $W := \{w_1, \dots, w_k\}$.

The definition requires a multiplication on $\mathrm{Lat}^{\mathrm{b}}\,\mathbf{F}(\mathfrak{G}K_{2k}(U,W))$ which involves an indeterminate x. For each balanced flat (equivalently, $\mathfrak{G}$-valued partition) α label its vertices $u_{\alpha i} := u_i$, $w_{\alpha i} := w_i$. Define $\gamma := \alpha \cdot \beta$ by identifying $w_{\alpha i}$ with u_{β_i} in $\alpha \mathbin{\dot\cup} \beta$ (call the result γ'), taking the closure in $\mathbf{F}(\mathfrak{G}K_{3k})$, multiplying by x^m where $m := \#$ of components of γ' contained completely within the identified vertices, and deleting the identified vertices $w_{\alpha i}$. Set $u_{\gamma i} := u_{\alpha i}$ and $w_{\gamma i} := w_{\beta i}$. [Annot. 20 Mar 2011.]
(**gg: matrd: Algeb**)

Avrim Blum

See N. Bansal.

Rafał Bocian, Mariusz Felisiak, & Daniel Simson

2013a On Coxeter type classification of loop-free edge-bipartite graphs and matrix morsifications. In: Nikolaj Björner *et al.*, eds., *15th International Symposium on Symbolic and Numeric Algorithms for Scientific Computing* (SYNASC 2013, Timisoara, Romania, 2013), pp. 115–118. IEEE, 2013. (**SG**)

2014a Numeric and mesh algorithms for the Coxeter spectral study of positive edge-bipartite graphs and their isotropy groups. *J. Comput. Appl. Math.* 259 (2014), part B, 815–827. MR3132846. Zbl 1314.05196. (**SG**)

F.T. Boesch, X. Li [Xiao Ming Li], & J. Rodriguez

1995a Graphs with most number of three point induced connected subgraphs. *Discrete Appl. Math.* 59 (1995), no. 1, 1–10. MR 1326713 (96b:05073) (*q.v.*). Zbl 835.05056.

Two-graphs and switching are mentioned. (**TG, Sw**)

Irina E. Bocharova, Florian Hug, Rolf Johannesson, Boris D. Kudryashov, & Roman V. Satyukov

2011a Some voltage graph-based LDPC tailbiting codes with large girth. *Information Theory (ISIT2011)* (Proc. 2011 IEEE Int. Sympos., St. Petersburg), pp. 732–736. IEEE, 2011. arXiv:1108.0840. (**GG: Cov**)

2012a Searching for voltage graph-based LDPC tailbiting codes with large girth. *IEEE Trans. Information Theory* 58 (2012), no. 4, 2265–2279. MR 2951330. arXiv:1108.0840.
(**GG: Cov**)

Sebastian Böcker & Jan Baumbach

2013a Cluster editing. In: Paola Bonizzoni, Vasco Brattka and Benedikt Löwe, eds., *The Nature of Computation: Logic, Algorithms, Applications* (Proc. 9th Conf. Computability in Europe (CiE 2013), Milan, 2013), pp. 33–44. LNCS, Vol. 7921. Springer, Berlin, 2013. MR 3102002.

l_{clu} Survey. Cluster editing is equivalent to finding the frustration index, cluster index, or p-cluster index of a signed complete graph $\Sigma = (K_n, \sigma)$, often with such restrictions as negating at most k edges at each vertex, or minimizing the total negated weight if edges are weighted. Dictionary: "editing" = negating some edge signs, "(p)-cluster editing" = changing the fewest signs to make Σ^+ into a union of (p) disjoint cliques; 2-cluster editing" = finding a minimum balancing edge set. [The field seems to be unaware of signed graphs, balance, and clustering and that this is the special case of Davis (1967a) with underlying complete graph.] [Annot. 4 Nov 2017.] (**sg: kg: Clu: Algor: Exp, Biol**)

Sebastian Böcker, Falk Hüffner, Anke Truss, & Magnus Wahlström

2009a A faster fixed-parameter approach to drawing binary tanglegrams. In: J. Chen and F.V. Fomin, eds., *Parameterized and Exact Computation* (4th Int. Workshop, IWPEC 2009, Copenhagen), pp. 38–49. Lect. Notes in Computer Sci., Vol. 5917. Springer, Berlin, 2009. MR 2773930 (no rev). Zbl 1273.68157.

The signed graph arises as a graph with edges labelled = (+) or ≠ (−). The "Balanced Subgraph" problem is to find a minimum balancing set. The algorithm of Hüffner, Betzler, and Niedermeier (2007a) is applied. [Annot. 6 Feb 2011.] (**sg: fr: Algor: Appl**)

Alexander Bockmayr

See also H. Siebert.

Alexander Bockmayr & Heike Siebert

2013a Bio-logics: Logical analysis of bioregulatory networks. In: Andrei Voronkov and Christoph Weidenbach, eds., *Programming Logics: Essays in Memory of Harald Ganzinger* (Workshop, Saarbrücken, 2013), pp. 19–34. Lect. Notes in Computer Sci., Vol. 7797. Springer-Verlag, Heidelberg, 2013. MR 3084884. Zbl 1383.92031. (**SD: Dyn**)

Hans L. Bodlaender, Michael R. Fellows, Pinar Heggernes, Federico Mancini, Charis Papadopoulos, & Frances Rosamond

2008a Clustering with partial information. In: Edward Ochmański and Jerzy Tyszkiewicz, eds., *Mathematical Foundations of Computer Science 2008* (Proc. 33rd Int. Symp., MFCS 2008, Torún, Poland, 2008), pp. 144–155. Lect. Notes in Computer Sci., Vol. , Vol. 5162. Springer, Berlin, 2008. MR 2539366 (2011e:05232). Zbl 1173.68596.

Short version, without most proofs, of (2010a). [Annot. 17 Nov 2017.] **(SG: Clu, KG: Algor)**

2010a Clustering with partial information. *Theor. Computer Sci.* 411 (2010), no. 7-9, 1202–1211. MR 2606055 (2011d:05345). Zbl 1213.05222.

The "fuzzy graphs" (not related to fuzzy graph theory) are, in essence, signed simple graphs; "fuzzy edges" are non-edges, which in a "normalization" become signed edges, resulting in a signed K_n. **(SG: Clu, KG: Algor)**

Hans L. Bodlaender & Jurriaan Hage

2012a On switching classes, NLC-width, cliquewidth and treewidth. *Theor. Computer Sci.* 429 (2012), 30–35. MR 2901392. Zbl 1242.05217.

NLC-width and clique-width of a graph vary only linearly by switching, but tree-width can vary arbitrarily. *Cf.* Hliněný *et al.* (2008a) for rank-width. [*Problem.* Generalize the widths to signed graphs and see how they behave on switching classes.] [Annot. 15 Jun 2022.] **(TG: Sw)**

Bernhard G. Bodmann & Vern I. Paulsen

2005a Frames, graphs and erasures. *Linear Algebra Appl.* 404 (2005), 118–146. MR 2149656 (2006a:42047). Zbl 1088.46009. arXiv:math/0406134.

Develops Strohmer and Heath (2003a) and Holmes and Paulsen (2004a). §4, "Two-uniform frames and graphs": In Def. 4.1, the "signature matrix" Q is $A(K_n, \varphi)$, (K_n, φ) = real or complex unit gain graph and (Thm. 4.2) has exactly 2 distinct eigenvalues. Ex. 4.4, 4.5: $Q = A(K_\Gamma)$, K_Γ = signed graph, from conference, skew-conference, and Hadamard matrices. Thm. 4.7: For real 2-uniform frames, $Q = A(K_\Gamma)$ where Γ has 2 eigenvalues. Frame equivalence = graph switching equivalence. Hence, # of inequivalent real 2-uniform frames = # of switching classes = # regular two-graphs. §5, "Graphs and error bounds". Def. 5.8: $\mathcal{G}_m^{(s)}$ = set of (K_n, σ) with frustration index s. Induced complete bipartite subgraphs (up to switching) hinder error bounds and have other significance. Lemma 5.19: E_3 = # of non-triples of a regular two-graph. §6: Specific examples.

[Much literature follows, e.g.: Bodmann–Paulsen–Tomforde (2009a), Duncan, Hoffman, and Solazzo (2010a), Hoffman and Solazzo (2012a), (2018a).] [Annot. 7 Aug 2018.] **(sg, gg: kg: TG: Adj: Geom, Appl)**

Bernhard G. Bodmann, Vern I. Paulsen, & Mark Tomforde

2009a Equiangular tight frames from complex Seidel matrices containing cube roots of unity. *Linear Algebra Appl.* 430 (2009), 396–417. MR 2460526 (2010b:42040). Zbl 1165.42007.

Adjacency matrices of cube-root-of-unity gain graphs on K_n. Dictionary: "Seidel matrix" = adjacency matrix of such a gain graph. [Annot. 27 Apr 2012.] **(gg: kg: Geom, adj)**

T.B. Boffey

1982a *Graph Theory in Operations Research.* Macmillan, London, 1982. Zbl 509.90053.

Ch. 10: "Network flow: extensions." 10.1(g): "Flows with gains," pp. 224–226. 10.3: "The simplex method applied to network problems," subsection "Generalised networks," pp. 246–250. **(GN: ma-**

trd(bases): Exp)

Kenneth P. Bogart

See M.K. Bennett, J.E. Bonin, and J.R. Weeks.

Jan Bok, Richard Brewster, Tomás Feder, Pavol Hell, & Nikola Jedličková

2020a List homomorphism problems for signed graphs. In: Javier Esparza and Daniel Král', eds., *45th International Symposium on Mathematical Foundations of Computer Science* (MFCS 2020, Prague, 2020), art. 20, 14 pp. LIPIcs. Leibniz Int. Proc. Inform., 170. Schloss Dagstuhl. Leibniz-Zentrum für Informatik, Wadern, 2020. MR 4140355. arXiv:2005.05547. (**SG: Hom: Algor**)

2022a List homomorphisms to separable signed graphs. In: Niranjan Balachandran and R. Inkulu, eds., *Algorithms and Discrete Applied Mathematics* (8th Int. Conf., CALDAM 2022, Puducherry, India), pp. 22–35. Lect. Notes in Computer Sci., Vol. 13179. Springer, Cham, 2022. MR 4471900.
Conference version of (20xxb). (**SG: Hom: Algor**)

2023a List homomorphism problems for signed trees. *Discrete Math.* 346 (2023), no. 3, art. 113257, MR 4517412. Zbl 1506.05082. arXiv:2005.05547. (**SG: Hom: Algor**)

20xxb List homomorphisms to separable signed graphs. arXiv:2306.06449. (**SG: Hom: Algor**)

Jan Bok, Richard Brewster, Pavol Hell, Nikola Jedličková, & Arash Rafiey

2022a Min orderings and list homomorphism dichotomies for signed and unsigned graphs. In: Armando Castañeda and Francisco Rodríguez-Henríquez, eds., *Latin 2022: theoretical informatics* (15th Latin American Sympos., Guanajuato, Mexico, 2022), pp. 510–526. Lect. Notes in Computer Sci., Vol. 13568. Springer, Cham, 2022. MR 4540186.
Conference version of (20xxa). (**SG: Hom: Algor**)

20xxa Min orderings and list homomorphism dichotomies for signed and unsigned graphs. arXiv:2206.01068. (**SG: Hom: Algor**)

Petre Boldescu

1970a Les théorèmes de Menelaus et Ceva dans un éspace affine de dimension n. [The theorems of Menelaus and Ceva in an n-dimensional affine space.] (In Romanian. French summary.) *An. Univ. Craiova Ser. a IV-a* 1 (1970), 101–106. MR 0333932 (48 #12251). Zbl 275.50008.
Generalized Ceva [strengthened via gain graphs in Zaslavsky (2003b) §2.6] and Menelaus theorems. [*Problem.* Formulate, explain, generalize Boldescu's Menelaus generalization in terms of gain graphs.] (**gg: Geom**)

Paolo Boldi & Sebastiano Vigna

1996a On some constructions which preserve sense of direction. In: *Proceedings of the 3rd International Colloquium on Structural Information and Communication Complexity* (SIROCCO'96, Siena, 1996), pp. 47–57. Carleton Scientific, Waterloo, Ont., Canada, 1996.
Preliminary version of (2000a). (**GG: Cov: Appl**)

2000a Coverings that preserve sense of direction. *Inform. Processing Lett.* 75 (2000), no. 4, 175–180. MR 1783445 (2001d:05143). Zbl 997.05076.

The sense-of-direction edge labelling is a gain function on a digraph whose covering graph preserves sense-of-direction under certain conditions. [Annot. 2 Apr 2022.] (**GG: Cov: Appl**)

Ethan D. Bolker

1977a Bracing grids of cubes. *Environment and Planning B* 4 (1977), 157–172.

The elementary 1-cycles associated with circuits of $\mathbf{F}(-\Gamma)$ ("bicycles") are crucial. [Their first publication, I believe.] (**ECyc, sg: matrd**)

1979a Bracing rectangular frameworks. II. *SIAM J. Appl. Math.* 36 (1979), 491–503. MR 0531610 (81j:73066b). Zbl 416.70010.

The elementary 1-cycles associated with circuits of $\mathbf{F}(\Sigma)$ ("bicycles"), mostly for $\Sigma = -\Gamma$. General signed graphs appear at Thm. 7, p. 505. Dictionary: "Signed bicycle" = elementary 1-cycle (circulation) associated with a circuit. (**ECyc, SG: Matrd, incid**)

Ethan D. Bolker & Thomas Zaslavsky

2006a A simple algorithm that proves half-integrality of bidirected network programming. *Networks* 48 (2006), no. 1, 36–38. MR 2243932 (2007b:05098). Zbl 1100.05046.

An idea of Bolker's (1979a), as developed by Bouchet (1983a), is turned into an algorithm slightly simpler than that of Appa and Kotnyek (2006a). (**SG: Ori, Incid, Algor, Sw**)

Béla Bollobás

1978a *Extremal Graph Theory.* L.M.S. Monographs, Vol. 11. Academic Press, London, 1978. MR 0506522 (80a:05120). Zbl 419.05031. Repr. Dover Publications, Mineola, N.Y., 2004. MR 2078877 (2005b:05124). Zbl 1099.05044

A rich source of problems: find interesting generalizations to signed graphs of questions involving even or odd circles, or bipartite graphs or subgraphs. (**par: Xtreml**)

§3.2, Thm. 2.2, is Lovász's (1965a) characterization of the graphs having no two vertex-disjoint circles. [*Problem.* Generalize to biased graphs having no two vertex-disjoint unbalanced circles, Lovász's theorem being the contrabalanced case.] (**GG: Circ**)

§6.6, Problem 47, is the theorem on biparticity (all-negative vertex frustration number) from Bollobás, Erdős, Simonovits, & Szemerédi (1978a). (**par: Fr**)

1998a *Modern Graph Theory.* Springer, New York, 1998. MR 1633290 (99h:05001). Zbl 902.05016.

Sign-colored plane graphs in Ch. X, "The Tutte polynomial", §6, "Polynomials of knots and links", pp. 368–370. Little use is made of the signs. (**SGc: Knot**)

B. Bollobás, P. Erdös, M. Simonovits, & E. Szemerédi

1978a Extremal graphs without large forbidden subgraphs. In: B. Bollobás, ed., *Advances in Graph Theory* (Proc. Cambridge Combin. Conf., 1977), pp. 29–41. Ann. Discrete Math., Vol. 3. North-Holland, Amsterdam, 1978. MR 0499108 (80a:05119). Zbl 375.05034.

Thm. 9 asymptotically estimates upper bounds on frustration index and vertex frustration number for all-negative signed graphs with fixed negative girth. [Sharpened by Komlós (1997a).] **(par: Fr)**

Béla Bollobás & András Gyárfás

2008a Highly connected monochromatic subgraphs. *Discrete Math.* 308 (2008), no. 9, 1722–1725. MR 2392611 (2009b:05183). Zbl 1137.05025.

See Łuczak (2016a). [Annot. 24 Jan 2016.] **(sg: Str)**

Bela Bollobás, Luke Pebody, & Oliver Riordan

2000a Contraction-deletion invariants for graphs. *J. Combin. Theory Ser. B* 80 (2000), 320–345. MR 1794697 (2001j:05055). Zbl 1024.05028.

§4, "Coloured graphs". **(SGc: Gen: Invar)**

Bela Bollobás & Oliver Riordan

1999a A Tutte polynomial for coloured graphs. Recent Trends in Combinatorics (Mátraháza, 1995). *Combin. Probab. Comput.* 8 (1999), 45–93. MR 1684623 (2000f:05033). Zbl 926.05017.

Discovers the fundamental relations for the commutative algebra underlying the parametrized Tutte polynomial of colored graphs. *Cf.* Zaslavsky (1992b). **(SGc: Gen: Invar, Knot)**

2002a A polynomial of graphs on surfaces. *Math. Ann.* 323 (2002), no. 1, 81–96. MR 1906909 (2003b:05052). Zbl 1004.05021.

The polynomial is a deletion-contraction invariant of signed graphs with rotation systems (called "ribbon graphs"). **(sg: Top: Incid)**

S. Bolouki

See S. Akbari.

Erik G. Boman, Doron Chen, Ojas Parekh, & Sivan Toledo

2005a On factor width and symmetric H-matrices. *Linear Algebra Appl.* 405 (2005), 239–248. MR 2148173 (2006e:15024). Zbl 1098.15014.

A real symmetric matrix $= \mathrm{H}(\Phi)\mathrm{H}(\Phi)^{\mathrm{T}}$ for a real gain graph Φ with a link (called "factor width 2"). Thm. 9. A has factor width 2 iff it is a symmetric H-matrix with diagonal ≥ 0. [Annot. 8 Mar 2011.] **(gg: Incid, Adj)**

Phillip Bonacich

1999a An algebraic theory of strong power in negatively connected exchange networks. *J. Math. Sociology* 23 (1999), no. 3, 203–224. Zbl 1083.91574.

P. 214: The distribution of power depends in part on whether $\mathrm{H}(-\Gamma)$ has full rank, i.e., Γ is bipartite (*cf.* van Nuffelen (1973a)), where Γ is the graph of potential exchanges. [Annot. 13 Aug 2012.] **(par: Incid, PsS)**

2007a Some unique properties of eigenvector centrality. *Social Networks* 29 (2007), 555–564.

§1.1.3, "Uses of $c(\beta)$ and x in signed graphs". [Annot. 12 Sept 2010.] **(SG, PsS: Eig)**

Phillip Bonacich & Paulette Lloyd

2004a Calculating status with negative relations. *Social Networks* 26 (2004), 331–338.

Compares the dominant-eigenvector measure of centrality in Σ, Σ^+, and dense induced subgraphs, in a standard example. [Annot. 22 Oct 2009.] (**SG: PsS: Eig**)

Valerio Boncompagni, Irena Penev, & Kristina Vušković

2017a Clique-cutsets beyond chordal graphs. IX Latin-American Algorithms, Graphs, and Optimization Sympos., Marseille (LAGOS'17). *Electronic Notes Discrete Math.* 62 (2017), 81–86. MR 3746703. Zbl 1383.05227.

Extended abstract of (2019a). (**sg: Str**)

2019a Clique-cutsets beyond chordal graphs. *J. Graph Theory* 91 (2019), no. 2, 192–246. MR 3948128. Zbl 1414.05126. arXiv:1707.03252. (**sg: Str**)

J.A. Bondy & L. Lovász

1981a Cycles through specified vertices of a graph. *Combinatorica* 1 (1981), 117–140. MR 0625545 (82k:05073). Zbl 492.05049.

If Γ is k-connected [and not bipartite], then any k [$k-1$] vertices lie on an even [odd] circle. [*Problem.* Generalize to signed graphs, this being the all-negative case.] (**sg: par, Circ**)

J.A. Bondy & M. Simonovits

1974a Cycles of even length in graphs. *J. Combin. Theory Ser. B* 16 (1974), 97–105. MR 0340095 (49 #4851). Zbl 283.05108.

If a graph has enough edges, it has even circles of all moderately small lengths. [*Problem* 1. Generalize to positive circles in signed graphs, this being the antibalanced (all-negative) case. For instance, *Problem* 2. If an unbalanced signed simple graph has positive girth $\geq l$ (i.e., no balanced circle of length $< l$), what is its maximum size? Are the extremal examples antibalanced? Balanced?] (**par: bal(Circ), Xtreml**)

Joseph E. Bonin

See also M.K. Bennett.

1989a (as Joseph Edmond Bonin) *Structural Properties of Dowling Geometries and Lattices.* Ph.D. thesis,. Dartmouth College, 1989. MR 2638328. (**gg: Matrd: Str**)

1993a Automorphism groups of higher-weight Dowling geometries. *J. Combin. Theory Ser. B* 58 (1993), 161–173. MR 1223690 (94k:51005). Zbl 733.05027, (Zbl 789.05017).

A weight-k higher Dowling geometry of rank n, $Q_{n,k}(\mathrm{GF}(q)^\times)$, is the union of all coordinate k-flats of $\mathrm{PG}(n-1,q)$: i.e., all flats spanned by k elements of a fixed basis. If $k>2$, the automorphism groups are those of $\mathrm{PG}(n-1,q)$ for $q>2$ and are symmetric groups if $q=2$. *Cf.* Ravagnani (2022a). (**gg: Gen: Matrd, Aut**)

1993b Modular elements of higher-weight Dowling lattices. *Discrete Math.* 119 (1993), 3–11. MR 1234055 (94h:05018). Zbl 808.06012.

See definition in (1993a). For $k>2$ the only nontrivial modular flats are the projective coordinate k-flats and their subflats. This gives some information about the characteristic polynomials [which, however, are

still only partially known]. [Kung (1996a), §6, has further results.]
(gg: Gen: Matrd: Invar)

1995a Automorphisms of Dowling lattices and related geometries. *Combin. Probab. Comput.* 4 (1995), 1–9. MR 1336651 (96e:05039). Zbl 950.37335.

The automorphisms of a Dowling geometry of a nontrivial group are the compositions of a coordinate permutation, switching, and a group automorphism. A similar result holds, with two exceptions, if some or all coordinate points are deleted. [A third exception is missed: the jointless Dowling geometry $Q_3^0(\mathbb{Z}_3)$.] [*Cf.* Schwartz (2002a).] **(gg: Matrd: Aut)**

1996a Open problem 6. A problem on Dowling lattices. In: Joseph E. Bonin, James G. Oxley, and Brigitte Servatius, eds., *Matroid Theory* (Proc., Seattle, 1995), pp. 417–418. Contemp. Math., Vol. 197. Amer. Math. Soc., Providence, R.I., 1996.

Problem 6.1. If a finite matroid embeds in the Dowling geometry of a group, does it embed in the Dowling geometry of some finite group? [No; see Brooksbank, Qin, Robertson, and Seress (2004a).] **(gg: Matrd)**

2003a Strongly inequivalent representations and Tutte polynomials of matroids. *Algebra universalis* 49 (2003), 289–303. MR 2021388 (2004i:05027). Zbl 1090.05009.

Dowling geometries are used to prove Prop. 1.1. [Annot. 27 May 2018.]
(gg: Matrd: Invar)

2006a Extending a matroid by a cocircuit. *Discrete Math.* 306 (2006), no. 8–9, 812–819. MR 2234987 (2006m:05045). Zbl 1090.05008.

§4 concerns Dowling lattices. **(GG: Matrd)**

Joseph E. Bonin & Kenneth P. Bogart

1991a A geometric characterization of Dowling lattices. *J. Combin. Theory Ser. A* 56 (1991), 195–202. MR 1092848 (92b:05019). Zbl 723.05033. **(gg: Matrd)**

Joseph E. Bonin & Joseph P.S. Kung

1994a Every group is the automorphism group of a rank-3 matroid. *Geom. Dedicata* 50 (1994), 243–246. MR 1286377 (95m:20005). Zbl 808.05029. **(gg: Matrd: Aut)**

2018a The $\mathcal{G}$-invariant and catenary data of a matroid. *Adv. Appl. Math.* 94 (2018), 39–70. MR 3739496. Zbl 1378.05020. arXiv:1510.00682.

Prop. 4.6. The Dowling matroids $Q_r(\mathfrak{G})$ are an example. [Annot. 10 Jan 2016.] **(gg: Matrd: Invar)**

Joseph E. Bonin & William P. Miller

1999a Characterizing combinatorial geometries by numerical invariants. *European J. Combin.* 20 (1999), 713–724. MR 1730820 (2001a:51007). Zbl 946.05020.

Dowling geometries are characterized amongst all simple matroids by numerical properties of large flats of ranks ≤ 7 (Thm. 3.4); amongst all matroids by their Tutte polynomials. **(gg: Matrd)**

Joseph E. Bonin & Hongxun Qin

2000a Size functions of subgeometry-closed classes of representable combinatorial geometries. *Discrete Math.* 224 (2000), 37–60. MR 1781284 (2001g:05031). Zbl 968.52009.

Extremal matroid theory. The Dowling geometry $Q_3(\mathrm{GF}(3)^\times) = \mathbf{F}(\pm K_3^\bullet)$ appears as an exceptional extremal matroid in Thm. 2.10. The extremal subset of $\mathrm{PG}(n-1,q)$ that does not contain the higher-weight Dowling geometry $Q_{m,m-1}(\mathrm{GF}(q)^\times)$ (see Bonin (1993a)) is found in Thm. 2.14. **(GG, Gen: Matrd: Xtreml, Invar)**

C. Paul Bonnington & Charles H.C. Little

1995a *The Foundations of Topological Graph Theory.* Springer, New York, 1995. MR 1367285 (97e:05071). Zbl 844.05002.

Signed-graph imbedding: see §2.3, §2.6 (esp. Thm. 2.4), pp. 44–48 (for the colorful 3-gem approach to crosscaps), §3.3, and Ch. 4 (esp. Thms. 4.5, 4.6). **(sg: Top, bal)**

Stefan Bornholdt

See J. Reichardt.

Bojana Borovićanin

See J.F. Wang.

E. Boros, Y. Crama, & P.L. Hammer

1992a Chvàtal cuts and odd cycle inequalities in quadratic 0—1 optimization. *SIAM J. Discrete Math.* 5 (1992), 163–177. MR 1157581 (93a:90043). Zbl 761.90069.

§4: "Odd cycles [i.e., negative circles] in signed graphs." Main problem: Find a minimum-weight deletion set in a signed graph with positively weighted edges. Related problems: A circle-covering formulation whose constraints correspond to negative circles. A dual circle-packing problem. **(SG: Fr, Geom, Circ, Algor)**

Endre Boros, Vladimir A. Gurvich, & Igor E. Zverovich

2010a Friendship two-graphs. *Graphs Combin.* 26 (2010), no. 5, 617–628. MR 2679935 (2011i:05106). Zbl 1228.05148.

Oriented all-negative graphs in which every two vertices are joined by a unique coherent path. (The authors describe this as alternating paths in an edge 2-colored graph. The "two-graph" is the pair of monocolored graphs.) [*Problem.* Generalize to arbitrary bidirected graphs.] [*Cf.* Bánkfalvi and Bánkfalvi (1968a) and Bang-Jensen and Gutin (1997a) for alternating walks.] [Annot. 25 Oct 2012.] **(sg: ori: Paths)**

Endre Boros & Peter L. Hammer

1991a The max-cut problem and quadratic 0—1 optimization; polyhedral aspects, relaxations and bounds. *Ann. Operations Res.* 33 (1991), 151–180. MR 1140978 (92j:90049). Zbl 741.90077.

Includes finding a minimum-weight deletion set (as in Boros, Crama, and Hammer (1992a)). **(SG, WG: Fr: Geom, Algor)**

Y.M. Borse

See G. Mundhe.

Bartłomiej Bosek

See M. Anholcer.

J.-P. Bouchaud, F. Krzakala, & O.C. Martin

2003a Energy exponents and corrections to scaling in Ising spin glasses. *Phys. Rev. B* 68 (2003), art. 224404, 11 pp.

Mostly, randomly weighted signed graphs (square and cubic lattices) with Gaussian signed weights. §VII, "Case of $+/-J$ couplings": Calculation experiments suggest unweighted signed graphs behave very differently from weighted ones. "[T]he local environment of a spin has no disorder out to finite distances: any sign of the Jij can be gauged [switched] away …". [That seems to mean imbalance can be switched away, which is wrong and casts doubt on the conclusions.] [Annot. 28 Jan 2015.] (**Phys: SG, WG**)

André Bouchet

1982a Constructions of covering triangulations with folds. *J. Graph Theory* 6 (1982), 57–74. MR 0644741 (83b:05057). Zbl 488.05032. (**sg: Ori, Appl(Top)**)

† 1983a Nowhere-zero integral flows on a bidirected graph. *J. Combin. Theory Ser. B* 34 (1983), 279–292. MR 0714451 (85d:05109). Zbl 518.05058.

Introduces nowhere-zero flows on signed graphs. A connected, coloop-free signed graph has a nowhere-zero integral flow with maximum weight ≤ 216. The value 216 cannot be replaced by 5, but: *Conjecture*(Bouchet): it can be replaced by 6. [The bidirection is inessential; it is a device to keep track of the flow.] [For progress see Khelladi (1987a), Zýka (1987a), Xu and Zhang (2005a), Raspaud and Zhu (2011a), Akbari, Daemi, *et al.* (2015a), Wei, Tang, and Dan (2014a), Schubert and Steffen (2015a), DeVos, Li, Lu, *et al.* (2021a). See Jensen and Toft (1995a) for other contributions.]

A topological application is outlined. [Annot. ca. 1983.]
(**SG: Matrd, Ori, Flows, Appl(Top)**)

Jean-Marie Bourjolly

1988a An extension of the König–Egerváry property to node-weighted bidirected graphs. *Math. Programming* 41 (1988), 375–384. MR 0955213 (90c:05161). Zbl 653.90083.

[See Sewell (1996a).] (**sg: Ori, GG: Algor**)

J.-M. Bourjolly, P.L. Hammer, & B. Simeone

1984a Node-weighted graphs having the König–Egerváry property. Mathematical Programming at Oberwolfach II (Oberwolfach, 1983). *Math. Programming Stud.* 22 (1984), 44–63. MR 0774233 (86d:05099). Zbl 558.05054. (**par: ori**)

Jean-Marie Bourjolly & William R. Pulleyblank

1989a König–Egerváry graphs, 2-bicritical graphs and fractional matchings. *Discrete Appl. Math.* 24 (1989), 63–82. MR 1011263 (90m:05069). Zbl 684.05036.

[It is hard to escape the feeling that we are dealing with all-negative signed graphs and that something here will generalize to other signed graphs. Especially see Thm. 5.1. Consult the references for related work.] (**Par; Ref**)

Imad Eddine Bousbaa

See H. Belbachir.

Nathan Bowler, Daryl Funk, & Daniel Slilaty

2020a Describing quasi-graphic matroids. *European J. Combin.* 85 (2020), art. 103062, 26 pp. MR 4037634. Zbl 1433.05066. arXiv:1808.00489.

Cf. Geelen–Gerards–Whittle (2018a). (**gg: Matrd**)

Garry S. Bowlin

2009a *Maximum Frustration of Bipartite Signed Graphs*. Doctoral dissertation, Binghamton University, 2009. MR 2713583 (no rev).

Strong results on structure, bounds, and asymptotics of the generalized Gale–Berlekamp switching game, i.e., maximum frustration of a signed $K_{r,s}$ (*cf.* Brown and Spencer (1971a)), by a linear programming method. Improves on Brown and Spencer (1971a) (*q.v.*), Gordon and Witsenhausen (1972a), Solé and Zaslavsky (1994a). [Annot. 9 Sept 2010, 30 Oct 2011.] **(SG: Fr: Geom)**

2012a Maximum frustration in bipartite signed graphs. *Electronic J. Combin.* 19 (2012), no. 4, art. P10, 13 pp. MR 3001647. Zbl 1266.05045.

Maximum frustration is $l_{\max}(\Gamma) := \max_\sigma l(\Gamma,\sigma)$. Thm. 27: $l_{\max}(K_{l,r} = \frac{1}{2}lr\big(1 - 2^{-(l-1)}\binom{l-1}{\lfloor (l-2)/2 \rfloor}\big)$. It is attained uniquely if $2^{l-1}|r$ and not at all otherwise.

Thm. 31: $l_{\max}(K_{5,r}) = \lfloor \frac{25}{16}r \rfloor - \varepsilon_r$ where $\varepsilon_r \in \{0,1\}$, $= 1$ iff $r \equiv 2,4,9,13 \bmod 16$. Thm. 33: $l_{\max}(K_{6,r}) = \lfloor \frac{66}{32}r \rfloor - \varepsilon_r$ where $\varepsilon_r \in \{0,1,2\}$ and depends on $r \bmod 32$ if $r > 6$. Thm. 33: $l_{\max}(K_{7,r}) = \lfloor \frac{154}{64}r \rfloor - \varepsilon_r$ where $\varepsilon_r \in \{0,1\}$ and depends on $r \bmod 64$ if $r > 49$. *Question.* Is ε_r for fixed l bounded by a linear function of l?

Cf. Brown and Spencer (1971a). [Annot. 21 Dec 2014.] **(SG: Fr: Geom)**

Garry Bowlin & Matthew G. Brin

2013a Coloring planar graphs via colored paths in the associahedra. *Int. J. Algebra Computation* 23 (2013), no. 6, 1337–1418. MR 3109450. Zbl 1273.05060. arXiv:1301.3984. **(SG: Bal)**

John Paul Boyd

1969a The algebra of group kinship. *J. Math. Psychology* 6 (1967), 139–167. Repr. in: Samuel Leinhardt, ed., *Social Networks: A Developing Paradigm*, pp. 319–346. Academic Press, New York, 1977. Zbl 172.45501 (172, p. 455a). Erratum. *Ibid.* 9 (1972), 339. Zbl 242.92010. **(SG: Bal)**

S.C. Boyd

See C. Benzaken.

Durmus Bozkurt

See S.B. Bozkurt.

S. Burcu Bozkurt & Durmus Bozkurt

2013a On the signless Laplacian spectral radius of digraphs. *Ars Combin.* 108 (2013), 193–200. MR 3060265. Zbl 1289.05270. **(par: Lap: Eig)**

John Bramsen

2002a Further algebraic results in the theory of balance. *J. Math. Sociology* 26 (2002), 309–319. Zbl 1014.05041.

Algorithmic ideas for estimating $l(\Sigma)$. Remarks on clusterability. **(SH)(SG: Fr: Algor; Clu)**

Franz J. Brandenburg

2002a Cycles in generalized networks. In: Luděk Kučera, ed., *Graph-Theoretic Concepts in Computer Science* (28th Int. Workshop, WG 2002, Český Krumlov, Czech Rep., 2002), pp. 47–56. Lect. Notes in Computer Sci., Vol. 2573. Springer, Berlin, 2002. MR 2062357 (no rev). Zbl 1022.90035.

The effects of gainy and lossy cycles and negative cycles on cheapest flow from source or between two nodes. [Annot. 21 Mar 2011.] (**GN: Algor**)

2003a Erratum: "Cycles in generalized networks". In: Hans L. Bodlaender, ed., *Graph-Theoretic Concepts in Computer Science* (29th Int. Workshop, WG 2003, Elspeet, The Netherlands, 2003), p. 383. Lect. Notes in Computer Sci., Vol. 2880. Springer-Verlag, Berlin, 2003. MR 2080096 (no rev). Zbl 1255.90119.

Results in (2002a) on cheapest flow from source are incorrect. [Annot. 21 Mar 2011.] (**GN: Algor**)

Franz J. Brandenburg & Mao-Cheng Cai

2009a Shortest path and maximum flow problems in networks with additive losses and gains. In: X. Deng, J.E. Hopcroft, and J. Xue, eds., *Frontiers in Algorithmics: Third International Workshop* (FAW 2009, Hefei, China), pp. 4–15. Lect. Notes in Computer Sci., Vol. 5598. Springer-Verlag, Berlin, 2009. Zbl 1248.68211 (no rev).

See (2011a). (**gg: incid: Algor, matrd**)

2011a Shortest path and maximum flow problems in networks with additive losses and gains. *Theor. Computer Sci.* 412 (2011), no. 4-5, 391–401. MR 2778472 (2011k:68052). Zbl 1230.90045.

Additive real gains. The lift matroid is implicit. Contrasts algorithmic complexity of additive with multiplicative gains. [Annot. 30 May 2012.] (**gg: incid: Algor, matrd**)

Benjamin Braun & Sarah Crown Rundell

2014a Hyperoctahedral Eulerian idempotents, Hodge decompositions, and signed graph coloring complexes. *Electronic J. Combin.* 21 (2014), no. 2, Paper 2.35, 21 pp. MR 3244801. Zbl 1300.05091. arXiv:1307.7323. (**SG: Col**)

A.J. Bray

See also G.J. Rodgers.

A.J. Bray, M.A. Moore, & P. Reed

1978a Vanishing of the Edwards-Anderson order parameter in two- and three-dimensional Ising spin glasses. *J. Phys. C: Solid State Phys.* 11 (1978), 1187–1202.

Random edge signs on a hypercubic lattice. [Annot. 12 Aug 2012.] (**Phys: SG: Rand, Fr**)

Richard C. Brewster

See also J. Bok.

Richard C. Brewster, Florent Foucaud, Pavol Hell, & Reza Naserasr

2017a The complexity of signed graph and edge-coloured graph homomorphisms. *Discrete Math.* 340 (2017), 223–235. MR 3578819. Zbl 1351.05099. arXiv:-1510.05502. HAL hal-01520743. (**SG, SGc: Hom, Algor**)

Richard C. Brewster & Timothy Graves

2009a Edge-switching homomorphisms of edge-coloured graphs. *Discrete Math.* 309 (2009), 5540–5546. MR 2567956 (2010k:05078). Zbl 1213.05066. (**gg: Sw, Hom**)

Richard C. Brewster, Arnott Kidner, & Gary MacGillivray

20xxa *k*-colouring (m, n)-mixed graphs with switching. Submitted. arXiv:2203.08070. **(gg: Sw, Col)**

Richard C. Brewster & Mark Siggers

2018a A complexity dichotomy for signed **H**-colouring. *Discrete Math.* 341 (2018), 2768–2773. MR 3843264. Zbl 1393.05108. **(SG: Hom: Algor)**

Matthew G. Brin

See G.S. Bowlin.

T. Britz, D.D. Olesky, & P. Van Den Driessche

2004a Matrix inversion and digraphs: the one factor case. *Electronic J. Linear Algebra* 11 (2004), 115–131. MR 2111518 (2005m:15008). Zbl 1063.05089. **(sd: QM)**

Hajo Broersma

See D. Hu.

Jared C. Bronski & Lee DeVille

2014a Spectral theory for dynamics on graphs containing attractive and repulsive interactions. *SIAM J. Appl. Math.* 74 (2014), no. 1, 83–105. MR 3158797. Zbl 1332.05086. arXiv:1303.0718.

Bounds on the positive index of inertia, n_+, of a weighted graph, in terms of edge signs. [Annot. 20 Mar 2016.] **(SG, WG: Adj: Eig)**

Jared C. Bronski, Lee Deville, & Paulina Koutsaki

2015a The spectral index of signed Laplacians and their structural stability. Manuscript, 2015. arXiv:1503.01069.

How the positive index of inertia of fixed Σ varies with edge weights. [Annot. 20 Mar 2016.] **(SG, WG: Adj: Eig)**

Peter Brooksbank, Hongxung Qin, Edmund Robertson, & Ákos Seress

2004a On Dowling geometries of infinite groups. *J. Combin. Theory Ser. A* 108 (2004), no. 1, 155–158. MR 2087311 (2005e:51014). Zbl 1056.51011.

Solution of Bonin (1996a). They produce a finite gain graph that has gains in no finite group. Dictionary: "Dowling geometry" = frame matroid of a gain graph [not an actual Dowling geometry, which would be impossible since a Dowling geometry determines its group; *cf.* Dowling (1973b)]. **(gg: Matrd)**

A.E. Brouwer, A.M. Cohen, & A. Neumaier

1989a *Distance-Regular Graphs.* Ergeb. Math., Third Ser., Vol. 18. Springer-Verlag, Berlin, 1989. MR 1002568 (90e:05001). Zbl 747.05073.

§1.5, "Taylor graphs and regular two-graphs": Signed complete graphs appear in the form of double covers of the complete graph. §3.8, "Graph switching, equiangular lines, and representations of two-graphs". §7.6C, "2-Transitive regular two-graphs". **(TG: kg, Geom: Exp, Ref)**

Andries E. Brouwer & Willem H. Haemers

2012a *Spectra of Graphs.* Universitext. Springer-Verlag, Berlin, 2012. MR 2882891. Zbl 1231.05001.

§1.1, "Matrices associated to a graph": "Laplace matrix" = Laplacian matrix $L(+\Gamma)$, from the "directed [i.e., oriented] incidence matrix" $\mathrm{H}(+\Gamma)$. "Signless Laplace matrix" = Laplacian matrix $L(-\Gamma)$, from

the "(undirected) [unoriented] incidence matrix" H($-\Gamma$) (with no -1s). Many results employ $L(-\Gamma)$, but signed graphs are ignored; e.g., see §§1.4.5, 14.4.3, "Line graphs" [*cf.* G.R. Vijayakumar *et al.*]. §1.8.2, "Seidel switching", defines the Seidel adjacency matrix $A(K_\Gamma)$ and its switching. Ch. 10, "Regular two-graphs".

$L(-\Gamma)$ appears in: Ch. 3: "Eigenvalues and Eigenvectors of Graphs", §15.3: "Other matrices with at most three eigenvalues". §15.3.1: "Few Seidel eigenvalues"; §15.3.3: "Three signless Laplace eigenvalues". [Annot. 19 Sept 2010, 23 Jan 2012.] (**Par: Eig, incid, TG, sw**)

Floor Brouwer & Peter Nijkamp

1983a Qualitative structure analysis of complex systems. In: P. Nijkamp, H. Leitner, and N. Wrigley, eds., *Measuring the Unmeasurable*, pp. 509–530. Martinus Nijhoff, The Hague, 1983. (**QM, SD: QSol, QSta: Exp**)

Edward M. Brown & Robert Messer

1979a The classification of two-dimensional manifolds. *Trans. Amer. Math. Soc.* 255 (1979), 377–402. MR 0542887 (80j:57007). Zbl 391.57010, (Zbl 414.57003).

Their "signed graph" we might call a type of Eulerian partially bidirected graph. That is, some edge ends are oriented (hence "partially bidirected"), and every vertex has even degree and at each vertex equally many edge ends point in and out ("Eulerian"). More specially, at each vertex all or none of the edge ends are oriented. (**sg: ori: gen: Appl**)

Gerald G. Brown & Richard D. McBride

1984a Solving generalized networks. *Management Sci.* 30 (1984), 1497–1523. MR 0878883 (no rev). Zbl 554.90032. (**GN: Matrd(bases)**)

Gerald G. Brown, Richard D. McBride, & R. Kevin Wood

1985a Extracting embedded generalized networks from linear programming problems. *Math. Programming* 32 (1985), no. 1, 11–31. MR 0787741 (86f:90090). Zbl 574.90060.

Identifying largest embedded generalized network matrices (i.e., incidence matrices of real multiplicative gain graphs) in a matrix is NP-complete. Heuristic algorithms for finding such embedded matrices and using them to speed up linear programming. [Annot. 2 Oct 2009.] (**GN: Incid: Algor**)

John Brown, Chris Godsil, Devlin Mallory, Abigail Raz, & Christino Tamon

2013a Perfect state transfer on signed graphs. *Quantum Information Comput.* 13 (2013), no. 5-6, 511–530. MR 3076338. arXiv:1211.0505. (**SG: Cov**)

Kenneth S. Brown & Persi Diaconis

1998a Random walks and hyperplane arrangements. *Ann. Probab.* 26 (1998), 1813–1854. MR 1675083 (2000k:60138). Zbl 938.60064.

The real hyperplane arrangement representing $-K_n$ is studied in §3D. It leads to a random walk on threshold graphs. (**par: Geom**)

Thomas A. Brown

See also F.S. Roberts.

T.A. Brown, F.S. Roberts, & J. Spencer

1972a Pulse processes on signed digraphs: a tool for analyzing energy demand. Rep. R-926-NSF, Rand Corp., Santa Monica, Calif., March, 1972. **(SDw)**

Thomas A. Brown & Joel H. Spencer

1971a Minimization of ± 1 matrices under line shifts. *Colloq. Math.* 23 (1971), 165–171. MR 0307944 (46 #7059). Zbl 222.05016.

Asymptotic estimates for the Gale–Berlekamp switching game, i.e., $l(K_{r,s})$, the maximum frustration index of signatures of $K_{r,s}$. [Improved by Gordon and Witsenhausen (1972a) and Bowlin (2009a), (2012a).] Also, exact values stated for $r \leq 4$ [extended by Solé and Zaslavsky (1994a) to $r = 5$, which was corrected and generalized by Bowlin (2009a), (2012a)]. [*Cf.* also Fishburn and Sloane (1989a), Carlson and Stolarski (2004a), and Roth and Viswanathan (2007a), (2008a) on Berlekamp's game, where $r = s$.] **(sg: Fr)**

William G. Brown, ed.

1980a *Reviews in Graph Theory.* 4 vols. American Math. Soc., Providence, R.I., 1980. Zbl 538.05001.

See esp.: §208: "Signed graphs (+ or − on each edge), balance" (undirected and directed), Vol. 1, pp. 569–571. **(SG, SD)**

Richard A. Brualdi

1976a Combinatorial properties of symmetric non-negative matrices. In: *Colloquio Internazionale sulle Teorie Combinatorie* (Rome, 1973), Tomo II, pp. 99–120. Atti dei Convegni Lincei, No. 17. Accademia Nazionale dei Lincei, Rome, 1976. MR 0485437 (58 #5275). Zbl 358.05013.

Thm. 8.2.1 of (2006a). [Annot. 13 Oct 2012.] **(sg: par: Adj)**

2006a *Combinatorial Matrix Classes.* Encyc. Math. Appl., Vol. 108. Cambridge Univ. Press, Cambridge, Eng., 2006. MR 2266203 (2007k:05038). Zbl 1106.05001.

§8.2, "Symmetric transportation polytopes": The vertices of the polytope of symmetric, non-negative matrices with given line sums (Thm. 8.2.1, due to Brualdi (1976a), Converse and Katz (1975a), Lewin (1977a)) or bounded line sums (Thms. 8.2.6–8) correspond to the independent sets in the frame matroid $\mathbf{F}(-K_n)$. [*Problem.* Generalize to a polytope whose vertices are associated with independent sets in $\mathbf{F}(\pm K_n)$. Possibly, the matrices have prescribed entry signs determining a signed graph.] [Annot. 13 Oct 2012.] **(sg: par: Adj)**

2011a *The Mutually Beneficial Relationship of Graphs and Matrices.* CBMS Reg. Conf. Ser. Math., No. 115. American Math. Soc., Providence, R.I., 2011 MR 2808017 (2012i:05159). Zbl 1218.05002.

§6.1, "Sign-nonsingular matrices": Signed digraphs, called "weighted digraphs" of $(0, \pm 1)$-matrices such that every matrix with that sign pattern is nonsingular. *Cf.* esp. Maybee *et al.*, van den Driessche *et al.*[Annot. 20 Nov 2011.] **(QM: QSol: sd: Exp)**

§9.4, "ASM patterns": Signed graphs appear in the study of patterns in alternating sign matrices. *Cf.* Brualdi, Kiernan, *et al.* (2013a). [Annot. 18 Nov 2011.] **(SG: Exp)**

Richard A. Brualdi, Kathleen P. Kiernan, Seth A. Meyer, & Michael W. Schroeder

2013a Patterns of alternating sign matrices. *Linear Algebra Appl.* 438 (2013), no. 10, 3967–3990. MR 3034511. Zbl 1281.15034. arXiv:1104.4075. (**SG**)

Richard A. Brualdi & Nancy Ann Neudauer

1997a The minimal presentations of a bicircular matroid. *Quart. J. Math. Oxford* (2) 48 (1997), 17–26. MR 1439695 (97m:05065). Zbl 938.05023.

Minimal transversal presentations of $\mathbf{F}(\Gamma, \varnothing)$, given Γ. (**Bic**)

Richard A. Brualdi & Herbert J. Ryser

1991a *Combinatorial Matrix Theory.* Encycl. Math. Appl., Vol. 39. Cambridge Univ. Press, Cambridge, Eng., 1991. MR 1130611 (93a:05087). Zbl 746.05002.

See §7.5. (**QM: QSol, SD, bal**)(**Exp, Ref**)

Richard A. Brualdi & Bryan L. Shader

1991a On sign-nonsingular matrices and the conversion of the permanent into the determinant. In: Peter Gritzman and Bernd Sturmfels, eds., *Applied Geometry and Discrete Mathematics: The Victor Klee Festschrift*, pp. 117–134. DIMACS Ser. Discrete Math. Theor. Comput. Sci., Vol. 4. American Math. Soc., Providence, R.I., 1991. MR 1116343 (92f:15003). Zbl 742.15001.

§1 reviews Seymour and Thomassen (1987a). Thm. 2.1: If two sign-nonsingular $(0, 1, -1)$-matrices have the same 0's (and total support), their signed digraphs are switching equivalent. [Annot. 12 Jun 2012.] (**QM, SD: QSol: Exp**)

1995a *Matrices of Sign-Solvable Linear Systems.* Cambridge Tracts in Math., Vol. 116. Cambridge Univ. Press, Cambridge, Eng., 1995. MR 1358133 (97k:15001). Zbl 833.15002.

Innumerable results and references on signed digraphs are contained herein. (**QM, SD: QSol, QSta**)(**Exp, Ref, Algor**)

Frank J. Bruggeman

See B.N. Kholodenko.

Jeroen Bruggeman

See V.A. Traag.

Michael Brundage

1996a *From the Even-Cycle Mystery to the L-Matrix Problem and Beyond.* M.S. thesis, Dept. of Mathematics, University of Washington, Seattle, 1996. URL (10/1997) `http://www.math.washington.edu/~brundage/evcy/`

A concise expository survey. Ch. 1: "Even cycles in directed graphs". Ch. 2: "L-matrices and sign-solvability", esp. § "Signed digraphs". Ch. 3: "Beyond", esp. § "Balanced labellings" (vertices labelled from the set $\{0, +1, -1\}$ so that from each vertex labelled $\varepsilon \neq 0$ there is an arc to a vertex labelled $-\varepsilon$) and § "Pfaffian orientations". (**SD, Par: Circ, QSol, Algor, VS: Exp, Ref**)

Maurizio Brunetti

See also A. Alazemi, F. Belardo, and J.-F. Wang.

2020a Laplacian spectral properties of signed circular caterpillars. *Theory Appl. Graphs* 7 (2020), no. 2, art. 1, 23 pp. MR 4119753. Zbl 1447.05120.

Unicycles with pendants that are K_2. For girth 3, extremal spectral radius and first and second Zagreb indices. [Annot. 15 Dec 2020.]

(**SG: Lap: Eig**)

Maurizio Brunetti, Matteo Cavaleri, & Alfredo Donno

2019a A lexicographic product for signed graphs. *Australasian J. Combin.* 74 (2019), no. 2, 332–343. MR 3949477. Zbl 1422.05060.

I.e., composition $\Sigma[\Lambda]$. Modified from Hameed and Germina's (2012b) definition and said to respect switching (but see (2019b)). It has good eigenvalue properties. [Annot. 20 Apr, 5 Jul 2019.] (**SG: Bal, Sw, Eig: Adj, Lap**)

2019b Erratum to the article [(2019a)]. *Australasian J. Combin.* 75 (2019), 256–258. MR 4008250. Zbl 1429.05178.

Their definition does not respect switching. *Problem*: Find a definition that does. [Annot. 2 Sept 2019.] (**SG: Bal, Sw**)

Maurizio Brunetti & Adriana Ciampella

2023a Signed bicyclic graphs with minimal index. *Commun. Combin. Optim.* 8 (2023), no. 1, 207–241. MR 4503662. (**SG: Adj: Eig**)

Maurizio Brunetti & Zoran Stanić

2022a Unbalanced signed graphs with extremal spectral radius or index. *Computational Appl. Math.* (2022) 41 (2022), art. 118, 13 pp. MR 4400622. (**SG: Adj: Eig**)

2022b Ordering signed graphs with large index. *Ars Math. Contemp.* 22 (2022), article P4.05, 14 pp. MR 4498576. Zbl 1497.05109.

Largest max eigenvalue of $A(\Sigma)$ within Σ that are connected, connected unbalanced, and signed complete. [Annot. 17 Jan 2022.] (**SG: Adj: Eig**)

Michael Brusco, Patrick Doreian, Andrej Mrvar, & Douglas Steinley

2011a Two algorithms for relaxed structural balance partitioning: linking theory, models, and data to understand social network phenomena. *Sociological Methods Res.* 40 (2011), no. 1, 57–87. MR 2758299 (no rev). (**SG: PsS, Str**)

Thomas H. Brylawski [Tom Brylawski]

1975a A note on Tutte's unimodular representation theorem. *Proc. Amer. Math. Soc.* 52 (1975), 499–502. MR 0419271 (54 #7294). Zbl 328.05017.

Implicitly, switching in the bipartite gain graph of a matrix. (**gg: sw**)

2000a A Möbius identity arising from modularity in a matroid bilinear form. *J. Combin. Theory Ser. A* 91 (2000), 622–639. MR 1780040 (2002a:05059). Zbl 966.05014.

§5, "q-Analogs from a Möbius identity": §5.1, "Dowling lattices" (an example): A complicated identity is derived from the Möbius function of $Q_n(\mathbb{F}_q^\times)$. [Annot. 26 Dec 2015.] (**gg: Matrd, Invar**)

Thomas Brylawski & James Oxley

1992a The Tutte polynomial and its applications. In: Neil White, ed., *Matroid Applications*, Ch. 6, pp. 123–225. Encycl. Math. Appl., Vol. 40. Cambridge Univ. Press, Cambridge, Eng., 1992. MR 1165543 (93k:05060). Zbl 769.05026.

§6.4, "The critical problem", §6.4.B, "Minimal and tangential blocks", pp. 171–172: Tangential blocks in Dowling geometries $Q_n(\mathrm{GF}(q)^\times)$, after

Whittle (1989a). [Annot. 16 Sept 2011.] **(gg: Matrd)**

J.A. Brzozowski
See C.J. Shi.

Changjiang Bu & Jiang Zhou
2012a Starlike trees whose maximum degree exceed 4 are determined by their Q-spectra. *Linear Algebra Appl.* 436 (2012), 143–151. MR 2859918 (2012j:05251). Zbl 1242.05160.
A subdivided star is determined, among all graphs, by Spec $L(-\Gamma)$. This completes work of Omidi (2009a) and of Omidi and Vatandoost (2010a). [Annot. 28 Nov 2012.] **(par: Lap: Eig)**

2012b Signless Laplacian spectral characterization of the cones over some regular graphs. *Linear Algebra Appl.* 436 (2012), no. 9, 3634–3641. MR 2900741. Zbl 1241.05069.
Let $\Gamma = \Delta \vee K_1$, the join of Δ and a point. Spec $L(-\Gamma)$ determines Γ if: Γ or Γ^c is a matching; 2-regular Γ has $n \geq 11$; 2-regular Γ^c is triangle-free. [This implies graphs with the same Spec $L(-\Gamma)$.]
Also, Thm. 3.6: Spec $L(-[K_{1,3} \vee \Gamma]) =$ Spec $L(-[(C_3 \cup K_1) \vee \Gamma])$ for any Γ. [Annot. 28 Nov 2012.] **(par: Lap: Eig)**

[Miguel Á. Valencia Bucio]
See M.Á. Valencia Bucio (under 'V').

M. Buckland, Brett Kolesnik, Rivka Mitchell, & Tomasz Przybyłowski
20xxa Random walks on Coxeter interchange graphs. In preparation.
Sequel to Kolesnik, Mitchell, and Przybyłowski (20xxa). **(SG: Invar)**

Fred Buckley, Lynne L. Doty, & Frank Harary
1988a On graphs with signed inverses. *Networks* 18 (1988), 151–157. MR 0953918 (89i:05222). Zbl 646.05061.
"Signed invertible graph" [i.e., sign-invertible graph] = graph Γ such that $A(\Gamma)^{-1} = A(\Sigma)$ for some signed graph Σ. Finds two classes of such graphs. Characterizes sign-invertible trees. [*Cf.* Godsil (1985a) and, for a different notion, Greenberg, Lundgren, and Maybee (1984b).] **(SG: Adj)**

Fred Buckley & Frank Harary
1990a *Distance in Graphs.* Addison–Wesley, Redwood City, Calif., 1990. MR 1045632 (90m:05002). Zbl 688.05017.
Signed graphs and sign-invertible graphs (Buckley, Doty, and Harary (1988a)): pp. 120–122. **(SG: Adj: Exp)**

N. Buckley
See C. Perina.

Vadim Bugaenko, Yonah Cherniavsky, Tatiana Nagnibeda, & Robert Shwartz
2015a Weighted Coxeter graphs and generalized geometric representations of Coxeter groups. *Discrete Appl. Math.* 192 (2015), 17–27. MR 3354815. Zbl 1319.05064. arXiv:1304.0692.

Weights are complex gains, or are signs. (**GG, SG: Bal**)

Francesco Bullo

See P. Cisneros-Velarde, P. Jia and W.-J. Mei.

Yurii Burman, Andrey Ploskonosov, & Anastasia Trofimova

2015a Matrix-tree theorems and discrete path integration. *Linear Algebra Appl.* 466 (2015), 64–82. MR 3278240. Zbl 1310.15010. (**gg: Lap**)

James R. Burns & Wayland H. Winstead

1982a Input and output redundancy. *IEEE Trans. Systems Man Cybernetics* SMC-12 (1982), no. 6, 785–793.

§ IV: "The computation of contradictory redundancy." Summarized in modified notation: In a signed graph, define $w_{ij}^{\varepsilon}(r)$ = number of walks of length r and sign ε from v_i to v_j. Define an adjacency matrix A by $a_{ij} = w_{ij}^{+}(1) + w_{ij}^{-}(1)\theta$, where θ is an indeterminate whose square is 1. Then $w_{ij}^{+}(r) + w_{ij}^{-}(r)\theta = (A^r)_{ij}$ for all $r \geq 1$. [We should regard this computation as taking place in the group ring of the sign group, where the sign group is treated as $\{+1, \theta\}$. The generalization to arbitrary gain graphs and digraphs is obvious.] Other sections also discuss signed digraphs [but have little mathematical content]. (**SD, gd: Adj, Paths**)

Eugene Burnstein

See R.B. Zajonc.

Martin Burtscher

See G. Alabandi.

Arthur H. Busch, Michael S. Jacobson, Timothy Morris, Michael J. Plantholt, & Shailesh K. Tipnis

2013a Improved sufficient conditions for the existence of anti-directed hamiltonian cycles in digraphs. *Graphs Combin.* 29 (2013), 359–364. MR 3053585. Zbl 1267.05159.

Improvement of Diwan, Frye, Plantholt, and Tipnis (2011a). [Annot. 5 Jun 2017.] (**gg: Str**)(**sg: par: Ori**)

Arthur Busch, Mohammed A. Mutar, & Daniel Slilaty

2022a Hamilton cycles in bidirected complete graphs. *Contrib. Discrete Math.* 17 (2022), no. 2, 137–149. MR 4531948. Zbl 1502.05083.

A coherent cycle generalizes an alternating circle in an edge-2-colored graph and a directed cycle in a directed graph (*cf.* Bankfalvi and Bankfalvi (1968a)). Strong connection is defined for a bidirected graph. Thm. 3.3: An oriented $\pm K_n$ ($n \geq 3$) has a cycle of every length $3, \ldots, n$ iff it is strongly connected. Thm. 4.4: It has a Hamiltonian cycle iff it is strongly connected and has an alternating 2-factor. *Cf.* Mutar (2017a). [Annot. 11 Aug 2018.] (**SG: Ori: Cyc**)

F.C. Bussemaker, P.J. Cameron, J.J. Seidel, & S.V. Tsaranov

1991a Tables of signed graphs. EUT Report 91-WSK-01. Dept. of Math. and Computing Sci., Eindhoven University of Technology, Eindhoven, 1991. MR 1131079 (92g:05001). (**SG: Sw**)

F.C. Bussemaker, D.M. Cvetković, & J.J. Seidel

1976a Graphs related to exceptional root systems. T.H.-Report 76-WSK-05, 91 pp. Dept. of Math., Technological University Eindhoven, Eindhoven, The Netherlands, 1976. Zbl 338.05116.

The 187 simple graphs with eigenvalues ≥ -2 that are not (negatives of) reduced line graphs of signed graphs are found, with computer aid. By Cameron, Goethals, Seidel, and Shult (1976a), all are represented by root systems E_d, $d = 6, 7, 8$. Most interesting is Thm. 2: each such graph is Seidel-switching equivalent to a line graph of a graph. [*Problem.* Explain this within signed graph theory.] (**LG: par: Eig**)

1978a Graphs related to exceptional root systems. In: A. Hajnal and Vera T. Sós, eds., *Combinatorics* (Proc. Fifth Hungar. Colloq., Keszthely, 1976), Vol. 1, pp. 185–191. Colloq. Math. Soc. János Bolyai, 18. North-Holland, Amsterdam, 1978. MR 0519264 (80g:05049). Zbl 392.05055.

Announces the results of (1976a). (**LG: par: Eig**)

F.C. Bussemaker, R.A. Mathon, & J.J. Seidel

1979a Tables of two-graphs. TH-Report 79-WSK-05. Dept. of Math., Technological University Eindhoven, Eindhoven, The Netherlands, 1979. Zbl 439.05032.

All $[K_n, \sigma]$, also cospectral pairs, and those that are regular, integral, or vertex-transitive, for $n \leq 9$, plus larger special types. [Annot. 29 Dec 2020.] (**TG, Sw, Exp**)(**TG: Adj: Eig**)

1981a Tables of two-graphs. In: S.B. Rao, ed., *Combinatorics and Graph Theory* (Proc. Sympos., Calcutta, 1980), pp. 70–112. Lect. Notes in Math., Vol. 885. Springer-Verlag, Berlin, 1981. MR 0655610 (84b:05055). Zbl 482.05024.

"The most important tables from" (1979a). (**TG, Sw**)(**TG: Adj: Eig**)

F.C. Bussemaker & A. Neumaier

1992a Exceptional graphs with smallest eigenvalue -2 and related problems. *Math. Comput.* 59 (1992), 583–608. MR 1134718 (93a:05089). Zbl 770.05060.

They are the antibalanced signed graphs with largest eigenvalue -2. Also, largest eigenvalue around -2. Two-graphs and work of Vijayakumar *et al.* are mentioned. [Annot. 29 Apr 2012.] (**TG, LG, Eig**)

Steve Butler

2010a Eigenvalues of 2-edge coverings. *Linear Multilinear Algebra* 58 (2010), 413–423. MR 2663442 (2011g:05173). Zbl 1187.05047.

Generalizing D'Amato (1979a) and Bilu and Linial (2006a). The "signed graph" G is vertex-signed; it is a branched double cover of a signed graph H whose edge signs are incorporated into weights. The interesting new idea is the branching, wherein a vertex may be singly covered. [May the branches correspond to half edges?] Adjacency and normalized Laplacian spectra of G are each obtained from those of H and a modified H. [Annot. 9 Mar 2011.] (**VS(Gen: Eig)**)(**SG: cov, Eig**)

Steve Butler, Minerva Catral, H. Tracy Hall, Leslie Hogben, Xavier Martínez-Rivera, Bryan Shader, & Pauline van den Driessche

2017a The enhanced principal rank characteristic sequence for Hermitian matrices. *Electronic J. Linear Algebra* 32 (2017), 58–75. MR 3627007. Zbl 1375.15057.

Example: Hermitian matrices of mixed graphs. Gain group $\{\pm 1, \pm \mathrm{i}\}$:

$\varphi(e) = 1$ for undirected, i for directed edges. **(gg: Adj)**

Jesper Makholm Byskov, Bolette Ammitzbøll Madsen, & Bjarke Skjernaa

2005a On the number of maximal bipartite subgraphs of a graph. *J. Graph Theory* 48 (2005), no. 2, 127–132. MR 2110582 (2005h:05099). Zbl 1059.05045.

Bounds on the number of maximal induced bipartite subgraphs. [*Problem*. Generalize to maximal induced balanced subgraphs, equivalently minimal balancing sets of vertices, especially in a signed graph.] **(par: bal)**

S. Cabasino, E. Marinari, P. Paolucci, & G. Parisi

1988a Eigenstates and limit cycles in the SK model. *J. Phys. A* 21 (1988), no. 22, 4201–4210. MR 0983779 (89k:82070). **(Phys: SG)**

He Cai

See Y. Jiang.

Hongyan Cai

See also C. Wen.

Hongyan Cai, Qiang Sun, Yuanpei Wang, & Guangjun Xu

20xxa Edge coloring of the signed Halin graph. Submitted.

Thm.: $\chi'(H,\sigma) = \Delta(H)$, $\forall\ \sigma$, where H = Halin graph. [Annot. 11 Feb 2022.] **(SG: ECol)**

Hongyan Cai, Qiang Sun, Guangjun Xu, & Shanshan Zheng

2022a Edge coloring of the signed generalized Petersen graph. *Bull. Malaysian Math. Sci. Soc.* 45 (2022), 647–661. MR 4391907. Zbl 1485.05052.

$\chi'(P(n,1)) = \Delta(P(n,1)) = 3$ for $n \geq 5$. For $n = 5, 6$, $\chi'(P(n,2)) = 3$ usually but $= 4$ in a few exceptions. [Annot. 12 Feb 2022.] **(SG: ECol)**

Leishen Cai & Baruch Schieber

1997a A linear-time algorithm for computing the intersection of all odd cycles in a graph. *Discrete Appl. Math.* 73 (1997), 27–34. MR 1431105 (97g:05149). Zbl 867.05066.

By the negative-subdivision trick (subdividing each positive edge into two negative ones), the algorithm will find the intersection of all negative circles of a signed graph. **(Par, sg: Fr, Circ: Algor)**

Mao-cheng Cai

See F.J. Brandenburg.

Qing Cai, Maoguo Gong, Lijia Ma, Shanfeng Wang, Licheng Jiao, & Haifeng Du

2015a A particle swarm optimization approach for handling network social balance problem. In: *2015 IEEE Congress on Evolutionary Computation (CEC)* (Sendai, Japan, 2015), pp. 3186–3991. IEEE, 2015.

"Balance problem" = clustering problem. Degree of inclusterability.

As do many "social balance" articles, wrongly attributes Cartwright–Harary (1956a) balance theory to Heider and incorrectly says the former only treats complete graphs. [Annot. 27 Nov 2018.] **(SG: Clu: Algor)**

Qing Cai, Maoguo Gong, Bo Shen, Lijia Ma, & Licheng Jiao

2014a Discrete particle swarm optimization for identifying community structures in signed social networks. *Neural Networks* 58 (2014), 4–13. **(SG: Clu: Adj, Algor)**

Grant Cairns & Yuri Nikolayevsky

2009a Generalized thrackle drawings of non-bipartite graphs. *Discrete Comput. Geom.* 41 (2009), no. 1, 119–134. MR 2470073 (2010a:05059). Zbl 1191.05032.

Thm. 2: Γ, connected and not bipartite, has a generalized thrackle drawing in the orientable surface of genus g iff $-\Gamma$ has an orientation embedding in the nonorientable surface with demigenus $2g-1$. [*Problem.* Generalize to all signed graphs.] **(sg: Par: Top)**

Tiziana Calamoneri, Angelo Monti, & Rossella Petreschi

2018a On dynamic threshold graphs and related classes. *Theor. Computer Sci.* 718 (2018), 46–57. MR 3775060. Zbl 1388.68212.

"Threshold signed graph" [not a signed graph]: a graph such that $(\exists\ S, T \in \mathbb{R}_{>0})(\exists\ a : V \to \mathbb{R})\ |a(v)| < \min(S, T)$ and $vw \in E \iff |a(v) + a(w)| \geq S$ or $|a(v) - a(w)| \geq T$. [*Cf.* Benzaken, Hammer, and de Werra (1985a), Calamoneri and Petreschi (2014a).] **(VS, sg)**

Uses the auxiliary signed graph of Hammer and Mahadev (1985a). [Annot. 22 Mar 2017.] **(SG: Appl: Bal)**

T. Calamoneri & R. Petreschi

2013a Graphs with Dilworth number two are pairwise compatibility graphs. *Electronic Notes Discrete Math.* 44 (2013), 31–38. **(VS)**

2014a On pairwise compatibility graphs having Dilworth number two. *Theor. Computer Sci.* 524 (2014), 34–40. MR 3163431 (*q.v.*). Zbl 1283.05142. Corrigendum. *Ibid.* 602 (2015), 158–159. MR 3399981 (no rev). Zbl 1283.05142.

Defines "threshold signed graphs"; *cf.* Calamoneri, Monti, and Petreschi (2018a). **(VS)**

Kyle David Calderhead

2002a *Variations on the Slope Problem.* Doctoral dissertation, University of Minnesota, 2002. MR 2703263 (no rev).

Ch. 6, "Type B analogs", introduces threshold signed graphs and applies signed graphs to the slopes problem (the minimum number of slopes of n points in the plane) for centrally symmetric points. A signed graph is threshold if its double cover is a threshold graph. **(SG)**

Laurence Calzone

See J.-P. Comet.

Verónica Cambiazo

See J. Aracena.

Peter J. Cameron

See also L. Babai and F.C. Bussemaker.

1977a Automorphisms and cohomology of switching classes. *J. Combin. Theory Ser. B* 22 (1977), 297–298. MR 0498227 (58 #16382). Zbl 331.05113, (Zbl 344.05128).

The first step towards (1977b), Thm. 3.1. **(TG: Aut)**

† 1977b Cohomological aspects of two-graphs. *Math. Z.* 157 (1977), 101–119. MR 0505778 (58 #21779). Zbl 353.20004, (Zbl 359.20004).

Introducing the cohomological theory of two-graphs. A two-graph τ is a 2-coboundary in the complex of GF(2)-cochains on $E(K_n)$. [The 1-cochains are the signed complete graphs, equivalently the graphs that are their negative subgraphs. *Cf.* D.E. Taylor (1977a).] Write Z_i, Z^i, B^i for the i-cycle, i-cocycle, and i-coboundary spaces. Switching a signed complete graph means adding a 1-cocycle to it; a switching class of signed complete graphs is viewed as a coset of Z^1 and is equivalent to a two-graph.

Take a group $\mathfrak{G}$ of automorphisms of τ. Special cohomology elements $\gamma \in H^1(\mathfrak{G}, B^1)$ and $\beta \in H^2(\mathfrak{G}, \tilde{B}^0)$ (where $\tilde{B}^0 = \{0, V(K_n)\}$, the reduced 0-coboundary group) are defined. Thm. 3.1: $\gamma = 0$ iff $\mathfrak{G}$ fixes a graph in τ. Thm. 5.1: $\beta = 0$ iff $\mathfrak{G}$ can be realized as an automorphism group of the canonical double covering graph of τ (viewing τ as a switching class of signed complete graphs). Conditions are explored for the vanishing of γ (related to Harries and Liebeck (1978a)) and β.

Z^1 is the annihilator of Z_1 = the space of even-degree simple graphs; the theorems of Mallows and Sloane (1975a) follow immediately. More generally: Lemma 8.2: Z^i is the annihilator of Z_i. Thm. 8.3: The numbers of isomorphism types of i-cycles and i-cocycles are equal, for $i = 1, \ldots, n-2$.

§8 concludes with discussion of possible generalizations, e.g., to oriented two-graphs (replacing GF(2) by $\mathrm{GF}(3)^\times$) and double coverings of complete digraphs (Thms. 8.6, 8.7). [*Cf.* Moorhouse (1995a). A full ternary analog is developed in Cheng and Wells (1984a)]

(**TG: Sw, Aut, Enum, Geom**)

1979a Cohomological aspects of 2-graphs. II. In: C.T.C. Wall, ed., *Homological Group Theory* (Proc. Sympos., Durham, 1977), Ch. 11, pp. 241–244. London Math. Soc. Lect. Note Ser. 36. Cambridge Univ. Press, Cambridge, 1979. MR 0564428 (81a:05061). Zbl 461.20001.

Exposition of parts of (1977b) with a simplified proof of the connection between β and γ. (**TG: Aut, Enum, Geom, Exp**)

1980a A note on generalized line graphs. *J. Graph Theory* 4 (1980), 243–245. MR 0570359 (81j:05089). Zbl 403.05048, (Zbl 427.05039).

[For generalized line graphs see Zaslavsky (1984c).] If two generalized line graphs are isomorphic, their underlying graphs and cocktail-party attachments are isomorphic, with small exceptions related to exceptional isomorphisms and automorphisms of root systems. The proof, along the lines of Cameron, Goethals, Seidel, and Shult (1976a), employs the canonical vector representation of the underlying signed graph.

(**sg: LG: Aut, Geom**)

1983a Automorphism groups of graphs. In: Lowell W. Beineke and Robin J. Wilson, eds., *Selected Topics in Graph Theory 2*, Ch. 4, pp. 89–127. Academic Press, London, 1983. MR 0797250 (86i:05079). Zbl 536.05037.

§8, "Switching": Graph switching, graph switching classes. Existence of a "representative": a graph in a switching class that has the same automorphism group as the switching class. §9, "Digraphs": Switching classes of tournaments on pp. 117–118. Switching a digraph means reversing all

edges between $X \subseteq V$ and X^c. [Annot. 27 Dec 2010.]
(**TG: Sw: Aut: Exp**)

1994a Two-graphs and trees. Graph Theory and Applications (Proc., Hakone, 1990). *Discrete Math.* 127 (1994), 63–74. MR 1273592 (95f:05027). Zbl 802.05042.

Let T be a tree. Construction 1 (simplifying Seidel and Tsaranov (1990a)): Take all triples of edges such that none separates the other two. This defines a two-graph on $E(T)$ [whose underlying signed complete graph is described by Tsaranov (1992a)]. Construction 2: Choose $X \subseteq V(T)$. Take all triples of end vertices of T whose minimal connecting subtree has its trivalent vertex in X. The two-graphs $(V, \mathcal{T})$ that arise from these constructions are characterized by forbidden substructures, namely, the two-graphs of (1) C_5 and C_6; (2) C_5. Also, trees that yield identical two-graphs are characterized. (**TG**)

1995a Counting two-graphs related to trees. *Electronic J. Combin.* 2 (1995), Research Paper 4. MR 1312733 (95j:05112). Zbl 810.05031.

Counting two-graphs of the types constructed in (1994a). (**TG: Enum**)

2007a Orbit counting and the Tutte polynomial. In: Geoffrey Grimmett and Colin McDiarmid, eds., *Combinatorics, Complexity, and Chance: A Tribute to Dominic Welsh*, pp. 1–10. Oxford Lect. Ser. Math. Appl., Vol. 34. Oxford Univ. Press, Oxford, 2007. MR 2314558 (2008a:05043). Zbl 1122.05022.

P.J. Cameron, J.M. Goethals, J.J. Seidel, & E.E. Shult

†† 1976a Line graphs, root systems, and elliptic geometry. *J. Algebra* 43 (1976), 305–327. MR 0441787 (56 #182). Zbl 337.05142. Repr. in Seidel (1991a), pp. 208–230.

The essential idea is that graphs with least eigenvalue ≥ -2 are represented by the angles of root systems. It follows that line graphs are so represented. [Similarly, signed graphs with largest eigenvalue ≤ 2 are represented by the inner products of root systems, as in Vijayakumar *et al.* These include the line graphs of signed graphs as in Zaslavsky (1984c), since simply signed graphs are represented by B_n or C_n with a few exceptions. The representation of ordinary graphs by all-negative signed graphs is motivated in Zaslavsky (1984c).]
(**LG: sg: Eig, Geom, Sw**)

Peter J. Cameron, Bill Jackson, & Jason D. Rudd

2008a Orbit-counting polynomials for graphs and codes. *Discrete Math.* 308 (2008), 920–930. MR 2378927 (2009e:05140). Zbl 1133.05030. (**sg: Invar: Flows**)

Peter J. Cameron & Charles R. Johnson

2006a The number of equivalence classes of symmetric sign patterns. Int. Workshop Combin., Linear Algebra, Graph Coloring. *Discrete Math.* 306 (2006), no. 23, 3074–3077. MR 2273136 (2007j:05105). Zbl 1105.05034.

The number of signatures of K_n°, the complete graph with loops, under symmetry, switching, and negation (treated as totally nonzero symmetric sign pattern matrices) equals the number of switching isomorphism classes of signed complete graphs. (*Cf.* Mallows and Sloane (1975a) and Cameron (1979a).) [Annot. 12 Aug 2012, 16 Nov 2015.]
(**sg: sw, tg: Adj: Enum**)

P.J. Cameron, J.J. Seidel, & S.V. Tsaranov

1994a Signed graphs, root lattices, and Coxeter groups. *J. Algebra* 164 (1994), 173–209. MR 1268332 (95f:20063). Zbl 802.05043.

A generalized Coxeter group Cox(Σ) and a Tsaranov group Ts(Σ) are defined via Coxeter relations and an extra relation for each negative circle in Σ. They generalize Coxeter groups of tree Coxeter graphs and the Tsaranov groups of a two-graph ($|\Sigma| = K_n$; see Seidel and Tsaranov (1990a)). A new operation of "local switching" is introduced, which changes the edge set of Σ but preserves the associated groups.

§2, "Signed graphs", proves some well-known properties of switching and reviews interesting data from Bussemaker, Cameron, Seidel, and Tsaranov (1991a). §3, "Root lattices and Weyl groups": The "intersection matrix" $2I + A(\Sigma)$ is a hyperbolic Gram matrix of a basis of $\mathbb{R}^n$ whose vectors form only angles $\pi/2, \pi/3, 2\pi/3$. To these vectors are associated the lattice $L(\Sigma)$ of their integral linear combinations and the Weyl group $W(\Sigma)$ generated by reflecting along the vectors. W is finite iff $2I + A(\Sigma)$ is positive definite (Thm. 3.1). *Problem* 3.6. Determine which Σ have this property. §4 introduces local switching to partially solve *Problem* 4.1: Which signed graphs generate the same lattice? Results and some experimental data are reported. All-negative signed graphs play a special role. Definition of *local switching at* v: (1) switch so the edges at v are positive, (2) divide the components of the negative subgraph of the neighborhood of v into two halves J, K, (3) add negative edges joining all vertices of J to all those of K, (4) negate all edges from v to J, (5) reverse the switching in step (1). [See Isihara (2007a) for more.] §6, "Coxeter groups": The relationship between the Coxeter and Weyl groups of Σ. Cox(Σ) is Cox($|\Sigma|$) with additional relations for antinegative (i.e., negative in $-\Sigma$) induced circles. §7: "Signed complete graphs". §8: "Tsaranov groups" of signed K_n's §9: "Two-graphs arising from trees" (as in Seidel and Tsaranov (1990a)).

Dictionary: "(Γ, f)" $= \Sigma = (\Gamma, \sigma)$. "Fundamental signing" = all-negative signing, giving the antibalanced switching class. "The balance" of a cycle (i.e., circle) = its sign $\sigma(C)$; "the parity" = $\sigma(-C)$ where $-C = C$ with all signs negated. "Even" = positive and "odd" = negative (referring to "parity"). "The balance" of Σ = the partition of all circles into positive and negative classes $\mathcal{C}^+$ and $\mathcal{C}^-$; this is the bias on $|\Sigma|$ due to the signing and should not be confused with the customary meaning of "balance", i.e., all circles are positive.

[A more natural definition of the intersection matrix would be $2I - A$. Then signs would be negative to those in the paper. The need for "parity" would be obviated, ordinary graphs would correspond to all-positive signings (and those would be "fundamental"), and the extra Coxeter relations would pertain to negative induced circles.]

(**SG: Adj, Eig, Geom, Sw(Gen), lg**)

Peter J. Cameron & Sam Tarzi

2004a Switching with more than two colours. *European J. Combin.* 25 (2004), no. 2, 169–177. MR 2070538 (2005j:05059). Zbl 033.05038.

The edges of K_n are colored by m colors. Thm.: For $m > 2$, the combined action of $\mathfrak{S}_n$ on vertices and $\mathfrak{S}_m$ on colors is transitive on

m-edge-colored complete graphs for finite n but not for infinite n. **(SGc: Gen: Sw)**

P.J. Cameron & Albert L. Wells, Jr.

† 1986a Signatures and signed switching classes. *J. Combin. Theory Ser. B* 40 (1986), 344–361. MR 0842999 (87m:05115). Zbl 591.05061. **(SG: TG: Gen)**

Federico Camia, Emilio De Santis, & Charles M. Newman

2002a Clusters and recurrence in the two-dimensional zero-temperature stochastic Ising model. *Ann. Appl. Probab.* 12 (2002), no. 2, 565–580. MR 1910640 (2003h:60144). Zbl 1020.60094. **(Phys, VS: Rand)**

Paul Camion

1963a Caracterisation des matrices unimodulaires. *Cahiers Centre Études Recherche Opér.* 5 (1963), 181–190. MR 0179101 (31 #3352). Zbl 124.00901.

Camion's signing algorithm (implicitly) finds a set of sign reversals to balance a bipartite signed graph. **(sg: Bal)**

1965a Characterization of totally unimodular matrices. *Proc. Amer. Math. Soc.* 16 (1965), 1068–1073. MR 0180568 (31 #4802). Zbl 134.25201.

1968a Modules unimodulaires. *J. Combin. Theory* 4 (1968), 301–362. MR 0327576 (48 #5918). Zbl 174.29504.

2006a Unimodular modules. *Discrete Math.* 306 (2006), no. 19-20, 2355–2382. MR 2261907 (2007e:05096). Zbl 1099.13021.

M. Campanino

1998a Strict inequality for critical percolation values in frustrated random-cluster models. MR 1670039 (2000b:60235). Zbl 926.60084.

Compares the critical percolation values and the critical temperatures of a finite, positively edge-weighted, signed graph to those of the corresponding all-positive ("unfrustrated", "ferromagnetic") weighted graph. The graph is $\Lambda \subset \mathbb{Z}^d$ with a partition π of the boundary $\partial\Lambda$ and all edges on each block of π. Dictionary: "frustrated path" = negative circle including a boundary edge; "frustrated configuration" = subgraph of Λ having a negative circle including at least one boundary edge. [Annot. 2 Apr 2013.] **(Phys, SG: WG: Fr)**

Sue Ann Campbell

See J. Bélair.

Manoel Campelo & Gérard Cornuéjols

2009a The Chvátal closure of generalized stable sets in bidirected graphs. LAGOS'09 *Electronic Notes Discrete Math.* 35 (2009), 89–95. MR 2579413 (no rev).

The generalized stable set polyhedron of B is (equivalent to) conv($\mathbb{Z}^n \cap \{0 \le x \in \mathbb{R}^n : \mathrm{H(B)}x \le b\}$ where $b \in \mathbb{Z}^m$, $m = \#E$. Dictionary: "directed edge" = positive, "undirected edge" = negative; "odd cycle" = negative circle. [Annot. 9 Jun 2011.] **(sg: ori, Incid, Geom)**

Yves Candau

See N. Ramdani.

E. Rodney Canfield

See E.A. Bender.

Ismail Naci Cangul
See K.N. Prakasha.

Chun Zheng Cao
See X.X. Zhu.

D.S. Cao
See R. Simion.

Fayun Cao, Han Ren, & Hanlin Chen
2019a New formulae for the bipartite vertex frustration and decycling number of graphs. *Appl. Math. Comput.* 347 (2019), 101–112. MR 3880070 (no rev). Zbl 1428.05148.
For $-\Gamma$. **(sg: Par: Fr)**

Meng-Yue Cao, Jack H. Koolen, Yen-Chi Roger Lin, & Wei-Hsuan Yu
2022a The Lemmens-Seidel conjecture and forbidden subgraphs. *J. Combin. Theory Ser. A* 185 (2022) [2021], art. 105538, 28 pp. MR 4316717. Zbl 1476.05109. arXiv:2003.07511.
Thm. 2.2: If least eigenvalue of $A(K_n, \sigma)$ is -5 and $n \geq 277$, then $\mathrm{rk}(A(K_n, \sigma) - 5I) > \lceil 2n/3 \rceil$. "Switching graph": a certain signed covering graph of (K_n, σ). Forbidden signed cliques for least eigenvalue -5. [Annot. 13 Oct 2021.] **(sg: KG: Adj: Eig, Sw)**

Ming Cao
See A. Proskurnikov and D. Xue.

Andrea Capocci
See V. Ciotti.

Chiara Cappello, Reza Naserasr, Eckhard Steffen, & Zhouningxin Wang
† 20xxa Critically 3-frustrated signed graphs. Submitted. arXiv:2304.10243.
Conjecture: $\forall\, k\ \exists$ finitely many irreducible critically k-frustrated signed graphs. (Definition at Cappello and Steffen (2022a).) Several more detailed conjectures. Partial results for $k = 3$. [Annot. 22 Apr 2023.] **(SG: Fr)**

Chiara Cappello & Eckhard Steffen
2022a Frustration-critical signed graphs. *Discrete Appl. Math.* 322 (2022), 183–193. MR 4481186. Zbl 1498.05121. arXiv:2112.02664.
Σ is k-critical if $k = l(\Sigma) > l(\Sigma \setminus e)$ for all edges. Structure of some critical signed graphs, especially 2-critical and certain signed cubic projective-planar graphs. [Continued in Cappello–Naserasr–Steffen–Wang (20xxa).] [Annot. 14 Feb 2022.] **(SG: Fr)**

Sergio Caracciolo
See also M Palassini.

S. Caracciolo, G. Parisi, & N. Sourlas
1982a Variational real space renormalization group and its application to frustrated systems. *Nuclear Phys. B* 205 (1982), no. 3, 345–354. MR 0668812 (83g:82069). Zbl 968.82513.
Approximation of physical quantities (free energy, critical temperature) on the triangular lattice with all-positive ("ferromagnetic"), all-negative ("antiferromagnetic"; "fully frustrated"), and arbitrary ("randomly frustrated") signs, by moving edges ("bonds", "couplings"), adjusting edge

weights, and coarsening the lattice to get recursive formulas. Also, a tentative analog for the square lattice with possible diagonals. [Annot. 28 Mar 2013.] **(Phys, sg: Fr)**

Alvaro Carbonero, Janelle Domantay, & Karen Guthrie

20xxa The optimization of signed trees. Submitted. arXiv:2109.01221.

Which signed trees have a given degree set? Some answers. [Annot. 14 Feb 2022.] **(SG)**

Domingos M. Cardoso

See also N.M.M. de Abreu and I. Gutman.

Domingos M. Cardoso, Dragoš Cvetković, Peter Rowlinson, & Slobodan K. Simić

2008a A sharp lower bound for the least eigenvalue of the signless Laplacian of a non-bipartite graph. *Linear Algebra Appl.* 429 (2008), no. 11-12, 2770–2780. MR 2455532 (2009i:05145). Zbl 1148.05046.

Thm.: $\min_\Gamma \lambda_{\max}(L(-\Gamma))$, for connected, nonbipartite Γ with $\#V = n$ is attained iff Γ is K_3 with an attached path. [*Problem.* Generalize to connected, unbalanced signed graphs.] [Annot. 4 Sept 2010.] **(sg: Par: Eig)**

Charles Carlson, Karthekeyan Chandrasekaran, Hsien-Chih Chang, Naonori Kakimura, & Alexandra Kolla

2019a Spectral aspects of symmetric matrix signings. In: Peter Rossmanith *et al.*, eds., *44th International Symposium on Mathematical Foundations of Computer Science* (MFCS 2019, Aachen), art. 81, 13 pp. Leibniz Int. Proc. Informatics, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany, 2019. MR 4008470. Zbl 1507.68225.

Extended abstract of (2020a). **(sg: QM: Str, Algor)**

2020a Spectral aspects of symmetric matrix signings. *Discrete Optimization* 37 (2020), art. 100582, 22 pp. MR 4095771. Zbl 1506.68068. arXiv:1611.03624v2.

Problem: sign a positively weighted graph (possibly with loops) to have a desired property such as invertibility or solvability, by forming Kronecker product $A(\Gamma, w) \circ A(\Sigma)$. Or, also add new edges to get the property. Algorithmic complexity, algorithms, and structural descriptions.

Simpler, weaker initial version at arXiv:1611.03624v1 [and with one result and proof duplicated independently by Akbari, Ghafari, Kazemian, and Nahvi (2020a)]. [Annot. 21 Dec 2020.] **(sg: QM: Str, Algor)**

Jordan Carlson & Daniel Stolarski

2004a The correct solution to Berlekamp's switching game. *Discrete Math.* 287 (2004), 145–150. MR 2094708 (2005d:05005). Zbl 1054.94023.

The minimum frustration index of a signed $K_{n,n}$ for $n = 10, 11, 12$ and bounds up to 20. Corrects and extends Fishburn and Sloane (1989a). **(sg: fr)**

Jaime Cartes

See W. Lebrecht, J.F. Valdés, and E.E. Vogel.

Dorwin Cartwright

See also T.C. Gleason, and Harary–Norman–Cartwright (1965a) *et al.*

Dorwin Cartwright & Terry C. Gleason

1966a The number of paths and cycles in a digraph. *Psychometrika* 31 (1966), 179–199. MR 0197195 (33 #5377). Zbl 143.43702.

Pp. 194ff., "A generalization of the method": Digraph edges have gains in a (commutative) semiring. A matrix method produces counts of paths and cycles of given lengths. Remarks on pp. 194, 199: The method can be applied to signed digraphs (unexplained) [the group ring $\mathbb{Z}[+,-]$ must be intended; matrix entry $= n_+(+) + n_-(-)$, $n_\varepsilon = \#$ of ε-paths/circles]. Dictionary: "generalized addition, multiplication" = operations in the semiring; "value matrix" = adjacency matrix of the gain graph. [Annot. 28 Apr 2017.] (**gd(Gen), sd: Adj, Paths, Circ**)

Dorwin Cartwright & Frank Harary

1956a Structural balance: a generalization of Heider's theory. *Psychological Rev.* 63 (1956), 277–293. Repr. in: Dorwin Cartwright and Alvin Zander, eds., *Group Dynamics: Research and Theory*, second ed., pp. 705–726. Harper and Row, New York, 1960. Also reprinted in: Samuel Leinhardt, ed., *Social Networks: A Developing Paradigm*, pp. 9–25. Academic Press, New York, 1977.

Expounds Harary (1953a), (1955a) with sociological discussion. Proposes to measure imbalance by the proportion of balanced circles (the "degree of balance") or balanced circles of length $\leq k$ ("degree of k-balance"). (**PsS, SG: Bal, Fr**)

1968a On the coloring of signed graphs. *Elem. Math.* 23 (1968), 85–89. MR 0233732 (38 #2053). Zbl 155.31703 (155, p. 317c).

"Coloring" is clustering as in Davis (1967a). Thm. 1 adds a bit to Davis (1967a) in Thm. 3: The clustering is unique $\iff$ all components of Σ^+ are adjacent. (**SG: Clu**)

1970a Ambivalence and indifference in generalizations of structural balance. *Behavioral Sci.* 15 (1970), no. 6, 497–513. (**SD: Gen: Bal**)

1977a A graph theoretic approach to the investigation of system-environment relationships. *J. Math. Sociology* 5 (1977), 87–111. MR 0444117 (56 #2477). Zbl 336.92026.

(**SD: Clu**)

1979a Balance and clusterability: an overview. In: Paul W. Holland and Samuel Leinhardt, eds., *Perspectives on Social Network Research* (Proc. Sympos., Dartmouth Coll., Hanover, N.H., 1975), Ch. 3, pp. 25–50. Academic Press, New York, 1979. (**SG, SD, VS: Bal, Fr, Clu, Adj: Exp**)

Adolfo Casari

See F. Barahona.

Federico Castillo

See F. Ardila.

Paul A. Catlin

1979a Hajós' graph-coloring conjecture: variations and counterexamples. *J. Combin. Theory Ser. B* 26 (1979), 268–274. MR 0532593 (81g:05057). Zbl 385.05033, Zbl 395.05033.

Thm. 2: If Γ is 4-chromatic, $[-\Gamma]$ contains a subdivision of $[-K_4]$ (an "odd-K_4"). [*Question.* Can this possibly be a signed-graph theorem? For instance, should it be interpreted as concerning the chromatic number of $-\Gamma$?] **(par: col)**

Minerva Catral

See also S. Butler.

M. Catral, D.D. Olesky, & P. van den Driessche

2009a Allow problems concerning spectral properties of sign pattern matrices: A survey. *Linear Algebra Appl.* 430 (2009), no. 11-12, 3080–3094. MR 2517861 (2010i:15066). Zbl 1165.15009.

D is the signed digraph of a square sign-pattern matrix S. Thm. 3.1: If the spectrum of A with signs S is arbitrary, D has positive and negative disjoint cycle unions of all orders. Thm. 4.1: If the inertia is arbitrary, D has a positive and a negative loop and a negative digon. [Annot. 4 Nov 2011.] **(SG: QM, Exp)**

Bogdan Cautis

See C. Giatsidis.

Matteo Cavaleri

See also F. Belardo.

Matteo Cavaleri, Daniele D'Angeli, & Alfredo Donno

† 2021a A group representation approach to balance of gain graphs. *J. Algebraic Combin.* 54 (2021), no. 1, 265–293. MR 4290425. Zbl 1470.05071. arXiv:2001.08490. **(GG: Adj: Algeb)**

2021b Gain-line graphs via G-phases and group representations. *Linear Algebra Appl.* 613 (2021), 241–270. MR 4199816. Zbl 1459.05163. arXiv:2007.14839. **(GG: LG: Adj: Algeb, Eig)**

2022a Characterizations of line graphs in signed and gain graphs. *European J. Combin.* 102 (2022), art. 103479, 19 pp. MR 4342436. Zbl 1486.05262. arXiv:-2101.09677. **(GG, SG: LG)**

Matteo Cavaleri & Alfredo Donno

2022a On cospectrality of gain graphs. *Special Matrices* 10 (2022), 343–365. MR 4436247. Zbl 1489.05092. arXiv:2111.12428. **(SG, GG: Adj: Eig)**

Matteo Cavaleri, Alfredo Donno, & Stefano Spessato

20xxa Godsil-McKay switchings for gain graphs. Submitted. arXiv:2207.10986. **(GG: Adj: Eig, Bal)**

Michael S. Cavers

2010a On reducible matrix patterns. *Linear Multilinear Algebra* 58 (2010), no. 2, 257–267. MR 2641538 (2011b:15072). Zbl 1189.15010. **(SD: QM)**

M. Cavers, S.M. Cioabă, S. Fallat, D.A. Gregory, W.H. Haemers, S.J. Kirkland, J.J. McDonald, & M. Tsatsomeros

2012a Skew-adjacency matrices of graphs. *Linear Algebra Appl.* 436 (2012), 4512–4529. MR 2917427. Zbl 1241.05070.

§4, "Characteristic polynomials of skew-adjacency matrices": From Γ form a signed digraph: each edge becomes a negative digon. An orientation Γ^τ is a choice of one arc from each digon; thus, it is a signed digraph. A "skew adjacency matrix" of Γ is a matrix $S := A(\Gamma^\tau)$. The characteristic polynomial $p_S(x)$ is odd/even for odd/even n. Eqs. (7),

(8) give formulas for the coefficients in terms of matchings and cycle signs. A "generic skew-adjacency matrix" of Γ has variables $x_{ij} = x_{ji}$ instead of 1's in S. Thm. 4.2: Spec S is unique iff Γ has no even circles (an "odd-circle graph"). Dictionary: "cycle" = circle, "dicycle" = cycle, "σ" = τ, "$\vec{G}(S)$" = Γ^τ.

More results about odd-circle and bipartite (even-circle) graphs. Lemma 6.3 implicitly switches Γ^τ through S. [*Problem.* Generalize to bidirected graphs, thereby unifying symmetric and skew-symmetric adjacency matrices. A negative edge becomes a positive digon, all-positive if oriented extraverted and all-negative if introverted. The all-negative case is symmetric, the all-positive case is skew-symmetric.] [Annot. 4 Jan 2013.] (**sd: Adj, sw**)

Michael S. Cavers & Shaun M. Fallat

2012a Allow problems concerning spectral properties of patterns. *Electronic J. Linear Algebra* 23 (2012), 731–754. MR 2966802. Zbl 1251.15034.

Signed digraphs are generalized to edges labelled by $0, +, -, +_0\,(\geq 0), -_0\,(\leq 0), *\,(\neq 0), \#$ (real). [Annot. 24 May 2013.] (**QM: SD(Gen)**)

Michael S. Cavers & Kevin N. Vander Meulen

2005a Spectrally and inertially arbitrary sign patterns. *Linear Algebra Appl.* 394 (2005), 53–72. MR 2100576 (2005f:15008). Zbl 1065.15009.

Lem. 5.1: An inertially arbitrary sign pattern contains a negative digon. [Annot. 5 Nov 2011.] (**QM: sd, sw**)

Nicolò Cesa-Bianchi, Claudio Gentile, Fabio Vitale, & Giovanni Zappella

2012a A correlation clustering approach to link classification in signed networks. In: Shie Mannor, Nathan Srebro, and Robert C. Williamson, eds., *Proceedings of the 25th Annual Conference on Learning Theory* (COLT 2012, Edinburgh, 2012), paper 34, 20 pp., electronic. JMLR: Workshop and Conf. Proc., Vol. 23. ACM, 2012. URL `http://jmlr.csail.mit.edu/proceedings/papers/v23/` arXiv:1301.4769 (Full version, 22 pp.). (**SG: Clu**)

2012b A linear time active learning algorithm for link classification. In: *Proceedings of the Workshop on Mining and Learning with Graphs* (10th, MLG-2012, Edinburgh, 2012). 6 pp., electronic. URL (9/2015) `https://dtai.cs.kuleuven.be/events/mlg2012/papers/7_Linear_Nicolo.pdf` arXiv:1301.4767 (full version).

Very short version of (2012c). (**SG: Clu**)

2012c A linear time active learning algorithm for link classification. In: F. Pereira, C.J.C. Burges, L. Bottou, and K.Q. Weinberger, eds., *Advances in Neural Information Processing Systems 25 (NIPS 2012)* (Lake Tahoe, Nev., 2012), 9 pp., electronic. URL (9/2015) `http://papers.nips.cc/paper/4598-a-linear-time-active-learning-algorithm-for-link-classification` arXiv:1301.4767 (full version).

Less short version of (2012b) and (2012d).

See arXiv:1301.4767 for the full version (16 pp.). (**SG: Clu**)

2012d A fast active learning algorithm for link classification. In: *13th Italian Conference on Theoretical Computer Science* (ICTCS 2012, Varese, Italy, 2012), paper 38, 4 pp., electronic. URL (9/2015) `http://ictcs.di.unimi.it/papers/`

paper_38.pdf

Shortest version of (2012c). **(SG: Clu)**

Seth Chaiken

1982a A combinatorial proof of the all minors matrix tree theorem. *SIAM J. Algebraic Discrete Methods* 3 (1982), 319–329. MR 0666857 (83h:05062). Zbl 495.05018.

§4: "Extension to signed graphs". Generalizing Zaslavsky (1982a), an all-minors matrix-tree theorem for weighted signed digraphs and a corollary for weighted signed graphs. Given: a signed graph on vertex set $[n]$. For a Laplace-type $n \times n$ matrix L (A in the paper), $L(\bar{U}, \bar{W})$ is L with the rows indexed by U and the columns indexed by W deleted. Take $U, W \subseteq V$ with $\#U = \#W = k \leq n$. Then $\det L(\bar{U}, \bar{W})$ is a sum of terms, one for each independent set F of rank $n-k$ in $\mathbf{F}(\Sigma)$ in which each tree component contains just one vertex from U and one from W. Each term has a sign depending partly on the number of negative paths by which F links U to W and partly on the linking pattern, and with magnitude $4^c \cdot$(weight product of F), where $c = \#$ of circles in F. [The credit to Zaslavsky is overly generous: only the case $U = W = \varnothing$ is his; the others are new.] The digraph version replaces 4 by 2 and imposes conditions on arc directions in the tree and nontree components of F.

A brief remark describes a gain-graphic ("voltage-graphic") generalization. **(SD, SG, GG: Lap, Incid, matrd)**

1996a Oriented matroid pairs, theory and an electrical application. In: Joseph E. Bonin, James G. Oxley, and Brigitte Servatius, eds., *Matroid Theory* (Proc., Seattle, 1995), pp. 313–331. Contemp. Math., Vol. 197. Amer. Math. Soc., Providence, R.I., 1996. MR 1411693 (97e:05058). Zbl866.05016.

Connects a problem on common covectors of two subspaces of $\mathbb{R}^m$, and more generally of a pair of oriented matroids, to the problem of sign-solvability of a matrix and the even-cycle problem for signed digraphs. **(QSol, sd: Par, Algor)**

1996b Open problem 5. A problem about common covectors and bases in oriented matroid pairs. In: Joseph E. Bonin, James G. Oxley, and Brigitte Servatius, eds., *Matroid Theory* (Proc., Seattle, 1995), pp. 415–417. Contemp. Math., Vol. 197. Amer. Math. Soc., Providence, R.I., 1996.

Possible generalizations to oriented matroids of sign-nonsingularity of a matrix. **(QSol, SD: Par)**

Seth Chaiken, Christopher R.H. Hanusa, & Thomas Zaslavsky

2010a Nonattacking queens in a rectangular strip. *Ann. Combin.* 14 (2010), 419–441. MR 2776757 (2012d:05034). Zbl 1233.05022. arXiv:1105.5087.

Affinographic hyperplanes and rooted integral gain graphs, from Forge–Zaslavsky (2007a), imply the structure of formulas counting nonattacking arrangements of identical chess pieces in an $m \times n$ strip, as a function of n. **(GG: Geom, Invar)**

2019a A q-queens problem. VI. The bishops' period. *Ars Math. Contemp.* 16 (2019), no. 2, 549–561. MR 3963222. Zbl 1416.05131. arXiv:1405.3001.

Hyperplane representation of sign-colored graphs with a new associated matroid, assisted by a related signed graph and its frame matroid. [Annot. 4 Apr 2011.] **(SGc: Geom, Matrd)(SG: matrd)**

Bikas K. Chakrabarti

See also S. Suzuki.

Bikas K. Chakrabarti, Amit Dutta, & Parongama Sen

1996a *Quantum Ising Phases and Transitions in Transverse Ising Models*. Lect. Notes in Phys., New Ser. m: Monographs, Vol. 41. Springer, Berlin, 1996. Zbl 872.60090.

Shows aspects of what physicists may ask about signed graphs. “Transverse” means external magnetic field(s), modellable as extra dominating vertices. Ch. 4, “ANNI model in transverse field”: *Cf.* Liebmann (1986a). Ch. 6, “Transverse Ising spin glass and random field systems”: The typical mountainous energy landscape. §6.1, “Classical Ising spin glasses: A summary”: Random signs. The $\pm J$ model is unweighted signed graphs. §6.2, “Quantum spin glasses”: Height of energy barriers between valleys may be less important than width due to quantum tunnelling.

Second edition: Suzuki, Inoue, and Chakrabarti (2013a). [Annot. 8 Aug 2018.] **(Phys: SG, wg: Fr)**

Ivy Chakrabarty

See M. Acharya.

Nilanjan Chakraborty

See D. Li.

Sudip Chakravarty

See R.R.P. Singh.

AtMa P.O. Chan, Jeffrey C.Y. Teo, & Shinsei Ryu

2015a Topological phases on non-orientable surfaces: twisting by parity symmetry. *New J. Phys.* 18 (2016), art. 035005, 22 pp. MR 3484878.

A physics approach to embedding a signed graph on a surface, via orientable surfaces with parity defects. Also, in part, the same for gain graphs. [Annot. 2 Apr 2016.] **(sg: Top)(gg: Top)**

Sarah Chand

See R. Farooq.

Karthekeyan Chandrasekaran

See C. Carlson.

Vijaya Chandru, Collette R. Coullard, & Donald K. Wagner

1985a On the complexity of recognizing a class of generalized networks. *Operations Res. Lett.* 4 (1985), 75–78. MR 0811167 (87a:90144). Zbl 565.90078.

Determining whether a gain graph with real multiplicative gains has a balanced circle, i.e., is not contrabalanced, is NP-hard. So is determining whether a real matrix is projectively equivalent to the incidence matrix of a contrabalanced real gain graph. **(GN, Bic: Incid, Algor)**

Chung-Chien Chang & Cheng-Ching Yu

1990a On-line fault diagnosis using the signed directed graph. *Industrial and Engineering Chem. Res.* 29 (1990), 1290–1299.

Modifies the method of Iri, Aoki, O’Shima, and Matsuyama (1979a) of constructing the diagnostic signed digraph, e.g. by considering transient and steady-state situations. **(SD: Appl, Ref)**

Gerard J. Chang
See J.H. Yan.

Hsien-Chih Chang
See C. Carlson.

Maw-Shang Chang
See L.-H. Chen.

Michael D. Chang
See M. Engquist.

Ting-Chung Chang [Ting-Jung Chang]
See T.J. Chang.

Ting-Jung Chang [Ting-Chung Chang]

Ting-Jung Chang & Bit-Shun Tam
2010a Graphs with maximal signless Laplacian spectral radius. *Linear Algebra Appl.* 432 (2010), no. 7, 1708–1733. MR 2592913 (2011e:15014). Zbl 1231.05166. **(par: Lap: Eig)**

Ting-Jung Chang (as Ting-Chung Chang) & Bit-Shun Tam
2011a Connected graphs with maximal Q-index: The one-dominating-vertex case. *Linear Algebra Appl.* 435 (2011), no. 10, 2451–2461. MR 2811129 (2012d:05220). Zbl 1222.05029. **(par: Lap: Eig)**

Ting-Jung Chang, Bit-Shun Tam, & Shu-Hui Wu
2011a Theorems on partitioned matrices revisited and their applications to graph spectra. *Linear Algebra Appl.* 434 (2011), 559–581. MR 2741241 (2012g:05131). Zbl 1225.05160. **(sg: Par: Eig)**

Yi Chang
See J.-L. Tang and S.H. Yang.

Claudine Chaouiya
See G. Didier and A. Naldi.

Guillaume Chapuy
See O. Bernardi.

Pierre Charbit
See P. Aboulker.

Moses Charikar
See also N. Ailon.

Moses Charikar, Neha Gupta, & Roy Schwartz
2017a Local guarantees in graph cuts and clustering. In: Friedrich Eisenbrand *et al.*, eds., *Integer Programming and Combinatorial Optimization* (19th Int. Conf., IPCO 2017, Waterloo, Ont., 2017), pp. 136–147. Lect. Notes in Computer Sci., Vol. 10328. Springer, Cham, 2017. MR 3678780. Zbl 06767445. arXiv:-1704.00355. **(SG: Clu: Algor)**

Moses Charikar, Venkatesan Guruswami, & Anthony Wirth

2003a Clustering with qualitative information. In: *Proceedings of the 44th Annual IEEE Symposium on Foundations of Computer Science* (FOCS'03), pp. 524–533. IEEE, 2003.

Conference version of (2005a). **(SG: WG: Clu: Algor)**

2005a Clustering with qualitative information. Learning Theory 2003. *J. Computer System Sci.* 71 (2005), no. 3, 360–383. MR 2168358 (2006f:68141). Zbl 1094.68075. **(SG: WG: Clu: Algor)**

Ankit Charls

See T. Sharma.

A. Charnes, M. Kirby, & W. Raike

1966a Chance-constrained generalized networks. *Operations Res.* 14 (1966), 1113–1120. Zbl 152.18302 (152, p. 183b). **(GN)**

A. Charnes & W.M. Raike

1966a One-pass algorithms for some generalized network problems. *Operations Res.* 14 (1966), 914–924. Zbl 149.38106 (149, p. 381f). **(GN: Incid)**

Clément Charpentier, Reza Naserasr, & ?Éric Sopena

2020a Homomorphisms of sparse signed graphs. *Electronic J. Combin.* 27 (2020), no. 3, art.¶3.6, 28 pp. MR 4245119. Zbl 1508.05073. HAL hal-03035030. **(SG: Hom)**

Gary Chartrand

See also M. Behzad.

1977a *Graphs as Mathematical Models.* Prindle, Weber and Schmidt, Boston, 1977. MR 0490611 (58 #9947). Zbl 384.05029.

[Repr. (1985a).] **(SG: Bal, Clu)**

1985a *Introductory Graph Theory.* Dover Publications, New York, 1985. MR 0783826 (86c:05001).

"Corrected reprint" of (1977a). **(SG: Bal, Clu)**

Gary Chartrand, Heather Gavlas, Frank Harary, & Michelle Schultz

1994a On signed degrees in signed graphs. *Czechoslovak Math. J.* 44(119) (1994), 677–690. MR 1295143 (95g:05084). Zbl 837.05110.

Net degree sequences (i.e., $d^{\pm} := d^{+} - d^{-}$; called "signed degree sequences") of signed simple graphs. A Havel–Hakimi-type reduction formula, but with an indeterminate length parameter; a determinate specialization to complete graphs. A necessary condition for a sequence to be a net degree sequence. Examples: paths, stars, double stars. [Continued in Yan, Lih, Kuo, and Chang (1997a). Solved in Michael (2002a).]

[This is a special case of weighted degree sequences of K_n with integer edge weights chosen from a fixed interval of integers. Here the interval is $[-1, +1]$. The theory of such degree sequences is due to V. Chungphaisan, Conditions for sequences to be r-graphic, *Discrete Math.* 7 (1974), 31–39. MR 0351903 (50 #4391). Michael (2002a) characterizes net degree sequences by noticing this connection.] **(SGw: Invar)**

[One can interpret net degrees as the net indegrees, $d^{\text{in}} - d^{\text{out}}$, of certain bidirected graphs. Change the positive (negative) edges to extroverted (resp., introverted). Then we have the net indegree sequence of an oriented $-\Gamma$. *Problem* 1. Generalize to all bidirected (simple, or simply signed) graphs, especially K_n's. *Problem* 2. Find an Erdős–Gallai-type characterization of net degree sequences of signed simple graphs. [Solved by Michael (2002a).] *Problem* 3. Characterize the separated signed degree sequences of signed simple graphs, where the separated signed degree is $(d^+(v), d^-(v))$. *Problem* 4. Generalize Problem 3 to edge k-colorings of K_n.] (**SG: ori: Invar**)

Gary Chartrand, Frank Harary, Hector Hevia, & Kathleen A. McKeon

1992a On signed graphs with prescribed positive and negative graphs. *Vishwa Int. J. Graph Theory* 1 (1992), 9–18. MR 1196220 (93m:05095).

What is the smallest order of an edge-disjoint union of two (isomorphism types of) simple graphs, Γ and Γ'? Bounds, constructions, and special cases. (The union is called a signed graph with Γ and Γ' as its positive and negative subgraphs.) Thm. 13: If Γ' is bipartite (i.e., the union is balanced) with color classes V_1' and V_2', the minimum order $= \min(\#V_1', \#V_2') + \max(\#V, \#V_1', \#V_2')$. (**wg**)(**SG: Bal**)

Bilal A. Chat

See H.A. Ganie.

Sourav Chatterjee

2007a Estimation in spin glasses: a first step. *Ann. Stat.* 38 (2007), no. 5, 1931–1946. MR 2363958 (2009k:62063). Zbl 1126.62128. arXiv:math/0604634.

Advanced statistics are applied to states $s : V \to \{+1, -1\}$ of weighted signed complete graphs to estimate the probability of s. Thm. 1.1 (too complicated to state here) gives estimators for some choices of weights. §1.4, "Application to the Sherrington–Kirkpatrick (S–K) model". [Annot. 31 Aug 2012.] (**SG, WG: State, Phys**)

2008a Chaos, concentration, and multiple valleys. Manuscript, 2008. arXiv:0810.4221.

§9, "Example: Generalized SK model of spin glasses": A general theory is applied to convex combinations $\sum_{p\geq 1} c_p(K_n^{(p)}, \sigma, w)$ of signed, weighted complete p-uniform hypergraphs, where $c_p \to 0$ slowly as $p \to \infty$. (Obviously, n also $\to \infty$.) [Published by inclusion in (2014a).] [Annot. 31 Aug 2012.] (**SH, WH: State(fr), Phys**)

2009a Disorder chaos and multiple valleys in spin glasses. Manuscript, 2009. arXiv:-0907.3381.

Weighted signed complete graph, (Σ, w) where $w : E \to \mathbb{R}_{>0}$. Near-ground states $s : V \to \{+1, -1\}$ (nearly smallest switched weight, $\sum_E \sigma^s(e)w(e)$) are highly dispersed in $\{+1, -1\}^V$, suggesting that $\{+1, -1\}^V$ has many near-minimal valleys. Perturbation of w can drastically change the valley structure. The same is not true on the square grid. [Published by inclusion in (2014a).] [Annot. 31 Aug 2012.] (**SG, WG: State(fr), Phys**)

2014a *Superconcentration and Related Topics*. Springer Monographs in Math. Springer, Cham, 2014. MR 3157205. Zbl 1288.60001.

Ch. 4: "Multiple valleys". **(Phys: sg:)**

Guy Chaty

1988a On signed digraphs with all cycles negative. *Discrete Appl. Math.* 20 (1988), 83–85. MR 0936899 (89d:05148). Zbl 647.05028.

Clarifies the structure of "free cyclic" digraphs and shows they include strong "upper" digraphs (see Harary, Lundgren, and Maybee (1985a)). **(SD: Str)**

Vaggos ChatziafratisVaggos Chatziafratis, Neha Gupta, & Euiwoong Lee

20xxa Inapproximability for local correlation clustering and dissimilarity hierarchical clustering. arXiv:2010.01459. **(sg: Clu: Algor)**

M. Chaves

See L. Tournier.

P.D. Chawathe & G.R. Vijayakumar

† 1990a A characterization of signed graphs represented by root system D_∞. *European J. Combin.* 11 (1990), 523–533. MR 1078708 (91k:05071). Zbl 764.05090.

A list of the 49 switching classes of signed simple graphs that are the forbidden induced subgraphs for a signed simple graph to be a reduced line graph of a simply signed graph without loops or half edges. The graphs have orders 4, 5, and 6. [See several other works of Vijayakumar *et al.*] **(SG: adj, LG, Geom, incid)**

Shuchi Chawla

See N. Bansal.

Charalampos Chelmis

See A. Srivastava.

Beifang Chen

2018a Conformal decomposition of integral tensions and potentials of signed graphs. *SIAM J. Discrete Math.* 31 (2018), 2457–2478. MR 3719012. Zbl 1373.05078.

Dual to Chen–Wang–Zaslavsky (2017a). **(SG: Flows(Du))**

2021a Conformal decomposition of integral flows on signed graphs with outer-edges. *Graphs Combin.* 37 (2021), 2207–2225. MR 4338725. Zbl 1479.05135.

Improves upon Chen–Wang–Zaslavsky (2017a). **(SG: Flows)**

Beifang Chen & Shuchao Li

2011a The number of nowhere-zero tensions on graphs and signed graphs. *Ars Combin.* 102 (2011), 47–64. MR 2847958. Zbl 1265.05527. **(SG)**

Beifang Chen & Jue Wang

† 2009a The flow and tension spaces and lattices of signed graphs. *European J. Combin.* 30 (2009), 263–279. MR 2460231 (2009i:05102). Zbl 1198.05085.

Introduces cuts, and directed circuits and cuts, of a signed graph; and the cycle (or circuit) and cut (or cocycle) spaces of a signed graph over a commutative, unital ring in which 2 is invertible. Definitions, basic theory, and graphical proofs. Orthogonal complementarity between real, or integral, circuit and cut spaces. Relationships between real and integral spaces. Interpretations in terms of flows and tensions.

A cut is an edge set $U := E\langle X, X^c\rangle \cup U_X$ where $X \subseteq V$ and U_X is a minimal balancing set of $E{:}X$. A minimal cut is a bond, i.e., a cocircuit in $\mathbf{F}(\Sigma)$. A circuit or cut has two possible "directions". A minimal directed cut need not be a directed bond. The indicator vectors of directed circuits generate the cycle ("circuit") space; the indicator vectors of directed cuts generate the cocycle ("cut") space.

The flow space or lattice is the real or integral null space of the incidence matrix. The tension space or lattice is the real or integral row space. The spaces equal lattices equal the real cycle and cut spaces and the lattices are their integral parts. Not every integral flow is in the integral span of circuit indicator vectors; but every integral tension is spanned by cut indicator vectors.

[Based upon and extending parts of J. Wang (2007a).]
(**SG: Str, Ori, Incid**)

2010a Torsion formulas for signed graphs. *Discrete Appl. Math.* 158 (2010), 1148–1157. MR 2629892 (2011j:05131). Zbl 1255.05090.

[Based upon part of J. Wang (2007a).] (**SG: Flows(Du)**)

2011a Classification of indecomposable integral flows on signed graphs. Manuscript, 2011, rev. 2013. arXiv:1112.0642. (**SG: Flows**)

Beifang Chen, Jue Wang, & Thomas Zaslavsky

2017a Resolution of irreducible integral flows on signed graphs. *Discrete Math.* 340 (2017), no. 6, 1271–1286. MR 3624612. Zbl 1369.05098. arXiv:1701.04494.

Irreducible integral flows include circuit flows as well as others of a complicated and unexpected nature. Resolved by lifting to the signed covering graph. [Based upon part of J. Wang (2007a) and also Chen and Wang (2011a), with a different method.] (**SG: Incid, Str**)

Chao-Yang Chen

See B. Hu.

Chen Chen, Jin Huang, & Shuchao Li

2018a On the relation between the H-rank of a mixed graph and the matching number of its underlying graph. *Linear Multilinear Algebra* 66 (2018), no. 9, 1853–1869. MR 3825702. Zbl 1392.05071.

For Φ with gain group $\{\pm 1, \pm \mathrm{i}\}$ ($\varphi(e) = 1$ for undirected, i for directed edges), $-2\xi(\|\Phi\|) \leq \operatorname{rk} A(\Phi) - \mu(\|\Phi\|) \leq \xi(\|\Phi\|)$. Characterizes equalities. [A different matching formula in Tian–Chen–Chu (2018a).] [Annot. 15 Dec 2020.] (**gg: Adj**)

Chen Chen, Shuchao Li, & Minjie Zhang

2019a Relation between the H-rank of a mixed graph and the rank of its underlying graph. *Discrete Math.* 342 (2019), no. 5, 1300–1309. *Discrete Math.* 342 (2019), no. 5, 1300–1309. MR 3905201. Zbl 1407.05144. arXiv:1812.05309.

Φ with gain group $\{\pm 1, \pm \mathrm{i}\}$: $\varphi(e) = 1$ for undirected, i for directed edges. $|\operatorname{rk} A(\Phi) - \operatorname{rk} A(\|\Phi\|)| \leq 2\xi(\|\Phi\|)$. Equality is characterized. [Annot. 15 Dec 2020.] (**gg: Adj**)

Doron Chen

See also E.G. Boman.

Doron Chen & Sivan Toledo

2005a Combinatorial characterization of the null spaces of symmetric H-matrices. *Linear Algebra Appl.* 392 (2004), 71–90. MR 2095908 (2005h:15016). Zbl

1061.65028.

Certain matrices are related to gain graphs and others to signed graphs. **(GG, SG: Incid, Matrd)**

Ge Chen

See W.-J. Mei.

Gina Chen, Vivian Liu, Ellen Robinson, Lucas J. Rusnak, & Kyle Wang

2018a A characterization of oriented hypergraphic Laplacian and adjacency matrix coefficients. *Linear Algebra Appl.* 556 (2018), 323–341. MR 3842586. Zbl 1394.05070. arXiv:1704.03599. **(SH: Ori: Adj, Lap)**

Haiyan Chen

See Y.-N. Zhang.

Hanlin Chen

See F.-Y. Cao.

Jia-Fen Chen

See also B.Y. Wu.

Jia-Fen Chen & Bang Ye Wu

2013a *Balancing a Complete Signed Graph by Editing Edges and Deleting Nodes*. Thesis, National Chung Cheng Univ., Taiwan, 2013.

Minimum number of edge and vertex deletions and edge additions to arrive at balance, by two algorithms. [This thesis is presumably by Chen.] [Annot. 5 Jun 2017.] **(SG: KG: Fr: Algor, Clu)**

Jianer Chen & Jonathan L. Gross

1995a Voltage graphs for parallel architecture layouts. In: Y. Alavi and A. Schwenk, eds., *Graph Theory, Combinatorics, Algorithms and Applications* (Proc. Seventh Quadren. Int. Conf. Theory Appl. Graphs, Kalamazoo, Mich., 1992), Vol. 1, pp. 455–466. Wiley, New York, 1995. MR 1405831 (97j:05023). Zbl 843.68085. **(GG: Cov, Top)**

Jianer Chen, Jonathan L. Gross, & Robert G. Rieper

1994a Overlap matrices and total imbedding distributions. *Discrete Math.* 128 (1994), 73–94. MR 1271857 (95f:05031). Zbl 798.05017. **(SG: Top, Sw)**

Jianer Chen & Jie Meng

2010a A $2k$ kernel for the cluster editing problem. In: *Computing and Combinatorics* (Proc. 16th Ann. Int. Conf., COCOON 2010, Nha Trang, Vietnam, 2010), pp. 459–468. Lect. Notes in Computer Sci., Vol. 6196. Springer, Berlin, 2010. MR 2720122 (no rev). Zbl 1286.05164.

See (2012a). **(sg: kg: Clu: Algor)**

2012a A $2k$ kernel for the cluster editing problem. *J. Computer Syst. Sci.* 78 (2012), no. 1, 211–220. MR 2896358. Zbl 1238.68062.

Equivalent: Is the clusterability index $l_{\text{clu}}(K_n, \sigma) \leq k$? Dictionary: "graph" = positive subgraph of (K_n, σ); "editing" = edge sign changes. [Definition: l_{clu} = smallest number of sign changes that give a clusterable (*cf.* Davis (1967a)) signed K_n.] [Annot. 13 Jun, 1 July 2017.] **(sg: kg: Clu: Algor)**

Jianwen Chen
See W.-Q. Duan.

Jie Chen
See Y. Jiang and H.W. Zhang.

Jing Chen & Genghua Fan

2018a Short signed circuit covers of signed graphs. *Discrete Appl. Math.* 235 (2018), 51–58. MR 3732594. Zbl 1375.05123. (**SG: matrd**)

2021a Circuit k-covers of signed graphs. *Discrete Appl. Math.* 294 (2021), 41–54. MR 4214950. Zbl 1459.05111. (**SG: matrd**)

Kuo-Chin Chen, Simon Apers, & Min-Hsiu Hsieh

20xxa (Quantum) complexity of testing signed graph clusterability. Submitted. arXiv:-2311.10480. (**SG: Clu: Algor**)

Li Chen
See F.-L. Tian.

Li-Hsuan Chen
See also B.Y. Wu.

Li-Hsuan Chen, Maw-Shang Chang, Chun-Chieh Wang, & Bang Ye Wu

2013a On the min-max 2-cluster editing problem. In: *Advances in Intelligent Systems and Applications*, Vol. 1, pp. 133–142. Springer, Berlin, 2013.
Cf. (2013b), Chen and Wu (2017a). (**sg: kg: Fr: Algor**)

2013b On the min-max 2-cluster editing problem. *J. Inform. Sci. Engineering* 29 (2013), no. 6, 1109–1120. MR 3137567. (**sg: kg: Fr: Algor**)

Li-Hsuan Chen & Bang Ye Wu

2017a Parameterized algorithms for min–max 2-cluster editing. *J. Combin. Optim.* 34 (2017), 47–63. MR 3661065. Zbl 1378.05197.
Equivalent to: Is $l(K_n, \sigma) \leq k$? Dictionary: "graph" = positive subgraph of (K_n, σ); "editing" = edge sign changes; "2-clustering" = bipartition of V. [Annot. 13 Jun 2017.] (**sg: kg: Fr: Algor**)

Ming-Zhu Chen, A-Ming Liu, & Xiao-Dong Zhang

20xxa The signless Laplacian spectral radius of graphs without intersecting odd cycles. arXiv:2108.03895.
The matrix is $L(-\Gamma)$ where Γ excludes a subgraph of odd circles with one common vertex. [*Problem.* Generalize to negative circles in a signed graph. That allows even circles; it should be easy.] [Annot. 17 Oct 2022.] (**sg: Par: Lap: Eig**)

Qiyao ChenQiyao Chen & Yichao Chen

2022a Parallel edges in ribbon graphs and interpolating behavior of partial-duality polynomials. *European J. Combin.* 102 (2022), art. 103492, 12 pp. MR 4350495. Zbl 1486.05152. arXiv:2106.00381.
Cf. Gross, Mansour, and Tucker (2021a). (**sg: Top: Du: Invar**)

Rong Chen
See also B. Bao.

2017a The excluded minors for the class of matroids that are graphic or bicircular lift. *Adv. Appl. Math.* 83 (2017), 97–114. MR 3573220. Zbl 1351.05048. (**GG: Matrd**)

Rong Chen, Matt DeVos, Daryl Funk, & Irene Pivotto

2015a Graphical representations of graphic frame matroids. *Graphs Combin.* 31 (2015), no. 6, 2075–2086. MR 3417216. Zbl 1327.05143. arXiv:1403.7733. (**GG: Matrd**)

Rong Chen & Zifei Gao

2016a Representations of bicircular lift matroids. *Electronic J. Combin.* 23 (2016), no. 3, Paper 3.42, 11 pp. MR 3558079. Zbl 1344.05037. arXiv:1510.02643. (**Bic**)

Rong Chen & Irene Pivotto

2018a Biased graphs with no two vertex-disjoint unbalanced cycles. *J. Combin. Theory Ser. B* 130 (2018), 207–245. MR 3772740. Zbl 1384.05102. arXiv:1403.1919. (**GG: Str**)

Rong Chen & Geoff Whittle

2018a On recognizing frame and lifted-graphic matroids. *J. Graph Theory* 86 (2018), no. 1, 72–76. MR 3729836. Zbl 06843119. arXiv:1601.01791. (**GG: Matrd: Algor**)

Rong Chen & Kai-nan Xiang

2012a Decomposition of 3-connected representable matroids. *J. Combin. Theory Ser. B* 102 (2012), no. 3, 647–670. MR 2900809. Zbl 1238.05046.

A "spike-like" or "swirl-like" matroid is $\mathbf{L}(\Gamma, \mathcal{B})$ or $\mathbf{F}(\Gamma, \mathcal{B})$ where Γ is a circle with all edges doubled or more. Thm. 1.1: A 3-connected vector matroid with $\#E \geq 9$ decomposes into spike-like, swirl-like, and "freely-placed-line" matroids and sequentially 4-connected matroids, assembled in a tree pattern along modular lines. [Annot. 18 Apr 2013.] (**gg: Matrd: Str**)

Siyuan Chen

See Y.F. Huang.

Tianran Chen, Robert Davis, & Evgeniia Korchevskaia

2023a Facets and facet subgraphs of symmetric edge polytopes. *Discrete Appl, Math.* 328 (2023), 139–153. MR 4526533. Zbl 1508.05040. arXiv:2107.12315.

Thm. 12 and Rem. 13 implicitly state the connection with balanced flats of an even-cycle matroid $\mathbf{F}(-\Gamma)$. [Annot. 20 Jun 2022.] (**sg: par: Matrd, Geom**)

Vinciane Chen, Angeline Rao, Lucas J. Rusnak, & Alex Yang

2015a A characterization of oriented hypergraphic balance via signed weak walks. *Linear Algebra Appl.* 485 (2015), 442–453. MR 3394156. Zbl 1322.05087. (**SH: Bal**)

Wei Chen

See also Y.-H. Li.

Wei Chen, Dan Wang, Ji Liu, Tamer Başar, & Li Qiu

2017a On spectral properties of signed Laplacians for undirected graphs. In: *2017 IEEE 56th Annual Conference on Decision and Control (CDC)* (CDC2017, Melbourne, 2017), pp. 1999–2002. IEEE, 2017. **(SG: Lap: Eig)**

Weisheng Chen

See J.S. Wu.

William Y.C. Chen, Larry X.W. Wang, & Arthur L.B. Yang

2011a Recurrence relations for strongly q-log-convex polynomials. *Canadian Math. Bull.* 54 (2011), no. 2, 217–229. MR 2884236 (2012k:05029). Zbl 1239.05190.

§3.4, "The Dowling polynomials": $D_m(n;x)$ and $F_{m,1}(n;x)$ are from Benoumhani (1997a). [Annot. 28 Jan 2015.] **(gg: Matrd: Invar)**

Xiaodan Chen & Yaoping Hou

2016a Upper bounds for the signless Laplacian spectral radius of graphs on surfaces. *Filomat* 30 (2016), no. 13, 3473–3481. MR 3593047. Zbl 1462.05214. **(sg: par: Lap: Eig, Top)**

Xiaolin Chen, Xueliang Li, & Yingying Zhang

2016a 3-Regular mixed graphs with optimum Hermitian energy. *Linear Algebra Appl.* 496 (2016), 475–486. MR 3464084. Zbl 1331.05139.

In defining the Hermitian adjacency matrix there seems to be a missing line $h_{lk} = +i$ if $l \to k$. **(gg: Adj: Eig)**

Xiaoyue Chen

See L.-G. Jin.

Xudong Chen, Mohamed-Ali Belabbas, & Tamer Başar

2017a Cluster consensus with point group symmetries. *SIAM J. Control Optim.* 55 (2017), no. 6, 3869–3889. MR 3732941. Zbl 1376.05036. arXiv:1601.06346.

Gain-graph generalization of Altafini (2012a) *et al.* **(GG: Clu: Dyn)**

Y. Chen, X.L. Wang, B. Yuan, & B.Z. Tang

2014a Overlapping community detection in networks with positive and negative links. *J. Stat. Mech.* 2014 (2014), art. P03021, 22 pp. arXiv:1310.4023. **(SG: Clu)**

Ya-Hong Chen, Rong-Ying Pan, & Xiao-Dong Zhang

2011a Two sharp upper bounds for the signless Laplacian spectral radius of graphs. *Discrete Math. Algorithms Appl.* 3 (2011), no. 2, 185–191. MR 2822283 (2012f:05173). Zbl 1222.05149. **(par: Lap: Eig)**

Yanqing Chen & Ligong Wang

2010a Sharp bounds for the largest eigenvalue of the signless Laplacian of a graph. *Linear Algebra Appl.* 433 (2010), no. 5, 908–913. MR 2658641 (2011h:05149). Zbl 1215.05100. **(par: Lap: Eig)**

Yichao Chen

See also Q.-Y. Chen.

20xxa Permutation-bipartition pairs. Submitted. arXiv:2204.04359. **(SG: Top)**

Yu Chen & Yaoping Hou

2021a Eigenvalue multiplicity in cubic signed graphs. *Linear Algebra Appl.* 630 (2021), 95–111. MR 4299098. Zbl 1473.05167.

Multiplicity of eigenvalue μ of $A(\Sigma)$ is $\leq \frac{1}{2}n$ (= iff $\mu = \pm\sqrt{3}$), except $\leq \frac{1}{2}n+1$ when $\mu = \pm 1$ (= iff $\Sigma \simeq (K_4, -\mu)$) or $\mu = 0$ (= iff $\Sigma \simeq +K_{33}$). [Annot. 1 Oct 2021.] **(SG: Adj: Eig)**

Yu Qing Chen, Anthony B. Evans, Xiaoyu Liu, Daniel C. Slilaty, & Xiangqian Zhou

20xxa Representations of signed graphs. Submitted. **(SG, GG: Geom)**

Zhibin Chen & Wenan Zang

2009a Odd-K_4's in stability critical graphs. *Discrete Math.* 309 (2009), no. 20, 5982–5985. MR 2552630 (2010j:05291). Zbl 1229.05134. **(sg: par: Str)**

Zhi-Hong Chen, Ying-Qiang Kuang, & Hong-Jian Lai

1999a Connectivity of cycle matroids and bicircular matroids. *Ars Combin.* 52 (1999), 239–250. MR 1705651 (2001d:05032). Zbl 977.05027.

The relationship between graph structure and the Tutte, verticial, and cyclic connectivities of the bicircular matroid. **(Bic: Str)**

Bo Cheng & Bolian Liu

2008a The base sets of primitive zero-symmetric sign pattern matrices. *Linear Algebra Appl.* 428 (2008), 715–73. MR 2382083 (2009c:15028). Zbl 1135.15014.

The Abelson–Rosenberg (1958a) algebra is employed, with symbols $0, 1, -1, \#$ for o, p, n, a. "Generalized sign pattern matrix": # entries are allowed. "Generalized signed digraph": #-arcs are allowed. **(QM: SD)**

2010a Primitive zero-symmetric sign pattern matrices with the maximum base. *Linear Algebra Appl.* 433 (2010), no. 2, 365–379. MR 2645090 (2011e:15058). Zbl 1193.15029. **(QM: SD)**

Feng Cheng & Li Hua You

2012a The base set of primitive anti-symmetric signed digraphs with no loops. (In Chinese.) *J. South China Normal Univ. Natural Sci. Ed.* 44 (2012), no. 4, 13–19. MR 3052990 (no rev). Zbl 1289.05188 (*q.v.*). **(SD: qm: Dyn)**

Jian Cheng

2017a *Integer Flows and Circuit Covers of Graphs and Signed Graphs*. Doctoral dissertation, West Virginia University, 2017. MR 3732040 (no rev). **(SG: Flows, flows)**

Jian Cheng, You Lu, Rong Luo, & Cun-Quan Zhang

2019a Shortest circuit covers of signed graphs. *J. Combin. Theory Ser. B* 134 (2019), 164–178. MR 3906634. Zbl 1402.05095. arXiv:1510.05717. **(SG: flows)**

Ying Cheng

1986a Switching classes of directed graphs and H-equivalent matrices. *Discrete Math.* 61 (1986), 27–40. MR 0850927 (88a:05075). Zbl 609.05039.

This article studies what are described as $\mathbb{Z}_4$-gain graphs Φ with underlying simple graph Γ. [However, see below.] They are regarded as digraphs D, the gains being determined by D as follows: $\varphi(u, v) = 1$ or 2 if (u, v) is an arc, 2 or 3 if (v, u) is an arc. [N.B. Γ is not uniquely determined by D.] Cheng's "switching" is gain-graph switching but only by switching functions $\eta : V \to \{0, 2\}$; I will call this "semiswitching". His "isomorphisms" are vertex permutations that are automorphisms of

Γ; I will call them "Γ-isomorphisms". The objects of study are equivalence classes under semiswitching (semiswitching classes) or semiswitching and Γ-isomorphism (semiswitching Γ-isomorphism classes). Prop. 3.1 concerns adjacency of vertex orbits of a Γ-isomorphism that preserves a semiswitching class (call it a Γ-automorphism of the class). Thm. 4.3 gives the number of semiswitching Γ-isomorphism classes. Thm. 5.2 characterizes those Γ-automorphisms of a semiswitching class that fix an element of the class; Thm. 5.3 characterizes the Γ-isomorphisms g that fix an element of every g-invariant semiswitching class.

[Likely the right viewpoint, as is hinted in §6, is that the edge labels are not $\mathbb{Z}_4$-gains but weights from the set $\{\pm1, \pm2, \ldots, \pm k\}$ with $k = 2$. Then semiswitching is ordinary signed switching, and so forth. However, I forbear to reinterpret everything here.]

In §6, $\mathbb{Z}_4$ is replaced by $\mathbb{Z}_{2k}$ [but this should be $\{\pm1, \pm2, \ldots, \pm k\}$]; semiswitching functions take values $0, k$ only. Generalizations of §§3, 4 are sketched and are applied to find the number of H-equivalent matrices of given size with entries $\pm1, \pm1, \ldots, \pm k$. ($H$- [or Hadamard] equivalence means permuting rows and columns and scaling them by -1.)

(**sg, wg, GG: Sw, Aut, Enum**)

Ying Cheng & Albert L. Wells, Jr.

1984a Automorphisms of two-digraphs. (Summary.) Proc. Fifteenth Southeastern Conf. on Combinatorics, Graph Theory and Computing (Baton Rouge, 1984), [Vol. 3]. *Congressus Numer.* 45 (1984), 335–336. MR 0777706 (86c:05004c) (volume).

A two-digraph is a switching class of $\mathbb{Z}_3$-gain graphs based on K_n.

(**gg, SD: Sw, Aut**)

† 1986a Switching classes of directed graphs. *J. Combin. Theory Ser. B* 40 (1986), 169–186. MR 0838217 (87g:05104). Zbl 565.05034, (Zbl 579.05027).

This exceptionally interesting paper treats a digraph as a ternary gain graph Φ (i.e., with gains in $\mathrm{GF}(3)^+$) based on K_n. A theory of switching classes and triple covering graphs, analogous to that of signed complete graphs (and of two-graphs) is developed. The approach, analogous to that in Cameron (1977b), employs cohomology. The basic results are those of general gain-graph theory specialized to the ternary gain group and graph K_n.

The main results concern a switching class [Φ] of digraphs and an automorphism group $\mathfrak{A}$ of [Φ]. §3, "The first invariant": Thm. 3.2 characterizes, by a cohomological obstruction γ, the pairs $([\Phi], \mathfrak{A})$ such that some digraph in [Φ] is fixed. Thm. 3.5 is an [interestingly] more detailed result for cyclic $\mathfrak{A}$. §4: "Triple covers and the second invariant". Digraph triple covers of the complete digraph are considered. Those that correspond to gain covering graphs of ternary gain graphs Φ are characterized ("cyclic triple covers", pp. 178–180). Automorphisms of Φ and its triple covering $\tilde{\Phi}$ are compared. Given $([\Phi], \mathfrak{A})$, Thm. 4.4 finds the cohomological obstruction β to lifting $\mathfrak{A}$ to $\tilde{\Phi}$. Thm. 4.7 establishes an equivalence between γ and β in the case of cyclic $\mathfrak{A}$.

§5: "Enumeration". Thm. 5.1 gives the number of isomorphism types of switching classes on n vertices, based on the method of Wells (1984a) for signed graphs. §6: "The fixed signing property". Thm. 6.1 characterizes the permutations of $V(K_n)$ that fix a gain graph in every invariant switching class, based on the method of Wells (1984a)

Dictionary: "Alternating function" on $X \times X$ = $\mathrm{GF}(3)^+$-valued gain function on K_X.

[See Babai and Cameron (2000a) for a treatment of tournaments as nowhere-zero ternary gain graphs based on K_n.]

(**gg: Sw, Aut, Enum, Cov**)

Cheng Zhiyun & Gao Hongzhu

2012a Some applications of planar graph in knot theory. *Acta Math. Sci. Ser. B, Eng. Ed.* 32B (2012), no. 2, 663–671. MR 2921907. Zbl 1265.57002.

Which planar sign-colored graphs from link diagrams correspond to knots. [Annot. 26 Jul 2013.] (**SGc: Knot**)

Zhiyun Cheng, Ziyi Lei, Yitian Wang, & Yanguo Zhang

2022a A categorification for the signed chromatic polynomial. *Electronic J. Combin.* 29 (2022), no. 2, art. P2.49. MR 4441085. Zbl 1492.05072. arXiv:2101.01661.

Cohomology theories for the chromatic polynomial and the balanced chromatic polynomial of Σ using a chain complex for each. Cohomology properties coordinate nicely with some polynomial properties. There are no categories. [*Question.* Is their cohomology related to that of (the complement of) the associated complex hyperplane arrangements of the signed graph?] Dictionary: "signed chromatic polynomial" = chromatic quasipolynomial. [Their history of signed graph coloring is somewhat misleading.] [Annot. 6 Jan 2021, 17 Jun 2022.]

(**SG: Col: Algeb, Invar**)

Gi-Sang Cheon & Ji-Hwan Jung

2012a r-Whitney numbers of Dowling lattices. *Discrete Math.* 312 (2012), no. 15, 2337–2348. MR 2926106. Zbl 1246.05009.

Cf. Mező (2010a). [Cheon and Jung seem to have made an independent discovery.] [Annot. 8 Apr 2016.] (**gg: Matrd: Invar**)

Danila Cherkashin & Pavel Prozorov

2022a On the minimal sum of edges in a signed edge-dominated graph. *Electronic J. Combin.* 29 (2022), no. 3, art. P3.38, 18 pp. MR 4467140. Zbl 1496.05062. arXiv:2012.09956.

Improves Akbari, Bolouki, *et al.* (2009a), i.e., $s \geq -n^2/25$, and $s = -(1+o(1))n^2/8(1+\sqrt{2})^2$ is attainable. [Annot. 15 Aug 2022.] (**SGw**)

T.C. Chern

See Kuo–Chern–Shih (1988a).

Yonah Cherniavsky

See also V. Bugaenko.

Yonah Cherniavsky, Avraham Goldstein, & Vadim E. Levit

2013a On the structure of the group of balanced labelings on graphs. In: Jaroslav Nešetřil and Marco Pellegrini, eds., *The Seventh European Conference on Combinatorics, Graph Theory and Applications* (EuroComb 2013, Pisa), pp. 117–122. CRM Ser., Vol. 16. Edizioni della Normale, Scuola Normale Superiore

Pisa, Pisa, Italy, 2013. MR 3184274 (no rev). Zbl 1293.05157.
(GG(Gen): Bal(Gen))

2014a Groups of balanced labelings on graphs. *Discrete Math.* 320 (2014), 15–25. MR 3147203. Zbl 1281.05117. arXiv:1301.4206. **(GG(Gen): Bal(Gen))**

2017a Balanced Abelian group-valued functions on directed graphs. *Ars Math. Contemp.* 13 (2017), no. 2, 307–315. MR 3720533. Zbl 1380.05093.
Abelian gains on a graph ("flexible case") and a digraph ("rigid case").
(GG, GD: Bal)

Yonah Cherniavsky, Avraham Goldstein, Vadim E. Levit, & Robert Shwartz

2016a Enumeration of balanced finite group valued functions on directed graphs. *Inform. Processing Lett.* 116 (2016), no. 7, 484–488. MR 3479183. Zbl 1353.05064. arXiv:1405.3686. **(GG(Gen): Bal(Gen))**

Yonah Cherniavsky & Robert Shwartz

2020a Quotients of Coxeter groups associated to signed line graphs. *Adv. Appl. Discrete Math.* 25 (2020), no. 2, 213–261. Zbl 1499.20093. **(SG: Algeb)**

Kshittiz Chettri & Biswajit Deb

2023a On characterization of balance and consistency preserving d-antipodal signed graphs. *Mathematics* 11 (2023), art. 2982, 15 pp.
For signed Smith graphs Σ, Σ^A is Σ with added edges uv if u, v are antipodal, signed $\sigma^A(uv) = \prod\{\sigma(P) : P = \min uv\text{-path}\}$. Balance and canonical consistency (*cf.* Sampathkumar (1984a)) of Σ^A. [Annot. 26 Jul 2023.] **(SG, VS: Bal, Bal(VS))**

William K. Cheung

See B. Yang.

Ming Chi

See B. Hu.

Kai-Yang Chiang

See also C.-J. Hsieh.

Kai-Yang Chiang, Cho-Jui Hsieh, Nagarajan Natarajan, Ambuj Tewari, & Inderjit S. Dhillon

2014a Prediction and clustering in signed networks: A local to global perspective. *J. Machine Learning Res.* 15 (2014), 1177–1213. MR 3195342. Zbl 1319.91134. arXiv:1302.5145. **(SG: Clu, Pred)**

Kai-Yang Chiang, Nagarajan Natarajan, Ambuj Tewari, & Inderjit S. Dhillon

2011a Exploiting longer cycles for link prediction in signed networks. In: *Proceedings of the 20th ACM Conference on Information and Knowledge Management* (CIKM '11, Glasgow, 2011), pp. 1157–1162. ACM, New York, 2011.
(SG, SD: Pred: Fr: Algor, PsS)

Kai-Yang Chiang, Joyce Jiyoung Whang, & Inderjit S. Dhillon

2012a Scalable clustering of signed networks using balance normalized cut. In: *Proceedings of the 21st ACM Conference on Information and Knowledge Management* (CIKM'12, Maui, 2012), pp. 615–624. ACM, New York, 2012.
(SG: Bal, Clu: Algor)

Shuya Chiba, Shinya Fujita, Ken-ichi Kawarabayashi, & Tadashi Sakuma

2009a Disjoint even cycles packing. European Conf. Combin. Graph Theory Appl. (EuroComb 2009). *Electronic Notes Discrete Math.* 34 (2009), 113–119. MR 2591427 (no rev). Zbl 1273.05169. (**Par: Circ**)

2014a Minimum degree conditions for vertex-disjoint even cycles in large graphs. *Adv. Appl. Math.* 54 (2014), 105–120. MR 3199776. Zbl 1284.05209. (**Par: Circ**)

K.P. Chithra

See N.K. Sudev.

Sergei Chmutov

2009a Generalized duality for graphs on surfaces and the signed Bollobás–Riordan polynomial. *J. Combin. Theory Ser. B* 99 (2009), no. 3, 617–638. MR 2507944 (2010f:05046). Zbl 1172.05015.

Sign-colored graphs embedded in a surface (Chmutov and Pak (2007a)). Duality with respect to an edge subset, applied to a sign-colored Bollobás–Riordan polynomial, gives a polynomial duality. [Further developments in Vignes-Tourneret (2009a) and Krushkal (2011a).] (**SGc: Top, Invar**)

Sergei Chmutov & Igor Pak

2007a The Kauffman bracket of virtual links and the Bollobás-Riordan polynomial. *Moscow Math. J.* 7 (2007), no. 3, 409–418. MR 2343139 (2008h:57006). Zbl 1155.57004.

Sign-colored graphs embedded in a surface (orientable or not, independently of the edge signs. [The orientation properties of the ribbons make a signed graph, independent of the sign-colors.] (**SGc, sg: Top, Invar**)

Kwang-Hyun Cho

See J.-R. Kim and Y.-K. Kwon.

Matthew Cho, Anton Dochtermann, Ryota Inagaki, Suho Oh, Dylan Snustad, & Bailee Zacovic

20xxa Chip-firing and critical groups of signed graphs. Submitted. arXiv:2306.09315. (**SG: Lap(Gen), Sw**)

Hyeong-ah Choi, Kazuo Nakajima, & Chong S. Rim

1989a Graph bipartization and via minimization. *SIAM J. Discrete Math.* 2 (1989), 38–47. MR 0976786 (89m:90132). Zbl 677.68036.

Vertex biparticity [i.e., vertex frustration number $l_0(-\Gamma)$] is compared to edge biparticity [frustration index $l(-\Gamma)$] (for cubic graphs) and studied algorithmically. Proved: $l_0(-\Gamma) = l(-\Gamma)$ for cubic graphs; thus, cubic "$l_0 \leq k$" is NP-complete because "$l \leq k$" is and $l_0 = l$. [Equality is generalized in Sivaraman and Zaslavsky (20xxa).] (**par: Fr: Algor**)

Timothy Y. Chow

2003a Symplectic matroids, independent sets, and signed graphs. *Discrete Math.* 263 (2003), 35–45. MR 1955713 (2004a:05033). Zbl 1014.05017.

§4, "From graphs to symplectic matroids": The matroid union of $\mathbf{F}(\Gamma, \sigma)$ over all signatures of a fixed graph yields a symplectic matroid. (**SG: Matrd**)

Debashish Chowdhury

1986a *Spin Glasses and Other Frustrated Systems.* Princeton Univ. Press, Princeton, and World Scientific, Singapore, 1986.

Includes brief survey of how physicists look upon frustration. See esp. §1.3, "An elementary introduction to frustration", where the signed square lattice graph illustrates balance vs. imbalance; Ch. 20, "Frustration, gauge invariance, defects and SG [spin glasses]", discussing planar duality (see e.g. Barahona (1982a), "gauge theories", where gains are in the orthogonal or unitary group (switching is called "gauge transformation" by physicists), and functions of interest to physicists; Addendum to Ch. 20, pp. 378–379, mentioning results on when the proportion of negative bonds is fixed, and on gauge theories.

(**Phys: SG, GG, VS, Fr: Exp, Ref**)

Nicholas A. Christakis

See D. Feng.

Dianhui Chu

See D. Li.

Rui Chu

See F.-L. Tian.

San Yan Chu

See S.L. Lee.

Ying Chu

See R.M. Gao.

Maria Chudnovsky

2005a Even hole free graphs. *Graph Theory Notes N.Y.* 49 (2005), 22–24. MR 2202297 (2006h:05185). (**SG: Circ: Algor**)

Maria Chudnovsky, William H. Cunningham, & Jim Geelen

2008a An algorithm for packing non-zero A-paths in group-labelled graphs. *Combinatorica* 28 (2008), no. 2, 145–161. MR 2399016 (2009a:05103). Zbl 1164.05029.

See Chudnovsky, Geelen, *et al.* (2006a). Structure theorem for optimal A-paths in terms of switching only vertices in A^c; algorithm for finding such. Lemma 3.1 generalizes the basis result of Chudnovsky, Geelen, *et al.* (2006a). [*Question.* $B(\Pi)$ is a subset of $V \times \mathfrak{G}$. How is this related to the covering graph? Can one simplify their proofs? A "non-zero" path is like a level-changing path in $\tilde{\Phi}$ (covering graph). This suggests modelling their picture by $\Phi' = \Phi \cup 1K_n$, i.e., with distinguished identity-gain complete subgraph. Or, by $\Omega \subseteq M{\cdot}\Delta$ = a biased expansion, with a distinguished maximal balanced subgraph.] (**GG: Paths: Str, Algor**)

Maria Chudnovsky, Jim Geelen, Bert Gerards, Luis Goddyn, Michael Lohman, & Paul Seymour

2006a Packing non-zero A-paths in group-labelled graphs. *Combinatorica* 26 (2006), no. 5, 521–532. MR 2279668 (2007j:05184). Zbl 1127.05050.

In a gain graph Φ, find the maximum number of vertex-disjoint paths with non-identity gain and with endpoints in $A \subseteq V$ (non-zero A-paths). Thm.: If max $< k$, there is a set X of up to $2k-2$ vertices such that every A-path in $\Phi \setminus X$ has identity gain. This is not best possible.

They prove: $\{B(\Pi) : \Pi \in \mathcal{P}^*(G, A)\}$ is the set of bases of a matroid.

Dictionary: "Group-labelled graph" = gain graph; Γ-labelled graph = Γ-gain graph (for a group Γ); "weight" = gain. "Shifting" = switching; "A-equivalent" = A^c-switching equivalent, i.e., obtained by switching vertices not in A. (**GG: Str, Paths**)

Maria Chudnovsky, Ken-ichi Kawarabayashi, & Paul Seymour

2005a Detecting even holes. *J. Graph Theory* 48 (2005), no. 2, 85–111. MR 2110580 (2006k:05197). Zbl 1062.05135.

Algorithm to detect positive holes (induced circles) in a signed graph. A polynomially equivalent problem is to decide whether a graph is negative-hole signable, i.e., has a signature in which every hole is negative. (**SG: Circ: Algor**)

S.T. Chui

See also B.W. Southern.

S.T. Chui, G. Forgacs, & D.M. Hatch

1982a Ground states and the nature of a phase transition in a simple cubic fully frustrated Ising model. *Phys. Rev. B* 25 (1982), no. 11, 6952–6958.

Physics of "fully frustrated" 3-dimensional cubic lattice, i.e., every square ("plaquette") is negative. Each square has one negative edge. This is the unique fully frustrated signature up to switching [short proof: the squares generate the cycle space], but there are many nonisomorphic ground states ($\zeta : V \to \{+1, -1\}$ such that $\min_\zeta \#(E^\zeta)^-$); they are said to form 12 mutually unreachable classes. App. A characterizes the ground states [and implies $l(\Sigma) = \frac{1}{4}\#V$ since each cube has one negative edge in each direction, neglecting boundary effects—or assuming toroidality]. The signed lattice is at times assumed to have a 2×2 fundamental domain; under that assumption there are 8 translational symmetry types of vertex, each forming a double-sized sublattice. Approximate clustering is discussed. [Annot. 18 Jun 2012.] (**Phys, SG: State(fr), sw, Clu**)

Deborah Chun, Tyler Moss, Daniel Slilaty, & Xiangqian Zhou

2015a Unavoidable minors of large 4-connected bicircular matroids. *Ann. Combin.* 19 (2015), no. 1, 95–105. MR 3319862. Zbl 1310.05043. (**Bic: Str**)

2016a Bicircular matroids representable over *GF* (4) or *GF* (5). *Discrete Math.* 339 (2016), 2239–2248. MR 3512338. Zbl 1338.05038.

Implies characterization of Γ with antivoltages in $\mathbb{Z}_3$. $\mathbb{Z}_4$. *Cf.* Zaslavsky (2007a). [Annot. 11 Jun 2019.] (**Bic: Geom**)

Yang Chun

See B. Jiao.

F.R.K. Chung, Wayne Goddard, & Daniel J. Kleitman

1994a Even cycles in directed graphs. *SIAM J. Discrete Math.* 7 (1994), 474–483. MR 1285584 (95e:05050). Zbl 809.05062.

A strongly connected digraph with $\#E \geq \lfloor (n + 1)^2/4 \rfloor$ has an even cycle. This is best possible. [This equals Petersdorf's (1966a) bound for

$l(K_{n+2}, \sigma)$. *Question.* Are they related?] [Annot. 12 Jun 2012.] (**sd: Par: Bal**)

Fan Chung & Mark Kempton

2013a A local clustering algorithm for connection graphs. In: Anthony Bonato *et al.*, eds., *Algorithms and Models for the Web Graph* (Proc. 10th Int. Workshop, WAW 2013, Cambridge, Mass., 2013), pp. 26–43. Lect. Notes in Computer Sci., Vol. 8305. Springer, Cham, 2013. MR 3163708. Zbl 1342.05158. (**GG, WG: Bal, Lap: Algor**)

2015a A local clustering algorithm for connection graphs. *Internet Math.* 11 (2015), no. 4-5, 333–351. MR 3373768.

Connection graph: A real-weighted $GL(\mathbb{R}, d)$-gain graph. (**GG, WG: Bal, Lap: Algor**)

Fan Chung, Wenbo Zhao, & Mark Kempton

2014a Ranking and sparsifying a connection graph. *Internet Math.* 10 (2014), no. 1-2, 87–115. MR 3274541. Zbl 1342.05182. (**GG, WG: Bal: Algor**)

Fu-Lai Chung

See X. Shen.

Taeyoung Chung, Jack Koolen, Yoshio Sano, & Tetsuji Taniguchi

2011a The non-bipartite integral graphs with spectral radius three. *Linear Algebra Appl.* 435 (2011), no. 10, 2544–2559. MR 2811137 (2012d:05224). Zbl 1222.05151.

§2.2, "Generalized line graphs and generalized signless Laplace matrices": The generalized signless Laplace matrix of (Γ, f), where $f : V \to \mathbb{Z}_{\geq 0}$, is $L(-\Gamma) + 2D(f)$. The incidence matrix of (Γ, f) is $\mathrm{H}(\Sigma)$ where Σ consists of $-\Gamma$ with $f(x)$ negative digons adjoined to $x \in V$. [See Zaslavsky (1984c), (2010b), (20xxa) for this construction, which is not stated here.] [Annot. 20 Dec 2011.] (**sg: Par: Eig, Incid, LG**)

V. Chvátal

See J. Akiyama.

Adriana Ciampella

See F. Belardo and M. Brunetti.

Olivier Cinquin & Jacques Demongeot

2002a Roles of positive and negative feedback in biological systems. *C. R. Biologies* 325 (2002), 1085–1095.

Stability of systems of nonlinear differential equations. Some mathematical treatment. (**SD: QSta, Appl**)

2002b Positive and negative feedback: Striking a balance between necessary antagonists. *J. Theor. Biol.* 216 (2002), 229–241. MR 1941484 (no rev). (**SD: Exp**)

S.M. Cioabă

See S. Akbari, F. Belardo, and M. Cavers.

Valerio Ciotti, Ginestra Bianconi, Andrea Capocci, Francesca Colaiori, & Pietro Panzarasa

2015a Degree correlations in signed social networks. *Physica A* 422, (2015), 25–39. arXiv:1412.1024. **(PsS: SG: Appl: Bal, Clu)**

Pedro Cisneros-Velarde

See also W.-J. Mei.

Pedro Cisneros-Velarde & Francesco Bullo

2020a Signed network formation games and clustering balance. *Dyn. Games Appl.* 10 (2020), no. 4, 783–797. MR 4181812. Zbl 1457.91105.

Players change edge signs to converge on a clustered signed graph. [Annot. 3 Jun 2022.] **(SG: Dyn: Clu)**

Lane Clark

2004a Limit theorems for associated Whitney numbers of Dowling lattices. *J. Combin. Math. Combin. Comput.* 50 (2004), 105–113. MR 2075859 (2005b:06007). Zbl 1053.06003.

Limit theorems and asymptotics for modified Whitney numbers (first kind) introduced by Benoumhani (1997a). [Annot. 12 Jul 2016.] **(gg: Matrd: Invar)**

F.W. Clarke, A.D. Thomas, & D.A. Waller

1980a Embeddings of covering projections of graphs. *J. Combin. Theory Ser. B* 28 (1980), 10–17. MR 0565507 (81f:05066). Zbl 351.05126, (Zbl 416.05069). **(gg: Top)**

Nancy E. Clarke, Samuel Fiorini, Gwenaël Joret, & Dirk Oliver Theis

2014a A note on the cops and robber game on graphs embedded in non-orientable surfaces. *Graphs Combin.* 30 (2014), no. 1, 19–124. MR 3143863. Zbl 1295.05153.

[Thm. 1 actually proves Thm. 1′: The cop number $c(\Gamma)$ satisfies $c(|\Sigma|) \leq c(\tilde{\Sigma})$, the cop number of the double cover of Σ.] Thm. 1 is stated and proved for Σ that is orientation embedded in a surface [but the proof uses only the double covering graph]. Lem. 1 includes Thm. 1′. [Annot. 30 Nov 2014.] **(sg: cov: Top)**

Katie Clinch, Anthony Nixon, Bernd Schulze, & Walter Whiteley

2020a Pairing symmetries for Euclidean and spherical frameworks. *Discrete Comput. Geom.* 64 (2020), no. 2, 483–518. MR 4131558. Zbl 1447.52024. arXiv:-1906.00578. **(GG: Geom)**

Nolan J. Coble

See F. Belardo.

A.M. Cohen

See A.E. Brouwer.

Bernard P. Cohen

See J. Berger.

Edith Cohen & Nimrod Megiddo

1989a Strongly polynomial-time and NC algorithms for detecting cycles in dynamic graphs. In: *Proceedings of the Twenty First Annual ACM Symposium on Theory of Computing* (Seattle, 1989), pp. 523–534.

Partial version of (1993a). **(GD: Bal: Algor)**

1991a Recognizing properties of periodic graphs. In: Peter Gritzmann and Bernd Sturmfels, eds., *Applied geometry and Discrete Mathematics: The Victor Klee*

Festschrift, pp. 135–146. DIMACS Ser. Discrete Math. Theor. Computer Sci., Vol. 4. Amer. Math. Soc., Providence, R.I., and Assoc. Computing Mach., 1991. MR 1116344 (92g:05166). Zbl 753.05047.

Given: a gain graph Φ with gains in $\mathbb{Z}^d$ (a "static graph"). Found: algorithms for (1) connected components and (2) bipartiteness of the covering graph $\tilde{\Phi}$ (the "periodic graph") and, (3) given costs on the edges of Φ, for a minimum-average-cost spanning tree in the covering graph. Many references to related work. **(GG: Cov: Algor, Ref)**

1992a New algorithms for generalized network flows. In: D. Dolev, Z. Galil, and M. Rodeh, eds., *Theory of Computing and Systems* (Proc., Haifa, 1992), pp. 103–114. Lect. Notes in Computer Sci., Vol. 601. Springer-Verlag, Berlin, 1992. MR 1233831 (no rev).

Preliminary version of (1994a), differing only slightly. **(GN: Algor)(sg: Ori: Algor)**

1993a Strongly polynomial-time and NC algorithms for detecting cycles in periodic graphs. *J. Assoc. Comput. Mach.* 40 (1993), 791–830. MR 1369189 (96h:05182). Zbl 782.68053.

Looking for a closed walk ("cycle") with gain 0 in a gain digraph with (additive) gains in $\mathbb{Q}^d$. [*Cf.* Kodialam and Orlin (1991a).] **(GD: Bal: Algor)**

1994a New algorithms for generalized network flows. *Math. Programming* 64 (1994), 325–336. MR 1286453 (95k:90111). Zbl 816.90057.

Maximize the fraction of demand satisfied by a flow on a network with gains. Positive real gains in §3. Bidirected networks with positive gains in §4; these are more general than networks with arbitrary non-zero real gains. **(GN: Algor)(sg: Ori: Algor)**

1994b Improved algorithms for linear inequalities with two variables per inequality. *SIAM J. Comput.* 23 (1994), 1313–1347. MR 1303338 (95i:90040). Zbl 833.90094. **(GN: Incid: Du: Algor)**

Olivier Cohen

See J. Aracena and J. Demongeot.

Francesca Colaiori

See V. Ciotti.

Charles J. Colbourn & Derek G. Corneil

1980a On deciding switching equivalence of graphs. *Discrete Appl. Math.* 2 (1980), 181–184. MR 0588697 (81k:05090). Zbl 438.05054.

Deciding switching isomorphism of graphs is polynomial-time equivalent to graph isomorphism. **(TG: Algor)**

Tom Coleman, James Saunderson, & Anthony Wirth

2008a A local-search 2-approximation for 2-correlation-clustering. In: D. Halperin and K. Mehlhorn, eds., *Algorithms – ESA 2008* (16th Ann. Europ. Symp. Algorithms, Karlsruhe, 2008), pp. 308–319. Lect. Notes in Computer Sci., Vol. 5193. Springer, Berlin, 2008. Zbl 1158.68549. **(SG: Clu)**

D.A. Coley

See T. Wanschura.

L. Collatz

1978a Spektren periodischer Graphen. *Resultate Math.* 1 (1978), 42–53. MR 0510149 (80b:05042). Zbl 402.05054.

Introducing periodic graphs: these are connected canonical covering graphs $\Gamma = \tilde{\Phi}$ of finite $\mathbb{Z}^d$-gain graphs Φ. The "spectrum" of Γ is the set of all eigenvalues of $A(\|\Phi\|)$ for all possible Φ. The spectrum, while infinite, is contained in the interval $[-r, r]$ where r is the largest eigenvalue of each $A(\|\Phi\|)$ [the "index" of von Below (1994a)]. The inspiration is tilings. (**GG: Cov: Eig**)

Barry E. Collins & Bertram H. Raven

1968a Group structure: attraction, coalitions, communication, and power. In: Gardner Lindzey and Elliot Aronson, eds., *The Handbook of Social Psychology*, second ed., Vol. 4, Ch. 30, pp. 102–204. Addison-Wesley, Reading, Mass., 1968.

"Graph theory and structural balance," pp. 106–109. (**PsS: SG: Exp, Ref**)

Luke Collins

See I. Sciriha.

Barbara Coluzzi, Enzo Marinari, Giorgio Parisi, & Heiko Rieger

2000a On the energy minima of the Sherrington-Kirkpatrick model. *J. Phys. A* 33 (2000), no. 21, 3851–3862. MR 1769547 (no rev). Zbl 945.82004. arXiv:cond-mat/0003287. (**Phys: SG**)

Ph. Combe & H. Nencka

1995a Non-frustrated signed graphs. In: J. Bertrand *et al.*, eds., *Modern Group Theoretical Methods in Physics* (Proc. Conf. in Honour of Guy Rideau, Paris, 1995), pp. 105–113. Math. Phys. Stud., Vol. 18. Kluwer, Dordrecht, 1995. MR 1361440 (96j:05105). Zbl 905.05071.

Σ is balanced iff a fundamental system of circles is balanced [as is well known; see *i.a.* Popescu (1979a), Zaslavsky (1981b)]. An algorithm [incredibly complicated, compared to the obvious method of tracing a spanning tree] to determine all vertex signings of Σ that switch it to all positive. Has several physics references. (**SG: Bal, Fr, Algor, Ref**)

1997a Cooperative networks and frustration on graphs. *Methods Funct. Anal. Topology* 3 (1997), 40–50. MR 1770677 (2001e:91135). Zbl 933.92005.

A signed-graphic model Σ of a neuron network. Obs.: A network is cooperative iff Σ has a non-frustrated state $s : V \to \{+1, -1\}$, i.e., the Hamiltonian ("energy") $H(s) := -\frac{1}{2}\sum_{uv\in E} \sigma(uv)s(u)s(v) = -\#E$. [Should be $-\frac{1}{2}\#E$.] H [i.e., Σ] is non-frustrated if some state is. Assertion: H is non-frustrated iff Σ is balanced. A proof idea (not a proof) is by setting up (real-valued) linear equations of positivity of generating circles; carried out for K_n. [See (1997b).] [Easy proof: $H(s) = -\frac{1}{2}\#E + \#E^-(\Sigma^s)$, hence H is non-frustrated iff Σ^s is all positive for some s iff Σ is balanced. See e.g. Zaslavsky (1982a), Cor. 3.3.] [Annot. 17 Jun, 17 Aug 2012.] (**SG: Bal, sw, Fr, Biol**)

1997b Frustration and overblocking on graphs. *Math. Computer Modelling* 26 (1997), no. 8-10, 307–309. MR 1492513 (no rev). Zbl 1185.05147.

No proofs. Prop. 1: The signatures of Γ are a "GF(2)-vector space". [Meaning: They are the points in $\{\pm 1\}^{\#E} \subset \mathbb{R}^{\#E}$.] Prop. 2: Nonfrustration corresponds to a large family of [real] linear systems. "Minimal" circles generalize plaquettes (girth circles) to arbitrary graphs. ["Minimal" =(?) minimum length, assuming such circles generate the cycle space. In general, choice of generating circles remains a good question.] "Fully frustrated": all minimal circles are negative. Prop. 3: Full frustration corresponds to another family of [real] linear systems. ["Overblocking": Fully frustrated and some nonminimal circles are negative.] Prop. 4: Linear system for overblocking in a fully frustrated signature. Cor. 5: K_5 is overblocking. $K_{3,2}$ cannot be fully frustrated. [Annot. 17 Jun 2012.] (**SG: Bal, sw, Fr, Phys**)

Jean-Paul Comet

See also A. Richard.

Jean-Paul Comet, Mathilde Noual, Adrien Richard, Julio Aracena, Laurence Calzone, Jacques Demongeot, Marcelle Kaufman, Aurélien Naldi, El Houssine Snoussi, & Denis Thieffry

2013a On circuit functionality in Boolean networks. *Bull. Math. Biol.* 75 (2013), 906–919. MR 3070304. Zbl 1272.92016. (**SD: Dyn**)

F.G. Commoner

1973a A sufficient condition for a matrix to be totally unimodular. *Networks* 3 (1973), 351–365. MR 0335550 (49 #331). Zbl 352.05012. (**SD: Bal**)

Michele Conforti

See also F. Barahona.

Michele Conforti & Gérard Cornuéjols

1995a Balanced 0, ±1-matrices, bicoloring and total dual integrality. *Math. Programming* 71 (1995), 249–258. MR 1378792 (97a:90103). Zbl 0849.90095.

Bicolorability means every square submatrix contains the incidence matrix of a balanced signed graph. (**SGw, sg: Bal(Gen)**)

Michele Conforti, Gérard Cornuéjols, Ajai Kapoor, & Kristina Vuškoviić

1994a Recognizing balanced $0, \pm 1$ matrices. In: *Proceedings of the 5th Annual ACM-SIAM Symposium on Discrete Algorithms* (Arlington, Va., 1994), pp. 103–111. Assoc. Computing Mach., New York, 1994. MR 1285156 (95e:05022). Zbl 867.05014. (**SGw, sg: Bal(Gen)**)

1995a A mickey-mouse decomposition theorem. In: Egon Balas and Jens Clausen, eds., *Integer Programming and Combinatorial Optimization* (4th Int. IPCO Conf., Copenhagen, 1995, Proc.), pp. 321–328. Lect. Notes in Computer Sci., Vol. 920. Springer, Berlin, 1995. MR 1367991 (96i:05139). Zbl 875.90002 (book).

The structure of graphs that are signable to be "without odd holes": that is, so that each triangle is negative and each chordless circle of length greater than 3 is positive. Proof based on Truemper (1982a). (**SG: Bal(Gen), Str**)

1997a Universally signable graphs. *Combinatorica* 17 (1997), 67–77. MR 1466576 (98g:05134). Zbl 980.00112.

Γ is "universally signable" if it can be signed so as to make every triangle negative and the holes independently positive or negative at will. Such graphs are characterized by a decomposition theorem which leads to a polynomial-time recognition algorithm. **(SG: Bal, Str)**

1999a Even and odd holes in cap-free graphs. *J. Graph Theory* 30 (1999), 289–308. MR 1669460 (99m:05155). Zbl 920.05028.

Recognition of graphs that are "strongly even-signable" f(signable so triangles are $-$ and longer circles with ≤ 1 chord are $+$) and "strongly odd-signable" (signable so quadrilaterals with a unique chord are $+$ and all other circles with ≤ 1 chord are $-$). [Description adapted from Trotignon and Vušković (2010a).] [Annot. 19 Jan 2015.] **(sg: Bal(Gen): Algor)**

2000a Triangle-free graphs that are signable without even holes. *J. Graph Theory* 34 (2000), 204–220. MR 1762021 (2001b:05188). Zbl 953.05061.

"Even hole" means a chordless circle, bigger than a triangle, that is positive in a given signing of the graph. The graphs of the title are characterized in several ways. Most of them have significant wheels. **(SG: Bal, Str, Algor)**

2001a Balanced 0, ±1 matrices. I. Decomposition. *J. Combin. Theory Ser. B* 81 (2001), no. 2, 243–274. MR 1814907 (2002c:05041). Zbl 1026.05016. **(SGw, sg: Bal(Gen))**

2001b Balanced 0, ±1 matrices. II. Recognition algorithm. *J. Combin. Theory Ser. B* 81 (2001), no. 2, 275–306. MR 1814908 (2002c:05042). Zbl 1026.05017. **(SGw, sg: Bal(Gen))**

2001c Perfect, ideal and balanced matrices. *European J. Oper. Res.* 133 (2001), 455–461. MR 1842697 (2002e:05062). Zbl 1053.15014.

§5, "Balanced matrices": Expounds part of Conforti and Cornuéjols (1995a). [Annot. 23 Aug 2014.] **(SGw, sg: Bal(Gen): Exp)**

2002a Even-hole-free graphs. I. Decomposition theorem. *J. Graph Theory* 39 (2002), 6–49. MR 1871344 (2003c:05189). Zbl 1005.05034. **(SG: Bal)**

2002b Even-hole-free graphs. II. Recognition algorithm. *J. Graph Theory* 40 (2002), 238–266. MR 1913849 (2004e:05182). Zbl 1003.05095. **(SG: Bal)**

Michele Conforti, Gérard Cornuéjols, & M.R. Rao

1999a Decomposition of balanced matrices. *J. Combin. Theory Ser. B* 77 (1999), 292–406. MR 1719340 (2001d:05126). Zbl 1023.05025. **(SGw, sg: Bal(Gen))**

Michele Conforti, Gérard Cornuéjols, & Klaus Truemper

1994a From totally unimodular to balanced 0, ±1 matrices: A family of integer polytopes. *Math. Oper. Res.* 19 (1994), no. 1, 21–23. MR 1290007 (96e:15023). Zbl 799.15010.

The forbidden matrix type A' in Rem. 3, par. 2 is the transpose of a signed-graph incidence matrix. [Annot. 23 Aug 2014.] **(sgw, sg: bal(gen))**

Michele Conforti, Gérard Cornuéjols, & Kristina Vuškoviić

1999a Balanced cycles and holes in bipartite graphs. *Discrete Math.* 199 (1999), 27–33. MR 1675908 (99j:05119). Zbl 939.05050. **(SGw, gg, sg: Bal)**

2006a Balanced matrices. *Discrete Math.* 306 (2006), 2411–2437. MR 2261909 (2007g:05131). Zbl 1102.05013.

Bipartite Γ is "balanceable" if it can be signed so each hole (chordless circle) is positive iff it is evenly even. [Truemper (1982a) implies a characterization by forbidden induced signed subgraphs.] "Strongly balancedable": also no circle has a unique chord. [Annot. 19 Jan 2015.] **(SGw, sg: Bal(Gen): Exp)**

Michele Conforti, Samuel Fiorini, Tony Huynh, Gwenaël Joret, & Stefan Weltge

2020a The stable set problem in graphs with bounded genus and bounded odd cycle packing number. In: Shuchi Chawla, ed., *Proceedings of the 2020 ACM-SIAM Symposium on Discrete Algorithms* (SODA 2020, Salt Lake City), pp. 2896–2915. Soc. Industrial Appl. Math. (SIAM), Philadelphia, 2020. MR 4141358. arXiv:1908.06300.

Orientably embedded [i.e., positive] odd circles in a nonorientable surface satisfy the Erdős–Posa property. [*Problem.* Generalize to negative circles in signed graphs.] [Annot. 25 Mar 2022.] **(sg: Par: Circ: Algor)**

Michele Conforti, Samuel Fiorini, Tony Huynh, & Stefan Weltge

2020a Extended formulations for stable set polytopes of graphs without two disjoint odd cycles. In: Daniel Bienstock and Giacomo Zambelli, eds., *Integer Programming and Combinatorial Optimization* (Proc. 21st Int. Conf., IPCO 2020, London), pp. 104–116. Lect. Notes in Computer Sci., Vol. 12125. Springer, Cham, 2020. MR 4139563. Zbl 1508.68263.

See (2022a). **(sg: par: Fr: Circ, Algor)**

2022a Extended formulations for stable set polytopes of graphs without two disjoint odd cycles. *Math. Programming* 192 (2022), no. 1-2, 547–566. MR 4391781. Zbl 1489.90151. arXiv:1911.12179.

[*Problem.* Generalize to negative circles in signed graphs.] [Annot. 25 Mar 2022.] **(sg: par: Fr: Circ, Algor)**

Michele Conforti & Bert Gerards

2007a Packing odd circuits. *SIAM J. Discrete Math.* 21 (2007), no. 2, 273–302. MR 2318666 (2008g:05162). Zbl 1139.05323.

The problem is to find the most vertex-disjoint negative circles in a signed graph (thus, odd-length circles in an ordinary graph). It is NP-hard but it can be solved in polynomial time for the signed graphs that exclude the switching classes $[-K_5]$, $[K_{3,3}^{1,1}]$, $[K_{3,3}^{1,2}]$, $[K_{3,3}^{2}]$, which are defined as: $K_{3,3}^{1,1} = +K_{3,3}$ with edge u_1v_1 made negative and the additional negative edge $-u_2v_2$, $K_{3,3}^{1,2} = +K_{3,3}$ with u_1v_1 made negative and added edges $-u_1u_2$ and $-u_1u_3$, and $K_{3,3}^{2} = +K_{3,3}$ with edges u_1v_1 and u_2v_2 made negative. **(SG: Circ: Str)**

Michele Conforti, Bert Gerards, & Ajai Kapoor

2000a A theorem of Truemper. *Combinatorica* 20 (2000), no. 1, 15–26. MR 1770518 (2001h:05085). Zbl 949.05071.

Full version of Conforti and Kapoor (1998a). **(SG: Bal)**

Michele Conforti & Ajai Kapoor

1998a A theorem of Truemper. In: Robert E. Bixby, E. Andrew Boyd, and Roger Z. Ríos-Mercado, eds., *Integer Programming and Combinatorial Optimization* (6th Int. IPCO Conf., Houston, 1998, Proc.), pp. 53–68. Lect. Notes in Computer Sci., Vol. 1412. Springer, Berlin, 1998. MR 1726335 (2000h:05184). Zbl 907.90269.

A new proof of Truemper's (1982a) theorem on prescribed hole signs. Discussion of applications. **(SG: Bal)**

Antonio Coniglio

1994a Frustrated percolation, spin glasses and glasses. *Nuovo Cimento D* 16 (1994), no. 8, 1027–1037.

The "site-frustrated percolation model": On a signed graph a vertex ("site") may be occupied or not, but a frustrated circle cannot be fully occupied. Unoccupied vertices are considered "defects or holes". Each possible configuration has a weight; the statistical properties are examined. [The unoccupied vertices constitute a balancing vertex set (i.e., its deletion leaves a balanced subgraph). Maximally occupied configurations correspond to minimum balancing vertex sets. *Question.* What does the physics mean for such sets, and vice versa?] [Annot. 22 Aug 2014.] **(sg, Phys: Fr, State(Gen))**

1998a Spin glasses, glasses and granular materials. *Philosophical Mag. B* 77 (1998), no. 2, 213– 219.

§2, "Frustrated lattice gas": a signed-graph ($\pm J$ Ising) model "diluted with lattice gas variables". §3, "Percolation in phase space": see xrefd1994aAntonio Coniglio. **(sg, Phys: Fr)**

1999a Frustrated percolation. *Physica A* 266 (1999), 379–389.

§2, "Clusters in the Ising spin glass model". §4, "Site-frustrated percolation": see (1994a). §6, "Hamiltonian formalism for FP": Eq. (24) shows a Potts–Ising model, corresponding to a signed graph with a variable vertex coloring as well as the spin state (i.e., vertex signs). [Annot. 22 Aug 2014.] **(sg, Phys: State)**

2002a Clusters in frustrated systems. *Physica A* 306 (2002), 76–89.

See esp. §3, "Clusters in the Ising spin-glass model". **(sg, Phys: State)**

A. Coniglio, F. di Liberto, G. Monroy, & F. Peruggi

1991a Cluster approach to spin glasses and the frustrated-percolation problem. *Phys. Rev. B* 44 (1991), no. 22, 12605–12608.

A cluster approach to the Ising spin glass model, i.e., vertex signs ("spins") on a signed graph. Dictionary: "NN" = "nearest-neighbor" = interactions only between adjacent vertices. [Annot. 22 Aug 2014.] **(sg, Phys: State)**

Joseph G. Conlon

2004a Even cycles in graphs. *J. Graph Theory* 45 (2004), no. 3, 163–223. MR 2037758 (2004m:05145). Zbl 1033.05062.

Main theorem: For 3-connected $G \neq K_4$, there is an even circle, deletion of whose vertices or edges leaves a 2-connected graph. [*Problem.*

Generalize to signed graphs. And see Voss (1991a).] (**par**)

S. Contreras

See A.J. Ramírez-Pastor, M.C. Salas-Solís, and E.E. Vogel.

George Converse & M. Katz

1975a Symmetric matrices with given row sums. *J. Combin. Theory Ser. A* 18 (1975), 171–176. MR 0363945 (51 #200). Zbl 297.05024.

An equivalent of Thm. 8.2.1 in Brualdi (2006a). [Annot. 13 Oct 2012.] (**sg: par: Adj**)

S.N. Coppersmith

See J.W. Landry.

Oscar Coppola, Jake Huryn, & Michael Reilly

20xxa An extension of Stanley's symmetric acyclicity theorem to signed graphs. Submitted. arXiv:2302.05992. (**SG: Invar: Col, Ori**)

Raul Cordovil

See P. Berthomé.

Denis Cornaz

2006a On co-bicliques. *RAIRO Oper. Res.* 41 (2006), 295–304. MR 2348004 (2009f:-90053). Zbl 1227.90043. (**SG**)

Denis Cornaz & A. Ridha Mahjoub

2007a The maximum induced bipartite subgraph problem with edge weights. *SIAM J. Discrete Math.* 21 (2007), no. 3, 662–675. MR 2353996 (2008j:05331). Zbl 1141.05076. (**SD**)

Derek G. Corneil

See C.J. Colbourn and Seidel (1991a).

Gérard Cornuéjols

See also M. Campelo and M. Conforti.

2001a *Combinatorial Optimization: Packing and Covering.* CBMS-NSF Reg. Conf. Ser. Appl. Math., Vol. 74. Soc. Indust. Appl. Math., Philadelphia, 2001. MR 1828452 (2002e:90004). Zbl 972.90059.

The topic is linear optimization over a clutter, esp. a "binary clutter", which is the class of negative circuits of a signed binary matroid. The class $\mathcal{C}^-(\Sigma)$ is an important example (see Seymour (1977a)), as is its blocker $b\big(\mathcal{C}^-(\Sigma)\big)$ [which is the class of minimal balancing edge sets; hence the frustration index $l(\Sigma)$ = minimum size of a member of the blocker].

Ch. 5: "Graphs without odd-K_5 minors", i.e., signed graphs without $-K_5$ as a minor. Some esp. interesting results: Thm. 5.0.7 (special case of Seymour (1977a), Main Thm.): The clutter of negative circles of Σ has the "Max-Flow Min-Cut Property" (Seymour's "Mengerian" property) iff Σ has no $-K_4$ minor. Conjecture 5.1.11 is Seymour's (1981a) beautiful conjecture (his "weak MFMC" is here called "ideal"). §5.2 reports the partial result of Guenin (2001a). (See also §8.4.)

Def. 6.2.6 defines a signed graph "$G(A)$" of a $0, \pm 1$-matrix A, whose transposed incidence matrix is a submatrix of A. §6.3.3: "Perfect $0, \pm 1$-

matrices, bidirected graphs and conjectures of Johnson and Padberg (1982a), associates a bidirected graph with a system of 2-variable pseudoboolean inequalities; reports on Sewell (1996a) (*q.v.*).

§8.4: "On ideal binary clutters", reports on Cornuéjols and Guenin (2002a), Guenin (1998a), and Novick and Sebö (1995a) (*qq.v.*).

(**Sgnd(Matrd), SG: Matrd, Geom, Incid(Gen), Ori: Exp, Ref, Exr**)

Gérard Cornuéjols & Bertrand Guenin

2002a Ideal binary clutters, connectivity, and a conjecture of Seymour. *SIAM J. Discrete Math.* 15 (2002), no. 3, 329–352. MR 1921026 (2003h:05057). Zbl 1035.90045.

A partial proof of Seymour's (1981a) conjecture. Main Thm.: A binary clutter is ideal if it has as a minor none of the circuit clutter of F_7, $\mathcal{C}^-(-K_5)$ or its blocker, or $\mathcal{C}^-(-K_4)$ or its blocker. Important are the lift and extended lift matroids, $\mathbf{L}(M, \sigma)$ and $\mathbf{L}_\infty(M, \sigma)$, defined as in signed graph theory. [See Cornuéjols (2001a), §8.4.]

(**Sgnd(Matrd), SG: Matrd, Geom**)

Hugo Corrales

See C.A. Alfaro.

Sylvie Corteel, David Forge, & Véronique Ventos

2015a Bijections between affine hyperplane arrangements and valued graphs. *European J. Combin.* 50 (2015), 30–37. MR 3361409. Zbl 1323.52012. arXiv:-1403.2573. HAL hal-01365957. (**gg: Geom, Matrd**)

S. Cosares

See L. Adler.

Collette R. Coullard

See also V. Chandru.

Collette R. Coullard, John G. del Greco, & Donald K. Wagner

†† 1991a Representations of bicircular matroids. *Discrete Appl. Math.* 32 (1991), 223–240. MR 1120878 (92i:05072). Zbl 755.05025.

§4: §4.1 describes 4 fairly simple types of "legitimate" graph operation that preserve the bicircular matroid. Thm. 4.11 is a converse: if Γ_1 and Γ_2 have the same connected bicircular matroid, then either they are related by a sequence of legitimate operations, or they belong to a small class of exceptions, all having order ≤ 4, whose bicircular matroid isomorphisms are also described. This completes the isomorphism theorem of Wagner (1985a). §5: If finitely many graphs are related by a sequence of legitimate operations (so their bicircular matroids are isomorphic), then they have contrabalanced real gains whose incidence matrices are row equivalent. These results are also found by a different approach in Shull, Orlin, *et al.* (1989a), Shull, Shuchat, *et al.* (1993a), (1997a).

(**Bic: Str, Incid**)

1993a Recognizing a class of bicircular matroids. *Discrete Appl. Math.* 43 (1993), 197–215. MR 1223421 (94i:05021). Zbl 777.05036. (**Bic: Algor**)

1993b Uncovering generalized-network structure in matrices. *Discrete Appl. Math.* 46 (1993), 191–220. MR 1243724 (95c:68179). Zbl 784.05044. (**GN: Bic: Incid, Algor**)

G. Coutinho, C. Godsil, H. Shirazi, & H. Zhan

2016a Equiangular lines and covers of the complete graph. *Linear Algebra Appl.* 488 (2016), 264–283. MR 3419786. Zbl 1409.05220. arXiv:1504.00085. **(sg: Lap, Cov)**

Gheorghe Craciun

See also M. Banaji and M. Mincheva.

Gheorghe Craciun & Martin Feinberg

2005a Multiple equilibria in complex chemical reaction networks: I. The injectivity property. *SIAM J. Appl. Math.* 65 (2005), 1526–1546. MR 2177713 (2006g:92075). Zbl 1094.80005. **(SG, Chem)**

2006a Multiple equilibria in complex chemical reaction networks: II. The species-reaction graph. *SIAM J. Appl. Math.* 66, no. 4, 1321–1338. MR 2246058 (2007e:92027). Zbl 1136.80306. **(SG, Chem)**

2006b Multiple equilibria in complex chemical reaction networks: extensions to entrapped species models. *IEEE Proc. Systems Biol.* 153 (2006), no. 4, 179–186. **(SG, Chem)**

Gheorghe Craciun, Casian Pantea, & Eduardo D. Sontag

2011a Graph-theoretic analysis of multistability and monotonicity for biochemical reaction networks. In: Heinz Koeppl, Douglas Densmore, Gianluca Setti, and Mario di Bernardo, eds., *Design and Analysis of Biomolecular Circuits: Engineering Approaches to Systems and Synthetic Biology*, pp. 63–72. Springer, New York, 2011. **(SG, Chem: Exp)**

Yves Crama

See also E. Boros.

1989a Recognition problems for special classes of polynomials in 0–1 variables. *Math. Programming* A44 (1989), 139–155. MR 1003557 (90f:90091). Zbl 674.90069.

Balance and switching are used to study pseudo-Boolean functions. (§§2.2 and 4.) **(SG: Bal, Sw)**

Yves Crama & Peter L. Hammer

1989a Recognition of quadratic graphs and adjoints of bidirected graphs. Combinatorial Math.: Proc. Third Int. Conf. *Ann. New York Acad. Sci.* 555 (1989), 140–149. MR 1018617 (91d:05044). Zbl 744.05060.

"Adjoint" = unoriented positive part of the line graph of a bidirected graph. "Quadratic graph" = graph that is an adjoint. Recognition of adjoints of bidirected simple graphs is NP-complete. **(sg: Ori: LG: Algor)**

Yves Crama, Peter L. Hammer, & Toshihide Ibaraki

1986a Strong unimodularity for matrices and hypergraphs. Appl. Combin. Methods Math. Programming (Gainesville, Fla., 1985). *Discrete Appl. Math.* 15 (1986), 221–239. MR 0865003 (88a:05105). Zbl 647.05042.

§7: Signed hypergraphs, with a surprising generalization of balance. **(SH: Bal)**

Y. Crama, M. Loebl, & S. Poljak

1992a A decomposition of strongly unimodular matrices into incidence matrices of digraphs. *Discrete Math.* 102 (1992), 143–147. MR 1170457 (93g:05097). Zbl

776.05071.

(**SG**)

R. Crowston, G. Gutin, M. Jones, & G. Muciaccia

2013a Maximum balanced subgraph problem parameterized above lower bound. *Theoretical Computer Sci.* 513 (2013), 53–64. MR 3128945. Zbl 1396.68054. arXiv:-1212.6848.

An algorithm for $l(\Sigma) \leq \frac{1}{2}\#E - \frac{1}{4}(n-1) - \frac{1}{4}k$ with time linear in n and exponential in k, assuming Σ is simple. (The upper bound $l \leq \frac{1}{2}\#E - \frac{1}{4}(n-1)$ is from Poljak and Turzík (1982a), (1986a).) [Annot. 2 Mar 2014.] (**SG: Fr: Algor**)

Anne Crumière & Paul Ruet

2008a Spatial differentiation and positive circuits in a discrete framework. Proc. Workshop (DCM 2007, Wrocław). *Electronic Notes Theor. Computer Sci.* 192 (2008), 85–100. Zbl 1277.92008. HAL hal-00692089.

Regulatory graph: a signed digraph. (**SD: Dyn, Biol**)

Anne Crumière & Mathieu Sablik

2008a Positive circuits and d-dimensional spatial differentiation: Application to the formation of sense organs in *Drosophila*. *BioSystems* 94 (2008), 102–108.

Regulatory graph: a signed digraph. (**Biol: SD: Dyn**)

Lin Cui & Yi-Zheng Fan

2010a The signless Laplacian spectral radius of graphs with given number of cut vertices. *Discuss. Math. Graph Theory* 30 (2010), no. 1, 85–93. MR 2676064 (2011j:05196). Zbl 1215.05101. (**par: Lap: Eig**)

Qing Cui

See W.-Z. Liu.

Shu-Yu Cui & Gui-Xian Tian

2012a The signless Laplacian spectrum of the (edge) corona of two graphs. *Utilitas Math.* 88 (2012), 287–297. MR 2975841. Zbl 1256.05137. (**par: Lap: Eig**)

G.J. Culos, D.D. Olesky, & P. van den Driessche

2016a Using sign patterns to detect the possibility of periodicity in biological systems. *J. Math. Biol.* 72 (2016), 1281–1300. MR 3464203. Zbl 1337.92015. (**QM: SD**)

William H. Cunningham

See J. Aráoz and M. Chudnovsky.

Dragoš M. Cvetković

See also R.A. Brualdi, F.C. Bussemaker, D.M. Cardoso, and M. Doob.

1978a The main part of the spectrum, divisors and switching of graphs. *Publ. Inst. Math. (Beograd) (N.S.)* 23 (37) (1978), 31–38. MR 0508125 (80h:05045). Zbl 423.05028.

1995a Star partitions and the graph isomorphism problem. *Linear Algebra Appl.* 39 (1995), 109–132. MR 1374474 (97b:05105). Zbl 831.05043.

Pp. 128–130 discuss switching-equivalent graphs. Some of the theory is invariant, hence applicable to two-graphs. [*Question*. How can this be generalized to signed graphs and their switching classes?] (**tg: Adj**)

2005a Signless Laplacians and line graphs. *Bull. Cl. Sci. Math. Nat. Sci. Math.* No. 30 (2005), 86–92. MR 2213761 (2006m:05152). Zbl 1119.05066.
(**par: Lap, LG: Eig**)

2008a New theorems for signless Laplacian eigenvalues. *Bull. Cl. Sci. Math. Nat. Sci. Math.* No. 33 (2008), 131–146. MR 2609604 (2011b:05145). Zbl 1199.05212.
(**par: Lap: Eig**)

2010a Spectral theory of graphs based on the signless laplacian (A quick outline). Res. report, 2010. URL `http://www.mi.sanu.ac.rs/projects/signless_L_reportJan28.pdf`

Surveys the spectral theory of $L(-\Gamma)$. [Annot. 23 Nov 2014.]
(**par: Lap: Eig: Exp**)

† 2010b Bibliography on the signless laplacian eigenvalues: First one hundred references. Manuscript, 2010. `http://www.mi.sanu.ac.rs/projects/signless_100.pdf`

Eigenvalues of $L(-\Gamma)$, with a brief history. $L(-\Gamma)$ is important because Spec $L(-\Gamma)$ "dominates" Spec $L(\Phi)$ for any signed or complex unit gain graph; *cf.* Reff (2012a). [Annot. 12 Dec 2020.] (**sg: Par: Lap: Eig**)

Dragoš M. Cvetković & Michael Doob

1984a Root systems, forbidden subgraphs, and spectral characterizations of line graphs. In: *Graph Theory* (4th Yugosl. Sem., Novi Sad, 1983), pp. 69–99. Univ. Novi Sad, Novi Sad, 1984. MR 0751442 (86a:05088). Zbl 533.05041.
(**sg: par: Geom, LG**)

Dragos M. Cvetković, Michael Doob, Ivan Gutman, & Aleksandar Torgašev

1988a *Recent Results in the Theory of Graph Spectra.* Ann. Discrete Math., 36. North-Holland, Amsterdam, 1988. MR 0926481 (89d:05130). Zbl 634.05034.

Signed graphs mentioned: P. 40 cites Zaslavsky (1981a). Pp. 44–45 (with unusual terminology) describe B.D. Acharya (1980a) and M.K. Gill (1981b). P. 100 cites B.D. Acharya (1979b). All-negative signatures are implicated in the infinite-graph eigenvalue theorem of Torgašev (1982a), Thm. 6.29 of this book. Möbius molecules (with signed molecular graphs) mentioned on p. 149. (**SG, par: Eig: Exp, Appl, Ref**)

Dragoš M. Cvetković, Michael Doob, & Horst Sachs

1980a *Spectra of Graphs: Theory and Application.* VEB Deutscher Verlag der Wissenschaften, Berlin, 1980. Copublished as: Pure and Appl. Math., Vol. 87. Academic Press, New York-London, 1980. MR 0572262 (81i:05054). Zbl 458.05042.

§4.6: Signed digraphs with multiple edges are employed to analyze the characteristic polynomial of a digraph. (Signed) switching, too. Pp. 187–188: Exercises involving Seidel switching and the Seidel adjacency matrix. Thm. 6.11 (Doob (1973a)): The even-cycle matroid determines the eigenvaluicity of -2. §7.3: "Equiangular lines and two-graphs." [Annot. $\leq$ 2000, rev 20 Sept 2010.]

[Russian ed.: Tsvetkovich, Dub, and Zakhs (1984a).]
(**SD, par, TG: Sw, Adj, Eig, Geom: Exp, Exr, Ref**)

1982a *Spectra of Graphs: Theory and Application.* VEB Deutscher Verlag der Wissenschaften, Berlin, 1982. MR 0690768 (84a:05046).

Update of (1980a).
(**SD, par, TG: Sw, Adj, Eig, Geom: Exp, Exr, Ref**)

1995a *Spectra of Graphs: Theory and Applications.* Third ed. Johann Ambrosius Barth, Heidelberg, 1995. MR 1324340 (96b:05108). Zbl 824.05046.
Appendices update (1982a), beyond the updating in Cvetković, Doob, Gutman, and Torgašev (1988a). App. B.3, p. 381 mentions work of Vijayakumar (*q.v.*). P. 422: Pseudo-inverse graphs (when $A(\Gamma)^{-1} = A(\Sigma)$ for some balanced Σ, $|\Sigma|$ is the "pseudo-inverse" of Γ).
(**SD, par, TG: Adj, Lap, Eig, Sw, Geom, Bal: Exp, Exr, Ref**)

Dragoš Cvetković, Michael Doob, & Slobodan Simić

1980a Some results on generalized line graphs. *C. R. Math. Rep. Acad. Sci. Canada* 2 (1980), 147–150. MR 0576993 (81f:05136). Zbl 434.05057.
Abstract of (1981a). (**sg: LG, Eig(LG), Aut(LG)**)

1981a Generalized line graphs. *J. Graph Theory* 5 (1981), 385–399. MR 0635701 (82k:05091). Zbl 475.05061. (**sg: LG, Eig(LG), Aut(LG)**)

Dragoš Cvetković, Peter Rowlinson, & Slobodan K. Simić

2004a *Spectral Generalizations of Line Graphs: On Graphs with Least Eigenvalue* −2. London Math. Soc. Lect. Note Ser., 314. Cambridge Univ. Press, Cambridge, Eng., 2004. MR 2120511 (2005m:05003). Zbl 1061.05057.
Generalized line graphs are the fundamental example. Pp. 190–191 mention signed graphs representable in root systems as in papers of G.R. Vijayakumar (*q.v.*) [but not mentioning line graphs of signed graphs]. [Annot. 13 Oct 2010.] (**LG: Gen, Geom, Eig**)(**SG: Geom: Exp**)

2007a Signless Laplacians of finite graphs. *Linear Algebra Appl.* 423 (2007), no. 1, 155–171. MR 2312332 (2008c:05105). Zbl 1113.05061.
"Signless Laplacian" $Q(\Gamma) :=$ Laplacian matrix $L(-\Gamma) = D(\Gamma) + A(\Gamma)$. Spectral properties; bounds for graph invariants; combinatorics of coefficients of characteristic polynomial of $L(-\Gamma)$. [*Problem.* Find all articles on "signless Laplacians", herein called $L(-\Gamma)$. Generalize to signed graphs, with nonbipartite graphs generalizing to unbalanced graphs.] [Annot. 14 Sept 2010.] (**sg: Par:Lap: Eig**)

2007b Eigenvalue bounds for the signless Laplacian. *Publ. Inst. Math. (Beograd) (N.S.)* 81(95) (2007), 11–27. MR 2401311 (2009e:05181). Zbl 1164.05038.
See (2007a). Thm.: For connected Γ with $\#V = n$ and $\#E = m$, $\lambda_{\max}(L(-\Gamma))$ is maximized when Γ is a nested split graph. Also, many computer-generated conjectures (*cf.* Aouchiche and Hansen (2010a)); some are proved (here or elsewhere) or disproved; some are difficult. [Annot. 4 Sept 2010, 22 Jan 2012.] (**Par: Lap: Eig, LG**)

2010a *An Introduction to the Theory of Graph Spectra.* London Math. Soc. Student Texts, 75. Cambridge Univ. Press, Cambridge, Eng., 2010. MR 2571608 (2011g:05004). Zbl 1211.05002.
Graph switching in §1.1. Reduced line graphs of simply signed graphs are implicit in the construction of generalized line graphs in §1.2. [Annot.

14 Sept 2010.] (**tg: Sw: Exp**)(**sg: LG: Exp**)
§7.8, "The signless Laplacian". (**Par, LG: Lap: Eig: Exp**)

Dragoš M. Cvetković & Slobodan K. Simić

1978a Graphs which are switching equivalent to their line graphs. *Publ. Inst. Math. (Beograd) (N.S.)* 23 (37) (1978), 39–51. MR 0508126 (80c:05108). Zbl 423.05035. (**sw: LG**)

2009a Towards a spectral theory of graphs based on the signless Laplacian. I. *Publ. Inst. Math. (Beograd) (N.S.)* 85(99) (2009), 19–33. MR 2536686 (2010i:05203). Zbl 224.05293.
See Cvetković, Rowlinson, and Simić (2007a). (**par: Lap: Eig**)

2010a Towards a spectral theory of graphs based on the signless Laplacian. II. *Linear Algebra Appl.* 432 (2010), no. 9, 2257–2272. MR 2599858 (2011d:05217). Zbl 1218.05089. (**par: Lap: Eig**)

2010b Towards a spectral theory of graphs based on the signless Laplacian. III. *Appl. Anal. Discrete Math.* 4 (2010), no. 1, 156–166. MR 2654936 (2011m:05169). Zbl 1265.05360. (**par: Lap: Eig**)

2011a Graph spectra in Computer Science. *Linear Algebra Appl.* 434 (2011), no. 6, 1545–1562. MR 2775765 (2011m:05170). Zbl 1207.68230. (**Par: Eig: Exp**)

D. Cvetković, S.K. Simić, & Z. Stanić

2010a Spectral determination of graphs whose components are paths and cycles. *Computers Math. Appl.* 59 (2010), 3849–3857. MR 2651858 (2011j:05197) (*q.v.*). Zbl 1198.05110.
Spec $L(-\Gamma)$ performs well to determine the graph from its spectrum. [Annot. 29 Apr 2022.] (**sg: Par: Eig**)

Marek Cygan, Marcin Pilipczuk, Michał Pilipczuk, & Jakub Onufry Wojtaszczyk

2012a Sitting closer to friends than enemies, revisited. In: Branislav Rovan, Vladimiro Sassone, and Peter Widmayer, eds., *Mathematical Foundations of Computer Science 2012* (37th MFCS, Bratislava, 2012), pp. 296–307. Lect. Notes in Computer Sci., Vol. 7464. Springer, Heidelberg, 2012. MR 3030440. Zbl 1365.68351. arXiv:1201.1869.
Same as (2015a). (**SG: KG: Bal, Algor**)

2015a Sitting closer to friends than enemies, revisited. *Theory Computing Systems* 56 (2015), no. 2, 394–405. MR 3311468. Zbl 1312.05067. arXiv:1201.1869.
Sequel to Kermarrec and Thraves (2011a). [Annot. 26 Apr 2012.] (**SG: KG: Bal, Algor**)

V. Jude Annie Cynthia & E. Padmavathy

2018a Signed product cordiality of circulant network. *Int. J. Pure Appl. Math.* 118 (2018), no. 23, 353–361.
More as in Baskar Babujee and Loganathan (2011a). (**Lab: VS: SG, Bal**)

2021a Signed cordial labeling and signed product cordial labeling of some interconnection networks. *Adv. Appl. Discrete Math.* 26 (2021), no. 1, 35–51. Zbl 1499.05547.

Cf. Delphy and Devaraj (2011a) and Babujee and Loganathan (2011a) for cordiality and product cordiality, resp. Examples. [Annot. 26 Sept 2022.] **(Lab: SG, VS)**

2021b On signed magic cordial labeling. *Adv. Appl. Discrete Math.* 27 (2021), no. 1, 15–30. Zbl 1499.05548. **(Lab: SG, VS)**

[Ilda P.F. da Silva]
See I.P.F. da Silva (under 'S').

Payal Dabas
See S. Kansal.

A. Daemi
See S. Akbari.

F. Scott Dahlgren
See M. Arav.

C. Dalf'o, M.A. Fiol, M. Miller, & J. Ryan [Joe Ryan]

2017a From expanded digraphs to lifts of voltage digraphs and line digraphs. *Australasian J. Combin.* 69 (2017), 323–333. MR 3714196. Zbl 1375.05113. arXiv:-1608.06233. **(GG: Cov)**

2017b On quotient digraphs and voltage digraphs. *Australasian J. Combin.* MR 3714200. Zbl 1375.05114. arXiv:1612.08855. **(GG: Cov)**

Mina Dalirrooyfard
See S. Akbari.

Soudabeh Dalvandi
See also S. Akbari.

S. Dalvandi, F. Heydari, & M. Maghasedi

2021a Signed complete graphs with negative paths. *J. Math. Extension* 15 (2021), no. 1, 127–136. Zbl 1473.05115.

Finds many eigenvalues when Σ^- consists of disjoint paths. [Annot. 26 Dec 2022.] **(SG: Adj: Eig)**

2022a Signed complete graphs with exactly m non-negative eigenvalues. *Bull. Malaysian Math. Sci. Soc.* 45 (2022), 2107–2122. MR 4489553. Zbl 1504.05120.

Adjacency eigenvalues of $K_n(-\Delta)$ with non-spanning Δ. Thm. 7: $c(\Delta) \leq m-1$. If $= m-1$, each component is complete bipartite. [I.e., bipartitionally induced in K_n. *Question.* Does that generalize to $\Gamma(-\Delta)$, for 2-connected Γ?] Thm. 11: $m = 3$ implies $c(\Delta) = 2$. Characterized: $m = 3$ with disconnected or (Thm. 15) tree Δ. Thm. 14: If connected Δ has exactly 2 nonnegative eigenvalues, then $K_n(-\Delta)$ has exactly 3. Lemmas: Spectra of examples such as $K_{2m}(-M_m)$, M_m an m-edge matching.

Thm. 8: A signed graph with exactly one positive eigenvalue is balanced. Thm. 10: For any Δ, if $m = 2$, Δ cannot be spanning. [Annot. 17 Oct, 26 Dec 2022, 13 Jan 2023.] **(SG: KG: Str, Adj: Eig)**

K.V. Dalvi
See G. Mundhe.

Edwin R. van Dam

See also P. Wissing.

E.R. van Dam & M.A. Fiol

2012a A short proof of the odd-girth theorem. *Electronic J. Combin.* 19 (2012), no. 3, art. P12, 5 pp. MR 2967217. Zbl 1253.05098. arXiv:1205.0153.

Odd Girth Thm.: A graph with $d+1$ eigenvalues and odd girth $\geq 2d+1$ is a generalized odd graph. [*Problem*: Generalize to signed graphs, odd girth becoming negative girth and distance-regular and generalized odd graphs becoming one wants to know what. Knowing what would indicate what a distance-regular or generalized-odd signed graph should be.] [Annot. 17 Dec 2014.] **(par: Eig: Str, Ref)**

Edwin R. van Dam & Willem H. Haemers

2003a Which graphs are determined by their spectrum? Special issue on the Combinatorial Matrix Theory Conference (Pohang, 2002). *Linear Algebra Appl.* 373 (2003), 241–272. MR 2022290 (2005a:05135). Zbl 1026.05079. **(par: Lap: Eig)(sg: par: Lap: Eig: Exp)**

2009a Developments on spectral characterizations of graphs. Int. Workshop Design Theory, Graph Theory, Comput. Methods – IPM Combinatorics II. *Discrete Math.* 309 (2009), no. 3, 576–586. MR 2499010 (2010h:05178). Zbl 1205.05156.

New and old results on $L(-\Gamma)$, the "signless Laplacian" of Γ. [Annot. 20 Dec 2011.] **(par: Lap: Eig)(Par: Eig: Exp)**

Susan S. D'Amato

1979a Eigenvalues of graphs with twofold symmetry. *Molecular Phys.* 37 (1979), 1363–1369. MR 0535191 (80c:05098).

Spectrum of signed covering graph. [See Butler (2010a).] [Annot. 9 Mar 2011.] **(sg: cov: Eig)**

1979b Eigenvalues of graphs with threefold symmetry. *Theor. Chim. Acta* 53 (1979), 319–326.

Ternary gain graphs: spectrum of covering graph, as with signed graphs in (1979a). [Annot. 9 Mar 2011.] **(gg: cov: Eig)**

Jeffrey M. Dambacher, Richard Levins, & Philippe A. Rossignol

2005a Life expectancy change in perturbed communities: Derivation and qualitative analysis. *Math. Biosciences* 197 (2005), no. 1, 1–14. MR 2167483 (2006d:92058). Zbl 1074.92037. **(SD: QM: QSta: Cyc, Ref)**

Jeffrey M. Dambacher, Hiram W. Li, & Philippe A. Rossignol

2003a Qualitative predictions in model ecosystems. *Ecological Modelling* 161 (2003), no. 1, 79–93.

Feedback predictions from signed digraph (D, σ) via "weighted predictions" $W_{ij} := |C_{ij}(-A(D, \sigma))|/P_{ij}(A(D))$, where C_{ij} is the cofactor and P_{ij} is the permanental cofactor. $W_{ij} = 1$ means perfect predictability, $= 0$ means no predictability. Numerical tests. Dictionary: "Community matrix" $= A(D, \sigma)$. [Annot. 9 Sept 2010.] **(SD: QM: QSta: Cyc, Ref)**

E. Damiani, O. D'Antona, & F. Regonati

1994a Whitney numbers of some geometric lattices. *J. Combin. Theory Ser. A* 65 (1994), 11–25. MR 1255260 (95e:06019). Zbl 793.05037.

E.g., log concavity of Whitney numbers of the second kind of Dowling lattices. [*Cf.* Stonesifer (1975a) and Benoumhani (1999a).] [Annot. rev. 30 Apr 2012.] (**gg: Matrd: Invar**)

A. Danielian

1961a Ground state of an Ising face-centered cubic lattice. *Phys. Rev. Lett.* 6 (1961), 670–671.

"Ground states", i.e. $\zeta : V \to \{+1, -1\}$ with smallest $\#(E^\zeta)^-$, of the all-negative (antiferromagnetic) $R \times R \times R$ face-centered cubic lattice graph [assumed toroidal to avoid boundary effects?]. Frustration index $l = 2\#V$; the number ("degeneracy") of ground states is $2^{A\sqrt[3]{\#V}}$ where $A > 0$; each ground state has 4-regular E^-. See (1964a) for more structure. [*Problem.* Determine the exact number and precise shape of all ground states ζ in terms of the graph. Is there something interesting about $(\Sigma^\zeta)^-$, e.g., in its circle decomposition, symmetries, or transformations from one to another?] [Annot. 21 Jun 2012.]

(**SG, Phys: Par: Fr, State(fr)**)

1964a Low-temperature behavior of a face-centered cubic antiferromagnet. *Phys. Rev.* 133 (1964), no. 5A, A1344–A1349.

§ II, "The ground state", continues (1961a) with more details on the structure of ground states ζ. The number of them is small compared to the all-negative triangular lattice [*Question*: and other all-negative, highly symmetric graphs?]. ζ on each x-, y-, or z-layer has a form described in the paper. Low-weight distance-2 edges will fix the ground state (p. A1346). § III, "The partition function", studies the effect of moving out of ground states. App. A derives a formula for the energy change from switching a cluster of vertices, in terms of frustrated and satisfied edges within and without the cluster. App. B estimates the effect of switching additional vertices. [*Problem.* Find rigorous treatments of such switchings; this means studying the energy landscape of state space $\{\zeta\} = \{+1, -1\}^{\{+1,-1\}^V}$.] Dictionary: "bond" = edge, "even/odd bond" = frustrated/satisfied edge = switches to + or −. [Annot. 21 Jun 2012.]

(**Phys, SG: Par: Fr, State(fr)**)

Daniele D'Angeli

See M. Cavaleri.

O. D'Antona

See E. Damiani.

George B. Dantzig

1963a *Linear Programming and Extensions.* Princeton Univ. Press, Princeton, N.J., 1963. MR 0201189 (34 #1073). Zbl 108.33103 (108, p. 331c).

Chapter 21: "The weighted distribution problem." 21-2: "Linear graph structure of the basis." (**GN: Matrd(Bases)**)

1966a *Linear Programming and Extensions.* (In Russian.) Transl. G.N. Andrianov, L.I. Gor'kov, A.A. Korbut, and A.N. Ljapunov. "Progress" Publishers, Moscow, 1966. Zbl 997.90504. (**GN: Matrd(Bases)**)

1998a *Linear Programming and Extensions.* Repr. of 1968 corr. ed. Princeton Landmarks in Math. Princeton Univ. Press, Princeton, N.J., 1998. MR 1658673

(99g:90004) (no rev). Zbl 997.90504. **(GN: Matrd(Bases))**

F.A. Dar

See S. Pirzada.

Richard D'Ari

See R. Thomas.

Avinandan Das, Lawqueen Kanesh, Jayakrishnan Madathil, & Saket Saurabh

2021a Odd cycle transversal in mixed graphs. In: Łukasz Kowalik *et al.*, eds., *Graph-Theoretic Concepts in Computer Science* (47th Int. Workshop, WG 2021, Warsaw), pp. 130–142. Lect. Notes in Computer Sci., Vol. 12911. Springer, Cham, 2021. MR 4364278.

"Odd cycle transversal" = cycle-balancing set of all-negative signed mixed graph. Problem: $\exists$ such set of size $\leq k$? [Annot. 25 Mar 2022.] **(SD: Fr: Algor)**

Kinkar Ch. Das [Kinkar Chandra Das]

See also J. Askari and S. Mandal.

2010a On conjectures involving second largest signless Laplacian eigenvalue of graphs. *Linear Algebra Appl.* 432 (2010), no. 11, 3018–3029. MR 2639266 (2011h:05151). Zbl 1195.05040. **(par: Lap: Eig)**

2011a Proof of conjecture involving the second largest signless Laplacian eigenvalue and the index of graphs. *Linear Algebra Appl.* 435 (2011), no. 10, 2420–2424. MR 2811126 (2012m:05204). Zbl 1223.05171. **(par: Lap: Eig)**

2012a Proof of conjectures involving the largest and the smallest signless Laplacian eigenvalues of graphs. *Discrete Math.* 312 (2012), 992–998. MR 2872940. Zbl 1237.05124.

Assume $n \geq 4$. Thm. 3.2: $\lambda_{\max}(-\Gamma) + \lambda_{\min}(-\Gamma) \leq 3n - 2 - 2\alpha(\Gamma)$, where $\alpha :=$ independence number; $=$ iff $\Gamma = K_{n-\alpha} \vee \bar{K}_\alpha$. Thm. 3.3: $\lambda_{\max}(-\Gamma) - \lambda_{\min}(-\Gamma) \geq 2 + 2\cos(/pi/n)$, with equality iff Γ is a path or odd circle. [Annot. 21 Jan 2012.] **(par: Lap: Eig)**

Prabir Das & S.B. Rao

1983a Alternating eulerian trails with prescribed degrees in two edge-colored complete graphs. *Discrete Math.* 43 (1983), 9–20. MR 0680299 (84k:05069). Zbl 494.05020.

Given an all-negative bidirected K_n and a positive integer $f_i = 2g_i$ for each vertex v_i. There is a connected subgraph having in-degree and out-degree $= g_i$ at v_i iff there is a g-factor of introverted and one of extroverted edges and the degrees satisfy a complicated degree condition. Generalizes Thm. 1 of Bánkfalvi and Bánkfalvi (1968a). [See Bang-Jensen and Gutin (1997a) for how to convert an edge 2-coloring to an orientation of an all-negative graph and for further developments on alternating walks.] **(par: ori)**

Sandip Das

See also J. Bensmail.

Sandip Das, Prantar Ghosh, Swathy Prabhu [Swathyprabhu Mj], & Sagnik Sen

2016a Relative clique number of planar signed graphs. In: Sathish Govindarajan and Anil Maheshwari, eds., *Algorithms and Discrete Applied Mathematics* (Proc.

2nd Int. Conf., CALDAM 2016, Thiruvananthapuram, India, 2016), pp. 326–336. Lect. Notes in Computer Sci., Vol. 9602, Springer, Cham, 2016. MR 3509769. Zbl 1437.05090. (**SG: Invar**)

2020a Relative clique number of planar signed graphs. *Discrete Appl. Math.* 280 (2020), 86–92. MR 4096165. Zbl 1439.05103. (**SG**)

Sandip Das, Soumen Nandi, Soumyajit Paul, & Sagnik Sen

2016a Chromatic number of signed graphs with bounded maximum degree. Manuscript, 2016. arXiv:1603.09557.

Cf. Duffy, Jacques, Montassier, and Pinlou (2009a) and Jacques and Pinlou (2022a). (**SG: Col**)

Sandip Das, Soumen Nandi, Sagnik Sen, & Ritesh Seth

2019a The relative signed clique number of planar graphs is 8. In: S.P. Pal and A. Vijayakumar, eds., *Algorithms and Discrete Applied Mathematics* (5th Int. Conf., CALDAM 2019, Kharagpur, India), pp. 245–253. Lect. Notes in Computer Sci., Vol. 11394. Springer, Cham, 2019. MR 3911531. Zbl 1444.05063. (**SG**)

Bhaskar DasGupta, German Andres Enciso, Eduardo Sontag, & Yi Zhang

2006a Algorithmic and complexity results for decompositions of biological networks into monotone subsystems. In: Carme Àlvarez and María Serna, eds., *Experimental Algorithms* (5th Int. Workshop, WEA 2006, Cala Galdana, Menorca, 2006), pp. 253–264. Lect. Notes in Computer Sci., Vol. 4007. Springer, Berlin, 2006. Zbl 196.92016.

Extended abstract of (2007a). (**SD(sg): Dyn, Algor, Biol**)

2007a Algorithmic and complexity results for decompositions of biological networks into monotone subsystems. *BioSystems* 90 (2007), no. 1, 161–178.

"…our problem amounts to finding ground states" $V \to \{+,-\}$ of a signed graph. Lemma 3: A dynamical system is monotone iff the associated signed graph is balanced. An algorithm to find $\#E - l(\Sigma)$ to within 7/8. Dictionary: "sign-consistency" = balance, "consistent edge" = satisfied edge (in a state). [Annot. 1 Jan 2012.] (**SD(sg): Dyn, Algor, Biol**)

Brian Davis

2019a Unlabeled signed graph coloring. *Rocky Mountain J. Math.* 49(4) (2019), 1111–1122. MR 3998912. Zbl 1419.05097. arXiv:1511.07730. (**SG: Col: Invar, matrd, Geom, Ori**)

James A. Davis

1963a Structural balance, mechanical solidarity, and interpersonal relations. *Amer. J. Sociology* 68 (1963), 444–463. Repr. with minor changes in: Joseph Berger, Morris Zelditch, Jr., and Bo Anderson, eds., *Sociological Theories in Progress, Vol. One*, Ch. 4, pp. 74–101. Houghton Mifflin, Boston, 1966. Also reprinted in: Samuel Leinhardt, ed., *Social Networks: A Developing Paradigm*, pp. 199–217. Academic Press, New York, 1977. (**PsS: SG, WG: Exp**)

1967a Clustering and structural balance in graphs. *Human Relations* 20 (1967), 181–187. Repr. in: Samuel Leinhardt, ed., *Social Networks: A Developing Paradigm*, pp. 27–33. Academic Press, New York, 1977.

Σ is "clusterable" if its vertices can be partioned so that each positive edge is within a part and each negative edge joins different parts. Thm.:

Σ is clusterable $\iff$ no circle has exactly one negative edge. [*Cf.* Cartwright and Harary (1968a).] [Further developed in Doreian and Mrvar (1996a).] [Complete-graph clustering begins (?) in Zahn (1964a) and Moon (1966a), now called "cluster editing" and focussed on algorithms; *cf.*, e.g., Böcker and Baumbach (2013a).] [Annot. rev. 18 Nov 2017, 11 Jan 2019.] (**SG: Clu**)

1979a The Davis/Holland/Leinhardt studies: An overview. In: Paul W. Holland and Samuel Leinhardt, eds., *Perspectives on Social Network Research* (Proc. Sympos., Dartmouth Coll., Hanover, N.H., 1975), Academic Press, New York, 1979.

Survey of triad analysis in signed complete digraphs; *cf.* e.g. Davis and Leinhardt (1972a), Wasserman and Faust (1994a). [Annot. 28 Apr 2009.] (**PsS, SD: Clu(Gen): Exp**)

James A. Davis & Samuel Leinhardt

1972a The structure of positive interpersonal relations in small groups. In: Joseph Berger, Morris Zelditch, Jr., and Bo Anderson, eds., *Sociological Theories in Progress, Vol. Two*, Ch. 10, pp. 218–251. Houghton Mifflin, Boston, 1972.

In "ranked clusterability" the vertices of a signed complete, symmetric digraph are divided into levels. The set of levels is totally ordered. A symmetric pair, $\{+vw, +wv\}$ or $\{-vw, -wv\}$, should be within a level. For an asymmetric pair, $\{+vw, -wv\}$, w should be at a higher level than v. Analysis in relation to both randomly generated and observational data. [Annot. 28 Apr 2009.] (**PsS, SD: Clu(Gen)**)

Robert Davis

See T.-R. Chen.

A.C. Day, R.B. Mallion, & M.J. Rigby

1983a On the use of Riemannian surfaces in the graph-theoretical representation of Möbius systems. In: R.B. King, ed., *Chemical Applications of Topology and Graph Theory* (Proc. Sympos., Athens, Ga., 1983), pp. 272–284. Stud. Phys. Theor. Chem., Vol. 28. Elsevier, Amsterdam, 1983. MR 0761923 (85h:05039).

A clumsy but intriguing way of representing some signed (or more generally, $\mathbb{Z}_n$-weighted) graphs: via 2-page (or, n-page) looseleaf book embedding (all vertices are on the spine and each edge is in a single page), with an edge in page k weighted by the "sheet parity index" $\alpha_k = (-1)^k$ (or, $e^{2\pi ik/n}$). Described in the [unnecessary] terminology of an n-sheeted Riemann surface. [A $\mathbb{Z}_n$-weighted) graph has such a representation iff the subgraph of edges with each weight is outerplanar.]

A variation to get switching classes of signed circles: replace α_k by the "connectivity parity index" $\alpha_k^{\sigma_k}$ where $\sigma_k =$ number of edges in page k. [The variation is valid only for circles.] [*Questions* vaguely suggested by these procedures: Which signed graphs can be switched so that the edges of each sign form an outerplanar graph? Also, the same for gain graphs. And there are many similar questions: for instance, the same ones with "outerplanar" replaced by "planar."]

(**SG: sw, Adj, Top, Chem: Exp, Ref**)(**WG: Adj, Top: Exp, Ref**)

Rajat K. De

See A. Bhattacharya.

[Nair Maria Maia de Abreu]

See N.M.M. Abreu (under 'A').

Biswajit Deb

See K. Chettri.

[Mathias Hudoba de Badyn]

See M.H. de Badyn (under 'B').

Marisa Debowsky

See D. Archdeacon.

Ernesto Dedò

1981a Sulla ricostruibilità del polinomio caratteristico del commutato di un grafo. [The reconstructibility of the characteristic polynomial of the line-graph of a graph.] *Boll. Un. Mat. Ital. A* (5) 18 (1981), no. 3, 423–429. MR 0633676 (82k:05078). Zbl 481.05050. (**par: Lap: Eig**)

C. De Dominicis

See G.J. Rodgers.

Matthias Dehmer

See G.H. Yu.

José F. De Jesús & Alexander Kelmans

2017a On graphs uniquely defined by their K-circular matroids. *Discrete Appl. Math.* 217 (2017), part 3, 474–487. MR 3579926. Zbl 1358.05047. arXiv:1508.07627. (**Bic: Gen**)

2017b k-circular matroids of graphs. *Discrete Appl. Math.* 225 (2017), 33–950. MR 3647486. Zbl 1361.05024. arXiv:1508.05364. (**Bic: Gen**)

Hidde de Jong

2002a Modeling and simulation of genetic regulatory systems: A literature review. *J. Comput. Biol.* 9 (2002), no. 1, 67–103. MR 3931957. (**SD: Dyn, Biol: Exp**)

I.J. Dejter & V. Neumann-Lara

1988a Unboundedness for generalized odd cyclic transversality. In: A. Hajnal, L. Lovász and V.T. Sós, eds., *Combinatorics* (Seventh Hungarian Colloq., Eger, 1987), pp. 195–203. Colloq. Math. Soc. János Bolyai, Vol. 52. North-Holland, Amsterdam, 1988. MR 1221557 (94b:05117). Zbl 705.05045.

Thm. 1: Frustration number $l_0(-\Gamma)$ is unbounded for graphs with no disjoint odd circles. Their examples are projective-planar antibalanced signed graphs. Generalized to circles of length $L \bmod N$ for many L, N. [Annot. 1 May 2017.] (**sg: Par: Fr, Circ, Str**)

1991a Voltage graphs and Hamilton cycles. In: V.R. Kulli, ed., *Advances in Graph Theory*, pp. 141–153. Vishwa International Publications, Gulbarga, 1991. MR 1218635 (94a:05135). Zbl 817.05002 (book). (**GG: Aut**)

[Pierre de la Harpe]

See P. de la Harpe (under 'H').

Anne Delandtsheer

1995a Dimensional linear spaces. In: F. Buekenhout, ed., *Handbook of Incidence Geometry: Buildings and Foundations*, Ch. 6, pp. 193–294. North-Holland, Amsterdam, 1995. MR 1360721 (96k:51012). Zbl 950.23458.

"Dimensional linear space" (DLS) = simple matroid. §2.7: "Dowling lattices," from Dowling (1973b). §6.7: "Subgeometry-closed and hereditary classes of DLS's," from Kahn and Kung (1982a). In §2.6, the "Enough modular hyperplanes theorem" from Kahn and Kung (1986a).

(**GG: Matrd: Exp**)

Patrick De Leenheer

See also D. Angeli and V.A. Traag.

Patrick De Leenheer, David Angeli, & Eduardo D. Sontag

2007a Monotone chemical reaction networks. *J. Math. Chem.* 41 (2007), no. 3, 295–314. MR 2343862 (2009c:92041). Zbl 1117.80309. (**SD, SG: Bal, Chem**)

2009a Monotone chemical reaction networks. (In Hungarian.) *Alkalmaz. Mat. Lapok* 26 (2009), no. 2, 381–402. MR 2583205 (no rev).

Transl. of (2007a) by Judit Várdai. (**SD, SG: Bal, Chem**)

[Gloria de Leon-Calio]

See G. de Leon-Calio (under 'L').

John G. del Greco

See also C.R. Coullard.

1992a Characterizing bias matroids. *Discrete Math.* 103 (1992), 153–159. MR 1171312 (93m:05050). Zbl 753.05021.

How to decide, given a matroid M and a biased graph Ω, whether $M = \mathbf{F}(\Omega)$. (**GG: Matrd**)

Leonardo Silva de Lima

See also S. Akbari, A. Oliveira, and C.S. Oliveira.

Leonardo de Lima, Vladimir Nikiforov, & Carla Oliveira

2016a The clique number and the smallest Q-eigenvalue of graphs. *Discrete Math.* 339 (2016), 1744–1752. MR 3477106. Zbl 1333.05192. arXiv:1508.01784. (**par: Adj: Eig**)

Leonardo Silva de Lima, Carla Silva Oliveira, Nair Maria Maia de Abreu, & Vladimir Nikiforov

2011a The smallest eigenvalue of the signless Laplacian. *Linear Algebra Appl.* 435 (2011), no. 10, 2570–2584. MR 2811139 (2012g:05140). Zbl 1222.05180. (**par: Lap: Eig**)

Prem Delphy P & Devaraj J

2011a On signed cordial graph. *Int. J. Math. Sci. Appl.* 1 (2011), no. 3, 1159–1167. MR 2844560 (n rev). Zbl 1266.05049.

Γ is "signed cordial" if $\exists$ edge signature σ such that $\#E^+ \approx \#E^-$ and $\mu_\sigma^{-1}(+1) \approx \mu_\sigma^{-1}(-1)$. Examples. [More in Cynthia and Padmavathy (2021a).]

[Babujee and Loganathan (2011a) define "signed product cordiality" based on vertex signs.] [Annot. 26 Sept 2022.] (**Lab: SG**)

Alberto Del Pia & Giacomo Zambelli

2009a Half-integral vertex covers on bipartite bidirected graphs: total dual integrality and cut-rank. *SIAM J. Discrete Math.* 23 (2009), no. 3, 1281–1296. MR 2538651 (2011b:05200). Zbl 1227.05209.

Dictionary: "Bipartite" = balanced. **(sg: Ori: Incid, Algor)**

Alberto Del Pia, Antoine Musitelli, & Giacomo Zambelli

2018a On matrices with the Edmonds–Johnson property arising from bidirected graphs. *J. Combin. Theory Ser. B* 130 (2018), 49–91. MR 3772734. Zbl 1384.05107.

(sg: Ori: Incid, Algor)

Ernesto W. De Luca

See J. Kunegis.

Emanuele Delucchi

2007a Nested set complexes of Dowling lattices and complexes of Dowling trees. *J. Algebraic Combin.* 26 (2007), no. 4, 477–494. MR 2341861 (2008i:05190). Zbl 1127.05107.

Studies Dowling trees (*cf.* Hultman (2007a)). **(gg: Matrd: Invar)**

Emanuele Delucchi, Noriane Girard, & Giovanni Paolini

2019a Shellability of posets of labeled partitions and arrangements defined by root systems. *Electronic J. Combin.* 26 (2019), no. 4, art. 4.14, 22 pp. MR 4025418. Zbl 1422.05108. arXiv:1706.06360.

Cf. Bibby and Gadish (2018a). **(gg: Matrd(Gen))**

Renata Raposo Del-Vecchio

See M.A.A. de Freitas and R.E. Mansano.

Erik D. Demaine, Dotan Emanuel, Amos Fiat, & Nicole Immorlica

2006a Correlation clustering in general weighted graphs. Approximation and Online Algorithms. *Theoretical Computer Sci.* 361 (2006), no. 2-3, 172–187. MR 2252576 (2008e:68157). Zbl 1099.68074.

Clustering in a weighted signed graph; *cf.* Bansal, Blum, and Chawla (2002a), (2004a). An $O(\log n)$-approximation algorithm based on linear-programming rounding and region growing. An $O(r^3)$-approximation algorithm for graphs without a $K_{r,r}$-minor [e.g., planar, if $r = 3$]. Equivalent to minimum multicut, hence hard to approximate better than $\Theta(\log n)$. [Annot. 13 Sept 2009.] **(SG: WG: Clu: Algor)**

Erik D. Demaine, MohammadTaghi Hajiaghayi, & Ken-ichi Kawarabayashi

2010a Decomposition, approximation, and coloring of odd-minor-free graphs. In: Moses Charikar, ed., *Proceedings of the Twenty-First Annual ACM-SIAM Symposium on Discrete Algorithms* (SODA '10, Austin, Tex., 2010), pp. 329–344. Society for Industrial and Appl. Math. Philadelphia, 2010. MR 2809679 (2012i:05276). Zbl 1288.05053.

Graphs excluding $-\Gamma$: structure and algorithms. Dictionary: "Γ has odd minor H" = $-\Gamma$ has minor $-H$. [Annot. 4 Feb 2021.]

(sg: Par: Str, Algor)

Erik Demaine & Nicole Immorlica

2003a Correlation clustering with partial information. In: *Proceedings of the 6th International Workshop on Approximation Algorithms for Combinatorial Optimization Problems and 7th International Workshop on Randomization and Approximation Techniques in Computer Science (RANDOM-APPROX 2003)*

(Princeton, N.J., 2003), pp. 1–13. Lect. Notes in Computer Sci., Vol. 2764. Springer, Berlin, 2003. MR 2080776 (2005c:68291). Zbl 1202.68479.

Conference version of Demaine, Emanuel, Fiat, and Immorlica (2006a). [Annot. 13 Sept 2009.] (**SG: WG: Clu: Algor**)

Jacques Demongeot

See also J. Aracena, O. Cinquin, J.-P. Comet, and L. Forest.

Jacques Demongeot, Julio Aracena, Samia Ben Lamine, Sylvain Meignen, Arnaud Tonnelier, & Rene Thomas

2001a Dynamical systems and biological regulations. In: Eric Goles and Servet Martínez, eds., *Complex Systems*, pp. 105–149. Nonlinear Phenomena and Complex Systems, Vol. 6. Kluwer, 2001. MR 1886355 (2003b:37040). Zbl 1333.37011.

Part I: Various definitions of an attractor tend not to be logically comparable. Part II, "Biological regulations": §7.1, "Positive regulation circuits and memory". §8, "Interaction matrices and ubiquitory genes". (**sd: Adj: Dyn: Exp**)

Jacques Demongeot, Julio Aracena, Florence Thuderoz, Thierry-Pascal Baum, & Olivier Cohen

2003a Genetic regulation networks: circuits, regulons and attractors. Réseaux de régulation génétique : circuits, régulons, attracteurs. *C.R. Biologies* 326 (2003), 171–188. (**SD, Biol: Dyn**)

Jacques Demongeot, Adrien Elena, Mathilde Noual, Sylvain Sené, & Florence Thuderoz

2011a "Immunetworks", intersecting circuits and dynamics. *J. Theor. Biol.* 280 (2011), 19–33. MR 2975039 (no rev). Zbl 1397.92249. HAL hal-00666298. (**SD: Dyn, Biol**)

Jacques Demongeot, Marcelle Kaufman, & René Thomas

2000a Positive feedback circuits and memory. *C.R. Acad. Sci. Paris, Sci. vie/Life Sci.* 323 (2000), 69–79. (**SD: Dyn**)

Jacques Demongeot, Mathilde Noual, & Sylvain Sené

2012a Combinatorics of Boolean automata circuits dynamics. *Discrete Appl. Math.* 160 (2012), 398–415. MR 2876323. Zbl 1238.37035.

The effect of cycles and intersecting cycles ("circuits") in signed digraphs representing the action of a Boolean automaton. ["Automata" should be "automaton".] Dictionary: "Boolean automata circuit" = digraph that is a signed cycle. "Double Boolean automata circuit" = digraph that is two signed cycles with one common vertex. [Annot. 16 Jan 2015.] (**SD: Dyn**)

Jacques Demongeot & René Thomas

1999a Positive regulation circuits and memory. Circuits de régulation positifs et mémoire. In: *Neuronal Information Processing: From Biological Data to Modelling and Applications*, pp. 148–163. 1999. (**SD: Dyn**)

Jacques Demongeot, René Thomas, & Michel Thellier

2000a A mathematical model for storage and recall functions in plants. *C.R. Acad. Sci. Paris, Sci. vie/Life Sci.* 323 (2000), 93–97.

§2, "The logical approach": Exposition of signed-digraph model of bioregulation. [Annot. 23 Aug 2017.] **(SD: Exp, Biol)**

Hanyuan Deng & He Huang

2013a On the main signless Laplacian eigenvalues of a graph. *Electronic J. Linear Algebra* 26 (2013), art. 25, 381–393. MR 3084649. Zbl 1282.05109. arXiv:-1208.5835. **(par: Lap: Eig)**

Hongzhong Deng & Peter Abell

2010a A study of local sign change adjustment in balancing structures. *J. Math. Sociology* 34 (2010), no. 4, 253–282. Zbl 1201.91166.

A random signed graph has edge vw with probability d, which is positive with probability α_0. Degree of balance is the proportion of triangles that are positive. A triangle of type T_i has i positive edges. They study the long-term proportions of triangle types in examples. §3, "Balance adjustment under a local rule": A triangle $\triangle uvw$ and edge uv are chosen at random; uv changes sign iff $\triangle uvw$ is negative. This "myopic adjustment rule" is iterated. For $0 < \alpha_0 < 1$, the proportions approach 37% each of 1 or 2 and 13% each of 0 or 3 negative edges. This contradicts the Cartwright–Harary (1956a) balance hypothesis. Convergence behavior in examples depends interestingly on n and d. §§4–7: The sign-change probability depends on the triangle type. Probabilities are suggested by models of Harary–Cartwright, Davis (1967a), and others, in which different sets of triangle types are "attractors". Analytical and example results are reported.

Model based on Antal, Krapivsky, and Redner (2006a). Successor to Abell and Ludwig (2009a) and Kujawski, Abell, and Ludwig (2010a). [See Barahona, Maynard, Rammal, and Uhry (1982a) for modelling of planar grid graphs.] [Annot. 6 Dec 2009.] **(SG: Bal)**

Hongzhong Deng, Peter Abell, Ji Li, & Jun Wu

2012a A study of sign adjustment in weighted signed networks. *Social Networks* 34 (2012), no. 2, 253–263. **(SG: WG, PsS)**

Hongzhong Deng, Peter Abell, Jun Wu, & Yuejin Tang

2016a The influence of structural balance and homophily/heterophobia on the adjustment of random complete signed networks. *Social Networks* 44 (2016), 190–201. **(SG: Bal, PsS: KG)**

Tristan Denley

1997a The odd girth of the generalised Kneser graph. *European J. Combin.* 18 (1997), 607–611. MR 1468332 (98g:05080). Zbl 908.05059.

"Odd girth" of Γ = negative girth of $-\Gamma$. [*Question.* Is there a generalization to (some, or many) signed Kneser and generalized Kenser graphs? The high symmetry of the graph suggests this.] [Annot. 19 Aug 2022.] **(sg: par: Circ)**

[Wouter de Nooy]

See W. de Nooy (under 'N').

[Arnout van de Rijt]

See A. van de Rijt (under 'V').

B. Derrida, Y. Pomeau, G. Toulouse, & J. Vannimenus

1979a Fully frustrated simple cubic lattices and the overblocking effect. *J. Physique* 40 (1979), 617–626.

Physics of the signed d-hypercube in which every plaquette is negative; specifically, [cleverly] construct $\Sigma_d = (Q_d, \sigma_d)$, $d > 0$, as $\Sigma_{d-1} \times (+Q_1)$ with the second copy of Σ_{d-1} negated. Invariants of physical interest are computed and compared to the balanced case. Dictionary: "plaquette" = square. (**Phys: SG**)

1980a Fully frustrated simple cubic lattices and phase transitions. *J. Physique* 41 (1980), 213–221. MR 0566063 (80m:82020). (**Phys: SG**)

Madhav Desai & Vasant Rao

1994a A characterization of the smallest eigenvalue of a graph. *J. Graph Theory* 18 (1994), no. 2, 181–194. MR 1258251 (95c:05084). Zbl 792.05096.

$\psi(\Gamma) := \min_S(l(-\Gamma{:}S) + \#E(S,S^c)/\#S)$, over $\varnothing \subset S \subseteq V$, is a measure of nonbipartiteness of Γ. $\mu_1 :=$ smallest eigenvalue of $L(-\Gamma)$ satisfies $\psi(\Gamma)^2/4\Delta(\Gamma) \leq \mu_1 \leq 4\psi(\Gamma)$. Their $e_{\min}(\Gamma) := l(-\Gamma)$. [See Fan and Fallat (2012a) for another eigenvalue connection with $l(-\Gamma)$.] [Annot. 19 Sept 2010, 29 Dec 2012.] (**Par: Eig, Fr**)

Emilio De Santis

See also F. Camia.

2001a Strict inequality for phase transition between ferromagnetic and frustrated systems. *Electronic J. Probab.* 6 (2001), Paper no. 6, 1–27. MR 1825713 (2002c:82026). Zbl 1050.82020. (**Phys, SG: Rand**)

E. De Santis & A. Gandolfi

1999a Bond percolation in frustrated systems. *Ann. Probab.* 27 (1999), no. 4, 1781–1808. MR 1742888 (2000k:60199). Zbl 0968.60092. (**SG: Phys: Fr**)

[L. de Sèze]

See L. de Sèze (under 'S').

C. De Simone, M. Diehl, M. Jünger, P. Mützel, G. Reinelt, & G Rinaldi

1995a Exact ground states of Ising spin glasses: New experimental results with a branch and cut algorithm. *J. Stat. Phys.* 80 (1995), 487–496. Zbl 1106.82323.

Improves the algorithm of Barahona, Grötschel, Jünger, and Reinelt (1988a) to find a switching with minimum $\#E^-$ ($= l(\Sigma)$) for signed toroidal square lattice graphs with an extra vertex (exterior magnetic field) and a fixed proportion of negative edges. Applied to many signatures in order to find statistical properties. Continued in (1996a). [Annot. 18 Aug 2012.] (**Phys, SG: State(fr): Algor**)

1996a Exact ground states of two-dimensional $\pm J$ Ising spin glasses. *J. Stat. Phys.* 84 (1996), 1363–1371.

Continuation of (1995a). [Annot. 18 Aug 2012.] (**Phys, SG: State(fr): Algor**)

A.H. Deutz, A. Ehrenfeucht, & G. Rozenberg

1994a Hyperedge channels are abelian. *Theor. Computer Sci.* 127 (1994), 367–393.

MR 1275824 (96b:68023). Zbl 824.68011. **(GH)**

Devaraj J

See P. Delphy.

Lee DeVille

See J.C. Bronski.

Vincent Devloo, Pierre Hansen, & Martine Labbé

2003a Identification of all steady states in large networks by logical analysis. *Bull. Math. Biol.* 65 (2003), 1025–1051. **(SD: Dyn)**

Matt DeVos

See also R. Chen.

2000a *Flows on Graphs.* Doctoral thesis, Princeton Univ., 2000.
See (2004a), (20xxa). [Annot. 23 March 2010.] **(SG: Ori, Flows)**

2004a Flows on bidirected graphs. Manuscript, 2004.
See (20xxa). Proves, maybe, that a nowhere-zero 12-flow exists (if a nowhere-zero flow exists). Corrected and extended in Raspaud and Zhu (2011a) (*q.v.*). [Annot. 23 March 2010.] **(SG: Ori, Flows)**

20xxa Flows on bidirected graphs. Submitted. arXiv:2013.8406
A nowhere-zero 12-flow exists if any nowhere-zero flow exists. [Annot. 18 Aug 2016.] **(SG: Ori, Flows)**

Matt DeVos & Daryl Funk

2018a Almost balanced biased graph representations of frame matroids. *Adv. Appl. Math.* 96 (2018), 139–175. MR 3767506. Zbl 1383.05218. arXiv:1606.07370. **(GG: Matrd)**

Matt DeVos, Daryl Funk, & Irene Pivotto

2014a When does a biased graph come from a group labelling? *Adv. Appl. Math.* 61 (2014), 1–18. MR 3267062. Zbl 1371.05113. arXiv:1403.7667. **(GG)**

2017a On excluded minors of connectivity 2 for the class of frame matroids. *European J. Combin.* 61 (2017), 167–196. MR 3588716. Zbl 1352.05099. arXiv:-1502.06896. **(GG: Matrd)**

Matt DeVos, Jiaao Li, You Lu, Rong Luo, Cun-Quan Zhang, & Zhang Zhang

2021a Flows on flow-admissible signed graphs. *J. Combin. Theory Ser. B* 149 (2021), 198–221. MR 4247027. Zbl 1466.05085. arXiv:1908.10853.
All such graphs admit a nowhere-zero 11-flow. *Cf.* Bouchet (1983a). [Annot. 6 Jul 2022.] **(SG: Flows)**

Matt DeVos & Kathryn Nurse

20xxa Cycles through two edges in signed graphs. Submitted. arXiv:2306.05574. **(SG: Circ)**

Matt DeVos, Edita Rollová, & Robert Šámal

† 2019a A note on counting flows in signed graphs. *Electronic J. Combin.* 26 (2019), no. 2, art. P2.38, 7 pp. MR 3962234. Zbl 1441.05096. arXiv:1701.07369.
For each $k \geq 0$, the number of nowhere-zero flows in an abelian group of order m that has k factors $\mathbb{Z}_2$ is a polynomial function $f_k(m/2^k)$.

(Generalizes $k = 0$ by Beck and Zaslavsky (2006b).) [Further theory in Ren and Qian (2019a).] [Annot. 14 Aug 2017.] (**SG: Flows: Invar**)

Sean Dewar

2021a Flexible placements of periodic graphs in the plane. *Discrete Comput. Geom.* 66 (2021), 1286–1329. MR 4333293. Zbl 1478.05032. arXiv:1911.05634. (**GG: Geom, Cov, Sw**)

M. Deza, V.P. Grishukhin, & M. Laurent

1991a The symmetries of the cut polytope and of some relatives. In: Peter Gritzman and Bernd Sturmfels, eds., *Applied Geometry and Discrete Mathematics: The Victor Klee Festschrift*, pp. 205–220. DIMACS Ser. Discrete Math. Theor. Comput. Sci., Vol. 4. American Math. Soc., Providence, R.I., 1991. MR 1116350 (92e:52019). Zbl 748.05061.

Switching (on coordinates) is an important symmetry of the cut polytope P_n (of K_n); see p. 206. [See Deza and Laurent (1997a).] Thm. 2.6: Aut $P_n = \mathfrak{D}_n$, the Weyl group [= SwAut($\pm K_n$), the switching automorphism group]. *Question* (p. 207): For the cut polytope $P_c(\Gamma)$, does Aut $P_c(\Gamma)$ = SwAut($\pm\Gamma$)? [Edge signs and SwAut are not stated as such.] [Annot. 12 Jun 2012.] (**sg: par: KG: Geom, sw**)

Michel Marie Deza & Monique Laurent

1997a *Geometry of Cuts and Metrics.* Algorithms and Combin., Vol. 15. Springer, Berlin, 1997. MR 1460488 (98g:52001). Zbl 885.52001.

A main object of interest is the cut polytope, which is the bipartite subgraph polytope (see Barahona, Grötschel, and Mahjoub (1985a)) of K_n, i.e., the balanced subgraph polytope (Poljak and Turzík (1987a)) of $-K_n$. §4.5, "An application to statistical physics", briefly discusses the spin glass application. §26.3, "The switching operation", discusses graph switching and its generalization to sets. §30.3, "Circulant inequalities", mentions Poljak and Turzík (1987a), (1992a). No explicit mention of signed graphs. (**sg: par: KG: fr, sw Geom: Exp**)

Ayushi Dhama

See also D. Sinha.

2013a *Contributions to the Theory of Signed Graphs.* Doctoral thesis, Banasthali University, 2013. (**SG**)

Dhananjayamurthy B.

See S. Shalini.

Inderjit S. Dhillon

See C.-J. Hsieh and K.-Y. Chiang.

Massimiliano Di Ventra

See H. Manukian and Y.R. Pei.

[F. di Liberto]

See F. di Liberto (under 'L').

Persi Diaconis

See K.S. Brown.

Yuanan Diao & Gábor Hetyei

2010a Relative Tutte polynomials for coloured graphs and virtual knot theory. *Combin. Probab. Computing* 19 (2010), no. 3, 343–369. MR 2607372 (2011f:05146). Zbl 1202.05064. **(SGc: Invar, Knot)**

Yuanan Diao, Gábor Hetyei, & Kenneth Hinson

2009a Tutte polynomials of tensor products of signed graphs and their applications in knot theory. *J. Knot Theory Ramifications* 18 (2009), no. 5, 561–589. MR 2527677 (2010c:57010). Zbl 1185.05083. arXiv:math/0702328. **(SGc: Invar, Knot)**

2011a A Tutte-style proof of Brylawski's tensor product formula. *European J. Combin.* 32 (2011), 775–781. MR 2821550. Zbl 1229.05067. **(SGc: Invar)**

Alicia Dickenstein & Mercedes Pérez Millán

† 2011a How far is complex balancing from detailed balancing? *Bull. Math. Biol.* 73 (2011), 811–828. MR 2785146 (2012d:92019). Zbl 1214.92036.

From a multiplicative gain digraph $\vec{\Phi} := (\vec{\Gamma}, \vec{\varphi}, \mathfrak{G})$ where $\vec{\Gamma}$ is a symmetric digraph, construct a gain graph $\Phi := (\Gamma, \varphi, \mathfrak{G})$ and $\mathfrak{G} = \mathbb{R}^{\times}_{>0}$, where Γ has an edge e_{ij} for each arc pair $(i,j),(j,i)$ and $\varphi_{ij}(e_{ij}) := \vec{\varphi}(i,j)/\vec{\varphi}(j,i)$. If Φ is balanced, $\vec{\Phi}$ is called "formally balanced". This property and related ones are studied. [The construction $\vec{\Phi} \mapsto \Phi$ is known in papers on chemical reaction graphs. This paper is more gain-graphic than most though the gain graph Φ is not explicit.]

[A circle $C = e_{12} \cdots e_{l-1,l} e_{l1}$ is balanced iff $\vec{\varphi}(\vec{C}) = \vec{\varphi}(\vec{C}^{-1})$, where $\vec{C} = \vec{e}_{12} \cdots \vec{e}_{l-1,l} \vec{e}_{l1}$. *Question.* What is the general theory of gain graphs derived from $\vec{\Phi}$ of the above type with a general abelian gain group? In general define gains on a symmetric digraph: let $\vec{\Gamma}$ have arcs $\vec{e}_{ij}$, possibly with multiple arcs and loops, with a pairing $^* : \vec{E} \leftrightarrow \vec{E}$ such that $\vec{e}_{ij}{}^*$ is an $\vec{e}_{ji}$. Example 1: For a gain graph, let $\vec{E} := \bigcup\{\vec{e}_{ij},\ \vec{e}_{ji} = \vec{e}_{ij}^{\,-1} : e_{ij} \in E\}$ and $\vec{\varphi}(\vec{e}_{ij}) := \varphi_{ij}(e_{ij})$; then each (nonloop) pair is a balanced digon. Example 2: Only one of each pair has identity gain; this seems inequivalent to Example 1 (*Question*: Is it?), so arbitrary gains on symmetric digraphs seem more general than such gains from gain graphs and more structured than general gain digraphs.] [Annot. 2 Apr 2016.] **(GD, gg: Bal, Chem; Ref)**

Gilles Didier & Elisabeth Remy

2012a Relations between gene regulatory networks and cell dynamics in Boolean models. *Discrete Appl. Math.* 160 (2012), no. 15, 2147–2157. MR 2954757. Zbl 1291.92065. **(SD: Dyn, Biol)**

Gilles Didier, Elisabeth Remy, & Claudine Chaouiya

2011a Mapping multivalued onto Boolean dynamics. *J. Theor. Biol.* 270 (2011), 177–184. MR 2974862 (no rev). Zbl 1331.92051. **(SD: Dyn, Biol)**

M. Diehl

See C. De Simone.

Hung T. Diep

See also O. Nagai.

H.T. Diep, ed.

2004a *Frustrated Spin Systems*. World Scientific, Hackensack, N.J., 2004. Zbl 1104.82002.

Expository. Any chapter might inspire interesting mathematics of signed graphs, esp. Ch. 1: Diep and Giacomini (2004a); Ch. 2: Nagai, Horiguchi, and Miyashita (2004a); Ch. 5: Misguich and Lhuillier (2004a). [Annot. 13 Aug 2018.] **(SG, Phys: Exp, Ref)**

H.T. Diep & H. Giacomini

2004a Frustration - Exactly solved frustrated models. In: H.T. Diep, ed., *Frustrated Spin Systems*, Ch. 1, pp. 1–58. World Scientific, Hackensack, N.J., 2004. Zbl 1104.82002 (book). arXiv:1912.13042.

Frustration on signed graph with Ising (± 1) and vector spins (implicitly). §§1.1–1.2 introduce frustration and physics concerns, e.g., "degeneracy" = multiple ground states. Later, various periodic signed lattice graphs (*cf.* Liebmann (1986a)) are solved and diagrammed for Ising and XY (S^1) spins, illustrating spin frustration. [Annot. 9 Aug 2018.] **(SG, Phys: Fr: Exp, Ref)**

Hung T. Diep, P. Lallemand, & O. Nagai

1985a Simple cubic fully frustrated Ising crystal by Monte Carlo simulations. *J. Appl. Phys.* 57 (1985), 3309–3311.

Physics of fully frustrated 3-dimensional cubic lattice (*cf.* Chui, Forgacs, and Hatch (1982a)), but the negative edges are specifically chosen to form three orthogonal families of straight lines, alternating along each plane. As signed lattice has a 2×2 fundamental domain, there are 8 translational symmetry types of vertex, each forming a double-sized sublattice. The sublattices exhibit somewhat differentiated behavior. [Annot. 18 Jun 2012.] **(Phys, SG, sw)**

1985b Critical properties of a simple cubic fully frustrated Ising lattice by Monte Carlo method. *J. Phys. C* 18 (1985), 1067–1078.

Simulations on the the signed graph of (1985a). The 8 sublattices are equivalent in pairs. [Annot. 18 Jun 2012.] **(Phys, SG: Fr)**

Jana Diesner

See L. Dinh.

V. Di Giorgio

1974a 2-modules dans un graphe: equilibre et coequilibre d'un bigraphe—application taxonomique. *Bull. Math. Soc. Sci. Math. R. S. Roumanie (N.S.)* 18 (66) (1974), 81–102 (1975). MR 0476564 (57 #16124). Zbl 324.05127. **(SG: Bal)**

Wil Dijkstra

1979a Response bias in the survey interview; an approach from balance theory. *Social Networks* 2 (1979–1980), no. 3, 285–304.

Extends signed graphs to sign set $\{\pm 1, 0\}$ and extends the notion of (degree of) cycle balance. A circle C is "balanced" if its sign product $\sigma(C) = +1$. Degree of balance = average sign product of all circles. Degree of local balance at $X \subseteq V$ is the average sign of all circles that contain X. Given a length weight function $1 \geq f(2) \geq f(3) \geq \cdots \geq 0$, the weighted degree of balance is the average value of $f(l(C))\sigma(C)$. [*Cf.* kinds of cycle balance in Cartwright and Harary (1956a), Morrissette

(1958a), Norman and Roberts (1972a), (1972b).] **(SG: Bal, Fr)**

It is assumed [!] that answers have probability dependent on weighted degree of local balance at $\{p, y\}$ where p = respondent and y = answer. Speculation about choice of functions f *et al.* One post-hoc application. **(SG: PsS)**

Genhong Ding

See X.B. Ma.

Ly Dinh, Rezvaneh Rezapour, Lan Jiang, & Jana Diesner

20xxa Structural balance in signed digraphs: considering transitivity to measure balance in graphs constructed by using different link signing methods. Submitted. arXiv:2006.02565. **(SD: Bal: Algor)**

Yvo M.I. Dirickx & M.R. Rao

1974a Networks with gains in discrete dynamic programming. *Management Sci.* 20 (1974), No. 11 (July, 1974), 1428–1431. MR 0359827 (50 #12279). Zbl 303.90052. **(GN: Matrd(bases))**

Divya T

See S. Hameed.

Ajit A. Diwan

See also M. Joglekar.

Ajit A. Diwan, Josh B. Frye, Michael J. Plantholt, & Shailesh K. Tipnis

2011a A sufficient condition for the existence of an anti-directed 2-factor in a directed graph. *Discrete Math.* 311 (2011), no. 21, 2556–2562. MR 2832152 (2012i:05118). Zbl 1238.05209.

An antidirected circle is a balanced circle in the poise gains of a digraph. [For early bidirected work see Andersen and Grant (1981a). For connected 2-factor see Busch, Jacobson, Morris, Plantholt, and Tipnis (2013a). For Hamilton paths in K_n see El Sahili and Abi Aad (2018a).] [*Question.* How does this generalize to bidirected graphs?] [Annot. 20, 30 May 2018.] **(gg: Str)(sg: par: Ori)**

Daniel B. Dix

2006a Polyspherical coordinate systems on orbit spaces with applications to biomolecular shape. *Acta Appl. Math.* 90 (2006), 247–306. MR 2248745 (2007k:92043). Zbl 1182.92028. **(GG: Appl)**

Vlastimil Dlab

1980a *Representations of Valued Graphs.* Sém. Math. Supérieures, 73. Dép. Math. Stat., Université de Montréal. Les Presses de l'Université de Montréal, Montréal, 1980. MR 0586769 (82k:16037). Zbl 478.16026.

A valued graph is a simple symmetric digraph with $\mathbb{Z}_{>0}$-gains, such that $\varphi(e_{ij})/\varphi(e_{ji}) = f(v_j)/f(v_i)$ for some $f : V \to \mathbb{Z}_{>0}$ [equivalently, cycle gains invert by reversing direction]. $\varphi(e_{ij})$ represents the dimension of an (F_i, F_j)-bimodule corresponding to e_{ij}, where F_i is an algebra associated to v_i. [There is no gain-graph theory.] [Based on 3 articles; see Zbl.] [Annot. 26 Dec 2015.] **(gd: Algeb)**

Duong D. Doan & Patricia A. Evans

2011a Haplotype inference in general pedigrees with two sites. 6th Int. Symp. Bioinformatics Res. Appl. (ISBRA'10, Storrs, Conn., 2010). *BMC Proc.* 5 (2011), Suppl. 2, 56, 10 pp.

A pedigree is a kind of signed graph with $< n$ edges, with 3-colored vertices. Frustration index ("line index") $l =$ minimum number of necessary recombinations. Elementary relations among l, vertex cuts, and switching. Reduction rules, including the negative-subdivision trick, to test $l \leq k$. [*Question.* Does sparseness reduce the hardness of testing $l \leq k$?] [Annot. 29 Apr 2012.] **(Biol: SG: Fr, Algor, sw)**

2011b An FPT haplotyping algorithm on pedigrees with a small number of sites. *Algorithms Molecular Biol.* 6 (2011), no. 8, 8 pp.

See (2011a). This problem adds parity constraints. [Annot. 29 Apr 2012.] **(Biol: SG: Fr, Algor)**

R.L. Dobrushin & S.B. Shlosman

1985a The problem of translation invariance of Gibbs states at low temperatures. Transl. Morton Hamermesh. In: S.P. Novikov, ed., *Mathematical Physics Reviews*, Vol. 5 (1985), pp. 53–195. Soviet Sci. Rev., Sect. C. Harwood, Chur, Switz., 1985. MR 0852217 (87j:82013). Zbl 0613.76010.

Partly expository. The problems are existence of nonperiodic ground states and stability for nonferromagnetic interactions, e.g., signed graphs that are not all positive. ("Ground state" means a state at temperature 0; a "state" is a probabilistic mixture of what are usually called states, here called "configurations".) The graphs are infinite hypercubic lattice graphs $\mathbb{Z}^\nu$, $\nu \geq 1$. The set of "configurations" $\sigma : \mathbb{Z}^\nu \to S$, where S is a fixed set (usually finite; $S = \{\pm 1\}$ for Ising model, etc.), is Ω. "State": a probability measure on Ω w.r.t. $\mathcal{B}$, the finitely cylindrical σ-algebra on $\mathbb{Z}^\nu$. "Interactions" between vertices have finite range, not necessarily only adjacent.

§2, "Ground states": §2.8, "The symmetric ferromagnetic Ising model", describes ground configurations in terms of the hypercubic lattice facets dual to frustrated edges of the configuration. 3-Dimensional examples. §2.9, "The antiferromagnetic Ising model"' with external field strength h. There are 1 and 2 ground configurations for $|h| > 2\nu$ and $< 2\nu$. Some nonperiodic ground states are described. At $h = \pm 2\nu$ ground states (not only configurations) exist, discussed in §§2.11–12. §2.11, "Random ground states–the antiferromagnetic case". §2.12, "Nonperiodic random ground states". App. I, "Ground states of the model of a one-dimensional lattice antiferromagnet", by S.E. Burkov and Ya.G. Sinai. App. II, "On random ground states of one-dimensional antiferromagnetic model", by A.A. Kerimov. Dictionary: "ferromagnetic" = all-positive; "antiferromagnetic" = all-negative; "periodic" = toroidal; "external field" = extra vertex v_0, positive, with $N(v_0) = V$ and arbitrary interaction strength. [Annot. 8 Mar, 23 May, 3 Jun 2015.]
(Phys, sg: bal: State(fr))(Phys, sg: par: State(fr))

Anton Dochtermann

See M. Cho.

Ebrahim Dodongeh

See S. Akbari.

Benjamin Doerr

2000a Linear discrepancy of basic totally unimodular matrices. *Electronic J. Combin.* 7 (2000), Research Paper R48, 4 pp. MR 1785144 (2001e:15017). Zbl 996.15012.

The linear discrepancy of the transposed incidence matrix of a balanced signed graph. (**sg: bal: Incid**)

B.G.S. Doman & J.K. Williams

1982a Low-temperature properties of frustrated Ising chains. *J. Phys. C* 15 (1982), 1693–1706.

§2, "The random bond model at low temperatures": A signed path with magnetic field B [interpretable as an extra all-positive dominating vertex with edge weights B; *cf.* Barahona (1982a)]. §3, "Frustrated periodic bond model": A path with edges signed $+---$ periodically. Describes ground states ("allowed states"), which depend on B. [Annot. 28 Aug 2012.] (**Phys, SG, WG: State(fr)**)

Janelle Domantay

See A. Carbonero.

Eytan Domany

See G. Hed and D. Kandel.

Mirela Domijan & Elisabeth Pécou

2012a The interaction graph structure of mass-action reaction networks. *J. Math. Biol.* 65 (2012), 375–402. MR 2944515. Zbl 1303.92038.

(**Biol, Chem: SD: Dyn**)

Bing-can Dong

See R.L. Li.

Chun-Long Dong

See Y.-Z. Fan.

Fengming Dong

See S.J. He.

Jiu-Gang Dong & Lin Lin

2016a Laplacian matrices of general complex weighted directed graphs. *Linear Algebra Appl.* 510 (2016), 1–9. MR 3551615. Zbl 1352.05112. (**GD: Lap**)

Wenkuan Dong

See D.-J. Wang.

Alfredo Donno

See F. Belardo and M. Cavaleri.

Michael Doob

See also D.M. Cvetković.

1970a A geometric interpretation of the least eigenvalue of a line graph. In: *Proceedings of the Second Chapel Hill Conference on Combinatorial Mathematics and Its Applications* (1970), pp. 126–135. University of North Carolina at Chapel Hill, Chapel Hill, N.C., 1970. MR 0268060 (42 #2959). Zbl 209.55403 (209, p. 554c).

A readable, tutorial introduction to (1973a) (without matroids).

(**ecyc: LG, Incid, Eig(LG)**)

1973a An interrelation between line graphs, eigenvalues, and matroids. *J. Combin. Theory Ser. B* 15 (1973), 40–50. MR 0439687 (55 #12573). Zbl 245.05125, (Zbl 257.05132).

Along with Simões-Pereira (1973a), introduces to the literature the even-cycle matroid $\mathbf{F}(-\Gamma)$ [previously invented by Tutte, unpublished]. The multiplicity of -2 as an eigenvalue (in characteristic 0) equals the number of independent even circles $= n - \operatorname{rk} \mathbf{F}(-\Gamma)$. In characteristic p there is a similar theorem, but the pertinent matroid is $\mathbf{M}(\Gamma)$ if $p = 2$ and, when $p|n$, the matroid has rank 1 greater than otherwise [a fact that mystifies me]. (**ECyc: LG, Incid, Eig(LG)**)

1974a Generalizations of magic graphs. *J. Combin. Theory Ser. B* 17 (1974), 205–217. MR 0364019 (51 #274). Zbl 271.05128, (Zbl 287.05124).

Thm. 3.2 is the theorem of van Nuffelen (1973a), supplemented by the observation that it remains true in any characteristic except 2. (**ECyc: Incid**)

1974b On the construction of magic graphs. In: F. Hoffman *et al.*, eds., *Proceedings of the Fifth Southeastern Conference on Combinatorics, Graph Theory and Computing* (Boca Raton, 1974), pp. 361–374. Utilitas Math. Publ. Inc., Winnipeg, Man., 1974. MR 0409279 (53 #13039). Zbl 325.05123. (**ecyc: Incid**)

1978a Characterizations of regular magic graphs. *J. Combin. Theory Ser. B* 25 (1978), 94–104. MR 0505855 (58 #21840). Zbl 384.05054. (**ecyc: Incid**)

Michael Doob & Dragoš Cvetković

1979a On spectral characterizations and embeddings of graphs. *Linear Algebra Appl.* 27 (1979), 17–26. MR 0545719 (81d:05050). Zbl 417.05025. (**sg: LG, Eig(LG)**)

Patrick Doreian

See also M. Brusco, N.P. Hummon and A. Mrvar.

1971a *Mathematics and the Study of Social Relations*. Schocken, New York, 1971.

Ch. 5, "Structural balance": Signed digraphs, sometimes ignoring direction. The Cartwright–Harary (1956a) model with signed digraphs. The Abelson–Rosenberg (1958a) model with ambiguous arcs [repeating Abelson–Rosenberg's error about balance via $R(\Sigma)$]. §5.1, "Balance theory". §5.2, "The structure theorems" (of balance and clusterability). §5.3, "Measures of balance". §5.4, "Construction of signed structures": distinguishes between balance of a signed digraph (as in Cartwright–Harary) and balance as perceived by a vertex (a person) (as in Heider (1946a)). §5.5, "Structural balance as a deductive theory". [Annot. 29 Aug 2013.] (**PsS, SD, sg: Bal, Fr, Clu, Adj: Exp**)

1985a Review of *Structural Models in Anthropology* by Per Hage and Frank Harary. *J. Math. Sociology* 11 (1985), 283–285.

Review of Hage and Harary (1983a). (**PsS: SG: Bal**)

2002a Event sequences as generators of social network evolution. *Social Networks* 24 (2002), 93–119. (**SG: Bal, PsS**)

2004a Evolution of signed human networks. *Metodološki Zvezki* 1 (2004), 277–293.

Reviews the development of balance and clustering theory for signed (di)graphs in social psychology, mainly Doreian and Mrvar (1996a), Doreian and Krackhardt (2001a), and especially Hummon and Doreian (2003a). The difference between Heider's (1946a) and Cartwright and Harary's (1956a) models, and the need to combine them. [Annot. 24 Apr 2009.]

(**PsS: Exp: SD, Bal, Clu, Algor**)

2008a A multiple indicator approach to blockmodeling signed networks. *Social Networks* 30 (2008), 247–258.

Signed graphs $\Sigma_1, \ldots$ ("multiple indicators") may be approximations of a hidden signed graph Σ. Goals: detect whether Σ exists, and find an optimal clustering of Σ. Methods: (1) Examine the Σ_j for compatibility via statistical tests. (2) Estimate Σ by $\sum_j \sigma_j$. (3) Applies the clusterability index and algorithm of Doreian and Mrvar (1996a). ((2) implies using weighted signed graphs.) This article treats examples, with analysis of the methods' success. [Annot. 27 Apr 2009.] (**PsS, SD: sg: Clu**)

2008b Clashing paradigms and mathematics in the social sciences. *Contemp. Sociology* 37 (2008), no. 6, 542–545.

Two books on and the philosophy of mathematics and sociology. [Annot. 27 Apr 2012.] (**PsS: SG, SD**)

Patrick Doreian, Vladimir Batagelj, & Anuška Ferligoj

2005a *Generalized Blockmodeling.* Structural Analysis in the Social Sciences, No. 25. Cambridge Univ. Press, Cambridge, Eng., 2005.

Ch. 10: "Balance theory and blockmodeling signed networks". Thm. (pp. 305–306; proof by Martin Everett): The sizes of the partitions of V that minimize the clustering index (Doreian and Mrvar (1996a)) are consecutive integers. (**PsS, SD: sg: Clu, Bal**)

Patrick Doreian, Roman Kapuscinski, David Krackhardt, & Janusz Szczypula

1996a A brief history of balance through time. *J. Math. Sociology* 21 (1996), 113–131. Repr. in Patrick Doreian and Frans N. Stokman, eds., *Evolution of Social Networks*, pp. 129–147. Gordon and Breach, Australia, Amsterdam, etc., 1997. Zbl 883.92034.

§2.3: "A method for group balance". Describes the negation-minimal index of clusterability (generalized imbalance) from Doreian and Mrvar (1996a). (**SG: Bal, Clu: Fr(Gen): Exp**)

§3.3: "Results for group balance". Describes results from analysis of data on a small (social) group, in terms of frustration index l and a clusterability index $\min_{k>2} 2P_{k,.5}$ (slightly different from the index in Doreian and Mrvar (1996a)), finding both measures (but more so the latter) decreasing with time. (**PsS: Bal, Clu: Fr(Gen)**)

Patrick Doreian & David Krackhardt

2001a Pre-transitive balance mechanisms for signed networks. *J. Math. Sociology* 25 (2001), no. 1, 43–67. Zbl 1017.91520.

In a signed digraph from empirical social-group data, a tendency to transitivity of signs on directed edges ij, ik, jk (i.e., $\sigma(ij)\sigma(jk)\sigma(ik) = +$) holds when $\sigma(ij) = +$ and fails when $\sigma(ij) = -$. This suggests that balance is not a primary tendency and Harary's (1953a) and Davis's

(1967a) theorems on balance and clusterability have limited relevance to social groups. [Also, that undirected signed graphs have limited relevance. Digraph sign transitivity properties are more relevant.] [A thoughtful article.] [Annot. 13 Apr 2009.] (**PsS, sd**)

Patrick Doreian, Paulette Lloyd, & Andrej Mrvar

2013a Partitioning large signed two-mode networks: Problems and prospects. *Social Networks* 35 (2013), 178–203. (**SG: Bal, Fr, PsS**)

Patrick Doreian & Andrej Mrvar

1996a A partitioning approach to structural balance. *Social Networks* 18 (1996), 149–168.

They propose indices for clusterability (as in Davis (1967a)) that generalize the frustration index of Σ. Fix $k \geq 2$ and $\alpha \in [0,1]$. For a partition π of V into k "clusters", they define $P(\pi) := \alpha n_- + (1-\alpha)n_+$, where $n_+ :=$ number of positive edges between clusters, $n_- :=$ number of negative edges within clusters, and $0 \leq \alpha \leq 1$ is fixed. The first proposed measure is $P_k := \min P(\pi)$, minimized over k-partitions. A second suggestion is the "negation-minimal index of generalized imbalance" [i.e., of clusterability], the smallest number of edges whose negation [equivalently, deletion] makes Σ clusterable. [Call it the 'clusterability index' $Q(\Sigma)$.] [Note that $P(\pi)$ effectively generalizes the Potts Hamiltonian as given by Welsh (1993a). *Question.* Does $P(\pi)$ fit into an interesting generalized Potts model?] [$P(\pi)$ also resembles the Potts Hamiltonian in Fischer and Hertz (1991a) (*q.v.* for a related research question).] [*Problem.* The data in Doreian (2008a), and common sense, suggest that clusters should be allowed to overlap. This is an open research direction.]

They employ a local optimization algorithm to evaluate $P_k(\alpha)$ and find an optimal partition: random descent from partition to neighboring partition, where π and π' are neighbors if they differ by transfer of one vertex or exchange of two vertices between two clusters. This was found to work well if repeated many times. [A minimizing partition into at most k clusters is equivalent to a ground state of the k-spin Potts model in the form given by Welsh (1993a), but not quite in that of Fischer and Hertz (1991a).]

Dictionary: $P(\pi)$ is the "criterion function". [More explicitly, call $Q(\Sigma, \pi; \alpha) := 2P(\pi)$ the 'α-weighted clusterability index of π', so the clusterability index is $Q(\Sigma) = \min_\pi Q(\Sigma, \pi; .5)$; and call $Q_k := 2P_k$ the 'k-clusterability index' of Σ.]. Clusterability is "k-balance" or "generalized balance". Clusters are "plus-sets". Signed digraphs are employed in the notation but direction is ignored.

[Further developments in Doreian *et al.* (various), Hummon and Doreian (2003a), Bansal *et al.* (2004a), Demaine *et al.* (2006a), Mrvar and Doreian (2009a).] [Annot. rev. 22 Sept 2009.]
(**SD: sg: Bal, Clu: Fr(Gen), Algor, PsS**)

1996b Structural balance and partitioning signed graphs. In: A. Ferligoj and A. Kramberger, eds., *Developments in Data Analysis*, pp. 195–208. Metodološki zvezki, Vol. 12. FDV, Ljubljana, Slovenia, 1996.

Similar to (1996a). Some lesser theoretical detail; some new examples. The k-clusterability index $P_k(\alpha)$ (1996a) is compared for different values of k, seeking the minimum. [But for which value(s) of α is not stated.] Interesting observation: optimal values of k were small. It is said that positive edges between parts are far more acceptable socially than negative edges within parts [thus, in the criterion function α should be rather near 1]. (**SD: sg: Bal, Clu: Fr(Gen), Algor, PsS**)

2009a Partitioning signed social networks. *Social Networks* 31 (2009), no. 1, 1–11.

Generalizes the ideas of (1996a) (*q.v.*). Given: A signed digraph $(\vec{\Gamma},\sigma)$; a "criterion function" $P(\pi,\rho) := \alpha n^+ + (1-\alpha)n^-$, where $\pi := \{B_1,\ldots,B_k\}$ partitions V into "clusters", $\rho : \pi\times\pi \to \{+,-\}$, $0 \leq \alpha \leq 1$ is fixed, and $n^\varepsilon :=$ number of edges $\overrightarrow{B_iB_j}$ with sign ε for which $\rho(B_i,B_j) = -\varepsilon$ (over all i,j). Objective: (π,ρ), or simply $k := \#\pi$, that minimizes $P(\pi,\rho)$. What is new and most interesting is ρ; also new is using the edge directions.

Call $(\vec{\Gamma},\sigma)$ "sign clusterable" if $\exists$ (π,ρ) with $P(\pi,\rho) = 0$. Clusterability is sign clusterability with $\rho = \rho_+$, where $\rho_+(B_i,B_j) := +$ if $i = j$, $-$ if $i \neq j$. Let $P(k) := \min\{P(\pi,\rho) : \#\pi = k\}$. Thm. 4: $P(k)$ is monotonically decreasing. [Thus, there is always an optimum π with singleton clusters. Why this does not vitiate the model is not addressed.] Thm. 5: If $(\vec{\Gamma},\sigma)$ is sign clusterable, then $(\vec{\Gamma},-\sigma)$ also is. If $(\vec{\Gamma},\sigma)$ is clusterable, then $(\vec{\Gamma},-\sigma)$ is not clusterable with the same π [provided $E \neq \varnothing$]. If $(\vec{\Gamma},\sigma)$ is sign clusterable with $\rho = -\rho_+$, then $(\vec{\Gamma},-\sigma)$ is clusterable with the same π. "Relocation": Shift one vertex, or exchange two vertices, between blocks so as to decrease P, as in (1996a). This is said (but not proved) to minimize P.

Refinements discussed: partially prespecified blocks; null blocks (without outgoing edges); criterion functions with special treatment of null blocks.

Applications to standard test examples of social psychology.

Dictionary: "balanced" = clusterable; "relaxed balanced" = sign clusterable; "k-balanced" = clusterable with $\#\pi = k$; "relaxed structural balance blockmodel" = this whole system. [Annot. 7 Feb 2009.]

(**SG: Bal, Clu, PsS**)

Florian Dörfler

See W.-J. Mei.

W. Dörfler

1977a Double covers of graphs and hypergraphs. In: *Beitrage zur Graphentheorie und deren Anwendungen* (Proc. Int. Colloq., Oberhof, D.D.R., 1977), pp. 67–79. Technische Hochschule, Ilmenau, 1977. MR 0599766 (82c:05074). Zbl 405.05055. (**SG: Cov, LG**)(**SD, SH: Cov**)

1978a Double covers of hypergraphs and their properties. *Ars Combin.* 6 (1978), 293–313. MR 0599766 (82d:05085). Zbl 423.050532. (**SH: Cov, LG**)

Tomislav Došlić

See also Z. Yarahmadi.

Tomislav Došlić & Damir Vukičevic

2007a Computing the bipartite edge frustration of fullerene graphs. *Discrete Appl. Math.* 155 (2007), 1294–1301. MR 2332321 (2008b:05086). Zbl 1118.05092. (**Par: Fr: Algor**)

[José Manuel dos Santos Simões-Pereira]

See J.M.S. Simões-Pereira.

Lynne L. Doty

See F. Buckley.

Peter Doubilet

1971a Dowling lattices and their multiplicative functions. In: *Möbius Algebras* (Proc. Conf., Waterloo, Ont., 1971), pp. 187–192. University of Waterloo, Ont., 1971, reprinted 1975. MR 0357137 (50 #9605). Zbl 385.05008. (**GG: Matrd**)

Peter Doubilet, Gian-Carlo Rota, & Richard Stanley

1972a On the foundations of combinatorial theory (VI): The idea of generating function. In: *Proceedings of the Sixth Berkeley Symposium on Mathematical Statistics and Probability* (Berkeley, Calif., 1970/71), Vol. II: *Probability Theory*, pp. 267–318. University of California Press, Berkeley, Calif., 1972. MR 0403987 (53 #7796). Zbl 267.05002. Repr. in: Gian-Carlo Rota, *Finite Operator Calculus*, pp. 83–134. Academic Press, New York, 1975. MR 0379213 (52 #119). Zbl 328.05007. Repr. again in: Joseph P.S. Kung, ed., *Gian-Carlo Rota on Combinatorics: Introductory Papers and Commentaries*, pp. 148–199. Birkhäuser, Boston, 1995. MR 1392961 (99b:01027). Zbl 841.01031.

§5.3: Brief treatment of Dowling lattices via symmetric gain digraphs (not quite the same as gain graphs). [Annot. rev 20 Jan 2021.] (**gg: Matrd**)

Thomas A. Dowling

1971a Codes, packings, and the critical problem. In: *Atti del Convegno di Geometria Combinatoria e sue Applicazioni (Perugia, 1970)*, pp. 209–224. Ist. Mat., Univ. di Perugia, Perugia, Italy, 1971. MR 0337669 (49 #2438). Zbl 231.05029.

Pp. 221–223: The first intimations of Dowling lattices/geometries/matroids, as in (1973a), (1973b), and their higher-weight relatives (see Bonin (1993a)). (**gg, Gen: Matrd**)

1973a A q-analog of the partition lattice. In: J.N. Srivastava *et al.*, eds., *A Survey of Combinatorial Theory* (Proc. Int. Sympos., Ft. Collins, Colo., 1971), Ch. 11, pp. 101–115. North-Holland, Amsterdam, 1973. MR 0366707 (51 #2954). Zbl 259.05023.

Linear-algebraic progenitor of (1973b). Treats the Dowling lattice of group $\mathrm{GF}(q)^\times$ as naturally embedded in $\mathrm{PG}^{n-1}(q)$. Interesting is p. 105, Remark: One might generalize some results to any ambient (simple) matroid. (**gg: Geom, Matrd: Invar**)

†† 1973b A class of geometric lattices based on finite groups. *J. Combin. Theory Ser. B* 14 (1973), 61–86. MR 0307951 (46 #7066). Zbl 247.05019. Erratum. *Ibid.* 15 (1973), 211. MR 0319828 (47 #8369). Zbl 264.05022.

$Q_n(\mathfrak{G})$ Introduces the Dowling lattices $Q_n(\mathfrak{G})$ of a group, treated as lattices of group-labelled partial partitions. Equivalent to the frame matroid of complete $\mathfrak{G}$-gain graph $\mathfrak{G}K_n^\bullet$. [The gain-graphic approach was known to Dowling (1973a), p. 109, but first published in Doubilet, Rota, and Stanley (1972a).] Isomorphism, vector representation, Whitney numbers and characteristic polynomial. [The first and still fundamental paper.]

[The Dowling Whitney numbers and polynomials have given rise to a little field of their own; *cf.* Mező (2010a) and references.]

(**gg: Matrd: Invar**)

Thomas Dowling & Hongxun Qin

2005a Reconstructing ternary Dowling geometries. *Adv. Appl. Math.* 34 (2005), no. 2, 358–365. MR 2110557 (2005j:05017). Zbl 1068.52017.

Thm. 1.5: The Dowling geometry $Q_r(\mathbb{Z}_2)$ is the only matroid of rank $r \geq 4$ such that every contraction by a point is $Q_{r-1}(\mathbb{Z}_2)$. (**sg: Matrd**)

Felix Dräxler

See I. Balla.

[Pauline van den Driessche]

See P. van den Driessche (under 'V').

See also J. Bensmail.

François Dross, Florent Foucaud, Valia Mitsou, Pascal Ochem, & Théo Pierron

2020a Complexity of planar signed graph homomorphisms to cycles. *Discrete Appl. Math.* 284 (2020), 166–178. MR 4115466. Zbl 1443.05084. arXiv:1907.03266. HAL hal-02990576. (**SG: Hom: Algor**)

J.M. Drouffe

See R. Balian.

K. Drühl & H. Wagner

1982a Algebraic formulation of duality transformations for Abelian lattice models. *Ann. Phys.* 141 (1982), 225–253. MR 0673981 (84h:82064).

(**SG, GG: Gen: Du, Fr, Phys**)

Lúcia Drummond

See M. Levorato.

Natasha D'Souza

See T. Singh.

Sabitha D'Souza [S. D'Souza]

See P.G. Bhat.

Donglei Du

See S. Ji.

Haifeng Du

See Q. Cai.

Hong Shan Du, Qing Jun Ren, Hou Chun Zhou, & Qing Yu Zheng

1998a The quasi-Laplacian permanental polynomial of a graph. (In Chinese.) *Qufu Shifan Daxue Xuebao Ziran Kexue Ban* [*J. Qufu Normal Univ., Nat. Sci.*] 24 (1998), no. 2, 59–62. MR 1655784 (no rev). (**par: Lap: Eig**)

Mingjun Du, Baoli Ma, & Deyuan Meng

2019a Algebraic criteria for structure identification and behaviour analysis of signed networks. *Int. J. Systems Sci.* 50 (2019), no. 12, 2333–2347. MR 4013308. Zbl 1483.93083.
(SD: Dyn)

2019b Edge convergence problems on signed networks. *IEEE Trans. Cybernetics* 49 (2019), no. 11, 4029–4041. **(SD, sg: Dyn: Incid, LG, Lap: Appl)**

2021a Further results for edge convergence of directed signed networks. *IEEE Trans. Cybernetics* 51 (2021), no. 12, 5659–5670. **(SD, sg: Dyn: Incid, LG, Lap: Appl)**

Wenxue Du
See also Y.-Z. Fan.

Wenxue Du, Xueliang Li, & Yiyang Li
2010a Various energies of random graphs. *MATCH Commun. Math. Comput. Chem.* 64 (2010), no. 1, 251–260. MR 2677586 (2011k:05133).
Including a "tight bound" on signless Laplacian energy, of $L(-\Gamma)$, and "exact estimate" of incidence energy, of $\mathrm{H}(-\Gamma)$. [Annot. 24 Jan 2012.]
(Par: Eig, Incid)

Hangen Duan
See S.C. Gong.

Wenqiang Duan, Qinma Kang, Yunfan Kang, Jianwen Chen, & Qingfeng Qin
2022a A simple and effective iterated greedy algorithm for structural balance in signed networks. *Int. J. Modern Phys. B* 36 (2022), No. 21, art. 2250129.
(SG: Fr: Algor)

M. Dub
See M. Doob.

Christopher Duffy, Fabien Jacques, Mickael Montassier, & Alexandre Pinlou
2009a The chromatic number of 2-edge-colored and signed graphs of bounded maximum degree. Manuscript, 2009. arXiv:2009.05439.
Cf. Jacques and Pinlou (2022a). Dictionary: "2-edge-colored" = signed, "signed graph" = switching class of signed graphs. **(SG: Col, Hom, Sw)**

P. Robert Duimering
See G. Adejumo.

Richard A. Duke, Paul Erdös, & Vojtěch Rödl
1991a Extremal problems for cycle-connected graphs. Proc. Twenty-Second Southeastern Conf. Combinatorics, Graph Theory, and Computing (Baton Rouge, La., 1991). *Congressus Numer.* 83 (1991), 147–151. MR 1152087 (93a:05073). Zbl 772.05052.
Results of the type in (1992a) for polygon lengths ≤ 8. Thm. 2: Given $d > 0$, constant. Every $-\Gamma$ with $\#V = n$ and $\#E = dn^2$ has a subgraph Σ' with $\#E' \geq d^2n^2(1 - o(1))$ in which every two edges belong to a balanced polygon of length at most 8. **(par: bal(Circ))**

1992a Cycle-connected graphs. *Discrete Math.* 108 (1992), 261–278. MR 1189849 (94a:05106). Zbl 776.05057.

All graphs are simple. This is one of four related papers (including (1991a)) that prove extremal results concerning subgraphs of $-\Gamma$ within which every two edges belong to a balanced circle of length at most $2k$, for all or particular k. Typical theorem: Let $F_l(n,m)$ = the largest number $m' = m'(n,m)$ such that every $-\Gamma$ with $\#V = n$ and $\#E \geq m$ has a subgraph Σ' with $\#E' = m'$ in which every two edges belong to a balanced circle of length at most l. For $m = m(n) \geq n^{3/2}$, there is a constant $c_3 > 0$ such that $F_l(n,m) \leq c_3 m^2 n^{-2}$ for all l. (§2, (2).) [*Problem.* Extend these extremal results in an interesting way to arbitrary signed simple graphs, or to simply signed graphs (no repeated edges with the same sign). (Merely allowing positive edges in addition to negative ones just makes the problem easier. Something more is required.)]
(par: bal(Circ): Xtreml)

David M. Duncan, Thomas R. Hoffman, & James P. Solazzo

2010a Equiangular tight frames and fourth root seidel matrices. *Linear Algebra Appl.* 432 (2010), 2816–2823. MR 2639246 (2011c:42081). Zbl 1223.05172.

Adjacency matrices of fourth-root-of-unity gain graphs on K_n. Dictionary: "Seidel matrix" = adjacency matrix of such a gain graph. [Annot. 20 Jun 2011.] **(gg: Geom, adj: kg)**

2011a Numerical measures for two-graphs. *Rocky Mountain J. Math.* 41 (2011), no. 1, 133–154. MR 2845937. Zbl 1213.05165. arXiv:0810.3189.

Some measures to distinguish nonisomorphic two-graphs, i.e., switching-nonisomorphic signatures of K_n. [Annot. 25 Oct 2012.]
(sg: kg, TG: Invar)

Yen Duong, Joel Foisy, Killian Meehan, Leanne Merrill, & Lynea Snyder

† 2012a Intrinsically linked signed graphs in projective space. *Discrete Math.* 312 (2012), 2009–2022. MR 2920861. Zbl 1243.05101. **(SG: Top)**

Arne Dür

1986a *Möbius Functions, Incidence Algebras and Power Series Representations.* Lect. Notes in Math., Vol. 1202. Springer-Verlag, Berlin, 1986. MR 0857100 (88m:05005). Zbl 592.05006.

Dowling lattices are an example of a categorial approach to incidence-algebra techniques in Ch. IV, §7. Computed are the characteristic polynomial and second kind of Whitney numbers. Binomial concavity, hence unimodality of the latter [*cf.* Stonesifer (1975a)] is proved by showing that a suitable generating polynomial has only distinct, negative zeros [*cf.* Benoumhani (1999a)]. **(gg: Matrd: Invar)**

Amit Dutta

See B.K. Chakrabarti.

Luke Duttweiler & Nathan Reff

2019a Spectra of cycle and path families of oriented hypergraphs. *Linear Algebra Appl.* 578 (2019) 251–271. MR 3953365. Zbl 1419.05122. **(SH: Adj, Lap: Eig)**

P.M. Duxbury

See M.J. Alava.

Zdeněk Dvořák & Luke Postle

2018a Correspondence coloring and its application to list-coloring planar graphs without cycles of lengths 4 to 8. *J. Combin. Theory Ser. B* 129 (2018), 38–54. MR 3758240. Zbl 1379.05034. arXiv:1508.03437.

A combinatorial variant of gains: partial injections between lists of a list coloring problem. [Short proof by Zając (20xxa).] [Annot. 29 Jul 2019.] **(gg(Gen): Col)**

Janusz Dybizbański

2020a 2-Edge-colored chromatic number of grids is at most 9. *Graphs Combin.* 36 (2020), no. 3, 831–837. MR 4090528. Zbl 1454.05037. **(SGc: Hom)**

Janusz Dybizbański, Anna Nenca, & Andrzej Szepietowski

2020a Signed coloring of 2-dimensional grids. *Inform. Processing Lett.* 156 (2020), art. 105918, 6 pp. MR 4056038. Zbl 1481.05044.

Dictionary: "signed coloring" = switching homomorphism. Bounds on target order. [Annot. 3 Sept 2020.] **(SG: Sw, Hom)**

I.E. Dzyaloshinskii & G.E. Volovik

1978a On the concept of local invariance in the theory of spin glasses. *J. Physique* 39 (1978), no. 6, 693–700.

[Early attempt to apply frustration in physics.] §2: Heisenberg spins $V(\Sigma) \to$ unit sphere S^2, applying "frustration lines" according to Toulouse (1977a). "Local discrete invariance" = switching. §8: Briefly, partial antiferromagnet (= signed graph); phase diagram varies with the proportion c of negative edges. [Annot. 6 Aug 2018.] **(Phys: SG: Fr, sw)**

David Easley & Jon Kleinberg

2010a *Networks, Crowds, and Markets: Reasoning About a Highly Connected World.* Cambridge Univ. Press, Cambridge, Eng., 2010. MR 2677125 (2011i:91001). Zbl 1205.91007.

Ch. 5, "Positive and negative relationships". §§5.1, "Structural balance", 5.2, "Balanced networks and the Cartwright-Harary Theorem": Balance by triangles in signed complete graphs. Proof of Harary's (1953a) bipartition theorem for complete graphs (§5.2). §5.3, "Applications of structural balance": Applications to history. Complete and incomplete graphs. Alternatives to structural balance. §5.4, "A weaker form of structural balance": Clusterability ("weak balance"). Proof of Davis's (1967a) clusterability theorem. §5.5, "Advanced material: Generalizing the definition of structural balance": Two parts. §5.5A, "Structural balance in arbitrary (non-complete) networks": Proof of Harary's bipartition theorem for general signed graphs by finding connected components of the positive subgraph, then applying breadth-first search to sign the components. [Annot. 22 March 2010.] **(SG: Bal: Exp, Exr)**

§5.5B, "Approximately balanced networks": Thm.: If the proportion of unbalanced triangles in a signed K_n is $\leq \varepsilon < \frac{1}{8}$, and if $\delta := \sqrt{[3]}\varepsilon$, then there are $(1-\delta)\#V$ vertices in which at most a fraction δ of the edges are negative, or there is a bipartition $V = X \cup Y$ such that at most a fraction δ of the edges in X and also in Y are negative and at most that fraction of the XY edges are positive. [Annot. 22 March 2010.]

(SG: Bal, fr)

M. Ebrahimi

See A. Kargaran.

Paul H. Edelman & Victor Reiner

1994a Free hyperplane arrangements between A_{n-1} and B_n. *Math. Z.* 215 (1994), 347–365. MR 1262522 (95b:52021). Zbl 793.05122.

Characterizes all $\Sigma \supseteq +K_n$ whose frame matroid $\mathbf{F}(\Sigma)$ is supersolvable, free, or inductively free. Essentially, iff the negative links form a threshold graph. [Continued in Bailey (20xxa). Generalized in part to arbitrary gain groups in Zaslavsky (2001a).] **(sg: Matrd, Geom, col)**

1996a Free arrangements and rhombic tilings. *Discrete Comput. Geom.* 15 (1996), no. 3, 307–340. MR 1380397 (97f:52019). Zbl 853.52013. Erratum. *Ibid.* 17 (1997), no. 3, 359. MR 1432070 (97k:52013). Zbl 853.52013.

Paul H. Edelman & Michael Saks

1979a Group labelings of graphs. *J. Graph Theory* 3 (1979), 135–140. MR 0530300 (80j:05071). Zbl 411.05059.

Given Γ and abelian group $\mathfrak{A}$. Vertex and edge labellings $\lambda : V \to \mathfrak{A}$ and $\eta : E \to \mathfrak{A}$ are "compatible" if $\lambda(v) = \sum_e \eta(e)$ for every vertex v, the sum taken over all edges incident with v. λ is "admissible" if it is compatible with some η. Admissible vertex labellings are characterized (differently for bipartite and nonbipartite graphs) and the number of edge labelings compatible with a given vertex labelling is computed. [Dual in a sense to Gimbel (1988a).] **(WG, VS: Bal(Du), Enum)**

Herbert Edelsbrunner, Günter Rote, & Emo Welzl

1987a Testing the necklace condition for shortest tours and optimal factors in the plane. In: T. Ottmann, ed., *Automata, Languages and Programming* (Proc., Karlsruhe, 1987), pp. 364–375. Lect. Notes in Computer Sci., Vol. 267. Springer, Berlin, 1987. MR 0912722 (88k:90065). Zbl 636.68042.

Summary of (1989a). **(par: ori, Geom: Algor)**

1989a Testing the necklace condition for shortest tours and optimal factors in the plane. *Theor. Computer Sci.* 66 (1989), 157–180. MR 1019083 (90i:90042). Zbl 686.68035.

§5.1, "Testing the feasibility of the linear program (2) or (1)": The dual linear program (4) belongs to an oriented all-negative signed graph. Treated by expanding it to the graphic LP belonging to the canonical covering graph. **(par: ori, Geom: Algor)**

Jack Edmonds

See also J. Aráoz and E.L. Lawler (1976a).

1965a Paths, trees, and flowers. *Canad. J. Math.* 17 (1965), 449–467. MR 0177907 (31 #2165). Zbl 132.20903 (132, p. 209c).

Followed up by much work, e.g., Witzgall and Zahn (1965a); see Ahuja, Magnanti, and Orlin (1993a) for some references.

(par: ori: incid, Algor)

1965b Maximum matching and a polyhedron with 0, 1-vertices. *J. Res. Nat. Bur. Standards (U.S.A.) Sect. B* 69B (1965), 125–130. MR 0183532 (32 #1012).

Zbl 141.21802 (141, p. 218b).

Alludes to the polyhedron of Edmonds and Johnson (1970a).

(par: ori: Incid, Geom)

Jack Edmonds & Ellis L. Johnson

†† 1970a Matching: a well-solved class of integral linear programs. In: Richard Guy *et al.*, eds., *Combinatorial Structures and Their Applications* (Proc. Calgary Int. Conf., Calgary, 1969), pp. 89–92. Gordon and Breach, New York, 1970. MR 0267898 (42 #2799). Zbl 258.90032.

Introduces "bidirected graphs". A "matching problem" is an integer linear program with nonnegative and possibly bounded variables and otherwise only equality constraints, whose coefficient matrix is the incidence matrix of a bidirected graph. No proofs. [See Aráoz, Cunningham, Edmonds, and Green-Krótki (1983a) for further work.]

(sg: Ori: Incid, Algor, Geom)

2003a Matching: a well-solved class of integral linear programs. In: Michael Jünger, Gerhard Reinelt, and Giovanni Rinaldi, eds., *Combinatorial Optimization—Eureka, You Shrink! Papers Dedicated to Jack Edmonds* (5th Int. Workshop, Aussois, France, 2001), pp. 27–30. Lect. Notes in Computer Sci., Vol. 2570, Springer-Verlag, Berlin, 2003. MR 2163946. Zbl 1024.90505.

Readably typeset reprint of (1970a). **(sg: Ori: Incid, Algor, Geom)**

S.F. Edwards & P.W. Anderson

1975a Theory of spin glasses. *J. Phys. F* 5 (1975), 965–974. Repr. in Marc Mézard, Giorgio Parisi, and Miguel Angel Virasoro, *Spin Glass Theory and Beyond*, pp. 89–98. World Scientific Lecture Notes in Physics, Vol. 9. World Scientific, Singapore, 1987.

Spins s_i are (unit) vectors; edge weights and signs are $J_{ij} \in \mathbb{R}$; the interaction between spins is $J_{ij} s_i \cdot s_j$ with the scalar product. [This seminal article leads to the appearance of signed graphs in physics. Later developments refine the definitions, e.g., to $J_{ij} = \pm 1$, J_{ij} Gaussian random variables, $s_i \in \{+1, -1\}$, etc. *Cf.* esp. Sherrington and Kirkpatrick (1975a).] **(Phys: sg, wg)**

Yoshimi Egawa

See N. Alon.

Richard Ehrenborg

2001a Counting faces in the extended Shi arrangement $\hat{\mathcal{A}}^r_n$. *Conference Proceedings of the 13th Int. Conference on Formal Power Series and Algebraic Combin.* (FPSAC, Tempe, Ariz., 2001), pp. 149–158.

Preliminary version of (2019a). [Annot. 11 Mar 2011.]

(gg: col, Invar, matrd, Geom)

2019a Counting faces in the extended Shi arrangement. *Adv. Appl. Math.* 109 (2019), 55–64. MR 3954085. Zbl 418.52010.

Calculates the characteristic (Cor. 2.5) and, implicitly, Whitney-number polynomials of $[-r+1, r]\vec{K}_n$ in terms of its affinographic hyperplane representation, the extended Shi arrangement. The object is to count faces of the latter by dimension and dimension of the infinite part. [Zaslavsky

(2003a) has a different generalization of Shi faces.] [Annot. 11 Mar 2011, 13 Jun 2020.] **(gg: col, Invar, matrd, Geom)**

Richard Ehrenborg & Margaret A. Readdy

1998a On valuations, the characteristic polynomial, and complex subspace arrangements. *Adv. Math.* 134 (1998), 32–42. MR 1612379 (98m:52018). Zbl 906.52004.

An abstract additive approach to the characteristic polynomial $p(\lambda)$, applied in particular (§3: "The divisor Dowling arrangement") to "divisor Dowling" hyperplane arrangements" $\mathcal{B}(m)$ and certain interpolating arrangements. [Let $\Phi = \mathfrak{G}_1 K_1 \cup \cdots \cup \mathfrak{G}_n K_n$, where $V(K_i) = \{v_1, \ldots, v_i\}$ and $\mathbb{Z}_m = \mathfrak{G}_1 \geq \cdots \geq \mathfrak{G}_n$. $\mathcal{B}(m)$ is the complex hyperplane representation of $\Phi^\bullet$. Thus, $p_{\mathcal{B}(m)}(\lambda) = \chi_{\Phi^\bullet}(\lambda)$, the chromatic polynomial. This is computable via gain-graph coloring when $\mathfrak{G}_1$ is any finite group. The same is true for the other arrangements treated herein.] [Annot. 25 Apr 2009.] **(gg: matrd: Geom, Invar)**

1999a On flag vectors, the Dowling lattice, and braid arrangements. *Discrete Comput. Geom.* 21 (1999), 389–403. MR 1672984 (2000a:52037). Zbl 941.52021.

Canonical complex hyperplane representation of the Dowling lattice of $\mathbb{Z}_k$. P. 395: an interesting EL-labelling of the Dowling lattice by a [disguised lexicographic] ordering of atoms. Thm. 4.9 is a recursive formula for its **ab**-index. Thm. 5.2: the **c**-2**d**-index of the face lattices in case $k = 1, 2$, i.e., those of the real root system arrangements A_n^* and B_n^*. §6 presents a combinatorial description of the face lattice of B_n^* [which it is interesting to compare with that in Zaslavsky (1991b)]. Dictionary: very confusingly, "region" = face. **(gg: Geom, Invar)**

2000a The Dowling transform of subspace arrangements. *J. Combin. Theory Ser. A* 91 (2000), 322–333. MR 1780026 (2001k:52038). Zbl 962.05005.

The group expansion of an ordinary graph is generalized to expansion of an $\mathbb{R}^\times_{>0}$-gain graph by a finite cyclic subgroup of $\mathbb{C}^\times$, with correspondingly generalized formulas for the chromatic polynomial. The computations are technically incorrect; they should be done by gain-graph coloring. [Dictionary: "directed cycle" = circle (not directed).] [Generalized in Koban (2004b).] **(GG: Geom, Invar)**

2009a Exponential Dowling structures. *European J. Combin.* 30 (2009), 311–326. MR 2460236 (2010a:06007). Zbl 1157.05002.

A generalization of Stanley's exponential structures, based on the partition lattice, to Dowling lattices. §2 defines Dowling lattices via partial partitions ("zero block" = set of non-partitioned elements). §3 defines Dowling exponential structures and gives compositional identities via generating functions. §4: generating-function identities for the Möbius invariant; structures with restricted block sizes—especially, block sizes divisible by r with K non-partitioned elements where $K \geq k$ and $K \equiv k \pmod r$. **(gg: matrd: Invar, Enum, Exp)**

Andrzej Ehrenfeucht

See also A.H. Deutz.

Andrzej Ehrenfeucht, Jurriaan Hage, Tero Harju, & Grzegorz Rozenberg

2000a Complexity issues in switching classes of graphs. In: Hartmut Ehrig *et al.*, eds., *Theory and Applications of Graph Transformations (TAGT'98)* (Proc. 6th Int.

Workshop, Paderborn, 1998), pp. 59–70. Lect. Notes in Computer Sci., Vol. 1764. Springer-Verlag, Berlin, 2000. MR 1794790 (no rev). Zbl 958.68133.
(TG: Sw: Algor)

2000b Pancyclicity in switching classes. *Inform. Processing Lett.* 73 (2000), 153–156. MR 1755051 (2001c:05081).

Every switching class of graphs except that of the edgeless graph contains a pancyclic graph. Thus Hamiltonicity is polynomial-time for graph switching classes. **(TG: Sw, Algor)**

2004a Embedding in switching classes with skew gains. In: Hartmut Ehrig *et al.*, eds., *Graph Transformations* (Proc. 2nd Int. Conf. (ICGT 2004), Rome, 2004), pp. 257–270. Lect. Notes in Computer Sci., Vol. 3256. Springer, Berlin, 2004. MR 2198112 (no rev). Zbl 1116.68553.

Cf. (2006a). **(GG(Gen): Sw)**

2006a The embedding problem for switching classes of graphs. Special issue on ICGT 2004. *Fund. Inform.* 74 (2006), no. 1, 115–134. MR 2282895 (2007h:68104). Zbl 1106.68053. **(GG: Sw)**

Andrzej Ehrenfeucht, Tero Harju, & Grzegorz Rozenberg

1996a Group based graph transformations and hierarchical representations of graphs. In: J. Cuny, H. Ehrig, G. Engels and G. Rozenberg, eds., *Graph Grammars and Their Application to Computer Science* (5th Int. Workshop, Williamsburg, Va., 1994), pp. 502–520. Lect. Notes in Computer Sci., Vol. 1073. Springer-Verlag, Berlin, 1996. MR 1422047 (97h:68097).

The "heierarchical structure" of a switching class of skew gain graphs based on K_n. **(gg(Gen): KG: Sw)**

1997a 2-Structures—A framework for decomposition and transformation of graphs. In: Grzegorz Rozenberg, ed., *Handbook of Graph Grammars and Computing by Graph Transformation. Vol. 1: Foundations*, Ch. 6, pp. 401–478. World Scientific, Singapore, 1997. MR 1480952 (99b:68006) (book). Zbl 908.68095 (book).

A tutorial (with some new proofs). The relevant sections, based on papers of Ehrenfeucht and Rozenberg with and without Harju, are those about dynamic labeled 2-structures, i.e., complete graphs with twisted gains. §6.7: "Dynamic labeled 2-structures". §6.8: "Dynamic ℓ2-structures with variable domains". §6.9: "Quotients and plane trees". §6.10: "Invariants", concerns certain switching invariants called "free invariants" when the gains are not twisted. **(gg: KG: sw: Exp, Ref)**

1997b Invariants of inversive 2-structures on groups of labels. *Math. Structures Computer Sci.* 7 (1997), 303–327. MR 1460397 (98g:20089). Zbl 882.05119.

Given a gain graph $(K_n, \varphi, \mathfrak{G})$, a word w in the oriented edges of K_n has a gain $\varphi(w)$; call this $\psi_w(\varphi)$. A "free invariant" is a ψ_w that is an invariant of switching classes. Thm.: There is a number $d = d(K_n, \mathfrak{G})$ such that the group of free invariants is generated by ψ_w with $w = z_1^d \cdots z_k^d u_1 \cdots u_l$ where w_i are triangular cycles (directed!) and u_i are commutators. [The whole paper applies *mutatis mutandis* to arbitrary graphs, the triangular cycles being replaced by any set of cycles containing a fundamental system.] Dictionary: "Inversive 2-structure"

= gain graph based on K_n. **(gg: KG: Sw, Invar)**

1999a *The Theory of 2-Structures: A Framework for Decomposition and Transformation of Graphs*. World Scientific, Singapore, 1999. MR 1712180 (2001i:05001). Zbl 981.05002. **(gg: KG: sw: Exp, Ref)**

2004a Transitivity of local complementation and switching on graphs. *Discrete Math.* 278 (2004), 45–60. MR 2035389 (2005d:05074). Zbl 1033.05052.

Let antilocal complementation at v mean reversing the edges except within the neighborhood of v. Let strictly antilocal complementation mean reversing the edges except within the closed neighborhood of v. Every simple graph of order n can be converted to every other one by antilocal complementations, and also by stricly antilocal complementations. **(TG)**

Andrzej Ehrenfeucht & Grzegorz Rozenberg

1993a An introduction to dynamic labeled 2-structures. In: Andrzej M. Borzyszkowski and Stefan Sokołowski, eds., *Mathematical Foundations of Computer Science 1993* (Proc., 18th Int. Sympos., MFCS '93, Gdańsk, 1993), pp. 156–173. Lect. Notes in Computer Sci., Vol. 711. Springer-Verlag, Berlin, 1993. MR 1265062 (95j:68126).

Extended summary of (1994a). **(GG(Gen): KG: Sw, Str)**

1994a Dynamic labeled 2-structures. *Math. Structures Comput. Sci.* 4 (1994), 433–455. MR 1322183 (96j:68144). Zbl 829.68099.

They prove that a complicated definition of "reversible dynamic labeled 2-structure" G amounts to a complete graph with a set, closed under switching, of twisted gains in a gain group Δ. The twist is a gain-group automorphism α such that $\lambda(e;x,y) = [\alpha\lambda(e;y,x)]^{-1}$, λ being the gain function. Dictionary: their "domain" D = vertex set, "labeling function" λ (or equivalently, g) = gain function, "alphabet" = gain group, "involution" $\delta = \alpha\circ$ inversion, "δ-selector" $\hat{S}$ = switching function, "transformation induced by $\hat{S}$" = switching by $\hat{S}$; a "single axiom" d.l. 2-structure consists of a single switching class.

Further, they investigate "clans" of G. Given g (i.e., λ), deleting identity-gain edges leaves isolated vertices ("horizons") and forms connected components, any union of which is a "clan" of g. A clan of G is any clan of any $g \in G$. **(GG(Gen): KG: Sw, Str)**

1994b Dynamic labeled 2-structures with variable domains. In: J. Karhumäki, H. Maurer, and G. Rozenberg, eds., *Results and Trends in Theoretical Computer Science* (Proc. Colloq. in Honor of Arto Salomaa, Graz, 1994), pp. 97–123. Lect. Notes in Computer Sci., Vol. 812. Springer-Verlag, Berlin, 1994. MR 1286959 (95m:68128).

Combinations and decompositions of complete graphs with twisted gains. **(GG(Gen): KG: Str, Sw)**

George C.M.A. Ehrhardt, Matteo Marsili, & Fernando Vega-Redondo

2005a On the rise and fall of networked societies. In: Joaquıın Marro, Pedro L. Garrido, and Miguel A. Muñoz, eds., *Modeling Cooperative Behavior in the Social Sciences* (Proc. 8th Granada Lect., Granada, Spain, 2005), pp. 96–103. AIP Conf. Proc., Vol. 779. Amer. Inst. Physics, Melville, N.Y., 2005. arXiv:-

physics/0505019.

§ III, "The effect of negative links": A random model where positive edges may appear, and may change to negative. Negative edges disappear over time. [Annot. 12 Aug 2012.] (**SG: PsS: Rand, Phys**)

K. Ehsani

See S. Akbari.

M. Einollahzadeh

See S. Akbari.

Kurt Eisemann

1964a The generalized stepping stone method for the machine loading model. *Management Sci.* 11 (1964/65), No. 1 (Sept., 1964), 154–176. Zbl 136.13901 (136, p. 139a). (**GN: Incid, Matrd(bases)**)

Joyce Elam, Fred Glover, & Darwin Klingman

1979a A strongly convergent primal simplex algorithm for generalized networks. *Math. Operations Res.* 4 (1979), 39–59. MR 0543608 (81g:90049). Zbl 422.90081. (**GN: Matrd(bases), Incid**)

Adrien Elena

See J. Demongeot.

David P. Ellerman

1984a Arbitrage theory: A mathematical introduction. *SIAM Rev.* 26 (1984), 241–261. MR 0738931 (85g:90024). Zbl 534.90014. (**GG: Bal, Incid, Flows: Appl, Ref**)

M.N. Ellingham

1991a Vertex-switching, isomorphism, and pseudosimilarity. *J. Graph Theory* 15 (1991), 563–572. MR 1133811 (92g:05136). Zbl 802.05057.

Main theorem (§2) characterizes, given two signings of K_n (where n may be infinite) and a vertex set S, when switching S makes the signings isomorphic. [*Problem* 1. Generalize to other underlying graphs. *Problem* 2. Prove an analog for bidirected K_n's.]

A corollary (§3) characterizes when vertices u, v of $\Sigma = (K_n, \sigma)$ satisfy $\Sigma^{\{u\}} \cong \Sigma^{\{v\}}$ and discusses when in addition no automorphism of Σ moves u to v. All is done in terms of Seidel (graph) switching (here called "vertex-switching") of unsigned simple graphs. (**kg: sw, TG**)

1996a Vertex-switching reconstruction and folded cubes. *J. Combin. Theory Ser. B* 66 (1966), 361–364. MR 1376057 (96i:05120). Zbl 856.05071.

Deepens the folded-cube theory of Ellingham and Royle (1992a). Nicely generalizing Stanley (1985a), the number of subgraphs of a signed K_n that are isomorphic to a fixed signed K_m is reconstructible from the s-vertex switching deck if the Krawtchouk polynomial $K^n_s(x)$ has no even zeros between 0 and m. (Closely related to Krasikov and Roditty (1992a), Theorems 5 and 6.) Remark 4: balance equations (Krasikov and Roditty (1987a)) and Krawtchouk polynomials both reflect properties of folded cubes. All is done in terms of Seidel switching of unsigned simple graphs. [It seems clear that the folded cube appears because it corresponds to the effect of switchings on signatures of K_n (or any connected graph), since switching by X and X^c have the same effect. For the bidirected case (Problem 2 under Stanley (1985a)), the unfolded cube should play a similar role. *Question.* When treating a general underlying graph Γ, will a polynomial influenced by $\operatorname{Aut}\Gamma$ replace the Krawtchouk polynomial?] **(kg: sw, TG)**

M.N. Ellingham & Gordon F. Royle

1992a Vertex-switching reconstruction of subgraph numbers and triangle-free graphs. *J. Combin. Theory Ser. B* 54 (1992), 167–177. MR 1152444 (93d:05112). Zbl 695.05053, (Zbl 748.05071).

Reconstruction of induced subgraph numbers of a signed K_n from the s-vertex switching deck, dependent on linear transformation and thence Krawtchouk polynomials as in Stanley (1985a). The role of those polynomials is further developed. Done in terms of Seidel switching of unsigned simple graphs, with the advantage of reconstructing arbitrary subgraph numbers as well. A gap is noted in Krasikov and Roditty (1987a), proof of Lemma 2.5. [Methods and results are closely related to Krasikov (1988a) and Krasikov and Roditty (1987a), (1992a).] **(kg: sw, TG)**

Joanna A. Ellis-Monaghan

See also L. Abrams.

Joanna A. Ellis-Monaghan & Iain Moffatt

2011a The Tutte–Potts connection in the presence of an external magnetic field. *Adv. Appl. Math.* 47 (2011), no. 4, 772–782. MR 2832375. Zbl 1232.05100. arXiv:-1005.5470 (extended version). **(sg: Top, Du)(SGc: Gen: Invar)**

2012a Twisted duality for embedded graphs. *Trans. Amer. Math. Soc.* 364 (2012), no. 3, 1529–1569. MR 2869185 (2012m:05102). Zbl 1238.05067. arXiv:0906.5557. **(sg: ori: Top, Du)**

2013a A Penrose polynomial for embedded graphs. *European J. Combin.* 34 (2013), no. 2, 424–445. MR 2994409. Zbl 1254.05080. arXiv:1106.5279. **(sg: Top, Du, Invar)**

2013b *Graphs on Surfaces: Dualities, Polynomials, and Knots.* SpringerBriefs in Mathematics. Springer, New York, 2013. MR 3086663. Zbl 1283.57001. **(SG: Top, Du, Invar)**

2015a Evaluations of topological Tutte polynomials. *Combin. Prob. Computing* 24 (2015), no. 3, 556–583. MR 3326433. Zbl 1371.05134. arXiv:1108.3321. **(SG: Top, Du, Invar)**

Joanna A. Ellis-Monaghan & Irasema Sarmiento

2011a A recipe theorem for the topological Tutte polynomial of Bollobás and Riordan. *European J. Combin.* 32 (2011), no. 6, 782–794. MR 2821551 (2012j:05220). Zbl 1223.05039. arXiv:0903.2643. **(sg: Top, Du, Invar)**

Joanna Ellis-Monaghan & Lorenzo Traldi

2006a Parametrized Tutte polynomials of graphs and matroids. *Combin. Probab. Comput.*, 15 (2006), no. 6, 835–854. MR 2271830 (2007j:05038). Zbl 1108.05024.

A slight generalization of the multiplicative property of the strong parametrized Tutte functions in Zaslavsky (1992b), and a necessary and sufficient condition for their existence on an arbitrary minor-closed class

of matroids inspired by Bollobás and Riordan (1999a). Also, specialization to graphs. Dictionary: "Tutte polynomial" means a function, not a polynomial. [Annot. rev 26 Jun 2022.] **(SGc: Gen: Invar)**

Abdelhakim El Maftouhi, Ararat Harutyunyan, & Yannis Manoussakis

2014a (as Hakim El Maftouhi, Ararat Harutyunyan, and Yannis Manoussakis) Balance in random signed graphs. In: *Bordeaux Graph Workshop 2014*, pp. 43–44. LaBRI, Bordeaux, 2014. URL `http://bgw.labri.fr/2014/bgw2014-booklet.pdf`

Extended abstract of El Maftouhi, Manoussakis, & Megalakaki (2012a) and (2015a). "Weak balance" = clusterability. [Annot. 19 Mar 2017.] **(SG: Rand: Bal, Clu)**

2015a Weak balance in random signed graphs. *Internet Math.* 11 (2015), no. 2, 143–154. MR 3316860 (no rev). **(SG: Rand: Clu)**

A. El Maftouhi, Y. Manoussakis, & O. Megalakaki

2012a Balance in random signed graphs. *Internet Math.* 8 (21012), no. 4, 364–380. MR 3009999. Zbl 1258.05110. **(SG: Bal, Fr: Rand)**

Amani Elrayes

See S. Nada.

Ashraf Elrokh

See S. Nada.

Amine El Sahili & Maria Abi Aad

2018a Antidirected Hamiltonian paths and directed cycles in tournaments. *Discrete Math.* 341 (2018), 2018–2027. MR 3802155. Zbl 1387.05136.

Antidirected means coherent in the poise gains of a digraph. [*Cf.* Diwan, Frye, Plantholt, and Tipnis (2011a).] [*Question.* How does this generalize to bidirected graphs?] [Annot. 30 May 2018.] **(gg: KG: Str)(sg: KG: par: Ori)**

D. Emanuel & A. Fiat

See also E. Demaine.

2003a Correlation clustering—Minimizing disagreements on arbitrary weighted graphs. In: *Algorithms—ESA 2003* (Budapest, 2003), pp. 208–220. Lect. Notes in Computer Sci., Vol. 2832. Springer, Berlin, 2003. MR 2085454. Zbl 1266.68228.

Conference version of Demaine, Emanuel, Fiat, and Immorlica (2006a). [Annot. 13 Sept 2009.] **(SG: WG: Clu: Algor)**

Frank Emmert-Streib

See G.H. Yu.

German Andres Enciso

See also B. DasGupta.

German Enciso & Eduardo D. Sontag

2005a Monotone systems under positive feedback: multistability and a reduction theorem. *Systems Control Lett.* 54 (2005), no. 2, 159–168. MR 2109582 (2005g:93055). Zbl 1129.93398. **(Dyn: SD)**

2006a Global attractivity, i/o monotone small-gain theorems, and biological delay systems. *Discrete Continuous Dynamical Systems* 14 (2006), no. 3, 549–578.

MR 2171727 (2006g:93128). Zbl 1111.93071. (**SD: Dyn, Biol**)

2008a Monotone bifurcation graphs. *J. Biol. Dynamics* 2 (2008), no. 2, 121–139. MR 2427522 (2009e:34092). Zbl 1141.92005. (**SD: Dyn, Biol**)

Mechthild Enderle

See J.D. Noh.

Shin-ichi Endoh

See T. Nakamura.

Gernot M. Engel & Hans Schneider

1973a Cyclic and diagonal products on a matrix. *Linear Algebra Appl.* 7 (1973), 301–335. MR 0323804 (48 #2160). Zbl 289.15006. (**gg: Sw**)

1975a Diagonal similarity and equivalence for matrices over groups with 0. *Czechoslovak Math. J.* 25(100) (1975), 389–403. MR 0396615 (53 #477). Zbl 329.15007. (**gg: Sw**)

1980a Matrices diagonally similar to a symmetric matrix. *Linear Algebra Appl.* 29 (1980), 131–138. MR 0562753 (81k:15017). Zbl 432.15014. (**gg: Sw**)

Michael Engquist & Michael D. Chang

1985a New labeling procedures for the basis graph in generalized networks. *Operations Res. Lett.* 4 (1985), no. 4, 151–155. MR 0821177 (87j:05137). Zbl 572.90095.

Generalizing pure-network procedures to get fast computations. [Annot. 4 Sept 2010.] (**GN: Matrd(Bases)**)

R.C. Entringer

1985a A short proof of Rubin's block theorem. In: B.R. Alspach and C.D. Godsil, eds., *Cycles in Graphs*, pp. 367–368. Ann. Discrete Math., Vol. 27. North-Holland Math. Stud., Vol. 115. North-Holland, Amsterdam, 1985. MR 0821538 (87f:05144). Zbl 576.05037.

See Erdős, Rubin, & Taylor (1980a). (**par: bal**)

H. Era

See J. Akiyama.

Paul Erdos [Paul Erdös, Paul Erdős, Pál Erdős]

See also B. Bollobás and R.A. Duke.

1996a On some of my favourite theorems. In: D. Miklós, V.T. Sós and T. Szőnyi, eds., *Combinatorics, Paul Erdős is Eighty* (Papers from the Int. Conf. on Combinatorics, Keszthely, 1993), Vol. 2, pp. 97–132. Bolyai Soc. Math. Studies, 2. János Bolyai Mathematical Society, Budapest, 1996. MR 1395856 (97g:00002). Zbl 0853.11001.

P. 119 mentions the theorem of Duke, Erdős, & Rödl (1991a) on even circles.

Pp. 120–121 mention (amongst similar problems) a theorem of Erdős and Hajnal (source not stated): Every all-negative signed graph with chromatic number $\aleph_1$ contains every finite bipartite graph [i.e., every finite, balanced, all-negative signed graph]. [*Problem.* Find generalizations to signed graphs. For instance: *Conjecture.* Every signed graph with chromatic number $\aleph_1$, that does not become antibalanced upon deletion of any finite vertex set, contains every finite, balanced signed

graph up to switching equivalence.] [MR: "this is one of the best collections of problems that Erdos has published."] (**par: bal: Exp, Ref**)

P. Erdös, R.J. Faudree, A. Gyárfás, & R.H. Schelp

1991a Odd cycles in graphs of given minimum degree. In: Y. Alavi, G. Chartrand, O.R. Oellermann, and A.J. Schwenk, eds., *Graph Theory, Combinatorics, and Applications* (Proc. Sixth Quadrennial Int. Conf. Theory Appl. Graphs, Kalamazoo, Mich., 1988), Vol. 1, pp. 407–418. Wiley, New York, 1991. MR 1170794 (93d:05085). Zbl 840.05050.

A large, nonbipartite, 2-connected graph with large minimum degree contains a circle of given odd length or is one of a single type of exceptional graph. [*Question.* Can this be generalized to negative circles in unbalanced signed graphs?] (**par, sg: Circ, Xtreml**)

P. Erdős, E. Győri, & M. Simonovits

1992a How many edges should be deleted to make a triangle-free graph bipartite? In: G. Halász, L. Lovász, D. Miklós, and T. Szönyi, eds., *Sets, Graphs and Numbers* (Proc., Budapest, 1991), pp. 239–263. Colloq. Math. Soc. János Bolyai, Vol. 60. János Bolyai Math. Soc., Budapest, and North-Holland, Amsterdam, 1992. MR 1218193 (94b:05104). Zbl 785.05052.

Assume $|\Sigma|$ simple of order n and $\not\supseteq$ a fixed graph Δ. Results on frustration index l of antibalanced Σ if Δ is 3-chromatic, esp. C_3. Thm.: If $\#E > n^2/5 - o(n^2)$, then $l(\Sigma) < n^2/25 - o(n^2)$. *Conjecture* (Erdős): For $\Delta = C_3$ the hypothesis on $\#E$ is unnecessary. [*Question* 1(a). Is the answer different when Σ need not be antibalanced? *Question* 2(a). Exclude a fixed signed graph whose signed chromatic number $= 1$. *Question* 3(a). In particular, exclude $-K_3$. *Question* 4(a). Exclude $-K_l$. *Question* 5(a). Exclude an unbalanced C_l. *Questions* 1–5(b). Even if $l(\Sigma)$ cannot be estimated, is there always an extremal graph that is antibalanced—as when no graph is excluded, by Petersdorf (1966a)?] (**par: Xtreml**)

P. Erdös & L. Pósa

1965a On independent circuits contained in a graph. *Canad. J. Math.* 17 (1965) 347–352. MR 0175810 (31 #86). Zbl 129.39904.

An upper bound on l_0, the vertex frustration number, in terms of vertex packing of unbalanced circles, in the contrabalanced case. *Problem.* Find an analog for signed graphs and a generalization to biased graphs. (**gg: fr, Circ**)

Paul Erdös, Arthur L. Rubin, & Herbert Taylor

1980a Choosability in graphs. In: *Proceedings of the West Coast Conference on Combinatorics, Graph Theory and Computing* (Arcata, Calif., 1979), pp. 125–157. Congressus Numer., XXVI. Utilitas Math. Publ. Inc., Winnipeg, Man., 1980. MR 0593902 (82f:05038). Zbl 469.05032.

Rubin's block theorem (Thm. R, p. 136): a block graph, not complete or an odd circle, contains an induced even circle with at most one chord. [*Cf.* Entringer (1985a).] [*Question.* Does this generalize to signed graphs, Rubin's block theorem being the antibalanced case? Rubin's 2-choosability theorem, p. 132, is also tantalizingly reminiscent of

antibalanced graphs, but in reverse.] **(par: Str, bal)**

Carolyn Eschenbach

See also Z.-S. Li and J. Stuart.

1993a Idempotence for sign-pattern matrices. *Linear Algebra Appl.* 180 (1993), 153–165. MR 1206414 (94b:15010). Zbl 777.05032.

Irreducible A is sign-idempotent iff every entry is $+$. Necessary and sufficient conditions for reducible A to be sign-idempotent; in particular, it need not have nonnegative entries, but V must partition into V_i inducing no arcs or an all-positive complete symmetric digraph with loops. [Counterexamples in Rong Huang, Sign idempotent sign patterns similar to nonnegative sign patterns, *Linear Algebra Appl.* 428 (2008), 2524–2535. MR 2416567 (2009c:15010). Zbl 1144.15014.] [Annot. 29 Sept 2012.] **(QM: SD: Adj)**

1993b Sign patterns that require exactly one real eigenvalue and patterns that require $n-1$ nonreal eigenvalues. *Linear Multilinear Algebra* 35 (1993), no. 3-4, 213–223. MR 1308691 (95k:15009). Zbl 787.15008. **(QM: sd)**

Carolyn A. Eschenbach, Frank J. Hall & Charles R. Johnson

1993a Self-inverse sign patterns. In: Richard A. Brualdi, Shmuel Friedland, and Victor Klee, eds., *Combinatorial and Graph-Theoretical Problems in Linear Algebra* (IMA Workshop, Minneapolis, 1991), pp. 245–256. IMA Volumes in Mathematics and its Applications, 50. Springer-Verlag, New York, 1993. MR 1240969 (94e:15004). Zbl 792.15008.

Sign matrices whose sign patterns are self-inverse are essentially adjacency matrices of signed graphs and are very few. [Annot. 13 Apr 2009.] **(sg, QM)**

Carolyn A. Eschenbach, Frank J. Hall, Charles R. Johnson, & Zhongshan Li

1997a The graphs of the unambiguous entries in the product of two $(+,-)$-sign pattern matrices. *Linear Algebra Appl.* 260 (1997), 95–118. MR 1448352 (98e:05075). Zbl 881.05089.

A, B are nowhere-zero sign-pattern matrices. $(AB)_{ij}$ may be necessarily $+$, $-$, or ambiguous [Abelson and Rosenberg's (1958a) p, n, a]. Let $\mathcal{R}$ = set of rows, $\mathcal{C}$ = set of columns. The graph $G(A,B) \subseteq K_{\mathcal{R}(A),\mathcal{C}(B)}$ has an edge ij for each unambigous entry in AB. The digraph $D(A^2)$ has an arc (i,j) for each unambiguous entry in A^2. Thm. 3.2: Γ is a $G(A,B)$ iff it is the disjoint union of bicliques and isolated vertices. Characterizing $D(A^2)$ seems hard. Results on special cases. [D and G are signed by p, n. Thm. 5.11: $D(A^2)$, if a circle, is balanced iff the circle is positive. §6, "Characterization of permutation graphs in D_n", i.e., $D(A^2)$ that are permutation graphs. [*Problem.* Investigate $D(A^2)$ and $G(A,B)$ signed by p, n. *Problem.* Generalize to allow 0 entries (thus working over Abelson–Rosenberg's algebra $\{p, n, a, o\}$.)] Dictionary: "signature similarity" of matrices = switching of digraph, "negative matching" = entry n in AB = negative edge in $G(A,B)$. [Annot. 4 Nov 2011.] **(QM: sd, sw)**

Carolyn A. Eschenbach, Frank J. Hall & Zhongshan Li

1998a From real to complex sign pattern matrices. *Bull. Austral. Math. Soc.* 5 (1998), 159–172. MR 1623848 (99d:15003). Zbl 951.15021.

$S := \{\alpha + i\beta : \alpha, \beta = 0, \pm\}$, the set of complex signs. A complex sign pattern matrix has complex signs as entries. If it is square it has a digraph $D(A)$ with complex signs as gains. §3, "Cyclic nonnegativity": Cycles with gain $\pm$. Switching (via matrices) by $\pm, \pm$i. Cor. 3.2: $D(A)$ is balanced iff it switches to all $+$. Thm. 3.3: iff D is balanced and $D(A + A^*)$ switches to all $+$. §4, "Stability": Lem. 4.1: If A is sign stable, every digon has real or purely imaginary gain. Lem. 4.2: If A is sign stable it is sign nonsingular. Thm. 4.4 (generalizing Quirk and Ruppert (1965a) and Maybee and Quirk (1969a)): Assume every vertex has a negative loop. Then A is sign stable iff all digons are negative and no longer cycles exist. Thm. 5.2: Similar, for ray stability. Dictionary: "cyclically nonnegative" = all cycle gains are $+$; "cyclically positive" = cyclically nonnegative and no zeros. [Annot. 4 Nov 2011.]

(**QM: Gen; gg, sw; QSta**)

Carolyn A. Eschenbach & Charles R. Johnson

† 1990a A combinatorial converse to the Perron-Frobenius theorem. *Linear Algebra Appl.* 136 (1990), 173–180. MR 1061544 (91d:15047). Zbl 739.15004.

"Perron property": spectral radius is an eigenvalue. Thm. 1.1: Sign pattern matrix A requires Perron property iff it is cyclically nonnegative. Some sign patterns that allow Perron property. Dictionary: "Cyclically nonnegative matrix": its signed digraph is cycle balanced. [Annot. 20 Oct 2020.] (**QM, SD: Bal(Cyc), sw**)

Carolyn A. Eschenbach & Zhongshan Li

1999a Potentially nilpotent sign pattern matrices. *Linear Algebra Appl.* 299 (1999), 81–99. MR 1723710 (2000i:15043). Zbl 941.15012.

Matrices with tree digraph. Cycle sign = sign product, p. 82. 2-cycle signs in Thms. 5.3 (proof), 5.5, 5.7 (proof). [Annot. 5 Nov 2011.]

(**QM: sd, sw**)

Pouya Esmailian, Seyed Ebrahim Abtahi, & Mahdi Jalili

2014a Mesoscopic analysis of online social networks: The role of negative ties. *Phys. Rev. E* 90 (2014), art. 042817, 11 pp. arXiv:1411.6057. (**SG: Clu**)

Pouya Esmailian & Mahdi Jalili

2015a Community detection in signed networks: the role of negative ties in different scales. *Sci. Reports* 5 (2015), art. 14339, 17 pp. (**SG: Clu: Algor**)

Ernesto Estrada

2019a Rethinking structural balance in signed social networks.. *Discrete Appl. Math.* 268 (2019), 70–90. MR 4007232. Zbl 1425.91370.

Axioms for measurement of imbalance. (**SG: Bal, Fr: PsS**)

Ernesto Estrada & Michele Benzi

2014a Walk-based measure of balance in signed networks: Detecting lack of balance in social networks. *Phys. Rev. E* 90 (2014), art. 042802, 10 pp. arXiv:1406.2132.

For signed graph or digraph, $K(\Sigma) := \operatorname{tr}(\exp A(\Sigma))/\operatorname{tr}(\exp A(|\Sigma|))$, "degree of balance" based on walk signs. Weighting a closed l-walk by $w(W) = 1/l!$, degree of balance is $K = w(\Sigma)/w(|\Sigma|)$; then $0 \leq K \leq 1$;

$K = 1$ iff Σ is cycle balanced. Thm 1: For $\Sigma_n := (K_n, \sigma)$ with $\Sigma_n^+ = C_n$, $K \to 0$, interpreted as Σ_n being "largely unbalanced". Degree of balance of v_i is $K_i := (\exp A(\Sigma))_{ii} / (\exp A(|\Sigma|))_{ii}$. §III: Replace A by βA, $\beta > 0$ (inverse "temperature"), giving $K(\beta)$, interpreted as equilibrium constant of a graph with fluctuating signs. §§IV-VII: Compares walk-based measure K to circle-based measures in some real examples including social networks. §VII, "Tuning balance in social networks": Varying β has interesting effects on K.

Dictionary: "the balance" = degree of balance; "balanced closed walk" = positive closed walk. The history is not entirely reliable. arXiv title: "Are social networks really balanced?" [Annot. 3 Feb 2018, rev 3 Dec 2020.] (**SG, SD: Fr, Adj, PsS, Phys**)

Ernesto Estrada & Naomichi Hatano

2008a Communicability in complex networks. *Phys. Rev. E* (3) 77 (2008), art. 036111, 12 pp. MR 2495430 (2010i:91171). (**SG: KG**)

Ernesto Estrada, Desmond J. Higham, & Naomichi Hatano

2008a Communicability and multipartite structures in complex networks at negative absolute temperatures. *Phys. Rev. E* 78 (2008), art. 026102, 7 pp. (**SG: KG: clu**)

Ernesto Estrada & Juan A. Rodríguez-Velázquez

2005a Spectral measures of bipartivity in complex networks. *Phys. Rev. E* (3) 72 (2005), no. 4, art. 046105, 6 pp. MR 2202758 (2006i:94124). (**par: Fr, Eig**)

Khasheyar Etemadi

See S. Akbari.

Anthony B. Evans

See Y.Q. Chen.

Patricia A. Evans

See D.D. Doan.

Cloyd L. Ezell

1979a Observations on the construction of covers using permutation voltage assignments. *Discrete Math.* 28 (1979), 7–20. MR 0542932 (81a:05040). Zbl 413.05005. (**GG: Top, Cov, sw**)

Peyman Ezzati

See S. Akbari.

Giuseppe Facchetti, Giovanni Iacono, & Claudio Altafini

2011a Computing global structural balance in large-scale signed social networks. *Proc. Nat. Acad. Sci.* 108 (2011), no. 52, 20953–20958. URL `http://www.pnas.org/cgi/doi/10.1073/pnas.1109521108` (**SG: Fr: Algor**)

François Fages

See also K. Sriram.

François Fages & Sylvain Soliman

2008a From reaction models to influence graphs and back: A theorem. In: Jasmin Fisher, ed., *Formal Methods in Systems Biology* (First Int. Workshop, FMSB 2008, Cambridge, Eng., 2008), pp. 90–102. Lect. Notes in Computer Sci., Vol.

5054. Springer, Berlin, 2008. MR 2497938 (2010e:92050). Zbl 1375.92021. **(sd: Chem)**

Ulrich Faigle & Rainer Schrader

1990a Orders and graphs. In: G. Tinhofer, E. Mayr, H. Noltemeier and M.M. Sysło, eds., *Computational Graph Theory*. Computing Supplementum, 7. Springer-Verlag, Vienna, 1990. MR 1059927 (91d:05085). Zbl 725.05045.

An example is threshold signed graphs (*cf.* Benzaken, Hammer, and de Werra (1985a)). [Annot. 16 Jan 2012.] **(SG)**

M. Falcioni, E. Marinari, M.L. Paciello, G. Parisi, & B. Taglienti

1981a Phase transition analysis in Z_2 and U(1) lattice gauge theories. *Phys. Lett. B* 105 (1981), no. 1, 51–54. **(SG: Phys)**

Shaun Fallat

See also M.S. Cavers and Y.-Z. Fan.

Shaun Fallat & Yi-Zheng Fan

2012a Bipartiteness and the least eigenvalue of signless Laplacian of graphs. *Linear Algebra Appl.* 436 (2012), no. 9, 3254–3267. MR 2900713. Zbl 1244.05142.

"Bipartiteness" of Γ [also known as biparticity] is $b(-\Gamma)$. "Algebraic bipartiteness" is $\lambda_{\min}(L(-\Gamma))$. Rephrased in terms of antibalanced signed graphs: Thm. 2.1. If $-\Gamma$ is unbalanced, $\lambda_{\min} \leq l_0(-\Gamma)$, the vertex frustration number. Thm. 2.4. (1) $\operatorname{Spec} L(\widetilde{-\Gamma}) = \operatorname{Spec} L(\Gamma) \cup \operatorname{Spec} L(-\Gamma)$. [A special case of Bilu and Linial (2006a), Lemma.] (2–4) Elementary properties of $\widetilde{-\Gamma}$ [found in Zaslavsky (1982a)]. (4) If $-\Gamma$ is connected and unbalanced, $\lambda_2(L(\widetilde{-\Gamma})) = \min\{\lambda_{\min}(\Gamma), \lambda_2(\Gamma)\} > 0$.

$\bar\psi(-\Gamma) := \min_S[2l(-\Gamma : S) + \#E(S, S^c)]/\#S$ $(S \neq \varnothing, V)$ (*cf.* Desai and Rao (1994a).) Thm. 2.6. If Γ is connected, $\Delta :=$ max degree, $\lambda_{\min} \geq \Delta - \sqrt{\Delta^2 - \bar\psi^2}$. Thm. 2.7. $\lambda_{\min} \leq 2\bar\psi \leq 4l(-\Gamma)/n$. (Strengthens Y.Y. Tan and Fan (2008a).) [*Conjecture*. The results must generalize to all (Γ, σ).] [Annot. 20 Jan 2012.] **(sg: Par: Eig, Cov)**

Genghua Fan

See B. Bao and J. Chen.

Qiong Fan

See Y.-P. Wu.

Yi-Zheng Fan

See also L. Cui, S. Fallat, S.C. Gong, B.S. Tam, Y.Y. Tan, Y. Wang, M.L. Ye, G.-D. Yu, and J. Zhou.

2003a On spectral integral variations of mixed graphs. *Linear Algebra Appl.* 374 (2003), 307–316. MR 2008794 (2005h:05133). Zbl 1026.05076.

The signed graphs (not necessarily simple) for which adding an edge changes only one eigenvalue of the Laplacian matrix $L(\Sigma)$ and increases that by an integer. [Dictionary: "mixed graph" = bidirected graph B where all negative edges are extraverted, in effect the signed graph $-\Sigma_{\mathrm{B}}$; "quasibipartite" = balanced; "e^{c}" = e with reversed sign. The article's sign $\operatorname{sgn}(e)$ equals $-\sigma_{\mathrm{B}}(e)$. The entire article is really about signed graphs Σ and $A(\Sigma)$ and $L(\Sigma)$ and uses signed-graph matrices and methods.] Thm. 1: This eigenvalue property holds iff the column $x(e)$ of e in H(Σ) is an eigenvector of $L(\Sigma)$. Corollaries give other criteria and identify the change in the one eigenvalue. Lemma 5: L is singular iff Σ is balanced [special case of Zaslavsky (1982a), Theorem 8A.4]. [Annot. 13 Apr 2009, 10 Feb 2012.]

(**SG: incid, Eig**)

2004a On structure of eigenvectors of mixed graphs. Sixth Int. Conf. Matrix Theory Appl. in China. *Heilongjiang Daxue Ziran Kexue Xuebao* (*J. Nat. Sci. Heilongjiang Univ.*) 21 (2004), no. 4, 50–54. MR 2129072 (no rev). Zbl 1077.05061.

Early version of (2007a). [Annot. 9 Jan 2013.] (**sg: Eig**)

2004b Largest eigenvalue of a unicyclic mixed graph. *Appl. Math. J. Chinese Univ. Ser. B* 19 (2004), no. 2, 140–148. MR 2063313 (no rev).

The "mixed graphs" are signed graphs with reversed signs; see (2003a). Graphs are simple. The eigenvalues are those of the Laplacian $L(\Sigma)$. Prop. 2.2: Laplacian spectrum of negative circle. [The first such proof. Equivalent to the adjacency spectrum because C_n is regular.] Thm. 2.8: The signed 1-trees with max and min $\lambda_{\min}(L(\Sigma))$. Thm. 2.9: Those with $\lambda_{\min} = n$. Thm. 2.10: Those with $\lambda_{\min} > n$. (N.B. Lem. 2.4: $\lambda_{\min} \leq n+1$ from Hou, Li, and Pan (2003a), Thm. 3.5(1), or X.D. Zhang and Li (2002a).) [Annot. 10 Feb 2012.] (**SG: incid, Eig**)

2005a On the least eigenvalue of a unicyclic mixed graph. *Linear Multilinear Algebra* 53 (2005), no. 2, 97–113. MR 2133313 (2005m:05145). Zbl 1062.05090.

The "mixed graphs" are signed graphs with reversed signs; see Y.-Z. Fan (2003a). Graphs are simple. The eigenvalue is that of $L(\Sigma)$. Eigenvector structure leads to results on minimum and maximum of the least eigenvalue, given order and girth. [Annot. 9 Jan 2013.] (**sg: Eig**)

2007a On eigenvectors of mixed graphs with exactly one nonsingular cycle. *Czechoslovak Math. J.* 57 (2007), no. 4, 1215–1222. MR 2357587 (2008i:05117). Zbl 1174.05075.

The "mixed graphs" are signed graphs with reversed signs; see Y.-Z. Fan (2003a). Graphs are simple. The eigenquantities are those of the Laplacian $L(\Sigma)$. The eigenvector of the smallest eigenvalue is similar to that of the second smallest Laplacian eigenvalue of a graph. [Annot. 9 Jan 2013.]

(**sg: Eig**)

Yi-Zheng Fan, Wen-Xue Du, & Chun-Long Dong

2014a The nullity of bicyclic signed graphs. *Linear Multilinear Algebra* 62 (2014), no. 2, 242–251. MR 3175412. Zbl 1297.05144. arXiv:1207.6765.

See Y.-Z. Fan, Y. Wang, and Y. Wang (2013a). (**SG: Eig**)

Yi-Zheng Fan & Shaun Fallat

2012a Edge bipartiteness and signless Laplacian spread of graphs. *Appl. Anal. Discrete Math.* 6 (2012), no. 1, 31–45. MR 2952601. Zbl 1289.05281.

Cf. M.H. Liu and Liu (2010a), Oliveira, de Lima, de Abreu, and Kirkland (2010a). Weak relations between $l(\Sigma)$ and spread of $L(\Sigma)$. Min spread of $L(\Sigma)$ is $2 + 2\cos(\pi/n)$, attained only by a path and $-C_{\text{odd}}$. Next smallest spread = 4, attained only by $-K_4$, $-\Gamma$ consisting of two triangles joined by an edge, $K_{1,3}$, C_{even}. Proofs by cases: $l(\Sigma) \leq 1$

or ≥ 2 (min spread is from $-K_4$). Dictionary: "edge bipartiteness" = frustration index $l(-\Gamma)$; "mixed graph" = (oriented) signed graph with reversed signs (oriented edges are called negative); Laplacian matrix of mixed graph $G = D(|\Sigma|) - A(\Sigma)$. [See Desai and Rao (1994a) for another eigenvalue connection with $l(\Sigma)$.] [Annot. 29 Dec 2012.]
(**Par: Eig, Fr, Cov**)

Yi-Zheng Fan, Shi-Cai Gong, Yi Wang, & Yu-Bin Gao

2009a First eigenvalue and first eigenvectors of a nonsingular unicyclic mixed graph. *Discrete Math.* 309 (2009), no. 8, 2479–2487. MR 2512565 (2010g:05212). Zbl 1182.05081.

The "mixed graphs" are signed graphs with reversed signs; see Fan (2003a). Graphs are simple. Eigenvalues are those of $L(\Sigma)$. (**sg: Eig**)

Yi-Zheng Fan, Shi-Cai Gong, Jun Zhou, Ying-Ying Tan, & Yi Wang

2007a Nonsingular mixed graphs with few eigenvalues greater than two. *European J. Combin.* 28 (2007), no. 6, 1694–1702. MR 2339495 (2008f:05115). Zbl 1122.05058.

The "mixed graphs" are signed graphs with reversed signs; see Fan (2003a). Assume Σ is connected. $m :=$ number of eigenvalues > 2. Thm. 2.2: $d :=$ longest path length, $\mu :=$ matching number. (i) $m \geq \lfloor d/2 \rfloor$, (ii) $m \geq \mu$ if $n > 2\mu$, (iii) $m \geq \mu - 1$ if $n = 2\mu$. Now assume Σ is unbalanced. Thm. 3.4. If $n \geq 7$, then $m = 2$ iff $|\Sigma|$ is one of two general types and Σ has a certain negative triangle. Thm. 3.5. If $n \geq 6$, then $m = 1$ iff $\Sigma \sim -K_4$ or an unbalanced subgraph. Dictionary: See Bapat, Grossman, and Kulkarni (1999a). [Annot. 13 Jan 2012.] (**sg: Eig**)

Yi-Zheng Fan, Hai-Yan Hong, Shi-Cai Gong, & Yi Wang

2007a Order unicyclic mixed graphs by spectral radius. *Australas. J. Combin.* 37 (2007), 305–316. MR 2284395 (2007j:05137). Zbl 1122.05059.

Finds the unicyclic signed graphs with first, second, and third largest spectral radii. The "mixed graphs" are signed graphs with reversed signs; see Fan (2003a). Graphs are simple. The eigenvalues are those of $L(\Sigma)$. [Annot. 9 Jan 2013.] (**sg: Eig**)

Yi-Zheng Fan, Bit-Shun Tam, & Jun Zhou

2008a Maximizing spectral radius of unoriented Laplacian matrix over bicyclic graphs of a given order. *Linear Multilinear Algebra* 56 (2008), no. 4, 381–397. MR 2434109 (2009e:15070). Zbl 1146.05032.

The maximal graphs are $K_4 \setminus e$ with $n-4$ pendant edges at one trivalent vertex. [Annot. 9 Sept 2010.] (**par: Incid, Eig**)

Yi-Zheng Fan, Yue Wang, & Yi Wang

2013a A note on the nullity of unicyclic signed graphs. *Linear Algebra Appl.* 437 (2013), no. 3, 1193–1200. MR 2997803. Zbl 1257.05083. arXiv:1107.0400.

For rk $A(\Sigma) \geq 2$, the cases where rk ≤ 3 are characterized. *Cf.* Y.-Z. Fan, W.-X. Du, and C.-L. Dong (2014a), X.-Z. Tan and B.-L. Liu (2006a). [Annot. 17 Dec 2011.] (**SG: Adj**)

Yi-Zheng Fan & Dan Yang

2009a The signless Laplacian spectral radius of graphs with given number of pendant vertices. *Graphs Combin.* 25 (2009), no. 3, 291–298. MR 2534887 (2010j:05233).

Zbl 1194.05085. **(par: Lap: Eig)**

E. Fanchon
See J. Aracena.

Mohammad Reza Farahani
See M.R. Rajesh Kanna.

Thomas J. Fararo
See N.P. Hummon.

Miriam Farber
See A. Berman.

Luerbio Faria, Sulamita Klein, & Matěj Stehlík

2012a Odd cycle transversals and independent sets in fullerene graphs. *SIAM J. Discrete Math.* 26 (2012), no. 3, 1458–1469. MR 3022147. Zbl 1256.05116.

If Γ is a fullerene graph (cubic, plane, no isthmi, all faces are pentagons and hexagons), $l(-\Gamma) \leq \sqrt{12n/5}$. Dictionary: "odd cycle transversal" = balancing set of $-\Gamma$. [Annot. 1 Oct 2012.] **(sg: par: Fr)**

Arthur M. Farley & Andrzey Proskurowski

1981a Computing the line index of balance of signed outerplanar graphs. Proc. Twelfth Southeastern Conf. on Combinatorics, Graph Theory and Computing (Baton Rouge, 1981), Vol. I. *Congressus Numer.* 32 (1981), 323–332. MR 0681891 (83m:68119). Zbl 489.68065.

Calculating frustration index is NP-complete, since it is more general than max-cut. However, for signed outerplanar graphs with bounded size of bounded faces, it is solvable in linear time. [It is quickly solvable for signed planar graphs. See Katai and Iwai (1978a), Barahona (1981a), (1982a), and more.] **(SG: Fr)**

Rashid Farooq
See also S. Hafeez and M. Khan.

Rashid Farooq, Mehtab Khan, & Sarah Chand

2019a On iota energy of signed digraphs. *Linear Multilinear Algebra* 67 (2019), no. 4, 705–724. MR 3914326. Zbl 1411.05138. **(SD: Adj: Eig)**

Rashid Farooq, Sarah Chand, & Mehtab Khan

2019a Iota energy of bicyclic signed digraphs. *Asian-European J. Math.* 12 (2019), no. 5, art. 1950078, 14 pp. MR 3999110. Zbl 1419.05123.

Cf. Hafeez and Khan (2018a). **(SD: Adj: Eig)**

M. Farzan

1978a Automorphisms of double covers of a graph. In: *Problemes Combinatoires et Theorie des Graphes* (Colloq. Int., Orsay, 1976), pp. 137–138. Colloques Int. du CNRS, 260. Editions du C.N.R.S., Paris, 1978. MR 0539960 (81a:05063). Zbl 413.05064.

A "double cover of a graph" means the double cover of a signing of a simple graph. **(sg: Cov, Aut)**

Gholam Hossein Fath-Tabar
See E. Ghasemian and F. Taghvaee.

R.J. Faudree

See also P. Erdős.

Ralph J. Faudree, Evelyne Flandrin, Michael S. Jacobson, Jenő Lehel, & Richard H. Schelp

2000a Even cycles in graphs with many odd cycles. *Graphs Combin.* 16 (2000), 399–410. MR 1804339 (2001k:05117). Zbl 984.05048. (**Par: Cyc**)

Katherine Faust

See also S. Wasserman.

2007a Very local structure in social networks. *Sociological Methodology* 37 (2007), Ch. 7, pp. 209–256.

A documented warning that properties of triads (vertex triples) in a graph or digraph tend to be heavily dependent on monad (single-vertex) or dyad (vertex pair) properties such as the density of edges or the degree distribution and therefore must be evaluated in comparison to expected triad properties given the distribution of dyad types. The focus is on digraphs; pp. 9–10 mention structural balance, i.e., signed-graph models. [*Problem* 1. Carry out a similar analysis for signed graphs, and in particular, signed complete graphs (equivalent to graphs). *Problem* 2. The same, for switching classes of the preceding, in which the meaning of a dyad census is unclear.] [Annot. 22 Aug 2014.] (**PsS**)

Siamak Fayyaz Shahandashti, Mahmoud Salmasizadeh, & Javad Mohajeri

2005a A provably secure short transitive signature scheme from bilinear group pairs. In: C. Blundo and S. Cimato, eds., *Security in Communication Networks* (4th Int. Conf., SCN 2004, Amalfi), pp. 60–76. Lect. Notes in Computer Sci., Vol. 3352. Springer-Verlag, Berlin, 2005. Zbl 1116.94320.

Edges have "signatures" for encryption. No edge signs! [Irresistible.] [Annot. 5 Mar 2011.] (**None**)

N.T. Feather

1971a Organization and discrepancy in cognitive structures. *Psychological Rev.* 78 (1971), 355–379.

A suggestion for defining balance in weighted digraphs: pp. 367–369. (**PsS: Bal: Exp**)(**WD: Bal**)

Tomás Feder

See J. Bok.

Martin Feinberg

See G. Craciun and G. Shinar.

Anna Felikson

See M.D. Sikirić.

Mariusz Felisiak

See also R. Bocian.

2013a Computer algebra technique for Coxeter spectral study of edge-bipartite graphs and matrix morsifications of Dynkin type $\mathbb{A}_n$. *Fundamenta Inform.* 125 (2013), no. 1, 21–49. MR 3114057. Zbl 1277.16040. (**SG**)

Mariusz Felisiak & Daniel Simson

2015a Applications of matrix morsifications to Coxeter spectral study of loop-free edge-bipartite graphs. *Discrete Appl. Math.* 192 (2015), 49–64. MR 3354818.

Zbl 1319.05142. (**SG**)

Michael R. Fellows

See H.L. Bodlaender.

Stefan Felsner & Kolja Knauer

2011a Distributive lattices, polyhedra, and generalized flows. *European J. Combin.* 32 (2011), 45–59. MR 2727459 (2012a:52022). Zbl 1205.06007.

“Generalized flows” are flows (conservative at each vertex, i.e., real 1-cycles) on a graph with positive real gains (“generalized network”). [Annot. 2 Apr 2013.] (**GG: GN: Incid**)

Paul Fendley & Vyacheslav Krushkal

2010a Link invariants, the chromatic polynomial and the Potts model. *Adv. Theor. Math. Phys.* 14 (2010), no. 2, 507–540. MR 2721654 (2011k:57015). Zbl 1207.82007. arXiv:0806.3484.

The Potts model treats a graph as all negative (“antiferromagnetic”; see the low-temperature expansion in §3). [Annot. 12 Jan 2012.] (**par: Invar**)

Derek Feng, Randolf Altmeyer, Derek Stafford, Nicholas A. Christakis, & Harrison H. Zhou

2022a Testing for balance in social networks. *J. Amer. Statist. Assoc.* 117 (2022), no. 537, 156–174. MR 4399076. arXiv:1808.05260.

A new, statistical model for measuring imbalance using triangles. [Hidden assumption: triangles largely determine the graph. Examples may not satisfy this.] [Annot. 19 Dec 2020.] (**SG: KG: PsS, Fr**)

Gang Feng

See Y.-Z. Wu.

Lihua Feng

See also L. Lu and G.H. Yu.

2010a The signless Laplacian spectral radius for bicyclic graphs with k pendant vertices. *Kyungpook Math. J.* 50 (2010), no. 1, 109–116. MR 2609079 (2011d:05221). Zbl 1205.05140. (**par: Lap: Eig**)

Lihua Feng & Guihai Yu

2009a On three conjectures involving the signless Laplacian spectral radius of graphs. *Publ. Inst. Math. (Beograd) (N.S.)* 85(99) (2009), 35–38. MR 2536687 (2010i:05204). Zbl 1265.05365. (**par: Lap: Eig**)

2009b The signless Laplacian spectral radius of unicyclic graphs with graph constraints. *Kyungpook Math. J.* 49 (2009), no. 1, 123–131. MR 2527378 (2011b:05148). Zbl 1201.05056. (**par: Lap: Eig**)

2010a The signless Laplacian spectral radius of graphs with given diameter. *Utilitas Math.* 83 (2010), 265–276. MR 2742294 (2011i:05129). Zbl 1242.05162.

The graphs with maximum spectral radius. [Annot. 19 Nov 2011.] (**par: Lap: Eig**)

Lihua Feng, Guihai Yu, & Aleksandar Ilić

2010a The Laplacian spectral radius for unicyclic graphs with given independence number. *Linear Algebra Appl.* 433 (2010), 934–944. MR 2658644 (2011f:05175).

Zbl 1215.05102. **(par: Lap: Eig)**

Lihua Feng, Guihai Yu, Aleksandar Ilić, & Dragan Stevanović

2013a The signless Laplacian spectral radius of graphs on surfaces. *Linear Multilinear Algebra* 61 (2013), no. 5, 573–581. MR 3005638. Zbl 1273.05134. **(sg: par: Lap: Eig, Top)**

Lin Feng, Yan Hong Yao, Ji Ming Guo, & Shang Wang Tan

2011a [On] The signless Laplacian spectral radius of unicyclic graphs with fixed girth. (In Chinese.) *Appl. Math. J. Chinese Univ. Ser. A* 26 (2011), no. 1, 121–126. MR 2807616 (no rev). Zbl 1240.05190. **(par: Lap: Eig)**

Shasha Feng, Li Wang, Yijia Li, Shiwen Sun, & Chengyi Xia

2018a A nonlinear merging protocol for consensus in multi-agent systems on signed and weighted graphs. *Physica A* 490 (2018), 653–663. MR 3716395 (no rev). **(SG, WG: Algor)**

Zouhaier Ferchiou & Bertrand Guenin

2020a A short proof of Shih's isomorphism theorem on graphic subspaces. *Combinatorica* 40 (2020), no. 6, 805–837. MR 4208098. Zbl 1474.05169.

Cf. C.-S. Shih (1982a). **(sg: matrd)**

Anuška Ferligoj

See P. Doreian.

Lori Fern [Lori Koban]

See also L. Koban.

Lori Fern, Gary Gordon, Jason Leasure, & Sharon Pronchik

2000a Matroid automorphisms and symmetry groups. *Combin. Probab. Comput.* 9 (2000), 105–123. MR 1762784 (2001g:05034). Zbl 960.05055.

Consider a subgroup W of the hyperoctahedral group Oc_n that is generated by reflections. Let $M(W)$ be the vector matroid of the vectors corresponding to reflections in W. The possible direct factors of any automorphism group of $M(W)$ are S_k, Oc_k, and Oc_k^+. The proof is stricly combinatorial, via signed graphs. **(SG: Matrd: Aut, Geom)**

Rosário Fernandes

2010a Location of the eigenvalues of weighted graphs with a cut edge. *Linear Multilinear Algebra* 58 (2010), no. 3, 305–322. MR 2663432 (2011d:15014). Zbl 1203.05092.

The "weights" are skew gains [*cf.* J. Hage (1999a) *et al.*] in $\mathbb{C}^\times$; the anti-involution is conjugation. Identities satisfied by the eigenvalues. [Annot. 11 Jan 2012.] **(gg(Gen): Gen: Eig)**

L.A. Fernández, V. Martin-Mayor, G. Parisi, & B. Seoane

2010a Spin glasses on the hypercube. *Phys. Rev. B* 81 (2010), #134403, 14 pp. arXiv:0911.4667.

Average behavior of random signed subhypercubes (Γ, σ), with spanning $\Gamma \subseteq Q_D$, with random spins $\zeta : V \to \{+1, -1\}$. Each (Γ, σ, ζ) is a "sample". To avoid irregularities Γ is z-regular ("connectivity z") for a fixed z (here, 6). [Annot. 19 Jun 2012.] **(Phys, SG: State)**

Daniela Ferrero

2008a Product line sigraphs. In: *The International Symposium on Parallel Architectures, Algorithms, and Networks* (i-span 2008), pp. 141–145. IEEE Computer Soc., 2008.

The product line graph [= $\Lambda_\times(\Sigma)$ in M. Acharya (2009a)] is balanced. [Immediate from Harary's (1953a) balance theorem or Sampathkumar's (1972a), (1984a) similar theorem.] [Annot. 2008, 20 Dec 2010.]

(**SG: LG, Bal**)

A. Fiat

See E. Demaine and D. Emanuel.

Miroslav Fiedler

1957a Uber qualitative Winkeleigenschaften der Simplexe. *Czechoslovak Math. J.* 7(82) (1957), 463–478. MR 0094740 (20 #1252). Zbl 093.33602 (93, p. 336b). (**SG: Geom**)

1957b Einige Satze aus der metrischen Geometrie der Simplexe in euklidischen Raumen. *Schr. Forschungsinst. Math.* 1 (1957), 157. MR 0087110 (19, 303d). Zbl 089.16706 (89, p. 167f). (**SG: Geom**)

1961a Uber die qualitative Lage des Mittelpunktes der ungeschriebenen Hyperkugel im n-Simplex [On the qualitative location of the center of circumscribed hyperspheres in the n-simplex]. *Comment. Math. Univ. Carolinae* 2 (1961), no. 1, 3–51. Zbl 101.13205 (101, p. 132e). (**SG: Geom**)

1964a Some applications of the theory of graphs in matrix theory and geometry. In: *Theory of Graphs and Its Applications* (Proc. Sympos., Smolenice, 1963), pp. 37–41. Publ. House Czechoslovak Acad. Sci., Prague, 1964. MR 0175109 (30 #5294). Zbl 163.45605 (163, 456e). (**SG: Geom**)

1967a Graphs and linear algebra. In: M. Fiedler, ed., *Theory of Graphs: International Symposium* (Rome, 1966), pp. 131–134. Gordon and Breach, New York; Dunod, Paris, 1967. MR 0223265 (36 #6313). Zbl 263.05124. (**SG: Geom**)

1969a Signed distance graphs. *J. Combin. Theory* 7 (1969), 136–149. MR 0242705 (39 #4034). Zbl 181.26001 (181, p. 260a). (**SG: Geom**)

1970a Poznámka o distancnich grafech [A remark on distance graphs] (in Czech). In: *Matematika (geometrie a teorie grafu)* [Mathematics (Geometry and Graph Theory)], pp. 85–88. Univ. Karlova, Prague, 1970. MR 0277410 (43 #3143). Zbl 215.50203. (**SG: Geom**)

1975a Eigenvectors of acyclic matrices. *Czechoslovak Math. J.* 25(100) (1975), 607–618. MR 0387308 (52 #8151). Zbl 325.15014. (**sg: Trees: Eig**)

1985a Signed bigraphs of monotone matrices. In: Horst Sachs, ed., *Graphs, Hypergraphs and Applications* (Proc. Int. Conf., Eyba, 1984), pp. 36–40. Teubner-Texte zur Math., B. 73. B.G. Teubner, Leipzig, 1985. MR 0869435 (87m:05121). Zbl 626.05023. (**SG: Eig: Exp**)

1993a A geometric approach to the Laplacian matrix of a graph. In: Richard A. Brualdi, Shmuel Friedland, and Victor Klee, eds., *Combinatorial and Graph-Theoretical Problems in Linear Algebra*, pp. 73–98. IMA Vols. Math. Appl., 50. Springer-Verlag, New York, 1993. MR 1240957 (94g:05055). Zbl 791.05073.

The signed bipartite graph of a normalized Gram matrix (pp. 85–86). This is applied to study the types of angles in a geometric simplex. Dictionary: $\Gamma(A)$ = the signed bipartite graph of a symmetric real matrix. (**SG**)

1998a Additive compound graphs. *Discrete Math.* 187 (1998), 97–108. MR 1630684 (99c:05131). Zbl 958.05091.

If $V(\Sigma) = [n]$, the k-th additive compound graph $\Sigma^{[k]}$ is defined for $k \in [n-1]$ via $A(\Sigma)$. It respects connection, also edge-disjoint union and induced subgraphs. Spec $\Sigma^{[k]} = \{$sums of k eigenvalues of $\Sigma\}$. Spectral radius: $\rho(\Sigma) \leq \rho(|\Sigma|)$ (Thm. 2.17). For a path, $\rho(+P_n^{[k]}) = -1 + \sin\frac{2k+1}{n+1}\pi / \sin\frac{1}{n+1}\pi$. $\Sigma^{[n-k]}$ is $(-\Sigma^{\zeta})^{[k]}$ where ζ switches odd-numbered vertices (Thm. 2.10, Rem. 2.11). $(+\Gamma)^{[k]}$ may be unbalanced, e.g. (Thm. 2.15, proof) $(+C_n)^{[2]}$ and $(+K_{1,3})^{[2]}$; however, $(\tilde{C}_n)^{[2]}$ is all-positive ($\tilde{C}_n$ = negative circle). Thm. 2.15: For connected Γ, $(+\Gamma)^{[2]}$ is balanced iff $\Gamma = P_n$. [Annot. 26 Jul 2013.] (**SG: LG(Gen): Adj, Sw**)

Miroslav Fiedler & Vlastimil Ptak

1967a Diagonally dominant matrices. *Czechoslovak Math. J.* 17(92) (1967), 420–433. MR 0215869 (35 #6704). Zbl 178.03402 (178, p. 034b). (**GG: Sw, bal**)

1969a Cyclic products and an inequality for determinants. *Czechoslovak Math. J.* 19(94) (1969), 428–451. MR 0215869 (40 #1409). Zbl 281.15014. (**gg: Sw**)

B. Fierro, F. Bachmann, & E.E. Vogel

2006a Phase transition in 2D and 3D Ising model by time-series analysis. *Physica B* 384 (2006), 215–217.

Physical parameters calculated on a signed square lattice ("Edwards–Anderson (1975a) model") with 1/32 of edges negative (p. 217). [Annot. 10 Jan 2015.] (**Phys: SG**)

Tara Fife, Dillon Mayhew, James Oxley, & Charles Semple

2020a The unbreakable frame matroids. *SIAM J. Discrete Math.* 34 (2020), no. 3, 1522–1537. MR 4121881. Zbl 1456.05028. (**GG: Matrd: Str**)

Rosa M.V. Figueiredo

See also N. Arınık, M. Levorato, I. Mendonça, and E. Queiroga.

Rosa Figueiredo & Yuri Frota

2013a An improved Branch-and-cut code for the maximum balanced subgraph of a signed graph. Manuscript, 2013. arXiv:1312.4345. HAL hal-02188356.

Max order of balanced induced subgraph. [Annot. 30 Mar 2021.] (**SG: Clu: Algor**)

2014a The maximum balanced subgraph of a signed graph: Applications and solution approaches. *European J. Oper. Res.* 236 (2014), 473–487. MR 3179876. Zbl 1317.90305. HAL hal-02181486. (**VS: Fr: Algor**)

Rosa Figueiredo, Yuri Frota, & Martine Labbé

2013a Solution of the maximum k-balanced subgraph problem. In: Giuseppe Nicosia and Panos Pardalos, eds., *Learning and Intelligent Optimization* (Proc. 7th Int.

Conf. Learning and Optimization, LION 7, Catania, Italy, 2013), pp. 266–271. Lect. Notes in Computer Sci., Vol. 7997. Springer, Berlin, 2013.

Preliminary version of (2019a). **(SG: Clu: Algor)**

2019a A branch-and-cut algorithm for the maximum k-balanced subgraph of a signed graph. *Discrete Appl. Math.* 261 (2019), 164–185. MR 3958238. Zbl 1410.05200. HAL hal-01937015. **(SG: Clu: Algor)**

Rosa M.V. Figueiredo, Martine Labbé, & Cid C. de Souza

2011a An exact approach to the problem of extracting an embedded network matrix. *Computers Oper. Res.* 38 (2011), no. 11, 1483–1492. MR 2781542 (2012f:90223) (*q.v.*). Zbl 1210.90038. **(SG: Incid: Algor)**

Rosa Figueiredo & Gisele Moura

2013a Mixed integer programming formulations for clustering problems related to structural balance. *Social Networks* 35 (2013), no. 4, 639–651. HAL hal-02171111. **(SG: Clu: Algor, Appl)**

Joseph Fiksel

1980a Dynamic evolution in societal networks. *J. Math. Sociology* 7 (1980), 27–46. MR 0572489 (81g:92023) (*q.v.*). Zbl 434.92022. **(SG: Clu, VS)**

Miguel Angel Fiol

See C. Dalf'o and E.R. van Dam.

Samuel Fiorini

See also N.E. Clarke and M. Conforti.

Samuel Fiorini, Nadia Hardy, Bruce Reed, & Adrian Vetta

2005a Approximate min-max relations for odd cycles in planar graphs. In: M. Jünger and V. Kaibel, eds., *Integer Programming and Combinatorial Optimization* (11th Int. IPCO Conf., IPCO 2005, Berlin), pp. 35–50. Lect. Notes in Computer Sci., Vol. 3509. Springer, Berlin, 2005. MR 2210011 (2006j:90108). Zbl 1119.90360.

See (2007a). **(SG: Fr, Circ)**

2007a Approximate min-max relations for odd cycles in planar graphs. *Math. Programming, Ser. B* 110 (2007), no. 1, 71–91. MR 2306131 (2008b:05087). Zbl 1113.05054.

$\nu :=$ max number of vertex-disjoint negative circles; $\nu' :=$ edge analog. $\rho :=$ min size of a transversal of negative face boundaries. Thm. 3 (Král and Voss (2004a)): frustration index $l(\Sigma) \leq 2\nu'$. (Here, a shorter proof.) Thm. 4: For an unbalanced signed plane graph, vertex frustration number $l_0(\Sigma) \leq 7\nu(\Sigma) + 3\rho(\Sigma) - 8$. [Improved by Král', Sereni, and Stacho (2012a).] Cor. 2: $l(\Sigma) \leq 10\nu(\Sigma)$. Dictionary: "odd" = negative, "even" = positive. [Annot. 6 Feb 2011.] **(SG: Fr, Circ)**

Samuel Fiorini & Gwenaël Joret

2009a On a theorem of Sewell and Trotter. *European J. Combin.* 30 (2009), 425–428. MR 2489274 (2010b:05131). Zbl 1229.05142.

Short proof of Sewell and Trotter (1993a). Dictionary: "totally odd K_4-subdivision" = "even subdivision of K_4" (Sewell and Trotter (1993a)). [Annot. 14 Feb 2013.] **(sg: par: Str)**

E. Fischer, J.A. Makowsky, & E.V. Ravve

2008a Counting truth assignments of formulas of bounded tree-width or clique-width. *Discrete Appl. Math.* 156 (2008), no. 4, 511–529. MR 2379082 (2009k:68090). Zbl 1131.68093.

The incidence graph of clauses is a signed bipartite graph. [Annot. 16 Jan 2012.] (**SG**)

Ilse Fischer & C.H.C. Little

2004a Even circuits of prescribed clockwise parity. *Electronic J. Combin.* 10 (2003), Research Paper 45, 20 pp. MR 2014532 (2004h:05071). Zbl 1031.05073. (**SG**)

K.H. Fischer & J.A. Hertz

1991a *Spin Glasses.* Cambridge Studies in Magnetism, Vol. 1. Cambridge Univ. Press, Cambridge, Eng., 1991. MR (93m:82019) (93m:82019).

An excellent introduction to many aspects of physics (mainly theoretical) that often seem to be signed graph theory or to generalize it, e.g., by randomly weighting the edges. (**Phys: sg: fr: Exp, Ref**)

§2.5, "Frustration", discusses the spin glass Ising model (essentially, signed graphs) in square and cubical lattices, including the "Mattis model" (a switching of all positive signs), as well as a vector analog, the "XY" model (planar spins) and (p. 46) even a general gain-graph model with switching-invariant Hamiltonian. (**Phys: SG: Fr, Sw: Exp, Ref**)

Ch. 3 concerns the Ising and Potts models. In §3.7: "The Potts glass", the Hamiltonian (without edge weights) is $H = -\frac{1}{2}\sum \sigma(e_{ij})(k\delta(s_i, s_j) - 1)$. [It is not clear that the authors intend to permit negative edges. If they are allowed, H is rather like Doreian and Mrvar's (1996a) $P(\pi)$. *Question.* Is there a worthwhile generalized signed and weighted Potts model with Hamiltonian that specializes both to this form of H and to P?] [Also *cf.* Welsh (1993a) on the Ashkin–Teller–Potts model.]
(**Phys: sg, clu: Exp**)

Steven D. Fischer

1993a *Signed Poset Homology and q-Analog Möbius Functions.* Doctoral thesis, University of Michigan, 1993.

§1.2: "Signed posets". Definition of signed poset: a positively closed subset of the root system B_n whose intersection with its negative is empty. (Following Reiner (1990).) Equivalent to a partial ordering of $\pm[n]$ in which negation is a self-duality and each dual pair of elements is comparable. [This is really a special type of signed poset. The latter restriction does not hold in general.]

Relevant contents: Ch. 2: "Cohen-Macaulay signed posets", §2.2: "EL-labelings of posets and signed posets", and shellability. Ch. 3: "Euler characteristics", and a fixed-point theorem. §5.1: "The homology of the signed posets S_Π" (a particular example). App. A: "Open problems", several concerning signed posets.

[Partially summarized by Hanlon (1996a).]
(**Sgnd: sg, ori, Geom, Invar**)

M. Hamit Fişek, Robert Z. Norman, & Max Nelson-Kilger

1992a Status characteristics and expectation states theory: A priori model parameters and test. *J. Math. Sociology* 16 (1992), no. 4, 285–303. Zbl 741.92024.

The "strength" of a path depends on the number of edges of each sign. [Annot. 10 Nov 2012.] (**PsS: SG**)

P.C. Fishburn & N.J.A. Sloane

1989a The solution to Berlekamp's switching game. *Discrete Math.* 74 (1989), 263–290. MR 0992740 (90e:90151). Zbl 664.94024.

The maximum frustration index of a signed $K_{t,t}$, which equals the covering radius of the Gale–Berlekamp code, is evaluated for $t \le 10$, thereby extending results of Brown and Spencer (1971a). See Table 1. [Corrected and extended by Carlson and Stolarski (2004a).] (**sg: Fr**)

Susanna Fishel

2019a A survey of the Shi arrangement. In: Hélène Barcelo *et al.*, eds., *Recent Trends in Algebraic Combinatorics*, pp. 75–113. Assoc. Women Math. Ser., Vol. 16. Springer, Cham, 2019. MR 3969572. Zbl 1417.52033. arXiv:1909.01257.

Much geometry, no gain graphs [though a gain graph is implicit; *cf.* e.g. Zaslavsky (2003a)]. [Annot. 22 Jan 2020.] (**Geom: gg: Invar**)

Michael E. Fisher & Rajiv R.P. Singh

1990a Critical points, large-dimensionality expansions, and the Ising spin glass. In: G.R. Grimmett and D.J.A. Welsh, eds., *Disorder in Physical Systems: A Volume in Honour of John M. Hammersley on the Occasion of His 70th Birthday*, pp. 87–111. Clarendon Press, Oxford, 1990. MR 1064557 (91j:82021). Zbl 725.60111.

Physics questions, e.g., phase transitions and high-temperature expansions, for signed lattice graphs ($\pm J$ spins) and with random weights (Gaussian edge weights). [Annot. 24 Aug 2012.] (**sg: Phys: Fr: Exp**)

Claude Flament

1958a L'étude mathématique des structures psycho-sociales. *L'Année Psychologique* 58 (1958), 119–131.

Signed graphs are treated on pp. 126–129. (**SG: Bal, PsS: Exp**)

1963a *Applications of Graph Theory to Group Structure.* Prentice-Hall, Englewood Cliffs, N.J., 1963. MR 0157785 (28 #1014). Zbl 141.36301 (141, p. 363a).

English edition of (1965a). Ch. 3: "Balancing processes." (**SG: KG: Bal, Algor: Exp**)

1965a *Théorie des graphes et structures sociales.* Math. et sci. de l'homme, Vol. 2. Mouton and Gauthier-Villars, Paris, 1965. MR 0221966 (36 #5018). Zbl 169.26603 (169, p. 266c).

Ch. III: "Processus d'équilibration." (**SG: KG: Bal, Algor: Exp**)

1970a Équilibre d'un graphe, quelques résultats algébriques. *Math. Sci. Humaines*, No. 30 (1970), 5–10. MR 0278978 (43 #4704). Zbl 222.05124. (**Bal**)

1979a Independent generalizations of balance. In: Paul W. Holland and Samuel Leinhardt, eds., *Perspectives on Social Network Research* (Proc. Sympos., Dartmouth Coll., Hanover, N.H., 1975), Ch. 10, pp. 187–200. Academic Press, New York, 1979. (**SG: Bal, PsS: Exp**)

Evelyne Flandrin

See R.J. Faudree.

Erica Flapan

1995a Intrinsic chirality. *J. Molecular Structure (Theochem)* 336 (1995), 157–164.

Intrinsic chirality means the graph cannot be embedded in 3-space without a twist. [*Question.* Can this be interpreted in terms of signed graphs?] *Cf.* (1998a), Flapan and Weaver (1996a), Hu and Qiu (2009a). [Annot. 4 Nov 2010.] (**sg: Top: Chem**)

1998a Knots and graphs in chemistry. *Chaos, Solitons & Fractals* 9 (415) (1998), 547–560. MR 1628741 (99c:57017). Zbl 933.57002.

A survey of chirality of 3-space embeddings. See (1995a). [Annot. 4 Nov 2010.] (**sg: Top: Chem: Exp**)

Erica Flapan & Nikolai Weaver

1996a Intrinsic chirality of 3-connected graphs. *J. Combinatorial Theory Ser. B* 68 (1996), 223–232. MR 1417798 (97k:05058). Zbl 861.05023.

See Flapan (1995a). [Annot. 4 Nov 2010.] (**sg: Top**)

T. Fleiner & G. Wiener

† 2016a Coloring signed graphs using DFS. *Optimization Lett.* 10 (2016), no. 4, 865–869. MR 3477382. Zbl 1336.05055. arXiv:1612.03280.

An elegant short proof by depth-first search of the signed-graph Brooks' theorem of Máčajová, Raspaud, and Škoviera (2016a). [More general short proof by Zając (20xxa).] [Annot. 17 Mar, 23 May 2017, rev 9 Feb 2020.] (**SG: Col: Algor**)

Herbert Fleischner

1991a *Eulerian Graphs and Related Topics.* Part 1, Vol. 2. Ann. Discrete Math., Vol. 50. North-Holland, Amsterdam, 1991. MR 1113484 (92f:05066). Zbl 792.05092. (**SG: Ori, Flows**)

Laura Floresc

See J. Spencer.

Rigoberto Flórez

See also M. Blair.

2005a *Four Studies in the Geometry of Biased Graphs.* Doctoral dissertation, State University of New York at Binghamton, 2005. MR 2707450.

Published as (2006a), (2009a), and Flórez and Forge (2007a), and in Flórez and Zaslavsky (2020a). (**GG: Matrd, Geom**)

2006a Lindström's conjecture on a class of algebraically non-representable matroids. *European J. Combin.* 27 (2006), no. 6, 896–905. MR 2226425 (2006m:05048). Zbl 1090.05010.

Lindström conjectured that a certain matroid $M(n)$ is algebraically nonrepresentable if n is nonprime. Proved by showing that $M(n)$ extends by harmonic conjugation to $\mathbf{L}_\infty(\mathbb{Z}_n K_3)$, which in turn extends to a contradiction if n is composite. (**gg: Matrd**)

2009a Harmonic conjugation in harmonic matroids. *Discrete Math.* 309 (2009), no. 8, 2365–2372. MR 2510362 (2010f:05038). Zbl 1207.05027.

In a harmonic matroid H, harmonic conjugates exist and are unique. If $\mathbf{L}_\infty(\mathfrak{G}K_3) \subseteq H$ and $\mathfrak{G} = \mathbb{Z}$ or $\mathbb{Z}_p$, then the closure of $\mathbf{L}_\infty$ under harmonic

conjugation is a projective plane over $\mathbb{Q}$ or $\mathrm{GF}(p)$, as appropriate.
(**gg: Matrd**)

Rigoberto Flórez & David Forge

2007a Minimal non-orientable matroids in a projective plane. *J. Combin. Theory Ser. A* 114 (2007), no. 1, 175–183. MR 2276967 (2007h:05031). Zbl 1120.52012.
The minimal matroids are contained in lift matroids of $\mathbb{Z}_n K_3$.
(**gg: Matrd**)

2023a Activity from matroids to rooted trees and beyond. *J. Combin. Theory Ser. A* 198 (2023), art. 104755, 21 pp. MR 4574103. arXiv:2209.03446.
(**Matrd: Invar: GG**)

Rigoberto Flórez & Thomas Zaslavsky

2019a Biased graphs. VI. Synthetic geometry. *European J. Combin.* 81 (2019), 119–141. MR 3953388. Zbl 1420.05030. arXiv:1608.06021. (**GG: Matrd, Geom**)

2020a Projective planarity of matroids of 3-nets and biased graphs. *Australasian J. Combin.* 76 (2020), no. 2, 299–338. MR 4055697. Zbl 1439.05046. arXiv:-1708.00095. (**GG: Matrd, Geom**)

20xxa Projective rectangles. Submitted. arXiv:2307.04079.
A projective rectangle is like a stripe in a projective plane. Examples arise by harmonic conjugation from extended lift matroids $\mathbf{L}_\infty(\mathbb{Z}_p{\cdot}K_3)$. [Annot. 13 Jul 2023.] (**GG: Geom, Matrd**)

Joel Foisy

See also Y. Duong.

20xxa A spatial version of Tutte's conflict graph. Submitted. arXiv:2207.06244.
Simplified description: Embed Γ in $\mathbb{R}^3$ with an S^2 that contains a maximal planar subgraph M. The conflict graph is Σ_M with $V(\Sigma_M) = E(\Gamma) \setminus E(M)$. $e, f \in V(\Sigma_M)$ may be obliged by certain criteria to be on the same side, or opposite sides, of S^2; then join them in Σ_M by a positive, or negative, edge. Thm.: If every conflict graph is unbalanced, Γ cannot be linklessly embedded in S^3. There are complications. [Annot. 19 Jul 2022.] (**SG: Top**)

Joel Foisy & Justin Raimondi

20xxa Conflict graphs of maximally planar subgraphs of Petersen family graphs. Submitted. arXiv:2207.06251.
Technical propositions about maximal planar subgraphs and their signed conflict graphs that are used in Foisy (20xxa). [Annot. 19 Jul 2022.] (**Top, SG**)

Wungkum Fong

2000a *Triangulations and Combinatorial Properties of Convex Polytopes*. Doctoral dissertation, Massachusetts Inst. of Technology, 2000.
A configuration consists of the vectors representing an acyclic orientation of a complete signed graph. The volume of the pyramid over the configuration with apex at the origin. [Ohsugi and Hibi (2003a) treats a similar problem. *Question*. Is there a connection with the chromatic

polynomial?] [Annot. 11 Apr 2011.] (**sg: Geom: Invar**)

Carlos M. da Fonseca

See A. Alazemi, M. Anđelić, and S.K. Simić.

Angela Fontan & Claudio Altafini

2018a Achieving a decision in antagonistic multiagent networks: frustration determines commitment strength. In: *2018 IEEE Conference on Decision and Control (CDC)* (Miami Beach, 2018), pp. 109–114. IEEE, 2018. (**SG: Dyn, Fr**)

2023a Pseudoinverses of signed Laplacian matrices. *SIAM J. Matrix Anal. Appl.* 44 (2023), no. 2, 622–647. MR 4589571. Zbl 1515.05082. (**SG: Lap**)

Loïc Forest, Nicolas Glade, & Jacques Demongeot

2007a Liénard systems and potential–Hamiltonian decomposition – Applications in biology. *C.R. Biologies* 330 (2007), 97–106.

P. 101 and Fig. 5 describe the "regulon", a signed digraph of order 2 with two stable states. [Annot. 23 Aug 2017.] (**Biol: SD: Exp**)

G. Forgacs

See also S.T. Chui and B.W. Southern.

1980a Ground-state correlations and universality in two-dimensional fully frustrated systems. *Phys. Rev. B* (3) 22 (1980), no. 9, 4473–4480. MR 0590596 (81i:82066).

Dictionary: "fully frustrated Ising model on a square lattice" = signed grid (square lattice) graph in which every quadrilateral is negative; "plaquette" = "square" = region boundary = quadrilateral. (**Phys: sg**)

G. Forgacs & E. Fradkin

1981a Anisotropy and marginality in the two-dimensional fully frustrated Ising model. *Phys. Rev. B* 23 (3) (1981), no. 7, 3442–3447. MR 0607834 (82c:82094). (**Phys: sg**)

David Forge

See also P. Berthomé, S. Corteel, and R. Flórez.

2014a Linial arrangements and local binary search trees. Manuscript, 2014. arXiv:1411.7834. (**GG: Geom**)

David Forge & Thomas Zaslavsky

2007a Lattice point counts for the Shi arrangement and other affinographic hyperplane arrangements. *J. Combin. Theory Ser. A* 114 (2007), no. 1, 97–109. MR 2275583 (2007i:52026). Zbl 1105.52014. arXiv:math/0609051.

The number of proper integral m-colorings of a rooted integral gain graph (root v_0 and a function $h : V \to \mathbb{Z}$ such that there are root edges ge_{0i} for all $g \in (-\infty, h_i]$; the rest of the gain graph is finite). (**GG: Geom, Invar, Matrd**)

2016a Lattice points in orthotopes and a huge polynomial Tutte invariant of weighted gain graphs. *J. Combin. Theory Ser. B* 118 (2016), 186–227. MR 3471850. Zbl 1317.05081. arXiv:1306.6132. HAL hal-01724088.

A weighted gain graph has lattice-ordered gain group and has vertex weights from an abelian semigroup acted upon by the gain group. The total dichromatic polynomial is a Tutte invariant (satisfying deletion-contraction and multiplicativity) with possibly uncountably many vari-

ables, but is not the universal one. *Problem.* Find the universal Tutte invariant. With integral gain group and integral weights, the integral chromatic function of (2007a) is an evaluation of the polynomial. Another special case is the polynomial of S.D. Noble and D.J.A. Welsh, A weighted graph polynomial from chromatic invariants of knots [Symposium à la Mémoire de François Jaeger (Grenoble, 1998). *Ann. Inst. Fourier (Grenoble)* 49 (1999), no. 3, 1057–1087]. (**GG: Invar, Matrd**)

Robin Forman

1993a Determinants of Laplacians on graphs. *Topology* 32 (1993), no. 1, 35–46. MR 1204404 (94g:58247). Zbl 780.05041. (**gg: Lap**)

C.M. Fortuin & P.W. Kasteleyn

1972a On the random cluster model. I. Introduction and relation to other models. *Physica* 57 (1972), 536–564. MR 0359655 (50 #12107).

Most of the paper recasts classical physical and other models (percolation, ferromagnetic Ising, Potts, graph coloring, linear resistance) in a common form that is generalized in §7, "Random cluster model". The "cluster (generating) polynomial" $Z(\Gamma; p, \kappa)$, where $p \in \mathbb{R}^E$ and $\kappa \in \mathbb{R}$, is a 1-variable specialization of the general parametrized dichromatic polynomial. In the notation of Zaslavsky (1992b) it equals $Q_\Gamma(q, p; \kappa, 1)$, where $q_e = 1 - p_e$. Thus it partially anticipates the general polynomials of Przytycka and Przytycki (1988a), Traldi (1989a), and Zaslavsky (1992b) that were based on Kauffman's (1989a) sign-colored Tutte polynomial. A spanning-tree expansion is given only for the resistance model. A feature [that seems not to have been taken up by subsequent workers] is the differentiation relation (7.7) connecting $\partial \ln Z/\partial q_e$ with [I think!] the expectation that the endpoints of e are disconnected in a subgraph. [Grimmett (1994a) summarizes subsequent work in the probabilistic direction.] (**sgc: Gen: Invar, Phys**)

Florent Foucaud

See also L. Beaudou, R.C. Brewster, and F. Dross.

Florent Foucaud, Hervé Hocquard, Dimitri Lajou, Valia Mitsou, & Théo Pierron

2019a Parameterized complexity of edge-coloured and signed graph homomorphism problems. In: *14th International Symposium on Parameterized and Exact Computation*, art. 15, 16 pp. LIPIcs. Leibniz Int. Proc. Informatics, Vol. 148. Schloss Dagstuhl – Leibniz-Zent. Inform., Wadern, 2019. MR 4042062. arXiv:1910.01099. (**SG, SGc: Hom**)

2022a Graph modification for edge-coloured and signed graph homomorphism problems: parameterized and classical complexity. *Algorithmica* 84 (2022), no. 5, 1183–1212. MR 4414088. (**SG, SGc: Hom**)

Florent Foucaud & Reza Naserasr

2014a The complexity of homomorphisms of signed graphs and signed constraint satisfaction. In: Alberto Pardo and Alfredo Viola, eds., *LATIN 2014: Theoretical Informatics* (Proc. 11th Latin American Symp., Montevideo, 2014), pp. 526–537. Lect. Notes in Computer Sci., Vol. 8392. Springer, Berlin, 2014. MR 3188136. Zbl 06276036. (**SG: Hom, Algor**)

Florent Foucaud, Reza Naserasr, & Rongxing Xu

2023a Extended double covers and homomorphism bounds of signed graphs. *Electronic J. Combin.* 30 (2023), no. 3, art. P3.31, 22 pp. HAL hal-03356591. (**SG: Hom**)

Louiza Fouli & Susan Morey

2012a Minimal reductions and cores of edge ideals. *J. Algebra* 364 (2012), 52–66. MR 2927047. Zbl 1279.13012. arXiv:1012.5746. (**Algeb: sg: par**)

J.-L. Fouquet

See C. Berge.

J.-C. Fournier

1979a *Introduction à la notion de matroïde (géométrie combinatoire)*. Publ. Math. d'Orsay, [No.] 79-03. Dép. Math., Université Paris-Sud, Orsay, 1979. MR 0551494 (81a:05027). Zbl 424.05018.

§3.12: "Matroïdes de Dowling" (p. 52). Definition by partial $\mathfrak{G}$-partitions; the linear representability theorem. (**gg: Matrd: Exp**)

Patrick W. Fowler

See also N. Basic.

2002a Hückel spectra of Möbius π systems. *Chem. Phys. Phys. Chem.* 4 (2002), no. 13, 2878–2883.

§3, "Double covers and quotient surfaces": A remarkable theorem: The spectrum of $A(\tilde{\Sigma})$ is $\operatorname{Spec} A(\Sigma) \cup \operatorname{Spec} A(|\Sigma|)$. Also, the eigenvectors of $A(\tilde{\Sigma})$ are (x, x) and $(x, -x)$ for x an eigenvector of $A(|\Sigma|)$ and of $A(\Sigma)$, respectively. A nice topological proof using orientation embedding of Σ in a surface S and the lift to an embedding of $\tilde{\Sigma}$ in the orientable double cover of S. [The graph theorem, not explicit, follows from the fact that every signed graph has an orientation embedding in some surface.] [Reproved independently, directly on the signed graph, by Bilu and Linial (2006a), Kalita and Pati (2012a), Gregory (2012a). Generalized to branched coverings in Butler (2010a).]

§4, "Chemical examples": (i), "Monocyclic rings": The eigenvalues and eigenvectors of a positive or negative circle, derived from the Frost–Musulin circle. [Reproved by Mathai and Zaslavsky (2012a) by a different method. Also proved by others.] An interesting discussion of the odd-order case. (ii), "In-plane π systems", (iii), "Cyclic polyacenes": Chemical applications. [Annot. 13 Jan 2015.]

(**sg: Cov, Adj: Eig, Chem**)

2011a Möbius systems and the Estrada index. *MATCH Commun. Math. Comput. Chem.* 66 (2011), no. 3, 751–764. MR 2884762 (2012m:05386). Zbl 1265.05366. (**sg: Eig, Cov, Chem**)

Eduardo Fradkin

See also G. Forgacs.

Eduardo Fradkin, B.A. Huberman, & Stephen H. Shenker

1978a Gauge symmetries in random magnetic systems. *Phys. Rev. B* 18 (1978), no. 9, 4789–4814.

Properties of physical interest of switching classes ("gauge symmetric" properties) of signed graphs. [Annot. 11 Jan 2015.] (**Phys, SG: Fr**)

Aviezri S. Fraenkel & Peter L. Hammer

1984a Pseudo-Boolean functions and their graphs. In: *Convexity and Graph Theory* (Jerusalem, 1981), pp. 137–146. North-Holland Math. Stud., 87. North-Holland, Amsterdam, 1984. MR 0791023 (87b:90147). Zbl 557.94019. (**sh: lg**)

F.A.M. França

See S. Akbari.

Elisa Franco

See F. Blanchini.

András Frank

1990a Packing paths, circuits, and cuts – a survey. In: B. Korte, L. Lovász, H.J. Prömel, and A. Schrijver, eds., *Paths, Flows, and VLSI-Layout*, pp. 47–100. Algorithms and Combinatorics, Vol. 9. Springer-Verlag, Berlin, 1990. MR 1083377 (91i:68116). Zbl 741.05042.

Pp. 89–91: Additively sign-weighted bipartite graphs. Thms. 8.1′, 8.5′: Criteria for negative circuit. [*Questions.* Is there a generalization to antibalanced signed graphs with additive sign-weights? Does the existence of minors help?] Thms. 8.1′w, 8.1″: Similar, for $\mathbb{Z}^+$-weighted or $\mathbb{Q}^+$-weighted graphs, not necessarily bipartite. Pp. 91–92 mention Gerards (1990a) and graphs with a bipartizing vertex. [Annot. 11 Jun 2012.] (**SGw, GGw: OG**)

1996a A survey on T-joins, T-cuts, and conservative weightings. In: D. Miklós, V.T. Sós, and T. Szőnyi, eds., *Combinatorics, Paul Erdős is Eighty*, Vol. 2, pp. 213–252. Bolyai Soc. Math. Stud., 2. János Bolyai Math. Soc., Budapest, 1996. MR 1395861 (97c:05115). Zbl 846.05062.

A “conservative ±1-weighting” of G is an edge labelling by $+1$’s and -1’s so that in every circle the sum of edge weights is nonnegative. It is a tool in several theorems. [Related: Ageev, Kostochka, and Szigeti (1995a), Sebö (1990a).] (**SGw: Str, Algor: Exp, Ref**)

Howard Frank & Ivan T. Frisch

1971a *Communication, Transmission, and Transportation Networks*. Addison-Wesley, Reading, Mass., 1971. MR 0347343 (49 #12063). Zbl 281.94012.

§6.12: “Graphs with gains,” pp. 277–288. (**GN: Exp**)

Ove Frank & Frank Harary

1979a Balance in stochastic signed graphs. *Social Networks* 2 (1979/80), 155–163. MR 0569277 (81e:05116).

An edge is present with probability α and positive with probability p. They compute the expected values of two kinds of measures of imbalance: the number of balanced triangles (whose variance is also given), and the number of induced subgraphs of order 3 having specified numbers of positive and negative edges. [Related: Škoviera (1992a), A.T. White (1994a).] (**SG: Rand, Fr**)

Giancarlo Franzese

1996a Cluster analysis for percolation on a two-dimensional fully frustrated system. *J. Phys. A* 29 (1996), 7367–7375. Zbl 904.60081.

The "fully frustrated" square lattice: alternate verticals are negative. Extending Kandel, Ben-Av, and Domany (1990a) by studying cluster properties in simulations, e.g., percolating clusters (that connect opposite sides of the lattice). Illuminating diagrams. [Annot. 18 Jun 2012.] (**Phys, SG: Clu**)

Maria Aguieiras A. de Freitas, Nair M.M. de Abreu, Renata R. Del-Vecchio, & Samuel Jurkiewicz

2010a Infinite families of Q-integral graphs. *Linear Algebra Appl.* 432 (2010), no. 9, 2352–2360. MR 2599865 (2011b:05150). Zbl 1219.05158. (**par: Lap: Eig**)

Maria Freitas, Renata Del-Vecchio, & Nair Abreu

2010a Spectral properties of KK_n^j graphs. *Mat. Contemp.* 39 (2010), 129–134. MR 2962586. Zbl 1251.05097.

The graph is $K_n \mathbin{\dot\cup} K_n$ with j additional edges. Spectral properties of $L(-\Gamma)$. [Annot. 20 Jan 2015.] (**par: Lap: Eig**)

Maria Aguieiras A. de Freitas, Renata R. Del-Vecchio, Nair M.M. de Abreu, & Steve Kirkland

2009a On Q-spectral integral variation. LAGOS'09—V Latin-Amer. Algor. Graphs Optim. Sympos. *Electronic Notes Discrete Math.* 35 (2009), 203–208. MR 2579431. Zbl 1268.05128. (**par: Lap: Eig**)

Maria Aguieiras A. de Freitas, Vladimir Nikiforov, & Laura Patuzzi

2013a Maxima of the Q-index: forbidden 4-cycle and 5-cycle. *Electronic J. Linear Algebra* 26 (2013), 905–916. MR 3192408. Zbl 1282.05166.

Thm. 1.1.: For $n \geq 4$, $C_4 \nsubseteq \Gamma \implies \lambda_{\max}(L(-\Gamma)) \leq \lambda_{\max}(F_n)$; $= \implies \Gamma = F_n$ (F_n is the windmill of $(n-1)/2$ triangles for odd n and is F_{n-1} with a pendant edge on the center for even n). Thm. 1.3: For $n \geq 6$, $C_5 \nsubseteq \Gamma \implies \lambda_{\max}(L(-\Gamma)) \leq \lambda_{\max}(L(-K_2 \vee \bar{K}_{n-2}))$; $= \implies \Gamma = K_2 \vee \bar{K}_{n-2}$. [Annot. 20 Jan 2015.] (**par: Lap: Eig**)

Christian Fremuth-Paeger & Dieter Jungnickel

1999a Balanced network flows. I: A unifying framework for design and analysis of matching algorithms. *Networks* 33 (1999), no. 1, 1–28. MR 1652254 (2000f:-90005). Zbl 999.90005. (**sg: par: Flows, cov**)

1999b Balanced network flows. II: Simple augmentation algorithms. *Networks* 33 (1999), no. 1, 29–41. MR 1652258 (2000g:90010). Zbl 999.90006. (**sg: par: Flows, cov**)

1999c Balanced network flows. III: Strongly polynomial augmentation algorithms. *Networks* 33 (1999), no. 1, 43–56. MR 1652262 (2000g:90011). Zbl 999.90007. (**sg: par: Flows, cov**)

2001a Balanced network flows. IV: Duality and structure theory. *Networks* 37 (2001), no. 4, 194–201. MR 1837197 (2002k:90010). Zbl 1038.90007. (**sg: par: Flows, cov**)

2001b Balanced network flows. V: Cycle-canceling algorithms. *Networks* 37 (2001), no. 4, 202–209. MR 1837198 (2002k:90011). Zbl 1038.90008. (**sg: par: Flows, cov**)

2001c Balanced network flows. VI: Polyhedral descriptions. *Networks* 37 (2001), no. 4, 210–218. MR 1837199 (2002k:90012). Zbl 1040.90002. (**sg: par: Flows, cov**)

2002a Balanced network flows. VII: Primal-dual algorithms. *Networks* 37 (2002), no. 1, 35–42. MR 1871705 (2003d:90007). Zbl 1040.90003. (**sg: par: Flows, cov**)

2002b An introduction to balanced network flows. In: K.T. Arasu and Á. Seress, eds., *Codes and Designs* (Columbus, Ohio, 2000), pp. 125–144. Ohio State Univ. Math. Res. Inst. Publ., 10. Walter de Gruyter, Berlin, 2002. MR 1948139 (2004b:05160). Zbl 1009.05113. (**sg: par: Flows, cov**)

2003a Balanced network flows. VIII: A revised theory of phase-ordered algorithms and the $O(\sqrt{n}m\log(n2/m)/\log n)$ bound for the nonbipartite cardinality matching problem. *Networks* 37 (2003), no. 3, 137–142. MR 1970119 (2004f:90015). Zbl 1106.90013. (**sg: par: Flows, cov**)

Noah E. Friedkin

See P. Jia, W.-J. Mei, and A. Proskurnikov.

Ivan T. Frisch

See H. Frank.

Tobias Fritz

2013a Velocity polytopes of periodic graphs and a no-go theorem for digital physics. *Discrete Math.* 313 (2013), 1289–1301. MR 3061113. Zbl 1279.05040. Corrigendum. *Ibid.* 313 (2013), 2380. MR 3084285. Zbl 1281.05081.

A periodic graph is the (infinite) $\mathbb{Z}^d$-covering graph of a (finite) $\mathbb{Z}^d$-gain graph. (**GG: Cov**)

Yuri Frota

See R.M.V. Figueiredo, M. Levorato, and E. Queiroga.

Josh B. Frye

See A.A. Diwan.

Toshio Fujisawa

1963a Maximal flow in a lossy network. In: J.B. Cruz, Jr., and John C. Hofer, eds., *Proceedings, First Annual Allerton Conference on Circuit and System Theory* (Monticello, Ill., 1963), pp. 385–393. Dept. of Electrical Eng. and Coordinated Sci. Lab., University of Illinois, Urbana, Ill., [1963]. (**GN: Matrd(bases)**)

Satoru Fujishige

See K. Ando.

See also Chiba.

Shinya Fujita & Ken-Ichi Kawarabayashi

2010a Non-separating even cycles in highly connected graphs. *Combinatorica* 30 (2010), no. 5, 565–580. MR 2776720 (2012b:05159). Zbl 1231.05146.

If Γ is k-connected ($k \geq 5$), $-\Gamma$ has a positive circle C such that $\Gamma \setminus V(C)$ is $(k-4)$-connected. If Γ has no triangles, we can say $(k-3)$-connected. (Thomassen (2001a) conjectured the odd-circle analog.) [*Problem.* Generalize to signed graphs such that $|\Sigma|$ is k-connected.] [Annot. 26 Dec 2012.] (**sg: Par: Circ**)

Shinya Fujita & Colton Magnant

2011a Note on highly connected monochromatic subgraphs in 2-colored complete graphs. *Electronic J. Combin.* 18 (2011), art. P15, 5 pp.. MR 2770120 (2012f:-05097). Zbl 1205.05090.

See Łuczak (2016a). [Annot. 24 Jan 2016.] **(sg: Str)**

D.R. Fulkerson, A.J. Hoffman, & M.H. McAndrew

1965a Some properties of graphs with multiple edges. *Canad. J. Math.* 17 (1965), 166–177. MR 0177908 (31 #2166). Zbl 132.21002.

The "odd-cycle condition" is that any two odd circles without a common vertex are joined by an edge. Assuming it, certain conditions are necessary and sufficient for a degree sequence to be realized by a submultigraph of K_n with prescribed multiplicities. The incidence matrix of $-K_n$ is employed in the geometrical proof. [*Problem.* Generalize to signed graphs.] [Annot. 30 May 2011.] **(sg: Par: incid)**

Edgar Fuller

See X.Q. Qi.

Atsushi Funato, Nan Li, & Akihiro Shikama

2014a Decomposable edge polytopes of finite graphs. Manuscript, 2014. arXiv:-1406.1942.

[This is the antibalanced case. *Problem.* Generalize to signed graphs, including balanced graphs.] **(sg: Par: Geom)**

Daryl Funk

See also N. Bowler, R. Chen, and M. DeVos.

2015a *On Excluded Minors and Biased Graph Representations of Frame Matroids.* Doctoral dissertation, Simon Fraser Univ., 2015. **(GG: Matrd)**

Daryl Funk & Dillon Mayhew

2018a On excluded minors for classes of graphical matroids. *Discrete Math.* 341 (2018), no. 6, 1509–1522. MR 3784773. Zbl 1384.05143. arXiv:1706.06265. **(GG: Matrd: Str)**

Daryl Funk, Dillon Mayhew, & Mike Newman

2022a Defining bicircular matroids in monadic logic. *Quart. J. Math.* 73 (2022), no. 1, 65–92. MR 4395073. arXiv:2005.04526. **(gg: Matrd: Bic)**

Daryl Funk, Irene Pivotto, & Daniel Slilaty

† 2022a Matrix representations of matroids of frame and lifted-graphic matroids correspond to gain functions. *J. Combin. Theory Ser. B* 155 (2022), 202–255. MR 4392273. Zbl 1487.05045. arXiv:1609.05574. **(GG: Matrd: Geom)**

Martin J. Funk

See M. Abreu.

H.N. Gabow

1983a An efficient reduction technique for degree-constrained subgraph and bidirected network flow problems. In: *Proceedings of the Fifteenth Annual ACM Symposium on Theory of Computing* (Boston, 1983), pp. 448–456. Assoc. for Computing Machinery, New York, 1983. MR 0842673 (87g:68004) (book).

$O(m^{3/2})$ algorithm for max integral flow. [See Babenko (2006b) for

improved time.] [Annot. 9 Sept 2010.] (**sg: Ori: Algor**)

Nir Gadish

See C. Bibby.

Stephen M. Gagola

1999a Solution to Problem 10606. *Amer. Math. Monthly* 106 (June–July, 1999), no. 6, 590–591.

Proposed by Zaslavsky (1997c), *q.v.* for statement of the problem and significance. (**gg**)

Ling Gai

See Sai Ji.

Giovanni Gaiffi

2016a Exponential formulas for models of complex reflection groups. *European J. Combin.* 55 (2016), 149–168. MR 3474798. Zbl 1333.05319. arXiv:1507.02090. (**Algeb: gg: Matrd**)

Anahí Gajardo

See M. Montalva.

David Gale

See also A.J. Hoffman.

David Gale & A.J. Hoffman

1982a Two remarks on the Mendelsohn–Dulmage theorem. In: Eric Mendelsohn, ed., *Algebraic and Geometric Combinatorics*, pp. 171–177. North-Holland Math. Stud., 65. Ann. Discrete Math., 15. North-Holland, Amsterdam, 1982. MR 0772593 (85m:05054). Zbl 501.05049. (**sg: Incid, Bal**)

Joseph A. Gallian

2009a A dynamic survey of graph labeling. *Electronic J. Combin.* Dynamic Surveys in Combinatorics, # DS6. URL `http://www.combinatorics.org/issue/view/Surveys/` MR 1668059 (99m:05141). Zbl 953.05067.

§3.7, "Cordial labelings"; §3.8, "The friendly index–balance index". From $f : V \to \mathbb{Z}_2$ obtain balanced edge gains $f^*(uv) = f(u) + f(v)$. f is "friendly" if it has essentially equal numbers of each label, i.e., equal or differing by 1. f is "cordial" if f and f^* have essentially equal numbers of each label. [*Cf.* Babujee and Loganathan (2011a).] A great many references. [(Γ, f) is like a balanced multiply signed graph but the questions are not gain-graphic.] [Annot. 9 Oct 2010.] (**vs: Exp, Ref**)

Vertex switching (switching one vertex) of some standard graph gives examples of many kinds of labellings. [Annot. 2 Jan 2015.] (**tg**)

Jean Gallier

2016a Spectral theory of unsigned and signed graphs. Applications to graph clustering: a survey. Manuscript, 2016, 122 pp. arXiv:1601.04692.

§5, "Signed graphs", uses Laplacian $L(\Sigma, w) := \mathrm{diag}(d) - A(\Sigma, w)$, $w =$ positive weight function, obtained from incidence matrix based on $\sqrt{w_{ij}}$ [= standard incidence matrix if $|w_{ij}| = 1$]. Dictionary: "signed degree" := unsigned degree $d_i := \sum_j w_{ij}$. *Cf.* alternatives in Kunegis, Schmidt, *et al.* (2010a), Knyazev (2018a). [Annot. 8 Feb 2021.]

(**SG, WG: Clu: Lap(Gen), Incid**)(**Exp**)

Anna Galluccio, Martin Loebl, & Jan Vondrák

See also J. Lukic.

2000a New algorithm for the Ising problem: Partition function for finite lattice graphs. *Phys. Rev. Lett.* 84 (2000), no. 26, 5924–5927.

Describes (2001a), emphasizing signed toroidal lattice graphs, i.e., toroidal lattice Ising models. [Annot. 18 Aug 2012.]

(SG, Phys: Fr: Algor)

2001a Optimization via enumeration: a new algorithm for the Max Cut Problem. *Math. Programming Ser. A* 90 (2001), 273–290. MR 1824075 (2002b:90057). Zbl 989.90127.

An algorithm for the generating function of weighted cuts (= partition function of Ising model), hence for $\sum_\zeta x^{|E^-(\Sigma^\zeta)|}$ and frustration index $l(\Sigma)$, in polynomial time for graphs of bounded genus. [Annot. 18 Aug 2012.] **(SG: Fr: Algor, Phys)**

Huiling Gan & Ellis L. Johnson

1989a Four problems on graphs with excluded minors. *Math. Programming* 45 (1989), 311–330. MR 1022812 (91a:90067). Zbl 681.90079. **(sg: Str, Algor)**

Alberto Gandolfi

See also E. De Santis.

A. Gandolfi, C.M. Newman, & D.L. Stein

2000a Zero-temperature dynamics of $\pm J$ spin glasses and related models. *Commun. Math. Phys.* 214 (2000), no. 2, 373–387. MR 1796026 (2001k:82072). Zbl 978.82098. **(SG, Phys: Fr)**

Robert Ganian, Petr Hliněný, & Jan Obdržálek

2010a Better algorithms for satisfiability problems for formulas of bounded rank-width. Kamal Lodaya and Meena Mahajan, eds., *IARCS Annual Conference on Foundations of Software Technology and Theoretical Computer Science* (FSTTCS 2010, 30th Int. Conf., Chennai, 2010), pp. 73–83. LIPICS: Leibniz Int. Proc. Informatics, Vol. 8. Schloss Dagstuhl and Leibniz-Zent. Inform., Wadern, 2010. MR 2577943 (no rev). Zbl 1245.68108. arXiv:1006.5621.

(SG: Appl: Algor)

Hilal A. Ganie & Bilal A. Chat

2019a Bounds for the energy of weighted graphs. *Discrete Math.* 268 (2019), 91–101. MR 4007233. Zbl 1419.05098. **(SG: Adj: Eig)**

Dongdong Gao

See D.-J. Wang.

Gao Hongzhu

See Cheng Z.Y.

Hua Gao, Zhijian Ji, & Ting Hou

2018a Equitable partitions in the controllability of undirected signed graphs. In: *2018 IEEE 14th International Conference on Control and Automation* (ICCA, Anchorage, Alaska, 2018), pp. 532–537. IEEE, 2018. **(SG: Lap: Eig)**

Jie Gao

See H.-T. Wang.

Jiyang Gao

20xxa Acyclic orientations and the chromatic polynomial of signed graphs. Submitted. (**SG: Col, Cov, Ori**)

Ruimei Gao & Ying Chu

2018a Freeness of arrangements between the Weyl arrangements of types $A_{n?1}$ and B_n. (In Chinese. English summary.) *J. Shandong Univ., Nat. Sci.* 53 (2018), no. 6, 70–75. Zbl 1424.51007.

Seems to be largely exposition of Edelman–Reiner (1994a). [Annot. 10 Jun 2020.] (**SG: Geom: Exp**)

Yu-Bin Gao

See also Y.Z. Fan, L.F. Huo, and Y.L. Shao.

Yubin Gao, Yihua Huang & Yanling Shao

2009a Bases of primitive non-powerful signed symmetric digraphs with loops. *Ars Combin.* 90 (2009), 383–388. MR 2489540 (2010c:05049). Zbl 1224.05208. (**SD, qm**)

Yubin Gao, Yanling Shao, & Jian Shen

2009a Bounds on the local bases of primitive nonpowerful nearly reducible sign patterns. *Linear Multilinear Algebra* 57 (2009), no. 2, 205–215. MR 2492103 (2010b:05103). Zbl 1166.15008. (**SD, QM**)

Marianne L. Gardner [Marianne Lepp]

See R. Shull.

T. Garel & J.M. Maillard

1983a Study of a two-dimensional fully frustrated model. *J. Phys. A* 16 (1983), 2257–2265. MR 0713188 (85b:82069).

Physics approach. Generalizes Southern, Chui, and Forgacs (1980a)'s square-lattice Ising model to four edge weights, symmetrically located, and reduces it to an all-positive graph with two weights. §3, "Application to the Villain model": All weights equal [hence a signed graph]; further results on Villain (1977a). [Annot. 16 Jun 2012.] (**Phys: sg: wg**)

Pravin Garg

See also D. Sinha.

2012a *An Excursion to Some Emerging Frontiers of the Theory of Signed Graphs.* Doctoral thesis, Banasthali University, 2012. (**SG**)

Vikas K. Garg

See P. Agrawal.

Michael Gargano & Louis V. Quintas

1985a A digraph generalization of balanced signed graphs. *Congressus Numerantium* 48 (1985), 133–143. MR 0830706 (87m:05095). Zbl 622.05027.

Characterizes balance in abelian gain graphs. [See Harary, Lindström, and Zetterström (1982a).] Very simple results on existence, for a given graph, of balanced nowhere-zero gains from a given abelian group. [Elementary, if one notes that such gains exist iff the graph is #G-colorable, G being the gain group]. Comparison with the approach of Sampathkumar and Bhave (1973a). Dictionary: "Symmetric G-weighted digraph"

= gain graph with gains in the (abelian) group G. "Weight" = gain. "Non-trivial" (of the gain function) = nowhere zero. (**GG: Bal**)

Michael L. Gargano, John W. Kennedy, & Louis V. Quintas

1998a Group weighted balanced digraphs and their duals. Proc. Twenty-ninth Southeastern Int. Conf. on Combinatorics, Graph Theory and Computing (Boca Raton, Fla., 1998). *Congressus Numer.* 131 (1998), 161–167. MR 1676483 (99j:05080). Zbl 951.05045.

An abelian gain graph Φ is cobalanced (here called "cut-balanced") if the sum of gains on the edges of each coherently oriented cutset is 0. [This generalizes Kabell (1985a).] Given Φ with $\|\Phi\|$ embedded in a surface, the surface dual graph is given gains by a right-rotation rule, thus forming a surface dual Φ^* of Φ. [This appears to require that the surface be orientable. Note that cobalance generalizes to nonabelian gains on orientably embedded graphs, since the order of multiplication for the gain product on a cutset is given by the embedding.] Thm. 3.2: For a plane embedding of Φ, Φ is cobalanced iff Φ^* is balanced. Thm. 3.4 restates as criteria for cobalance of Φ the standard criteria for balance of Φ^*, as in Gargano and Quintas (1985a). More interesting are "well-balanced" graphs, which are both balanced and cobalanced. *Problem.* Characterize them. Dictionary (also see Gargano and Quintas (1985a)): Balance is called "cycle balance". (**GG: Bal(Du)**)

Marcin Gąsiorek

2013a Efficient computation of the isotropy group of a finite graph: a combinatorial approach. In: Nikolaj Björner *et al.*, eds., *15th International Symposium on Symbolic and Numeric Algorithms for Scientific Computing* (SYNASC 2013, Timisoara, Romania, 2013), pp. 104–111. IEEE, 2013. (**SG**)

Marcin Gąsiorek, Daniel Simson & Katarzyna Zając

2012a On Coxeter spectral study of posets and a digraph isomorphism problem. In: *14th International Symposium on Symbolic and Numeric Algorithms for Scientific Computing* (SYNASC 2012, Timisoara, Romania), pp. 369–375. IEEE, 2012. (**SG: Adj**)

2014a On corank two edge-bipartite graphs and simply extended Euclidean diagrams. In: Franz Winkler *et al.*, eds., *The 16th International Symposium on Symbolic and Numeric Algorithms for Scientific Computing* (SYNASC 2014, Timisoara, Romania), pp. 66–73. IEEE, 2014. (**SG**)

2015a On Coxeter type study of non-negative posets using matrix morsifications and isotropy groups of Dynkin and Euclidean diagrams. *European J. Combin.* 48 (2015), 127–142. MR 3339018. Zbl 1318.06004. (**SG**)

2015b Structure and a Coxeter–Dynkin type classification of corank two non-negative posets. *Linear Algebra Appl.* 469 (2015), 76–113. MR 3299057. Zbl 1305.06002. (**SG**)

Gilles Gastou & Ellis L. Johnson

1986a Binary group and Chinese postman polyhedra. *Math. Programming* 34 (1986), 1–33. MR 0819872 (88e:90060). Zbl 589.52004.

§10 introduces the co-postman and "odd circuit" problems, treated more thoroughly in Johnson and Mosterts (1987a) (*q.v.*). "Odd" edges and

circuits are precisely negative edges and circles in an edge signing. The "odd circuit matrix" represents $\mathbf{L}(\Sigma)$ (p. 30). The "odd circuit problem" is to find a shortest negative circle; a simple algorithm uses the signed covering graph (pp. 30–31). The "Fulkerson property" may be related to planarity and K_5 minors [which suggests comparison with Barahona (1990a), §5].
(**SG: Fr(Gen), Circ, Incid, Matrd(Bases), cov, Algor**)

Atul Gaur

See Acharya, Pranjali, Gaur, and Kumar (2017a) and Pranjali.

Heather Gavlas [Heather Jordon]

See G. Chartrand, D. Hoffman, and H. Jordon.

Alexander L. Gavrilyuk, Akihiro Munemasa, Yoshio Sano, & Tetsuji Taniguchi

2021a Signed analogue of line graphs and their smallest eigenvalues. *J. Graph Theory* 98 (2021), no. 2, 309–325. MR 4371442. arXiv:2003.05578.
(**SG: LG: par: Adj: Eig**)

Premiysław Gawroński

See also F. Hassanibesheli and K. Kułakowski.

P. Gawroński, P. Gronek, & K. Kułakowski

2005a The Heider balance and social distance. *Acta Phys. Polonica B* 36 (2005), no. 8, 2549–2558. arXiv:physics/0501160.
Exposition of Kułakowski–Gawroński–Gronek (2005a), and some numerical results. [Annot. 24 Aug 2018.] (**SG, WG: Bal, Dyn, Exp**)

Przemysław Gawroński, Małgorzata J. Krawczyk, & Krzysztof Kułakowski

2015a Emerging communities in networks — a flow of ties. *Acta Phys. Polonica B* 46 (2015), no. 5, 911–921. arXiv:1505.06295.
§2, "Cognitive dissonance": A system of differential equations is used to find a switching with fewest negative edges. Repeats Gawroński and Kułakowski (2005a) with much more detail. [Annot. 19 Jan 2016.]
(**SG: fr: Algor**)

P. Gawroński & K. Kułakowski

2005a Heider balance in human networks. In: Joaquıın Marro, Pedro L. Garrido, and Miguel A. Muñoz, eds., *Modeling Cooperative Behavior in the Social Sciences* (Proc. 8th Granada Lect., Granada, Spain, 2005), pp. 93–95. AIP Conf. Proc., Vol. 779. Amer. Inst. Physics, Melville, N.Y., 2005. arXiv:physics/0503085.
See Gawroński, Krawczyk, and Kułakowski (2015a) [Annot. 21 Jan 2016.] (**SG: fr: Algor**)

2007a A numerical trip to social psychology: long-living states of cognitive dissonance. In: Y. Shi *et al.*, eds., *Computational Science – ICCS 2007* (7th Int. Conf., Beijing, 2007), Part IV, pp. 43–50. Lect. Notes in Computer Sci., Vol. 4490. Springer, Berlin, 2007. arXiv:physics/0611276. (**SG: fr: Algor**)

Gayathri H

See Reshma R.

Chuanyuan Ge & Shiping Liu

20xxa Symmetric matrices, signed graphs, and nodal domain theorems. Submitted. arXiv:2201.00198. **(SG: Lap, Bal, Sw)**

Jim [James F.] Geelen

See also M. Chudnovsky.

2008a Some open problems on excluding a uniform matroid. *Adv. Appl. Math.* 41 (2008), 628–637. MR 2459453 (2009k:05050). Zbl 1172.05013.

§6, "The basic classes": Frame matroids $\mathbf{F}(\Omega^\bullet)$ of full biased graphs are called "framed Dowling matroids" and the submatroids $\mathbf{F}(\Omega)$ without the extra unbalanced edges are called "Dowling matroids" [despite these generalizations of Dowling geometries $Q_n(\mathfrak{G})$ having been introduced in Zaslavsky (1977a), (1987a), (1991a)]. Bicircular matroids $\mathbf{F}(\Gamma, \varnothing)$ are mentioned as examples and are important in Conj. 6.1 due to Johnson, Robertson, and Seymour. Thm. 6.2 has as one important class $\mathbf{F}(K_k, \varnothing)$. There follows a nice list of four basic questions about frame matroids: Find the excluded minors. Can they be recognized in polynomial time? Find the excluded minors for the frame matroids that have canonical representations (in the sense of Zaslavsky (2003b)) or frame representations (in the sense of papers of Geelen, Gerards, & Whittle) over $\mathbb{R}$ [the projective representation Thm. 7.1 of Zaslavsky (2003b) might be relevant?]. Can those matroids be recognized in polynomial time? [Annot. 26 Jan 2015.] **(gg: Matrd: Incid, Geom)**

In Conj. 8.2 about the fewest flats that cover a matroid, one special class is the bicircular matroids. [Annot. 26 Jan 2015.] **(Bic)**

Spikes $\mathbf{F}(2C_n, \mathcal{B})$ are important (p. 630). [Annot. 29 Apr 2012.] **(gg: Matrd)**

2013a Graphical representations of matroids. *The Matroid Union* (2013), http://matroidunion.org/?p=266

Cf. Geelen–Gerards–Whittle (2018a). [Annot. 2 Jul 2019.] **(GG: Matrd(Gen): Exp)**

James F. [Jim] Geelen & A.M.H. [Bert] Gerards

2005a Regular matroid decomposition via signed graphs. *J. Graph Theory* 48 (2005), no. 1, 74–84. MR 2104582 (2005h:05037). Zbl 1055.05024.

The lift matroid. **(SG: Matrd: Str)**

2009a Excluding a group-labelled graph. *J. Combin. Theory Ser. B* 99 (2009), 247–253. MR 2467829 (2009k:05169). Zbl 1226.05213.

Given finite, abelian $\mathfrak{G}$ and $\mathfrak{G}' \leq \mathfrak{G}$, and a $\mathfrak{G}$-gain graph Φ with a minor $\Psi \cong \mathfrak{G}' K_{4t}$ where $t = 8n\#\mathfrak{G}^2$. Thm. 1.3: Either $\exists\ X \subseteq V$ with $\#X < t$ such that in $\Phi \setminus X$ the block containing most of Ψ is $\mathfrak{G}'$-balanced, or Ψ has a minor $\cong \mathfrak{G}'' K_n$ where $\mathfrak{G}' < \mathfrak{G}'' \leq \mathfrak{G}$. t may not be best possible. Thm. 1.4: $\forall\ n$, $\exists\ l(n)$ such that $\|\Phi\|$ has a $K_{l(n)}$ minor $\implies \Phi$ has a $0K_n$ minor. Dictionary: "Group-labelled graph" = gain graph; Γ means $\mathfrak{G}$; G means Φ; $\tilde{G}$ means $\|\Phi\|$; "shifting" means "switching"; $\mathfrak{G}'$-balanced means switchable so all gains are in $\mathfrak{G}'$. **(GG: Str)**

Jim Geelen, Bert Gerards, Bruce Reed, Paul Seymour, & Adrian Vetta

2009a On the odd-minor variant of Hadwiger's conjecture. *J. Combin. Theory Ser. B* 99 (2009), no. 1, 20–29. MR 2467815 (2010f:05149). Zbl 1213.05079.

An "odd K_m-expansion" in Γ is a homeomorph of $-K_m$ in $-\Gamma$. [*Problem.* Generalize to signed graphs.] **(sg: Par: Str)**

Jim Geelen, Bert Gerards, & Geoff Whittle

2002a (as James F. Geelen, A.M.H. Gerards, & Geoff Whittle) Branch-width and well-quasi-ordering in matroids and graphs. *J. Combin. Theory Ser. B* 84 (2002), no. 2, 270–290. MR 1889259 (2003f:05027). Zbl 1037.05013.

Vector representation of spikes $\mathbf{L}(2C_n, \mathcal{B})$ and tipped spikes $\mathbf{L}_\infty(2C_n, \mathcal{B})$. Thm.: Any representation of $\mathbf{L}(2C_n, \mathcal{B})$ extends to one of $\mathbf{L}_\infty(2C_n, \mathcal{B})$, if $n \geq 4$. [Special case of Zaslavsky (2003b), Thm. 7.1.] Thm.: All representations of $\mathbf{L}_\infty(2C_n, \mathcal{B})$ are of a specific form, up to projective transformations. [Annot. 18 Apr 2013.] **(gg: Matrd, Incid)**

2006a Matroid T-connectivity. *SIAM J. Discrete Math.* 20 (2006), no. 3, 588–596. MR 2272215 (2007j:05040). Zbl 1122.05023.

The full bicircular matroid $\mathbf{F}(\Gamma^\bullet, \varnothing)$ appears on p. 589. **(gg: bic)**

2006b Towards a structure theory for matrices and matroids. In: Marta Sanz-Solé *et al.*, eds., *Proceedings of the International Congress of Mathematicians* (ICM, Madrid, 2006), Vol. III: *Invited Lectures*, pp. 827–842. European Mathematical Society, Zürich, 2006. MR 2275708 (2008a:05045). Zbl 1100.05016.

See (2007a). **(gg: Matrd: Exp)**

2007a Towards a matroid-minor structure theory. In: Geoffrey Grimmett *et al.*, eds., *Combinatorics, Complexity, and Chance: A Tribute to Dominic Welsh*, pp. 72–82. Oxford Lect. Ser. Math. Appl., Vol. 34. Oxford Univ. Press, Oxford, 2007. MR 2314562 (2008d:05037) (*q.v.*). Zbl 1130.05015.

Conjecture. A minor-closed proper subclass of all GF q-representable matroids is essentially constructible from frame matroids and their duals. Dictionary: "Dowling matroid" = simple frame matroid, i.e., submatroid of Dowling's (1973a), (1973b) matroids $\mathbf{F}(\mathfrak{G}K_n^\bullet)$, for $\mathfrak{G} = \mathbb{F}_q^\times$. [Annot. 25 May 2009.] **(gg: Matrd: Exp)**

2013a Structure in minor-closed classes of matroids. In: Simon R. Blackburn, Stefanie Gerke, and Mark Wildon, eds., *Surveys in Combinatorics 2013*, Ch. 8, pp. 327–362. MR 3184116. Zbl 1318.05015. **(GG: Matrd: Exp)**

2015a The highly connected matroids in minor-closed classes. *Ann. Combin.* 19 (2015), no. 1, 107–123. MR 3319863. Zbl 1310.05044. arXiv:1312.5012.

Perturbations of representations of frame matroids are important. But see Grace and van Zwam (2018a) for counterexamples. [Annot. 27 Feb 2017, rev 15 Jul 2019.] **(gg: Matrd: Incid)(sg: Matrd)**

2018a Quasi-graphic matroids. *J. Graph Theory* 87 (2018), no. 2, 253–264. MR 3742182. Zbl 1387.05038. arXiv:1512.03005.

Cf. Bowler–Funk–Slilaty (2020a). **(GG: Matrd)**

2018b Retraction [of previous on-line version], *ibid.* 87 (2018), no. 2, 265.

The retraction is of a previous version. **(GG: Matrd)**

James F. Geelen & Bertrand Guenin

2002a Packing odd circuits in Eulerian graphs. *J. Combin. Theory Ser. B* 86 (2002), no. 2, 280–295. MR 1933464 (2004g:05129). Zbl 1023.05091.

Adds to Guenin's theorem (2001a): Thm.: If Σ has no $-K_5$ minor, then the dual linear program has a half-integral minimum (assuming f has nonnegative coefficients). Dictionary: "odd" = negative; the "Eulerian graph" is signed. (**SG, Du: Incid, Geom, Str**)

Jim Geelen & Tony Huynh

2006a Colouring graphs with no odd-K_n minor. Manuscript, 2002, 2006. URL `http://www.math.uwaterloo.ca/~jfgeelen/publications/colour.pdf` (**SG, Col**)

Jim Geelen & Kasper Kabell

2009a The Erdös–Pósa property for matroid circuits. *J. Combin. Theory Ser. B* 99 (2009), 407–419. MR 2482958 (2010c:05026). Zbl 1229.05071.

Bicircular matroid is an example of the property. §6.2, "Cleaning a nest". Lemma 6.1: A sufficiently large "nest" without a large uniform minor has a frame matroid $\mathbf{F}(K_n^\bullet, \mathcal{B})$ as a minor. §7, "Cliques": A sufficiently large $\mathbf{F}(K_n^\bullet, \mathcal{B})$ has $\mathbf{M}(K_n)$ or $\mathbf{F}(K_n, \varnothing)$ as a minor. Dictionary: "Dowling clique" = full frame matroid of biased K_n, "Dowling representation" = the biased K_n. [Annot. 12 Jul 2016.] (**gg: Matrd, Bic**)

James Geelen, James Oxley, Dirk Vertigan, & Geoff Whittle

2002a Totally free expansions of matroids. *J. Combin. Theory Ser. B* 84 (2002), no. 1, 130–179. MR 1877906 (2002j:05035). Zbl 1048.05020.

A rank-r swirl is $\mathbf{F}(2C_r, \varnothing)$. Free spikes and rank-$r$ swirls, also the latter with one unbalanced loop, are important. *Conjecture*: The 3-connected, rank-k matroids, representable over GF(q) and having no $\mathbf{L}(2C_k, \varnothing)$ or $\mathbf{F}(2C_k, \varnothing)$ minor, have a bounded number of inequivalent GF(q)-representations. [Annot. 25 May 2009, 29 Apr 2012.] (**gg: Matrd: Incid**)

2004a A short proof of non-GF(5)-representability of matroids. *J. Combin. Theory Ser. B* 91 (2004), 105–121. MR 2047534 (2005b:05055). Zbl 1050.05024.

The "free swirl" Δ_r is $\mathbf{F}(2C_k, \varnothing)$. The "free spike" Λ_r is $\mathbf{L}(2C_k, \varnothing)$. They play a main role re large totally free matroids (Thm. 3.4). [Annot. 8 Mar 2011.] (**gg: Matrd**)

Jim Geelen & Cynthia Rodriguez

2021a Disjoint non-balanced A-paths in biased graphs. *Adv. Appl. Math.* 126 (2021), art. 102014, 6 pp. MR 4224061. Zbl 1462.05297. (**GG: Paths**)

M.C. Geetha

See P.S.K. Reddy.

Laura Gellert & Raman Sanyal

2017a On degree sequences of undirected, directed, and bidirected graphs. *European J. Combin.* 64 (2017), 113–124. MR 3658823. Zbl 1365.05050. arXiv:1512.08448.

[*Cf.* Chartrand–Gavlas–Harary–Schultz (1994a), Yan–Lih–Kuo–Chang (1997a), Michael (2002a), Hoffman–Jordon (2006a), Pirzada–Naikoo–Dar (2007b), Pirzada–Dar (2007b), Tam–Fan–Zhou (2008a), Jordon–

McBride–Tipnis (2009a), Pirzada (2012a).] (**SG, Ori: Invar**)

[Joseph Ben Geloun]

See J. Ben Geloun (under 'B').

Xianya Geng, Shuchao Li, & Slobodan K. Simić

2010a On the spectral radius of quasi-k-cyclic graphs. *Linear Algebra Appl.* 433 (2010), no. 8-10, 1561–1572. MR 2718221 (2011f:05178). Zbl 1211.05074.

§2 mentions $L(-\Gamma)$. Quasi-k-cyclic means $\exists\ v$ such that $\Gamma \setminus v$ has cyclomatic number k. For $k \le 2$, Thm. 3.2 describes all Γ maximizing the largest eigenvalue of $L(-\Gamma)$. [Annot. 21 Jan 2012.] (**par: Lap: Eig**)

Claudio Gentile

See N. Cesa-Bianchi.

A.M.H. [Bert] Gerards

See also M. Chudnovsky, M. Conforti, and J. Geelen.

1988a Homomorphisms of graphs into odd cycles. *J. Graph Theory* 12 (1988), 73–83. MR 0928737 (89h:05045). Zbl 691.05013.

If an antibalanced, unbalanced signed graph has no homomorphism into its shortest negative circle, then it contains a subdivision of $-K_4$ or of a loose $\pm C_3$ (here called an "odd K_4" and an "odd K_3^2"). (A loose $\pm C_n$ consists of n negative digons in circular order, each adjacent pair joined either at a common vertex or by a link.) [*Question.* Do the theorem and proof carry over to any unbalanced signed graph?] Other results about antibalanced signed graphs are corollaries. Several interesting results about signed graphs are lemmas. (**Par, SG: Hom**)

1989a A min-max relation for stable sets in graphs with no odd-K_4. *J. Combin. Theory Ser. B* 47 (1989), 330–348. MR 1026068 (91c:05143). Zbl 691.05021.

Let Σ be antibalanced and without isolated vertices and contain no subdivision of $-K_4$. Then max. stable set size = min. cost of a cover by edges and negative circles. Also, min. vertex-cover size = max. profit of a packing of edges and negative circles. Also, weighted analogs. [*Question.* Do the theorem and proof extend to any Σ?] (**par, sg: Circ**)

1989b A short proof of Tutte's characterization of totally unimodular matrices. *Linear Algebra Appl.* 114/115 (1989), 207–212. MR 0986875 (90b:05033). Zbl 676.05028.

The proof of Lemma 3 uses a signed graph. (**SG: Bal**)

†† 1990a *Graphs and Polyhedra: Binary Spaces and Cutting Planes.* CWI Tract 73. Centrum voor Wiskunde en Informatica, Amsterdam, 1990. MR 1106635 (92f:52027). Zbl 727.90044.

[Very incomplete annotation.] Thm.: Given Σ, the set $\{x \in \mathbb{R}^n : d_1 \le x \le d_2, b_1 \le \mathrm{H}(\Sigma)^{\mathrm{T}} x \le b_2\}$ has Chvatal rank ≤ 1 for all integral vectors d_1, d_2, b_1, b_2, iff Σ contains no subdivided $-K_4$.
(**SG: Incid, Geom, Bal, Sw, Str**)

1992a On shortest T-joins and packing T-cuts. *J. Combin. Theory Ser. B* 55 (1992), 73–82. MR 1159855 (93d:05093). Zbl 810.05056. (**SG: Str**)

1992b Odd paths and circuits in planar graphs with two odd faces. CWI Report BS-R9218, September 1992. (**SG: Circ, top**)

1994a An orientation theorem for graphs. *J. Combin. Theory Ser. B* 62 (1994), 199–212. MR 1305048 (96d:05051). Zbl 807.05020. (**par, sg: Matrd, Ori**)

† 1995a On Tutte's characterization of graphic matroids—a graphic proof. *J. Graph Theory* 20 (1995), 351–359. MR 1355434 (96h:05038). Zbl 836.05017.

Signed graphs used to prove Tutte's theorem. The signed-graph matroid employed is the extended lift matroid $\mathbf{L}_\infty(\Sigma)$ ("extended even cycle matroid"). The main theorem (Thm. 2): Let Σ be a signed graph with no $-K_4$, $\pm K_3$, $-Pr_3$, or Σ_4 link minor; then Σ can be converted by Whitney 2-isomorphism operations ("breaking" = splitting a component in two at a cut vertex, "glueing" = reverse, "switching" = twisting across a vertex 2-separation) to a signed graph that has a balancing vertex ("blocknode"). Here Σ_4 consists of $+K_4$ with a 2-edge matching doubled by negative edges and one other edge made negative.

More translation: His "Σ" is our E^-. "Even, odd" = positive, negative (for edges and circles). "Bipartite" = balanced; "almost bipartite" = has a balancing vertex. (**SG: Matrd, Str, Incid**)

1995b Matching. In: M.O. Ball, T.L. Magnanti, C.L. Monma, and G.L. Nemhauser, eds., *Network Models*, Ch. 3, pp. 135–224. Handbooks Oper. Res. Management Sci., Vol. 7. North-Holland, Amsterdam, 1995. MR 1420868. Zbl 839.90131.

§7.2.2, "Network flows and bidirected graphs". Generalized matchings in bidirected graphs. [Annot. 9 Jun 2011.] (**sg: Ori: Incid**)

A.M.H. Gerards & M. Laurent

1995a A characterization of box $\frac{1}{d}$-integral binary clutters. *J. Combin. Theory Ser. B* 65 (1995), 186–207. MR 1358985 (96k:90052). Zbl 835.05017.

Thm. 5.1: The collection of negative circles of Σ is box $\frac{1}{d}$-integral for some/any integer $d \geq 2$ iff it does not contain $-K_4$ as a link minor. (**SG: Circ, Geom**)

A.M.H. Gerards, L. Lovász, A. Schrijver, P.D. Seymour, C.-S. Shi, & K. Truemper

1990a Manuscript to be prepared as of 1990.

Extension of Gerards and Schrijver (1986b) [same comments apply. The proliferating authorship is preventing this major contribution from ever being published as such—though one hopes not! See Seymour (1995a) for description of two main theorems]. (**SG: Str, Matrd, Top**)

A.M.H. Gerards & A. Schrijver

†† 1986b Signed graph – regular matroids – grafts. Research Memorandum, Faculteit der Economische Wetenschappen, Tilburg University, 1986, 57 pp. https://research.tilburguniversity.edu/en/publications/signed-graphs-regular-matroids-grafts

Essential, major theorems. The (extended) lift matroid of a signed graph is one of the objects studied. Some of this material is published in Gerards (1990a). This paper is in the process of becoming Gerards, Lovász, *et al.* (1990a) [was in process; that is unlikely to be written.]

[Annot. rev 18 May 2017.] (**SG: Str, Matrd**)

1986a Matrices with the Edmonds–Johnson property. *Combinatorica* 6 (1986), 365–379. MR 0879340 (88g:05087). Zbl 641.05039, (Zbl 565.90048).

A subsidiary result: If $-\Gamma$ contains no subdivided $-K_4$, then Γ is t-perfect. (**sg: Par: Geom, Str**)

A.M.H. Gerards & F.B. Shepherd

1998a Strong orientations without even directed circuits. *Discrete Math.* 188 (1998), 111–125. MR 1630434 (99i:05091). Zbl 957.05048. (**sd: Par: Cyc**)

1998b The graphs with all subgraphs t-perfect. *SIAM J. Discrete Math.* 11 (1998), 524–545. MR 1640924 (2000e:05074). Zbl 980.38493.

Extension of Gerards (1989a). An "odd-K_4" is a graph whose all-negative signing is a subdivided $-K_4$. A "bad-K_4" is an odd-K_4 which does not consist of exactly two undivided K_4 edges that are nonadjacent while the other edges are replaced by even paths. Thm. 1: A graph that contains no bad-K_4 as a subgraph is t-perfect. Thm. 2 characterizes the graphs that are subdivisions of 3-connected graphs and contain an odd-K_4 but no bad-K_4. [The fact that 'badness' is not strictly a parity property weighs against the possibility that Gerards (1989a) extends well to signed graphs.] (**par, sg: Str, Algor**)

K.A. Germina

See also Ashraf P K, S. Hameed, A. Mathew, R.T. Roy, T.V. Shijin, and N.K. Sudev.

K.A. Germina & P.K. Ashraf

2013a On open domination and domination in signed graphs. *Int. Math. Forum* 8 (2013), no. 38, 1863–1872. MR 3152954 (no rev). Zbl 1301.05247. (**SG: Dom**)

K.A. Germina & Shahul Hameed K (as K. Shahul Hameed)

2010a On signed paths, signed cycles and their energies. *Appl. Math. Sci. (Ruse)* 4 (2010), no. 70, 3455–3466. MR 2769200 (no rev). Zbl 1237.05125.

Eigenvalues and energies of $A(\Sigma)$ and Laplacian matrices $L(\Sigma)$ of signed paths and circles; also recurrences for the characteristic polynomials. Energy of $A := \sum |\lambda_i(A)|$; energy of $L := \sum |\lambda_i(L) - \bar{d}|$ where $\bar{d} :=$ average degree. [*Cf.* Mathai and Zaslavsky (2012a).] [Annot. 14 Nov 2010.] (**SG: Eig: Paths, Circ**)

K.A. Germina, Shahul Hameed K, & Thomas Zaslavsky

2011a On products and line graphs of signed graphs, their eigenvalues and energy. *Linear Algebra Appl.* 435 (2011), no. 10, 2432–2450. MR 2811128 (2012j:05254). Zbl 1222.05223. arXiv:1010.3884.

Adjacency matrix A and eigenvalues and energy for the general "Cvetković product", NEPS$(\Sigma_1, \ldots, \Sigma_k; \mathcal{B})$, and for a line graph $\Lambda(\Sigma)$ (as in Zaslavsky (2010b), (2012c), (20xxa)). Laplacian matrix $L(\Sigma)$; $L(+\Gamma) =$ Laplacian of graph Γ; $L(-\Gamma) =$ signless Laplacian; and its eigenvalues and energy for Cartesian product $\Sigma_1 \times \cdots \times \Sigma_r$. Also, $A(\Lambda(\Sigma))$. Thm.: The Cartesian product is balanced iff all Σ_i are balanced. Examples: Planar, cylindrical, and toroidal grids with product signatures; line graphs of those grids and of $+K_n$ and $-K_n$. [Annot. 19 Oct 2010.]

(**SG: Bal, Adj, Eig, LG**)

K.A. Germina & Sahariya

2015a On 2-path invariant signed graphs. *Adv. Appl. Discrete Math.* 15 (2015), no. 1, 21–32. MR 3242810. Zbl 1320.05056.

The "k-path graph" $(\Sigma)_k = k$-path graph $(|\Sigma|)_k$ of $|\Sigma|$ with $\sigma_k(vw) := \max \sigma(P_{vw})$ over vw-paths of length k. (Different from Gill and Patwardhan (1986a).) Question: $(\Sigma)_2 \sim \Sigma$? By Escalante and Montejano (1974), $|\Sigma| \cong K_n$ or C_{2m+1}. Thm. 7: $(C_{2m+1}, \sigma)_2$ is balanced. Thms. 11–13: Sufficient conditions for $(K_n, \sigma)_2 \sim (K_n, \sigma)$. [Annot. 4 Jan 2022.] (**SG, Sw**)

Mehrdad Ghadiri

See S. Akbari.

A. Ghafari

See S. Akbari.

Joobin Gharibshah

See M. Shahriari.

E. Ghasemian

See also S. Akbari.

E. Ghasemian & G.H. Fath-Tabar

2017a On signed graphs with two distinct eigenvalues. *Filomat* 31 (2017), no. 20, 6393–6400. MR 3746874. Zbl 1499.05360.

Classifies 3- and 4-regular, triangle-free Σ with two eigenvalues. Cor. 2.5 says for 3-regular, the unique example is the cube Q_3 with all C_4's negative. Thm. 2.7 states a 4-regular characterization [incomplete; *cf.* Hou, Tang, and Wang (2019a). Succeeded by Stanić (2020b).] [Annot. 29 May 2018.] (**SG: Adj: Eig**)

Anna Maria Ghirlanda

See L. Muracchini.

Ebrahim Ghorbani

See also R.B. Bapat and S. Akbari.

2017a On eigenvalues of Seidel matrices and Haemers' conjecture. *Designs Codes Cryptogr.* 84 (2017), no. 1-2, 189–195. MR 3654204. Zbl 1367.05128. arXiv:-1301.0075. (**sg: KG: Adj: Eig**)

Ebrahim Ghorbani, Willem H. Haemers, Hamid Reza Maimani, & Leila Parsaei Majd

2020a On sign-symmetric signed graphs. *Ars Math. Contemp.* 19 (2020), 83–93. MR 4173666. Zbl 1465.05072. arXiv:2003.09981.

"Sign-symmetric" means $\Sigma \simeq -\Sigma$. Nonbipartite examples are found. Sign symmetry implies symmetric spectrum but not conversely; shown by examples. [Annot. 23 Nov 2020, 23 May 2022.] (**SG: Str, Adj(Eig)**)

Ebrahim Ghorbani & Arezoo Majidi

2021a Signed graphs with maximal index. *Discrete Math.* 344 (2021), no. 8, art. 112463, 8 pp. MR 4262020. Zbl 1466.05087. arXiv:2101.01503.

Given (K_n, σ) and $m = \#E^- \leq n^2/4$ (about half the edges), finds $\max_\sigma \lambda_{\max}(K_n, \sigma)$ and the cases of equality. [Supersedes Akbari, Dalvandi, *et al.* (2019a), Kafai, Heydari, *et al.* (2021a).] [Annot. 27 May 2021.] (**SG: KG: Adj: Eig**)

Modjtaba Ghorbani

See also M. Hakimi-Nezhaad.

Modjtaba Ghorbani, Mardjan Hakimi-Nezhaad, & Bo Zhou

2023a Graphs with at most four Seidel eigenvalues. *Kragujevac J. Math.* 47 (2023), no. 2, 173–186 (to appear). MR 4592793.

Seidel matrix $S(\Gamma) := A(K_\Gamma)$. When does it have eigenvalue with multiplicity $\geq n-d$, $d \leq 3$? Partial solution. [Annot. 9 Nov 2020.] (**sg: KG: sw, Adj: Eig**)

Vahid Ghorbani, Ghodratollah Azadi, & Habib Azanchiler

2020a On the structure of spikes. *AKCE Int. J. Graphs Combin.* 17 (2020), no. 3, 883–886. MR 4181588. Zbl 1471.05018.

Spike = extended lift matroid $\mathbf{L}_\infty(2C_r, \mathcal{B})$ where $\mathcal{B} \subseteq \{C_r \subset 2C_r\}$. Construction of spikes by matroid operations from binary spikes (in which $\mathcal{B}$ is maximum). *Question.* Are these biased-graph operations? (**gg: Matrd**)

Prantar Ghosh

See S. Das.

A. Ghouila-Houri

See C. Berge.

H. Giacomini

See H.T. Diep.

Christos Giatsidis

See also F.D. Malliaros.

Christos Giatsidis, Bogdan Cautis, Silviu Maniu, Dimitrios M. Thilikos, & Michalis Vazirgiannis

2014a Quantifying trust dynamics in signed graphs, the S-Cores approach. In: *Proceedings of the 2014 SIAM International Conference on Data Mining* (Philadelphia, 2014). SIAM, 2014. (**SD: Str**)

Rick Giles

1982a Optimum matching forests. I: Special weights. II: General weights. III: Facets of matching forest polyhedra. *Math. Programming* 22 (1982), 1–11, 12–38, 39–51. MR 0637527–0637529 (82m:05075a,b,c). Zbl 468.90053, Zbl 468.90054, Zbl 468.90055.

In the author's "mixed" graphs, the undirected edges are really extraverted bidirected edges. (**sg: ori**)

Mukhtiar Kaur Gill [Mukti Acharya]

See also B.D. Acharya and Mukti Acharya.

1981a A graph theoretical recurrence formula for computing the characteristic polynomial of a matrix. In: S.B. Rao, ed., *Combinatorics and Graph Theory* (Proc. Sympos., Calcutta, 1980), pp. 261–265. Lect. Notes in Math., Vol. 885. Springer-Verlag, Berlin, 1981. MR 0655622 (83f:05047). Zbl 479.05030.

Introduces "quasicospectrality" of graphs or digraphs, i.e., they have cospectral signatures. See B.D. Acharya, Gill, and Pathwardhan (1984a) and M. Acharya (2012a). [Annot. 3 Feb 2012.] (**SG, SD: Eig**)

1981b A note concerning Acharya's conjecture on a spectral measure of structural balance in a social system. In: S.B. Rao, ed., *Combinatorics and Graph Theory* (Proc. Sympos., Calcutta, 1980), pp. 266–271. Lect. Notes in Math., Vol. 885. Springer-Verlag, Berlin, 1981. MR 0655623 (84d:05121). Zbl 476.05073.

Assume $|\Sigma_1| = |\Sigma_2|$. If Σ_1 and Σ_2 have the same value of B.D. Acharya's (1980a) measure of imbalance, $A(\Sigma_1)$ and $A(\Sigma_2)$ may have different spectra. [Not surprisingly.] (**SG: Bal, Eig**)

1982a *Contributions to Some Topics in Graph Theory and Its Applications*. Ph.D. thesis, Dept. of Mathematics, Indian Institute of Technology, Bombay, 1982.

Most of the results herein have been published separately. See M.K. Gill (1981a), (1981b), Gill and Patwardhan (1981a), (1982a), (1986a), M. Acharya (2009a). (**SG, SD: Bal, LG, Adj, Eig**)

M.K. Gill & B.D. Acharya

1980a A recurrence formula for computing the characteristic polynomial of a sigraph. *J. Combin. Inform. System Sci.* 5 (19 80), 68–72. MR 0586322 (81m:05097). Zbl 448.05048. (**SG: Eig**)

1980b A new property of two dimensional Sperner systems. *Bull. Calcutta Math. Soc.* 72 (1980), 165–168. MR 0669580 (83m:05121). Zbl 531.05058. (**SG: Bal, Geom**)

M.K. Gill & G.A. Patwardhan

1981a A characterization of sigraphs which are switching equivalent to their line sigraphs. *J. Math. Phys. Sci.* 15 (1981), 567–571. MR 0650430 (84h:05106). Zbl 488.05054.

The line graph is that of Behzad–Chartrand (1969a). (**SG: LG**)

1982a A characterization of sigraphs which are switching equivalent to their iterated line sigraphs. *J. Combin. Inform. System. Sci.* 7 (1982), 287–296. MR 0724371 (86a:05103). Zbl 538.05060.

The line graph is that of Behzad–Chartrand (1969a). (**SG: LG**)

1986a Switching invariant two-path signed graphs. *Discrete Math.* 61 (1986), 189–196. MR 0855324 (87j:05138). Zbl 594.05059.

The k-path signed graph of Σ [I write $D_k(\Sigma)$] is the distance-k graph on V with signs $\sigma_k(uv) = -$ iff every length-k path is all negative. The equation $\Sigma \simeq D_2(\Sigma)$ is solved. [Annot. 29 Apr 2009.] (**SG, Sw**)

Robert Gill

1998a (as Robert Voorhees Gill) *A Generalization of the Partition Lattice: Combinatorial Properties and the Action of the Symmetric Group*. Doctoral dissertation, University of Michigan, 1998. MR 2697157 (no rev). (**gg: matrd: Geom, Invar, Aut**)

1998b The number of elements in a generalized partition semilattice. *Discrete Math.* 186 (1998), 125–134. MR 1623892 (99e:52014). Zbl 956.52009.

The semilattice is the intersection semilattice of an affinographic hyperplane arrangement representing $[-k, k]K_n$ [and is therefore isomorphic to the geometric semilattice of all k-composed partitions of a set; see, e.g., Zaslavsky (2002a), Ex. 10.5]. The rank and the Whitney numbers of the first kind are calculated with the aid of species. See Kerr (1999a) for homology.

(**gg: matrd: Geom, Invar**)

2000a The action of the symmetric group on a generalized partition semilattice. *Electronic J. Combin.* 7 (2000), Research Paper 23, 20 pp. MR 1755612 (2001g:-05107). Zbl 947.06001.

See (1998a). (**gg: matrd: Geom, Invar, Aut**)

Ernst D. Gilles

See S. Klamt.

John Gimbel

1988a Abelian group labels on graphs. *Ars Combin.* 25 (1988), 87–92. MR 0944350 (89k:05046). Zbl 655.05034.

The topic is "induced" edge labellings, that is, $w(e_{uv}) = f(u)f(v)$ for some $f : V \to \mathfrak{A}$. The number of f that induce a given induced labelling, the number of induced labellings, and a characterization of induced labellings. All involve the 2-torsion subgroup of $\mathfrak{A}$, unless Γ is bipartite. The inspiration is dualizing magic graphs. [Somewhat dual to Edelman and Saks (1979a).] (**par: incid**)(**VS(Gen): Enum**)

Omer Giménez, Anna de Mier, & Marc Noy

2005a On the number of bases of bicircular matroids. *Ann. Combin.* 9 (2005), no. 1, 35–45. MR 2135774 (2005m:05049). Zbl 1059.05030.

The number of bases is bounded above by $C^n\cdot$(number of spanning trees) in a simple graph but not in a multigraph. More precise results for K_n and $K_{n,m}$. [See Neudauer, Meyers, and Stevens (2001a) and Neudauer and Stevens (2001a).] (**Bic: Incid**)

Omer Giménez & Marc Noy

2006a On the complexity of computing the Tutte polynomial of bicircular matroids. *Combin. Probab. Comput.* 15 (2006), no. 3, 385–395. MR 2216475 (2007a:05029). Zbl 1094.05013.

Known NP-hardness results for transversal matroids apply to their proper subclass, bicircular matroids, with a few possible exceptions.

(**Bic: Incid: Algor**)

Giulia Giordano

See F. Blanchini.

Ioannis Giotis & Venkatesan Guruswami

2006a Correlation clustering with a fixed number of clusters. In: *Proceedings of the Seventeenth Annual ACM-SIAM Symposium on Discrete Algorithms*, pp. 1167–1176. ACM, New York, 2006. MR 2373844 (2009f:62098). Zbl 1194.62087.

(**SG: WG: Clu: Algor**)

2006b Correlation clustering with a fixed number of clusters. *Theory Comput.* 2 (2006), 249–266. MR 2322880 (2009e:68118). (**SG: WG: Clu: Algor**)

Noriane Girard

See E. Delucchi.

Pierre-Louis Giscard, Paul Rochet, & Richard C. Wilson

2017a Evaluating balance on social networks from their simple cycles. *J. Complex Networks* 5 (2017), no. 5, 750–775. MR 3801709. arXiv:1606.03347.
(**SG: Fr, PsS**)

Nicolas Glade
See L. Forest.

Roland Glantz & Marcello Pelillo
2006a Graph polynomials from principal pivoting. *Discrete Math.* 306 (2006), no. 24, 3253–3266. MR 2279060 (2008d:05112) (*q.v.*). Zbl 1125.05073.
The polynomials arise from $A(\Phi)$ where Φ is an F^+-gain graph, F a field. (**GG: Invar, Adj**)

Terry C. Gleason
See also D. Cartwright.

Terry C. Gleason & Dorwin Cartwright
1967a A note on a matrix criterion for unique colorability of a signed graph. *Psychometrika* 32 (1967), 291–296. MR 0214509 (35 #5359). Zbl 184.49202 (184, p. 492b).
"Colorable" = clusterable. The adjacency matrices of Σ^+ and Σ^- are employed separately. The arithmetic is mostly "Boolean", i.e., $1+1=0$. A certain integral matrix T shows whether or not Σ is clusterable. [Annot. 11 Nov 2008.] (**SG: Clu, Adj**)

Fred Glover
See also J. Elam.

F. Glover, J. Hultz, D. Klingman, & J. Stutz
1978a Generalized networks: A fundamental computer-based planning tool. *Management Sci.* 24 (1978), 1209–1220. (**GN: Algor, Matrd(bases): Exp, Ref**)

Fred Glover & D. Klingman
1973a On the equivalence of some generalized network problems to pure network problems. *Math. Programming* 4 (1973), 269–278. MR 0317845 (47 #6393). Zbl 259.90012.
(**GN: Bal, Incid**)

1973b A note on computational simplifications in solving generalized transportation problems. *Transportation Sci.* 7 (1973), 351–361. MR 0418463 (54 #6502).
(**GN: Matrd(bases), geom**)

Fred Glover, Darwin Klingman, & Nancy V. Phillips
1992a *Network Models in Optimization and Their Applications in Practice.* Wiley-Interscience, New York, 1992.
Textbook. See especially Ch. 5: "Generalized networks."
(**GN: Algor: Exp**)

F. Glover, D. Klingman, & J. Stutz
1973a Extensions of the augmented predecessor index method to generalized network problems. *Transportation Sci.* 7 (1973), 377–384. (**GN: Matrd(bases), matrd**)

Herman Gluck

1975a Almost all simply connected closed surfaces are rigid. In *Geometric Topology* (Proc., Park City, Utah, 1974), pp. 225–239. Lect. Notes in Math., Vol. 438. Springer, Berlin, 1975. MR 0400239 (53 #4074). Zbl 315.50002.

Σ = signed plane graph. N = total number of sign changes in rotating around each vertex. "Lemma 5.2. (Cauchy)" $N \leq 4n - 8$. [Did Cauchy really put signs on graph edges?] [Annot. 24 Jul 2023.] (**SG**)

Luis Goddyn

See also M. Chudnovsky.

Luis Goddyn, Winfried Hochstättler, & Nancy Ann Neudauer

2016a Bicircular matroids are 3-colorable. *Discrete Math.* 339 (2016), 1425–1429. MR 3475555. Zbl 1333.05063.

(**Bic: Bic**)

C.D. Godsil

See also J. Brown and G. Coutinho.

1985a Inverses of trees. *Combinatorica* 5 (1985), 33–39. MR 0803237 (86k:05084). Zbl 578.05049.

If T is a tree with a perfect matching, then $A(T)^{-1} = A(\Sigma)$ where Σ is balanced and $|\Sigma| \supseteq \Gamma$. *Question.* When does $|\Sigma| = \Gamma$? [Solved by Simion and Cao (1989a).] [*Cf.* Buckley, Doty, and Harary (1988a), Tifenbach & Kirkland (2009a), and for a generalization to rings Bapat and Ghorbani (2014a). For a different notion, Greenberg, Lundgren, and Maybee (1984b).] [Annot. < 1988 *et seq.*]

Problem. A bipartite graph with a unique perfect matching has an inverse signed multigraph: $A(\Gamma)^{-1} = A(\Sigma)$. When is Σ balanced (i.e., switches to an unsigned graph)? Elegant solution by Yang and Ye (2018a). [Annot. 11 Dec 2018.] (**sg: Adj, Bal, sw**)

1996a Covers of complete graphs. In: *Progress in Algebraic Combinatorics* (Fukuoka, 1993, Kyoto, 1994–5), pp. 137–163. Adv. Stud. Pure Math., Vol. 24. Kinokuniya, Kyoto, 1996. MR 1414466 (97m:05201). Zbl 859.05076.

Covering graphs of complete permutation gain graphs. Dictionary: "arc function" = permutation gain function. [Annot. 7 Nov 2020.]

(**GG: KG: Cov**)

C.D. Godsil & I. Gutman

1981a On the matching polynomial of a graph. In: L. Lovász and Vera T. Sós, eds., *Algebraic Methods in Graph Theory* (Proc., Szeged, 1978), Vol. I, pp. 241–249. Colloq. Math. Soc. János Bolyai, Vol. 25. North-Holland, Amsterdam, 1981. MR 0642044 (83b:05101). Zbl 0476.05060. (**SG: Adj eig**)

Chris Godsil & Gordon Royle

2001a *Algebraic Graph Theory.* Graduate Texts in Math., Vol. 207. Springer-Verlag, New York, 2001. MR 1829620 (2002f:05002). Zbl 968.05002.

Ch. 11, "Two-graphs": Equiangular lines (van Lint and Seidel (1966a), Lemmens and Seidel (1973a)), graph switching (van Lint and Seidel (1966a), Seidel (1976a)), regular two-graphs (Taylor (1977a)).

(**TG: Adj, Eig, Geom, Sw**)

Ch. 12, "Line graphs and eigenvalues": Based on Cameron, Goethals, Seidel, and Shult (1976a). (**LG: sg: Eig, Geom, Sw**)

§15.3, "Signed matroids": Sign-colored matroids and graphs. Rank generating polynomial (see Kauffman (1989a)). §16.3, "Signed plane graphs", §16.5, "Reidemeister invariants", §16.6, "The Kauffman bracket", §16.8, "Connectivity": Properties of Kauffman's (1989a) "signed-graph" (really sign-colored graph) Tutte polynomial. §16.7, "The Jones polynomial" of a knot. (**Sc, SGc: Adj, Incid, Top**)

J.M. Goethals

See also P.J. Cameron.

J.M. Goethals & J.J. Seidel

1970a Strongly regular graphs derived from combinatorial designs. *Canad. J. Math.* 22 (1970) 597–614. MR 0282872 (44 #106). Zbl 198.29301.

A symmetric Hadamard matrix H with constant diagonal can be put in the form $A(K_n, \sigma) \pm I$ for some signed K_n that represents a regular two-graph [see D.E. Taylor (1977a)] of order $4s^2$ (Thm. 4.1). (**tg: Adj**)

Michael Goff

2003a Recovering networks with signed conductivities. REU paper, University of Washington, 2003. URL http://www.math.washington.edu/~reu/papers/2003/goff/mgoff.pdf

Partial treatment of the problem in W. Johnson (2012a). [Annot. 26 Dec 2012.] (**sg: WG: Adj**)

Gülistan Kaya Gök

2019a Some bounds on the Seidel energy of graphs. *TWMS J. Appl. Engineering Math.* 9 (2019), no. 4, 949–956. (**sg: KG: Adj: Eig**)

Andrew V. Goldberg & Alexander V. Karzanov

1994a Path problems in skew-symmetric graphs. In: *Proceedings of the 5th annual ACM-SIAM symposium on discrete algorithms* (Arlington, Va., 1994), pp. 526–535. New York, Assoc. Comput. Machinery (ACM), 1994. MR 1285193 (95c:05074). Zbl 867.90118. (**sd: Flows, Cov**)

1996a Path problems in skew-symmetric graphs. *Combinatorica* 16 (1996), no. 3, 353–382. MR 1417346 (97h:05099). Zbl 867.05037. (**sd: Flows, Cov**)

2004a Maximum skew-symmetric flows and matchings. *Math. Program., Ser. A* 100 (2004), no. 3, 537–568. MR 2129927 (2005m:90142). Zbl 1070.90090.

Techniques for digraph flows are extended to bidirected flows, treated via the double covering digraph (*cf.* Tutte (1967a)). [Annot. 9 Sept 2010.] (**sg: Ori: Flows, Cov**)

Felix Goldberg & Steve Kirkland

2014a On the sign patterns of the smallest signless Laplacian eigenvector. *Linear Algebra Appl.* 443 (2014), 66–85. MR 3148894. Zbl 1282.05114. arXiv:1307.7749.

The sign pattern of an eigenvector of the smallest eigenvalue of $L(-\Gamma)$ for a bipartite graph + some edges may be predictable. [*Problem.* Generalize to signed graphs.] [Annot. 23 Nov 2014.] (**sg: par: Eig**)

Andrew V. Goldberg, Éva Tardos, & Robert E. Tarjan

1990a Network flow algorithms. In: B. Korte, L. Lovász, H.J. Prömel, and A. Schrijver, eds., *Paths, Flows, and VLSI-Layout*, pp. 101–164. Algorithms and Combinatorics, Vol. 9. Springer-Verlag, Berlin, 1990. MR 1083378 (92i:90043). Zbl 728.90035.

§1.5, "The generalized flow problem": Max flow, conservative except at the source, in networks with (real, positive) gains; generalized augmenting paths. §1.6, "The restricted problem": Flows with gains, conservative except at source and sink, whose residual flow has no gainy cycles that avoid the source. §1.7, "Decomposition theorems" for flows with or without gains. §6, "The generalized flow problem": Combinatorial algorithms; connections between flow problems with and without gains [Annot. 11 Jun 2012.] (**GN: Algor**)

Jay R. Goldman & Louis H. Kauffman

1993a Knots, tangles, and electrical networks. *Adv. Appl. Math.* 14 (1993), 267–306. MR 1228742 (94m:57013). Zbl 806.57002. Repr. in Louis H. Kauffman, *Knots and Physics*, 2nd edn., pp. 684–723. Ser. Knots Everything, Vol. 1. World Scientific, Singapore, 1993. MR 1306280 (95i:57010). Zbl 868.57001.

The parametrized Tutte polynomial [as in Zaslavsky (1992b) *et al.*] of an $\mathbb{R}^\times$-weighted graph is used to define a two-terminal "conductance". Interpreting weights as crossing signs (± 1) in a planar link diagram with two blocked regions yields invariants of tunnel links. [Also see Kauffman (1997a).] (**SGw: Gen: Invar, Knot, Phys**)

Avraham Goldstein

See Y. Cherniavsky.

Richard Z. Goldstein & Edward C. Turner

1979a Applications of topological graph theory to group theory. *Math. Z.* 165 (1979), 1–10. MR 0521516 (80g:20050). Zbl 377.20027, (Zbl 387.20034). (**SG: Top**)

Eric Goles

See J. Aracena.

Harry F. Gollub

1974a The subject-verb-object approach to social cognition. *Psychological Rev.* 81 (1974), 286–321. (**PsS: vs**)

Martin Charles Golumbic

1979a A generalization of Dirac's theorem on triangulated graphs. In: Allan Gewirtz and Louis V. Quintas, eds., Second Int. Conf. on Combinatorial Mathematics (New York, 1978). *Ann. New York Acad. Sci.* 319 (1979), 242–246. MR 0556028 (81c:05077). Zbl 479.05055.

Further results on chordal bipartite graphs. Their properties imply standard properties of ordinary chordal graphs. [See (1980a) for more.] (The "only if" portion of Thm. 4 is false, according to (1980a), p. 267.) (**sg: bal, cov**)

1980a *Algorithmic Graph Theory and Perfect Graphs.* Academic Press, New York, 1980. MR 0562306 (81e:68081). Zbl 541.05054.

§12.3: "Perfect elimination bipartite graphs," and §12.4: "Chordal bipartite graphs," expound perfect elimination and chordality for bipartite graphs from Golumbic and Goss (1978a) and Golumbic (1979a). In particular, Cor. 12.11: A bipartite graph is chordal bipartite iff every induced subgraph has perfect edge elimination scheme. [*Problem.* Guided by these results, find a signed-graph generalization of chordality

that corresponds to supersolvability and perfect vertex elimination (*cf.* Zaslavsky (2001a)).] **(sg: bal, cov)**

Martin Charles Golumbic & Clinton F. Goss

1978a Perfect elimination and chordal bipartite graphs. *J. Graph Theory* 2 (1978), 155–163. MR 0493395 (80d:05037). Zbl 411.05060.

A perfect edge elimination scheme is a bipartite analog of a perfect vertex elimination scheme. A chordal bipartite graph is a bipartite graph in which every circle longer than 4 edges has a chord. Analogs of properties of chordal graphs, e.g., Dirac's separator theorem, are proved. In particular, a chordal bipartite graph has a perfect edge elimination scheme. [See Golumbic (1980a) for more.] **(sg: bal)**

Sergio Gómez, Pablo Jensen, & Alex Arenas

2009a Analysis of community structure in networks of correlated data. *Phys. Rev. E* 80 (2009), no. 1, 016114. arXiv:0812.3030.

Cf. Bansal, Blum, and Chawla (2004a). **(SG, WG: Clu)**

J.R. Gonçalves

See also J.A. Blackman.

J.R. Gonçalves, J. Poulter, & J.A. Blackman

1995a $\pm J$ Ising model in 2D and of general composition. *J. Magnetism Magnetic Materials* 140–144 (1995), 1701–1702. **(Phys: sg: Fr, Eig)**

1997a Bond and site defects in fully frustrated two-dimensional Ising systems. *J. Phys. A* 30 (1997), no. 9, 2947–2962. MR 1456894 (98d:82030). Zbl 920.60082.

Signed square and triangular lattice graphs with all face circles ("plaquettes") negative ("frustrated"). Entropy change due to changing an edge sign (creating a "bond defect"), or one or two vertex deletions ("site defects"). Eigenstates correspond to negative plaquettes, as in Blackman (1982a) and Blackman and Poulter (1991a). [Annot. 17 May 2013.] **(Phys, SG: Fr, Eig)**

Chen Gong

See W.-C. Liu.

Maoguo Gong

See Q. Cai and J.S. Wu.

Shi-Cai Gong

See also Y. Wang.

2011a The unicyclic graphs with extremal signless Laplacian spectral spread. In: *Proceedings of 2011 World Congress on Engineering and Technology* (CET 2011), Vol. 1, pp. ?. [This may not have been published.] **(par: Lap: Eig)**

Shicai Gong, Hangen Duan, & Yizheng Fan

2006a On eigenvalues distribution of mixed graphs. *J. Math. Study* 39 (2006), no. 2, 124–128. MR 2248100 (2007b:05136). Zbl 1104.05045.

The "mixed graphs" are signed graphs. "[R]elations between the eigenvalues and matching number, diameter, and the number of quasi-pendant vertices of mixed graphs." (From the abstract.) [Annot. 9 Jan 2013.]

(sg: Eig)

Shi-Cai Gong & Yi-Zheng Fan

2007a Nonsingular unicyclic mixed graphs with at most three eigenvalues greater than two. *Discuss. Math. Graph Theory* 27 (2007), no. 1, 69–82. MR 2321423 (2008b:05102). Zbl 1139.05033.

Characterizes such signed ("mixed") graphs, for $L(\Sigma)$, for $n \geq 9$. [Annot. 23 Mar 2009.] **(sg: incid, Eig)**

Shi-Cai Gong & Guang-Hui Xu

2012a The characteristic polynomial and the matchings polynomial of a weighted oriented graph. *Linear Algebra Appl.* 436 (2012), no. 9, 3597–3607. MR 2900738. Zbl 1244.05120.

A "weighted oriented graph" is an $\mathbb{R}^+$-gain graph. The "skew adjacency matrix" is the gain-graphic adjacency matrix. [Annot. 7 Feb 2012.] **(gg: Adj)**

Mauricio González

See J. Aracena.

Andrew Goodall, Bart Litjens, Guus Regts, & Lluís Vena

2017a A Tutte polynomial for non-orientable maps. Ninth European Conf. Combinatorics, Graph Theory Appl. (EuroComb 2017, Vienna, 2017). *Electronic Notes Discrete Math.* 61 (2017), 513–519. Zbl 1378.05094.

Extended abstract of (2020a). **(SG:Top, Invar)**

2019a The canonical Tutte polynomial for signed graphs. *Acta Math. Univ. Comenianae (N.S.)* 88 (2019), no. 3, 749–754. MR 4012875.

Early, shorter version of (2021a). **(SG: Invar, Matrd)**

2020a A Tutte polynomial for maps II: the non-orientable case. *European J. Combin.* 86 (2020), art. 103095, 32 pp. MR 078935. Zbl 1437.05108. arXiv:1804.01496. **(SG: Top, Invar, Matrd)**

2021a Tutte's dichromate for signed graphs. *Discrete Appl. Math.* 289 (2021), 153–184. MR 4164543. Zbl 1454.05045. arXiv:1903.07548. **(SG: Invar, Matrd)**

Gary Gordon

See also L. Fern.

1997a Hyperplane arrangements, hypercubes and mixed graphs. Proc. Twenty-eighth Southeastern Int. Conf. on Combinatorics, Graph Theory and Computing (Boca Raton, Fla., 1997). *Congressus Numer.* 126 (1997), 65–72. MR 1604975 (98j:-05038). Zbl 901.05055.

An explicit bijection between the regions of the real hyperplane arrangement corresponding to $\pm K_n^\circ$ and the set of "good signed [complete] mixed graphs" $G_{\mathbf{a}}$ of order n. The latter are a notational variant of the acyclic orientations τ of $\pm K_n^\circ$ [and are therefore in bijective correspondence with the regions, by Zaslavsky (1991b), Thm. 4.4]. Dictionary: a directed edge in $G_{\mathbf{a}}$ is an oriented positive edge in τ, while a positive or negative undirected edge in $G_{\mathbf{a}}$ is an introverted or extroverted negative edge of τ. The main result, Thm. 1, is an interesting and significant explicit description of the acyclic orientations of $\pm K_n^\circ$. Namely, one orders the vertices and directs all positive edges upward; then one steps inward randomly from both ends of the ordered vertex set, one vertex at a time, at each new vertex orienting all previously unoriented negative edges to be introverted if the vertex was approached from below, extroverted if from above in the vertex ordering. [This clearly guarantees acyclicity.] [*Problem.* Generalize to arbitrary signed graphs.]

Lemma 2, "a standard exercise", is that an orientation of $\pm K_n^\circ$ (with the loops replaced by half edges) is acyclic iff the magnitudes of its net degrees are a permutation of $\{1, 3, \ldots, 2n-1]\}$. [Similarly, an orientation of $\pm K_n^\circ$ is acyclic iff its net degree vector is a signed permutation of $\{2, 4, \ldots, 2n\}$ (Zaslavsky (1991b), p. 369, but possibly known beforehand in other terminology). Both follow easily from Zaslavsky (1991b), Cor. 5.3: an acyclic orientation has a vertex that is a source or sink.]

(**SG: ori: incid, Geom**)

1999a The answer is $2^n \cdot n!$ What's the question? *Amer. Math. Monthly* 106 (Aug.–Sept., 1999), no. 7, 636–645. MR 1720459 (2000j:05050). Zbl 982.05052.

§5 presents the (signed-graph) question: an appealing presentation of material from (1997a). (**SG: ori, Incid, Geom, N: Exp**)

Gary Gordon, Jennifer McNulty, & Nancy Ann Neudauer

2016a Fixing numbers for matroids. *Graphs Combin.* 32 (2016), 133–146. MR 3436955. Zbl 1332.05029. arXiv:1307.7460.

§4, "Cycle and bicircular matroids": Thm. 4.3: If Γ is 3-connected and $\#V \geq 5$, then $\mathbf{M}(\Gamma)$ and $\mathbf{F}(\Gamma, \varnothing)$ have the same fixing number. Thm. 4.4: If Γ is 2-connected, $\#V \geq 5$, min degree ≥ 3, then $\operatorname{Aut}\mathbf{F}(\Gamma, \varnothing) \cong \operatorname{Aut}\Gamma$. Cor. 4.6.2: Fixing number of $\mathbf{F}(K_n, \varnothing)$ is 5 for $n = 4$, $\lfloor 2n/3 \rfloor$ if $n > 4$. [Annot. 8 Jan 2016.] (**GG: Bic: Aut**)

Y. Gordon & H.S. Witsenhausen

1972a On extensions of the Gale–Berlekamp switching problem and constants of l_p-spaces. *Israel J. Math.* 11 (1972), 216–229. MR 0304078 (46 #3213). Zbl 238.46009.

Asymptotic estimates of $l(K_{r,s})$, the maximum frustration index of signatures of $K_{r,s}$, improving the bounds of Brown and Spencer (1971a).

(**sg: Fr**)

Clinton F. Goss

See M.C. Golumbic.

Eric Gottlieb

1998a (as Eric Inness Gottlieb) *Cohomology of Dowling Lattices and Lie Superalgebras.* Ph.D. thesis, University of Miami, 1998. MR 2698113.

See Gottlieb and Wachs (2000a). (**gg: Matrd: Invar**)

2003a On the homology of the h, k-equal Dowling lattice. *SIAM J. Discrete Math.* 17 (2003), no. 1, 50–71. MR 2033305 (2004k:05209). Zbl 1033.05098.

The lattice is the subposet of $\operatorname{Lat}\mathbf{F}(\mathfrak{G}K_n)$ consisting of the flats whose nontrivial balanced components have order $\geq k$ and whose unbalanced component, if any, has order $\geq h$. If $\#\mathfrak{G} = 2$ and $h \leq k$ we have the lattice of Björner and Sagan (1996a). (**gg: Matrd: Invar**)

2003b On EL-shelling for the nondecreasing partition lattice. Proc. Thirty-Fourth Southeastern Int. Conf. Combinatorics, Graph Theory and Computing. *Con-*

gressus Numer. 162 (2003), 119–127. MR 2050544 (2005e:05159). Zbl 1056.05147. (**gg, sg: Matrd**)

Eric Gottlieb & Michelle L. Wachs

2000a Cohomology of Dowling lattices and Lie (super)algebras. *Adv. Appl. Math.* 24 (2000), no. 4, 301–336. MR 1761776 (2001i:05161). Zbl 1026.05104.

Two monomorphisms of the cohomology of the order complex of the lattice of flats of $Q_n(\mathfrak{G})$, upon which $\mathfrak{S}_n \wr \mathfrak{G}$ acts as operators, into enveloping algebras of certain Lie algebras and Lie superalgebras.

(**gg: Matrd: Invar**)

Michael J. Gottstein & Thomas Zaslavsky

20xxa The Rhodes semilattice of a biased graph. *Aequationes Math.* (to appear).

The semilattice, due to John Rhodes, is reinterpreted as the lattice of all closed, balanced subgraphs of $\mathfrak{G}{\cdot}K_n$ and generalized to biased graphs. Possible lattices include the lattices of all frame-closed or all lift-closed subgraphs and the lattice of semiclosed (i.e., balance-closed) subgraphs. [Annot. 21 Jan 2024.] (**GG: Matrd(Gen)**)

20xxb Weakly negative circles versus best clustering in signed graphs. In preparation. (**SG: Str**)

Samira Goudarzi

See S. Akbari.

Ian P. Goulden, Jin Ho Kwak, & Jaeun Lee

2005a Enumerating branched orientable surface coverings over a non-orientable surface. *Discrete Math.* 303 (2005), 42–55. MR 2181041 (2006i:05089). Zbl 1079.05025. (**SG: Cov, Top, gg**)

Antoine Gournay

2016a An isoperimetric constant for signed graphs. *Expositiones Math.* 34 (2016), no. 3, 339–351. MR 3521482. Zbl 1342.05085. arXiv:1410.5995. (**SG: Invar, Cov**)

Jean-Luc Gouzé

1998a Positive and negative circuits in dynamical systems. *J. Biol. Systems* 6 (1998), 11–15. Zbl 1058.37537. (**SD: Dyn**)

Kevin Grace

2021a The templates for some classes of quaternary matroids. *J. Combin. Theory Ser. B* 146 (2021), 286–363. MR 4155284. Zbl 1478.05019. arXiv:1902.07136. (**GG: Matrd**)

Kevin Grace & Stefan H.M. van Zwam

2018a On perturbations of highly connected dyadic matroids. *Ann. Combin.* 22 (2018), 513–542. MR 3845746. Zbl 1396.05021. arXiv:1712.07702.

Counterexamples to Geelen, Gerards, and Whittle (2015a).

(**GG: Matrd**)

2019a The highly connected even-cycle and even-cut matroids. *SIAM J. Discrete Math.* 33 (2019), no. 1, 26–67. MR 3896635. Zbl 1402.05030. arXiv:1610.01106. (**SG: Matrd, Du**)

Jarosław Grytczuk

See M. Anholcer.

R.L. Graham

1968a On finite 0-simple semigroups and graph theory. *Math. Systems Theory* 2 (1968), 325–339. MR 0240228 (39 #1580). Zbl 177.03103 (177, 31c).

Bipartite gain graphs are essential in the representation theory of 0-simple semigroups. *Cf.* Rees (1940a), Houghton (1977a), Rhodes and Steinberg (2009a). [Annot. 19 Aug 2019.] (**gg: sw, Aut: Algeb**)

R.L. Graham & N.J.A. Sloane

1985a On the covering radius of codes. *IEEE Trans. Inform. Theory* IT-31 (1985), 385–401. MR 0794436 (87c:94048). Zbl 585.94012.

See Example b, p. 396 (the Gale–Berlekamp code). (**sg: Fr**)

M.J. Grannell & T.S. Griggs

2009a Embeddings and designs. In: Lowell W. Beineke and Robin J. Wilson, eds., *Topics in Topological Graph Theory*, Ch. 13, pp. 268–288. Encycl. Math. Appl., Vol. 128. Cambridge Univ. Press, Cambridge, Eng., 2009. MR 2581550 (no rev). Zbl 1225.05034.

The voltage graph (i.e., gain graph) construction is used to generate embeddings of combinatorial designs. [Annot. 12 Jun 2013.]

(**Top: GG, Cov: Exp**)

Douglas D. Grant

See L.D. Andersen.

Ante Graovac, Ivan Gutman, & Nenad Trinajstić

1977a *Topological Approach to the Chemistry of Conjugated Molecules*. Lect. Notes in Chem., Vol. 4. Springer-Verlag, Berlin, 1977. Zbl 385.05032.

§2.7. "Extension of graph-theoretical considerations to Mobius systems." (**SG: Adj, Eig, Chem**)

A. Graovac & N. Trinajstić

1975a Mobius molecules and graphs. *Croatica Chemica Acta (Zagreb)* 47 (1975), 95–104. (**SG: Adj, Eig, Chem**)

1976a Graphical description of Möbius molecules. *J. Molecular Structure* 30 (1976), 416–420.

The "Möbius graph" (i.e., signed graph of a suitably twisted ring hydrocarbon) is introduced with examples of the adjacency matrix and characteristic polynomial. (**Chem: SG: Adj, Eig**)

Timothy Graves

See Brewster and Graves (2009a).

Gary Greaves, Jack Koolen, Akihiro Munemasa, Yoshio Sano, & Tetsuji Taniguchi

2015a Edge-signed graphs with smallest eigenvalue greater than −2. *J. Combin. Theory Ser. B* 110 (2015), 90–111. MR 3279389. Zbl 1302.05074.

(**SG: Adj: Eig: Str**)

Gary Greaves, Jacobus H. Koolen, Akihiro Munemasa, & Ferenc Szöllősi

2016a Equiangular lines in Euclidean spaces. *J. Combin. Theory Ser. A* 138 (2016), 208–235. MR 3423477. Zbl 1330.51006.

§3, "Some structural results on Seidel matrices". Remarkable congruence results. Let $\Sigma := (K_n, \sigma)$. Thm. 3.5: $\det A(\Sigma) \equiv (-1)^n(1-n) +$

$4n(\#E) \pmod 8$. Cor. 3.6: $\det A(\Sigma) \equiv 1 - n \pmod 4$. Thm. 3.7 gives per $A(\Sigma) \pmod 8$. Prop. 3.8: $\Sigma \not\simeq -\Sigma$ if $n \equiv 3 \pmod 4$.

§4, "Small sets of equiangular lines", finds all $[\Sigma]$ for $n \leq 12$, thus proving Haemers' (2012a) conjecture for $n \leq 12$. Application to equiangular lines. [Annot. 10 Dec 2020.] (**SG: KG: Geom, sw, Adj: Eig**)

Gary Greaves, Bojan Mohar, & Suil O

2019a Interlacing families and the Hermitian spectral norm of digraphs. *Linear Algebra Appl.* 564 (2019), 201–208. MR 3885710. Zbl 1405.05069.

Gain group $\{\pm 1, \pm \mathrm{i}\}$: $\varphi(e) = 1$ for undirected, i for directed edges. (**gg: Adj**)

Gary R.W. Greaves & Zoran Stanić

2022a Signed $(0,2)$-graphs with few eigenvalues and a symmetric spectrum. *J. Combin. Designs* 30 (2022), no. 5, 332–353. MR 4403834. arXiv:2107.11556.

$(0,2)$-graph: all $\#[N(u) \cap N(v)] = 0, 2$. (**SG: Adj: Eig**)

[John G. del Greco]

See J.G. del Greco (under 'D').

F. Green

1987a More about NP-completeness in the frustration model. *OR Spektrum* 9 (1987), 161–165. MR 0908232 (88m:90053). Zbl 625.90070.

Proves polynomial time for the reduction employed in Bachas (1984a) and improves the theorem to: The frustration-index decision problem on signed (3-dimensional) cubic lattice graphs with 9 layers is NP-complete. [2 layers, in Barahona (1982a).] (**SG: Fr: Algor**)

Jan Green-Krótki

See J. Aráoz.

Harvey J. Greenberg, J. Richard Lundgren, & John S. Maybee

1983a Rectangular matrices and signed graphs. *SIAM J. Algebraic Discrete Methods* 4 (1983), 50–61. MR 0689865 (84m:05052). Zbl 525.05045.

From a matrix B, with row set R and column set C, form the "signed bipartite graph" BG^+ with vertex set $R \cup C$ and an edge $r_i c_k$ signed $\operatorname{sgn} b_{ik}$ whenever $b_{ik} \neq 0$. The "signed row graph" RG^+ is the two-step signed graph of BG^+ on vertex set R: that is, $r_i r_j$ is an edge if $\operatorname{dist}^{BG^+}(r_i, r_j) = 2$ and its sign is the sign of any shortest $r_i r_j$-path. If some edge has ill-defined sign, RG^+ is undefined. The "signed column graph" CG^+ is similar. The paper develops simple criteria for existence and balance of these graphs and the connection to matrix properties. It examines simple special forms of B. (**QM: SG, Bal, Appl**)

1984a Signed graphs of netforms. Proc. Fifteenth Southeastern Conf. on Combinatorics, Graph Theory and Computing. *Congressus Numer.* 44 (1984), 105–115. MR 0777533 (87c:05085). Zbl 557.05048.

Application of (1983a), (1984b). "Netform" = incidence matrix of a positive real gain graph (neglecting a minor technicality). Thm. 1: B is a netform iff $RG^+(B)$ exists and is all negative. (Then $CG^+(B)$ also exists.) Thm. 2: If the row set partitions so that all negative elements are in some rows and all positives are in the other rows, then $RG^+(B)$ is all negative and balanced. Thm. 3: If Σ is all negative and balanced, then B exists as in Thm. 2 with $RG^+(B) = \Sigma$. [Equivalent to theorem of Hoffman and Gale (1956a).] B is an "inverse" of Σ. Thm. 4 concerns "inverting" $-\Gamma$ in a minimal way. Then B will be (essentially) the incidence matrix of $+\Gamma$. (**SG, gg: incid, Bal, VS, Exp, Appl**)

1984b Inverting signed graphs. *SIAM J. Algebraic Discrete Methods* 5 (1984), 216–223. MR 0745440 (86d:05085). Zbl 581.05052.

See (1983a). "Inversion" means, given a signed graph Σ_R, or Σ_R and Σ_C, finding a matrix B such that $\Sigma_R = RG^+(B)$, or $\Sigma_R = RG^+(B)$ and $\Sigma_C = CG^+(B)$. The elementary solution is in terms of coverings of Σ_R by balanced cliques. It may be desirable to minimize the size of the balanced clique cover; this difficult problem is not tackled. (**QM: SG, VS, Bal**)

Harvey J. Greenberg & John S. Maybee, eds.

1981a *Computer-Assisted Analysis and Model Simplification* (Proc. First Sympos., Univ. of Colorado, Boulder, Col., 1980). Academic Press, New York, 1981. MR 0617930 (82g:00016). Zbl 495.93001.

Several articles relevant to signed (di)graphs. (**QM**)(**SD, SG: Bal**)

Curtis Greene & Thomas Zaslavsky

1983a On the interpretation of Whitney numbers through arrangements of hyperplanes, zonotopes, non-Radon partitions, and orientations of graphs. *Trans. Amer. Math. Soc.* 280 (1983), 97–126. MR 0712251 (84k:05032). Zbl 539.05024.

§9: "Acyclic orientations of signed graphs." Continuation of Zaslavsky (1991b), counting acyclic orientations with specified unique source; also, with edge e having specified orientation and with no termini except at the ends of e. The proof is geometric. (**SG: Matrd, Ori, Geom, Invar**)

David A. Gregory

See also M. Cavers.

2012a Spectra of signed adjacency matrices. Presented to Queen's-R.M.C. Discrete Mathematics Seminar. Ms., 2012. `http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.225.7817`

$\rho =$ spectral radius. *Conjecture* 2: Given simple Γ, for some σ we have $\rho(A(\Gamma,\sigma)) \leq 2\sqrt{\Delta - 1}$. Thus, focus is on small ρ—via matching polynomial of Γ. Lemma 3: $\rho(\Sigma) \geq \sqrt{\bar{d}(\Gamma)}$; $=$ iff $A(\Sigma)^2 = \bar{d}I_n$. §6, "2-covers and Ramanujan families": "2-cover" $= \tilde{\Sigma}$. Lemma 6 rediscovers Fowler's (2002a) theorem (*q.v.*). [Annot. 13 Jun 2019.] (**SG: Adj: Eig, sw, KG**)

David A. Gregory, Kevin N. Vandermeulen, & Bryan L. Shader

1996a Rank decompositions and signed bigraphs. *Linear Multilinear Algebra* 40 (1996), 283–301. MR 1384648 (97a:05147). Zbl 866.05042.

For bipartite Σ, $\mathcal{M}$:= class of matrices with weak sign pattern Σ. Every $A \in \mathcal{M}$ is the sum of rk A rank-1 matrices in $\mathcal{M}$ iff (*) $\sigma(C) = -(-1)^{\#C/2}$ for every circle with $\#C \geq 6$. Thm. 3.2: Σ has (*) for every circle iff it is a spanning subgraph of a signed 4-cockade. Thm. 3.7. Σ has (*) for circles with $\#C \geq 6$ iff, after switching, it is obtained by three constructions from a negative C_4, a subgraph of $+K_{3,n}$, or a signed graph R_n. [Annot. 6 Mar 2011.] (**SG: QM, Circ**)

Gary S. Grest

See also D. Blankschtein.

1985a Fully and partially frustrated simple cubic Ising models: a Monte Carlo study. *J. Phys. C* 18 (1985), 6239–6246.

Simulation of the cubic signed graph of Blankschtein, Ma, and Nihat Berker (1984a). [Annot. 18 Jun 2012.] (**Phys, SG: State(fr)**)

T.S. Griggs

See M.J. Grannell.

Will Grilliette, Josephine Reynes, & Lucas J. Rusnak

2022a Incidence hypergraphs: injectivity, uniformity, and matrix-tree theorems. *Linear Algebra Appl.* 634 (2022), 77–105. MR 4339605. Zbl 1479.05206. arXiv:1910.02305. (**SH: Incid**)

Will Grilliette & Lucas J. Rusnak

20xxa Incidence hypergraphs: Box products & the Laplacian. Submitted. arXiv:2007.01842. (**SH: Algeb**)

G. Grimmett

1994a The random-cluster model. In: F.P. Kelly, ed., *Probability, Statistics and Optimisation*, Ch. 3, pp. 49–63. Wiley, Chichester, 1994. MR 1320741 (96d:60154). Zbl 858.60093.

Reviews Fortuin and Kasteleyn (1972a) and subsequent developments esp. in multidimensional lattices. The viewpoint is mainly probabilistic and asymptotic. §3.7, "Historical observations," reports Kasteleyn's account of the origin of the model. (**sgc: Gen: Invar, Phys: Exp**)

Ya.R. Grinberg & A.M. Rappoport

2011a Configuration and minimal coloring of unbalanced graphs. (In Russian.) *Doklady Akad. Nauk* 439 (2011), no. 6, 743–745. MR 2883804 (no rev). Zbl 1238.05118.

See (2011b). (**SG: Fr, Str, Clu**)

2011b Configuration and minimal coloring of disbalanced graphs. *Doklady Math.* 84 (2011), no. 1, 579–581. MR 2883804 (no rev). Zbl 1238.05118.

Thm. 1: The contrabalanced signed graphs are the cactus forests (Husimi forests) in which every circle is negative. Dictionary: "disbalance" = contrabalance, "junction" = cutpoint, "cyclically splittable" = every block is a circle, "p-groupable" = p-clusterable. [Annot. 9 Jun 2012, 22 Jan 2015.] (**SG: Fr, Str, Clu**)

Richard C. Grinold

1973a Calculating maximal flows in a network with positive gains. *Operations Res.* 21 (1973), 528–541. MR 0351412 (50 #3900). Zbl 304.90043.

Objective: to find the maximum output for given input. Basic solutions correspond to bases of $\mathbf{F}(\Phi')$, Φ' being the underlying gain graph Φ together with an unbalanced loop adjoined to the sink. Onaga (1967a) also treats this problem. (**GN: Matrd(bases), Algor**)

Heinz Gröflin & Thomas M. Liebling

1981a Connected and alternating vectors: polyhedra and algorithms. *Math. Programming* 20 (1981), 233–244. MR 0607409 (83k:90061). Zbl 448.90035. (**sg, Geom**)

Piotr Gronek

See P. Gawroński and K. Kułakowski.

Jonathan L. Gross

See also J. Chen.

1974a Voltage graphs. *Discrete Math.* 9 (1974), 239–246. MR 0347651 (50 #153). Zbl 286.05106. (**GG: Top, Cov**)

2009a Distribution of embeddings. In: Lowell W. Beineke and Robin J. Wilson, eds., *Topics in Topological Graph Theory*, Ch. 3, pp. 45–61. Encycl. Math. Appl., Vol. 128. Cambridge Univ. Press, Cambridge, Eng., 2009. MR 2581540 (no rev). Zbl 1197.05032.

§3, "Total embedding distributions": "Twist" (= edge signature) is used to construct nonorientable embeddings, which increase the count of embeddings. [Annot. 12 Jun 2013.] (**Top: sg: Exp**)

Jonathan L. Gross, Toufik Mansour, & Thomas W. Tucker

2020a Partial-duality for ribbon graphs, I: Distributions. *European J. Combin.* 86 (2020), art. 103084, 20 pp. MR 4056111. Zbl 1437.05060. (**sg: Top: Du: Invar**)

2021a Partial-duality for ribbon graphs, II: Partial-twuality polynomials and monodromy computations. *European J. Combin* 95 (2021), art. 103329, 28 pp. MR 4248388. Zbl 1466.05050.

Cf. Chen and Chen (2022a). (**sg: Top: Du: Invar**)

2021b Partial-duality for ribbon graphs, III: a Gray code algorithm for enumeration. *J. Algebraic Combin.* 54 (2021), 1119–1135. MR 4348919. Zbl 1479.05076. (**sg: Top: Du: Invar: Algor**)

Jonathan L. Gross & Thomas W. Tucker

1977a Generating all graph coverings by permutation voltage assignments. *Discrete Math.* 18 (1977), 273–283. MR 0465917 (57 #5803). Zbl 375.55001. (**GG: Top, Cov**)

1979a Fast computations in voltage graph theory. In: Allan Gewirtz and Louis V. Quintas, eds., Second Int. Conf. on Combinatorial Mathematics (New York, 1978). *Ann. New York Acad. Sci.* 319 (1979), 247–253. MR 0556029 (80m:94111). Zbl 486.05027. (**GG: Top, Cov, Sw**)

1987a *Topological Graph Theory*. Wiley, New York, 1987. MR 0898434 (88h:05034). Zbl 621.05013. Repr. with minor additions: Dover Publications, Mineola, N.Y., 2001. MR 1855951. Zbl 991.05001.

Ch. 2: "Voltage graphs and covering spaces." Ch. 4: "Imbedded voltage graphs and current graphs." (**GG: Top, Cov**)

§3.2.2: "Orientability." §3.2.3: "Rotation systems." §4.4.5: "Nonorientable current graphs", discusses how to deduce, from the signs on a current graph, the signs of the "derived" graph of the dual voltage graph. [The same rule gives the signs on the surface dual of any orientation-embedded signed graph.] (The sign group here is $\mathbb{Z}_2$.) (**SG: Top**)

2009a Embedding graphs on surfaces. In: Lowell W. Beineke and Robin J. Wilson, eds., *Topics in Topological Graph Theory*, Ch. 1, pp. 18–33. Encycl. Math. Appl., Vol. 128. Cambridge Univ. Press, Cambridge, Eng., 2009. MR 2581538 (no rev). Zbl 1197.05033.

§4, "Rotation systems": Mentions signed graphs for orientation embedding. §5, "Covering spaces and voltage graphs": Voltage graphs for surface embedding of graphs. §6, "Enumeration": See Kwak and Lee (2009a). Dictionary: "voltage graph" = gain graph; edge sign − is "twisted", + is "flat". [Annot. 12 Jun 2013.]

(**Top: SG, sw, GG, Cov: Exp**)

Jerrold W. Grossman

See also R.B. Bapat.

Jerrold W. Grossman & Roland Häggkvist

1983a Alternating cycles in edge-partitioned graphs. *J. Combin. Theory Ser. B* 34 (1983), 77–81. MR 0701173 (84h:05044). Zbl 491.05039, (Zbl 506.05040).

They prove the special case in which B is all negative of the following generalization, which is an immediate consequence of their result. [*Theorem.* If B is a bidirected graph such that for each vertex v there is a block of B in which v is neither a source nor a sink, then B contains a coherent circle. ("Coherent" means that at each vertex, one edge is directed inward and the other outward.)] (**par: ori**)

Jerrold W. Grossman, Devadatta M. Kulkarni, & Irwin E. Schochetman

1994a Algebraic graph theory without orientation. *Linear Algebra Appl.* 212/213 (1994), 289–307. MR 1306983 (96b:05111). Zbl 817.05047.

Topics: The unoriented incidence matrix of Γ [i.e., the incidence matrix $\mathrm{H}(-\Gamma)$], the Laplacian matrix $L(-\Gamma)$, the even-cycle ("even circuit") matroid $\mathbf{F}(-\Gamma)$, a partial all-minors matrix-tree theorem [completed in Bapat, Grossman, and Kulkarni (1999a)]. [This part is not new. See van Nuffelen (1973a) for $\mathrm{rank}(\mathrm{H}(-\Gamma))$; Zaslavsky (1982a), §8 for both matrices; Tutte (1981a), Doob (1973a), and Simões-Pereira (1973a) for the matroid; Chaiken (1982a) for the whole matrix-tree theorem.]

§§4, 5: Vector spaces associated with $\mathbf{F}(-\Gamma)$ and its dual, expressed both combinatorially in terms of vectors associated with matroid circuits and cocircuits (of two kinds) and as null and row spaces of $\mathrm{H}(-\Gamma)$ and $\mathrm{H}(-\Gamma)^{\mathrm{T}}$. E.g., in §5 is the all-negative case of: A basis for $\mathrm{Nul}\,\mathrm{H}(\Sigma)^{\mathrm{T}}$ consists of one switching function positivizing each balanced component of Σ. [The viewpoint, going from matroids to vector spaces over fields, usually with characteristic $\neq 2$, contrasts sharply with that of Tutte (1981a), who starts with integral chain groups ($\mathbb{Z}$-modules) and ends with chain-group properties and matroids. This is the only thorough development I know of vector spaces of a signed graph before Chen and Wang (2009a), despite some aspects' having appeared e.g. in Bolker (1977a), (1979a), and Tutte (1981a). It will be still more valuable if it is extended to $\mathbb{R}^{\times}$-gain graphs and to $F^{\times}$-gain graphs for any field F.]

Dictionary: $M = \mathrm{H}(-\Gamma)$; "k-reduced spanning substructure" ≅ independent set of rank $n-k$ in $\mathbf{F}(-\Gamma)$; "quasi edge cut" = balancing set; "quasibond" = minimal balancing set; "even circuit" = positive closed walk; "bowtie" = contrabalanced handcuff; "marimba stick" = half edge. (**ECyc, par: Incid, Bal, Du**)

1995a On the minors of an incidence matrix and its Smith normal form. *Linear Algebra Appl.* 218 (1995), 213–224. MR 1324059 (95m:15020). Zbl 819.05043.

Rank of unoriented incidence matrix of Γ (i.e., $\mathrm{H}(-\Gamma)$) [as in van Nuffelen (1973a)]. Thm. 2.2: the nonzero minors of $\mathrm{H}(-\Gamma)$ are $\pm 2^k$ for all $0 \leq k \leq \tau_o(\Gamma)$, where $\tau_o :=$ max # disjoint odd circles. [Improves Zaslavsky (1982a), Lemma 8A.2, for $\Sigma = -\Gamma$. The latter implies that the nonzero minors of $\mathrm{H}(\Sigma)$ are $\pm 2^k$ for all $0 \leq k \leq l_0(\Sigma)$, $l_0 :=$ frustration number = max # disjoint negative circles.]

Consequences are Smith normal form of $\mathrm{H}(-\Gamma)$ (§3) and total integrality of some integer programs with $\mathrm{H}(-\Gamma)$ as coefficient matrix. [Annot. rev. 11 Mar 2022.] (**par: Incid, ecyc, Geom**)

Martin Grötschel

See also F. Barahona.

M. Grötschel, M. Jünger, & G. Reinelt

1987a Calculating exact ground states of spin glasses: a polyhedral approach. In: J.L. van Hemmen and I. Morgenstern, eds., *Heidelberg Colloquium on Glassy Dynamics* (Proc., 1986), pp. 325–353. Lect. Notes in Physics, Vol. 275. Springer-Verlag, Berlin, 1987. MR 0916885 (no rev).

§2, "The spin glass model": finding the weighted frustration index in a weighted signed graph (Σ, w), or finding a ground state in the corresponding Ising model, is equivalent to the weighted max-cut problem in $(-\Sigma, w)$. This article concerns finding the exact weighted frustration index. §3, "Complexity", describes previous results on NP-completeness and polynomial-time solvability. §4, "Exact methods", discusses previous solution methods. §5, "Polyhedral combinatorics", shows that finding weighted frustration index is a linear program on the cut polytope; also expounds related work. The remainder of the paper concerns a specific cutting-plane method suggested by the polyhedral combinatorics.

(**sg: fr(gen), State: Algor, Geom, Ref**)(**Phys, Ref: Exp**)

Martin Grötschel, László Lovász, & Alexander Schrijver

1988a *Geometric Algorithms and Combinatorial Optimization*. Algorithms and Combin., Vol. 2. Springer-Verlag, Berlin, 1988. MR 0936633 (89m:90135). Zbl 634.05001.

Ch. 8, "Combinatorial optimization: A tour d'horizon": Topics mentioned include odd circles, maximum-gain flow, odd cuts.

(**par, gg: Circ, Algor**)

1993a *Geometric Algorithms and Combinatorial Optimization*. Second corrected ed. Algorithms and Combin., Vol. 2. Springer-Verlag, Berlin, 1993. MR 1261419 (95e:90001). Zbl 837.05001.

Essentially the same as (1988a). (**par, gg: Circ, Algor**)

M. Grötschel & W.R. Pulleyblank

1981a Weakly bipartite graphs and the max-cut problem. *Operations Res. Lett.* 1 (1981/82), 23–27. MR 0643056 (83e:05048). Zbl 478.05039, Zbl 494.90078.

Includes a polynomial-time algorithm, which they attribute to "Waterloo folklore", for shortest (more generally, min-weight) even or odd path,

hence (in an obvious way) odd or even circle. [Attributed by Thomassen (1985a) to Edmonds (unpublished). Adapts to signed graphs by the negative subdivision trick: Subdivide each positive edge of Σ into two negative edges, each with half the weight. The min-weight algorithm applied to the subdivision finds a min-weight (e.g., a shortest) negative circle of Σ.] [This paper is very easy to understand. It is one of the best written I know.] [Weakly bipartite graphs are certain signed graphs. Further work: Barahona, Grötschel, and Mahjoub (1985a), Poljak and Tuza (1995a), and esp. Guenin (1998a), (2001a).]
(par: Algor, Geom, Paths, Circ)(sg: Geom)

D.A. Grundy, D.D. Olesky, & P. van den Driessche

2012a Constructions for potentially stable sign patterns. *Linear Algebra Appl.* 436 (2012), 4473–4488. MR 2917424. Zbl 1245.15030. **(QM: SD)**

Xiangbai Gu

See D. Peng.

Huanhuan Guan

See C. Wen.

Zhi-Hong Guan

See B. Hu.

Victor Guba & Mark Sapir

1997a *Diagram Groups.* Mem. Amer. Math. Soc., vol. 130 (1997), no. 620. MR 1396957 (98f:20013). Zbl 930.20033.

The "labelled oriented graph" (pp. 12–13) is a gain graph with a gain semigroup (instead of group) which is the semigroup generated by an alphabet and its inverse. **(gg(Gen))**

James E. Gubernatis

See N. Hatano.

Bertrand Guenin

See also A. Abdi, G. Cornuéjols, Z. Ferchiou, J.F. Geelen, and C. Heo.

1998a *On Packing and Covering Polyhedra.* Ph.D. dissertation, Grad. Sch. Industrial Engin., Carnegie–Mellon University, 1998.
(SG: Geom)(Sgnd(Matrd): Geom)

1998b A characterization of weakly bipartite graphs. In: Robert E. Bixby, E. Andrew Boyd, and Roger Z. Ríos-Mercado, eds., *Integer Programming and Combinatorial Optimization* (6th Int. IPCO Conf., Houston, 1998), pp. 9–22. Lect. Notes in Computer Sci., Vol. 1412. Springer, Berlin, 1998. MR 1726332 (2000i:05158). Zbl 909.90264.

Extended abstract of (2001a). **(SG: Geom)**

† 2001a A characterization of weakly bipartite graphs. *J. Combin. Theory Ser. B* 83 (2001), 112–168. MR 1855799 (2002h:05145). Zbl 1030.05103.

Σ is "weakly bipartite" (Grötschel and Pulleyblank (1981a)) if its clutter of negative circles is ideal (i.e., has the "weak MFMC" property of Seymour (1977a)). [This is a polyhedral property that can be equivalently stated: Define a "negative circle cover" to be an edge multiset that intersects every negative circle, and a "weighted negative circle cover" to

be an edge weighting by nonnegative real numbers such that the total weight of each negative circle is at least 1. Weak biparticity means that, for every linear functional $f : E \to \mathbb{R}$, the minimum value over all weighted negative circle covers is attained by a negative circle cover.] Thm.: Σ is weakly bipartite iff it has no $-K_5$ minor. This proves part of Seymour's (1981a) conjecture (see Cornuéjols (2001a)). [Short proof: Schrijver (2002a).] Dictionary: "odd" = negative, "even" = positive.
(SG: Geom, Circ, Str)

2001b Integral polyhedra related to even cycle and even cut matroids. In: Karen Aardal and Bert Gerards, eds., *Integer Programming and Combinatorial Optimization* (8th Int. IPCO Conf., Utrecht, 2001). Lect. Notes in Computer Sci., Vol. 2081, 196–209. Springer, Berlin, 2001. MR 1939172 (2003j:90090). Zbl 1010.90088.
See (2002a). **(sg: Par: Matrd, Du, Geom)**

2002a Integral polyhedra related to even-cycle and even-cut matroids. *Math. Operations Res.* 27 (2002), no. 4, 693–710. MR 1939172 (2003j:90090). Zbl 1082.90584.
In Σ distinguish a negative link e_{st}. An "unbalanced port" is $C \setminus e_{st}$ where C is an unbalanced circuit of $\mathbf{L}(\Sigma)$ that contains e_{st}. Replace "negative circle" by "negative port" in the definition of (2001a). Thm.: The minimum value over all weighted unbalanced port covers is attained by an unbalanced port cover, iff Σ has no $-K_5$ minor and $\mathbf{L}(\Sigma)$ has no F_7^* minor. [The latter can be replaced by: Σ has no $(\pm C_4 \setminus \text{edge})$ minor, by Zaslavsky (1990a).] Dictionary: "even-cycle matroid" = lift matroid $\mathbf{L}(\Sigma)$, not the even-cycle matroid $\mathbf{F}(-\Gamma)$ in W.T. Tutte (1981a), M. Doob (1973a); "odd st-walk" = unbalanced port.
(SG: Geom, Matrd, Du, Str)

2002b A short proof of Seymour's characterization of the matroids with the max-flow min-cut property. *J Combin. Theory Ser. B* 86 (2002), no. 2, 273–279. MR 1933463. Zbl 1023.05029.
The excluded minor is $\mathbf{L}(-K_4)$. **(SG, Sgnd(Matrd): Flows)**

2002c A short proof of Seymour's characterization of the matroids with the max-flow min-cut property. In: William J. Cook and Andreas S. Schulz, eds., *Integer Programming and Combinatorial Optimization* (9th Int. IPCO, Cambridge, Mass., 2002), pp. 1–12. Lect. Notes in Computer Sci., Vol. 2337. Springer, Berlin, 2002. MR 2061054. Zbl 1049.90529.
See (2002b). **(SG, Sgnd(Matrd): Flows)**

2016a A survey on flows in graphs and matroids. *Discrete Appl. Math.* 209 (2016), 122–132. MR 3510436. Zbl 1339.05165. **(SG: Flows, Str)**

Bertrand Guenin & Cheolwon Heo

2023a Recognizing even-cycle and even-cut matroids. *Math. Programming* 202 (2023), 515–542. **(SG: Matrd, Du: Algor)**

Bertrand Guenin, Cheolwon Heo, & Irene Pivotto

20xxa Signed graphs with the same even cycles. Preprint. **(SG: Matrd, Du, Sw)**

Bertrand Guenin, Irene Pivotto, & Paul Wollan

2011a Isomorphism for even cycle matroids - I. Manuscript, 2011. arXiv:1109.2978.

The lift matroids $\mathbf{L}(\Sigma)$ of signed graphs (not Doob's (1973a) even-cycle matroids $\mathbf{F}(-\Gamma)$). To appear, augmented, as several papers. Dictionary: See (2016b). [Annot. 7 Aug 2019.] (**SG: Matrd**)

2013a Relationships between pairs of representations of signed binary matroids. *SIAM J. Discrete Math.* 27 (2013), no. 1, 329–341. MR 3032922. Zbl 1268.05042. (**Sgnd(Matrd)**)

2016a Displaying blocking pairs in signed graphs. *European J. Combin.* 51 (2016), 135–164. MR 3398845. Zbl 1321.05102. (**SG: Matrd: Str**)

2016b Stabilizer theorems for even cycle matroids. *J. Combin. Theory Ser. B* 118 (2016), 44–75. MR 3471844. Zbl 1332.05030.

Dictionary: "Even" = positive, "odd" = negative, "cycle" = binary cycle = even subgraph, "even cycle matroid" = lift matroid $\mathbf{L}(\Sigma)$, not the even-cycle matroid defined in Doob (1973a) (*cf.* Tutte (1981a)). (**sg: Matrd: Str**)

Bertrand Guenin & Leanne Stuive

2013a Single commodity-flow algorithms for lifts of graphic and cographic matroids. In: Michel Goemans and José Correa, eds., *Integer Programming and Combinatorial Optimization* (16th Int. IPCO, Valparaíso, Chile, 2013), pp. 193–204. Lect. Notes in Computer Sci., Vol. 7801. Springer, Heidelberg, 2013. MR 3085546. Zbl 1372.05030.

Extended abstract of (2016a). (**gg, Du: Flows**)

2016a Single commodity-flow algorithms for lifts of graphic and cographic matroids. *SIAM J. Discrete Math.* 30 (2016), no. 3, 1775–1797. MR 3544848. Zbl 1344.05040. (**gg, Du: Flows**)

Sylvain Guillemot

2008a FPT algorithms for path-transversals and cycle-transversals problems in graphs. In: Martin Grohe and Rolf Niedermeier, eds., *Parameterized and Exact Computation* (Proc. 3rd Int. Workshop, IWPEC 2008, Victoria, B.C., 2008), pp. 129-140. Lect. Notes in Computer Sci., Vol. 5018. Springer, Berlin, 2008. MR 2503559 (2010e:68087). Zbl 1142.68599. (**sg, GG: fr: Algor**)

2011a FPT algorithms for path-transversal and cycle-transversal problems. *Discrete Optimization* 8 (2011), 61–71. MR 2772561 (2012h:68112). Zbl 1248.90072. (**sg, GG: fr: Algor**)

Basak Guler, Burak Varan, Kaya Tutuncuoglu, Mohamed Nafea, Ahmed A. Zewail, Aylin Yener, & Damien Octeau

2014a Optimal strategies for targeted influence in signed networks. In: *2014 IEEE–ACM International Conference on Advances in Social Networks Analysis and Mining* (ASONAM 2014, Beijing), pp. 906–911. IEEE Conf. Publications, 2014.

Extending ideas of Li, Chen, Wang, & Zhang (2013a). (**SG: PsS**)

N. Gülpinar, G. Gutin, G. Mitra, & A. Zverovitch

2004a Extracting pure network submatrices in linear programs using signed graphs. *Discrete Appl. Math.* 137 (2004), no. 3, 359–372. MR 2036638 (2004k:90145). Zbl 1095.90112.

Problem: Finding a largest embedded network matrix (up to "reflection" = row negation). Given a $0, \pm 1$-matrix AS, let Σ have for vertices the rows of A, with an edge εe_{ij} iff $\operatorname{sgn}(a_{ik}a_{jk}) = -\varepsilon$ in the k-th column for some k. Let $\alpha :=$ maximum size of a stable set in a graph. Thm.: The maximum height of a reflected network submatrix of A equals $\max_X \alpha((\Sigma^X)^-)$ over all switchings Σ^X. This implies a heuristic algorithm for finding a large embedded network matrix. [Annot. 30 Sept 2009.] (**SG: incid: Bal, Algor**)

Mahadevappa M. Gundloor

See H.S. Ramane.

Hangtian Guo

See also H.Q. Lin.

Hangtian Guo, Huiqiu Lin, & Yanhua Zhao

2021a A spectral condition for the existence of a pentagon in non-bipartite graphs. *Linear Algebra Appl.* 627 (2021), 140–149. MR 4278142. Zbl 1468.05153.

[*Problem.* Generalize to negative pentagons in unbalanced signed graphs.] [Annot. 6 Jul 2022.] (**par: Adj: Eig: bal**)

Heng Guo & Mark Jerrum

2021a Approximately counting bases of bicircular matroids. *Combin. Probab. Comput.* 30 (2021), no. 1, 124–135. MR 4205662. Zbl 1504.68282. arXiv:1808.09548. (**gg: Matrd: Bic: Algor**)

Ji Ming Guo

See also L. Feng, J.X. Li, S.W. Tan, X.L. Wu, and J.M. Zhang.

Ji-Ming Guo, Jianxi Li, & Wai Chee Shiu

2013a On the Laplacian, signless Laplacian and normalized Laplacian characteristic polynomials of a graph. *Czechoslovak Math. J.* 63 (2013), no. 3, 701–720. MR 3125650. Zbl 1299.05216.

Operations on the characteristic polynomial of $L(-\Gamma)$ are applied to construct cospectral graphs. [Annot. 23 Nov 2014.] (**sg: par: Eig**)

Jiong Guo, Hannes Moser, & Rolf Niedermeier

2009a Iterative compression for exactly solving NP-hard minimization problems. In: J. Lerner, D. Wagner, and K.A. Zweig, eds., *Algorithmics of Large and Complex Networks: Design, Analysis, and Simulation*, pp. 65–80. Lect. Notes in Computer Sci., Vol. 5515. Springer, Berlin, 2009. Zbl 1248.68380.

Iterative compression results in vast speed-up for, e.g., Graph Bipartization and Signed Graph Balancing (§3.1). *Cf.* Hüffner, Betzler, and Niedermeier (2007a). [Annot. 6 Feb 2011.] (**SG: Fr: Algor**)

Krystal Guo

2015a *Simple Eigenvalues of Graphs and Digraphs.* Doctoral thesis, Simon Fraser University, Burnaby, British Columbia, 2015. (**gg: Adj: Eig**)

Krystal Guo & Bojan Mohar

2017a Hermitian adjacency matrix of digraphs and mixed graphs. *J. Graph Theory* 85 (2017), no. 1, 217–248. MR 3634484. Zbl 1365.05173. arXiv:1505.01321.

"Digraph" = "mixed graph" = $\{1, \pm\mathrm{i}\}$-gain graph Φ. "Hermitian adjacency matrix" = $A(\Phi)$. Studies eigenvalue properties in terms of gain-graph properties. §8, "Cospectrality": Reversing arcs in a cut without undirected edges, replacing undirected edges in a cut without arcs by arcs in the same direction, and "four-way switching" retain the spectrum (because they are gain switching operations). So does reversing all arcs (it is conjugation). *Cf.* Liu and Li (2015a) and Mohar (2016a). [Annot. 4 Feb 2018, 23 Dec 2020.] **(gg: sw, Adj: Eig)**

2017b Digraphs with Hermitian spectral radius below 2 and their cospectrality with paths. *Discrete Math.* 340 (2017), no. 11, 2616–2632. MR 3689910. Zbl 1369.05092. **(gg: Adj: Eig)**

Qiao Guo, Yaoping Hou, & Deqiong Li

2020a The least Laplacian eigenvalue of the unbalanced unicyclic signed graphs with K pendant vertices. *Electronic J. Linear Algebra* 36 (2020), 390–399. MR 4118164. Zbl 1444.05091. **(SG: Lap: Eig)**

Qiumin Guo

See also W.L. Guo.

Qiumin Guo, Weili Guo, Wentao Hu, & Guangfeng Jiang

2017a The global invariant of signed graphic hyperplane arrangements. *Graphs Combin.* 33 (2017), no. 3, 527–535. MR 3654136. Zbl 1370.52070.

For the complex arrangement $\mathcal{H}[\Sigma]$, the third quotient of the lower central series of $\pi(\mathbb{C}^n \setminus \bigcup \mathcal{H}[\Sigma]$ has a combinatorial interpretation in terms of Σ. [Annot. 1 Nov 2014.] **(SG: Geom, Algeb)**

Shuguang Guo

See G.L. Yu.

Weili Guo

See also Q.-M. Guo.

Weili Guo, Qiumin Guo, & Guangfeng Jiang

2018a Falk invariants of signed graphic arrangements. *Graphs Combin.* 34 (2018), no. 6, 1247–1258. MR 3881264. Zbl 1406.52041. **(SG: Geom, matrd, Invar)**

Weili Guo & Michele Torielli

2018a On the Falk invariant of signed graphic arrangements. *Graphs Combin.* 34 (2018), no. 3, 466–488. MR 3784929. Zbl 1392.52012. arXiv:1703.09402. **(SG: Geom, matrd, Invar)**

2020a On the Falk invariant of hyperplane arrangements attached to gain graphs. *Australasian J. Combin.* 77 (2020), no. 3, 301–317. MR 4224497. Zbl 1444.05149. arXiv:1707.08449. **(GG: Geom, matrd, Invar)**

2021a On the Falk invariant of Shi and Linial arrangements. *Discrete Comput. Geom.* 66 (2021), no. 2, 751–768. MR 4292763. Zbl 1471.52021. arXiv:2101.04517. **(GG: Geom, matrd, Invar)**

Yihao Guo

See M. Zhu.

Anika Gupta

See D. Li.

G. Gupta

See F. Harary.

Neha Gupta

See M. Charikar and V. Chatziafratis.

Satyam Guragain

See also B. Sonar.

Satyam Guragain & Ravi Srivastava

20xxa Interlacing properties of eigenvalues of Laplacian and net-Laplacian matrix of signed graphs. arXiv:2310.11907. (**SG: Lap: Eig**)

Razvan Gurau

2010a Topological graph polynomials in colored group field theory. *Ann. Henri Poincaré* 11 (2010), 565–584. MR 2677739 (2011k:81098). Zbl 1208.81153. arXiv:-0911.1945. arXiv:0911.1945. (**sg: Top: Invar**)

Venkatesan Guruswami

See M. Charikar and I. Giotis.

Vladimir A. Gurvich

See E. Boros.

Karen Guthrie

See A. Carbonero.

Gregory Gutin

See also J. Bang-Jensen, R. Crowston, and N. Gülpinar.

G. Gutin & D. Karapetyan

2009a A selection of useful theoretical tools for the design and analysis of optimization heuristics. *Memetic Computing* 1 (2009), 25–34.

§2.1, "Preprocessing in linear programming": Exposition of Gülpinar, Gutin, Mitra, and Zverovitch (2004a). [Annot. 30 Sept 2009.]

(**SG: incid, Bal, Algor: Exp**)

Gregory Gutin, Daniel Karapetyan, & Igor Razgon

2009a Fixed-parameter algorithms in analysis of heuristics for extracting networks in linear programs. In: J. Chen and F.V. Fomin, eds., *Parameterized and Exact Computation* (4th Int. Workshop, IWPEC 2009, Copenhagen), pp. 222–233. Lect. Notes in Computer Sci., Vol. 5917. Springer, Berlin, 2009. MR 2773945 (no rev). Zbl 1273.68175. arXiv:0906.1359.

Algorithmics of finding a balanced signed-graph incidence matrix ("reflected network matrix") in a given matrix. [Annot. 26 Dec 2012.]

(**sg: Bal: Sw, Algor**)

Gregory Gutin, Benjamin Sudakov, & Anders Yeo

1998a Note on alternating directed cycles. *Discrete Math.* 191 (1998), 101–107. MR 1644876 (99d:05050). Zbl 956.05060.

Existence of a coherent circle with alternating colors in a digraph with an edge 2-coloring is NP-complete. However, if the minimum in- and out-degrees of both colors are sufficiently large, such a cycle exists. [This problem generalizes the undirected, edge-2-colored alternating-circle problem, which is a special case of the existence of a bidirected coherent circle—see Bang-Jensen and Gutin (1997a). *Question.* Is this alternating cycle problem also signed-graphic?] (**par: ori: Circ: Gen**)

Gregory Gutin & Alexei Zverovitch

2003a Extracting pure network sub-matrices in linear programs using signed graphs, part II. *Commun. Dependability Quality Management* 5 (2003), no. 1, 58–65. (**SG: Algor**)

Ivan Gutman

See also N.M.M. de Abreu, D.M. Cvetković, A. Graovac, S.-L. Lee, and H.S. Ramane.

1978a Electronic properties of Möbius systems. *Z. Naturforsch.* 33a (1978), 214–216. MR 0489372 (58 #8800). (**SG: Adj, Eig, Chem**)

1988a Topological analysis of eigenvalues of the adjacency matrices in graph theory: A difficulty with the concept of internal connectivity. *Chem. Phys. Lett.* 148 (1988), 93–94.

Points out an ambiguity in the definitions of Lee, Lucchese, and Chu (1987a) in the case of multiple eigenvalues. [See Lee and Gutman (1989a) for the repair.] (**VS, SGw**)

Ivan Gutman, Dariush Kiani, Maryam Mirzakhah, & Bo Zhou

2009a On incidence energy of a graph. *Linear Algebra Appl.* 431 (2009), no. 8, 1223–1233. MR 2547906 (2010k:05174). Zbl 1175.05084. (**par: Incid Eig**)

Ivan Gutman, Shyi-Long Lee, Yeung-Long Luo, & Yeong-Nan Yeh

1994a Net signs of molecular graphs: dependence of molecular structure. *Int. J. Quantum Chem.* 49 (1994), 87–95.

How to compute the balanced signing of Γ that corresponds to eigenvalue λ_i (see Lee, Lucchese, and Chu (1987a)), without computing the eigenvector X_i. Theorem: If $v_r v_s \in E$, then $X_{ir} X_{is} = \sum_P f(P; \lambda_i)$, where $f(P; \lambda) := \varphi(G - V(P); \lambda)/\varphi'(G; \lambda)$, $\varphi(G; \lambda)$ is the characteristic polynomial, and the sum is over all paths connecting v_r and v_s. Hence $\sigma_i(v_r v_s) = \operatorname{sgn}(X_{ir} X_{is})$ is determined. [An interesting theorem. *Questions.* Does it generalize if one replaces Γ by a signed graph, this being the balanced (all-positive) case? In such a generalization, if any, how will σ enter in—by restricting the sum to positive paths, perhaps? What about graphs with real gains, or weights?] (**VS, SGw**)

Ivan Gutman, Shyi-Long Lee, Jeng-Horng Sheu, & Chiuping Li

1995a Predicting the nodal properties of molecular orbitals by means of signed graphs. *Bull. Inst. Chem., Academica Sinica* No. 42 (1995), 25–31.

Points out some difficulties with the method of Lee and Li (1994a). "Net sign" of $\tau : V \to \{\pm 1\}$ means $\sum_v \tau(v)$ [$= \#V - 2(\text{cut size})$]. (**VS, Chem**)

Ivan Gutman, Shyi-Long Lee, & Yeong-Nan Yeh

1992a Net signs and eigenvalues of molecular graphs: some analogies. *Chem. Phys. Lett.* 191 (1992), 87–91.

A connected graph Γ has n eigenvalues and n corresponding balanced signings (see Lee, Lucchese, and Chu (1987a)). Let $S_1 \geq S_2 \geq \cdots \geq S_n$ be the net signs of these signings and $m = \#E$. The net signs satisfy analogs of properties of eigenvalues. (A) If $\Delta \subset \Gamma$, then $S_1(\Delta) < S_1$. (B) $S_1 = m \geq S_2 + 2$. (C, D) For bipartite Γ, $S_n = -m$. Otherwise, $S_n \geq -m + 2$. From (B, C, D) we have $\#S_i \leq m - 2$ for all $i \neq 1$ and, if Γ is bipartite, $i \neq n$. (E, F) If Γ is bipartite, then $S_i = -S_{n+1-i}$

and at least $a - b$ net signs equal 0, where $a \geq b$ are the numbers of vertices in the two color classes. The analogy is imperfect, since $S_1 + S_2 + \cdots + S_n \geq 0$, while equality holds for eigenvalues. [*Questions.* Some of these conclusions require Γ to be bipartite. Does that mean that they will generalize to an arbitrary balanced signed graph Σ in place of the bipartite Γ, the eigenvectors being those of Σ? Will the other results generalize with Γ replaced by any signed graph? How about real gains, or weights?] (**VS, SGw**)

Ivan Gutman, María Robbiano, Enide Andrade Martins, Domingos M. Cardoso, Luis Medina, & Oscar Rojo

2010a Energy of line graphs. *Linear Algebra Appl.* 433 (2010), no. 7, 1312–1323. MR 2680258 (2012a:05188). Zbl 1194.05137. (**Par: Eig, Incid, LG**)

Ivan Gutman & Oskar E. Polansky

1986a *Mathematical Concepts in Organic Chemistry.* Springer-Verlag, Berlin, 1986. MR 0861119 (87m:92102). Zbl 657.92024.

See pp. 54–55 for eigenvalues of adjacency matrices of positive and negative circles. [Annot. 4 Nov 2010.] (**Chem: Exp: SG: Eig**)

Pavol Gvozdjak & Jozef Širáň

1993a Regular maps from voltage assignments. In: Neil Robertson and Paul Seymour, eds., *Graph Structure Theory* (Proc., Seattle, 1991), pp. 441–454. Contemp. Math., Vol. 147. Amer. Math. Soc., Providence, R.I., 1993. MR 1224722 (94j:05047). Zbl 791.05025.

§3, "Voltage assignments and derived maps", defines gain graph and covering graph (and map). §4, "Lifting map automorphisms": A map automorphism lifts iff it preserves the class of identity-gain walks. [Initiates method developed in Nedela and Škoviera (1997b), Malnič, Nedela, & Škoviera (2000a), (2002a), *et al.*] Dictionary: "voltage" = gain, "derived graph" = gain covering graph, "map" = combinatorial definition of embedded graph, "local group" (at a vertex) = fundamental group (at the vertex). [Annot. 11 Jun 2012.] (**GG: Aut, Cov, Top**)

A. Gyárfás

See B. Bollobás and P. Erdős.

Eszter Gyimesi & Gábor Nyul

2018a A comprehensive study of r-Dowling polynomials. *Aequationes Math.* 92 (2018), 515–527. MR 3805869. Zbl 1420.11048.

Combinatorial interpretation of the r-Dowling numbers, generalizing Belbachir and Bousbaa (2013a). $D_{n,m,r} = \#$ of ways to partition $[n+r]$ into k blocks so $n+1, \ldots, n+r$ are in different blocks, then color nonmininal elements of each block $B \subseteq [n]$ with m colors.

[Let group $\mathfrak{G}$ have order m; use color set $\mathfrak{G}$, least elements are colored ε; then $W_m(n,k,1) = W_k(Q_n(\mathfrak{G})) =$ Whitney number of Dowling lattice ($=$ lattice of frame matroid $\mathbf{F}(\mathfrak{G}K_n^\bullet)$). Proved by Dowling (1973b). *Improvements*: (1) $W_m(n,k,r) = \#$ colorings by $\mathfrak{G}$ as above, but each minimal element has color ε (a Dowling-like special case of Gyimesi–Nyul's theorem). (2) *Conjecture.* Replacing partitions by permutations and blocks by cycles, $|w_m(n,k,r)| = \#$ colorings as above. See note at

Belbachir and Bousbaa (2013a) for similar interpretations of $|w_m(n,k,0)|$ and $W_m(n,k,0)$.] [Annot. 28 May 2018.]

[Independently, Conjecture (2) is proved in (2019a).] [Annot. 22 Sept 2018.] (**gg: matrd: Invar(Gen)**)

2019a New combinatorial interpretations of r-Whitney and r-Whitney-Lah numbers. *Discrete Appl. Math.* 255 (2019), 222–233. MR 3926342. Zbl 1459.11071.

["Whitney" should be "Dowling" or "Dowling–Whitney"; Whitney numbers are more general.]

Definitions in terms of permutations [= partitions into circularly ordered blocks] (r-Dowling of first kind), partitions (r-Dowling of second kind), and partitions into linearly ordered blocks (r-Dowling–Lah), all with r distinguished elements appearing in distinct blocks and an m-coloring rule for most elements of the other blocks. [*Cf.* (2018a) for Dowling-style reinterpretations.] Identities, formulas, with combinatorial proofs.

[*Cf.* notes on (2018a). This m-coloring is equivalent to Dowling's (1973b) homogeneously $\mathbb{Z}_m$-labelled partitions (or coloring by any $\mathfrak{G}$ of order m), applied also to circularly and linearly ordered blocks and generalized to multiple distinguished elements (Dowling being $r = 1$). Each such type forms a lattice in the natural partial ordering of homogeneously $\mathfrak{G}$-colored partitions *et al.* (ignoring distinguished elements). Partitions generalize Dowling lattices (geometric lattice); permutations generalize regions of a hyperplane representation (Eulerian lattice;). (3) *Question.* Is there similar geometry for $r > 1$? (4) *Question.* With $r = 1$, is there a geometry for linearly ordered blocks? (5) *Problem.* Determine the properties of these lattices and how they relate to each other. E.g., ranked, geometric, Eulerian? Effect of the natural homomorphisms?] [Annot. 22 Sept 2018.] (**gg: matrd: Invar(Gen)**)

Ervin Győri

See also P. Erdős.

Ervin Györi, Alexandr V. Kostochka, & Tomasz Łuczak

1997a Graphs without short odd cycles are nearly bipartite. *Discrete Math.* 163 (1997), 279–284. MR 1428582 (97g:05103). Zbl 871.05040.

Given $\Sigma = -\Gamma$ and positive ρ, suppose every negative circle has length $\geq n/\rho$. Then Σ has frustration index $\leq 200\rho^2(\ln(10\rho))^2$ (best possible up to a constant factor) and vertex frustration number $\leq 15\rho\ln(10\rho)$ (best possible up to a logarithmic factor). The proof is based on an interesting, refining lemma. [*Problem.* Generalize to arbitrary Σ.] (**sg: Par: Fr**)

M. Hachimori & M. Nakamura

2007a A factorization theorem of characteristic polynomials of convex geometries. *Ann. Combin.* 11 (2007), 39–46. MR 2311929 (2008b:52001). Zbl 1110.06006.

Signed graph coloring is mentioned as an example. [Annot. 10 Mar 2011.] (**SG: Invar: Exp**)

Willem H. Haemers

See also S. Akbari, A.E. Brouwer, M. Cavers, E. Ghorbani, and E.R. van Dam.

2012a Seidel switching and graph energy. *MATCH Commun. Math. Comput. Chem.* 68 (2012), 653–659. MR 3052170. Zbl 1289.05290.

Graphs whose energy is not altered by switching. §3, "Seidel matrix": Energy $\mathcal{E}(K_\Delta)$ of signed complete graphs K_Δ ("Seidel energy of Δ"). Thm. 3.1: $\mathcal{E}(K_\Delta) \leq n\sqrt{n-1}$, and $=$ iff Δ is a conference graph. Thm. 3.3: $\mathcal{E}(K_\Delta) > \sqrt{2n(n-1)}$ if $n > 2$. Fact: $\mathcal{E}(+K_n) = 2(n-1)$. *Conjecture.* $\min_\sigma \mathcal{E}(K_n, \sigma) = 2(n-1)$. [Proved in Akbari, Einollahzadeh, *et al.* (2020a), previously for $n \leq 12$ in Greaves *et al.* (2016a).] [Annot. 13 Jan 2015, 10 Dec 2020.] **(SG(KG): Sw, Adj: Eig)**

Willem H. Haemers & Mohammad Reza Oboudi

2020a Universal spectra of the disjoint union of regular graphs. *Linear Algebra Appl.* 606 (2020), 244–248. MR 4133572. Zbl 1447.05123. arXiv:2004.02499.

$U(\Gamma) := \alpha A + \beta I + \gamma J + \delta D$. E.g., characteristic polynomials of $A(K_\Gamma)$ and $A(-\Gamma)$ if each component is regular. [Annot. 30 Sep 2023.] **(sg: kg: Adj(Gen): Eig)**

W.H. Haemers & G.R. Omidi

2011a Universal adjacency matrices with two eigenvalues. *Linear Algebra Appl.* 435 (2011), no. 10, 2520–2529. MR 2811135 (2012e:05230). Zbl 1221.05233. **(par: Adj: Eig)**

Willem H. Haemers & Edward Spence

2004a Enumeration of cospectral graphs. *European J. Combin.* 25 (2004), 199–211. MR 2070541 (2005d:05102). Zbl 1033.05070.

"Sign-less Laplacian" $Q(\Gamma) :=$ Laplacian matrix $L(-\Gamma) = D(\Gamma) + A(\Gamma)$. $L(-\Gamma)$ seems ($n \leq 11$) to allow fewer cospectral graphs than does $A(\Gamma)$ or $L(\Gamma)$. [Annot. Sept 2010.] **(par: Lap: Eig)**

Willem H. Haemers & Hatice Topcu

2023a On signed graphs with at most two eigenvalues unequal to ± 1. *Linear Algebra Appl.* 670 (2023), 68–77. MR 4578671. arXiv:2109.02522.

Disconnected, bipartite, and complete underlying graphs. Examples that are neither balanced nor antibalanced. [Annot. 23 Jan 2022.] **(SG: Adj: Eig, Sw)**

2024a The signed graphs with two eigenvalues unequal to ± 1. *Appl. Math. Computation* 463 (2024), art. 128348, 14 pp. arXiv:2301.01623.

Completes (2023a) for all signed simple graphs. Here, the connected, incomplete, nonbipartite graphs that are neither balanced nor antibalanced. There are 19 types. [Annot. 26 Jul 2023.] **(SG: Adj: Eig, Sw)**

Sumaira Hafeez & Rashid Farooq

2020a On energy ordering of vertex-disjoint bicyclic sidigraphs. *AIMS Math.* 5 (2020), no. 6, 6693–6713. MR 4148974. Zbl 1485.05084. arXiv:2004.04032. **(SD: Adj: Eig)**

Sumaira Hafeez, Rashid Farooq, & Mehtab Khan

2019a Bicyclic signed digraphs with maximal energy. *Appl. Math. Comput.* 347 (2019), 702–711. MR 3881711 (no rev). Zbl 1428.05192. **(SD, WD: Adj: Eig)**

Sumaira Hafeez & Mehtab Khan

2018a Iota energy of weighted digraphs. *Trans. Combin.* 7 (2018), no. 3, 55–73. MR 3841401. Zbl 1463.05335.

Iota energy := $\sum |\operatorname{Im} \lambda_i|$, λ_i = adjacency eigenvalues. Weights from $\mathbb{R}^{\times}$. Results on cyclic and unicyclic weighted digraphs; sgn(weight product) affects the results. Thm. 3.5: General upper bound. §4, "Equienergetic weighted digraphs": Examples. [Annot. 3 Jul 2018.]

(**SD, WD: Adj: Eig**)

2020a Computing extremal energy of a class of bicyclic weighted digraphs. *Asian-European J. Math.* 13 (2020), no. 5, art. 2050090, 22 pp. MR 4134198. Zbl 1458.05121. (**SD, WD: Adj: Eig**)

2021a Total energy of signed digraphs. *Computer Sci. J. Moldova* 29 (2021), no. 1(85), 105–134. MR 4256598. Zbl 1469.05087.

Cf. Hafeez and Khan (2018a). (**SD: Adj: Eig**)

Jurriaan Hage

See also H.L. Bodlaender and A. Ehrenfeucht.

1999a The membership problem for switching classes with skew gains. *Fund. Inform.* 39 (1999), 375–387. MR 1823982 (2002b:05071). Zbl 944.68144.

An algorithm to decide whether two skew gain graphs are switching equivalent. (**GG(Gen): Sw, Algor**)

2001a *Structural Aspects of Switching Classes.* Doctoral dissertation, Universiteit Leiden, 2001. IPA Dissertation Ser., UL.2001-8. [Instituut voor Programmatuurkunde en Algoritmiek, 2001.]

Contains the material of the following papers, along with updates and improved results: Ehrenfeucht, Hage, Harju, and Rozenberg (2000a), (2000b), (2006a), Hage (1999a), and Hage and Harju (1998a), (2000a), (2004a).

Errata and a corrected version at URL (1/2002) `http://www.cs.uu.nl/people/jur/2s.html` (**TG: Sw, Algor**)(**GG(Gen): Sw, Algor**)

2003a Enumerating submultisets of multisets. *Inform. Processing Lett.* 85 (2003), no. 4, 221–226. MR 1950498 (2003m:05008). Zbl 1173.68511.

§3, "The computation of the switches of a graph": Using a Hamiltonian path of vertex switchings in a graph switching class seems to speed up generating the class. [Annot. 15 Jun 2022.] (**TG: Sw: Algor**)

2012a Subgroup switching of skew gain graphs. *Fund. Inform.* 116 (2012), 111–122. MR 2977837 (no rev). Zbl 1243.05205.

Skew gains reverse by an involutory antiautomorphism of the gain group (Hage and Harju (2000a)). Here switching is restricted by prescribing for each vertex a subgroup from which the switching value may be taken. Properties of ordinary switching generalize, or become more complicated, or become too difficult. Further research is needed. [Annot. 17 Dec, 5 Jan 2011–12.] (**GG(Gen): Sw: Gen**)

Jurriaan Hage & Tero Harju

1998a Acyclicity of switching classes. *European J. Combin.* 19 (1998), 321–327. MR 1621017 (99d:05051). Zbl 905.05057.

Classifies the switching-equivalent pairs of forests. Thm. 2.2: In a Seidel switching class of graphs there is at most one isomorphism type of tree; and there is at most one tree, with exceptions that are completely classified. Thms. 3.1 and 4.1: In a switching class that contains a disconnected forest there are at most 3 forests (not necessarily isomorphic); the cases in which there are 2 or 3 forests are completely classified. (Almost all are trees plus isolated vertices.) [*Question*. Regarding these results as concerning the negative subgraphs of switchings of signed complete graphs, to what extent do they generalize to switchings of arbitrary signed simple graphs?] [B.D. Acharya (1981a) asked which simple graphs switch to forests, with partial results.] (**TG: Sw**)

2000a The size of switching classes with skew gains. *Discrete Math.* 215 (2000), 81–92. MR 1746450 (2001d:05074). Zbl 949.05039.

Introducing "skew gain graphs", which generalize gain graphs (see Zaslavsky (1989a)) to incorporate dynamic labelled 2-structures (see Ehrenfeucht and Rozenberg). Inversion is replaced by a gain-group antiautomorphism δ of period at most 2. Thus $\varphi(e^{-1}) = \delta(\varphi(e))$, while in switching by τ, one defines $\varphi^\tau(e; v, w) = \delta(\tau(v))\varphi(e; v, w)\tau(w)$. The authors find the size of a switching class $[\varphi]$ in terms of the centralizers and/or δ-centralizers of various parts of the image of φ_T, that is, φ switched to be the identity on a spanning tree T. The exact formulas depend on whether Γ is complete, or bipartite, or general, and on the choice of T (the case where $T \cong K_{1,n-1}$ being simplest). (**GG(Gen): Sw**)

2004a A characterization of acyclic switching classes of graphs using forbidden subgraphs. *SIAM J. Discrete Math.* 18 (2004), no. 1, 159–176. MR 2112496 (2005k:05205). Zbl 1071.05063.

Solves the problem raised by B.D. Acharya (1981a). (**TG: Sw**)

2007a Towards a characterization of bipartite switching classes by means of forbidden subgraphs. *Discuss. Math. Graph Theory* 27 (2007), no. 3, 471–483. MR 2412359 (2009b:05126). Zbl 1142.05042.

Partial results on the forbidden induced subgraphs for graph switching classes with no bipartite member. [Annot. 9 Sept 2010.] (**TG: Sw**)

2009a On involutions arising from graphs. In: Anne Condon *et al.*, eds., *Algorithmic Bioprocesses*, pp. 623–630. Springer, Berlin, 2007. MR 2762065 (2012j:20157). Zbl 1183.68407 (book).

Algebra related to the skewness, i.e., involutory anti-automorphisms, of skew gains on a graph. [Annot. 12 Sept 2017.] (**gg(Gen): Algeb**)

Jurriaan Hage, Tero Harju, & Elmo Welzl

2002a Euler graphs, triangle-free graphs and bipartite graphs in switching classes. In: *Graph Transformation* (Proc. First Int. Conf., Rome, 2002), pp. 148–160. Lect. Notes in Computer Sci., Vol. 2505. Springer-Verlag, London, 2002. MR 2049362 (no rev). Zbl 1028.68101.

Preliminary version of (2003a). [Annot. 9 Sept 2010.] (**TG: Sw**)

2003a Euler graphs, triangle-free graphs and bipartite graphs in switching classes. Special issue on ICGT 2002. *Fund. Inform.* 58 (2003), no. 1, 23–37. MR 2056589 (2005b:05206). Zbl 1054.05092.

Polynomial-time algorithms for whether a graph switching class contains a triangle-free, or bipartite, or Eulerian, member. (**TG: Sw**)

Per Hage

1979a Graph theory as a structural model in cultural anthropology. *Annual Rev. Anthropology* 8 (1979), 115–136.

"Signed graphs", pp. 120–124. "Structural duality", pp. 132–133. Other examples. [Annot. 2 Aug 2010.] (**SG, PsS: Bal, Fr, Clu Exp**)

Per Hage & Frank Harary

1983a *Structural Models in Anthropology.* Cambridge Univ. Press, Cambridge, Eng., 1983. MR 0738630 (86e:92002).

Signed graphs are treated in Ch. 3 and 6, marked (i.e., vertex-signed) graphs in Ch. 6. [Reviewed in Doreian (1985a).]
(**SG, PsS: Bal: Exp**)(**VS: Exp**)

1986a Some genuine graph models in anthropology. *J. Graph Theory* 10 (1986), no. 3, 353–361. MR 0856121 (87i:92061). Zbl 605.05042. (**PsS, SG: Exp**)

1987a *Exchange in Oceania.* Routledge and Kegan Paul, London, 1987. (**SG: Appl**)

Roland Häggkvist

See J.W. Grossman.

Mohammad Hassan Shirdareh Haghighi

See F. Motialah.

MohammadTaghi Hajiaghayi

See E.D. Demaine.

Mardjan Hakimi-Nezhaad

See also M. Ghorbani.

Mardjan Hakimi-Nezhaad & Modjtaba Ghorbani

2020a On the Estrada index of Seidel matrix. *Math. Interdisciplinary Res.* 5 (2020), no. 1, 43–54.

Cf. Askari, Iranmanesh, Das (2016a). Bounds. Lem. 3.1: Seidel matrix of composition of two graphs. *Conjectures*: $[-K_n]$ maximizes and $[+K_n]$ minimizes Estrada index of signed K_n. K_{P_n} (path) minimizes and K_{S_n} (star) minimizes for K_T, T a tree. [Annot. 9 Nov 2020.]
(**sg: KG: sw, Adj: Eig**)

F.D.M. Haldane

See J. Vannimenus.

Frank J. Hall

See also M. Arav and C.A. Eschenbach.

Frank J. Hall & Zhongshan Li

2007a Sign pattern matrices. In: Leslie Hogben, ed., *Handbook of Linear Algebra*, pp. 33-1–33-21. Discrete Math. Appl. Chapman & Hall/CRC Press, Boca Raton, 2007. MR 2279160 (2007j:15001) (book). Zbl 1122.15001. (**QM: sd**)

H. Tracy Hall

See S. Butler.

Peter Hall

See B. Xiao.

Joshua Hallam, Jeremy L. Martin, & Bruce E. Sagan

2019a Increasing spanning forests in graphs and simplicial complexes. *European J. Combin.* 76 (2019), 178–198. MR 3886522. Zbl 1402.05183. arXiv:1610.05093.

§4, "ISFs in multigraphs": The matroid is the Dowling geometry $\mathbf{F}(\mathbb{C}^* K_n^\bullet)$ or a submatroid. Dictionary: "Labeled n-multigraph" = simple $\mathbb{C}^*$-gain graph; "0-edge" = half edge; "ISF"' (increasing spanning forest) = increasing basis of $\mathbf{F}(\mathbb{C}^* K_n^\bullet)$; "perfectly labeled" = closed in $\mathbf{F}(\mathbb{C}^* K_n^\bullet)$. [Annot. 19 Oct 2019.] (**GG: Geom, matrd, Invar**)

Maureen Hallinan & David D. McFarland

1975a Higher order stability conditions in mathematical models of sociometric or cognitive structure. *J. Math. Sociology* 4 (1975), 131–148. MR 0421740 (54 #9734). Zbl 322.92024.

§ "Signed directed graphs" (pp. 134–141): Tendency towards "transitivity" (balance or clusterability) in signed digraphs. The impetus for single-arc change (sign change, or deletion or introduction) cannot be determined by triangles alone (Props. 1.1–1.3). [Annot. 23 Nov 2012.] (**SD: bal, clu, PsS**)

Mark D. Halsey

1987a Line-closed combinatorial geometries. *Discrete Math.* 65 (1987), no. 3, 245–248. MR 0897649 (88g:05043). Zbl 613.05014.

Dowling lattices are line closed; thus line closure does not imply vector representability. [Annot. 8 Apr 2016.] (**gg: Matrd: Str**)

Ginji Hamano

2016a Existence of a regular unimodular triangulation of the edge polytopes of finite graphs. *Comment. Math. Univ. St. Pauli* 65 (2016), no. 2, 85–96. MR 3675806. Zbl 1379.52015. arXiv:1405.3341.

Disjoint odd circles are important for edge polytope of all-extraverted graph. [*Question.* How does this apply to edge polytopes of general bidirected graphs?] [Annot. 6 Jul 2022.] (**par: Circ, Geom**)

Saira Hameed

See U. Ahmad.

Shahul Hameed K

See also K.A. Germina, A. Mathew, K.O. Ramakrishnan, R.T. Roy, and T.V. Shijin.

Shahul Hameed K & K.A. Germina

† 2012a Balance in gain graphs – A spectral analysis. *Linear Algebra Appl.* 436 (2012), no. 5, 1114–1121. MR 2890908. Zbl 1236.05096.

Thm. 2.4: For a field F, an $F^\times$-gain graph Φ is balanced iff $\operatorname{Spec}_F A(\Phi) = \operatorname{Spec}_F A(\|\Phi\|)$—generalizing B.D. Acharya (1980a). Thms. 2.2, 3.1: a formula and a recurrence (involving circles) for the characteristic polynomial. Corollaries for signed graphs. [Annot. 12 Jul 2019.] (**GG: Eig, Bal, SG**)

2012b On composition of signed graphs. *Discuss. Math. Graph Theory* 32 (2012), no. 3, 507–516. MR 2974034. Zbl 1257.05056.

Balance and eigenvalues in a definition of composition (lexicographical product). Only the composition of gains of the type balanced[identity] is balanced. [*Cf.* Brunetti, Cavaleri, and Donno (2019a).] [Annot. 20 Apr 2019.] (**SG: Bal, Eig: Adj, Lap**)

2012c Balance in certain gain graph products. Set Valuations, Signed Graphs and Geometry (IWSSG-2011, Int. Workshop, Mananthavady, Kerala, 2011). *J. Combin. Inform. System Sci.* 37 (2012), no. 2-4, 205–216. Zbl 1301.05280.

Definitions of Cartesian, lexicographic, Cvetković (NEPS) products, with balance, adjacency matrices, eigenvalues. In part, generalizes to gain graphs portions of (2012b) and Germina, Hameed, and Zaslavsky (2011a). For balance of lex product see (2012b). Cvetković product NEPS$(\Phi_1, \ldots, \Phi_\nu)$ preserves balance. [Annot. 20 Apr 2019, rev 12 Jul 2019.] (**GG: Bal, Adj, Eig**)

20xxa On bounds for the eigenvalues and energy of signed graphs. Submitted. (**SG: Eig**)

Shahul Hameed K, Albin Mathew, Germina K A, & Thomas Zaslavsky

20xxa Vector valued switching in signed graphs. *Commun. Combin. Optim.* (in press and online). arXiv:2208.00149.

Vector-valued switching of Σ, or k-switching, means $\sigma(uv) \mapsto \sigma^\zeta(uv) = \operatorname{sgn}(\zeta(u) \cdot \zeta(v))$, where $\zeta : V \to \{\pm1, 0\}^k \setminus \mathbf{0}$. Main question: "balancing dimension" $\operatorname{bdim}(\Sigma) := \min\{k : (\exists\zeta)\ \Sigma^\zeta \text{ is balanced}\}$. Variants such as injective ζ and $\zeta : V \to \{\pm1\}^k$. Solved for examples, e.g. $\operatorname{bdim}(C_n^-) = 2$ $(n > 3)$, $= 3$ $(n = 3)$, where C_n^- = negative circle. A successor: Mathew and Germina (20xxa). [Annot. 16 Jun 2023.] (**SG: Sw(Gen), Bal**)

Shahul Hameed K., Viji Paul, & K.A. Germina

2015a On co-regular signed graphs. *Australasian J. Combin.* 62(1) (2015), 8–17. MR 3337173. Zbl 1321.05103.

See Viji P (2012a) for definitions. (**SG**)

Shahul Hameed Koombail & Ramakrishnan K O

2024a On transinverse of matrices and its applications. *J. Algebraic Sys.* 12 (2024), no. 1, 135–147. (**GG: Incid, Bal**)

20xxa Balance theory: An extension to conjugate skew gain graphs. *Commun. Combin. Optim.* (in press). (**GG(Gen): Bal: Adj, Lap**)

Shahul Hameed Koombail, Ramakrishnan K O, & Biju K

20xxa Distance matrices for conjugate skew gain graphs. In preparation. (**GG(Gen)**)

Shahul Hameed K, Remna K P, Divya T, Biju K, Rajeevan P, Santhosh G O, & Ramakrishnan K O

20xxa On the metric dimension of signed graphs. Submitted. arXiv:2106.12539.

For distance-compatible Σ (*cf.* Hameed, Shijin, *et al.* (2021a)), let $\dim \Sigma$ = min size of signed-distance resolving set. Solved for star, circle; partial results for K_n, wheels, trees. [Annot. 13 Aug 2021.] (**SG**)

Shahul Hameed K, Roshni T Roy, Soorya P, & Germina K A

2023a On the characteristic polynomial of skew gain graphs. *Southeast Asian Bull. Math.* 47 (2023), no. 1, 63–72. MR 4563099. arXiv:2009.08708. (**GG(Gen): Adj: Eig**)

Shahul K. Hameed, T.V. Shijin, P. Soorya, K.A. Germina, & Thomas Zaslavsky

2021a Signed distance in signed graphs. *Linear Algebra Appl.* 608 (2021), 236–247. MR 4143538. Zbl 1458.05054. arXiv:2005.06202.

Set $\sigma_{max}(u,v) := \max\{\sigma(P) : \text{shortest } uv\text{-path } P\}$, $\sigma_{\min}$ similarly. Signed distance: $d_{max}(u,v) := \sigma_{\max}(u,v)d(u,v)$, $d_{\min}$. Signed distance matrices $D^{\max}$, $D^{\min}$. If $d_{\max} = d_{\min}$ ("distance compatibility"), $D^{\pm} := D^{\max}$. Associated signed complete graphs. Thm. 3.5: Σ balanced iff any $D(\Sigma)$ cospectral with $D(|\Sigma|)$.

Σ is distance compatible if geodetic, balanced, or antibalanced. Thm. 5.1: Bipartite Σ is distance-compatible iff balanced. Ex. 5.4: Distance compatible but not geodetic, balanced, or antibalanced.

[*Cf.* Roy, Germina *et al.* (2024a), Shijin, Soorya, *et al.* (2020a). Generalized by Samanta and Kannan (2022a), Shijin and Germina (20xxa).] [Annot. 11 Oct 2020.]

(**SG: WG, Adj(Gen), Bal, Eig, Sw**)

Hasti Hamidzade & Dariush Kiani

2010a Erratum to "The lollipop graph is determined by its Q-spectrum". *Discrete Math.* 310 (2010), no. 10-11, 1649. MR 2601277 (2011c:05194). Zbl 1210.05080.

Corrected proof of Y.P. Zhang, Liu, Zhang, and Yong (2009a), Thm. 3.3. [Annot. 16 Oct 2011.] (**par: Lap: Eig**)

Peter L. Hammer

See also E. Balas, C. Benzaken, E. Boros, J.-M. Bourjolly, Y. Crama, and A. Fraenkel.

1974a Boolean procedures for bivalent programming. In: P.L. Hammer and G. Zoutendijk, eds., *Mathematical Programming in Theory and Practice* (Proc. NATO Adv. Study Inst., Figueira da Foz, Portugal, 1972), pp. 311–363. North-Holland, Amsterdam, and American Elsevier, New York, 1974. MR 0479387 (57 #18817). Zbl 335.90034 (book). (**sg: ori**)

1977a Pseudo-Boolean remarks on balanced graphs. In: L. Collatz, G. Meinardus, and W. Wetterling, eds., *Numerische Methoden bei Optimierungsaufgaben, Band 3: Optimierung bei graphentheoretischen und ganzzahligen Problemen* (Tagung, Oberwolfach, 1976), pp. 69–78. Int. Ser. Numer. Math., Vol. 36. Birkhäuser, Basel, 1977. MR 0465947 (57 #5833). Zbl 405.05054. (**SG: Bal**)

P.L. Hammer, C. Benzaken, & B. Simeone

1980a Graphes de conflit des fonctions pseudo-booléennes quadratiques. In: P. Hansen and D. de Werra, eds., *Regards sur la Théorie des Graphes* (Actes du Colloq., Cerisy, 1980), pp. 165–170. Presses Polytechniques Romandes, Lausanne, Switz., 1980. MR 0614299 (82d:05054) (book). (**sg**)

P.L. Hammer, T. Ibaraki, & U. Peled

1980a Threshold numbers and threshold completions. In: M. Deza and I.G. Rosenberg, eds., *Combinatorics 79* (Proc. Colloq., Montreal, 1979), Part II. *Ann. Discrete Math.* 9 (1980), 103–106. MR 0597360 (81k:05092). Zbl 443.05064.

(par: ori)

1981a Threshold numbers and threshold completions. In: Pierre Hansen, ed., *Studies on Graphs and Discrete Programming* (Proc. Workshop, Brussels, 1979), pp. 125–145. North-Holland Math. Studies, 59. Ann. Discrete Math., 11. North-Holland, Amsterdam, 1981. MR 0653822 (83m:90062). Zbl 0482.05060.

See description of Thm. 8.5.2 in Mahadev and Peled (1995a). **(par: ori)**

P.L. Hammer & N.V.R. Mahadev

1985a Bithreshold graphs. *SIAM J. Algebraic Discrete Methods* 6 (1985), 497–506. MR 0791177 (86h:05093). Zbl 579.05052.

An auxiliary signed graph on E is a proof tool. See description in §8.3 of Mahadev and Peled (1995a). [Later work by Mahadev and Peled (1988a), Hammer, Mahadev, and Peled, R. Petreschi and T. Calamoneri.] [Annot. rev. 22 Mar 2017.] **(SG: Appl: Bal)**

Peter L. Hammer, N.V.R. Mahadev, & Uri N. Peled

1989a Some properties of 2-threshold graphs. *Networks* 19 (1989), 17–23. MR 0973562 (89m:05096). Zbl 671.05059.

Uses the auxiliary signed graph of Hammer and Mahadev (1985a). [Annot. 22 Mar 2017.] **(SG: Appl: Bal)**

1993a Bipartite bithreshold graphs. *Discrete Math.* 119 (1993), 79–96. MR 1234060 (94e:05234). Zbl 790.05082.

Uses the auxiliary signed graph of Hammer and Mahadev (1985a). [Annot. 17 Mar 2017.] **(SG: Appl: Bal)**

Peter L. Hammer & Sang Nguyen

1977a APOSS. A partial order in the solution space of bivalent programs. In: J. Rose and C. Bilciu, eds., *Modern Trends in Cybernetics and Systems* (Proc. Third Int. Congress, Bucharest, 1975), Vol. I, pp. 869–883. Springer, Berlin, 1977. MR 0475847 (57 #15430). Zbl 414.90063. **(sg: ori)**

1979a A partial order in the solution space of bivalent programs. In: Nicos Christofides, Aristide Mingozzi, Paolo Toth, and Claudio Sandi, eds., *Combinatorial Optimization*, Ch. 4, pp. 93–106. Wiley, Chichester, 1979. MR 0557004 (82a:90099) (book). Zbl 414.90063. **(sg: ori)**

J. Hammann

See E. Vincent.

P.R. Hampiholi, H.S. Ramane, Shailaja S. Shirkol, Meenal M. Kaliwal, & Saroja R. Hebbar

2017a A note on signed semigraphs. *Int. J. Comput. Appl. Math.* 12 (2017), no. 3, 887–898.

Semigraph := (V, E); edge $e = (v_1, v_2, \ldots, v_k) = (v_k, \ldots, v_2, v_1)$, i.e., symmetrically ordered subset of V (due to E. Sampathkumar). Sign $\sigma(e) = (-1)^k$. Parity and sign are not distinguished. Balance results appear to be immediate corollaries of known signed-graph facts. Vertices may also be signed. [Annot. 11 May 2018.] **(Sgnd: Bal, VS)**

Miaomiao Han

See J.-B. Liu and X.Y. Yuan.

Wei Han

See S.Y. Wang.

Phil Hanlon

1984a The characters of the wreath product group acting on the homology groups of the Dowling lattices. *J. Algebra* 91 (1984), 430–463. MR 0769584 (86j:05046). Zbl 557.20009. **(gg: Matrd: Aut)**

1988a A combinatorial construction of posets that intertwine the independence matroids of B_n and D_n. Manuscript, 1988.

Computes the Möbius functions of posets obtained from Lat $\mathbf{F}(\pm K_n^\circ)$ by discarding those flats with unbalanced vertex set in a given lower-hereditary list. Examples include Lat $\mathbf{F}(\pm K_n^{(k)})$, the exponent denoting the addition of k negative loops. Generalized and superseded by Hanlon and Zaslavsky (1998a). **(sg: Matrd: Gen: Invar)**

1991a The generalized Dowling lattices. *Trans. Amer. Math. Soc.* 325 (1991), 1–37. MR 1014249 (91h:06011). Zbl 748.05043.

The lattices are based on a rank, n, a group, and a meet sublattice of the lattice of subgroups of the group. The Dowling lattices are a special case. **(gg: Matrd: Gen: Invar)**

1996a A note on the homology of signed posets. *J. Algebraic Combin.* 5 (1996), 245–250. MR 1394306 (97f:05194). Zbl 854.06004.

1394306 Partial summary of Fischer (1993a). **(Sgnd)**

Phil Hanlon & Thomas Zaslavsky

1998a Tractable partially ordered sets derived from root systems and biased graphs. *Order* 14 (1997–98), 229–257. MR 1634902 (2000a:06016). Zbl 990.03811.

Computes the characteristic polynomials (Thm. 4.1) and hence the Möbius functions (Cor. 4.4) of posets obtained from Lat $\mathbf{F}(\Omega)$, Ω a biased graph, by discarding those flats with unbalanced vertex set in a given lower-hereditary list. Examples include Lat $\mathbf{F}(\mathfrak{G}K_n^{(k)})$ where $\mathfrak{G}$ is a finite group, the exponent denoting the addition of k unbalanced loops. The interval structure, existence of a rank function, covering pairs, and other properties of these posets are investigated. There are many open problems. Simplifies, then generalizes, Hanlon (1988a).
(GG: Matrd, Gen: Invar, Str, Col)

Pierre Hansen

See also M. Aouchiche, V. Devloo, and C.S. Oliveira.

1978a Labelling algorithms for balance in signed graphs. In: *Problèmes combinatoires et théorie des graphes* (Colloq. Int., Orsay, 1976), pp. 215–217. Colloques Int. du CNRS, 260. Editions du C.N.R.S., Paris, 1978. MR 0539978 (80m:68057). Zbl 413.05060.

§1: Algorithm 1 labels vertices of a signed graph to detect imbalance and a negative circle if one exists. [It is equivalent to switching a maximal forest to all positive and looking for negative edges. Independently discovered by Harary and Kabell (1980a).] §2: Algorithm 2 is the unweighted case of the algorithm of (1984a). Path balance in a signed digraph is discussed. §3: The frustration index of a signed graph is bounded below by the negative-circle packing number, which can be crudely bounded by Alg. 1. (**SG, SD: Bal, Fr: Algor, sw**)

1979a Methods of nonlinear 0–1 programming. In: P.L. Hammer, E.L. Johnson, and B.H. Korte, eds., *Discrete Optimization II* (Proc., Banff and Vancouver, 1977), pp. 53–70. Ann. Discrete Math., Vol. 5. North-Holland, Amsterdam, 1979. MR 0558567 (no rev). Zbl 426.90063.

See pp. 58–59. (**SG: Bal: Exp**)

1983a Recognizing sign solvable graphs. *Discrete Appl. Math.* 6 (1983), 237–241. MR 0712924 (84i:68112). Zbl 524.05048.

Improves the characterization by Maybee (1981a) of sign-solvable digraphs with an eye to more effective algorithmic recognition. Thm. 2.2: A signed digraph D is sign solvable iff its positive subdigraph is acyclic and each strongly connected component has a vertex that is the terminus of no negative, simple directed path. §3: "An algorithm for sign solvability" in time $O(\#V\,\#E)$. (**SD: QSol: Algor**)

1984a Shortest paths in signed graphs. In: A. Burkard *et al.*, eds., *Algebraic Methods in Operations Research*, pp. 201–214. North-Holland Math. Stud., 95. Ann. of Discrete Math., 19. North-Holland, Amsterdam, 1984. MR 0780022 (86i:05086). Zbl 567.05032.

Algorithm to find shortest walks of each sign from vertex x_1 to each other vertex, in a signed digraph with positive integral(?) weights (i.e., lengths) on the edges. Applied to digraphs with signed vertices and edges; N-balance in signed graphs; sign solvability. The problem for (simple) paths is discussed [which is solvable by any min-weight parity path algorithm; see the notes on Grötschel and Pulleyblank (1981a)]. (**SD, WD: Paths, VS, Bal, QSol: Algor**)

Pierre Hansen & Claire Lucas

2009a An inequality for the signless Laplacian index of a graph using the chromatic number. *Graph Theory Notes N.Y.* 57 (2009), 39–42. MR 2666279 (2011c:-05195). (**par: Lap: Eig**)

2010a Bounds and conjectures for the signless Laplacian index of graphs. *Linear Algebra Appl.* 432 (2010), no. 12, 3319–3336. MR 2639286 (2011m:05173). Zbl 1214.05079.

[See Liu and Lu (2014a) for solution of a conjecture.] (**sg: par: Eig**)

Pierre Hansen & Bruno Simeone

1986a Unimodular functions. *Discrete Appl. Math.* 14 (1986), 269–281. MR 0848659 (88a:90138). Zbl 597.90058.

Three types of relatively easily maximizable pseudo-Boolean function ("unimodular" and two others) are defined. For quadratic pseudo-Boolean functions f, the three types coincide; f is unimodular iff an associated signed graph is balanced (Thm. 3). Thus one can quickly recognize unimodular quadratic functions, although not unimodular functions in general. If the graph is a tree, the function can be maximized in linear time. (**SG: Bal, Algor**)

Christopher R.H. Hanusa

See Chaiken, Hanusa, and Zaslavsky (2019a).

Fei Hao, Stephen S. Yau, Geyong Min, & Laurence T. Yang

2014a Detecting k-balanced trusted cliques in signed social networks. *IEEE Internet Computing* 18 (2014), no. 2, 24–31.

Dictionary: "k-balanced trusted clique" = all-positive k-clique in Σ. The problem: detect all such cliques. [Annot. 27 Nov 2018.] (**SG: Clu: Algor**)

Rong Xia Hao

See also S.-J. He.

Rong Xia Hao & Yan Pei Liu

2010a Auxiliary graphs of projective planar signed graphs. (In Chinese.) *J. Systems Sci. Math. Sci.* 30 (2010), no. 9, 1251–1258. MR 2785248 (2012c:05093). Zbl 1240.05139.

Σ is projective planar iff an auxiliary graph is balanced. [The auxiliary graph may be a tree. It may have order linear in that of Σ.] [Annot. 25 Apr 2012.] (**SG: Top**)

Xiao Hui Hao & Bao Feng Li

2008a The quasi-Laplacian spectral radius of a graph. (In Chinese.) *Math. Practice Theory* 38 (2008), no. 4, 158–160. MR 2435555 (no rev). Zbl 1174.05438. (**par: Lap: Eig**)

Xiao Hui Hao & Li Jun Zhang

2009a The largest eigenvalue of the quasi-Laplacian matrix of a connected graph. (In Chinese.) *Math. Pract. Theory* 39 (2009), no. 7, 178–181. MR 2553871 (no rev). Zbl 1212.05154. (**par: Lap: Eig**)

Masaaki Harada & Akihiro Munemasa

2012a On the classification of weighing matrices and self?orthogonal codes. *J. Combin. Designs* 20 (2012), no. 1, 40–57. MR 2864617. Zbl 1252.05026. arXiv:-1011.5382.

Signed bipartite graphs with $B(\Sigma)B^{\mathrm{T}}(\Sigma) = 5I$. [Annot. 14 Oct 2020.] (**sg: Adj**)

Frank Harary

See also L.W. Beineke, A. Blass, F. Buckley, D. Cartwright, G. Chartrand, O. Frank, and P. Hage.

†† 1953a On the notion of balance of a signed graph. *Michigan Math. J.* 2 (1953–1954), 143–146 and addendum preceding p. 1. MR 0067468 (16, 733). Zbl 056.42103 (56, p. 421c).

Σ The main theorem (Thm. 3) characterizes balanced signings as those for which there is a bipartition of the vertex set such that an edge is positive iff it lies within a part [I call this a Harary bipartition]. Thm. 2: A signing of a simple [or a loop-free] graph is balanced iff, for each pair of vertices, every path joining them has the same sign. The generating function for counting nonisomorphic signed simple graphs with n vertices by numbers of positive and negative edges is $g_n(x + y)$ where $g_n(x)$ is the g.f. of nonisomorphic simple graphs.

[The birth of signed graph theory. Although Thm. 3 was anticipated by König (1936a) (Thm. X.11, for finite and infinite graphs) without the terminology of signs, here is the first recognition of the crucial fact that labelling edges by elements of a group—specifically, the sign group—can lead to a general theory.] [Annot. ca. 1977. Rev. 20 Jan 2010.] [*Cf.* Whiteley (1991a).] [Annot. 12 Jun 2012.] (**SG: Bal, Enum**)

1955a On local balance and N-balance in signed graphs. *Michigan Math. J.* 3 (1955–1956), 37–41. MR 0073170 (17, 394). Zbl 070.18502 (70, p. 185b).

Σ is (locally) balanced at a vertex v if every circle on v is positive; then Thm. 3′: Σ is balanced at v iff every block containing v is balanced. Σ is N-balanced if every circle of length $\leq N$ is positive; Thm. 2 concerns characterizing N-balance. Lemma 3: For each circle basis, Σ is balanced iff every circle in the basis is positive. [This strengthens König (1936a) Thm. 13 for finite graphs.] (**SG: Bal**)

1957a Structural duality. *Behavioral Sci.* 2 (1957), 255–265. MR 0134799 (24 #B851).

"Antithetical duality" (pp. 260–261) introduces antibalance. Remarks on signed and vertex-signed graphs are scattered about the succeeding pages. (**SG: Bal, Par**)

1958a On the number of bi-colored graphs. *Pacific J. Math.* 8 (1958), 743–755. MR 0103834 (21 #2598). Zbl 084.19402 (84, p. 194b).

§6: "Balanced signed graphs". (**SG: Bal: Enum**)

1959a Graph theoretic methods in the management sciences. *Management Sci.* 5 (1959), 387–403. MR 0108387 (21 #7103). Repr. in: Samuel Leinhardt, ed., *Social Networks: A Developing Paradigm*, pp. 371–387. Academic Press, New York, 1977.

Pp. 400–401: List of characterizations of balance. (**SG: Bal: Exp**)

1959b On the measurement of structural balance. *Behavioral Sci.* 4 (1959), 316–323. MR 0112850 (22 #3696).

Proposes to measure imbalance by (i) $\beta(\Sigma)$, the proportion of positive circles ("degree of balance") from Cartwright and Harary (1956a), (ii) the frustration index $l(\Sigma)$ (here called "line index of balance") [*cf.* Abelson and Rosenberg (1958a)], i.e., the smallest number of edges whose deletion or equivalently (Thm. 7) negation results in balance, and (iii) the frustration number $l_0(\Sigma)$ ("point index"). For β restricted to unbalanced blocks with cyclomatic number ξ: Thm. 4: $\min\beta \leq (\xi-1)/(\xi-1+2^{\xi-1})$. Thm. 5: $\max\beta \geq 1-2/(\xi+1)$ (e.g., a ladder with $\xi+1$ rungs and one rung negative). Cors.: $\min\beta \to 0$, $\max\beta \to 1$ as $\xi \to \infty$. *Conjecture.* The bounds are best possible. [I know of no work on this.] Thm. 6 (contributed by J. Riordan): Asymptotically, $\beta(-K_n) - \frac{1}{2} \sim \frac{1}{2}(-1/e)^n$. [Annot. ≤1980. Rev. 20 Jan 2010.] (**SG: Fr**)

1960a A matrix criterion for structural balance. *Naval Res. Logistics Quarterly* 7, No. 2 (June, 1960), 195–199. Zbl 091.15904 (91, p. 159d).

First explicit appearance of the incidence matrix $\mathrm{H}(\Sigma)$, called J. Thm. 2 (Heller and Tompkins (1956a), Hoffman and Gale (1956a)): Σ is balanced iff $\mathrm{H}(\Sigma)$ is totally unimodular. Cor.: The unoriented incidence matrix of Γ is totally unimodular iff Γ is bipartite. [Annot. 10 Nov 2008.] (**SG: Bal, Incid: Exp**)

Thm. 3: A necessary and sufficient condition that a subdeterminant

is 0 in H(Σ), provided Σ is balanced. [Zaslavsky (1981a) §8A evaluates subdeterminants for any Σ.] [Annot. 20 Jan 2010.] (**SG: Bal, Incid**)

1970a Graph theory as a structural model in the social sciences. In: Bernard Harris, ed., *Graph Theory and Its Applications*, pp. 1–16. Academic Press, New York, 1970. MR 0263676 (41 #8277). Zbl 224.05129.

1971a Demiarcs: An atomistic approach to relational systems and group dynamics. *J. Math. Sociology* 1 (1971), 195–205. MR 0371738 (51 #7955).

Signed, oriented half edges, applied to represent interpersonal relations. (**PsS: SD**)

1979a Independent discoveries in graph theory. In: Frank Harary, ed., Topics in Graph Theory (Proc. Conf., New York, 1977). *Ann. New York Acad. Sci.* 328 (1979), 1–4. MR 0557880 (81a:05001). Zbl 465.05026.

1980a Some theorems about graphs from social sciences. In: *Proceedings of the West Coast Conference on Combinatorics, Graph Theory and Computing* (Arcata, Calif., 1979), pp. 41–47. Congressus Numerantium, XXVI. Utilitas Math. Publ. Inc., Winnipeg, Man., 1980. MR 0593895 (81m:05118). Zbl 442.92027.

(**SG: Bal: History, Exp**)

1980b Graph theoretic models. *Theor. Computer Sci.* 11 (1980), 117–121. MR 0572211 (81g:68093). Zbl 0452.05053.

Mentions the balance-detection algorithm of Harary and Kabell (1980a). [Annot. 15 Aug 2017.] (**SG: Bal: Algor: Exp**)

1981a Structural models and graph theory. In: Harvey J. Greenberg and John S. Maybee, eds., *Computer-Assisted Analysis and Model Simplification* (Proc. Sympos., Boulder, Col., 1980), pp. 31–58. Discussion, pp. 103–111. Academic Press, New York, 1981. MR 0617930 (82g:00016) (book). Zbl 495.93001 (book).

See remarks of Bixby (p. 111). (**SG, VS, SD: Bal, Algor: Exp**)

1983a Consistency theory is alive and well. *Personality and Social Psychology Bull.* 9 (1983), 60–64.

Historical remarks. E.g., it was Osgood and Tannenbaum (1955a) that inspired Harary to study vertex signings (Beineke and Harary (1978a), (1978b)). (**PsS, SG: Exp**)

1985a The reconstruction conjecture for balanced signed graphs. In: B.R. Alspach and C.D. Godsil, eds., *Cycles in Graphs*, pp. 439–442. Ann. Discrete Math., Vol. 27. North-Holland Math. Stud., Vol. 115. North-Holland, Amsterdam, 1985. MR 0821544 (87d:05122). Zbl 572.05048.

Reconstruction from the multiset of vertex-deleted subgraphs. Σ^+ is reconstructible if Σ is connected and balanced and not all positive or all negative. (**SG: Bal**)

F. Harary & G. Gupta

1997a Dynamic graph models. *Math. Computer Modelling* 25 (1997), no. 7, 79–87. MR 1452306 (98b:05092). Zbl 879.68085.

§3.9, "Signed graphs", mentions that deletion index = negation index

(Harary (1959b)). **(SG: Fr: Exp)**

Frank Harary & Jerald A. Kabell

1980a A simple algorithm to detect balance in signed graphs. *Math. Social Sci.* 1 (1980/81), 131–136. MR 0590724 (81j:05098). Zbl 497.05056.

Equivalent to switching so a spanning tree is all positive, then searching for a negative edge. [Independently discovered by Hansen (1978a)—and rather obvious by switching.] **(SG: Bal, Algor)**

1981a Counting balanced signed graphs using marked graphs. *Proc. Edinburgh Math. Soc.* (2) 24 (1981), 99–104. MR 0625282 (83a:05072). Zbl 476.05043.

Generating functions and partially explicit formulas for connected and all isomorphism types of vertex-signed, and balanced signed, graphs of order n. [Annot. 15 Aug 2017.] **(SG, VS: Enum)**

Frank Harary & Helene J. Kommel

1978a Matrix measures for transitivity and balance. *J. Math. Sociology* 6 (1978/79), 199–210. MR 0539378 (81a:05056). Zbl 408.05028.

§2, "Balance measures in signed graphs": Balance in Σ is measured by the proportion of positive triangles, or quadrilaterals, as computed from small powers of A and $\#A$. §3, "Balance in signed digraphs": Similar measures for a signed digraph using digons or triangles. §4, "Example": A signed digraph of order 5. [Annot. 10 Nov 2008, 2 Sept 2013.] **(SG: Fr, Adj)**

1979a The graphs with only self-dual signings. *Discrete Math.* 26 (1979), 235–241. MR 0535250 (80h:05047). Zbl 408.05045. **(SG, VS: Aut)**

Frank Harary, Meng-Hiot Lim, Amit Agarwal, & Donald C. Wunsch

2004a Algorithms for derivation of structurally stable Hamiltonian signed graphs. *Int. J. Computer Math.* 81 (2004), no. 11, 1349–1356. MR 2172923 (no rev). Zbl 1065.05049.

Thm. 1: The sizes of cuts in K_n. Thm. 2: A subgraph of a balanced signed graph is balanced. [Annot. 12 Sept 2009.] **(SG: Bal)**

Frank Harary, Meng-Hiot Lim, & Donald C. Wunsch

2002a Signed graphs for portfolio analysis in risk management. *IMA J. Management Math.* 13 (2002), no. 3, 201–210. Zbl 1065.91025.

"Assets" (vertices) have positive or negative correlation (edges of K_n). Switching is a means of hedging risk, which is highest with all edges positive. Imbalance indicates unpredictability; measured by the proportion of positive triangles. [An unconvincing system.] §5: "Balance analysis case study". *Cf.* Vasanthi, Arumugam, *et al.* (2015a), Perina, Buckley, and Nagar (2017a) Mansano, Allem, *et al.* (20xxa). [Annot. 10 Sept 2009, 12 Feb 2022.] **(SG: KG: Appl: Bal, Sw, Exp)**

Frank Harary & Bernt Lindström

1981a On balance in signed matroids. *J. Combin. Inform. System. Sci.* 6 (1981), 123–128. MR 0630233 (83i:05024). Zbl 474.05021.

Thm. 1: The number of balanced signings of matroid M is $\leq 2^{\mathrm{rk}(M)}$, with equality iff M is binary. Thm. 3: Minimal deletion and negation sets coincide for all signings of M iff M is binary. Thm. 5: For connected binary M, a signing is balanced iff every circuit containing a fixed point is balanced. **(Sgnd: Matrd: Bal, Fr)**

Frank Harary, Bernt Lindström, & Hans-Olov Zetterström

1982a On balance in group graphs. *Networks* 12 (1982), 317–321. MR 0671831 (84a:-05055). Zbl 496.05052.

Implicitly characterizes balance and balancing sets in a gain graph Φ by switching (proof of Thm. 1). [For balance, see also Acharya and Acharya (1986a), Zaslavsky (1977a) and ((1989a), Lemma 5.3. For abelian gains, see also Gargano and Quintas (1985a). In retrospect we can see that the characterization of balanced gains is as the 1-coboundaries with values in a group, which for abelian groups is essentially classical.] Thm. 1: The number of balanced gain functions. Thm. 2: Any minimal deletion set is an alteration set. Thm. 3: $l(\Phi) \leq m(1 - \#\mathfrak{G}^{-1})$. Thm. 4: $l(\Sigma) \leq \frac{1}{2}(m - \frac{n-1}{2})$, with strict inequality if not all degrees are even. [Compare with Akiyama, Avis, Chvátal, and Era (1981a), Thm. 1.]

(GG, SG: sw(Bal), Enum(Bal), Fr)

Frank Harary, J. Richard Lundgren, & John S. Maybee

1985a On signed digraphs with all cycles negative. *Discrete Appl. Math.* 12 (1985), 155–164. MR 0808456 (87g:05108). Zbl 586.05019.

Which digraphs D can be signed so that every cycle is negative? Three types of example. Type 1: The vertices can be numbered $1, 2, \ldots, n$ so that the downward arcs are just $(2,1), (3,2), \ldots, (n, n-1)$. (Strong "upper" digraphs; Thm. 2.) Type 2: No cycle is covered by the remaining cycles ("free cyclic" digraphs). This type includes arc-minimal strong digraphs. Type 3: A symmetric digraph, iff the underlying graph Γ is bipartite and no two points on a common circle and in the same color class are joined by a path outside the cycle (Thm. 10; proved by signing Γ via Zaslavsky (1981b)). [Further work in Chaty (1988a).]

(SD: Bal, SG)

Frank Harary, Robert Z. Norman, & Dorwin Cartwright

1965a *Structural Models: An Introduction to the Theory of Directed Graphs*. Wiley, New York, 1965. MR 0184874 (32 #2345). Zbl 139.41503 (139, p. 415c).

In Ch. 10, "Acyclic digraphs": "Gradable digraphs", pp. 275–280. That means a digraph whose vertices can be labelled by integers so that $f(w) = f(v) + 1$ for every arc (v, w). [Equivalently, the Hasse diagram of a graded poset.] [Characterized by Topp and Ulatowski (1987a).]

(GD: bal, Exr)

Ch. 13: "Balance in structures". "Criteria for balance", pp. 340–346 (*cf.* Harary (1953a)); local balance (Harary (1955a))). "Measures of structural balance", pp. 346–352: "degree of balance" (proportion of balanced circles; Cartwright and Harary (1956a)); "line-index for balance" [frustration index] (Abelson and Rosenberg (1958a), Harary (1959b)).

"Limited balance", pp. 352–355. Harary (1955a)); also: Adjacency matrix $A(D, \sigma)$ of a signed digraph: entries are $0, \pm 1$. The "valency matrix" is the Abelson–Rosenberg (1958a) adjacency matrix R. Thm. 13.8: Entries of $(R - pI)^l$ show the existence of (undirected) walks of length l of each sign between pairs of vertices. [Strengthened in Zaslavsky (2010b), Thm. 2.1.]

"Cycle-balance and path-balance", pp. 355–358: here directions of arcs are taken into account. E.g., Thm. 13.11: Every cycle is positive iff each strong component is balanced as an undirected graph.

(SG: Bal, Fr, Adj: Exp, Exr)(SD: Bal, Exr)

1968a *Introduction a la théorie des graphes orientés. Modèles structuraux.* Dunod, Paris, 1968. Zbl 176.22501 (176, p. 225a).

French edition of (1965a). **(GD: bal, Exr)**
(SG: Bal, Fr, Adj: Exp, Exr)(SD: Bal, Exr)

Frank Harary & Edgar M. Palmer

1967a On the number of balanced signed graphs. *Bull. Math. Biophysics* 29 (1967), 759–765. Zbl 161.20904 (161, p. 209d). **(SG: Bal: Enum)**

1973a *Graphical Enumeration.* Academic Press, New York, 1973. MR 0357214 (50 #9682). Zbl 266.05108.

Four exercises and a remark concern signed graphs, balanced signed graphs, and signed trees. **(SG: Enum, Bal)**

1977a (As "F. Kharari and È. Palmer") *Perechislenie grafov.* "Mir", Moscow, 1977. MR 0447038 (56 #5353).

Russian translation of (1973a). **(SG: Enum, Bal)**

Frank Harary, Edgar M. Palmer, Robert W. Robinson, & Allen J. Schwenk

1977a Enumeration of graphs with signed points and lines. *J. Graph Theory* 1 (1977), 295–308. MR 0465932 (57 #5818). Zbl 379.05035.

See Bender and Canfield (1983a). **(SG, VS: Enum)**

Frank Harary & Michael Plantholt

1983a The derived signed graph of a digraph. *Expositiones Math.* 1 (1983), no. 4, 343–347. MR 0782975 (86h:05056). Zbl 525.05030.

L_{HP} A digraph D gives a signed line graph $\Lambda_{HP}(D)$ with $V_{HP} := E(D)$ and edges $+ef$ if e, f have the same head, $-ef$ if e, f have the same tail. [The negative part of $\Lambda(+D)$ in Zaslavsky (2010b), (2012c), (20xxa) with extraverted edges made positive and introverted edges negative.] [Annot. 4 Sept 2010, 17 Jan 2012.] **(SG: LG, Bal)**

Frank Harary & Geert Prins

1959a The number of homeomorphically irreducible trees, and other species. *Acta Math.* 101 (1959), 141–162. MR 0101846 (21 #653). Zbl 084.19304 (84, p. 193d).

(SG: Enum)

Frank Harary & Robert W. Robinson

1977a Exposition of the enumeration of point-line-signed graphs enjoying various dualities. In: R.C. Read and C.C. Cadogan, eds., *Proceedings of the Second Carribean Conference in Combinatorics and Computing* (Cave Hill, Barbados, 1977), pp. 19–33. Dept. of Math., University of the West Indies, Cave Hill, Barbados, 1977.

Counts signed trees. [Annot. 28 May 2017.] (**SG, VS: Enum**)

Frank Harary & Bruce Sagan

1984a Signed posets. In: *Calcutta Mathematical Society Diamond-cum-Platinum Jubilee Commemoration Volume (1908–1983)*, Part I, pp. 3–10. Calcutta Math. Soc., Calcutta, 1984. MR 0845035 (87k:06003). Zbl 588.05048.

A signed poset is a (finite) partially ordered set P whose Möbius function takes on only values in $\{0, \pm 1\}$. $S(P)$ is the signed graph with $V = P$ and $E_\varepsilon = \{xy : x \leq y \text{ and } \mu(x, y) = \varepsilon 1\}$ for $\varepsilon = +, -$. Some examples are chains, tree posets, and any product of signed posets. Thm. 1 characterizes P such that $\#S(P) \cong H(P)$, the Hasse diagram of P. Thm. 3 characterizes posets for which $S(P)$ is balanced. Thm. 4 gives a sufficient condition for clusterability of $S(P)$. There are many unanswered questions, most basically *Question* 1. Which signed graphs have the form $S(P)$? [See Zelinka (1988a) for a partial answer.] (**SG, Sgnd**)

Frank Harary & Marcello Truzzi

1979a The graph of the zodiac: On the persistence of the quasi-scientific paradigm of astrology. *J. Combin. Inform. System Sci.* 4 (1979), 147–160. MR 0564189 (82e:00004) (*q.v.*). (**SG: Bal**)

Katsumi Harashima

See H. Kosako.

E. Harburg

See K.O. Price.

Mela Hardin

See M. Beck.

Nadia Hardy

See S. Fiorini.

Tero Harju

See also A. Ehrenfeucht and J. Hage.

2004a Tutorial on DNA computing and graph transformation. In: H. Ehrig *et al.*, eds., *ICGT 2004*, pp. 434–436. Lect. Notes in Computer Sci., Vol. 3256. Springer, Berlin, 2004.

[Vertex-]signed overlap graphs mentioned on p. 436. [Annot. 6 Feb 2011.] (**VS: Algor, Appl**)

2005a Combinatorial models of gene assembly. In: S.B. Cooper, B. Löwe, and L. Torenvliet, eds., *New Computational Paradigms* (First Conf. Computability in Europe, CiE 2005, Amsterdam, 2005), pp. 188–195. Lect. Notes in Computer Sci., Vol. 3526. Springer, Berlin, 2005. Zbl 1113.68400.

A vertex-signed graph (called a "signed graph") encodes the overlap of signed permutations (pp. 190ff.). [Annot. 6 Feb 2011.] (**VS: Algor, Appl**)

Tero Harju, Chang Li, & Ion Petre

2007a Examples on the parallel complexity of signed graphs. TUCS Tech. Rep. No. 811, Turku Centre for Computer Science, 20 pp. Turku, Finland, 2007. (**SG: Algor**)

2008a Graph theoretic approach to parallel gene assembly. *Discrete Appl. Math.* 156 (2008), no. 18, 3416–3429. MR 2467313 (2010c:92075). Zbl 1200.05238.

See Harju, Li, Petre, and Rozenberg (2005a). The "parallel complexity" of a vertex-signed graph is the minimum number of operations required to reduce it to ∅. The value for an all-positive or all-negative tree is low (≤ 3 and 2, resp.). *Conjecture.* That of an all-negative graph is ≤ 3. (**VS, Appl**)

2008b Parallel complexity of signed graphs for gene assembly in ciliates. *Soft Computing* 12 (2008), 731–737. Zbl 1137.92305.

See (2008a). The parallel complexity of various examples, e.g., complete tripartite graphs with constant sign (complexity ≤ 3), and an all-positive circle with two negative leaves hanging off each circle vertex (complexity ≤ 4 or 5). (**VS, Appl**)

Tero Harju, Chang Li, Ion Petre, & Gregorz Rozenberg

2005a Parallelism in gene assembly. In: C. Ferretti, G. Mauri, and C. Zandron, eds., *DNA Computing* (Proc. 10th Int. Workshop on DNA Computing, DNA10, Milan, 2004), pp. 138–148. Lect. Notes in Computer Sci., Vol. 3384. Springer, Berlin, 2005. MR 2179032 (no rev). Zbl 1116.68454.

The signs are on vertices. An operation is "local complementation" of a vertex v: in the neighborhood $N(v)$, negate the vertices and complement the edges. Molecular operations formalized for vertex-signed graphs are: (1) deletion of an isolated negative vertex, (2) local complementation of a positive vertex, then deletion of the vertex, (3) a complementation in the neighborhood of two adjacent negative vertices v, w: complement in $N(v) \cup N(w)$, then complement in $N(v) \cap N(w)$. (The paper has a misprint.) The objective is to reduce the graph to ∅ by these operations, if possible. One consideration is when operations can be performed "in parallel", i.e., independently of order of operations. (**VS, Appl**)

2007a Complexity measures for gene assembly. In: K. Tuyls *et al.*, eds., *Knowledge Discovery and Emergent Complexity in Bioinformatics* (First Int. Workshop, KDECB 2006, Ghent, 2006), pp. 42–60. Lect. Notes in Bioinformatics. Lect. Notes in Computer Sci., Vol. 4366. Springer, Berlin, 2007.

§7, "Fourth complexity measure: Parallelism": A definition of parallelism in terms of applying rules (operations) to vertex-signed graphs. [Annot. 6 Feb 2011.] (**VS: Algor**)

2006a Parallelism in gene assembly. *Nat. Computing* 5 (2006), no. 2, 203–223. MR 2259034 (2007h:68043). Zbl 1114.68043.

See (2005a). (**VS, Appl**)

Tero Harju, Ion Petre, & Gregorz Rozenberg

2004a Tutorial on DNA computing and graph transformation. In: H. Ehrig *et al.*, eds., *ICGT 2004*, pp. 434–436. Lect. Notes in Computer Sci., Vol. 3256. Springer, Berlin, 2004.

See Harju, Li, Petre, and Rozenberg (2005a) *et al.* (**VS, Appl: Exp**)

Pierre de la Harpe

1994a Spin models for link polynomials, strongly regular graphs and Jaeger's Higman-Sims model. *Pacific J. Math.* 162 (1994), no. 1, 57–96. MR 1247144 (94m:-

57014). Zbl 795.57002. **(SGc: Knot, Invar)**

David Harries & Hans Liebeck

1978a Isomorphisms in switching classes of graphs. *J. Austral. Math. Soc. (A)* 26 (1978), 475–486. MR 0520101 (80a:05109). Zbl 411.05044.

Given $\Sigma = (K_n, \sigma)$ and an automorphism group $\mathfrak{A}$ of the switching class $[\Sigma]$, is $\mathfrak{A}$ "exposable" on $[\Sigma]$ (does it fix a representative of $[\Sigma]$)? General techniques and a solution for the dihedral group. Done in terms of Seidel switching of unsigned simple graphs. (A further development from Mallows and Sloane (1975a). [Related work in M. Liebeck (1982a) and Cameron (1977a).]) **(kg: sw, TG: Aut)**

Matthew Hartley

See G.R. Walther.

Alexander K. Hartmann

See also C. Amoruso, G. Hed, O. Melchert, and M. Pelikan.

1998a Are ground states of 3d $\pm J$ spin glasses ultrametric? *Europhys. Lett.* 44 (1998), no. 2, 249–254.

The distances of ground states in a signed cubic lattice with side L, measured by overlap, tested on many samples with $L \leq 14$ for the appearance of approaching ultrametricity in the infinite limit of L. There is such an appearance. L is too small for quantitative statements. [Ultrametricity is a strong property that has been conjectured by Parisi *et al.* It is disavowed in Hed, Hartmann, Stauffer, and Domany (2001a).] Dictionary: *cf.* (2000a). [Annot. 11 Jan 2015.] **(SG, Phys: Fr: State)**

1999a Ground-state landscape of 2d $\pm J$ Ising spin glasses. *European Phys. J. B* 8 (1999), 619–626.

Examples of signed square lattice graphs. Evidence is against ultrametricity of ground states, contrary to prior findings in higher dimensions. Dictionary: *cf.* (2000a). [Annot. 10 Jan 2015.] **(SG: Fr: State: Algor, Phys)(Algor: Exp)**

1999b Ground-state behavior of the three-dimensional $\pm J$ random-bond Ising model. *Phys. Rev. B* 59 (1999), no. 5, 3617–3623.

Examples of signed cubic lattice graphs for varying concentrations $p := \#E^-/\#E$ of negative edges: ground states, frustration index ("ground state energy"), average overlap, etc. Dictionary: *cf.* (2000a). [Annot. 10 Jan 2015.] **(SG: Fr, State: Algor, Phys)**

1999c Scaling of stiffness energy for three-dimensional $\pm J$ Ising spin glasses. *Phys. Rev. E* 59 (1999), no. 1, 84–87.

"Stiffness" = ground-state domain wall energy. The graph is a cubic lattice. Many ground states are generated and compared. **(SG, Phys: Fr: State, Algor)**

† 2000a Ground-state clusters of two-, three-, and four-dimensional $\pm J$ Ising spin glasses. *Phys. Rev. E* 63 (2000), art. 016106, 7 pp.

The ground states in examples of signed square, cubic, and tesseractic (4-hypercubic) lattices are found to fall into relatively few clusters. An algorithmic method called "ballistic search" permits larger conclusions from smaller numbers of states. Dictionary: "ground state"

= switching with fewest negative edges, "ground state energy" = $l(\Sigma)$, "ground state graph" has ground states ζ for vertices and an edge between ground states that differ by switching a vertex (necessarily having $d^+ = d^-$), "cluster" = component of ground-state graph, "overlap" of states $q(\zeta, \zeta') := n^{-1}[\#(\zeta\zeta')^{-1}(+) - \#(\zeta\zeta')^{-1}(-)] = (1/n)\cdot$[number of vertices of agreement − number of vertices of disagreement]. [Annot. 10 Jan 2015.] (**SG: Fr: State: Algor, Phys**)

2008a Droplets in the two-dimensional $\pm J$ Ising spin glass. *Phys. Rev. B* 77 (2008), art. 144418, 5 pp. (**SG: Fr: State: Algor, Phys**)

2011a Ground states of two-dimensional Ising spin glasses: Fast algorithms, recent developments and a ferromagnet-spin glass mixture. *J. Stat. Phys.* 144 (2011), 519–540. MR 2826631 (2012g:82059). Zbl 1227.82086.

Review of the ground state problem, methods, and conclusions. (**SG, Phys: Fr: State, Algor: Exp, Ref**)

Alexander K. Hartmann & Federico Ricci-Tersenghi

2002a Direct sampling of complex landscapes at low temperatures: The three-dimensional $\pm J$ Ising spin glass. *Phys. Rev. E* 66 (2002), art. 224419, 8 pp.

The state landscape appears to be more complex at small positive temperatures than at zero temperature. (**SG: Fr: State: Algor, Phys**)

Alexander K. Hartmann & Heiko Rieger

2002a *Optimization Algorithms in Physics.* WILEY-VCH, Berlin, 2002. MR 1881155 (2004b:00006). Zbl 1003.82002. (**Phys: sg: Fr: Algor**)

Alexander K. Hartmann & Heiko Rieger, eds.

2004a *New Optimization Algorithms in Physics.* WILEY-VCH, Weinheim, 2004. MR 2089307 (2005d:90004). Zbl 1054.90002. (**Phys: sg: Fr: Algor**)

Alexander K. Hartmann & Martin Weigt

2005a *Phase Transitions in Combinatorial Optimization Problems: Basics, Algorithms and Statistical Mechanics.* Wiley-VCH, Weinheim, Germany, 2005. MR 2293999 (2009b:82028). Zbl 1094.82002.

"Example: Ising spin glasses": Frustration index of signed graphs on p. 6. §11.7, "Matchings and spin glasses": Outlines the matching theory method (*cf.* Katai and Iwai (1978a) and Barahona (1982a)) for planar graphs, for calculating $l(\Sigma)$ and locating ground states (switchings with fewest negative edges). Also, locating interesting excited states (states with more than the fewest unsatisfied edges), specifically, domain walls and droplets. A "domain" is generated by negating signs of a set of edges; the vertices whose spins remain the same form one domain and the complement is the other. The increased energy (the "domain wall energy") has thermodynamic implications. [How to choose the negation set, and what can be the shapes of domain walls, are not obvious.] A "droplet" in a state s, vis-á-vis a ground state s_0, is a component of the subgraph induced by $(ss_0)^{-1}(-1)$. The sizes of droplets appear to have consequences for thermodynamics. [Annot. 24 Aug 2012.] (**SG: WG, Fr, State: Phys, Algor: Exp, Ref**)

A.K. Hartmann & A.P. Young

2001a Lower critical dimension of Ising spin glasses. *Phys. Rev. B* 64 (2001), art. 180404, 4 pp.

For unweighted ($\pm J$) and randomly weighted (Gaussian) signed graphs, ground states are computed and compared. The lower critical dimension is different in the two types. [Annot. 22 Jan 2015.]

(**Phys, SG: Fr, State**)

Nora Hartsfield & Gerhard Ringel

1989a Minimal quadrangulations of nonorientable surfaces. *J. Combin. Theory Ser. A* 50 (1989), 186–195. MR 0989193 (90j:57003). Zbl 665.51007.

"Cascades": see Youngs (1968a). (**sg: Ori: Appl**)

Forough Hassanibesheli, Leila Hedayatifar, Przemysław Gawroński, Maria Stojkow, Dorota Żuchowska-Skiba, & Krzysztof Kułakowski

2017a Gain and loss of esteem, direct reciprocity and Heider balance. *Physica A* 468 (2017), 334–339. MR 3580677 (no rev). Zbl 1400.91423. arXiv:1609.03358.

(**SD: Dyn, Bal, Fr**)

K. Hassani Monfared, G. MacGillivray, D.D. Olesky, & P. van den Driessche

2019a Inertias of Laplacian matrices of weighted signed graphs. *Special Matrices* 7 (2019), 327–342. MR 4045536. Zbl 1431.05101. arXiv:1912.03844.

Dictionary: "flexibility" $\tau(\Gamma,\sigma) = r(E^+) + r(e^-) - r(E)$, r = rank in $\mathbf{F}(\Gamma)$. [Hence related to connectivity of $\mathbf{F}(\Gamma)$ via sign partition of E.] [Annot. 3 Sept 2019.] (**SG, WG: Lap: Eig, matrd**)

Keivan Hassani Monfared & Sudipta Mallik

2019a An analog of matrix tree theorem for signless Laplacians. *Linear Algebra Appl.* 560 (2019), 43–55. MR 3866544. Zbl 1402.05139. arXiv:1805.04759.

Matrix tree theorem is special case $-\Gamma$ of Zaslavsky (1982a), Thm. 8A.4. New: # negative circles in $-\Gamma$ is $\leq \det L(-\Gamma)/4$, with = iff $-\Gamma$ is balanced or unicyclic. [*Problem.* Generalize to signed graphs.] [Annot. 13 Oct 2022.] (**sg: Par: Lap**)

Kurt Hässig

1975a Theorie verallgemeinerter Flüsse und Potentiale. In: *Siebente Oberwolfach-Tagung uber Operations Research* (1974), pp. 85–98. Operations Research Verfahren, Band XXI. A. Hain, Meisenheim am Glan, 1975. MR 0450137 (56 #8434). Zbl 358.90070. (**GN: Incid**)

1979a *Graphentheoretische Methoden des Operations Research*. Leitfaden der angew. Math. und Mechanik, 42. B.G. Teubner, Stuttgart, 1979. MR 0528758 (80f:-90002). Zbl 397.90061.

Ch. 5: "Verallgemeinerte Fluss- und Potentialdifferenzen-probleme." The lift matroid arises from a side condition, i.e., extra row, added to the incidence matrix of the graph. [The side condition is expressed graphically by additive real gains.] (**GN: Incid, Matrd, Bal: Exp, Ref**)

Refael Hassin

1981a Generalizations of Hoffman's existence theorem for circulations. *Networks* 11

(1981), 243–254. MR 0636230 (83c:90055). Zbl 459.90026. **(GN)**

O. Hatami

See S. Akbari.

P. Hatami

See S. Akbari.

Naomichi Hatano

See also E. Estrada.

Naomichi Hatano & James E. Gubernatis

2000a A bivariate multicanonical Monte Carlo of the 3D $\pm J$ spin glass. In: David P. Landau *et al.*, eds., *Computer Simulation Studies in Condensed-Matter Physics XII* (Proc. Twelfth Workshop, Athens, Ga., 1999), pp. 149–161. Springer Proc. Phys., Vol. 85. Springer, Berlin, 2000.

Similar to Hatano and Gubernatis (2002a). [Annot. 28 Mar 2013.] **(Phys, sg: State(fr))**

2002a Evidence for the double degeneracy of the ground-state in the three-dimensional $\pm J$ spin glass. *Phys. Rev. B* 66 (2002), art. 054437, 14 pp. arXiv:cond-mat/0008115.

A Monte-Carlo investigation of infinite signed cubic lattice graphs at zero temperature, by means of large finite cubic lattices. Are there only two ground states, one the negative of the other, or are there many, unrelated ground states? The paper supports the former. *Cf.* Hatano and Gubernatis (2002b). [Having only one ground state (up to sign reversal) means there is only one switching that minimizes $\#E^-$. (*Conjecture.* That is not true of any signed graph, finite or infinite, except for very special, very regular graphs and signatures.) However, zero temperature may distort the normal behavior of a signed graph.] [Annot. 28 Mar 2013.] **(Phys, sg: State(fr))**

2002b Double degeneracy in the ground state of the 3D $\pm J$ spin glass. *Comput. Phys. Commun.* 147 (2002), no. 1-2, 414–418. Zbl 994.82557.

Reply to criticism of Hatano and Gubernatis (2002a). [Annot. 28 Mar 2013.] **(Phys, sg: State(fr))**

D.M. Hatch

See S.T. Chui.

Penny Haxell

See L. Beaudou.

Emilie Haynsworth & A.J. Hoffman

1969a Two remarks on copositive matrices. *Linear Algebra Appl.* 2 (1969), 387–392. MR 0248157 (40 #1411). Zbl 185.08004.

Matrix $A(K_n, \sigma) + I$: properties as quadratic form. Thm.: It is copositive iff (K_n, σ) is balanced. *Cf.* Hoffman and Pereira (1973a). [Annot. 28 May 2017.] **(sg: kg: adj)**

Bian He, Ya-Lei Jin, & Xiao-Dong Zhang

2013a Sharp bounds for the signless Laplacian spectral radius in terms of clique number. *Linear Algebra Appl.* 438 (2013), no. 10, 3851–3861. MR 3034503. Zbl 1282.05119. arXiv:1209.3214.

§4: The incidence energy (derived from $Q := L(-\Gamma)$) has a bound like that in Thm. 4.5. [Annot. 21 Jan 2012.] **(par: Lap: Eig)**

Chang-Xiang He

See also P.-K. Zhang.

Chang-Xiang He & Min Zhou

2014a A sharp upper bound on the least signless Laplacian eigenvalue using domination number. *Graphs Combin.* 30 (2014), 1183–1192. MR 3248498. Zbl 1298.05203.
(par: Lap: Eig: dom)

Jin-Ling He

See J.-Y. Shao.

Shengjie He & Rong-Xia Hao

20xxa The relation between the independence number and rank of a signed graph. Submitted. arXiv:1907.07837. **(SG: Adj)**

Shengjie He, Rong-Xia Hao, & Fengming Dong

2020a The rank of a complex unit gain graph in terms of the matching number. *Linear Algebra Appl.* 589 (2020), 158–185. MR 4046874. Zbl 1437.05141. arXiv:-1909.07555.

Bounds on $\operatorname{rk} A(\Phi)$ in terms of matching number μ and cyclomatic number ξ. Thm. 1.10: $2\mu - 2\xi \leq r \leq 2\mu + \xi$. Thms. 1.11–12: Equality characterized. Thm. 1.13: Better bounds in terms of edge bipartiticity and acyclic induced subgraphs. [Annot. 2 Jan 2020.] **(GG: Adj)**

Shengjie He, Rong-Xia Hao, & Hong-Jian Lai

2019a Bounds for the matching number and cyclomatic number of a signed graph in terms of rank. *Linear Algebra Appl.* 572 (2019), 273–291. MR 3926237. Zbl 1411.05107.

Bounds on rank: $2\mu - 2\xi \leq \operatorname{rk} A(\Sigma) \leq 2\mu + 2\xi$. Upper and lower bounds are attained by loose cacti with certain circle signs. [*Cf.* He, Hao, & Dong (2020a).] [Annot. 17 Jan 2019.] **(SG: Adj)**

Shengjie He, Rong-Xia Hao, & Aimei Yu

2022a Bounds for the rank of a complex unit gain graph in terms of the independence number. *Linear Multilinear Algebra* 70 (2022), no. 7, 1382–1402. MR 4413130. Zbl 1487.05162. arXiv:1909.08533.

Rank of $A(\Phi)$. **(GG: Adj)**

20xxa On the inertia index of a mixed graph with the matching number. Submitted. arXiv:1909.07146. **(gg: Adj)**

Shushan He & Shuchao Li

2012a On the signless Laplacian index of unicyclic graphs with fixed diameter. *Linear Algebra Appl.* 436 (2012), no. 1, 252–261. MR 2859926 (2012j:05257). Zbl 1229.05201. **(sg: par: Eig)**

Xiaocong He

See S. Zaman.

Xiping He

See S.-J. Yang.

Patrick Headley

1997a On a family of hyperplane arrangements related to the affine Weyl groups. *J. Algebraic Combin.* 6 (1997), 331–338. MR 1471893 (98e:52010). Zbl 911.52009.

The characteristic polynomials of the Shi hyperplane arrangements $\mathcal{S}(W)$ of type W for each Weyl group W, evaluated computationally. $\mathcal{S}(W)$ is obtained by splitting the reflection hyperplanes of W in two in a certain way; thus $\mathcal{S}(A_{n-1})$ splits the arrangement representing $\mathrm{Lat}\,\mathbf{F}(K_n)$—more precisely, it represents $\mathrm{Lat}^{\mathrm{b}}\{0,1\}\vec{K}_n$; that of type B_n splits the arrangement representing $\mathrm{Lat}\,\mathbf{F}(\pm K_n^{\bullet})$, and so on. [*Cf.* Athanasiadis (1996a).] (**gg: Geom, Matrd, Invar**)

Brian Healy & Arthur Stein

1973a The balance of power in international history: Theory and reality. *J. Conflict Resolution* 17 (1973), no. 1, 33–61.

Describes balance (incorrectly) and clusterability of a signed graph; examines the relevance of, *i.a.*, signed-graphic balance. [Annot. 9 Jun 2012.] (**PsS; SG: Bal, Clu: Exp**)

Robert W. Heath Jr.

See T. Strohmer.

Saroja R. Hebbar

See P.R. Hampiholi.

Guy Hed, Alexander K. Hartmann, Dietrich Stauffer, & Eytan Domany

2001a Spin domains generate hierarchical ground state structure in $J = \pm 1$ spin glasses. *Phys. Rev. Lett.* 86 (2001), no. 14, 3148–3151.

Proposes an intermediate structure of ground states (switchings with smallest $\#E^-$) of a signed graph ("Ising spin glass" with edge weights ± 1), not ultrametric (*cf.* Hartmann (1998a)) but "hierarchical". [Annot. 11 Jan 2015.] (**SG: State(fr), Phys**)

Leila Hedayatifar

See F. Hassanibesheli.

Rajneesh Hegde

See A. van Zuylen.

Pinar Heggernes

See H.L. Bodlaender.

Fritz Heider

1946a Attitudes and cognitive organization. *J. Psychology* 21 (1946), 107–112.

No mathematics, but a formative article. Precursor and inspiration of Harary (1953a) and Cartwright and Harary (1956a) (*q.v.*). Signed digraphs (at most 4 vertices) and signed vertices are unsystematically present but there is no graph theory. Complicated by multiple types of vertices and edges [dispensed with by Cartwright–Harary]. [Annot. 15 Oct 2018.] (**PsS, sd, vs: Bal**)

1979a On balance and attribution. In: Paul W. Holland and Samuel Leinhardt, eds., *Perspectives on Social Network Research* (Proc. Sympos., Dartmouth Coll., Hanover, N.H., 1975), Ch. 2, pp. 11–23. Academic Press, New York, 1979.

How Heider came to his balance theory (1946a) and what it means. Remarks on the Cartwright–Harary (1956a) theory. [Annot. 1 Oct 2018.] **(PsS)(SG: Bal)**

E. Heilbronner

1964a Hückel molecular orbitals of Möbius-type conformations of annulenes. *Tetrahedron Lett.* 5 (1964), no. 29, 1923–1928.

Introduces a negative sign into the bonds of a cyclic molecule [thus leading to Möbius molecules and chemical signed graph theory; *cf.* Zimmerman (1966a), Gutman (1978a) *et al.*, Trinajstić (1983a), (1992a), Rzepa (2005a), Herges (2006a), *et al.*]. Eigenvalues of adjacency matrices of negative circles. [Annot. 4 Nov 2010, 23 Nov 2012.] **(Chem: SG: bal, Eig)**

Matthias Hein

See P. Mercado.

Peter Christian Heinig

2011a Chio condensation and random sign matrices. Manuscript, 2011. arXiv:-1103.2717. **(SG: Rand)**

Richard V. Helgason

See J.L. Kennington.

Pavol Hell

See J. Bok and R.C. Brewster.

I. Heller

1957a On linear systems with integral valued solutions. *Pacific J. Math.* 7 (1957), 1351–1364. MR 0094381 (20 #899). Zbl 079.01903. **(sg: Incid)**

I. Heller & C.B. Tompkins

1956a An extension of a theorem of Dantzig's. In: H.W. Kuhn and A.W. Tucker, eds., *Linear Inequalities and Related Systems*, pp. 247–252. Annals of Math. Studies, No. 38. Princeton Univ. Press, Princeton, N.J., 1956. MR 0081871 (18, 459). Zbl 072.37804 (72, p. 378d).

Thm.: The incidence matrix of a signed graph where all edges are links is totally unimodular iff the signed graph is balanced. (Not stated in terms of signed graphs.) *Cf.* Hoffman and Gale (1956a), Hoffman (1960a), and Reichmeider (1984a). **(sg: Incid, Bal)**

Marc Hellmuth

See T. Biyikoğlu.

P.S. Hemavathi

See Lokesha–Hemavathi–Vijay (2018a), Reddy–Hemavathi–Rajendra (2014a), Reddy–Misra–Hemavathi (2014a).

J.L. van Hemmen

1983a Equilibrium theory of spin glasses: mean-field theory and beyond. In: J.L. van Hemmen and I. Morgenstern, eds., *Heidelberg Colloquium on Spin Glasses* (Proc., Heidelberg, 1983), pp. 203–233. Lect. Notes in Physics, Vol. 192. Springer-Verlag, Berlin, 1983. MR 0733800 (85d:82086).

§2.3, "Frustration": Physics of Ising models with edges ("bonds") that are positive, negative, or of undetermined sign. [Annot. 16 Jun 2012.] **(Phys: sg)**

Robert L. Hemminger & Joseph B. Klerlein

1979a Line pseudodigraphs. *J. Graph Theory* 1 (1977), 365–377. MR 0465926 (57 #5812). Zbl 379.05032.

An attempt, intrinsically unsuccessful, to represent the (signed) line graph of a digraph (see Zaslavsky (20xxa)) by a digraph. [Continued by Klerlein (1975a).] **(sg: LG, ori)**

Robert L. Hemminger & Bohdan Zelinka

1973a Line isomorphisms on dipseudographs. *J. Combin. Theory Ser. B* 14 (1973), 105–121. MR 0314679 (47 #3230). Zbl 263.05107. **(sg: LG, ori)**

Anthony Henderson

2006a Plethysm for wreath products and homology of sub-posets of Dowling lattices. *Electronic J. Combin.* 13 (2006), no. 1, Research art. R87, 25 pp. MR 2255429 (2007f:05187). Zbl 1113.05101.

The subposets are $Q_n^{1 \bmod d}(\mathfrak{G})$ where $d > 1$, whose elements are the flats $A \in \operatorname{Lat} \mathbf{F}(\mathfrak{G}K_n^\bullet)$ such that d divides the order of the unbalanced part and the number of vertices every balanced component is $\equiv 1 \bmod d$. **(gg: Matrd: Aut)**

Michael Henley

See F. Ardila.

Cheolwon Heo

See also B. Guenin.

2016a *Recognizing Even-Cycle and Even-Cut Matroids.* Master's thesis, Univ. of Waterloo, 2016. `https://uwspace.uwaterloo.ca/handle/10012/10407`

Dictionary: "even-cycle matroid" = signed graphic lift matroid; "even-cut" = dual matroid. **(SG: Matrd, Du: Algor)**

2021a *Representations of Even-cycle and Even-cut Matroids.* Doctoral thesis, Univ. of Waterloo, 2021. `https://uwspace.uwaterloo.ca/handle/10012/17297` **(SG: Matrd, Du: Algor)**

Cheolwon Heo & Bertrand Guenin

2020a Recognizing even-cycle and even-cut matroids. In: *Integer Programming and Combinatorial Optimization* (IPCO 2020, London), pp. 182–195. Lect. Notes in Computer Sci., Vol. 12125. Springer, Cham, 2020. MR 4139569. Zbl 1504.05052.

Dictionary: "Even-cycle matroid" = lift matroid of signed graph. **(SG: Matrd: Algor)**

Rainer Herges

2006a Topology in chemistry: Designing Möbius molecules. *Chem. Rev.* 106 (2006), 4820–4842.

A Möbius molecule has a half-twist in a ring structure [hence can be modeled by an unbalanced signed graph; *cf.* Heilbronner (1964a), Gutman (1978a) *et al.*, Trinajstić (1983a), (1992a)]. A survey of specific types of Möbius molecules. §3.1.4, "Other Möbius systems": Rzepa

(2005a) *et al.*, Craig, and also Fowler (2002a) have proposed that certain annulenes are intrinsically twisted (i.e., unbalanced) due to the d-orbital or p-orbital structure. [Annot. 23 Nov 2012.]
(**Chem: sg: bal: Exp, Ref**)

Patricia Hersh & Ed Swartz

2008a Coloring complexes and arrangements. *J. Algebraic Combin.* 27 (2008), 205–214. MR 2375492 (2008m:05109). Zbl 1154.05315.

Remark 19: Chromatic polynomials of signed graphs vis-á-vis subarrangements of the root system arrangement $\mathcal{B}_n$ in Thm. 18, which gives properties of an h-vector. [Annot. 1 Mar 2011.] (**SG: Invar**)

Daniel Hershkowitz & Hans Schneider

1993a Ranks of zero patterns and sign patterns. *Linear Multilinear Algebra* 34 (1993), no. 1, 3–19. MR 1334927 (96g:15004). Zbl 793.05027.

Bipartite Σ such that every matrix with sign pattern Σ has the same rank, over each field $\neq \mathbb{F}_2$. [Annot. 6 Mar 2011.] (**SG: QM**)

J.A. Hertz

See K.H. Fischer.

Ryan Hessert & Sudipta Mallik

2021a Moore-Penrose inverses of the signless Laplacian and edge-Laplacian of graphs. *Discrete Math.* 344 (2021), no. 8, art. 112451, 15 pp. MR 4254527. Zbl 1466.05120. arXiv:2005.03702.

[*Problem.* Combine both cases by generalizing to signed graphs.] [Annot. 28 Dec 2022.] (**sg: Par: Incid, Lap**)

2023a The inverse of the incidence matrix of a unicyclic graph. *Linear Multilinear Algebra* 71 (2023), no. 4, 513–527. MR 4577208. arXiv:2201.02580.

The Moore–Penrose inverse, if the matrix has not full rank. [*Problem.* Extend to a (balanced) signed unicyclic graph.] [Annot. 11 Feb 2022.] (**Par: Incid, Lap**)

Gábor Hetyei

See Y. Diao.

Chris Heunen & Vaia Patta

2018a The category of matroids. *Appl. Categorical Structures* 26 (2018), no. 2, 205–237. MR 3770908. Zbl 1403.18002. arXiv:1512.01390.

§7.3, "Graphs", mentions possible bicircular, frame, and lift functors. [Annot. 2 Oct 2018.] (**gg:Matrd, Bic**)

Hector Hevia

See G. Chartrand.

Farideh Heydari

See S. Akbari, S. Dalvandi, N. Kafai, and M. Souri.

Takayuki Hibi

See also T. Matsui and H. Ohsugi.

Takayuki Hibi, Nan Li, & Yan X Zhang

2013a Separating hyperplanes of edge polytopes. *J. Comhin. Theory Ser. A* 120 (2013), 218–231. MR 2971708. Zbl 1255.05123. arXiv:1112.5047.
(**sg: par: Geom**)

Takayuki Hibi, Aki Mori, Hidefumi Ohsugi, & Akihiro Shikama

2016a The number of edges of the edge polytope of a finite simple graph. *Ars Math. Contemp.* 10 (2016), no. 2, 323–332. MR 3529294. Zbl 1351.52007. arXiv:-1308.3530.

[This is the antibalanced case. *Problem.* Generalize to signed graphs, including balanced graphs.] (**sg: Par: Geom**)

Takayuki Hibi, Kenta Nishiyama, Hidefumi Ohsugi, & Akihiro Shikama

2014a Many toric ideals generated by quadratic binomials possess no quadratic Gröbner bases. *J. Algebra* 408 (2014), 138–146. MR 3197176. Zbl 1304.13040.

Antibalanced graph criteria. [*Problem.* Generalize to signed graphs.] [Annot. 5 Oct 2014.] (**sg: Par: Algeb**)

Akihiro Higashitani

See T. Matsui.

Desmond J. Higham

See E. Estrada.

Yusuke Higuchi & Iwao Sato

2015a A balanced signed digraph. *Graphs Combin.* 31 (2015), no. 6, 2215–2230. MR 3417229. Zbl 1330.05080.

"Balance" of (D, σ) = cycle balance (*cf.* B.D. Acharya (1980a)). Characterized by graph zeta function. Signed digraph covering. [Annot. 6 Jul 2022.] (**SD: Bal, Cov**)

Timo Hiller

2017a Friends and enemies: A model of signed network formation. *Theor. Economics* 12 (2017), 1057–1087. MR 3713674. Zbl 1396.91044.

Treats balance via game theory. [Annot. 6 Jul 2022.] (**SG: Dyn, Bal**)

Franziska Hinkelmann

See A. Veliz-Cuba.

K. Hinson

See Y. Diao.

Sandra Hirche

See D. Xue.

André Hirschowitz

See M. Hirschowitz.

Michel Hirschowitz, André Hirschowitz, & Tom Hirschowitz

2007a A theory for game theories. In: V. Arvind and S. Prasad, eds., *FSTTCS 2007: Foundations of Software Technology and Theoretical Computer Science* (27th Int. Conf., New Delhi, 2007), pp. 192–203. Lect. Notes in Computer Sci., Vol. 4855. Springer-Verlag, Berlin, 2007. MR 2480201 (2010h:91057). Zbl 1136.68035. (**SD: Appl**)

Tom Hirschowitz

See M. Hirschowitz.

Petr Hliněný

See also R. Ganian.

Petr Hlineňý, Sang-Il Oum, Detlef Seese, & Georg Gottlob

2008a Width parameters beyond tree-width and their applications. *Computer J.* 51 (2008), no. 3, 326–362.

§4.2.1: Switching changes rank-width of a graph by at most 1, contrasting with other widths in Bodlaender and Hage (2012a). [Annot. 15 Jun 2022.] (**TG: Sw**)

Tuyen-Thanh-Thi Ho, Hung Thanh Vu, & Bac Hoai Le

2013a A decision-making based feature for link prediction in signed social networks. In: *The 2013 RIVF International Conference on Computing and Communication Technologies – Research, Innovation, and Vision for Future (RIVF)* (Hanoi, 2013), pp. 169–174. IEEE, 2013.

Given $\vec{\Gamma}$ and signs on $E(\vec{\Gamma}) \setminus (u, v)$, $\sigma(u, v)$ is predicted by signed indegrees of v and signed outdegrees of u. Justified for social networks by appeal to psychological traits. [Annot. 22 Jan 2015.]

(**SD: Pred: Algor: PsS**)

Dorit S. Hochbaum

1998a Instant recognition of half integrality and 2-approximations. In: Klaus Jansen and José Rolim, eds., *Approximation Algorithms for Combinatorial Optimization* (Aalborg, 1998), pp. 99–110. Lect. Notes in Computer Sci., Vol. 1444. Springer, Berlin, 1998. MR 1677400. Zbl 911.90261.

Integer programs with constraints of a generalized real gain-graphic form, $\alpha x - \beta y - \gamma \leq z$, the gain being β/α. Slightly extends Hochbaum, Megiddo, Naor, and Tamir (1993a). (**gn: Incid(Du): Algor**)

1998b The t-vertex cover problem: extending the half integrality framework with budget constraints. In: Klaus Jansen and José Rolim, eds., *Approximation Algorithms for Combinatorial Optimization* (Aalborg, 1998), pp. 111–122. Lect. Notes in Computer Sci., Vol. 1444. Springer, Berlin, 1998. MR 1677404 (2000b:90032). Zbl 908.90213.

Integer programs as in (1998a) with "budget constraints".

(**gn: Incid(Du): Algor**)

2000a Instant recognition of polynomial time solvability, half integrality and 2-approximations. In: Klaus Jansen and Samir Khuller, eds., *Approximation Algorithms for Combinatorial Optimization* (Saarbrücken, 2000), pp. 2–14. Lect. Notes in Computer Sci., Vol. 1913. Springer, Berlin, 2000. MR 1850069 (no rev). Zbl 976.90123.

Integer programs as in (1998a). There is a polynomial-time solution via a minimum cut, or else a half-integral partial solution.

(**gn: Incid(Du): Algor**)

2002a Solving integer programs over monotone inequalities in three variables: a framework for half integrality and good approximations. O.R. for a United Europe (Budapest, 2000). *European J. Operational Res.* 140 (2002), no. 2, 291–321. MR 1899053 (2003e:90052). Zbl 1001.90050.

Constraints of a generalized positive-real gain-graphic form, $\alpha x - \beta y - \gamma \leq z$, the gain being β/α, contrasting $\alpha, \beta \geq 0$ to the intrinsically hard case where a negative coefficient is allowed but a half-integral approximate solution is easy. (**gn: Incid(Du): Algor**)

Dorit S. Hochbaum, Nimrod Megiddo, Joseph (Seffi) Naor, & Arie Tamir

1993a Tight bounds and 2-approximation algorithms for integer programs with two variables per inequality. *Math. Programming Ser. B* 62 (1993), 69–83. MR 1247607 (94k:90050). Zbl 802.90080.

Approximate solution of integer linear programs with real, dually gain-graphic coefficient matrix. [See Sewell (1996a).] (**GN: Incid(Du): Algor**)

Dorit S. Hochbaum & Joseph (Seffi) Naor

1994a Simple and fast algorithms for linear and integer programs with two variables per inequality. *SIAM J. Computing* 23 (1994), 1179–1192. MR 1303329 (95h:90066). Zbl 831.90089.

Linear and integer programs with real, dually gain-graphic coefficient matrix: feasibility for linear programs, solution of integer programs when the gains are positive ("monotone inequalities"), and identification of "fat" polytopes (that contain a sphere larger than a unit hypercube). (**GN: Incid(Du): Algor, Ref**)

Winfried Hochstättler

See also L. Goddyn.

Winfried Hochstättler, Robert Nickel, & Britta Peis

2006a Two disjoint negative cycles in a signed graph. CTW2006 – Cologne-Twente Workshop on Graphs and Combinatorial Optimization. *Electronic Notes Discrete Math.* 50 (2006), 107–111. MR 2307287 (no rev). Zbl 1134.05319.

Incidence matrix used to find the circles in slow polynomial time. Use of graphic structure is explored. (**SG: Str: Circ: Algor, Incid**)

Hervé Hocquard

See F. Foucaud.

Cornelis Hoede

1981a The integration of cognitive consistency theories. Memorandum nr. 353, Dept. of Appl. Math., Twente University of Tech., Enschede, The Netherlands, Oct., 1981. (**PsS: Gen**)(**SG, VS: Bal**)

1982a Anwendungen von Graphentheoretischen Methoden und Konzepten in den Socialwissenschaften. Memorandum nr. 390, Dept. of Appl. Math., Twente University of Tech., Enschede, the Netherlands, May, 1982.

Teil 4: "Kognitive Konsistenz." (**PsS: Gen: Exp**)

† 1992a A characterization of consistent marked graphs. *J. Graph Theory* 16 (1992), 17–23. MR 1147800 (93b:05141). Zbl 748.05081.

Characterizes when one can sign the vertices of a graph so every circle has positive sign product, solving the problem of Beineke and Harary (1978b). Given Γ, $\mu : V \to \{+, -\}$, and a spanning tree T: (Γ, μ) is consistent iff the fundamental circles with respect to T are positive and the endpoints of the intersection of two fundamental circles have the same sign. A polynomial-time algorithm ensues. [The definitive word until Joglekar, Shah, and Diwan (2010a). Does not include signed edges.] [Annot. rev. 11 Sept 2010, 2 May 2012.] (**VS: Bal: Str**)

Jan B. Hoek

See B.N. Kholodenko.

P. Hoever, W.F. Wolff, & J. Zittartz

1981a Random layered frustration models. *Z. Phys. B* 41 (1981), 43–53. MR 0600279 (81m:82027).

Physics of Ising models on a planar square lattice. Exact solutions for partition function, free energy, ground state energy. The transition temperature depends only on the average edge sign, $(\#E^+ - \#E^-)/\#E$. Switching is implicit ("substituting spins"). Model (a): all horizontal edges are + (attainable by switching); if horizontally periodic these are "random layered frustration" models. Model (b): Assumed switched to minimize $\#E^-$. Dictionary: "plaquette" = quadrilateral, "frustration index" = sign of plaquette.

They conjecture thermodynamic consequences if the ground states ($s : V \to \{+1, -1\}$ with $l(\Sigma)$ frustrated edges) are connected in the state graph $\{+1, -1\}^V$. [*Question*. For which Σ are the ground states connected?] [Annot. 16 Jun, 28 Aug 2012.] (**Phys: SG: sw**)

Peter D. Hoff

2005a Bilinear mixed-effects models for dyadic data. *J. Amer. Statistical Assoc.* 100, No. 469 (2005), 286–295. MR 2156838 (no rev). Zbl 1117.62353. (**SG, PsS: Bal**)

Alan J. Hoffman

See also D.R. Fulkerson, D. Gale, and E. Haynsworth.

1960a Some recent applications of the theory of linear inequalities to extremal combinatorial analysis. In: Richard Bellman and Marshall Hall Jr., eds., *Combinatorial Analysis*, pp. 113–127. Proc. Sympos. Appl. Math., Vol. 10. American Mathematical Soc., Providence, R.I., 1960. MR 0114759 (22 #5578). Zbl 096.00606 (96, p. 6f). (**sg: incid, bal: Exp**)

1970a $-1-\sqrt{2}$? In: Richard Guy *et al.*, eds., *Combinatorial Structures and Their Applications* (Proc. Calgary Int. Conf., 1969), pp. 173–176. Gordon and Breach, New York, 1970. Zbl 262.05133. (**LG**)

1972a Eigenvalues and partitionings of the edges of a graph. *Linear Algebra Appl.* 5 (1972), 137–146. MR 0300937 (46 #97). Zbl 247.05125. (**Par: Eig, Fr**)

1974a On eigenvalues of symmetric $(+1, -1)$ matrices. *Israel J. Math.* 17 (1974), 69–75. MR 0349709 (50 #2202). Zbl 281.15003.

Eigenvalues of signed complete graphs. (**sg: kg: Eig**)

1975a Spectral functions of graphs. In: *Proceedings of the International Congress of Mathematicians* (Vancouver, 1974), Vol. 2, pp. 461–463. Canad. Math. Congress, Montreal, 1975. MR 0434886 (55 #7850). Zbl 344.05164. (**TG, Eig**)

1976a On spectrally bounded signed graphs. (Abstract.) In: *Transactions of the Twenty-First Conference of Army Mathematicians* (White Sands, N.M., 1975), pp. 1–5. ARO Rep. 76-1. U.S. Army Research Office, Research Triangle Park, N.C., 1976. MR 0547323 (58 #27648).

Abstract of (1977b). Also, bounding the least eigenvalue in terms of principal submatrices. (**SG: LG**)

1977a On graphs whose least eigenvalue exceeds $-1-\sqrt{2}$. *Linear Algebra Appl.* 16 (1977), 153–165. MR 0469826 (57 #9607). Zbl 354.05048.

Introduces generalized line graphs. [They are the reduced line graphs of signed graphs of the form $-\Gamma$ with any number of negative digons attached to each vertex; see Zaslavsky (2010b), Ex. 7.6; (20xxa)]. (**LG**)

1977b On signed graphs and gramians. *Geometriae Dedicata* 6 (1977), 455–470. MR 0463211 (57 #3167). Zbl 407.05064.

Σ is a signed simple graph. Let λ be the least eigenvalue of $A(\Sigma)$. Can (*) $A(\Sigma) - \lambda I - KK^{\mathrm{T}}$ be zero for some K with all entries $0, \pm 1$? When $\lambda = -2$, K exists [equivalently, Σ is a reduced line graph of a signed graph; *cf.* Zaslavsky (2010b), (20xxa)], with finitely many exceptions; the proof uses root systems; *cf.* Cameron, Goethals, Seidel, and Shult (1976a). In general, no K may give zero, but the minimum, over all K, of the largest element of (*) is bounded by a function of λ.

(**SG: LG: Adj, Eig**)

[A.J. Hoffman & D. Gale]

1956a Appendix [to the paper of Heller and Tompkins (1956a)]. In: H.W. Kuhn and A.W. Tucker, eds., *Linear Inequalities and Related Systems*, pp. 252–254. Annals of Math. Studies., No. 38. Princeton Univ. Press, Princeton, N.J., 1956.

(**sg: Incid: bal**)

Alan J. Hoffman & Peter Joffe

1978a Nearest $\mathcal{S}$-matrices of given rank and the Ramsey problem for eigenvalues of bipartite $\mathcal{S}$-graphs. In: *Problèmes Combinatoires et Théorie des Graphes* (Colloq. Int., Orsay, 1976), pp. 237–240. Colloques Int. du CNRS, 260. Editions du C.N.R.S., Paris, 1978. MR 0539983 (81b:05080). Zbl 413.05031. (**SG: Eig**)

Alan J. Hoffman & Francisco Pereira

1973a On copositive matrices with $-1, 0, 1$ entries. *J. Combinatorial Theory Ser. A* 14 (1973), 302–309. MR 0316482 (47 #5029). Zbl 273.15019.

Matrices $A(\Sigma) + I$ for simple $|\Sigma|$: properties as quadratic forms. Generalizes Haynsworth and Hoffman (1969a). [Annot. 28 May 2017.]

(**sg: adj**)

Dean Hoffman & Heather Jordon

2006a Signed graph factors and degree sequences. *J. Graph Theory* 52 (2006), no. 1, 27–36. MR 2214439 (2006k:05174). Zbl 1117.05089.

The net degree of a vertex in Σ is $d^+(v) - d^-(v)$. [This is best viewed as degree in an all-negative bidirected graph; *cf.* p. 35.] Thms. 2.3 (for Σ) and 4.1 (for a bidirected graph B, called a "mixed signed graph") are an interesting f-factor theorem in terms of net degrees. Thm. 4.1: Given $f : V \to \mathbb{Z}$, an "f-factor" is a subgraph whose net in-degree vector $= f$. For disjoint $S, T \subseteq V$ and a component Q of $\mathrm{B} \setminus (S \cup T)$, $J(Q, S, T)$ is computed in terms of f and in-degrees and out-degrees of edges among Q, S, T. $q(S, T)$ is the number of J-odd components Q. An f-factor exists iff q satisfies an inequality. Thm. 3.2: Fixing the maximum edge multiplicity, an Erdős–Gallai-type characterization of net degree sequences—simplifying the theorem of Michael (2002a). Thm. 4.2: Net in-degree sequences of bidirected simple graphs. [More in Jordon, McBride, and Tipnis (2009a).] [Annot. 14 Oct 2009.]

(**SG: ori: Invar**)

Thomas R. Hoffman

See also D.M. Duncan.

Thomas R. Hoffman & James P. Solazzo

2012a Complex equiangular tight frames and erasures. *Linear Algebra Appl.* 437 (2012), no. 2, 549–558. MR 2921716. Zbl 1246.42028. arXiv:1107.2267. (**gg: kg: Adj**)

2018a Complex two-graphs. *Houston J. Math.* 44 (2018), no. 1, 283–300. MR 3796449. Zbl 1391.05165. arXiv:1408.0334.

(**gg: KG: Adj, TG**)

Karl Heinz Hoffmann

2002a The statistical physics of energy landscapes: From spin glasses to optimization. In: K.H. Hoffmann and M. Schreiber, eds., *Computational Statistical Physics*, Ch. 4, pp. 57–76. Springer, Berlin, 2002. Zbl 1160.82330.

Expository, accessible. Main example is the Ising model: fixed weighted Σ and variable $\zeta : V \to \{\pm 1\}$ with Hamming distance and energy function $e : \{\pm 1\}^V \to \mathbb{R}$ which is mountainous, i.e, many local minima (valleys) with low or high intermediate values $e(\zeta)$. Ground states (minimizing $e(\zeta)$) are therefore hard to find computationally. [*Cf.*, e.g., Vogel *et al.*] Random "thermal variation" of ζ leads to slow "relaxation" from higher to lower local minima, in theory and practice. [Annot. 7 Aug 2018.] (**sg, wg: VS: Str, Phys, Appl: Exp**)

Franz Höfting & Egon Wanke

1993a Polynomial algorithms for minimum cost paths in periodic graphs. In: Vijaya Ramachandran *et al.*, eds., *Proceedings of the Fourth Annual ACM-SIAM Symposium on Discrete Algorithms* (Austin, Tex., 1993), pp. 493–499. Assoc. for Computing Machinery, New York, and Soc. for Industrial and Appl. Math., Philadelphia, 1993. MR 1213262 (93m:05184). Zbl 801.68133.

Given a finite gain digraph Φ (the "static graph") with gains in $\mathbb{Z}^d$ and a rational cost for each edge, find a minimum-cost walk ("path") in its canonical covering graph $\tilde{\Phi}$ with given initial and final vertices.

(**GD(Cov): Algor**)

1994a Polynomial time analysis of toroidal periodic graphs. In: Serge Abiteboul and Eli Shamir, eds., *Automata, Languages and Programming* (Proc. 21st Int. Colloq., ICALP 94, Jerusalem, 1994), pp. 544–555. Lect. Notes in Computer Sci., Vol. 820. Springer-Verlag, Berlin, 1994. MR 1334129 (96c:05164).

Take a gain digraph Φ (the "static graph") with gains in $\mathbb{Z}_\alpha = \mathbb{Z}_{\alpha_1} \times \cdots \times \mathbb{Z}_{\alpha_d}$ (where $\alpha = (\alpha_1, \cdots, \alpha_d)$) and its canonical covering digraph $\tilde{\Phi}$ (the "toroidal periodic graph"). Treated algorithmically via integer linear programming and linear Diophantine equations: existence of directed paths (NP-complete, but polynomial-time if Φ is strongly connected) and number of strongly connected components of $\tilde{\Phi}$.

(**GD(Cov): Algor, Geom**)

1995a Minimum cost paths in periodic graphs. *SIAM J. Computing* 24 (1995), 1051–1067. MR 1350758 (96d:05061). Zbl 839.05063.

Full version of (1993a). The min-cost problem is expressed as an integer linear program. Various conditions under which the problem is NP-hard, even a very restricted version without costs (Thms. 3.3, 3.5), or polynomial-time solvable (e.g.: without costs, when Φ is an undirected gain graph: Thm. 3.4; with costs, when d is fixed: Thm. 4.5). (**GD, GG(Cov): Algor, Geom, Ref**)

2000a Polynomial-time analysis of toroidal periodic graphs. *J. Algorithms* 34 (2000), no. 1, 14–39. MR 1732196 (2001k:68111). Zbl 958.68129.

Full version of (1994a). (**GD(Cov): Algor, Geom**)

Leslie Hogben

See also S. Butler.

2005a Spectral graph theory and the inverse eigenvalue problem of a graph. *Electronic J. Linear Algebra* 14 (2005), 12–31. MR 2202430 (2006k:05133). Zbl 1162.05333. (**par: Adj: Eig**)

Paul W. Holland & Samuel Leinhardt

1970a A method for detecting structure in sociometric data. *Amer. J. Sociology* 76 (1970), no. 3, 492–513.

A formulation of structure in terms of weak partial ordering, i.e., transitivity, hence triads (triples of elements). Refers to (1970b) for specific structures, e.g., structural balance of Cartwright–Harary (1956a) and clustering of Davis (1967a). The types of triples allowed determine what specific model applies. P. 495 states conditions for structural balance or clustering. [Annot. 26 Dec 2012.] (**PsS: sg: Bal, Clu**)

1970b A unified treatment of some structural models for sociometric data. Tech. Rep., Carnegie-Mellon University, 1970.

See (1970a). [Annot. 26 Dec 2012.] (**PsS: sg: Bal, Clu**)

1971a Transitivity in structural models of small groups. *Comparative Group Studies* 2 (1971), 107–124. (**PsS: SG: Bal**)

Paul W. Holland & Samuel Leinhardt, eds.

1979a *Perspectives on Social Network Research* (Proc. Math. Soc. Sci. Board Adv. Res. Sympos. on Social Networks held at Dartmouth College, Hanover, N.H., September 18–21, 1975). Academic Press, New York, 1979. (**PsS, SG**)

Roderick B. Holmes & Vern I. Paulsen

2004a Optimal frames for erasures. *Linear Algebra Appl.* 377 (2004), 31–51. MR 2021601 (2004j:42028). Zbl 1042.46009.

Adjacency matrices of cube-root-of-unity gain graphs. [Annot. 20 Jun 2011.] (**gg: adj**)

[Hein van der Holst]

See H. van der Holst (under 'V').

Hai-Yan Hong

See Y.-Z. Fan.

Sungpyo Hong

See J.H. Kwak.

Yiguang Hong

See D.-Y. Meng.

Yuan Hong & Xiao-Dong Zhang

2005a Sharp upper and lower bounds for largest eigenvalue of the Laplacian matrices of trees. *Discrete Math.* 296 (2005), no. 2-3, 187–197. MR 2154712 (2006g:05127). Zbl 1068.05044.

Thm. 2: If some neighbors of v in Γ are regrafted onto u, forming Γ', and if $x_u \geq x_v$ in the Perron vector of $L(-\Gamma)$, then $\lambda_{\min}(L(-\Gamma)) < \lambda_{\min}(L(-\Gamma'))$. [Annot. 24 Jan 2012.] (**par: Lap: Eig**)

Shlomo Hoory, Nathan Linial, & Avi Wigderson

2006a Expander graphs and their applications. *Bull. Amer. Math. Soc. (N.S.)* 43 (2006), no. 4, 439–561. MR 2247919 (2007h:68055). Zbl 1147.68608.

§6, "Spectrum and expansion in lifts of graphs": covering graphs of permutation gain graphs, and from Bilu and linial (2006a) of signed graphs. §6.1, "Covering maps and lifts": Covering graphs of permutation gain graphs, presented as symmetric digraphs with invertible arc gains. §2.6, "Eigenvalues - old and new": Prop. 6.3. The covering graph's eigenvalues include those of the (underlying) base graph Γ and its eigenvectors sum to 0 on fibers. Prop. 6.4. The signed covering graph's eigenvalues are those of Γ and those of (Γ, σ). §6.4, "Nearly-Ramanujan graphs by way of 2-lifts": Conjectured and proven eigenvalue ranges when the base graph is a Ramanujan graph. Dictionary:"signing" of $A(\Gamma)$ means $A(\Gamma, \sigma)$ for any edge signature. "2-lift" = double covering graph. [Annot. 25 Aug 2011.] (**sg: Cov, Eig: Exp**)

John Hopcroft

See T. Joachims.

Carlos Hoppen

See R.E. Mansano.

Tsuyoshi Horiguchi

See also O. Nagai.

1986a Fully frustrated Ising model on a square lattice. *Progress Theor. Phys. Suppl.* No. 87 (1986), 33–42. MR 0884854 (88g:82063).

On the square lattice, physical quantities for periodic signed graphs with up to four edge weights. A fairly general model of which several previous ones are special cases. [Annot. 22 Jan 2015.] (**Phys: SG, WG**)

A. Hosseiny

See A. Kargaran.

Jiangyou Hou

See D. Hu.

Ting Hou

See H. Gao and X.-Z. Liu.

Yaoping Hou

See also X.D. Chen, Y. Chen, Q. Guo, L. Ou, D.-J. Wang, J.-J. Wang, and Z. Xiong.

2005a Bounds for the least Laplacian eigenvalue of a signed graph. *Acta Math. Sinica (Engl. Ser.)* 21 (2005), no. 4, 955–960. MR 2156977 (2006d:05120). Zbl 1080.05060. (**SG: Eig, Bal**)

Yaoping Hou, Jiongsheng Li, & Yongliang Pan

2003a On the Laplacian eigenvalues of signed graphs. *Linear Multilinear Algebra* 51 (2003), 21–30. MR 1950410 (2003j:05084). Zbl 1020.05044.

Properties of (mainly) largest eigenvalue $\lambda_{\max}(\Sigma)$ of the Laplacian matrix $L(\Sigma)$ of a signed simple graph. Thms. 2.5–2.6 repeat standard criteria for balance [with a sign error in (3) of each]. Main results:

Upper bounds, all in terms of underlying graph: Lemma 3.1: For connected Γ, $\lambda_{\max}(\Gamma, \sigma) \leq \lambda_{\max}(-\Gamma)$, = iff σ is antibalanced (e.g., $-\Gamma$). Thm. 3.4: $\lambda_{\max}(\Sigma) \leq 2(n-1)$, = iff $\Sigma \sim -K_n$. Thm. 3.5: $\lambda_{\max}(\Sigma) \leq$ (1) max edge degree + 2, (2) max(vertex degree + average neighbor degree), (3) a combination of these degrees; = iff Σ is antibalanced and $|\Sigma|$ is semiregular bipartite.

Lower bounds: Cor. 3.8: $\lambda_{\max}(\Sigma^+) + \lambda_{\max}(\Sigma^-) \geq \lambda_{\max}(\Sigma) \geq \lambda_{\max}(\Sigma^+)$, $\lambda_{\max}(\Sigma^-)$. Thm. 3.9: If Σ has a vertex of degree $n-1$, then $\lambda_{\max}(\Sigma) \geq \lambda_{\max}(+|\Sigma|)$, with equality iff Σ is balanced. Thm. 3.10: $\lambda_{\max}(\Sigma) \geq 1 + \max_v d_{|\Sigma|}(v)$.

Interlacing: Lemma 3.7 (special case): $\lambda_i(\Sigma) \geq \lambda_i(\Sigma \setminus e) \geq \lambda_{i+1}(\Sigma)$, where $\lambda_1 = \lambda_{\max} \geq \lambda_2 \geq \cdots$.

Problems about existence of cospectral unbalanced signed graphs. (**SG: Eig**)

Yaoping Hou, Zikai Tang, & Dijian Wang

2019a On signed graphs with just two distinct adjacency eigenvalues. *Discrete Math.* 342 (2019), no. 12, art. 111615. MR 3990025. Zbl 1422.05049.

Classifies those with maximum degree ≤ 4 (all are regular, by Ramezani (2020a)). For 3-regular there are $+K_4 < -K_4$, and Q_3 as in Ghasemian and Fath-Tabar (2017a). For 4-regular there are $+K_5$, $-K_5$, and triangle-free examples Q_4 with all C_4's negative, S_{14}, and for $n \geq 3$, T_{2n} (with eigenvalues ± 2), contrary to Ghasemian and Fath-Tabar. For eigenvalues in $[-2, 2]$ they give an elementary proof of part of McKee and Smyth (2007a). [Annot. 29 May 2018, rev 3 Feb 2021.] (**SG: Adj: Eig**)

2019b On signed graphs with just two distinct Laplacian eigenvalues. *Appl. Math. Comput.* 351 (2019), 1–7. MR 3903669 (no rev). Zbl 1428.05193.

Classifies those with maximum degree ≤ 4. The irregular ones are not obtained from (2019a). [Annot. 3 Feb 2021.] (**SG: Lap: Eig**)

Yao Ping Hou & Li Juan Wei

1999a Whitney numbers of the second kind for Dowling lattices. (In Chinese.) *Acta Sci. Natur. Univ. Norm. Hunan.* 22 (1999), No. 3, 6–10. MR 1746888 (2000k:-05017). Zbl 948.05004.

Combinatorial proof of an explicit formula for W_k [possibly the standard one?]. Studies "associated numbers" W_k^r. Proved: $W_{n-k} \leq W_k$ for $k \leq 3$ [this must be an error for $W_k \leq W_{n-k}$ and must have some restriction on n; well known for $k = 1$]. (**gg: Matrd: Invar**)

C.H. Houghton

1977a Completely 0-simple semigroups and their associated graphs and groups. *Semigroup Forum* 14 (1977), no. 1, 41–67. MR 0453897 (56 #12150). Zbl 358.20071.

Cf. Rees (1940a), Graham (1968a). **(gg: sw: Algeb)**

R.M.F. Houtappel

1950a Statistics of two-dimensional hexagonal ferromagnetics with "Ising"- interaction between nearest neighbours only. *Physica* 16 (1950), 391–392. Zbl 038.41903 (38, p. 419c).

Announcement of (1950b). [Annot. 19 Jun 2012.] **(Phys, WG, sg)**

1950b Order-disorder in hexagonal lattices. *Physica* 16 (1950), 425–455. MR 0039632 (12, 576j). Zbl 038.13903 (38, p. 139c).

Ising spins, i.e. $\zeta : V \to \{+1, -1\}$, in the triangular and honeycomb (hexagonal) lattice graphs on a torus. Different edge weights ("bond strengths") and signs are allowed in the three directions. The all-negative triangular signature (i.e., "antiferromagnetic" with equal weights) is an exceptional case. Switching the triangular lattice (p. 449, bottom) permits assuming that two chosen directions are positive. Exceptional weights are the antibalanced triangular lattice with equal smaller weights, e.g., all weights equal (p. 449, bottom). The honeycomb cannot be exceptional [because it is balanced] (p. 451). [See also Newell (1950a), I. Syôzi (1950a), Wannier (1950a).] [Annot. 20 Jun 2012.] **(Phys, WG, sg: sw)**

Cho-Jui Hsieh

See also K.-Y. Chiang.

Cho-Jui Hsieh, Kai-Yang Chiang, & Inderjit S. Dhillon

2012a Low rank modeling of signed networks. In: *Proceedings of the 18th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining* (KDD '12, Beijing, 2012), pp. 507–515. ACM, New York, 2012. **(SG: Bal, Clu: Algor)**

Min-Hsiu Hsieh

See K.-C. Chen.

L. Hsu

See E. Kaszkurwicz.

Bin Hu

See also C.-C. Huang.

Bin Hu, Zhi-Hong Guan, Xiao-Wei Jiang, Ming Chi, Rui-Quan Liao, & Chao-Yang Chen

2016a Event-driven multi-consensus of multi-agent networks with repulsive links. *Inform. Sci.* 173 (2016), 110–123. Zbl 1429.68310. **(SG: Clu: Algor)**

Chunyang Hu & Shiping Liu

20xxa Vertex isoperimetry on signed graphs and spectra of non-bipartite Cayley graphs. arXiv:2306.05306. **(SG: Adj, Lap: Eig)**

Dan Hu, Hajo Broersma, Jiangyou Hou, & Shenggui Zhang

2021a On the spectra of general random mixed graphs. *Electronic J. Combin.* 28 (2021), no. 1, art. P1.3. MR 4245236. Zbl 1468.05156.

Random $\{1, \pm\mathrm{i}\}$-gain graphs. Probabilistic upper bound. Approximation of normalized Laplacian spectrum. [Annot. 8 Feb 2021.] **(gg:Rand: Adj, Lap: Eig)**

Dan Hu, Xueliang Li, Xiaogang Liu, & Shenggui Zhang

2017a The spectral distribution of random mixed graphs. *Linear Algebra Appl.* 519 (2017), 343–365. MR 3606276. Zbl 1357.05082.

A random $\{1, \pm \mathrm{i}\}$-gain graph is Φ_n with probabilities on each edge of p^2 for gain 1, $p(1-p)$ for each gain $\pm\mathrm{i}$, and $(1-p)^2$ for no edge, where $p = p(n) \in (0, 1)$. Thm. 2: The limiting spectral distribution of $\Phi_n \cdot (1/\sqrt{np(2-p-p^3)})$ as $n \to \infty$ is the standard semicircle distribution. [Annot. 15 Dec 2020.] **(gg: Rand: Adj: Eig)**

Fu-Tao Hu & Mei-Yu Sun

2022a Induced subgraph and eigenvalues of some signed graphs. *AKCE Int. J. Graphs Combin.* 19 (2022), no. 3, 167–170. MR 4517963.

Certain Cartesian products $\Sigma_1 \times \Sigma_2$. **(SG: Adj: Eig)**

Guang Hu & Wen-Yuan Qiu

2009a Extended Goldberg polyhedral links with odd tangles. *MATCH Commun. Math. Comput. Chem.* 61 (2009), no. 3, 753–766. Zbl 1189.92027.

See Flapan (1995a). [Annot. 4 Nov 2010.] **(sg: Top, Chem)**

Jiangping Hu

See Y.-Z. Wu and Y.-L. Zhou.

Lili Hu & Xiangwen Li

2018a Every signed planar graph without cycles of length from 4 to 8 is 3-colorable. *Discrete Math.* 341 (2018), no. 2, 513–519. MR 3724119. Zbl 1376.05067. **(SG: Col)**

2018b Nowhere-zero flows on signed wheels and signed fans. *Bull. Malaysian Math. Sci. Soc.* 41 (2018), no. 4, 1697–1709. MR 3854484. Zbl 1404.05073. **(SG: Flows, Ori)**

Wentao Hu

See Q.M. Guo.

Zhixiang Hu

See J.-Y. Shao.

Bobo Hua

See F.M. Atay.

Hongbo Hua

2007a Bipartite unicyclic graphs with large energy. *MATCH Commun. Math. Comput. Chem.* 58 (2007), no. 1, 57–73. MR 2335478 (2008d:05101). Zbl 1224.05301.

Fix $n \geq 13$. For connected unicyclic Γ such that $-\Gamma$ is balanced, excluding circles and balloons ("tadpoles", "lollipops"), the maximum energy occurs for a hexagon attached by an edge to the third vertex of a path. [*Problem.* Replace "bipartite" by "signed", i.e., allow unbalanced signed graphs.] [Annot. 24 Jan 2012.] **(par: Lap: Eig)**

Chuanchao Huang, Bin Hu, Ruixian Yang, & Guangmei Wu

2018a SNMFP: A two-stage approach to community detection in signed networks. *Physica A* 510 (2018), 754–764. **(SG: Clu: Adj, Dyn: Algor)**

Hao Huang

2019a Induced subgraphs of hypercubes and a proof of the Sensitivity Conjecture. *Ann. Math.* (2) 190 (2019), no. 3, 949–955. MR 4024566. Zbl 1427.05116. arXiv:1907.00847.

Lemma 2.2 has $A(Q_n, \sigma)$ in which every quadrilateral is negative. Lemma 2.3 is the [previously known] bound $\lambda_{\max}(\Sigma) \leq \Delta(|\Sigma|)$. [Annot. 31 Jul 2019.] (**sg: Adj, Eig**)

He Huang

See H.Y. Deng.

Jin Huang

See C. Chen.

Jing Huang, Shuchao Li, & Hua Wang

2018a Relation between the skew-rank of an oriented graph and the independence number of its underlying graph. *J. Combin. Optim.* 36 (2018), 65–80. MR 3811192. Zbl 1398.05093. arXiv:1704.06867.

Skew adjacency matrix $= \mathrm{i}^{-1}A(\Phi)$ where gain graph Φ has $\varphi(\overrightarrow{v_iv_j}) = \mathrm{i}$. [Annot. 15 Jul 2019.] (**gg: Adj**)

Qiongxiang Huang

See L. Lu and J.F. Wang.

Rong Huang, Jianzhou Liu, & Li Zhu

2011a A structural characterization of real k-potent matrices. *Linear Multilinear Algebra* 59 (2011), no. 4, 433–439. MR 2802524 (2012d:15023). Zbl 1237.15029. (**QM: SD**)

Sumin Huang

See X.-Y. Ren.

Tao Huang

See S.-D. Zhai.

Ting-Zhu Huang

See G.X. Tian, J.M. Zhang, and L. Zhang.

Xueyi Huang

See J.-J. Wang.

Yihua Huang

See Y.-B. Gao.

Yufei Huang

See also C.H. Liang.

Yufei Huang, Bolian Liu, & Siyuan Chen

2012a The generalized τ-bases of primitive non-powerful signed digraphs with d loops. *Graphs Combin.* 28 (2012), 227–242. MR 2891644. Zbl 1256.05093. (**SD: qm**)

Yufei Huang, Bolian Liu & Yingluan Liu

2011a The signless Laplacian spectral radius of bicyclic graphs with prescribed degree sequences. *Discrete Math.* 311 (2011), no. 6, 504–511. MR 2799902 (2012a:05189). Zbl 1222.05130.

The largest spectral radius and the extremal graphs. [Annot. 19 Nov 2011.] **(par: Lap: Eig)**

Zexi Huang, Arlei Silva, & Ambuj Singh

2022a POLE: Polarized embedding for signed networks. In: *Proceedings of the Fifteenth ACM International Conference on Web Search and Data Mining* (WSDM '22, 2022, Tempe, Ariz.), 11 pp. ACM, New York, 2022. arXiv:2110.09899. **(SG: fr: Algor, PsS)**

B.A. Huberman

See E. Fradkin.

Falk Hüffner

See also S. Böcker.

Falk Hüffner, Nadja Betzler, & Rolf Niedermeier

2007a Optimal edge deletions for signed graph balancing. In: Camil Demetrescu, ed., *Experimental Algorithms* (6th Int. Workshop, WEA 2007, Rome, 2007), pp. 297–310. Lect. Notes in Computer Sci., Vol. 4525. Springer-Verlag, Berlin, 2007. Zbl 1203.68125.

An improved algorithm for frustration index of $-\Sigma$. Dictionary: "balanced" = antibalanced, i.e., balance of $-\Sigma$; "2-coloring that minimizes inconsistencies with given edge labels" = switching function that minimizes $|E^-(-\Sigma)|$. [Annot. 10 Sept 2011.] **(SG: Fr: Algor)**

2010a Separator-based data reduction for signed graph balancing. *J. Combin. Optim.* 20 (2010), no. 4, 335–360. MR 2734305 (2011j:05325). Zbl 1206.90201.

Dictionary: "balanced" = antibalanced, i.e., balance of $-\Sigma$. **(SG: Fr: Algor)**

Falk Hüffner, Christian Komusiewicz, & André Nichterlein

2015a Editing graphs into few cliques: Complexity, approximation, and kernelization schemes. In: Frank Dehne *et al.*, eds., *Algorithms and Data Structures* (Proc. 14th Int. Symp., WADS 2015, Victoria, B.C., 2015), pp. 410–421. Lect. Notes in Computer Sci., Vol. 9214. Springer, Cham, 2015. MR 3677580. Zbl 06502370. **(sg: kg: Clu, Fr: Algor)**

Florian Hug

See I.E. Bocharova.

JiSun Huh, Sangwook Kim, & Boram Park

2021a On toric ideals arising from signed graphs. *J. Algebraic Combin.* 53 (2021), 1265–1298. MR 4263651. Zbl 1478.13040. arXiv:1810.02082.

Unifies toric ideals of graphs and digraphs (*cf.* Villarreal (1995a), Ohsugi and Hibi (1999a), Reyes, Tatakis, and Thoma (2012a), *et al.*) by generalization to bidirected graphs B (Beta). A positive closed walk $W = e_1 \cdots e_l$ is broken at incoherent vertices into segments $W_1, \ldots, W_k$ ($k \geq 1$) and gives binomial $B_W := \prod E(W_{\text{odd}}) - \prod E(W_{\text{even}})$, where $W_{\text{odd}} := W_1 \cup W_3 \cup \cdots$ and $W_{\text{even}} := W_2 \cup W_4 \cup \cdots$. The toric ideal $I_{\mathrm{B}} \subseteq \mathbb{Z}[E]$ is generated by those binomials.

§3.1, "Primitive binomials". Thm. 3.1 (restated): B_W is primitive iff W is an edge-simple walk that consists of a cactus whose outer circles are negative and inner circles are positive.

[See Reyes, Tatakis, and Thoma (2012a) for two proposed signed-graph problems.]

Dictionary: "signed graph" = bidirected graph B; "balanced vertex" in a walk = coherent vertex; "even-, odd-signed" walk = positive, negative walk; $A(\mathrm{B})$ = incidence matrix H; $r(\mathrm{B}) = \mathrm{nul}\,\mathrm{H}$.

(**sg: Ori: Incid, Algeb**)

Axel Hultman

2002a Polygraph arrangements. *European J. Combin.* 23 (2002), 937–948. MR 1938350 (2003i:05137). Zbl 1018.52009.

§5, "A Dowling generalization". (**gg: Matrd**)

2007a The topology of spaces of phylogenetic trees with symmetry. *Discrete Math.* 307 (2007), no. 14, 1825–1832. MR 2316821 (2008a:05055). Zbl 1109.92031.

Introduces Dowling trees: "Natural Dowling analogues of the complex of phylogenetic trees". (**gg: Matrd: Invar**)

2007b Link complexes of subspace arrangements. *European J. Combin.* 28 (2007), no. 3, 781–790. MR 2300759 (2007m:52029). Zbl 1113.52038. arXiv:math/0507314.

Interprets chromatic polynomials of signed graphs in terms of Hilbert polynomials. (**SG: Invar**)

John Hultz

See also F. Glover.

John Hultz & D. Klingman

1979a Solving singularly constrained generalized network problems. *Appl. Math. Optim.* 4 (1978), 103–119. MR 0475831 (57 #15414). Zbl 373.90075.

(**GN: Matrd: bases**)

Norman P. Hummon & Patrick Doreian

† 2003a Some dynamics of social balance processes: bringing Heider back into balance theory. *Social Networks* 25 (2003), 17–49.

Presents a model for evolution of balance and clusterability (as in Davis (1967a)) of a signed digraph and explores it via computer simulations.

Definitions: Given a signed digraph $\vec{\Sigma}$ and a partition π of V, define the 'clusterability' $c(\vec{\Sigma}, \pi) :=$ (# negative edges within blocks of π) + (# positive edges between blocks). Define $\pi(\vec{\Sigma}) :=$ any π that minimizes $c(\vec{\Sigma}, \pi)$. Define $\vec{\Sigma}(v_i) := \{v_i\vec{v}_j \in \vec{E}(\vec{\Sigma})\}$ with signs. ($\vec{\Sigma}$ models relations in a social group V. $\vec{\Sigma}_i$ is the graph of relations perceived by v_i.)

Initial conditions: Fixed $\#V$, fixed "contentiousness" $p :=$ the probability that an initial edge is negative, a fixed "communication" rule, random $\vec{\Sigma}^0$ and, for each $v_i \in V$, $\vec{\Sigma}_i^0 := \vec{\Sigma}^0$. At time $t+1$, $\vec{\Sigma}_i^t(v_i)$ changes to $\vec{\Sigma}_i^{t+1}(v_i)$ to minimize $d(d(\vec{\Sigma}_i^{t+1}, \pi(\vec{\Sigma}^t))$. Then $\vec{\Sigma}_j^{t+1}(i)$ changes to $\vec{\Sigma}_i^{t+1}(v_i)$ for some v_j (depending on $\vec{\Sigma}_i$ and the communication rule).

Computer simulations examined the types of changes and emerging clusterability of $\vec{\Sigma}^t$ or $\vec{\Sigma}_i^t$ as t increases, under four different communication rules, random initial conditions with various p, and $\#V = 3, 5, 7, 10$. The outcomes are highly suggestive (see §4; p seems influential). [*Prob-*

lem. Predict the outcomes in terms of initial conditions through a mathematical analysis.] [Annot. 26 Apr 2009.]
(**SD, sg: Bal, Clu: Algor**)(**PsS**)

Norman P. Hummon & T.J. Fararo

1995a Assessing hierarchy and balance in dynamic network models. *J. Math. Sociology* 20 (1995), 145–159. Zbl 858.92032.

David J. Hunter

2012a *Essentials of Discrete Mathematics*. Second ed. Jones & Bartlett Learning, Sudbury, Mass., 2012. Third ed., 2017. Zbl 1236.00002.

§6.2.4, "Signed graphs and balance". Elementary. [Annot. 12 Jan 2018.]
(**SG: Bal: Exp**)

John E. Hunter

1978a Dynamic sociometry. *J. Math. Sociology* 6 (1978), 87–138. MR 0504069 (58 #20631). (**SG: Bal, Clu**)

Bofeng Huo, Shengjin Ji, Xueliang Li, & Yongtang Shi

2011a Solution to a conjecture on the maximal energy of bipartite bicyclic graphs. *Linear Algebra Appl.* 435 (2011), no. 4, 804–810. MR 2807234 (2012e:05232). Zbl 1220.05073.

[*Question*. Do the results generalize to $A(\Sigma)$ for antibalanced signed bicyclic graphs?] [Annot. 8 Sept 2016.] (**par: Adj: Eig**)

Bofeng Huo, Xueliang Li, & Yongtang Shi

2011a Complete solution to a problem on the maximal energy of unicyclic bipartite graphs. *Linear Algebra Appl.* 434 (2011), no. 5, 1370–1377. MR 2763594 (2011m:05176). Zbl 1205.05146.

[*Question*. Do the results generalize to $A(\Sigma)$ for antibalanced signed unicyclic graphs?] [Annot. 21 Mar 2011.] (**par: Adj: Eig**)

Li Fang Huo & Yu Bin Gao

2010a Local bases of two class of primitive nonpowerful signed digraphs with girth 2. In Chinese; English summary. *Math. Pract. Theory* 40 (2010), no. 10, 235–239. MR 2730313 (no rev). Zbl 1493.05144 (no rev).
(**SD: Adj, qm**)

C.A.J. Hurkens

1989a On the existence of an integral potential in a weighted bidirected graph. *Linear Algebra Appl.* 114/115 (1989), 541–553. MR 0986893 (90c:05142). Zbl 726.05050.

Given: a bidirected graph B (with no loose or half edges or positive loops) and an integer weight b_e on each edge. Wanted: an integral vertex weighting x such that $\mathrm{H}(B)^{\mathrm{T}}x \leq b$, where $\mathrm{H}(B)$ is the incidence matrix. Such x exists iff (i) every coherent circle or handcuff walk has nonnegative total weight and (ii) each doubly odd Korach walk (a generalization of a coherent handcuff that has a cutpoint dividing it into two parts, each with odd total weight) has positive total weight. This improves a theorem of Schrijver (1991a) and is best possible. Dictionary: "path" ("cycle") = coherent (closed) walk. **(sg: Ori: Incid)**

Jake Huryn

See O. Coppola.

David A. Huse

See C.K. Thomas.

Joan Hutchinson

See D. Archdeacon.

Daniel Huttenlocher

See J. Leskovec.

Tony Huynh

See also M. Conforti and J. Geelen.

2009a (As Tony Chi Thong Huynh) *The Linkage Problem for Group-labelled Graphs*. Doctoral thesis, University of Waterloo, 2009.

Gains are in $\mathrm{GF}(q)^\times$ or sometimes in a finite abelian group. Dictionary: "group-labelled graph" = gain graph, "Dowling matroid" = frame matroid (not Dowling geometry), "shifting" = switching. **(GG: Matrd)**

Tony Huynh, Felix Joos, & Paul Wollan

2019a A unified Erdős–Pósa theorem for constrained cycles. *Combintorica* 39 (2019), no. 2, 91–133. MR 3936194. Zbl 1438.05198. arXiv:1605.07082.

Double gain graph: gains in $\mathfrak{G}_1 \times \mathfrak{G}_2$. Doubly nonneutral circle: both gains $\neq 1$. **(GG: Circ: Str)**

Tony Huynh, Andrew D. King, Sang-Il Oum, & Maryam Verdian-Rizi

2017a Strongly even cycle decomposable graphs. *J. Graph Theory* 84 (2017), no. 2, 158–175. MR 3601124. arXiv:1209.0160. Zbl 1354.05073.

Γ is "strongly even cycle decomposable" iff every signed graph (Γ, σ) decomposes into positive circles (confusingly called "even cycles"). Dictionary: *cf.* Huynh, Oum, and Verdian-Rizi (2017a). [Annot. 26 Dec 2012.] **(SG: Circ)**

Tony Huynh, Sang-il Oum, & Maryam Verdian-Rizi

2017a Even cycle decompositions of graphs with no odd-K_4-minor. *European J. Combin.* 65 (2017), 1–14. MR 3679832. Zbl 1369.05172. arXiv:1211.1868.

Despite the name, decomposability of Σ inton(edge-disjoint) positive circles. Dictionary: "even" = positive, "odd" = negative (hence, unnecessary confusion), "even-length" = even, "re-signing" = switching, "parity" of vertex = parity of $d^-(v)$. [Annot. 26 Dec 2012.]

(SG: Circ)

Dawn Iacobucci

1994a Graphs and matrices. In: Stanley Wasserman & Katherine Faust, eds., *Social Network Analysis: Methods and Applications*, Ch. 4, pp. 92–166. Structural Anal. Soc. Sci., 8. Cambridge Univ. Press, Cambridge, 1994.

See Wasserman and Faust (1994a). [Annot. 21 Jan 2019.]

(PsS, SG, SD: Bal, Exp, Ref)

Giovanni Iacono

See also G. Facchetti and N. Soranzo.

Giovanni Iacono & Claudio Altafini

2010a Monotonicity, frustration, and ordered response: an analysis of the energy landscape of perturbed large-scale biological networks. *BMC Systems Biol.* 4 (2010), art. 83, 14 pp. + suppl. (**SD, SG: Fr, Sw, State, Algor, Biol**)

G. Iacono, F. Ramezani, N. Soranzo, & C. Altafini

2010a Determining the distance to monotonicity of a biological network: a graph-theoretical approach. *IET Systems Biol.* 4 (2010), no. 3, 223–235. (**SD, SG: Fr, Sw, Algor, Biol**)

Toshihide Ibaraki

See also Y. Crama and P.L. Hammer.

T. Ibaraki & U.N. Peled

1981a Sufficient conditions for graphs to have threshold number 2. In: Pierre Hansen, ed., *Studies on Graphs and Discrete Programming* (Proc. Workshop, Brussels, 1979), pp. 241–268. North-Holland Math. Studies, 59. Ann. Discrete Math., 11. North-Holland, Amsterdam, 1981. MR 0653829 (84f:05056). Zbl 479.05058. (**par: ori**)

Takashi Iino

See T. Yoshikawa.

Takeo Ikai

See H. Kosako.

Yoshiko T. Ikebe & Akihisa Tamura

2003a Polyhedral proof of a characterization of perfect bidirected graphs. *IEICE Trans. Fundam. Electronic Commun. Computer Sci.* E86-A (2003), no. 5, 1000–1007. (**sg: Ori: Geom**)

20xxa Perfect bidirected graphs. Manuscript.

A transitively closed bidirection of a simple graph is perfect iff its underlying graph is perfect. (See Johnson and Padberg (1982a) for definitions.) [Also proved by Sewell (1996a).] (**sg: Ori: Incid, Geom**)

Rintaro Ikeshita & Shin-Ichi Tanigawa

† 2018a Count matroids of group-labeled graphs. *Combinatorica* 38 (2018), no. 5, 1101–1127. MR 3884781. Zbl 1424.05039. arXiv:1507.01259. (**GG: Matrd**)

Victor Il'ev, Svetlana Il'eva, & Alexander Kononov

2016a Short survey on graph correlation clustering with minimization criteria. In: Yury Kochetov *et al.*, eds., *Discrete Optimization and Operations Research* (9th Int. Conf., DOOR 2016, Vladivostok, 2016), pp. 25–36. Lect. Notes in Computer Sci., Vol. 9869. Springer, [Cham], 2016. MR 3577727. Zbl 1380.68314. (**sg: Clu**)

Svetlana Il'eva

See V. Il'ev.

Aleksandar Ilić

See L.H. Feng, G.H. Yu, and B. Zhou.

Denis Petrovich Ilyutko

See V.O. Manturov.

Nicole Immorlica

See E. Demaine.

Ryota Inagaki

See M. Cho.

Takehiro Inohara

1999a On conditions for a meeting not to reach a recurrent argument. *Appl. Math. Comput.* 101 (1999), 281–298. MR 1677966 (99k:90010). Zbl 942.91019. **(SD, PsS)**

2000a Meetings in deadlock and decision makers with interperception. *Appl. Math. Comput.* 109 (2000), 121–133. MR 1738208 (2000m:91035). Zbl 1042.91010. **(SD, PsS)**

2002a Characterization of clusterability of signed graph in terms of Newcomb's balance of sentiments. *Appl. Math. Comput.* 133 (2002), no. 1, 93–104. MR 1923185 (2003i:05064). Zbl 1023.05072.

Assumption: all $\sigma(i,i) = +$. Thm. 3: A signed complete digraph is clusterable iff $\sigma(i,j) = -$ or $\sigma(j,k) = \sigma(i,k)$ for every triple $\{i,j,k\}$ of vertices (not necessarily distinct). [The notation is unnecessarily complicated.] **(SD: Clu, PsS)**

2003a Clusterability of groups and information exchange in group decision making with approval voting system. *Appl. Math. Comput.* 136 (2003), no. 1, 1–15. MR 1935595 (2004b:91059). Zbl 1042.91086. **(SD: KG: Bal, Clu, PsS)**

2004a Quasi-clusterability of signed graphs with negative self evaluation. *Appl. Math. Comput.* 158 (2004), no. 1, 201–215. MR 2091243 (2005f:05072). Zbl 1055.05074. **(SD: Clu, PsS)**

2004b Signed graphs with negative self evaluation and clusterability of graphs. *Appl. Math. Comput.* 158 (2004), no. 2, 477–487. MR 2094633 (2005f:05073). Zbl 1054.05048. **(SD: Clu, PsS)**

2007a Relational dominant strategy equilibrium as a generalization of dominant strategy equilibrium in terms of a social psychological aspect of decision making. *European J. Oper. Res.* 182 (2007), 856–866. Zbl 1121.90355. **(SD, PsS)**

Takehiro Inohara, Shingo Takahashi, & Bunpei Nakano

1998a On conditions for a meeting not to reach a deadlock. *Appl. Math. Comput.* 90 (1998), 1–9. MR 1485601. Zbl 907.90014. **(SD, PsS)**

2000a Credibility of information in 'soft' games with interperception of emotions. *Appl. Math. Comput.* 115 (2000), 23–41. MR 1779380 (2001e:91037). Zbl 1046.91004. **(SD, PsS)**

Jun-ichi Inoue

See S. Suzuki.

Yuri J. Ionin & Mohan S. Shrikhande

2006a *Combinatorics of Symmetric Designs.* Cambridge Univ. Press, Cambridge, Eng., 2006. MR 2234039 (2008a:05001). Zbl 1114.05001.

§7.3, "Switching in strongly regular graphs": Graph switching and two-graphs. (**TG, Sw: Exp**)

Jerad Ipsen & Sudipta Mallik

20xxa Incidence and Laplacian matrices of wheel graphs and their inverses. Submitted. arXiv:2201.02579.

Moore–Penrose inverse of $L(-\Gamma)$. [Annot. 13 Oct 2022.] (**sg: Par: Incid, Lap**)

Ali Iranmanesh

See J. Askari.

Masao Iri

See also J. Shiozaki.

Masao Iri & Katsuaki Aoki

1980a A graphical approach to the problem of locating the origin of the system failure. *J. Operations Res. Soc. Japan* 23 (1980), 295–312. MR 0606141 (82c:90041). Zbl 447.90036. (**SD, VS: Appl**)

Masao Iri, Katsuaki Aoki, Eiji O'Shima, & Hisayoshi Matsuyama

1976a [A graphical approach to the problem of locating the system failure.] (In Japanese.) [???] 76(135) (1976), 63–68. (**SD, VS: Appl**)

1979a An algorithm for diagnosis of system failures in the chemical process. *Computers and Chem. Eng.* 3 (1979), 489–493 (1981).

The process is modelled by a signed digraph with some nodes v marked by $\mu(v) \in \{+,-,0\}$. (Marks $+,-$ indicate a failure in the process.) Object: to locate the node which is origin of the failure. An oversimplified description of the algorithm: μ is extended arbitrarily to V. Arc (u,v) is discarded if $0 \neq \mu(u)\mu(v) \neq \sigma(u,v)$. If the resulting digraph has a unique initial strongly connected component S, the nodes in it are possible origins. Otherwise, this extension provides no information. (I have overlooked: special marks on "controlled" nodes; speedup by stepwise extension and testing of μ.) [Continued in Shiozaki, Matsuyama, O'Shima, and Iri (1985a).] [This article and (1976a) seem to be the origin of a whole literature. See e.g. Chang and Yu (1990a), Kramer and Palowitch (1987a)] (**SD, VS: Appl, Algor**)

Lucas Isenmann & Timothée Pecatte

2017a Möbius stanchion systems. *Electronic Notes Discrete Math.* 62 (2017), 177–182. MR 3746719. Zbl 1383.05069.

Signed plane graph Σ is orientation embedded via the plane vertex rotations in surface S. Thm.: All single-face embeddings are connected via two operations: negating certain edges, and simultaneously negating certain edge pairs. The edges involved depend on the 1-face walk. Dictionary: "Möbius edge" = negative edge; "painting walk" = face walk in S. "Möbius stanchion system (MSS)" = Σ with single-face embedding in S [*cf.* Širáň and Škoviera (1991a)]. [Annot. 2 Nov 2017.] (**sg: Top**)

Toru Ishihara

2000a Cameron's construction of two-graphs. *Discrete Math.* 215 (2000), 283–291. MR 1746466 (2000k:05090). Zbl 959.05099.

A new proof of Cameron (1994a). (**TG**)

2002a Signed graphs associated with the lattice A_n. *J. Math. Univ. Tokushima* 36 (2002), 1–6 (2003). MR 1974060 (2004c:05086). Zbl 1032.05061.

A signed graph corresponding to a base of A_n is a [signed] path of cliques and locally switches to a path. (For local switching see Cameron, Seidel, and Tsaranov (1994a).) (**SG: Geom**)(**SG: Sw: Gen**)

2004a Local switching of some signed graphs. *J. Math. Univ. Tokushima* 38 (2004), 1–7. MR 2123167 (2005m:05110). Zbl 1067.05032.

Which signed graphs locally switch to a tree? Examples only. (**SG: Sw: Gen**)

2005a Local switching of signed induced cycles. *J. Math. Univ. Tokushima* 39 (2005), 1–5. MR 2194305 (2006i:05077).

Converting an induced circle to a path by local switching. (**SG: Sw: Gen**)

2007a Signed graphs and Hushimi trees. *J. Math. Univ. Tokushima* 41 (2007), 13–23. MR 2380208 (no rev). Zbl 1138.05316.

Local switching between trees. [Annot. 28 Dec 2011.] (**CSG**)

Sorin Istrail

2000a Statistical mechanics, three-dimensionality and NP-completeness. I. Universality of intractability for the partition function of the Ising model across non-planar lattices. In: *Proceedings of the Thirty-Second Annual ACM Symposium on Theory of Computing* (STOC, Portland, Ore., 2000), pp. 87–96. ACM, New York, 2000. MR 2114521 (no rev).

Extends Barahona (1982a) on finite signed lattice graphs to the computational complexity of (a) ground states (i.e., frustration index) and (more difficult) (b) partition function (generating function of frustrated edges over all states), for signed infinite lattice graphs. [An infinite lattice graph is (apparently) a graph drawn in $\mathbb{E}^d$, crossings allowed, that has translational symmetry in d independent directions.] General conclusion: For nonplanar ones they are NP-hard. Thm. 1: A lattice graph in $d = 2, 3$ is planar iff it does not contain a certain $d = 2$ lattice graph K_0, the "Basic Kuratowskian". Lem. 2: Every 3-regular graph has a subdivision contained in K_0. §5, "Computational complexity of the 3D Ising models": Lattice graphs with signs, subgraphs thereof, all-positive subgraphs, all-negative subgraphs. Thm. 2: For every subgraph of a signed non-planar infinite lattice graph, computing $l(\Sigma)$ for finite sublattice graphs Σ is NP-hard. Thm. 3: For every subgraph of an all-negative non-planar infinite lattice graph, computing $l(\Sigma)$ for finite sublattice graphs Σ is NP-hard. §5.3, "Ising models with $\{-J, +J\}$ interactions": For every signed non-planar infinite lattice graph, computing $l(\Sigma)$ for finite sublattice graphs Σ is NP-hard; the proof is postponed to "the full version of the paper" [which has not appeared]. [Annot. 21 Aug 2012.] (**SG, Phys: Fr**)

Gabriel Istrate

2009a On the dynamics of social balance on general networks (with an application to XOR-SAT). Machines, Computations and Universality, Part II. *Fund. Inform.* 91 (2009), no. 2, 341–356. MR 2516378 (2010f:68140). Zbl 1181.91282.

Imbalance measured by triangles. Repeatedly change signs of edges of a fixed graph. Looks for recurrent states and time to become balanced. [Annot. 5 May 2010.] (**SG: Fr: Dyn**)

C. Itzykson

See R. Balian.

P.L. Ivanescu [P.L. Hammer]

See E. Balas and P.L. Hammer.

Sousuke Iwai

See also O. Katai.

Sousuke Iwai & Osamu Katai

1978a Graph-theoretic models of social group structures and indices of group structures. (In Japanese.) *Systems and Control* (*Shisutemu to Seigyo*) 22 (1978), 713–722. MR 0540551 (80d:92038). (**CPsS: Exp**)

Ravi Iyengar [Satteluri R.K. Iyengar]

See A. Ma'ayan.

Hiroshi Iyetomi

See T. Yoshikawa.

Slavik Jablan

See Kauffman, Jablan, Radović, and Sazdanović (2013a).

Mike Jackanich

See M̃Beck.

Bill Jackson

See P.J. Cameron and J. Bang-Jensen.

Michael S. Jacobson

See A.H. Busch and R.J. Faudree.

Fabien Jacques

See also C. Duffy.

Fabien Jacques & Alexandre Pinlou

2021a The chromatic number of signed graphs with bounded maximum average degree. In: Jaroslav Nešetřil *et al.*, eds., *Extended Abstracts EuroComb 2021* (European Conf. Combin. Graph Theory Appl., virtual), pp. 657–662. Trends Math., Vol. 14. Birkhäuser, Cham, 2021.

Extended abstract of (2022a). (**SG: Hom, Col**)

2022a The chromatic number of signed graphs with bounded maximum average degree. *Discrete Appl. Math.* 316 (2022), 43–59. MR 4413252. Zbl 1490.05074. arXiv:2104.11121. (**SG: Hom, Col**)

François Jaeger

1992a On the Kauffman polynomial of planar matroids. In: Jaroslav Nešetřil and Miroslav Fiedler, eds., *Fourth Czechoslovak Symposium on Combinatorics, Graphs and Complexity* (Prachatice, 1990), pp. 117–127. Ann. Discrete Math., Vol. 51. North-Holland, Amsterdam, 1992. MR 1206253 (94d:57016). Zbl 763.05021.

(This is not the colored Tutte polynomial of Kauffman (1989a).)

Jaeger shows that the Kauffman polynomial, originally defined for link diagrams and here transformed to an invariant of signed plane graphs,

depends only on the edge signs and the circle matroid. It can also be reformulated to be essentially independent of signs. *Problem.* Define a similar invariant for more general matroids.
(**SGc, Sgnd(Matrd): Invar, Knot**)

G.R. Jafari
See A. Kargaran and R. Masoumi.

N. Jafari Rad
See N. Kafai.

R. Jagadeesh
See M.R. Rajesh Kanna.

John C. Jahnke
See J.O. Morrissette.

Kamal Jain
See A. van Zuylen.

Rashmi Jain
See also M. Acharya.

Rashmi Jain, Mukti Acharya, & Sangita Kansal

2018a Vertex equitable labeling of signed bistars. *Nat. Acad. Sci. Lett.* 41 (2018), no. 4, 239–242. MR 3842237. (**SG**)

2021a Characterizations of line-cut signed graphs. *Nat. Acad. Sci. Lett.* 44 (2021), no. 2, 147–150. MR 4233614. (**SG: LG(Gen), VS**)

Rashmi Jain, Sangita Kansal, & Mukti Acharya

2015a $\mathcal{C}$-cycle compatible splitting signed graphs $\mathfrak{S}(S)$ and $\Gamma(S)$. *European J. Pure Appl. Math.* 8 (2015), no. 4, 469–477. MR 3418484. Zbl 1440.05110.
Splitting graph $|\Sigma|'$ is signed two ways. In $\Gamma(\Sigma)$, $\sigma(uv') = \sigma(uv)$. In $\mathfrak{S}(\Sigma)$, $E^- = E{:}\mu_\sigma^{-1}(-)$. Σ for which $\Gamma(\Sigma) \cong \mathfrak{S}(\Sigma)$, and for which each is harmonious. [Annot. 18 Oct 2023.] (**SG, VS**)

2017a $\mathcal{C}$-consistent and $\mathcal{C}$-cycle compatible dot-line signed graphs. Int. Conf. Current Trends in Graph Theory and Computation (CTGTC-2016, New Delhi). *Electronic Notes Discrete Math.* 63 (2017), 469–478. MR 3754837. Zbl 1383.05143. (**SG: LG**)

Mahdi Jalili
See P. Esmailian, A. Javari, and M. Shahriari.

Fareeha Jamal & Mehtab Khan

2020a Extremal iota energy of a subclass of bicyclic digraphs and sidigraphs. *Asian-European J. Math.* 13 (2020), no. 7, art. 2050127, 13 pp. MR 4140792. Zbl 1458.05122.
Cf. Hafeez and Khan (2018a). (**SD: Adj: Eig**)

Robert Janczewski, Krzysztof Turowski, & Bartłomiej Wróblewski

2023a Edge coloring of graphs of signed class 1 and 2. *Discrete Appl. Math.* 338 (2023), 311–319. MR 4613884. Zbl 1519.05078. arXiv:2205.15425. HAL hal-04156101.
$\chi'(\Sigma) = \Delta(|\Sigma|) + 0$ or 1 by Behr (2020a). Is $\chi'(\Gamma, \sigma)$ independent of σ? If so, Γ is said to be in class $1^\pm$ or $2^\pm$, respectively. Partial

characterizations. [Annot. 4 Jun 2022.] (**SG: ECol**)

20xxa Edge coloring of products of signed graphs. Submitted. arXiv:2312.02691. (**SG: ECol**)

Hye Jin Jang, Jack Koolen, Akihiro Munemasa, & Tetsuji Taniguchi

2014a On fat Hoffman graphs with smallest eigenvalue at least -3. *Ars Math. Contemp.* 7 (2014), no. 1, 105–121. MR 3047614. Zbl 1301.05219. arXiv:1211.3929. (**SG: Eig**)

Abdul Salam Jarrah

See E. Sontag.

John J. Jarvis & Anthony M. Jezior

1972a Maximal flow with gains through a special network. *Operations Res.* 20 (1972), 678–688. MR 0317739 (47 #6286). Zbl 241.90021. (**GN: Matrd(bases)**)

A. Javanmard

See S. Akbari.

Amin Javari & Mahdi Jalili

2014a Cluster-based collaborative filtering for sign prediction in social networks with positive and negative links. *ACM Trans. Intelligent Sys. Tech.* 5 (2014), no. 2, art. 24, 19 pp. (**SG: Clu, Pred: Algor**)

M. Javarsineh

See S. Akbari.

C. Jayaprakash

See J. Vannimenus.

C. Jayasekaran [Chelliah Jayasekaran]

See also G. Sumathy and V. Vilfred.

2007a *A Study on Self Vertex Switchings of Graphs*, Ph.D. dissertation, Manonmanium Sundaranar University, Tirunelveli, India, 2007.

A self vertex switching is a Seidel (graph) switching $\Gamma^v \cong \Gamma$ for $v \in V$, or it is v [better called a "self-switching vertex", *cf.* MR for Vilfred and Jayasekaran (2009a)]. [*Cf.* articles of J. Hage.] [Annot. 26 Sept 2012.] (**tg: Sw**)

2012a Self vertex switchings of unicyclic graphs. *Graph Theory Notes N.Y.* 62 (2012), 29–38. MR 3012272.

See Jayasekaran (2007a). Characterizes unicyclic graphs with a self-switching vertex. [Annot. 26 Sept 2012.] (**tg: Sw**)

2012b Self vertex switchings of connected unicyclic graphs. *J. Discrete Math. Sci. Cryptogr.* 15 (2012), no. 6, 377–388. MR 3060112 (no rev). Zbl 1350.05077. (**tg: Sw**)

2016a Self vertex switchings of trees. *Ars Combin.* 127 (2016), 33–43. MR 3559076. Zbl 1474.05047.

See Jayasekaran (2007a). Characterizes trees with a self-switching vertex. [Annot. 26 Sept 2012.] (**tg: Sw**)

2016b Self vertex switchings of disconnected unicyclic graphs. *Ars Combin.* 129 (2016), 51–62. MR 3560853. Zbl 1413.05318. (**tg: Sw**)

C. Jayasekaran & M. Ashwin Shijo

2021a Anti-duplication self vertex switching in some graphs. *Malaya J. Mat.* 9 (2021), no. 1, 338–342. MR 4229922. (**tg: Sw**)

C. Jayasekaran, J. Christabel Sudha, & M. Ashwin Shijo

2021a 2-Vertex self switching of forest. *Nonlinear Stud.* 28 (2021), no. 3, 749–759. MR 4318145. (**tg: Sw**)

C. Jayasekaran & G. Sumathy

2016a Characterization of disconnected two-cyclic graphs with a self vertex switching. *J. Combin. Inform. System Sci.* 41 (2016), no. 1-3, 123–140. MR 3676449. Zbl 1375.05258. (**tg: Sw**)

2014a Self vertex switching of connected two-cyclic graphs. *J. Discrete Math. Sci. Cryptogr.* 17 (2014), no. 2, 157–179. MR 3241015 (no rev). Zbl 1348.05114. (**tg: Sw**)

Nikola Jedličková

See J. Bok.

Clark Jeffries

1974a Qualitative stability and digraphs in model ecosystems. *Ecology* 55 (1974), 1415–1419.

Sufficient (and necessary) conditions for sign stability in terms of negative cycles and a novel color test. Proofs are sketched or (for necessity) absent. [Necessity is proved in Logofet and Ul'yanov (1982a), (1982b).] (**SD: QSta**)

1993a Some matrix patterns arising in queuing theory. In: Richard A. Brualdi, Shmuel Friedland, and Victor Klee, eds., *Combinatorial and Graph-Theoretical Problems in Linear Algebra*, pp. 165–174. IMA Vols. Math. Appl., 50. Springer-Verlag, New York, 1993. MR 1240961 (94e:15056). Zbl 789.60069.

In a weighted symmetric digraph, a cycle is "balanced" if the product of its weights equals the weight product of the inverse cycle (p. 171). If all cycles of length ≥ 3 are balanced, stability multipliers exist in an associated differential system (Thms. 9, 10). [For weights a_{ij}, define gains $\varphi(v_i, v_j) := a_{ij}/a_{ji}$. Then "balance" is balance in the gain graph. *Question*: What can be made of this?] [Annot. 13 Apr 2009.] (**gg: bal**)

Clark Jeffries, Victor Klee, & Pauline van den Driessche

1977a When is a matrix sign stable? *Canad. J. Math.* 29 (1977), 315–326. MR 0447288 (56 #5603). Zbl 383.15005. (**SD: QSta**)

Clark Jeffries & P. van den Driessche

1988a Eigenvalues of matrices with tree graphs. *Linear Algebra Appl.* 101 (1988), 109–120. MR 0941299 (89i:05198). Zbl 686.05037.

A is a real matrix whose bipartite graph is a forest. The signed digraph $\vec{\Sigma}(A)$ yields information about eigenvalues. Controllability of solutions of $\dot{x}(t) = Ax(t)$ may be deduced from $\vec{\Sigma}(A)$. [Annot. 24 July 2010.] (**QM: SD**)

1991a Qualitative stability and solvability of difference equations. *Linear Multilinear Algebra* 30 (1991), no. 4, 275–282. MR 1129184 (92m:39011). Zbl 777.39001.
(QM: SD, QSol, QSta)

Eva Jelínková & Jan Kratochvíl

2008a On switching to H-free graphs. In: Hartmut Ehrig *et al.*, eds., *Graph Transformations* (4th Int. Conf., ICGT 2008, Leicester, U.K., 2008), pp. 379–395. Lect. Notes in Computer Sci., Vol. 5214. Springer-Verlag, Berlin, 2008. Zbl 1175.68298.

Characterizing graph switching classes that contain a graph with no H subgraph, for some particular graph H. [An example of Kratochvíl, Nešetřil, & Zýka (1992a).] [Annot. 21 Mar 2011.] **(TG: Sw)**

Robin Jenkins

See P. Abell.

Kevin Jennings

See C. O'Brien.

Pablo Jensen

See S. Gómez.

Paul A. Jensen & J. Wesley Barnes

1980a *Network Flow Programming.* Wiley, New York, 1980. MR 0579183 (82f:90096). Zbl 502.90057. Repr.: Robert E. Krieger, Melbourne, Fla., 1987. MR 0934708 (89a:90152).

§1.4: "The network-with-gains model." §2.8: "Networks with gains—example applications." Ch. 9: "Network manipulation algorithms for the generalized network." Ch. 10: "Generalized minimum cost flow problems." **(GN: Matrd(bases))**

§5.5: "Negative cycles." **(OG: Matrd(bases))**

1984a *Potokovoe programmirovanie.* Radio i Svyaz, Moskva, 1984. Zbl 598.90035.

Russian translation of (1980a). **(GN: Matrd(bases))(OG: Matrd(bases))**

P.A. Jensen & Gora Bhaumik

1977a A flow augmentation approach to the network with gains minimum cost flow problem. *Management Sci.* 28 (1976/77), no. 6 (Feb., 1977), 631–643. MR 0441300 (55 #14163a). Zbl 352.90024. **(GN)**

T.R. Jensen & F.B. Shepherd

1995a Note on a conjecture of Toft. *Combinatorica* 15 (1995), no. 3, 373–377. MR 1357283 (96h:05073). Zbl 832.05038.

Proves Toft's (1975a) conjecture for a 4-critical graph with a degree-3 vertex, whence for line graphs. [Annot. 2 Nov 2017.] **(sg: par: Col)**

Tommy R. Jensen & Bjarne Toft

1995a *Graph Coloring Problems.* Wiley, New York, 1995. MR 1304254 (95h:05067). Zbl 950.45277.

§8.14: "t-perfect graphs." Related to $-\Gamma$ with no subgraph homeomorphic to $-K_4$ (no "odd-K_4"). *Cf.* Gerards and Schrijver (1986a), Gerards and Shepherd (1998b). **(sg: Par: Geom, Str)**

§13.4: "Bouchet's 6-flow conjecture" (for signed graphs). See Bouchet

(1983a), Khelladi (1987a). (**SG: Flows**)

§15.9: "Square hypergraphs." Related to nonexistence of even cycles in a digraph and to sign nonsingularity. See Seymour (1974a) and Thomassen (1985a), (1986a), (1992a). (**sd: Par: bal, QSol: Exp**)

See also H. Guo.

Mark Jerrum & Alistair Sinclair

1990a Polynomial-time approximation algorithms for the Ising model (extended abstract). In: Michael S. Paterson, ed., *Automata, Languages and Programming* (Proc. 17th Int. Colloq., Warwick, 1990), pp. 462–475. Lect. Notes in Computer Sci., Vol. 443. Springer-Verlag, Berlin, 1990. MR 1076810 (91e:68004) (book). Zbl 764.65091.

Extended abstract of (1993a). [Annot. 26 Jun 2011.] (**sg: Fr, Phys**)

1993a Polynomial-time approximation algorithms for the Ising model. *SIAM J. Comput.* 22 (1993), no. 5, 1087–1116. MR 1237164 (94g:82007). Zbl 782.05076.

§6, "Completeness results": The problem Ising is to find the partition function $\sum_{\zeta:V\to\{+1,-1\}} 2^{-\beta H(\Sigma^\zeta)}$ of a signed simple graph Σ, where $H(\Sigma^\zeta) = \sum_{vw\in E} \sigma^\zeta(vw)$. Thm. 14 suggests nonexistence of certain approximation algorithms. [Annot. 26 Jun 2011.] (**sg: Fr, Phys**)

R.H. Jeurissen

1975a Covers, matchings and odd cycles of a graph. *Discrete Math.* 13 (1975), 251–260. MR 0412039 (54 #168). Zbl 311.05129.

Involves the negative-circle edge-packing number of $-\Gamma$. (**par: Fr**)

1981a The incidence matrix and labellings of a graph. *J. Combin. Theory Ser. B* 30 (1981), 290–301. MR 0624546 (83f:05048). Zbl 409.05042, (Zbl 457.05047).

The rank of the incidence matrix of a signed graph, in arbitrary characteristic, generalizing the all-negative results of Doob (1974a). Employs column operations on the incidence matrix. Application to magic labellings, where at each vertex a number (in a ring) is specified; the value of an edge is added if it enters the vertex and subtracted if it departs. §5, "Generalizations": "Mixed" graphs, really signed graphs. §6: A new proof of Doob (1973a)'s theorem on the multiplicity of -2 as a line-graph eigenvalue in arbitrary characteristic. (**sg, ori: Incid, Eig(LG)**)

1983a Disconnected graphs with magic labellings. *Discrete Math.* 43 (1983), 47–53. MR 0680303 (84c:05064). Zbl 499.05053.

The graphs, called "mixed", are bidirected graphs without introverted edges. Dictionary: " 'bipartite' " = balanced (as a signed graph; the term "balanced" is herein used with another meaning). (**sg, ori: incid**)

1983b Pseudo-magic graphs. *Discrete Math.* 43 (1983), 207–214. MR 0685628 (84g:-05122). Zbl 514.05054.

Mostly, the graphs are all-negative signed graphs (oriented to be extroverted). §5, "Labelings of mixed graphs", discusses bidirected graphs without introverted edges; as in the undirected problem, the (signed-graphically) balanced and unbalanced cases differ. (**sg, ori: Incid**)

1988a Magic graphs, a characterization. *European J. Combin.* 9 (1988), 363–368. MR 0950055 (89f:05138). Zbl 657.05065.

Connected graphs with magic labellings are classified, separately for bipartite and nonbipartite graphs [as one might expect, due to the connection with the incidence matrix of $-\Gamma$; see Stewart (1966a)]. (**par: incid**)

William S. Jewell

1962a Optimal flow through networks with gains. *Operations Res.* 10 (1962), 476–499. MR 0144784 (26 #2325). Zbl 109.38203 (109, p. 382c). (**GN**)

P. Jeyalakshmi

2021a Domination in signed graphs. *Discrete Math. Algorithms Appl.* 13 (2021), article 2050094, 11 pp. MR 4201751. Zbl 1460.05079.

$S \subseteq V$ dominates if every $v \notin S$ has more positive than negative neighbors in S. Thm. 2.3: Upper bound on $\#E^-$ if $\exists$ dominating set. [Independent of $\#E$; should be strengthened.] Thm. 3.1: $\#S \geq n/(1+\Delta^+)$. Thm. 3.4: Upper bound on $\min \#S$. Characterizations for $\min \#S \leq 4$. Examples: paths and circles. [Annot. 27 Dec 2020.] (**Lab: SG: Dom**)

P. Jeyalakshmi & K. Karuppasamy

2023a Domination changing and unchanging signed graphs upon the vertex removal. *TWMS J. Appl. Engin. Math.* 3 (2023), no. 4, 1423–1433.

Cf. B.D. Acharya (2013a). (**SG: Dom**)

P. Jeyalakshmi, K. Karuppasamy, & S. Arockiaraj

2021a Independent domination in signed graphs. *Discrete Math. Algorithms Appl.* 13 (2021), no. 1, art. 2050094, 11 pp. MR 4324432. Zbl 1478.05065. (**Lab: SG: Dom**)

Anthony M. Jezior

See J.J. Jarvis.

Samuel Jezný & Marián Trenkler

1983a Characterization of magic graphs. *Czechoslovak Math. J.* 33(108) (1983), 435–438. MR 0718926 (85c:05030). Zbl 571.05030.

A weak characterization of magic graphs. [See Jeurissen (1988a) for a stronger characterization.] (**par: Incid**)

Sai Ji, Dachuan Xu, Donglei Du, & Ling Gai

2020a Approximation algorithm for the balanced 2-correlation clustering problem on well-proportional graphs. In: Zhao Zhang *et al.*, eds., *Algorithmic Aspects in Information and Management* (14th Int. Conf., AAIM 2020, Jinhua, China), pp. 97–107. Lect. Notes in Computer Sci., Vol. 12290. Springer, Cham, 2020. Zbl 1482.68176.

Two clusters of equal size in (K_n, σ) with fewest bad edges. (Not a balanced signed graph.) [Annot. 17 Sept 2022.] (**SG: KG: Clu: Algor**)

Sai Ji, Dachuan Xu, Min Li, & Yishui Wang

2019a Approximation algorithm for the correlation clustering problem with non-uniform hard constrained cluster sizes. In: Ding-Zhu Du, Lian Li, *et al.*, eds., *Algorithmic Aspects in Information and Management* (13th Int. Conf., AAIM

2019, Beijing), pp. 159–168. Lect. Notes in Computer Sci., Vol. 11640. Springer, Cham, 2019.

Part of (2022a). (**SG: KG: Clu: Algor**)

2022a Approximation algorithms for two variants of correlation clustering problem. *J. Combin. Optim.* 43 (2022), 933–952. MR 4452857. Zbl 1495.90155.

Clustering in signed K_n with fewest bad edge. Model 1: Random signs with specified probabilities. Model 2: Specified maximum cluster sizes. [Annot. 17 Sept 2022.]

(**SG: KG: Clu: Rand: Algor**)(**SG: KG: Clu: Algor**)

Shengjin Ji

See B. Huo.

Zhijian Ji

See H. Gao and X.-Z. Liu.

Peng Jia, Noah E. Friedkin, & Francesco Bullo

2016a The coevolution of appraisal and influence networks leads to structural balance. *IEEE Trans. Network Sci. Engin.* 3 (2016), no. 4, 286–298. MR 3589253 (no rev). (**SD: KG: Bal, Clu: Dyn**)

Guangfeng Jiang

See also Q.M. Guo and W.L. Guo.

Guangfeng Jiang & Jianming Yu

2004a Supersolvability of complementary signed-graphic hyperplane arrangements. *Australasian J. Combin.* 30 (2004), 261–276. MR 2080474 (2005j:05042). Zbl 1054.05049.

Characterizes supersolvability of $\mathbf{F}(K_n, \sigma)$. [A special case of Zaslavsky (2001a).] (**SG: Geom: matrd**)

Guangfeng Jiang, Jianming Yu, & Jianghua Zhang

2008a Poincaré polynomial of a class of signed complete graphic arrangements. In: Kazuhiro Konno and Viet Nguyen-Khac, eds., *Algebraic Geometry in East Asia—Hanoi 2005* (Proc. 2nd Int. Conf., Hanoi, 2005), pp. 289–297. Adv. Stud. Pure Math., Vol. 50. Math. Soc. of Japan, Tokyo, 2008. MR 2409562 (2009j:52024). Zbl 1144.52025.

The chromatic polynomial of $K_n(-K_3)$, i.e., $+K_n$ with a triangle made all negative, factors integrally except for a cubic factor. [See Zaslavsky (1982c), §7, for a graph-theoretic treatment of such examples. One expects a direct proof by adding positive vertices in sequence to $-K_3$. *Problem.* Evaluate $\chi_\Sigma(\lambda)$ where Σ is Σ_1 with a new vertex positively adjacent to all vertices of Σ_1.] [Annot. 25 Feb 2012.] (**SG: Geom, Invar**)

Huidi Jiang & Hongwei Zhang

2020a Output sign-consensus of heterogeneous multiagent systems over fixed and switching signed graphs. *Int. J. Robust Nonlinear Control* 30 (2020), no. 5, 1938–1955. MR 4086453 (no rev). Zbl 1465.93197.

Cf. Sun, Zhang, and Lewis (2021a). (**SD**)

Jing-Jing Jiang

See S.W. Tan and X.L. Wu.

Jonathan Q. Jiang

2015a Stochastic block model and exploratory analysis in signed networks. *Phys. Rev. E* 91 (2015), art. 062805, 11 pp. arXiv:1501.00594. (**SG**)

Lan Jiang

See L. Dinh.

Xiao-Wei Jiang

See B. Hu.

Ye Jiang, Hongwei Zhang, He Cai, & Jie Chen

2017a Sign-consensus of multi-agent systems over fast switching signed graphs. In: *2017 IEEE 56th Annual Conference on Decision and Control (CDC)* (CDC2017, Melbourne, 2017), pp. 1993–1998. IEEE, 2017. (**SG**)

Ye Jiang, Hongwei Zhang, & Jie Chen

2017a Sign-consensus of linear multi-agent systems over signed directed graphs. *IEEE Trans. Industrial Electronics* 64 (2017), no. 6, 5075–5083. (**SD: Bal: Dyn**)

2017b Sign-consensus of multi-agent systems over signed graphs with intermittent communications. In: *2017 IEEE 56th Annual Conference on Decision and Control (CDC)* (CDC2017, Melbourne, 2017), pp. 5973–5977. IEEE, 2017. (**SG: Bal: Dyn**)

Yiting Jiang

2022a *Divers aspects de la coloration de graphes/Many Aspects of Graph Coloring*. Doctoral thesis, Inst. de Recherche en Informatique Fondamentale, 2022. HAL tel-04005771.

"Generalized signed graph" = permutation gain graph. (**SG, GG: Col**)

Yiting Jiang, Daphne Der-Fen Liu, Yeong-Nan Yeh, & Xuding Zhu

2019a Colouring of generalized signed triangle-free planar graphs. *Discrete Math.* 342 (2019), no. 3, 836–843. MR 3885203. Zbl 1403.05049.

See Jin, Wong, and Zhu (20xxa). "Signs" are permutation gains. (**GG: Col**)

Yiting Jiang & Xuding Zhu

2020a 4-colouring of generalized signed planar graphs. *Electronic J. Combin.* 27 (2020), no. 3, art. 3.31, 23 pp. MR 4245144. Zbl 1446.05032.

"Good" gains in $\mathfrak{S}_4$ that use $\mathfrak{e}$ must be in $\mathfrak{V}_4$. Dictionary: "generalized signed" = permutation gains in $\mathfrak{S}_4$; 4-coloring = permutation gain coloring by [4]. [Annot. 21 May 2022.] (**GG: Col**)

Zilin Jiang & Alexandr Polyanskii

20xxa Forbidden induced subgraphs for graphs and signed graphs with eigenvalues bounded from below. Submitted. arXiv:2111.10366. (**SG: Adj: Eig**)

Zilin Jiang, Jonathan Tidor, Yuan Yao, Shengtong Zhang, & Yufei Zhao

2023a Spherical two-distance sets and eigenvalues of signed graphs. *Combinatorica* 43 (2023), 203–232. arXiv:2006.06633.

Fix $-1 \le \beta < 0 \le \alpha < 1$. A two-distance set is $X \subseteq S^d$ such that all $\cos\angle(x,y) \in \{\alpha,\beta\}$. Thm. 1.11: $\max \#X \le d + m$ where m

asymptotically$_{d\to\infty}$ = max multiplicity of eigenvalue $(1-\alpha)/(\alpha-\beta)$ in a certain family of signed graphs.

Dictionary: "Chromatic number $\chi^{\pm}(\Sigma)$" = cluster number, i.e., min # of clusters.

[*Questions* for clusterable Σ: How can it be switched to remain clusterable? How does the cluster number change?] [Annot. 7 Nov 2020.] **(SG: Clu, Adj: Eig)**

Bao Jiao, Yang Chun, & Tianyong Qiang (as Tianyongqiang)

2010a Signless Laplacians of finite graphs. In: *2010 International Conference on Apperceiving Computing and Intelligence Analysis* (ICACIA, Chengdu, 2010), pp. 440–443. IEEE, 2010. **(par: Lap: Eig)**

Licheng Jiao

See Q. Cai and J.S. Wu.

Qiang Jiao

See X. Lin.

Yang Jiao

See J.-S. Wu.

Jesús Arturo Jiménez González & Andrzej Mróz

20xxa Bidirected graphs, integral quadratic forms and some Diophantine equations. Submitted. arXiv:2304.12555. **(sg: Ori: Incid, Algeb)**

Raúl D. Jiménez

See O. Rojo.

Chao Jin

See J.S. Wu.

Ligang Jin

See also Y.-L. Kang.

Ligang Jin, Yingli Kang, & Eckhard Steffen

2016a Choosability in signed planar graphs. *European J. Combin.* 52A (2016), 234–243. MR 3425977. Zbl 1327.05082. arXiv:1502.04561. **(SG: Col)**

Ligang Jin, Tsai-Lien Wong, & Xuding Zhu

2021a Colouring of S-labelled planar graphs. *European J. Combin.* 92 (2021), art. 103198, 6 pp. MR 4142159. Zbl 1458.05072.

Dictionary: "S-labelled" = permutation gains in S. **(GG: Col)**

20xxa Colouring of generalized signed planar graphs. Submitted. arXiv:1811.08584.

Dictionary: "weak L-coloring" of Γ = L-list coloring of $+\Gamma$; "generalized signed graph" = $\mathfrak{S}_n$-gain graph. *Cf.* Jiang, Liu, Yeh, and Zhu (2019a), X.D. Zhu (2020a). **(GG: Col)**

Xian'an Jin & Fuji Zhang

2005a The Kauffman brackets for equivalence classes of links. *Adv. Appl. Math.* 34 (2005), no. 1, 47–64. MR 2102274 (2005j:57009). Zbl 1060.05041.

They compute the Read–Whitehead chain polynomial of a sign-colored graph in which, for each divalent vertex, the two incident edges have the same color. This is applied to get the Kauffman bracket of small link diagrams. [*Cf.* W.L. Yang and Zhang (2007a).] (**SGc: Invar, Knot**)

2007a The replacements of signed graphs and Kauffman brackets of link families. *Adv. Appl. Math.* 39 (2007), no. 2, 155–172. MR 2333646 (2009b:57005). Zbl 1129.57004. arXiv:math/0511326. (**SGc: Invar, Knot**)

Ya-Lei Jin

See B.A. He.

Thorsten Joachims & John Hopcroft

2005a Error bounds for correlation clustering. In: Luc De Raedt and Stefan Wrobel, eds., *ICML 2005: Proceedings, Twenty-Second International Conference on Machine Learning* (Bonn, 2005), pp. 385–392. ACM, New York, 2005. MR none. (**sg: Clu: Algor**)

Herbert Jodlbauer

See G.H. Yu.

Peter Joffe

See A.J. Hoffman.

Manas Joglekar, Nisarg Shah, & Ajit A. Diwan

2010a Balanced group labeled graphs. In: *International Conference on Recent Trends in Graph Theory and Combinatorics (ICRTBC-2010)* (Cochin, 2010) [Summaries], pp. 120–121. Dept. of Math., Cochin Univ. of Science and Technology, 2010.

Extended abstract of (2012a). [Annot. 13 Jan 2012.] (**SG, VS: Bal**)

†† 2012a Balanced group labeled graphs. Recent Trends in Graph Theory and Combinatorics (Cochin, 2010). *Discrete Math.* 312 (2012), no. 9, 1542–1549. MR 2899887. Zbl 1239.05162.

Γ has weights $w : V \cup E \to \mathfrak{A}$ where $\mathfrak{A}$ is an abelian group. ($\mathfrak{A} = \mathbb{Z}_2$ is signs.) "Balance" = harmony: the sum around every circle = 0. Thm. 1: There are $\#\mathfrak{A}^{\#V+t-c(\Gamma)}$ harmonious labellings, where $t :=$ number of edge 3-components of Γ. Lemma 2. If (Γ, w) is balanced and u, v are edge 3-connected in Γ, then $2w(P) = w(u) + w(v)$ for every uv-path. [Annot. 30 Aug 2010.] (**GGw: Bal**)

Thm. 3 is a construction for all edge 2-connected Γ such that $\exists$ harmonious sign labelling, not all $+$. [This is the definitive characterization of consistent vertex signatures as in Beineke and Harary (1978b), improving on Hoede (1992a).] [Annot. 30 Aug 2010.] (**SG, VS: Bal**)

Rolf Johannesson

See I.E. Bocharova.

Karl H. Johansson

See G.-D. Shi.

Mikael Johansson

See G.-D. Shi.

David John

1998a Minimal edge cuts to induce balanced signed graphs. Proc. Twenty-ninth Southeastern Int. Conf. Combinatorics, Graph Theory and Computing (Boca

Raton, Fla., 1998). *Congressus Numer.* 132 (1998), 5–8. MR 1676490 (99j:-05178). Zbl 991.53332.

Polynomial-time algorithms to decide balance of a signed graph [this has long been known; see e.g. Hansen (1978a)] and allegedly to find the minimum number of negative edges whose deletion makes the graph balanced [the frustration index]. Contract the positive edges, leaving a graph consisting of the negative edges. To detect balance, look for bipartiteness of the contraction. [Inferior to the standard algorithm.] For negative frustration index, find a maximum cut of the contraction. [Something is wrong, since Max Cut is NP-complete and frustration index contains Max Cut. I believe the algorithm finds a nonmaximum cut.] (**SG: Bal, Fr: Algor**)

Eugene C. Johnsen

1986a Structure and process: Agreement models for friendship formation. *Social Networks* 8 (1986), no. 3, 257–306. MR 0860770 (87m:92093).

Signed complete digraphs $(\overleftrightarrow{K}_n, \sigma)$. A list of permitted isomorphism types of triads (order-3 induced subgraphs) (a "microstructure") implies a list of possible σ's (a "macrostructure"). Ex. 2.1: Permitting symmetrically signed triads with 1 or 3 positive arc pairs gives balanced signed complete graphs as in Cartwright and Harary's model (1956a). Ex. 2.2: Symmetric signs with 0, 1, or 3 positive arc pairs give clusterable signed complete graphs as in Davis's model (1967a). Other examples are positive digraph models from the literature, with negative arcs inserted to complete the digraph. Throughout, empirical data are used to prune potential examples.

§3, "Submodels and their substantive interpretation: $\sigma = (\sigma_t)_t$ evolves in [my simplification] discrete time $t \in \mathbb{Z}$ according to some combination of four "processes". The corresponding equilibrium signatures are the macrostructure and give the permitted triads. The processes:

(1) $\sigma_t(ac) = \sigma_t(bc) \implies \sigma_{t+1}(ab) = \sigma_{t+1}(ba) = +.$
$\sigma_t(ab) = -$ or $\sigma_t(ba) = - \implies \sigma_{t+1}(ac) \neq \sigma_{t+1}(bc).$

(2) $\sigma_t(ac) = \sigma_t(bc) \implies \sigma_{t+1}(ab) = +$ or $\sigma_{t+1}(ba) = +.$
$\sigma_t(ab) = \sigma_t(ba) = - \implies \sigma_{t+1}(ac) \neq \sigma_{t+1}(bc).$

(3) $\sigma_t(ab) = +$ or $\sigma_t(ba) = + \implies \sigma_{t+1}(ac) = \sigma_{t+1}(bc).$
$\sigma_t(ac) \neq \sigma_t(bc) \implies \sigma_{t+1}(ab) = \sigma_{t+1}(ba) = -.$

(4) $\sigma_t(ab) = \sigma_t(ba) = + \implies \sigma_{t+1}(ac) = \sigma_{t+1}(bc).$
$\sigma_t(ac) \neq \sigma_t(bc) \implies \sigma_{t+1}(ab) = -$ or $\sigma_{t+1}(ba) = -.$

§4, "Agreement–friendship processes related to affect": The equilibrium triads for each process and some combinations ("microprocesses") and the corresponding macrostructure. E.g., (1) with (3) gives the balance model. Some combinations do not yield macrostructures. Five combinations are "core". Later §§: Further analysis of the core combinations ("core microprocesses"). [Annot. 27 Dec 2012.] (**PsS, SD: KG: Str**)

1989a The micro-macro connection: Exact structure and process. In: Fred Roberts, ed., *Applications of Combinatorics and Graph Theory to the Biological and Social Sciences*, pp. 169–201. IMA Vols. Math. Appl., Vol. 17. Springer-Verlag, New York, 1989. MR 1009376 (90g:92089) (*q.v.*). Zbl 725.92026 (*q.v.*).

An elaborate classificatory analysis of "triads" (signed complete directed graphs of 3 vertices) vis-á-vis "macrostructures" (signed complete directed graphs) with reference to structural interactions and implications of triadic numerical restrictions on "dyads" (s.c.d.g. of 2 vertices). Connections to certain models of affect in social psychology. ["Impenetrability! That's what *I* say!" "Would you tell me, please," said Alice, "what that means?"] (**KG, SD, SG: Bal, PsS: Exp**)

Eugene C. Johnsen & H. Gilman McCann

1982a Acyclic triplets and social structure in complete signed digraphs. *Social Networks* 3 (1982), 251–272.

Balance and clustering analyzed via triples rather than edges. [Possible because the digraph is complete. A later analysis via triples is in Doreian and Krackhardt (2001a).] (**SD: Bal, Clu**)

Charles R. Johnson

See also P.J. Cameron and C.A. Eschenbach.

1983a Sign patterns of inverse nonnegative matrices. *Linear Algebra Appl.* 55 (1983), 69–80. MR 0719863 (86i:15001). Zbl 519.15008. (**SG: QSol**)

Charles R. Johnson, Frank Thomson Leighton, & Herbert A. Robinson

1979a Sign patterns of inverse-positive matrices. *Linear Algebra Appl.* 24 (1979), 75–83. (**SG: QSol**)

Charles R. Johnson & John Maybee

1991a Qualitative analysis of Schur complements. In: *Applied Geometry and Discrete Mathematics: The Victor Klee Festschrift*, pp. 359–365. DiMACS Ser. Discrete Math. Theor. Computer Sci., Vol. 4. Amer. Math. Soc., Providence, 1991. MR 1116363 (92h:15004). Zbl 742.15009.

In square matrix A let $A[S]$ be the principal submatrix with rows and columns indexed by S. Thm. 1: Assume $A[S]$ is sign-nonsingular in standard form and $i, j \notin S$. Then the (i, j) entry of the Schur complement of $A[S]$ has sign determined by the sign pattern of A iff, in the signed digraph of A, every path $i \to j$ via S has the same sign. (**QM: sd**)

Charles R. Johnson, William D. McCuaig, & David P. Stanford

1995a Sign patterns that allow minimal semipositivity. *Linear Algebra Appl.* 223–224 (1995), 363–373. MR 1340701 (96g:15021). Zbl 829.15017. (**SG: QM: QSol**)

Charles R. Johnson, Michael Neumann, & Michael J. Tsatsomeros

1996a Conditions for the positivity of determinants. *Linear Multilinear Algebra* 40 (1996), 241–148. MR 1382081 (97a:15014). Zbl 866.15001. (**SD: QM**)

Charles R. Johnson, D.D. Olesky, Michael Tsatsomeros, & P. van den Driessche

1993a Spectra with positive elementary symmetric functions. *Linear Algebra Appl.* 180 (1993), 247–261. MR 1206420 (94a:15028). Zbl 778.15006.

Suppose the signed digraph D of an $n \times n$ matrix has longest cycle length k and all cycles of $-D$ are negative. Theorem: If $k = n - 1$, the eigenvalues lie in a domain subtending angle $< 2\pi/k$. This is known for $k = 2$ but false for $k = n - 3$. (**QM, SD**)

Charles R. Johnson, Frank Uhlig, & Dan Warner

1982a Sign patterns, nonsingularity, and the solvability of $Ax = b$. *Linear Algebra Appl.* 47 (1982), 1–9. MR 0672727 (84h:15005). Zbl 488.15002. (**SG: QSol**)

David S. Johnson

1983a The NP-completeness column: An ongoing guide. VI. *J. Algorithms* 4 (1983), 87–100. Zbl 509.68034.

§4, Problem 3, "Ground state of a spin glass": Is the "ground state spin energy" of a weighted signed graph $\leq K$? NP-complete for weights ± 1 on a two-layer cubic lattice; *cf.* Barahona (1982a). Related problems. [Annot. 18 Jun 2012.] (**SG: wg, Fr, Algor**)

Ellis L. Johnson

See also J. Edmonds, H.L. Gan, and G. Gastou.

1965a Programming in networks and graphs. Report ORC 65-1, Operations Research Center, University of California, Berkeley, Calif., Jan. 1965.

§7: "Flows with gains." §8: "Linear programming in an undirected graph." §9: "Integer programming in an undirected graph."
(**GN: Incid, Matrd(bases)**)(**ecyc: Incid, Matrd(bases), Algor**)

1966a Networks and basic solutions. *Operations Res.* 14 (1966), 619–623. (**GN**)

Ellis L. Johnson & Sebastiano Mosterts

1987a On four problems in graph theory. *SIAM J. Algebraic Discrete Methods* 8 (1987), 163–185. MR 0881177 (88d:05097). Zbl 614.05036.

Two of the problems: Given a signed graph (edges called "even" and "odd" rather than "positive" and "negative"). The co-postman problem is to find a minimum-cost deletion set (of edges). The "odd circuit" problem is to find a minimum-cost negative circle. The Chinese postman problem is described in a way that involves cobalance and "switching" around a circle. (**SG: Fr(Gen), Incid**)

Ellis L. Johnson & Manfred W. Padberg

1982a Degree-two inequalities, clique facets, and biperfect graphs. In: Achim Bachem, Martin Grotschel, and Bernhard Korte, eds., *Bonn Workshop on Combinatorial Optimization* (Fourth, 1980), pp. 169–187. North-Holland Math. Studies, 66. Ann. Discrete Math., 16. North-Holland, Amsterdam, 1982. MR 0686306 (84j:05085). Zbl 523.52009.

Geometry of the bidirected stable set polytope $P(\mathrm{B})$ (which generalizes the stable set polytope to bidirected graphs), defined as the convex hull of $0, 1$ solutions of $x_i + x_j \leq 1$, $-x_i - x_j \leq -1$, $x_i \leq x_j$ for extroverted, introverted, and directed edges of B. (Thus, undirected graphs correspond to extroverted bidirected graphs.) It suffices to treat transitively closed bidirections of simple graphs ([unfortunately] called "bigraphs"). [Such a bidirected graph must be balanced.] A "biclique" (S_+, S_-) is the Harary bipartition of a balanced complete subgraph (S_+, S_- are the source and sink sets of the subgraph). It is "strong" if no external vertex has an edge directed out of every vertex of S_+ and an edge directed into every vertex of S_-. Strong bicliques generate facet inequalities of the polytope. Call B perfect if these facets (and nonnegativity) determine $P(\mathrm{B})$. Γ is "biperfect" if every transitively closed bidirection B of Γ is perfect. Conjectures: Γ is biperfect iff it is perfect. Γ is perfect iff some transitively closed bidirection is perfect. [Both proved by Sewell (1996a) and independently by Ikebe and Tamura (20xxa). See e.g. Tamura (1997a) for further work.] **(sg: Ori: Incid, Geom, sw)**

Skyler J. Johnson

See L.J. Rusnak.

Will Johnson

2012a Circular planar resistor networks with nonlinear and signed conductors. Manuscript, 2012. arXiv:1203.4045.

Problem: Recover the conductances of branches of an electrical network, given boundary voltages and currents, when conductances can be nonlinear or negative. The "conductances" are gains. §11, "Applications of negative conductances": §11.1, "Removing a mild failure of circular planarity": A graphical transform of positive conductances can introduce negative conductances, using an idea from Schrøder (1995a) and Goff (2003a). §11.2, "Knot theory": Treating the colors ±1 as conductances, a corollary on pseudoline arrangements and tangles.

Suggests that the proof in Goff (2003a) is flawed (see fn. 4). [Annot. 26 Dec 2012.] **(gn: Adj: Appl)(SGc: Knot)**

M. Jones

See R. Crowston.

[Hidde de Jong]

See H. de Jong (under 'D').

Felix Joos

See T. Huynh.

Mohammadreza Jooyandeh, Dariush Kiani, & Maryam Mirzakhah

2009a Incidence energy of a graph. *MATCH Commun. Math. Comput. Chem.* 62 (2009), no. 3, 561–572. MR 2568740 (2010j:05238). Zbl 1274.05295. **(par: Incid)**

Tibor Jordán, Viktória E. Kaszanitzky, & Shin-ichi Tanigawa

2016a Gain-sparsity and symmetry-forced rigidity in the plane. *Discrete Comput. Geom.* 55 (2016), 314–372. MR 3458601. Zbl 1366.52021.

§3.2, "Gain-count matroids": Gain-graph generalization of White–Whiteley (1983a)'s count matroids. **(GG: Geom, Matrd)**

Heather Jordon [Heather Gavlas]

See also G. Chartrand and D. Hoffman.

Heather Jordon, Richard McBride, & Shailesh Tipnis

2009a The convex hull of degree sequences of signed graphs. *Discrete Math.* 309 (2009), no. 19, 5841–5848. MR 2551962 (2010k:05120). Zbl 1208.05043.

Consider signed simple graphs of order n. $P_n :=$ polytope determined by the inequalities from Hoffman and Jordon (2006a) that characterize net degree vectors. Thm. 2.7: $P_n =$conv(net degree vectors). Thm. 2.9: Each vertex of $P_n \leftrightarrow$ a unique signed graph, which is a signed K_n. §3: Comparison with net degree vectors of digraphs. [As in other papers on net degree sequences, the best viewpoint is that "signed" edges are oriented negative edges and "directed" edges are oriented positive edges.]

[Annot. 1 Oct 2009.] **(SG: ori: Invar: Geom)**

Gwenaël Joret

See N.E. Clarke, M. Conforti, and S. Fiorini.

Leif Kjær Jørgensen

1989a Some probabilistic and extremal results on subdivisions and odd subdivisions of graphs. *J. Graph Theory* 13 (1989), 75–85. MR 0982869 (90d:05186). Zbl 672.05070.

Let $\sigma_{\rm op}(\Gamma)$, or $\sigma_{\rm odd}(\Gamma)$, be the largest s for which $-\Gamma$ contains a subdivision of $-K_s$ (an "odd-path-K_sS"), or $[-\Gamma]$ contains an antibalanced subdivision of K_s (an "odd-K_sS"). Thm. 4: $\sigma_{\rm op}(\Gamma), \sigma_{\rm odd}(\Gamma) \approx \sqrt{n}$. Thms. 7, 8 (simplified): For $p = 4, 5$ and large enough $n = \#V$, $\sigma_{\rm odd}(\Gamma) \geq p$ or Γ is a specific exceptional graph. *Conjecture* 9. The same holds for all $p \geq 4$. [*Problem.* Generalize this to signed graphs.] **(par: Xtreml)**

J. Paulraj Joseph

See V. Vilfred.

James Joseph & Mayamma Joseph

2023a Roman domination in signed graphs. *Commun. Combin. Optim.* 8 (2023), no. 2, 349–358. MR 4570022.

D dominates if $N^+[D] = V$. Roman dominating function (RDF): $f \in \{0,1,2\}^V$ such that $(I + A(\Sigma))f \geq \mathbf{1}$ and $f^{-1}(0) \subseteq N^+(f^{-1}(2))$. Roman dominating number $:= \min_f \sum_u f(u)$. Lemma 2: 5 forbidden induced paths. Signed paths, cycles, stars admitting RDF are characterized. [Annot. 22 Apr 2023.] **(SG)**

2023b Roman domination number of signed graphs. *Commun. Combin. Optim.* 8 (2023), no. 4, 759–766. **(SG)**

Mayamma Joseph

See J. Joseph, P.B. Joshi and S.R. Shreyas.

Prajakta Bharat Joshi & Mayamma Joseph

2017a Further results on color energy of graphs. *Acta Univ. Sapientiae, Informatica* 9 (2017), no. 2, 119–133. Zbl 1380.05067.

Cf. Adiga, Sampathkumar, *et al.* (2013a). **(sg: Adj: Eig)**

Prajakta Bharat Joshi, Mayamma Joseph, & Mukti Acharya

20xxa Color energy of signed graphs. Submitted. **(SG: Adj: Eig)**

Shalini Joshi

See B.D. Acharya.

Jürgen Jost

See also R. Mulas.

Jürgen Jost & Raffaella Mulas

2019a Hypergraph Laplace operators for chemical reaction networks. *Adv. Math.* 351 (2019), 870–896. MR 3955999. Zbl 1415.05107. arXiv:1804.01474. **(SH(Gen): Lap: Eig)**

Simon Joyce

2017a *Interaction Graphs Derived From Activation Functions and Their Application to Gene Regulation.* Doctoral dissertation, Binghamton University, 2017. MR 3755540 (no rev). **(SD: Dyn, Biol)**

Tadeusz Józefiak & Bruce Sagan

1992a Free hyperplane arrangements interpolating between rootsystem arrangements. In: *Séries formelles et combinatoirealgébrique* (Actes du colloque, Montréal, 1992), pp. 265–270.Publl. Lab. Combin. Inform. Math., Vol. 11. Dép. de math. et d'informatique, Université de Québec à Montréal, 1992.

Summarizes the freeness results in (1993a).

(sg, gg: Geom, matrd, Invar)

1993a Basic derivations for subarrangements of Coxeter arrangements. *J. Algebraic Combin.* 2 (1993), 291–320. MR 1235282 (94j:52023). Zbl 798.05069.

The hyperplane arrangements (over fields with characteristic $\neq 2$) corresponding to certain signed graphs are shown to be "free". Explicit bases and the exponents are given. The signed graphs are: $+K_{n-1} \subseteq \Sigma_1 \subseteq +K_n$ (known), $\pm K_n \subseteq \Sigma_2 \subseteq \pm K_n^\circ$, $\pm K_n \subseteq \Sigma_3 \subseteq \pm K_n^\circ$; also, those obtained from $+K_n$ or K_n° by adding all negative links in the order of their larger vertex (assuming ordered vertices) (Thms. 4.1, 4.2) or smaller vertex (Thms. 4.4, 4.5); and those obtained from $\pm K_{n-1}$ by adding positive edges ahead of negative ones (Thm. 4.3). [For further developments see Edelman and Reiner (1994a).] Similar theorems hold for complex arrangements when the sign group is replaced by the complex s-th roots of unity (§5). The Möbius functions of Σ_2, known from Hanlon (1988a), are deduced in §6. **(sg, gg: Geom, matrd, Invar)**

[Michael Juenger]

See M. Jünger.

Ji-Hwan Jung

See G.-S. Cheon.

Michael Jünger

See F. Barahona, C. De Simone, M. Grötschel, F. Liers, and M. Palassini.

Mark Jungerman & Gerhard Ringel

1978a The genus of the n-octahedron: Regular cases. *J. Graph Theory* 2 (1978), 69–75. MR 0485485 (58 #5315). Zbl 384.05037.

"Cascades": see Youngs (1968a). **(sg: Ori: Appl)**

K. Jüngling

1975a Exact solution of a nonplanar two-dimensional Ising model with short range two-spin interaction. *J. Phys. C* 8 (1975), L169–L171.

Sequel to Jüngling and Obermair (1974a). Physics of signed diagonal square lattice with two bond strengths, reduced to Baxter model. [See Southern, Chui, and Forgacs (1980a), Garel and J.M. Maillard (1983a).] [Annot. 16 Jun 2012.] **(Phys: sg: wg)**

K. Jüngling & G. Obermair

1974a Note on universality and the eight-vertex model. *J. Phys. C* 7 (1974), L363–L365.

More general but less developed predecessor of Jüngling (1975a). [Annot. 16 Jun 2012.] (**Phys: sg: wg**)

Dieter Jungnickel

See C. Fremuth-Paeger.

Samuel Jurkiewicz

See M.A.A. de Freitas.

James Justus

2005a Qualitative scientific modeling and loop analysis. *Philosophy of Science* 72 (2005), 1272–1286. MR 2295282 (2007j:00008).

Philosophical discussion of qualitative differential equations with emphasis on Levins (1975a). [Annot. 9 Sept 2010.]
(**SD: QM: QSta: Exp**)

Jerald A. Kabell

See also Harary and Kabell (1980a).

1985a Co-balance in signed graphs. *J. Combin. Inform. System Sci.* 10 (1985), 5–8. MR 0959659 (89i:05232). Zbl 635.05028.

Cobalance means that every cutset has positive sign product. Thm.: Σ is cobalanced iff every vertex star has evenly many negative edges. For planar graphs, corollaries of this criterion and Harary's bipartition theorem result from duality. [The theorem follows easily by looking at the negative subgraph.] (**SG: Bal(Du), Bal**)

1988a An algorithmic look at cycles in signed graphs. 250th Anniversary Conf. on Graph Theory (Fort Wayne, Ind., 1986). *Congressus Numer.* 63 (1988), 229–230. MR 0988654 (90d:05143). Zbl 666.05046. (**SG, SD: Bal: Algor**)

Kasper Kabell

See Geelen and Kabell (2009a).

Navid Kafai & Farideh Heydari

2024a Maximizing the indices of a class of signed complete graphs. *Commun. Combin. Optim.* 9 (2024), no. 1, 169–175. (**SG: Adj: Eig**)

N. Kafai, F. Heydari, N. Jafari Rad, & M. Maghasedi

2021a On the signed complete graphs with maximum index. *Iranian J. Sci. Tech. Trans. A Sci.* (2021), no. 6, 2085–2090. MR 4329226. arXiv:2102.03308.

2021b Correction. *Iranian J. Sci. Tech. Trans. A Sci.* (2021), no. 6, 2249. MR 4329240.

Assuming A is unicyclic of order k, $\lambda_{\max}(K_n(-A))$ is maximized by $A =$ triangle with pendant star. [*Cf.* Ebrahim Ghorbani & Arezoo Majidi (2021a).] [Annot. 26 May 2021.] (**SG: KG: Adj: Eig**)

Jeff Kahn

1980a Varieties of combinatorial geometries. In: *Report on the XVth Denison-O.S.U. Math. Conf.* (Granville, Ohio, 1980), pp. 90–91. Dept. of Math., Ohio State University, Columbus, Ohio, 1980.

Brief (and slightly incomplete) announcement of Kahn and Kung (1982a). [Annot. 18 July 2014.] (**GG: Matrd**)

Jeff Kahn & Joseph P.S. Kung

1980a Varieties and universal models in the theory of combinatorial geometries. *Bull. Amer. Math. Soc. (N.S.)* 3 (1980), 857–858. MR 0578380 (81i:05051). Zbl 473.05025.

Announcement of (1982a). **(gg: Matrd)**

†† 1982a Varieties of combinatorial geometries. *Trans. Amer. Math. Soc.* 271 (1982), 485–499. MR 0654846 (84j:05043). Zbl 503.05010. Repr. in: Joseph P.S. Kung, *A Source Book in Matroid Theory*, pp. 395–409, with commentary, pp. 335–338. Birkhäuser, Boston, 1986. MR 0890330 (88e:05028). Zbl 597.05019.

A "variety" is a class closed under deletion, contraction, and direct summation and having for each rank a "universal model", a single member containing all others. There are two nontrivial types of variety of finite matroids: matroids representable over GF(q), and gain-graphic matroids with gains in a finite group $\mathfrak{G}$. The universal models of the latter are the Dowling geometries $Q_n(\mathfrak{G})$.

It is incidentally proved (§7, pp. 490–492) that Dowling geometries of non-group quasigroups cannot exist in rank $n \geq 4$. **(gg: Matrd)**

1986a A classification of modularly complemented geometric lattices. *European J. Combin.* 7 (1986), 243–248. MR 0862370 (87i:06026). Zbl 614.05018.

Such a geometric lattice of rank ≥ 4, if not a projective geometry with few points deleted, is a Dowling lattice. **(gg: Matrd)**

Jeff Kahn & Roy Meshulam

1998a On the number of group-weighted matchings. *J. Algebraic Combin.* 7 (1998), 285–290. MR 1616012 (99b:05113). Zbl 899.05042.

Continues Aharoni, Meshulam, and Wajnryb (1995a) (*q.v.* for definitions), generalizing its Thm. 1.3 (the case $k = \#\mathfrak{K} = 2$ of the following). Let m = number of 0-weight matchings, δ = minimum degree. Thm. 1.1: If $m > 0$ then $m \geq (\delta - k + 1)!$. *Conjecture* 1.2. k can be reduced. (See the paper for details.) [*Question.* Is there a generalization to weighted digraphs? One could have two kinds of arcs: some weighted from $\mathfrak{K}$, and some weighted 0. The perfect matching might be replaced by an alternating Hamilton cycle or a spanning union of disjoint alternating cycles.] **(WG)**

Thm. 2.1: In a $\mathfrak{K}$-weighted simple digraph with all outdegrees $> k$, there is a nonempty set of disjoint cycles whose total weight is 0. **(WD)**

Tomáš Kaiser, Robert Lukot'ka, Edita Máčajová, & Edita Rollová

2019a Shorter signed circuit covers of graphs. *J. Graph Theory* 92 (2019), no. 1, 39–56. MR 3984679. Zbl 1425.05128. arXiv:1706.03808. **(SG: flows)**

Tomáš Kaiser, Robert Lukot'ka, & Edita Rollová

2017a Nowhere-zero flows in signed graphs: A survey. In: Domenico Labbate, ed., *Selected Topics in Graph Theory and Its Applications*, pp. 85–104. Lect. Notes Sem. InterdiscipMat., 14. Seminario Interdisciplinare di Matematica (S.I.M.), Potenza, Italy, 2017. Zbl 06769482. arXiv:1608.06944. **(SG: Flows)**

Tomáš Kaiser & Edita Rollová

2016a Nowhere-zero flows in signed series-parallel graphs. *SIAM J. Discrete Math.* 30 (2016), no. 2, 1248–1258. MR 3510003. Zbl 1338.05107. arXiv:1411.1788.

(SG: Flows)

Naonori Kakimura

See also C. Carlson.

2010a Matching structure of symmetric bipartite graphs and a generalization of Pólya's problem. *J. Combin. Theory Ser. B* 100 (2010), 650–670. MR 2718684 (2011j:05265). Zbl 1208.05112.

Symmetric matching theory of a bipartite graph with left-right symmetry, with a symmetric Mendelsohn–Dulmage theorem. [A symmetrically bipartite graph Γ' is the signed covering graph of an all-negative signed graph $-\Gamma$, possibly with half edges. A symmetrical matching in Γ' corresponds to a subgraph of $-\Gamma$ with maximum degree 1. *Problem.* Develop the symmetric matching theory of any graph with an involutory, fixed-point-free automorphism in terms of a matching theory of signed graphs with half edges.] [Annot. 29 Sept 2011.] **(sg: cov: Str)**

Naonori Kakimura & Ken-ichi Kawarabayashi

2012a Packing cycles through prescribed vertices under modularity constraints. *Adv. Appl. Math.* 49 (2012), no. 2, 97–110. MR 2946427. Zbl 1245.05102. **(gg: Circ)**

2013a Half-integral packing of odd cycles through prescribed vertices. *Combinatorica* 33 (2013), no. 5, 549–572. MR 3132926. Zbl 1324.05148. **(sg: Par: Circ)**

Debajit Kalita

See also R.B. Bapat and S. Barik.

2012a *Spectra of Weighted Directed Graphs.* Doctoral dissertation, Indian Inst. of Technology Guwahati, Guwahati, India, 2012. **(SG: Cov, Adj: Eig)**

2012b On 3-colored digraphs with exactly one nonsingular cycle. *Electronic J. LInear Algebra* 23 (2012), 397–421. MR 2928567. Zbl 1252.05122.

"3-colored digraph" = gain graph with gains in $\{\pm 1, \pm \mathrm{i}\}$ [not a digraph]. **(SG: Cov, Adj)**

2013a Determinant of the Laplacian matrix of a weighted directed graph. In: Ravindra B. Bapat *et al.*, eds., *Combinatorial Matrix Theory and Generalized Inverses of Matrices* (Proc. Int. Workshop and Conf., CMTGIM 2012, Manipal), pp. 57–63. Springer India, New Delhi, 2013. MR 3075960. Zbl 1291.05123. **(SG: Cov, Lap, Eig)**

2013b Spectral integral variation and unicyclic 3-colored digraphs with second smallest eigenvalue 1. *Linear Algebra Appl.* 439 (2013), no. 1, 55–65. MR 3045222. Zbl 1282.05123. **(SG: Cov, Eig)**

2015a Properties of first eigenvectors and first eigenvalues of nonsingular weighted directed graphs. *Electronic J. Linear Algebra* 30 (2015), 227–242. MR 3358539. Zbl 1323.05079.

"Weighted directed graph" = complex unit gain graph [not a digraph]. **(GG: Lap: Eig)**

2016a Extremizing first eigenvalue of 3-colored digraphs made with given blocks. *Linear Algebra Appl.* 503 (2016), 83–99. MR 3492660. Zbl 1338.05162.

"Weighted directed graph" = complex unit gain graph [not a digraph]. **(gg: Lap: Eig)**

Debajit Kalita & Sukanta Pati

2012a On the spectrum of 3-colored digraphs. *Linear Multilinear Algebra* 60 (2012), no. 6, 743–756. MR 2929181. Zbl 1244.05145.

Reproves the eigenvalue theorem of Fowler (2002a). A "3-colored digraph" is a bidirected graph where all negative edges are extraverted. [Annot. 13 Jan 2012, 13 Jan 2015.] (**SG: Cov, Adj, Lap: Eig**)

2014a A reciprocal eigenvalue property for unicyclic weighted directed graphs with weights from $\{\pm1, \pm i\}$. *Linear Algebra Appl.* 449 (2014), 417–434. MR 3191876. Zbl 1286.05090.

The structure of $\{\pm1, \pm\mathrm{i}\}$-gain graphs with one circle C that have the reciprocal eigenvalue property (every λ and $1/\lambda$ have the same multiplicity). They have a perfect matching of pendant edges, or an exceptional structure depending on $\varphi(C)$. [Annot. 28 May 2013.]

(**gg: Adj: Eig, Str**)

Debajit Kalita & Kuldeep Sarma

2022a Unicyclic 3-colored digraphs with bicyclic inverses. *Electronic J. Linear Algebra* 38 (2022), 519–530. MR 4494144. Zbl 1497.05150.

Solved: $A(\Phi)^{-1} = A(\Phi')$ with Φ as in (2022b), where Φ' is bicyclic. [Annot. 19 Sept 2022.] (**gg: Adj**)

2022b On the inverse of unicyclic 3-coloured digraphs. *Linear Multilinear Algebra* 70 (2022), no. 21, 6223–6239. MR 4568549.

Solved: $A(\Phi)^{-1} = A(\Phi')$ where Φ is odd unicyclic with perfect matching and circle gain $= \pm\mathrm{i}$. Special cases of bipartite or unicyclic Φ'. $L(\Phi)$ must be nonsingular [because Φ is connected and unbalanced]. Strong reciprocal eigenvalue property. [Annot. 19 Sept 2022.] (**gg: Adj**)

Meenal M. Kaliwal

See P.R. Hampiholi.

M. Kamaraj

See M. Parvathi.

Hidehiko Kamiya, Akimichi Takemura, & Hiroaki Terao

2009a The characteristic quasi-polynomials of the arrangements of root systems and mid-hyperplane arrangements. In: Fouad El Zein, Alexander I. Suciu, *et al.*, eds., *Arrangements, Local Systems and Singularities* (CIMPA Summer School, Istanbul, 2007), pp. 177–190. Progr. Math., Vol. 283. Birkhäuser Verlag, Basel, 2010. MR 3025864. Zbl 1370.32011. arXiv:0707.1381. (**sg: Geom, Invar**)

2011a Periodicity of non-central integral arrangements modulo positive integers. *Ann. Combin.* 15 (2011), no. 3, 449–464. MR 2836451. Zbl 1233.32018. arXiv:-0803.2755.

§5, "Example": The affino-signed-graphic arrangement $\hat{\mathcal{B}}_m^{[0,a]}$ of Athanasiadis (1999a). [Annot. 26 May 2018.] (**gg: Geom, Invar**)

2012a Arrangements stable under the Coxeter groups. In: Anders Björner *et al.*, eds., *Configuration Spaces: Geometry, Combinatorics and Topology* (Pisa, 2010), pp. 327–354. Centro di Ricerca Mat. Ennio De Giorgi (CRM) Series, No. 14. Edizioni della Normale, Pisa, 2012. MR 3203646. Zbl 1276.14080. arXiv:-1103.5179.

§3.5, "Signed all-subset arrangement". (**SG: Geom, Invar**)

[Axel von Kamp]

See A. von Kamp (under 'V').

Daniel Kandel, Radel Ben-Av, & Eytan Domany

† 1990a Cluster dynamics for fully frustrated systems. *Phys. Rev. Lett.* 65 (1990), no. 8, 941–944.

A new probabilistic algorithm for clustering in a ground state (a function $s : V \to \{+1, -1\}$ such that $\#E^- = l(\Sigma)$) of an all-negative ("fully frustrated") square lattice Σ. A "cluster" in s is a partition of V such that switching any part does not change $\#E^-$. The objective is to join vertices connected by satisfied edges but not those joined by frustrated edges; this cannot be solved uniquely for any unbalanced Σ, so previous methods (used for balanced Σ), e.g., nearest-neighbor moves in state space ("single spin flips"), are ineffective (see p. 942, col. 1; p. 943, col. 2). The algorithm depends on the square lattice structure since it works on squares ("plaquettes"); it succeeds because it works through plaquettes instead of edges (p. 943, col. 2). [*Problem*: Do state-space algorithms help to approximate signed-graph clustering in the sense of Davis (1967a)? Finding a ground state is NP-hard in general, though not for planar signed graphs (*cf.* Katai and Iawi (1978a), Barahona (1982a)).] [Annot. 18 Jun 2012.] (**Phys, SG: Clu: Algor**)

Lawqueen Kanesh

See A. Das.

Qinma Kang

See W.-Q. Duan.

Rongrong Kang & Xiang Li

2022a Coevolution of opinion dynamics on evolving signed appraisal networks. Automatica 137 (2022), art. 110138, 10 pp. MR 4366235 (no rev). Zbl 1482.91164. (**SG: Dyn: Bal**)

Yingli Kang

See also L.-G. Jin.

2018a Hajós-like theorem for signed graphs. *European J. Combin.* 67 (2018), 199–207. MR 3707227. Zbl 1371.05114. arXiv:1702.08232. (**SG: Col: Str**)

Yingli Kang, Xiaoyue Chen, & Ligang Jin

2022a A study on parity signed graphs: the *rna* number. *Appl. Math. Comput.* 431 (2022), art. 127322, 9 pp. MR 4444383. Zbl 1510.05099. arXiv:2111.04956.

Answers questions raised in Acharya and Kureethara (2021a) and Acharya, Kureethara, and Zaslavsky (2021a), and more. Smart new techniques. [Annot. 15 Nov 2021.] (**Lab: SG, Sw**)

Yingli Kang & Eckhard Steffen

2016a The chromatic spectrum of signed graphs. *Discrete Math.* 339 (2016), 2660–2663. MR 3518416. Zbl 1339.05169. arXiv:1510.00614. (**SG: Col**)

2017a Circular coloring of signed graphs. *J. Graph Theory* 87 (2018), no. 2, 135–148. MR 3742174. Zbl 1383.05103. arXiv:1509.04488. (**SG: Col**)

Yunfan Kang

See W.-Q. Duan.

Mariusz Kaniecki, Justyna Kosakowska, Piotr Malicki, & Grzegorz Marczak

2015a A horizontal mesh algorithm for a class of edge-bipartite graphs and their matrix morsifications. *Fundamenta Inform.* 136 (2015), no. 4, 345–379. MR 3320020. Zbl 1335.05174. (**SG**)

[M.R. Rajesh Kanna]

See M.R. Rajesh Kanna (under 'R').

M. Rajesh Kannan

See also R. Mehatari and A. Samanta.

M. Rajesh Kannan, Navish Kumar, & Shivaramakrishna Pragada

2021a Bounds for the extremal eigenvalues of gain Laplacian matrices. *Linear Algebra Appl.* 625 (2021), 212–240. MR 4264826. Zbl 1465.05073. arXiv:2102.07560. (**GG: Fr, Lap: Eig**)

2022a Normalized Laplacians for gain graphs. *Amer. J. Combin.* 1 (2022), 20–39. arXiv:2009.13788. (**GG: Fr, Lap: Eig, Bal**)

M. Rajesh Kannan & Shivaramakrishna Pragada

2023a Signed spectral Turań type theorems. *Linear Algebra Appl.* 663 (2023), 62–79. MR 4538433. arXiv:2204.09870.

Let $m = \#E$. Thm. 3.3: $\lambda_{\max}^2 \leq 2(m - l(\Sigma))(1 - \omega_{\mathrm{b}}^{-1})$, where $\omega_{\mathrm{b}} =$ balanced clique number. Thm. 3.7: $m \leq l(\Sigma) + \frac{1}{2}n^2(1 - \omega_{\mathrm{b}}^{-1})$. Thm. 4.4: $\lambda_{\max}^2 \leq m + [6(t_+ - t_-)]^{2/3}$, where $t_\varepsilon = \#$ triangles with sign ε. §5, "Signed walks and largest eigenvalue"" Thm. 5.1 connects ω_{b} with the smallest number of negative r-walks in any switching Σ^ζ. *Cf.* Lan, Li, and Liu (2023a). [Annot. 20 Jun 2022.] (**SG: Adj: Eig**)

Sangita Kansal

See also M. Acharya, R. Jain, and Payal.

Sangita Kansal & Payal Dabas

2021a An introduction to signed Petri net. *J. Math.* 2021 (2021), art. 5595536, 8 pp. MR 4277469. Zbl 1477.68182. (**SG: Bal(Gen), Appl**)

Konstantinos Kaparis & Adam N. Letchford

2018a A note on the 2-circulant inequalities for the max-cut problem. *Operations Res. Lett.* 46 (2018), no. 4, 443–447. MR 3827655. Zbl 1452.90269.

Greater depth about Poljak and Turzik (1992a). [Annot. 4 Jun 2018.] (**Par: Fr, Geom: Algor**)

Vikram Singh Kapil

See R.P. Sharma.

Ajai Kapoor

See M. Conforti.

Roman Kapuscinski

See P. Doreian.

D. Karapetyan

See G. Gutin.

Mehran Kardar

See L. Saul.

František Kardoš & Jonathan Narboni

2021a On the 4-color theorem for signed graphs. *European J. Combin.* 91 (2021), art. 103215, 8 pp. MR 4161809. Zbl 1458.05073. arXiv:1906.05638.

A signed planar graph may have chromatic number > 4, disproving a conjecture of Máčajová, Raspaud, and Škoviera (2016a). [Annot. 26 Jul 2019.] **(SG: Col)**

A. Kargaran, M. Ebrahimi, M. Riazi, A. Hosseiny, & G.R. Jafari

2020a Quartic balance theory: Global minimum with imbalanced triangles. *Phys. Rev. E* 102 (2020), art. 012310, 8 pp. MR 4137484. arXiv:2001.01719.

Measuring imbalance by triples and quadruples. [Annot. 18 Dec 2020.] **(SG: Fr)**

M.M. Karkhaneei

See S. Akbari.

Richard M. Karp, Raymond E. Miller, & Shmuel Winograd

1967a The organization of computations for uniform recurrence equations. *J. Assoc. Computing Machinery* 14 (1967), 563–590. MR 0234604 (38 #2920). Zbl 171.38305 (171, p. 383e).

Implicitly, concerns the existence of nonpositive directed tours (closed trails) in a $\mathbb{Z}^d$-gain graph (the "dependence graph" of a system of recurrences). **(gd: cov)**

K. Karuppasamy

See P. Jeyalakshmi.

Alexander V. Karzanov

See M.A. Babenko and A.V. Goldberg.

Yasuhiro Kasai, Ayao Okiji, & Itiro Syozi

1981a Application of real replica method to Syozi model. *Progress Theor. Phys.* 65 (1981), no. 1, 140–153. **(Phys: SG)**

1981b The ground state of a replicated Ising system. *Progress Theor. Phys.* 65 (1981), no. 4, 1439–1442. MR 0620472 (82h:82030). Zbl 1074.82528.

Grand partition function $:= \sum_\theta \exp(\#E^- - \#E^+)$ over all edge signatures θ and all switchings of a lattice graph, investigated for a physical phase via multiple replicates and analytic continuation. [The relevance to signed graphs is obscured by summing over all signatures.] [Annot. 17 Aug 2012.] **(Phys: SG)**

1981c Ising replicated system of $\pm J$ model. *Progress Theor. Phys.* 66 (1981), no. 5, 1561–1573. MR 0642957 (83b:82081). Zbl 1074.82547.

Similar to (1981b), without analytic continuation. §2 recapitulates (1981b). §3 does calculations for the path graph. §4, "The ground state". [Annot. 17 Aug 2012.] (**Phys: SG**)

Yoshi Kashima

See G. Robins.

Stanisław Kasjan & Daniel Simson

2015a Mesh algorithms for Coxeter spectral classification of Cox-regular edge-bipartite graphs with loops, I. Mesh Root Systems. *Fundamenta Inform.* 139 (2015), 153–184. MR 3383583. (**SG**)

2015b Mesh algorithms for Coxeter spectral classification of Cox-regular edge-bipartite graphs with loops, II. Application to Coxeter spectral analysis. *Fundamenta Inform.* 139 (2015), 153–184. MR 3383584. Zbl 1335.05145. (**SG**)

2015c Algorithms for isotropy groups of Cox-regular edge-bipartite graphs. *Fundamenta Inform.* 139 (2015), 249–275. MR 3383587. (**SG**)

Eva Kaslik & Ileana Rodica Rădulescu

2022a Stability and bifurcations in fractional-order gene regulatory networks. *Appl. Math. Comput.* 421 (2022), art. 126916, 15 pp. MR 4364751 (no rev). Zbl 1510.92077.

From abstract: "oscillatory behavior ... is expected only in the case of an odd number of repressive genes." [Annot. 6 Feb 2022.] (**Biol: vs: Bal**)

Adrien Kassel & Thierry Lévy

2020a A colourful path to matrix-tree theorems. *Algebraic Combin.* 3 (2020), no. 2, p. 471–482. MR 4099004. Zbl 1436.05049. arXiv:1903.02491.

Broad generalization of the matrix-tree theorems of Zaslavsky (1982a) and Chaiken (1982a). [Annot. 11 Feb 2022.] (**SG, GG: Lap**)

P.W. Kasteleyn

See also C.M. Fortuin.

1963a Dimer statistics and phase transitions. *J. Math. Phys.* 4 (1963), 287–293. MR 0153427 (27 #3394).

§V, "The Ising problem": The ferromagnetic Ising model can be converted into a dimer-covering problem. The method has since been applied to signed graphs (the general Ising problem); *cf.* Thomas and Middleton (2009a), (2013a), and references therein. [Annot. 10 Jan 2015.] (**Phys, Algor**)

P.W. Kasteleyn & C.M. Fortuin

1969a Phase transitions in lattice systems with random local properties. In: *International Conference on Statistical Mechanics* (Proc., Kyoto, 1968), pp. 11–14. Supplement to *J. Physical Soc. Japan*, Vol. 26, 1969. Physical Society of Japan, [Tokyo?], 1969.

A specialization of the parametrized dichromatic polynomial of a graph: $Q_\Gamma(q, p; x, 1)$ where $q_e = 1 - p_e$. [Essentially, announcing Fortuin and Kasteleyn (1972a).] (**sgc: Gen: Invar, Phys**)

Viktória E. Kaszanitzky

See T. Jordán.

E. Kaszkurwicz & L. Hsu

1982a On qualitative equilibria in Lotka–Volterra models. In: *Proceedings of the 1982 American Control Conference* (Rio de Janeiro, 1982), pp. 481–483. IEEE, 1982. (**sd: QM: QSol**)

Osamu Katai

See also S. Iwai.

1979a Studies on aggregation of group structures and group attributes through quantification methods. D.Eng. dissertation, Kyoto University, 1979.

Osamu Katai & Sousuke Iwai

† 1978a Studies on the balancing, the minimal balancing, and the minimum balancing processes for social groups with planar and nonplanar graph structures. *J. Math. Psychology* 18 (1978), 140–176. MR 0515232 (83m:92072). Zbl 394.92027.

Balance and detecting balance are discussed at length. Finding the frustration index $l(\Sigma)$ is solved for planar graphs by converting it into a matching problem in the dual graph with signed vertices. This applies also when edges are weighted by positive reals. [Barahona (1982a) and Barahona, Maynard, Rammal, and Uhry (1982a) have a similar, later, but independent solution for the planar frustration index. Barahona (1981a), (1990a) solves toroidal graphs.]

The nonplanar problem is treated via $A(\Sigma)$, but amounts to finding $\min_\zeta(\#E^+(\Sigma^\zeta) - \#E^-(\Sigma^\zeta))$ [which is NP-hard]. This suggests an iterative procedure which consists of switching $v \in V$ that minimizes $d^\pm(v)$, and repeating; it may not attain the true minimum. [Mitra (1962a) also proposed this.] [Annot. 22 Jun 2012.]

(**SG, VS, WG, PsS: Bal, Fr, Algor, Adj, sw**)

1978b On the characterization of balancing processes of social systems and the derivation of the minimal balancing processes. *IEEE Trans. Systems Man Cybernetics* SMC-8 (1978), 337–348. MR 0479461 (57 #18886) (*q.v.*). Zbl 383.92025.

A shorter version of (1978a). Lem. 1 [restated]: Σ is balanced iff it switches to all positive. [Annot. 22 Jun 2012.]

(**SG, VS, WG, PsS: Bal, Fr, Algor, Adj, sw**)

1978c Characterization of social balance by statistical and finite-state systems theoretical analysis. In: *Proceedings of the International Conference on Cybernetics and Society* (Tokyo, 1978). IEEE, 1978. (**SG, WG, PsS: Bal, Fr**)

Priya Kataria

See D. Sinha.

M. [Moshe] Katz

See also G. Converse.

1970a On the extreme points of a certain convex polytope. *J. Combin. Theory* 8 (1970), 417–423. MR 0255582 (41 #243). Zbl 194.34102.

The doubly stochastic case (all line sums = 1) of Thm. 8.2.1 in Brualdi (2006a). [Annot. 13 Oct 2012.] (**sg: par: Adj**)

Louis H. Kauffman

See also J.R. Goldman.

1986a Signed graphs. Abstract 828-57-12, *Abstracts Amer. Math. Soc.* 7 (1986), no. 5, p. 307.

Announcement of (1989a). **(SGc: Knot: Invar)**

1988a New invariants in the theory of knots. *Amer. Math. Monthly* 95 (1988), 195–242. MR 0935433 (89d:57005). Zbl 657.57001.

A leisurely development of Kauffman's combinatorial bracket polynomial of a link diagram and the Jones and other knot polynomials, including the basics of (1989a). **(Knot, SGc: Invar: Exp)**

† 1989a A Tutte polynomial for signed graphs. *Discrete Appl. Math.* 25 (1989), 105–127. MR 1031266 (91c:05082). Zbl 698.05026.

The Tutte polynomial, also called "Kauffman's bracket of a signed graph" and equivalent to his bracket of a link diagram, is of an edge-sign-colored graph. It is defined by a sum over spanning trees of terms that depend on the signs and activities of the edges and nonedges of the tree. The point is that the deletion-contraction recurrence over an edge has parameters dependent on the color of the edge; also, the parameters of the two colors are related. The purpose is to develop the bracket of a link diagram combinatorially. §3.2, "Link diagrams": how link diagrams correspond to signed plane graphs. §4, "A polynomial for signed graphs", defines the general sign-colored graph polynomial $Q[\Sigma](A, B, d)$ by deletion-contraction, modified multiplication on components, and evaluation on graphs of loops and isthmi. §5, "A spanning tree expansion for $Q[G]$" [G means Σ], proves $Q[\Sigma]$ exists by producing a spanning-tree expansion, shown independent of the edge ordering by a direct argument. [No dichromatic form of $Q[\Sigma]$ appears; but see successor articles.] §6, "Conclusion", remarks that $Q[\Sigma]$ is invariant under signed-graphic Reidemeister moves II and III. [This significant work, inspired by Thistlethwaite (1988a), led to independent but related generalizations by Przytycka and Przytycki (1988a), Schwärzler and Welsh (1993a), Traldi (1989a), and Zaslavsky (1992b) that were partially anticipated by Fortuin and Kasteleyn (1972a). Also see (1997a).]

(SGc: Invar, Knot)

1991a *Knots and Physics.* Ser. Knots Everything, Vol. 1. World Scientific, River Edge, N.J., 1991. MR 1141156 (93b:57010). Zbl 733.57004.
2nd ed., 1993, with added appendices not related to sign-colored graphs. MR 1306280 (95i:57010). Zbl 868.57001.
3rd ed., 2001, with more appendices not related to sign-colored graphs. MR 1858113 (2002h:57012). Zbl 1057.57001.
4th ed., 2013, with more NEW appendices not related to sign-colored graphs. Ser. Knots Everything, Vol. 53. World Scientific, Hackensack, N.J., 2013. MR 3013186. Zbl 1266.57001. **(SGc: Invar: Exp)**

1997a Knots and electricity. In: S. Suzuki, ed., *Knots '96* (Proc. Fifth Int. Research Inst., Math. Soc. Japan, Tokyo, 1996), pp. 213–230. World Scientific, Singapore, 1997. MR 1664963 (99m:57006). Zbl 967.57007.

§2, "A state summation for classical electrical networks", uses a form of the parametrized dichromatic polynomial $Q_\Gamma(B, A; 1, 1)$ [as in Zaslavsky (1992b) *et al.*; *cf.*], where $A(e), B(e) \in \mathbb{C}^\times$, to compute conductances as

in Goldman and Kauffman (1993a). **(sgc: Gen: Invar: Exp)**

§3, "The bracket polynomial", discusses the connections with sign-colored graphs and electricity. *Problem*: Is there a signed graph, not reducible by signed-graphic Reidemeister moves (see (1989a)) to a tree with loops, whose sign-colored dichromatic polynomial is trivial? If not, the Jones polynomial detects the unknot.

(SGc: Invar: Exp)(SGc: Invar)

Louis H. Kauffman, Slavik Jablan, Ljiljana Radović, & Radmila Sazdanović

2013a Reduced relative Tutte, Kauffman bracket and Jones polynomials of virtual link families. *J. Knot Theory Ramifications* 22 (2013), no. 4, art. 1340003, 12 pp. MR 3055554. Zbl 1266.05069. arXiv:1106.2785. **(SGc: Knot, Invar)**

Marcelle Kaufman

See also J.-P. Comet, J. Demongeot and R. Thomas.

M. Kaufman, C. Soulé, & R. Thomas

2007a A new necessary condition on interaction graphs for multistationarity. *J. Theor. Biol.* 248 (2007), 675–685. MR 2899089 (no rev). Zbl 1451.92146. **(SD: Dyn)**

Marcelle Kaufman & René Thomas

2003a Emergence of complex behaviour from simple circuit structures. Émergence de comportements complexes à partir de structures de circuits simples. *C.R. Biologies* 326 (2003), 205–214. **(SD: Dyn)**

M. Kaufman, J. Urbain, & R. Thomas

1985a Towards a logical analysis of the immune response. *J. Theor. Biol.* 114 (1985), no. 4, 527–561. MR 0796984 (87d:92013). **(Biol, sd: Dyn)**

Tali Kaufman & Alexander Lubotzky

2014a High dimensional expanders and property testing. In: *Proceedings of the 5th Conference on Innovations in Theoretical Computer Science* (ITCS '14, Princeton, N.J., 2014), pp. 501–506. ACM, 2014. MR 3359501. Zbl 1365.68462. arXiv:1312.2367.

The focus is on "testability". §3.1, "Sum functions on graphs": "Sum function" = balanced sign function. Prop. 9: Balance is not testable on graphs whose girth $\to \infty$. Prop. 10: Balance is testable on complete graphs. Rem. 3: Balance of (K_n, σ) is determined by triangles. [Long known.] Prop. 11 connects triangle testing on the 1-skeleton of a simplicial complex to the first expansion constant of the complex. [*Question.* What does that mean for flag complexes?] §3.3: "Seidel switching": Switching equivalence of signed complete graphs is testable. [An easy corollary of Prop. 10.] [Annot. 1 Jul 2022.] **(sg: Bal, KG)**

Bableen Kaur

See D. Sinha.

Ken-ichi Kawarabayashi

See also Chiba, M. Chudnovsky, E.D. Demaine, S. Fujita, and N. Kakimura.

2008a A weakening of the odd Hadwiger's conjecture. *Combin. Probab. Computing* 17 (2008), no. 6, 815–821. MR 2463413 (2009j:05083). Zbl 1188.05062.

$\exists\ f(k)$ such that, if $-\Gamma$ has no $-K_k$ minor, then V partitions into $496k$ parts of order $\leq f(k)$. Based on Geelen, Gerards, Reed, Seymour, and

Vetta (2009a). [*Problem.* Generalize to signed graphs.] [Annot. 4 Feb 2021.] (**sg: Par: Col(Gen)**)

2009a Note on coloring graphs without odd-K_k-minors.. *J. Combin. Theory Ser. B* 99 (2009), no. 4, 728–731. MR 2518204 (2010e:05109). Zbl 1198.05058.

Short proof of Geelen, Gerards, Reed, Seymour, and Vetta (2009a): If $-\Gamma$ has no $-K_k$ minor, then Γ is $O(k\sqrt{\log k})$-colorable. [Annot. 4 Feb 2021.] (**Par: Col**)

2013a Totally odd subdivisions and parity subdivisions: Structures and coloring. In: Sanjeev Khanna, ed., *Proceedings of the Twenty-Fourth Annual ACM-SIAM Symposium on Discrete Algorithms* (SODA '13, New Orleans, 2013), pp. 1013–1029. Soc. for Industrial and Appl. Math., Philadelphia, 2013. MR 3202965. Zbl 1423.05175. (**par: Str, Algor**)

Ken-ichi Kawarabayashi & Yusuke Kobayashi

2012a Edge-disjoint odd cycles in 4-edge-connected graphs. In: Christoph Dürr and Thomas Wilke, eds., *29th International Symposium on Theoretical Aspects of Computer Science* (Proc. STACS'12, Paris, 2012), pp. 206–217. Leibniz Int. Proc. Informatics, Vol. 14. Leibniz-Zentrum für Informatik, Wadern, 2012. MR 2909315. Zbl 1244.05129.

Conference version of (2016a). (**Par: Str: Cyc**)

2016a Edge-disjoint odd cycles in 4-edge-connected graphs. *J. Combin. Theory Ser. B* 119 (2016), 12–27. MR 3486335. Zbl 1334.05072. (**Par: Str: Cyc**)

Ken-ichi Kawarabayashi, Orlando Lee, & Bruce Reed

2014a Removable paths and cycles with parity constraints. *J. Combin. Theory Ser. B* 106 (2014), 115–133. MR 3194198. Zbl 1297.05126. (**Par: Str**)

Ken-ichi Kawarabayashi, Zhentao Li, & Bruce Reed

2010a Recognizing a totally odd K_4-subdivision, parity 2-disjoint rooted paths and a parity cycle through specified elements. In: Moses Charikar, ed., *Proceedings of the Twenty-First Annual ACM-SIAM Symposium on Discrete Algorithms* (SODA '10, Austin, Tex., 2010), pp. 318–328. Society for Industrial and Appl. Math. Philadelphia, 2010. Zbl 1288.05279.

Extended abstract. A totally odd subdivision of Γ is a signed graph that is a (signed-graphic) subdivision of $-\Gamma$ and is itself all negative. Dictionary: "parity" = sign in $-\Gamma$; "parity path/circle" = path or circle of specified sign. (**sg: par: Str, Algor**)

Ken-ichi Kawarabayashi & Bojan Mohar

2006a Approximating the list-chromatic number and the chromatic number in minor-closed and odd-minor-closed classes of graphs. In: *STOC'06: Proceedings of the 38th Annual ACM Symposium on Theory of Computing* (Seattle, 2006), pp. 401–416. ACM, New York, 2006. MR 2277166 (2007m:68309). Zbl 1301.05334.

Odd minor: specifically, a $-K_k$ minor of $-\Gamma$. [Annot. 4 Feb 2021.] (**Par: Col: Algor**)

Ken-Ichi Kawarabayashi & Atsuhiro Nakamoto

2007a The Erdős–Pósa property for vertex- and edge-disjoint odd cycles in graphs on orientable surfaces. *Discrete Math.* 307 (2007), no. 6, 764–768. MR 2291454

(2007h:05084). Zbl 1112.05056. **(sg: Par: Circ, Top)**

Ken-ichi Kawarabayashi & Kenta Ozeki

2013a A simpler proof for the two disjoint odd cycles theorem. *J. Combin. Theory Ser. B* 103 (2013), 313–319. MR 3048156. Zbl 1301.05196.

Thm. 1 states the characterization for internally 4-connected graphs. For the generalization to disjoint negative circles in signed graphs of any connectivity see the earlier paper by Slilaty (2007a). [Annot. 25 Jun 2013.] **(sg: Par: Str)**

Ken-Ichi Kawarabayashi & Bruce Reed

2008a Fractional coloring and the odd Hadwiger's conjecture. *European J. Combin.* 29 (2008), no. 2, 411–417. MR 2388377 (2008k:05192. Zbl 1142.05027.

Thm.: If $-\Gamma$ has no $-K_p$ minor, then Γ has a fractional $2p$-coloring. *Cf.* Geelen, Gerards, Reed, Seymour, and Vetta (2009a). [*Problem.* Generalize to signed graphs.] [Annot. 4 Feb 2021.] **(sg: Par: Col((Gen))**

2009a A nearly linear time algorithm for the half integral parity disjoint paths packing problem. In: Claire Mathieu, ed., *Proceedings of the Twentieth Annual ACM-SIAM Symposium on Discrete Algorithms* (New York, 2009), pp. 1183–1192. SIAM, Philadelphia, 2009. MR 2807561 (no rev). Zbl 1423.68349. **(sg: Par: Algor)**

2009b Highly parity linked graphs. *Combinatorica* 29 (2009), no. 2, 215–225. MR 2520281 (2010k:05157). Zbl 1212.05143. **(sg: Par: Str)**

2010a An (almost) linear time algorithm for odd cycles transversal. In: Moses Charikar, ed., *Proceedings of the Twenty-First Annual ACM-SIAM Symposium on Discrete Algorithms* (SODA '10, Austin, Tex., 2010), pp. 365–378. Society for Industrial and Appl. Math. Philadelphia, 2010. MR 2809682 (2012j:68132). Zbl 1288.05280.

"Odd cycles transversal problem": Is $l_0(-\Gamma) \leq k$? An $O(m\alpha(m,n))$ algorithm for fixed k, where $\alpha =$ inverse Ackermann function; also solves edge form: Is $l(-\Gamma) \leq k$? Also, an $O(m\alpha(m,n) + n\log n)$ algorithm for 2-packing of k odd circles. Also, simplified proof of Bruce Reed (1999a). [Annot. 29 Dec 2020.] **(sg: par: fr, Algor)**

2010b Odd cycle packing. In: *STOC'10—Proceedings of the 2010 ACM International Symposium on Theory of Computing* (Cambridge, Mass., 2010), pp. 695–704. Association for Computing Machinery (ACM), New York, 2010. MR 2743319 (no rev). Zbl 1293.05177.

Extended abstract. **(sg: Par: Circ)**

Ken-ichi Kawarabayashi, Bruce Reed, & Orlando Lee

2009a Removable cycles in non-bipartite graphs. *J. Combin. Theory Ser. B* 99 (2009), no. 1, 30–38. MR 2467816 (2009k:05108). Zbl 1176.05041.

Γ 3-connected, $s,t \in V$, $\forall$ 3-cut X each component of $\Gamma \setminus X$ contains s or t. Then (1) $\exists$ induced st-path P such that $-\Gamma \setminus V(P)$ is unbalanced, or (2) Γ is planar such that every negative face contains one of s, t; but not both (1) and (2). [*Problem.* Generalize to signed graphs.] [Annot. 4

Feb 2021.] (**Par: Str**)

Ken-ichi Kawarabayashi, Bruce Reed, & Paul Wollan

2011a The graph minor algorithm with parity conditions. In: Rafail Ostrovsky, ed., *2011 IEEE 52nd Annual Symposium on Foundations of Computer Science—FOCS 2011* (Palm Springs, Calif., 2011), pp. 27–36. IEEE Computer Soc., Los Alamitos, Calif., 2011. MR 2932677. Zbl 1293.00031 (book).

The graph minors algorithm for a $-\Gamma$ minor, specifically $-K_n$ ("odd K_n minor"). Disjoint paths with parity conditions. Related problems. [Annot. 4 Feb 2021.] (**sg: Par: Algor**)

Ken-ichi Kawarabayashi & Zi-Xia Song

2007a Some remarks on the odd Hadwiger's conjecture. *Combinatorica* 27 (2007), no. 4, 429–438. MR 2359825 (2008h:05046). Zbl 1164.05025.

Thm.: $\exists\ f(k)$: If Γ is $(496k + 13)$-connected of order $\geq f(k)$, then $-\Gamma$ has a $-K_k$ minor or $-\Gamma \setminus 8k$ vertices is balanced. Dictionary: "odd complete minor of Γ" $= -K_k$ minor of $-\Gamma$. *Cf.* Geelen, Gerards, Reed, Seymour, and Vetta (2009a). [*Problem.* Generalize to signed graphs.] [Annot. 4 Feb 2021.] (**sg: Par: Str**)

Ken-ichi Kawarabayashi & Paul Wollan

2006a Non-zero disjoint cycles in highly connected group labelled graphs. *J. Combin. Theory Ser. B* 96 (2006), no. 2, 296–301. MR 2208356 (2006j:05108). Zbl 1090.05038. (**GG: Circ**)

Ken-ichi Kawarabayashi & David R. Wood

2012a Cliques in odd-minor-free graphs. In: *Proceedings of the Eighteenth Computing: The Australasian Theory Symposium* (CATS '12 , Melbourne, 2012), pp. 133–138. Australasian Theory Symp., Vol. 128. Australian Computer Soc., Darlinghurst, Australia, 2012. (**sg: par: Str, Algor**)

Yasushi Kawase, Yusuke Kobayashi, & Yutaro Yamaguchi

2015a Finding a path with two labels forbidden in group-labeled graphs. In: Magnús M. Halldórsson *et al.*, eds., *Automata, Languages, and Programming*, Part I (Proc. 42nd Int. Colloq., ICALP 2015, Kyoto), pp. 797–809. Lect. Notes Computer Sci., Vol. 9134. Springer, Heidelberg, 2015. MR 3382488. Zbl 1440.05176.

Extended abstract of (2020a). (**GG: Paths: Algor**)

2020a Finding a path with two labels forbidden in group-labeled graphs. *J. Combin. Theory Ser. B* 143 (2020), 65–122. MR 4089576. Zbl 1437.05208. arXiv:1807.00109.

Gain graphs with two forbidden gain values. (**GG: Paths: Algor**)

N. Kawashima & H. Rieger

1997a Finite-size scaling analysis of exact ground states for $\pm J$ spin glass models in two dimensions. *Europhys. Lett.* 39 (1997), no. 1, 85–90. arXiv:cond-mat/9612116. (**Phys, SG: Fr, State**)

2004a Recent progress in spin glasses. In: H.T. Diep, ed., *Frustrated Spin Systems*, Ch. 9, pp. 491–596. World Scientific, Hackensack, N.J., 2004. Zbl 1104.82002 (book).

Many subsections throughout on open problems about $\pm J$ models (signed graphs; ground state $\leftrightarrow$ switching to fewest negative edges $\leftrightarrow$

frustration index $l(\Sigma)$) and continuous models (stochastic edge signs and weights); esp., ground state computations, mostly on excessively small graphs, looking for phase transitions and critical points. For $\pm J$, e.g: §§9.2.1, 9.2.4, 9.3.3, 9.6.3. §9.6: XY, Heisenberg, Potts spins (in S^2, S^3, $[q]$). §9.7, "Weak disorder": "gauge invariance" (switching invariance) implies some properties. Ample references. [Annot. 15 Aug 2018.] (**SG, Phys: Fr, Sw: Exp, Ref**)

B. Kawecka-Magiera

See M.J. Krawczyk.

K. Kazemian

See S. Akbari.

Peter Keevash

See I. Balla.

Nataša Kejžar, Zoran Nikoloski, & Vladimir Batagelj

2008a Probabilistic inductive classes of graphs. *J. Math. Sociology* 32 (2008), no. 2, 85–109. Zbl 1135.91424.

§3.4, "Dynamic of social balance processes": Probabilistic rules for dynamics of balance processes. [Annot. 9 Jan 2022.] (**SG: Dyn: Rand**)

Christine A. Kelley & Joerg Kliewer

2010a Algebraic constructions of graph-based nested codes from protographs. In: *2010 IEEE International Symposium on Information Theory* (Proc., Austin, Tex., 2010), pp. 829–833. arXiv:1006.2977. (**GG: Cov: Appl**)

Alexander Kelmans

See J.F. De Jesús.

Dzh. Kemeni & Dzh. Snell

See J.G. Kemeny and J.L. Snell.

John G. Kemeny

1959a Mathematics without numbers. Quantity and Quality. *Daedelus* 88 (1959), no. 4, 577–591.

Model No. 1 expounds signed graphs in social psychology from Cartwright and Harary (1956a). [Annot. 27 Dec 2012.] (**SG: PsS, Bal: Exp**)

John G. Kemeny & J. Laurie Snell

1962a *Mathematical Models in the Social Sciences.* Introductions to Higher Mathematics. Ginn, Boston [Blaisdell, Waltham, Mass.], 1962. Repr.: MIT Press, Cambridge, Mass., 1972. MR 0140375 (25 #3797), repr. MR 0363521 (50 - #15959), repr. MR 0519512 (80a:92060). Zbl 256.92003 (256, p. 92c) (no rev).

Chapter VIII: "Organization theory: Applications of graph theory." See pp. 97–101 and 105–107. (**SG: Bal: Exp**)

1972a (As "Dzh. Kemeni & Dzh. Snell") *Kiberneticheskoe Modelriovanie. Nekotorye Prilozheniya.* Transl. B.G. Mirkin. Preface by I.B. Gutchin. "Sovetskoe Radio", Moscow, 1972. Zbl 256.92002.

Russian translation of (1962a). (**SG: Bal: Exp**)

Arnfried Kemnitz & Margit Voigt

2021a A note on complex-4-colorability of signed planar graphs. *Electronic J. Combin.* MR 4257780. Zbl 1464.05154.

For some signed planar graphs, proper coloration by $\pm 1, \pm \mathrm{i}$ is not possible. [Annot. 6 May 2021.] (**SG: Col(Gen)**)

B.K. Kempegowda

See M.R. Rajesh Kanna.

Mark Kempton

See F. Chung.

A. Joseph Kennedy

See also M. Parvathi.

2007a Class partition algebras as centralizer algebras of wreath products. *Commun. Algebra* 35 (2007), no. 1, 145–170. MR 2287557 (2008j:16072). Zbl 1151.20006. (**gg: matrd: Algeb**)

John W. Kennedy

See M.L. Gargano.

Jeff L. Kennington & Richard V. Helgason

1980a *Algorithms for Network Programming.* Wiley, New York, 1980. MR 0581251 (82a:90173). Zbl 502.90056.

Ch. 5: "The simplex method for the generalized network problem." (**GN: Matrd(Bases): Exp**)

Richard Kenyon

2011a Spanning forests and the vector bundle Laplacian. *Ann. Prob.* 39 (2011), no. 5, 1983–2017. MR 2884879 (2012k:82011). Zbl 1252.82029. (**gg: Lap**)

Anne-Marie Kermarrec & Afshin Moin

2011a Energy models for drawing signed graphs. Res. rep., 2011, 29 pp. HAL inria-00605924. (**SG: Clu: Geom**)

Anne-Marie Kermarrec & Christopher Thraves

2011a Can everybody sit closer to their friends than their enemies? In: Filip Murlak and Piotr Sankowski, eds., *Mathematical Foundations of Computer Science 2011* (36th Int. Symp., Warsaw), pp. 388–399. Lect. Notes in Computer Sci., Vol. 6907. Springer, Heidelberg, 2011. MR 2881711. Zbl 1343.68184.

Can (K_n, σ) be drawn in $\mathbb{R}^l$ so every positive neighbor is closer than every negative neighbor, for each vertex? Polynomial-time algorithm for $l = 1$. [Continued by Cygan, Pilipczuk, *et al.* (2012a).] [Annot. 26 Apr 2012.] (**SG: Geom: KG: Bal, Algor, Clu**)

2014a Signed graph embedding: when everybody can sit closer to friends than enemies. Manuscript, 2014. arXiv:1405.5023.

Cf. (2011a). (**SG: Geom: KG: Bal, Algor, Clu**)

Julie Kerr

1999a A basis for the top homology of a generalized partition lattice. *J. Algebraic Combin.* 9 (1999), 47–60. MR 1676728 (2000k:05265). Zbl 921.05063.

The lattice is isomorphic to the semilattice of k-composed partitions of a set with a top element adjoined. (See R. Gill (1998b).) (**gg: matrd: Geom, Top**)

Mehtab Khan

See also R. Farooq, S. Hafeez, and F. Jamal.

Mehtab Khan & Rashid Farooq

2017a On the energy of bicyclic signed digraphs. *J. Math. Inequalities* 11 (2017), no. 3, 845–862. MR 3732818. Zbl 1371.05164. (**SD: Adj: Eig**)

Muhammad Ali Khan

See S. Pirzada.

H. Kharaghani

2003a On a class of symmetric balanced generalized weighing matrices. *Designs Codes Cryptogr.* 30 (2003), no. 2, 139–149. MR 2006485 (2004j:05027). Zbl 1036.05016.

A "balanced generalized weighing matrix" is the group-ring adjacency matrix $\hat{A}$ of a gain digraph $\vec{\Phi}$, with finite gain group $\mathfrak{G}$, such that $\hat{A}\hat{A}^* = kI + ls(J - I)$ where $s := \sum_{g\in\mathfrak{G}} g$. Constructs examples of $\hat{A}$ where $\mathfrak{G}$ is cyclic and $\vec{\Phi}$ is symmetric with no loops. [The article does not mention gain digraphs.] (**gg: Adj**)

F. Kharari & È. Palmer [Frank Harary & Edgar M. Palmer]

See Harary and Palmer (1977a).

Abdelkader Khelladi

See also O. Bessouf.

1985a *Propriétés algébriques des structures combinatoires.* Thèse de doctorate des sciences mathématiques, Université des Sciences et de la Technologie Houari Boumediene, Alger, 1985.

Partie I, "Flots, potentiels et tensions dans les graphes biorientés": Bidirected (and signed) graphs. §2, "Graphes biorientés – Graphes signés". §3, "Flots dans les graphes biorientés, Conjecture de Bouchet". §4, "Potentiels et tensions, Facteurs dans les graphs biorientés".

Introduces quasibalance (called "m-balance"): Two negative circles have at least 2 common vertices. [Annot. 10 Jan 2019.]

(**SG: Ori: Str, Flows**)

1987a Nowhere-zero integral chains and flows in bidirected graphs. *J. Combin. Theory Ser. B* 43 (1987), 95–115. MR 0897242 (88h:05045). Zbl 617.90026.

Improves the result of Bouchet (1983a) about nowhere-zero integral flows on a signed graph. Σ has such an 18-flow if 4-connected, a 30-flow if 3-connected and without a positive triangle, and if quasibalanced ("m-balanced"; *cf.* (1985a)) a 6-flow (proving Bouchet's conjecture in that case). [Annot. rev 10 Jan 2019.] (**SG: Matrd, Flows, Ori**)

1999a Colorations généralisées, graphes biorientés et deux ou trois choses sur François. Symposium à la Mémoire de François Jaeger (Grenoble, 1998). *Ann. Inst. Fourier (Grenoble)* 49 (1999), 955–971.
MR 1703433 (2000h:05083). Zbl 917.05026.

Comments on the results of Bouchet (1983a) and Khelladi (1987a).

(**SG: Matrd, Flows, Ori**)

Boris N. Kholodenko, Anatoly Kiyatkin, Frank J. Bruggeman, Eduardo Sontag, Hans V. Westerhoff, & Jan B. Hoek

2002a Untangling the wires: A strategy to trace functional interactions in signaling and gene networks. *Proc. Nat. Acad. Sci.* 99 (2002), no. 20, 12841–12846.

A matrix-based method to infer the signs (and magnitudes) of an interaction digraph from measurement of the interactions between modules of the digraph. [Annot. 25 Jan 2015.] (**SD: Algor: Biol**)

Antonina P. Khramova

See A. Abiad.

Dariush Kiani

See I. Gutman, H. Hamidzade, M. Jooyandeh, and M. Mirzakhah.

Arnott Kidner

See R.C. Brewster.

Kathleen P. Kiernan

See R.A. Brualdi.

Dae San Kim

See T.K. Kim.

Dongseok Kim & Jaeun Lee

2008a The chromatic numbers of double coverings of a graph. *Discrete Math.* 308 (2008), no. 22, 5078–5086. MR 2450445 (2009k:05082). Zbl 1158.05026.

(**SG: Cov: Col**)

Eun Jung Kim

See N. Alon.

Jeong-Rae Kim, Yeoin Yoon, & Kwang-Hyun Cho

2008a Coupled feedback loops form dynamic motifs of cellular networks. *Biophys. J.* 94 (2008), 359–365.

Collects the effects of the three types of coupled cycles (signed $++$, $+-$, $--$) in an interaction signed digraph in biological examples modelled by differential equations. Observes that $++$ cycle pairs "enhance signal amplification and" bistability, $--$ enhance homeostasis, and $+-$ "enable reliable decision-making" by the biological system. [*Cf.* Sriram, Soliman, and Fages (2009a).] [Annot. 16 Jan 2015.] (**Biol: SD: Dyn**)

Jong-Jae Kim

See O. Nagai.

Ringi Kim, Seog-Jin Kim, & Xuding Zhu

2022a Signed colouring and list colouring of k-chromatic graphs. *J. Graph Theory* 99 (2022), no. 4, 637–650. MR 4429172.

(**SG: Col**)

Sangwook Kim

See J.-S. Huh.

Seog-Jin Kim

See also R.-G. Kim.

Seog-Jin Kim & Kenta Ozeki

2019a A note on a Brooks' type theorem for DP-coloring. *J. Graph Theory* 91 (2019), no 2, 148–161. MR 3948125. Zbl 1419.05076. arXiv:1709.09807. (**SG: Col**)

Taekyun Kim & Dae San Kim

2022a Degenerate Whitney numbers of first and second kind of Dowling lattices. *Russian J. Math. Phys.* 29 (2022), no. 3, 358–377. MR 4480315. Zbl 1507.11022. arXiv:2103.08904.

These "degenerate Whitney numbers" are a further generalization of "r-Whitney numbers", *cf.* Mező (2010a). [Their meaning is puzzling as they appear to be closely related to Dowling's characteristic polynomials.] [Dowling is incorrectly credited with defining Whitney numbers.] [Annot. 21 Jul 2022.] (**gg: Matrd: Invar**)

2022b Degenerate r-Whitney numbers and degenerate r-Dowling polynomials via boson operators. *Adv. Appl. Math.* 140 (2022), art. 102394, 21 pp. MR 4443761. Zbl 1510.11080. arXiv:2204.07843.

Dowling lattices mentioned at the beginning. [Annot. 25 Jul 2023.] (**gg: matrd: Invar**)

Andrew D. King

See T. Huynh.

Harunobu Kinoshita

See T. Yamada.

Shin'ichi Kinoshita

See also T. Yajima.

Shin'ichi Kinoshita & Hidetaka Terasaka

1957a On unions of knots. *Osaka Math J.* 9 (1957), 131–153. MR 0098386 (20 #4846). Zbl 080.17001.

Employs the sign-colored graph of a link diagram from Bankwitz (1930a) to form certain combinations of links. (**SGc: Knot**)

M. Kirby

See A. Charnes.

Steve Kirkland

See also M. Cavers, M.A.A. de Freitas, F. Goldberg, C.S. Oliveira, and J. Stuart.

2011a Sign patterns for eigenmatrices of nonnegative matrices. *Linear Multilinear Algebra* 59 (2011), no. 9, 999–1018. MR 2826068 (2012j:15049). Zbl 1239.15011. (**QM, SD**)

Steve Kirkland, J.J. McDonald, & M.J. Tsatsomeros

1996a Sign-patterns which require a positive eigenvalue. *Linear Multilinear Algebra* 41 (1996), no. 3, 199–210. MR 1430028 (97j:15009). Zbl 871.15009. (**QM, SD**)

Steve Kirkland & Debdas Paul

2011a Bipartite subgraphs and the signless Laplacian matrix. *Appl. Anal. Discrete Math.* 5 (2011), no. 1, 1–13. MR 2809028 (2012c:05191). Zbl 1289.05298. (**Par: Eig, incid**)

Scott Kirkpatrick

See also D. Sherrington and J. Vannimenus.

1977a Frustration and ground-state degeneracy in spin glasses. *Phys. Rev. B* 16 (1977), no. 10, 4630–4641.

Estimates the number of ground states of signed d-dimensional hypercubic lattices, $d \geq 2$. With random signs, of which the proportion x is negative, the expected proportion of negative ("frustrated") squares is computed to be $4x(1-x)[x^2+(1-x)^2] \leq 0.5$, $\approx$ for $.2 < x < .8$. [This assumes the squares' signs are independent, which is only true when $d = 2$.] Certain ice models are equivalent to signed graphs (p. 4632). §III, "Exact results": In $d = 2$ the strings pairing negative squares in a ground state are short on average. In $d = 3$ ground states are more difficult to find [a conclusion essentially proved by Barahona (1982a)] but there are interesting remarks on strings pairing negative squares. §IV, "Monte Carlo results": "carried out on fairly large samples" in $d = 2, 3$ for Ising spins (± 1) [with 1977 computing power]. Are there many ground states or only one (up to global spin reversal)? Evidence in $d = 2$ suggests signed graphs ("± 1") are quite different from randomly weighted signed graphs ("Gaussian"). Signed-graph behavior differs for very low vs. middling density of negative edges; there seem to be more ground states at middling density. There seem to be fewer ground states in $d = 3$ than $d = 2$ (p. 4637). Discussion of expected behavior of low-frustration states; a remarkable planar example in Fig. 14. Dictionary: "bond" = edge; "state" = $\zeta : V \to \{+1, -1\}$; frustrated bond = $\sigma^\zeta(e) = -1$; frustration = $\#(\sigma^\zeta)^{-1}(-1)$; "ground state" = switching with min frustration = $l(\Sigma)$; "degeneracy" = multiple states with same amount of frustration. [Annot. 22 Jan 2015.] (**Phys: SG, State(fr), Sw**)

Scott Kirkpatrick & David Sherrington

† 1978a Infinite-ranged models of spin-glasses. *Phys. Rev. B* 17 (1978), no. 11, 4384–4403. Repr. in M. Mézard, G. Parisi, and M.A. Virasoro, *Spin Glass Theory and Beyond*, pp. 109–128. World Scientific Lect. Notes in Physics, Vol. 9. World Scientific, Singapore, 1987.

Random edge weights and signs on K_n with vertex signs ± 1. Most interesting: § VI, "Statics for $T \neq 0$", where the "energy" (frustration index $l(\Sigma)$) landscape of random signs is described, based on computer experiments, as consisting of deep valleys, each having several local minima of l separated by slightly higher ridges, and with high-l barriers separating the valleys. [Presumably, the distance function is Hamming distance between reduced sign functions, i.e., those with $E^- = l$.] [A seminal successor to Edwards and Anderson (1975a). This picture, while convincing, has never been proved; it remains an object of intense curiosity. *Cf.* Marvel, Kleinberg, Kleinberg, and Strogatz (2011a), (2011b).] [Annot. 22 Aug 2012, 23 Jan 2015.] (**Phys: sg: Fr(State)**)

Nanao Kita

2020a Signed analogue of general Kotzig–Lovász decomposition. *Discrete Appl. Math.* 284 (2020), 61–70. MR 4115457. Zbl 1443.05085. arXiv:1709.07414. (**SG: Ori: Str**)

20xxb Bidirected graph II: Extension of basilica order. In preparation. (**SG: Ori: Str**)

20xxc Bidirected graph III: Algorithms for basilica decomposition. In preparation.

(**SG: Ori: Str**)

Ouail Kitouni & Nathan Reff

2019a Lower bounds for the Laplacian spectral radius of an oriented hypergraph. *Australasian J. Combin.* 74 (2019), no. 3, 408–422. MR 3969739. Zbl 1419.05127.
(**SH: Lap: Eig**)

Teeradej Kittipassorn & Gábor Mészáros

† 2015a Frustrated triangles. *Discrete Math.* 338 (2015), no. 12, 2363–2373. MR 3373339. Zbl 1318.05019. arXiv:1411.1749.

Thorough study of the number c_3^- of negative triangles in a signed K_n. Two-thirds of the numbers from 0 to $\binom{n}{3}$ cannot be $c_3^-(K_n,\sigma)$. Some numbers that are, are $0 = a_0 \leq b_0 \leq a_1 \leq \cdots \leq a_m \leq b_m \approx n^{3/2}$ where $b_i = a_i + i(i-1)$ and $a_{i+1} = b_i + (n-2) - i(i+1)$. For $i \leq m$, $c_3^-(K_n,\sigma) \in [a_i, b_i]$ iff $l(K_n,\sigma) = i$. $\exists\, f(n)$ such that if $n \gg 0$, all $j \in [f(n), \binom{n}{3} - f(n)]$ are $c_3^-(K_n,\sigma)$'s. Etc. [For other studies of negative circles *cf.* Tomescu (1976a), Popescu and Tomescu (1996a), Antal, Krapivsky, and Redner (2005a), Schaefer and Zaslavsky (2019a).] [Annot. 26 Sept 2015, 6 Jan 2017.]
(**sg: Fr: Circ**)

Anatoly Kiyatkin

See B.N. Kholodenko.

Ralf Klamma

See M. Shahriari.

Steffen Klamt

See also I.N. Melas.

Steffen Klamt, Julio Saez-Rodriguez, Jonathan A. Lindquist, Luca Simeoni, & Ernst D. Gilles

2006a A methodology for the structural and functional analysis of signaling and regulatory networks. *BMC Bioinformatics* 7 (2006), art. 56, 26 pp.
(**Biol, SD, SH: Algor**)

Steffen Klamt & Axel von Kamp

2009a Computing paths and cycles in biological interaction graphs. *BMC Bioinformatics* 10 (2009), art. 181, 11 pp. + 2 supplements.

Interaction graph: a signed digraph.
(**SD Dyn: Algor(Paths, Cyc), Biol**)

Victor Klee

See also C. Jeffries.

1971a The greedy algorithm for finitary and cofinitary matroids. In: Theodore S. Motzkin, ed., *Combinatorics*, pp. 137–152. Proc. Sympos. Pure Math., Vol. 19. Amer. Math. Soc., Providence, R.I., 1971. MR 0332538 (48 #10865). Zbl 229.05031.

Along with Simões-Pereira (1972a), invents the bicircular matroid (here, for infinite graphs).
(**Bic**)

1989a Sign-patterns and stability. In: Fred Roberts, ed., *Applications of Combinatorics and Graph Theory to the Biological and Social Sciences*, pp. 203–219. IMA Vols. Math. Appl., Vol. 17. Springer-Verlag, New York, 1989. MR 1009377 (90h:34081). Zbl 747.05057.

When are various forms of stability of a linear differential equation $\dot{x} = Ax$ determined solely by the sign pattern of A? A survey of elegant combinatorial criteria. Signed digraphs [alas] play but a minor role. (**QM: QSta, SD: Exp, Ref**)

1993a Open Problem 2. In: Richard A. Brualdi, Shmuel Friedland, and Victor Klee, eds., *Combinatorial and Graph-Theoretical Problems in Linear Algebra*, p. 257. IMA Vols. Math. Appl., Vol. 50. Springer-Verlag, New York, 1993. MR 1240954 (94d:00012) (book). Zbl 780.00017 (book).

A question about sign solvability that generalizes "the infamous even cycle problem." [Annot. 13 Apr 2009.] (**QM: sd: QSol, QSta**)

Victor Klee, Richard Ladner, & Rachel Manber

1984a Signsolvability revisited. *Linear Algebra Appl.* 59 (1984), 131–157. MR 0743051 (86a:15004). Zbl 543.15016. (**SD, QM: QSol, Algor**)

Victor Klee & Pauline van den Driessche

1977a Linear algorithms for testing the sign stability of a matrix and for finding Z-maximum matchings in acyclic graphs. *Numer. Math* 28 (1977), 273–285. MR 1553991 (no rev). Zbl 348.65032, (Zbl 352.65020). (**SD: QM, QSta, Algor**)

Sulamita Klein

See L. Faria.

Jon M. Kleinberg

See D. Easley, J. Leskovec, and S.A. Marvel.

Robert D. Kleinberg

See S.A. Marvel.

Peter Kleinschmidt & Shmuel Onn

1995a Oriented matroid polytopes and polyhedral fans are signable. In: Egon Balas and Jens Clausen, eds., *Integer Programming and Combinatorial Optimization* (4th Int. IPCO Conf., Copenhagen, 1995, Proc.), pp. 198–211. Lect. Notes in Computer Sci., Vol. 920. Springer, Berlin, 1995. MR 1367982 (97b:05040).

In a graded partially ordered set with 0 and 1, assign a sign to each covering pair (x, y) where y is covered by 1. This is an "exact signing" if in every upper interval there is just one y whose coverings are all positive. Then the poset is "signable". (**Sgnd: Geom**)

1996a Signable posets and partitionable simplicial complexes. *Discrete Comput. Geom.* 15 (1996), 443–466. MR 1384886 (97a:52014). Zbl 853.52010.

See (1995a) for definition. Signability is a generalization to posets of partitionability of a simplicial complex (Prop. 3.1). Shellable posets, and face lattices of spherical polytopes and oriented matroid polytopes, are signable. A stronger property of a simplicial complex, "total signability", which applies for instance to simplicial oriented matroid polytopes (Thm. 5.12), implies the upper bound property (Thm. 4.4). Computational complexity of face counting and of deciding shellability and partitionability are discussed in §6. (**Sgnd: Geom, Algor**)

Joseph B. Klerlein

See also R.L. Hemminger.

1975a Characterizing line dipseudographs. In: F. Hoffman *et al.*, eds., *Proceedings of the Sixth Southeastern Conference on Combinatorics, Graph Theory and Computing* (Boca Raton, 1975), pp. 429–442. Congressus Numerantium, XIV. Utilitas Math. Publ. Inc., Winnipeg, Man., 1975. MR 0396322 (53 #190). Zbl 325.05106.

Continues the topic of Hemminger and Kerlein (1979a). (**sg: LG, ori**)

Joerg Kliewer

See C.A. Kelley.

Darwin Klingman

See J. Elam, F. Glover, and J. Hultz.

Elizabeth Klipsch

20xxa Some signed graphs that are forbidden link minors for orientation embedding. Manuscript.

For each $n \geq 5$, either $-K_n$ or its 1-edge deletion, but not both, is a forbidden link minor. Which one it is, is controlled by Euler's polyhedral formula, provided $n \geq 7$. [A long version with excruciating detail is available.] (**SG: Top, Par**)

Ton Kloks, Haiko Müller, & Kristina Vušković

2009a Even-hole-free graphs that do not contain diamonds: A structure theorem and its consequences. *J. Combin. Theory Ser. B* 99 (2009), 733–800. MR 2522592 (2010j:05345). Zbl 1218.05160.

A decomposition theorem for graphs without induced even circles and $K_4 \setminus e$'s. [*Question.* Does it make sense to generalize to signed graphs without chordless balanced circles (longer than 3?) or $[K_4 \setminus e]$'s?] [Annot. 10 Mar 2011.] (**par: Str**)

T. Klotz

See also J.F. Valdés and E.E. Vogel.

T. Klotz & S. Kobe

1994a 'Valley structures' in the phase space of a finite 3D Ising spin glass with $\pm I$ interactions. *J. Phys. A* 27 (1994), L95–L100. MR 1269034 (no rev).

The energy (i.e., $\#E^-(\Sigma^\zeta)$) landscape of switchings of a signed graph, the underlying graph being a cubic lattice. [Annot. 4 Jan 2015.] (**SG: State(fr), Sw, Phys**)

Kolja Knauer

See S. Felsner.

Oliver Knill

2017a On the arithmetic of graphs. Manuscript, 2017. arXiv:1706.05767.

"Signed graph" = formal difference $G_1 - G_2$, roughly equivalent to defining $\Sigma = \Sigma^+ - \Sigma^-$. (**SG: Algeb**)

2019a More on numbers and graphs. Manuscript, 2019. arXiv:1905.13387. (**SG: Algeb**)

2021a Remarks about the arithmetic of graphs. Manuscript, 2021. arXiv:2106.10093. (**SG: Algeb**)

Klaus Knorr

See J.D. Noh.

Andrew Knyazev

2018a On spectral partitioning of signed graphs. In: Fredrik Manne, Peter Sanders, and Sivan Toledo, eds.,*2018 Proceedings of the SIAM Workshop on Combinatorial Scientific Computing* (CSC18, Bergen, Norway, 2018), pp. 11–22. SIAM ePub, 2018. arXiv:1701.01394.

Spectral clustering bipartitions V according to signs in an eigenvector of min eigenvalue of Laplacian. Argues that for Σ, Laplacian $L(\Sigma) = \mathrm{diag}(\deg) - A(\Sigma)$ is better than "signed Laplacian" $D - A(\Sigma)$ where D has large positive diagonal as in Kunegis, Schmidt, *et al.* (2010a). [Annot. 8 Feb 2021.] (**SG: Clu: Lap**)

Lori Koban [Lori Fern]

See also L. Fern.

2004a Comments on "Supersolvable frame-matroid and graphic-lift lattices" by T. Zaslavsky. *European J. Combin.* 25 (2004), 141–144. MR 2031808 (2004k:05054). Zbl 1031.05032.

Correction to Thm. 2.1; improved (and corrected) proof of Thm. 2.2 of Zaslavsky (2001a). (**GG: Matrd**)

2004b *Two Generalizations of Biased Graph Theory: Circuit Signatures and Modular Triples of Matroids, and Biased Expansions of Biased Graphs.* Doctoral dissertation, State University of New York at Binghamton, 2004. MR 2706325 (no rev).

Ch. 1: "Circuit signatures and modular triples." When can gains be applied to matroids, as they are to graphs in Zaslavsky (1991a), to produce a linear class of circuits and hence a lift matroid? Theorem 1.4.1: When the group has exponent > 2, one needs a ternary circuit signature, thus a ternary matroid. Theorem 1.4.5: When the group has exponent 2 the matroid must be binary (no circuit signature is required). (**Matrd: GG: Gen**)

Ch. 2: "Biased expansions of biased graphs." Generalizes group and biased expansions of a graph and the chromatic (and bias-matroid characteristic) polynomial formulas (Zaslavsky (1995b), (20xxj)) to expansions of a biased graph. Ch. 3: "When are biased expansions actually group expansions?" Partial results about characterizing biased expansions of biased graphs that are group expansions; counterexamples to several plausible conjectures. (**GG: Matrd, Invar, Geom**)

2008a A modular triple characterization of circuit signatures. *European J. Combin.* 29 (2008), no. 1, 159–170. MR 2368623 (2008k:05040). Zbl 1127.05021.

Four kinds of circuit signatures of a matroid can be characterized through modular triples of copoints or circuits. They are lift signatures as well as the previously known weak orientations, orientations, and ternary signatures. Lifting signatures are needed to define a matroid with gains and thereby a lift matroid determined by the gains. (**GG: Gen, Matrd**)

2012a Biased expansions of biased graphs and their chromatic polynomials. *Ann. Combin.* 16 (2012), no. 4, 781–788. MR 3000445.

Generalizes group and biased expansions of a graph (Zaslavsky (1995b), Ex. 3.8; Zaslavsky (2001a), Ex. 4.1; Zaslavsky (20xxj)) to biased expansions of a biased graph. The definition is similar but tricky. The chromatic polynomials follow similar formulas. [Annot. 20 Oct 2012.]
(**GG: Invar, Matrd**)

Yusuke Kobayashi
See K. Kawarabayashi and Y. Kawase.

S. Kobe
See T. Klotz, J.F. Valdés, and E.E. Vogel.

William Kocay & Douglas Stone

1993a Balanced network flows. *Bull. Inst. Combin. Appl.* 7 (1993), 17–32. MR 1206759 (93j:05148). Zbl 804.05057.
Balanced network = signed covering graph of $-\Gamma$ with edges vw lifted to arcs $(+v, -w)$, and with added source and sink. [Annot. 8 Mar 2011.]
(**sg: cov**)

1995a An algorithm for balanced flows. *J. Combin. Math. Combin. Comput.* 19 (1995), 3–31. MR 1358494 (96j:90087). Zbl 841.68098.
Continuation of (1993a). [Annot. 8 Mar 2011.] (**sg: cov: Algor**)

Muralidharan Kodialam & James B. Orlin

1991a Recognizing strong connectivity in (dynamic) periodic graphs and its relation to integer programming. In: *Proceedings of the Second Annual ACM-SIAM Symposium on Discrete Algorithms* (San Francisco, 1991), pp. 131–135. Assoc. for Computing Machinery, New York, 1991. Zbl 800.68639.
Linear programming methods to find the strongly connected components of a periodic digraph from the static graph: i.e., of the covering digraph of a gain digraph Φ with gains in $\mathbb{Q}^d$ by looking at Φ. *Cf.* Cohen and Megiddo (1993a), whose goals are similar but algorithms differ.
(**GD(Cov): Bal, Circ: Algor**)

Vijay Kodiyalam, R. Srinivasan, & V.S. Sunder

2000a The algebra of G-relations. *Proc. Indian Acad. Sci., Math. Sci.* 110 (2000), no. 3, 263–292. MR 1781906 (2001k:16019) (*q.v.*). Zbl 992.16015.
(**gg: Algeb, matrd**)

Yoshitaka Koga, Tatsuya Maruta, & Keisuke Shiromoto

2018a On critical exponents of Dowling matroids. *Designs Codes Cryptogr.* 86 (2018), 1947–1962. MR 3816208. Zbl 1394.05014.
Higher-weight Dowling matroids, *cf.* Kung (1996b), [Ravagni (2022a)]. Connections to linear codes. Thm. 17 and §4: Bounds for certain cases. [Annot. 17 Nov 2022.] (**Matrd: Invar, gg(Gen)**)

Shungo Koichi

2014a The Buneman index via polyhedral split decomposition. *Adv. Appl. Math.* 60 (2014), 1–24. MR 3256746. Zbl 1300.05053.
§§4 2–3: Signed partial partitions (treated as sign-symmetric partitions of $\pm[n] \cup \{0\}$). Two sets of signed partial partitions are equivalent if one is converted to the other by switching in $\pm K_n^\bullet$. A “signed bipartition” is

a signed partial partition with one block (that is, ignoring the 0 block). [Annot. 28 Jan 2015.] (**sg: Matrd**)

Johan Kok

See N.K. Sudev.

Tamara Koledin

See also M. Anđelić.

Tamara Koledin & Zoran Stanić

2017a Connected signed graphs of fixed order, size, and number of negative edges with maximal index. *Linear Multilinear Algebra* 65 (2017), no. 11, 2187–2198. MR 3740690.

Consider all Σ with simple, connected $|\Sigma|$ and fixed values $|V| = n$, $|E| = m$, $|E^-| = k$. Thm. 4.1: With some exceptions, if $\lambda_{\max}(\Sigma_0) = \max\{\lambda_{\max}(\Sigma) : \forall\ \Sigma\}$, then Σ_0^- is a bipartite chain graph and each $\Sigma_0^+{:}U_i$ is a threshold graph. [Annot. 6 Apr 2021.] (**SG: Adj: Eig: Str**)

2020a On a class of strongly regular signed graphs. *Publ. Math. Debrecen* 97 (2020), no. 3-4, 353–365. MR 4194066.

Cf. Stanić (2019d). All SRSG are sorted into 5 classes; Class 3 is where $c = \frac{1}{2}(a + b) \neq 0$. Thms. 4.1–2: Σ in Class 3 is net regular. $A(\Sigma)$ has exactly 3 eigenvalues. The net-regularity degree $d^{\pm}$ is a simple eigenvalue. For the other eigenvalues, $\lambda^2 + \frac{1}{2}(b - a)\lambda + \frac{1}{2}(a + b) = r$. Similarities to strongly regular graphs, e.g., Thm. 4.4: A connected, regular, net-regular, inhomogeneous signed graph with 3 eigenvalues, where $d^{\pm}$ is a simple eigenvalue, is strongly regular. Thm. 4.8: Σ in Class 3 with $d^{\pm} = 0$ is complete with $a + b = -2$. Thm. 4.9: If Σ is in Class 3 and the eigenvalues $\neq d^{\pm}$ have equal multiplicity, then Σ is complete, $d^{\pm} = 0$, and $a = b = -1$. Thm. 5.2. Σ in Class 3 with $r \leq 10$ is complete, with one possible exception. Two incomplete examples are constructed from 3-class Johnson schemes. [Annot. 4 Apr 2021.] (**SG: Adj: Eig**)

2022a Notes on Johnson and Hamming signed graphs. *Bull. Math. Soc. Sci. Math. Roumanie* 65 (113) (2022), no. 3, 303–315. MR 4488335.

Strongly regular Σ (*cf.* Stanić (2019d)) related to association schemes. Σ based on Johnson and Hamming graphs. NASC for strong regularity. The first known strongly regular Σ with > 4 eigenvalues. [Annot. 19 Sept 2022.] (**SG: Adj: Eig**)

Brett Kolesnik

See also M. Buckland.

Brett Kolesnik, Rivka Mitchell, & Tomasz Przybyłowski

20xxa Tournaments on signed graphs. Submitted. arXiv:2312.04532.

Sequel: Buckland, Kolesnik, Mitchell, and Przybyłowski (20xxa). (**SG: Invar**)

Alexandra Kolla

See C. Carlson.

János Komlós

1997a Covering odd cycles. *Combinatorica* 17 (1997), 393–400. MR 1606044 (99b:-05114). Zbl 902.05036.

Sharp asymptotic upper bounds on frustration index and vertex frustration number for all-negative signed graphs with fixed negative girth. Improves Bollobás, Erdős, Simonovits, & Szemerédi (1978a). [*Problem.* Generalize to arbitrary signed graphs or signed simple graphs.]

(**Par: Fr**)

Helene J. Kommel

See Harary and Kommel (1978a).

A. Kompišová & E. Máčajová

2019a Flow number and circular flow number of signed cubic graphs. *Acta Math. Univ. Comenianae (N.S.)* 88 (2019), no. 3, 877–883. MR 4014147.

(**SG: Flows: Invar**)

2022a Flow number and circular flow number of signed cubic graphs. *Discrete Math.* 345 (2022), no. 8, art. 112917, 12 pp. MR 4409974. Zbl 1490.05104.

(**SG: Flows: Invar**)

Christian Komusiewicz

See F. Hüffner.

Dénes König

1936a *Theorie der endlichen und unendlichen Graphen: Kombinatorische Topologie der Streckenkomplexe.* Mathematik und ihre Anwendungen, Band 16. Akademische Verlagsges., Leipzig, 1936. Repr.: Chelsea, New York, 1950. MR 0036989 (12, 195). Zbl 013.22803 (13, p. 228c).

§ X.3, "Komposition von Büsheln", contains Thms. 9–16 of Ch. X. I restate them in terms of a signature on the edge set; König says subgraph or p-subgraph ("p-Teilgraph") to mean what we would call the negative edge set of a signature or a balanced signature. Instead of signed switching, König speaks of set summation ("composition") with a vertex star ("Büschel"). His theorems apply to finite and infinite graphs except where stated otherwise. Thm. 9: The edgewise product of balanced signatures is balanced. Thm. 10: Every balanced signing of a finite graph is a switching of the all-positive signature. Thm. 11: A signature is balanced iff it has a Harary bipartition [see Harary (1953a)]. Thm. 12 (cor. of 11): A graph is bicolorable iff every circle has even length. [König makes this fundamental theorem a corollary of a signed-graph theorem!] Thm. 13: A signature is balanced if (not "only if", but that is obvious) every circle of a fundamental system is positive. Thm. 14: A graph with n vertices (a finite number) and c components has 2^{n-c} balanced signings. Thm. 16: The set of all vertex switchings, except for one in each finite component of Γ, forms a basis for the space of all finitely generated switchings.

[Switching is born here but not recognized until reinvented. König's one failing was not to see the role of edge and circle signs; thus, I regard Harary (1953a) as the true invention of signed graphs.] [Annot. rev. 26 Aug 2018.]

(**sg: Bal, sw, Enum**)

1986a *Theorie der endlichen und unendlichen Graphen. Mit einer Abhandlung von L. Euler* Ed. and introd. by H. Sachs, introd. by P. Erdos, biographical essay by T. Gallai [in English]. Teubner-Archiv zur Math., 6. BSB B. G. Teubner, Leipzig, 1986. MR 0886676 (88i:01168). Zbl 608.05002.

Reprint of (1936a) together with Euler's paper (in Latin and German) on the Königsberg bridges and supplementary material. **(sg: Bal, sw, Enum)**

1990a *Theory of Finite and Infinite Graphs.* Transl. Richard McCoart, commentary by W.T. Tutte, biographical sketch by T. Gallai. Birkhäuser, Boston, 1990. MR 1035708 (91f:01026). Zbl 695.05015.

English translation of (1936a). § X.3: "Composition of stars". ["Kreis" (circle, meaning circle) is unfortunately translated as "cycle"—one of the innumerable meanings of "cycle".] **(sg: Bal, sw, Enum)**

Alexander Kononov

See V. Il'ev.

Jack [Jacobus] H. Koolen

See S. Akbari, M.-Y. Cao, T.Y. Chung, G. Greaves, and H.J. Jang.

Justin Koonin

2014a Topology of eigenspace posets for imprimitive reflection groups. *J. Combin. Theory Ser. A* 127 (2014), 121–148. MR 3252658. Zbl 1301.06010. arXiv:-1208.4435.

An eigenspace poset is described in terms of "d-divisible, k-evenly colored Dowling lattices", which are subposets of Dowling lattices. [Annot. 12 Jul 2016.] **(gg: Matrd, Geom, Gen)**

Evgeniia Korchevskaia

See T.-R. Chen.

Hideo Kosako

See also S.J. Moon.

Hideo Kosako, Suck Joong Moon, Katsumi Harashima, & Takeo Ikai

1993a Variable-signed graph. *Bull. Univ. Osaka Pref. Ser. A* 42 (1993), 37–49. MR 1287466 (96e:05167). Zbl 798.05070.

"Variable-signed graph" = signed simple (di)graph Σ with switching function p and switched graph Σ^p. Known basic properties of switching are established. More interesting: planar duality when $|\Sigma|$ is planar. The planar dual $|\Sigma|*$ inherits the same edge signs; a dual vertex has sign of the surrounding primal face boundary. Property 9 is in effect the statements: (1) If a signed plane graph has f negative face boundaries, then $l(\Sigma) \geq f/2$. (2) If the negative faces fall into two connected groups with oddly many faces in each, (1) can be improved to $\geq f/2+1$. Finally, incidence matrices are studied that are only superficially related to signs. [The paper is hard to interpret due to mathematical imprecision and language difficulty.] **(SG: Sw, fr, Du, Incid)**

Justyna Kosakowska

See also M. Kaniecki.

2012a Inflation algorithms for positive and principal edge-bipartite graphs and unit quadratic forms. *Fundamenta Inform.* 119 (2012), no. 2, 149–162. MR 2977486. **(SG)**

George E. Kostakis

See K.C. Mondal.

Alexandr V. Kostochka

See A.A. Ageev and E. Györi.

Sven Kosub

See T. Akutsu.

Balázs Kotnyek

See also G. Appa and L.S. Pitsoulis.

2002a *A Generalization of Totally Unimodular and Network Matrices*. Doctoral thesis, London School of Economics, 2002.

Introducing binet matrices; *cf.* Appa and Kotnyek (2006a). A binet matrix is $A = \mathrm{H}B^{-1}\mathrm{H}(\mathrm{B})$ where B is a bidirected graph (which may be assumed to have no balanced components) and B is a basis for $\mathbf{F}(\Sigma(\mathrm{B}))$. *Problem*: To recognize a binet matrix. Thm.: If an $n \times m$ matrix A is an indecomposable binet matrix, then at most one component of B has no half edge (and the remaining component has a negative circle). [Further work in Appa and Kotnyek (2006a), Musitelli (2007a), (2007a).] [Annot. 15 January 2013.] (**SG: Ori: Incid, Algor**)

Manoshi Kotoky

See A.K. Baruah.

Jomon Kottarathil

2023a On pseudo-balancing of path-induced signed graphs. *Jordan J. Math. Stat.* 16 (2023), no. 3, 431–443.

Cf. Kottarathil and Naduvath (20xxa). $\Sigma_p(K_{mn})$ is homogeneous; $\Sigma_p(W_n)$ is balanced. "Pseudo-balance" means $|E^-|$ is even. [Not similar to balance.] Balanced Σ_p is pseudo-balanced. $\Sigma_p(K_n)$ is pseudo-balanced iff $n \not\equiv 1 \pmod 4$. If Γ is even-regular incomplete, Σ_p is pseudo-balanced. Minor results for Eulerian Γ. [Annot. 13 Oct 2023.] (**SG**)

Jomon Kottarathil & Sudev Naduvath

20xxa On path-induced signed graphs. Submitted.

For a path decomposition of Γ let $V_p = \{\text{endpoints of paths}\}$. The "path-induced signed graph" Σ_p has $E^+ := E{:}V_p$ [where $|V_p|$ is min?]. If Γ is odd-regular, Σ_p is balanced. [Annot. 13 Oct 2023.] (**SG: Bal**)

A. Kotzig

1968a Moves without forbidden transitions in a graph. *Mat. Časopis* 18 (1968), 76–80. MR 0242709 (39 #4038). Zbl 155.31901 (155, p. 319a). (**par: ori**)

Paulina Koutsaki

See J.C. Bronski.

István Kovács, Aleksander Malnič, Dragan Marušič, & Štefko Miklavič

2009a One-matching bi-Cayley graphs over abelian groups. *European J. Combin.* 30 (2009), 602–616. MR 2489254 (2010e:05247). Zbl 1204.05079.

Uses gain graphs ("voltage graphs") to construct graphs with certain kinds of automorphisms. [Annot. 28 Mar 2017.] (**GG: Cov: Algeb**)

Vladislav B. Kovchegov

1984a Markov's model of relations in small group and analysis of group structures. (In Russian.) In: *Mathematical Methods in Sociological Researches*. Inst. of

Sociology, Moscow, 1984.
See (1994a). **(SG: WG, Adj, Bal, Clu)**

1989a Balance and maximum unergodicity hypothesis for institutions. (In Russian.) In: *Mathematical Modelling of Social Processes*. Acad. of Social Sciences, Moscow, 1989.
See (1994a). **(SG: WG, Adj, Bal, Clu)**

1992a Modeling of human institutions by network of automatons with relations. In: *Proceedings of the Eleventh European Meeting of Cybernetics and Systems Research* (Vienna, 1992), pp. 989–995.
See (1994a). **(SG: WG, Adj, Bal, Clu)**

1994a A model of dynamics of group structures of human institutions. *J. Math. Sociology* 18 (1994), no. 4, 315–332. MR 1262516 (no rev). Zbl 829.92028.
A "model of the institution with relations" consists of a loopless digraph $D = (V, A)$ with $V = \{1, 2, \ldots, n\}$, a group $\mathfrak{G}$, sets X and Y, and functions $f : A \to Y$, $z_i : Y \to \mathcal{P}(\mathfrak{G})$ $\forall i \in V$. We consider $r : A \to \mathfrak{G}$ such that $r(i, j) \in z_i(f(i, j))$. [That is, (D, r) is a gain digraph with gain group $\mathfrak{G}$. The edges are colored by f and the gains are constrained by the list $z_i(y)$ for each vertex i and color y.] [Annot. 24 Nov 2012.]
(SG: WG, Adj, Bal, Clu, Geom)

Robin Koytcheff
See E. Ziv.

David Krackhardt
See P. Doreian.

Thomas Krajewski, Iain Moffatt, & Adrian Tanasa

2015a An extension of the Bollobás-Riordan polynomial for vertex partitioned ribbon graphs: definition and universality. EuroComb 2015 (Bergen, 2015). *Electronic Notes Discrete Math.* 49 (2015), 285–292. Zbl 1346.05124. **(sg: Top: Invar)**

2018a Hopf algebras and Tutte polynomials. *Adv. Appl. Math.* 95 (2018), 271–330. MR 3759218. Zbl 1379.05057. arXiv:1508.00814.
Surface-embedded signed graphs ("ribbon graphs") are a main example. [Annot. 17 Apr 2019.] **(sg: Top: Invar)**

Thomas Krajewski, Vincent Rivasseau, & Fabien Vignes-Tourneret

2011a Topological graph polynomial and quantum field theory. Part II: Mehler kernel theories. *Ann. Henri Poincaré* 12 (2011), 483–545. MR 2785137 (2012m:81060). Zbl 1217.81128. arXiv:0912.5438. **(sg: Top: Invar)**

Daniel Král', Jean-Sébastien Sereni, & Ladislav Stacho

2012a Min-max relations for odd cycles in planar graphs. *SIAM J. Discrete Math.* 26 (2012), no. 3, 884–895. MR 3022112. Zbl 1256.05119. arXiv:1108.4281.
$\nu :=$ max number of vertex-disjoint negative circles. The vertex frustration number $l_0(\Sigma) \le 6\nu(\Sigma)$ for planar $|\Sigma|$, improving on Fiorini, Hardy, Reed, and Vetta (2005a), (2007a). Dictionary: "odd" = negative, "even" = positive, "transversal" = $X \subseteq V$ such that $\Sigma \setminus X$ is balanced. [Annot. 1 Oct 2012, rev 14 Jan 2017.] **(sg: par: Fr, Circ)**

Daniel Král' & Heinz-Jürgen Voss

2004a Edge-disjoint odd cycles in planar graphs. *J. Combin. Theory Ser. B* 90 (2004), 107–120. MR 2041320 (2005d:05089). Zbl 1033.05064.

Thm. 1: For a signed plane graph Γ, the frustration index $l(\Sigma) \leq 2\nu'$, where $\nu' :=$ maximum number of edge-disjoint negative circles. Dictionary: "odd" = negative, "even" = positive. [Continued in Fiorini, Hardy, Reed, and Vetta (2007a), Thm. 3.] [Annot. 6 Feb 2011.]

(**sg: Par: Fr, Circ**)

Mark A. Kramer

See also O.O. Oyeleye.

M.A. Kramer & B.L. Palowitch, Jr.

1987a A rule-based approach to fault diagnosis using the signed directed graph. *AIChE J.* 33 (1987), 1067–1078. MR 0895873 (88j:94060).

Vertex signs indicate directions of change in vertex variables; signed directed edges describe relations among these directions.

Truth tables for a signed edge as a function of endpoint signs. Algorithms for deducing logical rules about states (assignments of vertex signs) from the signed digraph. Has a useful discussion of previous literature, e.g., Iri, Aoki, O'Shima, and Matsuyama (1979a).

(**SD, VS: Appl, Algor, Ref**)

P.L. Krapivsky

See T. Antal.

I. Krasikov

1988a A note on the vertex-switching reconstruction. *Int. J. Math. Math. Sci.* 11 (1988), 825–827. MR 0959466 (89i:05204). Zbl 663.05046.

Following up Stanley (1985a), a signed K_n is reconstructible from its single-vertex switching deck if its negative subgraph is disconnected [therefore also if its positive subgraph is disconnected] or if the minimum degree of its positive or negative subgraph is sufficiently small. All done in terms of Seidel switching of unsigned simple graphs. (**kg: sw, TG**)

1994a Applications of balance equations to vertex switching reconstruction. *J. Graph Theory* 18 (1994), 217–225. MR 1268771 (95d:05091). Zbl 798.05039.

Following up Krasikov and Roditty (1987a), (K_n, σ) is reconstructible from its s-vertex switching deck if $s = \frac{1}{2}n - r$ where $r \in \{0, 2\}$ and $r \equiv n \pmod 4$, or $r = 1 \equiv n \pmod 2$; also, if $s = 2$ and the minimum degree of the positive or negative subgraph is sufficiently small. Also, bounds on $\#E^-$ if (K_n, σ) is not reconstructible. Negative-subgraph degree sequence: reconstructible when $s = 2$ and $n \geq 10$. Done in terms of Seidel switching of unsigned simple graphs. (**kg: sw, TG**)

1996a Degree conditions for vertex switching reconstruction. *Discrete Math.* 160 (1996), 273–278. MR 1417580 (97f:05137). Zbl 863.05056.

If the minimum degrees of its positive and negative subgraphs obey certain bounds, a signed K_n is reconstructible from its s-switching deck. The main bound involves the least and greatest even zeros of the Krawtchouk polynomial $K_s^n(x)$. Done in terms of Seidel switching of unsigned simple graphs. [More details in Zbl.] (**kg: sw, TG**)

Ilia Krasikov & Simon Litsyn

1996a On integral zeros of Krawtchouk polynomials. *J. Combin. Theory Ser. A* 74 (1996), 71–99. MR 1383506 (97i:33005). Zbl 853.33008.

Among the applications mentioned (pp. 72–73): 2. "Switching reconstruction problem", i.e., graph-switching reconstruction as in Stanley (1985a) etc. 4. "Sign reconstruction problem", i.e., reconstructing a signed graph from its s-edge negation deck, which is the multiset of signed graphs obtained by separately negating each subset of s edges (here called "switching signs", but it is not signed-graph switching); this is a new problem. (**kg: sw, TG**)(**SG**)

I. Krasikov & Y. Roditty

1987a Balance equations for reconstruction problems. *Arch. Math. (Basel)* 48 (1987), 458–464. MR 0888875 (88g:05096). Zbl 594.05049.

§2: "Reconstruction of graphs from vertex switching". Corollary 2.3. If a signed K_n is not reconstructible from its s-vertex switching deck, a certain linear Diophantine system (the "balance equations") has a certain kind of solution. For $s = 1$ the balance equations are equivalent to Stanley's (1985a) theorem; for larger s they may or may not be. All is done in terms of Seidel switching of unsigned simple graphs. [Ellingham and Royle (1992a) note a gap in the proof of Lemma 2.5.] (**kg: sw, TG**)

1992a Switching reconstruction and Diophantine equations. *J. Combin. Theory Ser. B* 54 (1992), 189–195. MR 1152446 (93e:05072). Zbl 702.05062.

Main Theorem. Fix $s \geq 4$. If n is large and (for odd s) not evenly even, every signed K_n is reconstructible from its s-vertex switching deck. Different results hold for $s = 2, 3$. (This is based on and strengthens Stanley (1985a).) Theorems 5 and 6 concern reconstructing subgraph numbers. All done in terms of Seidel switching of unsigned simple graphs. (**kg: sw, TG**)

1994a More on vertex-switching reconstruction. *J. Combin. Theory Ser. B* 60 (1994), 40–55. MR 1256582 (94j:05090). Zbl 794.05092.

Based on (1987a) and strengthening Stanley (1985a): Theorem 7. A signed K_n is reconstructible if the Krawtchouk polynomial $K_s^n(x)$ "has one or two even roots [lying] far from $n/2$" (the precise statement is complicated). Numerous other partial results, e.g., a signed K_n is reconstructible if $s = \frac{1}{2}(n - r)$ where $r = 0, 1, 3,$ or $2, 4, 5, 6$ with side conditions. All is done in terms of Seidel switching of unsigned simple graphs. (**kg: sw, TG**)

Jan Kratochvíl

See also E. Jelínková.

1989a Perfect codes and two-graphs. *Comment. Math. Univ. Carolin.* 30 (1989), no. 4, 755–760. MR 1045906 (91a:05080). Zbl 693.05060.

A two-graph $\mathcal{T}$ has a perfect code if every graph in its switching class has a 1-perfect vertex code (a perfect dominating set). Thm. $\mathcal{T}$ has a perfect code iff one of its graphs is the union of up to 3 disjoint cliques iff $\mathcal{T}$ has no sub-pentagons and no sub-4-cocliques. [Annot. 21 Mar 2011.] (**TG: Sw, dom**)

2003a Complexity of hypergraph coloring and Seidel's switching. In: Hans L. Bod-

laender, ed., *Graph-Theoretic Concepts in Computer Science* (29th Int. Workshop, WG 2003, Elspeet, Neth., 2003), pp. 297–308. Lect. Notes in Computer Sci., Vol. 2880. Springer-Verlag, Berlin, 2003. MR 2080089 (no rev). Zbl 1255.68082.

Results about properties as in Kratochvíl, Nešetřil, & Zýka (1992a). E.g., switchability to a regular graph is NP-complete. [Annot. 21 Mar 2011.] **(TG: Sw)**

Jan Kratochvíl, Jaroslav Nešetřil, & Ondřej Zýka

1992a On the computational complexity of Seidel's switching. In: Jaroslav Nešetřil and Miroslav Fiedler, eds., *Fourth Czechoslovak Symposium on Combinatorics, Graphs and Complexity* (Prachatice, 1990), pp. 161–166. Ann. Discrete Math., Vol. 51. North-Holland, Amsterdam, 1992. MR 1206260 (93j:05156). Zbl 768.68047.

Is a given graph switching isomorphic to a graph with a specified property? (This is Seidel switching of simple graphs.) Depending on the property, this question may be in P or be NP-complete, whether the original property is in P or is NP-complete. Properties: containing a Hamilton path; containing a Hamilton circle; no induced P_2; regularity; etc. Thm. 4.1: Switching isomorphism and graph isomorphism are polynomially equivalent. **(TG: Sw: Algor)**

Stefan Kratsch & Magnus Wahlström

2012a Compression via matroids: a randomized polynomial kernel for odd cycle transversal. In: *Proceedings of the Twenty-Third Annual ACM-SIAM Symposium on Discrete Algorithms* (SODA '12), pp. 94–103. Soc. for Industrial and Appl. Math., Philadelphia, 2012. MR 3205199. **(sg: par: fr: Algor)**

2014a Compression via matroids: a randomized polynomial kernel for odd cycle transversal. *ACM Trans. Algorithms* 10 (2014), no. 4, art. 20, 15 pp. MR 3254508. **(sg: par: fr: Algor)**

M.J. Krawczyk, K. Malarz, B. Kawecka-Magiera, A.Z. Maksymowicz, & K. Kułakowski

2005a Spin-glass properties of an Ising antiferromagnet on the Archimedean $(3,12^2)$ lattice. *Phys. Rev. B* 72 (2005), art. 24445, 5 pp. **(par: State(fr))**

A. Krieger & B. O'Connor

2013a Tutte polynomial of signed graphs and its categorification. Slides from a lecture at Ohio State University, 2013. `https://people.math.osu.edu/chmutov.1/wor-gr-su13/pres.pdf`

Introduces trivariate Tutte polynomial. See Goodall, Litjens, Regts, and Vena (2017a) *et seq.* for further developments. **(SG: Invar)**

Matthias Kriesell

See J. Bang-Jensen.

D.S. Krotov

See also E. Bespalov.

2010a On connection between the switching separability of a graph and its subgraphs. (In Russian.) *Diskretn. Anal. Issled. Oper.* 17 (2010), no. 2, 46–56, 101.

2010b On a connection between the switching separability of a graph and its subgraphs. (English trans.) *J. Appl. Industrial Math.* 5 (2011), no. 2, 240–246. MR 2682089 (2011h:05119). Zbl 1249.05184. arXiv:1104.0003.

Γ is "switching separable" if $\exists$ Seidel switching that is nontrivially disconnected. (Trivially: all vertices but one are connected.) Thm.: If all $\Gamma \setminus v$ and $\Gamma \setminus \{u, v\}$ are, then Γ is. Deleting only single vertices is insufficient, for odd $n > 4$. [Annot. 31 Jul 2018.] (**tg: Sw: Str**)

Uffe Krusenstjerna-Hastrøm & Bjarne Toft

1980a Special subdivisions of K_4 and 4-chromatic graphs. *Monatsh. Math.* 89 (1980), no. 2, 101–109. MR 0572886 (81g:05058). Zbl 0421.05025.

Special case of Toft's (1975a) conjecture. (**sg: par: Col**)

Vyacheslav Krushkal

See also P. Fendley.

2011a Graphs, links, and duality on surfaces. *Combin. Prob. Computing* 20 (2011), 267–287. MR 2769192 (2012d:05190). Zbl 1211.05029.

§7, "A multivariate graph polynomial": A partially parametrized rank-generating polynomial ("multivariate Tutte polynomial") for graphs embedded in surfaces, with the somewhat awkward duality relation (7.3). *Cf.* Chmutov and Pak (2007a) and Chmutov (2009a). [Annot. 12 Jan 2012.] (**GGw: Invar**)

F. Krzakala

See J.-P. Bouchaud.

Ying-Qiang Kuang

See Z.H. Chen.

Boris D. Kudryashov

See I.E. Bocharova.

Christian Kuehn

See R. Mulas.

Lukas Kühne & Geva Yashfe

2019a Representability of matroids by c-arrangements is undecidable. Manuscript, 2019, 2020. MR 4526828. arXiv:1912.06123.

§4, "Generalized Dowling geometries": Rank 3 full frame matroids derived from finite group presentations. Representation by matrix gains. Dictionary: "triangle matroid" = rank-3 full frame matroid. *Cf.* (2020a). [Annot. 20 Oct 2020.]

(**gg: Matrd: Geom**)

2020a Undecidability of c-arrangement matroid representations. Proc. 32nd Int. Conf. Formal Power Seri. Algebraic Combin. (Online, 2020). *Sém. Lotharingien Combin.* 84B (2020), art. 87, 12 pp. MR 4138714. Zbl 1447.05049. (**gg: Matrd: Geom**)

Bernard Kujawski, Mark Ludwig, & Peter Abell

2010a Structural balance dynamics and group formation: An exploratory study. Manuscript, 2010. (**SG: Bal**)

Krzysztof Kułakowski

See also P. Gawroński, F. Hassanibesheli, A. Mańka-Krasoń, B. Tadić, and J. Tomkowicz.

2007a Some recent attempts to simulate the Heider balance problem. *Computing in Science and Engineering* 9 (July/Aug. 2007), no. 4, 86–91. arXiv:physics/0612197. (**SG, WG: Bal, Dyn: Exp**)

Krzysztof Kułakowski, Premiysław Gawroński, & Piotr Gronek

2005a The Heider balance: a continuous approach. *Int. J. Mod. Phys. C* 16 (2005), no. 5, 707–716. Zbl 1103.91405. arXiv:physics/0501073. (**SG, WG: Bal, Dyn**)

Devadatta M. Kulkarni

See J.W. Grossman.

Samir Khuller

See S. Ahmadi.

Atul Kumar

See B. Adhikari.

Navish Kumar

See M.R. Kannan.

R. Pradeep Kumar

See M.R. Rajesh Kanna.

Sandeep Kumar

See D. Sinha.

T.R. Vasanth Kumar

See P.S.K. Reddy.

[Vijaya Kumar]

See G.R. Vijayakumar.

[Anita Kumari Rao]

See A.K. Rao (under 'R').

Jérôme Kunegis

2011a *On the Spectral Evolution of Large Networks.* Doctoral dissertation, Univ. Koblenz-Landau, 2011.

Ch. 5, "Signed networks", pp. 79–106.

(**SG, SD: Lap: Bal, Clu, Fr, Eig, WG, Pred, PsS**)

2014a Applications of structural balance in signed social networks. Manuscript, 2014. arXiv:1402.6865.

§3, "Measuring structural balance: The signed clustering coefficient": A new definition; the coefficient for Σ is $3\big(\sum_{C_3\subseteq\Sigma}\sigma(C_3)\big)/\#\{(e,f) : e\sim f\}$. Also defined for signed digraphs. [Annot. 8 Jan 2016.] §4, "Visualizing structural balance: Signed graph drawing": Applies $L(\Sigma)$ to signed-graph drawing. §5, "Capturing structural balance: The signed Laplacian": I.e., $L(\Sigma)$. §5.3, "Balanced graphs": Then $\operatorname{Spec} L(\Sigma) = \operatorname{Spec} L(|\Sigma|)$ and the eigenvectors of Σ are switched (componentwise) from those of $|\Sigma|$. §6, "Measuring structural balance 2: Algebraic conflict": The smallest eigenvalue of $L(\Sigma)$ is dubbed "algebraic conflict" since it is > 0 iff Σ is unbalanced. *Cf.* Hou (2005a). §7, 'Maximizing structural balance: Signed spectral clustering": Uses $L(\Sigma)$, alternatively $D^{-1}A(\Sigma)$ (D = diagonal degree matrix). §8, "Predicting structural balance: Signed resistance distance": A way to compute resistance distance for "signed resistances" = weighted signed edges, with an adapted Kirchhoff's current law. Used for edge prediction.

Partly expository.

(**SG, SD: Lap: Bal, Clu, Fr, Eig, WG, Pred, PsS**)

Jérôme Kunegis, Andreas Lommatzsch, & Christian Bauckhage

2009a The slashdot zoo: mining a social network with negative edges. In: *Proceedings of the 18th International Conference on the World Wide Web* (Madrid, 2009), pp. 741–750. Assoc. for Computing Machinery, New York, 2009.

(**SG: WG: Clu: Algor**)

Jérôme Kunegis, Julia Preusse, & Felix Schwagereit

2013a What is the added value of negative links in online social networks? In: *WWW '13: Proceedings of the 22nd International Conference on World Wide Web* (Rio de Janeiro, 2013), pp. 727–736. ACM, 2013.

A fairly large partial positive subgraph tends to predict negative edges but only by using a combination of centrality and proximity predictors. [Annot. 29 Dec 2020.] (**PsS: SG, Pred: Algor**)

Jérôme Kunegis & Stephan Schmidt

2007a Collaborative filtering using electrical resistance network models with negative edges. In: Petra Perner, ed., *Advances in Data Mining: Theoretical Aspects and Applications* (Proc. 7th Industrial Conf., ICDM 2007, Leipzig), pp. 269–282. Lect. Notes in Computer Sci., Vol. 4597. Springer, Berlin, 2007.

(**sg, WG: Lap**)

Jérôme Kunegis, Stephan Schmidt, Şahin Albayrak, Christian Bauckhage, & Martin Mehlitz

2008a Modeling collaborative similarity with the signed resistance distance kernel. In: Malik Ghallab *et al.*, eds., *ECAI 2008 – 18th European Conference on Artificial Intelligence*, pp. 261–265. Frontiers in Artificial Intelligence and Applications, Vol. 178. IOS Press, Amsterdam, 2008. (**SG: Adj, Algor**)

Jérôme Kunegis, Stephan Schmidt, Andreas Lommatzsch, Jürgen Lerner, Ernesto W. De Luca, & Sahin Albayrak

2010a Spectral analysis of signed graphs for clustering, prediction and visualization. In: Srinivasan Parthasarathy *et al.*, eds., *Proceedings of the Tenth SIAM International Conference on Data Mining* (Columbus, Ohio, 2010), pp. 559–570. Soc. for Industrial and Appl. Math., 2010. (**SG: Eig, Clu, Geom, Pred, Algor**)

Joseph P.S. Kung

See also J.E. Bonin and J. Kahn.

1986a Numerically regular hereditary classes of combinatorial geometries. *Geom. Dedicata* 21 (1986), 85–105. MR 0850567 (87m:05056). Zbl 591.05019.

Examples include Dowling geometries, Ex. (6.2), and the frame matroids of full group expansions of graphs in certain classes; see pp. 98–99.

(**GG: Matrd**)

1986b Radon transforms in combinatorics and lattice theory. In: Ivan Rival, ed., *Combinatorics and Ordered Sets* (Proc., Arcata, Calif., 1985), pp. 33–74. Contemp. Math., Vol. 57. Amer. Math. Soc., Providence, R.I., 1986. MR 0856232 (88d:05024). Zbl 595.05006.

P. 41: Exposition of Stanley (1985a) from the viewpoint of the finite Radon transform. (**kg: sw, TG**)

1987a Research Problem 87. *Discrete Math.* 65 (1987), 105–106.

Conjecture: For every group $\mathfrak{G}$, $\exists\ k = k_{\mathfrak{G}}$ such that if M is a rank-n matroid ($n > k$) where every rank-k interval $[x, \hat{1}] \cong Q_k(\mathfrak{G})$, then $M \subseteq Q_n(\mathfrak{G})$. [This should be provable. k should be small.] [Annot. 9 Apr 1987.] (**gg: Matrd: Str**)

1990a Combinatorial geometries representable over GF(3) and GF(q). I. The number of points. *Discrete Comput. Geom.* 5 (1990), 83–95. MR 1018017 (90i:05028). Zbl 697.51007.

The Dowling geometry over the sign group is the largest simple ternary matroid not containing the "Reid matroid". (**sg: Matrd: Xtreml**)

1990b The long-line graph of a combinatorial geometry. II. Geometries representable over two fields of different characteristic. *J. Combin. Theory Ser. B* 50 (1990), 41–53. MR 1070464 (91m:51007). Zbl 645.05026.

Dowling geometries used in the proof of Prop. (1.2). (**gg: Matrd**)

1993a Extremal matroid theory. In: Neil Robertson and Paul Seymour, eds., *Graph Structure Theory* (Proc., Seattle, 1991), pp. 21–61. Contemp. Math., Vol. 147. Amer. Math. Soc., Providence, R.I., 1993. MR 1224696 (94i:05022). Zbl 791.05018.

Survey with new results; largely on size bounds and extremal matroids for certain minor-closed classes. §2.7: "Gain-graphic matroids," i.e., frame matroids of gain graphs. P. 30, top and fn. 9 on extremal gain-graph theory. §4.3: "Varieties." Conj. (4.9)(c) on growth rates. §4.5, "Framed gain-graphic matroids," i.e., cones over ("framed") frame matroids in projective space. §6.1: "Cones," i.e., unions of long lines on a common point: p. 47. Thm. (6.15) is a quadratic bound on matroids whose minors exclude (approximately) $q + 2$-point lines and non-frame planes. Conj. (7.1) on directions in $\mathbb{C}^n$-matroids proposes that cyclic Dowling matroids are extremal. §8: "Concluding remarks," on a possible ternary analog of Seymour's decomposition theorem.
(**GG: Matrd: Xtreml, Str, Exp, Ref**)

1993b The Radon transforms of a combinatorial geometry. II. Partition lattices. *Adv. Math.* 101 (1993), 114–132. MR 1239455 (95b:05051). Zbl 786.05018.

Dowling lattices are lower-half Sperner. The proof is given only for partition lattices. (**gg: Matrd**)

1996a Matroids. In: M. Hazewinkel, ed., *Handbook of Algebra*, Vol. 1, pp. 157–184. North-Holland (Elsevier), Amsterdam, 1996. MR 1421801 (98c:05040). Zbl 856.05001.

§6.2: "Gain-graphic matroids," i.e., frame matroids of gain graphs.
(**GG: Matrd: Exp**)

† 1996b Critical problems. In: Joseph E. Bonin, James G. Oxley, and Brigitte Servatius, eds., *Matroid Theory* (Proc., Seattle, 1995), pp. 1–127. Contemp. Math., Vol. 197. Amer. Math. Soc., Providence, R.I., 1996. MR 1411690 (97k:05049). Zbl 862.05019.

A remarkable more-than-survey with numerous new results and open problems. §4.5: "Abstract linear functionals in Dowling group geometries". §6: "Dowling geometries and linear codes", concentrates on higher-weight Dowling geometries, extending Bonin (1993b). §7.4: "Critical exponents of classes of gain-graphic geometries". §7.5: "Growth rates of classes of gain-graphic geometries". §8.5: "Jointless Dowling group geometries". Cor. 8.30. §8.11: "Tangential blocks in $\mathcal{Z}(A)$". Also see pp. 56, 61, 88, 92, 114. Dictionary: "Gain-graphic matroids" = frame matroids of gain graphs. **(GG, Gen: Matrd)**

1998a A geometric condition for a hyperplane arrangement to be free. *Adv. Math.* 135 (1998), 303–329. MR 1620842 (2000f:05023). Zbl 905.05017.

Delete from a Dowling geometry a subset S that contains no whole plane. Found: necessary and sufficient conditions for the characteristic polynomial to factor completely over the integers. When the geometry corresponds to a hyperplane arrangement, many more of the arrangements are not free than are free; however, if S contains no whole line, all are free (so the characteristic polynomial factors completely over $\mathbb{Z}$) while many are not supersolvable. **(gg: Matrd: Invar)**

2000a Critical exponents, colines, and projective geometries. *Combin. Probab. Comput.* 9 (2000), 355–362. MR 1786924 (2002f:05048). Zbl 974.51008.

Higher-weight Dowling geometries yield counterexamples to a conjecture. **(gg: Gen: Matrd: Invar)**

2001a Twelve views of matroid theory. In: Sungpyo Hong *et al.*, eds., *Combinatorial & Computational Mathematics* (Proc., Pohang, 2000), pp. 56–96. World Scientific, Singapore, 2001. MR 1868420 (2002i:05028). Zbl 1001.05038.

§5: "Graph theory and lean linear algebra". "Lean" means at most 2 nonzero coordinates, hence gain graphs. §6, "Varieties of finite matroids", summarizes Kahn and Kung (1982a). §7, "Secret-sharing matroids": *Question.* Is the Dowling matroid $Q_n(\mathfrak{G})$ a secret-sharing matroid? **(GG: Matrd)**

§11, "Generic rank-generating polynomials": The "Tugger polynomial" is a partially parametrized rank-generating polynomial (*cf.* Zaslavsky (1992b)). **(Sc(Matrd): Gen: Invar)**

2002a Curious characterizations of projective and affine geometries. Special issue in memory of Rodica Simion. *Adv. Appl. Math.* 28 (2002), 523–543. MR 1900006 (2003c:51008). Zbl 1007.51001.

Dowling geometries $\mathbf{F}(\mathfrak{G}K_n^\bullet)$ (if $\#\mathfrak{G} > 2$) and jointless Dowling geometries $\mathbf{F}(\mathfrak{G}K_n)$ (if $\#\mathfrak{G} > 4$) exemplify Lemma 3.4, which says that 5 numbers characterize the line sizes in a simple matroid with all lines of size 2, 3, or l. **(gg: Matrd: Invar)**

2006a Minimal blocks of binary even-weight vectors. *Linear Algebra Appl.* 416 (2006), 288–297. MR 2242730 (2008d:05038). Zbl 1115.05012.

§4, "Minimal blocks from graphs": $\mathrm{GF}(q)^\times \cdot \Gamma$ is a minimal k-block over $\mathrm{GF}(q)$ if Γ is minimally j-chromatic for a certain $j = f(k)$, and is a minimal 1-block if Γ is an odd circle. [Annot. 20 Jun 2011.] **(GG: Matrd)**

Joseph P.S. Kung & James G. Oxley

1988a Combinatorial geometries representable over GF(3) and GF(q). II. Dowling

geometries. *Graphs Combin.* 4 (1988), 323–332. MR 0965387 (90i:05029). Zbl 702.51004.

For $n \geq 4$, the Dowling geometry of rank n over the sign group is the unique largest simple matroid of rank n that is representable over GF(3) and GF(q). (**sg: Matrd: Xtreml**)

H. Kunze & D. Siegel

1994a A graph theoretical approach to monotonicity with respect to initial conditions. In: Xinzhi Liu and David Siegel, eds., *Comparison Methods and Stability Theory* (Proc., Waterloo, Ont., 1993). Lect. Notes Pure Appl. Math., Vol. 162. Marcel Dekker, New York, 1994. MR 1291622 (95g:34065). (**SD: Bal, Dyn**)

1999a A graph theoretical approach to monotonicity with respect to initial conditions II. *Nonlinear Analysis* 35 (1999), 1–20. MR 1634009 (99g:34032). (**SD: Bal, Dyn**)

David Kuo

See J.H. Yan.

Y.S. Kuo

See also W.-S. Shih.

Y.S. Kuo, T.C. Chern, & Wei-kuan Shih

1988a Fast algorithm for optimal layer assignment. In: *Proceedings of the 25th ACM/IEEE Design Automation Conference* (Anaheim, Calif., 1988), pp. 554–559.

Algorithm, by minimum perfect matching, for $l(\Sigma)$ for a weighted signed graph that is cubic and planar. See Kuo–Chern–Shih (1988a). [Authors are unaware of Katai and Iwai (1978a) or Barahona (1982a) etc.] [Annot. 21 Dec 2014.] (**WG, sg: fr: Algor**)

Yueh-Er Kuo

See G. de Leon-Calio.

Ranan D. Kuperman

See Z. Maoz.

Joseph Varghese Kureethara

See also and M. Acharya.

2020a New developments in the study of parity signed graphs. In: Samayan Narayanamoorthy, ed., *Advances In Applicable Mathematics– ICAAM2020* (Coimbatore, India, 2020). AIP Conf. Proc., Vol. 2261, art. 020001, 4 pp.

Cf. Acharya and Kureethara (2021a). [Annot. 13 Dec 2020.] (**SG: Lab: Exp**)

Jin Ho Kwak

See also I.P. Goulden.

Jin Ho Kwak, Sungpyo Hong, Jaeun Lee, & Moo Young Sohn

2000a Isoperimetric numbers and bisection widths of double coverings of a complete graph. *Ars Combin.* 57 (2000), 49–64. MR 1796626 (2001h:05083). Zbl 1064.05076. (**sg: KG: Cov**)

Jin Ho Kwak & Jaeun Lee

2001a Enumeration of graph coverings, surface branched coverings and related group theory. In: Sungpyo Hong *et al.*, eds., *Combinatorial & Computational Mathematics* (Proc., Pohang, 2000), pp. 97–161. World Scientific, Singapore, 2001. MR 1868421 (2003b:05083). Zbl 1001.05092.

Voltage graphs (i.e., gain graphs) and their covering graphs ("derived graphs") defined in §1; emphasis on groups and counting group covering graphs of a graph. (**gg: Cov, Top**)

2009a Enumerating coverings. In: Lowell W. Beineke and Robin J. Wilson, eds., *Topics in Topological Graph Theory*, Ch. 9, pp. 181–198. Encycl. Math. Appl., Vol. 128. Cambridge Univ. Press, Cambridge, Eng., 2009. MR 2581546 (no rev). Zbl 1225.05202.

Counting of various kinds of covering graphs via gain graphs ("voltage graphs"). §1, "Introduction": Definition of voltage graphs. §2 "Graph coverings" via voltage graphs. Then counting: §3, "Regular coverings"; §4, "Surface branched coverings";. §5, "Regular surface branched coverings". §6, "Distribution of surface branched coverings". [Annot. 12 Jun 2013.] (**Top: gg, Enum, Cov: Exp**)

J.H. Kwak, Jaeun Lee, & Young-hee Shin

2004a Balanced regular coverings of a signed graph and regular branched orientable surface coverings over a non-orientable surface. *Discrete Math.* 275 (2004), 177–193. MR 2026284 (2004i:05036). Zbl 1030.05034.

The number of isomorphism types of regular balanced coverings of a signed graph. A covering is a sign-preserving covering projection from one signed graph to another. (**SG: Top: Enum**)

Yung-Keun Kwon & Kwang-Hyun Cho

2007a Boolean dynamics of biological networks with multiple coupled feedback loops. *Biophys. J.* 92 (2007), 2975–2981 + suppl. 2 pp.

Simulations suggest that more positive cycles lead to more fixed points and more negative cycles lead to more non-fixed-point attractors (with a fixed number of variables [or genes]). [Annot. 16 Jan 2015.] (**SD: Dyn: Str**)

Ivan Kyrchei, Eran Treister, & Volodymyr Pelykh

20xxa The determinant of the Laplacian matrix of a quaternion unit gain graph. Submitted. (**GG: Lap**)

Vincent Labatut

See N. Arınık, R. Figueiredo, and I. Mendonça.

Domenico Labbate

See M. Abreu.

Martine Labbé

See V. Devloo and R.M.V. Figueiredo.

Nicholas Lacasse

20xxa Minimal and disjoint negation sets in signed graphs. Submitted. arXiv:2010.02276.

Cf. Naserasr and Yu. (**SG: Fr, Sw**)

20xxb The characteristic polynomial of the odd arrangement of hyperplanes. Submitted. (**GG: Geom: Invar**)

Richard Ladner

See V. Klee.

George M. Lady, Thomas J. Lundy, & John Maybee

1995a Nearly sign-nonsingular matrices. *Linear Algebra Appl.* 220 (1995), 229–248. MR 1334579 (96e:15007). Zbl 838.15013.

The signed digraph $S(A)$ of square matrix A. Thm. 1: A is NSNS iff the rows can be permuted so that $S(A)$ has a negative loop at each vertex and no other negative cycles, and no vertex-disjoint positive cycles. [Annot. 12 Jun, 24 Nov 2012.] (**SD: QM**)

George M. Lady & John S. Maybee

1983a Qualitatively invertible matrices. *Math. Social Sci.* 6 (1983), 397–407. MR 0747746 (85f:15005). Zbl 547.15002.

In terms of signed graphs, restates and completes the characterizations of sign-invertible matrices A due to Bassett, Maybee, and Quirk (1968a) and George M. Lady (The structure of qualitatively determinate relationships. *Econometrica* 51 (1983), 197–218. MR 0694457 (85c:90019). Zbl 517.15004) and reveals the sign pattern of A^{-1} in terms of path signs in the associated signed digraph. (**QM: QSol: SD**)

J.C. Lagarias

1985a The computational complexity of simultaneous diophantine approximation problems. *SIAM J. Computing* 14 (1985), 196–209. MR 0774939 (86m:11048). Zbl 563.10025.

Theorem F: Feasibility of integer linear programs with at most two variables per constraint is NP-complete. (**GN(Incid): Du: Algor**)

Hong-Jian Lai

See Z.H. Chen, S.-J. He, Y.T. Liang, and J.-B. Liu.

Dimitri Lajou

See also F. Foucaud.

2019a On the achromatic number of signed graphs. *Theor. Computer Sci.* 759 (2019), 50–60. MR 3913216. Zbl 1405.05063. arXiv:1902.04828. HAL hal-02014469.

Introduces achromatic numbers $\chi_a(\Sigma)$ for sign-colored simple graphs and $\chi_s([\Sigma]) := \max \chi_a(\Sigma')$ over $\Sigma' \in [\Sigma]$ for switching classes, leading to two variants for switching classes and four for unsigned graphs: max and min of $\chi_a(\Sigma')$ for $\Sigma' \in [\Sigma]$, max and min of $\chi_a(\Sigma)$ over $|\Sigma| = \Gamma$, max and min of $\chi_s([\Sigma])$ over $|\Sigma| = \Gamma$. Five of the eight, including χ_a and χ_s, are shown NP-complete; the other three are open. A "k-coloring" of Σ is $\alpha : V \to [k]$ such that $\forall \{i, j\}$ the set $\{uv \in E : \alpha(u) = i,\ \alpha(v) = j\}$ is homogeneously signed. A k-coloring of $[\Sigma]$ is a k-coloring of any $\Sigma' \in [\Sigma]$.

[This coloring is unrelated to (signed) coloring defined in Zaslavsky (1982b). Is there an achromatic number for the latter?]

Dictionary: "2-edge-colored graph" = sign-colored graph Σ, "re-signing" = switching, "signed graph" = switching class $[\Sigma]$, "2-edge-colored homomorphism" = edge-sign-preserving homomorphism, "signed homomorphism" = switching (circle-sign-preserving) homomorphism, "(un)balanced path/cycle" = (negative) positive path/circle. [Annot. 28 Dec 2019.] (**SGc, SG: Col(Gen)**)

2020a On Cartesian products of signed graphs. In: Manoj Changat and Sandip Das, eds., *Algorithms and Discrete Applied Mathematics* (Proc. 6th Int. Conf., CAL-

DAM 2020, Hyderabad, India, 2020), pp. 219–234. Lect. Notes in Computer Sci., Vol. 12016. Springer, Cham, 2020. MR 4074911. Zbl 1456.05133. HAL hal-02491883.

Product defined by Germina, Hameed, and Zaslavsky (2011a). *Cf.* (2019a). (**SG: Algor, Hom**)

2021a *On Various Graph Coloring Problems.* [*Sur divers problèmes de coloration de graphes.*] Doctoral thesis, Univ. de Bordeaux, 2021. HAL tel-03522406.

Part I, "Signed graphs". §2, "Introduction to signed graphs". §3, "Complexity of edge-colored and signed graphs modification problems". §4, "Coloring signed graphs with small cyclomatic number". §5, "Cartesian product of signed graphs". [Annot. 24 Mar 2022.] (**SG: Col, Hom, Algor**)

2022a On Cartesian products of signed graphs. *Discrete Appl. Math.* 319 (2022), 533–555. MR 4457822. Zbl 1494.05051.

Algorithm for Cartesian product decomposition. Homomorphic chromatic number defined via sign-preserving homomorphisms. [Annot. 1 May 2021, 8 Mar 2023.] (**SG: Algor, Hom**)

Aparna Lakshmanan S

See J.J. Palathingal.

R. Lakshmi

See M.V. Anusha.

P. Lallemand

See H.T. Diep.

[S. Ben Lamine]

See S. Ben Lamine (under 'B').

Kaiyang Lan, Jianxi Li, & Feng Liu

2023a Remarks on the largest eigenvalue of a signed graph. *Bull. Malaysian Math. Sci. Soc.* 46 (2023), art. 157, 9 pp. MR 4612760.

The graphs that attain upper bounds in Kannan and Pragada (2023a). Also, $\lambda_{\max}(\Sigma)$ compared to $\lambda_{\max}(\Sigma \setminus v)$. [Annot. 29 Sep 2023.] (**SG: Adj: Eig: Str**)

Yanhua Lan

See K.C. Mondal.

Kelvin Lancaster

1981a Maybee's "Sign solvability". In: Harvey J. Greenberg and John S. Maybee, eds., *Computer-Assisted Analysis and Model Simplification* (Proc. Sympos., Boulder, Col., 1980), pp. 259–270. Academic Press, New York, 1981. MR 0617930 (82g:00016) (book). Zbl 495.93001 (book).

Comment on Maybee (1981a). (**QM: QSol: SD**)

J.W. Landry & S.N. Coppersmith

2002a Ground states of two-dimensional $\pm J$ Edwards-Anderson spin glasses. *Phys. Rev. B* 65 (2002), art. 134404, 15 pp. arXiv:cond-mat/0109136.

Finds all ground states of small signed square lattice graphs, and their distribution, to investigate how physical properties vary with $x :=$

$\#E^+/\#E$. Dictionary: "ground state" = switching with fewest negative edges. [Annot. 10 Jan 2015.] (**SG: State(fr): Algor, Phys**)

2004a Quantum properties of a strongly interacting frustrated disordered magnet. *Phys. Rev. B* 69 (2004), art. 184416, 6 pp.

Similar to (2002a) with a "quantum term" added. The effect is that of an extra vertex v_0 added to Σ, positively adjacent to all of V with an arbitrary strength. Quantum ground and low-energy states are linear combinations of ground states of Σ in a single component of the ground=state graph. Dictionary: "low-energy" = relatively few negative edges, "ground state graph" has ground states ζ for vertices and an edge between ground states that differ by switching a vertex (necessarily having $d^+ = d^-$). [Annot. 10 Jan 2015.] (**SG: State(fr): Algor, Phys**)

Carsten Lange, Shiping Liu, Norbert Peyerimhoff, & Olaf Post

2015a Frustration index and Cheeger inequalities for discrete and continuous magnetic Laplacians. *Calc. Variations Partial Diff. Eqns.* 54 (2015), no. 4, 4165–4196. MR 3426108. Zbl 1330.05103. arXiv:1502.06299. (**Phys: gg, Bal**)

Jan-Hendrik Lange

20xxa Flow-partitionable signed graphs. Submitted. arXiv:2005.01536. (**SG: Flows**)

Steven Landy

1988a A generalization of Ceva's theorem to higher dimensions. *Amer. Math. Monthly* 95 (Dec., 1988), no. 10, 936–939. MR 0979140 (90c:51020). Zbl 663.51011.

The theorem characterizes concurrence of lines drawn from each vertex of a rectilinear simplex to a point in the opposite side. [*Problem.* Reformulate, maybe generalize, in terms of gain graphs. *Cf.* Boldescu (1970a), Zaslavsky (2003b) §2.6.] (**gg: Geom**)

Andrea S. LaPaugh & Christos H. Papadimitriou

1984a The even-path problem for graphs and digraphs. *Networks* 14 (1984), 507–513. MR 0767373 (86g:05057). Zbl 552.68059.

Fast algorithms for existence of even paths between two given vertices (or any two vertices) of a graph. The corresponding digraph problem is NP-complete. [Signed (di)graphs are similar, due to the standard reduction by negative subdivision.] [Also see, e.g., works by Thomassen.] (**Par: Paths: Algor**)(**sd: Par: Paths: Algor**)

Michel Las Vergnas

See A. Björner.

Martin Lätsch & Britta Peis

2008a On a relation between the domination number and a strongly connected bidirection of an undirected graph. *Discrete Appl. Math.* 156 (2008), 3194–3202. MR 2468789 (2010a:05139). Zbl 1176.05058.

A bidirected graph (Γ, τ) (where τ assigns + or − to each incidence) is "strongly connected" if there is a coherent walk from any vertex to any other vertex. The distance $\mathrm{dist}_{(\Gamma,\tau)}(u, v) :=$ the minimum length of a coherent $\vec{uv}$ walk. The diameter $\mathrm{diam}(\Gamma, \tau) := \max_{(u,v)\in V^2} \mathrm{dist}_{(\Gamma,\tau)}(u, v)$. In Γ define $i :=$ number of isthmi, $\gamma :=$ domination number. Thm. 5: Γ has a strongly connected bidirection iff $\#V = 1$ or Γ is connected and minimum degree ≥ 2. Thm. 10: If Γ has strongly connected bidirections

τ_j ($j = 1, \ldots, k$), then $\min_i \operatorname{diam}(\Gamma, \tau_j) \leq 2i + 2\min(i, 1) + 5\gamma - 1$. When $i = 0$, τ_j can be chosen so $\Sigma(\Gamma, \tau_j)$ is all positive. *Conjecture.* Also true when $i > 0$. Thm. 11: If Γ has a strongly connected bidirection, then $\min_j \operatorname{diam}(\Gamma, \tau_j) \leq 6\gamma + 3$. By Fig. 8 this bound must be at least $6\gamma + 1$ if isthmi are allowed. The proofs are constructive, esp. by extending to Γ a bidirection of a dominating subgraph. Dictionary: "path" = walk [not trail]. [Annot. 27 Apr 2007.] **(sg: Ori: Invar, dom)**

Reinhard Laubenbacher

See E. Sontag and A. Veliz-Cuba.

Monique Laurent

See M.M. Deza and A.M.H. Gerards.

Eugene L. Lawler

1976a *Combinatorial Optimization: Networks and Matroids.* Holt, Rinehart and Winston, New York, 1976. MR 0439106 (55 #12005). Zbl 413.90040. Repr.: Dover Publications, Mineola, N.Y., 2001. Zbl 1058.90057.

Ch. 6: "Nonbipartite matching." §3: Bidirected flows. **(sg: Ori)**

Ch. 4: "Network flows." §8: "Networks with losses and gains." §12: "Integrality of flows and the unimodular property."

(GN)(sg: Incid, Bal)

Ben Lawson

See C.E. Tsourakakis.

Bac Hoai Le

See T.T.T. Ho.

Jason Leasure

See L. Fern.

Walter Lebrecht

See also J.F. Valdés and E.E. Vogel.

W. Lebrecht & J.F. Valdés

2013a $\pm J$ Ising model on mixed Archimedean lattices: $(3^3, 4^2)$, $(3^2, 4, 3, 4)$, $(3, 12^2)$, $(4, 6, 12)$. *Physica A* 392 (2013), no. 19, 4549–4570. MR 3083111. Zbl 1395.82049.

(Phys: sg: Fr)

W. Lebrecht, J.F. Valdés, & E.E. Vogel

2003a Frustration in mixed two-dimensional $\pm J$ Ising lattices. *Physica A* 323 (2003), 466–486. Zbl 1050.82009.

Randomly signed Kagomé and five-point-star planar lattices with specified concentration x of positive edges: frustration index ("frustration length") *et al.*, with combinatorial and numerical results compared. Also, compared with results for homogeneous lattices like square and triangular to analyze effects of degree ("coordination number"), plaquette shape (degree of polygonal faces), *et al.*[Annot. 3 Jan 2015.]

(SG, Phys: Fr)

2008a Local analysis of frustration based on Kagomé lattices. *Physica A* 387 (2008), 5147–5158.

Ground state energy $l(\Sigma)$, *et al.*, as functions of $x := \#E^+/\#E$. Analytical, probabilistic, and computational results are largely consistent.

[Annot. 3 Jan 2015.] **(SG, Phys: Fr, Sw)**

W. Lebrecht & E.E. Vogel

1996a Order parameters and percolation for ground-state of honeycomb lattices. In: F. Leccabue and V. Sagredo, eds., *Magnetism, Magnetic Materials and Their Applications* (Proc., Mérida, Venezuela, 1995), pp. 304–309. World Scientific, Singapore, 1996. **(SG: Fr: State, Phys)**

W. Lebrecht, E.E. Vogel, J. Cartes, & J.F. Valdés

2004a Plaquette distributions for $\pm J$ Ising lattices. *Physica A* 342 (2004), 90–96.

In given Γ, $x := \#E^+/\#E$ implies an expected number of frustrated (negative) plaquettes. Γ is triangular, square, hexagonal, Kagomé, etc., with periodic boundary conditions (i.e., toroidal) or is the graph of a regular or semiregular polyhedron. Dictionary: *cf.* Vogel, Cartes, Contreras, Lebrecht, and Villegas (1994a). [Annot. 2 Jan 2015.]
(SG: State, Phys)

W. Lebrecht, E.E. Vogel, & J.F. Valdés

2002a Ising model on mixed two-dimensional lattices. *Physica B* 320 (2002), 343–347.

Probabilistic and computational analysis of average states on signed toroidal Kagomé and five-point-star lattices. Frustration, energy, *et al.* as functions of $x := \#E^+/\#E$. Comparison to honeycomb, square, and triangular lattices (*cf.* other papers of the authors). Dictionary: *cf.* Vogel, Cartes, Contreras, Lebrecht, and Villegas (1994a). [Annot. 3 Jan 2015.] **(SG, Phys: State, Fr)**

2004a Frustration in Archimedean $\pm J$ lattices. *J. Alloys Compounds* 369 (2004), 66–69.

Toroidal ("periodic boundary conditions") lattice $(3, 4, 6, 4)$ (Grünbaum–Shephard classification) with random signs having proportion x of positive edges. Distribution of frustrated (negative) plaquettes, proportion of satisfied edges, *et al.*, in ground states. Comparison to other lattices (*cf.* other papers of the authors). [Annot. 3 Jan 2015.]
(SG, Phys: State, Fr)

Bruno Leclerc

1981a Description combinatoire des ultramétriques. *Math. Sci. Humaines* No. 73 (1981), 5–37. MR 0623034 (82m:05083). Zbl 476.05079. **(SG: Bal)**

Etienne Leclerc, Gary MacGillivray, & Jacqueline M. Warren

20xxa Switching (m, n)-mixed graphs with respect to Abelian groups. Submitted. arXiv:2110.01576. **(gg: Sw)**

J. Leclercq & R. Thomas

1981a Analyses booléenne et continue de systèmes comportant des boucles de rétroaction. II. Système á deux attracteurs formé d'une boucle positive et d'une boucle négative conjuguées. (In French.) *Acad. Roy. Belg. Bull. Cl. Sci.* (5) 67 (1981), no. 3, 190–225 (1 foldout). MR 0652794 (84h:92020b). Zbl 493.92004.

Two signed digraph cycles of opposite signs, joined at a vertex, give a multiplicity of dynamics depending on the initial state and the delays, such as one or more stable states, an unstable cycle of states and related stable state cycles, etc. Part I: Richelle (1980a). [Annot. 10 Jul 2022.]

(sd: Dyn: Biol)

Euiwoong Lee

See V. Chatziafratis.

Gibaek Lee, Sang-Oak Song, & En Sup Yoon

2003a Multiple-fault diagnosis based on system decomposition and dynamic PLS. *Indust. Engin. Chem. Res.* 42 (2003), 6145–6154.

Combines signed digraphs and partial least squares for fault analysis in chemical engineering. **(SD: Appl)**

Jaeun Lee

See I.P. Goulden, D. Kim, and J.H. Kwak.

Jon Lee

1989a Subspaces with well-scaled frames. *Linear Algebra Appl.* 114/115 (1989), 21–56. MR 0986864 (90k:90111). Zbl 675.90061.

See §9. **(sg: Ori: Incid, Flows, Algor)**

Jon Lee & Matt Scobee

1999a A characterization of the orientations of ternary matroids. *J. Combin. Theory Ser. B* 77 (1999), no. 2, 263–291. MR 1719344 (2000k:05073). Zbl 1024.05016.

The results imply that a ternary matroid, such as the frame matroid of a signed graph, has at most three orientation classes. [Thanks to Stefan van Zwam.] [Annot. 2 Apr 2013.] **(sg: matrd)**

Orlando Lee

See K. Kawarabayashi.

Sang-Gu Lee & Jin-Woo Park

2015a Sign idempotent sign pattern matrices that allow idempotence. *Linear Algebra Appl.* 487 (2015), 232–241. MR 3405558. Zbl 1328.15042.

See §4, "Nonnegativity of sign idempotent sign pattern matrices". **(QM: SD, SG: sw, Bal)**

Sang-Gu Lee, Se-Won Park, & Han-Guk Seol

1998a Linear operators strongly preserving matrices whose sign patterns require the Perron property.. *Linear Algebra Appl.* 269 (1998), 53–63. MR 1483520 (98i:-15006). Zbl 887.15014.

Cf. Eschenbach and Johnson (1990a). [Annot. 20 Oct 2020.] **(QM, sd: bal(Cyc), sw)**

Shyi-Long Lee

See also I. Gutman and P.K. Sahu.

1989a Comment on 'Topological analysis of the eigenvalues of the adjacency matrices in graph theory: A difficulty with the concept of internal connectivity'. *J. Chinese Chem. Soc.* 36 (1989), no. 1, 63–65.

Response to Gutman (1988a). Proposes weighted net sign: divide by number of nonzero vertex signs. The goal is to have the ordering of net signs correlate more closely with that of eigenvalues. **(VS, SGw, Chem)**

1989b Net sign analysis of eigenvectors and eigenvalues of the adjacency matrices in graph theory. *Bull. Inst. Chem., Academica Sinica* No. 36 (1989), 93–104.

Expounds principally Lee, Lucchese, and Chu (1987a) and Lee and Gutman (1989a). Examples include all connected, simple graphs of order ≤ 4 and some aromatics. (**VS, SGw, Chem: Exp**)

1992a Topological analysis of five-vertex clusters of group IVa elements. *Theoretica Chimica Acta* 81 (1992), 185–199.

See Lee, Lucchese, and Chu (1987a). More examples; again, eigenvalue and net-sign orderings are compared. (**VS, SGw, Chem**)

Shyi-Long Lee & Ivan Gutman

1989a Topological analysis of the eigenvectors of the adjacency matrices in graph theory: Degenerate case. *Chem. Phys. Lett.* 157 (1989), 229–232.

Supplements Lee, Lucchese, and Chu (1987a) to answer an objection by Gutman (1988a), by treating vertex signs corresponding to multidimensional eigenspaces. (**VS, SGw, Chem**)

Shyi-Long Lee & Chiuping Li

1994a Chemical signed graph theory. *Int. J. Quantum Chem.* 49 (1994), 639–648.

Varies Lee, Lucchese, and Chu (1987a) by taking net signs of all balanced signings, instead of only those obtained from eigenvectors, for small paths, circles, and circles with short tails. The distribution of net sign, over all signings of each graph, is more or less binomial. (**VS, SGw, Chem**)

1994b On generating molecular orbital graphs: the first step in signed graph theory. *Bull. Inst. Chem., Academica Sinica* No. 41 (1994), 69–75.

Abbreviated presentation of (1994a). (**VS, SGw: Exp**)

Shyi-Long Lee & Feng-Yin Li

1990a Net sign approach in graph spectral theory. *J. Molecular Structure (Theochem)* 207 (1990), 301–317.

Similar topics to S.L. Lee (1989a), (1989b). Several examples of order 6. (**VS, SGw, Exp, Chem**)

1990b Net sign analysis of five-vertex chemical graphs. *Bull. Inst. Chem., Academica Sinica* No. 37 (1990), 83–97.

See Lee, Lucchese, and Chu (1987a). Treats all connected, simple graphs of order 5. (**VS, SGw, Chem**)

Shyi-Long Lee, Feng-Yin Li, & Friday Lin

1991a Topological analysis of eigenvalues of particle in one- and two-dimensional simple quantal systems: Net sign approach. *Int. J. Quantum Chem.* 39 (1991), 59–70.

See Lee, Lucchese, and Chu (1987a). § II: Net signs calculated for paths. §§ III, IV: Planar graphs with two different types of potential, yielding complicated results. (**VS, SG, Chem**)

Shyi-Long Lee, Robert R. Lucchese, & San Yan Chu

1987a Topological analysis of eigenvectors of the adjacency matrices in graph theory: The concept of internal connectivity. *Chem. Phys. Lett.* 137 (1987), 279–284. MR 0910752 (88i:05130).

Introduces the net sign of a (balanced) signed graph. A graph has vertices signed according to the signs of an eigenvector X_i of the adjacency matrix, $\mu(v_r) = \operatorname{sgn}(X_{ir})$, and $\sigma(v_r v_s) = \mu(v_r)\mu(v_s)$ [hence Σ is balanced]. Note that a vertex can have 'sign' 0. Net sign of a [hydrocarbon] chemical graph is applied to prediction of properties of molecular orbitals. (**VS, SGw, Chem**)

Shyi-Long Lee, Yeung-Long Luo, & Yeong-Nan Yeh

1991a Topological analysis of some special graphs. III. Regular polyhedra. *J. Cluster Sci.* 2 (1991), 105–116.

See Lee, Lucchese, and Chu (1987a). Net signs for the Platonic polyhedra (Table I). (**VS, SGw, Chem**)

Shyi-Long Lee & Yeong-Nan Yeh

1990a Topological analysis of some special classes of graphs. Hypercubes. *Chem. Phys. Lett.* 171 (1990), 385–388.

Follows up Lee, Lucchese, and Chu (1987a) and Lee and Gutman (1989a), calculating net signs of eigenspatially signed hypercube graphs of dimensions up to 6 by means of a general graph-product formula. (**VS, SGw, Chem**)

1993a Topological analysis of some special classes of graphs. II. Steps, ladders, cylinders. *J. Math. Chem.* 14 (1993), 231–241. MR 1262027 (95f:05079).

See Lee, Lucchese, and Chu (1987a). Net signs and eigenvalues are compared. (**VS, SGw, Chem**)

Géraud Le Falher & Fabio Vitale

2016a Even trolls are useful: Efficient link classification in signed networks. Manuscript, 2016. arXiv:1602.08986. (**SD: PsS: Algor**)

Hanno Lefmann

1990a On families in finite lattices. *European J. Combin.* 11 (1990),165–179. MR 1044456 (91i:06009). Zbl 734.06007.

Thm. 1.2 bounds the size of a family of lattice elements with prescribed meet ranks. Dowling lattices are an example of this and related results. [Annot. 9 Apr 2016.] (**gg: Matrd**)

Jenő Lehel

See R.J. Faudree.

Ziyi Lei

See Z.-Y. Cheng.

Frank Thomson Leighton

See C.R. Johnson.

Samuel Leinhardt

See also J.A. Davis and P.W. Holland.

Samuel Leinhardt, ed.

1977a *Social Networks: A Developing Paradigm.* Academic Press, New York, 1977.

An anthology reprinting some basic papers in structural balance theory, including some elementary signed-graph theory. (**PsS, SG: Bal, Clu**)

P.W.H. Lemmens & J.J. Seidel

1973a Equiangular lines. *J. Algebra* 24 (1973), 494–512. MR 0307969 (46 #7084). Zbl 255.50005. Repr. in Seidel (1991a), pp. 127–145.

Hints of graph switching; see van Lint and Seidel (1966a). (**Geom, sw**)

Gloria de Leon-Calio & Yueh-Er Kuo

2003a Signed digraphs and their applications. *Soochow J. Math.* 29 (2003), no. 1, 69–81. MR 1975279 (2004c:05082). Zbl 1032.05062.

Simple properties of pulse properties, as in Roberts (1976a). Seems largely expository. [Annot. 5 Jun 2019.] (**SDw: Exp**)

Marianne Lepp [Marianne L. Gardner]

See R. Shull.

Jürgen Lerner

See J. Kunegis.

Jure Leskovec, Daniel Huttenlocher, & Jon Kleinberg

2010a Signed networks in social media. In: *CHI '10: Proceedings of the 28th ACM Conference on Human Factors in Computing Systems* (Atlanta, 2010), pp. 1361–1370. Assoc. for Computing Machinery, New York, 2010. arXiv:1003.2424. (**SD, SG: Bal, Clu**)

2010b Predicting positive and negative links in online social networks. In: *WWW '10: Proceedings of the 19th International Conference on World Wide Web* (Raleigh, N.C., 2010). Assoc. for Computing Machinery, New York, 2010. arXiv:1003.2429. (**SD: Bal**)

Adam N. Letchford

See K. Kaparis.

Emily Leven, Brendon Rhoades, & Andrew Timothy Wilson

2014a Bijections for the Shi and Ish arrangements. *European J. Combin.* 39 (2014), 1–23. MR 3168512. Zbl 1284.05331. arXiv:1307.6523. (**gg: Geom**)

Richard Levins

See also J.M. Dambacher and C.J. Puccia.

1974a The qualitative analysis of partially specified systems. *Ann. N.Y. Acad. Sci.* 231 (1974), 123–138. Zbl 285.93028. (**SD: QM: QSta: Cyc**)

1975a Evolution in communities near equilibrium. In: M. Cody and J.M. Diamond, eds., *Ecology and Evolution of Communities*, pp. 16–50. Harvard Univ. Press, Cambridge, Mass., 1975. (**SD: QM: QSta: Cyc**)

Vadim E. Levit

See Y. Cherniavsky.

Mario Levorato, Rosa Figueiredo, Yuri Frota, & Lúcia Drummond

2017a Evaluating balancing on social networks through the efficient solution of correlation clustering problems. *EURO J. Computational Optim.* 5 (2017), no. 4, 467–498. MR 3736647. Zbl 1386.90161. HAL hal-02178542. Erratum. *ibid.* 5 (2017), no. 4, 499. MR 3736648 (no rev). (**SG: Clu: Algor**)

Thierry Lévy

See A. Kassel.

Mordechai Lewin

1977a On the extreme points of the polytope of symmetric matrices with given row sums. *J. Combin. Theory Ser. A* 23 (1977), no. 2, 223–231. MR 0444495 (56 #2846). Zbl 362.05040.

An equivalent of Thm. 8.2.1 in Brualdi (2006a). [Annot. 13 Oct 2012.] **(sg: par: Adj)**

Frank L. Lewis

See Z.-Y. Sun.

Torina Lewis, Jenny McNulty, Nancy Ann Neudauer, Talmage James Reid & Laura Sheppardson

2013a Bicircular matroid designs. *Ars Combin.* 110 (2013), 513–523. MR 3100270.

The connected bicircular matroids in which all circuits have the same size, i.e., which are duals of matroid designs, are certain uniform subdivisions of uniform matroids. [Annot. 9 Jun 2013.] (**Bic**)

David W. Lewit

See E.G. Shrader.

Josef Leydold

See T. Bıyıkoğlu.

Claire Lhuillier

See G. Misguich.

Bao Feng Li

See X.H. Hao.

Bo Li

See S.-J. Yang.

Cai Heng Li & Jozef Širáň

2007a Möbius regular maps. *J. Combin. Theory Ser. B* 97 (2007), no. 1, 57–73. MR 2278124 (2007h:05043). Zbl 1106.05033.

That is, signed expansion graphs $\pm\Gamma$, orientation embedded in a surface (Möbius), whose map automorphisms act transitively on flags (regularity). Properties of their automorphism groups. [Follows Wilson (1989a).] [Annot. rev. 31 Jul 2014.] (**SG: Top: Aut**)

Chang Li

See T. Harju.

Chiuping Li

See I. Gutman and S.L. Lee.

Chuandong Li

See S.-J. Yang.

Dan Li, Huiqiu Lin, & Jixiang Meng

2023a Extremal spectral results related to spanning trees of signed complete graphs. *Discrete Math.* 346 (2023), no. 2, art. 113250, 17 pp. MR 4510055. arXiv:-2201.06729.

First, max eigenvalue of $A(K_n, -T)$, $T =$ spanning tree. Second, upper bounds on min eigenvalue of distance matrix of incomplete Σ. [Annot.

23 Jan 2022.] **(SG: KG: Adj: Eig)(SG: Adj(Gen): Eig)**

Deqiong Li

See Q. Guo and D.-J. Wang.

Dong Li, Cuihua Wang, Shengping Zhang, Guanglu Zhou, Dianhui Chu, & Chong Wu

2017a Positive influence maximization in signed social networks based on simulated annealing. *Neurocomputing* 260 (2017), 69–78. **(SG: Algor)**

Dong Li, Zhi-Ming Xu, Nilanjan Chakraborty, Anika Gupta, Katia Sycara, & Sheng Li

2014a Polarity related influence maximization in signed social networks. *PLoS ONE* 9 (2014), no. 7, art. 102199, 12 pp. **(SG, PsS)**

Feng-Hin Li

See S.L. Lee.

Guangbin Li

2013a The signless Laplacian spectral radius of C_4-free graphs with even order. *Basic Sci. J. Textile Univ. / Fangzhi Gaoxia* 26 (2013), no. 2, 171–175. Zbl 1299.05224. **(par: Lap: Eig)**

Guojun Li & Aimei Yu

2015a A characterization of bicyclic signed graphs with nullity $n-7$. *J. Math. Res. Appl.* 35 (2015), no. 1, 1–10. MR 3328496. Zbl 1340.05114.

That is, adjacency rank 7. **(SG: Adj: Eig)**

Hao Li

See W.J. Ning.

Hiram W. Li

See J.M. Dambacher.

Hong-Hai Li

See also L. Su.

Hong-Hai Li & Jiong-Sheng Li

2008a An upper bound on the Laplacian spectral radius of the signed graphs. *Discuss. Math. Graph Theory* 28 (2008), no. 2, 345–359. MR 2477235 (2010a:05115). Zbl 1156.05035.

Dictionary: See X.D. Zhang and Li (2002a). [Annot. 23 Mar 2009.] **(SG: incid, Eig)**

2009a Note on the normalized Laplacian eigenvalues of signed graphs. *Australasian J. Combin.* 44 (2009), 153–162. MR 2527006 (2010i:05210). Zbl 1177.05050. **(SG: Eig)**

Hong-Hai Li, Bit-Shun Tam, & Li Su

2013a On the signless Laplacian coefficients of unicyclic graphs. *Linear Algebra Appl.* 439 (2013), no. 7, 2008–2028. MR 3090451.

Minimum and maximum magnitudes and associated graphs are found (for $n \geq 5$). Thms. 4.1, 5.1, 5.2, 5.3 on transforms of Γ have two cases depending on whether (connected) Γ is bipartite. [*Conjecture.* The results generalize to signed graphs with two (connected) cases: balanced or not.] [Annot. 20 Jan 2015.] **(par: Lap: Eig)**

Ji Li

See H.Z. Deng.

Jiaao Li

See also M. DeVos.

Jiaao Li, Yongtang Shi, Zhouningxin Wang, & Chunyan Wei

20xxa Homomorphisms to small negative even cycles. In preparation. **(SG: Hom, Col)**

Jianxi Li

See also J.M. Guo and K.-Y. Lan.

Jianxi Li & Ji-Ming Guo

2013a The signless Laplacian spectral radii of modified graphs. *Math. Commun.* 18 (2013), 67–73. MR 3085789. Zbl 1276.05070. **(par: Lap: Eig)**

Jing Li

See S.Y. Wang.

Ke Li

See also L.G. Wang.

Ke Li, Ligong Wang, & Guopeng Zhao

2011a The signless Laplacian spectral radius of tricyclic graphs and trees with k pendant vertices. *Linear Algebra Appl.* 435 (2011), no. 4, 811–822. MR 2807235 (2012f:05179). Zbl 1220.05075. **(par: Lap: Eig: Str)**

2011b The signless Laplacian spectral radius of unicyclic and bicyclic graphs with a given girth. *Electronic J. Combin.* 18 (2011), no. 1, Paper 183, 10 pp. MR 2836818 (2012g:05138). Zbl 1230.05200. **(par: Lap: Eig: Str)**

Jiong-Sheng Li

See Y.P. Hou, H.H. Li, and X.D. Zhang.

Lulu Li

See G.-H. Mu.

Min Li

See S. Ji.

Nan Li

See A. Funato and T. Hibi.

Qian Li & Bolian Liu

2008a Bounds on the kth multi-g base index of nearly reducible sign pattern matrices. *Discrete Math.* 308 (2008), 4846–4860. MR 2446095 (2010a:05037). Zbl 1167.15013. **(QM: SD)**

Qian Li, Bolian Liu, & Jeffrey Stuart

2010a Bounds on the k-th generalized base of a primitive sign pattern matrix. *Linear Multilinear Algebra* 58 (2010), no. 3, 355–366. MR 2663436 (2011c:15090). Zbl 1196.15030. **(SD: QM)**

Qingdu Li

See S.-D. Zhai.

Rao Li

2010a Inequalities on vertex degrees, eigenvalues and (signless) Laplacian eigenvalues of graphs. *Int. Math. Forum* 5 (2010), no. 37-40, 1855–1860. MR 2672449 (no rev). Zbl 1219.05088. **(par: Lap: Eig)**

Ruilin Li & Jinsong Shi

2010a The minimum signless Laplacian spectral radius of graphs with given independence number. *Linear Algebra Appl.* 433 (2010), no. 8-10, 1614–1622. MR 2718223 (2011m:05181). Zbl 1211.05075. **(par: Lap: Eig)**

Rui-lin Li, Jin-song Shi, & Bing-can Dong

2011a Maximal signless Laplacian spectral radius of bicyclic graphs with given independence number. (In Chinese?) *J. East China Norm. Univ. Natur. Sci. Ed.* 2011 (2011), no. 3, 73–84, 99. MR 2867304 (no rev). Zbl 1240.05194. **(par: Lap: Eig)**

Selena Li

See L.J. Rusnak.

Sheng Li

See D. Li.

Shuang-Dong Li

See Y. Wang.

Shuchao Li

See also B. Chen, C. Chen, X.Y. Geng, S.S. He, J. Huang, G.-F. Wang, W. Wei, and M.J. Zhang.

Shuchao Li, Wanting Sun, & Baogen Xu

20xxa Relation between the adjacency rank of a complex unit gain graph and some classical parameters of its underlying graph. Submitted. **(GG: Adj)**

Shuchao Li, Wanting Sun, & Yuantian Yu

2022a Adjacency eigenvalues of graphs without short odd cycles. *Discrete Math.* 345 (2022), no. 1, art. 112633, 13 pp. MR 4318317. Zbl 1480.05086. arXiv:-2109.04599.

Upper bound on $\lambda_1^2 + \lambda_2^2$ for odd girth $> 2k+1$. Spectral radius can imply odd girth $\leq 2k+1$. Also, proves theorem of Lin and Guo (2021a). [*Problem.* Generalize to negative girth of unbalanced signed graphs.] [Annot. 28 Jun 2022.] **(par: Adj: Eig)**

Shuchao Li & Yi Tian

2011a On the (Laplacian) spectral radius of weighted trees with fixed matching number q and a positive weight set. *Linear Algebra Appl.* 435 (2011), no. 6, 1202–1212. MR 2807144 (2012f:05180). Zbl 1222.05165.

Weights $w : E \to \mathbb{R}_{>0}$. Since $\operatorname{Spec} L(\Gamma, w) = \operatorname{Spec} L(-\Gamma, w)$, $L(-\Gamma, w)$ is used to find $\lambda_{\max}(L(\Gamma, w))$. [Annot. 21 Jan 2012.] **(par: Lap: Eig)**

2012a Some bounds on the largest eigenvalues of graphs. *Appl. Math. Lett.* 25 (2012), 326–332. MR 2855981 (2012h:05195). Zbl 1243.05152.

Laplacian matrix $L(-\Gamma)$. [Annot. 29 Jul 2019.] **(par: Lap: Eig)**

2015a Some results on the bounds of signless Laplacian eigenvalues. *Bull. Malaysian Math. Sci. Soc.* 38 (2015), no. 1, 131–141. MR 3394043. Zbl 1308.05074.

Upper and lower bounds on the sum of largest eigenvalues of $L(-\Gamma)$ and $L(-\Gamma^c)$ for a simple graph. [Also see Oliveira and de Lima (2016a).] Eigenvalue and eigenvector bounds from $L(-\Gamma)$ on the clique and stability numbers. [Annot. 7 Jan 2015.] **(par: Lap: Eig)**

Shuchao Li & Shujing Wang

2012a The least eigenvalue of the signless Laplacian of the complements of trees. *Linear Algebra Appl.* 436 (2012), no. 7, 2398–2405. MR 2890000. Zbl 1238.05162. **(par: Lap: Eig)**

2020a The energy of random signed graph. *Linear Algebra Appl.* 585 (2020), 227–240. MR 4019819. Zbl 1426.05103. arXiv:1812.11865.

The model: $\mathrm{Prob}(+e_{ij}) = p$, $\mathrm{Prob}(-e_{ij}) = q$, $\mathrm{Prob}(\text{no } e_{ij}) = 1 - p - q$. An integral expression for energy $\mathcal{E}$ and Wigner matrices. Thm. 2.7: $\mathcal{E} = n^{3/2}(\frac{8}{3\pi}\sqrt{p+q-(p-q)^2} + o(1))$. §3, "The energy of the random multipartite signed graph": bounds. [Annot. 12 Jul 2019.] **(SG: Rand: Eig)**

Shuchao Li & Wei Wei

2020a The multiplicity of an A_α-eigenvalue: A unified approach for mixed graphs and complex unit gain graphs. *Discrete Math.* 343 (2020), no. 8, art. 111916, 19 pp. MR 4082511. Zbl 1441.05142.

Via gain group $\mathbb{T} := \{z \in \mathbb{C} : |z| = 1\}$, generalizes results on graphs, signed graphs, mixed graphs. [Annot. 4 Mar 2022.] **(GG: Adj(Gen): Eig)**

Shuchao Li & Ting Yang

2022a On the relation between the adjacency rank of a complex unit gain graph and the matching number of its underlying graph. *Linear Multilinear Algebra* 70 (2022), no. 9, 1768–1787. MR 4429445. Zbl 1490.05126.

Complex unit gain graph Φ, matching number α', $r = \mathrm{rk}\, A(\Phi)$. Bounds on $r - 2\alpha'$, $r - \alpha'$, r/α'. Characterizes extreme cases. [Annot. 24 Jul 2023.] **(GG: Adj)**

Shuchao Li & Yuantian Yu

2022a Hermitian adjacency matrix of the second kind for mixed graphs. *Discrete Math.* 345 (2022), no. 5, art. 112798, 22 pp. MR 4368471. arXiv:2102.03760.

The graphs are (and are treated as) gain graphs with gains 1 for undirected, ω for directed edges, where ω is a complex unit chosen [seemingly arbitrarily] to be $\sqrt[6]{1}$ as in Mohar (2020a). [*Question*. How much is independent of the choice of ω?] [Annot. 25 Mar 2022.] **(gg: Adj)**

Shuchao Li & Li Zhang

2011a Permanental bounds for the signless Laplacian matrix of bipartite graphs and unicyclic graphs. *Linear Multilinear Algebra* 59 (2011), no. 2, 145–158. MR 2773647 (2012a:05194). Zbl 1239.05116.

Sharp upper and lower bounds for $\mathrm{per}(L(-\Gamma))$ when Γ is unicyclic or bipartite, with or without girth, and characterization of extremal graphs. [Bipartite Γ means they are doing $L(\Gamma)$; the truly signed part is for unicyclic graphs only.] [Annot. 19 Nov 2011.] **(par: Lap: Eig)**

2012a Permanental bounds for the signless Laplacian matrix of a unicyclic graph with diameter d. *Graphs Combin.* 28 (2012), no. 4, 531–546. MR 2944040.

See Li and Zhang (2011a). Here, the second minimum of, and a lower bound for, per $L(-\Gamma)$. [Annot. 24 Jan 2012.] **(par: Lap)**

Shuchao Li & Minjie Zhang

2012a On the signless Laplacian index of cacti with a given number of pendant vertices. *Linear Algebra Appl.* 436 (2012), no. 12, 4400–4411. MR 2917417. Zbl 1241.05082. **(par: Lap: Eig)**

Shuchao Li, Siqi Zhang, & Baogen Xu

2019a The relation between the H-rank of a mixed graph and the independence number of its underlying graph. *Linear Multilinear Algebra* 67 (2019), no. 11, 2230–2245. MR 4002346. Zbl 1422.05064.

Φ with gain group $\{\pm 1, \pm \mathrm{i}\}$: $\varphi(e) = 1$ for undirected, i for directed edges.

Xiang Li

See R.-R. Kang.

Xiangwen Li

See L.L Hu.

Xiaodi Li

See G.-H. Mu.

Xiao Ming Li

See F.T. Boesch and F.L. Tian.

Xiaowang Li

See X.-Y. Ren.

Xueliang Li

See also X.-L. Chen, W.X. Du, D. Hu, B.F. Huo, and J.-X. Liu.

Xueliang Li & Wen Xia

2019a Skew-rank of an oriented graph and independence number of its underlying graph. *J. Combin. Optim.* 38 (2019), 268–277. MR 3955753. Zbl 1420.05133.

Cf. Huang, Li, and Wang (2018a). [Annot. 15 Jul 2019.] **(gg: Adj)**

2020a Adjacency rank and independence number of a signed graph. *Bull. Malaysian Math. Sci. Soc.* 43 (2020), no. 1, 993–1007. MR 4044914. Zbl 1433.05141. **(SG: Adj)**

Xueliang Li, Jianbin Zhang, & Lusheng Wang

2009a On bipartite graphs with minimal energy. *Discrete Appl. Math.* 157 (2009), no. 4, 869–873. MR 2499503 (2010f:05116). Zbl 1226.05161.

[Bipartite energy is the energy of $A(-\Gamma)$ for bipartite Γ. *Problem* 1. Generalize to antibalanced signed graphs. *Problem* 2. Generalize to signed graphs.] [Annot. 24 Jan 2012.] **(sg: par: bal: Lap: Eig)**

Yadong Li, Jing Liu, & Chenlong Liu

2014a A comparative analysis of evolutionary and memetic algorithms for community detection from signed social networks. *Soft Computing* 18 (2014), 329–348. **(SG: Str: Algor)**

Yang Li

See B. Yang.

Yanhua Li, Wei Chen, Yajun Wang, & Zhi-Li Zhang

2013a Influence diffusion dynamics and influence maximization in social networks with friend and foe relationships. In: *Proceedings of the Sixth ACM International Conference on Web Search and Data Mining* (WSDM '13, 2013), pp. 657–666. ACM, New York, 2013. arXiv:1111.4729. (**SG: PsS**)

2015a Voter model on signed social networks. *Internet Math.* 11 (2015), no. 2, 93–133. MR 3316858. Zbl 1465.91082. (**SD**)

Yijia Li

See S.-S. Feng.

Yiyang Li

See W.X. Du.

Yong Li

See J.-S. Wu.

Yongtao Li & Yuejian Peng

2022a The maximum spectral radius of non-bipartite graphs forbidding short odd cycles. *Electronic J. Combin.* 29 (2022), no. 4, art. 4.2.

[*Problem.* Generalize to unbalanced signed graphs without short negative circles.] [For Laplacian analog *cf.* Liu, Miao, and Xue (20xxa).] [Annot. 27 Dec 2022.] (**Par: Adj: Eig; Str**)

Yu Li

See S.-J. Yang.

Yuemeng Li

See also L.T. Wu.

Yuemeng Li, Xintao Wu, & Aidong Lu

2017a On spectral analysis of directed signed graphs. In: *2017 IEEE International Conference on Data Science and Advanced Analytics* (DSAA, Tokyo), pp. 539–548. IEEE, 2017.

Extended abstract of (2018a). (**SD: Adj: Eig: Clu, Algor**)

Yuemeng Li, Shuhan Yuan, Xintao Wu, & Aidong Lu

2018a On spectral analysis of directed signed graphs. *Int. J. Data Sci. Analytics* 6 (2018), 147–162. arXiv:1612.08102. (**SD: Adj: Eig: Clu, Algor**)

Zhentao Li

See K. Kawarabayashi.

Zhongshan Li

See also M. Arav, C.A. Eschenbach, F.J. Hall and L. Zhang.

Zhongshan Li, Frank Hall, & Carolyn Eschenbach

1994a On the period and base of a sign pattern matrix. *Linear Algebra Appl.* 212-213 (1994), 101–120. MR 1306974 (95m:15026). Zbl 821.15017.

Chaohua Liang, Bolian Liu, & Yufei Huang

2010a The kth lower bases of primitive non-powerful signed digraphs. *Linear Algebra Appl.* 432 (2010), no. 7, 1680–1690. MR 2592910 (2011b:15076). Zbl 1221.05190. (**SD: qm**)

Yanting Liang, Bolian Liu, & Hong-Jian Lai

2009a Multi-g base index of primitive anti-symmetric sign pattern matrices. *Linear Multilinear Algebra* 57 (2009), no. 6, 535–546. MR 2543715 (2010i:05151. Zbl 1221.15019. (**QM: SD**)

Rui-Quan Liao
See B. Hu.

F. di Liberto
See A. Coniglio.

Hans Liebeck
See D. Harries.

Martin W. Liebeck

1980a Lie algebras, 2-graphs and permutation groups. *Bull. London Math. Soc.* 33 (1982), 76–85. MR 0565479 (81f:05095). Zbl 499.05031.

Examines the F Aut($[\Sigma]$)-module $F\,V(\Sigma)$, where Σ is a signed complete graph and F is a field of characteristic 2. (**TG: Aut**)

1982a Groups fixing graphs in switching classes. *J. Austral. Math. Soc. (A)* 33 (1982), 76–85. MR 0662362 (83h:05048). Zbl 499.05031.

Given an abstract group $\mathfrak{A}$, which of its permutation representations are exposable on every invariant switching class of signed complete graphs [see Harries and H. Liebeck (1978a) for definitions]? (**kg: sw, TG: Aut**)

Thomas M. Liebling
See H. Gröflin.

Rainer Liebmann

† 1986a *Statistical Mechanics of Periodic Frustrated Ising Systems*. Lect. Notes in Phys., Vol. 251. Springer-Verlag, Berlin, 1986. MR 0850837 (87k:82004).

Detailed and readable descriptions, often simplified and relatively combinatorial, of the state of knowledge about Ising systems in the form of signed graphs and weighted signed graphs. [Relatively accessible to combinatorists.] Dictionary: "model" = graph with signs and usually weights, "ferromagnetic" = positive edge, "antiferromagnetic" = negative edge, "fully frustrated" = all girth circles are negative, "state" = $s : V \to \{+1, -1\}$, "ground state" = state with fewest frustrated edges, "ground state degeneracy" = number of ground states (1 being nondegenerate), "excited state" = non-ground state. §2.1.1, "Ground state degeneracy of the ANNNI-chain", on chains of triangles with two bond signs and strengths, J_1 and J_2. (ANNI = Axial Next Nearest Neighbor Ising model.) The number and description of ground states are treated in detail, as well as less combinatorial physical quantities. §2.3.1, "Periodic frustrated chains": All weights equal, so this is signed graphs. Restates Doman and Williams (1982a) in terms of a path with distance-2 edges, signed with period 4. The path edges have constant sign (either $+$ or $-$ by switching) and weight B; the distance-2 edges are $+ - --$ with weight J.

§3.1.2b, "Star-triangle transformation": Edge signs and weights transform. The triangle-star transformation on a negative triangle gives imaginary signs. [*Question.* Does this indicate a use for complex unit gains?] §3.2, "Triangular lattice": Based on Houtappel (1950a), (1950b) and Wannier (1950a). §3.3.1, "Union Jack lattice": Square lattice, edges weighted J_1, with alternating diagonals in alternating squares weighted $J_2 < 0$. All triangles are negative. $|J_2|/J_1$ determines behavior. For ratio 1 (a signed graph), there are $\approx C^{\#V}$ ground states for a finite sublattice, where $C \geq \sqrt{(17/8)}$. §3.3.2, "Villain's odd model": *Cf.* Villain (1977a). §3.3.3, "Hexagon lattice": *Cf.* Wolff and Zittartz (1982a), (1983a). §3.3.4, "Pentagon lattice": *Cf.* Waldor, Wolff, and Zittartz (1985a). §3.3.5, "Kagomé lattice": Various periodic sign patterns; references. §3.3.6, "Connection between GS [ground state] degeneracy and existence of a phase transition at $T_c = 0$": The conjecture of Hoever, Wolff, and Zittartz (1981a). Also, a conjecture of Süto on the exact conditions under which the ground states are connected in the state graph. §3.4, "Frustrated Ising systems with crossing interactions": Several more complicated extensions of previous models, usually by adding distance-2 edges ("nnn interactions"). See (2) below.

§4.1, "fcc antiferromagnet": All-negative face-centered cubic lattice graph. Interesting remarks on how ground state and near-ground state structure might influence physical properties. §4.2, "Fully and partially frustrated simple cubic lattice": The fully frustrated planar square lattice can be stacked in various ways to produce differently frustrated cubic lattices. §4.3, "AF pyrochlore model": All-negative tetrahedra joined at corners. §4.4, "ANNNI-model": All-positive cubic lattice with negative distance-2 vertical edges.

Two frequent remarks: (1) An external magnetic field reduces the number of ground states. (2) Slightly more complicated graphs give models that are not exactly solvable. [Combinatorial explanations: The magnetic field corresponds to an extra vertex, positively adjacent to all $V(\Sigma)$; see Barahona (1982a). The more complicated graphs are nonplanar; Barahona (1982a) and Istrail (2000a) indicate that this is the obstacle to exact solution.] [Annot. 28 Aug 2012.]

(**Phys, SG, WG: Fr, State(fr): Exp, Ref**)

Magnhild Lien & William Watkins

2000a Dual graphs and knot invariants. *Linear Algebra Appl.* 306 (2000) 123–130. MR 1740436 (2000k:05187). Zbl 946.05061.

The Laplacian matrices of a signed plane graph and its dual have the same invariant factors. The proof is via the signed graphs of knot diagrams. (**SGc: Du, Lap, Eig, Knot**)

Frauke Liers

See also M. Palassini and G. Pardella.

Frauke Liers, Michael Jünger, Gerhard Reinelt, & Giovanni Rinaldi

2004a Computing exact ground states of hard Ising spin glass problems by branch-and-cut. In: Alexander K. Hartmann & Heiko Rieger, eds., *New Optimization Algorithms in Physics*, pp. 47–69. WILEY-VCH, Weinheim, 2004. Zbl 1059.90147.

Reduces frustration index to max cut. Algorithms for the latter. [Annot. 6 Jul 2022.] (**sg: Fr: Algor**)

Ko-Wei Lih

See J.H. Yan.

Bart Litjens

See A. Goodall.

Chjan C. Lim

1993a Nonsingular sign patterns and the orthogonal group. *Linear Algebra Appl.* 184 (1993), 1–12. MR 1209379 (94c:15036). Zbl 782.68098.

A family of bipartite signed wheels that prevent $A = (A^{-1})^{\mathrm{T}}$. A family of bipartite signed graphs which allow it. [Annot. 6 Mar 2011.]

(**SG: QM**)

Ee-Peng Lim

See D. Lo.

Meng-Hiot Lim

See Harary, Lim, *et al.*

Carlos V.G.C. Lima, Dieter Rautenbach, Uéverton S. Souza, & Jayme L. Szwarcfiter

2018a Bipartizing with a matching. In: Donghyun Kim, R.N. Uma, and Alexander Zelikovsky, eds., *Combinatorial Optimization and Applications* (COCOA 2018, 12th Int. Conf., Atlanta), pp. 198–213. Lect. Notes in Computer Sci., Vol. 11346. Springer, Cham, 2018. MR 3893518.

Slightly abbreviated version of (2022a). (**par: Fr, Algor**)

2022a On the computational complexity of the bipartizing matching problem. *Ann. Oper. Res.* 316 (2022), no. 2, 1235–1256. MR 4478058. Zbl 1497.05093. arXiv:-1710.07741.

It is NP-complete to decide which graphs become bipartite by deleting a matching, even for 3-colorable planar graphs with $\Delta(\Gamma) = 4$, but linear-time solvable for subcubic graphs. [*Question.* Which signed graphs have a balancing matching?] [Annot. 10 Sept 2022.] (**par: Fr, Algor**)

[Leonardo Silva de Lima]

See L.S. de Lima (under 'D').

Enzo M. Li Marzi

See F. Belardo and J.F. Wang.

Friday Lin

See S.L. Lee.

See also D. Li, H.T. Guo, and Y.-A. Wang.

Huiqiu Lin & Hangtian Guo

2021a A spectral condition for odd cycles in non-bipartite graphs. *Linear Algebra Appl.* 631 (2021), 83–93. MR 4308089. Zbl 1483.05095.

Max. spectral radius of Γ with order n and odd girth $\geq 2k+3$: only one graph. [Also in Li, Sun, and Yu (2022a).] [This is the case $\Sigma = -\Gamma$ of *Problem.* Generalize to unbalanced signed graphs with negative girth k.] [Annot. 28 Jun 2022.] (**par: Adj: Eig: bal**)

Lin Lin

See J.-G. Dong.

Shangwei Lin

See S.Y. Wang.

Xue Lin, Qiang Jiao, & Long Wang

2019a Opinion propagation over signed networks: Models and convergence analysis. *IEEE Trans. Automatic Control* 64 (2019), no. 8, 3431–3438. MR 3992885. Zbl 1482.91165. **(SG: PsS, Dyn)**

Yen-Chi Roger Lin

See M.-Y. Cao.

Jonathan A. Lindquist

See S. Klamt.

Bernt Lindström

See F. Harary.

Gabriele Lini

See C. Altafini.

Nathan Linial

See Y. Bilu and S. Hoory.

Sóstenes Lins

1981a A minimax theorem on circuits in projective graphs. *J. Combin. Theory Ser. B* 30 (1981), 253–262. MR 0624541 (82j:05074). Zbl 457.05057.

For Eulerian Σ in projective plane, max. number of edge-disjoint negative circles = min. number of edges cut by a noncontractible closed curve that avoids the vertices. [Generalized by Schrijver (1989a).] **(SG: Top, fr, Circ, Algor)**

1982a Graph-encoded maps. *J. Combin. Theory Ser. B* 32 (1982), 171–181. MR 0657686 (83e:05049). Zbl 465.05031, (Zbl 478.05040).

See §4. [*Cf.* Zaslavsky (1992a) for related work.] **(sg: Top: bal)**

1985a Combinatorics of orientation reversing circles. *Aequationes Math.* 29 (1985), 123–131. MR 0819300 (87c:05051). Zbl 592.05019. **(sg, par: Top, Bal, Fr)**

J.H. van Lint & J.J. Seidel

1966a Equilateral point sets in elliptic geometry. *Proc. Koninkl. Ned. Akad. Wetenschap. Ser. A* 69 (= *Indag. Math.* 28) (1966), 335–348. MR 0200799 (34 #685). Zbl 138.41702 (138, p. 417b). Repr. in Seidel (1991a), pp. 3–16.

Introduces graph switching. **(tg, Geom)**

Svante Linusson

See C.A. Athanasiadis.

Marc J. Lipman & Richard D. Ringeisen

1978a Switching connectivity in graphs. In: F. Hoffman *et al.*, eds., *Proc. of the Ninth Southeastern Conf. on Combinatorics, Graph Theory and Computing* (Boca Raton, 1978), pp. 471–478. Congressus Numerantium, XXI. Utilitas Math. Publ. Inc., Winnipeg, Man., 1978. MR 0527972 (80k:05073). Zbl 446.05033. **(TG)**

A. Lipshtat

See A. Ma'ayan.

C.H.C. Little

See I. Fischer.

Simon Litsyn
See I. Krasikov.

Charles H.C. Little
See C.P. Bonnington.

A-Ming Liu
See M.-Z. Chen.

Bolian Liu
See also B. Cheng, Y.F. Huang, C. Li, Q.A. Li, Y.T. Liang, J.P. Liu, M.H. Liu, and Z.F. You.

2007a The period and base of a reducible sign pattern matrix. *Discrete Math.* 307 (2007), 3031–3039. MR 2371074 (2009i:15043). Zbl 1127.15018. (**QM: SD**)

Bolian Liu, Muhuo Liu, & Zhifu You

2013a The majorization theorem for signless Laplacian spectral radii of connected graphs. *Graphs Combin.* 29 (2013), no. 2, 281–287. MR 3027603. Zbl 1263.05062.

For a degree sequence π, define $\mu_c(\pi) := \max_\Gamma \lambda_{\max}(L(-\Gamma))$ over connected Γ with degree sequence π and c circles. Let $\pi \preccurlyeq \pi'$ in the majorization ordering. Thm. 2: Under certain assumptions on c, π, π', $\mu(\pi) \leq \mu(\pi')$. For the special cases of unicyclic and bicyclic graphs: X.D. Zhang (2009a) and Huang, Liu, and Liu (2011a). For majorization also see Tam, Fan, and Zhou (2008a), M.H. Liu and Liu (2012a). [Annot. 24 Jan 2012.] (**par: Lap: Eig**)

Chenlong Liu
See Y.D. Li.

Daphne Der-Fen Liu
See Y.-T. Jiang.

Fang Liu
See J.S. Wu.

Feng Liu
See K.-Y. Lan and X.-J. Tian.

Gui Zhen Liu & Qiang Wu

1995a Applications of graph theory to social science. (In Chinese. English summary.) *Shandong Daxue Xuebao Ziran Kexue Ban* 30 (1995), no. 4, 361–366. MR 1387317 (97c:05149). Zbl 882.05116.

Describes some applications of and some results about balance in signed graphs. (**SG: Bal, PsS: Exp, Matrd**)

Henry Liu, Robert Morris, & Noah Prince

2009a Highly connected monochromatic subgraphs of multicolored graphs. *J. Graph Theory* 61 (2009), no. 1, 22–44. MR 2514097 (2010d:05083). Zbl 1202.05079.

See Łuczak (2016a). [Annot. 24 Jan 2016.] (**sg: Str**)

Huan Liu
See G. Beigi and J.-L. Tang.

Huiqing Liu
See also Q. Wen and Y.-P. Wu.

Huiqing Liu & Mei Lu

2014a A conjecture on the diameter and signless Laplacian index of graphs. *Linear Algebra Appl.* 450 (2014), 158–174. MR 3192475. Zbl 1286.05093.

Corrects and proves a conjecture of Hansen and Lucas (2010a) on $\max \lambda_{\max}(L(-\Gamma))D(\Gamma)$ for $\#V = n$, where $D =$ diameter. [Annot. 23 Nov 2014.] (**par: Lap: Eig**)

Ji Liu

See W. Chen.

Jia-Bao Liu & Shaohui Wang

2017a A note on "Extremal graphs with bounded vertex bipartiteness number". Manuscript, 2017. arXiv:1704.02867.

Counterexamples and correction to Robbiano, Morales, and San Martin (2016a). [*Problem.* Generalize to frustration number of Σ.] [Annot. 19 May 2018.] (**sg: Par: Fr, Eig**)

Jianbing Liu, Miaomiao Han, & Hong-Jian Lai

2022a Weighted modulo orientations of graphs and signed graphs. *Electronic J. Combin.* 29 (2022), no. 4, art. P4.47, 20 pp.

Despite the title this is about (existence of) modular flows with specified boundaries, not weights or orientations. [Annot. 17 Dec 2022.] (**SG: Flows**)

Jianping Liu & Bolian Liu

2008a The maximum clique and the signless Laplacian eigenvalues. *Czechoslovak Math. J.* 58(133) (2008), no. 4, 1233–1240. MR 2471179 (2010a:05116). Zbl 1174.05079.

Bounds on the clique number $\omega(\Gamma)$ based on the least and greatest eigenvalues of $L(-\Gamma)$. A similar lower bound on the stability number $\alpha(\Gamma)$. [Annot. 23 Nov 2014.] (**par: Lap: Eig**)

Jianxi Liu & Xueliang Li

2015a Hermitian-adjacency matrices and Hermitian energies of mixed graphs. *Linear Algebra Appl.* 466 (2015), 182–207. MR 3278246. Zbl 1302.05106.

"Hermitian adjacency matrix" $= A(\Phi)$ with gain group $\{\pm 1, \pm \mathrm{i}\}$: $\varphi(e) = 1$ for undirected, i for directed edges. Thm. 2.7: Sachs formula for $\det A(\Phi)$. Thm. 2.8: Coefficients of characteristic polynomial. Imaginary-gain circles contribute nothing to the formulas. Thm. 2.16: If $|\Phi_1| = |\Phi_2|$ and gains differ on some edge in a cut, then $\operatorname{Spec} A(\Phi_1) = \operatorname{Spec} A(\Phi_2)$ [obviously false as stated]. [*Cf.* Mohar (2016a).] §3, "Bounds of Hermitian energy". §4, "Mixed graphs that share the same spectrum with their underlying graphs". Thm. 4.1: $\operatorname{Spec} A(\Phi) = \operatorname{Spec} A(\|\Phi\|)$ iff Φ is balanced. Thm. 4.2: Cartesian product [as in] preserves balance. Dictionary: "value" = gain; "rank, corank" = those of matroid $\mathbf{F}(\|\Phi\|)$; "positive" = balanced, "generalized orientation" = partial orientation. §5, "Oriented graphs". §6, "Integral representation for Hermitian energy". [Annot. 15 Dec 2020.] (**gg: Adj, Sw: Eig**)

Jianzhou Liu

See R. Huang.

Jiming Liu
See B. Yang.

Jing Liu
See Y.D. Li.

Lele Liu
See X.Y. Yuan.

Lily L. Liu & Yi Wang
2007a A unified approach to polynomial sequences with only real zeros. *Adv. Appl. Math.* 38 (2007), 542–560. MR 2311051 (2008a:05013). Zbl 1123.05009.
§3.5., "Compositions of multisets and Dowling lattices". **(gg: Matrd: Invar)**

Lu Liu
See Y.-Z. Wu.

Mu Huo Liu
See also B.L. Liu.

Muhuo Liu & Bolian Liu
2010a The signless Laplacian spread. *Linear Algebra Appl.* 432 (2010), no. 2-3, 505–514. MR 2577696 (2011d:05226). Zbl 1206.05064.
Spread S = difference of largest and smallest eigenvalues, studied for $L(-\Gamma)$. Let $m(v)$:= average degree in $N(v)$. Thm. 2.1: $\Delta-\delta+1 \leq S \leq \max_v\{d(v)+m(v)\}$. Other lower bounds in terms of $\sum_v d(v)^2$, average degree of independent set of vertices. Thm. 2.5: Min spread of unicyclic graphs. [*Cf.* (2011a); Oliveira, de Lima, de Abreu, and Kirkland (2010a); Fan and Fallat (2012a).] **(par: Lap: Eig)**

2011a On the spectral radii and the signless Laplacian spectral radii of c-cyclic graphs with fixed maximum degree. *Linear Algebra Appl.* 435 (2011), no. 12, 3045–3055. MR 2831596 (2012h:05197). Zbl 1226.05138. **(par: Lap: Eig)**

2012a New method and new results on the order of spectral radius. *Computers Math. Appl.* 63 (2012), no. 3, 679–686. MR 2871667 (2012i:05171). Zbl 1238.05164.
Also see Liu, Liu, and You (2013a). **(par: Lap: Eig)**

2015a On the signless Laplacian spectra of bicyclic and tricyclic graphs. *Ars Combin.* 120 (2015), 169–180. MR 3363272. Zbl 1349.05213.
The 2 or 4 largest eigenvalues and spreads of $L(-\Gamma)$. [*Problem.* Generalize to signed graphs, or complex unit gain graphs. *Cf.* Reff (2012a).] [Annot. 19 May 2018.] **(par: Lap: Eig)**

Muhuo Liu, Bolian Liu, & Fuyi Wei
2011a Graphs determined by their (signless) Laplacian spectra. *Electronic J. Linear Algebra* 22 (2011), 112–124. MR 2781040 (2012g:05141). Zbl 1227.05185. **(par: Lap: Eig)**

Muhuo Liu, Xuezhong Tan, & Bolian Liu
2010a The (signless) Laplacian spectral radius of unicyclic and bicyclic graphs with n vertices and k pendant vertices. *Czechoslovak Math. J.* 60 (2010), no. 3, 849–867. MR 2672419 (2011f:05185). Zbl 1224.05311. **(par: Lap: Eig)**

2011a The largest signless Laplacian spectral radius of connected bicyclic and tricyclic graphs with n vertices and k pendant vertices. (In Chinese.) *Appl. Math. J. Chinese Univ. Ser. A* 26 (2011), no. 2, 215–222. MR 2838952 (2012e:05238). Zbl 1240.05197. (**par: Lap: Eig**)

Ning Liu & William J. Stewart

2011a Markov chains and spectral clustering. In: *Performance Evaluation of Computer and Communication Systems: Milestones and Future Challenges*, pp. 87–98. Lect. Notes in Computer Sci., Vol. 6821. Springer-Verlag, Berlin, 2011. (**Par: Eig: Appl**)

Ruifang Liu

See also M.Q. Zhai.

Ruifang Liu, Lu Miao, & Jie Xue

20xxa Maxima of the Q-index of non-bipartite C_3-free graphs. Submitted. arXiv:2209.00801.

[*Problem.* Generalize to unbalanced signed graphs without negative triangles.] [For a stronger adjacency analog *cf.* Yongtao Li & Yuejian Peng (2022a).] [Annot. 27 Dec 2022.] (**Par: Lap: Eig; Str**)

Shiping Liu

See also F.M. Atay, C.-Y. Ge, C.-Y. Hu, and C. Lange.

Shiping Liu, Norbert Peyerimhoff, & Alina Vdovina

2020a Signatures, lifts, and eigenvalues of graphs. In: Fatihcan M. Atay *et al.*, eds., *Discrete and Continuous Models in the Theory of Networks*, pp. 255–269. Operator Theory Adv. Appl., Vol. 281. Birkhäuser/Springer, Cham, 2020. MR 4181350. Zbl 1473.05176. arXiv:1412.6841.

Gains in the group $\mathbb{T}_3$ of cube roots of unity. Eigenvalues of the $\mathbb{T}_3$-covering graph in terms of those of the gain graph Φ and underlying graph $\|\Phi\|$. [Annot. 30 Oct 2017.] (**GG: Adj, Cov, Eig**)

Shuting Liu

See Z.-W. Wang.

Vivian Liu

See G. Chen.

Weichan LiuWeichan Liu, Chen Gong, Lifang Wu, & Xin Zhang

2018a A note on the vertex arboricity of signed graphs. *Utilitas Math.* 106 (2018), 251–258. MR 3753793. Zbl 1393.05139. arXiv:1708.03077.

Introduces vertex arboricity $\mathrm{va}(\Sigma) := \min m$ such that Σ has an m-coloring c whose improper edge set $I(c)$ satisfies: $I(c){:}c^{-1}(\pm i)$ is a forest $\forall i \geq 0$. c is a "signed tree coloring". Thm. 2: va is switching invariant. Lemma 3: Signed tree colorings combine under K_2- and K_3-unions. Thm. 4: A signed near-triangulation (of the plane) is list-tree-colorable under assumptions on the lists. Cor. 5: A balanced signed triangulation is tree 3-colorable [but easily proved by Thm. 2 and known theorem that $\mathrm{va}(\Gamma) \leq 3$ for planar Γ]. Thm. 11: $\mathrm{va}(\Gamma) \leq 3$ if Γ has no K_5 minor (expressed in terms of balanced signatures). [Annot. 28 Jun 2022.] (**SG: Col(Gen), Sw**)

Weijun Liu

See L. Lu.

Wenzhong Liu, Qing Cui, & Yunzhuo Wang

2022a On 4-connected 4-regular graphs without even cycle decompositions. *Discrete Math.* 345 (2022), no. 10, art. 113012. MR 4432158.

On decomposing $E(-\Gamma)$ into positive circles. *Cf.* Rizzi (2001a). [Annot. 11 Jul 2022.] (**Par: Str: Circ**)

Wenzhong Liu, Huazheng You, & Qing Cui

2020a On even cycle decompositions of line graphs of cubic graphs. *Discrete Math.* 343 (2020), no. 7, art. 111904, 15 pp. MR 4078918. Zbl 1440.05169.

Question of Markström (2012a) solved for oddness ≤ 4. [Annot. 11 Nov 2020.] (**par: LG: Str**)

Xianzhu Liu, Zhijian Ji, & Ting Hou

2019a Graph partitions and the controllability of directed signed networks. *Sci. China Inform. Sci.* 62 (2019), art. 042202, 11 pp. MR 3919757.

(**SD, SG, WG: Eig, Bal: Algor**)

Xiaogang Liu

See also D. Hu and Y.P. Zhang.

Xiaogang Liu, Suijie Wang, Yuanping Zhang, & Xuerong Yong

2011a On the spectral characterization of some unicyclic graphs. *Discrete Math.* 311 (2011), 2317–2336. MR 2832132. Zbl 1242.05165. (**par: Lap: Eig**)

Xiaoyu Liu

See Y.Q. Chen.

Xin Liu

See G.-H. Yu.

Xueyan Liu

See B. Yang.

Yan Pei Liu

See R.X. Hao.

Yingluan Liu

See Y.F. Huang.

Yu Liu & Lihua You

2014a Further results on the nullity of signed graphs. *J. Appl. Math.* 2014 (2014), art. 483735, 8 pp. MR 3173331. Zbl 1406.05064. arXiv:1309.0174.

Thm. 1: Sachs formula for coefficients of the characteristic polynomial of $A(\Sigma, w)$ where $w : E \to \mathbb{R}_{>0}$. [*Cf.* Trinajstić (1977a), (1983a) (no weights).] (**SG: Adj: Eig**)

Yue Liu

See also X.Y. Yuan.

Yue Liu, Jia-Yu Shao, & Ling-Zhi Ren

2011a Characterization of ray pattern matrix whose determinantal region has two components after deleting the origin. *Linear Algebra Appl.* 435 (2011), 3139–3150. MR 2831602 (2012f:15054). Zbl 1226.15021.

Dictionary: "arc-weighted digraph of A" = complex gain digraph whose adjacency matrix is A. (**QM: gg: Adj**)

Etera R. Livine
See R.C. Avohou.

Paulette Lloyd
See P. Bonacich and P. Doreian.

David Lo, Didi Surian, Philips Kokoh Prasetyo, Kuan Zhang, & Ee-Peng Lim
2013a Mining direct antagonistic communities in signed social networks. *Inform. Process. Management* 49 (2013), 773–791. (**SD: clu: Algor**)

Martin Loebl
See also Y. Crama and A. Galluccio.

Martin Loebl & Iain Moffatt
2008a The chromatic polynomial of fatgraphs and its categorification. *Adv. Math.* 217 (2008), no. 4, 1558–1587. MR 2382735 (2008j:05114). Zbl 1131.05036. arXiv:math/0511557. (**SGc: Top, Invar**)

[Shobana Loganathan]
See L. Shobana.

D.O. Logofet & N.B. Ul'yanov
1982a Necessary and sufficient conditions for the sign stability of matrices. (In Russian.) *Dokl. Akad. Nauk SSSR* 264 (1982), 542–546. MR 0659759 (84j:15018). Zbl 509.15008.
Necessity of Jeffries' (1974a) sufficient conditions. (**QSta**)

1982b (as D.O. Logofet and N.B. Ul'janov) Necessary and sufficient conditions for the sign stability of matrices. *Soviet Math. Dokl.* 25 (1982), 676–680. MR 0659759 (84j:15018). Zbl 509.15008.
English trans. of (1982a). (**QSta**)

Michael Lohman
See M. Chudnovsky.

V. Lokesha
See also P.S.K. Reddy.

V. Lokesha, P.S. Hemavathi, & S. Vijay
2018a Note on super radial signed graphs. *Ultra Sci. Phys. Sci., Sect. A* 30 (2018), no. 1, 40–44. Zbl 1416.05132. (**SG: Sw**)

V. Lokesha, P. Siva Kota Reddy, & S. Vijay
2009a The triangular line n-sigraph of a symmetric n-sigraph. *Adv. Stud. Contemp. Math. (Kyungshang)* 19 (2009), no. 1, 123–129. MR 2542128 (2010k:05121). Zbl 1213.05120.
Definitions and notation as in Sampathkumar, Reddy, and Subramanya (2008a). Generalization of Subramanya and Reddy (2009a) to symmetric n-signed graphs, with similar definitions and results. [The results remain true without assuming symmetry.] [Annot. 10 Apr 2009.]
(**SG(Gen), gg: Bal, LG(Gen), Sw**)

Andreas Lommatzsch
See J. Kunegis.

Bo Long

See S.H. Yang.

M. Loréa

1979a On matroidal families. *Discrete Math.* 28 (1979), 103–106. MR 0542941 (81a:-05029). Zbl 409.05050.

Discovers the "linearly bounded" (or "count") matroids of graphs. [See White and Whiteley (1983a), Whiteley (1996a), Schmidt (1979a).]
(**MtrdF: Bic, Gen**)

Martin Lotz & Johann A. Makowsky

2004a On the algebraic complexity of some families of coloured Tutte polynomials. *Adv. Appl. Math.* 32 (2004), 327–349. MR 2037634 (2004k:05116). Zbl 1041.05042. arXiv:none. (**SGw: Invar: Algor**)

E. Loukakis

2003a A dynamic programming algorithm to test a signed graph for balance. *Int. J. Computer Math.* 80 (2003), no. 4, 499–507. MR 1983308. Zbl 1024.05034.

Another algorithm for detecting balance [*cf.* Hansen (1978a), Harary and Kabell (1980a)]. Also, once again proves that all-negative frustration index [obviously equivalent to Max Cut] is NP-complete.
(**SG: Bal, Fr: Algor**)

Janice R. Lourie

1964a Topology and computation of the generalized transportation problem. *Management Sci.* 11 (1965) (Sept., 1964), no. 1, 177–187. (**GN: Matrd(bases)**)

László Lovász

See also J.A. Bondy, Gerards, Lovász, *et al.* (1990a), and M. Grötschel.

1965a On graphs not containing independent circuits. (In Hungarian.) *Mat. Lapok* 16 (1965), 289–299. MR 0211902 (35 #2777). Zbl 151.33403 (151, p. 334c).

Characterization of the graphs having no two vertex-disjoint circles. See Bollobás (1978a) for exposition in English. [*Major Problem.* Characterize the biased graphs having no two vertex-disjoint unbalanced circles. This theorem is the contrabalanced case. For the sign-biased case see Slilaty (2007a). McCuaig (1993a) might be relevant to the general problem.] (**GG: Circ: Str**)

1979a *Combinatorial Problems and Exercises.* North-Holland, Amsterdam, and Akadémiai Kiadó, Budapest, 1979. MR 0537284 (80m:05001). Zbl 439.05001.

Prob. 7.21 finds $\operatorname{rk} \mathrm{H}(-\Gamma)$ [*cf.* van Nuffelen (1973a)]. Prob. 10.18: The vertex frustration number of a contrabalanced graph vs. the circle edge-packing number. [Annot. 16 Jun 2012.] (**sg: par: Incid**)(**gg: fr**)

1983a Ear-decompositions of matching-covered graphs. *Combinatorica* 3 (1983), 105–117. MR 0716426 (85b:05143). Zbl 516.05047.

It is hard to escape the feeling that we are dealing with all-negative signed graphs and their $-K_4$ and $-K_2^\circ$ minors. [And indeed, see Gerards and Schrijver (1986a) and Gerards, Lovász, *et al.* (1990a) and the notes on Seymour (1995a).] (**Par: Str**)

1993a *Combinatorial Problems and Exercises*,. Second ed. Elsevier, Amsterdam, and Akadémiai Kiadó, Budapest, 1993. MR 1265492 (94m:05001). Zbl 785.05001.

See (1979a). [Annot. 16 Jun 2012.] **(sg: par: Incid)(gg: fr)**

2007a *Combinatorial Problems and Exercises.* Second ed., corr. reprint. AMS Chelsea Publ., American Mathematical Soc., Providence, R.I., 2007. MR 2321240 (no rev). Zbl 439.05001.

See (1979a). [Annot. 16 Jun 2012.] **(sg: par: Incid)(gg: fr)**

2011a Subgraph densities in signed graphons and the local Simonovits–Sidorenko conjecture. *Electronic J. Combin.* 18 (2011), #P127. MR 2811096 (2012f:05158). Zbl 1219.05084. arXiv:1004.3026. **(SG)**

L. Lovász & M.D. Plummer

1986a *Matching Theory.* North-Holland Math. Stud., Vol. 121. Ann. Discrete Math., Vol. 29. Akadémiai Kiadó, Budapest, and North-Holland, Amsterdam, 1986. MR 0859549 (88b:90087). Zbl 618.05001.

Pp. 247–248: Shortest odd/even uv-path problem in Γ. Lemma 6.6.9 reduces min length of odd path to a min-weight perfect matching problem in a modified graph. Exerc. 6.6.10–11 are similar for even paths and odd/even circles. [*Problem.* Generalize to negative/positive paths and circles in signed graphs.] §6.6, p. 252: $l(-\Gamma)$ [i.e., max cut in Γ], $l(\Sigma)$ for signed planar graphs. Cor. 6.19: For planar Γ, $l(-\Gamma) = \frac{1}{2}$(max number of circles in a 2-packing of negative circles). [*Question*: How does this generalize to signed planar graphs?] Pp. 252–253: Odd-circle packing and 2-packing. [Annot. 10 Nov 2010.]

(sg, par: fr, Paths, Circ: Exp)

§8.7, pp. 353–354: Weighted non-ferromagnetic Ising model. [Annot. 10 Nov 2010.] **(SG, WG: Phys, fr: Exp)**

2009a *Matching Theory.* AMS Chelsea Publ. (Amer. Math. Soc.), Providence, R.I., 2009. MR 2536865. Zbl 618.05001.

Reprint of (1986a) with errata and an appendix of updates. [Annot. 10 Nov 2010.]

(sg: par: Circ, Paths, fr: Exp)(sg, WG: Phys, fr: Exp)

L. Lovász, L. Pyber, D.J.A. Welsh, & G.M. Ziegler

1995a Combinatorics in pure mathematics. In: R.L. Graham, M. Grötschel, and L. Lovász, eds., *Handbook of Combinatorics*, Vol. II, Ch. 41, pp. 2039–2082. North-Holland (Elsevier), Amsterdam, and MIT Press, Cambridge, Mass., 1995. MR 1373697 (97f:00003). Zbl 851.52017.

§7: "Knots and the Tutte polynomial", considers the signed graph of a knot diagram (pp. 2076–77). **(SGc: Knot)**

Aidong Lu

See Y.-M. Li and L.T. Wu.

Lingfei Lu

See M. Zhu.

Lu Lu, Lihua Feng, & Weijun Liu

20xxa Signed zero-divisor graphs over commutative rings. *Commun. Math. Stat.* (in press).

Sign $\sigma(uv) = -$ iff u, v are nilpotent and $uv = 0$; allows $u = v$. Graph structure, balance, quotients, and adjacency spectrum. Examples: $\mathbb{Z}_n$

for some n. Open problems. [Annot. 3 Jul 2023.]
(**Algeb: SG: Str, Adj: Eig**)

Lu Lu, Lihua Feng, Weijun Liu, & Guihai Yu

2023a Zero-divisor graphs of rings and their Hermitian matrices. *Bull. Malaysian Math. Sci. Soc.* 46 (2023), art. 130, 19 pp.

Mixed graph for some product rings with $\{\pm1, \pm\mathrm{i}\}$ gains as in Liu and Li (2015a) and Mohar (2016a). Graph structure and adjacency eigenvalues. [Annot. 3 Jul 2023.] (**Algeb: gg: Str, Adj: Eig**)

Lu Lu, Jianfeng Wang, & Qiongxiang Huang

2021a Complex unit gain graphs with exactly one positive eigenvalue. *Linear Algebra Appl.* 608 (2021), 270–281. MR 4152791. Zbl 1458.05148. (**GG: Adj: Eig, Sw**)

Mei Lu

See H.Q. Liu and W.J. Ning.

Yong Lu

See also Q. Wu and L. Zhang.

Yong Lu, Ligong Wang, & Peng Xiao

2017a Complex unit gain bicyclic graphs with rank 2, 3 or 4. *Linear Algebra Appl.* 523 (2017) 169–186. MR 3624673. Zbl 1370.05139. arXiv:1511.07589.

Rank of $A(\Phi)$. (**GG: Adj**)

Yong Lu, Ligong Wang, & Qiannan Zhou

2017a Hermitian-Randić matrix and Hermitian-Randić energy of mixed graphs. *J. Inequalities Appl.* 2017 (2017), paper no. 54, 14 pp. MR 3619588. Zbl 1358.05177. arXiv:610.09783.

Normalized adjacency matrix with gain group $\{\pm1, \pm\mathrm{i}\}$: $\varphi(e) = 1$ for undirected, i for directed edges. (**gg: Adj**)

2018a The rank of a signed graph in terms of the rank of its underlying graph. *Linear Algebra Appl.* 538 (2018), 166–186. MR 3722834. Zbl 1374.05133.

Rank of $A(\Sigma)$ vs. $A(|\Sigma|)$. (**SG: Adj**)

2018b Skew-rank of an oriented graph in terms of the rank and dimension of cycle space of its underlying graph. *Filomat* 32 (2018), no. 4, 1303–1312. MR 3848106. (**gg: Adj**)

2019a The rank of a complex unit gain graph in terms of the rank of its underlying graph. *J. Combin. Optim.* 38 (2019), 570–588. MR 3969335. Zbl 1420.05087. arXiv:1711.11448.

Rank of $A(\Phi)$ vs. $A(\|\Phi\|)$. (**GG: Adj**)

Yong Lu & Jingwen Wu

2021a No signed graph with the nullity $\eta(G, \sigma) = |V(G)| - 2m(G) + 2c(G) - 1$. *Linear Algebra Appl.* 615 (2021), 175–193. MR 4200821. Zbl 1459.05112. arXiv:-2006.00002.

$\operatorname{rk} A(\Sigma) + 2\xi - 2\mu = s$ is impossible for $s = 1$, where $\mu :=$ matching number, $\xi :=$ cyclomatic number. Fixing ξ, infinitely many Σ exist for which $s \in \{0\} \cup [2, 3\xi]$. [Annot. 6 Feb 2021.] (**SG: Adj**)

2021b Bounds for the rank of a complex unit gain graph in terms of its maximum degree. *Linear Algebra Appl.* 610 (2021), 73–85. MR 4159284. Zbl 1458.05149.

rk $A(\Phi) \geq n/\Delta(G)$. Equality is characterized. [Annot. 4 Jun 2022.] (**GG: Adj**)

Yong Lu & Qi Wu

20xxa Inertia indices of a complex unit gain graph in terms of matching number. *Linear Multilinear Algebra* (to appear). arXiv:2108.01443.

Cf. He, Hao, and Dong (2020a). Four inequalities and characterizations of equality. [Annot. 4 Jun 2022.] (**GG: Adj**)

Yong Lu & Wei-Ru Xu

2021a An lower bound of the rank of a signed graph in terms of order and maximum degree. *ScienceAsia* 47 (2021), no. 6, 779–784.

Thms.: rk $A(K_{a,b}, \sigma) = 2$ iff $(K_{a,b}, \sigma)$ is balanced. rk $A(\Sigma) \geq n/\Delta$, $=$ iff $\Sigma \sim k$ copies of $+K_{\Delta,\Delta}$. rk $A(\Sigma) = (n+1)/\Delta$ iff Σ is the preceding with one missing vertex. [Annot. 4 Jun 2022.] (**SG: Adj**)

You Lu

See also J.-A. Cheng, M. DeVos, and X. Wang.

You Lu, Rong Luo, Michael Schubert, Eckhard Steffen, & Cun-Quan Zhang

2020a Flows on signed graphs without long barbells. *SIAM J. Discrete Math.* 34 (2020), no. 4, 2166–2182. MR 4164490. Zbl 1450.05037. arXiv:1908.11004.

In many ways they behave like flows on unsigned graphs. (**SG: Flows**)

You Lu, Rong Luo, & Cun-Quan Zhang

2018a Multiple weak 2-linkage and its applications on integer flows of signed graphs. *European J. Combin.* 69 (2018), 36–48. MR 3738139. Zbl 1376.05066.

Σ without edge-disjoint negative circles has a nowhere-zero 6-flow, satisfying Bouchet's (1983a) conjecture. [Annot. 26 Nov 2020.] (**SG: Flows**)

Alexander Lubotzky

See T. Kaufman.

Claire Lucas

See M. Aouchiche and P. Hansen.

Robert R. Lucchese

See S.L. Lee.

Henri Luchian

See A. Băutu.

Tomasz Łuczak

See also E. Györi.

2016a Highly connected monochromatic subgraphs of two-colored complete graphs. *J. Combin. Theory Ser. B* 117 (2016), 88–92. MR 3437613. Zbl 1329.05172.

Thm. If $2 \leq k \leq (n+3)/4$, then $\Sigma = (K_n, \sigma)$ contains a k-connected homogeneously signed subgraph of order $> n - 2(k-1)$ or k-connected all-positive and all-negative subgraphs of order $n - 2(k-1)$. Completes work of Bollobás and Gyárfás (2008a) (who conjectured most of this), Liu, Morris, and Prince (2009a), and Fujita and Magnant (2011a). [Annot. 24 Jan 2016.] (**sg: Str**)

Mark Ludwig

See also P. Abell and B. Kujawski.

M. Ludwig & P. Abell

2007a An evolutionary model of social networks. *European Phys. J. B* 58 (2007), 97–105.

Signed edges are added to and deleted from a fixed set of nodes under a balancing rule. Imbalance measured by frustrated triangles impels evolution, which converges under some conditions. [Annot. 20 Jun 2011.] (**SG: Bal, Fr: Dyn**)

J. Lukic, A. Galluccio, E. Marinari, O.C. Martin, & G. Rinaldi

2004a Critical thermodynamics of the two-dimensional $\pm J$ Ising spin glass. *Phys. Rev. Lett.* 92 (2004), no. 11, #117202. arXiv:cond-mat/0309238.

Physical properties of a signed toroidal square lattice graph, from computation of the exact partition function (energy distribution) via Galluccio, Loebl, and Vondrák (2000a), (2001a). E.g., the approximate proportion of negative edges is important. [Annot. 18 Aug 2012.] (**SG: Phys, Fr**)

Robert Lukot'ka

See T. Kaiser.

J. Richard Lundgren

See H.J. Greenberg and F. Harary.

Thomas J. Lundy

See also G.M. Lady.

Thomas J. Lundy, John Maybee, & James Van Buskirk

1996a On maximal sign-nonsingular matrices. *Linear Algebra Appl.* 247 (1996), 55–81. MR 1412740 (97k:15020) (*q.v.*). Zbl 862.15019.

Constructions of such matrices. A matrix definition of C_4-cockades. [Annot. 6 Mar 2011.] (**SG: QSol**)

Feng Luo

See H.-T. Wang.

Guoqiang Luo

See S.-D. Zhai.

Rong Luo

See J.-A. Cheng, M. DeVos, Y. Lu, X.Q. Qi, L. Zhang, and X.D. Zhang.

Yeung-Long Luo

See I. Gutman and S.L. Lee.

Bob Lutz

2019a *Electrical Networks, Hyperplane Arrangements and Matroids.* Ph.D. thesis, University of Michigan, 2019. MR 4071609 (no rev). (**gg: matrd, Geom**)

2019b Electrical networks and hyperplane arrangements. *Adv. Appl. Math.* 110 (2019), 375–402. MR 3990769. Zbl 1427.52017. arXiv:1709.01227.

"Dirichlet hyperplane arrangements" are a kind of gain-graphic arrangement; *cf.* (2022a). [Annot. 16 Nov 2018.] (**gg: matrd, Geom**)

2019c Koszulness and supersolvability for Dirichlet arrangements. *Proc. Amer. Math. Soc.* 147 (2019), no. 11, 4937–4947. MR 4011525. Zbl 1427.52016. arXiv:-

1810.03518. (**GG: Matrd, Geom**)

2022a Matroids arising from electrical networks. *Adv. Appl. Math.* 137 (2022), art. 102331, 32 pp. MR 4388465. Zbl 1486.05038. arXiv:1809.10100.

"Dirichlet matroids" are complete lift matroids of a kind of biased graph. Dirichlet arrangements (2019b) are hyperplane representations of them. [Annot. 16 Nov 2018, 25 Oct 2019.] (**GG: Matrd, Geom**)

Shengxiang Lv [Shengxiang Lyu]

2015a The largest demigenus over all signatures on $K_{3,n}$. *Graphs Combin.* 31 (2015), 169–181. MR 3293474. Zbl 306.05086.

For $n \geq 3$, $D(K_{3,n})) := \max_\sigma d(K_{3,n}, \sigma) = 2\lfloor \frac{1}{4}(n-2) \rfloor + 0$ if $n \equiv 3 \pmod 4$, $+1$ otherwise ($d =$ demigenus, "Euler genus"). Most interesting feature: The maximum is attained (not uniquely) with only a single negative edge. [Annot. 7 Nov 2017.]. (**SG: Top**)

Shengxiang Lv & Zihan Yuan

2018a The smallest surface that contains all signed graphs on $K_{4,n}$. *Discrete Math.* 341 (2018), 732–747. MR 3754386. Zbl 1378.05084.

Thm.: $D(K_{4,n}) := \max_\sigma d(K_{4,n}, \sigma) = d(K_{4,n}) + 1$ for $n > 4$, $D(K_{4,4}) = 4$. As with $K_{2,n}$ and $K_{3,n}$ (*cf.* Lv (2015a)), the maximum is attained with only one negative edge, except for $K_{4,4}$, where the maximum requires a negative perfect matching. *Question.* Does this pattern continue for $K_{m,n}$, $m > 4$? [Annot. 7 Nov 2017.] (**SG: Top**)

Shengxiang Lyu [Shengxiang Lv]

See S.-X. Lv.

Yu.I. Lyubich

See G.R. Belitskii.

Baoli Ma

See M.J. Du.

Hongping Ma

See also L.Q. Wang and W. Wei.

2009a Bounds on the local bases of primitive, non-powerful, minimally strong signed digraphs. *Linear Algebra Appl.* 430 (2009), no. 2-3, 718–731. MR 2473178 (2009i:05100). Zbl 1151.05020. (**SD: Adj, qm**)

Jun Ma

See S.-D. Zhai.

Lijia Ma

See Q. Cai.

Hongping Ma & Zhengke Miao

2011a Imprimitive non-powerful sign pattern matrices with maximum base. *Linear Multilinear Algebra* 59 (2011), no. 4, 371–390. MR 2802520 (2012k:05170). Zbl 1221.15042. (**QM: SD: Adj**)

M. Ma

See D. Blankschtein.

Xiaobin Ma, Genhong Ding, & Long Wang

20xxa On the nullity and the matching number of unicyclic signed graphs. Submitted.

Further develops Y.Z. Fan, Wang, and Wang (2013a). Employs the matching number to express the adjacency nullity and to characterize adjacency rank 6 and 7 of a signed unicyclic graph. [Annot. 17 Dec 2011.] **(SG: Eig)**

Xiaobin Ma, Dein Wong, & Fenglei Tian

2016a Skew-rank of an oriented graph in terms of matching number. *Linear Algebra Appl.* 495 (2019), 242–255. MR 3462998. Zbl 1331.05181.

Φ := gain graph with gains $\pm$i. Problem: $\operatorname{rk} A(\Phi)$ [special case of adjacency rank for complex unit gain graphs; *cf.* Reff (2012a)]. Thm. 3.1(i): $2\mu - 2\xi \leq \operatorname{rk} A(\Phi) \leq 2\mu$, where μ := matching number of $\|\Phi\|$, ξ := cyclomatic number. Thm. 4.1 characterizes $2\mu - 2\xi = \operatorname{rk} A(\Phi)$. [Generalized in Fenglei Tian, Li Chen, & Rui Chu (2018a).] Dictionary: $\mathrm{i} = \sqrt{-1}$, $G := \|\Phi\|$, $\vec{G}$:= orientation making all gains i, skew adjacency matrix of $\vec{G} := \mathrm{i}^{-1}A(\Phi)$, skew rank of $\vec{G} = \operatorname{rk} A(\Phi)$, $\operatorname{sgn}(C) = \varphi(C)/\mathrm{i}^{|C|}$. [Annot. 10 May 2019.] **(gg: Adj, SG)**

A. Ma'ayan, A. Lipshtat, R. Iyengar, & E.D. Sontag

2008a Proximity of intracellular regulatory networks to monotone systems. *ET Systems Biol.* 2 (2008), no. 3, 103–112. **(SD, Biol: Dyn: Fr: Algor)**

Edita Máčajová

See also T. Kaiser and A. Kompišová.

Edita Máčajová & Ján Mazák

2013a On even cycle decompositions of 4-regular line graphs. *Discrete Math.* 313 (2013), 1697–1699. MR 3061005. Zbl 1277.05067.

Includes positive-circle decomposition (called "even cycle decomposition") of a signed graph; *cf.* Markström (2012a). Thm. 2: An infinite class of 4-regular, 4-connected Σ without such a decomposition. *Question* 1: Does every signed line graph of a cubic graph without isthmus, with even $\#E^-$, have such a decomposition? [Annot. 4 Jun 2017.] **(SG: Circ: Str)**

Edita Máčajová, André Raspaud, Edita Rollová, & Martin Škoviera

2016a Circuit covers of signed graphs. *J. Graph Theory* 81 (2016), no. 2, 120–133. MR 3433634. Zbl 1332.05066.

Dictionary: "signed circuit" (not a circuit with signs) = sign circuit = frame circuit. Improved in Kaiser, Lukot'ka, *et al.* (2019a). **(SG: Matrd)**

Edita Máčajová, André Raspaud, & Martin Škoviera

2014a The chromatic number of a signed graph. In: *Bordeaux Graph Workshop 2014*, pp. 29–30. LaBRI, Bordeaux, 2014. URL `http://bgw.labri.fr/2014/bgw2014-booklet.pdf`

Extended abstract of (2016a). [Annot. 19 Mar 2017.] **(SG: Col)**

† 2016a The chromatic number of a signed graph. *Electronic J. Combin.* 23 (2016), no. 1, art. P1.14, 10 pp. MR 3484719. Zbl 1329.05116. arXiv:1412.6349.

$\chi(\Sigma)$ Coloring is exactly as in Zaslavsky (1982b) [despite authors' statement to the contrary]. Chromatic number $\chi(\Sigma)$:= size of smallest set of signed colors [a better definition than Zaslavsky's].

Main results: Thm. 6 (Brooks' Theorem for signed simple graphs): $\chi(\Sigma) \leq \Delta(\Sigma)$ except for balanced (K_n, σ) and (C_{odd}, σ) and unbalanced $(C_{\text{even}}, \sigma)$. [Fleiner and Wiener (2016a) have a short list-coloring proof. Zając (20xxa) has a short, more general proof.] Prop. 4(i): $\Sigma \not\geq K_4 \implies \chi(\Sigma) \leq 3$. Thm. 10: If $|\Sigma|$ is planar, $\chi(\Sigma) \leq 5$, ≤ 4 if C_3-free, ≤ 3 if girth ≥ 5. *Conjecture*: Every planar signed graph is 4-colorable. [Annot. 14 Sept 2015, 7 May 2018, rev 9 Feb, 2 Jul 2020, 5 Jan 2021.] (**SG: Col: Invar**)

Edita Máčajová & Edita Rollová

2011a On the flow numbers of signed complete and complete bipartite graphs. *Electronic Notes Discrete Math.* 38 (2011), 591–596. Zbl 1274.05207. (**SG: Flows**)

2015a Nowhere-zero flows on signed complete and complete bipartite graphs. *J. Graph Theory* 78 (2015), no. 2, 108–130. MR 3293079. Zbl 1307.05096. (**SG: Flows**)

Edita Máčajová & Martin Škoviera

2011a Determining the flow numbers of signed eulerian graphs. *Electronic Notes Discrete Math.* 38 (2011), 585–590. Zbl 1274.05283. arXiv:1408.1703. (**SG: Flows**)

2015a Remarks on nowhere-zero flows in signed cubic graphs. *Discrete Math.* 338 (2015), no. 5, 809–815. MR 3303859. Zbl 1306.05087. (**SG: Flows**)

2016a Characteristic flows on signed graphs and short circuit covers. *Electronic J. Combin.* 223 (2016), no. 3, art. P3.30, 10 pp. MR 3558067. Zbl 1344.05066. arXiv:1407.5268. (**SG: Flows, matrd**)

2017a Odd decompositions of Eulerian graphs. *SIAM J. Discrete Math.* 31 (2017), no. 3, 1923–1936. MR 3691723. Zbl 1370.05126. arXiv:1607.00053. (**SG: Flows**)

2017b Nowhere-zero flows on signed Eulerian graphs. *SIAM J. Discrete Math.* 31 (2017), no. 3, 1937–1952. MR 3691724. Zbl 1370.05085. arXiv:1408.1703. (**SG: Flows**)

Edita Máčajová & Eckhard Steffen

2015a The difference between the circular and the integer flow number of bidirected graphs. *Discrete Math.* 338 (2015), no. 6, 866–867. MR 3318624. Zbl 1371.05112.

The difference can be ≥ 3. It is at most 3 if Bouchet's (1983a) conjecture is true. [Annot. 20 Jun 2022.] (**SG: Flows**)

Ben D. MacArthur

See R. Mulas.

Enzo Maccioni

See F. Barahona.

Gary MacGillivray

See also R.C. Brewster, E. Leclerc, and K. Hassani Monfared.

Gary MacGillivray, Ben Tremblay, & Jacqueline M. Warren

20xxa Colourings of m-edge-coloured graphs and switching. Submitted.

Great generalization of Brewster and Graves (2009a). (**gg(Gen), Cov**)

Amila P. Macodi-Ringia

See M.M. Mangontarum.

Jayakrishnan Madathil

See A. Das.

Bolette Ammitzbøll Madsen

See J.M. Byskov.

S. Madhumitha & Sudev Naduvath

2023a Graphs defined on rings: A review. *Mathematics* 11 (2023), art. 3643, 79 pp.

§2.2, "Signed graphs based on the unitary Cayley graphs". Unitary Cayley signed graph, pp. 37–38. Unitary addition Cayley graphs, pp. 48–50. Unit graph of rings, p. 59. Generalities, p. 72. [Annot. 25 Aug 2023.] (**SG: Algeb: Bal, Eig, LG: Exp**)

K.V. Madhusudhan

See also P.S.K. Reddy.

K.V. Madhusudhan & S. Vijay

2018a Distance divisor signed graphs. *IOSR J. Engineering* 8 (2018), no. 10, 25–27.

$DD(|\Sigma|)$ is signed using the canonical vertex signature, by $\sigma_{DD}(uv) = \mu_\sigma(u)\mu_\sigma(v)$. [Annot. 26 May 2021.] (**SG, VS, Sw**)

2019a A note on detour radial signed graphs. *Int. J. Math. Combin.* 2 (2019), 112–116.

$DR(|\Sigma|)$ is signed by $\sigma_{DR}(uv) = \mu_\sigma(u)\mu_\sigma(v)$. [Annot. 12 Jul 2019.] (**SG, VS**)

[A. El Maftouhi, Abdelakim El Maftouhi]

See A. El Maftouhi (under 'E').

Mohammad Maghasedi

See S. Akbari, S. Dalvandi, N. Kafai, and M. Souri.

Colton Magnant

See S. Fujita.

Thomas L. Magnanti

See R.K. Ahuja.

N.V.R. Mahadev

See also P.L. Hammer.

N.V.R. Mahadev & U.N. Peled

1995a *Threshold Graphs and Related Topics*. Ann. Discrete Math., Vol. 56. North-Holland, Amsterdam, 1995. MR 1417258 (97h:05001). Zbl 950.36502.

§8.3: "Bithreshold graphs" (from Hammer and Mahadev (1985a)), and §8.4: "Strict 2-threshold graphs" (from Hammer, Mahadev, and Peled (1989a)), characterize two types of threshold-like graph. In each, a different signed graph H is defined on $E(\Gamma)$ so that Γ is of the specified type iff H is balanced. (The negative part of H is the "conflict graph", Γ^*.) The reason is that one wants Γ to decompose into two subgraphs, and the subgraphs, if they exist, must be the two parts of the Harary bipartition of H. [Thus one also gets a fast recognition algorithm, though not the fastest possible, for the desired type from the fast recognition of balance.] **(SG: Bal: Appl)**

§8.5: "Recognizing threshold dimension 2." Based on Raschle and Simon (1995a). Given: $\Gamma \subseteq K_n$ such that Γ^* is bipartite. Orient $-K_n$ so that Γ-edges are introverted and the other edges are extroverted. Their "alternating cycle" is a coherent closed walk in this orientation. Let us call it "black" (in a given black-white proper coloring of Γ^*) if its Γ-edges are all black. Thm. 8.5.2 (Hammer, Ibaraki, and Peled (1981a)): If there is a black coherent closed walk in E_0, then there is a coherent tour (closed trail) of length 6 (which is a pair of joined triangles or a hexagon—their AP_5 and AP_6). Thm. 8.5.4: Given that there is no black coherent hexagon, one can recolor quickly so there is no black coherent 6-tour. Thm. 8.5.9: Given that there is no 'double' coherent hexagon (the book's "double AP_6"), one can recolor quickly so there is no black coherent hexagon. Thm. 8.5.28: Any 2-coloring of Γ^* can be quickly transformed into one with no 'double' coherent hexagon. [*Question.* Can any of this, especially Thm. 8.5.2, be generalized to arbitrary oriented all-negative graphs B? Presumably, this would require first defining a conflict graph on the introverted edges of B. More remotely, consider generalizing to bidirected complete or arbitrary graphs.] **(par: ori, Algor)**

§9.2.1: "Threshold signed graphs." See Benzaken, Hammer, and de Werra (1981a), (1985a). In this version it's not clear where the signs are! (and their role is trivial). Real weights are assigned to the vertices and an edge receives the sign of the weight product of its endpoints. **(sg: bal)**

1988a Strict 2-threshold graphs. *Discrete Appl. Math.* 21 (1988), 113–131. MR 0959424 (89i:05234). Zbl 0658.05063.

Uses the auxiliary signed graph of Hammer and Mahadev (1985a). [Annot. 22 Mar 2017.] **(SG: Appl: Bal)**

John Maharry, Neil Robertson, Vaidy Sivaraman, & Daniel Slilaty

2017a Flexibility of projective-planar embeddings. *J. Combin. Theory Ser. B* 122 (2017), 241–300. MR 3575205. Zbl 1350.05020.

§1: The flexibility is connected to duality of signed-graphic frame matroids by Slilaty (2005a). [Annot. 20 Dec 2011.] **(SG: Top)**

Ali Ridha Mahjoub

See F. Barahona and D. Cornaz.

J.M. Maillard

See T. Garel and J. Vannimenus.

Hamid Reza Maimani

See S. Akbari and E. Ghorbani.

H.R. Maimani, L. Parsaei-Majd, M.R. Pournaki, & M. Poursoltani

2023a On the structure of matroids arising from the gain graphs. *Discrete Math.* 346 (2023), no. 12, art. 113637. MR 4632036. **(GG: Matrd)**

[Leila Parsaei Majd]

See L. Parsaei-Majd (under 'P').

Arezoo Majidi

See E. Ghorbani.

Konstantin Makarychev

See N. Alon.

Yury Makarychev

See N. Alon.

J.A. Makowsky

See also E. Fischer and M. Lotz.

2001a Colored Tutte polynomials and Kauffman brackets for graphs of bounded tree width. In: *Proceedings of the Twelfth Annual ACM-SIAM Symposium on Discrete Algorithms* (Washington, D.C., 2001), pp. 487–495. Soc. for Industrial and Appl. Math., Philadelphia, Pa., 2001. MR 1958441 (no rev). Zbl 988.05087.
See (2005a). (**SGc: Invar: Algor**)

2005a Coloured Tutte polynomials and Kauffman brackets for graphs of bounded tree width. *Discrete Appl. Math.* 145 (2005), no. 2, 276–290. MR 2113147 (2005m:05214). Zbl 1084.05505.
Polynomial-time computability for edge-colored graphs of bounded tree width. [Also see Traldi (2006a).] (**SGc: Gen: Invar: Algor, Knot**)

A.Z. Maksymowicz

See M.J. Krawczyk.

Krzysztof Malarz

See M.J. Krawczyk and B. Tadić.

H.A. Malathi & H.C. Savithri

2010a A note on jump symmetric n-sigraph. *Int. J. Math. Combin.* 2010 (2010), vol. 2, 65–67. Zbl 1216.05051. (**SG(Gen): LG**)

M. Malek-Zavarei & J.K. Aggarwal

1971a Optimal flow in networks with gains and costs. *Networks* 1 (1971), 355–365. MR 0295831 (45 #4896). Zbl 236.90026. (**GN: bal**)

Piotr Malicki

See M. Kaniecki.

Günther Malle

1982a On maximum bipartite subgraphs. *J. Graph Theory* 6 (1982), 105–113. MR 655195 (83i:05047). Zbl 493.05035.
Frustration index $l(-\Gamma)$. Criterion for unique minimum deletion set in $-\Gamma$. All-negative Petersen and dodecahedron characterized via minimum deletion sets. [*Problem*. Extend to other signed graphs.] [Annot. 28 Jun 2022.] (**sg: par: Fr**)

Fragkiskos D. Malliaros, Christos Giatsidis, Apostolos N. Papadopoulos, & Michalis Vazirgiannis

2020a The core decomposition of networks: theory, algorithms and applications. *VLDB J.* 29 (2020), 61–92. HAL hal-01986309.
§2.2.3, "Signed graphs", defines "core" := maximum subgraph with

$d^+_{\text{in,out}}$, $d^-_{\text{in,out}} \geq$ given bounds. [Annot. 2 Feb 2020.] (**SD**)

Sudipta Mallik

See also R. Hessert and J. Ipsen.

20xxa Matrix tree theorem for the net Laplacian matrix of a signed graph. *Linear Multilinear Algebra* (to appear). arXiv:2210.03711.

The theorem arises by introducing $\sqrt{-1}$ into the incidence matrix of an orientation of $|\Sigma|$, not Σ. Unusually for oriented $|\Sigma|$, it is not about digraph properties. [Annot. 28 Dec 2022.] (**SG: Incid, Lap(Gen)**)

R.B. Mallion

See A.C. Day.

Devlin Mallory

See also J. Brown.

Devlin Mallory, Abigail Raz, Christino Tamon, & Thomas Zaslavsky

2013a Which exterior powers are balanced? *Electronic J. Combin.* 20 (2013), no. 2, art. P43, 14 pp. MR 3084585. Zbl 1266.05047. arXiv:1301.0973. (**SG: Bal**)

C.L. Mallows & N.J.A. Sloane

1975a Two-graphs, switching classes and Euler graphs are equal in number. *SIAM J. Appl. Math.* 28 (1975), 876–880. MR 0427128 (55 #164). Zbl 275.05125, (Zbl 297.05129).

Thm. 1: For all n, the number of unlabelled two-graphs of order n [i.e., switching isomorphism classes of signed K_n's] equals the number of unlabelled even-degree simple graphs on n vertices. The key to the proof is that a permutation fixing a switching class fixes a signing in the class. (Seidel (1974a) proved the odd case, where the fixing property is simple.) Thm. 2: The same for the labelled case. [More in Cameron (1977b), Cameron and Wells (1986a), Cheng and Wells (1984a), (1986a)]

To prove the fixing property they find the conditions under which a given permutation π of $V(K_n)$ and switching set C fix some signed K_n. [More in Harries and Liebeck (1978a), Liebeck (1982a), and Cameron (1977b).] (**TG: Aut, Enum**)

Aleksander Malnič

See also I. Kovács.

1998 Group actions, coverings and lifts of automorphisms. *Discrete Math.* 182 (1998), 203–218. MR 1603687 (98k:57006). Zbl 893.57001.

Gain graphs ("voltage graphs") are the tool. (**GG: Cov, Aut**)

2002a Action graphs and coverings. *Discrete Math.* 244 (2002), 299–322. MR 1844040 (2003b:05081). Zbl 996.05067.

Gain graphs ("voltage graphs") and lifting automorphisms of their underlying graphs are a main example. [Annot. 11 Jun 2012.] (**GG: Aut, Cov**)

Aleksander Malnič, Roman Nedela, & Martin Škoviera

2000a Lifting graph automorphisms by voltage assignments. *European J. Combin.* 21 (2000), 927–947. MR 1787907 (2001i:05086). Zbl 966.05042.

Automorphisms of gain graphs that lift to the covering graph. [Annot. 18 Apr 2012.] (**GG: Cov, Aut**)

2002a Regular homomorphisms and regular maps. *European J. Combin.* 23 (2002), 449–461. MR 1914482 (2003g:05045). Zbl 1007.05062.

§6, "Invariance of voltage assignments", concerns automorphisms of a gain graph that preserve the gains, in connection with lifting automorphisms to the regular covering graph. The treatment is via maps as gain graphs with rotation systems. [Annot. 18 Apr 2012.] (**GG: Cov, Aut**)

John W. Mamer

See R.D. McBride.

Rachel Manber

See also R. Aharoni and V. Klee.

1982a Graph-theoretical approach to qualitative solvability of linear systems. *Linear Algebra Appl.* 48 (1982), 457–470. MR 0683238 (84g:68054). Zbl 511.15008. (**SD, QM: QSol**)

Rachel Manber & Jia-Yu Shao

1986a On digraphs with the odd cycle property. *J. Graph Theory* 10 (1986), 155–165. MR 0890220 (88i:05090). Zbl 593.05032. (**SD, SG: Par**)

Federico Mancini

See H.L. Bodlaender.

Santanu Mandal & Ranjit Mehatari

20xxa On the Seidel spectrum of threshold graphs. Submitted. arXiv:2101.03364.

[*Cf.* Milica Anđelić, Tamara Koledin, & Zoran Stanić (2019a).] (**SG: KG: Adj: Eig**)

Santanu Mandal, Ranjit Mehatari, & Kinkar Chandra Das

20xxa On the spectrum and energy of Seidel matrix for chain graphs. Submitted. arXiv:2205.00310. (**SG: KG: Adj: Eig**)

Tabitha Agnes Mangam

See D. Antoney.

Mahid M. Mangontarum, Amila P. Macodi-Ringia, & Normalah S. Abdulcarim

2014a The translated Dowling polynomials and numbers. *Int. Scholarly Res. Notices Discrete Math.* 2014 (2014), art. 678408, 8 pp. Zbl 1460.05014.

Introduces 0-Dowling polynomials $D_{n,m,0}(x) := \sum_k W_{m,0}(n,k)x^k$; *cf.* Belbachir and Bousbaa (2013a) and 0-Dowling numbers $D_{n,m,0}(1)$. Formulas, identities, integrals, etc., for 0-Whitney and 0-Dowling. Dictionary: "translated Dowling" = 0-Dowling [so named and coordinated with r-Whitney and r-Dowling numbers in Gyimesi and Nyul (2018a)]; polynomials and numbers $\widetilde{D}_{(m)}(x)$, $\widetilde{D}_{(m)}(n) = D_{n,m,0}(x)$, $D_{n,m,0}(1)$ [notation of Gyimesi–Nyul]. [Annot. 28 May 2018.] (**gg: matrd Invar**)

Silviu Maniu

See C. Giatsidis.

Anna Mańka

See A. Mańka-Krasoń.

Anna Mańka-Krasoń & Krzysztof Kułakowski

2009a Magnetism of frustrated regular networks. *Acta Phys. Polonica B* 40 (2009), no. 5, 1455–1461.

2010a Frustration and collectivity in spatial networks. In: Roman Wyrzykowski, *et al.*, eds., *Parallel Processing and Applied Mathematics* (8th Int. Conf., PPAM 2009, Wroclaw, 2009), Revised Selected Papers, Part II, pp. 539–546. Lect. Notes in Computer Sci., Vol. 6068. Springer, Berlin, 2010. arXiv:0904.4002. (**sg: par: Fr**)

Anna Mańka, Krzysztof Malarz, & Krzysztof Kullłakowski

2007a Clusterization, frustration and collectivity in random networks. *Int. J. Modern Phys. C* 18 (2007), no. 11, 1765–1773. Zbl 1170.82371.

Computer experiments on physics aspects of all-negative signed graphs. [Annot. 14 Feb 2011.] (**Par: Fr**)

R. Lawrence Joseph Manoharan

See P.L. Rozario Raj.

Y. Manoussakis

See A. El Maftouhi.

Rafael Esteves Mansano, Luiz Emilio Allem, Renata Raposo Del-Vecchio, & Carlos Hoppen

20xxa Balanced portfolio via signed graphs and spectral clustering in the Brazilian stock market. *Quality & Quantity* (to appear).

Cf.. Harary, Lim, and Wunsch (2002a). Applies B.D. Acharya (1980a). [Annot. 7 Feb 2022.] (**Appl: SG, WG: Adj: Eig, Clu**)

Toufik Mansour

See J.L. Gross.

Vassily Olegovich Manturov

2010a *Virtual Knots: The State of the Art.* (In Russian.) NIC, 2010.

Original Russian ed. of Manturov and Ilyutko (2013a). (**SGc, VSc: Exp**)

Vassily Olegovich Manturov & Denis Petrovich Ilyutko

2013a *Virtual Knots: The State of the Art.* Ser. Knots Everything, Vol. 51. World Scientific, New Jersey, 2013. MR 2986036. Zbl 1270.57003.

English trans. of Manturov (2010a). Ch. 9, "Theory of graph-links": 0/1-labelled and sign-colored chords in chord diagrams. 0/1-labelled and sign-colored vertices. 0/1-labelled and sign-colored graphs. [Annot. 13 Sept 2013.] (**SGc, VSc: Exp**)

Haik Manukian

See also Y.R. Pei.

Haik Manukian, Yan Ru Pei, Sean R.B. Bearden, & Massimiliano Di Ventra

2020a Mode-assisted unsupervised learning of restricted Boltzmann machines. *Commun. Phys.* 3 (2020), art. 105, 8 pp. arXiv:2001.05559. (**SG: Fr: Phys, State**)

Yanqi Mao

See X.Y. Yuan.

Zeev Maoz, Lesley G. Terris, Ranan D. Kuperman, & Ilan Talmud

2007a What is the enemy of my enemy? Causes and consequences of imbalanced international relations, 1816–2001. *J. Politics* 69 (2007), no. 1, 100–115.
(PsS, SG: Bal)

Dănuţ Marcu

[I cannot vouch for the authenticity of these articles. See MR 1324075 (97a:-05095) and Zbl 701.51004. Also see MR 1038400 (92a:51002), MR 1094344 (92b:51026), MR 1107637 (92h:11026), MR 1427830 (97k:05050); and Marcu (1981b).]

1980a On the gradable digraphs. *An. Ştiinţ. Univ. "Al. I. Cuza" Iaşi Secţ. I a Mat. (N.S.)* 26 (1980), 185–187. MR 0582484 (82k:05056) (*q.v.*). Zbl 438.05032.
See Harary, Norman, and Cartwright (1965a) for the definition.
(GD: bal)

1981a No tournament is gradable. *An. Univ. Bucureşti Mat.* 30 (1981), 27–28. MR 0643669 (83c:05069). Zbl 468.05028.
See Harary, Norman, and Cartwright (1965a) for the definition. The tournaments of order 3 are [trivially] not gradable, whence the titular theorem. **(GD: bal)**

1981b Some results concerning the even cycles of a connected digraph. *Studia Univ. Babeş-Bolyai Math.* 26 (1981), 24–28. MR 0654119 (83e:05058). Zbl 479.05032.
§1, "Preliminary considerations", appears to be an edited, unacknowledged transcription of parts of Harary, Norman, and Cartwright (1965a) (or possibly (1968a)), pp. 341–345. Wording and notation have been modified, a trivial corollary has been added, and some errors have been introduced; but the mathematics is otherwise the same down to details of proofs. §2, "Results", is largely a list of the corollaries resulting from setting all signs negative. The exception is Thm. 2.5, for which I am not aware of a source; however, it is simple and well known. **(sg(SD): Bal)**

1987a Note on the matroidal families. *Riv. Math. Univ. Parma* (4) 13 (1987), 407–412. MR 0977694 (89k:05025).
Matroidal families of (multi)graphs (see Simões-Pereira (1973a)) correspond to functions on all isomorphism types of graphs that are similar to matroid rank functions, e.g., submodular. This provides insight into matroidal families, e.g., it immediately shows there are infinitely many.
(MtrdF: Bic, ECyc: Gen)

Adam W. Marcus, Daniel A. Spielman, & Nikhil Srivastava

2013a Interlacing families I: Bipartite Ramanujan graphs of all degrees. In: *2013 IEEE 54th Annual Symposium on Foundations of Computer Science* (Proc., FOCS 2013, Berkeley, Calif.), pp. 529–537. IEEE Computer Soc., Los Alamitos, Calif., 2013. MR 3246256. Zbl 1334.68008 (book).
Preliminary version of (2015a); the latter has slight (and occasionally important) additions, deletions, and corrections. [Annot. 18 Oct 2015.]
(SG: Cov, Adj: Eig)

2015a Interlacing families I: Bipartite Ramanujan graphs of all degrees. *Ann. Math.* (2) 182 (2015), no. 1, 307–325. MR 3374962. Zbl 1316.05066.

Dictionary: "2-lift" of Σ = signed covering graph $\tilde{\Sigma}$. "Double-cover" of Γ = that of $-\Gamma$. **(SG: Cov, Adj: Eig)**

Grzegorz Marczak

See also M. Kaniecki.

Grzegorz Marczak, Daniel Simson & Katarzyna Zając

2013a On computing non-negative loop-free edge-bipartite graphs. In: Nikolaj Björner *et al.*, eds., *15th International Symposium on Symbolic and Numeric Algorithms for Scientific Computing* (SYNASC 2013, Timisoara, Romania, 2013), pp. 68–75. IEEE, 2013. **(SG)**

Stuart Margolis, John Rhodes, & Pedro V. Silva

† 2021a On the Dowling and Rhodes lattices and wreath products. *J. Algebra* 578 (2021), 55–91. MR 4234797. Zbl 1461.05026. arXiv:1710.05314.

The Dowling lattice (1973b) $Q_n(\mathfrak{G})$ of a group $\mathfrak{G}$ and the Rhodes lattice in semigroup theory are constructed from the same complete $\mathfrak{G}$-gain graph $\mathfrak{G}K_n^\bullet$ through somwhat opposite orderings. The frame matroid $\mathbf{F}(\mathfrak{G}K_n^\bullet)$ and lift matroid $\mathbf{L}(\mathfrak{G}K_n)$ are recovered from the Dowling and Rhodes lattices via "boolean representable simplicial complexes" (*cf.* Rhodes and Silva, *Boolean Representations of Simplicial Complexes and Matroids*, 2015). [Annot. 7 Apr 2021.] **(GG, Matrd)**

Enzo Marinari

See also S. Cabasino, B. Coluzzi, M. Falcioni, and J. Lukic.

Enzo Marinari, Giorgio Parisi, & Felix Ritort

1995a The fully frustrated hypercubic model is glassy and aging at large D. *J. Phys. A* 28 (1995), 327–334.

Ising (spins, i.e., vertex values, $\in \mathbb{S}^0 = \{+1, -1\}$) and XY (spins $\in \mathbb{S}^1$, i.e., complex units) models behave differently on a fully frustrated signed hypercube graph Q_D (all squares are negative). Numerical study of Ising spins of two such signatures: $\sigma_1(x, x + e_\mu) = (-1)^{x_1+\cdots+x_{\mu-1}}$, while σ_2 ["simplex" in the construction must mean hypercube] is from Derrida, Pomeau, Toulouse, and Vannimenus (1979a); "with identical results". [Reason: $\Sigma_1 \cong \Sigma_2$ under the coordinate transformation $i \leftrightarrow D + 1 - i$.] Based on simulations with $D \leq 47$, Ising ground states seem to be few and hard to find. Near-ground states are easier to find but, apparently, tend to be far from ground states.

For positive temperature T, as $A(\Sigma)$ ("interaction matrix $J_{x,y}$") is orthogonal [up to scaling], one can approximate by averaging over orthogonal adjacency matrices.

In simulations with XY spins the ground state is highly accessible.

Dictionary: "ground state" = switching with minimum $\#E^-$. [Annot. 19 Jun 2012.] **(Phys, SG)**

1995b Replica theory and large-D Josephson junction hypercubic models. *J. Phys. A* 28 (1995), 4481–4503. MR 1352169 (96g:82032). Zbl 925.82088. arXiv:cond-mat/9502067.

Physics on hypercube Q_D with complex unit gains and three types of spin, after Parisi (1994a), via simulations for $3 \leq D \leq 16$. [Annot. 19 Jun 2012.] **(Phys, gg)**

2000a On the 3D Ising spin glass. *J. Phys. A* 27 (1994), no. 8, 2687–2708. MR 1280826 (no rev). arXiv:cond-mat/9310041. **(Phys: SG)**

Fabrizio Marinelli & Angelo Parente

2016a A heuristic based on negative chordless cycles for the maximum balanced induced subgraph problem. *Computers Oper. Res.* 69 (2016), 68–78. MR 3458032. **(SG: Fr: VS, Algor)**

A.V. Markovskiĭ

1997a Analysis of the structure of signed directed graphs. (In Russian.) *Izv. Akad. Nauk Teor. Sist. Upr.* 1997 (1997), no. 5, 144–149. Eng. trans., *J. Comput. Systems Sci. Int.* 36 (1997), no. 5, 788–793. MR 1679025 (2000a:05099). Zbl 898.05078. **(SD: WG)**

Harry Markowitz

1955a Concepts and computing procedures for certain X_{ij} programming problems. In: H.A. Antosiewicz, ed., *Proceedings of the Second Symposium in Linear Programming* (Washington, D.C., 1955), Vol. II, pp. 509–565. Nat. Bur. Standards of U.S. Dept. of Commerce, and Directorate of Management Analysis, DCS Comptroller, HQ, U.S. Air Force, 1955. Sponsored by Office of Scientific Res., Air Res. and Develop. Command. MR 0075673 (17, 789).

Also see RAND Corporation Paper P-602, 1954. **(GN: matrd(bases))**

Klas Markström

2012a Even cycle decompositions of 4-regular graphs and line graphs. *Discrete Math.* 312 (2012), no. 17, 2676–2681. MR 2935419. Zbl 1246.05087.

Can $-\Gamma$ be decomposed into positive circles? Studied for 4-regular graphs and line graphs of cubic graphs. *Cf.* C.Q. Zhang (1994a), Máčajová and Mazák (2013a) (especially), and Liu–You–Cui (2020a). Dictionary: "oddness" of $\Gamma = \min \#$ negative circles in a 2-factor of $-\Gamma$. [Annot. 12 Jan 2012, rev 11 Nov 2020.] **(par: Str)**

Clifford W. Marshall

1971a *Applied Graph Theory.* Wiley-Interscience, New York, 1971. MR 0323595 (48 #1951). Zbl 226.05101.

"Consistency of choice" discusses signed graphs, pp. 262–266. **(SG: Bal, Adj: Exp)**

T.H. Marshall

See N. Alon.

Matteo Marsili

See G.C.M.A. Ehrhardt.

Florian Martin

2017a Frustration and isoperimetric inequalities for signed graphs. *Discrete Appl. Math.* 217, no. 2, 276–285. MR 3581352. Zbl 1358.05133. **(SG: Fr)**

O.C. Martin

See J.-P. Bouchaud and J. Lukic.

Jeremy L. Martin

See J. Hallam.

Samuel Martin

See S. Ahmadizadeh.

V. Martin-Mayor

See L.A. Fernández.

José Martínez-Bernal, Miguel A. Valencia-Bucio, & Rafael H. Villarreal

2020a Linear codes over signed graphs. *Designs Codes Cryptogr.* 88 (2020), no. 2, 273–296. MR 4055232. Zbl 1452.94117. arXiv:1904.09487. (**SG: Incid, Matrd, Fr**)

Xavier Martínez-Rivera

See S. Butler.

Enide Andrade Martins

See N.M.M. de Abreu and I. Gutman.

Dragan Marušič

See I. Kovács.

Tatsuya Maruta

See Y. Koga.

Seth A. Marvel, Jon Kleinberg, Robert D. Kleinberg, & Steven H. Strogatz

2011a Continuous-time model of structural balance. *Proc. Nat. Acad. Sci. (U.S.A.)* 108 (2011), no. 5, 1771–1776. arXiv:1010.1814.

A differential equation model of balancing processes, based on Kułakowski, Gawroński, & Gronek (2005a). Conclusion: Final state is balance. *Cf.* (2011b). [See commentary, Srinivasan (2011a).] [Annot. 6 Feb 2011.] (**SG: KG: Fr, Dyn**)

2011b Supporting information. *Proc. Nat. Acad. Sci. (U.S.A.)* 108 (2011), URL `http://www.pnas.org/cgi/doi/10.1073/pnas.1013213108`.

The mathematical support for (2011a). [Annot. 6 Feb 2011.] (**SG: KG: Fr, Dyn**)

Seth A. Marvel, Steven H. Strogatz, & Jon M. Kleinberg

2009a Energy landscape of social balance. *Phys. Rev. Lett.* 103 (2009), art. 198701, 4 pp.

Signed complete graphs under Antal, Krapivsky, and Redner's (2005a) "constrained triad dynamics": Imbalance measured by triangles; an edge is negated if it is in more negative than positive triangles. Paley graphs P give K_P with equally many positive and negative triangles on each edge (normalized "energy" $= 0$). Other such states exist. [Zyga (2009a) gives a popular exposition.] [*Questions.* Do unbalanced locally minimal regions with more than one point (graph) exist? How does the landscape look for switching classes?] [Annot. 5 May 2010, 26 Jan 2011.] (**SG: KG: Fr, State: Dyn**)

[Enzo M. Li Marzi]

See E.M. Li Marzi (under 'L').

Andrew J. Mason

See S. Aref.

J.H. Mason

1977a Matroids as the study of geometrical configurations. In: *Higher Combinatorics* (Proc. NATO Adv. Study Inst., Berlin, 1976), pp. 133–176. NATO Adv. Study

Inst. Ser., Ser. C: Math. Phys. Sci., Vol. 31. Reidel, Dordrecht, 1977. MR 0519783 (80k:05037). Zbl 358.05017.

§§2.5–2.6: "The lattice approach" and "Generalized coordinates", pp. 172–174, propose a purely matroidal and more general formulation of Dowling's (1973b) construction of his lattices. **(gg(Gen): Matrd)**

1981a Glueing matroids together: A study of Dilworth truncations and matroid analogues of exterior and symmetric powers. In: *Algebraic Methods in Graph Theory* (Proc., Szeged, 1978), Vol. II, pp. 519–561. Colloq. Math. Soc. János Bolyai, 25. North-Holland, Amsterdam, 1981. MR 0642060 (84i:05041). Zbl 477.05022.

Dowling matroids are an example in §1. **(gg: Matrd)**

R. Masoumi, F. Oloomi, S. Sajjadi, A.H. Shirazi, & G.R. Jafari

2022a Modified Heider balance on Erdös-Rényi networks. *Phys. Rev. E* 106 (2022), art. 034309. arXiv:2204.04469.

arXiv title: "Modified Heider balance on sparse random networks". **(SG: Fr: Phys)**

Nancy Matar, Lon H. Mitchell, & Sivaram K. Narayan

2022a On the minimum semidefinite rank of signed graphs. *Linear Algebra Appl.* 642 (2022), 73–85. MR 4381684.

Cf. Al-Aqtash (2014a). **(SG: Adj: QM)**

A.M. Mathai & Thomas Zaslavsky

2012a On adjacency matrices and descriptors of signed cycle graphs. Set Valuations, Signed Graphs and Geometry (IWSSG-2011, Int. Workshop, Mananthavady, Kerala, 2011). *J. Combin. Inform. System Sci.* 37 (2012), no. 2-4, 359–372. Zbl 1301.05157. arXiv:1303.3082.

Eigenvalues of $A(C_n, \sigma)$ (previously stated by Fowler (2002a); equivalent to Fan (2007a)'s Laplacian eigenvalues) by an elegant matrix method. [*Cf.* Germina and Hameed (2010a).] Some ways to partially or wholly distinguish different signatures of C_n are compared. [Annot. 6 Sept 2010, 7 Nov 2013, 13 Jan 2015.] **(SG: Eig)**

Albin Mathew

See also S. Hameed.

Albin Mathew & Germina K A

20xxa Mycielskian of signed graphs. Submitted. arXiv:2302.00946.

Does not respect balance or switching. Thm. 2.9: Chromatic number compared with that of Σ. §2.3: Matrices in terms of those of Σ. [Annot. 14 Feb 2023.] **(SG: Bal, Sw, Col, Adj, Incid, Lap)**

20xxb Vector valued switching in the products of signed graphs. *Commun. Combin. Optim.* (to appear). arXiv:2306.10132.

Cf. Hameed, Mathew, Germina, and Zaslavsky (20xxa). Inequalities for balancing dimension of Cartesian, tensor, strong, and lexicographic products. [Annot. 16 Jun 2023.] **(SG: Sw(Gen))**

Albin Mathew, T.V. Shijin, Roshni T Roy, P. Soorya, Shahul K. Hameed, & K.A. Germina

2023a Social balance - a signed detour distance analysis. *J. Math. Sociology* 47 (2023), no. 3, 244–254.

Similar to Hameed, Shijin, *et al.* (2021a) but for detour distance matrices, meaning longest vw-paths instead of shortest. [Annot. 31 Dec 2021.] **(SG: WG, Adj, Bal, Eig, Sw)**

Anisha Jean Mathias, V. Sangeetha, & Mukti Acharya

2020a Restrained domination in signed graphs. *Acta Univ. Sapientiae, Math.* 12 (2020), no. 1, 155–163. MR 4126665. Zbl 1452.05078.

Restrained dominating set D means $E \setminus (E{:}D^c)$ is balanced. [Annot. 3 Sept 2020.] **(SG: Dom)**

2023a Restrained critical and abundant signed graphs. *Adv. Appl. Discrete Math.* 28 (2023), no. 1, 49–68. **(SG)**

20xxa Critical concepts of restrained domination in signed graphs. *Discrete Math. Algorithms Appl.* (in press).

Cf. (2020a). Effect of edge deletion or addition, esp. in examples. [Annot. 1 Oct 2021.] **(SG: Dom)**

20xxb Restrained edge critical abundant signed graphs. Submitted. **(SG)**

R.A. Mathon

See F.C. Bussemaker and Seidel (1991a).

Tetsushi Matsui, Akihiro Higashitani, Yuuki Nagazawa, Hidefumi Ohsugi, & Takayuki Hibi

2011a Roots of Ehrhart polynomials arising from graphs. *J. Algebraic Combin.* 34 (2011), 721–749. MR 2842918 (2012i:52024). Zbl 1229.05122. arXiv:1003.5444.

Edge polytope (*cf.* Ohsugi and Hibi (1998a)), and introducing symmetric edge polytope of graph = convex hull of vectors for both directions of each edge. [Annot. 4 Jun 2022.] **(sg: par: Geom)**

Tatsuya Matsuoka & Shun Sato

2020a Making bidirected graphs strongly connected. *Algorithmica* 82 (2020), no. 4, 787–807. MR 406879. Zbl 1441.05127. arXiv:1709.00824. **(sg: Ori: Str)**

Hisayoshi Matsuyama

See M. Iri and J. Shiozaki.

Amelia R.W. Mattern

2020a *Deficiency in Signed Graphs.* Doctoral dissertation, Binghamton University, 2020. MR 4144528 (no rev).

Contains (2021a) and (20xxb). [Annot. 28 May 2020.] **(SG: Col: Algor, Sw)**

2021a The chromatic number of joins of signed graphs. *Graphs Combin.* 37 (2021), 2723–2735. MR 4338758. Zbl 1479.05138. arXiv:2005.14337.

$\chi(\Sigma_1 \vee_+ \Sigma_2)$ (chromatic number of all-positive join) derived from chromatic numbers and deficiencies of Σ_1 and Σ_2, with an exception that is partially characterized. [Annot. 27 May 2020.] **(SG: Col)**

20xxb Deficiency in signed graphs. Submitted. arXiv:2005.14336.

Coloring as in Zaslavsky (1982b), $\chi(\Sigma)$ as in Máčajová *et al.* (2014a). The deficiency of a proper coloration is the number of unused colors, which may be positive even for a minimal coloration. Deficiency set $\mathrm{Def}(\Sigma) := \{$deficiencies of minimal colorations$\}$. *Problems*: Find min, max deficiencies $m(\Sigma)$ and $M(\Sigma)$. Solved for $\chi = 2$. Solved algorithmically for M, complexity unknown for m, if $\chi = 3$. Open for $\chi > 3$.

Switching deficiency of Σ: easy. [Annot. 27 May 2020.]

(**SG: Col: Algor, Sw**)

Laurence R. Matthews

1977a Bicircular matroids. *Quart. J. Math. Oxford* (2) 28 (1977), 213–227. MR 0505702 (58 #21732). Zbl 386.05022.

Thorough study of bicircular matroids, introduced by Klee (1971a) and Simões-Pereira (1972a). [*Cf.* Zaslavsky (1982d), (2007a), Wagner (1985a), Coullard, del Greco, and Wagner (1991a), Shull, Shuchat, Orlin, and Lepp (1993a), Giménez, de Mier, and Noy (2005a), McNulty and Neudauer (2008a), Sivaraman (2014a).] (**Bic**)

1978a Properties of bicircular matroids. In: *Problèmes Combinatoires et Théorie des Graphes* (Colloq. Int., Orsay, 1976), pp. 289–290. Colloques Int. du CNRS, 260. Editions du C.N.R.S., Paris, 1978. MR 0539994 (81a:05030). Zbl 427.05021.

Summary of (1977a). (**Bic**)

1978b Matroids on the edge sets of directed graphs. In: *Optimization and Operations Research* (Proc. Workshop, Bonn, 1977), pp. 193–199. Lect. Notes in Economics and Math. Systems, 157. Springer-Verlag, Berlin, 1978. MR 0511340 (80a:05103). Zbl 401.05031.

Announcement of (1978c). (**gg: Matrd**)

1978c Matroids from directed graphs. *Discrete Math.* 24 (1978), 47–61. MR 0522733 (81e:05055). Zbl 388.05005.

Invents frame matroids of poise, modular poise, and antidirection bias on a digraph. (**gg: Matrd**)

1979a Infinite subgraphs as matroid circuits. *J. Combin. Theory Ser. B* 27 (1979), 260–273. MR 0554294 (81e:05056). Zbl 433.05018. (**Bic: Gen**)

Laurence R. Matthews & James G. Oxley

1977a Infinite graphs and bicircular matroids. *Discrete Math.* 19 (1977), 61–65. MR 0498193 (58 #16348). Zbl 386.05021. (**Bic**)

Alexey Matveev

See A. Proskurnikov.

Jean François Maurras

1972a Optimization of the flow through networks with gains. *Math. Programming* 3 (1972), 135–144. MR 0314441 (47 #2993). Zbl 243.90048. (**GN: Matrd**)

Mano Ram Maurya, Raghunathan Rengaswamy, & Venkat Venkatasubramanian

2003a A systematic framework for the development and analysis of signed digraphs for chemical processes. 1. Algorithms and analysis. *Indust. Eng. Chem. Res.* 42 (2003), 4789–4810. (**SD: QSta: Algor, Appl**)

2003b A systematic framework for the development and analysis of signed digraphs for chemical processes. 2. Control loops and flowsheet analysis. *Indust. Eng. Chem. Res.* 42 (2003), 4811–4827. (**SD: QSta: Algor, Appl**)

2006a A signed directed graph-based systematic framework for steady-state malfunction diagnosis inside control loops. *Chem. Engineering Sci.* 61 (2006), 1790–1810. **(SD: Appl)**

2007a A signed directed graph and qualitative trend analysis-based framework for incipient fault diagnosis. *Chem. Engineering Res. Design* 85 (2007), no. 10, 1407–1422. **(SD: Appl)**

John S. Maybee

See also L. Bassett, H.J. Greenberg, F. Harary, C.R. Johnson, G.M. Lady, and T.J. Lundy.

1974a Combinatorially symmetric matrices. *Linear Algebra Appl.* 8 (1974), 529–537. MR 0453583 (56 #11845). Zbl 438.15021 (no rev).

Survey and simple proofs. **(QM: sd, gg, QSta)(Exp)**

1980a Sign solvable graphs. *Discrete Appl. Math.* 2 (1980), 57–63. MR 0569486 (81g:05063). Zbl 439.05024. **(SD: QM: QSol)**

1981a Sign solvability. In: Harvey J. Greenberg and John S. Maybee eds., *Computer-Assisted Analysis and Model Simplification* (Proc. Sympos., Boulder, Col., 1980), pp. 201–257. Discussion, p. 321. Academic Press, New York, 1981. MR 0617930 (82g:00016) (book). Zbl 495.93001 (book).

For comments, see Lancaster (1981a). **(QM: QSol: SD)**

1989a Qualitatively stable matrices and convergent matrices. In: Fred Roberts, ed., *Applications of Combinatorics and Graph Theory to the Biological and Social Sciences*, pp. 245–258. IMA Vols. Math. Appl., Vol. 17. Springer-Verlag, New York, 1989. MR 1009379 (90h:34082). Zbl 708.15007.

Signed (di)graphs play a role in characterizations. See e.g. §7. Also see Roberts (1989a), §4. **(QM, SD)**

John S. Maybee & Stuart J. Maybee

1983a An algorithm for identifying Morishima and anti-Morishima matrices and balanced digraphs. *Math. Social Sci.* 6 (1983), 99–103. MR 0747560 (85f:05084). Zbl 567.05038.

A linear-time algorithm to determine balance or antibalance of the undirected signed graph of a signed digraph. The algorithm of Harary and Kabell (1980a) appears to be different. **(SG: Bal, Par: Algor)**

John Maybee & James Quirk

1969a Qualitative problems in matrix theory. *SIAM Rev.* 11 (1969), 30–51. MR 0247866 (40 #1127). Zbl 186.33503 (186, p. 335c).

An important early survey with new results.
(QM, SD: QSol, QSta, bal; Exp(in part), Ref)

John S. Maybee & Daniel J. Richman

1988a Some properties of GM-matrices and their inverses. *Linear Algebra Appl.* 107 (1988), 219–236. MR 0960147 (89k:15039). Zbl 659.15021.

Square matrix A is a GM-matrix if, for every positive and negative cycle P and N in its signed digraph, $V(P) \supseteq V(N)$. Classification of irreducible GM-matrices; connections with the property that each $p \times p$ principal minor has sign $(-1)^p$; some conclusions about the inverse.
(SD: QM)

1988b From qualitative matrices to quantitative restrictions. *Linear Multilinear Algebra* 22 (1988), no. 3, 229–248. MR 0937168 (89e:15032). Zbl 647.15001. (**QM: SD**)

John S. Maybee & Gerry M. Weiner

1987a L-functions and their inverses. *SIAM J. Algebraic Discrete Methods* 8 (1987), 67–76. MR 0872057 (88a:26021). Zbl 613.15005.

An L-function is a nonlinear generalization of a qualitative linear function. Signed digraphs play a small role. (**QM, SD**)

Stuart J. Maybee

See J.S. Maybee.

W. Mayeda & M.E. Van Valkenburg

1965a Properties of lossy communication nets. *IEEE Trans. Circuit Theory* CT-12 (1965), 334–338. (**GN**)

Dillon Mayhew

See also T. Fife and D. Funk.

2005a Inequivalent representations of bias matroids. *Combin. Probab. Comput.* 14 (2005), 567–583. MR 2160419 (2006j:05040). Zbl 1081.05021.

The number of inequivalent representations of a frame matroid over a fixed finite field is bounded, if the matroid does not have a free swirl $\mathbf{F}(2C_n, \varnothing)$ as a minor. (**GG: Matrd**)

Dillon Mayhew, Geoff Whittle, & Stefan H.M. van Zwam

2011a An obstacle to a decomposition theorem for near-regular matroids. *SIAM J. Discrete Math.* 25 (2011), no. 1, 271–279. MR 2801229 (2012d:05097). Zbl 1290.05057. arXiv:0905.3252. (**SG: Matrd**)

R. Maynard

See J.C. Angles d'Auriac, F. Barahona, and I. Bieche.

Ján Mazák

See E. Máčajová.

M.H. McAndrew

See D.R. Fulkerson.

Richard McBride

See H. Jordon.

Richard D. McBride

See also G.G. Brown.

1985a Solving embedded generalized network problems. *European J. Operational Res.* 21 (1985), 82–92. Zbl 565.90038.

Introducing the algorithm "EMNET", which employs embedded generalized-network matrices (i.e., incidence matrices of real multiplicative gain graphs) with side constraints (i.e., extra rows) to speed up linear programming. [Annot. 2 Oct 2009.] (**GN: Incid: Algor**)

1998a Progress made in solving the multicommodity flow problem. *SIAM J. Optim.* 8 (1998), no. 4, 947–955. MR 1641274 (99i:90110). Zbl 912.90128.

Employing embedded generalized-network matrices to speed up linear programming. [Annot. 2 Oct 2009.] (**GN: Incid: Algor: Exp**)

Richard D. McBride & John W. Mamer

1997a Solving multicommodity flow problems with a primal embedded network simplex algorithm. *INFORMS J. Comput.* 9 (1997), no. 2, 154–163. MR 1477311. Zbl 885.90040. (**GN: Incid: Algor**)

2004a Implementing an LU factorization for the embedded network simplex algorithm. *INFORMS J. Comput.* 16 (2004), no. 2, 109–119. MR 2063190.

Matrix factorization to speed up the method of McBride (1985a). [Annot. 2 Oct 2009.] (**GN: Incid: Algor**)

Richard D. McBride & Daniel E. O'Leary

1997a An intelligent modeling system for generalized network flow problems: With application to planning for multinational firms. *Ann. Operations Res.* 75 (1997), 355–372. Zbl 894.90060. (**GN: Incid: Algor**)

H. Gilman McCann

See E.C. Johnsen.

William McCuaig

See also C.R. Johnson.

1993a Intercyclic digraphs. In: Neil Robertson and Paul Seymour, eds., *Graph Structure Theory* (Proc., Seattle, 1991), pp. 203–245. Contemp. Math., Vol. 147. Amer. Math. Soc., Providence, R.I., 1993. MR 1224708 (94f:05062). Zbl 789.05042.

Characterizes the digraphs with no two disjoint cycles as well as those with no two arc-disjoint cycles. [Since cycles do not form a linear subclass of circles, this is not a biased-graphic theorem, but it might be of use in studying biased graphs that have no two disjoint balanced circles. See Lovász (1965a), Slilaty (2007a).] (**Str**)

2000a Even dicycles. *J. Graph Theory* 35 (2000), no. 1, 46–68. MR 1775794 (2001f:-05087). Zbl 958.05070. (**SD: par: Str**)

2001a Brace generation. *J. Graph Theory* 38 (2001), no. 3, 124–169. MR 1859786 (2002h:05136). Zbl 991.05086.

Results needed for (2004a). (**SD: par**)

† 2004a Pólya's permanent problem. *Electronic J. Combin.* 11 (2004), Research Paper 79, 83 pp. MR 2114183 (2005i:05004). Zbl 1062.05066.

See the description of Robertson, Seymour, and Thomas (1999a), who independently prove the main theorem. (**SD: par: Str**)(**SG**)

2005a When all dicycles have the same length. Manuscript, ca. 2005.

Uses the main theorem of (2004a) and Robertson, Seymour, and Thomas (1999a) to prove: A digraph has an edge weighting in which all cycles have equal nonzero total weight iff it does not contain a "double dicycle": a symmetric digraph whose underlying simple graph is a circle. There is also a structural description of such digraphs. (**SD: par: Str**)(**Sw**)

William McCuaig, Neil Robertson, P.D. Seymour, & Robin Thomas

1997a Permanents, Pfaffian orientations, and even directed circuits. Extended abstract. In: *Proceedings of the Twenty-Ninth Annual ACM Symposium on Theory of Computing* (STOC 97, El Paso, Tex., 1997), pp. 402–405. ACM Press, New York, 1997. Zbl 963.68153.

Extended abstract of McCuaig (2004a) and Robertson, Seymour, and Thomas (1999a). **(SD: par)**

W.D. McCuaig & M. Rosenfeld

1985a Parity of cycles containing specified edges. In: B.R. Alspach and C.D. Godsil, eds., *Cycles in Graphs*, pp. 419–431. Ann. Discrete Math., Vol. 27. North-Holland Math. Stud., Vol. 115. North-Holland, Amserdam, 1985. MR 0821542 (87g:05139). Zbl 583.05037.

In a 3-connected graph, almost any two edges are in an even and an odd circle. [By the negative-subdivision trick this generalizes to signed graphs.] **(Par, sg: Bal)**

Judith J. McDonald

See M. Cavers and S. Kirkland.

David D. McFarland

See M. Hallinan.

Sean McGuinness

2020a Frame matroids, toric ideals, and a conjecture of White. *Adv. Appl. Math.* 118 (2020), art. 102042, 46 pp. MR 4088420. Zbl 1440.05056. arXiv:1910.08587. **(GG: Matrd: Algeb)**

Brendan D. McKay, Mirka Miller, & Jozef Širáň

1998a A note on large graphs of diameter two and given maximum degree. *J. Combin. Theory Ser. B* 74 (1998), 110–118. MR 1644043 (99c:05108). Zbl 911.05031.

Also see Šiagiová (2001a). **(GG: Cov)**

James McKee & Chris Smyth

2007a Integer symmetric matrices having all their eigenvalues in the interval $[-2, 2]$. *J. Algebra* 317 (2007), 260–290. MR 2360149 (2008j:15038). Zbl 1140.15007. arXiv:0705.3599.

The matrices (except those of orders 1, 2) are signed-graph "adjacency" matrices A with diagonal entries $0, 1, -1$. There are 3 infinite families and a few sporadic examples of maximal such signed graphs; all of which satisfy $A^2 = 4I$. The proof uses "charged signed graphs", i.e., a signed graph with 0, +1, or −1 attached to each vertex (and appearing on the diagonal of the adjacency matrix). Switching a vertex negates the charge. Dictionary: "strongly equivalent" = switching isomorphic; "bipartite" = switching isomorphic to its negation (negation includes negating the charges). [The charged signed graphs are really oriented all-negative graphs with half edges. The adjacency matrix is not $A(\Sigma)$ but an oriented adjacency matrix $\vec{A}$ defined by $\vec{a}_{ij}$ = net in-degree of v_iv_j edges at v_i.] [Elementary proof for signed simple graphs: Hou, Tang, and Wang (2019a).] [Annot. 27 Jun 2008.] **(SG: par: ori: Eig)**

P. 265 says a signed graph (without charges) that is switching isomorphic to its negation must be a bipartite graph. [The Petersen graph with

$E^- = \{$alternating edges of a $C_6\}$ is a counterexample (Alex Schaefer). *Problem.* Characterize Σ such that $-\Sigma \simeq \Sigma$.] [Annot. 28 May 2016.] (**SG: Sw, Aut**)

2012a Integer symmetric matrices of small spectral radius and small Mahler measure. *Int. Math. Res. Not.* 2012 (2012), no. 1, 102–136. MR 2874929. Zbl 1243.15020. arXiv:0907.0371. (**SG: Eig**)

2020a Symmetrizable integer matrices having all their eigenvalues in the interval $[-2, 2]$. *Algebraic Combin.* 3 (2020), no. 3, 775–789. MR 4113606. Zbl 1446.15016. arXiv:2002.06082. (**Eig: sg, Sw**)

2020b Symmetrizable matrices, quotients, and the trace problem. *Linear Algebra Appl.* 600 (2020), 60–81. MR 4090841. Zbl 1442.15016. (**SG: Adj**)

Terry A. McKee

1984a Balance and duality in signed graphs. Proc. Fifteenth Southeastern Conf. on Combinatorics, Graph Theory and Computing (Baton Rouge, 1984). *Congressus Numer.* 44 (1984), 11–18. MR 0777525 (87b:05124). Zbl 557.05046. (**SG: Bal: Du**)

1987a A local analogy between directed and signed graphs. *Utilitas Math.* 32 (1987), 175–180. MR 0921647 (89a:05075). Zbl 642.05023. (**SG: Du, Clu, Bal**)

2002a Chordally signed graphs. *Discrete Appl. Math.* 119 (2002), 273–280. MR 1906865 (2003d:05101). Zbl 1003.05051.

A chordally signed graph is a chordal graph signed so every positive circle C of length at least 4 has a chord such that $C \cup e$ is balanced. Characterized in various ways. (**SG**)

2007a Chordal multipartite graphs and chordal colorings. *Discrete Math.* 307 (2007), 2309–2314. MR 2340631 (2008f:05063). Zbl 1123.05042.

P. 2312: An auxiliary graph can be treated as signed; chordal coloring is signed-graph clustering. [Annot. 11 Jul 2012.] (**SG: Clu**)

Kathleen A. McKeon

See G. Chartrand.

Jennifer McNulty

See also G. Gordon, T. Lewis and N.A. Neudauer.

Jennifer McNulty & Nancy Ann Neudauer

2008a On cocircuit covers of bicircular matroids. *Discrete Math.* 308 (2008), 4008–4013. MR 2418105 (2009e:05069). Zbl 1148.05022. (**Matrd: Bic**)

Luis Medina

See I. Gutman and G. Pastén.

Killian Meehan

See Y. Duong.

Yotsanan Meemark & Borworn Suntornpoch

2014a Balanced unitary Cayley sigraphs over finite commutative rings. *J. Algebra Appl.* 13 (2014), no. 5, art. 1350152, 12 pp. MR 3190081. Zbl 1291.05086.

(**SG: Algeb: Bal**)

O. Megalakaki
See A. El Maftouhi.

Nimrod Megiddo
See E. Cohen and D. Hochbaum.

Ranjit Mehatari
See also S. Mandal.

Ranjit Mehatari, M. Rajesh Kannan, & Aniruddha Samanta
2022a On the adjacency matrix of a complex unit gain graph. *Linear Multilinear Algebra* 70 (2022), no. 9, 1798–1813. MR 4429447. arXiv:1812.03747.
(**GG: Adj, Eig, Bal**)

Ritu Rani Meherwal
See G.N. Purohit.

Kurt Mehlhorn & Dimitrios Michail
2005a Implementing minimum cycle basis algorithms. In: S.E. Nikoletseas, ed., *Experimental and Efficient Algorithms* (4th Int. Workshop, WEA 2005, Santorini Island, 2005), pp. 32–43. Lect. Notes in Computer Sci., Vol. 3503. Springer, Berlin, 2005. Zbl 1121.05314.

The "signed graph G_i" is a signed covering graph $\tilde{\Sigma}_i$. Used to find minimum cycle basis in a positively weighted graph Γ. Σ_i has negative edge set S_i, the "witness set". [Annot. 6 Feb 2011.] (**SG: Algor, Cov**)

2006a Implementing minimum cycle basis algorithms. *ACM J. Exper. Algorithmics* 11 (2006), 14 pp. MR 2306622 (2007m:05139). Zbl 1143.05310.
See (2005a). (**SG: Algor, Cov**)

Martin Mehlitz
See J. Kunegis.

Seema Mehra & Manjeet Singh
2017a Single valued neutrosophic signedgraphs [*sic*]. *Int. J. Computer Appl.* 157 (2017), no. 9, 31–34.

Cf. Mishra and Pal (2016a). Same confused definitions and trivial results, but neutrosophic. [Annot. 30 Jul 2022.] (**SG, VS**)

A. Mehrabian
See S. Akbari.

Wenjun Mei, Ge Chen, Noah E. Friedkin, & Florian Dörfler
2022a Structural balance and interpersonal appraisals dynamics: Beyond all-to-all and two-faction networks. *Automatica* 140 (2022), art. 110239, 11 pp. MR 4395969. arXiv:2012.10151.
(**SD: KG: Bal(Gen), Dyn**)

Wenjun Mei, Pedro Cisneros-Velarde, Ge Chen, Noah E. Friedkin, & Francesco Bullo
2019a Dynamic social balance and convergent appraisals via homophily and influence mechanisms. *Automatica* 110 (2019), art. 108580, 11 pp. MR 4395969. Zbl

1429.91269. arXiv:1710.09498. **(SD: KG: Bal, Dyn)**

Sylvain Meignen
See J. Demongeot.

Ioannis N. Melas, Regina Samaga, Leonidas G. Alexopoulos, & Steffen Klamt
2013a Detecting and removing inconsistencies between experimental data and signaling network topologies using integer linear programming on interaction graphs. *PLoS Comput. Biol.* 9 (2013), no. 9, art. e1003204, 19 pp. MR 3131677 (no rev). **(SD, Biol: Fr: Algor)**

O. Melchert & A.K. Hartmann
2008a Ground states of 2D $\pm J$ Ising spin glasses via stationary Fokker–Planck sampling. *J. Stat. Mech.* 2008 (2008), art. P10019. Zbl 1459.82329.
In other words, finding the frustration index of a signed planar graph. The titular method seems to be less efficient than others. [Annot. 9 Jan 2015.] **(SG: Fr, Algor, Phys)**

Miguel A. Meléndez-Jiménez
See A. Parravano.

Avraham A. Melkman
See T. Akutsu.

C. Mendes Araújo & Juan R. Torregrosa
2009a Sign pattern matrices that admit M-, N-, P- or inverse M-matrices. *Linear Algebra Appl.* 421 (2009), 724–731. MR 2535545 (2010e:15035). Zbl 1170.15008.
The digraphs treated are acyclic or cycles. See Thm. 3.4 on signed cycles. Also, the "2-cycle property" means a negative 2-cycle. [Annot. 29 Sept 2012.] **(QM: sd: Adj)**

2011a Sign pattern matrices that admit P_0-matrices. *Linear Algebra Appl.* 435 (2011), no. 8, 2046–2053. MR 2810645 (2012e:15064). Zbl 1222.15034.
Almost identical to (2009a) with "P_0-matrices" replacing "M-, N-, P- or inverse M-matrices". Treats directed circles as well as cycles and acyclic digraphs. [Annot. 29 Sept 2012.] **(QM: sd: Adj)**

J.F.F. Mendes
See M. Ostilli.

Marco A. Mendez
See J. Aracena.

Miguel A. Méndez & José L. Ramírez
2019a A new approach to the r-Whitney numbers by using combinatorial differential calculus. *Acta Univ. Sapientiae Math.* 11 (2019), no. 2, 387–418. MR 4072595. Zbl 1472.11084. arXiv:1702.06519. **(gg: Matrd(Gen): Invar)**

Israel Mendonça, Rosa Figueiredo, Vincent Labatut, & Philippe Michelon
2015a Relevance of negative links in graph partitioning: A case study using votes from the European Parliament. In: *2nd European Network Intelligence Conference* (ENIC, Karlskrona, Sweden, 2015), pp. 122–129. arXiv:1507.04215. HAL hal-01176090. **(SG: Clu: Algor, Appl)**

2015b Informative Value of negative links in graph partitioning, with an application to European Parliament votes. In: *6éme Conférence Modèles et Analyse* *de Réseaux: approches mathématiques et informatiques (MARAMI)* (Nîmes, France, 2015), 13 pp. HAL hal-01176090. (**SG: Clu: Algor, Appl**)

Deyuan Meng

See also M.J. Du.

2017a Bipartite containment tracking of signed networks. *Automatica* 79 (2017), 282–289. MR 3627993. Zbl 1371.93010. (**SD: Algor**)

2018a Dynamic distributed control for networks with cooperative–antagonistic interactions. *IEEE Trans. Automatic Control* 63 (2018), no. 8, 2311–2326. MR 3845965. Zbl 1423.93025. (**SG: Dyn**)

Deyuan Meng, Ziyang Meng, & Yiguang Hong

2018a Uniform convergence for signed networks under directed switching topologies. *Automatica* 90 (2018), 8–15. MR 3764379. Zbl 1387.93130. (**SD: Dyn: Adj**)

Jie Meng

See J.-E. Chen.

Jixiang Meng

See D. Li and L.-L. Yuan.

Ziyang Meng

See D.-Y. Meng.

Mircea Merca

2013a A note on the r-Whitney numbers of Dowling lattices. *C.R. Acad. Sci. Paris, Ser. I* 351 (2013), no. 17-18, 649–655. MR 3124320. Zbl 1277.05167.
Cf. Mező (2010a). (**gg: Matrd: Invar**)

Pedro Mercado, Francesco Tudisco, & Matthias Hein

2016a Clustering signed networks with the geometric mean of Laplacians. In: D.D. Lee *et al.*, eds., *Advances in Neural Information Processing Systems 29* (NIPS 29, Barcelona, 2016), 9 pp. URL `https://papers.nips.cc/book/advances-in-neural-information-processing-systems-29-2016`
arXiv:1701.00757. (**SG: Clu: Lap, Algor**)

2019a Spectral clustering of signed graphs via matrix power means. In: Kamalika Chaudhuri and Ruslan Salakhutdinov, eds., *Proceedings of the 36th International Conference on Machine Learning* (ICML 2019, Long Beach, Calif.), 11 pp. + 17 suppl. Proc. Machine Learning Res., Vol. 97. Machine Learning Research Press, 2019. arXiv:1905.06230. (**SG: Clu: Adj**)

Valeriu Mereacre

See K.C. Mondal.

M.A. Mermet

See J. Aracena.

Leanne Merrill

See Y. Duong.

Russell Merris

1994a Laplacian matrices of graphs: a survey. *Linear Algebra Appl.* 197/198 (1994), 143–176. MR 1275613 (95e:05084). Zbl 802.05053.

Thm.: Spec $L(\Gamma)$ = Spec $L(-\Gamma)$ iff Γ is bipartite. [*Cf.* antibalanced case of B.D. Acharya (1980a).] [Annot. 21 Jan 2012.]
(Par: Lap: Eig, bal)

1995a A survey of graph Laplacians. *Linear Multilinear Algebra* 39 (1995), no. 1–2, 19–31. MR 1374468 (97c:05104). Zbl 832.05081. **(par: Lap: Eig)**

Mehran Mesbahi
See M.H. de Badyn.

Roy Meshulam
See R. Aharoni and J. Kahn.

Nacim Meslem
See N. Ramdani.

Robert Messer
See E.M. Brown.

Thomas Mestl
See E. Plahte.

Gábor Mészáros
See T. Kittipassorn.

Karola Mészáros

2011a Root polytopes, triangulations, and the subdivision algebra, II. *Trans. Amer. Math. Soc.* 363 (2011), no. 11, 6111–6141. MR 2817421 (2012g:52021). Zbl 1233.05216. arXiv:0904.3339.

A signed simple graph generates a polytope $P(\Sigma)$ whose volume is calculated. [Annot. 11 Sept 2010.] **(SG: Geom)**

2012a Demystifying a divisibility property of the Kostant partition function. *Pacific J. Math.* 260 (2012), no. 1, 215–225. MR 3001792. Zbl 1256.05015. arXiv:-1101.0388.

[Positive edges are called "negative" and vice versa.] Flows on signed graphs give combinatorial interpretations of and identities for the partition functions. Negative ("positive") edges are introverted, so flow goes in and disappears. [Annot. 25 Mar 2013.] **(SG: Flows: Appl)**

Karola Mészáros & Alejandro H. Morales

2012a Flow polytopes and the Kostant partition function for signed graphs (extended abstract). In: *24th International Conference on Formal Power Series and Algebraic Combinatorics (FPSAC 2012)* (Nagoya, 2012), pp. 947–959. D.M.T.C.S. Proc. Discrete Math. & Theor. Computer Sci. (DMTCS), Nancy, 2012. MR 2964267 (volume). Zbl 1412.05088.

See (2015a). **(SG: Flows, Geom)**

2015a Flow polytopes of signed graphs and the Kostant partition function. *Int. Math. Res. Notes* 2015 (2015), no. 3, 830–871. MR 3340339. Zbl 1307.05097. arXiv:-1208.0140.

[The edge signs are the opposite of what they ought to be.]
(SG: Flows, Geom)

Frédéric Meunier & András Sebő

2009a Paintshop, odd cycles and necklace splitting. *Discrete Appl. Math.* 157 (2009), 780–793. MR 2499492 (2010e:90102). Zbl 1163.90774.

The negative circles of a signed graph and its minimal and minimum balancing sets are crucial. Dictionary: "signed graph" = $(|\Sigma|, E^-)$, "odd cycle" = negative circle, "odd cycle clutter" = $\mathcal{B}^c(\Sigma)$, "transversal" = "uncut" = minimal balancing set, "BIP(G, F)" = MinBalSet(Σ) (the problem of finding a minimum balancing set), "resigning" = switching. [Annot. 22 Sept 2010.] (**SG: Fr**)

David A. Meyer

See D.J. Song.

Seth A. Meyer

See R.A. Brualdi.

Hildegard Meyer-Ortmanns

See F. Radicchi.

Andrew M. Meyers

See N.A. Neudauer.

Erika Meza

See M. Beck.

Marc Mézard, Giorgio Parisi, & Miguel Angel Virasoro

1987a *Spin Glass Theory and Beyond.* World Scientific Lect. Notes in Physics, Vol. 9. World Scientific, Singapore, 1987. MR 1026102 (91k:82066).

Focuses on the Sherrington–Kirkpatrick model, i.e., underlying complete graph, emphasizing the Parisi-type model (see articles reprinted herein), which posits numerous metastable states, separated by energy barriers of greatly varying heights and subdividing as temperature decreases (*cf.* Kirkpatrick and Sherrington (1978a)). Essentially heuristic (as noted in MR): that is, the ideas awaited [and still largely await] mathematical justification.

Many original articles on Ising and vector models (both of which are based on weighted signed graphs) are reprinted herein, though few are of general signed-graphic interest.

[Also see, i.a., Toulouse (1977a) *et al.*, Chowdhury (1986a), Fischer and Hertz (1991a), Vincent, Hammann, and Ocio (1992a) for physics, Barahona (1982a), etc., Grötschel, Jünger, and Reinelt (1987a) for mathematics.]

[Metastable states in the model appear to correspond to local minima of the state frustration function of the underlying weighted signed graph (Σ, w), with ultrametric distance function $d(s, s') := \min_P \max_{s'' \in P} H(s'')$, where P ranges over all paths $P : s \to s'$ in the graph of states and H is the Hamiltonian of a state of (Σ, w), equivalently the (weighted) frustration of the state. *Problem.* Study this metric on the state space of various signed K_N's and other signed graphs. Possibly this will shed light on the physics; it will certainly be interesting for signed graphs.]

(**Phys, SG: Fr, State: Exp, Ref**)

Ch. 0, "Introduction", briefly compares, in the obvious way, balance in social psychology [they neglect to mention the original paper, Cartwright and Harary (1956a)] with frustration in spin glasses.

(**Phys, PsS: SG: Bal: Exp**)

Pt. 1, "Spin glasses", Ch. II, "The TAP approach": pp. 19–20 describe 1-vertex switching of a weighted signed graph to reduce frustration, not however necessarily producing the frustration index (minimum frustration). *Question.* How does the valley ("basin of attraction") of a ground state (minimum frustration) compare with the valley of a metastable state (locally minimum frustration); in particular can it be much smaller? [Annot. rev. 15 Aug 2018.]
(**Phys: SG: Fr, Sw, Algor: Exp**)

István Mező

2010a A new formula for the Bernoulli polynomials. *Results. Math.* 58 (2010), 329–335. MR 2728160 (2011k:11036). Zbl 1237.11010.

Introduces the r-Whitney numbers [of Dowling lattices, not of geometric lattices in general]. $r = 1$ are the original numbers. [The numbers are popular. *Cf.* Cheon and Jung (2012a), Merca (2013a), Rahmani (2014a), Mező (2014a), Gyimesi and Nyul (2018a), Kim and Kim (2022a), etc.] [Annot. rev. 28 May 2018.] (**gg: Matrd: Invar**)

2014a A kind of Eulerian numbers connected to Whitney numbers of Dowling lattices. *Discrete Math.* 328 (2014), 88–95. MR 3199820. Zbl 1288.05016.

Cf. (2010a), Cheon and Jung (2012a). (**gg: Matrd: Invar**)

Lu Miao

See R.-F. Liu.

Zhengke Miao

See H.P. Ma, G.L. Yu, and L.Q. Wang.

Isaac B. Michael & Mark R. Sepanski

2016a Net regular signed trees. *Australasian J. Combin.* 66 (2016), no. 2, 192–204. MR 3556127. Zbl 1375.05125. (**SG: Str, Enum**)

T.S. Michael

2002a Signed degree sequences and multigraphs. *J. Graph Theory* 41 (2002), 101–105. MR 1926311 (2003g:05042). Zbl 1012.05052.

Characterizes net degree sequences of signed graphs with fixed maximum edge multiplicity. [See Chartrand, Gavlas, Harary, and Schultz (1994a) for explanation.] (**SGw: Invar, Algor**)

Dimitrios Michail

See K. Mehlhorn.

Philippe Michelon

See I. Mendonça.

Manuel Middendorf

See E. Ziv.

A. Alan Middleton

See C.K. Thomas.

Anna de Mier

See O. Giménez.

S. Migowsky

See T. Wanschura.

Štefko Miklavič
See I. Kovács.

Alexander R. Miller
2015a Foulkes characters for complex reflection groups. *Proc. Amer. Math. Soc.* 143 (2015), no. 8, 3281–3293. MR 3348771. Zbl 1314.05225. (**gg: Matrd**)

2015b Eigenspace arrangements of reflection groups. *Trans. Amer. Math. Soc.* 367 (2015), no. 12, 8543–8578. MR 3403065. Zbl 1333.20041. arXiv:1208.1944. (**gg: Matrd**)

Mirka Miller
See C. Dalf'o and B.D. McKay.

Raymond E. Miller
See R.M. Karp.

William P. Miller
See also J.E. Bonin.

1997a Techniques in matroid reconstruction. *Discrete Math.* 170 (1997), 173–183. MR 1452942 (98f:05039). Zbl 878.05020.
Dowling geometries are reconstructible from their hyperplanes, their deletions, and their contractions. (**gg: Matrd**)

Geyong Min
See F. Hao.

Maya Mincheva
See also G.R. Walther.

Maya Mincheva & Gheorghe Craciun
2008a Multigraph conditions for multistability, oscillations and pattern formation in biochemical reaction networks. *Proc. IEEE* 96 (2008), no. 8, 1281–1291. (**SD: Chem, Biol: Dyn: Exp**)

Edward Minieka
1972a Optimal flow in a network with gains. *INFOR* 10 (1972), 171–178. Zbl 234.90012. (**GN: Matrd(indep), Bal**)

1978a *Optimization Algorithms for Networks and Graphs.* Marcel Dekker, New York and Basel, 1978. MR 0517268 (80a:90066). Zbl 427.90058.
§4.6: "Flows with gains," pp. 151–174. Also see pp. 80–81. (**GN: Bal, Sw, matrd(indep): Exp**)

1981a *Algoritmy Optimizatsii na Setyakh i Grafakh.* Transl. M.B. Katsnel'son and M.I. Rubinshteĭn; ed. E.K. Maslovskiĭ. Mir, Moskva, 1981. MR 0641852 (83f:-90118). Zbl 523.90058.
Russian translation of (1978a). (**GN: Bal, Sw, matrd(indep): Exp**)

Maryam Mirzakhah
See also I. Gutman and M. Jooyandeh.

M. Mirzakhah & D. Kiani
2010a The Sun graph is determined by its signless Laplacian spectrum. *Electronic J. Linear Algebra* 20 (2010), 610–620. MR 2735977 (2011j:05209). Zbl 1205.05149. (**par: Lap: Eig**)

2012a Some results on signless Laplacian coefficients of graphs. *Linear Algebra Appl.* 437 (2012), 2243–2251. MR 2954486. Zbl 1247.05141. **(par: Lap: Eig)**

Grégoire Misguich & Claire Lhuillier

2004a Two-dimensional quantum antiferromagnets. In: H.T. Diep, ed., *Frustrated Spin Systems*, Ch. 5, pp. 229–306. World Scientific, Hackensack, N.J., 2004.

I.a., details of ground-state spin alignments in XY and Heisenberg models (S^2 and S^3 spins) on simple periodic lattices. [Annot. 13 Aug 2018.] **(SG, Phys: Fr: Exp, Ref)**

S.N. Mishra & A. Pal

2016a Intuitionistic fuzzy signed graphs. *Int. J. Pure Appl. Math.* 106 (2016), no. 6, 113–122.

Edges and possibly vertices of fuzzy graph are signed. Definitions are confused and incomplete. Trivial results. [More in Mehra and Singh (2017a) and Moudgalya and Permi (2022a).] [Annot. 30 Jul 2022.] **(SG, VS, Fr)**

V. Mishra

1974a *Graphs Associated With* $(0, +1, -1)$ *Arrays.* Doctoral thesis, Indian Institute of Technology, Bombay, 1974.

The arrays are matrices.

$\otimes$ Defines tensor product $\Sigma_1 \otimes \Sigma_2$ to have an edge $(v_1, v_2)(w_1, w_2)$ iff v_1w_1 and v_2w_2 are edges, with sign $\sigma((v_1, v_2)(w_1, w_2)) := \sigma_1(v_1w_1)\sigma_2(v_2w_2)$ (*cf.* Sinha and Garg (2014a)). [Annot. 23 Nov 2014.] **(SG)**

U.K. Misra

See P.S.K. Reddy.

Lon H. Mitchell

See N. Matar.

Rivka Mitchell

See M. Buckland and B. Kolesnik.

G. Mitra

See N. Gülpinar.

S. Mitra

1962a Letter to the editors. *Behavioral Sci.* 7 (1962), 107.

Treats signed simple graphs via the Abelson–Rosenberg (1958a) structure matrix R. Observes that balance holds iff $R = rr^{\mathrm{T}}$ for some vector $r \in \{p, n\}^V$; also, asserts that frustration index $l(\Sigma)$ = minimum number of negative edges over all switchings of Σ. [Proved in Barahona, Maynard, Rammal, and Uhry (1982a).] Asserts an algorithm for computing $l(\Sigma)$: switch vertices whose negative degree exceeds positive degree, one at a time, until no such vertices remain [incorrect: consider K_6, all positive except a negative C_6]. [Annot. corr. 20 Jan 2010.] **(sg: kg: Adj, sw, Fr)**

Sovan Mitra

See V. Vasanthi.

Michael Mitzenmacher

See C.E. Tsourakakis.

Valia Mitsou

See F. Dross and F. Foucaud.

Seiji Miyashita

See O. Nagai.

Hirobumi Mizuno & Iwao Sato

1997a Enumeration of finite field labels on graphs. *Discrete Math.* 176 (1997), 197–202. MR 1477289 (98e:05059). Zbl 893.05015.

Isomorphism types, under the action of a subgroup of $\operatorname{Aut}\Gamma$, of coboundaries of 1-chains $f : V \to \mathbb{F}_q^+$ in $-\Gamma$. (In other words, the edge labels are $\delta f(uv) = f(u) + f(v)$.) [*Question.* Does it generalize to signed graphs? The subgroup would be of $\operatorname{Aut}\Sigma$, or one can count isomorphism types of switching classes under a subgroup of $\operatorname{Aut}[\Sigma]$.] [Annot. 16 Jan 2012.] (**par: incid**)

2000a Zeta functions of graph coverings. *J. Combin. Theory Ser. B* 80 (2000), 247–257. MR 1794694. Zbl 1023.05114. (**GG**)

2004a Weighted zeta functions of graphs. *J. Combin. Theory Ser. B* 91 (2004), 169–183. MR 2064866. Zbl 1048.05044. (**GG**)

2010a Weighted scattering matrices of regular coverings of graphs. *Linear Multilinear Algebra* 58 (2010), no. 7, 927–940. MR 2742326 (2011k:05145). Zbl 1231.05170. (**GGw: Cov: Invar, Adj: Gen**)

[Swathyprabhu Mj]

See Swathyprabhu under 'S'.

Iain Moffatt

See also J.A. Ellis-Monaghan, T. Krajewski, and M. Loebl.

2010a Partial duality and Bollobás and Riordan's ribbon graph polynomial. *Discrete Math.* 310 (2010), no. 1, 174–183. MR 2558980 (2011b:05112). Zbl 1229.05123. arXiv:0809.3014. (**SG: Top, Invar, Du**)

2011a Unsigned state models for the Jones polynomial. *Ann. Combin.* 15 (2011), no. 1, 126–146. MR 2785760 (2012b:05087). Zbl 1235.05072. arXiv:0710.4152.

Vertex models (graphs with vertices labelled ± 1) and Potts models (edges labelled ± 1) can be replaced by unsigned models by converting an edge-labelled graph into an orientable ribbon graph. A limited parametrized rank-corank polynomial appears (in the standard way) as the Potts partition function. [Annot. 23 Apr 2009.] (**SGc: Invar**)

2013a Separability and the genus of a partial dual. *European J. Combin.* 34 (2013), 355–378. MR 2994404. Zbl 1254.05047. arXiv:1108.3526. (**sg: Top, Du**)

2016a Ribbon graph minors and low-genus partial duals. *Ann. Combin.* 20 (2016), 373–378. MR 3505325. Zbl 1339.05377. arXiv:1502.00269. (**SG: Top, Du**)

A.R. Moghaddamfar

See Y. Bagheri.

Javad Mohajeri

See S. Fayyaz Shahandashti.

Bojan Mohar

See also G. Greaves, K. Guo, and Kawarabayashi and Mohar (2006a).

1989a An obstruction to embedding graphs in surfaces. *Discrete Math.* 78 (1989), 135–142. MR 1020656 (90h:05046). Zbl 686.05019.

The "overlap matrix" of a signed graph with respect to a rotation system and a spanning tree provides a lower bound on the demigenus that sometimes improves on that from Euler's formula. (**SG: Top**)

2016a Hermitian adjacency spectrum and switching equivalence of mixed graphs. *Linear Algebra Appl.* 489 (2016), 324–340. MR 3421853. Zbl 1327.05215.

Φ with gain group $\{\pm 1, \pm \mathrm{i}\}$: $\varphi(e) = 1$ for undirected, i for directed edges [as in Liu and Li (2015a)]. Switching. Adjacency cospectrality, including spectrally unique families. *Cf.* Guo and Mohar (2017a). [Annot. 15 Dec 2020.] (**gg: Sw, Adj: Eig**)

2020a A new kind of Hermitian matrices for digraphs. *Linear Algebra Appl.* 584 (2020), 343–352. MR 4013179. Zbl 1426.05009. arXiv:1909.10878.

The graphs are treated as (and are) gain graph with gains $\sqrt[6]{1}$ in the edge direction, the inverse in the opposite direction. This is said to be more natural than gains $\sqrt[4]{1}$ as in (2016a), Guo and Mohar (2017a). [*Cf.* Li and Yu (2022a).] [Annot. 23 Mar 2022.] (**gg: Adj: Eig**)

Bojan Mohar & Svatopluk Poljak

1993a Eigenvalues in combinatorial optimization. In: Richard A. Brualdi, Shmuel Friedland, and Victor Klee, eds., *Combinatorial and Graph-Theoretical Problems in Linear Algebra*, pp. 107–151. IMA Vols. Math. Appl., 50. Springer-Verlag, New York, 1993. MR 1240959 (95e:90003). Zbl 806.90104.

Switching of a weight function on an unsigned graph (p. 119), from C. Delorme and S. Poljak, Combinatorial properties and the complexity of a max-cut approximation, Tech. Rep. 91687, Inst. Diskrete Math., Universität Bonn, 1991. [Annot. 13 Apr 2009.] (**Sw**)

Bojan Mohar & Paul D. Seymour

2002a Coloring locally bipartite graphs on surfaces. *J. Combin. Theory Ser. B* 84 (2002), no. 2, 301–310. MR 1889261 (2003b:05059). Zbl 1079.05027.

[Also see Nakamoto, Negami, and Ota (2002a), (2004a).] (**sg: Top: sw**)

Bojan Mohar & Carsten Thomassen

2001a *Graphs on Surfaces.* Johns Hopkins Stud. Math. Sci. Johns Hopkins Univ. Press, Baltimore, 2001. MR 1844449 (2002e:05050). Zbl 979.05002.

§3.3, "Embedding schemes", surveys rotation systems and edge signatures for embedding in nonorientable surfaces. *Cf.* Ringel (1977a) and Stahl (1978a). §4.1, "Embeddings combinatorially": Detailed treatment of embeddings from rotation systems and optionally an edge signature. [Lins (1982a), (1985a), Širáň and Škoviera (1991a), Zaslavsky (1992a), (1993a), *et al.* are regrettably never mentioned in this valuable book.] (**sg: Top**)

Afshin Moin

See A.-M. Kermarrec.

G.H. Mokashi
See K.S. Betageri.

Kartik Chandra Mondal, Valeriu Mereacre, George E. Kostakis, Yanhua Lan, Christopher E. Anson, Ion Prisecaru, Oliver Waldmann, & Annie K. Powell

2015a A strongly spin-frustrated $Fe^{III}{}_7$ complex with a canted intermediate spin ground state of $S = 7/2$ or $9/2$. *Chem. European J.* 2015 (2015), no. 21, 10835–10842.

Novel arrangements of frustrated signs with S^1 spins in a ground state. Especially see Figs. 1, 4. [Annot. 20 Mar 2016.] (**Phys: sg: State, Fr**)

[Keivan Hassani Monfared]
See K. Hassani Monfared (under 'H').

G. Monroy
See A. Coniglio.

Marco Montalva
See also J. Aracena.

Marco Montalva, Julio Aracena, & Anahí Gajardo

2008a On the complexity of feedback set problems in signed digraphs. IV Latin-American Algorithms, Graphs, and Optimization Sympos. (Puerto Varas, Chile, 2007). *Electronic Notes Discrete Math.* 30 (2008), 249–254. MR 2570648 (no rev). Zbl 1341.05112.

Complexity of finding a minimum set of vertices, or arcs, that covers all positive, or negative, cycles in a signed digraph. All are NP-complete, by polynomial-time reduction to the existence problems Even Cycle and Odd Cycle in the positive and negative problems, respectively. [Directed frustration index and directed vertex frustration number are the negative-cycle cover problems, which are said to be easier than the positive-cycle cover problems.] [Annot. 20 July 2009.]
(**SD: Fr: Gen, Algor**)

Mickael Montassier
See C. Duffy.

Amanda Montejano, Pascal Ochem, Alexandre Pinlou, André Raspaud, & Éric Sopena

2008a Homomorphisms of 2-edge-colored graphs. IV Latin-American Algorithms, Graphs, and Optimization Symposium. *Electronic Notes Discrete Math.* 30 (2008), 33–38. MR 2570612 (no rev). Zbl 1341.05085. HAL lirmm-00196757.

Preliminary version of (2010a). (**SGc: Hom**)

2010a Homomorphisms of 2-edge-colored graphs. *Discrete Appl. Math.* 158 (2010), no. 12, 1365–1379. MR 2652013. Zbl 1341.05085. HAL lirmm-00433039.
(**SGc: Hom**)

James D. Montgomery

2009a Balance theory with incomplete awareness. *J. Math. Sociology* 33 (2009), 69–96. Zbl 1169.91438.

Signed digraphs with possible multiple arcs of different sign, with two types of vertices ("actors" having positive and possibly negative loops, and "objects" having no loops), and with extra "awareness" arcs between actor vertices. Emphasis on directionality of arcs. "Boolean multiplication" [Boolean Hadamard product] of separate positive, negative, and awareness adjacency matrices to form mixed adjacency matrices. Assumption: Over time the signed digraph evolves towards sign-transitive closure constrained by the awareness arcs, whose absence impedes transitive closure. Four specific "mechanisms" are postulated for the evolution, of which two are essential (Lemma 1). Propositions present conclusions (no surprises) about intermediate and final (i.e., constrained sign-transitively closed) signed digraphs. Dictionary: "balance closure" = sign-transitive closure, i.e., arc-transitive closure with positive triple sign. [The idea of constrained closure is mathematically intriguing, though the notation is heavy.] [For more on sign-transitive closure in signed digraphs see Doreian and Krackhardt (2001a).] [Annot. 16 Apr 2009.] (**SD, PsS: Bal**)

Angelo Monti

See T. Calamoneri.

Elliott W. Montroll

1964a Lattice statistics. In: Edwin F. Beckenbach, ed., *Applied Combinatorial Mathematics*, Ch. 4, pp. 96–143. Wiley, New York, 1964. MR 0174486 (30 #4687) (book). Zbl 141.15503 (141, p. 155c).

§4.4: "The Pfaffian and the dimer problem". Exemplified by the square lattice, expounds Kasteleyn's method of signing edges to make the Pfaffian term signs all positive. Partial proofs. §4.7, "The Ising problem", pp. 127–129, explains application to the Ising model. Exceptionally readable. [Further development in, e.g., Vazirani and Yannakakis (1988a), (1989a).] (**SG, Phys: Exp**)

J.W. Moon

1966a A note on approximating symmetric relations by equivalence relations. *SIAM J. Appl. Math.* 14 (1966), no. 2, 226–227. MR 0205865 (34 #5691). Zbl 66.00802 (66, 8b).

Observes that $l_{\text{clu}}(K_n, \sigma)/\binom{n}{2} \leq \frac{1}{2}$. Thm.: Let $\varepsilon > 0$. When $n \gg 0$, $l_{\text{clu}}(K_n, \sigma)/\binom{n}{2} \geq \frac{1}{2} - \varepsilon$ for almost all σ. Sequel to Zahn (1964a). [Annot. 10 Nov 2017.] (**sg: Clu**)

J.W. Moon & L. Moser

1966a An extremal problem in matrix theory. *Mat. Vesnik* N.S. 3(18) (1966), 209–211. MR 0207570 (34 #7385). Zbl 146.01401 (146, p. 14a).

Studies the maximum frustration index of a signed $K_{r,s}$. (**sg: Fr**)

Suck Joong Moon

See also H. Kosako.

Suck-Joong Moon & Hideo Kosako

1993a The variable-signed graph and its application. In: *TENCON '93: IEEE Region 10 Conference on Computer, Communication, Control and Power Engineering* (Proc., 1993), Vol. 4, pp. 522–525.

See Kosako, Moon, *et al.* (1993a) (**SG, VS: Sw, fr, Du, Incid**)

M.A. Moore

See A.J. Bray.

G. Eric Moorhouse

1995a Two-graphs and skew two-graphs in finite geometries. *Linear Algebra Appl.* 226/228 (1995), 529–551. MR 1344584 (96f:51012). Zbl 839.05024.

Two-graphs = switching classes of signed K_n's (*cf.* Seidel (1976a)). Skew two-graphs = switching classes of $\mathbb{Z}_3$-gain graphs on K_n (*cf.* Cameron (1977b), §8; ["two-digraphs" in Cheng and Wells (1984a), Cheng (1986a)]). Applied to construct invariants of structures in finite geometry. [Annot. 5 Aug 2018.]

(**sg, gg: kg: TG, TG(Gen), Adj, Sw, Invar: Geom**)

Alejandro H. Morales

See K. Mészáros.

Katherine Tapia Morales

See M. Robbiano.

A. Moreira

See J. Aracena.

Susan Morey

See L. Fouli.

Aki Mori

See T. Hibi.

Michio Morishima

1952a On the laws of change of the price-system in an economy which contains complementary commodities. *Osaka Economic Papers* 1 (1952), 101–113.

§4: "Alternative expression of the assumptions (1)," can be interpreted with hindsight as proving that, for a signed K_n, every triangle is positive iff the signature switches to all positive. (Everything is done with sign-symmetric matrices, not graphs, and switching is not mentioned in any form.) (**sg: kg: bal, sw**)

Robert Morris

See H. Liu.

Timothy Morris

See A.H. Busch.

Julian O. Morrissette

1958a An experimental study of the theory of structural balance. *Human Relations* 11 (1958), 239–254.

Proposes that edges have strengths between −1 and +1 instead of pure signs. The Cartwright–Harary degree of balance (1956a), computed from circles, is modified to take account of strength. In addition, signed graphs are allowed to have edges of two types, say U and A, and only short mixed-type circles enter into the degree of balance. This is said to be more consistent with the experimental data reported herein.

(**PsS, SG, Gen: Fr**)

Julian O. Morrissette & John C. Jahnke

1967a No relations and relations of strength zero in the theory of structural balance. *Human Relations* 20 (1967), 189–195.

Reports an experiment; then discusses problems with and alternatives to the Cartwright–Harary (1956a) circle degree of balance. (**PsS: Fr**)

Julian O. Morrissette, John C. Jahnke, & Keith Baker

1966a Structural balance: A test of the completeness hypothesis. *Behavioral Sci.* 11 (1966), no. 2, 121–125.

Proposes to measure degree of balance by $c^+(\Sigma)/c(K_n)$ instead of $c^-(\Sigma)/c(|\Sigma|)$ as in Cartwright and Harary (1956a), to overcome logical incompatibility between the latter measure, the principle of increasing balance, and an assumed tendency towards completeness in a (signed) graph of social relations; as well as for experimental reasons. [Annot. 3 Sept 2013.] (**SG, PsS: Bal**)

Hannes Moser

See J. Guo.

L. Moser

See J.W. Moon.

Tyler Moss

See D. Chun.

Sebastiano Mosterts

See E.L. Johnson.

Satish V. Motammanavar

See H.B. Walikar.

Fatemeh Motialah & Mohammad Hassan Shirdareh Haghighi

2019a Laplacian spectral characterization of signed sun graphs. *Theory Appl. Graphs* 6 (2019), no. 2, art. 3, 9 pp. MR 4042255. Zbl 1429.05130.

Sun = circle with one pendant edge at each vertex. Determined by Laplacian spectrum iff unbalanced or has odd girth. [Annot. 15 Dec 2020.] (**SG: Lap: Eig**)

Madhwesha Moudgalya R & Kavita Permi

2022a Connectivity indices of labeled signed graphs using domination degree. *J. Algebraic Stat.* 13 (2022), no. 3, 3280 - 3287

Cf. Mishra and Pal (2016a). Same confused definitions, with badly defined "topological indices"; obvious results. [Annot. 30 Jul 2022.] (**SG, VS**)

C.F. Moukarzel

See M.J. Alava.

Gisele Moura

See R. Figueiredo.

Nazanin Movarrae

See P. Ochem.

[Eunice Gogo Mphako]

See E. Mphako-Banda.

Eunice Mphako-Banda [Eunice Gogo Mphako]

2002a (as Eunice Gogo Mphako) The component number of links from graphs. *Proc. Edinburgh Math. Soc.* 45 (2002), 723–730. MR 1933752 (2003g:05046). Zbl 009.05048.

§5: Bracket polynomials of signed matroids, after Schwärzler and Welsh (1993a). [Annot. 21 May 2013.] **(Sgnd(Matrd): Invar)**

2004a (as Eunice Gogo Mphako) H-lifts of tangential k-blocks. *Discrete Math.* 285 (2004), 201–210. MR 2062843 (2005c:05046). Zbl 1044.05032.

H is a group. "H-lift" $L_r(M)$ of $M := \mathbf{F}(\Phi)$ means $\mathbf{F}(\Phi \cup (H{\cdot}S_r)^{(v_r)})$ where Φ is an H-gain graph with $V(\Phi) = \{v_1, \ldots, v_{r-1}\}$ and S_r is the star with center v_r and pendant vertices $v_1, \ldots, v_{r-1}$. Main result: Thm. 3.13: $p_{L_r(M)}(\lambda) = (\lambda - 1)p_M(\lambda - \#H)$. §5, "$H$-lifts of tangential blocks": $L_r(M^\bullet)$ is a tangential block if $M^\bullet := \mathbf{F}(\Phi^\bullet)$ is one. [This lift is not the lift matroid.] [Annot. 20 Oct 2020.] **(gg: Matrd)**

2015a H-Trees, restrictions of Dowling group geometries. *Bull. Korean Math. Soc.* 52 (2015), no. 3, 955–962. MR 3353304. Zbl 1328.05038.

H is a finite group. "Complete H-tree", "-path", "-star", "-cycle", "fan", "wheel" = full frame matroid $\mathbf{F}(H{\cdot}\Delta^\bullet)$ where Δ is a tree, path, etc. [The stated definition of H-tree is incomplete.] Spanning submatroids are called "H-trees", etc. Props. 3.1, 5.1 list simple properties. Thm. 3.2 evaluates $\#E(H{\cdot}\Delta^\bullet)$. Thms. give the characteristic polynomials [they are special cases of Zaslavsky (1995b), §§4, 5]. [Annot. 20 Oct 2020.]
(gg: Matrd, Invar)

Andrzej Mróz

See J.A. Jimenez G.

Andrej Mrvar

See also M. Brusco, P. Doreian and W. de Nooy.

Andrej Mrvar & Patrick Doreian

2009a Partitioning signed two-mode networks. *J. Math. Sociology* 33 (2009), no. 3, 196–221. Zbl 1168.91511.

§2, "Formalization of block-modeling signed two-mode data": A signed two-mode network is a signed simple bipartite graph with color classes V_1, V_2. The objective is partitions π_1, π_2 of V_1, V_2 that minimize a "criterion function" $P := \alpha i_- + (1-\alpha)i_+$; usually $\alpha = .5$. $k_1 := \#\pi_1$ and $k_2 := \#\pi_2$, or other restrictions, may be specified. Definitions: $\pi_i := \{V_{i1}, \ldots, V_{ik_i}\}$. A "block" is a nonvoid set $E(V_{1i}, V_{2j})$. Its sign is the sign of the majority of edges, + if a draw. e is "consistent" with (π_1, π_2) if it is in a block of sign $\sigma(e)$. $i_\varepsilon :=$ number of inconsistent edges of sign ε. [Annot. 17 Aug 2009.] **(SG: Clu, PsS)**

Guohong Mu, Lulu Li, & Xiaodi Li

2020a Quasi-bipartite synchronization of signed delayed neural networks under impulsive effects. *Neural Networks* 129 (2020), 31–42. Zbl 1478.93658.

Balanced signed digraphs, not switched to all positive [which would be simpler]. [Annot. 19 Oct 2020.] **(SD: Bal: Dyn)**

Lili Mu & Richard P. Stanley

2015a Supersolvability and freeness for ψ-graphical arrangements. *Discrete Comput. Geom.* 53 (2015), 965–970. MR 3341588. Zbl 1314.05076. arXiv:1501.07612.

Cf. Stanley (2015a). [Annot. 16 Nov 2018.] **(gg: Geom)**

G. Muciaccia
See R. Crowston.

Antara Mukherjee
See M. Blair.

Raffaella Mulas
See also A. Abiad, E. Andreotti, and J. Jost.

20xxa Sharp bounds for the largest eigenvalue of the normalized hypergraph Laplace operator. Submitted. arXiv:2004.02154. **(SH: Lap: Eig)**

20xxb Spectral classes of hypergraphs. Submitted. arXiv:2007.04273. **(SH, lg: Adj, Lap, Incid: Eig)**

Raffaella Mulas, Christian Kuehn, & Jürgen Jost

2020a Coupled dynamics on hypergraphs: Master stability of steady states and synchronization. *Phys. Rev. E* 101 (2020), art. 062313, 6 pp. MR 4121260. arXiv:-2003.13775. **(SH: Lap: Eig)**

Raffaella Mulas & Nathan Reff

20xxa Spectra of complex unit hypergraphs. Submitted. arXiv:2011.10458. **(SH(Gen): Adj, Lap, Incid: Eig)**

Raffaella Mulas, Rubén J. Sánchez-García, & Ben D. MacArthur

20xxa Hypergraph automorphisms. Submitted. arXiv:2010.01049. **(SH: Aut, Lap: Eig)**

Raffaella Mulas & Zoran Stanić

2022a Star complements for ± 2 in signed graphs. *Special Matrices* 10 (2022), 258–266. MR 4381526. Zbl 1484.05136. **(SG: Adj: Eig)**

Raffaella Mulas & Dong Zhang

2021a Spectral theory of Laplace operators on oriented hypergraphs. *Discrete Math.* 344 (2021), no. 6, art. 112372, 22 pp. MR 4236395. arXiv:2004.14671. **(SH: Lap: Eig)**

Haiko Müller
See T. Kloks.

Ganesh Mundhe, Y.M. Borse, & K.V. Dalvi

2022a On graphic elementary lifts of graphic matroids. *Discrete Math.* 345 (2022), no. 10, art. 113014. MR 4435894. Zbl 1491.05048. arXiv:1910.05689.

Lifts via binary coextensions. The 6 forbidden minors for graphic matroids all of whose binary coextensions are graphic. [Annot. 20 Jun 2022.] **(gg: Matrd)**

Akihiro Munemasa
See also A.L. Gavrilyuk, G. Greaves, M. Harada, and H.J. Jang.

Akihiro Munemasa, Yoshio Sano, & Tetsuji Taniguchi

2014a Fat Hoffman graphs with smallest eigenvalue at least $-1-\tau$. *Ars Math. Contemp.* 7 (2014), no. 1, 247–262. MR 3084550. Zbl 1301.05221. arXiv:1111.7284. **(SG: Eig)**

Luigi Muracchini & Anna Maria Ghirlanda

1965a Sui grafi segnati ed i grafi commutati. *Statistica* (Bologna) 25 (1965), 677–680. MR 0199122 (33 #7272).

A partially successful attempt to use unoriented signed graphs to define a line graph of a digraph. [See Zaslavsky (2010b), (20xxa), (2012c) for the correct signed-graph approach.] The Harary–Norman line digraph is also discussed. (**SG: Bal, LG**)

Kunio Murasugi

1988a On the signature of a graph. *C.R. Math. Rep. Acad. Sci. Canada* 10 (1988), 107–111. MR 0933223 (89h:05056).

The signature of a sign-colored graph (see (1989a)) is an invariant of the sign-colored graphic matroid. (**SGc: Incid, matrd**)

1989a On invariants of graphs with applications to knot theory. *Trans. Amer. Math. Soc.* 314 (1989), 1–49. MR 0930077 (89k:57016). Zbl 726.05051.

Studies a dichromatic form, $P_\Sigma(x, y, z)$, of Kauffman's (1989a) Tutte polynomial of a sign-colored graph. The deletion-contraction parameters are $a_\varepsilon = 1$, $b_\varepsilon = x^\varepsilon$ for $\varepsilon = \pm 1$; the initial values are such that $P_\Sigma(x, y, z) = y^{-1}Q_\Sigma(a, b; y, z)$ of Zaslavsky (1992b). The polynomial is shown to be, in effect, an invariant of the sign-colored graphic matroid.

Much unusual graph theory is in here. A special focus is the degrees of the polynomial. First Main Thm. 3.1: Formulas for the maximum and minimum combined degrees of $P_\Sigma(x, y, z)$. §7, "Signature of a graph", studies the signature (σ in the paper, s here) of the Laplacian matrix $L(\Sigma)$ (B_Σ in the paper) obtained by changing the diagonal of $A(\Sigma)$ so the row sums are 0. Prop. 7.2 is a matrix-tree theorem [entirely different from that of Zaslavsky (1982a)]. The Second Main Thm. 8.1 bounds the signature: $\#V - 2\beta_0(\Sigma^-) + 1 \leq s \leq \#V - 2\beta_0(\Sigma^+) + 1$ ($\beta_0 = c(\Sigma)$), with equality characterized. The Laplacian matrix is further examined later on. §9, "Dual graphs": Differing from most studies, here the dual of a sign-colored plane graph is the planar dual with same edge signs [however, negating all colors is a triviality]. §10, "Periodic graphs": These graphs might be called branched covering graphs of signed gain graphs with finite cyclic gain group. [Thus they generalize the periodic graphs of Collatz (1978a) and others.] §§12–15 concern applications to knot theory.

(**SGc: Invar, Incid, GG(Cov), Du, Knot**)

1991a Invariants of graphs and their applications to knot theory. In: S. Jackowski, B. Oliver, and K. Pawałowski, eds., *Algebraic topology Poznań 1989* (Proc., Poznań, 1989), pp. 83–97. Lect. Notes in Math., Vol. 1474. Springer-Verlag, Berlin, 1991. MR 1133894 (92m:57015). Zbl 751.57007.

§§1–3 expound results from (1989a) on the dichromatic polynomial and the signature of a sign-colored graph and knot applications. §5 discusses the signed Seifert graph of a link diagram.

(**SGc: Invar, Incid, Knot: Exp**)

1993a *Musubime riron to sono ~~ono~~.* [Knot Theory and Its Applications.] (In Japanese.) 1993.

See (1996a). (**SGc: Knot**)

1996a *Knot Theory and Its Applications.* Birkhäuser, Boston, 1996. MR 1391727 (97g:57011). Zbl 864.57001.

Updated translation of (1993a) by Bohdan Kurpita. Pp. 36–37: Construction of signed plane graph from link diagram, and conversely.
(**SGc: Knot**)

Kunio Murasugi & Jozef H. Przytycki

1993a *An Index of a Graph with Applications to Knot Theory.* Mem. Amer. Math. Soc., Vol. 106, No. 158. Amer. Math. Soc., Providence, R.I., 1993. MR 1171835 (94d:57025). Zbl 792.05047.

Ch. I, "Index of a graph". The "index" is the largest number of "independent" edges, where "independent" has a complicated recursive definition (unrelated to matchings), one of whose requirements is that the edges be "singular" (= simple). The positive or negative index of a sign-colored graph is similar except that the independent edges must all be positive or negative. [The general notion is that of the index of a graph-subgraph pair. The signs pick out complementary subgraphs.] Thm. 2.4: Each of these indices is additive on blocks of a bipartite graph. The main interest, because of applications to knot theory, is in bipartite plane graphs. Ch. II, "Link theory": Pp. 26–27 define the sign-colored Seifert graph of an oriented link diagram and apply the graphical index theory.
(**SGc: Invar, Du, Knot**)

Tadao Murata

1965a Analysis of lossy communication nets by modified incidence matrices. In: M.E. Van Valkenburg, ed., *Proceedings, Third Annual Allerton Conference on Circuit and System Theory* (Monticello, Ill., 1965), pp. 751–761. Dept. of Electrical Eng. and Coordinated Sci. Lab., University of Illinois, Urbana, Ill.; and Circuit Theory Group, Inst. of Electrical and Electronics Engineers, [1965].
(**GN: Incid**)

Antoine Musitelli

See also A. Del Pia.

2007a *Recognition of Generalized Network Matrices.* Doctoral thesis, École Polytechique Fedéral Lausanne, 2007. arXiv:0807.3541.

A polynomial-time algorithm for recognizing binet matrices in time $O(n^6\#E)$. See (2010a). [Annot. 15 January 2013.]
(**SG: Ori: Incid, Algor**)

2010a Recognizing binet matrices. *Math. Programming* 124 (2010), no. 1-2, 349–381. MR 2679995 (2011g:68121). Zbl 1206.68149.

Cf. Kotnyek (2002a). Description of the algorithm of (2007a), which involves reduction to the cases of "cyclic" and "bicyclic" matrices. These are the incidence matrices of bidirected graphs with, respectively, one or two components that are without half edges. [Annot. 15 January 2013, rev 16 Oct 2017.] (**SG: Ori: Incid, Algor**)

Mohammed A. Mutar

See also A.H. Busch.

2017a *Hamiltonicity in Bidirected Signed Graphs and Ramsey Signed Numbers.* M.S. thesis, Wright State University, 2017.

Necessary and sufficient conditions for coherent Hamiltonian cycles in oriented $\pm K_n$, $\pm K_{n,n}$, $-K_{n,n}$. *Cf.* Busch–Mutar–Slilaty (2022a).
(**SG: Ori: Cyc**)

Some values of $r^*(n,m) := \min\{r : \forall(K_r,\sigma) \supseteq [+K_n] \text{ or } [-K_m]\}$, opening a new Ramsey theory. [Annot. 29 Jun 2022.] (**SG: Ramsey**)

Mohammed A. Mutar, Vaidy Sivaraman, & Daniel Slilaty

2024a Signed Ramsey numbers. *Graphs Combin.* 40 (2024), no. 1, art. 9, 9 pp. MR 4682857.

Introduces signed-graph Ramsey theory. Signed Ramsey number $r_\pm(s,t) := \min\{n : \text{every } (K_n,\sigma) \text{ contains balanced } K_t \text{ or antibalanced } K_s\}$. [Annot. 17 Jan 2024.] (**SG: Sw: Invar**)

A. Muthaiyan & A. Nesamathi

2016a Some new face and total face signed product cordial graphs. *IJISET Int. J. Innovative Sci. Engin. Tech.* 3 (2016), no. 8, 399–407.

More examples as in Rozario Raj and Manoharan (2016a).
(**Lab: VS: SG, Bal**)

P. Mützel

See C. De Simone.

Shokry Nada, Amani Elrayes, Ashraf Elrokh, & Aya Rabie

2019a Signed product cordial of the sum and union of two fourth power of paths and cycles. *Machine Learning Res.* 4 (2019), no. 4, 45–50.

Cf. Babujee and Loganathan (2011a). [Annot. 12 Aug 2021.]
(**Lab: VS, SG**)

S. Nada, A. Elrokh, A. Elrayes, & A. Rabie

2019a The signed product cordial for corona of paths and fourth power of paths. *Int. J. Math. Combin.* 4 (2019), 102–111

Cf. Babujee and Loganathan (2011a). [Annot. 26 Sept 2022.]
(**Lab: VS, SG**)

Sudev Naduvath

See J. Amreen, A. Aniyan, J. Kottarathil, S. Madhumitha, and for same author also N.K. Sudev (under 'S').

Mohamed Nafea

See B. Guler.

Ojiro Nagai

See also H.T. Diep.

Ojiro Nagai, Tsuyoshi Horiguchi, & Seiji Miyashita

2004a Properties and phase transitions in frustrated Ising systems. In: H.T. Diep, ed., *Frustrated Spin Systems*, Ch. 2, pp. 59–105. World Scientific, Hackensack, N.J., 2004. Zbl 1104.82002 (book).

Physics questions on various periodic signed lattice graphs, dim $= 2, 3$. [*Question*. Do the phenomena treated here for periodic signed lattices suggest interesting mathematics, possibly for more general Σ?]

§2.5, "Ising model with large S on antiferromagnetic triangular lattice": Spin 1/2 generalizes to (integral) spin S = spin $\in \{-S, -(S-1), \ldots, 0, \ldots, S-1, S\}$ [as in coloring Σ with $2S+1$ colors] on all-negative triangular lattice. Large S gives new kinds of ground state (min energy). [*Question.* Are these new signed-graph coloring problems?] §2.6, "Ising model with infinite-spin on antiferromagnetic triangular lattice": "Infinite spin" means $S \to \infty$.

Dictionary: "local gauge transformation" = switching, "spin 1/2" (often assumed in physics) = Ising spins ± 1. [Annot. 9 Aug 2018.]

(**SG, Fr, Sw: Phys: Exp, Ref**)

Ojiro Nagai, Koichi Nishino, Jong-Jae Kim, & Yuuzi Yamada

1988a Magnetic properties of a three-dimensional Ising crystal with zero-point entropy. *Phys. Rev. B* 37 (1988), no. 10, 5448–5451.

[Partly from previous works cited?] Cubic lattice with $E^- = \{(i,j,k)(i+1,j,k) : j+k \text{ even}\}$. Edge weights ("bond strengths") $a, 1, 1$ in x, y, z directions, $a > 0$. Ising ground states $\psi : V \to \{\pm 1\}$ (i.e., least weight of unsatisfied edges): For $a < 2$, each yz-plane is satisfied (ψ is constant). For $a > 2$, each x line is satisfied (constant ψ on all-+ lines, alternating on all-− lines). For $a = 2$, both are ground states. Some results are for multivalued spins $-S, \ldots, S-1, S$. Consequence: At $a = 2$ there are "free"' spins ($\psi(v) = \pm 1$ arbitrarily in ground states) at some vertices. [*Question.* Does this suggest interesting mathematics, possibly for more general Σ?] [Annot. 14 Aug 2018.] (**Phys: SG: Fr**)

O. Nagai, Y. Yamada, & H.T. Diep

1985a Linear-chain-like excitations in a three-dimensional Ising lattice with frustration: Monte Carlo simulations. *Phys. Rev. B* 32 (1985), no. 1, 480–483.

Computer simulations on a cubic lattice with $E^- = \{(i,j,k)(i+1,j,k) : j \text{ even}\}$. In Ising ground states $\psi : V \to \{\pm 1\}$ (i.e., fewest unsatisfied edges), each z-line has constant ψ along the line; in $1/4$ of them the spin constant varies with time. [*Question.* Does this suggest interesting mathematics for more general Σ?] [Annot. 14 Aug 2018.]

(**Phys: SG: Fr**)

Atulya K. Nagar

See C. Perina and V. Vasanthi.

K.M. Nagaraja

See P.S.K. Reddy.

Yuuki Nagazawa

See T. Matsui.

P. Nageswari & P.B. Sarasija

2014a Seidel energy and its bounds. *Int. J. Math. Anal.* 8 (2014), no. 58, 2869–2871.

(**sg: KG: Adj: Eig**)

Tatiana Nagnibeda

See V. Bugaenko.

Mina Nahvi

See S. Akbari.

T.A. Naikoo

See S. Pirzada.

Takeshi Naitoh
See K. Ando.

Kazuo Nakajima
See H. Choi.

Atsuhiro Nakamoto
See also D. Archdeacon.

Atsuhiro Nakamoto, Seiya Negami, & Katsuhiro Ota

2002a Chromatic numbers and cycle parities of quadrangulations on nonorientable closed surfaces. Ninth Quadrennial Int. Conf. Graph Theory, Combinatorics, Algorithms Appl. *Electronic Notes Discrete Math.* 11 (2002), 509–518. MR 2155788 (no rev). Zbl 1075.05532.

A "cycle parity" on surface S = homomorphism $\rho : \pi(S) \to \mathbb{Z}_2 \cong \{+,-\}$, equivalently $\rho : H_1(S; \mathbb{Z}_2 \to \mathbb{Z}_2 \cong \{+,-\}$. ρ implies a signature (actually, a switching class) of any embedded graph Γ. There are one nontrivial type of cycle parity on an orientable surface and three on a nonorientable surface N_d, different for odd and even d, except two on N_2 and one on N_1. If $\Gamma \hookrightarrow S$ so every face boundary is even ("even embedding"), then $\rho(W) = \#W \bmod 2$ for closed walks is a cycle parity. Thm. 9: For three of the six types on N_d's, there is a negative cut that opens N_d to an orientable surface. [Also see Mohar and Seymour (2002a).] [Annot. 11 Jun 2012.] **(sg: Top: sw)**

2004a Chromatic numbers and cycle parities of quadrangulations on nonorientable closed surfaces. *Discrete Math* 285 (2004), 211–218. MR 2062844 (2005k:05101). Zbl 1044.05034. **(sg: Top: sw)**

Daishin Nakamura & Akihisa Tamura

1998a The generalized stable set problem for claw-free bidirected graphs. In: Robert E. Bixby, E. Andrew Boyd, and Roger Z. Ríos-Mercado, eds., *Integer Programming and Combinatorial Optimization* (6th Int. IPCO Conf., Houston, 1998, Proc.), pp. 69–83. Lect. Notes in Computer Sci., Vol. 1412. Springer, Berlin, 1998. MR 1726336 (2000h:05209). Zbl 907.90272.

The problem of the title is solvable in polynomial time. See Johnson and Padberg (1982a), Tamura (1997a) for definitions. They reduce to simple graphs, transitively bidirected with no sink or introverted edge (called "canonical" bidirected graphs). **(sg: Ori: Geom, Sw, Algor)**

1998b Generalized stable set problems for claw-free bi-directed graphs. (In Japanese.) Theory and Applications of Mathematical Optimization (Kyoto, 1998). *Sūrikaisekikenkyūsho Kōkyūroku* No. 1068 (1998), 100–109. Zbl 939.05506 (no rev). **(sg: Ori: Geom, Sw, Algor)**

2000a A linear time algorithm for the generalized stable set problem on triangulated bidirected graphs. New Trends in Mathematical Programming (Kyoto, 1998). *J. Operations Res. Soc. Japan* 43 (2000), 162–175. MR 1768393 (2001c:90093). Zbl 1138.90494. **(sg: Ori: Geom: Algor)**

M. Nakamura
See M. Hachimori.

Tota Nakamura, Shin-ichi Endoh, & Takeo Yamamoto

2003a Weak universality of spin-glass transitions in three-dimensional $\pm J$ models. *J. Phys. A* 36 (2003), 10895–10906. MR 2025232 (no rev). Zbl 1075.82508.

Physics of Ising, XY, and Heisenberg spin-glass models on a signed square lattice graph with 3-dimensional spin vectors. The Hamiltonian of state $S : V \to \mathbb{S}^2$ (the sphere) is $\sum_{uv \in E} \sigma(uv) S_u \cdot S_v$. [Annot. 17 Jun 2012.] **(Phys: SG)**

Bunpei Nakano

See T. Inohara.

Norihiro Nakashima & Shuhei Tsujie

2021a Enumeration of flats of the extended Catalan and Shi arrangements with species. *J. Integer Seq.* 24 (2021), no. 9, art. 21.9.2, 21 pp. MR 4336094. Zbl 1489.52016. arXiv:1904.09748. **(GG: Geom: Invar)**

Preetum Nakkiran

See C.E. Tsourakakis.

Vasileios Nakos

See C.E. Tsourakakis.

Aurélien Naldi

See also J.-P. Comet.

Aurélien Naldi, Elisabeth Remy, Denis Thieffry, & Claudine Chaouiya

2011a Dynamically consistent reduction of logical regulatory graphs. *Theor. Computer Sci.* 412 (2011), no. 21, 2207–2218, MR 2809505 (2012a:92077). Zbl 1211.92024. **(SD, Dyn)**

Soumen Nandi

See J. Bensmail and S. Das.

L. Nanjundaswamy

See E. Sampathkumar.

Assaf Naor

See N. Alon.

Joseph (Seffi) Naor

See D. Hochbaum.

Vito Napolitano

See M. Abreu.

Sivaram K. Narayan

See N. Matar.

Ramasuri Narayanam

See P. Agrawal.

Jonathan Narboni

See F. Kardoš.

Reza Naserasr

See also L. Beaudou, R.C. Brewster, C. Cappello, C. Charpentier, and F. Foucaud.

20xxa Not $\{\pm 1, \pm 2\}$-colorable planar signed graphs. Submitted.

A simpler, smaller proof of Kardoš and Narboni (2021a), with remarks and questions on related homomorphisms. [Annot. 17 Nov 2019.] (**SG: Col, Hom**)

Reza Naserasr & Lan Anh Pham

2022a Complex and homomorphic chromatic number of signed planar simple graphs. *Graphs Combin.* 38 (2022), art. 58, 22 pp. MR 4393983. HAL hal-03000542.

Color set $C_{k,l} = \{\pm 1, \ldots, \pm k, \pm 1\mathrm{i}, \ldots, \pm l\mathrm{i}\}$, $\mathrm{i} = \sqrt{-1}$. Coloration κ is proper when $|\kappa(u)| \neq |\kappa(v)|$ or $\operatorname{sgn}\kappa(u) \cdot \operatorname{sgn}\kappa(v) \neq \sigma(uv)$, where $\operatorname{sgn} z := z/|z|$. [Annot. 25 Oct 2020.] (**SG: Col, Hom**)

Reza Naserasr, Lan Anh Pham, & Zhouningxin Wang

2022a Density of C_{-4}-critical signed graphs. *J. Combin. Theory Ser. B* 153 (2022), 81–104. MR 4345281. Zbl 1481.05086. arXiv:2101.08612. HAL hal-03000545.
C_{-4} = negative quadrilateral C_4. (**SG: Hom**)

Reza Naserasr, Edita Rollová, & Éric Sopena

2013a Homomorphisms of planar signed graphs to signed projective cubes. *Discrete Math. Theor. Computer Sci.* 15 (2013), no. 3, 1–12. MR 3119638. Zbl 1283.05186. HAL hal-00876298. (**SG: Hom**)

2013b On homomorphisms of planar signed graphs to signed projective cubes. In: Jaroslav Nešetřil and Marco Pellegrini, eds., *The Seventh European Conference on Combinatorics, Graph Theory and Applications* (EuroComb 2013, Pisa), pp. 271–276. CRM Ser., Vol. 16. Edizioni della Normale, Scuola Normale Superiore Pisa, Pisa, Italy, 2013. MR 3185818 (no rev). Zbl 06302999. HAL hal-00829117. (**SG: Hom**)

2013c Homomorphisms of signed bipartite graphs. In: Jaroslav Nešetřil and Marco Pellegrini, eds., *The Seventh European Conference on Combinatorics, Graph Theory and Applications* (EuroComb 2013, Pisa), pp. 345–350. CRM Ser., Vol. 16. Edizioni della Normale, Scuola Normale Superiore Pisa, Pisa, Italy, 2013. MR 3185829 (no rev). Zbl 1292.05191. HAL hal-00829116. (**SG: Hom**)

2015a Homomorphisms of signed graphs. *J. Graph Theory* 79 (2015), no. 3, 178–212. MR 3346138. Zbl 1322.05069. HAL hal-00991796. (**SG: Hom**)

Reza Naserasr, Sagnik Sen, & Éric Sopena

2021a The homomorphism order of signed graphs. *J. Combin. Math. Combin. Computing* 116 (2021), 169–182. MR 4260818. Zbl 1466.05088. HAL hal-02969878. (**SG: Hom**)

Reza Naserasr, Sagnik Sen, & Qiang Sun

2016a Walk-powers and homomorphism bounds of planar signed graphs. *Graphs Combin.* 32 (2016), no. 4, 1505–1519. MR 3514981. Zbl 1342.05059. (**SG: Hom, Top**)

Reza Naserasr, Riste Škrekovski, Zhouningxin Wang, & Rongxing Xu

20xxa Mapping sparse signed graphs to (K_{2k}, M). Submitted. arXiv:2101.08619. HAL hal-03109532. (**SG: Hom**)

Reza Naserasr, Éric Sopena, & Thomas Zaslavsky

2021a Homomorphisms of signed graphs: An update. *European J. Combin.* 91 (2021), art. 103222, 20 pp. MR 4161815. Zbl 1458.05098. arXiv:1909.05982. HAL hal-02429415.

Several aspects of homorphisms. Aims to set correct terminology, give proofs of fundamental properties, raise open questions. [Annot. 16 Sept 2019.] (**SG, SGc: Hom: Exp**)

Reza Naserasr & Zhouningxin Wang

2021a Bounding signed series-parallel graphs and cores of signed K_4-subdivisions. *J. Combin. Math. Combin. Comput.* 116 (2021), 27–51. MR 4260812. Zbl 1466.05089. HAL hal-02969873. (**SG: Hom**)

2023a Signed bipartite circular cliques and a bipartite analogue of Grötzsch's theorem. *Discrete Math.* 346 (2023), no. 12, art. 113604, 12 pp. MR 4623406. HAL hal-03356580. (**SG: Col**)

Reza Naserasr, Zhouningxin Wang, & Xuding Zhu

2021a Circular chromatic number of signed graphs. *Electronic J. Combin.* 28(2) (2021), no. 2, art. P2.44, 40 pp. MR 4281210. Zbl 1466.05090. arXiv:-2010.07525. HAL hal-02969872.

[*Cf.* Pan and Zhu (2022a).] (**SG: Col**)

Reza Naserasr & Weiqiang Yu

2023a Packing signatures in signed graphs. *SIAM J. Discrete Math.* 37(2023), no. 4, 2365–2381. MR 4654108. HAL hal-03960731.

"Signature packing number" or negative-edge-set packing number $\rho(\Sigma)$ = maximum number of disjoint sets $E^-(\Sigma^\zeta)$ of switchings of Σ. Connections to homomorphisms and the 4-Color Theorem. Dictionary: "signature" = negative edge set. *Cf.* Lacasse (20xxa). [Annot. 15 May 2021.] (**SG: Sw: Hom, Col, Algor**)

2023b Separating signatures in signed planar graphs. *Discrete Appl. Math.* 338 (2023), 302–310. MR 4611134.

Generalizes (2023a) to disjoint sets of switchings of several arbitrary signatures. Dictionary: "signature" = negative edge set. [Annot. 2 Apr 2022.] (**SG: Sw, Col, Hom**)

20xxa On the packing number of antibalanced signed simple planar graphs of negative girth at least 5. Submitted. HAL hal-04080407. (**SG: Par**)

C.St.J.A. Nash-Williams

1960a On orientations, connectivity, and odd-vertex-pairings in finite graphs. *Canad. J. Math.* 12 (1960), 555–567. MR 0118684 (22 #9455). Zbl 096.38002 (96, p. 380b).

1969a Well-balanced orientations of finite graphs and unobtrusive odd-vertex-pairings. In: W.T. Tutte, ed., *Recent Progress in Combinatorics* (Proc. Third Waterloo Conf., 1968), pp. 133–149. Academic Press, New York, 1969. MR 0253933 (40 #7146). Zbl 209.55701 (209, p. 557a).

Nagarajan Natarajan

See K.-Y. Chiang.

Nutan G. Nayak

2014a Equienergetic net-regular signed graphs. *Int. J. Contemp. Math. Sci.* 9 (2014), no. 14, 685–693.

Eigenvalues of $K_{-\overline{C}_n}$. These lead to non-cospectral pairs of net-regular signed K_n's with equal adjacency energy. [Annot. 20 Apr 2019.]
(**SG: KG: Adj: Eig**)

2016a On net-regular signed graphs. *Int. J. Math. Combin.* 1 (2016), 57-64.

Elementary facts of net regularity. Eigenvalues of $K_{-\overline{C}_n}$. [Annot. 20 Apr 2019.] (**SG: KG: Adj: Eig**)

2017a On net-Laplacian energy of signed graphs. *Commun. Combin. Optim.* 2 (2017), no. 1, 11–19. MR 3657818. Zbl 386.05081.

Net Laplacian $L^{\pm}(\Sigma) := \mathrm{diag}(d^{\pm}_{\Sigma}) - A(\Sigma)$. (**SG: Lap, Eig**)

2017b Spectra and energy of signed graphs. *Int. J. Math. Combin.* 1 (2017), 10–21.
(**SG: Adj: Eig**)

Zachary Neal

See S. Aref.

Roman Nedela

See also A. Malnič.

Roman Nedela & Martin Škoviera

1996a Regular embeddings of canonical double coverings of graphs. *J. Combin. Theory Ser. B* 67 (1996), 249–277. MR 1399678 (97e:05078). Zbl 856.05029.

By "canonical double covering" of Γ they mean the signed covering graph $\tilde{\Sigma}$ of $\Sigma = -\Gamma$, but without reversing orientation at the negative covering vertex [as one would do in a signed covering graph (*cf.* e.g. Zaslavsky (1992a))], because orientable embeddings of Γ are being lifted to orientable embeddings of $\tilde{\Sigma}$. [Thus these should be thought of not as signed graphs but rather as voltage (i.e., gain) graphs with 2-element gain group.] Instead of reversal they twist the negative-vertex rotations by taking a suitable power. In some cases this allows classifying the orientable, regular embeddings of $\tilde{\Sigma}$. (**Par: Cov, Top, Aut**)

1997a Exponents of orientable maps. *Proc. London Math. Soc.* (3) 75 (1997), 1–31. MR 1444311 (98i:05059). Zbl 877.05012.

Main topic: the theory of twisting of rotations as in (1996a).
(**GG: Cov, Top, Aut**)

Portions concern double covering graphs of signed graphs. §7: "Antipodal and algebraically antipodal maps". A map is "antipodal" if it is the orientable double covering of a nonorientable map; that is, as a graph it is the canonical double covering of an unbalanced signed graph. A partial algebraic criterion for a map to be antipodal. §9: "Regular embeddings of canonical double coverings of graphs". See (1996a).
(**Par: Cov, Top, Aut**)

1997b Regular maps from voltage assignments and exponent groups. *European J. Combin.* 18 (1997), 807–823. MR 1478826 (98j:05061). Zbl 908.05036.

Cases in which the classification of (1996a) is necessarily incomplete are studied by taking larger voltage (i.e., gain) groups and twisting the

rotations at covering vertices by taking a power that depends on the position of the vertex in its fiber. Main result: the (very special) conditions on twisting under which a regular map lifts to a regular map.
(**GG: Cov, Top, Aut**)

Seiya Negami
See D. Archdeacon and A. Nakamoto.

Peter Nelson & Jorn van der Pol

2019a On the number of biased graphs. *SIAM J. Discrete Math.* 33 (2019), no. 1, 373–382. MR 3915407. Zbl 1405.05084. arXiv:1807.11383.
Almost all biased simple graphs are complete graphs with Hamiltonian bias. [Annot. 20 Aug 2019.] (**GG: Enum**)

Max Nelson-Kilger
See M.H. Fişek.

Mohammad Ali Nematollahi
See U. Ahmad and S. Akbari.

Toshio Nemoto
See K. Ando.

Anna Nenca
See J. Dybizbański.

Hanna Nencka [H. Nencka-Fisek]
See H. Nencka-Fisek.

H. Nencka-Fisek [H. Nencka]
See also Ph. Combe.

1982a Can frustrations arise in Ising systems with multi-spin interactions of different signs? *J. Appl. Phys.* 53 (1982), 7969–7970. (**SG(Gen), Phys: sh: Fr**)

1984a Necessary and sufficient conditions for the overblocking effect. In: A. Pękalski and J. Sznajd, eds., *Static Critical Phenomena in Inhomogeneous Systems* (Proc. XX Karpacz Winter School Theor. Phys., Karpacz, Poland, 1984), pp. 337–343. Lect. Notes in Physics Vol. 206. Springer-Verlag, Berlin, 1984. MR 0839663 (87i:82096).
Signs are defined for arbitrary proper subhypercubes of the hypercube Q_d [thus giving a signed hypergraph]. A "plaquette" (this is nonstandard) is a $k-1$-dimensional band around a k-subhypercube; its sign is the product of signs of its $k-1$-faces. Overblocking means not all plaquettes can simultaneously be negative ("frustrated"). The interesting proof is by the adjacency graph of 2-faces of a 3-cube in Q_d. Identify opposite 2-faces to a single vertex whose sign is the product of 2-face signs; the faces of a 3-cube form a triangle whose vertices alternate in sign, if all plaquettes were negative. Conclusion: All 2-faces cannot be negative, if $d > 2$. [Presumably a similar argument should be applied to plaquettes of $k-1$-faces of a k-cube, $k > 3$, but it is not. There would be one plaquette per dimension. *Question.* Is there such a generalization?] [Annot. 19 Jun 2012.] (**SG(Gen), Phys: sh: Fr**)

1985a Topological closure as the necessary condition for frustration or phase transitions. *J. Math. Phys.* 26 (1985), 1597–1599. MR 0793301 (87f:82026).

A higher-dimensional Ising model with a sign attached to each "plaquette" (see (1985a)). A plaquette is frustrated if the spin product (spin $= \pm 1$) of its sites (vertices) fails to match the attached sign. A necessary condition for frustration is said to be an "umbrella" (a topological construction, possibly a cap on the plaquette?). An example is a triangular lattice. The main example is $\mathbb{Z}^d$. The theorem implies that only $k < d$ can have a frustrated plaquette [obvious, if a plaquette lives in the boundary of a subcube]. [The article seems imprecise. The idea could be worth pursuing.] [Annot. 26 Dec 2014.] (**SG(Gen), Phys: sh: Fr**)

A. Nesamathi

See A. Muthaiyan.

Jaroslav Nešetřil

See also J. Kratochvíl.

Jaroslav Nešetřil & André Raspaud

2000a Colored homomorphisms of colored mixed graphs. *J. Combin. Theory Ser. B* 80 (2000), 147–155. MR 1778206 (2001k:05078). Zbl 1026.05047. (**sg, sd: Hom**)

Nancy Ann Neudauer

See also R.A. Brualdi, L. Goddyn, G. Gordon, and J. McNulty.

1998a *The Transversal Presentations and Graphs of Bicircular Matroids*. Doctoral dissertation, University of Wisconsin–Madison, 1998. MR 2697849.

The matroids are $\mathbf{F}(\Gamma, \varnothing)$. (**Bic**)

2002a Graph representations of a bicircular matroid. *Discrete Appl. Math.* 118 (2002), no. 3, 249–262. MR 1892972 (2003b:05047). Zbl 990.05025.

Survey of parts of Brualdi and Neudauer (1997a), Wagner (1985a), and Coullard, del Greco, and Wagner (1991a), with supplementary results on nice graphs whose bicircular matroid, $\mathbf{F}(\Gamma, \varnothing)$, equals M. (**Bic**)

Nancy Ann Neudauer, Andrew M. Meyers, & Brett Stevens

2001a Enumeration of the bases of the bicircular matroid on a complete graph. Proc. Thirty-second Southeastern Int. Conf. Combinatorics, Graph Theory and Computing (Baton Rouge, La., 2001). *Congressus Numer.* 149 (2001), 109–127. MR 1887396 (2002m:05054). Zbl 1003.05031.

Counts bases and connected bases. Very complicated formulas. [The results count labelled simple 1-trees and 1-forests. A 1-*tree* is a tree with one extra edge forming a circle. A 1-*forest* is a disjoint union of 1-trees. A connected basis of the bicircular matroid $\mathbf{F}(K_n, \varnothing)$ for $n \geq 3$ is a labelled simple 1-tree; a basis is a labelled simple 1-forest. Riddell (1951a) has a less complicated formula for 1-trees.] (**Bic: Invar(Bases)**)

Nancy Ann Neudauer & Brett Stevens

2001a Enumeration of the bases of the bicircular matroid on a complete bipartite graph. *Ars Combin.* 66 (2003), 165–178. MR 1961484 (2004a:05034). Zbl 1075.05510.

Bases are counted and their structure compared to the spanning trees of the graph. [A basis is a simple, labelled 1-forest (*cf.* Neudauer, Meyers, and Stevens (2001a)) whose circles are even.] (**Bic: Invar(Bases)**)

Nancy Ann Neudauer & Daniel Slilaty

2017a Bounding and stabilizing realizations of biased graphs with a fixed group. *J. Combin. Theory Ser. B* 122 (2017), 149–166. MR 3575200. Zbl 1350.05073. (**GG**)

A. Neumaier

1982a Completely regular twographs. *Arch. Math. (Basel)* 38 (1982), 378–384. MR 0658386 (83g:05066). Zbl 475.05045.

In the signed graph (K_n, σ) of a two-graph (see D.E. Taylor (1977a)), a "clique" is a vertex set that induces an antibalanced subgraph. A two-graph is "completely regular" if every clique of size i lies in the same number of cliques of size $i+1$, for all i. Thm. 1.4 implies there is only a small finite number of completely regular two-graphs. (**TG**)

Michael Neumann

See C.R. Johnson.

Víctor Neumann-Lara

See I.J. Dejter.

Bryan Nevarez

See M. Beck.

T.M. Newcomb

See also K.O. Price.

1968a Interpersonal balance. In: R.P. Abelson *et al.*, eds., *Theories of Cognitive Consistency: A Sourcebook*. Rand-McNally, Chicago, Ill., 1968. (**PsS**)

G.F. Newell

1950a Crystal statistics of a two-dimensional triangular Ising lattice. *Phys. Rev.* (2) 79 (1950), 876–882. MR 0039631 (12, 576i). Zbl 038.13902 (38, p. 139b).

The same physics conclusions as Houtappel's (1950a), (1950b) for a signed, weighted triangular lattice. [Also see I. Syôzi (1950a), Wannier (1950a).] [Annot. 20 Jun 2012.] (**Phys, WG, sg: Fr**)

Alantha Newman

See N. Ailon.

Charles M. Newman

See also F. Camia and A. Gandolfi.

Charles M. Newman & Daniel L. Stein

1997a Metastate approach to thermodynamic chaos. *Phys. Rev. E* (3) 55 (1997), no. 5, part A, 5194–5211. MR 1448389 (98k:82098). arXiv:cond-mat/9612097.

A technical paper supporting (1998a). [Annot. 26 Aug 2012.] (**Phys: sg: State, fr**)

1998a Simplicity of state and overlap structure in finite-volume realistic spin glasses. *Phys. Rev. E* 57 (1998), no. 2, 1356–1366. MR 1606015 (99b:82057). arXiv:-cond-mat/9711010. (**Phys**)

Mike Newman

See D. Funk.

1998a Thermodynamic chaos and the structure of short-range spin glasses. In: Anton Bovier and Pierre Picco, eds., *Mathematical Aspects of Spin Glasses and Neural Networks*, pp. 243–287. Progress in Prob., Vol. 41. Birkhäuser, Boston, 1998. MR 1601751 (99b:82056). Zbl 896.60078.

See especially §3, "The standard SK picture". The Hamiltonian $H_\sigma(s) = -\sum_{vw\in E} \sigma(vw)s(v)s(w)$ is standard. Criticizes the typical physics application of randomly signed (and possibly weighted) K_n (Sherrington–Kirkpatrick model) to $\mathbb{Z}^d$-lattice graphs by limits of finite (cubical) subgraphs. Raises the question of a "pure state" (*cf.* Mézard, Parisi, and Virasoro (1987a) *et al.*) of a signed K_n, where a state is $s : V \to \{+1, -1\}$ and a pure state is apparently a linear combination of or probability distribution on states, especially in the $\mathbb{Z}^d$ limit. A pure state is not well defined but is related to states of low frustration (and high probability). [*Question.* Is there a graphical meaning of a pure state, based on the (ambiguous) physics definition? It should involve states with low frustration, because they dominate the partition function $Z(\sigma) = \sum_s e^{H_\sigma(s)}$, and on the qualities desired for computing quantities of physical interest, especially in terms of H and Z.]

A "metastate" is a measure on states, essentially a linear combination with explicit coefficients. Pure states on $\mathbb{Z}^d$ should be metastates. See (1997a). [*Question.* Is there a graph-theory meaning to all this? Does it lead to a definition of frustration in an infinite signed (or gain) graph?] [Annot. 26 Aug 2012.] (**Phys: sg: State, fr: Exp, Ref**)

2010a Distribution of pure states in short-range spin glasses. *Int. J. Modern Phys. B* 24 (2010), no. 14, 2091–2106. MR 2659908 (2011g:82055). Zbl 1195.82093.

Further development of (1997a); *cf.* (1998a). [Annot. 26 Aug 2012.] (**Phys: sg: State, fr**)

Sang Nguyen

See P.L. Hammer.

Aidin Niaparast

See S. Akbari.

André Nichterlein

See F. Hüffner.

Robert Nickel

See W. Hochstättler.

Rolf Niedermeier

See F. Hüffner.

F. Nieto

See A.J. Ramírez-Pastor and F. Romá.

Juhani Nieminen

1976a Weak balance: A combination of Heider's theory and cycle and path-balance. *Control Cybernet.* 5 (1976), 69–73. MR 0429628 (55 #2639).

S^c is the "signed closure" of a signed digraph S. S is "weakly balanced" if in S^c all directed digons and all induced transitive triangles are positive. Thm.: S is weakly balanced iff it is path- and cycle-balanced. Also, the degree of weak balance.

(**SD: Bal**)(**SD: Fr: Algor**)

Peter Nijkamp

See F. Brouwer.

Vladimir Nikiforov

See also N.M.M. de Abreu, M.A.A. de Freitas, and L.S. de Lima.

2008a A spectral condition for odd cycles in graphs. *Linear Algebra Appl.* 428 (2008), no. 7, 1492–1498. MR 2388633 (2008k:05130). Zbl 1152.05045. arXiv:0707.4499.

For order $n \gg 0$, if $-\Gamma$ has least eigenvalue $< \sqrt{\lfloor n^2/4 \rfloor}$, then it has negative (i.e., odd) circles of all lengths $\leq n/320$. [*Question.* Does this property generalize to signed graphs as: eigenvalue bound $\Longrightarrow$ all negative circles of lengths $\leq$ upper limit?] [Annot. 20 Sept 2015.]

(**sg: par: Adj: Eig**)

2014a Maxima of the Q-index: degenerate graphs. *Electronic J. Linear Algebra* 27 (2014), art. 15, 250–257. MR 3194954. Zbl 1288.05164. arXiv:1309.4837.

Assume Γ is "k-degenerate": every (induced) subgraph has a vertex of degree $\leq k$. Thm. 1.2: $\lambda_{\max}(-\Gamma) \leq \lambda_{\max}(-(K_k \vee \bar{K}_{n-k}))$, $=$ iff $\Gamma = K_k \vee \bar{K}_{n-k}$. ($\lambda_{\max}$ = max eigenvalue of $L(\Sigma) :=$ Laplacian matrix.) Thm. 1.3: $\lambda_{\max}(-\Gamma) \leq$ function of n, $\#E$, Δ, and δ. [Annot. 20 Jan 2015.]

(**par: Lap: Eig**)

2014b An asymptotically tight bound on the Q-index of graphs with forbidden cycles. *Publ. Inst. Math. (Beograd)* (N.S.) 95(109) (2014), 189–199. MR 3221226. Zbl 1433.05204. arXiv:1310.1430.

(**par: Lap: Eig**)

Vladimir Nikiforov & Xiying Yuan

2014a Maxima of the Q-index: graphs without long paths. *Electronic J. Linear Algebra* 27 (2014), art. 32, 504–514. MR 3266163. Zbl 1320.05078. arXiv:1308.4341.

Thm. 1.4: For large n, with $\lambda_{\max} = \lambda_{\max}(L(\Sigma))$: (i) $\Gamma \not\supseteq P_{2k+1} \Longrightarrow \Gamma = K_k \vee \bar{K}_{n-k}$ or $\lambda_{\max}(-\Gamma) < \lambda_{\max}(-(K_k \vee \bar{K}_{n-k}))$. (ii) $\Gamma \not\supseteq P_{2k+2} \Longrightarrow \Gamma = K_k \vee \bar{K}_{n-k} \cup e$ or $\lambda_{\max}(-\Gamma) < \lambda_{\max}(-(K_k \vee \bar{K}_{n-k} \cup e))$. ($P_l$ = path of length l.) [Annot. 20 Jan 2015.] (**par: Lap: Eig**)

2015a Maxima of the Q-index: forbidden even cycles. *Linear Algebra Appl.* 471 (2015), 636–653. MR 3314357. Zbl 1307.05149. arXiv:1410.2142. (**par: Lap: Eig**)

Yuri Nikolayevsky

See G. Cairns.

Zoran Nikoloski

See N. Kejžar.

Wenjie Ning, Hao Li, & Mei Lu

2013a On the signless Laplacian spectral radius of irregular graphs. *Linear Algebra Appl.* 438 (2013), no. 5, 2280–2288. MR 3005290. Zbl 1258.05074.

(**par: Lap: Eig**)

Koichi Nishino

See O. Nagai.

Kenta Nishiyama

See T. Hibi.

Anthony Nixon

See K. Clinch.

M. Nogala

See E.E. Vogel.

Kenta Noguchi

2017a Even embeddings of the complete graphs and their cycle parities. *J. Graph Theory* 85 (2017), no. 1, 187–206. MR 3634482. Zbl 1377.05042.

Surface embedding of $-\Gamma$ so every face boundary is positive. [*Cf.* Archdeacon, Hutchinson, *et al.* (2001a), Nakamoto, Negami, and Ota (2002a), (2004a).] (**sg: Top: sw**)

J.D. Noh, H. Rieger, M. Enderle, & K. Knorr

2002a Critical behavior of the frustrated antiferromagnetic six-state clock model on a triangular lattice. *Phys. Rev. E* 66 (2002), art. 026111, 7 pp. MR 1922222 (2003f:82030). arXiv:cond-mat/0204087.

The all-negative triangular lattice with 6th-root-of-unity spins. [Annot. 24 Mar 2013.] (**Phys, sg: Par: Fr**)

Rafidah MD Noor

See S.R. Shahriary.

Wouter de Nooy

1999a The sign of affection: Balance-theoretic models and incomplete signed digraphs. *Social Networks* 21 (1999), 269–286.

Vertex ranking (a partial ordering) based on arc signs. Thm. 3 characterizes equality of rank. Thm. 6 characterizes strict inequality. [Annot. 11 Sept 2010.] (**SD: PsS, Bal, Clu**)

2008a Signs over time: statistical and visual analysis of a longitudinal signed network. *J. Social Structure* 9 (2008), art. 1, 32 pp. (**SG: Fr, PsS: Dyn**)

Wouter de Nooy, Andrej Mrvar, & Vladimir Batagelj

2005a *Exploratory Social Network Analysis with Pajek.* Structural Anal. Soc. Sci., No. 27. Cambridge Univ. Press, Cambridge, Eng., 2005.

Pajek is a computer package that analyzes networks, i.e., graphs, including signed graphs. Ch. 4: "Sentiments and friendship." Computation of balance and clusterability of signed (di)graphs. §4.2: "Balance theory." Introductory. §4.4: "Detecting structural balance and clusterability." How to use Pajek to optimize clustering. §4.5: "Development in time." Pajek can look for evolution towards balance or clusterability.

§10.3: "Triadic analysis." Types of balance and clusterability, with the triads (order-3 induced subgraphs) that do or do not occur in each. Table 16, p. 209, "Balance-theoretic models", is a chart of 6 related models. §§10.7, 10.10: "Questions" and "Answers." Some are on balance models. §10.9: "Further reading." [Annot. 28 Apr 2009.]

(**SG, SD, PsS: Bal, Clu, Algor: Exp**)

Robert Z. Norman

See also M.H. Fişek and F. Harary.

Robert Z. Norman & Fred S. Roberts

1972a A derivation of a measure of relative balance for social structures and a characterization of extensive ratio systems. *J. Math. Psychology* 9 (1972), 66–91. MR 0293041 (45 #2121). Zbl 233.92006.

Circle ("cycle") indices of imbalance: the proportion of circles that are unbalanced, with circles weighted nonincreasingly according to length. (**SG: Fr(Circ)**)

1972b A measure of relative balance for social structures. In: Joseph Berger, Morris Zelditch, Jr., and Bo Anderson, eds., *Sociological Theories in Progress, Vol. Two*, Ch. 14, pp. 358–391. Houghton Mifflin, Boston, 1972.

Exposition and application of (1972a). (**SG: Fr(Circ): Exp, PsS**)

Mathilde Noual

See also J. Aracena, J.-P. Comet, and J. Demongeot.

Mathilde Noual, Damien Regnault, & Sylvain Sené

2013a About non-monotony in Boolean automata networks. *Theor. Computer Sci.* 504 (2013), 12–25. MR 3107548. Zbl 1297.68179. (**SD: Dyn**)

Beth Novick & András Sebö

1995a On combinatorial properties of binary spaces. In: Egon Balas and Jens Clausen, eds., *Integer Programming and Combinatorial Optimization* (4th Int. IPCO Conf., Copenhagen, 1995, Proc.), pp. 212–227. Lect. Notes in Computer Sci., Vol. 920. Springer-Verlag, Berlin, 1995. MR 1367983 (96h:05039).

The clutter of negative circuits of a signed binary matroid (M, σ). Important are the lift and extended lift matroids, $\mathbf{L}(M, \sigma)$ and $\mathbf{L}_\infty(M, \sigma)$, defined as in signed graph theory. An elementary result: the clutter is signed-graphic iff $\mathbf{L}_\infty(M, \sigma)/e_0$ is graphic (which is obvious). There are also more substantial but complicated results. [See Cornuéjols (2001a), §8.4.] (**Sgnd(Matrd), SG: Matrd**)

1996a On ideal clutters, metrics and multiflows. In: William H. Cunningham, S. Thomas McCormick, and Maurice Queyrann, eds., *Integer Programming and Combinatorial Optimization* (5th Int. IPCO Conf., Vancouver, 1996, Proc.), pp. 275–287. Lect. Notes in Computer Sci., Vol. 1084. Springer-Verlag, Berlin, 1996. MR 1441807 (98i:90075). (**Sgnd(Matrd): Matrd**)

Marc Noy

See O. Giménez.

Cyriel van Nuffelen

1973a On the rank of the incidence matrix of a graph. Colloque sur la Theorie des Graphes (Bruxelles, 1973). *Cahiers Centre Etudes Rech. Oper.* 15 (1973), 363–365. MR 0347660 (50 #162). Zbl 269.05116.

Theorem restated: the unoriented incidence matrix has rank $\operatorname{rk} \mathbf{F}(-\Gamma)$. [Because the matrix represents $\mathbf{F}(-\Gamma)$: see Zaslavsky (1982a). In retrospect, partially implicit in Stewart (1966a) and completely so in Stanley (1973a).] (**par: Incid, ecyc**)

1976a On the incidence matrix of a graph. *IEEE Trans. Circuits Systems* CAS-23 (1976), 572. MR 0441791 (56 #186).

Summarizes (1973a). **(par: Incid, ecyc)**

Koji Nuida

See also T. Abe.

2010a A characterization of signed graphs with generalized perfect elimination orderings. *Discrete Math.* 310 (2010), no. 4, 819–831. MR 2574831 (2011a:05140. Zbl 1209.05119. arXiv:0712.4118. **(SG: Str, Geom)**

Yasuhide Numata

See T. Abe.

Kathryn Nurse

See L. Beaudou and M. DeVos.

Gábor Nyul

See E. Gyimesi.

Suil O

See G. Greaves.

Jan Obdržálek

See R. Ganian.

G. Obermair

See K. Jüngling.

Mohammad Reza Oboudi

See also S. Akbari and W.H. Haemers.

2016a Energy and Seidel energy of graphs. *MATCH Commun. Math. Comput. Chem.* 75 (2016), 291–303. MR 3496590. Zbl 1461.05133.

Cf. Haemers (2012a) for Seidel energy $\mathcal{E}(K_\Delta)$. **(sg: KG: Adj: Eig)**

2021a Seidel energy of complete multipartite graphs. *Special Matrices* 9 (2021), 212–216. MR 4241490. Zbl 1476.05124.

Thm. 5: $\mathcal{E}(K_{K_{n_1,\dots,n_t}}) = 2[n - t - \min_i \lambda_i(A(K_\Delta))]$ where $\Delta = K_{n_1\cdots n_t}$, hence proving Thm. 6: Haemers' (2012a) minimality conjecture over all such Δ. [Annot. 30 Sep 2023.] **(sg: KG: Adj: Eig)**

Cian O'Brien, Kevin Jennings, & Rachel Quinlan

2020a Alternating signed bipartite graphs and difference-1 colourings. *Linear Algebra Appl.* 604 (2020), 370–398. MR 4122676. Zbl 1447.05098. arXiv:1911.10869. **(SG: ECol)**

Pascal Ochem

See also F. Dross and A. Montejano.

Pascal Ochem & Nazanin Movarrae

2017a Oriented, 2-edge-colored, and 2-vertex-colored homomorphisms. *Inform. Processing Lett.* 123 (2017), 42–46. MR 3635310. Zbl 1405.05120. HAL irmm-02083721. **(SGc: Hom)**

Pascal Ochem, Alexandre Pinlou, & Sagnik Sen

2014a Homomorphisms of signed planar graphs. Manuscript, 2014. arXiv:1401.3308. **(SGc: Hom)**

2017a Homomorphisms of 2-edge-colored triangle-free planar graphs. *J. Graph Theory* 85 (2017), no. 1, 258–277. MR 3634486. Zbl 1365.05097. HAL irmm-01376130. **(SG: Hom)**

M. Ocio
See E. Vincent.

B. O'Connor
See A. Krieger.

Damien Octeau
See B. Guler.

Suho Oh
See M. Cho.

Hidefumi Ohsugi
See also T. Matsui.

1999a Unimodular regular triangulations of (0,1)-polytopes associated with finite graphs. In: Chrystopher L. Nehaniv *et al.*, eds., *Algebraic Engineering* (Proc. Int. Workshop Formal Languages Computer Sys., Kyoto, 1997, and First Int. Conf. Semigroups Algebraic Engineering, Aizu, 1997), pp. 159–171. World Scientific, Singapore, 1999. Zbl 1027.52008. **(sg: par: Geom, Algeb)**

2000a Compressed polytopes, initial ideals and complete multipartite graphs. In: *Proceedings of the Third Symposium on Algebra, Languages and Computation* (Osaka, 1999), pp. 45–54. Shimane Univ., Matsue, 2000. MR 1774200 (no rev). Zbl 943.13016.
Extended abstract of Ohsugi and Hibi (2000a) [Annot. 24 Jan 2016.] **(sg: Par: Geom, Algeb)**

Hidefumi Ohsugi & Takayuki Hibi

1998a Normal polytopes arising from finite graphs. *J. Algebra* 207 (1998), 409–426. MR 1644250 (2000a:13010). Zbl 926.52017.
The odd-cycle condition of Fulkerson, Hoffman, and McAndrew (1965a) is employed in polynomial algebra. "Graph polytope" [later named "edge polytope"] $P_{-\Gamma} := \operatorname{conv} \mathbf{x}(E(-\Gamma))$, where $\mathbf{x}(E(-\Gamma)) = \{$columns of incidence matrix $\mathrm{H}(-\Gamma)\}$. Dictionary: "$\mathcal{P}_G$" $= P_{-\Gamma}$. [This is antibalanced. *Problem.* Generalize to signed graphs, including balanced graphs.] [Annot. 30 May 2011.] **(sg: Par: Geom, Algeb)**

1999a Toric ideals generated by quadratic binomials. *J. Algebra* 218 (1999), 509–527. MR 1705794 (2000f:13055). Zbl 0943.13014.
§1, "Binomial ideals arising from finite graphs": The edge ring of $-\Gamma$ (negative edges because the analysis is antibalanced—even and odd graph circles are different) is $K[x_i x_j : ij \in E(\Gamma)]$. Thm. 1.2: The toric ideal I_Γ is generated by quadratic binomials iff in $-\Gamma$, each positive circle has certain chords, each contrabalanced tight handcuff has an edge between its circles, and each two disjoint negative circles are joined by at least 2 edges [i.e., antibalanced criteria]. [*Problem.* Understand this via signed graphs.] §4: Edge polytope $P_{-\Gamma}$ properties such as minimal volume. §5, "Simple edge polytopes": E.g., Cor. 5.4: $P_{-\Gamma}$ is simple iff $\Gamma = K_{p,q}$. [*Problem.* Generalize to signed graphs, including ordinary graphs Γ (i.e., all positive). The edge ring would be bidirected:

$K[\mathbf{x}^{\vec{e}_{ij}} : \vec{e}_{ij} \in E(\mathrm{B})]$ for a bidirection B of Σ. E.g., one expects $P_{+\Gamma}$ to be simple iff $\Gamma = +K_n$.] [Annot. 5 Oct 2014, 3 Jun 2015.] **(sg: Par: Algeb, Geom)**

1999b A normal $(0,1)$-polytope none of whose regular triangulations is unimodular. *Discrete Comput. Geom.* 21 (1999), 201–204. MR 1668090 (99k:52023). Zbl 927.52018.

The polytope is the edge polytope conv $\mathrm{H}(-\Gamma)$ (*cf.* (1998a)) where $\Gamma = C_5$ with a triangle on each edge. [Annot. 18 Aug 2018.] **(sg: par: Geom)**

2000a Compressed polytopes, initial ideals and complete multipartite graphs. *Illinois J. Math.* 44 (2000), no. 2, 391–406. MR 1775328 (2001e:05092). Zbl 943.13016.

Edge polytope $P_{-K_{n_1,\ldots,n_k}}$ and edge ring. [This is antibalanced. *Problem*. Generalize to signed graphs, including balanced graphs.] [Annot. 5 Oct 2014.] **(sg: Par: Geom, Algeb)**

2003a Normalized volumes of configurations related with root systems and complete bipartite graphs. *Discrete Math.* 268 (2003), 217–242. MR 1983280 (2004m:-52018). Zbl 1080.14059.

A configuration consists of the vectors representing an acyclic orientation of a complete signed bipartite graph. The volume is that of the pyramid over the configuration with apex at the origin. (Successor to Fong (2000a).) [*Question*. Is there a connection with the chromatic polynomial?] **(sg: Geom: Invar)**

2008a Simple polytopes arising from finite graphs. In: Matthias Dehmer, Michael Drmota, and Frank Emmert-Streib, eds., *Proceedings of the 2008 International Conference on Information Theory and Statistical Learning* (ITSL 2008, Las Vegas, 2008), pp. 73–79. CSREA Press, 2008. URL `https://dblp1.uni-trier.de/db/conf/itsl/itsl2008` arXiv:0804.4287. **(sg: par: Geom)**

Hidefumi Ohsugi & Kazuki Shibata

2012a Smooth Fano polytopes whose Ehrhart polynomial has a root with large real part. *Discrete Comput. Geom.* 47 (2012), 624–628. MR 2891253. Zbl 1238.52005. arXiv:1109.0791. **(sg: par: Geom)**

Nacim Oijid

See J. Bensmail.

Ayao Okiji

See Y. Kasai.

E. Olaru

See St. Antohe.

Marián Olejár

See J. Širáň.

D.D. Olesky

See also B.D. Bingham, T. Britz, M. Catral, G.J. Culos, D.A. Grundy, C.R. Johnson, and K. Hassani Monfared.

D.D. Olesky, M.J. Tsatsomeros, & P. van den Driessche

2012a Sign patterns with a nest of positive principal minors. *Linear Algebra Appl.* 436 (2012), 4392–4399. MR 2917416. Zbl 1244.15022. **(QM: SD)**

[

Aroldo Oliveira, Leonardo Silva de Lima, & Nair Maria Maia de Abreu

2012a On the spread and the chromatic number of a graph. Proc. Forty-Third Southeastern Int. Conf. Combinatorics, Graph Theory and Computing. *Congressus Numer.* 212 (2012), 57–64. MR 3024414. Zbl 1278.05105.

For $L(-\Gamma)$, spread = (largest − smallest eigenvalue) $\leq \chi(\Gamma)$. [Annot. 20 Jan 2015.] **(par: Lap: Eig)**

Carla Silva Oliveira

See also L.S. de Lima.

Carla Oliveira & Leonardo de Lima

2016a A lower bound for the sum of the two largest signless Laplacian eigenvalues. 14th Cologne-Twente Workshop on Graphs and Combinatorial Optimization (CTW'16, Gargnano, Italy, 2016). *Electronic Notes Discrete Math.* 55 (2016), 173–176. Zbl 1356.05082. arXiv:1412.0323.MR none 7/19.

A degree bound for $L(-\Gamma)$. *Cf.* Li and Tian (2015a). [Annot. 8 Jan 2015.] **(par: Lap: Eig)**

Carla Silva Oliveira, Leonardo Silva de Lima, Nair Maria Maia de Abreu, & Pierre Hansen

2010a Bounds on the index of the signless Laplacian of a graph. *Discrete Appl. Math.* 158 (2010), no. 4, 355–360. MR 2588119 (2011d:05228). Zbl 1225.05174. **(par: Lap: Eig)**

Carla Silva Oliveira, Leonardo Silva de Lima, Nair Maria Maia de Abreu, & Steve Kirkland

2010a Bounds on the Q-spread of a graph. *Linear Algebra Appl.* 432 (2010), no. 9, 2342–2351. MR 2599864 (2011k:05146). Zbl 1214.05082.

Cf. M.H. Liu and Liu (2010a). **(par: Lap: Eig)**

F. Oloomi

See R. Masoumi.

Stig W. Omholt

See E. Plahte.

G.R. Omidi

See also F. Ayoobi and W.H. Haemers.

2009a On a signless Laplacian spectral characterization of T-shape trees. *Linear Algebra Appl.* 431 (2009), no. 9, 1607–1615. MR 2555062 (2010m:05181). Zbl 1169.05351.

A subdivided $K_{1,3}$ is determined by Spec $L(-\Gamma)$. [Continued in Omidi and Vatandoost (2010a) and Bu and Zhou (2012a).] [Annot. 28 Nov 2012.] **(par: Lap: Eig)**

Gholam R. Omidi & Ebrahim Vatandoost

2010a Starlike trees with maximum degree 4 are determined by their signless Laplacian spectra. *Electronic J. Linear Algebra* 20 (2010), 274–290. MR 2653539 (2011c:05205). Zbl 1205.05151.

A subdivided $K_{1,4}$ is determined by Spec $L(-\Gamma)$. Continuation of Omidi (2009a). [Continued in Bu and Zhou (2012a).] [Annot. 28 Nov 2012.] (**par: Lap: Eig**)

Kenji Onaga

1966a Dynamic programming of optimum flows in lossy communication nets. *IEEE Trans. Circuit Theory* CT-13 (1966), 282–287. (**GN**)

1967a Optimal flows in general communication networks. *J. Franklin Inst.* 283 (1967), 308–327. MR 0218100 (36 #1189). Zbl 203.22402 (203, p. 224b). (**GN**)

Shmuel Onn

See also P. Kleinschmidt.

1997a Strongly signable and partitionable posets. *European J. Combin.* 18 (1997), 921–938. MR 1485377 (99d:06007). Zbl 887.06003.

For "signability" see Kleinschmidt and Onn (1995a). A strong signing is an exact signing that satisfies a recursive condition on lower intervals. (**Sgnd, Geom**)

Rikio Onodera

1968a On signed tree-graphs and cotree-graphs. *RAAG Res. Notes* (3) No. 133 (1968), ii + 29 pp. MR 0237383 (38 #5671). Zbl 182.58201 (182, p. 582a).

The adjacency graph of trees of a graph is signed from a vertex signature and is shown to be balanced. [Trivial.] [Annot. 24 July 2010.] (**SG: Bal**)

The Open University

1981a Graphs and Digraphs. Unit 2 in Course TM361: Graphs, Networks and Design. The Open Univ. Press, Walton Hall, Milton Keynes, England, 1981.

Social sciences (pp. 21–23). Signed digraphs (pp. 50–52). [Published version: see Wilson and Watkins (1990a).] (**SG, PsS, SD: Exp**)

Peter Orlik & Louis Solomon

1980a Unitary reflection groups and cohomology. *Invent. Math.* 59 (1980), 77–94. MR 0575083 (81f:32017). Zbl 452.20050.

Thm. (4.8): The characteristic polynomials of the Dowling lattices and jointless Dowling lattices of $\mathbb{Z}_r$, computed via group theory as part of the general treatment of finite unitary reflection groups. (**gg: matrd, Geom**)

1982a Arrangements defined by unitary reflection groups. *Math. Ann.* 261 (1982), 339–357. MR 0679795 (84h:14006). Zbl 491.51018.

In the intersection lattice of reflection hyperplanes of a finite unitary reflection group, the characteristic polynomial of an upper interval has an integral factorization. The proofs involve detailed study of the group actions on $\mathbb{C}^l$. Dictionary: $\mathcal{A}_l(r)$ and $\mathcal{A}_l^k(r)$ are the arrangements corresponding to the rank-l Dowling lattices and partially jointless Dowling lattices of $\mathbb{Z}_r$. Relevant results: §2: "Monomial groups": Cor. (2.4) counts the flats, Prop. (2.5) and Cor. (2.7) gives the polynomials for $\mathcal{A}_l(r)$ [all known from Dowling (1973b)]. Cor. (2.10) counts the flats, Prop. (2.13) gives the polynomial of $\mathcal{A}_l^k(r)$, Prop. (2.14) notes that proper upper intervals are Dowling lattices [all fairly obvious via gain graphs and coloring (Zaslavsky (1995b))]. (**gg: matrd, Geom, Invar**)

1983a Coxeter arrangements. In: Peter Orlik, ed., *Singularities* (Arcata, Calif., 1981), Part 2, pp. 269–291. Proc. Sympos. Pure Math., Vol. 40. Amer. Math. Soc., Providence, R.I., 1983. MR 0713255 (85b:32016). (**gg: matrd, Geom, Invar**)

James B. Orlin

See also R.K. Ahuja, M. Kodialam, and R. Shull.

1984a Some problems on dynamic/periodic graphs. In: *Progress in combinatorial optimization* (Proc. Conf., Waterloo, Ont., 1982), pp. 273–293. Academic Press, Toronto, 1984. MR 0771882 (86m:90058). Zbl 547.05060.

Problems on 1-dimensional periodic graphs (i.e., covering (di)graphs of $\mathbb{Z}$-gain graphs Φ) that can be solved in Φ: connected components, strongly connected components, directed path from one vertex to another, Eulerian trail (directed or not), bicolorability, and spanning tree with minimum average cost.
(**GG, GD: Cov: Paths, Circ, Col: Algor**)

1985a On the simplex algorithm for networks and generalized networks. *Math. Programming Study* 24 (1985), 166–178. MR 0820998 (87k:90102). Zbl 592.90031.
(**GN: Matrd(Bases): Algor**)

Charles E. Osgood & Percy H. Tannenbaum

1955a The principle of congruity in the prediction of attitude change. *Psychological Rev.* 62 (1955), 42–55. (**VS: PsS**)

Eiji O'Shima

See M. Iri and J. Shiozaki.

M.A. Osorio

See E.E. Vogel.

Patric R.J. Östergård

See F. Szöllősi.

M. Ostilli & J.F.F. Mendes

2009a Small-world of communities: communication and correlation of the meta-network. *J. Stat. Mech.* 2009 (2009), art. L08004. arXiv:0812.0608.

Short version of (2009b). (**Phys: Fr**)

2009b Communication and correlation among communities. *Phys. Rev. E* 80 (2009), art. 011142, 23 pp. MR 2552037 (2010h:91167). arXiv:0902.0888.
(**Phys: Fr, sg**)

Katsuhiro Ota

See D. Archdeacon and A. Nakamoto.

Li Ou, Yaoping Hou, & Zhuang Xiong

2021a The net Laplacian spectra of signed complete graphs. *Contemp. Math.* 2 (2021), no. 4, 409–417. (**SG: KG: Adj: Eig**)

Sang-Il Oum

See T. Huynh.

James G. Oxley

See also T. Brylawski, T. Fife, J. Geelen, J.P.S. Kung, and L.R. Matthews.

1992a Infinite matroids. In: Neil White, ed., *Matroid Applications*, Ch. 3, pp. 73–90. Encycl. Math. Appl., Vol. 40. Cambridge Univ. Press, Cambridge, Eng., 1992. MR 1165540 (93f:05027). Zbl 766.05016.

See Exer. 3.20. (**Bic: Exp**)

1992b *Matroid Theory*. Oxford Univ. Press, Oxford, 1992. MR 1207587 (94d:05033). Zbl 784.05002.

Thm. 6.6.3: proof from Brylawski's (1975a). (**gg: sw: Exp**)

§10.3: Exer. 3 concerns the Dowling lattices of $\mathrm{GF}(q)^\times$. §12.2: Exer. 13 concerns $\mathbf{F}(\Omega)$. (**gg: Matrd: Exp**)

2011a *Matroid Theory*, 2nd ed. Oxford Grad. Texts Math., 21. Oxford Univ. Press, Oxford, 2011. MR 2849819 (2012k:05002). Zbl 1254.05002.

§6.10, "Dowling geometries": Frame (i.e., bias) matroid theory of biased graphs. Examples: gain and signed graphs, Dowling (1973a), (1973b) geometries, bicircular, even-circle (even-cycle, factor), and poise and antidirection matroids. Representability of Dowling geometries. Kahn and Kung's (1982a) varieties. Other mentions of Dowling geometries in Prop. 14.10.22, §15.3 p. 590, §15.9 p. 605, and Appendix, "Some interesting matroids", p. 663; of bicircular matroids in Exer. 10.4.12, Prop. 11.1.6, Exer. 11.1.7, Conj. 14.3.12, Thm. 14.10.19. [Annot. 21 Mar 2011.]

Spikes (with tips) and swirls, important in matroid structure theory, are the lift (extended lift) and frame matroids of biased $2C_n$'s. Spikes: pp. 40–42, 72–74, 111–112, 197–202, 545–548, 568, 662, *et al.* Swirls: pp. 552, 568, 664, *et al.* [The biased-graph representation could simplify some of the descriptions.] [Annot. 7 Feb 2013.]

(**GG: Matrd, Bic, ECyc: Exp, Exr**)

James Oxley, Dirk Vertigan, & Geoff Whittle

1996a On inequivalent representations of matroids over finite fields. *J. Combin. Theory Ser. B* 67 (1996), 325–343. MR 1399683 (97d:05052). Zbl 856.05021.

§5: Free swirls, $\mathbf{F}(2C_n, \varnothing)$ ($n \geq 4$), mentioning their relationship to Dowling lattices, and complete free spikes, $\mathbf{L}_\infty(2C_n, \varnothing)$. (**GG: Matrd**)

Olayiwola O. Oyeleye & Mark A. Kramer

1988a Qualitative simulation of chemical process systems: Steady-state analysis. *AIChE J.* 34 (1988), no. 9, 1441–1454.

Extends the signed digraph of Iri, Aoki, O'Shima, and Matsuyama (1979a) "to account for complex dynamics". [Annot. 17 Feb 2013.]

(**SD, VS: Appl, Algor**)

Kenta Ozeki

See K. Kawarabayashi and S.-J. Kim.

Meltem Öztürk

See O. Bessouf.

M.L. Paciello

See M. Falcioni.

Manfred W. Padberg

See E.L. Johnson.

E. Padmavathy

See V.J.A. Cynthia.

Carles Padró

See A. Beimel.

Steven R. Pagano

† 1998a *Separability and Representability of Bias Matroids of Signed Graphs*. Doctoral thesis, State University of New York at Binghamton, 1998. MR 2697393 (no rev).

Ch. 1: "Separability". Graphical characterization of bias-matroid k-separations of a biased graph. Also, some results on the possibility of k-separations in which one or both sides are connected subgraphs. (**GG: Matrd: Str**)

Ch. 2: "Representability". The frame matroid of every signed graph is representable over all fields with characteristic $\neq 2$. For which signed graphs is it representable in characteristic 2 (and therefore representable over GF(4), by the theorem of Geoff Whittle, A characterization of the matroids representable over GF(3) and the rationals. *J. Combin. Theory Ser. B* 65 (1995), 222–261. MR 1358987 (96m:05046). Zbl 835.05015.)? Solved (for 3-connected signed graphs having vertex-disjoint negative circles and hence nonregular matroid). There are two essentially different types: (i) two balanced graphs joined by three independent unbalanced digons; (ii) a cylindrical signed graph, possibly with balanced graphs adjoined by 3-sums. [See notes on Seymour (1995a) for definition of (ii) and for Lovász–Slilaty's structure theorem in the case without vertex-disjoint negative circles.]

Furthermore, the representations of these graphs in characteristic not 2 are all canonical signed-graphic, while any representations over GF(4) are canonical $\mathbb{Z}_3$-gain graphic. (**SG: Matrd: Incid, Str, Top**)

Ch. 3: "Miscellaneous results". (**SG: Matrd: Incid, Str**)

1999a Binary signed graphs. Manuscript, ca. 1999. (**SG: Matrd: Incid, Str**)

1999b Signed graphic GF(4) forbidden minors. Manuscript, ca. 1999. (**SG: Matrd**)

1999c GF(4)-representations of bias matroids of signed graphs: The 3-connected case. Manuscript, ca. 1999. (**SG: Matrd: Incid, Str, Top**)

Igor Pak

See S. Chmutov.

Anita Pal

See D. Banerjee and S.N. Mishra.

Matteo Palassini & Sergio Caracciolo

2000a Monte Carlo simulation of the three-dimensional Ising spin glass. In: David P. Landau *et al.*, eds., *Computer Simulation Studies in Condensed-Matter Physics XII* (Proc. Twelfth Workshop, Athens, Ga., 1999), pp. 162–166. Springer Proc. Phys., Vol. 85. Springer, Berlin, 2000. arXiv:cond-mat/9911449.

Physical quantities on the $\pm J$ cubic lattice model, i.e., a signed cubic lattice graph. [Annot. 28 Mar 2013.] (**Phys, sg: Fr, State**)

Matteo Palassini, Frauke Liers, Michael Juenger, & A.P. Young

2003a Low-energy excitations in spin glasses from exact ground states. *Phys. Rev. B* 68 (2003), art. 064413, 16 pp. arXiv:cond-mat/0212551.

§§II–III: Take a ground state ζ_0. Add $(\varepsilon/\#E)\zeta_0(v_i)\zeta_0(v_j)$ to J_{ij}, raising energy of ζ_0 by $\varepsilon > 0$ and of any other state ζ by less, the amount depending on the edges whose signs differ in ζ_0 and ζ [i.e., negative edges in $\Sigma^{\zeta_0\zeta}$]. This may change the relative energies of states. See if a near-ground state becomes a ground state. [This interesting approach makes sense only for weighted Σ.]

§§IV–V: Algorithm for frustration index (equivalently, weighted frustration index) of a weighted signed graph, tested on small cubic lattice graphs. It uses a branch-and-cut method that requires solving many linear programs. [This part makes sense for unweighted Σ.]

Dictionary: J_{ij} = signed weight of edge e_{ij}; "state" = vertex signing ζ; "energy" of ζ = weighted frustration of $(\Sigma^\zeta, \#J)$:= total unsigned weight of $E^-(\Sigma^\zeta)$; "ground state" = any ζ that minimizes energy; minimum energy = energy of ground state = weighted frustration index $l(\Sigma, \#J)$; "free" or "periodic" boundary conditions = nontoroidal ([path]^3) or toroidal ([circle]^3). [Annot. 22 Dec 2014.]

(**SG, WG: State(fr), Fr: Algor**)

Jeepamol J. Palathingal

2020a *Studies on Graph Operators.* Doctoral thesis, Mahatma Gandhi University, Kottayam, 2020.

Ch. 4: Signed Gallai graph. Ch. 5: Signed anti-Gallai graph. *Cf.* Palathingal and Lakshmanan (2017a), (20xxa). (**SG: LG(Gen)**)

Jeepamol J. Palathingal & Aparna Lakshmanan S

2017a Forbidden subgraph characterizations of extensions of Gallai graph operator to signed graph. *Ann. Pure Appl. Math.* 14 (2017), no. 3, 437–448.

Gallai graph $\Gamma(\Sigma)$: $V(\Gamma) := E(\Sigma)$, $ef \in E(\Gamma)$ when $e, f \notin$ triangle, and three different sign functions modelled on Λ_{BC}, $\Lambda_\times$, $\Lambda_\bullet$. Thms.: All forbidden induced subgraphs for each type (but the type is not determined by forbidden induced subgraphs). [Annot. 17 Dec 2019.]

(**SG: LG(Gen): Str**)

20xxa Forbidden subgraph characterizations of extensions of anti-Gallai graph operator to signed graph. Preprint.

Anti-Gallai graph $\Delta(\Sigma)$: $V(\Delta) := E(\Sigma)$, $ef \in E(\Delta)$ when $e, f \in$ triangle, and the three sign functions in (2017a). Thms.: The same kind as in (2017a). [Annot. 17 Dec 2019.] (**SG: LG(Gen): Str**)

Edgar M. Palmer

See F. Harary and F. Kharari.

B.L. Palowitch, Jr.

See M.A. Kramer.

Rong-Ying Pan
See Y.H. Chen.

Yongliang Pan
See Y.P. Hou.

Zhishi Pan & Xuding Zhu

2022a The circular chromatic numbers of signed series-parallel graphs. *Discrete Math.* 345 (2022), no. 3, art. 112733, 10 pp. MR 4349877.
Cf. Naserasr, Wang, and Zhu (2021a). (**SG: Col**)

Casian Pantea
See D. Angeli and G. Craciun.

Pietro Panzarasa
See V. Ciotti.

Giovanni Paolini
See also E. Delucchi.

2020a Shellability of generalized Dowling posets. *J. Combin. Theory Ser. A* 171 (2020), art. 105159, 19 pp. MR 4019110. Zbl 1477.06011. arXiv:1811.08403.
The marked Dowling posets of Bibby and Gadish (2018a) and Delucchi–Girard–Paolini (2019a). (**gg: Matrd(Gen)**)

P. Paolucci
See S. Cabasino.

Gyula Pap

2005a Packing non-returning A-paths algorithmically. In: Stefan Felsner, ed., *2005 European Conference on Combinatorics, Graph Theory and Applications (EuroComb '05)* (Berlin, 2005), pp. 139–144, electronic. Discrete Math. & Theor. Computer Sci. Proceedings AE, 2005. URL `http://www.dmtcs.org/dmtcs-ojs/index.php/proceedings/issue/view/77`. Zbl 1192.05123.
See (2008a). (**GG: Str, Paths, Algor**)

2007a Packing non-returning A-paths. *Combinatorica* 27 (2007), no. 2, 247–251. MR 2321928 (2008c:05148). Zbl 1136.05060.
Given: Φ with gain group $\mathfrak{S}(\Omega)$, the symmetric group of a set Ω, $A \subseteq V$, and $\omega : A \to \Omega$. An A-path is a path P with endpoints $v, w \in A$ and internally disjoint from A; it is "returning" if $\omega(v)\varphi(P) = \omega(w)$. Thm. The largest number of disjoint returning A-paths equals the minimum, over all satisfied edge subsets F, of the maximum number of disjoint $[A \cup V(F)]$-paths in $\|\Phi\| \setminus F$. [For "satisfied" edges see Zaslavsky (2009a).] Generalizes and simplifies Chudnovsky, Geelen, *et al.* (2006a), which is the case where the gains act regularly and $\omega =$ constant. (**GG: Str, Paths**)

2008a Packing non-returning A-paths algorithmically. *Discrete Math.* 308 (2008), no. 8, 1472–1488. MR 2392063 (2009e:05160). Zbl 1135.05060. (**GG: Str, Paths, Algor**)

Christos H. Papadimitriou
See also E.M. Arkin and A.S. LaPaugh.

Christos H. Papadimitriou & Kenneth Steiglitz

1982a *Combinatorial Optimization: Algorithms and Complexity.* Prentice-Hall, Englewood Cliffs, N.J., 1982. MR 0663728 (84k:90036). Zbl 503.90060.

Repr. with minor additions and corrections: Dover Publications, Mineola, N.Y., 1998. MR 1637890 (no rev). Zbl 944.90066.

See Ch. 10, Problems 6–7, p. 244, for bidirected graphs and flows in relation to the matching problem. (**sg: Ori: Flows**)

1985a *Kombinatornaya optimiztsiya. Algoritmy i Slozhnost'.* Transl. V.B. Alekseev. Mir, Moskva, 1985. MR 0801895 (86i:90067). Zbl 598.90067.

Russian translation of (1982a). (**sg: Ori: Flows**)

Apostolos N. Papadopoulos

See F.D. Malliaros.

Charis Papadopoulos

See H.L. Bodlaender.

Konstantinos Papalamprou

See also G. Appa and L.S. Pitsoulis.

2009a *Structural and Decomposition Results for Binet Matrices, Bidirected Graphs and Signed-Graphic Matroids.* Doctoral thesis, London School of Economics, 2009. MR 3301606 (no rev). (**SG: Ori, Incid, Matrd**)

2021a On the basis pair graphs of signed-graphic matroids. *Discrete Math. Algorithms Appl.* 13 (2021), no. 4, art. 2150033, 5 pp. MR 4284036. (**SG: Matrd: Du, Bases**)

Konstantinos Papalamprou & Leonidas Pitsoulis

2012a On the basis pair graphs of signed-graphic matroids. In: *Combinatorial Optimization* (Second Int. Sympos., ISCO 2012, Athens), pp. 463–474. Lect. Notes in Computer Sci., Vol. 7422. Springer, Heidelberg, 2012. MR 3006050. Zbl 06101806. (**SG: Matrd: Algor**)

2013a Decomposition of binary signed-graphic matroids. *SIAM J. Discrete Math.* 27 (2013), no. 2, 669–692. MR 3040957. Zbl 1272.05021. arXiv:1011.6497.

A binary matroid is signed-graphic iff, for some copoint H, all the bridges of H (in the sense of Tutte) are graphic aside from one that is signed-graphic (and possibly graphic). [Annot. 5 Feb 2010.]

(**SG: Matrd, Str**)

2017a Signed-graphic matroids with all-graphic cocircuits. *Discrete Math.* 340 (2017), no. 12, 2889–2899. MR 3698077. Zbl 1370.05032.

Frame matroids $\mathbf{F}(\Sigma)$ where all copoints are graphic matroids.

(**SG: Matrd**)

Konstantinos Papalamprou, Leonidas Pitsoulis, & Balász Kotnyek

20xxa A mathematical programming approach for recognizing binet matrices. *Optim. Lett.* (in press). (**SG: Ori: incid: Algor; Matrd**)

Konstantinos Papalamprou, Leonidas S. Pitsoulis, & Eleni-Maria E. Vretta

2021a On characterizing the class of cographic signed-graphic matroids. *Discrete Appl. Math.* 296 (2021), 90–102. MR 4243323. Zbl 1461.05027. (**SG: Matrd**)

G. Pardella & F. Liers

2008a Exact ground states of large two-dimensional planar Ising spin glasses. *Phys. Rev. E* 78 (2008), art. 056705, 10 pp. MR 2551393 (2011c:82040). arXiv:-0801.3143.

A new algorithm for the frustration index of a planar signed graph. *Cf.* Bieche *et al.* (1980a) and Barahona (1981a), (1982a). [Annot. 23 Nov 2014.] (**sg: Fr: Algor**)

Eduardo G. Pardo, Mauricio Soto, & Christopher Thraves

2015a Embedding signed graphs in the line: Heuristics to solve MinSA problem. *J. Combin. Optim.* 29 (2015), 451–471. MR 3297933. Zbl 1316.90058. (**SG: Geom: Algor**)

Ojas Parekh

See E.G. Boman.

Angelo Parente

See F. Marinelli.

Giorgio Parisi

See also S. Cabasino, S. Caracciolo, B. Coluzzi, M. Falcioni, L.A. fndez, E. Marinari, and M. Mézard.

1987a Spin glass theory. In: R. Pynn and T. Riste, eds., *Time Dependent Effects in Disordered Materials*, pp. 317–329. Plenum, 1987. Repr. in Giorgio Parisi, *Field Theory, Disorder and Simulations*, Ch. 14, pp. 285–297. World Scientific, Singapore, 1992.

Relatively (a careful word) simple explanation of physics of "Ising spin glasses" = spin states on signed graphs, and of the replica method for studying them. [Annot. 9 Aug 2018.] (**sg: Phys, Fr: Exp**)

1991a On the emergence of tree-like structures in complex systems. In: O.T. Solbrig and G. Nicolis, eds., *Perspectives on Biological Complexity*, pp. 77–111. Int. Union of Biological Sci., 1991. Repr. in Giorgio Parisi, *Field Theory, Disorder and Simulations*, Ch. 15, pp. 298–335. World Scientific, Singapore, 1992.

§5.3, "Spin glasses": Friendly treatment of balance and frustration in signed graphs with a distinct physics slant, e.g., asymptotic behaviors. [Annot. 9 Aug 2018.] (**SG: Bal, Fr, Phys: Exp**)

1994a D-dimensional arrays of Josephson junctions, spin glasses and q-deformed harmonic oscillators. *J. Phys. A* 27 (1994), 7555–7568. MR 1312271 (95m:82070). Zbl 844.60095. arXiv:cond-mat/9410088.

Physics on hypercube Q_D with complex unit gains φ (φ is a "$U(1)$ gauge field"). Spins $\zeta(v)$ can be (i) complex units or (ii) Gaussian random complex numbers, or (iii) ζ can be a unit vector $\in \mathbb{C}^n$; mainly, (ii). Assumed: each square ("plaquette") $C_{\alpha,\beta}$ (with vertices $x, x+e_\alpha, x+e_\alpha+e_\beta, x+e_\beta, x$ for any $x \in V(Q_D)$) has gain $e^{iB\sigma_{\alpha,\beta}}$ in a fixed orientation, where $\sigma_{\alpha,\beta} \in \{+1,-1\}$ determines which orientations have gains e^{iB} and e^{-iB}. $B = 0$ gives balance; $B = \pi$ gives all plaquette gains -1 (full frustration). If $D \leq 3$, but not if $D > 3$, the choices of σ are equivalent by switching in the gain group $\mathbb{C}^\times$. The statistics of random σ are investigated. [Annot. 19 Jun 2012.] (**Phys, gg**)

1996a A mean field theory for arrays of Josephson junctions. *J. Math. Phys.* 37 (1996), no. 10, 5158–5170. MR 1411624 (97i:82029). Zbl 872.60038.

Complex unit gain graphs. The Hamiltonian is the quadratic form $\bar{z}A(\Phi)z$. [Annot. 12 Aug 2012.] **(GG: Phys)**

Boram Park

See J.S. Huh.

Jin-Woo Park

See also S.-G. Lee.

Jin-Woo Park & Sung-Soo Pyo

2011a Construction and nonnegativity of sign idempotent sign pattern matrices. *Linear Algebra Appl.* 435 (2011), 2860–2869. MR 2825287 (2012h:15010). Zbl 1230.15004. **(QM: SD, bal, sw)**

Se-Won Park

See S.-G. Lee.

Antonio Parravano, Ascensión Andina-Díaz, & Miguel A. Meléndez-Jiménez

2016a Bounded confidence under preferential flip: a coupled dynamics of structural balance and opinions. *PLoS ONE* 1 (2016), no. 10, art. 164323, 23 pp. arXiv:-1610.02660. **(SG: Bal, Dyn)**

Leila Parsaei-Majd

See also S. Akbari, E. Ghorbani, and H.R. Maimani.

Leila Parsaei-Majd, Zoran Stanić, & Behruz Tayfeh-Rezaie

20xxa On weight-symmetric 3-coloured digraphs. Submitted.

"3-coloured digraph" = gain graph Φ with gain group $\{\pm 1, \pm \mathrm{i}\}$. '1Weight-symmetric" means $\Phi \simeq -\Phi$. Singularity of $A(\Phi)$. Constructions of $\Phi \simeq -\Phi$; constructions of $\Phi \not\simeq -\Phi$ with symmetric adjacency spectrum. [Annot. 19 Sept 2022.] **(gg: Adj: Eig; Sw)**

M. Parvathi

2004a Signed partition algebras. *Commun. Algebra* 32 (2004), no. 5, 1865–1880. MR 2099708 (2005g:16060). Zbl 1081.20008.

They are the special case of Bloss (2003a) where $\mathfrak{G} = \{+, -\}$. [Annot. 21 Mar 2011.] **(gg: Algeb, matrd)**

M. Parvathi & M. Kamaraj

1998a Signed Brauer's algebras. *Commun. Algebra* 26 (1998), no. 3, 839–855. MR 1606174 (99c:16028). Zbl 944.16015.

The algebra is generated by multiplying two-layer signed graphs ("Brauer graphs"). In the product $\Sigma_1\Sigma_2$ the bottom layer of Σ_1 cancels with the top layer of Σ_2 using edge-sign product. (Signs are represented by arrows [!].) [Annot. 5 Jun 2012.] **(gg: Algeb, matrd)**

2002a Matrix units for signed Brauer's algebras. *Southeast Asian Bull. Math.* 26 (2002), no. 2, 279–297. MR 2047807 (2005b:16055). Zbl 1066.16014. **(gg: Algeb, matrd)**

M. Parvathi & A. Joseph Kennedy

2004a *G*-vertex colored partition algebras as centralizer algebras of direct products. *Commun. Algebra* 32 (2004), no. 11, 4337–4361. MR 2102453 (2005i:16068). Zbl 1081.20009. **(gg: Algeb, matrd)**

2004b Representations of vertex colored partition algebras. *Southeast Asian Bull. Math.* 28 (2004), no. 3, 493–518. MR 2084740 (2006c:16051). Zbl 1081.20010. **(gg: Algeb, matrd)**

2005a Extended G-vertex colored partition algebras as centralizer algebras of symmetric groups. *Algebra Discrete Math.* 2005, no. 2, 58–79. MR 2238218 (2007b:-16068). Zbl 1091.20005. **(gg: Algeb, matrd)**

M. Parvathi & D. Savithri

2002a Representations of G-Brauer algebras. *Southeast Asian Bull. Math.* 26 (2002), no. 3, 453–468. MR 2047837 (2005b:16056). Zbl 1065.20017. **(gg: Algeb, matrd)**

M. Parvathi & C. Selvararj

1999a Signed Brauer's algebras as centralizer algebras. *Commun. Algebra* 27 (1999), no. 12, 5985–5998. MR 1726289 (2000j:16051). Zbl 944.16016. **(gg: Algeb, matrd)**

2004a Note on signed Brauer's algebras. *Southeast Asian Bull. Math.* 27 (2004), no. 5, 883–898. MR 2175793 (2006i:16047). Zbl 1071.16010. **(gg: Algeb, matrd)**

2006a Characters of signed Brauer's algebras. *Southeast Asian Bull. Math.* 30 (2006), no. 3, 495–514. MR 2243691 (2007d:16068). Zbl 1150.16303. **(gg: Algeb, matrd)**

M. Parvathi & B. Sivakumar

2008a The Klein-4 diagram algebras. *J. Algebra Appl.* 7 (2008), no. 2, 231–262. MR 2417044 (2009b:16032). Zbl 1167.16012. **(gg: Algeb, matrd)**

2008b R-S correspondence for $(Z_2 \times Z_2) \wr S_n$ and Klein-4 diagram algebras. *Electronic J. Combin.* 15 (2008), no. 1, Research Paper R98, 28 pp. MR 2426161 (2009i:05233). Zbl 1163.05300. **(gg: Algeb, matrd)**

M. Parvathi, B. Sivakumar, & A. Tamilselvi

2007a R-S correspondence for the hyper-octahedral group of type B_n—a different approach. *Algebra Discrete Math.* 2007 (2007), no. 1, 86–107. MR 2367517 (2008k:05203). Zbl 1164.05465. **(gg: Algeb, matrd)**

M. Parvathi & A. Tamilselvi

2007a Robinson-Schensted correspondence for the signed Brauer algebras. *Electronic J. Combin.* 14 (2007), no. 1, Research Paper 49, 26 pp. MR 2336326 (2008e:-05143). Zbl 1163.05336. **(gg: Algeb, matrd)**

2008a Robinson-Schensted correspondence for the G-Brauer algebras. In: S.K. Jain and S. Parvathi, eds., *Noncommutative Rings, Group Rings, Diagram Algebras and Their Applications* (Proc. Int. Conf., Chennai, 2006), pp. 137–150. Contemp. Math., Vol. 456. Amer. Math. Soc., Providence, R.I., 2008. MR 2416147 (2009m:16060). Zbl 1187.05085. **(gg: Algeb, matrd)**

M. Siva Parvathi

See M.V. Anusha.

Germain Pastén, Oscar Rojo, & Luis Medina

2021a On the A_α-eigenvalues of signed graphs. *Mathematics* 2021 (2021), no. 9, art. 1990, 14 pp. (**SG: Adj(Gen): Eig**)

Sukanta Pati
See R.B. Bapat, S. Barik, and D. Kalita.

Vaia Patta
See C. Heunen.

Philippa Pattison
1993a *Algebraic Models for Social Networks.* Structural Analysis in the Social Sciences, 7. Cambridge Univ. Press, Cambridge, 1993.
Ch. 8, pp. 258–9: "The balance model. The complete clustering model." They are embedded in a more general framework.
(**SG, Sgnd: Adj, Bal, Clu: Exp**)

Laura Patuzzi
See M.A.A. de Freitas.

G.A. Patwardhan
See B.D. Acharya and M.K. Gill.

Debdas Paul
See S. Kirkland.

Soumyajit Paul
See S. Das.

[Viji Paul]
See Viji Paul (under 'V').

Loïc Paulevé & Adrien Richard
2010a Topological fixed points in Boolean networks. Points fixes topologiques dans les réseaux booléens. *C. R. Acad. Sci. Paris, Ser. I* 348 (2010), 825–828. MR 2677973 (2011g:05305). Zbl 1194.94209. HAL hal-00510892. (**SD: Dyn**)

2012a Static analysis of boolean networks based on interaction graphs: A survey. *Electronic Notes Theor. Computer Sci.* 284 (2012), 93–104. MR 2956839. Zbl 1283.92044. HAL hal-00714476. (**SD: Dyn, Biol**)

Vern I. Paulsen
See B.G. Bodmann and R.B. Holmes.

Payal [Payal Dabas]
See also S. Kansal.

Payal & Sangita Kansal
2022a Structural matrices for signed Petri net. *AKCE Int. J. Graphs Combin.* 19 (2022), no. 2, 102–107. MR 4452559. (**SG: Incid**)
20xxa Analysis of signed Petri net. *Int. J. Computing Sci. Math.* (to appear). (**SG: Incid**)

20xxb Domination in signed Petri net. Submitted. arXiv:2001.04374. (**SG**)

20xxc Logic signed Petri net. Submitted. arXiv:2008.11585. (**SG**)

Charles Payan

1983a Perfectness and Dilworth number. *Discrete Math.* 44 (1983), no. 2, 229–230. MR 0689816 (84e:05090). Zbl 518.05053.
See Benzaken, Hammer, and de Werra (1985a). (**SGc**)

Edmund R. Peay

1977a Matrix operations and the properties of networks and directed graphs. *J. Math. Psychology* 15 (1977), 89–101. MR 0444333 (56 #2690). (**SD, WD: Adj: Gen**)

1977b Indices for consistency in qualitative and quantitative structures. *Human Relations* 30 (1977), 343–361.
Proposes an index of inclusterability for signed graphs and generalizes to edges weighted by a linearly ordered set. (**SG, Gen: Clu: Fr(Gen)**)

1980a Connectedness in a general model for valued networks. *Social Networks* 2 (1980), 385–410. MR 0602317 (82h:92053) (*q.v.*).
Real-number edge weights; the value of a path is the minimum absolute weight. [Annot. 11 Sept 2010.] (**WG**)

1982a Structural models with qualitative values. *J. Math. Sociology* 8 (1982), 161–192. MR 0655909 (83d:92107). Zbl 486.05060.
See mainly §3: "Structural consistency." (**sd: Gen: Bal, Clu**)

Luke Pebody

See B. Bollobás.

Timothée Pecatte

See L. Isenmann.

Elisabeth Pécou

See M. Domijan.

Yan Ru Pei

See also H. Manukian.

Yan Ru Pei & Massimiliano Di Ventra

20xxa A finite-temperature phase transition for the Ising spin-glass in $d \geq 2$. Submitted. arXiv:2105.01188. (**SG: Fr: Phys, State**)

Yan Ru Pei, Haik Manukian, & Massimiliano Di Ventra

2020a Generating weighted MAX-2-SAT instances with frustrated loops: an RBM case study. *J. Machine Learning Res.* 21 (2020), 1–55. MR 4209445. arXiv:1905.05334.
Weighted MAX-2-SAT means weighted frustration index $l(\Sigma, w) := \min\{w(D) : D \subseteq E,\ \Sigma \setminus D \text{ balanced}\}$ for given weights $w : E \to \mathbb{R}$.
(**SG: WG: Fr: Phys, State, Algor**)

Britta Peis

See W. Hochstättler and M. Lätsch.

David B. Peizer

See P.J. Runkel.

Uri N. Peled

See S.R. Arikati, A. Bhattacharya, P.L. Hammer, T. Ibaraki, and N.V.R. Mahadev.

Martin Pelikan & Alexander K. Hartmann

2007a Obtaining ground states of Ising spin glasses via optimizing bonds instead of spins. (Extended abstract.) In: *GECCO '07: Genetic and Evolutionary Computation Conference* (GECCO 2007, London), p. 628. ACM, New York, 2007.
Announcement of (2007b). (**SG, Phys: State(fr): Algor**)

2007b Obtaining ground states of Ising spin glasses via optimizing bonds instead of spins. Report, Missouri Estimation of Distribution Algorithms Laboratory, Dept. of Mathematics and Computer Science, University of Missouri–St. Louis, 2007. URL `http://medal-cs.umsl.edu/` (**SG, Phys: State(fr): Algor**)

Marcello Pelillo
See R. Glantz.

Volodymyr Pelykh
See I. Kyrchei.

R.A. Pendavingh & S.H.M. van Zwam
2010a Confinement of matroid representations to subsets of partial fields. *J. Combin. Theory Ser. B* 100 (2010), 510–545. MR 2718674 (2011i:05043). Zbl 1231.05062. arXiv:0806.4487.
Dowling's (1973b), (1973a) $Q_n(\mathrm{GF}(q)^\times)$ is an example. [Annot. 1 Sept 2017.] (**Matrd: gg**)

2013a Skew partial fields, multilinear representations of matroids, and a matrix tree theorem. *Adv. Appl. Math.* 50 (2013), 201–227. MR 2996392. Zbl 1256.05047.
Introduces representation of Dowling geometries $Q_n(\mathfrak{G})$ over a skew partial field (*cf.* van Zwam (2009a), §3.2). [Continued in Vertigan (2015a).] [Annot. 28 Jan 2015.] (**Matrd: gg: Incid**)

Irena Penev
See V. Boncompagni.

Di Peng, Xiangbai Gu, Yuan Xu, & Qunxiong Zhu
2015a Integrating probabilistic signed digraph and reliability analysis for alarm signal optimization in chemical plant. *J. Loss Prevention Process Industries* 33 (2015), 279–288. (**SD: Rand, Appl**)

Yuejian Peng
See Y.-T. Li.

Francisco Pereira
See A.J. Hoffman.

Jessica Pereira, Tarkeshwar Singh, & S. Arumugam
2023a Additively graceful signed graphs. *AKCE Int. J. Graphs Combin.* 20 (2023), no. 3, 300–307. (**SG: Lab**)

Mercedes Pérez Millán
See A. Dickenstein.

C. Perina, N. Buckley, & A.K. Nagar
2017a Application of metaheuristics algorithms and signed graphs to portfolio turnover management. *Int. J. Innovation Management Tech.* 8 (2017), no. 2, 161–165.
Same error in balance testing as in Vasanthi *et al.* (2015a). Confusing exposition. Genetic, simulated annealing, and ant-colony algorithms

with example. (**Appl: SG: Bal: Algor**)

Kavita S. Permi
See M. Moudgalya R and P.S.K. Reddy.

F. Peruggi
See A. Coniglio.

Paweł Petecki
See F. Belardo.

M. Petersdorf
1966a Einige Bemerkungen über vollständige Bigraphen. *Wiss. Z. Techn. Hochsch. Ilmenau* 12 (1966), 257–260. MR 0225682 (37 #1275). Zbl 156.44302.

Treats signed K_n's. Satz 1: $\max_\sigma l(K_n, \sigma) = \lfloor (n-1)^2/4 \rfloor$ with equality iff (K_n, σ) is antibalanced. [From which follows easily the full Thm. 14 of Abelson and Rosenberg (1958a).] Also, some further discussion of antibalanced and unbalanced cases. [For extensions of this problem see notes on Erdős, Győri, & Simonovits (1992a).] (**SG: Fr**)

Ion Petre
See A. Alhazov and T. Harju.

Rossella Petreschi
See also T. Calamoneri.

Rossella Petreschi & Andrea Sterbini
1995a Recognizing strict 2-threshold graphs in $O(m)$ time. *Inform. Processing Lett.* 54 (1995), no. 4, 193–198. MR 1337823 (96i:68034a). Zbl 0875.68453. Erratum. *Ibid.* 56 (1995), no. 1, 65. MR 1361260 (96i:68034b).

Uses the auxiliary signed graph of Mahadev and Peled (1988a). [Annot. 22 Mar 2017.] (**SG: Appl: Bal, Algor**)

[Rossella Petreschim & Andrea Sterbini]
Misprint for R. Petreschi & A. Sterbini.

Norbert Peyerimhoff
See C. Lange and S.-P. Liu.

Nathan Pflueger
2011a Graph reductions, binary rank, and pivots in gene assembly. *Discrete Appl. Math.* 159 (2011), no. 17, 2117–2134. MR 2832336 (2012j:05421). Zbl 1237.05181. arXiv:1103.4334. (**SG: Algor, Appl**)

Lan Anh Pham
See R. Naserasr.

Geevarghese Philip, Ashutosh Rai, & Saket Saurabh
2015a Generalized pseudoforest deletion: Algorithms and uniform kernel. In: Giuseppe F. Italiano *et al.*, eds., *Mathematical Foundations of Computer Science 2015* (40th Int. Symp., MFCS 2015, Milan, 2015), Part II, pp. 517–528. Lect. Notes in Computer Sci., Vol. 9235. Springer, Berlin, 2015. MR 3419512. Zbl 1400.05242.

"Pseudoforest" [or 1-forest] = independent set in the bicircular matroid $\mathbf{F}(\Gamma, \varnothing)$. Problem: Can $\Gamma \setminus (\leq k$ vertices) be a pseudoforest? Generally,

"l-pseudoforest" = forest + l edges. Problem: Can $\Gamma \setminus (\leq k$ vertices) be an l-pseudoforest? [Annot. 22 Dec 2017.] (**bic: Algor**)

2018a Generalized pseudoforest deletion: Algorithms and uniform kernel. *SIAM J. Discrete Math.* 32 (2018), no. 2, 882–901. MR 3787778. Zbl 1384.05148.

Extended version of (2015a). (**bic: Algor**)

J.L. Phillips

1967a A model for cognitive balance. *Psychological Rev.* 74 (1967), 481–495.

Proposes to measure imbalance of a signed (di)graph by largest eigenvalue of a matrix close to $I + A(\Sigma)$. (*Cf.* Abelson 1967a.) Possibly, means to treat only graphs that are complete aside from isolated vertices. [Somewhat imprecise.] Summary of Ph.D. thesis. (**SG: Bal, Fr, Adj**)

Nancy V. Phillips

See F. Glover.

[Alberto Del Pia]

See A. Del Pia.

Jean-Claude Picard & H. Donald Ratliff

1973a A graph-theoretic equivalence for integer programs. *Operations Res.* 21 (1973), 261–269. MR 0359788 (50 #12240). Zbl 263.90021.

A minor application of signed switching to a weighted graph arising from an integer linear program. (**sg: sw**)

Théo Pierron

See J. Bensmail, F. Dross, and F. Foucaud.

Oleg Pikhurko

2001a Uniform families and count matroids. *Graphs Combin.* 17 (2001), 729–740. MR 1876580 (2003c:05120). Zbl 987.05060.

Generalizes graphic count matroids (White and Whiteley (1983a), Whiteley (1996a)) to r-hypergraphs and more general linear functions as rank bounds. [Annot. 27 Jul 2022.] (**Matrd: Bic: Gen**)

Vincent Pilaud

2013a Signed tree associahedra. Manuscript, 2013, 50 pp. arXiv:1309.5222.

A signed tree is a tree with vertices in $\pm[n]$. Nesting structure of induced subgraphs, etc. [Annot. 7 Feb 2022.] (**VS**)

2014a Signed tree associahedra. In: *26th International Conference on Formal Power Series and Algebraic Combinatorics* (FPSAC 2014, Chicago), pp. 309–320. DMTCS Proc., Vol. AT. URL `https://doi.org/10.46298/dmtcs.2402`. MR 3466354 (book). Zbl 1394.52012. HAL hal-01207614.

Condensed version of (2013a). [Annot. 7 Feb 2022.] (**VS**)

Marcin Pilipczuk

See M. Cygan.

Michał Pilipczuk

See M. Cygan.

P. Pincus

See S. Alexander.

Alexandre Pinlou

See also C. Duffy, F. Jacques, A. Montejano, and P. Ochem.

2016a *Some Problems in Graph Coloring: Methods, Extensions and Results*. Habilitation á Diriger des Recherches, Univ. Montpellier, 2016. HAL tel-01376199.

§2.2, "Homomorphisms of signed graphs". (**SG: Col, Hom**)

Shariefuddin Pirzada

See also M.A. Bhat and T. Shamsher.

2012a Signed degree sequences in signed graphs. Set Valuations, Signed Graphs and Geometry (IWSSG-2011, Int. Workshop, Mananthavady, Kerala, 2011). *J. Combin. Inform. System Sci.* 37 (2012), no. 2-4, 179–204. Zbl 1300.05068.

Dictionary: "Signed degree" = net degree $d^{\pm}(v)$ [*cf.* Chartrand, Gavlas, Harary, and Schultz (1994a)]. (**SG: ori: Invar: Exp**)(**SG: ori: Invar**)

S. Pirzada & Mushtaq A. Bhat

2014a Energy of signed digraphs. *Discrete Appl. Math.* 169 (2014), 195–205. MR 3175069. Zbl 1288.05166. arXiv:1309.6266. (**SD: Adj: Eig**)

S. Pirzada & F.A. Dar

2007a Signed degree sets in signed 3-partite graphs. *Mat. Vesnik* 59 (2007), no. 3, 121–124. MR 2361920 (2008k:05095). Zbl 1224.05222. (**SG: ori: Invar**)

2007b Signed degree sequences in signed 3-partite graphs. *J. Korean Soc. Ind. Appl. Math.* 11 (2007), no. 1, 9–14. (**SG: ori: Invar**)

S. Pirzada, Muhammad Ali Khan, & E. Sampathkumar

2013a Coloring of signed graphs. In: Kamiel Cornelissen, Ruben Hoeksma, *et al.*, eds., *Proceedings of the 12th Cologne-Twente Workshop on Graphs and Combinatorial Optimization* (CTW-2013, Enschede, 2013), pp. 191–195. Centre for Telematics and Information Technology, Univ. of Twente, Enschede, Neth., 2013.

σ bipartitions $V(\Gamma)$ as $V_{\text{odd}} \cup V_{\text{even}}$ by parity of $d^{-}(v)$. Coloring = properly coloring $\Gamma_c := \Gamma{:}V_{\text{odd}} \cup \Gamma{:}V_{\text{even}}$. [Why not an arbitrary bipartition? Signs seem superfluous.] Results: chromatic polynomial [the usual one of Γ_c], examples like paths, circles. [Annot. 1 Apr 2019.] (**SGc: Col**)

S. Pirzada, T.A. Naikoo, & F.A. Dar

2007a Signed degree sets in signed graphs. *Czechoslovak Math. J.* 57 (2007), no. 3, 843–848. MR 2356284 (2008g:05088). Zbl 1174.05059. arXiv:math/0609121.

The set, as opposed to sequence, of net degrees [*cf.* Chartrand, Gavlas, Harary, and Schultz (1994a)] of a signed simple graph can be any finite set of integers. Also, the smallest order of a signed graph with given net degree set. (**SG: ori: Invar**)

2007b Signed degree sequences in signed bipartite graphs. *AKCE Int. J. Graphs Combin.* 4 (2007), no. 3, 301–312. MR 2384886 (no rev). Zbl 1143.05307. arXiv:-math/0609122.

Characterization of net degree sequences of signed, simple, bipartite graphs. [Annot. 15 Nov 2011.] (**SG: ori: Invar**)

2008a A note on signed degree sets in signed bipartite graphs. *Appl. Anal. Discrete Math.* 2 (2008), no. 1, 114–117. MR 2396733 (2009a:05092). Zbl 1199.05159. arXiv:math/0609122.

Every finite set of integers is the net degree set of a connected signed bipartite graph. [Annot. 10 Sept 2010.] (**SG: ori: Invar**)

S. Pirzada, Tahir Shamsher, & Mushtaq A. Bhat

20xxa On ordering of minimal energies in bicyclic signed graphs. Submitted.

Among connected signed graphs with cyclomatic number 2 and order n, the 20 with least energy for $n \geq 30$ and the 16 for $n \geq 17$, via extensive computations. [Annot. 3 Feb 2021.] (**SG: Adj: Eig**)

20xxb On the eigenvalues of signed complete bipartite graphs. Submitted. arXiv:-2111.07262.

Interesting relations between Σ^{-} structure and eigenvalues. [Annot. 23 Jan 2022.] (**SG: Adj: Eig**)

Tomaž Pisanski

See also N. Basic, V. Batagelj, and F. Belardo.

Tomaž Pisanski & Primož Potočnik

2004a Graphs on surfaces. In: Jonathan L. Gross and Jay Yellen, eds., *Handbook of Graph Theory*, pp. 611–624. Discrete Math. Appl. (Boca Raton). CRC Press, Boca Raton, Fla., 2004. MR 2035186 (2004j:05001) (book). Zbl 1036.05001 (book).

Cryptic. Dictionary (my best guess): "signed edge" = oriented edge; "signed boundary walk" (of a face) = directed face boundary walk; "signature" = set of negative edges of an embedding; "switch" = negative (= orientation-reversing) edge of an embedding. (**sg: Top**)

Tomaž Pisanski & Jože Vrabec

1982a Graph bundles. Preprint Ser., Dept. Math., University of Ljubljana, 1982.

Definition (see Pisanski, Shawe-Taylor, and Vrabec (1983a)), examples, superimposed structure, classification. (**GG: Cov(Gen)**)

Tomaž Pisanski, John Shawe-Taylor, & Jože Vrabec

1983a Edge-colorability of graph bundles. *J. Combin. Theory Ser. B* 35 (1983), 12–19. MR 0723566 (85b:05086). Zbl 505.05034, (Zbl 515.05031).

A graph bundle is, roughly, a covering graph with an arbitrary graph F_v (the "fibre") over each vertex v, so that the edges covering $e{:}vw$ induce an isomorphism $F_v \to F_w$. (**GG: Cov(Gen): ECol**)

Tomaž Pisanski & Arjana Žitnik

2009a Representing graphs and maps. In: Lowell W. Beineke and Robin J. Wilson, eds., *Topics in Topological Graph Theory*, Ch. 8, pp. 151–180. Encycl. Math. Appl., Vol. 128. Cambridge Univ. Press, Cambridge, Eng., 2009. MR 2581545 (no rev). Zbl 1225.05185.

Signed graphs are implicit in pp. 170–171. Dictionary: "flat" = +, "twisted" = −. [Annot. 26 Aug 2108.] (**sg: Top: Exp**)

Leonidas S. Pitsoulis

See also G. Appa and K. Papalamprou.

2014a *Topics in Matroid Theory*. SpringerBriefs in Optimization. Springer, New York, 2014. MR 3154793. Zbl 1319.05033.

Ch. 6, "Signed-graphic matroids": Signed graphs; signed-graphic matroids; binary signed-graphic matroids; decomposition. (**SG: Matrd**)

Leonidas Pitsoulis, Konstantinos Papalamprou, Gautam Appa, & Balázs Kotnyek

2009a On the representability of totally unimodular matrices on bidirected graphs. *Discrete Math.* 309 (2009), no. 16, 5024–5042. MR 2548904 (2010m:05182). Zbl 1182.05120.

Tour matrices of bidirected graphs are closed under 1-, 2-, and 3-sums. Possibly, every totally unimodular matrix is a tour matrix. (**Ori: Incid(Gen)**)

Leonidas Pitsoulis & Eleni-Maria Vretta

2015a Decomposition of quaternary signed-graphic matroids. Manuscript, 2015. arXiv:-1510.06727. (**SG: Matrd: Str**)

Irene Pivotto

See also R. Chen, M. DeVos, D. Funk, and B. Guenin.

2006a *On Excluded Minors for Even Cut Matroids.* Master's thesis, Univ. of Waterloo, 2006. URL `http://hdl.handle.net/10012/2680` (**SG: Matrd**)

2011a *Even Cycle and Even Cut Matroids.* Doctoral dissertation, Univ. of Waterloo, 2011. URL `http://hdl.handle.net/10012/5956` (**SG: Matrd**)

2013a Biased graphs and their matroids I, II, III. *The Matroid Union*, 2013, `http://matroidunion.org/?p=161`, `http://matroidunion.org/?p=279`, `http://matroidunion.org/?p=357` (**GG: Matrd: Exp**)

Erik Plahte, Thomas Mestl, & Stig W. Omholt

1995a Feedback loops, stability and multistationarity in dynamical systems. *J. Biol. Systems* 3 (1995), no. 2, 409–413. (**sd: QM: Dyn**)

Michael Plantholt

See A.H. Busch, A.A. Diwan, and F. Harary.

Andrey Ploskonosov

See Y. Burman.

M.D. Plummer

See L. Lovász.

Jorn van der Pol

See P. Nelson.

Agnieszka Polak & Daniel Simson

2013a Algorithms computing $O(n, \mathbb{Z})$-orbits of P-critical edge-bipartite graphs and P-critical unit forms using Maple and C#. *Algebra Discrete Math.* 16 (2013), no. 2, 242–286. MR 3186088. Zbl 1310.05218. (**SG: Algor**)

2013b On Coxeter spectral classification of P-critical edge-bipartite graphs of Euclidean type $\tilde{\mathbb{A}}_n$. Combinatorics 2012 (Perugia, 2012). *Electronic Notes in Discrete Math.* 40 (2013), 311–316. MR 3155275 (volume). Zbl 1292.05005 (volume). (**SG**)

2013c Algorithmic experiences in Coxeter spectral study of P-critical edge-bipartite graphs and posets. In: Nikolaj Björner *et al.*, eds., *15th International Sympo-*

sium on Symbolic and Numeric Algorithms for Scientific Computing (SYNASC 2013, Timisoara, Romania, 2013), pp. 375–382. IEEE, 2013. (**SG**)

2014a Coxeter spectral classification of almost TP-critical one-peak posets using symbolic and numeric computations. *Linear Algebra Appl.* 445 (2014), 223–255. MR 3151272. Zbl 1290.16014. (**SG**)

Oskar E. Polansky

See I. Gutman.

Svatopluk Poljak

See also Y. Crama and B. Mohar.

Svatopluk Poljak & Daniel Turzík

1982a A polynomial algorithm for constructing a large bipartite subgraph, with an application to a satisfiability problem. *Canad. J. Math.* 34 (1982), 519–524. MR 0663301 (83j:05048). Zbl 471.68041, (Zbl 487.68058).

Main Theorem: For a simple, connected signed graph of order n and size $\#E = m$, the frustration index $l(\Sigma) \leq \frac{1}{2}m - \frac{1}{4}(n-1)$. The proof is algorithmic, by constructing a (relatively) small deletion set. Dictionary: Σ is an "edge-2-colored graph" (G, c), E^+ and E^- are called E_1 and E_2, a balanced subgraph is "generalized bipartite", and $m - l(\Sigma)$ is what is calculated. [This gives an upper bound on $D(\Gamma) := \max_\sigma l(\Gamma, \sigma)$ for a connected, simple graph, whereas Akiyama, Avis, Chvátal, and Era (1981a) has a lower bound on D.] (**SG: Fr, Algor**)

1986a A polynomial time heuristic for certain subgraph optimization problems with guaranteed worst case bound. *Discrete Math.* 58 (1986), 99–104. MR 0820844 (87h:68131). Zbl 585.05032.

Generalizes (1982a), with application to signed graphs in Cor. 3. (**SG: Fr, Algor**)

1987a On a facet of the balanced subgraph polytope. *Časopis Pěst. Mat.* 112 (1987), 373–380. MR 0921327 (89f:05155). Zbl 643.05059.

The polytope $P_B(\Sigma)$ (the authors write P_{BL}) is the convex hull in $\mathbb{R}^E$ of characteristic vectors of balanced edge sets. It generalizes the bipartite subgraph polytope $P_B(\Gamma) = P_B(-\Gamma)$ (see Barahona, Grötschel, and Mahjoub (1985a)), but is essentially equivalent to it according to Prop. 2: The negative-subdivision trick preserves facets of the polytope. Thm. 1 gives new facets, corresponding to certain circulant subgraphs. (They are certain unions of two Hamilton circles, each having constant sign.) (**SG: Fr, Geom**)

1992a Max-cut in circulant graphs. *Discrete Math.* 108 (1992), 379–392. MR 1189859 (93k:05101).

Further development of (1987a) for all-negative Σ. The import for general signed graphs is not discussed. [Developed more in Kaparis and Letchford (2018a).] (**Par: Fr, Geom**)

Svatopluk Poljak & Zsolt Tuza

1995a Maximum cuts and large bipartite subgraphs. In: W. Cook, L. Lovász, and P. Seymour, eds., *Combinatorial Optimization* (DIMACS Special Year, New Brunswick, N.J., 1992–1993), pp. 181–244. DIMACS Ser. Discrete Math. Theor.

Computer Sci., Vol. 20. Amer. Math. Soc., Providence, R.I., 1995. MR 1338615 (97a:90106). Zbl 819.00048.

Surveys max-cut and weighted max-cut [that is, max size balanced subgraph and max weight balanced subgraph in all-negative signed graphs]. See esp. §2.9: "Bipartite subgraph polytope and weakly bipartite graphs". [The weakly bipartite classes announced by Gerards suggested that a signed-graph characterization of weakly bipartite graphs is called for. This is provided by Guenin (2001a).]

§1.2, "Lower bounds, expected size, and heuristics", surveys results for all-negative signed graphs that are analogous to results in Akiyama, Avis, Chvátal, and Era (1981a) (*q.v.*), etc. [*Problem*. Generalize any of these results, that are not already generalized, to signed simple graphs and to simply signed graphs.] (**par: Fr, tg(Sw): Exp, Ref**)

Albert Pollatchek

1975a *Relationships Between Combinatorics and 0-Simple Semigroups.* Ph.D. thesis, Univ. of California, Berkeley, 1975 MR 2940578 (no rev). (**gg: aut, Algeb**)

1976a Relationships between combinatorics and 0-simple semigroups. *Semigroup Forum* 12 (1976), no. 1, 78–82. MR 0486249 (58 #6017). Zbl 322.20029.

Announcement of (1977a). (**gg: aut, Algeb**)

1977a Relationships between combinatorics and 0-simple semigroups. *J. Pure Appl. Algebra* 9 (1976/77), no. 3, 301–334. MR 0453905 (56 #12158). Zbl 421.20029. (**gg: aut, Algeb**)

Alexandr Polyanskii

See Z.L. Jiang.

Y. Pomeau

See B. Derrida.

Kalpesh M. Popat

See S.K. Vaidya.

Dragos Popescu [Dragoş-Radu Popescu]

See D.-R. Popescu.

Dragoş-Radu Popescu [Dragos Popescu]

1979a Proprietati ale grafurilor semnate. [Properties of signed graphs.] (In Romanian. French summary.) *Stud. Cerc. Mat.* 31 (1979), 433–452. MR 0560478 (82b:05111). Zbl 426.05048.

A signed K_n is balanced or antibalanced or has a positive and a negative circle of every length $k = 3, \ldots, n$. For odd n, the signed K_n if not balanced has at least $\frac{n-1}{2}$ negative Hamiltonian circles. For even n, $-K_n$ does not maximize the number of negative circles. A "circle basis" is a set of the smallest number of circles whose signs determine all circle signs. This is proved to have $\binom{n-1}{2}$ members. Furthermore, there is a basis consisting of k-circles for each $k = 3, \ldots, n$. [A circle basis in this sense is the same as a basis of circles for the binary cycle space. See Zaslavsky (1981b), Topp and Ulatowski (1987a).] (**SG: Fr, Circ**)

1991a Cicluri în grafuri semnate. [Cycles in signed graphs.] (In Romanian; French summary.) *Stud. Cerc. Mat.* 43 (1991), no. 3/4, 85–219. MR 1138705 (92j:05114). Zbl 751.05060.

Ch. 1: "A-balance" (p. 91). Let F be a spanning subgraph of K_n and A a signed K_n. The "product" of signed graphs is $\Sigma_1 * \Sigma_2$ whose underlying graph is $|\Sigma_1| \cup |\Sigma_2|$, signed as in Σ_i for an edge in only one Σ_i but with sign $\sigma_1(e)\sigma_2(e)$ if in both. Let $\mathcal{G}_F$ denote the group of all signings of F; let $\mathcal{G}_F(A)$ be the group generated by the set of restrictions to F of isomorphs of A. A member of $\mathcal{G}_F(A)$ is "A-balanced"; other members of $\mathcal{G}_F$ are A-unbalanced. We let $\hat{\Sigma}$ denote the coset of Σ and $\approx$ the "isomorphism" of cosets induced by graph isomorphism, i.e., cosets are isomorphic if they have isomorphic members. Let $\dot{\Sigma}$ be the isomorphism class of Σ, $\dot{\hat{\Sigma}}$ the isomorphism class of $\hat{\Sigma}$, and $\overset{\circ}{\rightarrow} \Sigma := \bigcup \dot{\hat{\Sigma}}$. Now choose a system of representatives of the coset isomorphism classes, $R = \{\Sigma_1, \ldots, \Sigma_l\}$. Prop. 1.4.1: Each $\dot{\Sigma}$ intersects exactly one $\hat{\Sigma}_i$. Let $R_i = \{\Sigma_{i1}, \ldots, \Sigma_{ia_i}\}$ be a system of representatives of $\hat{\Sigma}_i/\cong$, arranged so that $\#E^-(\Sigma_{ij})$ is a minimum when $j = 1$. This minimum value is the "[line] index of A-imbalance" of each $\Sigma \in \overset{\circ}{\rightarrow} \Sigma_i$ and is denoted by $\delta_A(\Sigma)$. (§2.1: Taking A to be K_n with one vertex star all negative makes this equal the frustration index $l(\Sigma)$.) Prop. 1.5.1: $\delta_A(\Sigma)$ is the least number of edges whose sign needs to be changed to make Σ A-balanced. Prop. 1.5.2. $\delta_A(\Sigma) = \#E^-(\Sigma)$ iff $\#(E^-(\Sigma) \cap E^-(F,\beta)) \leq \frac{1}{2}\#E^-(F,\beta)$ for every signing β) of F. Finally, for each $\Sigma \in \mathcal{G}_F$ define the "Σ-relation" on coset isomorphism classes $\dot{\hat{\Sigma}}_i$ to be the relation generated by negating in Σ_1 all the edges of $E^-(\Sigma)$, extended by isomorphism and transitivity. This is well defined (Prop. 1.6.1) and symmetric (Prop. 1.6.2) and is preserved under negation of coset isomorphism classes (Prop. 1.6.4, 1.6.5). Self-negative classes, such that $\dot{\hat{\Sigma}} \approx -\dot{\hat{\Sigma}}$, are the subject of Prop. 1.6.3.

Ch. 2: "Signed complete graphs" (p. 106). §2.5: "H-graphs". If H is a signed K_h, a "standard H-graph" Σ is a signed K_n such that $\Sigma^- \cong H^- \cup K^c_{n-h}$. Prop. 2.5.3. Assume certain hypotheses on n, $\#X_0$ for $X_0 \subseteq V(\Sigma)$, and a quantity $D^-(H)$ derived from negative degrees. Then $\#E^- = l(\Sigma) \Rightarrow$ the induced subgraph $G{:}X_0$ is a standard H-graph with $\#E^-(\Sigma{:}X_0) = l(\Sigma{:}X_0)$. The cases $H^- = K_1$, K_2, and a 2-edge path are worked out. For the former, Prop. 2.5.3 reduces to Sozański's (1976a) Thm. 3.

Ch. 3: "Frustration index" (p. 158). Some upper bounds.

Ch. 4: "Evaluations, divisibility properties" (p. 174). Similar to parts of (1996a) and Popescu and Tomescu (1996b).

Ch. 5: "Maximal properties" (p. 198). §5.1: "Minimum number and maximum number of negative stars, resp. 2-stars". §5.2 is a special case of Popescu and Tomescu (1996a), Thm. 2. §5.3: "On the maximum number of negative cycles in some signed complete graphs". Shows that Conjecture 1 is false for even $n \geq 6$. Some results on the odd case.

Conjecture 1 (Tomescu). A signed complete graph of odd order has the most negative circles iff it is antibalanced. (Partial results are in

§5.3.) [This example maximizes $l(\Sigma)$. A somewhat related conjecture is in Zaslavsky (1997b).] *Conjecture* 2. See (1993a). *Conjecture* 3. Given k and m, there is $n(k,m)$ so that for any $n \geq n(k,m)$, a signed K_n with m negative edges has (a) the most negative k-circles iff the negative edges are pairwise nonadjacent; (b) the fewest iff the negative edges form a star. (**SG: Bal(Gen), KG, Fr, Enum: Circ, Paths**)

1993a Problem 17. *Research Problems* at the Int. Conf. on Combinatorics (Keszthely, 1993). Unpublished manuscript. János Bolyai Math. Soc., Budapest, 1993.

Conjecture. An unbalanced signed complete graph has the minimum number of negative circles iff its frustration index equals 1. [This has been proved.] (**SG: Circ, Fr**)

1996a Une méthode d'énumération des cycles négatifs d'un graphe signé. *Discrete Math.* 150 (1996), 337–345. MR 1392742 (97c:05077). Zbl 960.39919.

The numbers of negative subgraphs, especially circles and paths of length k, in an arbitrarily signed K_n. Complicated formulas; divisibility and congruence properties. Extends part of Popescu and Tomescu (1996a).

(**SG: KG, Enum: Circ, Paths**)

1999a Balance in systems of finite sets. Proc. Annual Meeting Fac. Math. (Bucharest, 1999). *An. Univ. Bucureşti Mat. Inform.* 48 (1999), no. 2, 29–40. MR 1829295 (2002c:05082).

A generalization of signed-graph frustration index. Let $\mathcal{F} \subseteq \mathcal{P}(E)$; let $\delta_{\min}(S|\mathcal{F}) := \min[\{\#(S \oplus F) : F \in \mathcal{F}\}$ and similarly $\delta_{\max}$. Application to signed graphs, where $\mathcal{F} = \mathcal{B}(\Sigma)$ and $\delta_{\min}(S|\mathcal{F}) = l(\Sigma|S)$. [Annot. 3 Oct 2014.] (**SG: Bal, Gen**)

2001a An inequality on the maximum number of negative cycles in complete signed graphs. *Math. Rep. (Bucur.)* 3(53) (2001), no. 1, 53–60. MR 1887184 (2002m:-05193). Zbl 1017.05099.

Similar to (1999a). (**SG: Fr**)

2007a Balance in systems of finite sets with applications. *J.UCS* 13 (2007), no. 11, 1755–1766. MR 2390248 (2009c:05245).

Similar to (1999a) but [*cf.* MR] with improved results. (**SG: Fr**)

Dragoş-Radu Popescu & Ioan Tomescu

1996a Negative cycles in complete signed graphs. *Discrete Appl. Math.* 68 (1996), 145–152. MR 1393315 (98f:05098). Zbl 960.35935.

The number c_p^- of negative circles of length p in a signed K_n with s negative edges. Thm. 1: For n sufficiently large compared to p and s, c_p^- is minimized if E^- is a star (iff, when $s > 3$) and is maximized iff E^- is a matching. Thm. 2: c_p^- is divisible by $2^{p-2-\lfloor \log_2(p-1) \rfloor}$. Thm. 3: If $s \sim \lambda n$ and $p \sim \mu n$ and the negative-subgraph degrees are bounded (this is essential), then asymptotically the fraction of negative p-circles is $\frac{1}{2}(1 - e^{-4\lambda\mu})$. [Kittipassorn & Mészáros (2015a) performs a detailed study of the number of negative triangles.] (**SG: KG: Fr, Enum: Circ**)

1996b Bonferroni inequalities and negative cycles in large complete signed graphs. *European J. Combin.* 17 (1996), 479–483. MR 1397155 (97d:05177). Zbl 861.05036.

A much earlier version of (1996a) with delayed publication. Contains part of (1996a): a version of Thm. 1 and a restricted form of Thm. 3.

(**SG: KG: Fr, Enum: Circ**)

L. Pósa

See P. Erdős.

Olaf Post

See C. Lange.

Luke Postle

See Z. Dvořák.

Alexander Postnikov See also F. Ardila.

1997a Intransitive trees. *J. Combin. Theory Ser. A* 79 (1997), 360–366. MR 1462563 (98b:05036). Zbl 876.05042.

§4.2 mentions the lift matroid of $\{1\}\vec{K}_n$, i.e., the integral poise gains of a transitively oriented complete graph, represented by the Linial arrangement. [*Cf.* Stanley (1996a).] (**GG: Matrd, Geom**)

Alexander Postnikov & Richard P. Stanley

2000a Deformations of Coxeter hyperplane arrangements. *J. Combin. Theory Ser. A* 91 (2000), 544–597. MR 1780038 (2002g:52032). Zbl 962.05004.

The arrangements are the canonical affine-hyperplane lift representations of certain additive real gain graphs. Characteristic polynomials of the former, equalling zero-free chromatic polynomials of the latter, are calculated. And much more. (**gg: Geom, Matrd, Invar**)

J. Poulter

See also A. Aromsawa, J.A. Blackman, and J.R. Goncalves.

J. Poulter & J.A. Blackman

2001a Properties of the $\pm J$ Ising spin glass on the triangular lattice. *J. Phys. A* 34 (2001), 7527–7539. Zbl 982.82032.

Triangular lattice graph with a definite proportion of negative edges. Successor to Blackman and Poulter (1991a). [Annot. 16 Aug 2018.]

(**Phys: sg: Fr**)

2005a Exact algorithm for spin-correlation functions of the two-dimensional $\pm J$ Ising spin glass in the ground state. *Phys. Rev. B* 72 (2005), art. 104422, 8 pp.

(**Phys: sg: Fr, State**)

M.R. Pournaki

See H.R. Maimani.

M. Poursoltani

See H.R. Maimani.

Swathy Prabhu [Swathyprabhu Mj]

See Swathyprabhu (under 'S').

Shivaramakrishna Pragada

See M.R. Kannan.

U.M. Prajapati & K.K. Raval

2019a Wheel related some signed product cordial graphs. *Int. J. Sci. Res. Rev.* 8 (2019), no. 2, 2934–2940.

More examples as in Baskar Babujee and Loganathan (2011a).

(**Lab: VS: SG, Bal**)

K.N. Prakasha, P. Siva Kota Reddy, & Ismail Naci Cangul

2017a Partition Laplacian energy of a graph. *Adv. Stud. Contemp. Math., Kyungshang* Zbl 1386.05114. (**sgw(Gen): Adj: Eig**)

Pranjali [Pranjali Sharma]

See also M. Acharya.

2023a Line signed graph of a signed unit graph of commutative rings. *Commun. Combin. Optim.* 8 (2023), no. 2, 313–326. MR 4570020.

Line graph $\Lambda_{BC}(G_\Sigma(R))$ (Behzad and Chartrand (1969a)) for ring R with unity. Thm. 5 characterizes R with balance. Thm. 14 characterizes R with canonical consistency. [Annot. 22 Apr 2023.]

(**Algeb: SG: LG: Bal**)

Pranjali Sharma & Mukti Acharya

2016a Balanced signed total graphs of commutative rings. *Graphs Combin.* 32 (2016), no. 4, 1585–1597. MR 3514985. Zbl 1342.05060. (**SG: Algeb**)

Pranjali, Atul Gaur, & Mukti Acharya

2016b $\mathcal{C}$-Consistency in signed total graphs of commutative rings. *Discrete Math. Algorithms Appl.* 8 (2016), no. 3, art. 1650041, 14 pp. MR 3531893. Zbl 1354.13008.

$\mathcal{C}$-Consistency = canonical consistency. (**SG: Algeb: LG(Gen), VS**)

Viktor K. Prasanna

See A. Srivastava.

Philips Kokoh Prasetyo

See D. Lo.

B. Prashanth

See P.S.K. Reddy.

Primož Potočnik

See also T. Pisanski.

Primož Potočnik & Mateja Šajna

2007a Self-complementary two-graphs and almost self-complementary double covers. *European J. Combin.* 28 (2007), 1561–1574. MR 2339485 (2008g:05177). Zbl 1123.05079.

Γ is "almost self-complementary" if $\Gamma \cong K_n \setminus M \setminus \Gamma$ where M is a perfect matching in $K_{2n} \setminus \Gamma$. Such graphs are double coverings $\widetilde{(\Delta, \sigma)}$ that are $\cong \widetilde{(\Delta^c, \sigma^c)}$ for some σ^c. Dictionary: "$\mathbb{Z}_2$-voltage assignment" = signature. [Annot. 22 Aug 2013.] (**TG: Cov, Aut**)

2009a Brick assignments and homogeneously almost self-complementary graph. *J. Combin. Theory Ser. B* 99 (2009), 185–201. MR 2467825 (2009i:05107). Zbl

1200.05182. (sg: Cov, Aut)

Annie K. Powell
See K.C. Mondal.

Julia Preusse
See J. Kunegis.

K.O. Price, E. Harburg & T.M. Newcomb
1966a Psychological balance in situations of negative interpersonal attitudes. *J. Personality Social Psychol.* 3 (1966), 265–270. (**PsS**)

Noah Prince
See H. Liu.

Geert Prins
See F. Harary.

Ion Prisecaru
See K.C. Mondal.

G.S. Shanmuga Priya
See M.V. Anusha.

Sharon Pronchik
See L. Fern.

James Propp
2001a A reciprocity theorem for domino tilings. *Electronic J. Combin.* 8 (2001), no. 1, Res. paper 18, 5 pp. MR 1855859 (2003e:05032). Zbl 982.05012. arXiv:math/0104011. (**SG: Appl**)

Anton V. Proskurnikov & Ming Cao
2017a Modulus consensus in discrete-time signed networks and properties of special recurrent inequalities. In: *2017 IEEE 56th Annual Conference on Decision and Control (CDC)* (CDC2017, Melbourne, 2017), pp. 2003–2008. IEEE, 2017. (**SG**)

Anton Proskurnikov, Alexey Matveev, & Ming Cao
2014a Consensus and polarization in Altafini's model with bidirectional time-varying network topologies. In: *53rd IEEE Conference on Decision and Control* (Los Angeles, 2014), pp. 2112–2117. IEEE, 2014. (**SG: Lap: Eig, Dyn**)

Anton V. Proskurnikov & Roberto Tempo
2018a A tutorial on modeling and analysis of dynamic social networks. Part II. *Annual Rev. Control* 45 (2018), 166–190. MR 3880766.

§6, "Disagreement via negative influence": Expounds Altafini's (2012a), (2013a) linear model of signed (weighted) (di)graph dynamics. §6.1, "Balance theory" for signed graphs including Altafini's Laplacian. §6.2, "Altafini's model of opinion formation": Static and dynamic, balanced and unbalanced. [Annot. 24 Dec 2018.]

(**PsS: SD, WG: Bal, Dyn: Exp**)

Anton V. Proskurnikov, Roberto Tempo, Ming Cao, & Noah E. Friedkin
2017a Opinion evolution in time-varying social influence networks with prejudiced agents. *IFAC-PapersOnLine* 50 (2017), no. 1, 11896–11901. arXiv:1704.06900.

(**SD: KG: Bal(Gen), Dyn**)

Andrzey Proskurowski
See A.M. Farley.

Alexandre Proutiere
See G.-D. Shi.

J. Scott Provan

1983a Determinacy in linear systems and networks. *SIAM J. Algebraic Discrete Methods* 4 (1983), 262–278. MR 0699780 (84g:90061). Zbl 558.93018. (**QSol, GN**)

1987a Substitutes and complements in constrained linear models. *SIAM J. Algebraic Discrete Methods* 8 (1987), 585–603. MR 0918061 (89c:90072). Zbl 645.90049.
§4: "Determinacy in a class of network models." [Fig. 1 and Thm. 4.7 hint at a possible digraph version of the signed-graph or gain-graph frame matroid.] (**sg?, gg: matrd(bases?): gen**)

Pavel Prozorov
See D. Cherkashin.

Tomasz Przybyłowski
See M. Buckland and B. Kolesnik.

Teresa M. Przytycka & Józef H. Przytycki

1988a Invariants of chromatic graphs. Tech. Rep. No. 88-22, University of British Columbia, Vancouver, B.C., 1988.
Generalizing concepts from Kauffman (1989a). [*Cf.* Traldi (1989a) and Zaslavsky (1992b).] (**SGc: Gen: Invar, Knot**)

1993a Subexponentially computable truncations of Jones-type polynomials. In: Neil Robertson and Paul Seymour, eds., *Graph Structure Theory* (Proc., Seattle, 1991), pp. 63–108. Contemp. Math., Vol. 147. Amer. Math. Soc., Providence, R.I., 1993. MR 1224697 (95c:57016). Zbl 812.57010.
A "chromatic graph" is a graph with edges weighted from the set $Z \times \{d, l\}$, Z being [apparently] an arbitrary set of "colors". A "dichromatic graph" has $Z = \{+, -\}$. Such graphs have general dichromatic polynomials [see Przytycka and Przytycki (1988a), Traldi (1989a), and Zaslavsky (1992b)], as [partially] anticipated by Fortuin and Kasteleyn (1972a). I will not attempt to summarize this paper.
(**SGc: Invar, Knot, Ref**)

Józef H. Przytycki
See K. Murasugi and T.M. Przytycka.

Vlastimil Ptak
See M. Fiedler.

Charles J. Puccia & Richard Levins

1986a *Qualitative Modeling of Complex Systems: An Introduction to Loop Analysis and Time Averaging.* Harvard Univ. Press, Cambridge, Mass., 1986.
(**SD: QM: QSta: Cyc**)

P. Simin Pulat

1989a A decomposition algorithm to determine the maximum flow in a generalized network. *Computers Oper. Res.* 10 (1989), no. 2, 161–172. MR 985143 (90a:90203).

Zbl 674.90023.

Decomposing an $\mathbb{R}_{>0}$-gain graph. [Annot. 8 Jan 2016.] (**gg: Algor**)

William R. Pulleyblank

See J.-M. Bourjolly and M. Grötschel.

G.N. Purohit & Ritu Rani Meherwal

2012a Signed graph partitioning by spectral rounding. *Int. J. Contemp. Math. Sci.* 7 (2012), no. 43, 2117–2124. MR 2980861 (no rev). Zbl 1255.05150. (**SG: WG: Eig, Algor**)

L. Pushpalatha

See E. Sampathkumar.

L. Pyber

See L. Lovász.

Sung-Soo Pyo

See J.-W. Park.

Hao Qi

2019a Critical permutation sets for generalized signed graph coloring. In: *Taipei International Workshop on Combinatorics and Graph Theory* (Taipei, 2019). Academia Sinica, Taipei, 2019. 1 p.

Abstract. Fix Γ and $S \subseteq \mathfrak{S}_k$. Γ is "S-colorable" if every S-permutation gain function φ makes (Γ, φ) $[k]$-colorable. "Critical" means $\forall\ S' \supset S$, $\exists\ \Gamma$ that is S- but not S'-colorable. *Cf.* Jin, Wong, and Zhu (20xxa). [Annot. 29 Jul 2019.] (**GG, SG: Col**)

Hao Qi, Tsai-Lien Wong, & Xuding Zhu

2019a Chromatic number and orientations of graphs and signed graphs. *Taiwanese J. Math.* 23 (2019), no. 4, 767–776. MR 3982060. Zbl 1417.05078. (**SG: Col**)

Jian Qi

See S.W. Tan.

Xingqin Qi, Huimin Song, Jianliang Wu, Edgar Fuller, Rong Luo, & Cun-Quan Zhang

2017a Eb&D: A new clustering approach for signed social networks based on both edge-betweenness centrality and density of subgraphs. *Physica A* 482 (2017), 147–157. MR 3651415 (no rev). (**SG: Clu: Algor**)

Jianguo Qian

See X.-Y. Ren, W. Wang, and W. Wang.

Yi Qian & Sibel Adalı

2013a Extended structural balance theory for modeling trust in social networks. In: Jordi Castellà-Roca *et al.*, eds., *2013 Eleventh Annual Conference on Privacy, Security and Trust* (PST2013, Tarragona, Catalonia, 2013), pp. 283–290. IEEE, 2013. (**SG: Bal: Geom, Dyn**)

Tianyong Qiang

See B. Jiao.

Hongxun Qin

See also J.E. Bonin, P. Brooksbank, T. Dowling, and D.C. Slilaty.

2004a Complete principal truncations of Dowling lattices. *Adv. Appl. Math.* 32 (2004), no. 1-2, 364–379. MR 2037636 (2005e:06003). Zbl 1041.05019.

These matroids are determined by their Tutte polynomials, except that only the order of the group can be determined. (**gg: Matrd: Incid**)

Hongxun Qin, Daniel C. Slilaty, & Xiangqian Zhou

2009a The regular excluded minors for signed-graphic matroids. *Combin. Prob. Computing* 18 (2009), 953–978. MR 2550378 (2010m:05062). Zbl 1231.05063.

The complete list of 31 forbidden minors that are regular matroids. [Annot. 10 Sept 2010.] (**SG: Matrd: Str**)

Qingfeng Qin

See W.-Q. Duan.

Li Qiu

See W. Chen.

Lihong Qiu

See Wang, Qiu, Qian, and Wang (2020a).

Wen-Yuan Qiu

See G. Hu.

Hui Qu

See G.-H. Yu.

Eduardo Queiroga, Anand Subramanian, Rosa Figueiredo, & Yuri Frota

2021a Integer programming formulations and efficient local search for relaxed correlation clustering. *J. Global Optim.* 81 (2021), 919–966. MR 4337982. Zbl 1481.90278.

Dictionary: "balance" = clusterability. (**SD(sg), WG: Clu: Algor**)

Rachel Quinlan

See C. O'Brien.

Louis V. Quintas

See M. Gargano.

James P. Quirk

See also L. Bassett and J.S. Maybee.

1974a A class of generalized Metzlerian matrices. In: George Horwich and Paul A. Samuelson, eds., *Trade, Stability, and Macroeconomics: Essays in Honor of Lloyd A. Metzler*, pp. 203–220. Academic Press, New York, 1974. (**QM: QSta: sd**)

1981a Qualitative stability of matrices and economic theory: a survey article. In: Harvey J. Greenberg and John S. Maybee, eds., *Computer-Assisted Analysis and Model Simplification* (Proc. Sympos., Boulder, Col., 1980), pp. 113–164. Discussion, pp. 193–199. Academic Press, New York, 1981. MR 0617930 (82g:00016) (book). Zbl 495.93001) (book).

Comments by W.M. Gorman (pp. 175–189) and Eli Hellerman (pp. 191–192). Discussion: see pp. 193–196. (**QM: QSta: sd, bal: Exp**)

James Quirk & Richard Ruppert

1965a Qualitative economics and the stability of equilibrium. *Rev. Economic Stud.* 32 (1965), 311–326. **(QM: QSta: sd)**

Aya Rabie

See S. Nada.

[N. Jafari Rad]

See N. Jafari Rad (under "J").

Nicole Radde

2010a Fixed point characterization of biological networks with complex graph topology. *Bioinformatics* 26 (2010), no. 22, 2874–2880. **(SD: Dyn)**

2011a The role of feedback mechanisms in biological network models - a tutorial. *Asian J. Control* 13 (2011), no. 5, 597–610. MR 2860960 (no rev). Zbl 1303.93087. **(SD: Dyn)**

Nicole Radde, Nadav S. Bar, & Murad Banaji

2010a Graphical methods for analysing feedback in biological networks – a survey. *Int. J. Systems Sci.* 41 (2010), no. 1, 35–46. MR 2599706 (no rev). Zbl 1291.92060. **(SD, Biol: Dyn: Exp)**

Filippo Radicchi, Daniele Vilone, & Hildegard Meyer-Ortmanns

2007a Universality class of triad dynamics on a triangular lattice. *Phys. Rev. E* 75 (2007), 021118. **(SG: Bal)**

Filippo Radicchi, Daniele Vilone, Sooeyon Yoon, & Hildegard Meyer-Ortmanns

2007a Social balance as a satisfiability problem of computer science. *Phys. Rev. E (3)* 75 (2007), no. 2, 026106, 17 pp. MR 2354025 (2008g:91190).

Antal, Krapivsky, and Redner (2005a) is generalized to k-cycle dynamics. [Annot. 20 Jun 2011.] **(SG: Bal: Algor)**

Marko Radovanović

See P. Aboulker.

Ljiljana Radović

See Kauffman, Jablan, Radović, and Sazdanović (2013a).

Ileana Rodica Rădulescu

See E. Kaslik.

Arash Rafiey

See J. Bok.

Fahimeh Rahimi

2016a *Covering Graphs and Equiangular Tight Frames.* Master's thesis, Univ. of Waterloo, 2016.

§0.1: Coverings of permutation gain graphs. §0.27, "Two-graphs". *Cf.* Coutinho–Godsil–Shirazi–Zhan (2016a), Godsil (1996a). [Annot. 7 Nov 2020.] **(gg: Cov: Exp)(TG: Cov: Exp)**

Mourad Rahmani

2014a Some results on Whitney numbers of Dowling lattices. *Arab J. Math. Sci.* 20 (2014), no. 1, 11–27. MR 3148044. Zbl 1377.11032. arXiv:1212.0954.

Cf. Mező (2010a), Cheon and Jung (2012a). Many formulas and identities for numbers and polynomials associated with Dowling lattices $Q_n(\mathfrak{G})$ and the r-Whitney and r-Dowling numbers and polynomials of Mező and Cheon–Jung. [Annot. 28 May 2018.] (**gg: matrd: Invar**)

Ashutosh Rai
See G. Philip.

W.M. Raike
See A. Charnes.

Justin Raimondi
See J. Foisy.

M.A. Rajan
See P. Balamuralidhar and H.K. Rath.

K.R. Rajanna
See P.S.K. Reddy.

Rajeevan P
See S. Hameed.

R. Rajendra
See also P.S.K. Reddy.

R. Rajendra, P. Siva Kota Reddy, & V.M. Siddalingaswamy
2018a On some signed graphs of finite groups. *South East Asian J. Math. Math. Sci.* 14 (2018), no. 3, 57–62. MR 3964934. Zbl 1428.05127.
Very elementary. [Annot. 19 Jan 2020.] (**SG: Algeb**)

Supriya Rajendra
See Reshma R.

M.R. Rajesh Kanna, R. Jagadeesh, & Mohammad Reza Farahani
2016a Minimum covering Seidel energy of a graph. *J. Indonesian Math. Soc.* 22 (2016), no. 1, 71–82. MR 3503081. Zbl 1463.05350. (**sg: KG: Adj: Eig**)

M.R. Rajesh Kanna, R. Jagadeesh, & B.K. Kempegowda
2016a Minimum dominating Seidel energy of a graph. *Int. J. Sci. Eng. Res.* 7 (2016), no. 5, 10–14. (**sg: KG: Adj: Eig**)

M.R. Rajesh Kanna, R. Pradeep Kumar, & Mohammad Reza Farahani
2016a Milovanović bounds for Seidel energy of a graph. *Adv. Theor. Appl. Math.* 10 (2016), no. 1, 37–44. (**sg: KG: Adj: Eig**)

M.R. Rajesh Kanna, R. Pradeep Kumar, & R. Jagadeesh
2015a Minimum covering color energy of a graph. *Int. J. Math. Anal.* 9 (2015), no. 8, 351–364.
From Γ define Σ as in Adiga, Sampathkumar, *et al.* (2013a). Fix C a minimum vertex cover of Γ; let I_C have 1 in (i,i) for $v_i \in C$, otherwise 0. Minimum covering color energy $\mathcal{E}^C :=$ energy of $A(\Sigma) + I_C$. [Annot. 22 Dec 2018.] (**sg: Adj: Eig**)

Ramakrishnan K O
See also S. Hameed.

Ramakrishnan K O, Shahul Hameed K, & Roshni T Roy

20xxa Distance Laplacian matrices for conjugate skew gain graphs. (**GG(Gen):Lap(Gen): Eig**)

Harishchandra S. Ramane

See also P.R. Hampiholi.

Harishchandra S. Ramane & Mahadevappa M. Gundloor

2017a More equienergetic signed graphs. *Math. Interdisciplinary Res.* 2 (2017), 169–179. (**SG: Adj: Eig**)

Harishchandra S. Ramane, Mahadevappa M. Gundloor, & Sunilkumar M. Hosamani

2016a Seidel equienergetic graphs. *Bull. Math. Sci. Appl.* 16 (2016), 62–69. (**sg: KG: Adj: Eig**)

Harishchandra S. Ramane, Ivan Gutman, & Mahadevappa M. Gundloor

2015a Seidel energy of iterated line graphs of regular graphs. *Kragujevac J. Math.* 39 (2015), no. 1, 7–12. MR 3361336 (no rev). Zbl 1464.05249. (**sg: LG: KG: Adj: Eig**)

Nacim Ramdani, Nacim Meslem, & Yves Candau

2010a Computing reachable sets for uncertain nonlinear monotone systems. *Nonlinear Analysis: Hybrid Systems* 4 (2010), 263–278. MR 2607907 (2011b:93054). Zbl 1201.93019. (**SD: QM**)

Fahimeh Ramezani

See G. Iacono and N. Soranzo.

Farzaneh Ramezani

See also Y. Bagheri.

2016a Cospectral lifts of graphs. Manuscript, 2016. arXiv:1601.02354.

Cospectrality of covering graphs of (apparently) abelian gain graphs. [Annot. 27 Jul 2022.] (**gg: Cov**)

2018a Constructing signed strongly regular graphs via star complement technique. *Math. Sci.* 12 (2018), no. 3, 157–161. MR 3861443. Zbl 1417.05125. (**SG: Adj, Eig**)

2019a Coloring problem of signed interval graphs. *Trans. Combin.* 8 (2019), no. 4, 1–9. MR 4023079. Zbl 1474.05170. arXiv:1612.03280. (**SG: Col**)

2020a On the signed graphs with two distinct eigenvalues. *Utilitas Math.* 114 (2020), 33–48. MR 4230326. Zbl 1453.05063. arXiv:1511.03511. (**SG: Adj: Eig**)

2022a Some regular signed graphs with only two distinct eigenvalues. *Linear Multilinear Algebra* 70 (2022), no. 3, 517–530. MR 4388842. Zbl 1484.05091. arXiv:1909.06817. (**SG: Adj, Eig**)

20xxa Some non-sign-symmetric signed graphs with symmetric spectrum. Submitted. arXiv:1909.06821. (**SG: Adj, Eig**)

Farzaneh Ramezani, Peter Rowlinson, & Zoran Stanić

2020a On eigenvalue multiplicity in signed graphs. *Discrete Math.* 343 (2020), no. 10, art. 111982, 8 pp. MR 4107756. Zbl 1445.05062. arXiv:1911.01113.

Eigenvalue $\lambda \neq 0, \pm 1$ of multiplicity m satisfies $n \leq \binom{n-m+2}{3}$. [Annot. 30 May 2020.] (**SG: Adj: Eig**)

2022a Signed graphs with at most three eigenvalues. *Czechoslovak Math. J.* 72(147) (2022), no. 1, 59–77. MR 4389106. (**SG: Adj: Eig**)

2022b More on signed graphs with at most three eigenvalues. *Discuss. Math. Graph Theory* 42 (2022), no. 4, 1313–1331. MR 4445854. (**SG: Adj: Eig**)

Farzaneh Ramezani & Zoran Stanić

2021a An upper bound for the Laplacian index of a signed graph. *Discrete Math. Lett.* 5 (2021), 24–28. MR 4227397. (**SG: Lap: Eig**)

2022a Some upper bounds for the net Laplacian index of a signed graph. *Bull. Iranian Math. Soc.* 48 (2022), no. 1, 243–250. MR 4391959. Zbl 1483.05072. (**SG: Lap(Gen): Eig**)

José L. Ramírez

See M.A. Méndez.

A.J. Ramirez-Pastor [Antonio José Ramirez Pastor]

See also F. Romá.

A.J. Ramírez-Pastor, F. Nieto, S. Contreras, & E.E. Vogel

2000a Site order parameters for $\pm J$ Ising lattices. *Physica A* 283 (2000), 94–99. (**SG: Fr, Phys**)

A.J. Ramírez-Pastor, F. Nieto, & E.E. Vogel

1997a Ising lattices with $\pm J$ second-nearest-neighbor interactions. *Phys. Rev. B* 55 (1997), no. 21, 14323–14329.

Randomly signed square lattice with half positive and half negative edges plus randomly signed second-neighbor edges, added according to various schemes and calculated for random examples. Compares properties to 3-dimensional cubic and planar lattices, in particular to the pure square lattice in Vogel, Cartes, Contreras, Lebrecht, and Villegas (1994a) (*q.v.* for dictionary). [Annot. 3 Jan 2015.] (**SG, Phys: Fr, Sw**)

A.J. Ramirez-Pastor, F. Romá, F. Nieto, & E.E. Vogel

2004a Effect of the ground-state structure on order parameters in $\pm J$ Ising lattices. *Physica A* 336 (2004), 454–460. (**SG: State(fr), Sw, Phys**)

R. Rammal

See F. Barahona and I. Bieche.

K. Ranganathan

See R. Balakrishnan.

R. Rangarajan

See also P.S.K. Reddy.

R. Rangarajan & P. Siva Kota Reddy

2008a Notions of balance in symmetric n-sigraphs. *Proc. Jangjeon Math. Soc.* 11 (2008), no. 2, 145–151. MR 2482598 (2010h:05143). Zbl 1205.05102.

S_n is a symmetric n-signed graph. Further definitions as in the notes to Sampathkumar, Reddy, and Subramanya (2008a), (2010c). §2, "Balance in an n-sigraph $S_n = (G, \sigma)$." Prop. 1 (generalizing Harary (1953a) for signed graphs): S_n is balanced iff for each pair $u, v \in V$, every uv-path has the same gain. [The simple proof of $\Longrightarrow$, which depends on the fact that the gain group has exponent 2, is the best I have seen. The proof of $\Leftarrow$ is incorrect.] Prop. 4: Σ_{S_n} is balanced iff $V = V_1 \cup V_2$ such that an edge has identity gain iff it lies within V_1 or V_2. Good proof via min as defined in the cited notes. §3, "Clustering in an n-sigraph $S_n = (G, \sigma)$." S_n is "clusterable" if V has a partition π such that an edge has identity gain iff it lies within a part of π. Prop. 5 generalizes Davis (1967a) to n-signed graphs. §3.1: "Local balance (Local i-balance) in an n-sigraph $S_n = (G, \sigma)$." Prop. 6 generalizes Harary (1955a)) on local balance [with a good proof]. Prop. 8: A complete S_n is balanced iff every triangle on one vertex is balanced. Prop. 9 [incorrect]: The same for imbalance. Prop. 10 gives the number of balanced $S_n = (K_k, \sigma)$ [incorrect; the correct value is $2^{\lceil k/2 \rceil (n-1)}$]. [The results are equally true, *mutatis mutandis*, without assuming symmetry.] [Minor typos require correction.] [Annot. 9 July 2009.] (**SG(Gen), gg: Bal**)

2008b Switching invariant 2-path sigraphs. *mySCIENCE* III (2008), no. 1, 20–25. (**SG: Sw**)

2008c Identity and non-identity graphs on n-sigraphs. *Int. J. Math. Sci. Engg. Appl.* 2 (2008), no. 3, 111–117. Zbl 1210.05058. (**SG(Gen)**)

2009a Notions of balance and consistency on symmetric n-marked graphs. *Bull. Pure Appl. Math.* 3 (2009), no. 1, 1–8. MR 2537685 (2010i:05156). Zbl 1200.05097. (**VS(Gen), SG(Gen), gg: Bal**)

2010a The edge C_4 signed graph of a signed graph. *Southeast Asian Bull. Math.* 34 (2010), 1066–1082. MR 2746741 (2011k:05100). Zbl 1240.05141.

Definitions as at Sampathkumar, Reddy, and Subramanya (2008a), (2010c). The edge C_4 signed graph $E_4(\Sigma) := (V', E', \sigma_S)$ where $V' := E$ and $E' := \{ef : \exists\ C_4 \ni e, f \text{ in } |\Sigma|\}$. Prop. 2.1: $E_4(\Sigma)$ is balanced. Cor. 2.5: $E_4(\Sigma) = E_4(-\Sigma)$. Prop. 2.3: $\Sigma \simeq E_4(\Sigma)$ iff Σ is a balanced signing of C_n, $n \geq 5$. Prop. 3.1: Σ' is an $E_4(\Sigma)$ iff it is balanced and $|\Sigma'|$ is an $E_4(\Gamma)$. [Annot. 2 Aug 2009, 20 Dec 2010.] (**SG: Bal, Sw, LG(Gen)**)

R. Rangarajan, P. Siva Kota Reddy, & N.D. Soner

2009a Switching equivalence in symmetric n-sigraphs. II. *J. Orissa Math. Soc.* 28 (2009), no. 1–2, 1–12. MR 2664129 (2011k:05099). Zbl 1244.05109.

Continuation of Rangarajan, Reddy, and Subramanya (2009a) and Reddy and Prashanth (2009a). Definitions as at Sampathkumar, Reddy, and Subramanya (2008a), (2010c). Φ is a symmetric n-signed graph. Prop. 4: Φ is the $(\leq m)$-distance graph $D_m(\Phi')$ of some Φ' iff it is balanced and $\|\Phi\|$ is a $(\leq m)$-distance graph. [Sufficiency is incorrect.] Solved [possibly incorrectly]: Φ^c or $\Lambda_S(\Phi^c) \simeq D_m(\Lambda_S(\Phi))$; $\Lambda_S(\Phi)$ or $\Lambda_S^2(\Phi) \simeq D_m(\Phi^{[c]})^{[c]}$ (except $\Lambda_S^2(\Phi) \simeq D_m(\Phi^c)^c$). [The results are equally true without requiring symmetry.] [Annot. 3 Aug 2009.] (**SG(Gen), gg: Sw, LG**)

2012a m^{th} Power symmetric n-sigraphs. *Italian J. Pure Appl. Math.* No. 29 (2012), 87–92. MR 3009596. Zbl 1328.05074. (**SG(Gen)**)

R. Rangarajan, P. Siva Kota Reddy, & M.S. Subramanya

2009a Switching equivalence in symmetric n-sigraphs. *Adv. Stud. Contemp. Math. (Kyungshang)* 18 (2009), no. 1, 79–85. MR 2479750 (2011a:05141). Zbl 1183.05033.

Continuation of Reddy, Vijay, and Lokesha (2009a), (2010a). Definitions as at Sampathkumar, Reddy, and Subramanya (2008a). Prop. 4 characterizes $C_E(\Phi)$. Solved: $\Lambda_S(\Phi) \simeq \Phi$; $\Phi \simeq \Lambda_S(\Phi)$; $\Lambda_S(\Phi) \simeq C_E(\Phi)$; $J(\Phi) \simeq C_E(\Phi)$. [The results remain true without assuming symmetry.] [Annot. 2 Aug 2009.] (**SG(Gen), gg: Sw**)

R. Rangarajan, M.S. Subramanya, & P. Siva Kota Reddy

2010a The H-line signed graph of a signed graph. *Int. J. Math. Combin.* 2010 (2010), no. 2, 37–43. Zbl 1216.05052.

H is a connected graph of order ≥ 3. $HL(\Sigma) \subseteq \Lambda_S(\Sigma)$ (defined at Sampathkumar, Reddy, and Subramanya (2010c)); $ef \in E(\Lambda_S(\Sigma))$ is in $HL(\Sigma)$ iff e, f are in a copy of H in $|\Sigma|$. Σ' is an $HL(\Sigma)$ iff it is balanced and $|\Sigma'|$ is an H-line graph. Solved: $HL(\Sigma) \simeq \Sigma$ for $H = C_k, P_k, K_r$ $L(\Sigma) \simeq \Lambda_S(\Sigma)$. Connections with graphs derived from matrices. [Annot. 7 Jan 2011.] (**SG: LG(Gen), Bal, Adj**)

2012a Neighborhood signed graphs. *Southeast Asian Bull. Math.* 36 (2012), no. 3, 389–397. MR 3005090 (no rev). Zbl 1274.05208.

Definitions as at Sampathkumar, Reddy, and Subramanya (2008a), (2010c). The neighborhood signed graph or 2-path graph $P_2(\Sigma)$ is (V, E_2, σ^c) where $E_2 := \{vw : \exists\ vw\text{-path of length } 2\}$. Thm. 5: $P_2(\Sigma)$ is balanced and the signature can be any balanced signature (by appropriate choice of σ). Solved: Σ, $P_2(\Sigma) \simeq \Sigma$; $P_2(\Sigma) \simeq \Sigma^c$; $P_2(\Sigma) \simeq \Lambda_S(\Sigma)$. For connected Σ: $P_2^r(\Sigma) \simeq \Lambda_S(\Sigma)$; $P_2(\Sigma) \simeq J_S(\Sigma)$. Also, $P_2^r(\Sigma) \simeq \Lambda_S^s(\Sigma)$ when $|\Sigma|$ is unicyclic with circle length l and $r, s < l/2$. [Annot. 2 Aug 2009.] (**SG: Bal, Sw, LG(Gen)**)

§5, "$(-1, 0, 1)$-Matrices and neighborhood signed graphs": Given a $(-1, 0, 1)$-matrix A with columns $a_1, \ldots, a_n$. Let $V_A := [n]$, $E_A := \{ij : (\exists\ k)\ a_{ki}a_{kj} \neq 0\}$, and $\sigma_A(ij) := \mu_i\mu_j$ where $\mu_i :=$ product of nonzero entries in a_i. Thm. 20: This signed graph of $A(\Sigma)$ is $P_2(\Sigma)$. [Annot. 10 Apr, 2 Aug 2009.] (**SG: Adj: Bal**)

Athira P Ranjith & Joseph Varghese Kureethara

2020a Sum signed graphs - I. In: Samayan Narayanamoorthy, ed., *Advances In Applicable Mathematics– ICAAM2020* (Coimbatore, India, 2020). AIP Conf. Proc., Vol. 2261, art. 030047, 4 pp.

"Sum signed graph": $\sigma_f(uv) = \operatorname{sgn}(n + \frac{1}{2} - f(u) - f(v))$ for a fixed bijection $f : [n] \to V$. Thms.: $\exists$ negative edge [if $n > 1$]. rna number (*cf.* Acharya and Kureethara (2021a)) $rna(\Sigma) := \min_{f:\sigma_f=\sigma} \#E^- = 1$ for path and star, 2 for circle, $\lfloor n^2/4 \rfloor$ for K_n. [Annot. 13 Dec 2020.] (**Lab: SG**)

A.R. Rao

See Acharya, Joshi, Rao, and Rao (2003a).

Angeline Rao

See V. Chen.

Anita Kumari Rao

See D. Sinha.

M.R. Rao

See Y.M.I. Dirickx.

S.B. Rao

See also B.D. Acharya, P. Das, and [G.R.] Vijaya Kumar.

1984a Characterizations of harmonious marked graphs and consistent nets. *J. Combin. Inform. System Sci.* 9 (1984), 97–112. MR 0959057 (89h:05048). Zbl 625.05049.

A complicated solution, with a polynomial-time algorithm, to the problem of characterizing consistency in vertex-signed graphs (*cf.* Beineke and Harary (1978b)). Thm. 4.1 points out that graphs with signed vertices and edges can be easily converted to graphs with signed vertices only; thus harmony in graphs with signed vertices and edges is characterized as well. [This paper was independent of and approximately simultaneous with B.D. Acharya (1983b), (1984a).] [See Joglekar, Shah, and Diwan (2010a) for the last word.] (**SG, VS: Bal, Algor**)

S.B. Rao, B.D. Acharya, T. Singh, & Mukti Acharya

2005a Graceful complete signed graphs. In: S. Arumugam, B.D. Acharya, and S.B. Rao, eds., *Graphs, Combinatorics, Algorithms and Applications* (Proc. Nat. Conf., Anand Nagar, Krishnankul, India, 2004), pp. 123–124. Narosa Publishing House, New Delhi, 2005.

Extended abstract without proofs. "Graceful" means $(1,1)$-graceful, $r = 1$, as at M. Acharya and Singh (2004a). Thm. 1: (K_n, σ) is graceful iff $n \leq 3$, $n = 4$ and $\#E^- \neq 3$, or $n = 5$ and $\#E^- \neq 5$ is odd and neither Σ^+ nor Σ^- is $K_{1,3}$. The proof involves a recursive labelling procedure. [Annot. 21 July 2010.] (**SGc: Lab**)

S.B. Rao, N.M. Singhi, & K.S. Vijayan

1981a The minimal forbidden subgraphs for generalized line graphs. In: S.B. Rao, ed., *Combinatorics and Graph Theory* (Proc. Sympos., Calcutta, 1980), pp. 459–472. Lect. Notes in Math., Vol. 885. Springer-Verlag, Berlin, 1981. MR 0655644 (83i:05062). Zbl 494.05053.

These are the minimal forbidden induced subgraphs for an all-negative signed simple graph to be the reduced line graph of a signed graph. (**sg: LG, par**)

Vasant Rao

See M. Desai.

Anatol Rapoport

1963a Mathematical models of social interaction. In: R.D. Luce, R.R. Bush, and E. Galanter, eds., *Handbook of Mathematical Psychology*, Vol. II, pp. 493–579. Wiley, New York, 1963.

§3.3, "The theory of structural balance". Nontechnical exposition. [Annot. 12 Nov 2018.] **(PsS: SG: Bal: Exp)**

A.M. Rappoport

See Ya.R. Grinberg.

Thomas Raschle & Klaus Simon

1995a Recognition of graphs with threshold dimension two. In: *Proceedings of the Twenty-Seventh Annual ACM Symposium on the Theory of Computing* (Las Vegas, 1995), pp. 650–661.

Expounded by Mahadev and Peled (1995a), §8.5 (*q.v.*).

(par: ori, Algor)

André Raspaud

See also E. Máčajová, A. Montejano, and J. Nešetřil.

Andre Raspaud & Xuding Zhu

2011a Circular flow on signed graphs. *J. Combin. Theory Ser. B* 101 (2011), 464–479. MR 2832812 (2012j:05191). Zbl 05987722.

Thm. 1: Σ has a nowhere-zero integral and circular [i.e., real] 4-flow if it is edge 4-connected. It has a nowhere-zero circular r-flow with $r < 4$ if it is edge 6-connected. A signed cut D [*cf.* Chen and Wang (2009a)] is described by a signed subset $X = X^+ \dot\cup X^-$ of V. Lemma 3: Σ has a circular r-flow iff it has an orientation such that $1/(r-1) \leq \#\partial^+(X)/\#\partial^-(X) \leq r-1$ for every X. Here $\partial^\varepsilon(X)$ is the set of ends in X, of $e \in D$, that have a certain sign. [Annot. 23 March 2010.]

(SG: Ori, Flows)

Hemant Kumar Rath, M. Rajan, & P. Balamuralidhar

2011a Monotonic signed graph approach for cross-layer congestion control in wireless ad-hoc networks. In: *2011 IEEE GLOBECOM Workshops* (GC Wkshps, Bangalore, 2011), pp. 309–314. IEEE, 2011. **(SD: Appl)**

H. Donald Ratliff

See J.-Cl. Picard.

Alberto Ravagnani

† 2022a Whitney numbers of combinatorial geometries and higher-weight Dowling lattices. *SIAM J. Appl. Algebra Geom.* 6 (2022), no. 2, 156–189. MR 4406906. arXiv:1909.10249.

The first serious computational methods and formulas for characteristic polynomials of higher-weight Dowling lattices (*cf.* Bonin (1993a)). Many results, such as Thm. 5.1: For fixed weight k and size q, the Whitney number w_i for higher ranks n are known in terms of their values for $n < ki$. Also, some explicit formulas for $k = 3$. Etc. [*Problem.* These lattices might be thought of as hypergraphic analogs of frame matroids of complete gain graphs. If that interpretation can be implemented, the polynomials might be chromatic polynomials.] [Related: Koga, Maruta, and Shiromoto (2018a).] [Annot. 22 Jul 2022.] **(Matrd: Invar, gg(Gen))**

Dieter Rautenbach

See also C.V.G.C. Lima.

Dieter Rautenbach & Bruce Reed

2001a The Erdös–Pósa property for odd cycles in highly connected graphs. Paul Erdös and His Mathematics (Budapest, 1999). *Combinatorica* 21 (2001), no. 2, 267–278. MR 1832451 (2002i:05073). Zbl 981.05066.

The smallest sufficient connectivity in Thomassen (2001a) is about $976k$. [For more, see Hochstättler, Nickel, and Peis (2006a).]

(par: Fr: Circ)

K.K. Raval

See U.M. Prajapati.

Bertram H. Raven

See B.E. Collins.

E.V. Ravve

See E. Fischer.

D.K. Ray-Chaudhuri, N.M. Singhi, & G.R. Vijayakumar

1992a Signed graphs having least eigenvalue around -2. *J. Combin. Inform. System Sci.* 17 (1992), 148–165. MR 1216974 (94g:05056). **(SG: Eig, Geom: Exp)**

Abigail Raz

See J. Brown and D. Mallory.

Igor Razgon

See G. Gutin.

Margaret A. Readdy

See also R. Ehrenborg.

2001a The Yuri Manin ring and its $\mathcal{B}_n$-analogue. *Adv. Appl. Math.* 26 (2001), 154–167. MR 1808445 (2001k:13034). Zbl 989.13016.

P. 164: Lattice of signed compositions ("ordered signed partitions"), from Ehrenborg and Readdy (1999a), §6. Pp. 164–165: Signed permutahedron [equivalent to acyclotope of $\pm K_n^\bullet$]. **(Sgnd)(sg: kg: Geom)**

S. Redner

See T. Antal.

A. Sashi Kanth Reddy

See Reddy, Vaidya, and Reddy (2011a).

P. Siva Kota Reddy

See also V. Lokesha, K.N. Prakasha, R. Rajendra, R. Rangarajan, E. Sampathkumar, and M.S. Subramanya.

2010a t-path sigraphs. *Tamsui Oxford J. Math. Sci.* 26 (2010), no. 4, 433–441. MR 2840769 (2012g:05096). Zbl 1236.05097. **(SG)**

2013a Smarandache directionally n-signed graphs — A survey. *Int. J. Math. Combin.* 2013 (2013), no. 2, 34–43. Zbl 1288.05123.

[Smarandache is irrelevant.] *Cf.* Sampathkumar, Reddy, and Subramanya (2010b). **(SG: Gen)**

20xxa Switching invariant t-path sigraphs. Manuscript.

In the t-path signed graph $(\Sigma)_t$, u, v are adjacent when joined by a path of length t, with signature σ^c (see Sampathkumar, Reddy, and Subramanya (2010c)). (The signature differs from that of Gill and Patwardhan

(1986a) and M. Acharya (1988a).) Solved: $\Sigma \simeq (\Sigma)_2$, $(\Sigma)_3$. [Annot. 10 Apr 2009.] **(SG: Sw, LG(Gen))**

20xxb A note on characterization of jump signed graphs. Manuscript. **(SG: LG)**

P. Siva Kota Reddy, M.C. Geetha, & K.R. Rajanna

2011a Switching equivalence in symmetric n-sigraphs–IV. *Scientia Magna* 7 (2011), no. 3, 34–38.

See definitions at Sampathkumar, Reddy, and Subramanya (2008a), (2010c). The antipodal graph $A(\Phi)$ has $V(A) := V$, $E(A) := \{uv : d(u,v) = \max\}$, $\sigma_A(uv) = \mu_\varphi(u)\mu_\varphi(v)$ where $\mu_\varphi =$ canonical vertex labelling. Solved: $A(\Phi) \simeq \Phi$ [trivial], $A(\Phi) \simeq \Phi^c$, etc. [elementary]. [Cors. 3.2, 3.3 are wrongly stated.] [Annot. 13 Jul 2013.] **(SG(Gen): Sw)**

2012a Switching equivalence in symmetric n-sigraphs. V. *Int. J. Math. Combin.* 2012 (2012), no. 3, 58–63. **(SG(Gen): Sw)**

P. Siva Kota Reddy, P.S. Hemavathi, & R. Rajendra

2014a Radial signed digraphs. *Math. Aeterna* 4 (2014), no. 3, 191–196. MR 3214737 (no rev).

Radial digraph of (D, σ) with certain balanced signs. [Annot. 22 Jan 2020.] **(SD, sg: Bal, Sw, VS)**

P. Siva Kota Reddy, V. Lokesha, B. Prashanth, & S. Vijay

2011a Some properties on super line signed graph $\mathcal{L}_3(S)$. *J. Analysis Comput.* 7 (2011), no. 1, 45–48. **(SG: LG(Gen))**

P. Siva Kota Reddy, V. Lokesha, & Gurunath Rao Vaidya

2010a A note on Smarandachely consistent symmetric n-marked graphs. *Int. J. Math. Combin.* 2010 (2010), no. 3, 41–44. Zbl 1238.05120.

[The name Smarandache is used for no reason.] Symmetric n-marking means $\mu : V \to H_n :=$ {symmetric sign n-vectors}. Consistent: product around every circle is $(+, \dots, +)$. Complement: replace μ by $t\mu$ where $t \in H_n$. Trivial results. [Annot. 13 Jan 2023.] **(VS(Gen))**

2010b The line n-sigraph of a symmetric n-sigraph. II. *Proc. Jangjeon Math. Soc.* 13 (2010), no. 3, 305–312. MR 3890295 . Zbl 1223.05109.

Definitions as at Sampathkumar, Reddy, and Subramanya (2008a). **(GG(Gen): LG)**

2010b The line n-sigraph of a symmetric n-sigraph. III. *Int. J. Open Problems Computer Sci. Math.* 3 (2010), no. 5, 172–178.

Definitions as at Sampathkumar, Reddy, and Subramanya (2008a). **(GG(Gen): LG)**

2011a Switching equivalence in symmetric n-sigraphs. III. *Int. J. Math. Sci. Engg. Appl.* 5 (2011), no. 1, 95–101. MR 2791536 (2012a:05141).

Definitions as at Sampathkumar, Reddy, and Subramanya (2008a). **(GG(Gen): Sw)**

P. Siva Kota Reddy & U.K. Misra

2012a The common minimal equitable dominating signed graphs. *Notes Number Theory Discrete Math.* 18 (2012), no. 4, 40–46. **(SG: Dom)**

2013a The equitable associate signed graphs. *Bull. Int. Math. Virtual Inst.* 3 (2013), no. 1, 15–20. **(SG)**

2013b Directionally n-signed graphs-III: The notion of symmetric balance. *Trans. Combin.* 2 (2013), no. 4, 53–62. MR 3150452 (no rev). Zbl 1301.05160.

Directional n-signing: see Sampathkumar–Reddy–Subramanya (2008a). Symmetric balance: the gain of every circle is symmetric. Main result: Thm. 4.1: Φ is symmetically balanced iff every circle has an even number of unsymmetric edge gains. [False. Let $\varphi(v_iv_j) = (a_1, \ldots, a_n)$, $b_i := a_ia_{n+1-i}) = +$, and $\varphi'(v_iv_j) := (b_1, \ldots, b_{\lceil n/2 \rceil})$. Note that φ' is a nondirectional n-signing. Then $\varphi(C)$ is symmetrically balanced iff φ' is a balanced gain graph. This contradicts Thm. 4.1.] [Annot. 3 Feb 2014.] **(GG(Gen): SG(Gen): Bal(Gen))**

2013c Graphoidal signed graphs. *Adv. Stud. Contemp. Math. (Kyungshang)* 23 (2013), no. 3, 451–460. MR 3113160. **(SG)**

2013d Restricted super line signed graph $\mathcal{RL}_r(S)$. *Notes Number Theory Discrete Math.* 19 (2013), no. 4, 86–92.

Cf. Reddy and Vijay (2012a). $\mathcal{RL}_r(\Sigma)$ has vertex set $V(\mathcal{RL}_r) = \mathcal{P}_r(E)$, edge set $E(\mathcal{RL}_r) = \{PQ \in \mathcal{P}_r(E) : \exists!$ adjacent $p \in P,\ q \in Q\}$, and signs $\sigma'(PQ) = \sigma(P)\sigma(Q)$. Solved: $\mathcal{RL}_r(\Sigma) \simeq \mathcal{L}_r(\Sigma)$. Prop. 10, characterizing signed graphs of the form $\mathcal{RL}_r(\Sigma)$, has a wrong proof and appears to be false. [Annot. 30 Apr 2022.] **(SG: LG(Gen))**

2014a Edge neighborhood signed graphs. *Appl. Math. Sci. (Ruse)* 8 (2014), no. 29–32, 1473–1482. MR 3200146 (no rev). **(SG)**

P. Siva Kota Reddy, U.K. Misra, & P. S. Hemavathi

2014a Note on neighborhood signed graphs and jump signed graphs. *Adv. Studies Contemp. Math., Kyungshang* 24, no. 2, 217–226 (2014). **(SG)**

P. Siva Kota Reddy, U.K. Misra, & P.N. Samanta

2013a The minimal equitable dominating signed graphs. *Scientia Magna* 9 (2013), no. 4, 64–70.

The intersection graph of minimal equitable dominating sets P of $|\Sigma|$, with $\operatorname{sgn}(PP') = \mu_\sigma(P)\mu_\sigma(P')$. [Results are trivial or (Prop. 3.1) wrong.] Dictionary: "complement" of Σ is $(|\Sigma|^c, \mu'_\sigma)$ where $\mu'_\sigma(uv) = \mu_\sigma(u)\mu_\sigma(v)$. [Annot. 4 Oct 2019.] **(SG, Bal: Dom)**

P. Siva Kota Reddy, K.M. Nagaraja, & M.C. Geetha

2012a The line n-sigraph of a symmetric n-sigraph. IV. *Int. J. Math. Combin.* 2012 (2012), no. 1, 106–112. Zbl1272.05069. **(SG(Gen): LG)**

2014a The line n-sigraph of a symmetric n-sigraph. V. *Kyungpook Math. J.* 54 (2014), no. 1, 95–101. MR 3190412. Zbl 1295.05118. **(SG(Gen): LG)**

P. Siva Kota Reddy, K.M. Nagaraja, & V.M. Siddalingaswamy

2015a Edge C_k signed graphs. *Int. J. Pure Appl. Math.* 98 (2014), no. 2, 231–238.

$E_k(\Sigma)$ has $V_k = E(\Gamma)$, $ef \in E_k$ iff e, f are adjacent or opposite in a C_k, $\sigma_k(ef) = \sigma(e)\sigma(f)$. Generalizes $k = 3, 4$ Subramanya–Reddy (2009a), Rangarajan–Reddy (2010a). Thm. 7: $\Sigma \sim E_k(\Sigma)$ iff $\Sigma =$ balanced C_n, $n \geq 5$. Minor graph equations solved. [Annot. 2 Jul 2019.] (**SG: LG(Gen): Bal, Sw**)

P. Siva Kota Reddy & Kavita S. Permi

2014a Signed graph equations: $N(\Sigma) \sim CMD$; $CMD(\Sigma) \sim MD(\Sigma)$; $MD(\Sigma) \sim L(\overline{\Sigma})$. *Bull. Int. Math. Virtual Institute* 4 (2014), 27–35.

$N =$ neighborhood signed graph (Rangarajan, Subramanya, and Reddy (2012a)), $(C)MD =$ (common) minimal dominating signed graph (Reddy and Prashanth (2012a), (2013a)). [Annot. 9 Apr 2020.] (**SG**)(**SG: Dom**)

P. Siva Kota Reddy, Kavita S. Permi, & K.R. Rajanna

2012a Combinatorial aspects of a measure of rank correlation due to Kendall and its relation to complete signed digraphs. *Int. J. Math. Combin.* 2012 (2012), no. 1, 74–77. (**SD: Appl**)

P. Siva Kota Reddy & B. Prashanth

2009a Switching equivalence in symmetric n-sigraphs. I. *Adv. Appl. Discrete Math.* 4 (2009), no. 1, 25–32. MR 2555623 (2010k:05122). Zbl 1176.05034.

Continuation of Rangarajan, Reddy, and Subramanya (2009a). Definitions as at Sampathkumar, Reddy, and Subramanya (2008a). Solved for an n-signed graph Φ: $\Lambda_S(\Phi) \simeq \Phi^c$; $\Lambda^k_S(\Phi) \simeq \Phi^c$. [The results remain true without assuming symmetry.] (**SG(Gen): Sw, LG**)

2012a The common minimal dominating signed graph. *Trans. Combin.* 1 (2012), no. 3, 39–46. (**SG**)

2012b $\mathcal{S}$-Antipodal signed graphs. *Tamsui Oxford J. Inform. Math. Sci.* 28 (2012), no. 2, 165–174. (**SG**)

2013a A note on minimal dominating signed graphs. *Int. J. Math. Combin.* 2013 (2013), no. 4, 96–102. (**SG: Dom**)

P. Siva Kota Reddy, B. Prashanth, & T.R. Vasanth Kumar

2011a Antipodal signed digraphs. *Adv. Stud. Contemp. Math. (Kyungshang)* 21 (2011), no. 4, 355–360. MR 2884999 (2012k:05172). Zbl 1250.05054. (**SD**)

P. Siva Kota Reddy, B. Prashanth, & V. Lokesha

2010a Smarandachely t-path step signed graphs. *Scientia Magna* 6 (2010), no. 3, 89–92.

[The name Smarandache is used for no reason.] (**SG**)

P. Siva Kota Reddy, B. Prashanth, & Kavita S. Permi

2011a A note on antipodal signed graphs. *Int. J. Math. Combin.* 2011 (2011), no. 1, 107–112. MR 2829740 (2012d:05173). Zbl 1230.05146.

The antipodal graph A of $|\Sigma|$ with $\sigma_A(uv) = \mu_\sigma(u)\mu_\sigma(v)$, hence balanced. Solved: $\Sigma \sim (A, \sigma_A)$, $-\Sigma \sim (A, \sigma_A)$. [The name Smarandache is used for no reason.] [Annot. 3 Sept 2019.] (**SG: LG(Gen)**)

2011b A note on switching in symmetric n-sigraphs. *Notes Number Theory Discrete Math.* 17 (2011), no. 3, 22–25. Zbl 1250.05055.

Switching multiple signs $\sigma(e) \in \{+,-\}^k$ by signs $\mu(v) \in \{+,-\}$. [Equivalent to restricted switching, i.e., $\mu(v) \in \{\pm(+,\ldots,+)\}$.] Characterized by cutset negation. [Annot. 7 Jan 2011.] (**SG(Gen): Sw**)

P. Siva Kota Reddy, B. Prashanth, & M. Ruby Salestina

2010a Smarandachely antipodal signed digraphs. *Scientia Magna* 6 (2010), no. 3, 84–88.

[The name Smarandache is used for no reason.] (**SD**)

P. Siva Kota Reddy, K.R. Rajanna, & Kavita S. Permi

2013a The common minimal common neighborhood dominating signed graphs. *Trans. Combin.* 2 (2013), no. 1, 1–8. MR 3150480. Zbl 1320.05057. (**SG**)

P. Siva Kota Reddy, R. Rangarajan, & M.S. Subramanya

2011a Switching invariant neighborhood signed graphs. *Proc. Jangjeon Math. Soc.* 14 (2011), no. 2, 249–258. MR 2829740 (2012d:05173). Zbl 1238.05121. (**SG, VS: LG**)

P. Siva Kota Reddy, R. Rajendra, & M.C. Geetha

2016a Boundary n-signed graphs. *Int. J. Math. Sci. Engg. Appl.* 10 (2016), no. II, 161–168. (**SG(Gen)**)

P. Siva Kota Reddy, E. Sampathkumar, & M.S. Subramanya

2010a Common-edge signed graph of a signed graph. *J. Indonesian Math. Soc.* 16 (2010), no. 2, 105–113. MR 2752773 (no rev). Zbl 1236.05098. (**SG**)

P. Siva Kota Reddy, K. Shivashankara, & K.V. Madhusudhan

2010a Negation switching equivalence in signed graphs. *Int. J. Math. Combin.* 2010 (2010), no. 3, 85–90. Zbl 1238.05122.

Solved: $-\Sigma$, $\Lambda^k_\times(\Sigma) \simeq \Lambda^2_\times(\Sigma)$, based on existing solutions for unsigned isomorphism. (See M. Acharya (2009a) for $\Lambda_\times$.) [Annot. 6 Feb 2011.] (**SG: LG, Sw**)

P. Siva Kota Reddy & M.S. Subramanya

2007a A characterization of symmetric 3-sigraphs whose line symmetric 3-sigraphs are switching equivalent. *J. Appl. Math. Anal. Appl.* 3 (2007), no. 1, 23–31. MR 2479512 (2009m:05080). (**SG(Gen): Sw, LG**)

2008a Jump symmetric 3-sigraphs. *Int. J. Phys. Sci.* 20 (2008), no. 2, 431–434. (**SG(Gen): LG**)

2009a Signed graph equation $L^L(S) \sim \bar{S}$. *Int. J. Math. Combin.* 4 (2009), 84–88 (2010). MR 2598675 (no rev). Zbl 1238.05123.

Definitions as at Sampathkumar, Reddy, and Subramanya (2008a). Solved: $\Sigma^c \simeq \Lambda^2_\times(\Sigma)$; $\Lambda^k_\times(\Sigma) \simeq \Sigma^c$. [$\Lambda_\times$ as in M. Acharya (2009a).] [Continued in Reddy, Vijay, and Lokesha (2009a), (2010a)]. [Annot. 3 Aug 2009.] (**SG: Bal, Sw, LG**)

2009b Note on path signed graphs. *Notes Number Theory Discrete Math.* 15 (2009), no. 4, 1–6.

$V(P_k(\Sigma)) :=$ {paths}, $PP' \in E(P_k(\Sigma))$ iff $P \cup P'$ is a path of order $k+1$ or a C_k, $\sigma(PP') = \sigma(P)\sigma(P')$. This is balanced. Solved: $\Sigma \simeq$

$P_3(\Sigma), P_4(\Sigma)$. [Annot. 7 Jan 2011.] **(SG: LG(Gen), Bal)**

P. Siva Kota Reddy, M.S. Subramanya, & R. Rajendra

2007a Symmetric 3-sigraphs. *J. Intelligent Systems Res.* 1 (2007), no. 2, 155–159. **(SG(Gen))**

P. Siva Kota Reddy, Gurunath Rao Vaidya, & A. Sashi Kanth Reddy

2011a Neighborhood symmetric n-sigraphs. *Scientia Magna* 7 (2011), no. 2, 99–105. **(SG(Gen))**

P. Siva Kota Reddy & S. Vijay

2010a Total minimal dominating signed graph. *Int. J. Math. Combin.* 3 (2010), 11–16. Zbl 1238.05124.

The intersection graph M_t of all total minimal dominating sets of $|\Sigma|$ is signed to be balanced using the canonical vertex signature of Σ. Such signed graphs are characterized. $M_t \simeq \Sigma, -\Sigma$ are solved, based on existing solutions for unsigned isomorphism. [Annot. 6 Feb 2011.] **(SG: Dom)**

2011a A note on characterization of jump signed graphs. *Bull. Pure Appl. Math.* 5 (2011), no. 2, 276–277. MR 3027214 (no rev). **(SG: LG)**

2012a The super line signed graph $\mathcal{L}_r(S)$ of a signed graph. *Southeast Asian Bull. Math.* 36 (2012), no. 6, 875–882. MR 3057818.

$V(\mathcal{L}_r(\Sigma)) := \mathcal{P}_r(E)$ with edge $PQ_{e,f} \in E(\mathcal{L}_r(\Sigma))$ for each adjacent $e \in P$, $f \in Q$ and $\sigma_{\mathcal{L}}(PQ_{e,f}) = \sigma(P)\sigma(Q)$. This is balanced. Solved: $\Sigma, \Lambda_\times(\Sigma) \simeq \mathcal{L}_2(\Sigma)$, $\Sigma \cong \mathcal{L}_2(\Sigma)$, *et al.*[*Cf.*Reddy and Misra (2013d).] [Annot. 7 Jan 2011.] **(SG: LG(Gen), Bal)**

P. Siva Kota Reddy, S. Vijay, & V. Lokesha

2009a n^{th} power signed graphs. *Proc. Jangjeon Math. Soc.* 12 (2009), no. 3, 307–313. MR 2582796 (2011e:05109). Zbl 1213.05121.

Definitions and notation as in Sampathkumar, Reddy, and Subramanya (2008a), (2010c).

D_m The "mth power signed graph" Σ^m [I will say "$\leq m$-distance signed graph" $D_m(\Sigma)$] is the graph of distance $\leq m$ in $|\Sigma|$ with signature σ^c. Prop. 5: Σ has the form $D_m(\Sigma')$ iff it is balanced and $|\Sigma|$ is a $(\leq m)$-distance graph. [Sufficiency is incorrect.] Solved [possibly incorrectly]: Σ^c or $\Lambda_\times(\Sigma^c) \simeq D_m(\Lambda_\times(\Sigma))$; $\Lambda_\times(\Sigma)^c \simeq D_m(\Sigma^c)$; $\Lambda^2_\times(\Sigma) \simeq D_m(\Sigma)$, $D_m(\Sigma)^c$, $D_m(\Sigma^c)$. [$\Lambda_\times$ as in M. Acharya (2009a).] [Annot. 12 Apr 2009.] **(SG: Bal, Sw, LG)**

2010a The n^{th} power signed graphs. II. *Int. J. Math. Combin.* 1 (2010), 74–79. MR 2662418 (no rev). Zbl 1207.05074.

Continuing (2009a) with: $\Lambda_\times(\Sigma) \simeq D_m(\Sigma^{[c]})$; $\Lambda_\times(\Sigma)^c \simeq D_m(\Sigma)$. [Annot. 10 Apr 2009.] **(SG: Bal, Sw, LG)**

P. Siva Kota Reddy, S. Vijay, & Kavita S. Permi

2013a Note on equitable signed graphs. *Math. Aeterna* 3 (2013), no. 3, 169–177. MR 3056111 (no rev). **(SG)**

P. Siva Kota Reddy, S. Vijay, & Kavita S Permi

2016a Radial signed graphs. *Math. Combin. Book Ser.* 4 (2016), 128–134.
Radial graph of Σ with certain balanced signs. [Annot. 22 Jan 2020.]
(SG: Bal, Sw, VS)

P. Siva Kota Reddy, S. Vijay, & B. Prashanth

2009a The edge C_4 n-sigraph of a symmetric n-sigraph. *Int. J. Math. Sci. Engg. Appl.* 3 (2009), no. 2, 21–27. **(SG(Gen): LG(Gen))**

P. Siva Kota Reddy, S. Vijay, & H.C. Savithri

2010a A note on path signed digraphs. *Int. J. Math. Combin.* 2010 (2010), vol. 1, 42–46. MR 2662415 (no rev). Zbl 1203.05065. **(SD: LG(Gen))**

Bruce Reed

See also C. Berge, S. Fiorini, J. Geelen, K. Kawarabayashi, and D. Rautenbach.

1999a Mangoes and blueberries. *Combinatorica* 19 (1999), no. 2, 267–296. MR 1723044 (2000i:05160). Zbl 0928.05059.
Given k, $\exists\ f(k)$ such that Γ has a 2-packing of $2k$ odd circles, or $l_0(-\Gamma) \leq f(k)$. [Simpler proof in Kawarabayashi and Reed (2010a).] [Annot. 29 Dec 2020.] **(par: fr)**

Bruce Reed, Kaleigh Smith, & Adrian Vetta

2004a Finding odd cycle transversals. *Oper. Res. Lett.* 32 (2004), 299–301. MR 2057781 (no rev). Zbl 1052.05061.
Algorithm, superseded by faster Kawarabayashi and Reed (2010a). [Annot. 29 Dec 2020.] **(sg: par: fr: Algor)**

P. Reed

See A.J. Bray.

David Rees

1940a On semi-groups. *Proc. Cambridge Philos. Soc.* 36 (1940), no. 4, 387–400. MR 0002893 (2, 127g). Zbl 28.00401 .
A fundamental paper introducing bipartite gain graphs and their switching automorphisms (all not recognized as such) into the foundations of semigroup theory. *Cf.* Graham (1968a), Houghton (1977a), Pollatchek (1977a), Rhodes and Steinberg (2009a). [Annot. 21 Aug 2019.]
(gg: aut, sw: Algeb)

Nathan Reff

See also F. Belardo, L. Duttweiler and O. Kitouni.

2011a New bounds for the Laplacian spectral radius of a signed graph. Manuscript, 2011. arXiv:1103.4629. **(SG: Lap: Eig)**

2012a Spectral properties of complex unit gain graphs. *Linear Algebra Appl.* 436 (2012), no. 9, 3165–3176. MR 2900705. Zbl 1241.05085. arXiv:1110.4554.
$\mathbb{T}$ Complex unit gain graphs have gain group $\mathbb{T} := \{z \in \mathbb{C} : |z| = 1\}$. Bounds on largest and smallest eigenvalues of $A(\Phi)$ (§3, "Eigenvalues of the adjacency matrix") and $L(\Phi)$ (§4, "Eigenvalues of the Laplacian matrix"). Most (except Thm. 4.9, where the edge gains affect the bounds) generalize known bounds for graphs, the signless Laplacian $L(-\Gamma)$, or signed graphs. Some generalizations are not obvious. Lemmas 3.1, 4.1: The spectrum of A or L depends only on the switching class. Lemmas

3.2, 4.2: If Φ is balanced, the spectra are the same as those of $\|\Phi\|$. [*Problem.* Generalize B.D. Acharya (1980a) by proving the converses.] Thm. 5.1: Exact eigenvalues for circle graphs. Thm. 5.4: Lemma 3.1 of Hou, Li, and Pan (2003a) generalized to complex unit gain graphs. [Annot. 30 Oct 2011, rev 20 Jan 2017.] (**GG: Eig, Incid**)

2012b *Gain Graphs, Group-Oriented Hypergraphs, and Matrices.* Doctoral dissertation, Binghamton University, 2012. MR 3078448 (no rev).
(**SG, GG, GH: Adj, Lap, Eig, Incid**)

2014a Spectral properties of oriented hypergraphs. *Electronic J. Linear Algebra* 27 (2014), no. 1, Art. 24, 373–391. MR 3218780. Zbl 1327.05220. arXiv:1506.05054.
(**SH: Ori: Eig**)

2016a Intersection graphs of oriented hypergraphs and their matrices. *Australasian J. Combin.* 65 (2016), 108–123. MR 3490209. Zbl 1344.05103. arXiv:1509.02132.
(**SH: Ori**)

2016b Oriented gain graphs, line graphs and eigenvalues. *Linear Algebra Appl.* 506 (2016), 316–328. MR 3530682. Zbl 1346.05103. arXiv:1506.05059.
(**GG: Ori: Eig, LG**)

Nathan Reff & Lucas J. Rusnak

2012a An oriented hypergraphic approach to algebraic graph theory. *Linear Algebra Appl.* 437 (2012), no. 9, 2262–2270. MR 2954488. Zbl 1247.05164. arXiv:-1506.05051.

Walks, the adjacency matrix and its powers, and the incidence matrix and the Laplacian matrix L of an oriented hypergraph have the same relationships as with graphs. [Annot. 19 Oct 2012.]
(**SH: Ori: Incid, Adj, Eig**)

Nathan Reff & Howard Skogman

2017a A connection between Hadamard matrices, oriented hypergraphs and signed graphs. *Linear Algebra Appl.* 529 (2017), 115–125. MR 3659795. Zbl 1365.05184.
(**SG, SH, Sw**)

Damien Regnault

See M. Noual.

F. Regonati

See E. Damiani.

Guus Regts

See A. Goodall.

Jörg Reichardt & Stefan Bornholdt

2006a Statistical mechanics of community detection. *Phys. Rev. E* 74 (2006), 016110, 14 pp. MR 2276596 (2007h:82089). (**sg: kg: Phys, Clu**)

Philip F. Reichmeider

1984a *The Equivalence of Some Combinatorial Matching Theorems.* Polygonal Publ. House, Washington, N.J., 1984. MR 0781348 (86j:05001). Zbl 562.05020.

Thm. 7.6, p. 107, attributed to Hoffman (1960a) (who credits Heller and Tompkins (1956a)): in effect the incidence matrix of a balanced signed

graph is totally unimodular. König's and Hall's theorems are corollaries, per Hoffman. [Annot. 8 Nov 2015.] (**sg: incid, bal: Exp**)

Talmage James Reid

See also T. Lewis.

Talmage James Reid & Lee Inmon Virden

1997a On rounded five-element lines of matroids. *DiscreteMath.* 163 (1997), 119–127. MR 1428563 (97m:05068). Zbl 873.05032.

The Dowling geometry $Q_3(\mathbb{F}_3^\times)$ is one of two crucial matroids. [Annot. 9 Apr 2016.] (**Matrd: Str: gg**)

Michael Reilly

See O. Coppola.

Gerhard Reinelt

See F. Barahona, C. De Simone, M. Grötschel, and F. Liers.

Victor Reiner

See also P.H. Edelman.

1993a Signed posets. *J. Combin. Theory Ser. A* 62 (1993), 324–360. MR 1207741 (94d:06011). Zbl 773.06008.

They are equivalent to acyclic bidirected graphs. (**Sgnd, sg: Ori: Str, geom**)

1997a Non-crossing partitions for classical reflection groups. *Discrete Math.* 177 (1997), 195–222. MR 1483446 (99f:06005). Zbl 892.06001.

"Noncrossing partitions [actually, partial partitions] of type B" are elements of the Dowling lattice $Q_n(\{+,-\})$ that, regarded as sign-symmetric partitions of $\pm[n]$, are noncrossing when drawn on the circular arrangement $[1,2,\ldots,n,-1,-2,\ldots,-n]$. [Annot. 28 Jan 2015.] (**sg: Matrd**)

Victor Reiner & Dennis Tseng

2014a Critical groups of covering, voltage and signed graphs. *Discrete Math.* 318 (2014), 10–40. MR 3141623. Zbl 1281.05072. arXiv:1301.2977. (**GG, SG: Cov**)

Timothy C. Reluga

See A. Saadatpour.

J.B. Remmel & Michelle L. Wachs

2004a Rook theory, generalized Stirling numbers and $(p;q)$-analogues. *Electronic J. Combin.* 11 (2004), no. 1, art. #R84, 48 pp. MR 2114188 (2005k:05007). Zbl 1065.05018.

§5, "Permutation statistics, colored partitions, and restricted growth functions": Connection with Dowling lattices $Q_n(\mathfrak{G})$. E.g., their $S_{n,k}^{1,j}(1,1) = W_{n-k+1}(Q_n(\mathbb{Z}_j))$. [Annot. 28 May 2018.] (**gg: Matrd: Invar**)

Remna K P

See S. Hameed.

Élisabeth Remy

See also G. Didier and A. Naldi.

Élisabeth Remy & Paul Ruet

2007a On differentiation and homeostatic behaviours of Boolean dynamical systems. In: Corrado Priami, ed., *Transactions on Computational Systems Biology VIII*, pp. 92–101. Lect. Notes in Bioinformatics. Lect. Notes in Computer Sci., Vol. 4780. Springer, Berlin, 2007. MR 2303765. Zbl 1141.92330. (**SD: Dyn**)

2008a [From elementary signed circuits to the dynamics of Boolean regulatory networks.] Manuscript, 2008.
(2008b) under a previous title; often cited as such. (**SD: Dyn**)

2008b From minimal signed circuits to the dynamics of Boolean regulatory networks. *Bioinformatics* 24 (2008), i220–i226. (**SD: Dyn**)

Élisabeth Remy, Paul Ruet, & Denis Thieffry

2006a Positive or negative regulatory circuit inference from multilevel dynamics. In: Christian Commault *et al.*, eds., *Positive Systems* (Proceedings of the second multidisciplinary international symposium on positive systems: Theory and applications, POSTA 06, Grenoble, 2006), pp. 263–270. Lect. Notes in Control and Inform. Sci., Vol. 341. Springer, Berlin, 2006. MR 2250264 (2007e:92032). Zbl 1132.93305. (**SD: Dyn**)

2008a Graphic requirements for multistability and attractive cycles in a Boolean dynamical framework. *Adv. Appl. Math.* 41 (2008), no. 3, 335–350. MR 2449595 (2010j:37018). Zbl 1169.05333. (**SD: Dyn**)

Han Ren

See F.-Y. Cao.

Ling-Zhi Ren

See Y. Liu.

Qing Jun Ren

See also H.S. Du.

2001a A note on the quasi-Laplacian spectra of graphs. (In Chinese.) *J. Nanjing Normal Univ. Nat. Sci. Ed.* 24 (2001), no. 2, 23–25. MR 1849157 (no rev). Zbl 984.05059. (**par: Lap: Eig**)

Xiangyu Ren & Xiaowang Li

2023a Note on a zero net-regular signed graph. *Discrete Appli. Math.* 327 (2023), 28–32. MR 4521314. Zbl 1506.05087.
Thm. 3.1: Connected Σ has $d^{\pm}(v) \equiv 0$ iff $\exists$ alternating Eulerian tour. [Annot. 28 Dec 2022.] (**SG: Str**)

Xiangyu Ren & Jianguo Qian

2019a Flow polynomials of a signed graph. *Electronic J. Combin.* 26 (2019), no. 3, art. 3.37, 16 pp. MR 4014607. Zbl 1419.05095. arXiv:1805.07878.
Builds on DeVos, Rollová, and Šámal (2019a). (**SG: Flows: Invar**)

20xxa On group-valued potential difference and flow in a signed graph. Submitted. (**SG: Flows: Gen(GG)**)

Xiangyu Ren, Jianguo Qian, Sumin Huang, & Junxia Zhang

2022a The odd-valued chromatic polynomial of a signed graph. *Discrete Math.* 345

(2022), no. 11, art. 113042, 10 pp. MR 4440102. (**SG: Hom, Col: Invar**)

Raghunathan Rengaswamy

See M. Bhushan and M.R. Maurya.

Reshma R, Gayathri H, & Supriya Rajendra

2021a On some parameters of parity signed graphs. *Turkish J. Computer Math. Educ.* 12 (2021), no. 13, 1992–1998.

Cf. Acharya–Kureethara–Zaslavsky (2021a). Additional examples, with partial proofs. [Annot. 20 Jun 2021.] (**Lab: SG**)

Enrique Reyes, Christos Tatakis, & Apostolos Thoma

2012a Minimal generators of toric ideals of graphs. *Adv. Appl. Math.* 48 (2012), 64–78. MR 2845507. Zbl 1266.14041.

Cor. 3.3 (restated): A closed walk in Γ is primitive iff its graph is a cactus tree in which each cutpoint is in exactly 2 blocks, the outer blocks are odd circles, and the inner blocks are even circles or isthmi. [Generalization: A closed walk in Σ is primitive iff its graph is a cactus tree in which each cutpoint is in exactly 2 blocks, the outer blocks are negative circles, and the inner blocks are positive circles or isthmi. *Problem*: Prove it.]

Thm. 4.13: Characterizes closed walks that give a minimal binomial of the ideal.

Thm. 4.16 (restated): A closed walk in Γ is fundamental iff it is a minimal walk of an even circle C with at most one chord, which creates two odd circles (i.e., $\Gamma{:}V(C)$ contains no other even circle, equivalently no other frame circuit), or of a handcuff circuit, of $\mathbf{F}(-\Gamma)$. [Generalization: A closed walk in Σ is fundamental iff it is a minimal walk of an positive circle C with at most one chord, which creates two negative circles (i.e., $\Sigma{:}V(C)$ contains no other positive circle, equivalently no other frame circuit), or of a handcuff circuit, of $\mathbf{F}(\Sigma)$. *Problem*: Prove it.] [Annot. 22 Oct 2020.] (**ecyc: Algeb, incid**)

Josephine Reynes

See W. Grilliette and L.J. Rusnak.

Rezvaneh Rezapour

See L. Dinh.

Brendon Rhoades

See D. Armstrong and E. Leven.

John Rhodes

See also S. Margolis.

John Rhodes & Benjamin Steinberg

2009a *The $\mathfrak{q}$-Theory of Finite Semigroups.* Springer Monographs in Math. Springer, Boston, 2009. MR 2472427 (2010h:20132). Zbl 1186.20043.

§4.13.1, "Topology of labeled graphs". §4.13.2. *Cf.* Rees (1940a), Graham (1968a), Houghton (1977a), Pollatchek (1977a).

(**gg: sw, aut: Algeb: Exp**)

Ricardo Riaza

2015a Some qualitative problems in network dynamics. Manuscript, 2015. arXiv:-1501.01904.

Preliminary version of (2017a). §4.5, "Signed graphs". (**SG: Dyn**)

2017a Structure and stability of the equilibrium set in potential-driven flow networks. *J. Math. Anal. Appl.* 454 (2017), no. 1, 335–360. MR 3649856. Zbl 1373.37196. §5.1, "Signed graphs". (**SG: Dyn**)

M. Riazi

See A. Kargaran.

Federico Ricci-Tersenghi

See A.K. Hartmann.

Adrien Richard

See also J. Aracena, J.-P. Comet, and L. Paulevé.

2009a Positive circuits and maximal number of fixed points in discrete dynamical systems. *Discrete Appl. Math.* 157 (2009), no. 15, 3281–3288. MR 2554797 (2010m:05132). Zbl 1193.05085. (**SD: Dyn**)

2010a Negative circuits and sustained oscillations in asynchronous automata networks. *Adv. Appl. Math.* 44 (2010), 378–392. MR 2600786 (2011a:92007). Zbl 1201.37117. (**SD: Dyn**)

2011a Local negative circuits and fixed points in non-expansive Boolean networks. *Discrete Appl. Math.* 159 (2011), no. 11, 1085–1093. MR 2794310 (2012d:94121). Zbl 1253.94082. (**SD: Dyn**)

2015a Fixed point theorems for Boolean networks expressed in terms of forbidden subnetworks. *Theor. Computer Sci.* 583 (2015), 1–26. MR 3337943. Zbl 1310.94244. (**SD: Dyn**)

2018a Fixed points and connections between positive and negative cycles in Boolean networks. *Discrete Appl. Math.* 243 (2018), 1–10. MR 3804734. Zbl 1387.05244. (**SD: Dyn**)

Adrien Richard & Jean-Paul Comet

2007a Necessary conditions for multistationarity in discrete dynamical systems. *Discrete Appl. Math.* 155 (2007), no. 18, 2403–2413. MR 2365052 (2008m:37032). Zbl 1125.37062. (**SD: Dyn**)

2011a Stable periodicity and negative circuits in differential systems. *J. Math. Biol.* 63 (2011), no. 3, 593–600. MR 2824982 (2012h:34096). Zbl 1311.34096. Erratum. *Ibid.* 70 (2015), no. 4, 957–958. MR 3306622. Zbl 1311.34097. (**SD: Dyn**)

Adrien Richard, Jean-Paul Comet, & Gilles Bernot

2005a R. Thomas' modeling of biological regulatory networks: Introduction of singular states in the qualitative dynamics. *Fundamenta Inform.* 65 (2005), 373–392. MR 2348098 (no rev). Zbl 1092.92001. "Regulatory network" = signed digraph. (**SD: Gen: Dyn, Biol**)

2005b Graph-based modeling of biological regulatory networks: Introduction of singular states. In: Vincent Danos *et al.*., eds., *Computational methods in systems biology* (Int. Conf. CMBS 2004, Paris), pp. 58–72. Lect. Notes in Computer Sci., Vol. 3082. Springer, Berlin, 2005. MR 2278705. Zbl 1088.93506.

(SD: Gen: Dyn, Biol)

Adrien Richard & Paul Ruet

2013a From kernels in directed graphs to fixed points and negative cycles in Boolean networks. *Discrete Appl. Math.* 161 (2013), no. 7-8, 1106–1117. MR 3030595. Zbl 1315.05068. **(SD: Dyn)**

Jean Richelle

See also D. Thieffry and R. Thomas.

1979a Comparative analysis of negative loops by continuous, boolean and stochastic approaches. In: René Thomas, ed., *Kinetic Logic: A Boolean Approach to the Analysis of Complex Regulatory Systems* (Proc. EMBO Course, Brussels, 1977), Ch. XIV, pp. 281–325. Lect. Notes in Biomath., Vol. 29. Springer-Verlag, Berlin, 1979.

The dynamics of a negative cycle differ under different models: continuous (differential equations), boolean (2-valued states), and stochastic, with or without the time delays found in biology. [Annot. 4 Aug 2018.] **(sd: Dyn, Biol)**

1980a Analyses booléenne et continue de systèmes comportant des boucles de rétroaction. I. Analyse continue des boucles de rétroaction simples. (In French.) *Acad. Roy. Belg. Bull. Cl. Sci.* (5) 66 (1980), no. 11, 890–912. MR 0637745 (84h:92020a). Zbl 493.92003.

A positive signed digraph cycle allows a stationary state while a negative cycle allows multiple stationary states. Part II: Leclercq and Thomas (1981a). [Annot. 10 Jul 2022.] **(sd: Dyn: Biol)**

Daniel J. Richman

See J.S. Maybee.

R.J. Riddell

1951a *Contributions to the Theory of Condensation.* Dissertation, University of Michigan, Ann Arbor, 1951.

Includes the number of labelled simple 1-trees of order n, i.e., bases of bicircular matroid $\mathbf{F}(K_n, \varnothing)$. [Sequence A057500 in N.J.A. Sloane, *The On-Line Encyclopedia of Integer Sequences*, URL `http://oeis.org/A057500`. [*Cf.* Neudauer, Meyers, and Stevens (2001a).] **(bic: Invar(bases))**

Heiko Rieger

See also M.J. Alava, B. Coluzzi, A.K. Hartmann, N. Kawashima, and J.D. Noh.

1998a Frustrated systems: Ground state properties via combinatorial optimization. In: János Kertész and Imre Kondor, eds., *Advances in Computer Simulation* (Budapest, 1996), pp. 122–158. Lect. Notes in Phys., Vol. 501. Springer-Verlag, Berlin, 1998. Zbl 898.65089. arXiv:cond-mat/9705010. **(Phys, sg: State(fr), Algor: Exp)**

Robert G. Rieper

See J. Chen.

M.J. Rigby

See A.C. Day.

Simone Righi & Károly Takács

2013a Signed networks, triadic interactions and the evolution of cooperation. In: Alex Graudenzi *et al.*, eds., *Proceedings Wivace 2013 - Italian Workshop on Artificial Life and Evolutionary Computation* (Wivace 2013, Milan, 2013), pp. 104–107. Electronic Proc. Theor. Computer Sci., Vol. 130, 2013. arXiv:1309.7698.
Extended abstract. See (2014b). **(SG: Dyn, PsS)**

2014a Triadic balance and closure as drivers of the evolution of cooperation. In: F.J. Miguel Quesada *et al.*, eds., *Advances in Computational Social Science and Social Simulation* (Barcelona, 2014), 4 pp. Social Simulation Conference, Bellaterra, Cataluña, 2014. URL `http://ddd.uab.cat/record/125597`
Extended abstract. See (2014b). **(SG: Dyn, PsS)**

2014b Emotional strategies as catalysts for cooperation in signed networks. *Adv. Complex Systems* 17 (2014), no. 2, art. 1450011, 23 pp. MR 3224041. arXiv:-1401.1996.
Random pairs of adjacent nodes (= persons) of Σ play Prisoner's Dilemma. Each node is a Cooperator, Defector, or Conditional player who cooperates iff the edge is positive. Σ and (in some papers) the node strategies evolve depending on the outcome of each round, which is either a single play (dyadic) or a round robin in a triangle (triadic). Some conditions evolve into universal cooperation, some into universal defection; conditions for each outcome are explored in these articles. [Annot. 8 Jan 2016.] **(SG: Dyn, PsS)**

2014c Parallel versus sequential update and the evolution of cooperation with the assistance of emotional strategies. Manuscript, 2014. arXiv:1401.4672.
See (2014b). **(SG: Dyn, PsS)**

2014d Degree variance and emotional strategies catalyze cooperation in dynamic signed networks. In: Flaminio Squazzoni *et al.*, eds., *Proceedings 28th European Conference on Modelling and Simulation* (ECMS 2014, Brescia), 7 pp. European Council for Modelling and Simulation (ECMS), 2014. arXiv:1404.6097.
See (2014b). **(SG: Dyn, PsS)**

[Arnout van de Rijt]
See A. van de Rijt (under 'V').

James E. Riley

1969a An application of graph theory to social psychology. In: G. Chartrand and S.F. Kapoor, eds., *The Many Facets of Graph Theory* (Proc., Kalamazoo, 1968), pp. 275–280. Lect. Notes in Math., Vol. 110. Springer, 1969. MR 0252266 (40 #5487). Zbl 198.52802. **(PsS: SG: Exp)**

Chong S. Rim
See H. Choi.

Giovanni Rinaldi
See C. De Simone, F. Liers, and J. Lukic.

R.D. Ringeisen
See also M.J. Lipman.

1974a Isolation, a game on a graph. *Math. Mag.* 47 (May, 1974), no. 3, 132–138. MR 0373642 (51 #9842).

Mathematical examination of a game based on switching vertices until a player creates an isolated vertex and wins. [Annot. 26 May 2017.] **(tg: Sw)**

Gerhard Ringel

See also N. Hartsfield and M. Jungerman.

1974a *Map Color Theorem.* Grundl. math. Wiss., B. 209. Springer-Verlag, Berlin, 1974. MR 0349461 (50 #1955). Zbl 287.05102.

"Cascades" (§8.3): see Youngs (1968a).

[Nonorientable embedding of K_n means an orientation embedding (*cf.* Zaslavsky (1992a)) of some signed K_n. *Question* (minor). What signs? For K_{16}, $\Sigma^- = K_{8,8}$ with C_8 in each side of the bipartition (found with S.-X. Lyu).] [Annot. 8 Oct 2016.] **(sg: Ori: Appl)**

1974b *Teorema o Raskraske Kart.* Transl. V.B. Alekseev. Ed. G.P. Gavrilov. "Mir", Moskva, 1977. MR 0465923 (57 #5809). Zbl 439.05019.

Russian translation of (1974a). **(sg: Ori: Appl)**

1977a The combinatorial map color theorem. *J. Graph Theory* 1 (1977), 141–155. MR 0444509 (56 #2860). Zbl 386.05030.

Signed rotation systems for graphs. Thm. 12: Signed rotation systems describe all cellular embeddings of a graph; an embedding is orientable iff its signature is balanced. *Cf.* Stahl (1978a). Dictionary: "Triple" = graph with signed rotation system. "Orientable" triple = balanced signature. "Oriented" = all positive. **(SG: Top, Sw)**

Oliver Riordan

See B. Bollobás.

S. Risau-Gusman

See F. Romá.

F. Ritort

See E. Marinari.

Vincent Rivasseau

See T. Krajewski.

Nicolas Rivier

1990a Gauge theory and geometry of condensed matter. In: J.F. Sadoc, ed., *Geometry in Condensed Matter Physics*, Ch. I, pp. 1–88. World Scientific, Singapore, 1990. MR 1090337 (no rev).

Engaging if physics-intensive exposition of continuous and discrete gauge transformations via fiber bundles. E.g., mixed Ising models (= signed graphs Σ) with discrete ± 1 or continuous S^3 spins. "Odd ring" defects (p. 21) treated via all-negative graph $-\Gamma$; "odd lines" (pp. 22 ff.) = negative paths. $\mathbb{Z}_2$ is implicit in sign of gauge-invariant physical configuration (§3.3), explicit in magnetization (p. 38). §4, "Gauge invariance in discrete space". §4.1, "Discrete gauge invariance in spin glasses": Ising spin glass = general Σ. $\mathbb{Z}_2$ gauge transformation (switching) "cannot be meaningfully generalized to ... XY or Heisenberg ... spins [*Question.* Is that true?] because [signs ± 1] are real numbers". §4.2.1, "Potential valleys in configuration space": "The configuration space is a direct product of ... odd [= negative?] line[s]." "Tunnelling" between valleys (regions closer to ground state potential). §4.2.2, "Elasticity of random networks": Fiber bundle = covering graph of permutation gain graph with $\mathfrak{S}_N$ action. Fig. 6: Signed graph on torus, cut along negative edges to become planar all-positive. §5.2, "Theory of surfaces": ¶¶1–3 must be read. §7, "Conclusions": Ising spin glass (i.e., Σ), p. 82. [Annot. 7 Aug 2018.] (**SG, gg: VS: Phys, Bal, Fr: Exp**)

Romeo Rizzi

2001a On 4-connected graphs without even cycle decompositions. *Discrete Math.* 234 (2001), 181–186. MR 1826832 (2001m:05156). Zbl 983.05067.

On decomposing $-\Gamma$ into positive circles. *Cf.* C.-Q. Zhang (1994a), Markström (2012a), Máčajová and Mazák (2013a), Liu, Cui, and Wang (2022a), etal [Annot. 13 Aug 2013.] (**Par: Str: Circ**)

María Robbiano

See also N.M.M. de Abreu and I. Gutman.

María Robbiano, Katherine Tapia Morales, & Bernardo San Martín

2016a Extremal graphs with bounded vertex bipartiteness number. *Linear Algebra Appl.* 493 (2016), 28–36. MR 3452724. Zbl 1329.05198.

They find simple Γ with "vertex bipartiteness number" $l_0(-\Gamma) \leq k$ that maximize spectral radii of $A(-\Gamma), L(-\Gamma)$. [*Problem*: Generalize to frustration number of Σ.] [*Cf.* Liu and Wang (2017a).] [Annot. 8 May 2017.] (**sg: Par: Fr, Eig**)

Jakayla R. Robbins

2003a *On Orientations of the Free Spikes.* Doctoral dissertation, University of Kentucky, 2003. MR 2704379 (no rev). (**gg: Matrd: Invar**)

2007a Enumerating orientations of the free spikes. *European J. Combin.* 28 (2007), 868–875. MR 2300767 (2007k:05047). Zbl 1112.05022.

The number of orientations of the free spike matroid $\mathbf{L}(2C_n, \varnothing)$ is $2^{n-1}D_n$, $D_n :=$ Dedekind number. [Annot. 29 Sept 2011.] (**gg: Matrd: Invar**)

2008a Representable orientations of the free spikes. *Discrete Math.* 308 (2008), no. 22, 5174–5183. MR 2450452 (2009h:05052). Zbl 1157.05020.

In general, not all orientations of the free spike matroid $\mathbf{L}(2C_n, \varnothing)$ have a real vector representation. Also, bounds on the number of representable orientations. [Annot. 29 Sept 2011.] (**gg: Matrd: Geom, Invar**)

Jakayla Robbins, Daniel Slilaty, & Xiangqian Zhou

2016a Clones in 3-connected frame matroids. *Discrete Math.* 339 (2016), 1329–1334. MR 3442541. Zbl 1329.05060. (**gg: Matrd**)

Fred S. Roberts

See also T.A. Brown and R.Z. Norman.

1974a Structural characterizations of stability of signed digraphs under pulse processes. In: Ruth A. Bari and Frank Harary, eds., *Graphs and Combinatorics* (Proc. Capital Conf., Geo. Washington Univ., 1973), pp. 330–338. Lect. Notes in Math., Vol. 406. Springer-Verlag, Berlin, 1974. MR 0360342 (50 #12792).

Zbl 302.05107. (**SDw**)

1976a *Discrete Mathematical Models, With Applications to Social, Biological, and Environmental Problems.* Prentice-Hall, Englewood Cliffs, N.J., 1976. Zbl 363.90002.

§3.1: "Signed graphs and the theory of structural balance." Many topics are developed in the exercises. Exercise 4.2.7 (from Phillips (1967a)).
(**SG, SD: Bal, Algor, Adj, Clu, Fr, PsS: Exp, Exr**)

Ch. 4: "Weighted digraphs and pulse processes." Signed digraphs here are treated as unit-weighted digraphs. Note esp.: §4.3: "The signed or weighted digraph as a tool for modelling complex systems." Conclusions about models are drawn from very simple properties of their signed digraphs. §4.4: "Pulse processes." §4.5: "Stability in pulse processes." Stability is connected to eigenvalues of $A(\Sigma)$.
(**SDw, SD, WD: Bal, Eig, PsS: Exp, Exr, Ref**)

1978a *Graph Theory and Its Applications to Problems of Society.* CBMS-NSF Regional Conf. Ser. in Appl. Math., 29. Soc. Indust. Appl. Math., Philadelphia, 1978. MR 0508050 (80g:90036). Zbl 452.05001.

Ch. 9: "Balance theory and social inequalities." Ch. 10: "Pulse processes and their applications." Ch. 11: "Qualitative matrices."
(**SG, SD, SDw: Bal, PsS, QM: Exp, Ref**)

1979a Graph theory and the social sciences. In: Robin J. Wilson and Lowell W. Beineke, eds., *Applications of Graph Theory*, Ch. 9, pp. 255–291. Academic Press, London, 1979. MR 0567125 (81h:05050) (book). Zbl 444.92018.

§2: "Balance and clusterability." Basics in brief. §7: "Signed and weighted digraphs as decision-making models." Cursory.
(**SG, PsS, SD, SDw: Bal, Clu, KG: Exp, Ref**)

1986a *Diskretnye matematicheskie modeli s prilozheniyami k sotsialnym, biologicheskim i ekologicheskim zadacham.* Transl. A.M. Rappoport and S.I. Travkin. Ed. and preface by A.I. Teĭman. Teoriya i Metody Sistemnogo Analiza [Theory and Methods of Systems Analysis]. "Nauka", Moscow, 1986. MR 0889897 (88e:00020). Zbl 662.90002.

Russian edition of (1976a).
(**SG, SD: Bal, Algor, Adj, Clu, Fr, PsS: Exp, Exr**)
(**SDw, SD, WD: Bal, Eig, PsS: Exp, Exr, Ref**)

1989a Applications of combinatorics and graph theory to the biological and social sciences: Seven fundamental ideas. In: Fred Roberts, ed., *Applications of Combinatorics and Graph Theory to the Biological and Social Sciences*, pp. 1–37. IMA Vols. Math. Appl., Vol. 17. Springer-Verlag, New York, 1989. MR 1009370 (91c:92001).

§4: "Qualitative stability." A fine, concise basic survey.
(**QM: SD: Exp, Ref**)

§5: "Balanced signed graphs." Another concise basic survey, and two open problems (p. 20). (**SG: Bal: Exp, Ref**)

1995a On the problem of consistent marking of a graph. *Linear Algebra Appl.* 217 (1995), 255–263. MR 1322554 (95k:05157). Zbl 830.05059.

Several characterizations of consistent vertex signatures of a graph. Γ is "markable" iff it has a consistent vertex signature that is not all $+$. Thm.:

3-connected Γ is markable iff it is bipartite. Thm.: A classification of markable 2-connected graphs with girth ≤ 5. [See Hoede (1992a).] [Annot. 27 Apr 2009.] (**VS: Bal**)

1999a On balanced signed graphs and consistent marked graphs. Ordinal and Symbolic Data Analysis (OSDA '98, University of Massachusetts, Amherst, Mass., 1998). *Electronic Notes Discrete Math.* 2 (1999), 12 pp. MR 1990307 (2004e:-05088). Zbl 971.68115.

A survey of balance in signed graphs and consistency in vertex-signed graphs and their supposed applications in social psychology and elsewhere. Results from Xu (1998a) and Roberts and Xu (2003a). §4: "Connections among balance, consistency, and other graph-theoretical notions". Lists some special and general equivalences, esp., with bipartiteness, or with all circle lengths divisible by 4. §5: "Coherent paths". Characterizations of consistency or balance from Beineke and Harary (1978b), Roberts and Xu (2003a), Acharya (1983a), Rao (1984a). §6: "Fundamental cycles and cycle bases". Hoede's (1992a) characterization of consistency, a variant, and one from Roberts and Xu (2003a) in terms of a circle basis. §7: "Markable graphs". Γ is "markable" iff it has a consistent vertex signature that is not all +. Thm. (Roberts). 3-connected Γ is markable iff it is bipartite. See Roberts (1995a) and S. Xu (1998a). §8: "Open questions". [Annot. 27 Apr 2009.]
(**SG, VS: Bal, PsS, Appl: Exp**)

2001a Discrete mathematics. In: Neil J. Smelser and Paul B. Baltes, eds., *International Encyclopedia of the Social & Behavioral Sciences*, pp. 3743–3746. Pergamon (Elsevier), 2001.

§5: "Signed and marked graphs". (**SG, VS, PsS: Exp**)

Fred S. Roberts & Thomas A. Brown

1975a Signed digraphs and the energy crisis. *Amer. Math. Monthly* 82 (June–July, 1975), no. 6, 577–594. MR 0368957 (51 #5195). Zbl 357.90070. (**SD, SDw**)

1977a [Reply to Waterhouse (1977a)]. *Amer. Math. Monthly* 84 (1977), 27.

Fred S. Roberts & Shaoji Xu

2003a Characterizations of consistent marked graphs. 1998 Conf. Ordinal and Symbolic Data Analysis (OSDA '98) (Amherst, Mass.). *Discrete Appl. Math.* 127 (2003), 357–371. MR 1984094 (2004b:05097). Zbl 1026.05054.

Several characterizations of consistent vertex-signed graphs, and algorithms to determine consistency, are surveyed or proved. Thm.: A vertex-signed graph is consistent iff every circle in some circle basis is positive and every two 3-connected vertices have the same sign. [Annot. 26 Apr 2009.] (**SG, VS: Bal, Algor**)

Edmund Robertson

See P. Brooksbank.

Neil Robertson, P.D. Seymour, & Robin Thomas

See also W. McCuaig and J. Maharry.

† 1999a Permanents, Pfaffian orientations, and even directed circuits. *Ann. of Math.* (2) 150 (1999), no. 3, 929–975. MR 1740989 (2001b:15013). Zbl 947.05066.

Question 1. Does a given digraph D have an even cycle? Question 2. Can a given digraph D be signed so that every cycle is negative? (These problems are easily seen to be equivalent.) The main theorem (the "Even Dicycle Thm.") is a structural characterization of digraphs that have a signing in which every cycle is negative. (These were previously characterized by forbidden minors in Seymour and Thomassen (1987a).)

The main theorem is proved also in McCuaig (2004a). See the joint announcement, McCuaig, Robertson, Seymour, and Thomas (1997a). (**SD: par: Str: Cyc**)

Garry Robins & Yoshi Kashima

2008a Social psychology and social networks: Individuals and social systems. *Asian J. Social Psychology* 11 (2008), 1–12.

Pp. 9–10: a critical review of signed-graph balance theory in social psychology. [Also see that of R.C. Roistacher1974aRichard C. Roistacher.] [Annot. 21 Aug 2014.] (**PsS: SG**)

Ellen Robinson

See L. Rusnak and G. Chen.

Herbert A. Robinson

See C.R. Johnson.

Robert W. Robinson

See also Harary, Palmer, Robinson, and Schwenk (1977a) and Harary and Robinson (1977a).

1981a Counting graphs with a duality property. In: H.N.V. Temperley, ed., *Combinatorics* (Proc. Eighth British Combinatorial Conf., Swansea, 1981), pp. 156–186. London Math. Soc. Lect. Note Ser., 52. Cambridge Univ. Press, Cambridge, England, 1981. MR 0633654 (83c:05071). Zbl 462.05035.

The "bilayered digraphs" of §7 are identical to simply signed, loop-free digraphs (where multiple arcs are allowed if they differ in sign or direction). Thm. 1: Their number b_n = number of self-complementary digraphs of order $2n$. Cor. 1: Equality holds if the vertices are signed and k-colored. In §8, Cor. 2 concerns vertex-signed and 2-colored digraphs; Cor. 3 concerns vertex-signed tournaments. Assorted remarks on previous signed enumerations, mainly from Harary, Palmer, Robinson, and Schwenk (1977a), are scattered about the article. (**SD, VS, SG: Enum**)

Paul Rochet

See P.-L. Giscard.

G.J. Rodgers & A.J. Bray

1988a Density of states of a sparse random matrix. *Phys. Rev. B* 37 (1988), no. 7, 3557–3562. MR 0932406 (89d:82054).

Physical quantities via spectrum of random $A(\Sigma)$ with $n \to \infty$ assuming expected degree $d_\Sigma(v) \sim pn$ where $0 < p < 1$. [Annot. 29 Dec 2012.] (**Phys: sg: Rand: Eig**)

G.J. Rodgers & C. De Dominicis

1990a Density of states of sparse random matrices. *J. Phys. A* 23 (1990), 1567–1573. MR 1048785 (91a:82043).

§3, "Solutions": Random $A(\Sigma)$ where Σ is sparse. [Annot. 29 Dec 2012.] (**Phys: sg: Rand: Eig**)

Y. Roditty

See I. Krasikov.

Vojtěch Rödl

See R.A. Duke.

Jose Antonio Rodriguez

See R.T. Boesch.

Elisabeth Rodríguez-Heck

2018a *Linear and Quadratic Reformulations of Nonlinear Optimization Problems in Binary Variables*. Doctoral thesis, Liége Université, 2018. (**SH, SG: Bal, Geom**)

2019a Linear and quadratic reformulations of nonlinear optimization problems in binary variables. *4OR* 17 (2019), no. 2, 221–222.

Abstract of (2018a). (**SH, SG: Bal, Geom**)

Juan A. Rodríguez-Velázquez

See E. Estrada.

Vladimir Rogojin

See A. Alhazov.

Richard C. Roistacher

1974a A review of mathematical methods in sociometry. *Sociological Methods Res.* 3 (1974), no. 2, 123–171.

Pp. 20–22: a critical review of the use of signed graphs in social psychology. [Also see that of Robins and Kashima (2008a).] [Annot. 21 Aug 2014.] (**PsS: SG**)

Oscar Rojo

See also I. Gutman and G. Pastén.

2009a Spectra of copies of a generalized Bethe tree attached to any graph. *Linear Algebra Appl.* 431 (2009), 863–882. MR 2535558 (2011c:05209). Zbl 1168.05328. (**par: Adj: Eig**)

2011a Line graph eigenvalues and line energy of caterpillars. *Linear Algebra Appl.* 435 (2011), 2077–2086. MR 2810648 (2012e:05243). Zbl 1222.05177. (**par: LG: Adj: Eig**)

Oscar Rojo & Raúl D. Jiménez

2011a Line graph of combinations of generalized Bethe trees: Eigenvalues and energy. *Linear Algebra Appl.* 435 (2011), no. 10, 2402–2419. MR 2811125 (2012f:05185). Zbl 1222.05178. (**par: Adj: Eig, LG**)

Oscar Rojo & Luis Medina

2010a Spectral characterization of some weighted rooted graphs with cliques. *Linear Algebra Appl.* 433 (2010), no. 7, 1388–1409. MR 2680266 (2011h:05163). Zbl 1194.05095. (**par: Adj: Eig**)

Edita Rollová

See also M. DeVos, T. Kaiser, E. Máčajová, and R. Naserasr.

Edita Rollová, Michael Schubert, & Eckhard Steffen

2014a Signed graphs with two negative edges. In: *Bordeaux Graph Workshop 2014*, pp. 41–42. LaBRI, Bordeaux, 2014.
URL http://bgw.labri.fr/2014/bgw2014-booklet.pdf

Extended abstract of (2018a). [Annot. 19 Mar 2017.] (**SG: Flows**)

2018a Flows in signed graphs with two negative edges. *Electronic J. Combin.* 25 (2018), no. 2, art. #P2.40, 18 pp. MR 3814274. Zbl 1388.05079. arXiv:-1604.08053. (**SG: Flows**)

F. Romá

See also A.J. Ramírez-Pastor.

F. Romá, F. Nieto, A.J. Ramirez-Pastor, & E.E. Vogel

2005a Thermodynamic integration method applied to $\pm J$ Ising lattices. *Physica A* 348 (2005), 216–222.

A signed toroidal square lattice with random signs. Analytical and numerical methods to study functions of $x := \#E^+/\#E$ such as proportion of frustrated (negative) plaquettes (face boundaries). (*Cf.* other papers of the authors.) [Annot. 3 Jan 2015.] (**SG, Phys: State**)

2006a Novel order parameter to describe the critical behavior of Ising spin glass models. *Physica A* 363 (2006), 327–333. Zbl 1073.82565. arXiv:cond-mat/0506767.

Assuming (with little loss of generality) equally many edges of each sign, randomly distributed, in a square lattice. [Annot. 3 Jan 2015.] (**Phys: SG: Rand: State, Sw**)

F. Romá, F. Nieto, E. Vogel, & A.J. Ramirez-Pastor

2004a Ground-state entropy of $\pm J$ Ising lattices by Monte Carlo simulations. *J. Stat. Phys.* 114 (2004), no. 5-6, 1325-1341. Zbl 1073.82565. (**Phys: SG: Rand: State**)

F. Romá, S. Risau-Gusman, A.J. Ramirez-Pastor, F. Nieto, & E.E. Vogel

2009a The ground state energy of the Edwards–Anderson spin glass model with a parallel tempering Monte Carlo algorithm. *Physica A* 88 (2009), 2821–2838. arXiv:0806.1054.

The efficiency of a Monte Carlo method for finding ground state energies *et al.*, with either random pure signs (i.e., $\pm J$ with $J = 1$, half the edges having each sign) or Gaussian randomly weighted random signs. [Annot. 3 Jan 2015.] (**SG: Fr, Sw, Phys: Algor**)

S.V. Roopa

See E. Sampathkumar.

P. Lawrence Rozario Raj & R. Lawrence Joseph Manoharan

2016a Face and total face signed product cordial labeling of planar graphs. *Asia Pacific J. Res.* 1 (2016), no. 41, 16–23.

See Baskar Babujee and Shobana (2011a). For a face of plane Γ, $\zeta^{**}(F) :=$ product of boundary vertex signs. ζ is a "face signed product cordial labeling" if signed product cordial and $\#\zeta^{**-1}(+) \approx \#\zeta^{**-1}(-)$. ζ is a "total face signed product cordial labeling" if total # positive vertices, edges, and faces $\approx$ # negative ones. $K_{1,1,n-2}$, $n \geq 5$, and another plane family are both. [Annot. 26 Dec 2020.] **(Lab: VS: SG, Bal)**

Frances Rosamond
See H.L. Bodlaender.

Milton J. Rosenberg
See also R.P. Abelson.

Milton J. Rosenberg & Robert P. Abelson

1960a An analysis of cognitive balancing. In: Milton J. Rosenberg *et al.*, eds., *Attitude Organization and Change: An Analysis of Consistency Among Attitude Components*, Ch. 4, pp. 112–163. Yale Univ. Press, New Haven, 1960.

An attempt to test structural balance theory experimentally. The test involves, in effect, a signed K_4 [an unusually large graph for such an experiment]. Conclusion: there is a tendency to balance but it competes with other forces. **(PsS: SG: kg)**

Seymour Rosenberg

1968a Mathematical models of social behavior. In: Gardner Lindzey and Elliot Aronson, eds., *The Handbook of Social Psychology*, second ed., Vol. 1, Ch. 3, pp. 179–244. Addison-Wesley, Reading, Mass., 1968.

"Balance model," pp. 196–199. "Congruity model," pp. 199–203. **(PsS, SG: Bal: Exp, Ref)**

M. Rosenfeld
See W.D. McCuaig.

Elissa Ross

2011a *Geometric and Combinatorial Rigidity of Periodic Frameworks as Graphs on the Torus.* Doctoral dissertation, York University, 2011. MR 2941979 (no rev).

Extending Whiteley (1988a). *Cf.* (2014a), (2015a). **(GG: sw, Cov, Top, Geom)**

2014a The rigidity of periodic frameworks as graphs on a fixed torus. *Contrib. Discrete Math.* 9 (2014), no. 1, 11–45. MR 3265750. Zbl 1317.52028. arXiv:1202.6652.

$\mathbb{Z}^d$-gain graphs. Dictionary: "net gain" = gain of closed walk, "T-gain procedure" = switching so $\varphi^\zeta|_T \equiv 0$, "local gain group" = $\langle \operatorname{Im} \varphi^\zeta \rangle$, "derived graph" = gain covering graph. **(GG: Geom, sw, Cov)**

2015a Inductive constructions for frameworks on a two-dimensional fixed torus. *Discrete Comput. Geom.* 54 (2015), 78–109. MR 3351759. Zbl 1327.52038. arXiv:-1203.6561.

$\mathbb{Z}^2$-gain graphs. Dictionary: (2014a). **(GG: Geom, sw, Cov)**

Philippe A. Rossignol
See J.M. Dambacher.

Gian-Carlo Rota
See P. Doubilet.

Günter Rote
See H. Edelsbrunner.

Ron M. Roth & Krishnamurthy Viswanathan

2007a On the hardness of decoding the Gale–Berlekamp code. In: *Information Theory, 2007* (ISIT 2007, IEEE Int. Symp., Nice, 2007), pp. 1356–1360. IEEE Press, 2007.

See (2008a). **(sg: fr, Algor)**

2008a On the hardness of decoding the Gale–Berlekamp code. *IEEE Trans. Inform. Theory* 54 (2008), no. 3, 1050–1060. MR 2445050 (2010h:94267).

Section III, "Relaxed problem": The frustration index of a bipartite signed graph is NP-complete. Thm. 4.1: The frustration index of a signed $K_{n,n}$ (where n is a variable) is NP-complete. The proofs use the bipartite adjacency matrix of the signed graph. The latter problem is polynomially reduced to the former by a construction using Kronecker product and a Hadamard matrix. The problems are interpreted as nearest-neighbor decoding of the Gale–Berlekamp code of order n.

Section V, "Decoding algorithm over the BSC": A polynomial-time approximate decoding algorithm that is asymptotically reliable. [Annot. 2 Sept 2009.] **(sg: fr, Algor)**

Uriel G. Rothblum & Hans Schneider

1980a Characterizations of optimal scalings of matrices. *Math. Programming* 19 (1980), 121–136. MR 0583274 (81j:65064). Zbl 437.65038. **(gg: matrd, Sw)**

1982a Characterizations of extreme normalized circulations satisfying linear constraints. *Linear Algebra Appl.* 46 (1982), 61–72. MR 0664696 (84d:90047). Zbl 503.05032. **(gg: matrd)**

Jianling Rou

See F.L. Tian.

Peter Rowlinson

See also D.M. Cardoso, D.M. Cvetković, and F. Ramezani.

Peter Rowlinson & Zoran Stanić

2021a Signed graphs with three eigenvalues: Biregularity and beyond. *Linear Algebra Appl.* 621 (2021), 272–295. MR 4234244. Zbl 1462.05240. **(SG: Adj: Eig)**

2022a Signed graphs whose spectrum is bounded by -2. *Appl. Math. Computation* 423 (2022), art. 126991, 8 pp. MR 4381063.

Builds on Yuan, Mao, and Liu (2021a). **(SG, LG: Adj: Eig)**

20xxa Trees as foundations for signed graphs. Submitted. **(SG: KG: LG: Adj: Eig)**

Bernard Roy

1959a Contribution de la théorie des graphes à l'étude de certains problèmes linéaires. *C.R. Acad. Sci. Paris* 248 (1959), 2437–2439. MR 0111631 (22 #2493).

Given real arc weights a on a digraph, there exists $t : V \to \mathbb{R}$ such that $t(w) - t(v) \geq a(e)$ for every arc $e{:}(v, w)$ iff every cycle has nonpositive weight sum. [*Cf.* Afriat (1963a).] **(WD: OG)**

1970a *Algèbre moderne et théorie des graphes, orientées vers les sciences économiques et sociales. Tome II: Applications et problèmes spécifiques.* Dunod, Paris, 1970. MR 0260413 (41 #5039). Zbl 238.90073.

§ IX.B.3.b: "Flots multiplicatifs et non conditionnels, ou k-flots." § IX.E.1.b: "Extension du problème central aux k-flots." § IX.E.2.c: "Quelques utilisations concretes des k-flots." **(GN: matrd(circuit): Exp)**

Mithun Roy

See J. Bensmail.

Roshni T Roy

See also A. Mathew, S. Hameed, and K.O. Ramakrishnan.

2023a *A Study on Matrices and Spectra of Signed Graphs and Skew Gain Graphs.* Doctoral dissertation, Central University of Kerala, 2023.

Ch. 1: Basics of signed, gain, and skew gain graphs. Ch. 2, "Signed distance Laplacian matrices for signed graphs": Roy, Germina, Hameed, and Zaslavsky (2024a). Ch. 3, "Normalized distance Laplacian matrices for signed graphs": Roy, Germina, and Hameed (20xxa). Ch. 4, "On the characteristic polynomial of skew gain graphs": Hameed, Roy, Soorya, and Germina (2023a). Ch. 5, "On two Laplacian matrices for skew gain graphs": Roy, Hameed, and Germina (2021a). Ch. 6, "On product of skew gain graphs": Roy, Germina, and Hameed (20xxb). §7.2: Open problems. [Annot. 16 Nov 2022.] (**SG, GG(Gen): Adj: Eig**)

Roshni T. Roy, K.A. Germina, & Shahul Hameed K

2023a Normalized distance Laplacian matrices for signed graphs. *Commun. Combin. Optim.* 8 (2023), no. 3, 445–456. MR 4602128. (**SG: Lap(Gen)**)

Roshni T Roy, Germina K A, & Shahul Hameed K

20xxb On product of skew gain graphs. Submitted. (**GG(Gen): Adj, Lap: Eig**)

Roshni T Roy, K A Germina, K Shahul Hameed, & Thomas Zaslavsky

2024a Signed distance Laplacian matrices for signed graphs. *Linear Multilinear Algebra* 72 (2024), no. 1, 106–117. MR 4685145. arXiv:2010.04204.

Cf. Hameed, Shijin, *et al.* (2021a). Laplacian $L(\Sigma, w)$ and $\det L(\Sigma, w)$ of positively weighted signed graph. Balance iff $\operatorname{nul} L(\Sigma, w) > 0$. Applied to $w :=$ min, max signed distance. Balance iff $L(\Sigma, w)$ cospectral with $L(|\Sigma|, d)$, $d =$ distance in $|\Sigma|$. [Annot. 11 Oct 2020.] (**SG: WG, Incid, Lap, Bal, Eig, Sw**)

Roshni T Roy, Shahul Hameed K., & Germina K.A.

2021a On two Laplacian matrices for skew gain graphs. *Electronic J. Graph Theory Appl.* 9 (2021), no 1, 125–135. MR 4253601. Zbl 1468.05164. arXiv:2009.10487.

Skew gains (*cf.* Hage (1999a) *et al.*) in $F^\times$ for field F. Sachs formula for characteristic polynomial of $L(\Phi)$. Modified "g-Laplacian" $L_g(\Phi)$ if F is ordered; incidence matrix, matrix-tree theorem. [Annot. 14 Jun 2022.] (**GG(Gen): Lap, Incid**)

Gordon F. Royle

See M.N. Ellingham and C.D. Godsil.

G. Rozenberg

See A.H. Deutz, A. Ehrenfeucht, and T. Harju.

Arthur L. Rubin

See P. Erdős.

Jason D. Rudd

See P.J. Cameron.

Paul Ruet

See also A. Crumière and É. Remy, and A. Richard.

2017a Negative local feedbacks in Boolean networks. *Discrete Appl. Math.* 221 (2017), 1–17. MR 3612582. Zbl 1357.05141. arXiv:1512.01573. **(SD: Dyn)**

Sarah Crown Rundell

See B. Braun.

Philip J. Runkel & David B. Peizer

1968a The two-valued orientation of current equilibrium theory. *Behavioral Sci.* 13 (1968), no. 1, 56–65. **(PsS: sg: Bal)**

Richard Ruppert

See J. Quirk.

Lucas J. Rusnak

See also G. Alabandi, G. Chen, V. Chen, W. Grilliette, and N. Reff.

†† 2010a *Oriented Hypergraphs.* Doctoral dissertation, Binghamton University, 2010. MR 2941411 (no rev).

Oriented hypergraphs generalize bidirected graphs: Each incidence gets a direction, or sign. Main interest: The linear dependencies of columns of a $(0, \pm 1)$-matrix, treated as the incidence matrix of an oriented hypergraph. Techniques are generalizations of those of signed graphic matroids (Zaslavsky (1982a)) but more complicated. The methods are most applicable to the matrices known as "balanceable".

(SH: Incid, Str, SG, Ori)

2013a Oriented hypergraphs: Introduction and balance. *Electronic J. Combin.* 20 (2013), no. 3, art. P48, 29 pp. MR 3118956. Zbl 1295.05169. arXiv:1210.0943. **(SH: Incid, Str, SG, Ori)**

Lucas J. Rusnak, Selena Li, Brian Xu, Eric Yan, & Shirley Zhu

2022a Oriented hypergraphs: Balanceability. *Discrete Math.* 345 (2022), no. 6, art. 112832, 14 pp. MR 4385165. Zbl 1486.05218. arXiv:2005.07722. **(SH: Ori, SG, Incid)**

Lucas J. Rusnak, Josephine Reynes, Skyler J. Johnson, & Peter Ye

2021a Generalizing Kirchhoff laws for signed graphs. *Australasian J. Combin.* 81 (2021), no. 3, 388–411. MR 4333849. Zbl 1483.05073. arXiv:2009.12680. **(SG, SH, Ori: Lap, Flows(Gen))**

Lucas J. Rusnak, Ellen Robinson, Martin Schmidt, & Piyush Shroff

2019a Oriented hypergraphic matrix-tree type theorems and bidirected minors via Boolean order ideals. *J. Algebraic Combin.* 49 (2019), no. 4, 461–473. MR 3954431. Zbl 1416.05179. arXiv:1709.04011. **(SH: Ori, SG: Adj, Lap, SG)**

Carrie Rutherford

See M. Banaji.

Joe Ryan

See C. Dalf'o.

K. Rybnikov [K.A. Rybnikov, Jr.; Konstantin Rybnikov]

See also S.S. Ryshkov.

1999a Stresses and liftings of cell-complexes. *Discrete Comput. Geom.* 21 (1999), 481–517. MR 1681885 (2001a:52016). Zbl 941.52008.

§4, "Quality transfer", concerns the existence of a satisfied state (called "quality translation" in Ryshkov and Rybnikov (1997a)) in a permutation gain graph Φ, where $\mathfrak{G}$ acts on a set Q. P. 487, top: A satisfied state exists iff Φ is balanced [Ryshkov and Rybnikov (1997a); but necessity is incorrect]. Lemma 4.1 appears to mean that a satisfied state exists iff it exists on each member of an arbitrary basis of the binary cycle space. [Not true, but interesting. The following special case, also invalid in general, was the author's intention (as I was told, Oct. 2000): Φ is balanced iff every member of a circle basis is balanced. The special case Lemma 4.2 is correct, because it is essentially homotopic.] [See Rybnikov and Zaslavsky (2005a), (2006a).] (**gg: bal, Cov**)

Konstantin Rybnikov & Thomas Zaslavsky

2005a Criteria for balance in abelian gain graphs, with an application to piecewise-linear geometry. *Discrete Comput. Geom.* 34 (2005), no. 2, 251–268. MR 2155721 (2006f:05086). Zbl 1074.05047.

§§1–4: A condition on binary cycles that implies balance but does not depend on having a fundmental system of circles; it requires an abelian gain group. §5: Satisfied states and balance of a permutation gain graph. (**GG: Bal**)

§6: The criterion is applied to calculate the dimension of the space of liftings of a piecewise-linear immersion of a d-cell complex in Euclidean d-space. (**GG: Bal, Geom**)

2006a Cycle and circle tests of balance in gain graphs: Forbidden minors and their groups. *J. Graph Theory* 51 (2006), no. 1, 1–21. MR 2184346 (2006i:05078). Zbl 1085.05033.

The class of Γ such that the criterion of (2005a) works for any gains on Γ is minor-closed. Some forbidden minors are given. Which ones they are depends on the class of permitted gain groups in a way that is not understood. (**GG: Bal: Str**)

S.S. Ryshkov & K.A. Rybnikov, Jr.

1996a Generatrissa. The Maxwell and Voronoĭ problems. (In Russian.) *Dokl. Akad. Nauk* 349 (1996), no. 6, 743–746. MR 1441201 (98b:52027). Zbl 906.51007.

Announcement of results in (1997a). ["Generatrissa" corresponds to English "generatrix".] (**gg: Geom**)

1996b Generatrissa: The problems due to Maxwell and Voronoi. *Dokl. Math.* 54 (1996), no. 1, 614–617. Zbl 906.51007.

Translation of (1996a). (**gg: Geom**)

1997a The theory of quality translations with applications to tilings. *European J. Combin.* 18 (1997), 431–444. MR 1444253 (98d:52031). Zbl 881.52015.

Let Φ be a permutation gain graph, with gain group $\mathfrak{G}$ acting on a set $\mathfrak{Q}$, and with underlying graph the d-cell adjacency graph of a kind of simply connected polyhedral d-cell complex. A "quality translation" is a satisfied state: a mapping $s : V \to \mathfrak{Q}$ such that $s(w) = s(v)\varphi(e; v, w)$ for every edge. A "circuit" is a closed walk that is not trivially reducible. Call a "$d-2$-circle" any circle contained in the star of a $d-2$-cell. Assume

$\mathfrak{G}$ and $\mathfrak{Q}$ fixed. Thms. 1–2 can be stated: In the free group on the edge set, the $d-2$-circles generate all circuits. Also, Φ is balanced iff all $d-2$-circles have identity gain. Thm. 3: Identity gain of all $d-2$-circles is necessary and sufficient for the existence of a satisfied state. [Necessity is incorrect because the action may have nontrivial kernel.] The idea of quality transfer goes back to Voronoĭ in 1908. [See Rybnikov (1999a) and Rybnikov and Zaslavsky (2005a) for more.]

Sufficiency in Thm. 3 is applied to lifting of tilings of Euclidean and spherical space. Thm. 4 ((1996a), Thm. 9): Balance ("canonical definition") of Φ is sufficient for lifting a tiling of $\mathbb{R}^d$. Here the qualities are affine functions. Thms. 5–6 ((1996a), Thm. 10): Balance within each $d-3$-cell star implies lifting. [See Rybnikov (1999a) and Rybnikov and Zaslavsky (2005a) for more.]

§8: "Applications to the colouring of tilings". **(gg: bal, Geom)**

Herbert J. Ryser

See R.A. Brualdi.

Shinsei Ryu

See A.P.O. Chan.

Henry S. Rzepa

2005a Möbius aromaticity and delocalization. *Chem. Rev.* 105 (2005), no. 10, 3697–3715. **(Chem: sg: bal: Exp, Ref)**

Rachid Saad

1996a Finding a longest alternating cycle in a 2-edge-coloured complete graph is in RP. *Combin. Probab. Computing* 5 (1996), 297–306. MR 1411089 (97g:05156). Zbl 865.05054.

Thm.: In a bidirected all-negative complete graph with a suitable extra hypothesis, the maximum length of a coherent circle equals the maximum order of a coherent degree-2 subgraph. More or less generalizes part of Bánkfalvi and Bánkfalvi (1968a) (*q.v.*). [Generalized in Bang-Jensen and Gutin (1998a).] [*Problem.* Generalize to signed complete graphs or further.] **(par: ori: Paths, Algor)**

Assieh Saadatpour, István Albert, & Réka Albert

2010a Attractor analysis of asynchronous Boolean models of signal transduction networks. *J. Theor. Biol.* 266 (2010), 641–656. MR 2981575 (no rev). Zbl 1407.92058.

(SD, Biol: Dyn)

Assieh Saadatpour, Réka Albert, & Timothy C. Reluga

2013a A reduction method for Boolean network models proven to conserve attractors. *SIAM J. Appl. Dyn. Syst.* 12 (2013), no. 4, 1997–2011. MR 3131470. Zbl 1308.92040. **(SD, Biol: Dyn)**

Eminjan Sabir

See L.-L. Yuan.

Mathieu Sablik

See A. Crumière.

Horst Sachs
See D.M. Cvetković.

Faiza Saeed
See U. Ahmad.

Julio Saez-Rodriguez
See S. Klamt.

Bruce Sagan
See also C. Bennett, A. Björner, A. Blass, J. Hallam, F. Harary, and T. Józefiak.

1995a Why the characteristic polynomial factors. *Sém. Lotharingien Combin.* 35 (1995) [1998], art. B35a, iii + 20 pp. MR 1399505 (98a:06006). Zbl 855.05012. arXiv:math/9812136.

A shorter predecessor of (1999a). (**SG, Gen: N: Col, G: Exp**)

1999a Why the characteristic polynomial factors. *Bull. Amer. Math. Soc. (N.S.)* 36 (1999), 113–133. MR 1659875 (2000a:06021). Zbl 921.06001.

In §4, coloring of a signed graph Σ, especially of $\pm K_n^\bullet$ and $\pm K_n$, is used to calculate and factor the characteristic polynomial of $\mathbf{F}(\Sigma)$. Presents the geometrical reinterpretation and generalization by Blass and Sagan (1998a). In §§5 and 6, other methods of calculation and factorization are applied to some signed graphs (in their geometrical representation). (**SG, Gen: N: Col, G: Exp**)

Barna Saha
See S. Ahmadi.

Sahariya
See K.A. Germina.

[Amine El Sahili]
See A. El Sahili (under 'E').

G. Sahoo
See S. Barik.

Prabhat K. Sahu & Shyi-Long Lee

2008a Net-sign identity information index: A novel approach towards numerical characterization of chemical signed graph theory. *Chem. Phys. Lett.* 454 (2008), 133–138.

The "net-sign identity information index" I_s is expressed [obscurely] in terms of $\#E^+$ and $\#E^-$ in a molecular structure graph. The purpose is to correlate with chemical phenomena. I_s and $\sqrt{I_s}$ are compared with other indices. [Annot. 6 Feb 2011.] (**SG: Chem**)

Shaik Sajana, D. Bharathi, & K.K. Srimitra

2017a Signed intersection graph of ideals of a ring. *Int. J. Pure Appl. Math.* 113 (2017), no. 10, 175–183.

Balance and homogeneity for ring $\mathbb{Z}_n$. [Annot. 25 Apr 2019.] (**SG: Algeb: Bal**)

Sina Sajjadi
See R. Masoumi.

Mateja Šajna
See P. Potočnik.

Michael Saks
See P.H. Edelman.

Tadashi Sakuma
See Chiba.

M.C. Salas-Solís, F. Aguilera-Granja, E.E. Vogel, & S. Contreras

2003a Order parameters in anisotropic two-dimensional $\pm J$ Ising lattices. *Physica A* 327 (2003), 477–490. Zbl 1031.82510.

Toroidal square lattice with fixed weights $f_x J$ horizontally and $f_y J$ vertically ($f_x, f_y, J > 0$), randomly signed with $\#E^+ = \frac{1}{2}\#E$ or with variable $x := \#E^+/\#E$. Studies dependence of "order parameters" on $f := f_x/f_y$ and x. [Annot. 3 Jan 2015.] (**SG, WG, Phys: Fr**)

Nicolau C. Saldanha

2002a Singular polynomials of generalized Kasteleyn matrices. *J. Algebraic Combin.* 16 (2002), no. 2, 195–207. MR 1943588 (2004c:05051). Zbl 1017.05077.

A generalized Kasteleyn matrix is the left-right adjacency matrix B of a bipartite gain graph with the complex units as gain group. (A Kasteleyn matrix has for gain group the sign group.) The object is to interpret combinatorially the coefficients of the characteristic polynomial, or the eigenvalues, of BB^*. The approach is cohomological (*cf.* Cameron (1977b). [Annot. rev. 10 Jun 2017.] (**GG, SG: Adj, Eig, Sw**)

Anwar Saleh
See S. Shalini.

M. Ruby Salestina
See P.S.K. Reddy.

Lillian Salinas
See J. Aracena.

Mahmoud Salmasizadeh
See S. Fayyaz Shahandashti.

Regina Samaga
See I.N. Melas.

Robert Šámal
See M. DeVos.

Aniruddha Samanta
See also R. Mehatari.

Aniruddha Samanta & M. Rajesh Kannan

2021a Bounds for the energy of a complex unit gain graph. *Linear Algebra Appl.* 612 (2021), 1–29. MR 4188336. Zbl 1459.05185. arXiv:2005.08634. (**GG: Adj: Eig**)

2022a Gain distance matrices for complex unit gain graphs. *Discrete Math.* 345 (2022) [2021], no. 1, art. 112634, 12 pp. MR 4318814. Zbl 1476.05127. arXiv:-2101.11558.

Generalizes Hameed, Shijin, *et al.* (2021a) to gain group of complex units. [Annot. 20 Oct 2021.] (**GG: Str, Adj(Gen): Eig**)

20xxa On the spectrum of complex unit gain graphs. Submitted. arXiv:1908.10668. (**GG: Adj: Eig**)

20xxd On the multiplicity of A_α-eigenvalues and the rank of complex unit gain graphs. Submitted. arXiv:2101.03752. (**GG: Adj(Gen): Eig**)

P.N. Samanta

See P.S.K. Reddy.

U. Samee

See M.A. Bhat.

E. Sampathkumar

See also C. Adiga and S. Pirzada.

1972a Point-signed and line-signed graphs. Karnatak University Graph Theory Res. Rep. 1, 1972.

See *Graph Theory Newsletter* 2 (Nov., 1972), no. 2, Abstract No. 7. (**SG, VS: Bal**)

1984a Point signed and line signed graphs. *Nat. Acad. Sci. Lett. (India)* 7 (1984), no. 3, 91–93. Zbl 552.05051.

μ_σ, $\partial\mu$ Consider a simple graph, an edge signature σ, and a vertex signature μ. Define $\mu_\sigma(v) := \prod\{\sigma(e) : e \text{ incident with } v\}$ [later dubbed "canonical marking"] and, for each component X, $\partial\mu(X) := \prod_{v\in X}\mu(v)$. μ is "p-balanced" if $\partial\mu \equiv +$. Thm. 1: $\partial\mu \equiv +$ iff $\mu = \mu_\sigma$ for some σ. [An early homology theorem.] Thm. 2: If σ is balanced and $\partial\mu_\sigma \equiv +$, then there exist all-negative, pairwise edge-disjoint paths connecting the μ_σ-negative vertices in pairs. [Quick proof: $\partial\mu \equiv +$ iff μ has evenly many negative vertices in each component. Negative vertices of μ_σ are odd-degree vertices of Σ^-. Apply Listing's Theorem (independently discovered in stronger form by Sampathkumar) to Σ^-.]

[It is interesting to base homology 0-chains like $\partial\mu$ on the components.] [Annot. rev. 27 Dec 2010.] (**SG, VS: Bal**)

2006a Generalized graph structures. *Bull. Kerala Math. Assoc.* 3, no. 2 (Dec., 2006), 67–123. MR 2290946 (no rev).

Within the class of simple graphs, what is a complement of a signed graph? An approach is to partition the edges of K_n into 3 classes: E^+, E^-, and E^c (the set of non-edges), and apply a specific permutation of these sets. Each permutation of order 2 implies a kind of complementation. Examination of self-complementarity. Generalizations of balance. Generalized to a graph Γ with k edge classes R_i [i.e., k-edge-colored graphs].

§10, "Balanced graph structures": "R_i-balance": $(\exists X)$ $R_i = E[X, X^c]$ (the cut between X and X^c). "$R_1 \cdots R_r$-balance": Similar for $R_1 \cup \cdots \cup R_r$. "Complete balance": R_i balance for all i. "Arbitrary balance": $R_{i_1} \cdots R_{i_r}$-balance for every $i_1, \ldots, i_r \in [k]$. *Problem* 11: Characterize this property. "r-relation balance": The same for fixed r. *Problem* 12: Characterize this property. Other, similar concepts based on partitioning

V. [Annot. 4 Sept 2010.] (**SG, SGc: Gen: Bal**)

2006b 4-Sigraphs. In: *International Conference on Discrete Mathematics, ICDM 2006* (Lect. Notes, Bangalore, 2006), p. 288. (**SG(Gen): GG**)

2011a Two new characterizations of consistent marked graph. *Adv. Stud. Contemp. Math. (Kyungshang)* 21 (2011), no. 4, 437–439. MR 2885007 (2012j:05192). Zbl 1250.05056. (**VS: Bal, SG**)

E. Sampathkumar & V.N. Bhave

1973a Group valued graphs. *J. Karnatak Univ. Sci.* 18 (1973), 325–328. MR 0347675 (50 #177). Zbl 284.05113.

Group-weighted graphs, both in general and where the group has exponent 2 (so all $x^{-1} = x$). Analogs of elementary theorems of Harary and Flament. Here balance of a circle means that the weight product around the circle, taking for each edge either $w(e)$ or $w(e)^{-1}$ arbitrarily, equals 1 for some choice of where to invert. [Hence, the graphs are not gain graphs.] (**WG, GG: Bal**)

E. Sampathkumar & L. Nanjundaswamy

1973a Complete signed graphs and a measure of rank correlation. *J. Karnatak Univ. Sci.* 18 (1973), 308–311. MR 0423674 (54 #11649) (*q.v.*). Zbl 291.62066.

Given a permutation of $\{1, 2, \ldots, n\}$, sign K_n so edge ij is negative if the permutation reverses the order of i and j and is positive otherwise. Kendall's measure τ of correlation of rankings (i.e., permutations) A and B equals $(\#E^+ - \#E^-)/\#E$ in the signature due to AB^{-1}. (**SG: KG**)

E. Sampathkumar, L. Pushpalatha, & M.A. Sriraj

2016a Color matrices. *Indian J. Discrete Math.* 2 (2016), no. 2, 103–108. MR 4317596.

Definitions at Sampathkumar and Sriraj (2013b) and *Cf.* Adiga, Sampathkumar, *et al.* (2013a). The colors are unlabelled. G colored by $\pi(\Sigma^-)$ is "color balanced" if Σ is balanced. "Color balanced path number" [not clear]. [Annot. 27 Jul 2018, 4 Jan 2019.] (**SG: clu, Adj, Bal**)

E. Sampathkumar, P. Siva Kota Reddy, & M.S. Subramanya

2008a Jump symmetric n-sigraph. *Proc. Jangjeon Math. Soc.* 11 (2008), no. 1, 89–95. MR 2429334 (2009j:05107). Zbl 1172.05028.

In the n-fold sign group $\{+, -\}^n$ an element is "symmetric" if it is its own reverse. A (symmetric) n-signed graph is a gain graph $\Phi = (\Gamma, \varphi)$ which has (symmetric) gains $\varphi(e) \in \{+, -\}^n$. [Equivalent to having arbitrary gains in $\{+, -\}^{\lceil n/2 \rceil}$.] Only symmetric n-signed graphs are treated.

Σ_Φ [The mapping $\min : \{+, -\}^n \to \{+, -\}$ by $\min(a_1, \ldots, a_n) = +$ if all $a_i = +$ and $= -$ otherwise gives a signed graph Σ_Φ with signs $\sigma_\Phi(e) := \min(\varphi(e))$.]

Def.: $\Phi_1 \simeq \Phi_2$ ("cycle isomorphism") if there is an isomorphism $\|\Phi_1\| \cong \|\Phi_2\|$ that preserves circle gains. Prop. 3: Symmetric n-signed graphs are cycle isomorphic iff they are switching isomorphic—generalizing $n = 1$ due to Sozański (1980a), Zaslavsky (1981b). [The proof (omitted) applies iff the gain group has exponent 2.]

φ_S Let $\varphi_S(ef) := \varphi(e)\varphi(f)$ for $e, f \in E$. [Generalizing $\sigma_\times$ of M. Acharya (2009a).]

J_S The jump graph is $J_S(\Phi) := (\Lambda(\Gamma)^c, \varphi_S)$. Solutions of $\Phi \simeq J_S(\Phi)$, $\Phi^t \simeq J_S(\Phi)$, $J_S(\Phi^t) \simeq J_S(\Phi)$, where $a^t := at$ for $a, t \in \{+,-\}^n$ and t is one of three special n-signs. [The last solution extends to arbitrary $t \in \{+,-\}^n$.]

Dictionary: "identity balance", "i-balance" = balance in Φ; "balance" = balance in Σ_Φ; $P(\vec{C}) := \varphi(C)$ in the indicated direction.

[The results remain true without assuming symmetry.] [Continued in (2010c), (2010d), Sampathkumar, Subramanya, and Reddy (2011a), and papers of P.S.K. Reddy.] [Annot. 2 Aug 2009, 20 Dec 2010.]
(**SG(Gen), gg: LG, Sw, Bal**)

2008b $(3, d)$-Sigraph and its applications. *Adv. Stud. Contemp. Math. (Kyungshang)* 17 (2008), no. 1, 57–67. MR 2428537 (2009g:05073).
The $n = 3$ case of (2010b). [Annot. 10 Apr 2009.]
(**SG(Gen), gg: Bal, Sw**)

2009a Directionally n-signed graphs. II. *Int. J. Math. Combin.* 2009 (2009), vol. 4, 89–98 (2010). MR 2598676 (no rev). Zbl 1238.05125.
Cf. (2010b). (**GG(Gen): Bal**)

2010a $(4, d)$-Sigraph and its applications. *Adv. Stud. Contemp. Math. (Kyungshang)* 20 (2010), no. 1, 115–124. MR 2597997 (2011i:05089). Zbl 1192.05067.
The $n = 4$ case of (2010b). [Annot. 9 Sept 2010.]
(**SG(Gen), gg: Bal, Sw**)

2010b Directionally n-signed graphs. In: B.D. Acharya, G.O.H. Katona, and J. Nesetril, eds., *Advances in Discrete Mathematics and Applications: Mysore, 2008* (Proc. Int. Conf. Discrete Math., ICDM-2008, Mysore, India, 2008), pp. 153–160. Ramanujan Math. Soc. Lect. Notes Ser., No. 13. Ramanujan Mathematical Soc., Mysore, India, 2010. MR 2766915 (2012g:05097). Zbl 1231.05119.
The gain group is the n-fold sign group $\{+,-\}^n$, with reversing automorphism $(a_1, \ldots, a_n)^r := (a_n, \ldots, a_1)$. The gains satisfy $\varphi(e^{-1}) = \varphi(e)^r$. For $t \in \{+,-\}^n$, the t-complement of Φ is $\|\Phi\|$ with gains $\varphi^t(e) := t\varphi(e)$. Elementary results on balance, t-complementation, switching, and isomorphism. Dictionary: "identity balance" = "i-balance" = balance in Φ; "balance" = balance in Σ_Φ defined at (2008a); $P(\vec{C}) := \varphi(C)$ in the indicated direction. [An interesting form of skew gain graph. The ideas should be pursued in directions suggested by Hage and Harju (2000a) and Hage (1999a).] [Annot. 10 Apr 2009.] (**SG(Gen), gg(Gen): Bal, Sw**)

2010c The line n-sigraph of a symmetric n-sigraph. *Southeast Asian Bull. Math.* 34 (2010), no. 5, 953–958. MR 2746762 (2012a:05142). Zbl 1240.05142.

Λ_S The line graph is $\Lambda_S(\Phi) := (\Lambda(\Gamma), \varphi_S)$ [generalizing $\Lambda_\times$ of M. Acharya (2009a)]. For other definitions and notation see (2008a).

Line graphs and jump graphs in the sense of (2008a) are characterized, respectively, as balanced symmetric n-signings of (unsigned) line graphs and their complements. [The characterizations remain true for unsymmetric n-signatures.] There are remarks about the t-complement $t\varphi$ (2010b) for three $t \in \{+,-\}^n$.

μ_φ, Φ^c The "complement" Φ^c is (Γ^c, φ^c) defined by $\mu_\varphi(v) := \prod_{uv\in E} \varphi(uv)$ ("canonical marking") (*cf.* Sampathkumar (1984a)) and $\varphi^c(uv) := \mu_\varphi(u)\cdot\mu_\varphi(v)$ [= product of gains of all edges incident in Φ to u or v but not both]. [Gains φ^c are clearly balanced.] Prop. 7: A symmetric n-signed graph is a line graph iff it is a balanced, symmetric n-signature of an unsigned line graph. [Because φ_S is arbitrary balanced gains.] Prop. 9: $\Lambda_S(\Phi)^c \sim J_S(\Phi)$. [Because both are balanced and the underlying graphs are the same.] Prop. 10 solves $\Lambda_S(\Phi) \simeq J_S(\Phi)$, generalizing M. Acharya and Sinha (2003a). [The solutions to such graph equations, here and in related papers of Rangarajan, Sampathkumar, Siva Kota Reddy, *et al.*, are easy corollaries of the similar results for unsigned graphs.] [All results remain true without assuming symmetry.] [Annot. 10 Apr, 1 Aug 2009, 20 Dec 2010.] (**SG(Gen), gg: LG, Sw, Bal**)

2010d Common-edge signed graph of a signed graph. *J. Indones. Math. Soc.* 16 (2010), no. 2, 105–112. MR 2752773 (no rev). Zbl 1236.05098.

C_E See (2008a), Sampathkumar, Subramanya, and Reddy (2011a) for definitions. The common-edge signed graph $C_E(\Sigma)$ is $\Lambda^2_\times(\Sigma)$. Prop. 4: Σ_0 is a common-edge signed graph iff it is balanced and $|\Sigma_0|$ is a common-edge graph. [Incorrect. $\Lambda^2_\times(\Sigma)$ does not have arbitrary balanced signs. E.g., $|\Sigma| = C_4$.] Equations solved [possibly incorrectly]: $\Sigma \simeq C^k_E(\Sigma)$ and $\Sigma \simeq \Lambda^k_\times(\Sigma)$ [this includes the preceding]. $\Lambda^k_\times(\Sigma) \simeq C^r_E(\Sigma)$. The jump graph (2008a) $J_S(\Sigma) \simeq C_E(\Sigma)$. [$\Lambda_\times$ as in M. Acharya (2009a).] [Annot. 12 Apr 2009.]

"Smarandanchely k-signed/marked graphs" are defined as k-signed/-marked graphs [and not used]. Signed/marked graphs are the case $k = 2$ [correctly: $k = 1$]. [Smarandanche has nothing to do with this.] [Annot. 7 Jan 2011.] (**SG: Bal, Sw, LG**)

E. Sampathkumar, S.V. Roopa, K.A. Vidya, & M.A. Sriraj

2015a Partition energy of a graph. *Proc. Jangjeon Math. Soc.* 18 (2015), no. 4, 473–493. MR 3444734. Zbl 1332.05115.

Definitions at Sampathkumar and Sriraj (2013b). Eigenvalues and energy of A. Examples. [Annot. 4 Jan 2019.] (**sgw(Gen): Adj: Eig**)

2018a Partition energy of complete product of circulant graphs and some new class of graphs. *Adv. Stud. Contemp. Math., Kyungshang* 28 (2018), no. 2, 269–283. Zbl 1400.05203.

Definitions at Sampathkumar and Sriraj (2013b). Equienergetic examples. Circulant-type examples. [Annot. 4 Jan 2019.] (**sgw(Gen): Adj: Eig**)

2019a Partition Laplacian energy of a graph. *Palestine J. Math.* 8 (2019), no. 1, 272–284. MR 3884324. Zbl 1406.05066.

Definitions at Sampathkumar and Sriraj (2013b). Eigenvalues and energy of Laplacian modification of A. Examples. [Annot. 4 Jan 2019.]

(**sgw(Gen): Adj: Eig**)

E. Sampathkumar & M.A. Sriraj

2013a $(2, d)$-Sigraphs. *Notes Number Theory Discrete Math.* 19 (2013), no. 4, 16–27. Zbl 1314.05091.

$(2, d)$-sigraph = directionally 2-signed graph = bidirected graph B. A notable new observation is Prop. 13, refined in Sampathkumar, Sriraj, and Zaslavsky (2012a). §3, "Directional adjacency matrix": $= A$ of the naturally corresponding signed digraph. §7, "Clusterable $(2, d)$-sigraphs". §9, "Induced $(2, d)$-sigraph of an (n, d)-sigraph G": In a directionally n-signed graph, multiply the first $\lceil n/2 \rceil$ directional signs to get a single directional sign. Dictionary: Signed graph is(B) $:= -\Sigma$(B). [Annot. 31 Jan, 29 Sept 2012.] (**SG(Gen): ori, SD: Gen: Bal, Adj**)

2013b Vertex labeled/colored graphs, matrices and signed graphs. *J. Combin. Inform. System Sci.* 38 (2013), no. 1-2, 113–120. Zbl 302.05162.

Decompose K_n into graphs Γ_i, $i = 1, 2, 3, 4$. Let $w(e) = \alpha_i \in \mathbb{Z}$ for $e \in E_i$ and $A := \sum_i \alpha_i A(\Gamma_i)$. In this paper, Γ is given, has labels $l : V \to S$ and partition π of V into color classes, or any partition π; $\alpha_i = 1, -1, 2, 0$ for $i = 1, 2, 3, 4$; $\Gamma_1 = \Gamma(\pi)$, $\Gamma_2 = \Gamma^c(\pi)$, $\Gamma_3 = \Gamma{:}\pi$, and $\Gamma_4 = \Gamma^c{:}\pi$. The "$L$-matrix" of (Γ, l) is A. Thm. 1 characterizes L-matrices. §3: If l properly colors Γ, then $A = A(\Sigma)$ where $\Sigma := +\Gamma_1 \cup -\Gamma_2$. Prop. 5 restated: Σ^- is a disjoint union of cliques. Alternatively, $-\Sigma$ is clusterable so the clusters are all-positive cliques. *Cf.* Adiga, Sampathkumar *et al.* (2013a), Sampathkumar, S.V. Roopa *et al.* (2015a), (2019a), Prakasha, Reddy, and Cangul (2017a). [The purpose of choosing $w_3 = 2$ is not clear.] [Annot. 14 Oct 2014, 4 Jan 2019.]

(**sgw, SG: Col, Clu, Adj**)

E. Sampathkumar, M.A. Sriraj, & L. Pushpalatha

2017a Strong signed graph structures labeled graph structures and vertex labeled graphs. *Indian J. Discrete Math.* 3 (2017), no. 3, 15–24. MR 4317598.

(**SG: Adj, clu**)

2017b Notions of balance in signed and marked graphs. *Indian J. Discrete Math.* 3 (2017), no. 3, 25–32. MR 4317599.

Given Σ and $\mu : V \to \{pm\}$, let $\bar{\sigma}(uv) :=$ $sigma^\mu(uv)$. Defs.: "e-balance": $\sigma^\mu \equiv +$. "Weak e-balance": σ is balanced. "Total balance": $\sigma^\mu \equiv +$ and $\mu(x) = \sigma(ux)\sigma(vx)$ $\forall x \in V$ and distinct $u, v \in N(x)$; characterized in Prop. 5.7. [Annot. 11 Jul 2022.]

(**SG, VS: Bal(Gen), Clu**)

E. Sampathkumar, M.A. Sriraj, & Thomas Zaslavsky

2012a Directionally 2-signed graphs and bidirected graphs. Set Valuations, Signed Graphs and Geometry (IWSSG-2011, Int. Workshop, Mananthavady, Kerala, 2011). *J. Combin. Inform. System Sci.* 37 (2012), no. 2-4, 373–377. Zbl 1301.05161. arXiv:1303.3084.

A formal treatment of Sampathkumar and Sriraj (2013a), Prop. 13, and the connection with sources, sinks, and switching in bidirected graphs. Given a bidirected graph B, its corresponding signed graph Σ(B) (*cf.* Zaslavsky (1991b)) is antibalanced iff every vertex in B is a source or sink. [Annot. 29 Sept 2012.] (**SG, Gen: Ori: Bal, Sw**)

E. Sampathkumar, M.S. Subramanya, & P. Siva Kota Reddy

2011a Characterization of line sidigraphs. *Southeast Asian Bull. Math.* 35 (2011), no. 2, 297–304. MR 2866547 (2012j:05193). Zbl 1240.05143.

The line signed graph is $\Lambda_\times(\Sigma)$ [see M. Acharya (2009a)]. Prop. 3: A signed graph is a line signed graph of this kind iff it is a line graph with balanced signs.

The line signed digraph is $\Lambda_S(\vec{\Gamma}, \sigma)$:= the Harary–Norman line digraph of $\vec{\Gamma}$, signed by σ^c defined as φ^c in Sampathkumar, Reddy, and Subramanya (2010c). Prop. 11: A signed digraph is a line signed digraph of this kind iff it is a Harary–Norman line digraph with (undirectedly) balanced signs. $(\vec{\Gamma}, \sigma)$ is switching isomorphic to $\Lambda_S(\vec{\Gamma}, \sigma)$ iff each component is a balanced directed cycle. [Annot. 4 Sept 2010.]

(**SG, SD: LG**)

Rubén J. Sánchez-García

See R. Mulas.

V. Sangeetha

See A.J. Mathias.

Bernardo San Martín

See N.M.M. Abreu and M. Robbiano.

Yoshio Sano

See T.Y Chung, A.L. Gavrilyuk, G. Greaves, and A. Munemasa.

Santhi. M & J. James Albert

2015a Signed product cordial in cycle related graphs. *Int. J. Math. Comp. Appl. Res.* 5 (2015), no. 1, 29–36.

More, as in Baskar Babujee and Loganathan (2011a). [Annot. 11 Mar 2017.] (**Lab: VS: SG, Bal**)

Santhosh G O

See S. Hameed.

[Emilio De Santis]

See E. De Santis (under 'D').

Raman Sanyal

See L. Gellert.

Mark Sapir

See V. Guba.

S.V. Sapunov

2002a Equivalence of marked graphs. [*Or*: Equivalence of labeled graphs.] (In Russian.) *Proceedings of the Institute of Applied Mathematics and Mechanics* [*Tr. Inst. Prikl. Mat. Mekh.*], Vol. 7, pp. 162–167. Nats. Akad. Nauk Ukrainy Inst. Prikl. Mat. Mekh., Donetsk, 2002. MR 2141811 (2006c:05070). Zbl 1081.68074.

Equivalence of signed graphs (?) that model languages. [Annot. 28

Dec 2011.] **(SG?)**

P.B. Sarasija
See P. Nageswari.

Hina Saraswat
See Sinha, Garg, and Saraswat (2013a).

Kuldeep Sarma
See D. Kalita.

Irasema Sarmiento
See also J.A. Ellis-Monaghan.

1999a A characterisation of jointless Dowling geometries. 16th British Combinatorial Conf. (London, 1997). *Discrete Math.* 197/198 (1999), 713–731. MR 1674899 (99m:51020). Zbl 929.05016.

They are 4-closed (determined by their flats of rank 4). They are characterized, among all matroids, by the statistics of flats of rank ≤ 7 and therefore by their Tutte polynomials. There are exceptions in rank 3. **(GG: Matrd: Invar)**

Iwao Sato
See also Y. Higuchi and H. Mizuno.

2006a Weighted zeta functions of graph coverings. *Electronic J. Combin.* 13 (2006), art. R91, 12 pp. MR 2274306 (2007k:05138). Zbl 1114.05084. **(GG, SG: Cov)**

2006b Edge zeta functions of graph coverings. *Ars Combin.* 81 (2006), 225–233. MR 2267814 (2007g:05082). Zbl 1174.05304. **(gg: Cov)**

2008a The stochastic weighted complexity of a group covering of a digraph. *Linear Algebra Appl.* 429 (2008), 1905–1914. MR 2446628 (2009h:05137). Zbl 1144.05322.

§3, "Weighted zeta functions of group covering of digraphs": The covering graphs ("derived graphs") of gain graphs ("voltage graphs"). **(GG: Cov)**

Shun Sato
See T. Matsuoka.

Roman V. Satyukov
See I.E. Bocharova.

Lawrence Saul & Mehran Kardar

1993a Exact integer algorithm for the two-dimensional $\pm J$ Ising spin glass. *Phys. Rev. E* 48 (1993), no. 5, R3221–R3224.

Announcement of (1994a) with some details, observations, and conclusions. [Annot. 18 Aug 2012.] **(SG: Phys, Fr, state: Algor)**

1994a The 2D $\pm J$ Ising spin glass: exact partition functions in polynomial time. *Nuclear Phys. B* 432 [FS] (1994), 641–667.

Algorithm for the energy distributions (the partition function) of the states of a randomly signed square, toroidal lattice graph. Applied to find statistical properties of such a signed graph. [Annot. 17 Aug 2012.] **(SG: Phys, Fr, state: Algor)**

B. David Saunders

See also A. Berman.

B. David Saunders & Hans Schneider

1978a Flows on graphs applied to diagonal similarity and diagonal equivalence for matrices. *Discrete Math.* 24 (1978), 205–220. MR 0522929 (80e:15008). Zbl 393.94046. **(gg: Sw)**

1979a Cones, graphs and optimal scalings of matrices. *Linear Multilinear Algebra* 8 (1979), 121–135. MR 0552356 (80k:15036). Zbl 433.15005. **(gg: Sw)(Ref)**

James Saunderson

See T. Coleman.

Saket Saurabh

See A. Das and G. Philip.

D. Savithri

See M. Parvathi.

H.C. Savithri

See H.A. Malathi and P.S.K. Reddy.

Radmila Sazdanović

See Kauffman, Jablan, Radović, and Sazdanović (2013a).

Alex Schaefer

2017a *Permutable Matchings and Negative Cycle Vectors.* Doctoral dissertation, Binghamton University, 2017. MR 3781741 (no rev).

Ch. 1, "Signed graphs: background and miscellaneous results". **(SG, Sw: Exp)**

Ch. 2, "The dimension of the negative cycle vectors of a signed graph": Same as Schaefer and Zaslavsky (2019a).

Ch. 3, "Graphs that contain multiply transitive matchings": Same as Schaefer and Swartz (2021a). [Annot. 15 Nov 2017.] **(SG: Invar, Sw, Geom)**

Alex Schaefer & Eric Swartz

2021a Graphs that contain multiply transitive matchings. *European J. Combin.* 92 (2021), art. 103236, 22 pp. MR 4149160. Zbl 1458.05219. arXiv:1706.08964. **(SG: Aut)**

Alex Schaefer & Thomas Zaslavsky

2019a The dimension of the negative cycle vectors of a signed graph. *Ars Math. Contemp.* 16 (2019), no. 2, 625–639. MR 3963226. Zbl 1416.05133. arXiv:-1706.09041.

Negative cycle vector $c^-(\Sigma) := (c_3^-(\Sigma), \ldots, c_n^-(\Sigma))$, $c_l^-(\Sigma) :=$ # negative l-circles in Σ. $\mathrm{NCV}(\Gamma) := \{c^-(\Gamma,\sigma) : \sigma \text{ signs } \Gamma\}$. Trivially, $\dim \mathrm{NCV} \leq$ # circle lengths in Γ. Theorem: Equality for graphs with a strongly permutable matching, including K_n, $K_{r,s}$. [Annot. 30 Jul 2020.] **(SG: Invar, Sw, Geom)**

R.H. Schelp

See P. Erdős and R.J. Faudree.

Baruch Schieber

See L. Cai.

Frank Schmidt

2004a Problems related to type-A and type-B matrices of chromatic joins. *Adv. Appl. Math.* 32 (2004), no. 1-2, 380–390. MR 2037637 (2004m:06007). Zbl 1050.06003. **(sg: Matrd)**

Martin Schmidt

See L. Rusnak.

Rüdiger Schmidt

1979a On the existence of uncountably many matroidal families. *Discrete Math.* 27 (1979), 93–97. MR 0534956 (80i:05029). Zbl 427.05024.

The "count" matroids of graphs (see Whiteley (1996a)) and an extensive further generalization of bicircular matroids that includes frame matroids of biased graphs. His "partly closed set" is a linear class of circuits in an arbitrary "count" matroid. (**GG: MtrdF, Bic, ECyc: Gen**)

Stephan Schmidt

See J. Kunegis.

Hans Schneider

See G.M. Engel, D. Hershkowitz, U.G. Rothblum, and B.D. Saunders.

Irwin E. Schochetman

See J.W. Grossman.

Rainer Schrader

See U. Faigle.

Alexander Schrijver

See also A.M.H. Gerards.

1986a *Theory of Linear and Integer Programming.* Wiley, Chichester, 1986. MR 0874114 (88m:90090). Zbl 665.90063.

Remark 21.2 (p. 308) cites Truemper's (1982a) definition of balance of a $0, \pm 1$-matrix. (**sg: par: Incid: Exp**)

1989a The Klein bottle and multicommodity flows. *Combinatorica* 9 (1989), 375–384. MR 1054013 (92b:90083). Zbl 708.05019.

Assume Σ embedded in the Klein bottle. If Σ is bipartite, negative girth = max. number of disjoint balancing edge sets. If Σ is Eulerian, frustration index = max. number of edge-disjoint negative circles. Proved via polyhedral combinatorics. (**SG: Top, Geom, Fr**)

1990a Applications of polyhedral combinatorics to multicommodity flows and compact surfaces. In: William Cook and P.D. Seymour, eds., *Polyhedral Combinatorics* (Morristown, NJ., 1989), pp. 119–137. DIMACS Ser. Discrete Math. Theor. Comp. Sci., Vol. 1. Amer. Math. Soc. and Soc. Indust. Appl. Math., Providence, R.I., 1990. MR 1105122 (92d:05057). Zbl 727.90025.

§2: "The Klein bottle," surveys (1989a). (**SG: Top, Geom, Fr: Exp**)

1990b Homotopic routing methods. In: B. Korte, L. Lovász, H.J. Prömel, and A. Schrijver, eds., *Paths, Flows, and VLSI-Layout*, pp. 329–371. Algorithms and Combinatorics, Vol. 9. Springer-Verlag, Berlin, 1990. MR 1083385 (92f:68139). Zbl 732.90087.

§4: "Edge-disjoint paths in planar graphs," pp. 342–345, "The projective plane and the Klein bottle," surveys (1989a).
(**SG: Top, Geom, Fr: Exp**)

§3: "Edge-disjoint paths and multicommodity flows," pp. 334 ff. [This work suggests there may be a signed-graph generalization with the theorems discussed corresponding to all-negative signatures.]
(**par: Paths: Exp**)

1991a Disjoint circuits of prescribed homotopies in a graph on a compact surface. *J. Combin. Theory Ser. B* 51 (1991), 127–159. MR 1088630 (92a:05048). Zbl 723.05050.

§2: "An auxiliary theorem on linear inequalities," concerns feasibility of inequalities with coefficient matrix containing incidence matrix of $-\Gamma$. [See Hurkens (1989a).] (**ecyc: Incid**)

1991b (As "A. Skhreĭver") *Teoriya lineĭnogo i tselochislennogo programmirovaniya*, Vols. 1 and 2. Mir, Moscow, 1991. MR 1224001 (94c:90003), MR 1240318 (94g:90005).

Russian translation of (1986a). (**sg: par: Incid: Exp**)

2002a A short proof of Guenin's characterization of weakly bipartite graphs. *J. Combin. Theory Ser. B* 85 (2002), 255–260. MR 1912966 (2003e:05119). Zbl 1024.05079.

A streamlined proof of the theorem of Guenin (2001a).
(**SG: Geom, Str**)

2003a *Combinatorial Optimization: Polyhedra and Efficiency*. Vol. A, *Paths, Flows, Matchings*. Vol. B, *Matroids, Trees, Stable Sets*. Vol. C, *Disjoint Paths, Hypergraphs*. Algor. Combin., Vol. 24 A, B, C. Springer, Berlin, 2003. MR 1956924 (2004b:90004a), MR 1956925 (2004b:90004b), MR 1956926 (2004b:90004c). Zbl 1041.90001, Zbl 1072.90030.

Vol. A, Ch. 36, "Bidirected graphs".

Vol. B, §68.6b, "Bidirected graphs", includes incidence matrix and signed graph. §68.6c, "Characterizing odd-K_4-free graphs by mixing stable sets and vertex covers": "Odd-K_4" $= -K_4$ minor of $-\Gamma$.

Vol. C, Ch. 75, "Cuts, odd circuits, and multiflows". Signed graphs, weakly and strongly balanced ("bipartite") signed graphs. §75.2, "Signed graphs". Ch. 78, "Ideal hypergraphs". §80.4, "On characterizing binary ideal hypergraphs".

Dictionary: "positive, negative" edge = extraverted or introverted negative edge, "directed" edge = positive edge, "even, odd" = positive, negative (edge or circle), "bipartite" = balanced. [Annot. 9 Jun 2011, 31 Dec 2020.] (**sg: Ori: Incid, Geom**)

Vol. C, Ch. 76, "Homotopy and graphs on surfaces". [Annot. 9 Jun 2011.] (**gg: Top**)

Konrad Schrøder

1995a Mixed-sign conductor networks. REU paper, University of Washington, 1995. URL `http://www.math.washington.edu/~reu/papers/1993/schroder/sign.pdf`

Partial treatment of the problem in W. Johnson (2012a). [Annot. 26

Dec 2012.] **(sg: WG: Adj)**

Michael W. Schroeder
See R.A. Brualdi.

Michael Schubert
See also Y. Lu, E. Rollová, and E. Steffen.

Michael Schubert & Eckhard Steffen
2015a Nowhere-zero flows on signed regular graphs. *European J. Combin.* 48 (2015), 34–47. MR 3339010. Zbl 1315.05070. arXiv:1307.1562. **(SG: Ori, Flows)**

Michelle Schultz
See G. Chartrand.

Bernd Schulze
See K. Clinch.

Felix Schwagereit
See J. Kunegis.

Gary K. Schwartz
2002a On the automorphism groups of Dowling geometries. *Combin. Probab. Comput.* 11 (2002), no. 3, 311–321. MR 1909505 (2004c:20005). Zbl 1008.06007.

Aut $Q_n(\mathfrak{G})$ factors in a certain natural way if, but also only if, $\mathfrak{G}$ factors. [Succeeds Bonin (1995a). *Cf.* Sikirić, Felikson, and Tumarkin (2011a) for (mostly) more restricted related results.] [Annot. rev. 9 Apr 2016.] **(gg: Matrd: Aut)**

Roy Schwartz
See M. Charikar.

W. Schwärzler & D.J.A. Welsh
1993a Knots, matroids and the Ising model. *Math. Proc. Cambridge Philos. Soc.* 13 (1993), 107–139. MR 1188822 (94c:57019). Zbl 797.57002.

Tutte and dichromatic polynomials of signed matroids, generalized from Kauffman (1989a); this is the 2-colored case of Zaslavsky's (1992b) strong Tutte functions of colored matroids. [For terminology see Zaslavsky (1992b).] Applications to knot theory.

§2, "A matroid polynomial", is foundational. Prop. 2.1 characterizes strong Tutte functions of signed matroids by two equations connecting their parameters and their values on signed coloops and loops. [If the function is 0 on positive coloops, the proof is incomplete and the functions $= 0$ except on $M = \varnothing$ are missed.] Prop. 2.2: The Tutte (basis-expansion) polynomial of a function W of signed matroids is well defined iff W is a strong Tutte function. Eq. (2.8) says $W =$ the rank generating polynomial Q_Σ (here also called W) if certain variables are nonzero; (2.9) shows there are only 3 essential variables since, generically, only the ratio of parameters is essential [an observation that applies to general strong Tutte functions]. Prop. 2.5 computes Q_Σ of a 2-sum.

§3 adapts Q_Σ to Kauffman's (1989a) and Murasugi's (1989a) signed-graph polynomials and simplifies some of the latter's results (esp. his chromatic degree). §4, "The anisotropic Ising model", concerns the Hamiltonian of a state of a signed graph. The partition function is essentially an evaluation of Q_Σ. §5, "The bracket polynomial", and §6, "The span

of the bracket polynomial": Certain substitutions reduce Q_Σ to 1 variable; its properties are examined, esp. in light of knot-theoretic questions. Thm. 6.4 characterizes signed matroids with "full span" (a degree property). §7, "Adequate and semi-adequate link diagrams", generalizes those notions to signed matroids. §8, "Zero span matroids": when does span(bracket) $= 0$? Yes if $M = M(\Sigma)$ where Σ reduces by Reidemeister moves to K_1, but the converse is open (and significant if true).
(**Sc(Matrd), SGc: Invar, Knot, Phys**)

Allen J. Schwenk
See Harary, Palmer, Robinson, and Schwenk (1977a).

Thomas Schweser & Michael Stiebitz
2017a Degree choosable signed graphs. *Discrete Math.* 340 (2017), no. 5, 882–891. MR 3612419. Zbl 1357.05055. arXiv:1507.04569. (**SG: Col**)

Irene Sciriha
See also N. Basic and F. Belardo.

Irene Sciriha & Luke Collins
2019a Two-graphs and NSSDs: An algebraic approach. *Discrete Appl. Math.* 266 (2019), 92–102. MR 3991601. Zbl 1464.05250.
Signed K_n with two eigenvalues, especially negatives of each other, leading to properties of conference graphs. "NSSD" = non-singular graph with a singular deck. [Annot. 4 Jun 2018, 11 Jul 2022.]
(**sg: kg, Adj: TG, Sw, Eig**)

Matt Scobee
See J. Lee.

Alexander D. Scott & Alan D. Sokal
2009a Some variants of the exponential formula, with application to the multivariate Tutte polynomial (alias Potts model). *Sém. Lotharingien Combin.* 61A (2009), art. B61Ae, 33 pp. MR 2529396 (2010i:05167). Zbl 1283.05138.
Cf. Sokal (2005a). (**SGw: Gen: Invar**)

András Sebö
See also F. Meunier and B. Novick.
1990a Undirected distances and the postman-structure of graphs. *J. Combin. Theory Ser. B* 49 (1990), 10–39. MR 1056818 (91h:05049). Zbl 638.05032.
See A. Frank (1996a). (**SGw: Str**)

Etsuo Segawa & Yusuke Yoshie
2021a Quantum search of matching on signed graphs. *Quantum Inform. Process.* 20 (2021), no. 5, art. 182, 23 pp. MR 4259185. arXiv:2007.07223.
Cf. Brown, Godsil, Mallory, Raz, Tamon (2013a). (**SG: KG**)

Deepak Sehrawat & Bikash Bhattacharjya
2019a Some bounds on the double domination of signed generalized Petersen graphs and signed I-graphs. *Indian J. Discrete Math.* 5 (2019), no. 2, 63–75. MR 4317623. arXiv:1907.11099.
Cf. Ashraf and Germina (2016a). Cubic: $\gamma_{\times 2}(\Sigma) \geq n/2$. k-step generalized Petersen graph $P_{m,k}$: upper bounds for $k = 1$, $k > 1$ and $\gcd(m,k) = 1$ or > 1, are $\approx m+2$, $\frac{3}{2}m$, $\frac{4}{3}m$. Similar bounds for I-

graphs. "I-graph": a bipartite union of 3 particular matchings. [Annot. 26 Dec 2020.] **(SG: Sw, Dom)**

2019b Maximum frustration in signed generalized Petersen graphs. *Indian J. Discrete Math.* 5 (2019), no. 2, 77–93. MR 4317624. arXiv:1905.05548.

$D(P_{m,k}) \leq 1 + \lfloor m/2 \rfloor$ if $d := \gcd(m,k) = 1$, $\leq 1 + d(1 + \lfloor m/2d \rfloor)$ if $d > 1$, thus improving Sivaraman (2012a) for these graphs. [Annot. 26 Dec 2020.] **(SG: Fr)**

2020a Signed complete graphs on six vertices and their frustration indices. *Adv. Appl. Discrete Math.* 24 (2020), no. 2, 129–142. Zbl 1499.05261. arXiv:1812.08383.

Thm. 4.2: There are 16 switching isomorphism types. [Previously found as "two-graphs", *cf.* Bussemaker, Mathon, and Seidel (1979a), (1981a).] New: Proof, unique minimal representatives, and (Thms. 5.1, 5.2) frustration indices l and frustration numbers l_0. E.g., $\max l = 6$ and $\max l_0 = 4$, uniquely for $E^- = C_3 \cup C_3$. [This is useful information.] [Annot. 16 Apr 2019, 28 Dec 2020.] **(SG: tg: Sw, Fr)**

2021a Non-isomorphic signatures on some generalised Petersen graph. *Electronic J. Graph Theory Appl.* 9 (2021), no. 2, 235–255. MR 4331324. Zbl 1482.05141. arXiv:1812.11360.

Number of switching isomorphism types of $(P_{2n+1,1}, \sigma)$. [Annot. 16 Apr 2019.] **(SG: Sw: Enum)**

2022a Chromatic polynomials of signed book graphs. *Theory Appl. Graphs* 9 (2022), no. 1, art. 4, 11 pp. MR 4398619. Zbl 1486.05103. arXiv:2206.08580 (arXiv:-1812.08382).

Book graph $B(m,n) :=$ union of n circles C_m at a single common edge. # switching isomorphism types $= n+1$ (depending on number of negative C_m's). Explicit formulas for chromatic and zero-free chromatic polynomials. [Annot. 16 Apr 2019, rev 24 Mar 2022.] **(SG: Sw, Invar)**

20xxb RNA number of some parity signed generalized Petersen graphs. Submitted. arXiv:2110.03264.

See Acharya and Kureethara (2021a) for *rna* number. **(Lab: SG)**

20xxc On the *rna* number of generalized Petersen graphs. *Commun. Combin. Optim.* (in press).

See Acharya and Kureethara (2021a) for *rna* number. **(Lab: SG)**

20xxd Enumeration of switching non-isomorphic signed wheels. arXiv:2106.01265. **(SG: Sw: Enum)**

J.J. Seidel

See also F.C. Bussemaker, P.J. Cameron, P.W.H. Lemmens, and J.H. van Lint.

1968a Strongly regular graphs with $(-1, 1, 0)$ adjacency matrix having eigenvalue 3. *Linear Algebra Appl.* 1 (1968), 281–298. MR 0234861 (38 #3175). Zbl 159.25403 (159, p. 254c). Reprinted in (1991a), pp. 26–43. **(tg)**

1969a Strongly regular graphs. In: W.T. Tutte, ed., *Recent Progress in Combinatorics* (Proc. Third Waterloo Conf. on Combinatorics, 1968), pp. 185–198. Academic Press, New York, 1969. MR 0253935 (54 #10047). Zbl 191.55202 (191, p.

552b). (**TG**)

1974a Graphs and two-graphs. In: F. Hoffman *et al.*, eds., *Proceedings of the Fifth Southeastern Conference on Combinatorics, Graph Theory, and Computing* (Boca Raton, 1974), pp. 125–143. Congressus Numerantium X. Utilitas Math. Publ. Inc., Winnipeg, Man., 1974. MR 0364028 (51 #283). Zbl 308.05120.
Cf. Mallows and Sloane (1975a). (**TG**)

† 1976a A survey of two-graphs. In: *Colloquio Internazionale sulle Teorie Combinatorie* (Roma, 1973), Tomo I, pp. 481–511. Atti dei Convegni Lincei, No. 17. Accad. Naz. Lincei, Rome, 1976. MR 0550136 (58 #27659). Zbl 352.05016. Reprinted in (1991a), pp. 146–176. (**TG: Adj, Eig, Cov, Aut**)

1978a Eutactic stars. In: A. Hajnal and Vera T. Sós, eds., *Combinatorics* (Proc. Fifth Hungar. Colloq., Keszthely, 1976), Vol. 2, pp. 983–999. Colloq. Math. Soc. János Bolyai, 18. North-Holland, Amsterdam, 1978. MR 0519322 (80d:05016). Zbl 391.05050.

1979a The pentagon. In: Allan Gewirtz and Louis V. Quintas, eds., Second Int. Conf. Combin. Math. (New York, 1978). *Ann. New York Acad. Sci.* 319 (1979), 497–507. MR 0556060 (81e:05004). Zbl 417.51005. (**TG: Adj, Eig**)

1979b The pentagon. In: P.C. Baayen *et al.*, eds., *Proceedings, Bicentennial Congress, Wiskundig Genootschaap* (Amsterdam, 1978), Part I, pp. 80–96. Mathematical Center Tracts, 100. Mathematisch Centrum, Amsterdam, 1979. MR 0541389 (80f:51008). Zbl 417.51005.
Same as (1979a), with photograph. (**TG: Adj, Eig**)

1991a *Geometry and Combinatorics: Selected Works of J.J. Seidel.* D.G. Corneil and R. Mathon, eds. Academic Press, Boston, 1991. MR 1116326 (92m:01098). Zbl 770.05001.
Reprints many articles on two-graphs and related systems. (**TG: Sw, Adj, Eig, Geom**)

1992a More about two-graphs. In: Jaroslav Nešetřil and Miroslav Fiedler, eds., *Fourth Czechoslovakian Symposium on Combinatorics, Graphs and Complexity* (Prachatice, 1990), pp. 297–308. Ann. Discrete Math., Vol. 51. North-Holland, Amsterdam, 1992. MR 1206283 (94h:05040). Zbl 764.05036. (**TG: Exp, Ref**)

1995a Geometric representations of graphs. *Linear Multilinear Algebra* 39 (1995), 45–57. MR 1374470 (97e:05149a). Zbl 832.05079. Errata. *Ibid.* 39 (1995), 405. MR 1399446 (97e:05149b). Zbl 843.05078.
§4, "Signed graphs": The "intersection matrix" $A+2I$ of a signed simple graph is the Gram matrix of a set of "root vectors" with respect to an "inner product" that may not be positive definite. Explains origin of local switching (*cf.* Cameron, Seidel, and Tsaranov (1994a) and Bussemaker, Cameron, Seidel, and Tsaranov (1991a)). For a signed complete graph, $A+3I$ represents lines at angles $\cos^{-1} 1/3$; it is positive semidefinite only for few graphs, which are classified (implicit in Lemmens and Seidel (1973a)). (**SG: Adj, Eig, Geom: Exp**)

1995b Discrete non-Euclidean geometry. In: F. Buekenhout, ed., *Handbook of Incidence Geometry: Buildings and Foundations*, Ch. 15, pp. 843–920. North-Holland (Elsevier), Amsterdam, 1995. MR 1360730 (96m:52001). Zbl 826.51012.

§3.2: "Equidistant sets in elliptic $(d-1)$-space." §3.3: "Regular two-graphs." **(TG: Adj, Eig, Geom: Exp)**

J.J. Seidel & D.E. Taylor

1981a Two-graphs, a second survey. In: L. Lovász and Vera T. Sós, eds., *Algebraic Methods in Graph Theory* (Proc. Int. Colloq., Szeged, 1978), Vol. II, pp. 689–711. Colloq. Math. Soc. János Bolyai, 25. János Bolyai Math. Soc., Budapest, and North-Holland, Amsterdam, 1981. MR 0642068 (83f:05070). Zbl 475.05073. Reprinted in (1991a), pp. 231–254. **(TG)**

J.J. Seidel & S.V. Tsaranov

1990a Two-graphs, related groups, and root systems. Algebra, Groups and Geometry. *Bull. Soc. Math. Belg. Ser. A* 42 (1990), 695–711. MR 1316218 (95m:20046). Zbl 736.05048.

A group $\mathrm{Ts}(\Sigma)$ is defined from a signed complete graph Σ: its generators are the vertices and its relations are $(uv^{-\sigma(uv)})^2 = 1$ for each edge uv. It is invariant under switching, hence determined by the two-graph of Σ. A certain subgraph of a Coxeter group of a tree T is isomorphic to $\mathrm{Ts}(\Sigma)$ for suitable Σ_T constructed from T. [Generalized in Cameron, Seidel, and Tsaranov (1994a). More on Σ_T under Tsaranov (1992a). The construction of Σ_T is simplified in Cameron (1994a).] **(TG: Adj, Geom)**

Chelliah Selvaraj

See also M. Parvathi.

2007a Factor algebras of signed Brauer's algebras. *Kyungpook Math. J.* 47 (2007), no. 4, 549–568. MR 2397479 (2009b:16076). Zbl 1187.16013. **(gg: Algeb, matrd)**

See also T. Fife.

Charles Semple & Geoff Whittle

1996a Partial fields and matroid representation. *Adv. Appl. Math.* 17 (1996), 184–208. MR 1390574 (97g:05046). Zbl 859.05035.

§7: "Dowling group geometries". A Dowling geometry of a group $\mathfrak{G}$ has a partial-field representation iff G is abelian and has at most one involution. [The condition is necessary but insufficient; see Vertigan (2015a), or Pendavingh and van Zwam (2013a), p. 225.] **(gg: Matrd: Incid)**

Parongama Sen

See B.K. Chakrabarti.

Sagnik Sen

See also J. Bensmail, L. Beaudou, S. Das, P. Ochem and R. Naserasr.

2014a *A Contribution to the Theory of Graph Homomorphisms and Colorings*. Doctoral dissertation, Univ. de Bordeaux, 2014. HAL tel-00960893.

Ch. 5, "Signified graphs". §5.3, "Signified coloring" [coloring is strange; unrelated to Zaslavsky (1982b)]. Ch. 6, "Signed graphs". §6.3, "Signed coloring" (mostly homomorphisms). Dictionary: Very strange terminology due in part to avoiding use of signs. "Signified" = signed, "signed" = switching class, "unbalanced path" = negative path, "odd" = antibalanced, "even" = bipartite, etc. [Annot. 13 Oct 2021.] **(SG: Col, Hom, Sw)**

Sylvain Sené

See J. Demongeot and M. Noual.

Masakazu Sengoku

1974a On hybrid tree graphs. *Electronic Commun. Japan* 57 (1974), no. 5, 18–23. MR 0456991 (56 #15210).

A signed graph derived from trees and cotrees is balanced. [Annot. 24 July 2010.] **(SG: Bal)**

Seunghyun Seo

2012a Shi threshold arrangement. *Electronic J. Combin.* 19 (2012), no. 3, art. P39, 9 pp. MR 2988861. Zbl 1257.52009.

The characteristic polynomial of the Shi threshold arrangement $\{x_i + x_j = 0, 1 : i < j\}$, computed modulo a large prime (the "finite field method", Athanasiadis (1996a)). [Annot. 14 Mar 2013, corr 22 Jan 2020.] **(gg: Geom, Invar)**

2017a The Catalan threshold arrangement. *J. Integer Seq.* 20 (2017), art. 17.1.1, 12 pp. MR 3606971. Zbl 1354.52026.

The characteristic polynomial of the Catalan threshold arrangement $\{x_i + x_j = 0, \pm 1 : i \neq j\}$, computed modulo a large prime. [Annot. 20 Feb 2020.] **(gg: Geom, Invar)**

B. Seoane

See L.A. Fernández.

Han-Guk Seol

See S.-G. Lee.

Mark R. Sepanski

See I.B. Michael.

Jean-Sébastien Sereni

See D. Král'.

Ákos Seress

See P. Brooksbank.

Ritesh Seth

See S. Das.

Anshu Sethi

See D. Sinha.

James P. Sethna

2006a *Statistical Mechanics: Entropy, Order Parameters, and Complexity*. Oxford Master Ser. in Physics, Vol. 14. Oxford Univ. Press, Oxford, 2006. Zbl 1140.82004.

Textbook. P. 12, fn. 16: Frustration index ("spin-glass ground states") is polynomially equivalent to graph coloring. §12.3.4, "Glassy systems: random but frozen", mentions frustration due to negative circles ("a loop with an odd number of antiferromagnetic couplings"). It is not yet known how many equilibrium [ground?] states exist. Fig. 12.17, "Frustration": An all-negative triangle with Ising spins (± 1). [Annot. 28 Aug 2012.]

(**Phys: SG: Fr, State(fr): Exp**)

E.C. Sewell

1996a Binary integer programs with two variables per inequality. *Math. Programming* 75 (1996), Ser. A, 467–476. MR 1422181 (97m:90059). Zbl 874.90138.

See Johnson and Padberg (1982a) for definitions. §2, "Equivalence to stable set problem": Optimization on the bidirected stable set polytope is reduced to optimization on a stable set polytope with no more variables. Results of Bourjolly (1988a) and Hochbaum, Megiddo, Naor, and Tamir (1993a) can thereby be explained. §3, "Perfect bigraphs", proves the conjectures of Johnson and Padberg (1982a): a transitively closed bidirection of a simple graph is perfect iff its underlying graph is perfect. [Also proved by Ikebe and Tamura (20xxa).] Dictionary: "Bigraph" = bidirected graph B. "Stable" set in B = vertex set inducing no introverted edge. (**SG: Ori: Incid, Geom, sw**)

E.C. Sewell & L.E. Trotter, Jr.

1993a Stability critical graphs and even subdivisions of K_4. *J. Combin. Theory Ser. B* 59 (1993), 74–84. MR 1234384 (94f:05122). Zbl 793.05133.

"Even subdivision of K_4" = $|\Sigma|$ where Σ is an all-negative subdivision of $-K_4$. (**sg: par: Str**)

1995a Stability critical graphs and ranks facets of the stable set polytope. *Discrete Math.* 147 (1995), 247–255. MR 1364517 (96g:05077). Zbl 838.05068. (**sg: par: Str**)

P.D. Seymour

See also M. Chudnovsky, J. Geelen, Gerards, Lovász, *et al.* (1990a), W. McCuaig, B. Mohar, and N. Robertson.

1974a On the two-colouring of hypergraphs. *Quart. J. Math. Oxford* (2) 25 (1974), 303–312. MR 0371710 (51 #7927). Zbl 299.05122. (**sd: Par: bal**)

1977a The matroids with the max-flow min-cut property. *J. Combin. Theory Ser. B* 23 (1977), 189–222. MR 0462996 (57 #2960). Zbl 375.05022.

The central example is $Q_6 = \mathcal{C}^-(-K_4)$, the clutter of (edge sets of) negative circles in $-K_4$. P. 199: the extended lift matroid $\mathbf{L}_\infty(-K_4) = F_7^*$, the dual Fano matroid. Result (3.4) readily generalizes (by the negative-subdivision trick) to: every $\mathcal{C}^-(\Sigma)$ is a binary clutter, that is, a port of a binary matroid. [This is also immediate from the construction of $\mathbf{L}_\infty(\Sigma)$.]

P. 200, (i)–(iii): Amongst minor-minimal binary clutters without the "weak MFMC property" are the circuit clutter of F_7^* and $\mathcal{C}^-(-K_5)$ and its blocker.

Main Thm. (§5): A binary clutter is "Mengerian" (I omit the definition) iff it does not have $\mathcal{C}^-(-K_4)$ as a minor. (See p. 200 for the antecedent theorem of Gallai.)

[See Cornuéjols (2001a), Guenin (2001a) for more.]

(**sg, Par: Matrd, Geom**)

1980a Decomposition of regular matroids. *J. Combin. Theory Ser. B* 28 (1980), 305–359. MR 0579077 (82j:05046). Zbl 443.05027.

The crucial matroid $R_{10} = \mathbf{F}(-K_5)$. [Annot. 3 Aug 2019.]
(sg: par: Matrd)

1981a Matroids and multicommodity flows. *European J. Combin.* 2 (1981), 257–290. MR 0633121 (82m:05030). Zbl 479.05023.

Conjecture (based on (1977a)). A binary clutter has the weak MFMC property iff no minor is either the circuit clutter of F_7 or $\mathcal{C}^-(-K_5)$ or its blocker. **(sgnd(Matrd), sg: Matrd)**

† 1995a Matroid minors. In: R.L. Graham, M. Grötschel, and L. Lovász, eds., *Handbook of Combinatorics*, Vol. I, Ch. 10, pp. 527–550. North-Holland (Elsevier), Amsterdam, and MIT Press, Cambridge, Mass., 1995. MR 1373666 (97a:05055). Zbl 960.24825.

In Thm. 6.6, p. 546, interpreting G as a signed graph and an "odd-K_4" as a subdivision of $-K_4$ gives the signed graph generalization, due to Gerards and Schrijver (1986a) [also Gerards (1990a), Thm. 3.2.3]. Let Σ be a signed simple, 3-connected graph in which no 3-separation has > 4 edges on both sides. Then Σ has no $-K_4$ minor iff either (i) deleting some vertex makes it balanced (the complete lift matroid of this type is graphic); or (ii) it is cylindrical: it can be drawn on a cylindrical surface that has a lengthwise red line so that an edge is negative iff it crosses the red line an odd number of times [Note: the extended lift matroid of this type is cographic, as observed by, I think, Gerards and Schrijver or by Lovász]. [See Pagano (1998a) for another use of cylindrical signed graphs.] [*Problem.* Find the forbidden topological subgraphs, link minors, and $Y\Delta$ graphs for cylindrical signed graphs.] [*Question.* Embed a signed graph in the plane with k distinguished faces so that a circle's sign is the parity of the number of distinguished faces it surrounds. Cylindrical embedding is $k = 1$. For each k, which signed graphs are so embeddable?] **(SG: Str, Top)**

Thm. 6.7, pp. 546–547, generalizes to signed graphs, interpreting G as a signed graph and an "odd cycle" as a negative circle. Take a signed simple, 3-connected, internally 4-connected graph. It has no two vertex-disjoint negative circles iff it is one of four types: (i) deleting some vertex makes it balanced; (ii) deleting the edges of an unbalanced triangle makes it balanced; (iii) it has order ≤ 5; (iv) it can be orientation-embedded in the projective plane. This is due to Lovász; see, if you can, Gerards, Lovász, *et al.* (1990a). [A 2-connected Σ has no vertex-disjoint negative circles iff $\mathbf{F}(\Sigma)$ is binary iff $\mathbf{F}(\Sigma)$ is regular iff the lift matroid $\mathbf{L}(\Sigma)$ is regular. See Pagano (1998a) for classification of Σ with vertex-disjoint negative circles according to representability of the frame matroid.]
(SG: Str, matrd, Top)

Paul Seymour & Carsten Thomassen

1987a Characterization of even directed graphs. *J. Combin. Theory Ser. B* 42 (1987), 36–45. MR 0872406 (88c:05089). Zbl 607.05037.

"Even" means every signing contains a positive cycle. A digraph is even iff it contains a subdigraph that is obtained from a symmetric odd-circle digraph by subdivision and a vertex-splitting operation. [*Cf.* Thomassen

(1985a).] **(sd: par: Str)**

L. de Sèze
See J. Vannimenus.

Bryan L. Shader
See R.A. Brualdi, S. Butler, and D.A. Gregory.

Nisarg Shah
See M. Joglekar.

[Siamak Fayyaz Shahandashti]
See S. Fayyaz Shahandashti (under 'F').

Mohsen Shahriari
See also S.R. Shahriary.

Mohsen Shahriari & Ralf Klamma

2015a Signed social networks: Link prediction and overlapping community detection. In: *Proceedings of the 2015 IEEE/ACM International Conference on Advances in Social Networks Analysis and Mining* (ASONAM '15, Paris, 2015), pp. 1608–1609. ACM, New York, 2015. **(SG: Clu(Gen), Fr(Gen), Pred: Algor)**

Mohsen Shahriari, Omid Askari Sichani, Joobin Gharibshah, & Mahdi Jalili

2016a Sign prediction in social networks based on users reputation and optimism. *Social Network Analysis Mining* 6 (2016), art. 91, 10 pp.

Ranking vertices by "reputation", $\mathrm{RR}(v_i)) := (d^+_{\text{in}}(v_i) - d^-_{\text{in}}(v_i))/(d^+_{\text{in}}(v_i) + d^-_{\text{in}}(v_i))$, and "optimism", $\mathrm{OP}(v_i)) := (d^+_{\text{out}}(v_i) - d^-_{\text{out}}(v_i))/(d^+_{\text{out}}(v_i) + d^-_{\text{out}}(v_i))$, produces better predictions of arc signs. [Annot. 22 Sept 2018.] **(SD: Pred: Algor)**

Saeed Reza Shahriary, Mohsen Shahriari, & Rafidah MD Noor

2015a A community-based approach for link prediction in signed social networks. *Sci. Programming* 2015 (2015), art. 602690, 10 pp. **(SG: Pred: Algor, PsS)**

Naomi Shaked-Monderer
See also A. Berman.

2016a SPN graphs: when copositive = SPN. *Linear Algebra Appl.* 509 (2016), 82–113. MR 3546402. Zbl 1345.05111. arXiv:1604.02172, arXiv:1712.05115. **(SG: QM)**

2018a Corrigendum to "SPN graphs: when copositive = SPN". *Linear Algebra Appl.* 541 (2018), 285–286. MR 3742623. Zbl 1430.05127. arXiv:1712.05115. **(SG: QM)**

Sophia Shalini G.B.Sophia Shalini G.B., Anwar Saleh, & Dhananjayamurthy B.V

2019a On the Seidel energy of certain mesh derived networks. *Int. J. Engineering Appl. Tech.* 9 (2019), no. 2, 1905–1910. **(sg: KG: Adj: Eig)**

Tahir Shamsher
See also S. Pirzada.

Tahir Shamsher, Mushtaq A. Bhat, & S. Pirzada

20xxa Unicyclic signed graphs with first eleven minimal energies. Submitted. **(SG: Adj: Eig)**

Tahir Shamsher, S. Pirzada, & Mushtaq A. Bhat

2023a On adjacency and Laplacian cospectral switching non-isomorphic signed graphs. *Ars Math. Contemp.* 23 (2023), no. 3, art. 9, 20 pp. MR 4548748. Zbl 1509.05086. arXiv:2205.08705.

Cospectrality of $A(\Sigma_1)$ with $L(\Sigma_2)$. (**SG: Adj, Lap: Eig**)

Hai-Ying Shan

See also J.-Y. Shao and L. You.

Hai-Ying Shan & Jia-Yu Shao

2004a Matrices with totally signed powers. *Linear Algebra Appl.* 376 (2004), 215–224. MR 2015535 (2004i:15030). Zbl 1055.15036. (**QM: SD**)

Ronghua Shang

See J.S. Wu.

Jia-Yu Shao

See also Y. Liu, R. Manber, H.Y. Shan, and L. You.

1998a On digraphs and forbidden configurations of strong sign nonsingular matrices. *Linear Algebra Appl.* 282 (1998), 221–232. MR 1648336 (99h:05086). Zbl 940.05044.

"Strong sign nonsingular digraph": every cycle is positive and all vw-paths have the same sign. [A natural class in signed digraph theory. *Problem.* How close are their properties to those of balanced signed graphs?] [Annot. 17 Oct 2023.] (**SD, QSol: Str**)

2000a On the digraphs of sign solvable linear systems. *Linear Algebra Appl.* 313 (2000), 115–126. MR 1770361 (2001e:05083). Zbl 958.15003.

Forbidden subgraphs are used to characterize the signed digraphs. [Annot. 6 Mar 2011.] (**SD: QSol**)

Jia-Yu Shao, Jin-Ling He, & Hai-Ying Shan

2003a Number of nonzero entries of S^2NS matrices and matrices with signed generalized inverses. *Linear Algebra Appl.* 373 (2003), 223–239. MR 1648336 (99h:05086). Zbl 1036.15005.

Cf. Jia-Yu Shao (1998a). (**SG: QSol**)

Jiayu Shao & Zhixiang Hu

2000a Characterizations of some classes of strong sign nonsingular digraphs. *Discrete Appl. Math.* 105 (2000), no. 1-3, 159–172. MR 1780469 (2001d:05072). Zbl 965.05050.

Cf. Jia-Yu Shao (1998a). Forbidden configurations for this property to hold for signed digraphs of two types. [Annot. 17 Oct 2023.] (**SD, QSol: Str**)

Yanling Shao

See also Y.-B. Gao.

Yanling Shao & Yubin Gao

2009a The local bases of primitive non-powerful signed symmetric digraphs with loops. *Ars Combin.* 90 (2009), 357–369. MR 2489538 (2010c:05054). Zbl 1224.05223. (**SD, sg: qm**)

Yanling Shao, Jian Shen, & Yubin Gao

2009a The kth upper bases of primitive non-powerful signed digraphs. *Discrete Math.* 309 (2009), no. 9, 2682–2686. MR 2523775 (2010h:05144). Zbl 1207.05073. **(SD: qm)**

2012a The kth upper and lower bases of primitive nonpowerful minimally strong signed digraphs. *Linear Multilinear Algebra* 60 (2012), no. 9, 1093–1113. MR 2966153. Zbl 1252.05077. **(SD: qm)**

Zhenan Shao & Xiying Yuan

2022a Some signed graphs whose eigenvalues are main. *Appl. Math. Comput.* 423 (2022), art. 127014, 9 pp. MR 4385150. arXiv:2106.07878. **(SG: Adj: Eig)**

Deepakshi Sharma

See D. Sinha.

[Pranjali Sharma]

See Pranjali.

Ram Parkash Sharma & Vikram Singh Kapil

2011a Irreducible $\overrightarrow{S_n}$-modules and a cellular structure of the signed Brauer algebras. *Southeast Asian Bull. Math.* 35 (2011), no. 3, 497–522, MR 2856396 (2012h:16003). Zbl 1240.20010. **(gg: Algeb, matrd)**

Tushar Sharma, Ankit Charls, & P.K. Singh

2011a Community mining in signed social networks –An automated approach. In: *2009 International Conference on Computer Engineering and Applications* (ICCEA 2009, Manila), art. 28, pp. 152–157. Int. Proc. Computer Sci. Inform. Tech. (IPCSIT), Vol. 2. IACSIT Press, Singapore, 2011. URL `http://www.ipcsit.com/vol2/28-A217.pdf` **(SG: Clu: Algor)**

John Shawe-Taylor

See T. Pisanski.

Bo Shen

See Q. Cai.

Jian Shen

See Y.-B. Gao and Y.L. Shao.

Xiao Shen & Fu-Lai Chung

2020a Deep network embedding for graph representation learning in signed networks. *IEEE Trans. Cybernetics* 50 (2020), no. 4, 1556–1568. arXiv:1901.01718.

Algorithm for reconstruction assuming few negative edges. [Annot. 5 Feb 2022.] **(SG: Algor)**

Yi-Huang Shen & Guangjun Zhu

2023a Regularity of powers of (parity) binomial edge ideals. *J. Algebraic Combin.* 57 (2023), 75–100. MR 4544259. Zbl 1509.13036. arXiv:2111.14175.

A hint of $\mathbf{F}(-\Gamma)$ in the parity binomial edge ideal. [Annot. 5 Feb 2023.] **(Algeb: sg: par)**

Jia Sheng & MiaoLin Ye

2010a The spectral radius of signless Laplacian of a connected graph with given independence number. *Math. Appl. (Wuhan)* 23 (2010), no. 4, 709–712. MR

2765865 (no rev). **(par: Lap: Eig)**

Stephen H. Shenker
See E. Fradkin.

F.B. Shepherd
See A.M.H. Gerards and T.R. Jensen.

Laura Sheppardson
See T. Lewis.

R. Sherkati
See S. Akbari.

Steven J. Sherman
See R.B. Zajonc.

David Sherrington & Scott Kirkpatrick
See also S. Kirkpatrick.

1975a Solvable model of a spin-glass. *Phys. Rev. Lett.* 35 (1975), no. 26, 1792–1796. Repr. in M. Mézard, G. Parisi, and M.A. Virasoro, *Spin Glass Theory and Beyond*, pp. 104–108. World Scientific Lect. Notes in Physics, Vol. 9. World Scientific, Singapore, 1987.

Introduces the Sherrington–Kirkpatrick spin-glass model, a randomly signed and (usually) weighted K_n. Announcement of part of Kirkpatrick and Sherrington (1978a) (*q.v.*). [Annot. 22 Aug 2012.] **(Phys: sg: Fr)**

Ronald G. Sherwin

1975a Structural balance and the sociomatrix: Finding triadic valence structures in signed adjacency matrices. *Human Relations* 28 (1975), 175–189.

A very simple [but not efficient] matrix algorithm for counting different types of circles in a signed (di)graph. ["Valence" means sign, unfortunately.] **(sg, SD: Bal, Circ: Invar: Algor)**

Jeng-Horng Sheu
See I. Gutman.

Chuan-Jin Shi

1992a A signed hypergraph model of constrained via minimization. In: *VLSI, 1992. Proceedings of the Second Great Lakes Symposium on VLSI* (Kalamazoo, Mich., 1992), pp. 159–166. IEEE, 1992. **(SH: Appl)**

1992b A signed hypergraph model of constrained via minimization. *Microelectronics J.* 23 (1992), no. 7, 533–542. **(SH: Appl)**

1993a Constrained via minimization and signed hypergraph partitioning. In: D.T. Lee and M. Sarrafzadeh, eds., *Algorithmic Aspects of VLSI Layouts*, pp. 337–356. World Scientific, Singapore, 1993. **(SH: Appl: Exp)**

1993b *Optimum Logic Encoding and Layout Wiring for VLSI Design: A Graph-Theoretic Approach*. Ph.D. thesis, University of Waterloo, 1993.
(SH: Incid, Bal, Algor, SG, Appl)

C.-J. Shi & J.A. Brzozowski

1999a A characterization of signed hypergraphs and its applications to VLSI via minimization and logic synthesis. *Discrete Appl. Math.* 90 (1999), no. 1-3, 223–243. MR 1666019 (99m:68155). Zbl 913.68104.

A signed hypergraph $H = (V, E, \psi)$ is a hypergraph (V, E) with an incidence signature $\psi : V \times E \to \{-1, 0, 1\}$. "Underlying graph" = bipartite incidence graph with edge signs ψ. Sign of a path [or walk] = product of incidence signs. Motivation: via minimization, i.e., minimize the number of connections between different planar layers of a two-layer circuit. [See Rusnak (2010a) for a different development of the same definitions. Path signs are different; the normal sign for signed graphs has an extra factor -1 for each edge.] e is "balanced" by a bipartition $V = V_1 \uplus V_2$ when incidences of e are in the same V_i iff they have the same sign. H is "balanced" if some bipartition balances every edge. Thm. 3.1: H is balanced iff every circle is positive. [I.e., antibalance, since walk signs are different from the norm.] Proof: Constructive [similar to but less exact than algorithms for signed graphs as in Harary and Kabell (1980a)], yielding Cor. 3.1: Testing balance takes linear time. Thm. 3.2: H is balanced iff its incidence dual is balanced. "Maximum balance problem": Minimize the number of unbalanced edges. Thm. 4.1: This is NP-complete, even for cubic graphs. [Known, as it contains the max-cut problem.] Thm. 4.2: NP-complete for planar signed hypergraphs with maximum degree > 3. (For max degree ≤ 3, polynomial-time algorithms are given in Shi (1993b).) *Problem*: Minimum Covering: Find the minimum number of bipartitions of V such that every edge is balanced by one of the bipartitions. Equivalently, decompose H into the smallest number of balanced subhypergraphs. [See Zaslavsky (1987b) for signed graphs.] Thm. 5.1: NP-complete. Proof: Reduction to graph colorability via decomposability of a graph into bipartite subgraphs [special case of signed-graph decomposition as in Zaslavsky (1987b)].

§6, "Constrained via minimization", summarizes connection with signed hypergraphs, based on Shi (1992a), (1992b). §7, "Constrained logic encoding".

§8, "Related notions: Signed graphs and $(0, \pm 1)$-matrices". §8.1, "Harary's signed graphs", compares their work with Harary (1953a) [no mention of Harary and Kabell (1980a)]. §8.2, "Restricted unimodularity and balanced $(0, \pm 1)$ matrices": The incidence matrix of $\mathrm{H}(H)$ if H is a graph [$\mathrm{H}(-H)$ in the normal definition] is totally unimodular iff $-H$ is balanced [essentially, Heller and Tompkins (1956a)].

[All problems and methods are equivalent to the similar problems for the signed graph derived by replacing each hyperedge by a balanced complete graph with Harary bipartition given by the sign bipartition of the hyperedge's incidences.] [Annot. 4 Nov 2010.]

(**SH: Incid, Bal, Algor, SG**)

C.-J. Shi, A. Vannelli, & J. Vlach

1990a A hypergraph partitioning approach to the via minimization problem. In: *Proceedings of the Canadian Conference on VLSI* (1990), pp. 2.7.1–2.7.8.

(**SH: sg: Bal**)

1997a Performance-driven layer assignment by integer linear programming and path-constrained hypergraph partitioning. *J. Heuristics* 3 (1997), no. 3, 225–243.

Zbl 1071.90584. **(SH: sg: Bal, Algor: Appl)**

Guodong Shi, Claudio Altafini, & John S. Baras

2017a Algebraic-graphical approach for signed dynamical networks. In: *2017 IEEE 56th Annual Conference on Decision and Control (CDC)* (CDC2017, Melbourne, 2017), pp. 2009–2014. IEEE, 2017. **(SG: Dyn)**

Guodong Shi, Alexandre Proutiere, Mikael Johansson, John S. Baras, & Karl H. Johansson

2013a Emergent behaviors over signed random networks in dynamical environments. Manuscript, 2013. arXiv:1309.5488. **(SG: Rand)**

2015a Emergent behaviors over signed random dynamical networks: State-flipping model. *IEEE Trans. Control Network Syst.* 2 (2015), no. 2, 142–153. MR 3361856. Zbl 1370.90061. arXiv:1411.0074. **(SG: Rand)**

2016a The evolution of beliefs over signed social networks. *Operations Res.* 64 (2016), no. 3, 585–604. MR 3515199. Zbl 1348.91239. arXiv:1307.0539. **(SG: Rand)**

2017a Emergent behaviors over signed random dynamical networks: Relative-state-flipping model. *IEEE Trans. Control Network Syst.* 4 (2017), no. 2, 369–379. MR 3667952. Zbl 1370.90062. arXiv:1412.1990. **(SG: Rand)**

Jinsong Shi

See R.L. Li.

Lingsheng Shi & Zhang Zhang

2018a Signed cycle double covers. *Electronic J. Combin.* 25 (2018), no. 4, art. P4.63, 12 pp. MR 3907794. Zbl 1409.05097. **(SG: Circ)**

Yongtang Shi

See B.F. Huo and J.-A. Li.

Kazuki Shibata

See H. Ohsugi.

V.S. Shigehalli & Kenchappa S. Betageri

2015a Color Laplacian energy of graphs. *J. Computer Math. Sci.* 6 (2015), no. 9, 485–494.

Laplacian energy of the signed graph of Adiga, Sampathkumar, *et al.* (2013a). **(sg: Lap: Eig)**

2016a A note on color energy and color Laplacian energy of graphs. *Int. J. Math. Archive* 7 (2016), no. 10, 199–204.

Cf. Adiga, Sampathkumar, *et al.* (2013a). **(sg: Lap: Eig)**

2016b A note on color energy of graphs. *Bull. Math. Stat. Res.* 4 (2016), no. 4, 47–50.

Cf. Adiga, Sampathkumar, *et al.* (2013a). **(sg: Adj: Eig)**

Ching-Hsien Shih

† 1982a *On Graphic Subspaces of Graphic Spaces.* Doctoral dissertation, Ohio State University, 1982. MR 2632480 (no rev).

Cf. Ferchiou and Guenin (2020a). **(sg: matrd)**

Wei-Kuan Shih

See also Kuo–Chern–Shih (1988a).

Wei-Kuan Shih, Sun Wu, & Y.S. Kuo

1990a Unifying maximum cut and minimum cut of a planar graph. *IEEE Trans. Computers* 39 (1990), no. 5, 694–697. MR 1059768 (91h:05117). Zbl 1395.05173.

Real edge weights make max and min cut equivalent. A faster algorithm than before. [This problem includes frustration index $l(\Sigma)$.] Dictionary: "positive", "negative" cut means total weight. [Annot. 19 Dec 2014.]

(**WG, sg: fr: Algor**)

Shijin T V [T.V. Shijin]

See also A. Mathew and S. Hameed.

2023a *The Notion of Signed Distance in Signed Graphs.* Doctoral dissertation, Central University of Kerala, 2023.

§1.2, "Signed graphs". Ch. 2, "Signed distance and distance compatibility in signed graphs": Hameed, Shijin, Soorya, Germina, and Zaslavsky (2021a). Ch. 3, "On the distance compatibility in product of signed graphs": Shijin, Soorya, Hameed, and Germina (2023a). Ch. 4, "On the distance spectra of product of signed graphs": Shijin, Soorya, Hameed, and Germina (2023b). Ch. 5, "On the powers of signed graphs": Shijin, Germina, and Hameed (2022a). Ch. 6, "Signed distance in weighted signed graphs": Shijin and Germina (20xxa). §7.2, "Scope for further research and some open problems": Problems about distance compatibility, signed distance matrices, signed-graph products and powers, and their connections. [Annot. 16 Nov 2022.] (**SG: Str, Eig, WG**)

Shijin T V & Germina K A

20xxa Signed distance in weighted signed graphs. Submitted.

Weights treated as edge lengths. Generalizes Hameed, Shijin, *et al.* (2021a). [Annot. 2 Nov 2022.]

(**SG: WG, Adj(Gen), Bal, Eig, Sw**)

Shijin T V, Germina K A, & Shahul Hameed K

2022a On the powers of signed graphs. *Commun. Combin. Optim.* 7 (2022), no. 1, 45–51. MR 4337200. arXiv:2009.10486.

Cf. Hameed, Shijin, *et al.* (2021a). $\Sigma^k_{\max}$ ($\Sigma^k_{\min}$) has edge uv signed $\sigma_{\max\,(\min)}(u,v)$ when $d(u,v) \leq k$. $\exists\ \Sigma^k$ if $\Sigma^k_{\max} = \Sigma^k_{\min}$. Theorems on balance and distance compatibility. [Annot. 27 Sept 2020.]

(**SG: Bal, Adj(Gen)**)

Shijin T V, Soorya P, Shahul Hameed K, & Germina K A

2020a On signed distance in product of signed graphs. Manuscript, 2020. arXiv:-2009.08707.

Cf. Hameed, Shijin, *et al.* (2021a). Thm. 2.2 characterizes distance-compatible signed graphs, somewhat more simply than the definition. §3 characterizes distance compatibility for Cartesian and lexicographical products and partially for tensor product. §4: Formulas for signed distance matrices of the former two. §5: Examples. [Annot. 19 Mar 2021.]

Superseded by (2023a), (2023b). (**SG**)(**SG: Adj(Gen), Eig**)

T.V. Shijin, P. Soorya, K. Shahul Hameed, & K.A. Germina

2023a On the distance compatibility in product of signed graphs. *Asian-European J. Math.* 16 (2023), no. 2, art. 2350015, 10 pp. MR 4526099. (**SG**)

2023b On the distance spectra of the product of signed graphs. *Commun. Combin. Optim.* 8 (2023), no. 1, 67–76. MR 450365.

Only for distance-compatible signed graphs and products (Hameed, Shijin, Soorya, *et al.* (2021a)). Thms. 4, 5: Distance matrices of Cartesian and lexicographic products. §3, "Distance spectra of some compatible signed graphs and their products": Signed Petersen graphs; Cartesian products of (K_n, σ_1) with (K_n, σ_2) or with $-\Gamma$; $\Sigma[(K_2, \sigma)]$. [Annot. 22 Apr 2023.] (**SG: Adj(Gen), Eig**)

M. Ashwin Shijo

See C. Jayasekaran.

Akihiro Shikama

See A. Funato and T. Hibi.

Young-hee Shin

See J.H. Kwak.

Guy Shinar & Martin Feinberg

2013a Concordant chemical reaction networks and the species-reaction graph. *Math. Biosci.* 241 (2013), 1–23. MR 3019690. Zbl 1309.92094. arXiv:1203.6560.

The "c-pair" ("complex pair") edges act like a negative edge. [Annot. 21 Jan 2015.] (**sd, sg: Dyn, Chem**)

Alana Shine

See M. Beck.

J. Shiozaki, H. Matsuyama, E. O'Shima, & M. Iri

1985a An improved algorithm for diagnosis of system failures in the chemical process. *Computers and Chem. Eng.* 9 (1985), 285–293.

Continuation of Iri, Aoki, O'Shima, and Matsuyama (1979a). (**SD, VS: Appl, Algor**)

A.H. Shirazi

See R. Masoumi.

H. Shirazi

See G. Coutinho.

Shailaja S. Shirkol

See P.R. Hampiholi.

Keisuke Shiromoto

See Y. Koga.

Wai Chee Shiu

See J.M. Guo.

[Shivakumar Swamy C.S.]

See S. Swamy C.S..

K. Shivashankara

See P.S.K. Reddy.

S.B. Shlosman

See Dobrushin and Shlosman (1985a).

L. Shobana [Shobana Loganathan]

See also J. Baskar Babujee and B. Vasuki.

L. Shobana & B. Vasuki

2017a Signed product cordial labeling on some connected graphs. *Int. J. Pure Appl. Math.* 113 (2017), no. 13, 198–207.

More, as in Baskar Babujee and Loganathan (2011a).

(Lab: VS: SG, Bal)

Elizabeth G. Shrader & David W. Lewit

1962a Structural factors in cognitive balancing behavior. *Human Relations* 15 (1962), 265–276.

For $\Gamma \subset K_n$ and signing σ of Γ, "plausibility" = mean and "differentiability" = standard deviation of $f(K_n, \sigma')$ over all extensions of σ to K_n, where f is any function that measures degree of balance. Proposed: tendency toward balance is high when plausibility and differentiability are high. A specific f, based on triangles and quite complicated, is studied for $n = 4$, with experiments. **(sg, fr, PsS)**

S.R. Shreyas & M. Joseph

2020a Characterization of signed paths and cycles admitting minus dominating function. *Commun. Combin. Optim.* 5 (2020), no. 1, 61–68. MR 4048279. Zbl 1449.05129.

Cf. B.D. Acharya (2012a). Characterizations are largely in terms of the lengths of positive and negative sections. [Annot. 25 Dec 2020.]

(SG: Lab, Dom)

A.S. Shrikanth

See C. Adiga.

Mohan S. Shrikhande

See Y.J. Ionin.

Shrikanth A.S.

See C. Adiga.

Piyush Shroff

See L. Rusnak.

Jinlong Shu

See G.L. Yu and M.Q. Zhai.

Alan Shuchat

See R. Shull.

Randy Shull, James B. Orlin, Alan Shuchat, & Marianne L. Gardner

1989a The structure of bases in bicircular matroids. *Discrete Appl. Math.* 23 (1989), 267–283. MR 0996138 (90h:05040). Zbl 698.05022.

[See Coullard, del Greco, and Wagner (1991a).] **(Bic: Bases)**

Randy Shull, Alan Shuchat, James B. Orlin, & Marianne Lepp

1993a Recognizing hidden bicircular networks. *Discrete Appl. Math.* 41 (1993), 13–53. MR 1197249 (94e:90122). Zbl 781.90089. **(GN: Bic: Incid, Algor)**

1997a Arc weighting in hidden bicircular networks. Proc. Twenty-eighth Southeastern Int. Conf. on Combinatorics, Graph Theory and Computing (Boca Raton, Fla., 1997). *Congressus Numer.* 125 (1997), 161–171. MR 1604964 (98m:05181). Zbl 902.90157. **(GN: Bic: Incid, Algor)**

E.E. Shult

See P.J. Cameron.

Robert Shwartz

See M. Amram, V. Bugaenko, and Y. Cherniavsky.

Jana Šiagiová

See also J. Širáň.

2001a A note on the McKay–Miller–Širáň graphs. *J. Combin. Theory Ser. B* 81 (2001), 205–208. MR 1814904 (2001k:05110). Zbl 1024.05039.

Cf. McKay, Miller, and Širáň (1998a). **(GG: Cov)**

M. Siami

See S. Akbari.

Omid Askari Sichani

See M. Shahriari.

V.M. Siddalingaswamy

See Rajendra, Reddy, and Siddalingaswamy (2018a) and Reddy, Nagaraja, and Siddalingaswamy (2015a).

Heike Siebert

See also A. Bockmayr.

2008a Local structure and behavior of Boolean bioregulatory networks. In: Katsuhisa Horimoto *et al.*, eds., *Algebraic Biology* (Third Int. Conf., AB 2008, Castle of Hagenberg, Austria, 2008), pp. 185–199. Lect. Notes in Computer Sci., Vol. 5147. Springer, Berlin, 2008. Zbl 1171.92303. **(SD: Dyn, Biol)**

2009a Deriving behavior of Boolean bioregulatory networks from subnetwork dynamics. *Math. Computer Sci.* 2 (2009), no. 3, 421–442. MR 2507427 (2010g:92009). Zbl 1205.37097. **(SD: Dyn, Biol)**

2009b Dynamical and structural modularity of discrete regulatory networks. In: Ralph-Johan Back, Ion Petre, and Erik de Vink, eds., *Proceedings Second International Workshop on Computational Models for Cell Processes* (CompMod 2009, Eindhoven, Netherlands), pp. 109–124. Electronic Proc. Theoretical Computer Sci., Vol. 6, 2009. arXiv:0910.1412.

The local and global interaction graphs are signed digraphs (p. 111). **(SD: Dyn, Biol)**

2011a Analysis of discrete bioregulatory networks using symbolic steady states. *Bull. Math. Biol.* 73 (2011), no. 4, 873–898. MR 2785148 (2012c:92006). Zbl 1214.92033. **(SD: Dyn, Biol)**

Heike Siebert & Alexander Bockmayr

2006a Incorporating time delays into the logical analysis of gene regulatory networks. In: Corrado Priami, ed.,*Computational Methods in Systems Biology* (Proc. Int. Conf. CMSB 2006, Trento, Italy), pp. 169–183. Lect. Notes in Computer Sci.,

Vol. 4210. Springer, Berlin, 2006. MR 2288350 (2007k:92067).
(**SD: Dyn, Biol**)

2007a Context sensitivity in logical modeling with time delays. In: Muffy Calder and Stephen Gilmore, eds., *Computational Methods in Systems Biology* (Proc. Int. Conf. CMSB 2007, Edinburgh, 2007), pp. 64–79. Lect. Notes in Computer Sci., Vol. 4695. Springer, Berlin, 2007. (**SD: Dyn, Biol**)

2008a Temporal constraints in the logical analysis of regulatory networks. *Theoretical Computer Sci.* 391 (2008), no. 3, 258–275. MR 2386791 (2008k:92035). Zbl 1133.68041. (**SD: Dyn, Biol**)

2008b Relating attractors and singular steady states in the logical analysis of bioregulatory networks. In: Hirokazu Anai *et al.*, eds., *Algebraic Biology* (Second Int. Conf., AB 2007, Castle of Hagenberg, Austria, 2007), pp. 36–50. Lect. Notes in Computer Sci., Vol. 4545. Springer, Berlin, 2008. Zbl 1127.92002.
(**SD: Dyn, Biol**)

David Siegel
See H. Kunze.

Mark Siggers
See R.C. Brewster.

Mathieu Dutour Sikirić, Anna Felikson, & Pavel Tumarkin

2011a Automorphism groups of root system matroids. *European J. Combin.* 32 (2011), 383–389. MR 2764801 (2012a:05063). Zbl 1229.05068. arXiv:0711.4670.
Schwartz (2002a) is more general but excepts the exceptional root systems. [Annot. 9 Apr 2016.] (**gg: Matrd: Aut**)

Arlei Silva
See Z.-X. Huang.

Ilda P.F. da Silva

1997a Note on inseparability graphs of matroids having exactly one class of orientations. *Discrete Math.* 171 (1997), 77–87. MR 1454442 (98c:05044). Zbl 874.05016. (**SG, Matrd**)

Pedro V. Silva
See S. Margolis.

Bruno Simeone
See also C. Benzaken, J.-M. Bourjolly, P.L. Hammer, and P. Hansen.

2011a Quadratic functions. In: Yves Crama and Peter L. Hammer, eds., *Boolean Functions: Theory, Algorithms, and Applications*, Ch. 5. Encyc. Math. Appl., Vol. 142. Cambridge University Press, Cambridge, 2011. MR 2742439 (book). Zbl 1237.06001 (book). (**SG: Bal, Algor: Exp**)

Luca Simeoni
See S. Klamt.

Slobodan K. Simić
See also M. Anđelić, F. Belardo, D.M. Cardoso, D.M. Cvetković, and X.Y. Geng.

1980a Graphs which are switching equivalent to their complementary line graphs I. *Publ. Inst. Math. (Beograd) (N.S.)* 27(41) (1980), 229–235. MR 0621954

(82m:05077). Zbl 531.05050. **(TG: LG)**

1982a Graphs which are switching equivalent to their complementary line graphs II. *Publ. Inst. Math. (Beograd) (N.S.)* 31(45) (1982), 183–194. MR 0710958 (85d:05207). Zbl 531.05051. **(TG: LG)**

Slobodan K. Simić, Milica Anđelić, Carlos M. da Fonseca, & Dejan Živković

2015a Notes on the second largest eigenvalue of a graph. *Linear Algebra Appl.* 465 (2015), 262–274. MR 3274675. Zbl 1302.05112.

Let H_i' denote the bridges of a cutpoint u in Γ with each edge subdivided once. Order so $\varepsilon_i := \lambda_{\max}(H_i' \backslash u)$ is decreasing. Cor. 3.4 (restated): $\varepsilon_2^2 \leq \lambda_2(\Gamma) \leq \varepsilon_1^2$, with $=$ iff $\varepsilon_1 = \varepsilon_2$. [Annot. 20 Jan 2015.] **(par: Lap: Eig)**

Slobodan K. Simić & Zoran Stanić

2008a *Q*-integral graphs with edge-degrees at most five. *Discrete Math.* 308 (2008), 4625–4634. MR 2438168 (2010g:05229). Zbl 1156.05037.

Determines all Γ with edge degree ≤ 4 and integral Spec $L(-\Gamma)$, and partially for edge degree 5. An important paper for integral spectra of $L(-\Gamma)$. [Annot. 29 Apr 2022.] **(par: Lap: Eig)**

2009a On some forests determined by their Laplacian or signless Laplacian spectrum. *Computers Math. Appl.* 58 (2009), no. 1, 171–178. MR 2535979 (2010j:05252). Zbl 1189.05106.

§4, "Determination by the signless Laplacian spectrum". Thm. 4.1: Among the forests whose trees are Smith trees (excluding a few), the three minimal graphs not determined amongst all graphs by Spec $L(-\Gamma)$. [Annot. 20 Jan 2015.] **(par: Lap: Eig)**

2016a Polynomial reconstruction of signed graphs. *Linear Algebra Appl.* 501 (2016), 390–408. MR 3485074. Zbl 1334.05056.

Reconstructing the characteristic polynomial (of $A(\Sigma)$) from vertex-deleted subgraphs: solved for cyclomatic number 0 [known: same as for graphs] and 1. A trivial counterexample: positive and negative C_n. *Question*: Is there a counterexample with nonisomorphic $|\Sigma_1|, |\Sigma_2|$? [Annot. 18 Dec 2016.] **(SG: Adj)**

2016b Polynomial reconstruction of signed graphs whose least eigenvalue is close to -2. *Electronic J. Linear Algebra* 31 (2016), Article 52, 740–753. MR 3603936. Zbl 1352.05119. **(SG: Adj)**

Rodica Simion

1995a On *q*-analogues of partially ordered sets. *J. Combin. Theory Ser. A* 72 (1995), no. 1, 135–183. MR 1354971 (97h:06011). Zbl 834.06006.

§6, "Dowling lattices": They are an example, thus having an EL-labelling induced from Π_n. [Annot. 9 Apr 2016.] **(gg: Matrd, Invar)**

2000a Combinatorial statistics on type-B analogues of noncrossing partitions and restricted permutations *Electronic J. Combin.* 7 (2000), Research Paper R9, 27 pp. MR 1741331 (2000k:05013). Zbl 938.05003.

"Type-B noncrossing partitions" are certain signed partial partitions of the ground set; i.e., certain elements of the Dowling lattice of $\{\pm\}$. **(gg: Matrd)**

R. Simion & D.-S. Cao

1989a Solution to a problem of C. D. Godsil regarding bipartite graphs with unique perfect matching. *Combinatorica* 9 (1989), 85–89. MR 1010303 (90f:05113). Zbl 688.05056.

Answering Godsil (1985a): $|\Sigma| = \Gamma$ iff Γ consists of a bipartite graph with a pendant edge attached to every vertex. [Surely there is a signed-graphic generalization of Godsil's and this theorem in which bipartiteness becomes balance or something like it.] (**sg: Adj, bal**)

Aron Simis, Wolmer V. Vasconcelos, & Rafael H. Villarreal

1999a The integral closure of subrings associated to graphs. *J. Algebra* 199 (1998), 281–289. MR 1489364 (99c:13004). Zbl 902.13004.

(**sg: Par: Incid Algeb, matrd**)

J.M.S. Simões-Pereira

1972a On subgraphs as matroid cells. *Math. Z.* 127 (1972), 315–322. MR 0317973 (47 #6522). Zbl 226.05016, (Zbl 243.05022).

"Cell" = circuit. Along with Klee (1971a), invents the bicircular matroid (here, for finite graphs) (Thm. 1). Suppose we have matroids on the edge sets of all [simple] graphs, such that the class of circuits is a [nonempty] union of homeomorphism classes of connected graphs. Thm. 2: The circle and bicircular matroids [and free matroids] are the only such matroids. (**MtrdF, Bic**)

1973a On matroids on edge sets of graphs with connected subgraphs as circuits. *Proc. Amer. Math. Soc.* 38 (1973), 503–506. MR 0314663 (47 #3214). Zbl 241.05114, Zbl 264.05126.

A family of (isomorphism types of) [simple] connected graphs is "matroidal" if for any Γ the class of subgraphs of Γ that are in the family constitute the circuits of a matroid on $E(\Gamma)$. Bicircular and even-cycle matroids are the two nicest examples. A referee contributes the even-cycle matroid [*cf.* Tutte (1981a), Doob (1973a)]. Thm.: The family cannot be finite [unless it is void or consists of K_2]. [See Marcu (1987a) for a valuable new viewpoint.] (**MtrdF, Bic, ECyc, Gen**)

1975a On matroids on edge sets of graphs with connected subgraphs as circuits II. *Discrete Math.* 12 (1975), 55–78. MR 0419275 (54 #7298). Zbl 307.05129.

Partial results on describing matroidal families of simple, connected graphs. Five basic types: free [omitted in the paper], cofree, circle, bicircular, and even-cycle. If the family does not correspond to one of these, then every member has ≥ 3 independent circles and minimum degree ≥ 3. (**MtrdF, Bic, ECyc: Gen**)

1978a A comment on matroidal families. In: *Problèmes Combinatoires et Théorie des Graphes* (Colloq. Int., Orsay, 1976), pp. 385–387. Colloques Int. du CNRS, 260. Editions du C.N.R.S., Paris, 1978. MR 0540020 (81b:05031). Zbl 412.05023.

Two small additions to (1973a), (1975a); one is that a matroidal family not one of the five basic types must contain $K_{p,q(p)}$ for each $m \geq 3$, with $q(p) \geq p$. (**MtrdF, Bic, ECyc: Gen**)

1992a Matroidal families of graphs. In: Neil White, ed., *Matroid Applications*, Ch. 4, pp. 91–105. Encycl. Math. Appl., Vol. 40. Cambridge Univ. Press, Cambridge, Eng., 1992. MR 1165541 (93c:05036). Zbl 768.05024.

"Count" matroids (see N. White (1986a)) in §4.3; Schmidt's (1979a) remarkable generalization in §4.4.
(**GG: MtrdF, Bic, ECyc: Gen: Exp, Exr, Ref**)

2016a (as José Manuel dos Santos Simões-Pereira) An existence problem for matroidal families. In: Karim Adiprasito *et al.*, eds., *Convexity and Discrete Geometry Including Graph Theory* (Mulhouse, 2014), pp. 261–262. Springer Proc. Math. Stat., Vol. 148. Springer, Cham, 2016. MR 3516717. Zbl 1379.05023.
Cf. Schmidt (1979a). Are there unknown matroidal families? [Annot. 6 Jul 2022.] (**MtrdF**)

Klaus Simon
See T. Raschle.

[C. De Simone]
See C. De Simone (under 'D').

M. Simonovits
See B. Bollobás, J.A. Bondy, and P. Erdős.

Daniel Simson
See also R. Bocian, M. Felisiak, M. Gąsiorek, S. Kasjan, G. Marczak, and A. Polak.

2013a A Coxeter-Gram classification of positive simply laced edge-bipartite graphs. *SIAM J. Discrete Math.* 27 (2013), no. 2, 827–854. MR 3048204. Zbl 1272.05072. (**SG**)

2013b A framework for Coxeter spectral analysis of edge-bipartite graphs, their rational morsifications and mesh geometries of root orbits. *Fundamenta Inform.* 124 (2013), no. 3, 309–338. MR 3100347. Zbl 1269.05073. (**SG**)

2013c Toroidal algorithms for mesh geometries of root orbits of the Dynkin diagram $\mathbb{D}_4$. *Fundamenta Inform.* 124 (2013), no. 3, 339–364. MR 3100348. Zbl 1269.05105. (**SG: Algor**)

Daniel Simson & Katarzyna Zając

2013a An inflation algorithm and a toroidal mesh algorithm for edge-bipartite graphs. Combinatorics 2012 (Perugia, 2012). *Electronic Notes in Discrete Math.* 40 (2013), 377–383. MR 3155275 (volume). Zbl 1292.05005 (volume). (**SG: Algor**)

2017a Inflation algorithm for loop-free non-negative edge-bipartite graphs of corank at least two.. *Linear Algebra Appl.* 524 (2017), 109–152. MR 3630181. Zbl 1361.05061. (**SG: Algor**)

Alistair Sinclair
See M. Jerrum.

Amit Singer
See A.S. Bandeira.

Ambuj Singh
See Z.-X. Huang.

Amrik Singh
See B. Adhikari.

Manjeet Singh

See S. Mehra.

P.K. Singh

See T. Sharma.

Rajiv R.P. Singh

See also M.E. Fisher.

Rajiv R.P. Singh & Sudip Chakravarty

1986a Critical behavior of an Ising spin-glass. *Phys. Rev. Lett.* 56 (1986), no. 2, 245–248. **(Phys, SG: Fr)**

Ranveer Singh

See also A. Berman.

2019a On eigenvector structure of weakly balanced networks. *Physica A* 527 (2019), art. 121093, 10 pp. MR 3947751.

"Weakly balanced" = clusterable. Examples. [Annot. 11 Jul 2022.]

(SG: Clu: Adj: Eig)

Ranveer Singh & Bibhas Adhikari

2017a Measuring the balance of signed networks and its application to sign prediction. *J. Stat. Mech.* 2017 (2017), no. 6, art. 063302, 16 pp. MR 3673443. Zbl 1457.90046. **(SG: Fr, Pred: Adj: Eig)**

Ranveer Singh & Ravindra B. Bapat

2017a Eigenvalues of weakly balanced signed graphs and graphs with negative cliques. Manuscript, 2017. arXiv:1702.06322. **(SG: Adj: Eig)**

2018a $\mathcal{B}$-partitions, determinant and permanent of graphs. *Trans. Combin.* 7 (2018), no. 3, 29–47. MR 3841400. Zbl 1474.05325. arXiv:1705.02517. **(SG, WG: Adj)**

Tarkeshwar Singh

See also M. Acharya, J. Pereira, and S.B. Rao.

2003a *Advances in the Theory of Signed Graphs.* Doctoral dissertation, University of Delhi, India, 2003.

Fairly complete accounts of M. Acharya & Singh (various) and Singh (2008b), supplemented with background, appendix, etc. Ch. II, "Graceful signed graphs", is in M. Acharya and Singh (2003a), (2004a), (2005a), (2004b). Ch. III, "Skolem graceful sigraphs": Announced in M. Acharya and Singh (2003b). Thm. 3.12: See M. Acharya and Singh (2010a). Thm. 3.13: A necessary condition for Skolem-gracefulness of signed multiple stars. Thm. 3.14: A sufficient condition for two signed stars. Ch. IV, "Negation-switching invariant sigraphs": See M. Acharya and Singh (2004a). Also: A binary encoding of signed circles. App., "A catalog of assorted labelled sigraphs". [Annot. 20 July 2009.]

(SGc: Lab)(SG: Sw, LG)

2008a Skolem and hooked Skolem graceful sigraphs. In: B.D. Acharya, S. Arumugam, and Alexander Rosa, eds., *Labelings of Discrete Structures and Applications* (Mananthavady, Kerala, 2006), pp. 155–164. Narosa, New Delhi, 2008. MR 2391786 (2009e:05281) (book). Zbl 1161.05340.

[*Cf.* M. Acharya and Singh (2004a), (2003b). Generalizing the definition: Given: a graph with r-colored edges, m_i of color i; a list

L of n integers. Required: A bijection $\lambda : V \to L$ such that, if $f(vw) := |\lambda(v) - \lambda(w)|$, then f restricted to color class i is a bijection to $[m_i]$.] Signed graphs are the case $r = 2$. Skolem gracefulness is the case where λ exists for $L = [n]$. Hooked Skolem gracefulness is the case where λ exists for $L = [n+1] \setminus \{n\}$. Results from M. Acharya and Singh (2010a) and Singh (2008b), examples, some proofs. (**SGc: Lab: Exp**)

2009a Graceful signed graphs on C_3^k. Fifth Int. Workshop on Graph Labelings (IWOGL 2009) (Krishnankoil, 2009). *AKCE Int. J. Graphs Combin.* 6 (2009), no. 1, 201–208. MR 2533200 (2010g:05330). Zbl 1210.05155.

"Graceful" means $(1,1)$-graceful, $r = 1$, as at M. Acharya and Singh (2004a). C_3^k is the windmill with k blades. Let Σ have ν negative rim edges, $1 \leq \nu \leq k/2$, and no other negative edges. Thm. 10: Σ is graceful if $k \equiv 0 \bmod 4$ and ν is even. Thms. 11, 12: Σ is graceful if $k \equiv 1, 2 \bmod 4$. [Annot. 21 July 2010.] (**SGc: Lab**)

2008b A note on hooked Skolem graceful sigraphs and its application. Manuscript, n.d.

See (2008a). Thm.: A signed k-edge matching is hooked Skolem graceful iff $k \equiv 0 \pmod 4$ and $\#E^-$ is odd, or $k \equiv 2 \pmod 4$ and $\#E^-$ is even, or $k \equiv 3 \pmod 4$. Curiously complementary to the theorem of M. Acharya and Singh (2010a). (**SGc: Lab**)

Tarkeshwar Singh & Natasha D'Souza

2010a Some results in graceful signed tree. [Abstract.] In: *International Conference on Recent Trends in Graph Theory and Combinatorics (ICRTBC-2010)* (Cochin, 2010) [Summaries], p. 169. Dept. of Math., Cochin Univ. of Science and Technology, 2010.

Some graceful signed trees (see M. Acharya and Singh (2004a)). Every signed tree is an induced subgraph of a graceful signed tree. [Annot. 31 Aug 2010.] (**SGc: Lab**)

N.M. Singhi

See also S.B. Rao, D.K. Ray-Chaudhuri, and G.R. Vijayakumar.

N.M. Singhi & G.R. Vijayakumar

1992a Signed graphs with least eigenvalue < -2. *European J. Combin.* 13 (1992), 219–220. MR 1164766 (93e:05069). Zbl 769.05065.

A short proof that every such signed simple graph contains an induced subgraph with least eigenvalue $= -2$. [Their $M := 2I + A(\Sigma)$ is the Laplacian matrix of $-\Sigma$.] (**SG: adj**)

Deepa Sinha

See also M. Acharya.

2005a *New Frontiers in the Theory of Signed Graphs.* Doctoral dissertation, University of Delhi, 2005.

[Partial description] Σ is "sign compatible" if $\exists\ X \subseteq V$ such that $E^- = E{:}X$. It is "canonically sign compatible" if $X = \mu_\sigma^{-1}(-)$ (*cf.* Sampathkumar (1984a)). [Annot. 12 Oct 2010.] (**SG: Bal(Gen)**)

Deepa Sinha & Mukti Acharya

2016a Characterization of signed graphs whose iterated signed line graphs are balanced or S-consistent. *Bull. Malaysian Math. Sci. Soc.* 39 (2016), no. 1, suppl., S297–S306. MR 3509081. Zbl 1339.05171.

Extensions to $\Lambda^k_{BC}(\Sigma)$ of M. Acharya and Sinha's (2002a) characterization of balance in the Behzad–Chartrand (1969a) line graph $\Lambda_{BC}(\Sigma)$ and to $\Lambda^k(\Sigma)$ of Acharya, Acharya, and Sinha's (2009a) criterion for consistency of $\Lambda(\Sigma)$. (**SG, VS: LG(Gen): Bal**)

Deepa Sinha & Ayushi Dhama

2012a Sign-compatibility of some derived signed graphs. *Mapana J. Sci.* 11 (2012), no. 4, 1–14. MR 3086508.

Cf. Sinha (2005a). (**SG: VS, LG(Gen): Bal(Gen)**)

2013a Sign-compatibility of some derived signed graphs. *Indian J. Math.* 55 (2013), no. 1, 23–40. MR 3086508. Zbl 1274.05209.

Cf. Sinha (2005a). (**SG: LG(Gen): Bal(Gen)**)

2013b Canonical sign-compatibility of some signed graphs. *J. Combin. Inform. System Sci.* 38 (2013), 129–138. Zbl 1316.05056.

Cf. Sinha (2005a). (**SG: Bal(Gen)**)

2013c Sign-compatibility of common-edge sigraphs and 2-path sigraphs. *Graph Theory Notes N.Y.* 65 (2013), 55–61. MR 3204940.

Cf. Sinha (2005a). (**SG: LG(Gen): Bal(Gen)**)

2013d On the unitary Cayley ring signed graphs $S_n^{\oplus}$. *J. Interconnection Networks* 14 (2013), no. 4, art. 1350020, 20 pp.

Characterizes balance, clusterability, and for some n also canonical consistency and sign compatibility. [Annot. 11 Apr 2016.] (**Algeb: SG: Bal, Clu, VS**)

2014a Negation switching invariant signed graphs. *Electronic J. Graph Theory Appl.* 2 (2014), no. 1, 32–41. MR 3199369. Zbl 1306.05088.

Thm. 2.2: Fairly elementary characterization of $\Sigma \simeq -\Sigma$. Thm. 2.3: Same for $\Sigma \cong -\Sigma$. [*Problem.* Find complete structural characterizations.] [Annot. 5 May 2014, 15 May 2018.] (**SG: Sw, Aut(Gen): Str**)

2014b Canonical sign compatibility of semi-total and total signed graphs. *Bull. Calcutta Math. Soc.* 106 (2014), no. 1, 55–64. MR 3380949. Zbl 1318.05034.

Define (semi)total signed graphs via the ×-line signed graph of M. Acharya (2009a). Characterizes those that are compatible (*cf.* Sinha (2005a)) with the canonical marking of the (semi)total signed graphs. [Annot. 9 Apr 2014.] (**SG: LG(Gen): Bal(Gen)**)

2015a On •-line signed graphs $L_\bullet(S)$. *Discuss. Math. Graph Theory* 35 (2015), no. 2, 215–227. MR 3338747. Zbl 1311.05078.

Definition *cf.* B.D. Acharya (2010a). Notation: $S = \Sigma$. Thm. 4: balance of $L_\bullet(\Sigma)$. Thm. 7: sign compatibility; *cf.* Sinha (2005a). Thm. 9: canonical sign compatibility; *cf.* Sinha (2005a). Thm. 13 characterizes connected •-line signed graphs. [Annot. 2 Oct 2019.] (**SG: LG: Bal, Bal(Gen)**)

2015b Unitary addition Cayley ring signed graphs $\sum_n^{\oplus *}$. *J. Discrete Math. Sci. Cryptography* 18 (2015), no. 5, 559–579. MR 3399713.
Cf. Sinha and Garg (2011e). [Annot. 7 Jan 2016.]
(SG: Bal, Clu, Bal(Gen))

2017a Unitary Cayley meet signed graphs. Int. Conf. Current Trends in Graph Theory and Computation (CTGTC-2016, New Delhi). *Electronic Notes Discrete Math.* 63 (2017), 425–434. MR 3754832. Zbl 1383.05160. **(SG: Bal, Clu, Bal(Gen))**

2017b Negation switching invariant 3-path signed graphs. *J. Discrete Math. Sci. Cryptography* 20 (2017), no. 3, 703–716. MR 3691461. **(SG: LG(Gen): Sw)**

20xxa On the unitary Cayley meet signed graphs $S_n^{\wedge}$. Submitted.
(SG: Bal, Clu, Bal(Gen))

20xxb Negation-switching invariant t-path signed graphs, $t \leq 3$. Submitted.
(SG: LG(Gen): Sw)

Deepa Sinha, Ayushi Dhama, & B.D. Acharya

2013a Unitary addition Cayley signed graphs. *European J. Pure Appl. Math.* 6 (2013), no. 2, 189–210. MR 3053442. Zbl 1389.05066.
Cf. Sinha and Garg (2011e). **(SG: Bal, Clu, Bal(Gen))**

Deepa Sinha & Pravin Garg

2010a Consistency of semi-total signed graphs. [Abstract.] In: *International Conference on Recent Trends in Graph Theory and Combinatorics (ICRTGC-2010)* (Cochin, 2010) [Summaries], p. 153. Dept. of Mathematics, Cochin University of Science and Technology, 2010.
Consistency of the canonical vertex signature of certain graphs related to the line graph and total graph of Σ; see e.g. (2011f), (2015a), (2015b). [Annot. 31 Aug 2010.] **(SG: LG(Gen): Bal(Gen))**

2011a Canonical consistency of signed line structures. *Graph Theory Notes N.Y.* 59 (2011), 22–27. MR 2849400 (2012g:05098).
Consistency of the canonical vertex signature of two kinds of line graph: (Thm. 2) $\Lambda_{BC}(\Sigma)$ of Behzad–Chartrand (1969a) and (Thm. 8) $\Lambda_\times(\Sigma)$ of M. Acharya (2009a). [Annot. 25 Mar 2011.] **(SG: LG: VS: Bal(Gen))**

2011b Balance and consistency of total signed graphs. *Indian J. Math.* 53 (2011), no. 1, 71–81. MR 2809572 (2012d:05174). Zbl 1238.05126.
T_{SG} Defines a vertex- and edge-signed total graph $\mathrm{T}_{SG}(\Sigma)$. Vertex signs $\mu_1(v) := \mu_\sigma(v)$, $\mu_1(e) = \sigma(e)$. Edge signs $\sigma_{\mathrm{T}}(uv) := \sigma(e_{uv})$, $\sigma_{\mathrm{T}}(ue) := \sigma(e)\mu_1(u)$, $\sigma_{\mathrm{T}}(ef) := \sigma(e)\sigma(f)$ [thus $\mathrm{T}_{SG}(\Sigma) \supseteq \Lambda_\times(\Sigma)$ of M. Acharya (2009a)]. Balance and consistency characterized. [For spectrally suitable total graphs *cf.* Belardo–Stanić–Zaslavsky (2023a).] [Annot. 13 Oct 2009, 20 Dec 2010, 1 Apr 2022.] **(SG, VS: LG(Gen): Bal, Bal(Gen))**

2011c On the regularity of some signed graph structures. *AKCE Int. J. Graphs Combin.* 8 (2011), no. 1, 63–74. MR 2839176 (2012f:05126). Zbl 1238.05127.
Σ is "regular" if Σ^+ and Σ^- are regular. For the edge signs of line graphs and total graph see (2011b). Characterizes Σ such that Λ_{BC} or $\Lambda_\times$ or T_{SG} is regular. Dictionary: "signed-regular" = regular. [Annot.

25 July 2011.] **(SG: LG, LG(Gen))**

2011d Characterization of total signed graph and semi-total signed graphs. *Int. J. Contemp. Math. Sci.* 6 (2011), no. 5-8, 221–228. MR 2797063 (no rev). Zbl 1235.05058.

Thm. 2.3 characterizes semi-total signed graphs. Thm. 3.2 characterizes semi-total point signed graphs. Thm. 4.4 characterizes total signed graphs. Each result applies a pre-existing characterization of underlying graphs. [Annot. 23 Nov 2014.] **(SG: LG(Gen))**

2011e On the unitary Cayley signed graphs. *Electronic J. Combin.* (2011), art. P229, 13 pp. MR 2861408 (2012k:05173). Zbl 1243.05110.

The unitary Cayley graph $X_n = (\mathbb{Z}_n, \{ab : \exists\ (b-a)^{-1}\})$. $S_n = (X_n, \sigma)$ where $\sigma(ab) = -$ iff $\nexists\ a^{-1}, b^{-1}$. Thm. 4: S_n is balanced iff n is even or a prime power. Cor. 5: S_n is antibalanced iff n is even. Cor. 7: $\Lambda_{BC}(S_n)$ is balanced iff n is a prime power. Thm. 20: Let n have at most 2 distinct odd prime factors. S_n is canonically consistent iff n is odd, evenly even, 2, or 6. [Annot. 16 Jan 2012.] **(SG: Bal, LG, Bal(Gen): Algeb)**

2011f Some results on semi-total signed graphs. *Discuss. Math. Graph Theory* 31 (2011), no. 4, 625–638. MR 2952233. Zbl 1255.05091.

Similar to (2011b), but for $T_2(\Sigma) := T_{SG}(\Sigma)$ without line-graph edges. [Annot. 13 Oct 2009.] **(SG, VS: LG(Gen): Bal)**

2013a A characterization of canonically consistent total signed graphs. *Notes Number Theory Discrete Math.* 19 (2013), no. 3, 70–77. Zbl 1314.05092.

Cf. Sinha and Garg (2011a). **(SG: LG(Gen): Bal(Gen))**

2014a Balance and antibalance of tensor product of two signed graphs. *Thai J. Math.* 12 (2013), no. 2, 303–311. MR 3217341. Zbl 1307.05100.

Tensor product defined by Mishra (1974a). For connected signed graphs: Thm. 2.6: $\Sigma_1 \otimes \Sigma_2$ is balanced iff Σ_1 and Σ_2 are both balanced or both antibalanced. Thm. 3.1: It is antibalanced iff one is balanced and the other is antibalanced. [Annot. 23 Nov 2014.] **(SG: Bal)**

2015a Canonical consistency of semi-total line signed graphs. *Nat. Acad. Sci. Lett. (India)* 38 (2015), no. 5, 429–432. MR 3417353.

Cf. Sinha and Garg (2011a). **(SG: LG(Gen): Bal(Gen))**

2015b Canonical consistency of semi-total point signed graphs. *Nat. Acad. Sci. Lett. (India)* 38 (2015), no. 6, 497–500. MR 3433367.

Cf. Sinha and Garg (2011a). **(SG: LG(Gen): Bal(Gen))**

Deepa Sinha, Pravin Garg, & Hina Saraswat

2013a On the splitting signed graphs. *J. Combin. Inform. System Sci.* 38 (2013), no. 1-4, 103–111. Zbl 1304.05067. **(SG, VS: Bal, Bal(Gen))**

Deepa Sinha & Sandeep Kumar

20xxa On the relation between index and Sombor index of signed graph. In preparation. **(SG)(SG: Adj: Eig)**

Deepa Sinha & Anita Kumari Rao

2017a On co-maximal meet signed graphs of commutative rings. Int. Conf. Current Trends in Graph Theory and Computation (CTGTC-2016, New Delhi). *Elec-*

tronic Notes Discrete Math. 63 (2017), 497–502. MR 3754840. Zbl 1383.05146. (**SG: Algeb**)

2018a Co-maximal signed graphs of commutative rings. *Turkish J. Math.* 42 (2018), 1203–1220. MR 3804981. Zbl 1424.05126. (**SG: Algeb, LG: Bal, Clu**)

2018b Embedding of signed regular graphs. *Notes Number Theory Discrete Math.* 24 (2018), no. 3, 131–141. (**SG**)

2022a Embedding of sign-regular signed graphs and its spectral analysis. *Linear Multilinear Algebra* 70 (2022), no. 8, 1496–1512. MR 4420912. (**SG**)(**SG: Adj: Eig**)

Deepa Sinha, Anita Kumari Rao, & Ayushi Dhama

2019a Spectral analysis of t-path signed graphs. *Linear Multilinear Algebra* 67 (2019), no. 9, 1879–1897. MR 3980671. Zbl 1418.05071.

Main result is Thm. 3.4: Spec $A(\Sigma)$ is sign-symmetric iff $\Sigma \simeq -\Sigma$. [N.B. $\Sigma \simeq -\Sigma$ is unsolved.] $(\Sigma)_t :=$ a kind of signed t-path graph. Other results, e.g., Thm. 2.8: For signed K_n, $(\Sigma)_2 \simeq -\Sigma$ iff Σ is balanced or clusterable in a limited way. [Annot. 27 May 2018.] (**SG: Adj: Eig**)(**SG: LG(Gen), Clu**)

Deepa Sinha, Anita Kumari Rao, & Pravin Garg

2016a Embedding of (i, j)-regular signed graphs in $(i+k, j+l)$-regular signed graphs. In: *2016 International Workshop on Computational Intelligence* (IWCI, Dhaka, 2016), pp. 215–218. IEEE, 2016.

Σ is (i, j)-regular if Σ^+ is i-regular and Σ^- is j-regular. Embedding is as a subgraph. The aim is to minimize the order of the supergraph. [Annot. 13 Mar 2018.] (**SG**)

Deepa Sinha & Anshu Sethi

2014a An optimal algorithm to detect balancing in common-edge sigraph. *Int. J. Computer Appl.* 93 (2014), no. 10, 19–25. (**SG: LG(Gen): Bal: Algor**)

2014b An algorithmic characterization of line signed graph. Manuscript, 2014.

For a signed simple graph Σ, algorithms to construct (§3) the line graph $\Lambda(|\Sigma|)$; (§4) Γ' such that $|\Sigma| = \Lambda(\Gamma')$, if it exists; (§5) the Behzad–Chartrand (1969a) line graph $\Lambda_{BC}(\Sigma)$; (§6) Σ' such that $\Sigma = \Lambda_{BC}(\Sigma')$, if it exists. *Cf.* M. Acharya and Sinha (2005a). [Annot. 24 Dec 2014.] (**SG: LG: Algor**)

2015a An optimal algorithm to detect sign compatibility of a given sigraph. *Nat. Acad. Sci. Lett. (India)* 38 (2015), no. 3, 235–238. MR 3366153.

Definition: Sinha (2005a). The algorithm detects the forbidden subgraphs: a path with edges $-, +, -$ and a triangle with edges $-, +, -$. [Annot. 6 Jan 2016.] (**SG: Algor**)

2015b An algorithmic characterization of sigraphs whose second iterated line sigraphs and common-edge sigraphs are switching equivalent.. *J. Discrete Math. Sci. Cryptography* 18 (2015), no. 5, 581–603. MR 3399714. (**SG: LG, LG(Gen): Sw: Algor**)

2015c An algorithm to detect balancing of iterated line sigraph. *SpringerPlus* 4 (2015), art. 704, 19 pp. (**SG: LG: Bal: Algor**)

2015d An algorithm to detect S-consistency in line sigraph. *J. Combin. Inform. System Sci.* 40 (2015), 61–91. MR 3586362. Zbl 1358.05134. (**SG: LG: Algor**)

2016a Encryption using network and matrices through signed graphs. *Int. J. Computer Appl.* 138 (2016), no. 4, 6–13. (**SG: Adj, LG, Algor: Appl**)

2017a An algorithmic characterization of splitting signed graph. Int. Conf. Current Trends in Graph Theory and Computation (CTGTC-2016, New Delhi). *Electronic Notes Discrete Math.* 63 (2017), 323–332. MR 3754821. Zbl 1383.05147. (**SG: Algor**)

Deepa Sinha & Deepakshi Sharma

2014a Signed graphs whose 2-path signed graphs are isomorphic to their square signed graphs. Manuscript, 2014.
Full version of (2014b). (**SG: LG(Gen), Algor**)

2014b Algorithmic characterization of signed graphs whose two path signed graphs and square signed graphs are isomorphic. In: *2014 International Conference on Soft Computing Techniques for Engineering and Technology* (ICSCTET-2014, Bhimtal, India, 2014), 5 pp. IEEE, [2014].
Extended abstract of (2014a). (**SG: LG(Gen), Algor**)

2016a On square and 2-path signed graph. *J. Interconnection Networks* 16 (2016), no. 1, art. 1550011, 19 pp. (**SG: LG(Gen), Algor**)

2016b On 2-path signed graphs. In: *2016 International Workshop on Computational Intelligence* (IWCI, Dhaka, 2016), pp. 218–220. IEEE, 2016.
Extended abstract. (**SG: LG(Gen): Bal, Clu, Str**)

2017a Characterization of 2-path product signed graphs with Its properties. *Comput. Intell. Neurosci.* 2017 (2017), art. 1235715, 8 pp. (**SG: LG(Gen): VS, Sw**)

2017b Transitivity model on signed graphs. Int. Conf. Current Trends in Graph Theory and Computation (CTGTC-2016, New Delhi). *Electronic Notes Discrete Math.* 63 (2017), 455–460. MR 3754835. Zbl 1383.05148. (**SG**)

2018a Iterated local transitivity model for signed social networks. *Appl. Algebra Eng. Commun. Comput.* 29 (2018), 149–167. MR 3769265. Zbl 1384.05093. (**SG: Bal, Clu, VS**)

2018b On the properties of square signed graph. *Nat. Acad. Sci. Lett. (India)* 41 (2018), 233–238. MR 3842236. (**SG**)

2019a Square signed graph. *Nat. Acad. Sci. Lett. (India)* 42 (2019), 513–518. MR 4039031. (**SG**)

2020a Characterization of 2-path signed network. *Complexity* 2020 (2020), art. 1028941, 13 pp. Zbl 1454.05119.
$(\Sigma)_2 := (V, E', \sigma')$, where $uv \in E'$ iff $d(u,v) = 2$, and $\sigma'(uv) = -$ iff every 2-path uwv is all negative. Thm. 1 characterizes $(\Sigma)_2$'s. Thms. 2–4: balance, clusterability, sign-regularity of $(\Sigma)_2$. [Some seem too complicated. Some are not clear.] [Annot. 9 Nov 2020.] (**SG: VS, Bal, Clu, Algor**)

Deepa Sinha, Deepakshi Sharma, & Bableen Kaur

2017a Signed zero-divisor graph. Int. Conf. Current Trends in Graph Theory and Computation (CTGTC-2016, New Delhi). *Electronic Notes Discrete Math.* 63 (2017), 517–524. MR 3754842. Zbl 1383.05149. (**SG: Algeb**)

Deepa Sinha, Somya Upadhyaya, & Priya Kataria

2013a Characterization of common-edge sigraph. *Discrete Appl. Math.* 161 (2013), no. 9, 1275–1285. MR 3030620. Zbl 1277.05080.

For definition *cf.* M. Acharya and Sinha (2006a). Thm. 6: Σ is a common-edge signed graph iff $|\Sigma|$ is a common-edge graph and its edges decompose into homogeneously signed complete graphs. §4, "Algorithm to output C_E-root sigraph of a given common-edge sigraph". §5, "Complexity of COMMON-EDGE SIGRAPH": It is $O(n^2\#E)$. [Annot. 23 Nov 2014.] (**SG: LG(Gen)**)

John Sinkovic

See M. Arav.

Jozef Širáň

See also D. Archdeacon, P. Gvozdjak, C.H. Li, and B.D. McKay.

1991a Characterization of signed graphs which are cellularly embeddable in no more than one surface. *Discrete Math.* 94 (1991), 39–44. MR 1141052 (92i:05086). Zbl 742.05035.

A signed graph orientation-embeds in only one surface iff any two circles are vertex disjoint. (**SG: Top**)

1991b Duke's theorem does not extend to signed graph embeddings. *Discrete Math.* 94 (1991), 233–238. MR 1138602 (92j:05065). Zbl 742.05036.

Richard A. Duke (The genus, regional number, and Betti number of a graph. *Canad. J. Math.* 18 (1966), 817–822. MR 0196731 (33 #4917).) proved that the (orientable) genus range of a graph forms a contiguous set of integers. Stahl (1978a) proved the analog for nonorientable embeddings. Širáň shows this need not be the case for the demigenus range of an unbalanced signed graph. However, any gaps consist of a single integer each. The main examples with gaps are vertex amalgamations of balanced and uniquely embeddable unbalanced signed graphs, but a 3-connected example is $+W_6$ together with the negative diameters of the rim. *Question* 1 (Širáň). Do all gaps occur at the bottom of the demigenus range? [*Question* 2. Can one in some way derive almost all signed graphs with gaps from balanced ones?] (**SG: Top**)

Jozef Širáň, Jana Šiagiová, & Marián Olejár

2009a Graph coverings and graph labellings. Special Issue on Graph Labelings. Fifth Int. Workshop on Graph Labelings (IWOGL 2009) (Krishnankoil, 2009). *AKCE Int. J. Graphs Combin.* 6 (2009), no. 1, 127–133. MR 2533240 (2010g:05331). Zbl 1210.05129.

Connectivity and automorphisms of a covering graph of a gain graph ("voltage graph"). [Annot. 21 July 2010.] (**GG: Cov: Aut, Exp**)

Jozef Širáň & Martin Škoviera

†† 1991a Characterization of the maximum genus of a signed graph. *J. Combin. Theory Ser. B* 52 (1991), 124–146. MR 1109428 (92b:05033). Zbl 742.05037.

The maximum demigenus $d_M(\Sigma)$ = the largest demigenus of a closed surface in which Σ orientation embeds. Two formulas are proved for $d_M(\Sigma)$: one a minimum and the other a maximum of readily computable numbers. Thus $d_M(\Sigma)$ has a "good" (polynomial) characterization. Along the way, several results are proved about single-face embeddings. *Problem* (§11). Characterize those edge-2-connected Σ such that Σ and all $\Sigma \backslash e$ have single-face embeddings. [A complex and lovely paper.]

[Re single-face embeddings, *cf.* Bernardi and Chapuy (2011a) and Isenmann and Pecatte (2017a).] (**SG: Top**)

Jozef Širáň & Thomas W. Tucker

2009a Symmetric maps. In: Lowell W. Beineke and Robin J. Wilson, eds., *Topics in Topological Graph Theory*, Ch. 10, pp. 199–224. Encycl. Math. Appl., Vol. 128. Cambridge Univ. Press, Cambridge, Eng., 2009. MR 2581547 (no rev). Zbl 1201.05022.

P. 203 mentions edge signs. P. 218 says the Petrie dual of a rotation system on Σ is the same rotation system on $-\Sigma$. [Annot. 26 Aug 2018.] (**sg: Top, sw: Exp**)

[P. Siva Kota Reddy]

See P.S.K. Reddy (under 'R').

B. Sivakumar

See also M. Parvathi.

2009a Matrix units for the group algebra $kG_f = k((Z_2 \times Z_2) \wr S_f)$. *Asian-European J. Math.* 2 (2009), no. 2, 255–277. MR 2532703 (2010g:16043). Zbl 1198.20013. (**gg: matrd: Algeb**)

Vaidy Sivaraman

See also J. Maharry and M.A. Mutar.

2012a *Some Topics Concerning Graphs, Signed Graphs, and Matroids*. Doctoral thesis, Ohio State University, 2012. MR 3130834 (no rev).

Thm.: $D(\Sigma) := \min_X l(\Sigma^X) \leq \frac{3}{8}n$ for cubic Σ. [*Cf.* Sehrawat and Bhattacharjya (2019b).] (**SG: Sw, Matrd**)

2014a Bicircular signed-graphic matroids. *Discrete Math.* 328 (2014), 1–4. MR 3199809. Zbl 1288.05035.

The graphs for which $\mathbf{F}(\Gamma, \varnothing)$ is a frame matroid of a signed graph: iff $\mathbf{F}(\Gamma, \varnothing)$ is ternary, and other characterizations including forbidden subgraphs. Successor to Matthews (1977a), and implicitly Zaslavsky (2007a) for group $\mathbb{Z}_2$. [Annot. 1 Oct 2017, rev 11 Jun 2019.] (**Bic, SG: Matrd**)

Vaidy Sivaraman & Daniel Slilaty

2019a The graphs that have antivoltages using groups of small order. *Discrete Math.* 342 (2019), 2951–2965. MR 3996733. Zbl 1419.05189.

Γ characterized for $|\mathfrak{G}| \leq 6$ by structure, forbidden minors, and algorithmic recognition. Successor to Zaslavsky (2007a), Sivaraman (2014a), and Chun, Moss, Slilaty, and Zhou (2016a). [Annot. 11 Jun 2019.] (**GG:Bic: Geom, Algor**)

2022a The family of bicircular matroids closed under duality. *Graphs Combin.* 8 (2020), art. 24, 20 pp. MR 4356265. Zbl 1481.05025. arXiv:2012.11712.

The 3-connected matroids that are bicircular and dual bicircular are the free swirls $\mathbf{F}(2C_n, \varnothing)$ and their minors, with exceptions of rank and dual rank ≤ 5. [Annot. 29 Dec 2020.] (**Bic, Du**)

Vaidy Sivaraman & Thomas Zaslavsky

20xxa The seven signed Heawood graphs. In preparation.

Successor to Zaslavsky (2012b), with several general theorems. There are 7 switching isomorphism classes of signatures of the Heawood graph H. l, l_0, χ, Q (inclusterability index) are computed. General thm.: For subcubic $|\Sigma|$, $l_0(\Sigma) = l(\Sigma)$ (for $-\Gamma$ see Choi, Nakajima, and Rim (1989a)). If $\#E^-(\Sigma) = l(\Sigma)$, then $Q(\Sigma) = l(\Sigma)$. [Annot. 1 Oct 2017.] (**SG: Sw, Str, Fr, Clu, Col**)

A. Skhreĭver [A. Schrijver]

See A. Schrijver.

D.B. Skillicorn

See Q. Zheng.

Bjarke Skjernaa

See J.M. Byskov.

Howard Skogman

See F. Belardo and N. Reff.

Martin Škoviera

See also E. Máčajová, A. Malnič, R. Nedela, and J. Širáň.

1983a Equivalence and regularity of coverings generated by voltage graphs. In: Miroslav Fiedler, ed., *Graphs and Other Combinatorial Topics* (Proc. Third Czechoslovak Sympos. on Graph Theory, Prague, 1982), pp. 269–272. Teubner-Texte Math., 59. Teubner, Leipzig, 1983. MR 0737050 (85e:05064). Zbl 536.05019. (**GG: Top, Cov, Sw**)

1986a A contribution to the theory of voltage graphs. *Discrete Math.* 61 (1986), 281–292. MR 0855333 (88a:05060). Zbl 594.05029.

Automorphisms of covering projections of canonical covering graphs of gain graphs. (**GG: Top, Cov, Aut, Sw**)

1992a Random signed graphs with an application to topological graph theory. In: Alan Frieze and Tomasz Luczak, eds., *Random Graphs, Vol. 2* (Proc., Poznań, 1989), Ch. 17, pp. 237–246. Wiley, New York, 1992. MR 1166619 (93g:05126). Zbl 817.05059.

The model: each edge is selected with probability p, positive with probability s. Under mild hypotheses on p and s, Σ is almost surely unbalanced and almost surely has a 1-face orientation embedding. [Related: Frank and Harary (1979a).] (**SG: Rand, Enum, Top**)

Riste Škrekovski

See R. Naserasr.

Daniel C. Slilaty

See also L. Abrams, N. Bowler, A.H. Busch, Y.Q. Chen, D. Chun, J. Maharry,

M.A. Mutar, N.A. Neudauer, H. Qin, and J. Robbins.

2000a *Orientations of Biased Graphs and Their Matroids.* Doctoral dissertation, State University of New York at Binghamton, 2000. MR 2701091 (no rev).

Introducing orientation of biased graphs and biased signed graphs by means of proper circle orientations and their generalization, "graphical orientation schemes". The definition is chosen so as to produce orientations of the bias and complete lift matroids and (though not in the thesis) to model the orientation of the bias or complete lift matroid of, respectively, an $\mathbb{R}^{\times}$- or $\mathbb{R}^{+}$-gain graph induced by its canonical bias or lift representation (Zaslavsky (2003b)). Characterizations of equivalence of different orientation schemes. The completeness question: when do graphical orientation schemes yield all orientations of the frame matroid? Always, for additively biased (i.e., signed) graphs and for some other kinds of biased graphs. (**GG: Ori, Matrd, OG, SG**)

† 2002a Matroid duality from topological duality in surfaces of nonnegative euler characteristic. *Combin. Probab. Computing* 11 (2002), no. 5, 515–528. MR 1930356 (2003i:05034). Zbl 1009.05036.

Duality of matroids of biased graphs, especially signed graphs, obtained by defining gains through embedding in a surface and dualizing the graph in the surface, especially the plane, projective plane, and torus. [Annot. rev 15 Dec 2020.] (**GG, SG: Matrd, Du, Top**)

2005a On cographic matroids and signed-graphic matroids. *Discrete Math.* 301 (2005), no. 12, 207–217. MR 2171313 (2007c:05049). Zbl 1078.05017. (**SG: Matrd, Top**)

2006a Bias matroids with unique graphical representations. *Discrete Math.* 306 (2006), no. 12, 1253–1256. MR 2245651 (2007b:05044). Zbl 1093.05015.

When does the frame matroid $\mathbf{F}(\Omega)$ determine the biased graph Ω? Given Ω and Ω_0, without isolated vertices, loose or half edges, or balanced loops. Assume Ω is 3-connected and contains three vertex-disjoint unbalanced circles, at most one of which is a loop. Thm. 2: $\mathbf{F}(\Omega) \cong \mathbf{F}(\Omega_0)$ iff $\Omega \cong \Omega_0$. [Annot. 14 Feb 2013.] (**GG: Matrd: Str, Circ**)

† 2007a Projective-planar signed graphs and tangled signed graphs. *J. Combin. Theory Ser. B* 97 (2007), no. 5, 693–717. MR 2344133 (2008j:05161). Zbl 1123.05046.

Thm.: The signed graphs with no two vertex-disjoint negative circles are those with a balancing vertex, or obtained from a projective-planar signed graph (*cf.* Zaslavsky (1993a)) or from $[-K_5]$ by t-summation with balanced signed graphs for $t = 1, 2, 3$. (Previously announced in less general form by Lovász [see Seymour (1995a)] but the proof was incorrect.) [*Major Problem.* Characterize the biased graphs having no two vertex-disjoint unbalanced circles. Lovász (1965a), *q.v.*, solved the contrabalanced case.] (**SG: Circ: Top, Str**)

2010a Integer functions on the edges and cycle space of a graph. *Graphs Combin.* 20 (2010), no. 2, 293–299. MR 2606501 (2011b:05092). Zbl 1230.05142.

Integral gains $\varphi : E \to \mathbb{Z}$ induce a cycle-space homomorphism $\hat{\varphi} : Z_1(\Gamma) \to \mathbb{Z}$. Let $f : Z_1(\Gamma) \to \mathbb{Z}$. Thm. 3: $f(W) \leq k\#W$ for every walk W iff $f = \hat{\varphi}$ for some φ satisfying $\max|\varphi(e)| \leq k$. Thm. 2: For odd k, if

also $f(W) \equiv \#W \bmod 2$, there is φ which assumes only odd values; and conversely. [Annot. 5 Sept 2010.] **(GG)**

2023a On colorings and orientations of signed graphs. *Discrete Math. Lett.* 12 (2023), 98–102. MR 4617325. **(SG: Col, Ori)**

2024a Graphs without a $2C_3$-minor and bicircular matroids without a $U_{3,6}$-minor. *Trans. Combin.* 13 (2024), no. 2, 165–167. **(Bic: Str)**

20xxa Connectivity in signed-graphic matroids. Submitted. **(SG: Matrd: Str)**

20xxc On colorings and orientations of signed graphs II. Submitted. **(SG: Col, Ori)**

Daniel C. Slilaty & Hongxun Qin

2007a Decompositions of signed-graphic matroids. *Discrete Math.* 307 (2007), no. 17-18, 2187–2199. MR 2340600 (2008f:05032). Zbl 1121.05055. **(SG: Matrd: Str)**

2008a The signed-graphic representations of wheels and whirls. *Discrete Math.* 308 (2008), no. 10, 1816–1825. MR 2394450 (2009c:05043). Zbl 1173.05311.

All frame matroids (of biased graphs) that are wheels and whirls, characterized topologically by embeddings in the projective plane (wheels) and the cylinder (whirls). **(GG: Matrd: Str)**

2008b Connectivity in frame matroids. *Discrete Math.* 308 (2008), no. 10, 1994–2001. MR 2394467 (2009e:05139). Zbl 1170.05323.

Graphical biconnectivity of Ω vs. matroid connectivity of $\mathbf{F}(\Omega)$, generalizing concepts developed by Wagner (1985a) for the bicircular matroid. **(GG: Matrd: Str)**

Daniel C. Slilaty & Thomas Zaslavsky

2015a Characterization of line-consistent signed graphs. *Discuss. Math. Graph Theory* 35 (2015), 589–594. MR 3368992. Zbl 1317.05081. arXiv:1404.1651.

A constructive proof of Acharya, Acharya, and Sinha's (2009a) criterion for consistency of $\Lambda(\Sigma)$. [Annot. 14 Oct 2009.] **(SG, VS: LG: Bal)**

20xxa Cobiased graphs: Single-element extensions and elementary quotients of graphic matroids. *Electronic J. Combin.* (to appear). arXiv:2401.17616.

Cobias means choosing a linear class of bonds in Γ. Axiomatics of the titular matroids. [Annot. 16 Jun 2023.] **(GG(Du): Matrd: Str)**

Daniel Slilaty & Xiangqian Zhou

2013a Some minor-closed classes of signed graphs. *Discrete Math.* 313 (2013), 313–325. MR 3004465. Zbl 1259.05157. **(SG: Top)**

N.J.A. Sloane

See P.C. Fishburn, R.L. Graham, and C.L. Mallows.

Kaleigh Smith

See B. Reed.

Alex Smola

See S.H. Yang.

Chris Smyth

See J. McKee.

J. Laurie Snell

See J. Berger and J.G. Kemeny.

El Houssine Snoussi

See also J.-P. Comet and D. Thieffry.

1998a Necessary conditions for multistationarity and stable periodicity. *J. Biol. Systems* 6 (1998), no. 1, 3–9. Zbl 0982.92001. (**SD: Dyn**)

El Houssine Snoussi & Rene Thomas

1993a Logical identification of all steady states: The concept of feedback loop characteristic states. *Bull. Math. Biol.* 55 (1993), no. 5, 973–991. Zbl 0784.92002. (**Dyn: SD**)

Dylan Snustad

See M. Cho.

Lynea Snyder

See Y. Duong.

Moo Young Sohn

See J.H. Kwak.

Alan D. Sokal

See also A.D. Scott.

2005a The multivariate Tutte polynomial (alias Potts model) for graphs and matroids. In: Bridget S. Webb, ed., *Surveys in Combinatorics 2005*, pp. 173–226. Cambridge Univ. Press, Cambridge, Eng., 2005. MR 2187739 (2006k:05052). Zbl 1110.05020.

The parametrized dichromatic polynomial with parameters $d_e = 1$, called the "multivariate Tutte polynomial". Partly expository, partly new. [See Zaslavsky (1992b).] (**SGw: Gen: Invar**)

James P. Solazzo

See D.M. Duncan and T.R. Hoffman.

Patrick Solé & Thomas Zaslavsky

1994a A coding approach to signed graphs. *SIAM J. Discrete Math.* 7 (1994), 544–553. MR 1299082 (95k:94041). Zbl 811.05034.

Among other things, improves some results in Akiyama, Avis, Chvátal, and Era (1981a). Thm. 1: For a loopless graph, $D(\Gamma) \geq \frac{1}{2}m - \sqrt{\frac{1}{2}\ln 2}\sqrt{m(n-c(\Gamma))}$. Thm. 2: For a simple, bipartite graph, $D(\Gamma) \leq \frac{1}{2}(m - \sqrt{m})$. *Conjecture.* The best general asymptotic lower bound is $D(\Gamma) \geq \frac{1}{2}m - c_1\sqrt{mn} + o(\sqrt{mn})$ where c_1 is some constant between $\sqrt{\frac{1}{2}\ln 2}$ and $\frac{1}{2}\pi$. *Question.* What is c_1 for, e.g., k-connected graphs? Thm. 4 gives girth-based upper bounds on $D(\Gamma)$. §5, "Embedded graphs", has bounds for several examples obtained by surface duality. All proofs are via covering radius of the cutset code of Γ. (**SG: Fr, Top**)

Extends to $r = 5$ the exact values of $D(K_{r,s})$ for $r \leq 4$ in Brown and Spencer (1971a). [But $r = 5$ has errors. Extended correctly to all r by

Bowlin (2009a), (2012a).] [Annot. rev. 14 Feb 2011.] (**SG: Fr**)

Sylvain Soliman
See F. Fages and K. Sriram.

Louis Solomon
See P. Orlik.

Bishal Sonar, Satyam Guragain, & Ravi Srivastava
20xxa On spectrum of neighbourhood corona product of signed graphs. arXiv:-2310.12814. (**SG: Adj, Lap: Eig**)

N.D. Soner
See R. Rangarajan.

Dongjin Song & David A. Meyer
2014a A model of consistent node types in signed directed social networks. In: *2014 IEEE/ACM International Conference on Advances in Social Networks Analysis and Mining* (ASONAM, Beijing, 2014), pp. 72–80. IEEE, 2014. arXiv:-1408.6822. (**SD: PsS**)

2015a Recommending positive links in signed social networks by optimizing a generalized AUC. In: *Proceedings of the Twenty-Ninth AAAI Conference on Artificial Intelligence* (AAAI15, Austin, Tex., U.S.A.), pp. 290–296. AAAI Press, Palo Alto, Calif., 2015. URL `http://www.aaai.org/ocs/index.php/AAAI/AAAI15/paper/view/9436` (**SG: Algor**)

2015b Link sign prediction and ranking in signed directed social networks. *Social Network Anal. Mining* 5 (2015), art. 52, 24 pp. (**SG: Pred: PsS**)

Huimin Song
See X.Q. Qi.

Joungmin Song
2017a On certain hyperplane arrangements and colored graphs. *Bull. Korean Math. Soc.* 54 (2017), no. 2, 375–382. MR 3632442. Zbl 1373.32022. arXiv:1606.07874.
Regions of the [Linial-like threshold] hyperplane arrangement $\mathcal{J}_n := \{x_i + x_j = 1 \text{ and } x_i = 0, 1\}$ are counted via graph theory related to $-K_n$. [The hyperplanes are translates of the hyperplanes in $\mathcal{H}[-K_n^\bullet]$. This calls for generalization via signed graphs.] [Annot. 14 Apr 2017.]
(**sg: par: Geom: Invar**)

2017b Enumeration of graphs and the characteristic polynomial of the hyperplane arrangements $\mathcal{J}_n$. *J. Korean Math. Soc.* 54 (2017), no. 5, 1595–1604. MR 3691940. Zbl 06853526. arXiv:1701.07313.
The characteristic polynomial; *cf.* (2017a). [Annot. 14 Apr 2017.]
(**sg: par: Geom: Invar**)

2017c Characteristic polynomial of certain hyperplane arrangements through graph theory. Manuscript, 2017. arXiv:1701.07330. (**sg: par: Geom: Invar**)

2018a Characteristic polynomial of the hyperplane arrangements $\mathcal{J}_n$ via finite field method. *Commun. Korean Math. Soc.* 33 (2018), no. 3, 759–765. MR 3846025. Zbl 1401.32023.

Cf. (2017a), (2017b). [Annot. 1 Aug 2018.] (**sg: par: Geom: Invar**)

Sang-Oak Song
See G. Lee.

Song Yi-Zhe
See B. Xiao.

Zi-Xia Song
See K. Kawarabayashi.

Eduardo D. Sontag
See also D. Angeli, B.N. Kholodenko, G. Craciun, B. DasGupta, and G.A. Enciso.

2004a Some new directions in control theory inspired by systems biology. *Systems Biol.* 1 (2004), no. 1, 9–18.

P. 13 describes how a signed digraph arises from differential equations, and that it is "monotone" [= isotone] iff it has no negative cycles. [Annot. 25 Jan 2015.] (**Biol: Dyn: SD: Exp**)

2005a Molecular systems biology and control. *European J. Control* 11 (2005), no. 4-5, 396–435. MR 2201569 (no rev).

§5.1, "Consistent graphs and monotone systems". §5.4, "Almost-monotonicity". Dictionary: "parity" = sign, "consistent" = balanced, "consistency deficit" = frustration index, "almost-consistency" = small $l(\Sigma)/\#E$, "monotone" = isotone (monotone weakly increasing). [Annot. 1 Jan 2012.] (**SD(sg): Bal, Fr: Dyn, Biol**)

2007a Monotone and near-monotone systems. In: I. Queinnec *et al.*, eds., *Biology and Control Theory: Current Challenges*, pp. 79–122. Lect. Notes in Control and Inform. Sci., Vol. 357. Springer-Verlag, Berlin, 2007. MR 2352229 (2008k:92021). *Cf.* arXiv:q-bio/0612032, arXiv:q-bio/0612033.

Conference version of (2007b); almost the same. [Annot. 23 Jan 2015.] (**SD, SG: Bal, Fr, Dyn, Biol: Exp, Ref**)

2007b Monotone and near-monotone biochemical networks. *Systems Synthetic Biol.* 1 (2007), 59–87.

Dictionary: "graph" = signed signed digraph; "spin assignment" = state = function $\zeta : V \to \{+1, -1\}$; edge "consistent" with ζ = satisfied edge ($\sigma(e) = \zeta_i\zeta_j$); "consistent spin assignment" Σ = potential function ζ (edge directions are ignored); "monotone" = balanced (undirected); "consistency deficit" = frustration index (undirected). (**SD, SG: Bal, Fr, Dyn, Biol: Exp, Ref**)

Eduardo Sontag, Alan Veliz-Cuba, Reinhard Laubenbacher, & Abdul Salam Jarrah

† 2008a The effect of negative feedback loops on the dynamics of Boolean networks. *Biophys. J.* 95 (2008), 518–526 + suppl. 9 pp. arXiv:0707.3468.

The directed frustration index $l(\vec{\Gamma}, \sigma)$ (called the "PF-distance") of a signed digraph is the smallest number of edges whose signs should be changed to eliminate all negative cycles. This index is a measure of the number of independent negative cycles. Then $l_{\max}(\vec{\Gamma}) := \max_\sigma l(\vec{\Gamma}, \sigma)$. P. 522: An algorithm "Distance to PF" for $l(\vec{\Gamma}, \sigma)$ with strongly connected

$\vec{\Gamma}$ (sufficient, since l is additive on strong components and a negative loop adds 1). Dictionary: "directed cycle" = cycle; "cycle" = circle; "odd parity" = negative sign; "positive feedback" (PF) = cycle balance (no negative cycles); "$|\vec{\Gamma}|$" = $l_{\max}(\vec{\Gamma})$.

A unate Boolean function $f : \mathbb{F}_2^n \to \mathbb{F}_2^n$ has a (signed) interaction digraph $\mathcal{D}(f)$. A computational experiment tests the connection between $l(\mathcal{D}(f))$ and the number and length of attractors (limit cycles) of f in $\mathbb{F}_2^n$, which appear to be direct and inverse, respectively. Dictionary: "unate" = each component function is monotone (isotone or antitone); "monotone" = isotone (monotone weakly increasing); "signed dependency graph" = (signed) interaction digraph, "distance-to-positive-feedback", "PF-distance" = frustration index. [Annot. 16 Jan 2015.] (**SD, SG: Fr, Algor**)

Soorya P

See also S. Hameed, A. Mathew, and T.V. Shijin.

2022a *A Study on Choosability in Graphs Signed Graphs and Related Parameters*. Doctoral dissertion, Central University of Kerala, 2022, (**SG: Col(Gen)**)

Éric Sopena

See J. Bensmail, C. Charpentier, A. Montejano, and R. Naserasr.

Nicola Soranzo

See also G. Iacono.

Nicola Soranzo, Fahimeh Ramezani, Giovanni Iacono, & Claudio Altafini

2012a Decompositions of large-scale biological systems based on dynamical properties. *Bioinformatics* 28 (2012), no. 1, 76–83. (**SD**)

Mauricio Soto

See E.G. Pardo.

C.M. Soukoulis

See D. Blankschtein.

Christophe Soulé

See also M. Kaufman.

2003a Graphic requirements for multistationarity. *ComPlexUs* 1 (2003), 123–133. arXiv:q-bio/0403033. (**SD: Dyn, Biol**)

2006a Mathematical approaches to differentiation and gene regulation. *C.R. Biologies* 329 (2006), 13–20. arXiv:q-bio/0510027. (**SD: Dyn, Biol**)

Mona Souri

See also S. Akbari.

M. Souri, F. Heydari, & M. Maghasedi

2020a Maximizing the largest eigenvalues of signed unicyclic graphs. *Discrete Math. Algorithms Appl.* 12 (2020), no. 2, art. 2050016, 8 pp. MR 4099663. Zbl 1456.05071. (**SG: Adj: Eig**)

2021a On the eigenvalues of some signed graphs. *Iranian J. Sci. Technol. Trans. A Sci.* 45 (2021), no. 2, 635–639. MR 4230645.

Spec $A(K_n(-K_{n_1,\ldots,n_k}))$. If three n_i are equal, Spec determines $K_{n_1,\ldots,n_k}$. [Annot. 6 Sept 2022.] (**SG: KG: Adj: Eig**)

N. Sourlas

See S. Caracciolo.

B.W. Southern, S.T. Chui, & G. Forgacs

1980a Non-universality for two-dimensional frustrated lattices? *J. Phys. C* 13 (1980), L827–L830.

Physics of signed square lattice graph, fully frustrated (all positive except for all-negative alternating vertical lines). Reduced to the "8-vertex" physics model by taking alternating sites (vertices) and observing they are 4-valent and all or half positive. [*Cf.* Garel and J.M. Maillard (1983a).] [Annot. 16 Jun 2012.] (**Phys: sg**)

Uéverton S. Souza

See C.V.G.C. Lima.

Cid C. de Souza

See R.M.V. Figueiredo.

[Natasha D'Souza]

See N. D'Souza (under 'D').

Tadeusz Sozański

1976a Processus d'équilibration et sous-graphes équilibrés d'un graphe signé complet. *Math. Sci. Humaines*, No. 55 (1976), 25–36, 83. MR 0543817 (58 #27613).

Σ denotes a signed K_n. The "level of balance" ("indice du niveau d'équilibre") $\rho(\Sigma)$:= maximum order of a balanced subgraph. [Complement of the vertex deletion number $l_0(\Sigma)$.] Define distance $d(\Sigma_1, \Sigma_2) := \#(E_{1+} \triangle E_{2+})$. Say Σ is p-clusterable if Σ^+ consists of p disjoint cliques [its "clusters"]. Thm. 1 evaluates the frustration index of a p-clusterable Σ. Thm. 2 bounds $l(\Sigma)$ in terms of n and $\rho(\Sigma)$. A negation set U for Σ "conserves" a balanced induced subgraph if they are edge-disjoint; it is "(strongly) conservative" if it conserves some (resp., every) maximum-order balanced induced subgraph. Thm. 3: Every minimum negation set conserves every balanced induced subgraph of order $> \frac{2}{3}n$. Thm. 4: A minimum negation set can be ordered so that, successively negating its edges one by one, ρ never decreases. (**SG: KG: Fr, Clu**)

1980a Enumeration of weak isomorphism classes of signed graphs. *J. Graph Theory* 4 (1980), 127–144. MR 0570348 (81g:05070). Zbl 434.05059.

"Weak isomorphism" = switching isomorphism. Principal results: The number of switching nonisomorphic signed K_n's. (*Cf.* Mallows and Sloane (1975a).) The number that are switching isomorphic to their negations. The number of nonisomorphic (not switching nonisomorphic!) balanced signings of a given graph. §2.3, "Space of signed graphs over a fixed graph", implicitly contains the theorem that two signed graphs are switching isomorphic iff there is an isomorphism of underlying graphs that preserves circle signs [*cf.* Zaslavsky (1982a), Prop. 3.2; (1981b), Thm. 7]. [Annot. rev. 22 Oct 2015.] (**SG, KG: Sw: Enum**)

1982a *Model rownowagi strukturalnej. Teoria grafow oznakowanych i jej zastosowania*

w naukach społecznych. [The structural balance model. The theory of signed graphs and its applications in the social sciences.] (In Polish.) Doctoral thesis, Jagellonian University, Krakow, 1982.
(**SG, PsS: Bal, Fr, Clu, Aut, Adj, Ref**)

Quico Spaen, Christopher Thraves Caro, & Mark Velednitsky

2020a The dimension of valid distance drawings of signed graphs. *Discrete Comput. Geom.* 63 (2020), 158–168. MR 4045743. Zbl 1429.05085.

Valid distance drawing: $V \hookrightarrow \mathbb{R}^k$ such that $(\forall v)$ all positive neighbors of v are closer than all negative neighbors. Problem: Find $L(n) := \min k$ with a valid drawing $\forall\ \Sigma$ such that $\#V = n$. Thm.: $\lfloor \log_5(n-3) \rfloor + 1 \leq L(n) \leq n-2$. [Annot. 28 Dec 2019.] (**SG: Geom**)

Edward Spence

See W.H. Haemers.

Joel Spencer

See T.A. Brown.

Stefano Spessato

See M. Cavaleri.

Daniel A. Spielman

See A.S. Bandeira and A.W. Marcus.

Joel Spencer with Laura Florescu

2014a *Asymptopia.* Student Math. Library, Vol. 71. American Mathematical Society, Providence, R.I., 2014. MR 3185739. Zbl 1331.00002.

§6,8, "An exact formula for unicyclic graphs": The number of bases of $\mathbf{L}(K_n, \varnothing)$, the bicircular lift matroid of K_n. §6.4, "Counting unicyclic graphs in Asymptopia": Asymptotics. [N.B. $\mathbf{L}_\infty(K_n, \varnothing)$ has n^{n-2} additional bases.] [Annot. 3 Oct 2014.] (**bic: matrd: Invar**)

Sam Spiro

2022a The Wiener index of signed graphs. *Appl. Math. Comput.* 416 (2022), art. 126755, 10 pp. MR 4337738. arXiv:2106.11869.

The graphs are sign-weighted, i.e., edges weighted by $\pm 1 \in \mathbb{Z}^+$ (the additive group). Weiner index $W_\sigma(G) := \sum_{u,v} d_\sigma(u,v)$ where $d_\sigma(u,v)$ ("signed distance") $:= \min_{P_{uv}} |\sum_{e \in P_{uv}} \sigma(e)|$. Thm. 1.3(b): If $W_\sigma(G) = 0$, then G contains an odd cycle. [*Problem.* This hints at generalization to a sign-weighted signed graph (G, σ, σ_1) where Thm. 1.3(b) becomes (G, σ_1) is not antibalanced.] [Annot. 25 Nov 2021.] (**SGw: Invar**)

K.K. Srimitra

See S. Sajana.

Aravind Srinivasan

2011a Local balancing influences global structure in social networks. *Proc. Nat. Acad. Sci. (U.S.A.)* 108 (2011), no. 5, 1751–1752.

Summary of and commentary on Marvel, Kleinberg, Kleinberg, and Strogatz (2011a). [Annot. 7 Feb 2011.] (**SG: KG: Fr, Dyn**)

Murali K. Srinivasan

See also A. Bhattacharya.

1998a Boolean packings in Dowling geometries. *European J. Combin.* 19 (1998), 727–731. MR 1642742 (99i:05059). Zbl 990.10387.

Decomposes the Dowling lattice $Q_n(\mathfrak{G})$ into Boolean algebras, indexed in part by integer compositions, that are cover-preserving and centered above the middle rank. (**GG: Matrd**)

R. Srinivasan

See V. Kodiyalam.

M.A. Sriraj

See C. Adiga and E. Sampathkumar.

K. Sriram, Sylvain Soliman, & François Fages

2009a Dynamics of the interlocked positive feedback loops explaining the robust epigenetic switching in *Candida albicans*. *J. Theor. Biol.* 258 (2009), 71–88.

The effect of a pair of positive cycles sharing a single vertex, with a biological example. *Cf.* Kim, Yoon, and Cho (2008a). [Annot. 16 Jan 2015.] (**SD: Bal: Dyn, Biol**)

Ajitesh Srivastava, Charalampos Chelmis, & Viktor K. Prasanna

2016a Computing competing cascades on signed networks. *Social Network Analysis Mining* 6 (2016), art. 82, 13 pp. (**SG: Dyn**)

Nikhil Srivastava

See A.W. Marcus.

Ravi Srivastava

See B. Sonar and S. Guragain.

Ladislav Stacho

See D. Král'.

Derek Stafford

See D. Feng.

Saul Stahl

1978a Generalized embedding schemes. *J. Graph Theory* 2 (1978), 41–52. MR 0485488 (58 #53180. Zbl 396.05013.

A generalized embedding scheme for a graph is identical to a rotation system for a signing of the graph. Thm. 2: Signed rotation systems describe all cellular embeddings of a graph. Thm. 4: Embeddings are homeomorphic iff their signed rotation systems are switching equivalent. Thm. 5: An embedding is orientable iff its signature is balanced. Compare Ringel (1977a). Dictionary: λ is the signature. "λ-trivial" means balanced. (**sg: Top, Sw**)

1978b The embeddings of a graph—a survey. *J. Graph Theory* 2 (1978), 275–298. MR 0512799 (80a:05085). Zbl 406.05027. (**sg: Top**)

2005a *Introduction to Topology and Geometry.* Wiley-Interscience, Hoboken, N.J., 2005. MR 2102439 (2005g:57001). Zbl 1063.57001.

§4.4, " Voltage graphs and their coverings": Rotation system for surface embedding, embedded covering of embedded voltage graph, branched covering graph and embedding. [Annot. 25 Apr 2014.]

(**GG: Top: Exp, Exr**)

Saul Stahl & Catherine Stenson

2013a *Introduction to Topology and Geometry*, 2nd ed. Wiley-Interscience, Hoboken, N.J., 2013. MR 3235588. Zbl 1286.57001.
See Stahl (2005a). (**GG: Top: Exp, Exr**)

David P. Stanford

See C.R. Johnson.

Zoran Stanić

See also M. Anđelić, F. Belardo, M. Brunetti, G. Greaves, R. Mulas, L. Parsaei-Majd, F. Ramezani, P. Rowlinson, and S.K. Simić.

2007a *Neke rekonstrukcije u spektralnoj teoriji grafova i grafovi sa integralnim Q-spektrom.* [*Some Reconstructions in Spectral Graph Theory and Graphs with Integral Q-Spectrum.*] (In Serbian.) Doctoral Thesis, Faculty of Math., University of Belgrade, 2007. (**par: Lap: Eig**)

2007b There are exactly 172 connected Q-integral graphs up to 10 vertices. *Novi Sad J. Math.* 37 (2) (2007), 193–205. MR 2401613 (no rev). Zbl 1164.05046.
I.e., Spec $L(-\Gamma)$ is integral. [Annot. 19 Feb 2021.] (**par: Lap: Eig**)

2009a On determination of caterpillars with four terminal vertices by their Laplacian spectrum. *Linear Algebra Appl.* 431 (2009), 2035–2048. MR 2567810 (2010j:05253). Zbl 226.05165.
§5: Spec $L(-\Gamma)$ is mentioned. [Annot. 16 Jan 2012.]
(**par: bal: Lap: Eig**)

2018a Perturbations in a signed graph and its index. *Discuss. Math. Graph Theory* 38 (2018), 841–852. MR 3811969. Zbl 1391.05178.
Perturbation by adding an edge or vertex or negating an edge. Effect on $\lambda_{\max}(\Sigma)$. Eg., a case of Cor. 8: Negating e in a nontrivial balanced block reduces $\lambda_{\max}$. §4, "Representatives of small switching equivalent signed graphs": Methodology for (2018b). Dictionary: "simple" = unsigned (treated as all positive). (**SG: Adj: Eig**)

2018b Signed graphs of small order. URL `www.math.rs/~zstanic/siggr.htm`.
Counts and lists of signed graphs for $n \leq 8$ and cospectral ones for $n \leq 7$. Data for (2018a). [Annot. 19 Feb 2021.] (**SG: Sw, Adj, Eig**)

2019a Some bounds for the largest eigenvalue of a signed graph. *Bull. math. Soc. sci. math. Roumanie* 62(100) (2019), no. 2, 183–189. MR 4007428. Zbl 1463.05237.
(**SG: Adj: Eig**)

2019b Bounding the largest eigenvalue of signed graphs. *Linear Algebra Appl.* 573 (2019), 80–89. MR 3933292. Zbl 1411.05109.
Upper bounds such as $\lambda_{\max} \leq \max_i \frac{1}{2}\big(\sqrt{5d_i^2 + 4(d_i m_i - 4t_i^-)} - d_i\big)$, where d_i = degree, m_i = average 2-degree, t_i^- = # negative triangles on v_i. [Annot. 15 Feb 2021.] (**SG: Adj: Eig**)

† 2019c Integral regular net-balanced signed graphs with vertex degree at most four. *Ars Math. Contemp.* 17 (2019), 103–114. MR 3998150. Zbl 1433.05142.
Important result, "Lemma" 2.1: If Σ is connected, then $\lambda_{\max}(\Sigma) = \lambda_{\max}(|\Sigma|)$ iff Σ is balanced. [This strengthens Acharya's (1980a) spectral characterization of balance in signed graphs.]

Also, all net-regular Σ that are 3-regular with integral Spec $A(\Sigma)$, and some 4-regular ones. [Annot. 19 Dec 2020, 21 Feb 2021.] (**SG: Adj: Eig**)

† 2019d On strongly regular signed graphs. *Discrete Appl. Math.* 271 (2019), 184–190. MR 4030312. Zbl 1428.05331. Corrigendum. *Discrete Appl. Math.* 284 (2020), 640. MR 4115513. Zbl 1443.05185.

Introduces SRSG. The definition is combinatorial. Assume simple, r-regular $|\Sigma|$. Let $w_2^{\pm}(u,v) = \#$ positive $-$ $\#$ negative 2-edge uv-walks. Let $w_2^{\pm}(uv) = a,\ b,\ c$ according as $uv \in E^+,\ \in E^-$, or $\notin E$. Σ is "strongly regular", unless $E = \varnothing$ or $\Sigma = (K_n,+),\ (K_n,-)$.

The balanced SRSG are the same as ordinary strongly regular graphs. The disconnected ones are characterized in Thm. 4.2. Zaslavsky's (2010b) "very SRSG" are a special case.

The connection with matrices is not straightforward. Thm. 4.1: Σ is strongly regular if $A(\Sigma)$ has two eigenvalues. Thm. 5.3: An unbalanced, net-regular $(K_{r,s},\sigma)$ is an SRSG iff Σ^+, equivalently Σ^-, is the incidence graph of a symmetric block design. Thm. 5.4 combinatorially characterizes bipartite SRSG with 2, 3, and 4 eigenvalues.

Cf. Koledin and Stanić (2020a). [Annot. 4 Apr 2021.] (**SG: Adj: Eig**)

2020a Lower bounds for the least Laplacian eigenvalue of unbalanced blocks. *Linear Algebra Appl.* 584 (2020), 145–152. MR 4010177. Zbl 1426.05058.

$\lambda_{\min}(L(\Sigma)) > 4/c^- n$ and $> \pi^2 \frac{12n^2-\pi^2}{12n^4} h^-$, where c^- = negative circumference, h^- = packing number of negative Hamiltonian circles (if any exist). [Annot. 4 Apr 2021.] (**SG: Adj: Eig**)

2020b Spectra of signed graphs with two eigenvalues. *Appl. Math. Comput.* 364 (2020), art. 124627, 9 pp. MR 3996372. Zbl 1433.05210.

Cf. Ramezani (20xxa). Completes list of 3-, 4-regular such Σ, after Ghasemian and Fath-Tabar (2017a), Hou, Tang, and Wang (2019a). §4, "Relations to line systems": Line systems at angles 90° and θ (*cf.*, e.g., Zaslavsky (2012c)). Includes reduced line graphs $\bar\Lambda(\Sigma)$. Thm. 4.3: Connected $\bar\Lambda(\Sigma)$ has 2 eigenvalues iff $\Sigma \simeq +K_n,\ +K_{1,s},\ \pm\Delta$ where Δ is regular, or two other graphs. §5, "Computational results": The 9 connected, incomplete Σ with $n \leq 10$ that have 2 eigenvalues. §6, "Constructions" of some regular examples with $n \leq 24$; related (non)existence results. [Annot. 4 Apr 2021.] (**SG: Adj: Eig, LG**)

2020c Controllability of certain real symmetric matrices with application to controllability of graphs. *Discrete Math. Lett.* 3 (2020), 9–13. MR 4131137. Zbl 1463.05355.

Cor 2.1 applies to signed graphs. [Annot. 19 Jan 2020.] (**Adj, Lap: SG**)

2020d Notes on exceptional signed graphs. *Ars Math. Contemp.* 18 (2020), 105–115. MR 4154730. Zbl 1464.05172.

Determines the maximal signed graphs with $\lambda_{\min}(\Sigma) \geq -2$ that are not negated line graphs of signed graphs. [Annot. 4 Oct 2019, 4 Apr 2021.] (**SG: Adj: Eig, Geom**)

2020e Net Laplacian controllability for joins of signed graphs. *Discrete Appl. Math.* 285 (2020), 197–203. MR 4111701. Zbl 1447.05099.

Net Laplacian $L^{\pm}(\Sigma) := \mathrm{diag}(d^{\pm}_{\Sigma}) - A(\Sigma)$. Especially for particular signed graphs. Application to control theory. (**SG: Lap: Eig, Appl**)

2020f On the spectrum of the net Laplacian matrix of a signed graph. *Bull. Math. Soc. Sci. Math. Roumanie* 63(111) (2020), no. 2, 205–213. MR 4202394.
Cf. (2020e). (**SG: Lap: Eig**)

2020g Main eigenvalues of real symmetric matrices with application to signed graphs. *Czechoslovak Math. J.* 70 (2020), no. 4, 1091–1102. MR 4181798.
(**SG: Adj: Eig**)

2020h A decomposition of signed graphs with two eigenvalues. *Filomat* 34 (2020), no. 6, 1949–1957. MR 4207727.
Connected Σ with 2 eigenvalues that admit vertex partition whose induced signed graphs also each have 2 eigenvalues. Independently proves a result used in Huang (2019a). [Annot. 29 Apr 2022.] (**SG: Adj: Eig**)

2020i Oriented graphs whose skew spectral radius does not exceed 2. *Linear Algebra Appl.* 603 (2020), 359–367. MR 4117880. Zbl 1446.05041.
All such digraphs are determined, using signed graphs as in (2022a). *Cf.* McKee and Smyth (2007a). [Annot. 29 Apr 2022.] (**SG: Adj: Eig**)

2021a A note on a walk-based inequality for the index of a signed graph. *Special Matrices* 9 (2021), 19–21. MR 4203316. Zbl 1457.05045.
The adjacency and Laplacian indices $\lambda_{\max}$ of Σ satisfy, for a certain vertex v, $\lambda_{\max}(n^{+}_{r,v} + n^{-}_{r,v} + \lambda^{r}_{\max}) \leq$ complicated bounds in terms of r-walks, where $n^{\varepsilon}_{r,v}$ counts signed r-walks. [Annot. 12 Dec 2020.]
(**SG: Adj, Lap: Eig**)

2021b Signed graphs with totally disconnected star complements. *Rev. Unión Mat. Argentina* 62 (2021), no. 1, 95–104. MR 4281073. Zbl 1473.05119.
(**SG: Adj: Eig**)

2021c Connected non-complete signed graphs which have symmetric spectrum but are not sign-symmetric. *Examples Counterex.* 1 (2021), art. 100007, 3 pp.
(**SG: Adj: Eig**)

2022a Some relations between the skew spectrum of an oriented graph and the spectrum of certain closely associated signed graphs. *Rev. Unión Mat. Argentina* 63 (2022), no. 1, 41–50. MR 4451124.
Sequel to (2023a). (**SG, gg: Adj: Eig**)

2022b Star complements in signed graphs with two symmetric eigenvalues. *Kuwait J. Sci.* 49 (2022), no. 2, 1–8. (**SG: Adj: Eig**)

2022c Notes on the polynomial reconstruction of signed graphs. *Bull. Malaysian Math. Sci. Soc.* 45 (2022), 1301–1314. MR 4412323. Zbl 1487.05134.
Reconstruction of the characteristic polynomial of $A(\Sigma)$ from those of vertex-deleted subgraphs, resolved for several types of graph. [Annot. 26 Apr 2022.] (**SG: Adj: Eig**)

2022d Signed graphs with two eigenvalues and vertex degree five. *Ars Math. Contemp.* 22 (2022), art. P1.10, 13 pp. MR 4424011.

Classifies 5-regular such graphs, based on weighing matrices as in Harada and Munemasa (2012a). For (≤ 4)-regular ones *cf.* e.g. (2020b). [Annot. 14 Oct 2020.] (**SG: Adj: Eig**)

2022e Some properties of the eigenvalues of the net Laplacian matrix of a signed graph. *Discuss. Math. Graph Theory* 42 (2022), no. 3, 893–903. MR 4431630.
Cf. (2020e). (**SG: Lap: Eig**)

2022f Some relations between the largest eigenvalue and the frustration index of a signed graph. *Amer. J. Combin.* 1 (2022), 65–72. (**SG: Fr, Adj: Eig**)

2022g On cospectral oriented graphs and cospectral signed graphs. *Linear Multilinear Algebra* 70 (2022), no. 19, 3689–3701.
Infinite families of cospectral signed regular graphs with different underlying graphs; cospectral digraphs are derived via (2023a). Techniques include the signed covering graph and line graphs of signed expansions.
Surprising Thm. 3.6: For line graph of signed expansion of r-regular Γ, $\operatorname{Spec}\Lambda(\pm\Gamma) = \{(2)^{n(r-1)}, (-2[r-1])^n\}$, independent of Γ. [Annot. 5 Apr 2021.] (**SG: Adj: Eig, Sw, Cov, Ori, LG**)

2023a Relations between the skew spectrum of an oriented graph and the spectrum of an associated signed graph. *Linear Algebra Appl.* 676 (2023), 241–250. MR 4620772.
For each digraph $\vec{G}$ there is Σ whose spectrum contains full information about $\operatorname{Spec}\vec{G}$. (**SG, gg: Adj: Eig**)

2023b Estimating distance between an eigenvalue of a signed graph and the spectrum of an induced subgraph. *Discrete Appl. Math.* 340 (2023) 32–40. MR 4615389. (**SG: Adj: Eig**)

2023c Notes on upper bounds for the largest eigenvalue based on edge-decompositions of a signed graph. *Kuwait J. Sci.* 50 (2023), 200–203. (**SG: Adj: Eig**)

2023d Walks and eigenvalues of signed graphs. *Special Matrices* 11 (2023), art. spma–2023–0104, 8 pp. MR 4644693. (**SG: Adj: Eig**)

Zoran Stanić & Ambat Vijayakumar

2021a On spectral radius of signed graphs without negative even cycles. *Bull. Math. Soc. Sci. Math. Roumanie* 64 (112) (2021), no. 1, 89–96. MR 4243509. (**SG: Adj: Eig**)

Richard P. Stanley

See also P. Doubilet, L.L. Mu, and A. Postnikov.

1973a Linear homogeneous diophantine equations and magic labelings of graphs. *Duke Math. J.* 40 (1973), 607–632. MR 0317970 (47 #6519). Zbl 269.05109.
P. 630 restates Stewart (1966a), Cor. 2.4 in a clear way and observes that, if Γ is bipartite, then $\dim V = \#E - n + 2$. These two statements are equivalent to van Nuffelen (1973a). (**par: incid, ecyc**)

1985a Reconstruction from vertex-switching. *J. Combin. Theory Ser. B* 38 (1985), 132–138. MR 0787322 (86f:05096). Zbl 572.05046.
From the 1-vertex switching deck (the multiset of isomorphism types of signed graphs resulting by separately switching each vertex) of $\Sigma =$

(K_n, σ), Σ can be reconstructed, provided that $4 \nmid n$. The same for i-vertex switchings, provided that the Krawtchouk polynomial $K_i^n(x)$ has no even zeros from 0 to n. When $i = 1$, the negative-subgraph degree sequence is always reconstructible. All done in terms of Seidel (graph) switching of unsigned simple graphs. [See Ellingham; Ellingham and Royle; Krasikov; Krasikov and Roditty for further developments. *Problem* 1. Generalize to signings of other highly symmetric graphs. *Problem* 2. Prove a similar theorem for switching of a bidirected K_n.]
(kg: sw, TG)

1986a *Enumerative Combinatorics, Volume I.* Wadsworth and Brooks/Cole, Monterey, Calif., 1986. MR 0847717 (87j:05003). Zbl 608.05001.
Ch. 3, "Partially ordered sets": Exercise 51, pp. 165 and 191, concerns the Dowling (1973a), (1973b) lattices of a group and mentions Zaslavsky's generalizations [signed and biased graphs].
(GG: Matrd, Invar: Exr, Exp)

1990a (As "R. Stenli") *Perechislitel'naya kombinatorika.* "Mir", Moscow, 1990. MR 1090542 (91m:05002).
Russian translation of (1986a). **(GG: Matrd, Invar: Exr, Exp)**

1991a A zonotope associated with graphical degree sequences. In: Peter Gritzmann and Bernd Sturmfels, eds., *Applied Geometry and Discrete Mathematics: The Victor Klee Festschrift*, pp. 555–570. DIMACS Ser. Discrete Math. and Theor. Computer Sci., Vol. 4. American Mathematical Soc. and Assoc. for Computing Machinery, Providence and Baltimore, 1991. MR 1116376 (92k:52020). Zbl 737.05057.
All-negative complete graphs (implicit in §3) and signed colorings (§4) are used to find the number of ordered degree sequences of n-vertex graphs and to study their convex hull. **(SG: Geom, Col)**

1996a Hyperplane arrangements, interval orders, and trees. *Proc. Nat. Acad. Sci. USA* 93 (1996), 2620–2625. MR 1379568 (97i:52013). Zbl 848.05005.
Deformed braid hyperplane arrangements, i.e., canonical affine hyperplanar representations of $\mathrm{Lat}^{\mathrm{b}}(K_n, \varphi)$ where $\varphi(ij) = l_i \in \mathbb{Z}$ when $i < j$. In particular, Thm. 3.2 for contrabalance (generic gains) [*cf.* Zaslavsky (1982d)] and (§4), all $l_i = 1$. Also (§5), the Shi arrangement, which represents $\mathrm{Lat}^{\mathrm{b}}\{0,1\}\vec{K}_n$. [Annot. rev. 21 Jul 2022.]
(gg: Geom, Matrd, Invar: Exp)

1997a *Enumerative Combinatorics, Volume 1.* Corrected reprint. Cambridge Stud. Adv. Math., Vol. 49. Cambridge Univ. Press, Cambridge, Eng., 1997. MR 1442260 (98a:05001). Zbl 970.29805, Zbl 945.05006.
Additional exercises, some updating, some corrections to (1986a).
(GG: Matrd, Invar: Exr, Exp)

1998a Hyperplane arrangements, parking functions and tree inversions. In: B.E. Sagan and R. Stanley, eds., *Mathematical Essays in Honor of Gian-Carlo Rota*, Progress in Math., Vol. 161, pp. 359–375. Birkhäuser, Boston, 1998. MR 1627378 (99f:05006). Zbl 980.39546. **(gg: Geom, Matrd, Invar: Exp)**

1999a *Enumerative Combinatorics, Volume 2.* Cambridge Stud. Adv. Math., Vol. 62. Cambridge Univ. Press, Cambridge, Eng., 1999. MR 1676282 (2000k:05026).

Zbl 928.05001, Zbl 978.05002.

Exercise 5.50: The Shi arrangement [the affinographic hyperplane representation of $\{0,1\}\vec{K}_n$ with gain group $\mathbb{Z}^+$]. Exercise 5.41(h–i): The Linial arrangement and its characteristic polynomial [$= \chi^*_{\{1\}\vec{K}_n}(\lambda)$]. Exercise 6.19(lll) conceals the Catalan arrangement [which represents $\{0,\pm1\}\vec{K}_n$]. Exercise 5.40(b): Counts two-graphs that $\not\supseteq [C_5]$.
(gg: Geom, matrd, Invar, TG: Exr, Exp)

2012a *Enumerative Combinatorics, Volume 1.* Second ed. Cambridge Stud. Adv. Math., Vol. 49. Cambridge Univ. Press, Cambridge, Eng., 2012. MR 2868112. Zbl 1247.05003.

Vastly enlarged from (1986a), (1997a). Ch. 3, "Partially ordered sets": Exercise 115b, solution, p. 434, mentions Zaslavsky (1981a). Exercise 117, solution, p. 435, mentions Zaslavsky (2002a). Exercise 131, pp. 385 and 439–440, concerns the Dowling (1973a), (1973b) lattices of a group and mentions Zaslavsky's generalizations to signed and gain [and biased] graphs. [Annot. 14 Jun 2012.] **(GG: Matrd, Invar: Exr, Exp)**

2015a Valid orderings of real hyperplane arrangements. *Discrete Comput. Geom.* 53 (2015), 951–964. MR 3341587. Zbl 1322.52018. arXiv:1306.1838.

Introduces "ψ-graphical" hyperplane arrangements, which are affine cross-sections of gain-graphic arrangements according to Lutz (2019a). Also see Mu–Stanley (2015a) and Suyama–Tsujie (2019a). [Annot. 16 Nov 2018.] **(gg: Geom)**

Dietrich Stauffer

See G. Hed.

Eckhard Steffen

See also C. Cappello, L.-G. Jin, Y.-L. Kang, Y. Lu, E. Rollová, and M. Schubert.

2014a Circular flow numbers of (signed) regular graphs. In: *Bordeaux Graph Workshop 2014*, pp. 37–38. LaBRI, Bordeaux, 2014. URL `http://bgw.labri.fr/2014/bgw2014-booklet.pdf`

Extended abstract. **(SG: Flows)**

Eckhard Steffen & Michael Schubert

2013a Nowhere-zero flows on signed regular graphs. In: Jaroslav Nešetřil and Marco Pellegrini, eds., *The Seventh European Conference on Combinatorics, Graph Theory and Applications* (EuroComb 2013, Pisa), pp. 621–622. CRM Ser., Vol. 16. Edizioni della Normale, Scuola Normale Superiore Pisa, Pisa, Italy, 2013. Zbl 1291.05084.

Abstract of Schubert and Steffen (2015a). **(SG: Ori, Flows)**

Eckhard Steffen & Alexander Vogel

2021a Concepts of signed graph coloring. *European J. Combin.* 91 (2021), art. 103226, 19 pp. MR 4161819. Zbl 1458.05100. arXiv:1909.09381.

A survey of many concepts, from the direct generalization of ordinary graph coloring in Zaslavsky (1982b), to variants such as circular coloring as in Kang and Steffen (2017a), to homomorphic "coloring" as expounded in Naserasr, Sopena, and Zaslavsky (2021a). [Annot. 5 Jan 2021.]
(SG: Col: Invar, Exp)

Matěj Stehlík
See L. Faria.

Kenneth Steiglitz
See C.H. Papadimitriou.

Arthur Stein
See B. Healy.

Daniel L. Stein
See also A. Gandolfi and C.M. Newman.

1989a Spin glasses. *Scientific American* 261 (July, 1989), no. 1, 52–59.

Informally describes frustration in spin glasses in terms of randomly ferromagnetic and antiferromagnetic interactions (see Toulouse (1977a)) and gives some history and applications. (**Phys: sg: bal, Rand: Exp**)

Benjamin Steinberg
See J. Rhodes.

Raphael Steiner

2022a Asymptotic equivalence of Hadwiger's conjecture and its odd minor-variant. *J. Combin. Theory Ser. B* 155 (2022), 45–51. MR 4379303. arXiv:2109.02302.

Cf. Geelen, Gerards, Reed, Seymour, and Vetta (2009a). [*Problem.* Generalize to signed graphs.] (**sg: Par: Str**)

Douglas Steinley
See M. Brusco.

[R. Stenli (Richard P. Stanley)]
See R.P. Stanley.

Catherine Stenson
See S. Stahl.

Andrea Sterbini
See R. Petreschi.

Dragan Stevanović
See also L.H. Feng and G.H. Yu.

2007a Research problems from the Aveiro Workshop on Graph Spectra. *Linear Algebra Appl.* 423 (2007), no. 1, 172–181. MR 2312333.

Two problems by Krzysztof Zwierzyński on the "signless Laplacian" matrix $L(-\Gamma)$ (see Cvetković, Rowlinson, and Simić (2007a) are: Problem AWGS.1, "The maximum clique and the signless Laplacian". Compare the clique number with the min eigenvalue. Problem AWGS.2, "Integral graphs". For which graphs are all eigenvalues (of $L(-\Gamma)$, in particular) integral? [Annot. 15 Sept 2010.] (**par: Lap: Eig**)

Brett Stevens
See N.A. Neudauer.

B.M. Stewart

1966a Magic graphs. *Canad. J. Math.* 18 (1966), 1031–1059. MR 0197358 (33 #5523). Zbl 149.21401 (149, p. 214a).

In $\mathbb{R}^{1+E} = \mathbb{R} \times \mathbb{R}^E$ with x_0 the first coordinate, let $\sigma_v(x) = \sum\{x_e : e \text{ is incident to } v\}$, and let $V = \{x \in \mathbb{R}^E : \sigma_v(x) = x_0, \forall v \in V\}$.

Cor. 2.4 (p. 1059): If Γ is connected and contains an odd circle, then $\dim V = \#E - n + 1$. [Restated as in Stanley (1973a). Since $V \cap \{x_0 = 0\}$ = null space of the incidence matrix $\mathrm{H}(-\Gamma)$, this cryptically and partially anticipates the first calculation of $\mathrm{rank}(\mathrm{H}(-\Gamma))$, by van Nuffelen (1973a).] **(par: incid, ecyc)**

William J. Stewart
See N. Liu.

Michael Stiebitz
See T. Schweser.

Allen H. Stix
1974a An improved measure of structural balance. *Human Relations* 27 (1974), 439–455. **(SG: Fr)**

Maria Stojkow
See F. Hassanibesheli.

Daniel Stolarski
See J. Carlson.

Douglas Stone
See W. Kocay.

J. Randolph Stonesifer
1975a Logarithmic concavity for a class of geometric lattices. *J. Combin. Theory Ser. A* 18 (1975), 216–218. MR 0357169 (50 #9637). Zbl 312.05019.

The second kind of Whitney numbers of a Dowling lattice are binomially concave, hence strongly logarithmically concave, hence unimodal. [*Cf.* Damiani, D'Antona, and Regonati (1994a) and Benoumhani (1999a).] [*Famous Problem* (Rota). Generalize this.] [Annot. rev. 30 Apr 2012.] **(gg: Matrd: Invar)**

Steven H. Strogatz
See also S.A. Marvel.

2010a The enemy of my enemy. *New York Times*, online edition, February 14, 2010, the Opinionator blog. URL `http://opinionator.blogs.nytimes.com/2010/02/14/the-enemy-of-my-enemy/`

A gentle explanation of negatives and negation, with special reference to balance in signed graphs. [Annot. 21 March 2010.] **(SG: Bal: Exp)**

Thomas Strohmer & Robert W. Heath Jr.
2003a Grassmannian frames with applications to coding and communication. *Appl. Comput. Harmonic Analysis* 14 (2003), 257–275. MR 1984549 (2004d:42053). Zbl 1028.42020. arXiv:math/0301135.

Notices connection with regular two-graphs via Seidel adjacency matrix (*cf.* Seidel (1976a)), because tight Grassmannian frames are equiangular. [Foundational, as explained in Bodmann and Paulsen (2005a), esp. §4.] [Annot. 6 Aug 2018.] **(sg: kg: TG: Adj: Geom, Appl)**

Jeffrey Stuart
See also Q.A. Li.

Jeffrey Stuart, Carolyn Eschenbach, & Steve Kirkland

1999a Irreducible sign k-potent sign pattern matrices. *Linear Algebra Appl.* 294 (1999), 85–92. MR 1693935 (2000f:15017). Zbl 935.15008.

Leanne Stuive

See B. Guenin.

Bernd Sturmfels

See A. Björner.

J. Stutz

See F. Glover.

Li Su

See also H.-H. Li.

Li Su, Hong-Hai Li, & Jing Zhang

2014a The minimum spectral radius of signless Laplacian of graphs with a given clique number. *Discuss. Math. Graph Theory* 34 (2014), 95–102. MR 3149820. Zbl 1292.05180.

[*Questions*. Does this apply to signed graphs, and what is the appropriate definition of a clique? Does it apply to complex unit gain graphs (*cf.* Reff (2012a))?] [Annot. 18 May 2018.] **(par: Lap: Eig)**

Ting Su

See L. Anderson.

C.K. Subbaraya

See C. Adiga.

S.P. Subbiah

2008a *A Study of Graph Theory: Topology, Steiner Domination and Semigraph Concepts*. Ph.D. thesis, Madurai Kamaraj University, 2008.

Contains material summarized in Subbiah and Swaminathan (2009a). [Annot. 2 Aug 2010.] (**SG**)

S.P. Subbiah & V. Swaminathan

2009a Properties of topological spaces associated with sigraphs. In: K. Somasundaram, ed., *Graph Theory and its Applications* (Proc.), pp. 233–241. Macmillan Publishers India, Delhi, 2009. MR 2574613 (no rev).

Topologies τ_+, τ_- on $V \longleftrightarrow \Sigma^\varepsilon, \varepsilon = +, -$ for a signed graph Σ [not necessarily simple or finite]. $\Sigma \mapsto (\tau_\pm)$: $\tau_\varepsilon = \{$unions of subsets of $\pi(\Sigma^\varepsilon)\}$, $\pi(\Gamma) :=$ connected-component partition of V in Γ. "Exclusive property": If $u, v \in$ same component of Σ^ε, they are not in the same component of $\Sigma^{-\varepsilon}$, for $\varepsilon = \pm$. "Transitivity": Every component of $\Sigma^\pm$ is a clique. Thm. 1: Bijection between topology pairs (τ_+, τ_-) and transitive signed graphs on a set V (Subbiah (2008a)). Further results [made elementary by observing that topology pairs are equivalent to partitions π_+, π_- of V. Exclusivity is $\pi_+ \wedge \pi_- = 0_V$ and is equivalent to simplicity of $|\Sigma|$. Topology is an epiphenomenon]. [It is not always clear when $|\Sigma|$ is meant to be simple.] [Annot. 2 Aug 2010.] (**SG**)

2009b Properties of topological spaces associated with sigraphs. Int. Conf. Graph Theory Appl. (Coimbatore, 2008). *Electronic Notes Discrete Math.* 33 (2009), 59–66. MR 2574613.

Shorter version of (2009a). [Annot. 2 Aug 2010.] (**SG**)

Anand Subramanian

See E. Queiroga.

M.S. Subramanya

See also R. Rangarajan, E. Sampathkumar, and P.S.K. Reddy.

M.S. Subramanya & P. Siva Kota Reddy

2008a On balance and clusters in graph structures. *Ultra Sci.* 20(1)M (2008), 159–162.

A "graph structure" (due to E. Sampathkumar in 2005) is $G := (V, \mathcal{R})$ where $\mathcal{R} = \{R_1, \ldots, R_k\}$, $k \geq 2$, $R_i \subseteq \mathcal{P}^{(2)}(V)$, and the R_i are disjoint. Let $\mathcal{S} \subseteq \mathcal{R}$ and $\|\mathcal{S}\| := \bigcup\{R : R \in \mathcal{S}\}$. Define $\|G\| := (V, \|\mathcal{S}\|)$ and let $\Sigma(\mathcal{S})$ be the signed $\|G\|$ with negative edge set $\|\mathcal{S}\|$. [$\Sigma(\mathcal{S})$ is not defined but is implicit.] G is "$\mathcal{S}$-balanced" if $\Sigma(\mathcal{S})$ is balanced, and "$\mathcal{S}$-clusterable" if $\Sigma(\mathcal{S})$ is clusterable. Prop. 3 [hard to interpret] seems to be Harary's (1953a) theorem for $\Sigma(\mathcal{S})$. Thm. 4: G is $\mathcal{S}$-balanced for all $\mathcal{S}$ iff it is $\{R_i\}$-balanced for all i. Thm. 7 is Davis's (1967a) characterization of clusterability applied to $\mathcal{S}$-clusterability. Thm. 8 has three conditions equivalent to $\mathcal{S}$-clusterability, assuming $\bigcup_1^k R_i = \mathcal{P}^{(2)}(V)$ and no $R_i = \varnothing$. [$k = 2, \#\mathcal{S} = 1$ is signed K_n.] Thm. 9: G is $\mathcal{S}$-clusterable for all $\mathcal{S}$ iff it is $\{R_i\}$-clusterable [the paper says "balanced"] for all i. [Annot. 1 Aug 2009.] (**sg, SG(Gen), gg: Bal, Sw, Clu**)

2009a Triangular line signed graph of a signed graph. *Adv. Appl. Discrete Math.* 4 (2009), no. 1, 17–23. MR 2555622 (2010m:05136). Zbl 1176.05036.

Definitions as at Sampathkumar, Reddy, and Subramanya (2008a), (2010c). Let $T(\Gamma) := (E, E_T)$ where $E_T := \{ef : e, f \in C_3 \text{ in } \Gamma\}$. The triangular line signed graph is $T(\Sigma) := (T(|\Sigma|), \sigma^c)$. Solved: $T(\Sigma) \simeq \Lambda_\times(\Sigma)$, $T^k(\Sigma) \simeq T^2(\Sigma)$. [$\Lambda_\times$ as in M. Acharya (2009a).] [Annot. 3 Aug 2009.] (**SG: Bal, Sw, LG(Gen), LG**)

Benjamin Sudakov

See I. Balla and G. Gutin.

Naduvath K. Sudev [Sudev Naduvath]

See also P.K. Ashraf, and for same author also S. Naduvath (under 'N').

N.K. Sudev, P.K. Ashraf, & K.A. Germina

2019a Some new results on integer additive set-valued signed graphs. *TWMS J. App. Eng. Math.* 9 (2019), no. 3, 638–645. arXiv:1609.00295. HAL hal-02263299. (**SG: Lab**)

N.K. Sudev, K.P. Chithra, & K.A. Germina

2017a Switched signed graphs of integer additive set-valued signed graphs. *Discrete Math. Algorithms Appl.* 9 (2017), no. 4, art. 1750043, 10 pp. MR 3686920. Zbl 1373.05169. (**SG: Lab, Sw**)

N.K. Sudev & K.A. Germina

2015a A study on integer additive set-valuations of signed graphs. *Carpathian Math. Publ.* 7 (2015), no. 2, 236–246. MR 3457909. Zbl 1329.05141. arXiv:1511.00678.

HAL hal-02099245. (**SG: Lab**)

Sudev Naduvath & Germina Augustine [K.A. Germina]

2018a *An Introduction to Sumset Valued Graphs (Theory and Applications)*, Lambert Academic Publishing, 2018.

§1.3, "Signed graphs": expository. Fix $X \subseteq \mathbb{Z}$. §6.2, "Sumset labeled signed graphs": A sumset labelling of Σ is an injective $f : V \to \mathcal{P}(X) \setminus \{\varnothing\}$ such that $\sigma(uv) = (-1)^{\#(f(u)+f(v))}$, $\forall uv \in E$. It is strong if $\forall uv \in E$, $f(u) + f(v)$ has no duplicate sums. Thm. 6.2.4 [6.2.5]: Let f be strong and $\#(f(u) + f(v)) = k$, $\forall uv$. Then Σ is balanced [clusterable] iff $|\Sigma|$ is bipartite or $\sqrt{k} \in 2\mathbb{Z}$ [k is odd]. More results for strong and weak f. §6.3, "Arithmetic sumset labeled signed graphs": Here $f(v)$ is an arithmetic progression. [Annot. 23 Oct 2019.] (**SG: Lab: Bal, Clu**)

Sudev Naduvath & Johan Kok

2020a On certain topological indices of signed graphs. *J. Math. Comput. Sci.* 10 (2020), no. 2, 248–261. arXiv:2002.10240.

Definitions and simple properties of positive, negative, mixed, and net Zagreb, Schultz, Gutman indices, obtained from positive, negative, and net degrees. [Annot. 29 Dec 2020.] (**SG: Invar**)

J. Christabel Sudha

See C. Jayasekaran.

N. Sudharsanam

See R. Balakrishnan.

R. Sujatha

See R. Sundareswaran.

G. Sumathy

See also C. Jayasekaran.

G. Sumathy & C. Jayasekaran

2018a A characterization of bicyclic but not two-cyclic graphs with a cut vertex as self vertex switching. *Utilitas Math.* 107 (2018), 87–101. MR 3793045. Zbl 1395.05176. (**tg: Sw**)

Mei-Yu Sun

See F.-T. Hu.

Qiang Sun

See H.-Y. Cai, R. Naserasr, and C. Wen.

Shiwen Sun

See S.-S. Feng.

Wanting Sun

See S.-C. Li.

Zhenyu Sun, Hongwei Zhang, & Frank L. Lewis

2021a Output sign-consensus of heterogeneous multiagent systems over fixed and switching signed graphs: an observer-based approach. *Int. J. Robust Nonlinear Control* 31 (2021), no. 12, 5849–5864. MR 4329717 (no rev).

Cf. Jiang and Zhang (2020a). (**SD**)

Zhi Ren Sun

See X.X. Zhu.

Zhongyao Sun

2015a *Analysis and Logical Modeling of Biological Signaling Transduction Networks*. Doctoral dissertation, Pennsylvania State University, 2015.

Ch. 5, "Determining the attractors of a boolean network using an elementary signaling mode approach", employs signed digraphs. (**SD: Dyn**)

R. Sundareswaran, R. Sujatha, & Goksen Bacak-Turan

2020a A study on vulnerability parameters of signed fuzzy graphs. In: Cengiz Kahraman *et al.*, eds., *Intelligent and Fuzzy Techniques in Big Data Analytics and Decision Making* (INFUS 2019 Conf., Istanbul), pp. 24–32. Adv. Intell. Syst. Comput., Vol. 1029. Springer Nature, 2020. (**SG: Invar**)

V.S. Sunder

See V. Kodiyalam.

Borworn Suntornpoch

See Y. Meemark.

Didi Surian

See D. Lo.

Daisuke Suyama

See also T. Abe.

Daisuke Suyama, Michele Torielli, & Shuhei Tsujie

2019a Signed graphs and the freeness of the Weyl subarrangements of type B_l. *Discrete Math.* 342 (2019), no. 1, 233–249. MR 3886267. Zbl 1400.05106. arXiv:1707.01967. (**SG: Geom: Algeb**)

Daisuke Suyama & Shuhei Tsujie

2019a Vertex-weighted graphs and freeness of ψ-graphical arrangements. *Discrete Comput. Geom.* 61 (2019), no. 1, 185–197. MR 3925550. Zbl 1412.52016. arXiv:1511.04853.

Cf. Stanley (2015a). [Annot. 16 Nov 2018.] (**gg: Geom**)

Masuo Suzuki

1991a Lee-Yang complex-field systems and frustrated Ising models. *J. Phys. Soc. Japan* 60 (1991), no. 2, 441–449. MR 1104390 (92h:82034) (*q.v.*).

§2, "Equivalence of Villain's frustrated system to Lee-Yang's complex-field systems": (2.3) summarizes Villain's (1977a) "fully frustrated" signed-graphic Ising model. [Annot. 17 Jun 2012.] (**Phys: SG: Exp**)

Sei Suzuki, Jun-ichi Inoue, & Bikas K. Chakrabarti

2013a *Quantum Ising Phases and Transitions in Transverse Ising Models*, second edition. Lect. Notes Physics, Vol. 862. Springer, Berlin, 2013. MR 3222781. Zbl 1268.82001.

Cf. first edition: Chakrabarti, Dutta, and Sen (1996a). (**Phys: SG, wg: Fr**)

V. Swaminathan

See S.P. Subbiah.

Chaitanya Swamy

2004a Correlation clustering: Maximizing agreements via semidefinite programming. In: *Proceedings of the Fifteenth Annual ACM-SIAM Symposium on Discrete Algorithms* (SODA, New Orleans, 2004), pp. 526–527. Assoc. for Computing Machinery, New York, and SIAM, Philadelphia, 2004. MR 2291092.
(**SG: WG: Clu: Algor**)

Shivakumar Swamy C.S.

See C. Adiga.

Ed Swartz

See P. Hersh.

Eric Swartz

See A. Schaefer.

Swathyprabhu Mj [Swathy Prabhu]

See S. Das.

Robert H. Swendsen

See J.-S. Wang.

Katia Sycara

See D. Li.

Itiro Syôzi

See also Y. Kasai.

1950a The statistics of honeycomb and triangular lattice. II. *Progress Theor. Phys.* 5 (1950), 341–351. MR 0039629 (12, 576g).

Physics of the all-negative ("antiferromagnetic") toroidal honeycomb (§7) and triangular (§9) lattices. The former is similar to all-positive ("ferromagnetic") [because balanced] while the latter is not [because unbalanced]. [Also see Houtappel (1950a), (1950b), Newell (1950a), Wannier (1950a).] [Annot. 21 Jun 2012.] (**Phys, sg: Par: Fr**)

Edward Szczerbickl

1996a Signed directed graphs and reasoning for agents and multi-agent systems. *Int. J. Systems Sci.* 27 (1996), no. 10, 1009–1015. Zbl 860.90071.

A state ("pattern") is $s : V \to \{+,-,0\}$. An arc uv is "consistent" if $s(v) = \sigma(uv)s(u)$. In a "solution" ζ all arcs are consistent. A state is propagated by $s'(v) = \sigma(uv)s(u)$. Rules for simplification to get a signed digraph with equivalent solutions. [The model is incomplete. The role of propagation is unclear.] [Annot. 24 Nov 2012.] (**SD: Appl, bal**)

Janusz Szczypula

See P. Doreian.

Andrzej Szepietowski

See also J. Dybizbański.

2019a Negative closed walks in signed graphs: A note. Manuscript, 2019. arXiv:-1910.06032.

Corrects an error in manuscript of Naserasr, Sopena, and Zaslavsky

(2021a). [Annot. 6 Jul 2022.] (**SG**)

Stefan Szeider
See N. Alon.

Endre Szemerédi
See B. Bollobás.

Zoltán Szigeti
See A.A. Ageev.

Ferenc Szöllősi
See also G. Greaves.

Ferenc Szöllősi & Patric R.J. Östergård

2018a Enumeration of Seidel matrices. *European J. Combin.* 69 (2018), 169–184. MR 3738150. Zbl 1376.05026.
Seidel matrix $= A(K_n, \sigma)$. Spectrum *et al.* for $n \leq 13$. Classification of those with 3 distinct eigenvalues for $n \leq 23$. Application to equiangular lines. [Annot. 22 Dec 2017.] (**sg: KG: Adj, Geom**)

Jayme L. Szwarcfiter
See C.V.G.C. Lima.

Bosiljka Tadić, Krzysztof Malarz, & Krzysztof Kułakowski

2005a Magnetization reversal in spin patterns with complex geometry. *Phys. Rev. Lett.* 94 (2005), art. 137204, 4 pp. (**sg: par: Fr**)

Fatemeh Taghvaee & Gholam Hossein Fath-Tabar

2024a Note on skew-eigenvalues of digraphs. *Trans. Combin.* 13 (2024), no. 3, 225–234.
Gain group $\{\pm 1, \pm \mathrm{i}\}$ implicit in skew adjacency matrix. (**gg: Adj: Eig**)

B. Taglienti
See M. Falcioni.

Artin Tajdini
See S. Akbari.

Martin Takáč

1997a Fixed point classification method for qualitative simulation. In: Ernesto Coasta and Amilcar Cardoso, eds., *Progress in Artificial Intelligence* (8th Portuguese Conf., EPIA-97, Coimbra, Portugal, 1997), pp. 255–266. Lect. Notes in Computer Sci., Vol. 1323. Springer, Berlin, 1997. MR 1703015 (no rev). Zbl 1044.68883. (**SD: QM: QSta**)

Károly Takács
See S. Righi.

Shingo Takahashi
See T. Inohara.

Akimichi Takemura
See H. Kamiya.

Lynn Takeshita

20xxa Coloring signed graphs. Manuscript, [n.d.] (**SG: Col: Exp**)

Michel Talagrand

1998a Huge random structures and mean field models for spin glasses. Proc. Int. Congress of Mathematicians, Vol. I (Berlin, 1998). *Documenta Math.*, Extra Vol. ICM 1998 (1998), Vol. I, pp. 507–536. MR 1648045 (2000c:60164). Zbl 902.60089. (**sg: Gen: fr:'Exp**)

Irving Tallman

1967a The balance principle and normative discrepancy. *Human Relations* 20 (1967), 341–355. (**PsS: ECol**)

Ilan Talmud

See Z. Maoz.

Bit-Shun Tam, Yi-Zheng Fan, & Jun Zhou

See also T.-J. Chang [T.-C. Chang], Y.Z. Fan, H.-H. Li, and Q. Wu.

2008a Unoriented Laplacian maximizing graphs are degree maximal. *Linear Algebra Appl.* 429 (2008), 735–758. MR 2428127 (2009c:05143). Zbl 1149.05034.

The matrix is $L(-\Gamma)$. "Maximizing" graphs are those whose degree sequences are maximal in the majorization ordering. [For majorization also see Liu, Liu, and You (2013a).] [Annot. 23 Mar 2009.] (**Par: Lap**)

Bit-Shun Tam & Shu-Hui Wu

2010a On the reduced signless Laplacian spectrum of a degree maximal graph. *Linear Algebra Appl.* 432 (2010), no. 7, 1734–1756. MR 2592914 (2011c:15041). Zbl 1230.05202. (**par: Lap: Eig**)

A. Tamilselvi

See also M. Parvathi.

2010a Robinson-Schensted correspondence for the G-vertex colored partition algebra. *Asian-European J. Math.* 3 (2010), no. 2, 369–385. MR 2669040 (2011j:16057). Zbl 1230.05010. (**gg: Algeb, matrd**)

Arie Tamir

See also D. Hochbaum.

1976a On totally unimodular matrices. *Networks* 6 (1976), 373–382. MR 0472865 (57 #12553). Zbl 356.15020. (**SD: Bal**)

Christino Tamon

See J. Brown and D. Mallory.

Akihisa Tamura

See also Y.T. Ikebe and D. Nakamura.

1997a The generalized stable set problem for perfect bidirected graphs. *J. Operations Res. Soc. Japan* 40 (1997), 401–414. MR 1476832 (99e:05063). Zbl 894.90156.

Problem: maximize an integral weight function over the bidirected stable set polytope (*cf.* Johnson and Padberg (1982a)). §3 concerns the effect on perfection of deleting all incoming edges at a vertex. §4 reduces the "generalized stable set problem" for bidirected graphs to the maximum weighted stable set problem for ordinary graphs, whence the problem for perfect bidirected graphs is solvable in polynomial time. (**sg: Ori: Incid, Geom, Sw, Algor**)

2000a Perfect $(0, \pm 1)$-matrices and perfect bidirected graphs. Combinatorics and Optimization (Okinawa, 1996). *Theor. Comput. Sci.* 235 (2000), no. 2, 339–356. MR 1756130 (2001i:15019). Zbl 938.68061.

The stable set problem associated with bidirected graphs.
(sg: Ori: Geom, Algor)

Shigetaro Tamura

2023a Postnikov–Stanley Linial arrangement conjecture. *J. Algebraic Combin.* 58 (2023), 651–679. **(gg, gg(Gen): Geom, Invar)**

Takeyuki Tamura

See T. Akutsu.

Jinsong Tan

2008a A note on the inapproximability of correlation clustering. *Inform. Processing Lett.* 108 (2008), 331–335. MR 2456610 (2009k:05167). Zbl 1189.05158. arXiv:-0704.2092. **(sg: Clu: Algor)**

Shang Wang Tan

See also L. Feng, X.L. Wu, and D.L. Zhang.

2010a On the Laplacian spectral radius of weighted trees with a positive weight set. *Discrete Math.* 310 (2010), no. 5, 1026–1036. MR 2575820 (2011e:05156). Zbl 1230.05147.

The results on $L(\Gamma, w)$ with edge weights $w : E \to \mathbb{R}_{>0}$ are deduced from results on $L(-\Gamma, w)$. [*Problem.* Show the same reasoning applies to all signatures of Γ.] [Annot. 20 Jan 2012.] **(par: WG: Eig)**

2010b On the weighted trees with given degree sequence and positive weight set. *Linear Algebra Appl.* 433 (2010), no. 2, 380–389. MR 2645091 (2011e:05157). Zbl 1209.05054.

Similar to (2010a). [Annot. 20 Jan 2012.] **(par: WG: Eig)**

Shang-wang Tan, Ji-ming Guo, & Jian Qi

2003a The spectral radius of Laplacian matrices and quasi-Laplacian matrices of graphs. *Gongcheng Shuxue Xuebao* [*Chinese J. Engineering Math.*] 20 (2003), no. 6, 69–74. MR 2031534 (2004k:05137). **(par: Lap: Eig)**

Shang-Wang Tan & Jing-Jing Jiang

2011a On the Laplacian spectral radius of weighted trees with fixed diameter and weight set. *Linear Multilinear Algebra* 59 (2011), no. 2, 173–192. MR 2773649 (2012a:05202). Zbl 1226.05169.

The "(signless) Laplacian" of a graph with positive edge weights, (Γ, w) where $w : E \to \mathbb{R}_{>0}$, is $L(-\Gamma, w) := D(\Gamma, w) + A(\Gamma, w)$ (called R). The spectral radius is that of $L(-\Gamma, w)$. [*Problem.* Generalize to all weighted signed graphs.] [Annot. 11 Jan 2011, 21 Jan 2012.] **(par: WG, Eig)**

Shang Wang Tan & Xing Ke Wang

2009a On the largest eigenvalue of signless Laplacian matrix of a graph. *J. Math. Res. Exposition* 29 (2009), no. 3, 381–390. MR 2510212 (2010h:05183). Zbl 1212.05164. **(par: Lap: Eig)**

Xuezhong Tan

See also M.H. Liu.

Xuezhong Tan & Bolian Liu

2006a On the spectrum of the quasi-Laplacian matrix of a graph. *Australasian J. Combin.* 34 (2006), 49–55. MR 2195309 (2006i:05106). Zbl 1102.05039. [Annot. 25 Oct 2014.]

(**Par: Eig, ecyc**)

Ying-Ying Tan

See also Y.-Z. Fan.

Ying Ying Tan & Yi Zheng Fan

2008a On edge singularity and eigenvectors of mixed graphs. *Acta Math. Sinica (Engl. Ser.)* 24 (2008), no. 1, 139–146. MR 2384238 (2008k:05134). Zbl 1143.05058.

Relations between least Laplacian eigenvalue, its eigenvector, and $l(\Sigma)$. Properties of the eigenvector when $l(\Sigma) = 1$, e.g., $\lambda_{\min} \leq (4/n)l(\Sigma)$. Dictionary: "mixed graph" = signed graph, "edge singularity" = frustration index $l(\Sigma)$. [Generalized in Bapat, Kalita, and Pati (2012a).] [Annot. 28 Oct 2011, 20 Jan 2012.] (**sg: Fr, Eig**)

Adrian Tanasa

See T. Krajewski.

B.Z. Tang

See Y. Chen.

Jiliang Tang

See also G. Beigi.

Jiliang Tang, Yi Chang, Charu Aggarwal, & Huan Liu

2015a Negative link prediction in social media. In: *Proceedings of the Eighth ACM International Conference on Web Search and Data Mining* (WSDM'15, Shanghai, 2015), pp. 87–96. ACM, New York, 2015. arXiv:1412.2723. (**SG: Pred: Algor**)

2016a A survey of signed network mining in social media. *ACM Computing Surveys* 49 (2016), no. 3, art. 42, 39 pp. arXiv:1511.07569. (**SG: Algor**)

Wen Tang

See E.L. Wei.

Wenliang Tang

See E.L. Wei

Zikai Tang

See Y.-P. Hou.

Shin-ichi Tanigawa

See also R. Ikeshita and T. Jordán.

2015a Matroids of gain graphs in applied discrete geometry. *Thans. Amer. Math. Soc.* 367 (2015), no. 12, 8597–8641. MR 3403067. Zbl 1325.05048. arXiv:1207.3601.

(**GG: Matrd: Gen**)

Tetsuji Taniguchi

See T.Y. Chung, A.L. Gavrilyuk, G. Greaves, Hye Jin Jang, and A. Munemasa.

Percy H. Tannenbaum

See C.E. Osgood.

Éva Tardos

See also A.V. Goldberg.

Èva Tardos & Kevin D. Wayne

1998a Simple generalized maximum flow algorithms. In: Robert E. Bixby, E. Andrew Boyd, and Roger Z. Ríos-Mercado, eds., *Integer Programming and Combinatorial Optimization* (6th Int. IPCO Conf., Houston, 1998, Proc.), pp. 310–324. Lect. Notes in Computer Sci., Vol. 1412. Springer, Berlin, 1998. MR 1726354 (2000i:90111). Zbl 911.90156.

Max flow in a network with positive rational gains. Multiple sources and sinks are allowed. "Relabeling" is switching the gains. Useful references to previous work. (**GN: Sw, Algor, Ref**)

Robert E. Tarjan

See A.V. Goldberg.

Christos Tatakis

See E. Reyes.

U. Tatt [W.T. Tutte]

See W.T. Tutte.

Behruz Tayfeh-Rezaie

See F. Ayoobi and L. Parsaei-Majd.

D.E. Taylor

See also J.J. Seidel.

1977a Regular 2-graphs. *Proc. Lond. Math. Soc.* (3) 35 (1977), 257–274. MR 0476587 (57 #16147). Zbl 362.05065.

Introducing two-graphs and regular two-graphs (defined by G. Higman, unpublished). [See Seidel (1976a) etc. for more.] A "two-graph" is the class $\mathcal{C}_3^-$ of negative triangles of a signed complete graph (K_n, σ). (See §2, p. 258, where the group is $\mathbb{Z}_2 \cong \{+, -\}$ and the definition is in terms of the 2-coboundary operator.) Two-graphs and switching classes of signed complete graphs are equivalent concepts (stated in terms of Seidel switching in §2, p. 260). A two-graph is "regular" if every edge lies in the same number of negative triangles. Thm.: $\mathcal{C}_3^-$ is regular iff $A(K_n, \sigma)$ has at most two eigenvalues. Various parameters of regular two-graphs are calculated. (**TG: Eig. Geom**)

Graeme Taylor

2010a *Cyclotomic Matrices and Graphs.* Doctoral dissertation, University of Edinburgh, 2010.

See (2011a). (**SG**)

2011a Cyclotomic matrices and graphs over the ring of integers of some imaginary quadratic fields. *J. Algebra* 331 (2011), no. 1, 523–545. MR 2774674 (2012b:15058). Zbl 1238.05166. arXiv:1011.2737. (**SG**)

Herbert Taylor

See P. Erdős.

Howard F. Taylor

1970a *Balance in Small Groups.* Van Nostrand Reinhold, New York, 1970.

A thorough and pleasantly written survey of psychological theories of balance, including formalizations by signed graphs (Chs. 3 and 6), experimental tests and critical evaluation of the formalisms, and so forth. Ch. 2, "Substantive models of balance", takes the perspective of social psychology. §2.2, "Varieties of balance theory", reviews the theories of Heider (1946a) (the source of Harary's (1953a) invention of signed graphs), Osgood and Tannenbaum (1955a), and others. §2.2e, "The Rosenberg-Abelson modifications", discusses their introduction of the "cost" of change of relations, which led them (Abelson and Rosenberg (1958a)) to propose the frustration index as a measure of imbalance. (**PsS: SG, WG: Exp, Ref**)

Ch. 3, "Formal models of balance", reviews various graph-theoretic models: signed and weighted signed, different ways to weigh imbalance, etc., the relationship to theories in social psychology being constantly kept in mind. §3.1, "Graph theory and balance theory", presents the basics of balance, measures of degree of balance by circles (Cartwright and Harary (1956a)), circles with strengths of edges (Morrissette (1958a)), local balance and N-balance (Harary (1955a))), edge deletion and negation (Abelson and Rosenberg (1958a), Harary (1959b)), vertex frustration number (Harary (1959b)). §3.2, "Evaluation of formalizations: strong points", and §3.3, "Evaluation of formalizations: weak points", judged from the applied standpoint. §3.3a, "Discrepancies between cycles or subsets of cycles", suggests that differing degrees of imbalance among certain different subsets of the vertices may be significant [Is this reasonable?] and proposes measures, e.g., a variance measure (p. 71), of this "discrepancy". (**PsS: SG, WG: Bal, Fr: Exp**)

Ch. 6, "Issues involving formalization", goes into more detail. §6.1, "Indices of balance", compares five indices, in particular Phillips' (1967a) eigenvalue index (also in Abelson (1967a)) with examples to show that the index differentiates among different balanced signings of the same graph. §6.2, "Extrabalance properties", discusses Davis's (1967a) clustering (§6.2b) and indices of clustering (§6.2c). §6.3, "The problem of cycle length and non-local cycles". Are long circles less important? Do circles at a distance from an actor (that is, a vertex) have less effect on the actor in balancing processes? (**PsS: SG: Fr, Adj: Exp**)

Siamak Tazari

2010a Faster approximation schemes and parameterized algorithms on H-minor-free and odd-minor-free graphs. In: Petr Hliněný and Antonín Kučera, eds., *Mathematical Foundations of Computer Science 2010* (35th Int. Sympos., Brno), pp. 641–652. Lect. Notes in Computer Sci., Vol. 6281. Springer, Berlin, 2010. MR 2727265 (2012g:68143). Zbl 1287.68183.

Extended abstract of (2012a). (**sg: par: fr: Algor**)

2012a Faster approximation schemes and parameterized algorithms on (odd-)H-minor-free graphs. *Theor. Computer Sci.* 417 (2012), 95–107. MR 2885892 (2012m:68486). Zbl 1241.05027. arXiv:1004.3392 (prelim version). (**sg: par: fr: Algor**)

Mina Teicher

See M. Amram.

Roberto Tempo
See A.V. Proskurnikov.

Jeffrey C.Y. Teo
See A.P.O. Chan.

Hiroaki Terao
See H. Kamiya.

Hidetaka Terasaka
See S. Kinoshita.

Lesley G. Terris
See Z. Maoz.

Evimaria Terzi & Marco Winkler

2011a A spectral algorithm for computing social balance. In: Alan Frieze, Paul Horn, and Paweł Prałat, eds., *Algorithms and Models for the Web Graph* (WAW 2011, 8th Int. Workshop, Atlanta, 2011), pp. 1–13. Lect. Notes in Computer Sci., Vol. 6732. Springer, Berlin, 2011. MR 2842308 (2012i:68186). Zbl 1327.68035.

$\hat{b}$:= fraction of balanced triangles, calculated via cubes of adjacency eigenvalues. [Annot. 15 Jul 2019.] (**SG: Fr: Adj**)

Jelena Tešić
See G. Alabandi.

Ambuj Tewari
See K.-Y. Chiang.

Dirk Oliver Theis
See N.E. Clarke.

Michel Thellier
See J. Demongeot.

Denis Thieffry

See also J.-P. Comet, A. Naldi, É. Remy, and R. Thomas.

2007a Dynamical roles of biological regulatory circuits. *Briefings Bioinformatics* 8 (2007), no. 4, 220–225.

Survey of positive and negative cycles in biological regulation. [Annot. 25 Jan 2015.] (**SD: Dyn: Exp**)

D. Thieffry, E.H. Snoussi, J. Richelle, & R. Thomas

1995a Positive loops and differentiation. *J. Biol. Systems* 3 (1995), no. 2, 157–466. (**SD**)

Dimitrios M. Thilikos
See C. Giatsidis.

Morwen B. Thistlethwaite

1988a On the Kauffman polynomial of an adequate link. *Invent. Math.* 93 (1988), 285–296. MR 0948102 (89g:57009). Zbl 645.57007.

A 1-variable Tutte-style polynomial Γ_Σ of a sign-colored graph. Fix an edge ordering. For each spanning tree T and edge e, let $\mu_T(e) = -A^{3\tau_T(e)\sigma(e)}$ if e is active with respect to T, $A^{\tau_T(e)\sigma(e)}$ if it is inactive, where $\tau_T(e) = +1$ if $e \in T$, -1 if $e \notin T$. Then $\Gamma_\Sigma(A) = \sum_T \prod_{e\in T} \mu_T(e)$.

[In the notation of Zaslavsky (1992a), $\Gamma_\Sigma(A) = Q_\Sigma$ with $a_\varepsilon = A^{-\varepsilon}, b_\varepsilon = A^\varepsilon$ for $\varepsilon = \pm 1$ and $u = v = -(A^2 + A^{-2})$.] §§3 and 4 show Γ_Σ is independent of the ordering. Other sections derive consequences for knot theory. [This marks the invention of a Tutte-style polynomial of a colored, or parametrized or weighted, graph or matroid, developed in Kauffman (1989a) and successors.] (**SGc: Knot: Invar**)

Apostolos Thoma

See E. Reyes.

A.D. Thomas

See F.W. Clarke.

Creighton K. Thomas, David A. Huse, & A. Alan Middleton

2011a Zero- and low-temperature behavior of the two-dimensional $\pm J$ ising spin glass. *Phys. Rev. Lett.* 107 (2011), art. 047203, 4 pp. arXiv:1103.1946.

A droplet model of a signed square lattice shows long-range correlations (spin-glass behavior) in the ground state. [Annot. 3 Jan 2015.] (**Phys, SG: State(fr)**)

Creighton K. Thomas & A. Alan Middleton

2009a Exact algorithm for sampling the two-dimensional Ising spin glass. *Phys. Rev. E* 80 (2009), art. 046708, 16 pp.

Both pure signed graphs ($\pm J$ model) and randomly weighted ones (Gaussian model), using the Kasteyn and Temperley–Fisher decoration and Pfaffian method. [Annot. 10 Jan 2015.] (**SG, WG: State(fr), Phys, Algor**)

2013a Numerically exact correlations and sampling in the two-dimensional Ising spin glass. *Phys. Rev. E* 87 (2013), art. 043303, 16 pp. arXiv:1301.1252.

Both pure signed graphs ($\pm J$ model) and randomly weighted ones (Gaussian model), using the Kasteyn and Temperley–Fisher decoration and Pfaffian method. [Annot. 10 Jan 2015.] (**SG, WG: State(fr), Phys, Algor**)

René Thomas

See also J. Demongeot, M. Kaufman, J. Leclercq, E.H. Snoussi, and D. Thieffry.

1973a Boolean formalization of genetic control circuits. *J. Theor. Biol.* 42 (1973), no. 3, 563–585. Errata. *Ibid.* 44 (1974), no. 2, 44.

A main progenitor of a large field of inquiry about biological and chemical regulatory systems with positive and negative feedback. [See, e.g., J. Aracena, É. Remy, A. Richard, H. Siebert, R. Thomas, and their many coauthors.] The diagrams show the embryonic appearance of signed digraphs. [Annot. 25 Apr 2014.] (**Biol: sd: Dyn**)

1978a Logical analysis of systems comprising feedback loops. *J. Theor. Biol.* 73 (1978), no. 4, 631–656. (**sd: Dyn, Biol**)

1979a The dynamic behavior of boolean systems comprising feedback loops. In: René Thomas, ed., *Kinetic Logic: A Boolean Approach to the Analysis of Complex Regulatory Systems* (Proc. EMBO Course, Brussels, 1977), Ch. VII, pp. 127–142. Lect. Notes in Biomath., Vol. 29. Springer-Verlag, Berlin, 1979.

Describes dynamics of very simple signed digraphs with up to two cycles. E.g.: One positive cycle leads to one of two steady states. One negative cycle implies cycling states. With two cycles having one common vertex, both positive are like one positive cycle. Both negative allow for multiple cyclic states. One of each sign allow both a steady state and cyclic states. [Annot. 4 Aug 2018.] (**SD: Dyn, Chem**)

1981a On the relation between the logical structure of systems and their ability to generate multiple steady states and sustained oscillations. In: J. Della Dora, Jacques Demongeot, & B. Lacolle, eds., *Numerical Methods in the Study of Critical Phenomena* (Proc. Colloq., Carry-le-Rouet, 1980), pp. 180–193. Springer Ser. Synergetics, Vol. 9. Springer, Berlin, 1981. MR 0660499 (83g:92037). Zbl 0489.92025. (**SD: Dyn**)

1994a The role of feedback circuits: Positive feedback circuits are a necessary condition for positive real eigenvalues of the Jacobian matrix. *Berichte Bunsenges. phys. Chem.* 98 (1994), no. 9, 1148–1151. (**SD: Dyn**)

1996a Analyse et synthèse de systèmes á dynamique chaotique en termes de circuits de rétroaction (feedback circuits). (In French.) *Acad. Roy. Belg. Bull. Cl. Sci.* (6) 7 (1996), no. 1-6, 101–124 (1997). MR 1475761 (98h:58121). Zbl 1194.94211. (**sd: Dyn**)

2006a Nullclines and nullcline intersections. *Int. J. Bifurcation Chaos* 16 (2006), no. 10, 3023–3033. MR 2283557 (2007g:34092). Zbl 1146.34308. (**SD: Dyn**)

René Thomas & Richard D'Ari

1990a *Biological Feedback.* CRC Press, Boca Raton, 1990. Zbl 743.92003. (**SD: Dyn, Biol**)

René Thomas & Marcelle Kaufman

2005a Frontier diagrams: Partition of phase space according to the signs of eigenvalues or sign patterns of the circuits. *Int. J. Bifurcation Chaos* 15 (2005), no. 10, 3051–3074. MR 2192633 (2006m:37037). Zbl 1093.37502. (**SD: Dyn**)

R. Thomas & J. Richelle

1986a Boolean and continuous analyses of systems containing feedback loops. IV. Positive feedback and multistationarity. *Acad. Roy. Belg. Bull. Cl. Sci.* (5) 72 (1986), no. 11, 435–453. MR 0637745 (84h:92020a). (**sd: Dyn**)

1988a Positive feedback loops and multistationarity. *Discrete Appl. Math.* 19 (1988), 381–386. MR 0936224 (89g:92007). Zbl 639.92003. (**sd: Dyn**)

René Thomas, Denis Thieffry, & Marcelle Kaufman

1995a Dynamical behaviour of biological regulatory networks—I. Biological role of feedback loops and practical use of the concept of the loop-characteristic state. *Bull. Math. Biol.* 57 (1995), no. 2, 247–276. Zbl 821.92009. (**SD: Dyn, Biol**)

Robin Thomas

See also W. McCuaig and N. Robertson.

Robin Thomas & Peter Whalen

2016a Odd $K_{3,3}$ subdivisions in bipartite graphs. *J. Combin. Theory Ser. B* 118 (2016), 76–87. MR 3471845. Zbl 1332.05116.

An "odd $K_{3,3}$" is an all-negative subdivision of $-K_{3,3}$, treated as unsigned. (**sg: Par: Str**)

Andrew Thomason

1988a A graph property not satisfying a "zero-one law". *European J. Combin.* 9 (1988), 517–521. MR 0970386 (90e:05051). Zbl 675.05057.

The property is the existence of an Eulerian cut. The asymptotic probability is .57 . . . . [*Problem.* Generalize to gain graphs with finite gain group, esp. to signed graphs. The property is that of being switchable so that the identity-gain edges form an Eulerian subgraph. (This has various meanings.) Variation: The property is that of having a maximal balanced subgraph that is Eulerian. One expects the asymptotic probabilities to be the same for both problems and to depend only on the group's order.] (**par: Rand**)

Carsten Thomassen

See also P.D. Seymour.

1985a Even cycles in directed graphs. *European J. Combin.* 6 (1985), 85–89. MR 0793491 (86i:05098). Zbl 606.05039.

It is an NP-complete problem to decide whether a given signed digraph has a positive but not all-positive cycle, even if there are only 2 negative arcs. This follows from Lemma 3 of Steven Fortune, John Hopcroft, and James Wyllie, The directed subgraph homeomorphism problem (see *Theor. Computer Sci.* 10 (1980), 111–121. MR 0551599 (81e:68079). Zbl 419.05028.) by the simple argument in the proof of Prop. 2.1 here.

To decide whether a specified arc of a digraph lies in an even cycle, or in an odd cycle, are NP-complete problems (Prop. 2.1). To decide existence of an even cycle [hence, by the negative subdivision trick, of a positive cycle in a signed digraph] is difficult [but is solvable in polynomial time; see Robertson, Seymour, and Thomas (1999a)], although existence of an odd cycle [resp., of a negative cycle] is easy, by a trick here attributed to Edmonds (unpublished). Prop. 2.2: Deciding existence of a positive cycle in a signed digraph is polynomial-time solvable if $\#E^-$ is bounded. Thm. 3.2: If the outdegrees of a digraph are all $> \log_2 n$, then *every signing* has a positive cycle, and this bound is best possible; restricting to the all-negative signature, the lower bound might (it's not known) go down by a factor of up to 2, but certainly (Thm. 3.1) a constant minimum on outdegree does not imply existence of an even cycle. [See (1992a) for the effect of connectivity.] (**SD, Par: Bal, Algor**)

1986a Sign-nonsingular matrices and even cycles in directed graphs. *Linear Algebra Appl.* 75 (1986), 27–41. MR 0825397 (87k:05120). Zbl 589.05050. Erratum. *Ibid.* 240 (1996), 238. MR 1387301. (**QM, sd: par: QSol, bal, Algor**)

1988a Paths, circuits and subdivisions. In: Lowell W. Beineke and Robin J. Wilson, eds., *Selected Topics in Graph Theory 3*, Ch. 5, pp. 97–131. Academic Press, London, 1988. MR 1205394 (93h:05003) (book). Zbl 659.05062.

§8: "Even directed circuits and sign-nonsingular matrices." (**SD, QM: Bal, QSol: Exp**)

§§8–10 treat even cycles in digraphs. (**SD: Bal: Exp**)

[*General Problem.* Generalize even-cycle and odd-cycle results to positive and negative cycles in signed digraphs, the unsigned results corresponding to all-negative signatures.]

1988b On the presence of disjoint subgraphs of a specified type. *J. Graph Theory* 12 (1988), 101–111. MR 0928740 (89e:05174). Zbl 662.05032.

There is an algorithm for detecting a balanced circle in a $\mathbb{Z}_m$-gain graph. Balance of such a gain graph is characterized. (**gg: Bal, Circ: Algor**)

1989a When the sign pattern of a square matrix determines uniquely the sign pattern of its inverse. *Linear Algebra Appl.* 119 (1989), 27–34. MR 1005232 (90f:05099). Zbl 673.05067. (**QM, SD: QSol, Adj**)

1990a Embeddings of graphs with no short noncontractible cycles. *J. Combin. Theory Ser. B* 48 (1990), 155–177. MR 1046752 (91b:05069). Zbl 704.05011.

§5 describes the "fundamental cycle method", a simple algorithm for a shortest unbalanced circle in a biased graph (Thm. 5.1). Thus the method finds a shortest noncontractible circle (Thm. 5.2). A noteworthy linear class: the surface-separating ("Π-separating") circles (p. 166). Dictionary: "3-path-condition" on a class F of circles = property that F^c is a linear class. "Möbius cycle" = negative circle in the signature induced by a nonorientable embedding (only on p. 166).

(**gg, sg: Circ: Algor, Top**)

1992a The even cycle problem for directed graphs. *J. Amer. Math. Soc.* 5 (1992), 217–229. MR 1135027 (93b:05064). Zbl 760.05051.

A digraph that is strongly connected and has all in- and out-degrees ≥ 3 contains an even cycle. (**sd: par: Cyc**)

1993a The even cycle problem for planar digraphs. *J. Algorithms* 15 (1993), 61–75. MR 1218331 (94d:05077). Zbl 784.68045.

A polynomial-time algorithm for deciding the existence of an even cycle in a planar digraph. (**sd: par: Cyc: Algor**)

1994a Embeddings of graphs. Graphs and Combinatorics (Qawra, 1990). *Discrete Math.* 124 (1994),217–228. MR 1258855 (95f:05035). Zbl 797.05035.

P. 225 and Thm. 6.3: the "3-path-condition" and shortest unbalanced circle algorithm from (1990a). Examples mentioned (under other names) are parity bias (all-negative signs) [underlying the even-circle matroid of Tutte (1981a) and Doob (1973a) via Zaslavsky (1989a)], poise bias [underlying a matroid of Matthews (1978c)], and noncontractible or orientation-reversing embedded circles [for the latter see esp. Lins (1985a) and Zaslavsky (1992a)]. (**gg, par: Exp**)

2001a The Erdös–Pósa property for odd cycles in graphs of large connectivity. Paul Erdös and His Mathematics (Budapest, 1999). *Combinatorica* 21 (2001), no. 2, 321–333. MR 1832455 (2002c:05108). Zbl 989.05062.

Given k, there exists K such that every sufficiently connected graph has k vertex-disjoint odd circles or K vertices whose deletion leaves a bipartite graph. [*Problem.* Given k, there exists K such that every sufficiently connected signed graph has k vertex-disjoint negative circles or K vertices whose deletion leaves a balanced graph.] [Annot. rev. 26

Dec 2012.] **(par: Fr: Circ)**

2001b Totally odd K_4-subdivisions in 4-chromatic graphs. *Combinatorica* 21 (2001), no. 3, 417–443. MR 1848060 (2002e:05058). Zbl 1012.05064.

Re-proves Zang (1998b) (Toft's (1975a) conjecture). [*Question*. What is the signed-graph generalization?] [Annot. rev. 26 Dec 2012, 29 Oct 2017.] **(sg: par: Col)**

G.L. Thompson

See V. Balachandran.

Christopher Thraves Caro [Christopher Thraves]

See F. Benitez, A.-M. Kermarrec, E.G. Pardo, and Q. Spaen.

Florence Thuderoz

See J. Demongeot.

Fenglei Tian

See also X.B. Ma, S. Wang, F. Xu, and M. Zhu.

Fenglei Tian, Li Chen, & Rui Chu

2018a Rank of the Hermitian-adjacency matrix of a mixed graph in terms of matching number. *Ars Combin.* 37 (2018), 221–232. MR 3790972. Zbl 06890323.

Generalizes Ma–Wong–Tian (2016a) to $2\mu(\Gamma) - 2\xi(\Gamma) \leq \operatorname{rk} A(\Phi) \leq 2\mu(\Gamma) + \xi(\Gamma)$ for gain graph Φ with gains in $\{\pm 1, \pm \mathrm{i}\}$. Characterizes equalities. [A different matching formula is in Chen–Huang–Li (2018a).] [Annot. 10 May 2019, rev 16 Oct 2020.] **(gg: Adj)**

Fenglei Tian, Xiaoming Li, & Jianling Rou

2014a A note on the signless Laplacian and distance signless Laplacian eigenvalues of graphs. *J. Math. Res. Appl.* 34 (2014), no. 6, 647–654. MR 3288067. Zbl 1324.05120. **(par: Lap: Eig)**

Fenglei Tian, Dengyin Wang, & Min Zhu

2016a A characterization of signed planar graphs with rank at most 4. *Linear Multilinear Algebra* 64 (2016), no. 5, 807–817. MR 3479381. Zbl 1335.05079.

Characterized: All signed graphs with $\operatorname{rk} A(\Sigma) = 2, 3$ and signed planar graphs with $\operatorname{rk} A(\Sigma) = 4$. [Annot. 22 Jan 2016.] **(SG: Adj)**

Fenglei Tian & Dein Wong

2017a Relation between the skew energy of an oriented graph and its matching number. *Discrete Appl. Math.* 222 (2017), 179–184. Zbl 1396.05078.

Given a digraph. In an oriented circle C, an edge has sign $+1$ or -1 depending on whether it is consistent or inconsistent with C. An even circle is "evenly/oddly oriented" if its sign product is $+1$ or -1. [Annot. 15 Aug 2022.] **(sg)**

2018a Nullity of Hermitian-adjacency matrices of mixed graphs. *J. Math. Res. Appl.* 38 (2018), no. 1, 23–33. MR 3752028. Zbl 1413.05249.

For Φ with gain group $\{\pm 1, \pm \mathrm{i}\}$ ($\varphi(e) = 1$ for undirected, i for directed edges), characterizes the spectrally unique ones with $\operatorname{rk} A(\Phi) = 3$. [Annot. 15 Dec 2020.] **(SG: Adj: Eig)**

Gui-Xian Tian

See also S.-Y. Cui.

Gui-Xian Tian, Ting-Zhu Huang, & Bo Zhou

2009a A note on sum of powers of the Laplacian eigenvalues of bipartite graphs. *Linear Algebra Appl.* 430 (2009), no. 8-9, 2503–2510. MR 2508309 (2010e:05191). Zbl 1165.05020.

A lower bound on $\sum_i \lambda_i(L(\Gamma))^\alpha$, over nonzero eigenvalues, for bipartite Γ and $\alpha \in \mathbb{R}^\times$. [*Question.* Is there a nonbipartite generalization involving $L(-\Gamma)$?] [Annot. 23 Jan 2012.] (**par: bal: Lap: Eig**)

Xiao-Jun Tian, Xiao-Peng Zhang, Feng Liu, & Wei Wang

2009a Interlinking positive and negative feedback loops creates a tunable motif in gene regulatory networks. *Phys. Rev. E* 80 (2009), no. 1, 011926. (**SD: Dyn, Biol**)

Yi Tian

See S.C. Li.

Jonathan Tidor

See Z.-L. Jiang.

R.M. Tifenbach

2011a Strongly self-dual graphs. *Linear Algebra Appl.* 435 (2011), no. 12, 3151–3167. MR 2831603 (2012h:05202). Zbl 1226.05170.

Cf. Tifenbach and Kirkland (2009a). An h-graph Γ is "self-dual" if it has inverse Σ and $\Gamma \cong |\Sigma|$, "strongly self-dual" if $\Gamma = \Sigma$. Thm. 3.2 is Tifenbach and Kirkland (2009a) Thm. 2.5 with strong self-duality instead of duality. §4, "Constructions of strongly self-dual graphs". §5, "Eigenvalues of self-dual h-graphs": For an eigenvalue λ of a self-dual h-graph, $-\lambda$ and $\pm 1/\lambda$ are eigenvalues. $\lambda = \pm 1$ if rational. ± 1 has multiplicity $\equiv m \pmod 2$. Thm. 5.5: Let $k^- := \#$ of vertices switched in changing Σ to Γ^+; then $\lambda = \pm 1$ has multiplicity $\geq |m - 2k^-|$. Examples. [Annot. 4 May 2017.] (**sg: Adj, Eig**)

R.M. Tifenbach & S.J. Kirkland

2009a Directed intervals and the dual of a graph. *Linear Algebra Appl.* 431 (2009), nos. 5-7, 792–807. MR 2535551 (2010m:05185). Zbl 1226.05171.

Inspired by Godsil (1985a) *et al.* Graphs are simple. An "h-graph" is bipartite with left set $\{u_1, \ldots, u_m\}$, right set $\{v_1, \ldots, v_m\}$, and a unique perfect matching $M = \{u_i v_i\}_i$. Thm. 1.1: $\exists$ labelling so every edge $u_i v_j$ has $i \leq j$ (Simion and Cao (1989a)). Thus, $\exists$ vertex labelling and partial order P_Γ on $\{v_1, \ldots, v_p\}$ so every edge $u_i v_j$ has $i \leq j$; this gives an acyclic digraph. If $A(\Gamma)^{-1} = A(\Sigma)$ for some Σ, then $\Gamma^+ := |\Sigma|$ ("dual" of Γ) is an h-graph, Σ is balanced, and every covering edge of P_{Γ^+} is negative in Σ (Thm. 2.3). Thm. 2.4: $P_\Gamma \cong P_{\Gamma^+}$. Thm. 2.5: Intervals of P_Γ have duals; intervals respect duality. Thm. 2.6: Γ^+ exists iff P_Γ has bipartite Hasse diagram and all intervals have duals. [The former is due to balance and the negative covering edges (i.e., antibalance).] §§3–4: Examples. [Annot. 4 May 2017.] (**sg: Adj**)

[*Problem.* Generalize this and other h-graph research to bipartite signed graphs with unique perfect matching, so having a signed-graphic inverse is natural. What do balance and the negative covering edges (antibalance) of the inverse digraph become?] [Annot. 4 May 2017.] (**SG: Adj**)

Shailesh K. Tipnis

See A.H. Busch, A.A. Diwan, and H. Jordon.

R.L. Tobin

1975a Minimal complete matchings and negative cycles. *Networks* 5 (1975), 371–387. MR 0395786 (52 #16578). Zbl 348.90151. (**sg: vs**)

Bjarne Toft

See also T.R. Jensen and U. Krusenstjerna-Hastrøm.

1975a Problem 10. In: M. Fiedler, ed., *Recent Advances in Graph Theory* (Proc. Second Czechoslovak Symp., Prague, 1974), pp. 543–544. Academia Praha, 1975. MR 0363962 (51 #217) (book). Zbl 316.00007 (book).

Proposes that for every 4-chromatic graph Γ, $-\Gamma$ contains a subdivision of $-K_4$ (that means every K_4 edge subdivides into an odd path). [Proved by Zang (1998b), Thomassen (2001b). *Cf.* Krusenstjerna-Hastrøm and Toft (1980a) and Jensen and Shepherd (1995a).] [Annot. 29 Oct 2017.] (**sg: par: Col**)

Sivan Toledo

See E.G. Boman and D. Chen.

Ioan Tomescu

See also D.R. Popescu.

1973a Note sur une caracterisation des graphes dont le degré de deséquilibre est maximal. *Math. Sci. Humaines*, No. 42 (1973), 37–40. MR 0366757 (51 #3003). Zbl 266.05115.

Independent proof of Petersdorf's (1966a) Satz 1. Also, treats similarly a variation on the frustration index. (**SG: Fr**)

1974a La réduction minimale d'un graphe à une réunion de cliques. *Discrete Math.* 10 (1974), 173–179. MR 0363992 (51 #247). Zbl 288.05127.

The fewest sign changes needed to make (K_n, σ) clusterable is $\leq \lfloor n/2 \rfloor \lfloor (n-1)/2 \rfloor$. [Annot. 6 Jan 2017.] (**SG: Bal, Clu**)

1976a Sur le nombre des cycles négatifs d'un graphe complet signé. *Math. Sci. Humaines*, No. 53 (1976), 63–67. MR 0457285 (56 #15493). Zbl 327.05119.

Consider (K_n, σ) with $\#E^- = p$. The parity of the number of negative triangles = that of np. The number of negative t-gons, for $t \geq 4$, is even [strengthened in Popescu (1991a), (1996a)]. [Kittipassorn & Mészáros (2015a) performs a detailed study of the number of negative triangles.] (**SG: Bal**)

1978a Problem 2. In: A. Hajnal and Vera T. Sós, eds., *Combinatorics* (Proc. Fifth Hungarian Colloq., Keszthely, 1976), Vol. II, p. 1217. Colloq. Math. Soc. János Bolyai, 18. János Bolyai Math. Soc., Budapest, and North-Holland, Amsterdam, 1978. MR 0519295 (80a:05002b) (book). Zbl 378.00007. (**SG: Bal**)

Mark Tomforde

See B.G. Bodmann.

Joanna Tomkowicz & Krzysztof Kułakowski

2010a Scaling of connected spin avalanches in growing networks. *Phys. Rev. E* 81 (2010), art. 052101, 4 pp. arXiv:0904.2697. **(par: State(fr))**

C.B. Tompkins
See I. Heller.

Arnaud Tonnelier
See J. Demongeot.

Hatice Topcu
See W.H. Haemers.

J. Topp & W. Ulatowski

1987a On functions which sum to zero on semicycles. *Zastosowanie Mat.* (*Applicationes Math.*) 19 (1987), 611–617. MR 0951376 (89i:05138). Zbl 719.05044.

An additive real gain graph is balanced iff every circle in a circle basis is balanced, iff the gains are induced by a vertex labelling [in effect, switch to 0], iff every two paths with the same endpoints have the same gains. A digraph is gradable (Harary, Norman, and Cartwright (1965a); also see Marcu (1980a)) iff φ_1 is balanced, where for each arc e, $\varphi_1(e) = 1 \in \mathbb{Z}$ (Thm. 3). The Windy Postman Problem (Thms. 4, 5). **(GG, GD: Bal)**

Aleksandar Torgašev
See also D.M. Cvetković.

1982a The spectrum of line graphs of some infinite graphs. *Publ. Inst. Math. (Beograd) (N.S.)* 31(45) (1982), 209–222. MR 0710960 (85d:05175). Zbl 526.05039.

An infinite analog of Doob's (1973a) characterization via the even-cycle matroid of when a line graph has -2 as an eigenvalue. [*Problem.* Generalize to line graphs of infinite signed graphs.] **(par: Eig(LG))**

1983a A note on infinite generalized line graphs. In: D. Cvetković *et al.*, eds., *Graph Theory* (Proc. Fourth Yugoslav Seminar, Novi Sad, 1983), pp. 291–297. Univ. Novom Sadu, Inst. Mat., Novi Sad, 1984. MR 0751456 (85i:05168). Zbl 541.05042.

An infinite graph is a generalized line graph iff its least "limit" eigenvalue ≥ -2. [*Problem.* Generalize to line graphs of infinite signed graphs.] **(par: Eig(LG))**

Michele Torielli
See also W.-L. Guo and D. Suyama.

Michele Torielli & Shuhei Tsujie

2020a Freeness of hyperplane arrangements between boolean arrangements and Weyl arrangements of type B_l. *Electronic J. Combin.* 27 (2020), no. 3, art. P3.10, 15 pp. MR 4245123. Zbl 1444.52011. arXiv:1807.02432. **(SG: Geom: Algeb)**

Juan R. Torregrosa
See C. Mendes Araújo.

[Núria Ballber Torres]
See N. Ballber Torres (under 'B').

Dejan V. Tošić
See M. Anđelić.

Gérard Toulouse

See also B. Derrida and J. Vannimenus.

1977a Theory of the frustration effect in spin glasses: I. *Commun. Phys.* 2 (1977), 115–119. Repr. in M. Mézard, G. Parisi, and M.A. Virasoro, *Spin Glass Theory and Beyond*, pp. 99–103. World Scientific Lect. Notes in Physics, Vol. 9. World Scientific, Singapore, 1987.

Introduces the notion of imbalance ("frustration") of a signed graph to account for inherent disorder in an Ising model (here synonymous with a signed graph, usually a lattice graph). (Positive and negative edges are called "ferromagnetic and antiferromagnetic bonds".) Observes that switching the edge signs from all positive (the model of D.D. Mattis, *Phys. Lett.* 56A (1976), 421–?) makes no essential difference. In a planar lattice [or any plane graph] frustration of face boundaries ("plaquettes") can be thought of as curvature, i.e., failure of flatness. Proposes two kinds of asymptotic behavior of frustration as a circle encloses more plaquettes. The planar-duality approach for finding the states with minimum frustration (i.e., switchings with fewest negative edges); the number of such states is the "ground-state degeneracy" and is important. Ideas are sketched; no proofs.

[A foundational paper. See Wannier (1950a) and, e.g., Villain *et al.* (1977a) *et al.*, Hoever, Wolff, and Zittartz (1981a), Barahona, Maynard, Rammal, and Uhry (1982a), van Hemmen (1983a), Wolff and Zittartz (1983a), Mézard, Parisi, and Virasoro (1987a), Fischer and Hertz (1991a), Schwärzler and Welsh (1993a), *et al.*] (**SG: Phys, Sw, Bal**)

1979a Symmetry and topology concepts for spin glasses and other glasses. Nonperturbative Aspects in Quantum Field Theory (Proc. Les Houches Winter Adv. Study Inst., 1978). *Phys. Rep.* 49 (1979), no. 2, 267–272. MR 0518399 (82j:82063).

Mainly for signed lattice graphs, with spins $s(v) \in S^{n-1}$ having symmetry group SO(n); $n = 1$ (Ising model) gives SO$\{+1, -1\}$; $n = 2$ is planar spins; $n = 3$ is Heisenberg spins. Two symmetry groups: $\mathbb{Z}_2^{\#V}$ acts on Σ (the "microscopic level"); SO(n) or O(n) acts on states s (the "macroscopic level"). [An edge is satisfied if $s(w) = \sigma(vw)s(v)$, otherwise frustrated.] A "ground state" (where the most edges are satisfied) has a topology of frustrated plaquettes [negative girth circles], whose nature, depending on the lattice dimension, is described intuitively. Regions ("packets") of relatively fixed spins can be identified. Topology of frustrated plaquettes leads to the homotopy groups of O(n). The effect on thermodynamic phases is discussed. Dictionary: "Local transformation" = switching. [Annot. 20 Aug 2012.] (**Phys, SG: Fr, Sw: Exp, Ref**)

1981a Spin glasses with special emphasis on frustration effects. In: Claudio Castellani *et al.*, eds., *Disordered Systems and Localization* (Rome, 1981), pp. 166–173. Lect. Notes in Phys., Vol. 149. Springer, Berlin, 1981.

§3, "Frustration": in signed graphs [after normalization to bond strength 1]. "Frustration function" of circles [= $\sigma(C)$] determines physical properties because they are "gauge [= switching] invariant", if no external magnetic field. §3.i, "Periodic frustrated models" [= toroidally embedded graphs]. §3.ii, "Fully frustrated models", where every "plaquette" [girth circle] is negative: overblocking effect, i.e., positive density of plaquettes with more than one negative edge. [A mathematically interesting concept, not understood today.] §3.iii, "Systems with finite residual entropy": e.g., antiferromagnetic [all-negative] Potts models. §3.iv, "Approach to spin glasses, by dilution of periodic frustrated systems" [embedding an unbalanced toroidal graph in a larger balanced graph?]. §3.v, "Connections with gauge theories; topological defects and their hydrodynamics": *cf.*, e.g., (1979a). §3.vi, "Random frustration (edge weights ± 1) models, in various space dimensions": comparing random signs ± 1 with Gaussian random edge weights (centered at 0, hence with signs and magnitudes). For signed K_n's ("Sherrington–Kirkpatrick model"), "in the thermodynamic limit [both] have the same physics." [Annot. 20 Aug 2012.] (**Phys: sg, Fr, Sw: Exp, Ref**)

Gérard Toulouse & Jean Vannimenus

1977a La frustration: un monde semé de contradictions. *La Recherche*, No. 83, Vol. 8 (Nov., 1977), 980–981.

Popular exposition of the elements of frustration in relation to the Ising model [evidently based on Toulouse (1977a)]. Briefly mentions the social psychology application. (**Phys: SG, Bal: Exp**)(**SG: PsS: Exp**)

1980a On the connection between spin glasses and gauge field theories. *Phys. Rep.* 67 (1980), no. 1, 47–54. MR 0600878 (no rev).

Annealed and quenched models on a square lattice are compared. Annealed: edge weights J_{ij} ("bond strengths") are random variables; this is randomly weighted, randomly signed graphs. Quenched, edge weights $= \pm J$; this is signed graphs. The annealed model "grossly underestimates frustration effects." Proposed corrective: introduce Lagrange multipliers for the plaquettes. This leads to unexplored theory. App. (c), "The frustration model": randomly signed graphs, especially regular graphs; compared to models with Gaussian random edge weights and signs. [Annot. 20 Aug 2012.] (**Phys: sg, Fr**)(**Phys: sg, Fr: Exp**)

L. Tournier & M. Chaves

2009a Uncovering operational interactions in genetic networks using asynchronous Boolean dynamics. *J. Theor. Biol.* 260 (2009), 196–209. MR 2973075 (no rev). Zbl 1402.92207. HAL hal-00554625. (**SD: Dyn, Biol**)

V.A. Traag & Jeroen Bruggeman

2009a Community detection in networks with positive and negative links. *Phys. Rev. E* 80 (2009), art. 036115, 6 pp. arXiv:0811.2329.

Generalizes a Potts model for positive links to signed graphs. Method is more general than the clustering model for signed graphs. [Applied in Yoshikawa, Iino, and Iyetomi (2012a).] (**SG: Clu, PsS**)

Vincent Antonio Traag, Paul Van Dooren, & Patrick De Leenheer

2013a Dynamical models explaining social balance and evolution of cooperation. *PLOS One* 8 (2014), no. 4, art. e60063, 7 pp. + 4 supplements. arXiv:1207.6588. (**SG, WG: Bal: KG: Dyn**)

Lorenzo Traldi

See also J. Ellis-Monaghan.

1989a A dichromatic polynomial for weighted graphs and link polynomials. *Proc. Amer. Math. Soc.* 106 (1989), 279–286. MR 0955462 (90a:57013). Zbl 713.57003.

Generalizing Kauffman's (1989a) Tutte polynomial of a sign-colored graph, Traldi's "weighted dichromatic polynomial" $Q(\Gamma; t, z)$ is Zaslavsky's (1992b) $Q_\Gamma(1, w; t, z)$, in which the deletion-contraction parameters $a_e = 1$ and $b_e = w(e)$, the weight of e. Thm. 2 gives the Tutte-style spanning-tree expansion. Thm. 4: Kauffman's Tutte polynomial $Q[\Sigma](A, B, d) = d^{-1}A^{\#E^+}B^{\#E^-}Q_{|\Sigma|}(1, w; d, d)$ for connected Σ, with $w(e) = (AB^{-1})^{\sigma(e)}$. [See Kauffman (1989a) for other generalizations. Traldi gives perhaps too much credit to Fortuin and Kasteleyn (1972a).]

P. 284: Invariance under Reidemeister moves of type II constrains the weighted dichromatic polynomial to, in essence, equal Kauffman's. Thus no generalization is evident in connection with general link diagrams. There is an interesting application to special link diagrams.

(**SGc: Gen: Invar, Knot**)

2004a A subset expansion of the coloured Tutte polynomial. *Combin. Probab. Comput.* 13 (2004), no. 2, 269–275. MR 2047240 (2004k:05095). Zbl 1049.05024.

The corank-nullity expansion of the usual Tutte polynomial generalizes to colored Tutte polynomials in the universal sense of Bollobás and Riordan (1999a). (**SGc: Gen: Matrd: Invar**)

2005a Parallel connections and coloured Tutte polynomials. *Discrete Math.* 290 (2005), no. 2–3, 291–299. MR 2123398 (2005j:05033). Zbl 1069.05021.

The Tutte polynomial of a parallel connection of colored graphs or matroids. (**SGc: Gen: Matrd: Invar**)

2006a On the colored Tutte polynomial of a graph of bounded treewidth. *Discrete Appl. Math.* 154 (2006), no. 6, 1032–1036. MR 2212555 (2006j:05199). Zbl 1091.05027.

Polynomial-time computability for colored graphs of bounded tree width. [Also see Makowsky (2005a).]

(**SGc: Gen: Invar: Algor, Knot**)

2015a The transition matroid of a 4-regular graph: An introduction. *European J. Combin.* 50 (2015), 180–207. MR 3361421. Zbl 1319.05034. arXiv:1307.8097.

§8, "Topological Tutte polynomials", defines the Bollobás–Riordan (2002a) ribbon polynomial via edge signs, then via transition circuits. [Annot. 3 Nov 2015.] (**SG: Top: Invar**)

Tan Nhat Tran

2019a Characteristic quasi-polynomials of ideals and signed graphs of classical root systems. *European J. Combin.* 79 (2019), 179–192. MR 3926514. Zbl 1412.52018. arXiv:1805.00179. (**SG: Geom, Invar, Algeb**)

Tuan Tran & Günter M. Ziegler

2014a Extremal edge polytopes. *Electronic J. Combin.* 21 (2014), no. 2, art. P2.57, 16 pp. MR 3244823. Zbl 1300.05145. arXiv:1307.6708.

Edge polytope $P_{-\Gamma}$ (*cf.* Ohsugi and Hibi (1998a)). [This is the antibalanced case. *Problem.* Generalize to signed graphs, including balanced

graphs.] (**sg: Par: Geom**)

Eran Treister
See I. Kyrchei.

Ben Tremblay
See G. MacGillivray.

Marián Trenkler
See S. Jezný.

Vilmar Trevisan
See F. Belardo.

Irene Triantafillou

2022a Spectra of signed graphs. In: Nicholas J. Daras & Themistocles M. Rassias, eds., *Approximation and Computation in Science and Engineering*, pp. 861–873. Springer Optim. Appl., Vol. 180. Springer Nature, Cham, 2022. MR 4436994.

Surveys adjacency and Laplacian spectra. For "sign-symmetric" *cf.* Ghorbani–Haemers–Maimani–Majd (2020a). [Annot. 23 May 2022.]
(**SG: Exp: Bal, Sw, Adj, Lap: Eig; Ref**)

Nenad Trinajstić
See also A. Graovac.

1977a Computing the characteristic polynomial of a conjugated system using the Sachs theorem. *Croatica Chemica Acta* 49 (1977), No. 4, 593–633.

"Extension of the Sachs formula to Möbius systems", pp. 608–613. "Möbius system" = signed graph. [Annot. 17 Mar 2020.]
(**Chem: SG: Adj**)

1983a *Chemical Graph Theory*. 2 vols. CRC Press, Boca Raton, Florida, 1983. MR 0772570 (86g:92044a), MR 0772571 (86g:92044b).

Vol. I: Ch. 3, § VI: " Möbius graphs." Ch. 5, § VI: "Extension of Sachs formula to Möbius systems." § VII: "The characteristic polynomial of a Möbius cycle." Ch. 6, § VIII: "Eigenvalues of Möbius annulenes."
(**SG: Chem, Eig: Exp**)

1992a *Chemical Graph Theory*. Second ed. CRC Press, Boca Raton, Florida, 1992. MR 1169298 (93g:92034).

Ch. 3, § V.B: "Möbius graphs." Ch. 4, § I: "The adjacency matrix": see pp. 42–43. Ch. 5: "The characteristic polynomial of a graph", § II.B: "The extension of the Sachs formula to Möbius systems"; § III.D: "Möbius cycles". Ch. 6, § VIII: "Eigenvalues of Möbius annulenes" (i.e., unbalanced circles); § IX: "A classification scheme for moncyclic systems" (i.e., characteristic polynomials of circles). (**SG: Eig, Chem**)

Ch. 7: "Topological resonance energy," § V.C: "Möbius annulenes"; § V.G: "Aromaticity in the lowest excited state of annulenes".
(**Chem; sg: bal**)

Anastasia Trofimova
See Y. Burman.

Nicolas Trotignon
See also P. Aboulker.

Nicolas Trotignon & Kristina Vušković

2010a A structure theorem for graphs with no cycle with a unique chord and its consequences. *J. Graph Theory* 63 (2010), no. 1, 31–67. MR 2590324 (2011g:05260). Zbl 1186.05104. arXiv:1309.0979.

Pp. 35–36: Brief description of graphs having special kinds of signatures. *Cf.* Conforti, Cornuéjols, and Vuškoviić (2006a) *et al.*[Annot. 19 Jan 2015.] **(SGw, sg: Bal(Gen): Exp)**

L.E. Trotter, Jr.

See E.C. Sewell.

Klaus Truemper

See also Conforti, Cornuéjols, and Truemper (1994a) and Gerards, Lovász, *et al.* (1990a), Tseng and Truemper (1986a).

1976a An efficient scaling procedure for gain networks. *Networks* 6 (1976), 151–159. MR 0452603 (56 #10882). Zbl 331.90027. **(gg: GN, sg: Bal, Sw)**

1977a On max flows with gains and pure min-cost flows. *SIAM J. Appl. Math.* 32 (1977), 450–456. MR 0432208 (55 #5197). Zbl 352.90069. **(GG, OG, GN, Bal)**

1977b Unimodular matrices of flow problems with additional constraints. *Networks* 7 (1977), 343–358. MR 0503664 (58 #20352). Zbl 373.90023. **(sg: Incid: Bal)**

1978a Optimal flows in nonlinear gain networks. *Networks* 8 (1978), 17–36. MR 0465133 (57 #5041). Zbl 381.90039. **(GN)**

†† 1982a Alpha-balanced graphs and matrices and GF(3)-representability of matroids. *J. Combin. Theory Ser. B* 32 (1982), 112–139. MR 0657681 (83i:05025). Zbl 478.05026.

A $0, \pm 1$-matrix is called "balanced" if it contains no submatrix that is the incidence matrix of a negative circle. More generally, α-balance of a $0, \pm 1$-matrix corresponds to prescribing the signs of holes in a signed graph. Main theorem characterizes the sets of holes (chordless circles) in a graph that can be the balanced holes in some signing. [See Conforti and Kapoor (1998a) for a new proof and discussion of applications.] **(sg: Bal, Incid)**

1992a *Matroid Decomposition.* Academic Press, San Diego, 1992. MR 1170126 (93h:-05046). Zbl 760.05001.

§12.1: "Overview." §12.2: "Characterization of alpha-balanced graphs," exposition of (1982a). **(sg: Bal, Sw)**

1992b A decomposition theory for matroids. VII. Analysis of minimal violation matrices. *J. Combin. Theory Ser. B* 55 (1992), 302–335. MR 1168967 (93e:05021). Zbl 809.05024.

According to Cornuéjols (2001a), this paper contains the following theorem: A bipartite graph is "balanceable" (has a ± 1-weighting $\pmod 4$ in which all polygons have sum $0 \pmod 4$) iff it does not contain an induced subgraph that is a subdivided odd wheel or a theta graph with nodes in opposite color classes. [The weights are not gains because they are not oriented. However, this has major applications to signed hypergraphs; *cf.* Rusnak (2010a).] [*Problem.* Generalize to arbitrary graphs.]

[In a bipartite graph the sum around a polygon has to be 0 or 2 (mod 4) and therefore belongs to a group $\cong \mathbb{Z}_2$ so can be considered a sign. However, it may not be possible to relabel the edges from $\mathbb{Z}_2$ so as to get the same polygon sums. I.e., the polygon signing may not be derivable from a signed graph.] (**SGw: bal**)

Théophile Trunck

See P. Aboulker.

Anke Truss

See S. Böcker.

Marcello Truzzi

See F. Harary.

S.V. Tsaranov

See also F.C. Bussemaker, P.J. Cameron, and J.J. Seidel.

1992a On spectra of trees and related two-graphs. In: Jaroslav Nešetřil and Miroslav Fiedler, eds., *Fourth Czechoslovak Symposium on Combinatorics, Graphs and Complexity* (Prachatice, 1990), pp. 337–340. Ann. Discrete Math., Vol. 51. North-Holland, Amsterdam, 1992. MR 1206289 (no rev). Zbl 776.05077.

A two-graph whose points are the edges of a tree T and whose triples are the nonseparating triples of edges of T (from Seidel and Tsaranov (1990a) via Cameron (1994a)). An associated signed complete graph Σ_T on vertex set $E(T)$ is obtained by orienting T arbitrarily, then taking $\sigma_T(ef) = +$ or $-$ depending on whether e and f are similarly or oppositely oriented in the path of T that contains both. Reorienting edges corresponds to switching Σ_T. Thm.: Letting $n = \#V(T)$, the matrices $3I_n + A(\Sigma_T)$ and $2I_{n+1} - A(T)$ have the same numbers of zero and negative eigenvalues. (**TG: Eig, Geom**)

1993a Trees, two-graphs, and related groups. In: D. Jungnickel and S.A. Vanstone, eds., *Coding Theory, Design Theory, Group Theory* (Proc. Marshall Hall Conf., Burlington, Vt., 1990), pp. 275–281. Wiley, New York, 1993. MR 1227141 (94j:05062).

New proof of theorem on the group (Seidel and Tsaranov (1990a)) of the two-graph (Tsaranov (1992a)) of a tree. (**TG: Eig, Geom**)

Michael J. Tsatsomeros

See M. Cavers, C.R. Johnson, S. Kirkland, and D.D. Olesky.

Dennis Tseng

See V. Reiner.

F.T. Tseng & K. Truemper

1986a A decomposition of the matroids with the max-flow min-cut property. *Discrete Appl. Math.* 15 (1986), 329–364. MR 0865011 (88b:05046). Zbl 679.90052. Addendum. *Discrete Appl. Math.* 20 (1988), 87–88. MR 0936900 (89b:05070). Zbl 818.05031.

A special case is decomposition of $\mathbf{L}_\infty(\Sigma)$. [Annot. 18 Jan 2021.] (**sg**)

Alexis Tsoukiàs

See O. Bessouf.

Charalampos E. Tsourakakis, Michael Mitzenmacher, Jarosław Błasiok, Ben Lawson, Preetum Nakkiran, & Vasileios Nakos

2020a Predicting positive and negative links with noisy queries: theory & practice. *Internet Math.* (2020), 17 pp. MR 4196797. arXiv:1709.07308v3. (**SG: Clu**)

Shuhei Tsujie

See T. Abe, N. Nakashima, D. Suyama, and M. Torielli.

D. Tsvetkovich, M. Dub, & Kh. Zakhs

1984a *Spektry grafov. Teoriya i primenenie.* (In Russian.) Transl. V.V. Strok, ed. V.S. Korolyuk. Preface by Strok and Korolyuk. Naukova Dumka, Kiev, 1984. MR 0746475 (85c:05025).

Russian ed. of Cvetković, Doob, and Sachs (1980a).
(**SD, par, TG: Sw, Adj, Eig, Geom: Exp, Exr, Ref**)

Jianhua Tu

See G.-H. Yu.

Thomas W. Tucker

See also J.L. Gross and J. Širáň.

1983a Finite groups acting on surfaces and the genus of a group. *J. Combin. Theory Ser. B* 34 (1983), no. 1, 82–98. MR 0701174 (85b:20055). Zbl 521.05027.
(**GG: Top**)

2009a The genus of a group. In: Lowell W. Beineke and Robin J. Wilson, eds., *Topics in Topological Graph Theory*, Ch. 11, pp. 225–244. Encycl. Math. Appl., Vol. 128. Cambridge Univ. Press, Cambridge, Eng., 2009. MR 2581548 (no rev). Zbl 1225.05130.

§3, "Quotient embeddings and voltage graphs". [Annot. 12 Jun 2013.]
(**Top: GG, sg, Cov: Exp**)

Francesco Tudisco

See P. Mercado.

Vanda Tulli

See A. Bellacicco.

Pavel Tumarkin

See M.D. Sikirić.

Hande Tunçel

See F.M. Atay.

Edward C. Turner

See R.Z. Goldstein.

Krzysztof Turowski

See R. Janczewski.

Daniel Turzík

See S. Poljak.

W.T. Tutte

† 1967a Antisymmetrical digraphs. *Canad. J. Math.* 19 (1967), 1101–1117. MR 0214512 (35 #5362). Zbl 161.20905 (161, p. 209e).

Integral (u, u)-flows on a signed graph with edge capacities, presented in the language of integral $(\tilde{u}, \tilde{u}^*)$-flows on a digraph with edge capacities,

with an orientation-reversing, fixed-point free, capacity-preserving involution *. [Such a digraph is the double covering digraph of a bidirected graph, thus the capacities and flows are equivalent to (u, u)-flows on a capacitated signed graph.] Analog of the Min-Flow Max-Cut Theorem (see 3.3). Structure of flows. Application to undirected graph factors. [*Problem.* Convert the entire paper to the language of signed graphs. Express the structure of (u, u)-flows in terms of signed-graphic objects such as unbalanced unicyclic subgraphs. Extract the implicit matroid theory, including the structure of cocircuits (*cf.* Chen and Wang (2009a)).] [Annot. 9 Sept 2010, 12 Jan 2012.] **(sg: ori, cov: Flows)**

† 1981a On chain-groups and the factors of graphs. In: L. Lovász and Vera T. Sós, eds., *Algebraic Methods in Graph Theory* (Proc. Colloq., Szeged, 1978), Vol. 2, pp. 793–818. Colloq. Math. Soc. János Bolyai, 25. János Bolyai Math. Soc., Budapest, and North-Holland, Amsterdam, 1981. MR 0642073 (83b:05104). Zbl 473.05023.

The chain-group approach to the dual even-cycle matroid, $\mathbf{F}(-\Gamma)^*$. Developed entirely in terms of the group $\Delta(\Gamma)$ [topologically, $B^1(\Gamma, \mathbb{Z})$] of integral 1-coboundaries. Assuming Γ connected: "Dendroids of $\Delta(\Gamma)$" = bases of $\mathbf{F}(-\Gamma)$; Thms. 8.6–7 give their structure in the bipartite and nonbipartite cases. Support of an elementary coboundary = circuit of $\mathbf{F}(-\Gamma)^*$; this is a bond of Γ if Γ is bipartite (Thm. 7.5) and a minimal balancing set otherwise (Thm. 7.6). Thm. 7.8: Any coboundary times some power of 2 is a sum of primitive coboundaries. [*Problem.* Explain how this is related to total dyadicity of the incidence matrix.] "Rank of $\Delta(\Gamma)$" = $\operatorname{rk} \mathbf{F}(-\Gamma)$; its value is given at the end of §8. §9 develops a relationship between "homomorphisms" of $\Delta(\Gamma)$ (linear functionals) and graph factors. §10: The dual chain group; characterization of circuits of $\operatorname{rk} \mathbf{F}(-\Gamma)$. [It is amazing what can be done with nothing but integral 1-coboundaries. *Problem* 1. Extend Tutte's theory of integral chain groups to all signed graphs. Grossman, Kulkarni, and Schochetman (1994a) have a development over a field but this is very different, even aside from their opposite viewpoint that goes from matroids to vector spaces. *Problem* 2. Extend to signed hypergraphs, where each hyperedge has a function $\tau_e : V(e) \to \{+, -\}$ (not distinguished from $-\tau_e$; as with bidirected graphs, choosing one of them corresponds to orienting e).]

[Tutte knew and lectured on $\mathbf{F}(-\Gamma)^*$ and/or $\mathbf{F}(-\Gamma)$ before anyone (Doob (1973a), Simões-Pereira (1973a)) published it.—information from Neil Robertson.] **(sg: ECyc, Du, incid)**

1984a *Graph Theory.* Encycl. Math. Appl., Vol. 21. Addison-Wesley, Menlo Park, Calif., 1984. MR 0746795 (87c:05001). Zbl 554.05001. Repr. Cambridge Univ. Press, Cambridge, Eng., 2001. MR 1813436 (2001j:05002). Zbl 964.05001.

Note VIII.12.1, "Unoriented coboundaries", mentions the work of (1981a). [Annot. 9 May 2014.] **(par: matrd)**

1988a (as "U. Tatt") *Teoriya grafov.* Transl. G.P. Gavrilov. "Mir", Moscow, 1988. MR 0977974 (89i:05093).

Russian trans. of (1984a). **(par: matrd)**

Kaya Tutuncuoglu
See B. Guler.

Zsolt Tuza
See S. Poljak.

Ilya Tyomkin
See A. Beimel.

Frank Uhlig
See C.R. Johnson.

J.P. Uhry
See F. Barahona and I. Bieche.

Włodzimierz Ulatowski
See also J. Topp.

1991a On Kirchhoff's voltage law in Z_n. *Discuss. Math.* 11 (1991), 35–50. MR 1178357 (93g:05121). Zbl 757.05058.

Examines injective, nowhere zero, balanced gains (called "graceful labellings") from Z_{m+1}, $m = \#E$, on arbitrarily oriented circles and variously oriented paths. [*Question*. Does this work generalize to bidirected circles and paths?] **(GD: bal: Circ, Paths)**

[N.B. Ul'janov]
See N.B. Ul'yanov.

N.B. Ul'yanov
See D.O. Logofet.

Somya Upadhyaya
See D. Sinha.

Gurunath Rao Vaidya
See P.S.K. Reddy.

Samir K. Vaidya & Kalpesh M. Popat

2019a Some new results on Seidel equienergetic graphs. *Kyungpook Math. J.* 59 (2019), no. 2, 335–340. MR 3987713. Zbl 1439.05145. **(sg: KG: Adj: Eig)**

J.F. Valdés
See also W. Lebrecht and E.E. Vogel.

J.F. Valdés, J. Cartes, & E.E. Vogel

2000a Polyhedra as $\pm J$ closed Ising lattices. *Rev. Mexicana Fís.* 46 (2000), no. 4, 348–356. MR 1783780 (2001g:82035). Zbl 1291.82030.

Physics and signed graph theory on a signed polyhedral graph, esp. properties of ground states as functions of $x := \#E^+/\#E$. Effects of vertex and face degrees. [Annot. 17 Jun 2012, 9 Jan 2015.]
(Phys, SG: State(fr))

J.F. Valdés, J. Cartes, E.E. Vogel, S. Kobe, & T. Klotz

1998a Relationship between the structure of the ground level and frustration in $\pm J$ Ising lattices. *Physica A* 257 (1998), 557–562.

Maps the ground states of a 6×6 toroidal square lattice with various signatures. Dictionary: *cf.* Vogel, Cartes, Contreras, Lebrecht, and Villegas (1994a). [Annot. 2 Jan 2015.] (**SG: State(fr), Sw, Phys**)

J.F. Valdés, W. Lebrecht, & E.E. Vogel

2007a $\pm J$ Ising model on Dice lattices. *Physica A* 385 (2007), 551–557.

Randomly signed dice lattice (planar, with rhombic faces) with specified $x := \#E^+/\#E$: frustration index, distribution of frustrated plaquettes (rhombi), *et al.*, as functions of x. This lattice is interesting because the average degree ("coordination number") is not integral; *cf.* Lebrecht, Vogel, and Valdés (2004a) *et al.* [Annot. 3 Jan 2015.] (**SG, Phys: Fr State, Algor**)

2012a $\pm J$ Ising model on homogeneous Archimedean lattices. *Physica A* 391 (2012), 2585–2599. MR 2882041. [Annot. 3 Jan 2015.] (**SG, Phys: Fr, State**)

Carlos E. Valencia

See also C.A. Alfaro.

Carlos E. Valencia & Rafael H. Villarreal

2006a Explicit representations of the edge cone of a graph. *Int. J. Contemp. Math. Sci.* 1 (2006), no. 2, 53–66. MR 2288999 (2007m:05164). Zbl 1118.05068.

As intersection of half-spaces. [Annot. 7 Jul 2022.] (**sg: Geom, Algeb**)

Miguel A. Valencia Bucio

See also J. Martínez-Bernal.

Miguel Ángel Valencia Bucio

2019a *Sobre Códigos y Demimatroides* [*On Codes and Demimatroids*]. Doctoral dissertation, CINVESTAV, México, D.F., 2019. (**SG, Matrd(Gen): Invar, Appl**)

James Van Buskirk

See T.J. Lundy.

[Edwin R. van Dam]

See E.R. van Dam (under 'D').

Pauline van den Driessche

See J. Bélair, B.D. Bingham, T. Britz, S. Butler, M. Catral, G.J. Culos, D.A. Grundy, C. Jeffries, C.R. Johnson, V. Klee, K. Hassani Monfared, and D.D. Olesky.

Hein van der Holst

See M. Arav.

[Jorn van der Pol]

See J. van der Pol (under 'P').

Arnout van de Rijt

2011a The micro-macro link for the theory of structural balance. *J. Math. Sociology* 35 (2011), no. 1-3, 94–113. MR 2844982 (2012i:91238). Zbl 1214.91094. (**SG: Fr**)

Kevin N. Vandermeulen

See M.S. Cavers and D.A. Gregory.

Paul Van Dooren
See V.A. Traag.

[J.L. van Hemmen]
See J.L. van Hemmen (under 'H').

Marc A.A. van Leeuwen
1996a The Robinson-Schensted and Schützenberger algorithms, an elementary approach. *Electronic J. Combin.* 3 (1996), no. 2, #R15, 32 pp. MR 1392500 (97e:05200). Zbl 852.05080.

Elements of the hyperoctahedral group $\mathfrak{O}_d$ (signed permutations) of even degree $d = 2n$ permute $\pm[n]$ and of odd degree $d = 2n+1$ permute $[-n, n]$ (pp. 22f. The natural involution is $\pi \mapsto -\bar{\pi}$, where $\bar{\pi}$ is the reverse of π [reminiscent of signed graph coloring]. [*Cf.* Bloss (2003a) and Parvathi (2004a).] [Annot. 19 Mar 2011.] (**sg: Algeb**)

A. Vannelli
See C.J. Shi.

Jean Vannimenus
See also B. Derrida and G. Toulouse.

J. Vannimenus, S. Kirkpatrick, F.D.M. Haldane, & C. Jayaprakash
1989a Ground-state morphology of random frustrated *XY* systems. *Phys. Rev. B* 39 (1989), no. 7, 4634–4643. MR 0986455 (89m:82087).

XY means signed graphs with complex-unit vertex spins. (**Phys: sg**)

J. Vannimenus, J.M. Maillard, & L. de Sèze
1979a Ground-state correlations in the two-dimensional Ising frustration model. *J. Phys. C: Solid State Phys.* 12 (1979), 4523–4532. (**Phys: SG**)

J. Vannimenus & G. Toulouse
1977a Theory of the frustration effect: II. Ising spins on a square lattice. *J. Phys. C: Solid State Phys.* 10 (1977), L537–L541. (**SG: Phys**)

[Cyriel van Nuffelen]
See C. v. Nuffelen (under 'N').

M.E. Van Valkenburg
See W. Mayeda.

Anke van Zuylen, Rajneesh Hegde, Kamal Jain, & David P. Williamson
2007a Deterministic pivoting algorithms for constrained ranking and clustering problems. In: *Proceedings of the Eighteenth Annual ACM-SIAM Symposium on Discrete Algorithms* (SODA '07, New Orleans, 2007), pp. 405–414. Assoc. for Computing Machinery, New York, and Soc. for Industrial and Appl. Math., Philadelphia, 2007. MR 2482866 (no rev). Zbl 1302.68326. (**sg: Clu: Algor**)

Anke van Zuylen & David P. Williamson
2009a Deterministic pivoting algorithms for constrained ranking and clustering problems. *Math. Oper. Res.* 34 (2009), no. 3, 594–620. MR 2555338 (2010j:68136). Zbl 1216.68343. (**sg: Clu: Algor**)

[Stefan H.M. van Zwam]
See S.H.M. van Zwam (under 'Z').

Burak Varan

See B. Guler.

Patricio Vargas

See E.E. Vogel.

V. Vasanthi, S. Arumugam, Atulya K. Nagar, & Sovan Mitra

2015a Applications of signed graphs to portfolio turnover analysis. 2nd Global Conf. Business and Social Sci. (Bali, 2015). *Procedia Social Behavioral Sci.* 211 (2015), 1203–1209.

Largely an imprecise exposition of Harary–Lim–Wunsch (2002a) with possible missing edges [hence their reduction of balance testing to triangles is invalid]. The claim about computational speed is wrong. [Annot. 12 Feb 2022.] (**Appl: SG: Bal**)

[T.R. Vasanth Kumar]

See P.S.K. Reddy.

Wolmer V. Vasconcelos

See A. Simis.

B. Vasuki

See also L. Shobana.

B. Vasuki & L. Shobana

2018a Signed product cordial labeling for some families of graphs. *Int. J. Simulation Systems Sci. Technol.* 19 (2018), no. 4, art. 23, 6 pp.

More examples as in Baskar, Babujee, and Loganathan (2011a). (**Lab: VS: SG, Bal**)

Ebrahim Vatandoost

See G.R. Omidi.

Vijay V. Vazirani & Mihalis Yannakakis

1988a Pfaffian orientations, 0/1 permanents, and even cycles in directed graphs. In: Timo Lepistö and Arto Salomaa, eds., *Automata, Languages and Programming* (Proc. 15th Int. Colloq., Tampere, Finland, 1988), pp. 667–681. Lect. Notes in Computer Sci., Vol. 317. Springer-Verlag, Berlin, 1988. MR 1023669 (90k:68078). Zbl 648.68060.

Slightly abridged version of (1989a). (**SD: Adj, Bal: Algor**)

1989a Pfaffian orientations, 0–1 permanents, and even cycles in directed graphs. *Discrete Appl. Math.* 25 (1989), 179–190. MR 1031270 (91e:05080). Zbl 696.68076.

"Evenness" of a digraph (i.e., every signing contains a positive cycle) is polynomial-time equivalent to evaluability of a certain 0–1 permanent by a determinant and to parts of the existence and recognition problems for Pfaffian orientations of a graph. Briefly expounded in Brundage (1996a).] (**SD: Adj, Bal: Algor**)

Michalis Vazirgiannis

See C. Giatsidis and F.D. Malliaros.

Alina Vdovina

See S.-P. Liu.

[Renata R. Del-Vecchio]

See R.R. Del-Vecchio (under 'D').

J.J.P. Veerman

2018a Social balance and the Bernoulli equations. *Amer. Math. Monthly* 125 (2018), 724–732. MR 3859644. Zbl 1403.37099. arXiv:1701.06946.

Differential equation $\dot{X}(t) = X(t)^2$ with $X(0) = X_0$, an $n \times n$ invertible matrix, with $\lambda :=$ max real eigenvalue > 0 having right and left eigenvectors v and w. Thm. 2: $\exists!\ X(t)$ for $t \in [0, \lambda^{-1})$; and $(\lambda^{-1} - 1)X(t) \to vw^{\mathrm{T}}$ as $t \uparrow \lambda^{-1}$. Meaning (§4): For $t \approx \lambda^{-1}$, $[n]$ develops up to 4 factions, two "cohesive" as in balance and (if X_0 is not symmetric) two "dispersive" that have antisymmetric relations with the first two. [The paper's Thm. 2 is unnecessarily constrained by a probabilistic statement that requires $2|n$.] *Cf.* Marvel, Kleinberg, *et al.* (2011a) and Traag, Dooren, *et al.* (2013a). [Annot. 12 Jun 2019.] (**SD, SG, WD: Bal, Dyn**)

Fernando Vega-Redondo

See G.C.M.A. Ehrhardt.

Guido Veiner

1995a Some problems of approximating symmetric relations by equivalence relation. *Proc. Estonian Acad. Sci. Eng.* 1 (1995), no. 2, 105–112. MR 1640812 (99d:-05083). URL `http://bit.ly/2j7tP4M`

Clustering some types of signed K_n. [Annot. 10 Nov 2017.] (**sg: Clu**)

Mark Velednitsky

See Q. Spaen.

Alan Veliz-Cuba

See also E.D. Sontag.

2011a Reduction of Boolean network models. *J. Theor. Biol.* 289 (2011), 167–172. MR 2973921 (no rev). Zbl 1397.92265. (**SD: Dyn: Str**)(**SD: Dyn: Biol**)

Alan Veliz-Cuba, Boris Aguilar, Franziska Hinkelmann, & Reinhard Laubenbacher

2014a Steady state analysis of Boolean molecular network models via model reduction and computational algebra. *BMC Bioinformatics* 21 (2014), art. 221, 8 pp. (**SD: Dyn**)

Alan Veliz-Cuba & Reinhard Laubenbacher

2012a On the computation of fixed points in Boolean networks. *J. Appl. Math. Comput.* 39 (2012), 145–153. MR 2914469. Zbl 1381.94145. (**SD: Dyn**)

Lluís Vena

See A. Goodall.

Venkat Venkatasubramanian

See M.R. Maurya.

Véronique Ventos

See P. Berthomé, S. Corteel, and D. Forge.

Maryam Verdian-Rizi

See T. Huynh.

Dirk Vertigan

See also J. Geelen and J. Oxley.

2015a Dowling geometries representable over rings. *Ann. Combin.* 19 (2015), 225–233. MR 3319870. Zbl 1310.05054.

Characterizes representability of the Dowling geometry $Q_n(\mathfrak{G})$ over a ring, or equivalently a skew partial field, for any finite group $\mathfrak{G}$, thereby solving Pendavingh and van Zwam (2013a), Problem 6.5. [Annot. 28 Jan 2015.] **(gg: Matrd)**

Adrian Vetta

See S. Fiorini, J. Geelen, and B. Reed.

K.A. Vidya

See E. Sampathkumar.

Sebastiano Vigna

See P. Boldi.

Fabien Vignes-Tourneret

See also T. Krajewski.

2009a The multivariate signed Bollobás–Riordan polynomial. *Discrete Math.* 309 (2009), no. 20, 5968–5981. MR 2552629 (2011a:05162). Zbl 1228.05183. arXiv:0811.1584.

Multivariate version of the Chmutov and Pak (2007a) and Chmutov (2009a) signed ribbon-graph polynomials for orientation-embedded signed graphs. [*Cf.* Krushkal (2011a).] [Annot. 12 Jan 2012.] **(SGc: Top, Invar)**

2011a Non-orientable quasi-trees for the Bollobás–Riordan polynomial. *European J. Combin.* 32 (2011), no. 4, 510–532. MR 2780852 (2012c:57007). Zbl 1226.05104. arXiv:1102.1627. **(sgc: Top, Invar)**

S. Vijay

See V. Lokesha, K.V. Madhusudhan, and P.S.K. Reddy.

Ambat Vijayakumar

See Z. Stanić.

[G.K. Vijayakumar]

See G.R. Vijayakumar.

G.R. Vijayakumar

See also P.D. Chawathe, D.K. Ray-Chaudhuri, and N.M. Singhi.

1984a (As "G.K. Vijayakumar") A characterization of generalized line graphs and classification of graphs with eigenvalues at least 2 [misprint for −2]. *J. Combin. Inform. System Sci.* 9 (1984), 182–192. MR 0959067 (89g:05055). Zbl 629.05046. **(sg: Eig, lg)**

† 1987a Signed graphs represented by D_∞. *European J. Combin.* 8 (1987), 103–112. MR 0884068 (88b:05111). Zbl 678.05058.

The finite signed simple graphs represented (see (1993a)) by a root system D_n have a characterization by forbidden induced subgraphs, the largest of which has order 6. [The complete list is given by Chawathe and Vijayakumar (1990a). The representable signed graphs are the reduced line graphs of simply signed graphs without loops or half edges; see Zaslavsky (2010b), (2012c), (20xxa).] Also, remarks on signed graphs representable by $\mathbb{R}^\infty$. **(SG: adj, Geom, incid, lg)**

1992a Signed graphs represented by root system E_8. Combinatorial Math. and Appl. (Proc., Calcutta, 1988). *Sankhya Ser. A* 54 (1992), 511–517. MR 1234728 (94d:05072). Zbl 882.05118.

The finite signed simple graphs represented (see (1993a)) by the root system E_8 are characterizable by forbidden induced subgraphs. The largest of these subgraphs has order 10. **(SG: adj, Geom, incid)**

1993a Algebraic equivalence of signed graphs with all eigenvalues ≥ -2. *Ars Combin.* 35 (1993), 173–191. MR 1220518 (93m:05134). Zbl 786.05059.

Finite signed simple graphs only. Σ is "represented" by $W \subseteq \mathbb{R}^\infty$ if the vertices can be mapped into W so that inner products equal signs of edges, where we interpret $\sigma(u, v) \in \{+1, -1, 0\}$. Thm. 1: $\lambda_{\min}(\Sigma) \geq -2 \iff \Sigma$ is represented by $\mathbb{R}^\infty \iff \Sigma$ is represented by D_∞ or E_8 (the root systems).

Σ' is "algebraically equivalent" to Σ if it is obtained from Σ by a sequence of switchings and algebraic transforms. The latter means taking two positively adjacent vertices a, b contained in no negative triangle, switching b, removing edges from b to all common neighbors of a and b, and adding an edge xb, for each $x \in N(a) \setminus N(b)$, with the same sign as xa. Thm. 5: If Σ is connected and $\lambda_{\min}(\Sigma) > -2$, then Σ is algebraically equivalent to the Dynkin diagram of A_n, D_n, or E_k ($k = 6, 7, 8$).

There are other, similar results. **(SG: adj, Geom, incid)**

1994a Representation of signed graphs by root system E_8. *Graphs Combin.* 10 (1994), 383–388. MR 1307045 (96a:05128). Zbl 821.05040.

An algebraic characterization of the forbidden induced subgraphs for simple signed graphs represented by the root system E_8. *Cf.* (1992a) and for definitions (1993a). **(SG: adj, Geom, incid)**

2007a Partitions of the edge set of a graph into internally disjoint paths. *Australas. J. Combin.* 39 (2007), 241–245. MR 2351204 (2008i:05156). Zbl 1134.05088.

A connected, contrabalanced biased graph $(\Gamma, \varnothing)$ has a covering by $\xi+1$ internally disjoint paths, where ξ = cyclomatic number, iff every $(\Gamma \setminus v, \varnothing)$ has no balanced components. [*Question* 1. Can this generalize to all connected biased graphs? Paths should become balanced subgraphs that are "path-like" (have at most two vertices of attachment). ξ should become a measure of the number of independent unbalanced circles. *Question* 2. Is there a recursive decomposition of a 2-connected biased graph into ξ path-like balanced subgraphs, generalizing the standard ear decomposition of a 2-connected, (contrabalanced biased) graph?] [Annot. 8 Mar 2008.] **(gg: Str, circ)**

2008a A graph labeling related to root lattices. In: B.D. Acharya, S. Arumugam, and Alexander Rosa, eds., *Labelings of Discrete Structures and Applications* (Mananthavady, Kerala, 2006), pp. 175–179. Narosa, New Delhi, 2008. MR 2391786 (2009e:05281) (book). Zbl 1180.05111.

A "2-fold labeling" of a signed simple graph Σ is a nonzero $f \in \mathrm{Nul}[2I+$

$A(\Sigma)$]. Thm. 3: If Σ has a 2-fold labeling, it has one such that, for some $v \in V$, $f(v) = 1$ and every $f(w)$ is an integer. Applied to prove via signed graphs the classification of root systems with root length $\sqrt{2}$. (**SG: Eig, Geom**)

2009a A method of classifying all simply laced root systems. *J. Algebra Appl.* 8 (2009), no. 4, 533–537. MR 2555519 (2010k:17016). Zbl 1172.05337.

Thm. 6: For a connected signed simple graph with eigenvalues ≤ 2, $2I - A(\Sigma)$ is the Gram matrix of a subset of root system D_n or E_8. (**SG: Adj, Geom**)

2011a A property of weighted graphs without induced cycles of nonpositive weights. *Discrete Math.* 311 (2011), no. 14, 1385–1387. MR 2795549 (2012f:05127). Zbl 1238.05128.

Let Γ be 2-connected. Thm.: If $\varphi : E \to \mathbb{Z}_{\leq 1}$, extended to $S \subset E$ by $\varphi(S) := \sum_{e \in S} f(e)$, satisfies $\varphi(C) > 0$ for every induced circle, then $\varphi(E) > 0$. Cor.: If $\varphi : E \to \mathbb{R}$ satisfies $\varphi(C) > 0$ for every circle (not necessarily induced), then $\varphi(E) > 0$. Cor.: If Σ has $\#E^+(C) > \#E^-(C)$ for every induced circle, then $\#E^+(\Sigma) > \#E^-(\Sigma)$, a conjecture from B.G. Xu (2009a). [*Cf.* Balakrishnan and Sudharsanam (1982a) where equality is treated.] [Annot. 16 Oct 2011.] (**SGw: Gen**)

2011b Equivalence of four descriptions of generalized line graphs. *J. Graph Theory* 67 (2011), no. 1, 27–33. MR 2809559 (2012e:05318). Zbl 1226.05185. (**sg: LG**)

2012a Spectral numbers related to signed graphs. *Utilitas Math.* 87 (2012), 33–40. MR 2919984. Zbl 1264.05083.

Let $\mathrm{Spec}(\Sigma)$:= spectrum of $A(\Sigma)$, $I(\Sigma) := [\min \mathrm{Spec}(\Sigma), \max \mathrm{Spec}(\Sigma)]$. If $\rho \in \mathbb{R}$ is such that $\rho \in I(\Sigma) \implies \rho \in \mathrm{Spec}(\Sigma')$ for some $\Sigma' \subseteq \Sigma$, then $\rho = 0, \pm 1, \pm 2$. If Σ' must be an induced subgraph, $\rho = 0, \pm 1, \pm\sqrt{2}, \pm 2$. (Also, similar results for graphs.) (**SG: Eig**)

2013a Some results on weighted graphs without induced cycles of nonpositive weights. *Graphs Combin.* 29 (2013), no. 4, 1101–1111. MR 3070078. Zbl 1268.05101. (**SGw: Gen**)

2014a From finite line graphs to infinite derived signed graphs. *Linear Algebra Appl.* 453 (2014), 84–98. MR 3201686. Zbl 1314.05174. (**SG: LG, Eig**)

G.R. Vijayakumar (as "Vijaya Kumar"), S.B. Rao, & N.M. Singhi

1982a Graphs with eigenvalues at least -2. *Linear Algebra Appl.* 46 (1982), 27–42. MR 0664693 (83m:05099). Zbl 494.05044.

A minimal forbidden induced graph has order at most 10, which is best possible. [Annot. 2 Aug 2010.] (**sg: adj, Geom, lg**)

G.R. Vijayakumar & N.M. Singhi

1990a Some recent results on signed graphs with least eigenvalues ≥ -2. In: Dijen Ray-Chaudhuri, ed., *Coding Theory and Design Theory Part I: Coding Theory* (Proc. Workshop IMA Program Appl. Combin., Minneapolis, 1987–88), pp. 213–218. IMA Vol. Math. Appl., Vol. 20. Springer-Verlag, New York, 1990.

MR 1047882 (91e:05069). Zbl 711.05033. (**SG: Geom, lg, adj: Exp**)

K.S. Vijayan

See S.B. Rao.

Viji Paul

See also S. Hameed K.

2012a *Labeling and Set-Indexing Hypergraphs of a Graph and Related Topics*. Doctoral thesis, Kannur University, 2012.

Ch. 5, "Co-regular signed graphs": Σ is (r,k)-"co-regular" if $|\Sigma|$ is r-regular and Σ^+ is $\frac{1}{2}(r+k)$-regular. An r-regular Γ has an (r,k)-co-regular signing iff Γ has a $\frac{1}{2}(r+k)$-factor. Examples are treated. Thm. 5.4.2: Every Σ is an induced subgraph of an (r,k)-co-regular signed graph for all r,k satisfying $r \geq \Delta(\Sigma^+)+\Delta(\Sigma^-)$, $r \equiv k \mod 2$, and $2\Delta(\Sigma^+) - r \leq k \leq r - 2\Delta(\Sigma^-)$ (Δ = max degree). [Annot. 22 Nov 2012.] (**SGc: Lab**)

V. Vilfred & C. Jayasekaran

2009a Interchange similar self vertex switchings in graphs. *J. Discrete Math. Sci. Cryptogr.* 12 (2009), no. 4, 467–480. MR 2589065 (2011f:05206). Zbl 1180.05089.

See Jayasekaran (2007a). Examines self-switching vertices that are interchanged by an automorphism of Γ ("interchange similar"). [Annot. 26 Sept 2012.] (**tg: Sw**)

V. Vilfred, J. Paulraj Joseph, & C. Jayasekaran

2008a Branches and joints in the study of self switching of graphs. *J. Combin. Math. Combin. Comput.* 67 (2008), 111–122. MR 2457789 (2010b:05165). Zbl 1184.05127.

See Jayasekaran (2007a). Examines when a cut vertex x is self switching, assuming all self-switching vertices are interchanged by automorphisms. [Annot. 26 Sept 2012.] (**tg: Sw**)

2010a Self vertex switchings of trees. In: T. Tamizh Chelvam *et al.*, eds., *Algebra, Graph Theory and Their Applications*, pp. 118–128. Narosa Publishing House, New Delhi, 2010. Zbl 1220.05019.

See Jayasekaran (2007a). Characterizes forests with a self-switching vertex. [Annot. 26 Sept 2012.] (**tg: Sw**)

Jacques Villain

1959a La structure des substances magnetiques. *J. Phys. Chem. Solids* 11 (1959), 303–309.

Some physics spin models do, or may, or do not, have negative edges ($J < 0$). Precedes recognition of frustration and switching. [Annot. 11 Aug 2018.] (**Phys: sg**)

1977a Spin glass with non-random interactions. *J. Phys. C: Solid State Phys.* 10 (1977), no. 10, 1717–1734.

Partition function of "odd model" on signed lattice graph, i.e., all "elementary polygons" ("plaquettes" in Toulouse (1977a): small chordless circles: squares, triangles, or hexagons in different lattices) are unbalanced. Spins $S_i := S(v_i)$ may be Ising (in $\{\pm 1\} = S^0$), XY (in S^1), or Heisenberg (in S^2). §3, "The Ising version of the odd model on the

two-dimensional, square lattice". §4, "XY version of the odd model on the two-dimensional, square lattice". Unique ground state up to a ± 1 variable. §5, "The odd model on the diamond lattice": Ising vs. higher-dimensional ground states. §6, "Magnetic susceptibility of the odd model on the diamond lattice (XY version)". §7, "Modified odd models": A few balanced plaquettes. Or, slightly varying spin magnitudes. §8, "The odd model on other types of lattice". App. 1: The odd model switches to periodic form for the lattices treated herein. [Annot. 17 Jun 2012, 10 Aug 2018.] (**SG: Phys, Sw, VS(Gen)**)

1977b Two-level systems in a spin-glass model: I. General formalism and two-dimensional model. *J. Phys. C: Solid State Phys.* 10 (1977), 4793–4803. (**Phys: SG**)

1978a Two-level systems in a spin-glass model: II. Three-dimensional model and effect of a magnetic field. *J. Phys. C: Solid State Phys.* 11 (1978), no. 4, 745–752. (**Phys: SG**)(**GG: Phys, Sw, Bal**)

Rafael H. Villarreal

See also J. Martínez-Bernal, A. Simis, and C.E. Valencia.

1995a Rees algebras of edge ideals. *Commun. Algebra* 23 (1995), no. 9, 3513–3524. MR 1335312 (96e:13005). Zbl 836.13014.

The algebra for Γ is closely related to the frame and lift matroids of $-\Gamma$. Cor. 3.2: The edge ideal $I(\Gamma)$ is of linear type iff $\mathbf{F}(-\Gamma)$ is a free matroid. Lem. 3.1: The even cycle space has codimension $\operatorname{rk} \mathbf{F}(-\Gamma)$ [as found in van Nuffelen (1973a)]. Prop. 4.2: The elementary vectors of $\operatorname{Nul} \mathrm{H}(-\Gamma)$, i.e., the integral cycle space, are the indicator vectors of frame circuits [as in Doob (1973a)]. [Annot. 3 Jun 2015, rev 18 Aug 2018, 22 Oct 2020.] (**sg: Par: Incid Algeb, matrd**)

1998a On the equations of the edge cone of a graph and some applications. *manuscripta math.* 97 (1998), 309–317. MR 1654776 (99i:05143). Zbl 920.13015. (**ecyc: Geom, Incid**)

J. Villegas

See E.E. Vogel.

Daniele Vilone

See F. Radicchi.

Andrew Vince

1983a Combinatorial maps. *J. Combin. Theory Ser. B* 34 (1983), 1–21. MR 0701167 (84i:05048). Zbl 505.05054.

See Theorem 6.1. (**sg: bal: Top**)

E. Vincent, J. Hammann, & M. Ocio

1992a Slow dynamics in spin glasses and other complex systems. In: D.H. Ryan, ed., *Recent Progress in Random Magnets*, pp. 207–236. World Scientific, Singapore, 1992.

Surveys experiments with spin glass materials, especially aging behavior. Observations tend to support a landscape of graph signatures with numerous metastable states, subdividing as temperature decreases. [Presumably, the states correspond to clusters of low-frustration states, separated by high-frustration barriers, subdivided into smaller clusters

by lower-frustration barriers, and so on. Mathematical examination is needed.] [Annot. 27 Aug 1998, 24 Aug 2012.] (**Phys: sg: State: Dyn**)

Miguel Angel Virasoro

See M. Mézard.

Lee Inmon Virden

See T.J. Reid.

Krishnamurthy Viswanathan

See R.M. Roth.

Fabio Vitale

See N. Cesa-Bianchi and G. Le Falher.

J. Vlach

See C.J. Shi.

Alexander Vogel

See E. Steffen.

E.E. Vogel [Eugenio E. Vogel Matamala]

See also B. Fierro, W. Lebrecht, A.J. Ramírez-Pastor, F. Romá, M.C. Salas-Solís, and J.F. Valdés.

E.E. Vogel, J. Cartes, S. Contreras, W. Lebrecht, & J. Villegas

1994a Ground-state properties of finite square and triangular Ising lattices with mixed exchange interactions. *Phys. Rev. B* 49 (1994), no. 9, 6018–6027.

Many small toroidal lattice examples are computed to estimate large-scale behavior of ground states: frustration index, ground state energy E_g, etc. The signs are random, half positive and half negative. Also near-ground states (energy $E_g + 4$), obtained either by switching one vertex of a ground state or as a local minimum in the state space.

Dictionary (for papers of Vogel *et al.*): "periodic boundary conditions" = toroidal; "plaquette" = face boundary circle; "curved" plaquette = frustrated = negative; "frustration length" = frustration index; "ground state" = minimizing switching function ζ of Σ (i.e.; $\#E^-(\Sigma^\zeta) = \min_\zeta = l$); "ground state degeneracy" W = number of ground states; "magnetization per spin" (of ground state) = $[\#\zeta^{-1}(+) - \#\zeta^{-1}(-)]/nW$; "remnant entropy" = $(\ln W)/n$. [Annot. 4 Jan 2015.] (**Phys, SG: State(fr), Fr**)

Eugenio E. Vogel, Jaime Cartes, Patricio Vargas, & Dora Altbir

2000a Simulation of hysteresis for $\pm J$ triangular lattices. *Physica B* 284–288 (2000), 1211–1212. (**SG, Phys: State(fr), Fr**)

E.E. Vogel, J. Cartes, P. Vargas, D. Altbir, S. Kobe, T. Klotz, & M. Nogala

1999a Hysteresis in $\pm J$ Ising square lattices. *Phys. Rev. B* 59 (1999), no. 5, 3325–3328. (**Phys, SG: State(fr)**)

E.E. Vogel, S. Contreras, J. Cartes, & M.A. Osorio

1996a Non-frustrated domains in Ising spin glasses with competing interactions. In: F. Leccabue and V. Sagredo, eds., *Magnetism, Magnetic Materials and Their Applications* (Proc., Mérida, Venezuela, 1995), pp. 152–160. World Scientific, Singapore, 1996.

In a signed, toroidal square lattice graph keep the edges that are satisfied in every ground state. 500 6×6 examples were computed. $h :=$

proportion of retained edges tends to $\approx .5$. The distribution of component sizes is quite different from that of random subgraphs of similar size. For large h there is a boundary-linking giant component. For $h \approx .5$ there tends to be a definite proportion of small components of particular shapes. [Annot. 8 Feb 2015.] (**SG: Fr: State, Phys**)

E.E. Vogel, S. Contreras, W. Lebrecht, & J. Cartes

1995a Order parameters for various Ising lattices with competing $\pm J$ interactions. *J. Magnetism Magnetic Materials* 140–144 (1995), 1793–1794.
(**SG, Phys: Fr: State**)

E.E. Vogel, S. Contreras, F. Nieto, & A.J. Ramírez-Pastor

1998a Percolation in $\pm J$ Ising lattices after removing frustration. *Physica A* 257 (1998), 256–263. (**SG, Phys: State(fr), Sw**)

E.E. Vogel, S. Contreras, M.A. Osorio, & J. Cartes

1998a Bond percolation in $\pm J$ Ising square lattices diluted by frustration. *Phys. Rev. B* 58 (1998), no. 13, 8475–8480.

Square lattices on the torus. "Diluted" means removing every edge that is frustrated in one or more ground states. Dictionary: *cf.* Vogel, Cartes, Contreras, Lebrecht, and Villegas (1994a). [Annot. 3 Jan 2015.]
(**SG, Phys: State(fr), Sw**)

E.E. Vogel, S. Contreras, M.A. Osorio, A.J. Ramírez-Pastor, & F. Nieto

1999a Percolating spin-glass domains in diluted $\pm J$ square lattices. *Physica A* 266 (1999), 425–429.

Square lattices on the torus. "Diluted" means removing every edge that is frustrated in one or more ground states. Dictionary: *cf.* Vogel, Cartes, Contreras, Lebrecht, and Villegas (1994a). [Annot. 3 Jan 2015.]
(**SG, Phys: State(fr), Sw**)

E.E. Vogel & W. Lebrecht

1997a Rapidly converging asymptotic expansions in $\pm J$ Ising lattices. *Z. Physik* 102 (1997), 145–155.

The expansions are for functions of signed toroidal triangular, square, and honeycomb lattices such as the "frustration length" = minimum length of a T-join in the dual graph connecting frustrated plaquettes in pairs, and the proportion of edges that are positive in Σ^ζ for every ground state ζ . Applies two theoretical methods and compares to computed examples. [Annot. 9 Jan 2015.] (**SG, Phys: State, State(fr): Invar**)

E.E. Vogel, A.J. Ramirez-Pastor, & F. Nieto

2002a Detailed structure of configuration space and its importance on ergodic separation of $\pm J$ Ising lattices. *Physica A* 310 (2002), 384–396. MR 1946320 (2003k:82024). Zbl 995.82018.

"All" signed square lattices studied for physical properties like order parameters and grouping of ground states (minimal switchings). Related numerical results in Lebrecht, Vogel, and Valdés (2002a). [Annot. 3 Jan 2015.] (**SG, Phys: State(fr)**)

E.E. Vogel, J.F. Valdés, & W. Lebrecht

2006a $\pm J$ Ising model on Archimedean $(4, 8^2)$ lattices. *Physica A* 371 (2006), 150–154. **(SG: State(fr), Phys)**

Margit Voigt
See A. Kemnitz.

G.E. Volovik
See I.E. Dzyaloshinskii.

Jan Vondrák
See A. Galluccio.

Axel von Kamp
See S. Klamt.

Heinz-Jürgen Voss
See also D. Král'.

1991a *Cycles and Bridges in Graphs.* Math. Appl. (E. Europ. Ser.), Vol. 49. Kluwer, Dordrecht, and Deutscher Verlag der Wissenschaften, Berlin, 1991. MR 1131525 (92m:05118). Zbl 731.05031.

§3.4, "The length of bridges of longest odd and even cycles." §3.9, "The 'odd circumference' in bridges of longest odd cycles." §7.6, "Longest odd and even cycles, ..." §8.4, "Odd and even cycles with a given number of diagonals." §10.1, "Long cycles and long even cycles with many diagonals." §10.3, "Long odd cycles with many diagonals in non-bipartite graphs." [*Problem.* Generalize results on even and odd circles to signed graphs. *Cf.*, e.g., Conlon (2004a).] **(sg: par: Circ)**

Jože Vrabek
See T. Pisanski.

Eleni-Maria E. Vretta
See K. Papalamprou and L. Pitsoulis.

Hung Thanh Vu
See T.T.T. Ho.

Damir Vukičevic
See T. Došlić.

Kristina Vušković
See also P. Aboulker, V. Boncompagni, M. Conforti, T. Kloks, and N. Trotignon.

2010a Even-hole-free graphs: a survey. *Appl. Anal. Discrete Math.* 4 (2010), no. 2, 219–240. MR 2724633. Zbl 1265.05518.

[*Question.* How does the concept of an even (or odd) hole generalize to signed graphs?] [Annot. 11 Jul 2022.] **(SG: Bal(Gen): Str, Algor: Exp)**

2013a The world of hereditary graph classes viewed through Truemper configurations. In: Simon R. Blackburn, Stefanie Gerke, and Mark Wildon, eds., *Surveys in Combinatorics 2013* (24th British Combin. Conf., London, 2013), Ch. 7, pp. 265–325. London Math. Soc. Lect. Note Ser., Vol. 409. Cambridge Univ. Press, Cambrige, Eng., 2013. MR 3184115. Zbl 1301.05279. **(sg: Eig: Exp)**

Michelle L. Wachs
See also A. Björner, E. Gottlieb, and J.B. Remmel.

2007a Poset topology: tools and applications. In: E. Miller, V. Reiner, and B. Sturmfels, eds., *Geometric Combinatorics*, pp. 497–615. American Math. Soc. and Institute for Advanced Study (IAS), Providence, R.I., and Princeton, N.J., 2007. MR 2383132 (no rev). Zbl 1135.06001. arXiv:math/0602226.
(sg: sg: Top, Algeb: Exp)

Donald K. Wagner

See also V. Chandru and C.R. Coullard.

†† 1985a Connectivity in bicircular matroids. *J. Combin. Theory Ser. B* 39 (1985), 308–324. MR 0815399 (87c:05041). Zbl 584.05019.

Prop. 1 and Thm. 2 show that n-connectivity of the bicircular matroid $BG(\Gamma)$ is equivalent to "n-biconnectivity" of Γ.

When do two 3-biconnected graphs have isomorphic bicircular matroids? §5 proves that 3-biconnected graphs with > 4 vertices have isomorphic bicircular matroids iff one is obtained from the other by a sequence of operations called "edge rolling" and "3-star rotation". This is the bicircular analog of Whitney's circle-matroid isomorphism theorem, but it is complicated. [An important theorem, generalized to all bicircular matroids in Coullard, del Greco, and Wagner (1991a). *Major Research Problems.* Generalize to frame matroids of biased graphs. Find the analog for lift matroids.] **(Bic: Str)**

1988a Equivalent factor matroids of graphs. *Combinatorica* 8 (1988), 373–377. MR 0981894 (90d:05071). Zbl 717.05022.

"Factor matroid" = even-cycle matroid $\mathbf{F}(-\Gamma)$. Decides when $\mathbf{F}(-\Gamma) \cong \mathbf{M}(\mathrm{B})$ where B is a given bipartite, 4-connected graph. **(ECyc: Str)**

H. Wagner

See K. Drühl.

Magnus Wahlström

See S. Böcker and S. Kratsch.

Bronislaw Wajnryb

See R. Aharoni.

Oliver Waldmann

See K.C. Mondal.

M.H. Waldor, W.F. Wolff, & J. Zittartz

1985a Ising models on the pentagon lattice. *Z. Phys. B* 59 (1985), no. 1, 43–51. MR 0788876 (86e:82054).

Physics of all-positive and all-negative ("fully frustrated": all face boundary circles are negative) signs as examples. § II, "Thermodynamics", b) "Homogeneous case": The all-negative signature has multiple ground states that have energy $-J$ (J = bond strength) per vertex because the frustrated edges form a perfect matching in a ground state. [I.e., frustration index $l(-\Gamma) = \#V$ for these pentagonal lattice graphs, assuming no boundary as, e.g., when the lattice is toroidal.] [Annot. 18 Jun 2012.] **(Phys: SG: Par: State(fr))**

H.B. Walikar, Satish V. Motammanavar, & B.D. Acharya

2015a Signed domination in signed graphs. *J. Combin. Inform. System Sci.* 40 (2015), 107–128. Zbl 1358.05135.

Like B.D. Acharya (2012b), but a signed domination function is $\mathbf{f} \in \{\pm 1\}^V$. NASC for a signed path, star, circle, or caterpillar to admit such a function. Every signed graph is an induced subgraph of a signed graph that admits one. [Annot. 18 May 2018, 29 Dec 2020.] (**SG: Lab, Dom**)

Derek A. Waller

See also F.W. Clarke.

† 1976a Double covers of graphs. *Bull. Austral. Math. Soc.* 14 (1976), 233–248. MR 0406876 (53 #10662). Zbl 318.05113.

The prehistory of signed covering graphs (*cf.* Zaslavsky (1982a), §6); the language has changed.

Signed covers $\tilde{\Sigma}$ in terms of covering projections $\downarrow$ viewed as graph homomorphisms $\to$. §2: Examples "Kronecker double cover" = signed cover of $-\Gamma$; disjoint union = signed cover of $+\Gamma$. General definition. §3, "Antipodal double covers", i.e., the pair covering a vertex are the unique pair at max distance in $\tilde{\Sigma}$. [*Problem.* If Γ has an antipodal cover, what are the signs?] Thm. 3.1: If Δ_i antipodally covers Γ_i ($i = 1, 2$), then $\Delta_1 \,\Box\, \Delta_2$ antipodally covers a graph.

§4, "Pullbacks of double covers": Prop. 4.1: If $\Gamma_2 \to \Gamma_1$ and $\Delta_1 \downarrow \Gamma_1$, then $\exists\ \Delta_2 \to \Delta_1$ that $\downarrow \Gamma_2$. Choice of spanning tree T means fixing $\sigma|_T \equiv +$. ["Nontrivial" edge = negative.] Structural results for connected base graphs. §5, "Classification of double covers": Hyperoctahedron $K_{n(2)} \downarrow \pm K_n$ (with "type 1" [positive] and "type 2" [negative] edges) and is "universal" for n-coloring. Prop. 5.3: $\Delta \downarrow \Gamma \implies \chi(\Delta) \leq \chi(\Gamma)$. Thm. 5.5: All double covers are pullbacks of $K_{n(2)} \downarrow \pm K_n$. Monomorphisms $\alpha', \beta' : C_n \to |\pm K_n|$ are equivalent iff they give the same sign in $\pm K_n$. Thm. 5.6: Double covers are isomorphic iff they induce switching equivalent signatures [which seems obvious in signed-graph language]. [Annot. 30 Jan 2022.] (**sg: Cov: sw, col**)

Zach Walsh

2022a A new matroid lift construction and an application to group-labeled graphs. *Electronic J. Combin.* 29 (2022), no. 1, art. P1.6, 18 pp. MR 4395251. arXiv:-2104.08257. (**GG: Matrd**)

Georg R. Walther, Matthew Hartley, & Maya Mincheva

2014a GraTeLPy: graph-theoretic linear stability analysis. *BMC Systems Biol.* 8 (2014), art. 22, 11 pp. (**SD: QM: Dyn, Chem, Biol**)

O. Walther

See Q. Zheng.

Changping Wang

2008a The signed matchings in graphs. *Discuss. Math. Graph Theory* 28 (2008), no. 3, 477–486. MR 2514204 (2010f:05147). Zbl 1175.05114.

Fix Γ. Signed matching: σ_m such that all $d^{\pm}(v) \leq 1$. Signed matching number $\beta_1'(\Gamma) := \max_{\sigma_m}(\#E^+ - \#E^-)$. Finding a max σ_m is strongly polynomial. Bounds on β_1'; exact values for P_n, C_n, K_n, $K_{p,q}$. Signed edge cover: σ^c such that all $d^{\pm}(v) \geq 1$. Signed edge cover number

$\rho_1'(\Gamma) := \min(\#E^+ - \#E^-)$. $\beta_1'(\Gamma) + \rho_1'(\Gamma)$ need not $= n$. [Annot. 17 Dec 2020.] **(sg, Algor)**

2010a Signed b-matchings and b-edge covers of strong product graphs. *Contrib. Discrete Math.* 5 (2010), no. 2, 1–10. MR 2791285 (2012h:05255). Zbl 1317.05156.

Generalizing (2008a), b- means $\geq b$ or $\leq b$. Bounds for strong product. Gallai theorem for K_n and $K_{p,q}$. [Annot. 17 Dec 2020.] **(sg)**

2013a The signed k-submatchings in graphs. *Graphs Combin.* 29 (2013), 1961–1971. MR 3119952. Zbl 1282.05191.

Generalizing (2008a), σ_S such that $d^{\pm}(v) \leq 1$ for $\geq k$ vertices. Sharp bounds on $\beta_S^k(\Gamma) := \max_{\sigma_S}(\#E^+ - \#E^-)$ and some exact values. [More in S. Akbari, M. Dalirrooyfard, K. Ehsani, & R. Sherkati (2016a).] [Annot. 17 Dec 2020.] **(sg)**

Chong-Jun Wang

See Y. Wang.

Chun-Chieh Wang

See L.-H. Chen.

Cuihua Wang

See D. Li.

Dan Wang

See W. Chen.

Dengyin Wang

See F.L. Tian and S. Wang.

Dijian Wang

See also Y.-P. Hou.

Dijian Wang, Wenkuan Dong, Yaoping Hou, & Deqiong Li

† 2023a On signed graphs whose spectral radius does not exceed $\sqrt{2+\sqrt{5}}$. *Discrete Math.* 346 (2023), no. 6, art. 113358, 21 pp. MR 4548954. Zbl 1512.05173. arXiv:2203.01530.

All Σ with $\max|\lambda_i(A(\Sigma))| \leq \sqrt{2+\sqrt{5}} = \sqrt{\tau}+\sqrt{\tau}^{-1}$ ($\tau =$ golden ratio), a Hoffman spectral limit number. *Cf.* Belardo and Brunetti (2024a). [Annot. 4 Mar 2022.] **(SG: Adj: Eig)**

Dijian Wang & Dongdong Gao

2023a Laplacian integral signed graphs with few cycles. *AIMS Math.* 8 (2023), no. 3, 7021–7031. MR 4550631.

Classifies connected Σ with cyclomatic numbers $\xi \leq 3$ and integral Spec $L(\Sigma)$. **(SG: Lap: Eig)**

Dijian Wang & Yaoping Hou

2018a On the sum of Laplacian eigenvalues of a signed graph. *Linear Algebra Appl.* 555 (2018), 39–52. MR 3834190. Zbl 1393.05184. **(SG: Lap: Eig)**

2019a Bicyclic signed graphs with at most one odd cycle and maximal energy. *Discrete Appl. Math.* 260 (2019), 244–255. MR 3944625. Zbl 1409.05133. **(SG: Adj: Eig)**

2020a Integral signed subcubic graphs. *Linear Algebra Appl.* 593 (2020), 29–44. MR 4066149. Zbl 1436.05066.

There are one infinite family and 14 sporadic signed graphs with integral Spec $A(\Sigma)$. [Annot. 9 Dec 2020.] **(SG: Adj: Eig)**

2021a Laplacian integral subcubic signed graphs. *Electronic J. Linear Algebra* 37 (2021), 163–176. MR 4246522. Zbl 1464.05171. **(SG: Lap: Eig)**

20xxa Unicyclic signed graphs with maximal energy. Submitted. arXiv:1809.06206. **(SG: Adj: Eig)**

Dijian Wang, Yaoping Hou, & Deqiong Li

2023a Signed graphs with all but two eigenvalues equal to ± 1. *Graphs Combin.* 39 (2023), no. 4, art. 87, 18 pp. MR 4619026. **(SG: Adj: Eig)**

20xxa Extremed [[*sic*]] signed graphs for triangle. Submitted. arXiv:2212.11460.
A Turán-type theorem for excluded negative triangles. Also, upper bound on spectral radius. **(SG: Str, Adj: Eig)**

Guangfu Wang, Shuchao Li, Wei Wei, & Siqi Zhang

2019a On the characteristic polynomials and H-ranks of the weighted mixed graphs. *Linear Algebra Appl.* 581 (2019), 383–404. MR 3987452. Zbl 1419.05107.
Φ with gain group $\{\pm 1, \pm \mathrm{i}\}$: $\varphi(e) = 1$ for undirected, i for directed edges. Formulas for coefficients of weighted characteristic polynomial. $\operatorname{rk} A(\Phi, w)$ determined in some cases. [Annot. 15 Dec 2020.] **(gg, WG: Adj)**

Hai-Feng Wang

See M.L. Ye.

Haotian Wang, Feng Luo, & Jie Gao

20xxa Co-evolution of opinion and social tie dynamics towards structural balance. Submitted. (SODA 2022, virtual). **(SD: KG: Bal, Dyn)**

Hua Wang

See J. Huang.

Jianfeng Wang

See also F. Belardo and L. Lu.

Jianfeng Wang & Francesco Belardo

2011a A note on the signless Laplacian eigenvalues of graphs. *Linear Algebra Appl.* 435 (2011), no. 10, 2585–2590. MR 2811140 (2012d:05242). Zbl 1225.05176. **(par: Lap: Eig)**

JianFeng Wang, Francesco Belardo, QiongXiang Huang, & Bojana Borovićanin

2010a On the two largest Q-eigenvalues of graphs. *Discrete Math.* 310 (2010), no. 21, 2858–2866. MR 2677645 (2011j:05218). Zbl 1208.05079. **(par: Lap: Eig)**

Jianfeng Wang, Francesco Belardo, QiongXiang Huang, & Enzo M. Li Marzi

2013a On graphs whose Laplacian index does not exceed 4.5. *Linear Algebra Appl.* 438 (2013), no. 4, 1541–1550. MR 3005240. Zbl 1259.05119.
See Wang, Huang, Belardo, and Li Marzi (2009a). [Annot. 20 Dec 2011.] **(par: Lap: Eig)**

JianFeng Wang, Francesco Belardo, Wei Wang, & QiongXiang Huang

2013a On graphs with exactly three Q-eigenvalues at least two. *Linear Algebra Appl.* 438 (2013), 2861–2879. MR 3018045. Zbl 1259.05118. **(par: Lap: Eig)**

Jianfeng Wang & Qiongxiang Huang

2011a Maximizing the signless Laplacian spectral radius of graphs with given diameter or cut vertices. *Linear Multilinear Algebra* 59 (2011), no. 7, 733–744. MR 2871248. Zbl 1222.05182.

Thm. 3.1: Fixing diameter d, the largest eigenvalue $q_1(L(-\Gamma))$ is maximized uniquely by a path of length d with a clique joined to its three middle vertices (K_n if $d \leq 2$). Thm. 3.2: Fixing $\nu := n-d$, $q_1 \nearrow 4\nu^2/(2\nu-1)$. Thm. 4.1: Fixing the number k of cutpoints, q_1 is maximized uniquely by K_{n-k} with paths of nearly equal length attached to every vertex. Fix $\mu := n-k$. Thm. 4.2: If $k \leq n/2$, $q_1 < 2(\mu-1) + k\mu/[2(\mu-1)^2 - n]$. Thm. 4.3: If $n/2 < k \leq n-3$, $q_1 \nearrow 2\mu - 1 + 1/(2\mu-3)$. [*Question.* Can these results generalize to signed graphs?] [Annot. 16 Jan 2012.] **(par: Lap: Eig)**

Jianfeng Wang, Qiongxiang Huang, Xinhui An, & Francesco Belardo

2010a Some results on the signless Laplacians of graphs. *Appl. Math. Lett.* 23 (2010), no. 9, 1045–1049. MR 2659136 (2011e:05161). Zbl 1209.05148. **(par: Lap: Eig)**

JianFeng Wang, QiongXiang Huang, Francesco Belardo, & Enzo M. Li Marzi

2009a On graphs whose signless Laplacian index does not exceed 4.5. *Linear Algebra Appl.* 431 (2009), no. 1-2, 162–178. MR 2522565 (2010g:05238). Zbl 1171.05035.

The signless Laplacian $Q = L(-\Gamma)$ is employed to derive results on the Laplacian $L(\Gamma)$. Continued in Wang, Belardo, Huang, and Li Marzi (2013a). [Annot. 9 Feb 2013.] **(par: Lap: Eig)**

2010a On the spectral characterizations of ∞-graphs. *Discrete Math.* 310 (2010), 1845–1855. MR 2629903 (2011m:05188). Zbl 1231.05174.

The spectra of $L(-\Gamma)$ and $L(+\Gamma)$ are compared, where Γ is a tight handcuff (a figure-eight graph). [*Conjecture.* The results certainly hold for $L(-\Gamma)$ vs. all $L(\Gamma,\sigma)$ since only the circle signs differentiate $+\Gamma$ from (Γ,σ).] [Annot. 20 Jan 2012.] **(par: Lap: Eig)**

Jian-Sheng Wang & Robert H. Swendsen

1988a Low-temperature properties of the $\pm J$ Ising spin glass in two dimensions. *Phys. Rev. B* 38 (1988), no. 7, 4840–4844. **(Phys: sg: Fr)**

Jianfeng Wang, Jing Wang, & Maurizio Brunetti

20xxa The Hoffman program of graphs: old and new. Submitted. arXiv:2012.13079.

Survey and new results on graphs with spectral radius bounded by a spectral limit point. §6, "Signed-adjacency matrix" [i.e., signed-graph adjacency matrix]: $A(\Sigma)$. §4, "Signless Laplacian matrix": $L(-\Gamma)$. §5, "Hermitian adjacency matrix": $A(\Phi)$ where Φ has gains in $\{\pm1, \pm\mathrm{i}\}$. §7, "Skew-adjacency matrix": Essentially, gains $\pm$i. [Annot. 4 Mar 2022.] **(SG, gg: Adj, Lap: Eig)(SG, gg: Adj, Lap: Eig: Exp)**

Jing Wang

See J.-F. Wang.

Jue Wang

See also B. Chen.

2007a *Algebraic Structures of Signed Graphs.* Doctoral dissertation, Hong Kong University of Science and Technology, 2007.

Cycle and cocycle spaces, interpretations and relationships. New formulation of matrix-tree generalization. (**SG: Incid, Str**)

Junjie Wang, Yaoping Hou, & Xueyi Huang

20xxa Turán problem for C^{-}_{2k+1}-free signed graph. Submitted. arXiv:2310.11061. (**SG: Adj: Eig, Xtreml**)

Kyle Wang

See G. Chen.

Larry X.W. Wang

See W.Y.C. Chen.

Li Wang

See S.-S. Feng.

Ligong Wang

See also Y.Q. Chen, K. Li, Y. Lu, and X.-W. Yang.

Ligong Wang, Guopeng Zhao, & Ke Li

2014a Seidel integral complete r-partite graphs. *Graphs Combin.* 30 (2014), 479–493. MR 3167023. Zbl 1298.05213.

I.e., $\Sigma = K_n(-K_{n_1,\ldots,n_r})$. (**SG: KG: Adj: Eig**)

Long Wang

See X. Lin and X.B. Ma.

Longqin Wang, Zhengke Miao, & Chao Yan

2009a Local bases of primitive non-powerful signed digraphs. *Discrete Math.* 309 (2009), no. 4, 748–754. MR 2502184 (2010f:05092). Zbl 1168.05029. (**SD: qm**)

Longqin Wang, Lihua You, & Hongping Ma

2011a Primitive non-powerful sign pattern matrices with base 2. *Linear Multilinear Algebra* 59 (2011), no. 6, 693–700. MR 2801362 (2012j:05272). Zbl 1223.05104. (**SD: QM**)

Lusheng Wang

See X.L. Li.

Qingwen Wang

See G.H. Yu.

Sai Wang, Dengyin Wang, & Fenglei Tian

2021a Relations between the positive inertia index of a $\mathbb{T}$-gain graph and that of its underlying graph. *J. Math. Res. Appl.* 41 (2021), no. 3, 221–237. MR 4267167. (**GG: Adj: Eig**)

Sai Wang, Dein Wong, & Fenglei Tian

2021a Graphs determined by their T-gain spectra. *Bull. Austral. Math. Soc.* 103 (2021), no. 2, 195–203. MR 4229487. Zbl 1460.05119. (**GG: Adj: Eig**)

Shanfeng Wang

See Q. Cai.

Shaohui Wang
See J.-B. Liu.

Shilin Wang & Bo Zhou
2013a The signless Laplacian spectra of the corona and edge corona of two graphs. *Linear Multilinear Algebra* 61 (2013), no. 2, 197–204. MR 3003050. Zbl 1259.05120.
(**par: Lap: Eig**)

Shiying Wang, Jing Li, Wei Han, & Shangwei Lin
2010a The base sets of quasi-primitive zero-symmetric sign pattern matrices with zero trace. *Linear Algebra Appl.* 433 (2010), 595–605. MR 2653824 (2011g:15056). Zbl 1195.15031. (**QM: SD**)

Shujing Wang
See S.C. Li.

Suijie Wang
See X.G. Liu.

Tianfei Wang
2007a Several sharp upper bounds for the largest Laplacian eigenvalue of a graph. *Sci. in China Ser. A Math.* 50 (2007), no. 12, 1755–1764. MR 2390486 (2009a:05131) (*q.v.*). Zbl 1134.05064.
See MR for the formulas, which apply to $q_1(L(-\Gamma))$ [hence to signed simple graphs]. The proofs use a normalized Laplacian. [Annot. 16 Jan 2012.] (**par: Lap: Eig**)

Wei Wang
See X.-J. Tian.

Wei Wang [mr36]
See also Wang, Qiu, Qian, and Wang (2020a).

Wei Wang & Jianguo Qian
2020a A generalization of Noel-Reed-Wu Theorem to signed graphs. *Discrete Math.* 343 (2020), no. 6, art. 111833, 12 pp. MR 4062291. Zbl 1437.05092. arXiv:-1810.09741. (**SG: Col: Gen**)

Wei Wang, Jianguo Qian, & Toshiki Abe
2019a Alon-Tarsi number and modulo Alon-Tarsi number of signed graphs. *Graphs Combin.* 35 (2019), no. 5, 1051–1064. MR 4003656. Zbl 1426.05059. arXiv:-1809.02907.
Previous version arXiv:1809.02907v1. (**SG: Col: Gen**)

Wei Wang, Zhidan Yan, & Jianguo Qian
2021a Eigenvalues and chromatic number of a signed graph. *Linear Algebra Appl.* 619 (2021), 137–145. MR 4223364. Zbl 1466.05125.
Thm.: $\chi(\Sigma) \geq 1 + \lambda_{\max}/|\lambda_{\min}|,\ n/(n - \lambda_{\max})$. [Annot. 19 Mar 2021.]
(**SG: Col, Adj: Eig**)

Wei Wang, Lihong Qiu, Jianguo Qian, & Wei Wang
2020a Generalized spectral characterization of mixed graphs. *Electronic J. Combin.* 27 (2020), no. 4, art. 4.55, 31 pp. MR 4245230. Zbl 1472.05102. arXiv:1911.13004.

Φ with gains in $\{1, \pm \mathrm{i}\}$ as in Liu and Li (2015a) and Mohar (2016a). "Generalized spectrum" = $(\operatorname{Spec} A(\Phi), \operatorname{Spec}(J - I - A(\Phi)))$. Characterization up to isomorphism, not switching isomorphism or conjugation (thus must be self-conjugate = digraph self-converse). [Conjugation preserves spectra. Switching preserves $\operatorname{Spec} A(\Phi)$ but not $\operatorname{Spec}(J - I - A(\Phi))$.] [For gain graphs, this is unnatural; spectral characterization up to switching and conjugation is natural.] [Annot. 23 Dec 2020.] (**gg: Adj: Eig**)

X.L. Wang

See Y. Chen.

Xiao Wang, You Lu, Cun-Quan Zhang, & Shenggui Zhang

2019a Six-flows on almost balanced signed graphs. *J. Graph Theory* 92 (2019), 394–404. MR 4030965. Zbl 1443.05087. (**SG: Flows**)

Xiao Wang, You Lu, & Shenggui Zhang

2021a Flow number of signed Halin graphs. *Appl. Math. Comput.* 393 (2021), art. 125751, 12 pp. MR 4172776.

If it has a nowhere-zero flow, it has a nowhere-zero 5-flow, thus satisfying Bouchet's (1983a) conjecture. [Annot. 26 Nov 2020.] (**SG: Flows**)

Xin Wang

See S.-D. Zhai.

Xing Ke Wang

See S.W. Tan.

Xioafeng Wang

See E.L. Wei.

Yajun Wang

See Y.-H. Li.

Yi Wang

See also Y.Z. Fan, L.L. Liu, G.-D. Yu, B.-J. Yuan, and J. Zhou.

Yi Wang & Yi-Zheng Fan

2012a The least eigenvalue of signless Laplacian of graphs under perturbation. *Linear Algebra Appl.* 436 (2012), no. 7, 2084–2092. MR 2889977. Zbl 1238.05168. (**par: Lap: Eig**)

Yi Wang, Shi-Cai Gong, & Yi-Zheng Fan

2018a On the determinant of the Laplacian matrix of a complex unit gain graph. *Discrete Math.* 341 (2018), no. 1, 81–86. MR 3713385. Zbl 1372.05097. (**GG: Lap**)

Yi Wang & Yeong-Nan Yeh

2005a Polynomials with real zeros and Pólya frequency sequences. *J. Combin. Theory Ser. A* 109 (2005), 63–74. MR 2110198 (2005i:05013). Zbl 1057.05007.

The Whitney numbers $W_m(n,k) := W_k(\operatorname{Lat} \mathfrak{Q} K_n^\bullet)$ ($m = \#\mathfrak{G}$) are Example 5. [Annot. 9 Apr 2016.] (**gg: Matrd: Invar**)

Yi Wang, Bo-Jun Yuan, Shuang-Dong Li, & Chong-Jun Wang

2017a Mixed graphs with H-rank 3. *Linear Algebra Appl.* 524 (2017), 22–34. MR 363017. Zbl 1361.05076.

Φ with gain group $\{\pm 1, \pm \mathrm{i}\}$: $\varphi(e) = 1$ for undirected, i for directed edges. "Rank" = rk $A(\Phi)$. [Annot. 15 Dec 2020.] (**gg: Adj**)

Yishui Wang

See S. Ji.

Yitian Wang

See Z.-Y. Cheng.

Yongang Wang

20xxa Spectral Turán problem for $\mathcal{K}_5^?$-free signed graphs. arXiv:2309.15434. (**SG: Adj: Eig: Str**)

Yongang Wang & Huiqiu Lin

20xxa The largest eigenvalue of $\mathcal{C}_4^-$-free signed graphs. arXiv:2309.04101. (**SG: Adj: Eig**)

Yuanpei Wang

See H.-Y. Cai.

Yue Wang

See Y.Z. Fan.

Yunzhuo Wang

See W.-Z. Liu.

Zhiwen Wang & Shuting Liu

2023a The index of signed graphs with forbidden subgraphs. *Bull. Malaysian Math. Sci. Soc.* 46 (2023), art. 160, 16 pp. MR 4616094.

Max $\lambda_{\max}(\Sigma)$ excluding subgraph $+K_s$, or $+C_k$ for all $k \leq n$ or all even $k \leq n$. [Annot. 29 Sep 2023.] (**SG: Adj: Eig: Str**)

Zhouningxin Wang

See L. Beaudou, C. Cappello, J.-A. Li, and R. Naserasr.

Egon Wanke

See also F. Höfting.

1993a Paths and cycles in finite periodic graphs. In: Andrzej M. Borzyszkowski and Stefan Sokołowski, eds., *Mathematical Foundations of Computer Science 1993* (Proc., 18th Int. Sympos., MFCS '93, Gdańsk, 1993), pp. 751–760. MR 1265106 (95c:05077). Zbl 925.05038.

Broadly resembles Höfting and Wanke (1994a) but omits those edges of $\tilde{\Phi}$ that are affected by the modulus α. (**GD(Cov): Algor**)

Ian M. Wanless

2005a Permanents of matrices of signed ones. *Linear Multilinear Algebra* 52 (2005), no. 6, 427–433. MR 2162064 (2006g:15013). Zbl 1085.15006.

per$(B) := \sum_M \sigma(M)$, summed over all perfect matchings $M \subseteq \Sigma = (K_{n,n}, \sigma)$, where $A(\Sigma) = \left(\begin{smallmatrix} B & O \\ O & B \end{smallmatrix}\right)$. [Annot. 22 Aug 2012.] (**sg: Adj**)

G.H. Wannier

1950a Antiferromagnetism. The triangular Ising net. *Phys. Rev.* (2) 79 (1950), 357–364. MR 0039627 (12, 576). Zbl 038.41904 (38, p. 419d). Errata. *Phys. Rev. B* 7 (1973), 5017.

Physical consequences of frustration with Ising spins, i.e. $\zeta : V \to \{+1, -1\}$, in the all-negative triangular lattice graph. [Also see Houtappel (1950a), (1950b), Newell (1950a), I. Syôzi (1950a).] [Annot. 16 Jun 2012.] (**Phys: SG: par: State(fr)**)

T. Wanschura, D.A. Coley, & S. Migowsky

1996a Ground-state energy of the $\pm J$ spin glass with dimension greater than three. *Solid State Commum.* 99 (1996), no. 4, 247–248.

Calculation by genetic algorithm for certain lattice graphs. [Annot. 3 Jan 2015.] (**Phys, SG: State(fr): Algor**)

Dan Warner

See C.R. Johnson.

Jacqueline M. Warren

See E. Leclerc and G. MacGillivray.

Stanley Wasserman & Katherine Faust, eds.

1994a *Social Network Analysis: Methods and Applications*. Structural Anal. Soc. Sci., 8. Cambridge Univ. Press, Cambridge, Eng., 1994. Zbl 980.24676.

§1.2: "Historical and theoretical foundations." A brief summary of various network methods in sociometry, signed graphs and digraphs among them. Ch. 4 by Dawn Iacobucci (1994a). Elementary. §4.4: "Signed graphs and signed directed graphs." Mathematical basics. §4.5: "Valued graphs and valued directed graphs." Mentions unweighted and positively weighted signed (di)graphs. Ch. 6: "Structural balance and transitivity." Application of balance of signed (di)graphs and of ensuing notions like clusterability, historically evolving into transitivity of unsigned digraphs. History and evaluation. §6.1: "Structural balance." Balance, indices of imbalance. §6.2: "Clusterability." All graphs, and complete graphs, as in Davis (1967a). §6.3: "Generalizations of clusterability." §6.3.2: "Ranked clusterability." As in Davis and Leinhardt (1972a). [Annot. 28 Apr 2009.] (**PsS, SG, SD: Bal, Fr, Clu, Gen: Exp, Ref**)

Yusuke Watanabe

2011a Uniqueness of belief propagation on signed graphs. In: J. Shawe-Taylor *et al.*, eds., *Advances in Neural Information Processing Systems 24* (Proc., NIPS 2011, Granada, Spain), art. 4422, 9 pp. Neural Information Processing Systems Foundation, 2011. (**SG**)

William C. Waterhouse

1977a Some errors in applied mathematics. *Amer. Math. Monthly* 84 (January, 1977), no. 1, 25–27. Zbl 376.9001 (*q.v.*).

Criticizes Roberts and Brown (1975a), (1977a). See rebuttal in the Zbl review.

John J. Watkins

See R.J. Wilson.

William Watkins

See M. Lien.

Kevin D. Wayne

See È. Tardos.

Nikolai Weaver

See E. Flapan.

Jeffrey R. Weeks & Kenneth P. Bogart

1979a Consensus signed digraphs. *SIAM J. Appl. Math.* 36 (1979), 1–14. MR 0519178 (81i:92026). Zbl 411.05042. (**SD**)

Chunyan Wei

See J.-A. Li.

Er Ling Wei, Wen Tang, & Dong Ye

2014a Nowhere-zero 15-flow in 3-edge-connected bidirected graphs. *Acta Math. Sinica (Engl. Ser.)* 30 (2014), no. 4, 649–660. MR 3176918. Zbl 1294.05088.

A nowhere-zero 15-flow exists. (*Cf.* Wei, Tang, and Wang (2011a)) [Annot. 16 Apr 2014.] (**SG: Ori, Flows**)

Erling Wei, Wenliang Tang, & Xioafeng Wang

2011a Flows in 3-edge-connected bidirected graphs. *Frontiers Math. China* 6 (2011), no. 2, 339–348. MR 2780896 (2012b:05137). Zbl 1226.05130.

A nowhere-zero 25-flow exists. (Progress on Bouchet's conjecture, Bouchet (1983a).) [Annot. 6 Jun 2011.] (**SG: Ori, Flows**)

Fuyi Wei

See F.-Y. Wei.

Fi-yi Wei

See also M.H. Liu.

Fi-yi Wei & Muhuo Liu

2011a Ordering of the signless Laplacian spectral radii of unicyclic graphs. *Australasian J. Combin.* 49 (2011), 255–264. MR 2790977 (2011m:05189). Zbl 1228.05208.

Unicyclic Γ with $\lambda^1(L(-\Gamma)) \geq n-2$. [*Problem.* Generalize to signed graphs, or complex unit gain graphs. *Cf.* Reff (2012a).] [Annot. 19 May 2019.] (**par: Lap: Eig**)

Li Juan Wei

See Y.P. Hou.

Po-Sun Wei & Bang Ye Wu

2012a Balancing a complete signed graph by changing minimum number of edge signs. In: *Proceedings of the 29th Workshop on Combinatorial Mathematics and Computation Theory* (Taipei, 2012), pp. 1–8. National Taipei College of Business, Taipei, Taiwan, 2012. URL `http://par.cse.nsysu.edu.tw/~algo/paper/paper_list12.htm`

Algorithm for frustration index $l(K_n, \sigma)$ by changing edge signs. [Annot. 5 Jun 2017.] (**SG: KG: Fr: Algor**)

Wei Wei

See also S.-L. Li and G.-F. Wang.

Wei Wei & Shuchao Li

2020a Relation between the Hermitian energy of a mixed graph and the matching number of its underlying graph. *Linear Multilinear Algebra* 68 (2020), no. 7, 1395–1410. MR 4137053. Zbl 1459.05194. **(gg: Adj: Eig)**

Wei Wei, Shuchao Li, & Hongping Ma

2021a Bounds on the nullity, the H-rank and the Hermitian energy of a mixed graph. *Linear Multilinear Algebra* 69 (2021), no. 13, 2469–2490. MR 4301424. Zbl 1472.05104.

Φ with gain group $\{\pm 1, \pm \mathrm{i}\}$: $\varphi(e) = 1$ for undirected, i for directed edges. **(gg: Adj, Eig)**

Martin Weigt

See A.K. Hartmann.

Gerry M. Weiner

See J.S. Maybee.

Volkmar Welker

1997a Colored partitions and a generalization of the braid arrangement. *Electronic J. Combin.* 4 (1997), no. 1, art. R4, 12 pp. MR 1435130 (98b:57026). Zbl 883.52010.

The arrangement is the affine part (that is, where $x_0 = 1$) of the projective representation of $\mathbf{F}(\Phi)$, where Φ is the complex multiplicative gain graph $\Phi = \{1\}K_{n+1} \cup \{re_{0i} : 1 \leq i \leq n \text{ and } 2 \leq r \leq s\}$. Here the vertex set is $\{0, 1, \ldots, n\}$, s is any positive integer, and re_{0i} (in the paper, $e_{0i}(r)$) denotes an edge v_0v_i with gain r. The topics of interest are those related to the complex complement. The study is based on the combinatorics of the intersection semilattice [that is, the geometric semilattice $\mathrm{Lat}^{\mathrm{b}}\,\Phi$ of balanced flats], including the Poincaré polynomial of the arrangement [equivalent to the balanced chromatic polynomial of Φ]. **(gg: Matrd, Geom, Invar)**

Albert L. Wells, Jnr.

See also P.J. Cameron and Y. Cheng.

1982a *Regular Generalized Switching Classes and Related Topics*. D.Phil. thesis, Oxford University, 1982. **(SG: Sw, Adj, Eig, Enum, TG, Geom, Cov, Aut)**

† 1984a Even signings, signed switching classes, and $(-1, 1)$-matrices. *J. Combin. Theory Ser. B* 36 (1984), 194–212. MR 0746549 (85i:05206). Zbl 527.05007. **(SG: Sw, Enum, Aut)**

D.J.A. Welsh [Dominic Welsh]

See also L. Lovász and W. Schwärzler.

1976a *Matroid Theory.* L.M.S. Monographs, Vol. 8. Academic Press, London, 1976. MR 0427112 (55 #148). Zbl 343.05002. Repr.: Dover Publications, Mineola, N.Y., 2010.

§11.4: "Partition matroids determined by finite groups", sketches the most basic parts of Dowling (1973b). **(gg: Matrd: Exp)**

1992a On the number of knots and links. In: G. Halász, L. Lovász, D. Miklós, and T. Szönyi, eds., *Sets, Graphs and Numbers* (Proc., Budapest, 1991), pp. 713–718. Colloq. Math. Soc. János Bolyai, Vol. 60. János Bolyai Math. Soc., Budapest, and North-Holland, Amsterdam, 1992. MR 1218230 (94f:57010). Zbl 799.57001.

The signed graph of a link diagram is employed to get an upper bound. **(SGc: Enum)**

1993a *Complexity: Knots, Colourings and Counting*. London Math. Soc. Lect. Note Ser., 186. Cambridge Univ. Press, Cambridge, Eng., 1993. MR 1245272 (94m:-57027). Zbl 799.68008.

Includes very brief treatments of some appearances of signed graphs.

§2.2, "Tait colourings", defines the signed graph of a link diagram, mentioned again in observation (2.3.1) on alternating links and Prop (5.2.16) on "states models" (from Schwärzler and Welsh (1993a)). §5.6, "Thistlethwaite's nontriviality criterion": the criterion depends on the signed graph.

§2.5, "The braid index and the Seifert index of a link", defines the Seifert graph, a signed graph based on splitting the link diagram. **(SGc, Knot)**

§5.7, "Link invariants and statistical mechanics", defines a relatively simple spin model for signed graphs, with an arbitrary finite number of possible spin values. The partition function is related to link diagrams.

§4.2, "The Ising model", introduces the basic concepts in mathematical terms. §6.4, "The complexity of the Ising model", "Computing ground states of spin systems", pp. 105–107, discusses finding a ground state of the Ising model. This is described as the min-weight cut problem with weights the negatives [this is an error] of the Ising bond interaction values: that is, the weighted frustration index problem in the negative [erroneous] of the Ising graph. It is the max-cut problem when the Ising graph is balanced (ferromagnetic) [should be antibalanced (antiferromagnetic)]. For external magnetic field, follows Barahona (1982a). **(sg: State(fr), Fr, Phys)**

§3.6, "Ice models", counts "ice configurations" (certain graph orientations) via poise gains modulo 3, although the counting function is not gain-graphic. **(gg, Invar, Phys)**

§4.4: "The Ashkin–Teller–Potts model". This treatment of the Potts model has a different Hamiltonian from that of Fischer and Hertz (1991a). [It does not seem that Welsh intends to admit edge signs. If they are allowed then the Hamiltonian (without edge weights) is $-\sum \sigma(e_{ij})(\delta(s_i, s_j) -1)$. Up to halving and a constant term, this is Doreian and Mrvar's (1996a) clusterability measure $P(\pi)$, with $\alpha = .5$, of the vertex partition induced by the state.] [Also *cf.* Fischer and Hertz (1991a).] **(clu, Phys)**

1993b The complexity of knots. In: John Gimbel, John W. Kennedy and Louis V. Quintas, eds., *Quo Vadis, Graph Theory?*, pp. 159–171. Ann. Discrete Math., Vol. 55. North-Holland, Amsterdam, 1993. MR 1217989 (94c:57021). Zbl 801.68086.

Link diagrams $\leftrightarrow$ dual pairs of sign-colored plane graphs: based on Yajima and Kinoshita (1957a). Unsolved algorithmic problems about knots based on link diagrams; in particular, triviality of diagrams is equivalent to *Problem* 4.2: A polynomial-time algorithm to decide whether the graphical Reidemeister moves can convert a given signed plane graph to one with edges all of one sign. **(SGc: Du, Knot: Algor, Exp)**

1993c Knots and braids: some algorithmic questions. In: Neil Robertson and Paul Seymour, eds., *Graph Structure Theory* (Proc., Seattle, 1991), pp. 109–123. Contemp. Math., Vol. 147. Amer. Math. Soc., Providence, R.I., 1993. MR 1224698 (94g:57014). Zbl 792.05058.

§1 presents the sign-colored graph of a link diagram and §5, "Reidemeister graphs", describes Schwärtzler and Welsh (1993a). §3 defines the sign-colored Seifert graph. (**SGc. Sc(Matrd): Invar, Algor, Knot: Exp**)

1994a The computational complexity of knot and matroid polynomials. *Discrete Math.* 124 (1994), 251–269. MR 1258858 (95d:57008). Zbl 816.57008.

Pp. 258–259: Sign-colored graph of an alternating link diagram. [Annot. 27 Feb 2017.] (**Knot: SGc: Exp**)

1997a Knots. In: Lowell W. Beineke and Robin J. Wilson, eds., *Graph Connections: Relationships between Graph Theory and other Areas of Mathematics*, Ch. 12, pp. 176–193. The Clarendon Press, Oxford, 1997. MR 1634542 (99a:05001) (book). Zbl 878.57001.

Mostly describes the signed graph of a link diagram and its relation to knot theory, including knot properties deducible directly from the signed graph, the Kauffman bracket and two-variable polynomials, etc. Similar to relevant parts of (1993a). (**SGc: Knot: Invar: Exp**)

Michael Welsh

2011a *Golden-mean and Secret Sharing Matroids.* Master's thesis, Victoria Univ. of Wellington, 2011.

Examples include: Def. 2.0.9: Dowling geometry $Q_n(\mathrm{GF}(3)^times)$. §3.1.2, "Spikes", i.e., $\mathbf{L}_\infty(2C_n, \mathcal{B})$. [Annot. 27 Feb 2017.] (**gg: Matrd: Exp**)

Stefan Weltge

See M. Conforti.

Emo Welzl

See H. Edelsbrunner and J. Hage.

Chao Wen, Qiang Sun, Shunzhe Zhang, Huanhuan Guan, & Chao Zhang

20xxa The homomorphism and colouring of the direct product of signed graphs. Submitted.

The weak Hedetniemi-type conjecture $\chi(\Sigma_1 \times \Sigma_2) \leq \min\{\chi(\Sigma_1), \chi(\Sigma_2)\}$ is disproved. It is valid if Σ_2 is balanced, with $=$ if also $|\Sigma_1|$ maps into $|\Sigma_2|$. [Annot. 5 Feb 2023.] (**SG: Col, Hom**)

Chao Wen, Qiang Sun, Hongyan Cai, & Chao Zhang

20xxa The edge coloring of the Cartesian product of signed graphs. Submitted.

Cf. Behr (2020a). Partial results distinguishing Class 1 from Class 2 in the products. [Annot. 27 Aug 2023.] (**SG: ECol, ori**)

Qin Wen, Qin Zhao, & Huiqing Liu

2015a The least signless Laplacian eigenvalue of non-bipartite graphs with given stability number. *Linear Algebra Appl.* 476 (2015), 148–158. MR 3327137. Zbl

1314.05120. **(Par: Lap: Eig)**

D. de Werra
See C. Benzaken.

Hans V. Westerhoff
See B.N. Kholodenko.

Peter Whalen
See R. Thomas.

Joyce Jiyoung Whang
See K.-Y. Chiang.

Arthur T. White

1984a *Graphs, Groups and Surfaces*. Completely revised and enlarged edn. North Holland Math. Stud., Vol. 8. North-Holland, Amsterdam, 1984. MR 0780555 (86d:05047). Zbl 551.05037.

Ch. 10: "Voltage graphs". **(GG: Top, Cov)**

1994a An introduction to random topological graph theory. *Combinatorics, Probability and Computing* 3 (1994), 545–555. MR 1314074 (95j:05083). Zbl 815.05027.

Take a graph Γ with cyclomatic number k and randomly sign it so that each edge is negative with probability p. The probability that (Γ, σ) is balanced $= 2^{-k}$ if $p = \frac{1}{2}$ [obvious] and $\leq [\max(p, 1-p)]^k$ in general [not obvious] (this has an interesting asymptotic consequence due to Gimbel, given in this paper). [Related: Frank and Harary (1979a).]
(SG: Rand, Bal)

2001a *Graphs of Groups on Surfaces: Interactions and Models*. North-Holland Math. Stud., 188. North-Holland (Elsevier), Amsterdam, 2001. MR 1852593 (2002k:-05001). Zbl 1054.05001.

§10-2, "Voltage graphs": Voltage graphs and the covering graph. Thm. 10-8 is similar to Biggs (1974a), Thm. 19.5. Construction of surface embeddings. §11-3, "Nonorientable voltage graph imbeddings": Rotation schemes supplemented by edge signatures as in Ringel (1977a), Stahl (1978a), and Zaslavsky (1992a). **(GG, SG: Top, Cov)**

2009a Embeddings and geometries. In: Lowell W. Beineke and Robin J. Wilson, eds., *Topics in Topological Graph Theory*, Ch. 12, pp. 245–267. Encycl. Math. Appl., Vol. 128. Cambridge Univ. Press, Cambridge, Eng., 2009. MR 2581549 (no rev). Zbl 1225.05086.

The voltage graph (i.e., gain graph) construction is used to generate embeddings of finite geometries. [Annot. 12 Jun 2013.]
(Top: GG, Cov: Exp)

Jacob A. White

20xxa Burnside chromatic polynomials of group-invariant graphs. *Discuss. Math. Graph Theory* (to appear).

Unlabelled coloring of a gain graph, in the form of its group covering graph, in the more general form of a graph with a group action. Incidentally, develops a renark in Zaslavsky (()009a)Thomas Zaslavsky. [Annot.

24 Nov 2020.] (**GG: Cov, Gen: Col, Invar**)

Neil L. White

See also A. Björner.

1986a A pruning theorem for linear count matroids. *Congressus Numerantium* 54 (1986), 259–264. MR 0885285 (88c:05047). Zbl 621.05009. (**Matrd: Bic: Gen**)

Neil White & Walter Whiteley

† 1983a A class of matroids defined on graphs and hypergraphs by counting properties. Unpublished manuscript, 1983.

See Whiteley (1996a) for an exposition and extension. [Generalization in Pikhurko (2001a).] (**Matrd: Bic: Gen**)

Walter Whiteley

See also K. Clinch and N.L. White.

1988a The union of matroids and the rigidity of frameworks. *SIAM J. Discrete Math.* 1 (1988), no. 2, 237–255. MR 0941354 (89d:05055). Zbl 671.05026. (**gg: Geom, Matrd**)

1991a The combinatorics of bivariate splines. In: Peter Gritzman and Bernd Sturmfels, eds., *Applied Geometry and Discrete Mathematics: The Victor Klee Festschrift*, pp. 587–608. DIMACS Ser. Discrete Math. Theor. Comput. Sci., Vol. 4. American Math. Soc., Providence, R.I., 1991. MR 1116378 (92m:41038). Zbl 741.41014.

"Balance" used for circles with identity gain (in a gain graph with additive matrix gains), independently of Harary (1953a). §3, "Splines and matrices on graphs": The matrix gains are L_{hi}^{r+1} (p. 592) and the balance equation is $(*)$ (p. 593). [Annot. 12 Jun 2012.] (**gg: bal**)

1996a Some matroids from discrete applied geometry. In: Joseph E. Bonin, James G. Oxley, and Brigitte Servatius, eds., *Matroid Theory* (Proc., Seattle, 1995), pp. 171–311. Contemp. Math., Vol. 197. Amer. Math. Soc., Providence, R.I., 1996. MR 1411692 (97h:05040). Zbl 860.05018.

Appendix: "Matroids from counts on graphs and hypergraphs", which expounds and extends Loréa (1979a), Schmidt (1979a), and especially White and Whiteley (1983a), describes matroids on the edge sets of graphs (and hypergraphs) that generalize the bicircular matroid. The definition: given $m \geq 0$ and $k \in \mathbb{Z}$, S is independent iff $\varnothing \subset S' \subseteq S$ implies $\#S' \leq m\#V(S') + k$. [Suggested name: "Linearly bounded matroids," since they are defined by a linear bound on the rank.] (**Matrd: Bic: Gen**)(**Ref**)

Geoff Whittle

See also R. Chen, J. Geelen, D. Mayhew, J. Oxley, and C. Semple.

1989a Dowling group geometries and the critical problem. *J. Combin. Theory Ser. B* 47 (1989), 80–92. MR 1007716 (90g:51008). Zbl 628.05018.

A Dowling-lattice version of Crapo and Rota's critical problem, with new structure of Dowling's (1973b) rank-n matroid $Q_n(\mathfrak{G}) = \mathbf{F}(\mathfrak{G}K_n^\bullet)$ of a finite group $\mathfrak{G}$. §3, "Tangential k-blocks over finite groups": Some minimal submatroids of $Q_n(\mathfrak{G})$ whose critical exponent is k ("tangential k-blocks over $\mathfrak{G}$"), such as $Q_n(\{+,-\})$ and, for $\mathfrak{H} \leq \mathfrak{G}$ and some values of n, $\mathbf{F}(\mathfrak{H}K_n)$ (Thm. 3.8, Cor. 3.9). Thm. 3.10 on structure of $\mathbf{F}(\mathfrak{G}K_n)$. [Annot. 25 May 2009, 13 Oct 2022.] (**GG: Matrd: Invar**)

1989b A generalisation of the matroid lift construction. *Trans. Amer. Math. Soc.* 316 (1989), 141–159. MR 0957084 (90b:05038). Zbl 684.05014.

Examples include bicircular and frame matroids. (**GG: Matrd, Bic**)

2005a Recent work in matroid representation theory. *Discrete Math.* 302 (2005), 285–296. MR 2179649 (2006m:05053). Zbl 1076.05022. annot P. 288: The "free spike Φ_r" is $\mathbf{L}(2C_r, \varnothing)$. Pp. 290–291: Biased graphs and the bias [i.e., frame] matroid. *Conjecture* 5.2: With few exceptions, a highly connected matroid that is representable over more than one characteristic is a frame or dual frame matroid. P. 294: The "free swirl Ψ_k" is $\mathbf{F}(2C_k, \varnothing)$. $U_{3,6} = \mathbf{L}(2C_3, \varnothing) = \mathbf{F}(2C_3, \varnothing)$ [the latter because there are no vertex-disjoint unbalanced circles]. [Annot. 25 May 2009.] (**gg: Matrd: Exp**)

Gábor Wiener

See T. Fleiner.

Avi Wigderson

See S. Hoory.

Chris Wiggins

See E. Ziv.

J.K. Williams

See also B.G.S. Doman.

1981a Ground state properties of frustrated Ising chains. *J. Phys. C* 14 (1981), 4095–4107.

§2, "The random-bond Ising chain in a uniform field: $(T = 0)$": A path with random edge signs, weighted J, magnetic field B [interpretable as an extra all-positive vertex, as in ??]. Continued in Doman and Williams (1982a), §2. [Annot. 28 Aug 2012.] (**Phys, SG, WG: State(fr)**)

David P. Williamson

See A. van Zuylen.

Andrew Timothy Wilson

See E. Leven.

Mark C. Wilson

See S. Aref.

Richard C. Wilson

See P.-L. Giscard.

Robin J. Wilson

See also L.W. Beineke.

Robin J. Wilson & John J. Watkins

1990a *Graphs: An Introductory Approach. A First Course in Discrete Mathematics.* Wiley, New York, 1990. MR 1038804 (91b:05001). Zbl 712.05001.

§3.2: "Social Sciences" (pp. 51–53) applies signed graphs. §5.1: "Signed digraphs" (pp. 96–98) discusses positive and negative feedback (i.e., positive and negative cycles) in applications. Based on Open University

(1981a). **(SG, PsS, SD: Exp)**

Steve Wilson

1989a Cantankerous maps and rotary embeddings of K_n. *J. Combin. Theory Ser. B* 47 (1989), no. 3, 262–273. MR 1026064 (90j:05115). Zbl 687.05018.

Cantankerous map: signed expansion graph $\pm\Gamma$, orientation embedded in a surface, whose map automorphisms act transitively on flags. Rotary map: map with automorphisms that are cyclic permutations around a face and around a vertex on the face. Thm.: A rotary map is either cantankerous or a kind of branched covering. [See C.H. Li and Širáň (2007a) for more on cantankerous maps.] [Annot. rev. 31 Jul 2014.] **(sg: Top: Aut)**

Marco Winkler

See E. Terzi.

Shmuel Winograd

See R.M. Karp.

Wayland H. Winstead

See J.R. Burns.

Anthony Wirth

See M. Charikar and T. Coleman.

Pepijn Wissing & Edwin R. van Dam

2022a Unit gain graphs with two distinct eigenvalues and systems of lines in complex space. *Discrete Math.* 345 (2022), art. 112827, 28 pp. MR 4379244. arXiv:-2105.09149. **(gg: Geom, Adj: Eig)**

2022b Spectral fundamentals and characterizations of signed directed graphs. *J. Combin. Theory Ser. A* 187 (2022), art. 105573, 40 pp. MR 4352598. Zbl 1480.05087. arXiv:2009.12181. **(SG: GG: Adj: Eig)**

H.S. Witsenhausen

See Y. Gordon.

C. Witzgall & C.T. Zahn, Jr.

1965a Modification of Edmonds' maximum matching algorithm. *J. Res. Nat. Bur. Standards (U.S.A.) Sect. B* 69B (1965), 91–98. MR 0188107 (32 #5548). Zbl 141.21901. **(par: ori)**

Jakub Onufry Wojtaszczyk

See M. Cygan.

W.F. Wolff

See also P. Hoever and M.H. Waldor.

W.F. Wolff & J. Zittartz

1982a Correlations in inhomogeneous Ising models. I. General methods, the "fully-frustrated square lattice" and the "chessboard" model. *Z. Phys. B* 47 (1982), no. 4, 341–352. MR 0675258 (85d:82104).

§ III, "The fully-frustrated square lattice model (FFS)": Square lattice graph signed so every square ("plaquette") is negative. § IV, "The chess-board model": Square lattice graph with alternate squares negative and positive. [Annot. 28 Aug 2012.] **(Phys, SG: State(fr))**

1983a Spin glasses and frustration models: Analytical results. In: J.L. van Hemmen and I. Morgenstern, eds., *Heidelberg Colloquium on Spin Glasses* (Proc., Heidelberg, 1983), pp. 252–271. Lect. Notes in Physics, Vol. 192. Springer-Verlag, Berlin, 1983.

Early theoretical physics study of frustrated graphs based on Toulouse (1977a). Signed square lattice with translational sign symmetry and limited variation of signs and edge weights. § II, "Layered Ising models". Dictionary: "plaquette" = square, "frustration index" = sign of a plaquette. [Annot. 24 May 2012.] (**Phys, SG: State(fr)**)

Paul Wollan

See B. Guenin, T. Huynh, and K. Kawarabayashi.

Dein Wong

See X.B. Ma, F.L. Tian, S. Wang, and F. Xu.

Tsai-Lien Wong

See L.-G. Jin and H. Qi.

R. Kevin Wood

See G.G. Brown.

Bartłomiej Wróblewski

See R. Janczewski.

Bang Ye Wu

See also C.-H. Bai, J.-F. Chen, L.-H. Chen, and P.-S. Wei.

2013b Balancing a complete signed graph by editing edges and deleting nodes. In: R.-S. Chang *et al.*, eds., *Advances in Intelligent Systems and Applications* (Proc. ICS 2012, Hualien, Taiwan, 2012), Vol. 1, pp. 79–88. Smart Innovation, Systems and Technologies, Vol. 20. Springer, Berlin, 2013. (**SG: KG: Fr: Algor**)

Bang Ye Wu & Li-Hsuan Chen

2015a Parameterized algorithms for the 2-clustering problem with minimum sum and minimum sum of squares objective functions. *Algorithmica* 72 (2015), 818–835. MR 3355837. Zbl 1328.68098.

Equivalent: Find minimizing bipartition for (K_n, σ). Objective: minimize $\sum_v c(v)$ (thus finding $l(K_n, \sigma)$) or $\sum_v c(v)^2$.

Dictionary: "graph" = positive subgraph of (K_n, σ); "editing" = edge sign changes; "2-clustering" = bipartition $V = X \cup Y$, "conflict number" $c(x)$, for $x \in X$ (say), $= \#(\text{edges } {+xx'}) + \#(\text{edges } {-xy})$. [Annot. 13 Jun 2017.] (**sg: kg: Fr(Gen): Algor**)

Baofeng Wu

See P.-K. Zhang.

Chai Wah Wu

2005a On Rayleigh–Ritz ratios of a generalized Laplacian matrix of directed graphs. *Linear Algebra Appl.* 402 (2005), 207–227. MR 2141085 (2005m:05108). Zbl 1063.05065.

The graphs are weighted mixed graphs, i.e., bidirected graphs without introverted edges, and the matrices are digraph matrices, i.e., (weighted) outdegree matrices. The "Laplacian" is $D - A$ where A is the adjacency matrix and D is the diagonal outdegree matrix. [Annot. 23 Mar 2009.] (**sg, sd: ori: incid, Eig**)

Chong Wu

See D. Li.

Guangmei Wu

See C.-C. Huang.

Jianliang Wu

See X.Q. Qi.

Jianshe Wu, Licheng Jiao, Chao Jin, Fang Liu, Maoguo Gong, Ronghua Shang, & Weisheng Chen

2012a Overlapping community detection via network dynamics. *Phys. Rev. E* 85 (2012), art. 016115, 7 pp.

Detects possibly overlapping clusters in Γ via (K_n, σ) with $E^+ = E(\Gamma)$. Each vertex gets a randomly phased oscillator. Edge weights $w_+ > 0$, $w_- \geq 0$. Oscillators $\to$ in-phase on $+$ edges, out-of-phase on $-$ edges, revealing clusters. [Annot. 16 Jun 2018.] (**sg: kg, WG: Clu: Dyn**)

Jianshe Wu, Long Zhang, Yong Li, & Yang Jiao

2016a Partition signed social networks via clustering dynamics. *Physica A* 443 (2016), 568–582. MR 3416828. Zbl 1400.91496. (**SG: Clu: Algor**)

Jingwen Wu

See Y. Lu.

Leting Wu, Xintao Wu, Aidong Lu, & Yuemeng Li

2014a On spectral analysis of signed and dispute graphs. In: *2014 IEEE International Conference on Data Mining*, pp. 1049–1054. IEEE, 2014.

Extended abstract of (2017a). (**SG: Clu: Adj: Eig, Geom**)

2017a On spectral analysis of signed and dispute graphs: Application to community structure. *IEEE Trans. Knowledge Data Engineering* 29 (2017), no. 7, 1480–1493. (**SG: Clu: Adj: Eig, Geom**)

Leting Wu, Xiaowei Ying, Xintao Wu, Aidong Lu, & Zhi-Hua Zhou

2011a Spectral analysis of k-balanced signed graphs. In: Joshua Zhexue Huang, Longbing Cao and Jaideep Srivastava, eds., *Advances in Knowledge Discovery and Data Mining* (Proc. 15th Pacific-Asia Conf., PAKDD 2011, Shenzen, Part II), pp. 1–12. Lect. Notes in Computer Sci., Vol. 6635. Springer, Berlin, 2011.

Spectral analysis of clusterable signed graphs. Dictionary: "k-balance" = k-clusterability. [Annot. 26 Apr 2012.] (**SG: Clu Eig**)

20xxa Examining spectral space of complex networks with positive and negative links. *Int. J. Social Network Mining* (to appear). (**SG: Eig: Clu Bal**)

Lifang Wu

See W.-C. Liu.

Qi Wu

See also Y. Lu.

Qi Wu, Yong Lu, & Bit-Shun Tam

2022a On connected signed graphs with rank equal to girth. *Linear Algebra Appl.* 651 (2022), 90–115. MR 4445167. Zbl 1493.05148. **(SG: Adj)**

Qiang Wu
See G.Z. Liu.

Shu-Hui Wu
See B.S. Tam.

Sun Wu
See W.-S. Shih.

Xiao Li Wu, Jing Jing Jiang, Ji Ming Guo, & Shang Wang Tan
2011a The minimal signless Laplacian spectral radius of graphs with diameter $n-4$. (In Chinese.) *Acta Math. Sinica (Chin. Ser.)* 54 (2011), no. 4, 601–608. MR 2868198 (2012i:05176). Zbl 1265.05423. **(par: Lap: Eig)**

Xintao Wu
See Y.-M. Li and L.T. Wu.

Yaping Wu, Qiong Fan, Huiqing Liu, & Weisheng Zhao
2024a On the skew spectral moments of trees with a given bipartition. *Trans. Combin.* 13 (2024), no. 2, 127–136.
Gain group $\{\pm 1, \pm \mathrm{i}\}$ implicit in skew adjacency matrix. **(gg: Adj: Eig)**

Yarong Wu
See G.L. Yu.

Yanzhi Wu, Lu Liu, Jiangping Hu, & Gang Feng
2018a Adaptive antisynchronization of multilayer reaction–diffusion neural networks. *IEEE Trans. Neural Networks Learning Sys.* 29 (2018), no. 4, 807–818. MR 3789355.
Multilayer signed digraph. **(SD, sg: Bal, Dyn)**

Yezhou Wu & Dong Ye
2018a Circuit covers of cubic signed graphs. *J. Graph Theory* 89 (2018), 40–54. MR 828127. Zbl 1398.05101. arXiv:1609.03620. **(SG: Str)**

2020a Minimum T-Joins and signed-circuit covering. *SIAM J. Discrete Math.* 34 (2020), no. 2, 1192–1204. MR 4101365. Zbl 1441.05098. arXiv:1803.03696. **(SG: matrd)**

Yezhou Wu, Dong Ye, Wenan Zang, & Cun-Quan Zhang
2014a Nowhere-zero 3-flows in signed graphs. *SIAM J. Discrete Math.* 28 (2014), no. 3, 1628–1637. MR 3264564. Zbl 1408.05069. **(SG: Flows)**

Yuhan Wu
See S.Y. Yi and L.H. You.

Zhaoyang Wu
2003a On the number of spikes over finite fields. *Discrete Math.* 265 (2003), 261–296. MR 1969378 (2004b:05057). Zbl 1014.05015.
A spike is $\mathbf{L}_\infty(\Omega)$ where $\|\Omega\| = 2C_n$. **(gg: Matrd: Enum)**

Donald C. Wunsch
See Harary, Lim, *et al.*

Chengyi Xia

See S.-S. Feng.

Wen Xia

See X.-L. Li.

Kai-nan Xiang

See R. Chen.

Bai Xiao, Song Yi-Zhe, & Peter Hall

2011a Learning invariant structure for object identification by using graph methods. *Computer Vision Image Understanding* 115 (2011), 1023–1031.

Empirical tests of usefulness of the eigenvalues (the "feature vector") of $L(-\Gamma)$. [Annot. 24 Jan 2012.] (**Par: Eig: Appl**)

Min Xiao

See S.-D. Zhai.

Peng Xiao

See Y. Lu.

Mengmeng Xie & Chuixiang Zhou

2020a Signed circuit cover of bridgeless signed graphs. *Graphs Combin.* 36 (2020), 1423–1443. MR 4148421. Zbl 1458.05101. (**SG: Matrd**)

Rundan Xing & Bo Zhou

2015a Laplacian and signless Laplacian spectral radii of graphs with fixed domination number. *Math. Nachrichten* 288 (2015), no. 4, 476–480. MR 3320461. Zbl 1396.05080. (**par: Lap: Eig**)

Zhuang Xiong

See also L. Ou.

20xxa The nullity of the net Laplacian matrix of a signed graph. Submitted. arXiv:-2310.12784. (**SG: Lap(Gen)**)

Zhuang Xiong & Yaoping Hou

2022a Eigenvalue-free interval for Seidel matrices of threshold graphs. *Applied Math. Comput.* 427 (2022), art. 127177. MR 4411535.

Esp., $(-\sqrt{2}, \sqrt{2})$ contains no eigenvalues of $A(\Sigma)$ other than ± 1. [*Cf.* Milica Anđelić, Tamara Koledin, & Zoran Stanić (2019a).] [Annot. 20 Jun 2022.] (**sg: KG: Adj: Eig**)

Baogen Xu

See also S.-C. Li.

2001a On signed edge domination numbers of graphs. *Discrete Math.* 239 (2001), 179–189. MR 1850997 (2002e:05104). Zbl 979.05081.

Let $s := \sum_e \sigma(e) = \#E^+ - \#E^-$ and $s_e := \sum\{\sigma(f) : f \in N[e]\}$. The signed edge domination number of Γ is $\min\{s : \sigma \text{ s.t. } s_e > 0,\ \forall e\}$. Question: What is $g(n) := \min\{s : \#V = n\}$? *Cf.* Akbari, Bolouki, *et al.* (2009a). [Annot. 17 Feb 2011.] (**SGw**)

2009a On signed cycle domination in graphs. *Discrete Math.* 309 (2009), no. 4, 1007–1012. MR 2502161 (2010e:05236). Zbl 1180.05082.

See Vijayakumar (2011a). [Annot. 10 Feb 2013.] **(SGw)**

Brian Xu

See L.J. Rusnak.

Dachuan Xu

See S. Ji.

Feng Xu, Qi Zhou, Dein Wong, & Fenglei Tian

2020a Complex unit gain graphs of rank 2. *Linear Algebra Appl.* 597 (2020), 155–169. MR 4080071. Zbl 1437.05157.

Rank of $A(\Phi)$. **(GG: Adj)**

Guang-Hui Xu

See S.C. Gong.

Guangjun Xu

See H.-Y. Cai.

Jing Xu

See B.-J. Yuan.

Meiling Xu

See G.L. Yu.

Rongxing Xu

See F. Foucaud and R. Naserasr.

Rui Xu & Cun-Quan Zhang

2005a On flows in bidirected graphs. *Discrete Math.* 299 (2005), 335–343. MR 2168714 (2006e:05081). Zbl 1073.05033.

Σ has a nowhere-zero 6-flow if it is coloop-free and edge 6-connected, thus solving Bouchet's (1983a) conjecture in this case. [Annot. 5 Feb 2010.] **(SG: Flows)**

Shaoji Xu

See also F.S. Roberts.

1998a *Cycle Space: Cycle Bases, Signed Graphs and Marked Graphs.* Doctoral dissertation, Rutgers Center for Operations Research, Rutgers University, 1998. **(SG, VS: Bal, fr, Algor, PsS)**

1998b The line index and minimum cut of weighted graphs. *European J. Operational Res.* 109 (1998), no. 3, 672–685. Zbl 972.05026. **(WG, SG: fr: Algor)**

Wei-Ru Xu

See Y. Lu.

Yuan Xu

See D. Peng.

Zhi-Ming Xu

See D. Li.

Dong XueDong Xue, Sandra Hirche, & Ming Cao

2020a Evolution of social power over influence networks containing antagonistic interactions. *Inform. Sci.* 540 (2020), 449–468. MR 4124416. Zbl 1479.91287.

(SG: Dyn)

Jie Xue

See R.-F. Liu.

Sandeep Kumar Yadav

See B. Adhikari.

Takeshi Yajima & Shin'ichi Kinoshita

1957a On the graphs of knots. *Osaka Math. J.* 9 (1957), 155–163. MR 0098385 (20 #4845). Zbl 080.17002 (80, p. 170b).

Examines the relationship between the two dual sign-colored graphs, Σ and Σ', of a link diagram (Bankwitz (1930a)), translating the Reidemeister moves into graph operations and showing that they will convert Σ into Σ'. **(SGc: Knot)**

Takeo Yamada

1988a A note on sign-solvability of linear system of equations. *Linear Multilinear Algebra* 22 (1988), no. 4, 313–323. MR 0943759 (89g:15005). Zbl 723.05111. **(QM: QSol: SD)**

Takeo Yamada & Harunobu Kinoshita

2002a Finding all the negative cycles in a directed graph. *Discrete Appl. Math.* 118 (2002), 279–291. MR 1892974 (2002m:05187). Zbl 999.05057.

In a real-weighted digraph, "negative" means the sum of weights is negative. **(WG)**

Yuuzi Yamada

See O. Nagai.

Chiaki Yamaguchi

2013a Conjectured exact percolation thresholds of the Fortuin–Kasteleyn cluster for the $\pm J$ Ising spin glass model. *Physica A* 392 (2013), no. 6, 1263–1268. MR 3019857 (no rev). arXiv:1004.0654. **(Phys: SG)**

2014a Analytical estimates of the locations of phase transition points in the ground state for the bimodal Ising spin glass model in two dimensions. *Progress Theor. Exper. Phys.* 2014 (2014), art. 053A02, 9 pp. arXiv:1309.6554. **(Phys: SG, State)**

Yutaro Yamaguchi

See Y. Kawase.

Takeo Yamamoto

See T. Nakamura.

Chao Yan

See G.L. Yu and L.Q. Wang.

Eric Yan

See L.J. Rusnak.

Jing-Ho Yan, Ko-Wei Lih, David Kuo, & Gerard J. Chang

1997a Signed degree sequences of signed graphs. *J. Graph Theory* 26 (1997), 111–117. MR 1469358 (98i:05160). Zbl 980.04848.

Net degree sequences of signed simple graphs. Thm. 2 improves the Havel–Hakimi-type theorem from Chartrand, Gavlas, Harary, and

Schultz (1994a) by determining the length parameter. Thm. 7 characterizes the net degree sequences of signed trees. [There seems to be room to strengthen the characterization and generalize to weighted degree sequences: see notes on Chartrand, Gavlas, *et al.* (1994a).]
(**SGw: ori: Invar**)

Zhidan Yan
See W. Wang.

Aimei Yang & Josh Bentley
2017a A balance theory approach to stakeholder network and apology strategy. *Public Relations Rev.* 43 (2017), no. 2, 267–277. (**PsS: SG: Bal**)

Alex Yang
See V. Chen.

Arthur L.B. Yang
See W.Y.C. Chen.

Bo Yang, William K. Cheung, & Jiming Liu
2007a Community mining from signed social networks. *IEEE Trans. Knowledge Data Engineering* 19 (2007), no. 10, 1333–1348.

An algorithm for an approximate clustering of a (weighted) signed (di)graph. Input: The graph and a length parameter l. Step 1: Construct transition probabilities $pij := [\sigma ij w_{ij}]^{+}/d(v_i)$. Step 2: Apply the probabilities in a random walk of length $\leq l$ on positive edges; the matrix of l-step probabilities is $(p_{ij})^l$. Combine in a cluster the vertices that have high probabilities from a given starting point. "High" and l are based on the network structure.

Also, a cut algorithm for approximate clustering. A cluster is $X \subset V$ such that the total net degree $d^{\pm}(\Sigma{:}X) \geq d^{\pm}(X, X^c)$ and $d^{\pm}(X^c, X) \leq d^{\pm}(\Sigma{:}X^c)$. [Annot. 11 Feb 2009.] (**SG: WG: Clu: Algor**)

Bo Yang, Xueyan Liu, Yang Li, & Xuehua Zhao
2017a Stochastic blockmodeling and variational Bayes learning for signed network analysis. *IEEE Trans. Knowledge Data Eng.* 29 (2017), no. 9, 2026–2039. (**SG: Clu**)

Bo Yang, Xuehua Zhao, & Xueyan Liu
2015a Bayesian approach to modeling and detecting communities in signed network. In: *Proceedings of the Twenty-Ninth AAAI Conference on Artificial Intelligence* (AAAI-15, Austin, 2015), pp. 1952–1958. AAAI Press, Palo Alto, Calif., 2015. URL https://www.aaai.org/ocs/index.php/AAAI/AAAI15/paper/view/9376 (**SG: Clu**)

Dan Yang
See Y.Z. Fan.

Fan Yang & Sanming Zhou
2016a Nowhere-zero 9-flows in 3-edge-connected signed graphs. Manuscript, 2016. arXiv:1508.04620.

Withdrawn due to incomplete proof. [Can someone complete it?] [Annot. 20 Jul 2020.] **(SG: Flows)**

Laurence T. Yang

See F. Hao.

Ruixian Yang

See C.-C. Huang.

Shiju Yang, Chuandong Li, Xiping He, & Wanli Zhang

2022a Variable-time impulsive control for bipartite synchronization of coupled complex networks with signed graphs. *Appl. Math. Computation* 420 (2022), art. 126899, 16 pp. MR 4361759. **(SD: Bal: Dyn)**

Shiju Yang, Chuandong Li, Yu Li, Ting Yang, & Bo Li

20xxa The fast fixed-time bipartite synchronization of coupled delayed neural networks with signed graphs. Submitted.

$\{V_1, V_2\}$ = Harary bipartition of balanced signed digraph $\vec{\Sigma}$. Nodes v_i have states $z_i(t) \in \mathbb{R}^m$ for time $t \geq 0$. Arc weights and a control function govern the evolution of z. "Bipartite synchronization" means $\exists\ y(t),\ t_0$ for which $z_i(t) = y(t), -y(t)$ when $v_i \in V_1, V_2$ respectively, $\forall\ t > t_0$. [One of many papers on the topic.] [Annot. 30 Jan 2022.] **(SD: Bal: Dyn)**

Shuang Hong Yang, Alex Smola, Bo Long, Hongyuan Zha, & Yi Chang

2012a Friend or frenemy? Predicting signed ties in social networks. In: *Proceedings of the 35th International ACM SIGIR Conference on Research and Development in Information Retrieval* (SIGIR '12), pp. 555–564. ACM, New York, 2012. **(SG: PsS)**

Ting Yang

See S.-C. Li and S.-J. Yang.

Weiling Yang & Fuji Zhang

2007a The Kauffman bracket polynomial of links and universal signed plane graph. In: Jin Akiyama *et al.*, eds., *Discrete Geometry, Combinatorics and Graph Theory* (7th China-Japan Conf., CJCDGCGT 2005, Tianjin and Xi'an, China, 2005), pp. 228–244. Lect. Notes in Computer Sci., Vol. 4381. Springer, Berlin, 2007. MR 2364767 (2009b:57031). Zbl 1149.05308.

The "chain polynomials" of sign-colored plane graphs with cyclomatic number ≤ 5 are obtained systematically. [*Cf.* Jin and Zhang (2005a), (2007a).] [Annot. 5 July 2009.] **(SGc: Invar)**

Xiuwen Yang & Ligong Wang

2021a Iota energy orderings of bicyclic signed digraphs. *Trans. Combin.* 10 (2021), no. 3, 187–200. MR 4257184. arXiv:2004.01412.

Cf. Hafeez and Khan (2018a). **(SD: Adj: Eig)**

Yujun Yang & Dong Ye

2018a Inverses of bipartite graphs. *Combinatorica* 38 (2018), no. 5, 1251–1263. MR 3884787. Zbl 1424.05127. arXiv:1611.06535.

Elegant solution to problem of Godsil (1985a): Iff Γ excludes an "odd flower". Dictionary: "diagonally similar to a non-negative matrix" =

balanced. *Question.* Is the answer the same if Γ is allowed to be a signed graph? [Annot. 11 Dec 2018.] **(sg: Adj, Bal, sw)**

Mihalis Yannakakis

See also V.V. Vazirani.

1985a On a class of totally unimodular matrices. *Math. Operations Res.* 10 (1985), 280–304. MR 0793885 (87h:90245). Zbl 565.90042.

"Restricted totally unimodular matrices": Bipartite. Structural characterization by decomposition into incidence matrices (and transposes) of balanced bidirected graphs. **(sg: Str)**

Structure of "restricted balanceable graphs", defined as bipartite and signable so a circle is positive iff it is evenly even. [Annot. 19 Jan 2015.] **(SGw, sg: Str)**

Milhalis Yannakakis [Mihalis Yannakakis]

See Mihalis Yannakakis.

Yan Hong Yao

See L. Feng.

Yuan Yao

See Z.-L. Jiang.

Zahra Yarahmadi

2010a The bipartite edge frustration of extension of splice and link graphs. *Appl. Math. Lett.* 23 (2010), no. 9, 1077–1081. MR 2659141 (2011e:05219). Zbl 1210.05115.

Dictionary: "bipartite edge frustration" of Γ = frustration index $l(-\Gamma)$. **(par: Fr)**

Zahra Yarahmadi & Ali Reza Ashrafi

2011a The bipartite edge frustration of graphs under subdivided edges and their related sums. *Computers Math. Appl.* 62 (2011), no. 1, 319–325. MR 2821848 (no rev). Zbl 1228.05132. **(par: Fr)**

2011b Extremal properties of the bipartite vertex frustration of graphs. *Appl. Math. Lett.* 24 (2011), 1774–1777. MR 2812210 (2012h:05163). Zbl 1234.05134.

Dictionary: "bipartite vertex frustration" of Γ = frustration number $l_0(-\Gamma)$. **(par: Fr)**

Z. Yarahmadi, T. Došlić, & A.R. Ashrafi

2010a The bipartite edge frustration of composite graphs. *Discrete Appl. Math.* 158 (2010), no. 14, 1551–1558. MR 2659170 (2011g:05301). Zbl 1215.05094. **(par: Fr)**

Geva Yashfe

See L. Kühne.

T. Yasuda

2015a *Inferring Chromosome Structures With Bidirected Graphs Constructed From Genomic Structural Variations.* Ph.D. Thesis, University of Tokyo, 2015. **(sg: Ori: Biol)**

Stephen S. Yau

See F. Hao.

Dong Ye

See E.L. Wei, Y.-J. Yang, Y.-Z. Wu, and L. Zhang.

Miao-Lin Ye, Yi-Zheng Fan, & Hai-Feng Wang

See also J. Sheng.

2010a Maximizing signless Laplacian or adjacency spectral radius of graphs subject to fixed connectivity. *Linear Algebra Appl.* 433 (2010), no. 6, 1180–1186. MR 2661684 (2011i:05142). Zbl 1207.05125. (**par: Lap: Eig**)

Peter Ye

See L.J. Rusnak.

Yeong-Nan Yeh

See I. Gutman, Y.-T. Jiang, S.L. Lee, and Y. Wang.

Aylin Yener

See B. Guler.

Anders Yeo

See N. Alon and G. Gutin.

Shu Yong Yi & Li Hua You

See also L.H. You.

2011a The local bases and bases of primitive anti-symmetric signed digraphs with no loops. *J. South China Normal Univ. Natur. Sci. Ed.* 2011 (2011), no. 1, 39–42. MR 2839257 (no rev). Zbl 1249.05262. (**SD: Adj, qm**)

Shuyong Yi, Lihua You, & Yuhan Wu

2012a The bases and base set of primitive symmetric loop-free signed digraphs. *J. Math. Res. Appl.* 32 (2012), no. 3, 313–326. MR 2985369. Zbl 1265.05287. (**SD, Dyn, qm**)

Xiaowei Ying

See L.T. Wu.

Xuerong Yong

See X.G. Liu and Y.P. Zhang.

En Sup Yoon

See G. Lee.

Yeoin Yoon

See J.-R. Kim.

Young-Jin Yoon

1997a A characterization of supersolvable signed graphs. *Commun. Korean Math. Soc.* 12 (1997), 1069–1073. MR 1643925 (99j:05165). Zbl 945.05051.

An attempt to characterize supersolvability of $\mathbf{F}(\Sigma)$ in terms of [bias-]simplicial vertices. [Fundamental conceptual and technical errors vitiate the entire paper; see Koban (2004a). For correct results see Zaslavsky (2001a) and Koban (2004a).] (**SG: Matrd: Str**)

Yusuke Yoshie

See E. Segawa.

Takeo Yoshikawa, Takashi Iino, & Hiroshi Iyetomi

2011a Market structure as a network with positively and negatively weighted links. In: Junzo Watada, Gloria Phillips-Wren, Lakhmi C. Jain, and Robert J. Howlett, eds., *Intelligent Decision Technologies* (Proc. 3rd Int. Conf., IDT'2011), pp. 511–518. Smart Innovation, Systems and Technologies, Vol. 10. Springer-Verlag, Berlin, 2011.

Preliminary report of (2012a). [Annot. 26 Jun 2012.]

(**SG, WG: Clu: Appl**)

2012a Observation of frustrated correlation structure in a well-developed financial market. *Progress Theor. Phys.* Suppl. No. 194 (2012), 55–63.

Application of correlation clustering to the Tokyo stock market. The "frustration" of a clustering $\pi = \{B_1, \ldots, B_k\} \in \Pi_V$ in a weighted signed graph (Σ, w) is $F(\pi) := -\sum_i \sum_{e \in E:B_i} w_e$ (*cf.* Traag and Bruggeman (2009a)). [Annot. 26 Jun 2012.] (**SG, WG: Clu: Appl**)

Huazheng You

See W.-Z. Liu.

Lihua You

See also F. Cheng, Y. Liu, L.Q. Wang,and S.Y. Yi.

Lihua You, Jiayu Shao, & Haiying Shan

2007a Bounds on the bases of irreducible generalized sign pattern matrices. *Linear Algebra Appl.* 427 (2007), 285–300. MR 2351360 (2008g:15009). Zbl 1179.15034.

(**QM: SD**)

Lihua You & Yuhan Wu

2011a Primitive non-powerful symmetric loop-free signed digraphs with given base and minimum number of arcs. *Linear Algebra Appl.* 434 (2011), no. 5, 1215–1227. MR 2763581 (2012c:05201). Zbl 1204.05051. (**SD: qm**)

Lihua You & Shuyong Yi

2012a The characterization of primitive symmetric loop-free signed digraphs with the maximum base. *Electronic J. Linear Algebra* 23 (2012), 122–136. MR 2889576. Zbl 1252.05078. (**SD: Adj, QM**)

Zhifu You

See also B.L. Liu.

Zhifu You & Bolian Liu

2011a The signless Laplacian separator of graphs. *Electronic J. Linear Algebra* 22 (2011), 151–160. MR 2781043 (2012a:05209). Zbl 1226.05174. (**par: Lap: Eig**)

A.P. Young

See R.N. Bhatt, K. Binder and M. Palassini.

Michael Young

See M. Beck.

J.W.T. Youngs

1968a Remarks on the Heawood conjecture (nonorientable case). *Bull. Amer. Math. Soc.* 74 (1968), 347–353. MR 0220623 (36 #3675). Zbl 161.43303.

Introducing "cascades": current graphs with bidirected edges. A "cascade" is a bidirected graph, not all positive, that is provided with both a rotation system (hence it is orientation embedded in a surface) and a current (which is a special kind of bidirected flow). Dictionary: "broken" = negative edge. [Also see Ringel (1974a).] **(sg: Ori: Appl, Flows)**

1968b The nonorientable genus of K_n. *Bull. Amer. Math. Soc.* 74 (1968), 354–358. MR 0220624 (36 #3676). Zbl 161.43304.
"Cascades": see (1968a). **(sg: Ori: Appl)**

Aimei Yu
See S.J. He G.J. Li, and W.J. Zhang.

Cheng-Ching Yu
See C.C. Chang.

Guanglong Yu, Shuguang Guo, & Meiling Xu
2013a On the least signless Laplacian eigenvalues of some graphs. *Electronic J. Linear Algebra* 26 (2013), no. 1, art. 37, 560–573. MR 3104546. Zbl 1282.05162.
The graphs minimizing $\lambda_{\min}(L(-\Gamma))$ given n and the matching number μ or edge cover number. Thm. 3.5 finds the (few) connected, unbalanced, antibalanced signed graphs $-\Gamma$ with given n, μ that minimize $\lambda_{\min}(L(-\Gamma))$. [*Problem.* Generalize to connected, unbalanced signed graphs.] [Annot. 20 Jan 2015.] **(par: Lap: Eig)**

Guanglong Yu, Zhengke Miao, Chao Yan, & Jinlong Shu
2013a Gaps in the base set of primitive nonpowerful sign patterns. *Linear Multilinear Algebra* 61 (2013), no. 6, 801–810. MR 3005658. Zbl 1273.05136. **(SD, QM)**

Guanglong Yu, Yarong Wu, & Jinlong Shu
2011a Signless Laplacian spectral radii of graphs with given chromatic number. *Linear Algebra Appl.* 435 (2011), no. 8, 1813–2096. MR 2810629 (2012e:05247). Zbl 1221.05244. **(par: Lap: Eig)**

2011b Sharp bounds on the signless Laplacian spectral radii of graphs. *Linear Algebra Appl.* 434 (2011), no. 3, 683–687. MR 2746075 (2012e:05246). Zbl 1225.05178. **(par: Lap: Eig)**

Gui-Dong Yu, Yi-Zheng Fan, & Yi Wang
2014a Quadratic forms on graphs with application to minimizing the least eigenvalue of signless Laplacian over bicyclic graphs. *Electronic J. Linear Algebra* 27 (2014), no. 1, art. 13, 213–236. MR 3194952. Zbl 1288.05168. **(par: Lap: Eig)**

Guihai Yu
See also L.H. Feng and L. Lu.

2008a On the maximal signless Laplacian spectral radius of graphs with given matching number. *Proc. Japan Acad. Ser. A Math. Sci.* 84 (2008), no. 9, 163–166. MR 2483600 (2009m:15012). Zbl 1175.05090. **(par: Lap: Eig)**

Guihai Yu, Matthias Dehmer, Frank Emmert-Streib, & Herbert Jodlbauer
2019a Hermitian normalized Laplacian matrix for directed networks. *Inform. Sci.* 495 (2019), 175–184. MR 3948704. Zbl 1451.05150.
It is a normalized Laplacian for digraph gains given by 1 if $e_{ij}, e_{ji} \in E$, gain i if only $e_{ij} \in E$. [*Cf.* Liu and Li (2015a), Guo and Mohar (2017a).] Results about eigenvalues, e.g., symmetry, special values 0 and 2, edge

interlacing, and expressions for coefficients of characteristic polynomial. [Annot. 15 Dec 2020.] **(gg: Lap: Eig)**

Guihai Yu, Lihua Feng, Aleksandar Ilić, & Dragan Stevanović

2015a The signless laplacian spectral radius of bounded degree graphs on surfaces. *Appl. Anal. Discrete Math.* 9 (2015), 332–346. MR 3444911. Zbl 1464.05258. **(sg: par: Lap: Eig, Top)**

Guihai Yu, Lihua Feng, & Hui Qu

2016a Signed graphs with small positive index of inertia. *Electronic J. Linear Algebra* 31 (2016), art. 18, 232–243. MR 3504407. Zbl 1339.05173. **(SG: Adj: Eig)**

Guihai Yu, Lihua Feng, Qingwen Wang, & Aleksandar Ilić

2014a The minimal positive index of inertia of signed unicyclic graphs. *Ars Combin.* 117 (2014), 245–255. MR 3243844. Zbl 1349.05153. **(SG: Adj: Eig)**

Guihai Yu, Xin Liu, & Hui Qu

2017a Singularity of Hermitian (quasi-)Laplacian matrix of mixed graphs. *Appl. Math. Comput.* 293 (2017), 287–292. MR 3549669 (no rev). Zbl 1411.05176.

Φ with gain group $\{\pm 1, \pm \mathrm{i}\}$ ($\varphi(e) = 1$ for undirected, i for directed edges). $L(\Phi)$ and $L(-\Phi)$ are positive semidefinite and singularity is characterized [*cf.* Guihai Yu & Hui Qu (2015a)]. Formula for determinant. [Annot. 15 Dec 2020.] **(gg: Lap)**

Guihai Yu & Hui Qu

2015a Hermitian Laplacian matrix and positive of mixed graphs. *Appl. Math. Comput.* 269 (2015), 70–76. MR 3396760 (no rev). Zbl 1410.05145.

Based on Liu and Li (2015a), Guo and Mohar (2017a), and Harary (1953a). §2, "The positive of mixed graphs". Thms. 2–3: For complete $\|\Phi\|$ [an unnecessary restriction], Φ is balanced iff $V = V_1 \cup V_2$ so all edges in V_i are positive and all V_1V_2 edges have gain i [note: $\nexists$ negative edges], iff all triangles are balanced. Thm. 4: Φ is balanced iff all uv-paths have the same gain. §3, "Hermitian Laplacian matrix of mixed graphs". Switching equivalence. Incidence matrix [special case of Zaslavsky (2003b), §2]. Laplacian matrix from incidence matrix, hence positive semidefinite (Thm. 9). Elementary eigenvalue properties and nice bounds [compare Reff (2012a)] for A and L. Dictionary: "positive" = balanced. [Annot. 15 Dec 2020.] **(gg: Bal, Adj, Incid, Lap: Eig)**

2018a More on spectral analysis of signed networks. *Complexity* 2018 (2018), art. 3467158, 6 pp. Zbl 1407.05154.

Spectral symmetry around 1 and meaning of eigenvalue 2 of normalized Laplacian matrix.

Major error: Their incidence matrix S is that of $-\Sigma$, i.e., $\mathrm{H}(-\Sigma)$ (Zaslavsky (1982a)); thus their Laplacian $L(\Sigma)$ is really $L(-\Sigma)$. Thm. 2: For connected Σ, $L(\Sigma)$ is singular iff all uv-walks have the same sign [I.e., Σ is balanced; known from Zaslavsky (1982a) in matroid form]. Wrong proof; the calculation reverses signs. Their Laplacian should be $L(-\Sigma)$ throughout. Further results are known or wrong; e.g., Thm. 9 to the extent correct is Zaslavsky (1982a), Thm. 8A.4. [The new results can and should be corrected.] [Annot. 10 Nov 2018.] (**SG: Lap: Eig**)

Guihai Yu, Hui Qu, & Jianhua Tu

2015a Inertia of complex unit gain graphs. *Appl. Math. Comput.* 265 (2015), 619–629. MR 3373510 (no rev). Zbl 1410.05174. (**GG: Adj: Eig**)

Guihai Yu, Xiying Yuan, & Hui Qu

2019a Signed k-uniform hypergraphs and tensors. *Linear Algebra Appl.* 580 (2019), 1–13. MR 3975082. Zbl 1420.05108. (**SH: Adj, Lap: Eig**)

Jianming Yu

See G. Jiang.

Wei-Hsuan Yu

See M.-Y. Cao.

Weiqiang Yu

See R. Naserasr.

Yuantian Yu

See S.-C. Li.

B. Yuan

See Y. Chen.

Bo-Jun Yuan

See also Y. Wang.

Bo-Jun Yuan, Yi Wang, & Jing Xu

2020a Characterizing the mixed graphs with exactly one positive eigenvalue and its application to mixed graphs determined by their H-spectra. *Appl. Math. Comput.* 380 (2020), art. 125279, 7 pp. MR 4085149. Zbl 1460.05120.

Φ with gain group $\{\pm 1, \pm \mathrm{i}\}$: $\varphi(e) = 1$ for undirected, i for directed edges. Examples where spectrum determines Φ [*cf.* Mohar (2016a)]. [Annot. 15 Dec 2020.] (**gg: Adj: Eig**)

Lili Yuan, Jixiang Meng, & Eminjan Sabir

2021a The antistrong property for special digraph families. *Graphs Combin.* 37 (2021), 2511–2519. MR 4338743. Zbl 1479.05133.

"Antistrong" means there is an antidirected trail from any vertex to any other vertex. Antistrong Cartesian and lexicographic products and tournaments. [*Problem.* Generalize to bidirected graphs.] [Annot. 22 Nov 2021.] (**gg: Paths(Gen)**)

Shuhan Yuan

See Y.-M. Li.

Xiying Yuan

See also V. Nikiforov, Z.-N. Shao, and G.H. Yu.

2014a Maxima of the Q-index: forbidden odd cycles. *Linear Algebra Appl.* 458 (2014), 207–216. MR 3231816. Zbl 1295.05147. arXiv:1401.4363. Corrigendum. *Ibid.* 65 (2015), 426–429. MR 3274687. Zbl 1304.05097. arXiv:1401.4363. (**par: Lap: Eig**)

Xi-Ying Yuan, Yue Liu, & Miaomiao Han

2011a The Laplacian spectral radius of trees and maximum vertex degree. *Discrete Math.* 311 (2011), no. 8-9, 761–768. MR 2774232 (2011m:05191). Zbl 1216.05013.

§3: $Q := L(-\Gamma)$ is used to prove results about trees. [Annot. 21 Jan 2012.] **(par: bal: Lap: Eig)**

Xiying Yuan, Yanqi Mao, & Lele Liu

2021a Maximal signed graphs with odd signed cycles as star complements. *Appl. Math. Comput.* 408 (2021), art. 126367, 11 pp. MR 4264737.

[See Rowlinson and Stanić (2022a).] **(SG: Adj: Eig)**

Zihan Yuan

See S.-X. Lv.

Raphael Yuster & Uri Zwick

1994a Finding even cycles even faster. In: Serge Abiteboul and Eli Shamir, eds., *Automata, Languages and Programming* (Proc. 21st Int. Colloq., ICALP 94, Jerusalem, 1994), pp. 532–543. Lect. Notes Computer Sci., Vol. 820. Springer-Verlag, Berlin, 1994. MR 1334128. Zbl 844.00024 (book).

Abbreviated version of (1997a). **(par: Circ: Algor)**

1997a Finding even cycles even faster. *SIAM J. Discrete Math.* 10 (1997), 209–222. MR 1445033 98d:05137. Zbl 867.05065.

For fixed *even* k, a very fast algorithm for finding a k-gon. Also, one for finding a shortest even circle. [*Question.* Are these the all-negative cases of similarly fast algorithms to find positive k-gons, or shortest positive circles, in signed graphs?] **(par: Circ: Algor)**

Sergey Yuzvinsky

2004a Realization of finite abelian groups by nets in $\mathbb{P}^2$. *Compos. Math.* 140 (2004), no. 6, 1614–1624. MR 2098405 (2005g:52057). Zbl 1066.52027.

Prop. 3.3: A k-net in $\mathbb{CP}^2$ whose classes are pencils is the canonical representation of the jointless Dowling geometry $Q^\dagger(\mathbb{Z}_m) = \mathbf{F}(\mathbb{Z}_m K_3)$ of a finite cyclic group. If a k-net in $\mathbb{CP}^2$ represents $\mathbf{F}(\mathfrak{A}K_3)$ for a finite abelian group $\mathfrak{A}$, then $\mathfrak{A}$ is a subgroup of a 2-torus (Thm. 4.4) or has small invariant factors (Thm. 5.4); in particular it cannot be $\mathbb{Z}_2^3$ (Thm. 4.2). The author conjectures more definitive characterizations. **(gg: Geom)**

Bailee Zacovic

See M. Cho.

C.T. Zahn, Jr.

See also C. Witzgall.

1964a Approximating symmetric relations by equivalence relations. *J. Soc. Industrial Appl. Math.* 12 (1964), no. 4, 840–847. MR 0172276 (30 #2496). Zbl 129.16003 (129, 160c).

Let $l_{\text{clu}}(K_n, \sigma) := \min_{\sigma^*} \rho(\sigma, \sigma^*)$ over all clusterable σ^*, where $\rho(\sigma, \sigma^*) := \#(E^+(\sigma^*) \oplus E^+(\sigma))$. §3 finds a minimizing σ^* when Σ^+ is a graph with a nontrivial clique attached to each vertex. §4 attaches cliques to the preceding clique vertices in certain cases. Dictionary: "symmetric relation" = graph $G = \Sigma^+$, "equivalence relation" = σ^*, "approximating equivalence relation" = "optimal partition" = σ^* minimizing ρ. [Sequel: Moon (1966a).] [This appears to be the first paper that is implicitly on clustering a signed complete graph, before Davis (1967a).] [Annot. 10 Nov 2017.] **(sg: Clu)**

1973a Alternating Euler paths for packings and covers. *Amer. Math. Monthly* 80 (1973), 395–403. MR 0373937 (51 #10137). Zbl 274.05112. **(par: ori)**

Katarzyna Zając

See also M. Gąsiorek, G. Marczak, and D. Simson.

2017a Numeric algorithms for corank two edge-bipartite graphs and their mesh geometries of roots. *Fundamenta Inform.* 152 (2017), no. 2, 185–222. MR 3636158. Zbl 1375.05167. **(SG: Algor)**

2019a On the structure of loop-free non-negative edge-bipartite graphs. *Linear Algebra Appl.* 579 (2019), 262–283. MR 3959735. Zbl 1419.05101. **(SG)**

2020a On polynomial time inflation algorithm for loop-free non-negative edge-bipartite graphs. *Discrete Appl. Math.* 283 (2020), 28–43. MR 4114879. Zbl 1442.05228. **(SG: Algor)**

Mariusz Zając

20xxa A short proof of Brooks' theorem. Submitted. arXiv:1805.11176.

Simple proof and algorithm, also applied to signed graphs and list coloring, generalizing Máčajová–Raspaud–Škoviera (2016a), Fleiner–Wiener (2016a), Dvořák–Postle (2018a). [Annot. 2 Jul 2020.] **(SG: Col, Algor)**

Robert B. Zajonc

1968a Cognitive theories in social psychology. In: Gardner Lindzey and Elliot Aronson, eds., *The Handbook of Social Psychology*, second ed., Vol. 1, Ch. 5, pp. 320–411. Addison-Wesley, Reading, Mass., 1968.

"Structural balance," pp. 338–353. "The congruity principle," pp. 353–359. **(PsS: SD, SG, Bal: Exp, Ref)**

Robert B. Zajonc & Eugene Burnstein

1965a Structural balance, reciprocity, and positivity as sources of cognitive bias. *J. Personality* 33 (1965), no. 4, 570–583.

Test of the relative importance of balance, reciprocity (= digon balance), and number of positive arcs on experimental subjects memorizing a [simple] signed digraph (represented by a sociological story). [The question raised is mathematically intriguing, but thus far undeveloped.] [Annot. 24 Nov 2012.] **(PsS: SD: Bal)**

Robert B. Zajonc & Steven J. Sherman

1967a Structural balance and the induction of relations *J. Personality* 35 (1967), no. 4, 635–650.

Experimental test of importance of balance in subjects' attitudes towards signed graphs of order 3 suggests that balance is a weak criterion. Also, concise survey of several previous experiments. [Annot. 24 Nov 2012.] **(PsS: SG: Bal)(PsS: SG, SD: Bal: Exp)**

[Kh. Zakhs]

See H. Sachs.

Shahid Zaman & Xiaocong He

2022a Relation between the inertia indices of a complex unit gain graph and those of its underlying graph. *Linear Multilinear Algebra* 70 (2022), no. 5, 843–877. MR 4395818. Zbl 1485.05034. (**GG: Adj: Eig**)

Giacomo Zambelli

See also A. Del Pia.

2005a A polynomial recognition algorithm for balanced matrices. *J. Combin. Theory Ser. B* 95 (2005), 49–67. MR 2154177 (2006g:05041). Zbl 1071.05019. (**SG: Bal, Algor**)

Wenan Zang

See also Z.B. Chen and Y.-Z. Wu.

1998a Proof of Toft's conjecture: Every graph containing no fully odd K_4 is 3-colorable (extended abstract). In: Wen-Lian Hsu and Ming-Yang Kao, eds., *Computing and Combinatorics* (Proc., 4th Ann. Int. Conf., COCOON'98, Taipei, 1998), pp. 261–268. Lect. Notes in Computer Sci., Vol. 1449. Springer-Verlag, Berlin, 1998. MR 1683343 (no rev). Zbl 914.05030.

Summary of (1998b). (**sg: par: Col**)

1998b Proof of Toft's conjecture: Every graph containing no fully odd K_4 is 3-colorable. *J. Combin. Optim.* 2 (1998), 117–188. MR 1631313. Zbl 914.05031.

Proves Toft's (1975a) conjecture: For every 4-chromatic graph Γ, −Γ contains a subdivided $-K_4$. [*Cf.* Thomassen (2001b).] [*Question.* What is the signed-graph generalization?] [Annot. 29 Oct 2017.] (**sg: par: Col**)

1998c Coloring graphs with no odd-K_4. *Discrete Math.* 184 (1998), 205–212. MR 1609310 (99e:05056). Zbl 957.05046.

An algorithm, based in part on Gerards (1994a), that, given Γ, finds a subdivided $[-K_4]$ in Γ or a 3-coloring of Γ. [*Question.* Is there a generalization to all signed graphs?] [Annot. rev. 29 Oct 2017.] (**sg: par: Col, Algor, Ref**)

Giovanni Zappella

See N. Cesa-Bianchi.

Thomas Zaslavsky

See also L. Anderson, M. Acharya, M. Beck, F. Belardo, P. Berthomé, O. Bessouf, E.D. Bolker, S. Chaiken, R. Flórez, D. Forge, K.A. Germina, M.J. Gottstein, C. Greene, S. Hameed, P. Hanlon, D. Mallory, A.M. Mathai, R. Naserasr, R.T. Roy, K. Rybnikov, E. Sampathkumar, A. Schaefer, V. Sivaraman, D.C. Slilaty, and P. Solé.

1977a Biased graphs. Unpublished manuscript, 1977.

Published, greatly expanded, as (1989a), (1991a), (1995b) and more; as well as (but restricted to signed graphs) (1982a), (1982b). (**GG: Bal, Matrd**)

1979a Line graphs of digraphs. Abstract 768-05-3, *Notices Amer. Math. Soc.* 26 (August, 1979), no. 5, A-448. (**SG: LG: Ori, Incid, Eig(LG). Sw**)

1980a Voltage-graphic geometry and the forest lattice. In: *Report on the XVth Denison-O.S.U. Math. Conf.* (Granville, Ohio, 1980), pp. 85–89. Dept. of

Math., Ohio State University, Columbus, Ohio, 1980. (**GG: Matrd, Bic**)

1981a The geometry of root systems and signed graphs. *Amer. Math. Monthly* 88 (1981), 88–105. MR 0606249 (82g:05012). Zbl 466.05058.

Signed graphs correspond to arrangements of hyperplanes in $\mathbb{R}^n$ of the forms $x_i = x_j$, $x_i = -x_j$, and $x_i = 0$. Consequently, one can compute the number of regions of the arrangement from graph theory, esp. for arrangements corresponding to "sign-symmetric" graphs, i.e., having both or none of each pair $x_i = \pm x_j$. Simplified account of parts of (1982a), (1982b), (1982c), emphasizing geometry. (**SG: Matrd, Geom, Invar**)

1981b Characterizations of signed graphs. *J. Graph Theory* 5 (1981), 401–406. MR 0635702 (83a:05122). Zbl 471.05035.

Notably, Thm. 6: A set $\mathcal{B}$ of circles in Γ is the set of positive circles in some signing of Γ iff every theta subgraph contains an even number of circles in $\mathcal{B}$. [*Cf.* Sozański (1980a).] [Annot. rev. 22 Oct 2015, 26 Mar 2022.] (**SG: Bal**)

1981c Is there a theory of signed graph embedding? In: *Report on the XVIth Denison-O.S.U. Math. Conf.* (Granville, Ohio, 1981), pp. 79–82. Dept. of Math., Ohio State University, Columbus, Ohio, 1981.

See (1997a). (**SG: Top, Matrd**)

†† 1982a Signed graphs. *Discrete Appl. Math.* 4 (1982), no. 1, 47–74. MR 0676405 (84e:05095a). Zbl 476.05080. Erratum. *Discrete Appl. Math.* 5 (1983), no. 2, 248. MR 0683518 (84e:05095b). Zbl 503.05060.

F(Σ) Basic results on: Switching (§3) (based on graph switching of Seidel (1976a) *et al.*). Prop. 3.2: $\Sigma_1 \sim \Sigma_2$ iff $\mathcal{B}(\Sigma_1) = \mathcal{B}(\Sigma_2)$, i.e., signed graphs are switching equivalent iff they have the same circle signs. [*Cf.* Sozański (1980a).] Minors (§4). The frame matroid $\mathbf{F}(\Sigma)$ in many cryptomorphisms (§5) (some erroneous: Thm. 5.1(f,g); partly corrected in the Erratum and fully in (1991a)), consistency of matroid with signed-graph minors; separators of $\mathbf{F}(\Sigma)$. The signed covering graph $\tilde{\Sigma}$ (§6).

In §8A, incidence and Laplacian matrices $\mathrm{H}(\Sigma)$ and $L(\Sigma)$, and matrix-tree theorem [different from that of Murasugi (1989a)] [generalized by Chaiken (1982a) to a weighted, all-minors version, both directed and undirected]. [Grossman, Kulkarni, and Schochetman (1995a) improve Lemma 8A.2 (for special case $-\Gamma$) to *Thm.*: The nonzero minors of $\mathrm{H}(\Sigma)$ are $\pm 2^k$ for all $0 \leq k \leq l_0(\Sigma)$, $l_0 :=$ frustration number $= \max \#$ disjoint negative circles.] In §8B, vector representation of the matroid $\mathbf{F}(\Sigma)$ by the incidence matrix [as multisubsets of root systems $B_n \cup C_n$].

Conjectures about the interrelation between representability in characteristic 2 and unique representability in characteristic 0 [since answered by Geoff Whittle, A characterisation of the matroids representable over GF(3) and the rationals. *J. Combin. Theory Ser. B* 65 (1995), 222–261. MR 1358987 (96m:05046). Zbl 0835.05015) as developed by Pagano (1998a), (1999c)].

Examples (§7) include: Sign-symmetric graphs and signed expansions $\pm\Gamma$. The all-negative graph $-\Gamma$, whose matroid (Cor. 7D.3; partially corrected in the Erratum) is the even-circle matroid (see Doob (1973a))

and whose incidence matrices include the unoriented incidence matrix of Γ. Signed complete graphs.

Generalizations to gain graphs (called "voltage graphs") mentioned in §9.

Dictionary: "matroid $G(\ )$" $=$ $\mathbf{F}(\ )$, the frame matroid. [Annot. rev. 22 Oct 2015, 11 Mar 2022.]

(**SG, GG: Matrd, Bal, Sw, Cov, Incid, Geom; ECyc, KG**)

† 1982b Signed graph coloring. *Discrete Math.* 39 (1982), 215–228. MR 0675866 (84h:-05050a). Zbl 487.05027.

$\chi_\Sigma(\lambda)$ A "proper k-coloring" of Σ partitions V into a special "zero" part, possibly void, that induces a stable subgraph, and up to k other parts (labelled from a set of k colors), each of which induces an antibalanced subgraph. A "zero-free proper k-coloring" is similar but without the "zero" part. [The suggestion is that a signed analog of a stable vertex set is one that induces an antibalanced subgraph. *Problem.* Use this insight to develop generalizations of stable-set notions, such as cliques and perfection. *Example.* Let $\alpha(\Sigma)$, the "antibalanced vertex set number", be the largest size of an antibalance-inducing vertex set. Then $\alpha(\Gamma) = \alpha(+\Gamma \cup -K_n)$.] §2, "Counting the coloring ways": One gets two related chromatic polynomials. The chromatic polynomial, $\chi_\Sigma(2k+1)$, counts all proper k-colorings; it is essentially the characteristic polynomial of the frame matroid. It can often be most easily computed via the zero-free chromatic polynomial, $\chi^*_\Sigma(2k)$, which counts proper zero-free colorings: see (1982c). Contraction-deletion formulas; subset expansions, where the zero-free polynomial sums only over balanced edge sets. §3, "Pairs of colorings and orientations": Compatible and proper pairs. Contraction and improper pairs. Counting formulas. (Generalizing R.P. Stanley, Acyclic orientations of graphs, *Discrete Math.* 5 (1973), 171–178. MR 0317988 (47 #6537). Zbl 258.05113.)

Continued in (1982c).

[k-coloring and zero-free k-coloring are usually better called $(2k+1)$- and $2k$-coloring, as in Macajova–Raspaud–Skoviera (2014a).] [Annot. rev. 26 Aug 2018.] (**SG, GG: Matrd, Col, Invar, Cov, Ori, Geom**)

1982c Chromatic invariants of signed graphs. *Discrete Math.* 42 (1982), 287–312. MR 0677061 (84h:05050b). Zbl 498.05030.

Continuation of (1982b). §1, "Balanced expansion formulas": The fundamental balanced expansion formulas, that express the chromatic polynomial in terms of the zero-free chromatic polynomial. §2, "Counting by color magnitudes and signs". More complicated expansion formulas. §§2–7: Many special cases, treated in great detail: antibalanced graphs, signed graphs that contain $+K_n$ or $-K_n$, signed K_n's (a.k.a. two-graphs), etc. §3, "Sign-symmetric graphs": i.e., signed expansions, optionally with added half edges and loops. §4, "Addition and deletion formulas". §5, "All-negative graphs; the even-circle chromatic polynomial". §6, "Partial matching numbers and ordinary chromatic coefficients". §7, "Signed complete graphs". §8, "Orientations": formulas for numbers of acyclic orientations in the examples (*cf.* (1991b)). [Annot. rev. 26 Feb 2012.]

(**SG, GG: Matrd, Invar, Col, Cov, Ori, Geom; ECyc, KG**)

1982d Bicircular geometry and the lattice of forests of a graph. *Quart. J. Math. Oxford* (2) 33 (1982), 493–511. MR 0679818 (84h:05050c). Zbl 519.05020.

The set of all forests in a graph forms a geometric lattice. The set of spanning forests forms a geometric semilattice. The characteristic polynomials count (spanning) forests. (**GG: Matrd, Bic, Geom, Invar**)

1982e Voltage-graphic matroids. In: Adriano Barlotti, ed., *Matroid Theory and Its Applications* (Proc. Session of C.I.M.E., Varenna, Italy, 1980), pp. 417–423. Liguore Editore, Naples, 1982. MR 0863015. Zbl 1225.05002. Repr.: C.I.M.E. Summer Schools, Vol. 83, Springer, Heidelberg, and Fondazione C.I.M.E., Florence, 2010. MR 2768789. Zbl 1225.05001.

The frame matroid of a gain graph. (**GG: Matrd, ECyc, Bic, Invar: Exp**)

1984a How colorful the signed graph? *Discrete Math.* 52 (1984), 279–284. MR 0772289 (86m:05045). Zbl 554.05026.

Studies zero-free [or balanced] chromatic number $\chi^* := \min(k : \exists$ proper coloring to $\{\pm1, \ldots, \pm k\}\}$, and in particular that of a complete signed graph (which may have parallel edges). The signed graphs whose χ^* is largest or smallest.

[Updated notation: unbalanced chromatic number $\chi^0 :=$ similar for $\{0, \pm1, \ldots, \pm k\}$, chromatic number (from Máčajová–Raspaud–Škoviera (2016a)) $\chi := \min(2\chi^0 + 1,\ 2\chi^*) =$ min size of color set.] [Annot. rev 2 Jul 2020.] (**SG: Col**)

1984b Multipartite togs (analogs of two-graphs) and regular bitogs. In: Proc. Fifteenth Southeastern Conf. on Combinatorics, Graph Theory and Computing (Baton Rouge, 1984), Vol. III. *Congressus Numer.* 45 (1984), 281–293. MR 0777728 (86d:05109). Zbl 625.05044.

A modestly successful attempt to generalize two-graphs along the cohomological lines of Cameron and Wells (1986a) [Annot. 6 July 2011.] (**SG: TG: Gen: Adj, Sw**)

1984c Line graphs of switching classes. In: *Report of the XVIIIth O.S.U. Denison Maths Conference* (Granville, Ohio, 1984), pp. 2–4. Dept. of Math., Ohio State University, Columbus, Ohio, 1984.

$\Lambda(\Sigma)$ The line graph of a switching class $[\Sigma]$ of signed graphs is a switching class of signed graphs; call it $[\Lambda'(\Sigma)]$. The reduced line graph Λ is formed from Λ' by deleting parallel pairs of oppositely signed edges. Then $A(\Lambda) = A(\Lambda') = 2I - \mathrm{H}^{\mathrm{T}}\,\mathrm{H}$, where H is an incidence matrix of Σ. Thm. 1: $A(\Lambda)$ has all eigenvalues ≤ 2. Examples: For an ordinary graph Γ, $\Lambda(-\Gamma) = -\Lambda(\Gamma)$. Example: taking $-\Gamma$ and attaching any number of pendant negative digons to each vertex yields (the negative of) Hoffman's generalized line graph. Additional results are claimed but there are no proofs. [Also see (20xxa).] [This work is intimately related to that of Vijayakumar *et al.*, which was then unknown to the author, and to Cameron (1980a) and Cameron, Goethals, Seidel, and Shult (1976a).] (**SG: LG: Sw, Eig, Incid**)

1987a The biased graphs whose matroids are binary. *J. Combin. Theory Ser. B* 42 (1987), 337–347. MR 0888686 (88h:05082). Zbl 667.05015.

For the frame (bias), lift, and extended lift matroids: forbidden-minor and structural characterizations. The latter for signed-graphiic frame matroids is superseded by a result of Pagano (1998a).

[Error in Cor. 4.3: In the last statement, omit "$\mathbf{F}(\Omega) = \mathbf{L}(\Omega)$." That is true when Ω has no loops, but may not be if Ω has a loop e (because Theorem 3(3) applies with unbalanced block e, but $(E \setminus e, e)$ is not a 2-separation).] (**GG: Matrd: Str**)

1987b Balanced decompositions of a signed graph. *J. Combin. Theory Ser. B* 43 (1987), 1–13. MR 0897236 (89c:05058). Zbl 624.05056.

Decompose $E(\Sigma)$ into the fewest balanced subsets (generalizing the decomposition biparticity of an unsigned graph), or the fewest balanced connected subsets. These minimum numbers are δ_0 and δ_1. Thm. 1: $\delta_0 = \lceil \log_2 \chi^*(-\Sigma) \rceil + 1$, where χ^* is the zero-free chromatic number. Thm. 2: $\delta_0 = \delta_1$ if Σ is complete. *Conjecture* 1. Σ partitions into δ_0 balanced, connected, and spanning edge sets (whence $\delta_0 = \delta_1$) if it has δ_0 edge-disjoint spanning trees. [Solved and generalized to basepointed matroids by D. Slilaty (unpublished).] *Conjecture* 2 is a formula for δ_1 in terms of δ_0 of subgraphs. [Thoroughly disproved by Slilaty (unpublished).] (**SG: Fr**)

1987c Vertices of localized imbalance in a biased graph. *Proc. Amer. Math. Soc.* 101 (1987), 199–204. MR 0897095 (88f:05103). Zbl 622.05054.

Such a vertex u (also, a "balancing vertex") is a vertex of an unbalanced graph Ω whose removal leaves a balanced graph [i.e., frustration number $l_0 =$]. Some elementary results, e.g., $\Omega = \Omega'/e$ where $\Omega \setminus e$ is balanced and e contracts to u. [Annot. rev. 19 Dec 2014.] (**GG: Fr**)

1987d The Möbius function and the characteristic polynomial. In: Neil White, ed., *Combinatorial Geometries*, Ch. 7, pp. 114–138. Encycl. Math. Appl., Vol. 29. Cambridge Univ. Press, Cambridge, 1987. MR 0921064 (88g:05048) (book). Zbl 632.05017.

Pp. 134–135 expound the geometrical version of Dowling lattices as in Dowling (1973a). (**gg: Geom, matrd, Invar: Exp**)

1988a Togs (generalizations of two-graphs). In: M.N. Gopalan and G.A. Patwardhan, eds., *Optimization, Design of Experiments and Graph Theory* (Proc. Sympos. in Honour of Prof. M.N. Vartak, Bombay, 1986), pp. 314–334. Indian Inst. of Technology, Bombay, 1988. MR 0998807 (90h:05112). Zbl 689.05035.

An attempt to generalize two-graphs (here [alas?] called "unitogs") in a way similar to that of Cameron and Wells (1986a) although largely independently. The notable new example is "Johnson togs", based on the Johnson graph of k-subsets of a set. "Hamming togs" are based on a Hamming graph (that is, a Cartesian product of complete graphs) and generalize examples of Cameron and Wells. Other examples are as in (1984b). (**SG: TG: Gen**)

1988b The demigenus of a signed graph. In: *Report on the XXth Ohio State-Denison Mathematics Conference* (Granville, Ohio, 1988). Dept. of Math., Ohio State University, Columbus, Ohio, 1988. (**SG: Top, Matrd**)

†† 1989a Biased graphs. I. Bias, balance, and gains. *J. Combin. Theory Ser. B* 47 (1989), 32–52. MR 1007712 (90k:05138). Zbl 714.05057.

Ω, Φ Fundamental concepts and lemmas of biased graphs. Bias from gains; switching of gains; characterization of balance [for which see also Harary, Lindström, and Zetterström (1982a)]. (**GG: Bal, Sw**)

1990a Biased graphs whose matroids are special binary matroids. *Graphs Combin.* 6 (1990), 77–93. MR 1058551 (91f:05097). Zbl 786.05020.

A complete list of the biased graphs Ω such that $\mathbf{F}(\Omega)$, $\mathbf{L}_\infty(\Omega)$, or $\mathbf{L}(\Omega)$ is one of the traditional special binary matroids, $\mathbf{m}(K_5), \mathbf{M}(K_{33}), F_7$, their duals, and $\mathbf{M}(K_m)$ (for $m \geq 4$) and R_{10}. [Unfortunately omitted are nonbinary matroids like the non-Fano plane and its dual.]

[Error: The graphs $\langle +K_n^\circ \rangle$ were overlooked in the last statement of Lemma 1H—due to an oversight in (1987a) Cor. 4.3—and thus in Props. 2A and 5A. A corrected last statement of Lemma 1H: "If Ω has no two vertex-disjoint negative circles, then $\mathbf{F}(\Omega) = M \iff \mathbf{L}(\Omega) = M$." In Prop. 2A, add $\Omega = \langle +K_3^\circ \rangle$ to the list for $\mathbf{M}(K_4)$. In Prop. 5A, add $\Omega = \langle +K_{m-1}^\circ \rangle$ to the list for $\mathbf{M}(K_m)$. Thanks to Stefan van Zwam (25 July 2007).] (**GG: Matrd**)

†† 1991a Biased graphs. II. The three matroids. *J. Combin. Theory Ser. B* 51 (1991), 46–72. MR 1088626 (91m:05056). Zbl 763.05096.

$\mathbf{F}, \mathbf{L}, \mathbf{L}_\infty$ Basic theory of the bias [or better, "frame"] matroid G (now called $\mathbf{F}$) (§2) and the lift and complete lift matroids, $\mathbf{L}$ and $\mathbf{L}_\infty$ (§3), of a gain graph or biased graph. Infinite graphs. Matroids that are intermediate between the bias and lift matroids. Several questions and conjectures. (**GG: Matrd**)

† 1991b Orientation of signed graphs. *European J. Combin.* 12 (1991), 361–375. MR 1120422 (93a:05065). Zbl 761.05095.

Oriented signed graph = bidirected graph. The oriented matroid of an oriented signed graph. A "cycle" in a bidirected graph is a bias circuit (a balanced circle, or a handcuff with both circles negative) oriented to have no source or sink. Cycles in Σ are compared with those in its signed (i.e., derived) covering graph $\tilde{\Sigma}$. The correspondences among acyclic orientations of Σ and regions of the hyperplane arrangements of Σ and $\tilde{\Sigma}$, and dually the faces of the acyclotope of Σ. Thm. 4.1: the net degree vector $d(\tau)$ of an orientation τ belongs to the face of the acyclotope that is determined by the union of all cycles. Cor. 5.3 (easy): a finite bidirected graph has a source or sink. (**SG: Ori, Matrd, Cov, Geom**)(**SGw: Invar**)

1992a Orientation embedding of signed graphs. *J. Graph Theory* 16 (1992), 399–422. MR 1185006 (93i:05056). Zbl 778.05033.

Positive circles preserve orientation, negative ones reverse it. The minimal embedding surface of a one-point amalgamation of signed graphs. The formula is almost additive. *Cf.* Lins (1982a), (1985a) for related work in different formalism. (**SG: Top**)

1992b Strong Tutte functions of matroids and graphs. *Trans. Amer. Math. Soc.* 334 (1992), 317–347. MR 1080738 (93a:05047). Zbl 781.05012.

Suppose that a function of matroids with labelled points is defined that is multiplicative on direct sums and satisfies a Tutte–Grothendieck recurrence with coefficients (the "parameters") that depend on the element being deleted and contracted, but not on the particular minor from which it is deleted and contracted: specifically, $F(M) = a_e F(M \backslash e) + b_e F(M/e)$ if e is not a loop or coloop in M. Thm. 2.1 completely characterizes such "strong Tutte functions" for each possible choice of parameters: there is one general type, defined by a rank generating polynomial $R_M(a, b; u, v)$ (the "parametrized rank generating polynomial") involving the parameters $a = (a_e)$, $b = (b_e)$ and the variables u, v, and there are a few special types that exist only for degenerate parameters. All have a Tutte-style basis expansion; indeed, a function has such an expansion iff it is a strong Tutte function (Thms. 7.1, 7.2). The Tutte expansion is a polynomial within each type. If the points are colored and the parameters of a point depend only on the color, one has a multicolored matroid generalization of Kauffman's (1989a) Tutte polynomial of a sign-colored graph. Kauffman's particular choices of parameters are shown to be related to matroid and color duality.

For a graph, "parametrized dichromatic polynomial" $Q_\Gamma = u^{c(\Gamma)} R_{\mathbf{M}(\Gamma)}$, where $\mathbf{M}$ = graphic matroid. A "portable strong Tutte function" of graphs is multiplicative on disjoint unions, satisfies the parametrized Tutte–Grothendieck recurrence, and has value independent of the vertex set. Thm. 10.1: Such a function either equals Q_Γ or is one of two degenerate exceptions. Prop. 11.1: Kauffman's (1989a) polynomial of a sign-colored graph equals $R_{\mathbf{M}(|\Sigma|),\sigma}(a, b; d, d)$ for connected Σ, where $a_+ = b_- = B$ and $a_- = b_+ = A$. [*Cf.* Traldi (1989a).]

[This paper differs from other generalizations of Kauffman's polynomial, by Przytycka and Przytycki (1988a) and Traldi (1989a) (and partially anticipated by Fortuin and Kasteleyn (1972a)), who also develop the parametrized dichromatic polynomial of a graph, principally in that it characterizes *all* strong Tutte functions; also in generalizing to matroids and in having little to say about knots. Schwärzler and Welsh (1993a) generalize to signed matroids (and characterize their strong Tutte functions) but not to arbitrary colors. Bollobás and Riordan (1999a) initiate the study of the underlying commutative algebra.]

(**Sc(Matrd), SGc: Gen: Invar, Du, Knot**)

† 1993a The projective-planar signed graphs. *Discrete Math.* 113 (1993), 223–247. MR 1212880 (94d:05047). Zbl 779.05018.

$\mathbb{P}^2$ Characterized by six forbidden minors or eight forbidden topological subgraphs, all small. A close analog of Kuratowski's theorem; the proof even has much of the spirit of the Dirac–Schuster proof of the latter, and all but one of the forbidden graphs are simply derived from the Kuratowski graphs. [Paul Seymour showed me an alternative proof from Kuratowski's theorem that explains this; but it uses sophisticated results, as yet unpublished, of Robertson, Seymour, and Shih.] [*Problem.* Find a proof using the signed covering graph and Kuratowski's theorem.]

(**SG: Top**)

[Related: "projective outer-planarity" (POP): embeddable in the projective plane with all vertices on a common face. I found most of the 40 or so forbidden topological subgraphs for POP of signed graphs (finding the rest would be routine); the proof is long and tedious and will not be published. *Problem.* Find a reasonable proof.] (**SG: Top**)

1994a Frame matroids and biased graphs. *European J. Combin.* 15 (1994), 303–307. MR 1273951 (95a:05021). Zbl 797.05027.

A simple matroidal characterization of the frame (or "bias") matroids of biased graphs. (**GG: Matrd**)

1995a The signed chromatic number of the projective plane and Klein bottle and antipodal graph coloring. *J. Combin. Theory Ser. B* 63 (1995), 136–145. MR 1309361 (95j:05099). Zbl 822.05028.

Introducing the signed Heawood problem: what is the largest signed, or zero-free signed, chromatic number of any signed graph that orientation embeds in the sphere with h crosscaps? Solved for $h = 1, 2$. (**SG: Top, Col**)

†† 1995b Biased graphs. III. Chromatic and dichromatic invariants. *J. Combin. Theory Ser. B* 64 (1995), 17–88. MR 1328292 (96g:05139). Zbl 857.05088.

Polynomials of gain and biased graphs: the fundamental object is a four-variable polynomial, the "polychromial" ("polychromatic polynomial"), that specializes to the chromatic, dichromatic, and Whitney-number polynomials. The polynomials come in two flavors: unrestricted and balanced, depending on the edge sets that appear in their defining sums. (They can be defined in the even greater abstraction of "two-ideal graphs", which clarifies the most basic properties.)

§4: "Gain-graph coloring". In $\Phi = (\Gamma, \varphi, \mathfrak{G})$, a "zero-free k-coloring" is a mapping $f : V \to [k] \times \mathfrak{G}$; it is "proper" if, when $e{:}vw$ is a link or loop and $f(v) = (i, g), f(w) = (i, h)$, then $h \neq g\varphi(e; v, w)$. A "k-coloring" is similar but the color set is enlarged by inclusion of a color 0; propriety requires the additional restriction that $f(v)$ and $f(w)$ are not both 0 (and $f(v) \neq 0$ if v supports a half edge). In particular, a "group-coloring" of Φ is a zero-free 1-coloring (ignoring the irrelevant numerical part of the color). A "partial group-coloring" is a group-coloring of an induced subgraph [which can only be proper if the uncolored vertices form a stable set]. The unrestricted and balanced chromatic polynomials count, respectively, unrestricted and zero-free proper k-colorings; the two Whitney-number polynomials count all colorings, proper and improper, by their improper edge sets.

§5: "The matroid connection". The various polynomials are, in essence, frame matroid invariants and closely related to corresponding lift matroid and extended lift matroid invariants.

Almost infinitely many identities, some of them (esp., the balanced expansion formulas in §6) essential. Innumerable examples worked in detail. [The first half, to the middle of §6, is fundamental. The rest is more or less ornamental. Most of the results are, intentionally, generalizations of properties of ordinary graphs.] (**GG: Invar, Matrd, Col**)

1996a The order upper bound on parity embedding of a graph. *J. Combin. Theory Ser. B* 68 (1996), 149–160. MR 1405709 (98f:05055). Zbl 856.05030.

The smallest surface that holds K_n with loops, if odd circles reverse orientation, even ones preserve it (this is parity embedding). I.e., the demigenus $d(-K_n^\circ)$. **(Par: Top)**

1997a Is there a matroid theory of signed graph embedding? *Ars Combin.* 45 (1997), 129–141. MR 1447764 (97m:05084). Zbl 933.05067. **(SG: Matrd, Top)**

1997b The largest parity demigenus of a simple graph. *J. Combin. Theory Ser. B* 70 (1997), 325–345. MR 1459877 (99e:05043). Zbl 970.37744.

Like (1996a), but without loops. *Conjecture* 1. The minimal surface for parity embedding K_n is sufficient for orientation embedding of any signed K_n. *Conjectures* 3–4. The minimal surfaces of $\pm K_n^\circ$ and $\pm K_n$ are the smallest permitted by the lower bound obtained from Euler's polyhedral formula. **(Par: KG: Top)**

1997c Avoiding the identity. Problem 10606, *Amer. Math. Monthly* 104 (Aug.–Sept., 1997), no. 7, 664.

Find an upper bound on $f(m)$ = largest r such that any group of order $\geq r$ has m elements such that no product of any subset, possibly with inverted elements, equals the identity. Solution by Gagola (1999a).

[The solution implies that $(*)$ $f_1(m) \leq \lceil 2^{m-1}(m-1)!\sqrt{e}\rceil$, where $f_1(m)$ = smallest r such that every group of order $\geq r$ is a possible gain group for every contrabalanced gain graph of cyclomatic number m. *Problem* 1. Find a good upper bound on f_1. $(*)$ is probably weak. *Problem* 2. Find a good lower bound. *Problem* 3. Estimate f_1 asymptotically.] **(gg: bic)**

1998a Signed analogs of bipartite graphs. *Discrete Math.* 179 (1998), 205–216. MR 1489083 (2000b:05067). Zbl 980.06737.

Basically, they are the antibalanced and bipartite signed graphs; but the exact description depends on the characterization one chooses for biparticity: whether it is evenness of circles, closed walks, face boundaries in surface embeddings, etc. Characterization by chromatic number leads to a slightly more different list of analogs. **(SG: Str, Top)**

1998b A mathematical bibliography of signed and gain graphs and allied areas. *Electronic J. Combin.*, Dynamic Surveys in Combinatorics (1998), No. DS8.
URL http://www.combinatorics.org/issue/view/Surveys
MR 1744869 (2000m:05001a). Zbl 898.05001.

Complete and annotated—or as nearly so as I can make it (est. 50%). In preparation in perpetuum. Hurry, hurry, write an article!

Published edns.: Edn. 6a (Edition 6, Revision a), 20 July 1998 (iv + 124 pp.). Edn. 7, 22–26 Sept. 1999 (vi + 151 pp.). Edn. 8, 8 Sept. 2012 (vi + 341 pp.). Edn. 9, 21 Dec. 2018 (vi + 518 pp.).
(SG, Ori, GG, GN, SD, VS, TG, . . . , Chem, Phys, Biol, PsS, Appl)

1998c Glossary of signed and gain graphs and allied areas. *Electronic J. Combin.*, Dynamic Surveys in Combinatorics (1998), No. DS9.
URL http://www.combinatorics.org/issue/view/Surveys
MR 1744870 (2000m:05001b). Zbl 898.05002.

A complete (or so it is intended) terminological dictionary of signed, gain, and biased graphs and related topics; including necessary special terminology from ordinary graph theory and mathematical interpretations of the special terminology of applications.

Published edns.: 21 July 1998 (25 pp.). Second ed. 18 September 1998 (41 pp.).

(**SG, Ori, GG, GN, SD, VS, TG, . . . , Chem, Phys, PsS, Appl**)

2001a Supersolvable frame-matroid and graphic-lift lattices. *European J. Combin.* 22 (2001), 119–133. MR 1808091 (2001k:05051). Zbl 966.05013.

Biased graphs whose bias and lift matroids are supersolvable are characterized by a form of simplicial vertex ordering—with a few exceptions. As preliminary results, modular copoints are characterized [but incompletely in the bias-matroid case, as observed by Koban (2004a)]. §4: "Examples": 4a: "Group expansions and biased expansions"; 4b: "Near-Dowling and Dowling lift lattices"; 4c: "An extension of Edelman and Reiner's theorem" to general gain groups (see Edelman and Reiner (1994a)); 4d: "Bicircular matroids". [Written in 1992 and long delayed. Correction in Koban (2004a). Independently, Yoon (1997a) incorrectly attempted the case of $\mathbf{F}(\Sigma)$. Jiang and Yu (2004a) rediscovered the case of $\mathbf{F}(K_n, \sigma)$.] [Annot. rev. 20 Oct 2012.] (**GG, SG: Matrd, Geom**)

2001b The largest demigenus of a bipartite signed graph. *Discrete Math.* 232 (2001), 189–193. MR 1823637 (2001m:05100). Zbl 982.05041.

The smallest surface for orientation embedding of $\pm K_{r,s}$. (**SG: Top**)

2002a Perpendicular dissections of space. *Discrete Comput. Geom.* 27 (2002), 303–351. MR 1921558 (2003i:52026). Zbl 1001.52011.

Given an additive real gain graph Φ on n vertices and n reference points Q_i in $\mathbb{E}^d$, use Φ to specify perpendicular hyperplanes to each of the lines Q_iQ_j by means of the "Pythagorean coordinate" along Q_iQ_j. For generic points, the number of regions is computable based on the fact that the generic hyperplane intersection lattice is $\mathrm{Lat}^{\mathrm{b}}\,\Phi$. Modifications of Pythagorean coordinates give intersection lattice $\mathrm{Lat}^{\mathrm{b}}(\|\Phi\|, \varnothing)$ or a slightly more complex variant, still for generic reference points.

(**GG: Geom, Matrd, Invar**)

2003a Faces of a hyperplane arrangement enumerated by ideal dimension, with application to plane, plaids, and Shi. *Geom. Dedicata* 98 (2003), 63–80. MR 1988424 (2004f:52025). Zbl 1041.52021.

§6, "Affinographic arrangements": hyperplane arrangements that represent the extended lift matroid $\mathbf{L}_\infty(\Phi)$ where Φ is an additive real gain graph. Examples: the weakly-composed-partition, extended Shi, and extended Linial arrangements. The faces are counted in terms of dimension and dimension of the infinite part. Ehrenborg (2019a) has more explicit formulas for Shi. (**GG: matrd, Geom, Invar**)

†† 2003b Biased graphs IV: Geometrical realizations. *J. Combin. Theory Ser. B* 89 (2003), no. 2, 231–297. MR 2017726 (2005b:05057). Zbl 1031.05034.

§§2–4: Various ways in which to represent the bias and lift matroids of a gain or biased graph over a skew field F. Bias matroid: canonical vector and hyperplanar representations (generalizing those of a graph) based on a gain group $\subseteq F^\times$, Menelæan and Cevian representations (gen-

eralizations of theorems of Menelaus and Ceva), switching vs. change of ideal hyperplane, equational logic. Lift matroid: canonical vector and hyperplanar representations (the latter generalizing the Shi and Linial arrangements among others) based on a gain group $\subseteq F^+$, orthographic representation (an affine variation on canonical representation), Pythagorean representation ((2002a)). Both: effect of switching, nonunique gain-group embedding. §5: Effect of Whitney operations, separating vertex. §6: Matroids characterized by restricted general position. §7, "Thick graphs": A partial unique-representation theorem for biased graphs with sufficient edge multiplicity. §8: The 7 biased K_4's. (**GG: Matrd, Geom, Invar**)

2006a Quasigroup associativity and biased expansion graphs. *Electronic Res. Announc. Amer. Math. Soc.* 12 (2006), 13–18. MR 2200950 (2006i:20081). Zbl 1113.05044.

Summary of (2012a). (**GG: Str**)

2007a Biased graphs. VII. Contrabalance and antivoltages. *J. Combin. Theory Ser. B* 97 (2007), no. 6, 1019–1040. MR 2354716 (2008h:05025). Zbl 1125.05048.

Contrabalanced graphs, whose gains are called antivoltages. Emphasis on the existence of antivoltages in groups $\mathfrak{G} = \mathbb{Z}_\mu$, $\mathbb{Z}$, and $\mathbb{Z}_p^k$ for application to canonical representation of the contrabalanced bias and lift matroids. The number of such antivoltages is a polynomial function of the group order or (for $\mathbb{Z}$) the bound on circle gains. [*Cf.* Sivaraman (2014a), Chun, Moss, Slilaty, and Zhou (2016a), and especially Sivaraman–Slilaty (2019a).] [Annot. rev 11 Jun 2019.] (**GG: Matrd, bic, Geom, Invar**)

2009a Totally frustrated states in the chromatic theory of gain graphs. *European J. Combinatorics* 30 (2009), 133–156. MR 2460223 (2009k:05100). Zbl 1125.05048.

Given a set Q of "spins", a state is $s : V \to Q$. The gain group $\mathfrak{G}$ acts on the spin set. In a permutation gain graph Φ with gain group $\mathfrak{G}$, edge $e{:}vw$ is "satisfied" if $s(w) = s(v)\varphi(e)$, otherwise "frustrated". A totally frustrated state (every edge is frustrated) generalizes a proper coloring. Enumerative theory, including deletion/contraction, a monodromy formula for the number of totally frustrated states, and a multivariate chromatic polynomial. An abstract partition function in the edge algebra. (**GG: Col: Gen: Invar, Matrd**)

2010a Six signed Petersen graphs. In: *International Conference on Recent Trends in Graph Theory and Combinatorics (ICRTBC-2010)* (Cochin, 2010) [Summaries], pp. 75–76. Dept. of Math., Cochin Univ. of Science and Technology, 2010.

Extended abstract of (2012b) [but not entirely correct]. There are six ways to sign the Petersen graph P up to switching isomorphism. Their frustration indices, automorphism and switching automorphism groups, chromatic numbers, and clusterability indices. [Annot. 30 Aug, 26 Dec 2010.] (**SG: Fr, Aut, Col, Clu**)

2010b Matrices in the theory of signed simple graphs. In: B.D. Acharya, G.O.H. Katona, and J. Nesetril, eds., *Advances in Discrete Mathematics and Applications: Mysore, 2008* (Proc. Int. Conf. Discrete Math. 2008, ICDM-2008, Mysore, India, 2008), pp. 207–229. Ramanujan Math. Soc. Lect. Notes Ser., No. 13. Ramanujan Mathematical Soc., Mysore, India, 2010. MR 2766941 (2012d:05017). Zbl 1231.05120. arXiv:1303.3083.

The adjacency, incidence, and Laplacian matrices, along with the adjacency matrices of line graphs. Balance, vertex degrees, eigenvalues, line graphs, strong regularity, etc. A survey, emphasizing work of Seidel, Vijayakumar, and Zaslavsky.

Abelson and Rosenberg's (1958a) adjacency matrix is mentioned.

(**SG: Adj, Lap, Eig, Incid, LG: Exp**)

2012a Associativity in multiary quasigroups: The way of biased expansions. *Aequationes Math.* 83 (2012), no. 1, 1–66. MR 2885498. Zbl 1235.05059. arXiv:-0411268.

An n-ary quasigroup $(\mathfrak{Q}, f)$ is essentially equivalent, up to isotopy, to a biased expansion $m{\cdot}C_{n+1}$. Factorizations of f appear as chords in a maximal extension of $m{\cdot}C_{n+1}$. Thm.: A biased expansion of a 3-connected graph (of order ≥ 4) is a group expansion. Cor.: If $n \geq 3$ and the factorization graph of $(\mathfrak{Q}, f)$ is 3-connected, $(\mathfrak{Q}, f)$ is isotopic to an iterated group. Thm.: For a biased expansion of a 2-connected graph of order ≥ 4, if all minors of order 4 are group expansions, so is the whole expansion. Cor.: If in $(\mathfrak{Q}, f)$ $(n \geq 3)$ all ternary residual quasigroups are iterated group isotopes, so is $(\mathfrak{Q}, f)$. Cor.: $(\mathfrak{Q}, f)$ is an iterated group isotope if $\#\mathfrak{Q} = 3$.

Other results: complete structural decomposition of nongroup biased expansions, or equivalently partially reducible multiary quasigroups, in terms of groups and, respectively, irreducible expansions or irreducible multiary quasigroups.

The matroids of maximal nongroup biased expansions are the nearest generalization of Dowling's (1973b) geometries. (**GG: Str, Matrd**)

2012b Six signed Petersen graphs, and their automorphisms. Recent Trends in Graph Theory and Combinatorics (Cochin, 2010). *Discrete Math.* 312 (2012), no. 9, 1558–1583. MR 2899889. Zbl 1239.05086. arXiv:1303.3347.

There are six ways to sign the Petersen graph P up to switching isomorphism. The frustration indices and numbers, automorphism and switching automorphism groups (in extensive detail), chromatic numbers, and clusterability indices of them and their negatives. All but automorphisms and clusterability are switching invariant, thus are solved for all signed Petersens. [Annot. 26 Dec 2010.] (**SG: Fr, Aut, Col, Clu**)

2012c Signed graphs and geometry. Set Valuations, Signed Graphs and Geometry (IWSSG-2011, Int. Workshop, Mananthavady, Kerala, 2011). *J. Combin. Inform. System Sci.* 37 (2012), no. 2-4, 95–143. Zbl 1301.05162. arXiv:1303.2770.

(**SG: Bal, Fr, Geom, Incid, Adj, Eig, Matrd, Exp**)

2016a Consistency in the naturally vertex-signed line graph of a signed graph. *Bull. Malaysian Math. Sci. Soc.* 39 (2016), no. 1, suppl., S307–S314. MR 3509082. Zbl 1339.05174. arXiv:1404.1652.

Constructive answers to the question about consistency of the vertex-signed line graph treated in Acharya, Acharya, and Sinha (2009a) and Slilaty and Zaslavsky (2015a). [Annot. 3 Nov 2013.]

(**SG: LG: VS: Bal**)

2017a Negative circles in signed graphs: A problem collection. Int. Conf. Current Trends in Graph Theory and Computation (CTGTC-2016, New Delhi). *Electronic Notes Discrete Math.* 63 (2017), 41–47. MR 3754789. Zbl 1383.05152. arXiv:1610.04691.

Short version of (2018a). **(SG: Circ, Bal, Fr)**

2018a Negative (and positive) circles in signed graphs: A problem collection. *AKCE Int. J. Graphs Combin.* 15 (2018), no. 1, 31–48. MR 3803228. Zbl 1390.05085. arXiv:1701.07963.

A collection of mostly open questions, some with solutions, about circles of specified sign and their interactions: e.g., detection, uniqueness, intersection, packing, covering. [Annot. 15 Jan 2018.] **(SG: Circ, Bal, Fr)**

2024a Whitney numbers of partial Dowling lattices. *Trans. Combin.* 13 (2024), no. 2, 169–178. MR 4683434. arXiv:2209.01775.

The Whitney numbers of the first kind of group expansions $\mathfrak{G}\Gamma^{(X)}$, also biased expansions $\gamma{\cdot}\Gamma^{(X)}$ ($\gamma \in \mathbb{Z}_{>0}$), are polynomial functions of $\#\mathfrak{G}$, or γ. Very detailed. [Annot. 6 Sept 2022.] **(GG: Matrd: Invar)**

20xxa Line graphs of signed graphs and digraphs. In preparation.

Λ Line graphs of signed graphs are, fundamentally, (bidirected) line graphs of bidirected graphs, $\Lambda(\mathrm{B})$. Then the line graph $\Lambda(\Sigma)$ of a signed graph is a polar graph, i.e., a switching class of bidirected graphs; the line graph of a polar graph is a signed graph; and the line graph of a sign-biased graph, i.e., of a switching class of signed graphs, is a sign-biased graph, $[\Lambda[\Sigma]]$. In particular, the line graph of an antibalanced switching class is an antibalanced switching class. (Partly for this reason, ordinary graphs should usually be regarded as antibalanced, i.e., all negative, in line graph theory.) Since a digraph is an oriented all-positive signed graph, its line graph is a bidirected graph whose positive part is the Harary–Norman line digraph. Among the line graphs of signed graphs, some reduce by cancellation of parallel but oppositely signed edges to all-negative graphs; these are precisely Hoffman's generalized line graphs of ordinary graphs, a fact which explains their line-graph-like behavior. [See (1984c), (2010b), (2012c). Also see Zelinka (1976a) *et al.*] [Annot. $\leq$ 1998; rev.]

[Attempts at a completely descriptive line graph $\Lambda(\vec{\Gamma})$ of a digraph have been Muracchini and Ghirlanda (1965a) and Hemminger and Klerlein (1979a).] [Annot. 1999.]

[The geometry of line graphs and signed graphs has been developed by Vijayakumar *et al.* (*q.v.*).] [Annot. 1999 *et seq.*]

A thorough definition (two ways) with matrix properties appears (at last) in Belardo, Stanić, and Zaslavsky (2023a). [Annot. 20 Jan 2024.]

The competition graph of a digraph $\vec{\Gamma}$ is the extraverted part of $\Lambda(\vec{\Gamma})$. [Annot. 16 Aug 2016.] **(SG: LG: Ori, Incid, Eig(LG), Sw)**

20xxd Geometric lattices of structured partitions: I. Gain-graphic matroids and group-valued partitions. Manuscript, 1985 *et seq.* **(GG: Matrd, Invar, col)**

20xxe Geometric lattices of structured partitions: II. Lattices of group-valued partitions based on graphs and sets. Manuscript, 1985 *et seq.*
(GG: Matrd, Invar, col)

20xxg Universal and topological gains for biased graphs. In preparation. **(GG: Top)**

20xxi Big flats in a box. In preparation.
The naive approach to characteristic polynomials via lattice point counting (in characteristic 0) and Möbius inversion (as in Blass–Sagan (1998a)) can only work when one expects it to. (This is a theorem.)
(GG: Geom, Matrd, Invar, col)

20xxj Biased graphs. V. Group and biased expansions. In preparation.
(GG: Matrd, Geom, Invar)

20xxk Petersen signed graphs. In preparation.
There are 6 signatures of the Petersen graph P, up to switching isomorphism; *cf.* (2012b). For four of them ($+P$, $-P$, P_I = the antipodal quotient of the icosahedral graph, P_1 with one negative edge), many facets are examined closely. Other examples: $\pm P$, $\widetilde{(P,\sigma)}$ = the double cover of a signed P. May generate more articles. [Annot. rev 20 Jan 2024.] **(SG: Sw, Bal, Fr, Clu, Cov, Top, Col, Matrd: Exp)**

20xxm Biased graphs. VIII. A cornucopia of examples. In preparation.
Numerous types of examples of biased graphs, many having particular theory of their own, e.g., Hamiltonian bias. **(GG: Matrd, Geom)**

Morris Zelditch, Jr.
See J. Berger.

Bohdan Zelinka
See also R.L. Hemminger.

1973a Polare und polarisierte Graphen. In: *XVIII. Int. Wiss. Kolloqu.* (Ilmenau, 1973), Vol. 2, Vortragsreihe "Theorie der Graphen und Netzwerke", pp. 27–28. Technische Hochschule, Ilmenau, 1973. Zbl 272.05102.
See (1976a). [This appears to be a very brief abstract of a lecture.]
(sg: Ori, sw)

1974a Polar graphs and railway traffic. *Aplikace Mat.* 19 (1974), 169–176. MR 0347346 (49 #12066). Zbl 283.05116.
See (1976a) for definitions. Railway tracks and switches modeled by edges and vertices of a polar graph. Forming its derived graph (see (1976d)), thence a digraph obtained therefrom by splitting vertices into two copies and adjusting arcs, the time for a train to go from one segment to another is found by a shortest path calculation in the digraph. A similar method is used to solve the problem for several trains.
(sg: Ori, sw: LG: Appl)

1976a Isomorphisms of polar and polarized graphs. *Czechoslovak Math. J.* 26(101) (1976), 339–351. MR 0498288 (58 #16429). Zbl 341.05121.
Basic definitions (Zítek (1972a)): "Polarized graph" B = bidirected graph (with no negative loops and no parallel edges sharing the same bidirection). "Polar graph" $P \cong$ switching class of bidirected graphs

(that is, we forget which direction at a vertex is in and which is out—here called "north" and "south" poles—but we remember that they are different).

Thms. 1–6. Elementary results about automorphisms, including finding the automorphism groups of the "complete polarized" and polar graphs. (The "complete polarized graph" has every possible bidirected link and positive loop, without repetition.) Thm. 7: With small exceptions, any (ordinary) graph can be made polar as, say, P so that Aut P is trivial.

Thms. 8–10. Analogs of Whitney's theorem that the line graph almost always determines the graph. The "pole graph" B* of B or [B]: Split each vertex into an "in" copy and an "out" copy and connect the edges appropriately. [Generalizes splitting a digraph into a bipartite graph. It appears to be a "twisted" signed double covering graph.] Thm. 8. The pole graph is determined, with two exceptions, by the edge relation $e \sim_1 f$ if both enter or both leave a common vertex. (A trivial consequence of Whitney's theorem.) Thm. 9. A polar graph [B] with enough edges going in and out at each vertex is determined by the edge relation $e \sim_2 f$ if one enters and the other exits a common vertex. (Examples show that too few edges going in and out leave [B] undetermined.) Thm. 10. Knowing $\sim_1$, $\sim_2$, and which edges are parallel with the same sign, and if no component of the simplified underlying graph of B is one of twelve forbidden graphs, then [B] is determined. [*Problem* 1. Improve Thm. 10 to a complete characterization of the bidirected graphs that are reconstructible from their line graphs (which are to be taken as bidirected; see Zaslavsky (2010b), (20xxa)). In connection with this, see results on characterizing line graphs of bidirected (or signed) graphs by Vijayakumar (1987a). *Problem* 2. It would be interesting to improve Thm. 9.] **(sg: Ori, sw: Aut, lg)**

1976b Analoga of Menger's theorem for polar and polarized graphs. *Czechoslovak Math. J.* 26(101) (1976), 352–360. MR 0498289 (58 #16430). Zbl 341.05122.

See (1976a) for basic definitions. Here is the framework of the 8 theorems. Given a bidirected or polar graph, B or P, vertices a and b, and a type X of walk, let s_X $[s'_X]$ = the fewest vertices [edges] whose deletion eliminates all (a, b) walks of type X, and let d_X $[d'_X]$ = maximum number of suitably pairwise internally vertex-disjoint [or, suitably pairwise edge-disjoint] walks of type X from a to b. [My notation.] By "suitably" I mean that a common internal vertex or edge is allowed in P (but not in B) if it is used oppositely by the two walks using it. (See the paper for details.) Thms. 1–4_1 (there are two Theorems 4) concern all-positive and all-introverted walks in a bidirected ("polarized") graph, and are simply the vertex and edge Menger theorems applied to the positive and introverted subgraphs. Thms. 4_2–7 concern polar graphs and have the form $s_X \leq d_X \leq 2s_X$ $[s'_X \leq d'_X \leq 2s'_X]$, which is best possible. Thms. 4_2–5 concern type "heteropolar" (equivalently, directed walks in a bidirected graph). The proofs depend on Menger's theorems in the double covering graph of the polar graph. [Since this has 2 vertices for each 1 in the polar graph, the range of d_X $[d'_X]$ is explained.] Thms. 6–7 concern type "homopolar" (i.e., antidirected walks). The proofs employ

the pole graph (see (1976a)). (**sg: Ori, sw: Paths**)

1976c Eulerian polar graphs. *Czechoslovak Math. J.* 26(101) (1976), 361–364. MR 0505895 (58 #21869). Zbl 341.05123.

See (1976a) for basic definitions. An Eulerian trail in a bidirected graph is a directed trail containing every edge. [Equivalently, a heteropolar trail that contains all the edges in the corresponding polar graph.] It is closed if the endpoints coincide and the trail enters at one end and departs at the other. The fewest directed trails needed to cover a connected bidirected graph is $\frac{1}{2}$ the total of the absolute differences between in-degrees and out-degrees at all vertices, or 1 if in-degree = out-degree everywhere. (**sg: Ori, sw: Paths**)

1976d Self-derived polar graphs. *Czechoslovak Math. J.* 26(101) (1976), 365–370. MR 0498290 (58 #16431). Zbl 341.05124.

See (1976a) for basic definitions. The "derived graph" of a bidirected graph [this is equivalent to the author's terminology] is essentially the positive part of the bidirected line graph. The theorem can be restated, somewhat simplified: A finite connected bidirected graph B is isomorphic to its derived graph iff B is balanced and contains exactly one circle. (**sg: Ori, sw: LG**)

1976e Groups and polar graphs. *Časopis Pěst. Mat.* 101 (1976), 2–6. MR 0505793 (58 #21790). Zbl 319.05118.

See (1976a) for basic definitions. A polar graph $PG(\mathfrak{G}, A)$ of a group and a subset A is defined. [It is the Cayley digraph.] In bidirected language: a (bi)directed graph is "homogeneous" if it has automorphisms that are transitive on vertices, both preserving and reversing the orientations of edges, and that induce an arbitrary permutation of the incoming edges at any given vertex, and similarly for outgoing edges. It is shown that the Cayley digraph $PG(\mathfrak{G}, A)$, where $\mathfrak{G}$ is a group and A is a set of generators, is homogeneous if A is both arbitrarily permutable and invertible by $\operatorname{Aut}\mathfrak{G}$. [Bidirection—i.e., the polarity—seems to play no part here.] (**sg: Ori, sw: Aut**)

1982a On double covers of graphs. *Math. Slovaca* 32 (1982), 49–54. MR 0648219 (83b:05072). Zbl 483.05057.

Is a simple graph Γ a double cover of some signing of a simple graph? An elementary answer in terms of involutions of Γ. Further: if there are two such involutions α_0, α_1 that commute, then Γ/α_i has involution induced by α_{1-i}, so is a double cover of $\Gamma/\langle\alpha_0, \alpha_1\rangle$, which is not necessarily simple. [No properties of particular interest for signed covering are treated.] (**sg: Cov**)

1983a Double covers and logics of graphs. *Czechoslovak Math. J.* 33(108) (1983), 354–360. MR 0718920 (85k:05098a). Zbl 537.05070.

The double covers here are those of all-negative simple graphs (hence are bipartite). Some properties of these double covers are proved, then connections with a certain lattice (the "logic") of a graph. (**par: Cov: Aut**)

1983b Double covers and logics of graphs II. *Math. Slovaca* 33 (1983), 329–334. MR 0720501 (85k:05098b). Zbl 524.05058.

The second half of (1983a). **(par: Cov: Aut)**

1988a A remark on signed posets and signed graphs. *Czechoslovak Math. J.* 38(113) (1988), 673–676. MR 0962910 (90g:05157). Zbl 679.05067 (*q.v.*).

Harary and Sagan (1984a) asked: which signed graphs have the form $S(P)$ for some poset P? Zelinka gives a rather complicated answer for all-negative signed graphs, which has interesting corollaries. For instance, Cor. 3: If $S(P)$ is all negative, and P has $\hat{0}$ or $\hat{1}$, then $S(P)$ is a tree. **(SG, Sgnd)**

Hans-Olov Zetterström

See Harary, Lindström, and Zetterström (1982a).

Ahmed A. Zewail

See B. Guler.

Hongyuan Zha

See S.H. Yang.

Mingqing Zhai, Ruifang Liu, & Jinlong Shu

2011a An edge-grafting theorem on Laplacian spectra of graphs and its application. *Linear Multilinear Algebra* 59 (2011), no. 3, 303–315. MR 2774085 (2012c:-05202). Zbl 1226.05175. **(par: Lap: Eig)**

Shidong Zhai

2016a Modulus synchronization in a network of nonlinear systems with antagonistic interactions and switching topologies. *Commun. Nonlinear Sci. Numerical Simulation* 33 (2016), 184–193. MR 3417178 (no rev). **(SD: Bal: Dyn)**

Shidong Zhai, Tao Huang, Guoqiang Luo, Xin Wang, & Jun Ma

2022a Pinning bipartite synchronization for coupled nonlinear systems with antagonistic interactions and time delay. *Physica A* 593 (2022), art. 126954, 13 pp. MR 4376488 (no rev). **(SG: Dyn, Bal)**

Shidong Zhai & Qingdu Li

2016a Pinning bipartite synchronization for coupled nonlinear systems with antagonistic interactions and switching topologies. *Systems Control Lett.* 94 (2016), 127–132. MR 3530606. Zbl 1344.93011. **(SD: Bal: Dyn)**

2016b Bipartite synchronization in a network of nonlinear systems: A contraction approach. *J. Franklin Inst.* 353 (2016), no. 17, 4602–4619. MR 3562993 (n rev). Zbl 1349.93026. **(SD: Bal: Dyn)**

Shidong Zhai, Min Xiao, & Qingdu Li

2017a Synchronization analysis of coupled identical linear systems with antagonistic interactions and time-varying topologies. *Neurocomputing* 244 (2017), 53–62. **(SG: Bal: Dyn: Algor)**

Shidong Zhai & Wei Xing Zheng

2021a Dynamic behavior for social networks with state-dependent susceptibility and antagonistic interactions. *Automatica* 129 (2021), art. 109652, 14 pp. MR 4255370. Zbl 1478.91151. **(SG: Dyn)**

2021b Generalized dynamics in social networks with antagonistic interactions. *Automatica* 129 (2021), art. 109652, 14 pp. Zbl 1478.91151. arXiv:1801.08713.

(SG: Dyn)

H. Zhan

See G. Coutinho.

Bingyan Zhang

See Y.P. Zhang.

Chao Zhan

See C. Wen.

Cun-Quan Zhang

See also J.-A. Cheng, M. DeVos, Y. Lu, X.Q. Qi, R. Xu, X. Wang, and Y.-Z. Wu.

1993a Even-cycle decomposition. Problem 4.2, p. 681, in Nathaniel Dean, Open problems. In: Neil Robertson and Paul Seymour, eds., *Graph Structure Theory* (Proc., Seattle, 1991), pp. 677–688. Contemp. Math., Vol. 147. Amer. Math. Soc., Providence, R.I., 1993. MR 1224693 (93m:05004) (book). Zbl 789.05080.

Conj. 12 is a sufficient condition for $-\Gamma$ to decompose into balanced circles. [*Problem.* Solve the obvious generalization to signed graphs. Is that easier because minors exist?] [Annot. 11 Jun 2012.] **(sg: par: Str)**

1994a On even circuit decompositions of Eulerian graphs. *J. Graph Theory* 18 (1994), no. 1, 51–57. MR 1248485 (95e:05080). Zbl 804.05053.

On decomposing $E(-\Gamma)$ into positive circles. Thm.: It is possible if Γ is 2-connected and Eulerian and has no K_5 minor. [*Problem.* The same, for any signed graph. Is a $-K_5$ minor an obstruction? Also see R. Rizzi (2001a) and K. Markström (2012a).] [Annot. 13 Aug 2013.] **(Par: Str: Circ)**

De Long Zhang & Shang Wang Tan

2003a On the strongly regular graphs and the Seidel switching. (In Chinese.) *Math. Appl. (Wuhan)* 16 (2003), no. 2, 145–148. MR 1979481 (no rev). Zbl 1030.05076 (no rev). **(tg, Sw)**

Dong Zhang

See A. Abiad and R. Mulas.

Fuji Zhang

See X.A. Jin and W. Yang.

Guang-Jun Zhang & Xiao-Dong Zhang

2011a The p-Laplacian spectral radius of weighted trees with a degree sequence and a weight set. *Electronic J. Linear Algebra* 22 (2011), 267–276. MR 2788647 (2012i:05051). Zbl 1227.05190.

Generalizes Biyikoğlu, Hellmuth, & Leydold (2009a) to positively edge-weighted graphs. [*Problem.* Generalize to signed graphs.] [Annot. 21 Jan 2012.] **(Par: Eig: Gen)**

Hongwei Zhang

See also H.-D. Jiang, Y. Jiang, and Z.-Y. Sun.

2014a Dynamic output feedback control of multi-agent systems over signed graphs. In: *Proceedings of the 33rd Chinese Control Conference* (Nanjing, 2014), pp. 1327–1332. IEEE, 2014. **(SD: Bal: Dyn)**

2015a Output feedback bipartite consensus and consensus of linear multi-agent systems. In: *2015 IEEE 54th Annual Conference on Decision and Control* (CDC, Osaka, 2015), pp. 1731–1735. IEEE, 2015. (**SD: Bal: Dyn**)

Hongwei Zhang & Jie Chen

2014a Bipartite consensus of general linear multi-agent systems. In: *2014 American Control Conference* (ACC, Portland, Ore., 2014), pp. 808–812. IEEE, 2014. (**SD: Bal: Dyn**)

2014b Bipartite consensus of linear multi-agent systems over signed digraphs: An output feedback control approach. In: *Proceedings of the 19th World Congress, The International Federation of Automatic Control* (Cape Town, 2014), pp. 4681–4686. IEEE, 2014. (**SD: Bal: Dyn**)

2017a Bipartite consensus of multi-agent systems over signed graphs: State feedback and output feedback control approaches. *Int. J. Robust Nonlinear Control* 27 (2017), no. 1, 3–14. MR 3589235. Zbl 1353.93011. (**SG: Bal: Dyn**)

Jianbin Zhang

See X.L. Li.

Jianghua Zhang

See G. Jiang.

Jie Zhang & Xiao-Dong Zhang

2013a The signless Laplacian coefficients and incidence energy of bicyclic graphs. *Linear Algebra Appl.* 439 (2013), 3859–3869. MR 3133462. Zbl 1282.05163. (**par: Lap: Eig**)

Jing Zhang

See L. Su.

Jingming Zhang & Jiming Guo

2012a The signless Laplacian spectral radius of tricyclic graphs with k pendant vertices. *J. Math. Res. Appl.* 32 (2012), no. 3, 281–287. MR 2985366. Zbl 1265.05430.

Given n and k, the unique tricyclic graph with largest max eigenvalue of $L(-\Gamma)$. [*Question.* Is it the same for arbitrarily signed tricyclic graphs?] [Annot. 7 Jul 2022.] (**par: Lap: Eig**)

Jing-Ming Zhang, Ting-Zhu Huang, & Ji-Ming Guo

2014a On the signless Laplacian spectral radius of bicyclic graphs with perfect matchings. *Sci. World J.* 2014 (2014), art. 374501, 6 pp.

The graph maximizing $\lambda_{\max}(L(-\Gamma))$. [Annot. 20 Jan 2015.] (**par: Lap: Eig**)

Jing-Yue Zhang

See L. Zhang.

Junxia Zhang

See X.-Y. Ren.

Kuan Zhang

See D. Lo.

Li Zhang

See also S.C. Li.

Li Zhang, You Lu, Rong Luo, Dong Ye, & Shenggui Zhang

2020a Edge coloring of signed graphs. *Discrete Appl. Math.* 282 (2020), 234–242. MR 4107751. Zbl 1442.05081.

Partial proof of Vizing theorem $\chi' \leq \Delta + 1$ for signed simple graphs. [Independent full proof by Behr (2020a).] $\chi' \leq \Delta$ for some planar graphs. Dictionary: "signed line graph" $= -\Lambda(\Sigma)$ (*cf.* Zaslavsky (20xxa) (2010b), (2012c)); "$\chi'_{\pm}$" $= \chi'$ (chromatic index). [Annot. 2 Jul 2020.]

(**SG: ECol: LG, Ori**)

Li Jun Zhang

See X.H. Hao.

Ling Zhang, Ting-Zhu Huang, Zhongshan Li, & Jing-Yue Zhang

2013a Several spectrally arbitrary ray patterns. *Linear Multilinear Algebra* 61 (2013), no. 4, 543–564. MR 3005636. Zbl 1273.05093. (**GG: QM**)

Long Zhang

See J.-S. Wu.

Minjie Zhang

See also C. Chen and S.C. Li.

Minjie Zhang & Shuchao Li

2015a On the signless Laplacian spectra of k-trees. *Linear Algebra Appl.* 467 (2015), 136–148. MR 3284805. Zbl 1304.05098. Corrigendum. *Ibid.* 485 (2015), 527–530. MR 3394162 (no rev). Zbl 1409.05134. arXiv:1507.02536. (**par: Lap: Eig**)

Peikang ZhangPeikang Zhang, Baofeng Wu, & Changxiang He

2021a Balancedness and spectra of signed graphs obtained by $\dot{H}$-join operation. *Comput. Appl. Math.* 40 (2021), no. 3, art. 90, 11 pp. MR 4233561. Zbl 1476.05068.

(**SG: Bal, Eig: Adj, Lap**)

Ping Zhang

1997a The characteristic polynomials of subarrangements of Coxeter arrangements. *Discrete Math.* 177 (1997), 245–248. MR 1483448 (98i:52016). Zbl 980.06614.

Blass and Sagan's (1998a) geometrical form of signed-graph coloring is used to calculate (I) characteristic polynomials of several versions of k-equal subspace arrangements (these are the main results) and (II) [also in (2000a)] the chromatic polynomials (in geometrical guise) of ordinary graphs extending K_n by one vertex, signed graphs extending $\pm K_n^\circ$ by one vertex, and $\pm K_n$ with any number of negative loops adjoined.

(**sg: Invar, Geom, col**)

2000a The characteristic polynomials of interpolations between Coxeter arrangements. *J. Combin. Math. Combin. Comput.* 34 (2000), 109–117. MR 1772789 (2001b:-05220). Zbl 968.32017.

Uses signed-graph coloring (in geometrical guise) to evaluate the chromatic polynomials (in geometrical guise) of all signed graphs interpolating between (1) $+K_n$ and $+K_{n+1}$ [i.e., ordinary graphs extending a complete graph by one vertex]; (2) $\pm K_{n-1}^\circ$ and $\pm K_n^\circ$; (3) $\pm K_n$ and $\pm K_n^\circ$ [known already by several methods, including this one]; (4a) $\pm K_{n-1}$ and

$\pm K_{n-1} \cup +K_n$; (4b) $\pm K_{n-1} \cup +K_n$ and $\pm K_n$; and certain signed graphs interpolating (by adding negative edges one vertex at a time, or working down and removing them one vertex at a time) between (5) $+K_n$ and $\pm K_n^\circ$; (6) $+K_n$ and $\pm K_n$. In cases (1)–(3) the chromatic polynomial depends only on how many edges are added [which is obvious from the coloring procedure, if it were not disguised by geometry].
(**sg: Invar, col, Geom**)

Shenggui Zhang
See D. Hu, X. Wang, and L. Zhang.

Shengping Zhang
See D. Li.

Shengtong Zhang
See Z.-L. Jiang.

Shunzhe Zhang
See C. Wen.

Siqi Zhang
See S.-C. Li and G.-F. Wang.

W.J. Zhang & A.M. Yu

2017a On the rank of weighted graphs. *Linear Multilinear Algebra* 65 (2017), no. 3, 635–652. MR 3589626. Zbl 1356.05067.
Characterizes signed graphs with adjacency rank 4, separately for bipartite and non-bipartite graphs. [Annot. 27 Dec 2017.] (**SG, WG: Adj**)

Wanli Zhang
See S.-J. Yang.

Xiao-Dong Zhang
See also A. Berman M.-Z. Chen, Y.H. Chen, B.A. He, Y. Hong, G.J. Zhang, and J. Zhang.

2004a Two sharp upper bounds for the Laplacian eigenvalues. *Linear Algebra Appl.* 376 (2004), 207–213. MR 2015534 (2004m:05173). Zbl 1037.05032.
§4, Remark 2: The main results extend to signed graphs ("mixed graphs"). [Annot. 23 Mar 2009.] (**sg: Lap: Eig**)

2004b Bipartite graphs with small third Laplacian eigenvalue. *Discrete Math.* 278 (2004), no. 1-3, 241–253. MR 2035402 (2004m:05172). Zbl 1033.05073.
[*Problem.* Explain in terms of signed graphs, generalizing to $L(-\Gamma)$.]
(**par: bal: Lap: Eig**)

2009a The signless Laplacian spectral radius of graphs with given degree sequences. *Discrete Appl. Math.* 157 (2009), no. 13, 2928–2937. MR 2537494 (2011a:05210). Zbl 1213.05153. (**par: Lap: Eig**)

Xiao-Dong Zhang & Jiong-Sheng Li

2002a The Laplacian spectrum of a mixed graph. *Linear Algebra Appl.* 353 (2002), 11–20. MR 1918746 (2003d:05138). Zbl 1003.05073.
Spectrum and spectral radius of the Laplacian matrix of a signed simple graph. [For this topic, orientation is irrelevant so the results apply

to all signed simple graphs, although they are stated for oriented signed graphs in the guise of mixed graphs.] Dictionary: "mixed graph" = bidirected graph where all negative edges are extraverted; "quasibipartite" = balanced; "line graph" = $-\Lambda(\Sigma)$ (the negative of the line graph of Σ). [Annot. 23 Mar 2009.] (**sg: Eig: Lap, LG**)

Xiao-Dong Zhang & Rong Luo

2003a The Laplacian eigenvalues of mixed graphs. *Linear Algebra Appl.* 362 (2003), 109–119. MR 1955457 (2003m:05128). Zbl 1017.05078.

Σ is a signed simple graph. $\lambda_{\max}(L(\Sigma)) \leq$ max edge degree + 2 (same as Hou, Li, and Pan (2003a), Thm. 3.5(1)). Also, other bounds on $\lambda_{\max}$. Thm. 2.5: The second smallest eigenvalue is $\leq \kappa(|\Sigma|)$ if there exists a minimum separating vertex set X such that $\Sigma \setminus X$ is balanced. Dictionary: See X.D. Zhang and Li (2002a). [Annot. 23 Mar 2009.] (**sg: Eig**)

2006a Non-bipartite graphs with third largest Laplacian eigenvalue less than three. *Acta Math. Sinica (Engl. Ser.)* 22 (2006), no. 3, 917–934. MR 2220184 (2007b:05141). Zbl 1102.05040.

[*Problem.* Generalize to connected, unbalanced signed graphs.] (**par: Lap: Eig**)

Xiao-Peng Zhang

See X.-J. Tian.

Xin Zhang

See W.-C. Liu.

Y. Zhang

See B. DasGupta.

Yan X Zhang

See T. Hibi.

Yanan Zhang & Haiyan Chen

2020a The average Laplacian polynomial of a graph. *Discrete Appl. Math.* 283 (2020), 737–743. MR 4114935. Zbl 1442.05105. (**SG: Lap**)

Yanguo Zhang

See Z.-Y. Cheng.

Yingying Zhang

See X.-L. Chen.

Yuanping Zhang

See also X.G. Liu.

Yuan-ping Zhang & Xiao-gang Liu

2012a Some eigenvalue relations of signed graphs. *J. Guangzhou Univ. (Nat. Sci. Ed.)* 11 (2012), no. 1, 83–86. Zbl 1265.05291. (**SG: Adj, Lap: Eig**)

Yuanping Zhang, Xiaogang Liu, Bingyan Zhang, & Xuerong Yong

2009a The lollipop graph is determined by its Q-spectrum. *Discrete Math.* 309 (2009), 3364–3369. MR 2526754 (2010g:05239). Zbl 1182.05084.

[Correction by Hamidzade and Kiani (2010a).] **(par: Lap: Eig)**

Zhang Zhang
See M. DeVos and L.-S. Shi.

Zhi-Li Zhang
See Y.-H. Li.

Guopeng Zhao
See K. Li and L.G. Wang.

Qin Zhao
See Q. Wen.

Weisheng Zhao
See Y.-P. Wu.

Xuehua Zhao
See B. Yang.

Yanhua Zhao
See H.T. Guo.

Yufei Zhao
See Z.-L. Jiang.

Q. Zheng, D.B. Skillicorn, & O. Walther

2015a Signed directed social network analysis applied to group conflict. In: Peng Cui *et al.*, eds., *IEEE International Conference on Data Mining Workshop* (ICDMW 2015, Atlantic City, N.J.), pp. 1007–1014. IEEE, 2015. **(SD: Adj, PsS)**

Qing Yu Zheng
See also H.S. Du.

Qing Yu Zheng & Qing Jun Ren

2001a Quasi-Laplacian characteristic polynomials of graphs [or, The quasi-Laplacian characteristic polynomial]. (In Chinese.) *Qufu Shifan Daxue Xuebao Ziran Kexue Ban* [*J. Qufu Normal Univ., Nat. Sci.*] 27 (2001), no. 4, 40–43. MR 1873005 (no rev). Zbl 1013.05050. **(par: Lap: Eig)**

Shanshan Zheng
See H.-Y. Cai.

Wei Xing Zhen
See S.-D. Zhai.

Zhehui Zhong
See G. Adejumo.

Bo Zhou
See also M. Ghorbani, I. Gutman, G.X. Tian, S.L. Wang, and R.D. Xing.

2008a A connection between ordinary and Laplacian spectra of bipartite graphs. *Linear Multilinear Algebra* 56 (2008), no. 3, 305–310. MR 2384657 (2008k:05139). Zbl 1163.05043.

Γ is bipartite. Subdivide every edge of Γ once. The eigenvalues are the square roots of the Laplacian eigenvalues of Γ, and 0. [*Problem* 1. Generalize to all graphs and the "signless Laplacian". *Problem* 2. Generalize to signed graphs via negative subdivision (every positive edge is subdivided into two negative edges).] [Annot. 28 Aug 2011.] (**par: bal: Lap: Eig**)

2010a Signless Laplacian spectral radius and Hamiltonicity. *Linear Algebra Appl.* 432 (2010), no. 2-3, 566–570. MR 2577702 (2011c:05219). Zbl 1188.05086. (**par: Lap: Eig**)

Bo Zhou & Aleksandar Ilić

2010a On the sum of powers of Laplacian eigenvalues of bipartite graphs. *Czechoslovak Math. J.* 60(135) (2010), no. 4, 1161–1169. MR 2738977 (2011m:05192). Zbl 1224.05333.

See Tian, Huang, and Zhou (2009a). Lower and upper bounds in terms of sums of squared degrees. Thus, bounds on incidence energy *et al.*[Annot. 24 Jan 2012.] (**par: bal: Lap: Eig**)

Chuixiang Zhou

See M.-M. Xie.

Guanglu Zhou

See D. Li.

Haijun Zhou

2005a Long-range frustration in a spin-glass model of the vertex-cover problem. *Phys. Rev. Lett.* 94 (2005), no. 21, art. 217203, 4 pp.

"Long-range frustration" means correlation between spins (± 1) of vertices at considerable distance, within the same "state" (a configuration domain separated by energy barriers). [This should be generalized to signed graphs.] [Annot. 12 Sept 2010.] (**sg: Phys: State**)

Harrison H. Zhou

See D. Feng.

Hou Chun Zhou

See H.S. Du.

Jiang Zhou

See C.J. Bu.

Jun Zhou, Yi-Zheng Fan & Yi Wang

See also Y.Z. Fan.

2007a On the second largest eigenvalue of a mixed graph. *Discuss. Math. Graph Theory* 27 (2007), no. 2, 373–384. MR 2355728 (2008j:05223). Zbl 1134.05067.

Sufficient condition for $\lambda_2(L(\Sigma)) \geq d_2$, the second largest eigenvalue and degree, respectively. Dictionary: See X.D. Zhang and Li (2002a). [Annot. 28 Oct 2011.] (**sg: Lap: Eig**)

Min Zhou

See C.-X. He.

Qi Zhou

See F. Xu.

Qiannan Zhou

See Y. Lu.

Sanming Zhou

See F. Yang.

Xiangqian Zhou
See Y.Q. Chen, D. Chun, H. Qin, J. Robbins, and D. Slilaty.

Yue Zhou
See F. Belardo.

Yulong Zhou & Jiangping Hu
2013a Event-based bipartite consensus on signed networks. *Proceedings of the 2013 IEEE International Conference on Cyber Technology in Automation, Control and Intelligent Systems* (Nanjing, 2013), pp. 326–330. IEEE, 2013. arXiv:1310.8377. (**SG: Lap: Eig**)

Zhi-Hua Zhou
See L.T. Wu.

Bao-Xuan Zhu
2010a On the signless Laplacian spectral radius of graphs with cut vertices. *Linear Algebra Appl.* 433 (2010), no. 5, 928–933. MR 2658643 (2011e:05163). Zbl 1215.05108. (**par: Lap: Eig**)

2011a Bounds on the eigenvalues of graphs with cut vertices or edges. *Linear Algebra Appl.* 434 (2011), 2030–2041. MR 2780398 (2012g:05150). Zbl 1216.05082. (**par: Lap: Eig**)

Guangjun Zhu
See Y.-H. Shen.

Jianming Zhu
2020a Unicyclic signed graphs with the first $\lfloor (n+1)/2 \rfloor$ largest energies. *Discrete Appl. Math.* 285 (2020), 350–363. MR 4114942. Zbl 1447.05100. (**SG: Adj: Eig**)

Li Zhu
See R. Huang.

Min Zhu
See also F.L. Tian.

Min Zhu, Yihao Guo, Fenglei Tian, & Lingfei Lu
2013a The least eigenvalues of the signless Laplacian of non-bipartite graphs with fixed diameter. *Int. J. Math. Engineering Sci.* 2 (2013), no. 6, 12 pp. (**par: Lap: Eig**)

Qunxiong Zhu
See D. Peng.

Shirley Zhu
See L.J. Rusnak.

Xiao Xin Zhu, Zhi Ren Sun, & Chun Zheng Cao
2008a A bound for the quasi-Laplacian spectral radius of connected graphs [or, A bound on quasi-Laplacian spectral radius of connected graphs]. (In Chinese.) *J. Nanjing Normal Univ. Nat. Sci. Ed.* 31 (2008), no. 2, 27–30. MR 2444764 (2009e:05196). Zbl 1174.05448. (**par: Lap: Eig**)

Xuding Zhu
See also Y.-T. Jiang, L.-G. Jin, R.-G. Kim, Z.-S. Pan, H. Qi, and A. Raspaud.

2015a Circular flow number of highly edge connected signed graphs. *J. Combin. Theory Ser. B* 112 (2015), 93–103. MR 3323036. Zbl 1310.05112. arXiv:-1211.3179. **(SG: Invar)**

2017a A note on two conjectures that strengthen the four colour theorem. Manuscript, 2017. arXiv:1711.02848.

Thm.: If every signed planar graph is 4-colorable (*cf.* Máčajová, Raspaud, and Škoviera (2016a)), then every planar graph is 2-list-bipartite-colorable. [Annot. 10 Nov 2017.] **(SG: Col)**

2020a A refinement of choosability of graphs. *J. Combin. Theory Ser. B* 141 (2020), 143–164. MR 4046290. Zbl 1430.05046. arXiv:1811.08587.

See Jin, Wong, and Zhu (20xxa). **(SG. GG: Col)**

Zhongxun Zhu

2011a The signless Laplacian spectral radius of bicyclic graphs with a given girth. *Electronic J. Linear Algebra* 22 (2011), 378–388. MR 2788652 (2012i:05178). Zbl 1227.05191. **(par: Lap: Eig)**

Günter M. Ziegler

See A. Björner, L. Lovász, and T. Tran.

Howard E. Zimmerman

1966a On molecular orbital correlation diagrams, the occurrence of Möbius systems in cyclization reactions, and factors controlling ground- and excited-state reactions. I. *J. Amer. Chem. Soc.* 88 (1966), no. 7, 1564–1565.

Energies of Möbius cycles with one negative sign, inspired by Heilbronner (1964a). [Annot. 23 Nov 2012.] **(Chem: sg: bal)**

František Zítek

1972a Polarisované grafy. [Polarized graphs.] Lecture at the Czechoslovak Conf. on Graph Theory, Štiřín, May, 1972.

For definitions see Zelinka (1976a). For work on these objects see many papers of Zelinka. **(sg: Ori, sw)**

1975a Some remarks on polar graphs. In: *Recent Advances in Graph Theory* (Proc. Second Czechoslovak Symp., Prague, 1974), pp. 537–539. Academia, Prague, 1975. MR 0398890 (53 #2741). Zbl 324.05113.

Cf. Zelinka (1976a) for definitions. The number of labelled polar trees; the same with given degrees. [Annot. 27 Jul 2013.] **(sg: Ori)**

Arjana Žitnik

See T Pisanski.

J. Zittartz

See P. Hoever, M.H. Waldor, and W.F. Wolff.

Etay Ziv, Robin Koytcheff, Manuel Middendorf, & Chris Wiggins

2005a Systematic identification of statistically significant network measures. *Phys. Rev. E* 71 (2005), no. 1, art. 016110, 8 pp. arXiv:cond-mat/0306610.

Statistical analysis on a space of graphs. Mentions easy generalization to signed (and weighted) graphs. [Annot. 8 Sept 2010.] (**SGw: Algor**)

Dejan Živković

See S.K. Simić.

Dorota Żuchowska-Skiba

See F. Hassanibesheli.

Mark Zuckerberg

2016a Geometric proofs for convex hull defining formulations. *Operations Res. Lett.* 44 (2016), no. 5, 625–629. MR 3546882. Zbl 1408.90193.

Example 3: The constraint matrix is the incidence matrix of a balanced signed graph. [Annot. 16 July 2016.] (**SG: Bal: Incid**)

Alejandro Zuñiga

See J. Aracena.

[Anke van Zuylen]

See A. van Zuylen (under 'V').

Alexei Zverovich

See G. Gutin.

Igor E. Zverovich

See also E. Boros.

2002a Arc signed graphs of oriented graphs. *Ars Combin.* 62 (2002), 289–297. MR 1881967 (2002k:05104). Zbl 1073.05539.

The arc signed graph $\Lambda_Z(\vec{\Gamma})$ of a digraph $\vec{\Gamma}$ (simple Γ) is the line graph $\Lambda(\Gamma)$ with $\sigma_Z(\vec{uv}\vec{wv}), \sigma_Z(\vec{vu}\vec{vw}) := +$ and $\sigma_Z(\vec{uv}\vec{vw}) := -$. [Thus, it is $-\Lambda(+\Gamma)$ where $+\Gamma$ has orientation $\vec{\Gamma}$; *cf.* Zaslavsky (2012c), (20xxa).] Thm. 1: A Krausz-type characterization of Λ_Z. Cor. 1: Λ_Z determines $\vec{\Gamma}$ up to isolated vertices and reversing the orientation. Thm. 2: Characterization by induced subgraphs: a finite list plus antibalanced circles of length ≥ 4. Cor. 2: $\Lambda_Z(\vec{\Gamma})$ graphs can be recognized and $\vec{\Gamma}$ reconstructed in polynomial time. Dictionary: "(+)-complete" means $+K_n$; "bicomplete" means complete and balanced. [Antibalanced circles are forbidden due to having all-positive base graphs.] (**SG: LG**)

A. Zverovitch

See N. Gülpinar.

Stefan H.M. van Zwam

See also K. Grace, D. Mayhew and R.A. Pendavingh.

2009a *Partial Fields in Matroid Theory.* Doctoral dissertation, Technische Universiteit Eindhoven, 2009.

§3.2, "The Dowling lift of a partial field". [Expanded in Pendavingh & van Zwam (2013a).] (**Matrd: gg**)

Uri Zwick

See R. Yuster.

Krzysztof Zwierzyński

See D. Stevanović (2007a).

Lisa Zyga

2009a Physics model determines dynamics of friends and enemies. *PhysOrg.com*, December 2, 2009. URL http://www.physorg.com/news178954961.html

Popular account of Marvel, Strogatz, and Kleinberg (2009a). [Annot. 26 Jan 2011.] (**SG: Phys, Fr, State: Dyn: Exp**)

Ondřej Zýka

See also J. Kratochvíl.

1987a Nowhere-zero 30-flow on bidirected graphs. KAM-DIMATIA Series 87-26, Charles University, Praha, 1987.

Cf. Bouchet (1983a). There is a nowhere-zero 30-flow [presumably, if the signed-graphic matroid has no coloop; there might be other restrictions]. [Annot. 15 Jan 2015.] (**SG: Flows**)