A Mathematical Bibliography of
Signed and Gain Graphs and Allied Areas

Compiled and Annotated by
THOMAS ZASLAVSKY

Department of Mathematical Sciences
Binghamton University (SUNY)
Binghamton, New York, U.S.A. 13902-6000
E-mail: zaslav@math.binghamton.edu

Submitted: March 19, 1998; Accepted: July 20, 1998.

Preliminary Tenth Edition
(Working copy as of April 5, 2021.)

Mathematics Subject Classifications (2000):
Primary 05-00, 05-02, 05C22;
Secondary 05B20, 05B35, 05C07, 05C10, 05C15, 05C17, 05C20, 05C25, 05C30, 05C35, 05C38, 05C40, 05C45, 05C50, 05C60, 05C62, 05C65, 05C70, 05C75, 05C80, 05C83, 05C85, 05C90, 05C99, 05E25, 05E30, 06A07, 15A06, 15A15, 15A99, 20B25, 20F55, 34C99, 51D20, 51D35, 51E20, 52B25, 52B35, 57M27, 68Q15, 68Q25, 68R10, 82B20, 82D30, 90B10, 90C08, 90C27, 90C35, 90C57, 90C60, 91B14, 91C20, 91D30, 91E10, 92D40, 92E10, 94B75.

Index

| A | 1 | H | 225 | O | 391 | V | 532 |
| B | 40 | I | 264 | P | 397 | W | 544 |
| C | 92 | J | 269 | Q | 418 | X | 562 |
| D | 131 | K | 282 | R | 419 | Y | 563 |
| E | 156 | L | 318 | S | 450 | Z | 572 |
| F | 169 | M | 345 | T | 510 |
| G | 189 | N | 379 | U | 532 |

Colleagues:
HELP!

If you have any suggestions whatever for items to include in this bibliography, or for other changes, please let me hear from you. Thank you.

Preface

[I]t should be borne in mind that incompleteness is a necessary concomitant of every collection of whatever kind. Much less can completeness be expected in a first collection, made by a single individual, in his leisure hours, and in a field which is already boundless and is yet expanding day by day.


A signed graph is a graph whose edges are labeled by signs. This is a bibliography of signed graphs and related mathematics.

Several kinds of labeled graph have been called “signed” yet are mathematically very different. I distinguish four types:

- **Group-signed graphs**: the edge labels are elements of a 2-element group and are multiplied around a circle (or along any walk). Among the natural generalizations are larger groups and vertex signs.

- **Sign-colored graphs**, in which the edges are labelled from a two-element set that is acted upon by the sign group: − interchanges labels, + leaves them unchanged. This is the kind of “signed graph” found in knot theory. The natural generalization is to more colors and more general groups—or no group.

- **Weighted graphs**, in which the edge labels are the elements +1 and −1 of the integers or another additive domain. Weights behave like numbers, not signs; thus I regard work on weighted graphs as outside the scope of the bibliography—except (to some extent) when the author calls the weights “signs”.

- Labelled graphs where the labels have no structure or properties but are called “signs” for any or no reason.

Each of these categories has its own theory or theories, generally very different from the others, so in a logical sense the topic of this bibliography is an accident of terminology. However, narrow logic here leads us astray, for the study of true signed graphs, which arise in numerous areas of pure and applied mathematics, forms the great majority of the literature. Thus I regard as fundamental for the bibliography the notions of balance of a circle (sign product equals +, the sign group identity) and the vertex-edge incidence matrix (whose column for a negative edge has two +1’s or two −1’s, for a positive edge one +1 and one −1, the rest being zero); this has led me to include work on gain graphs (where the edge labels are taken from any group) and “consistency” in vertex-signed graphs, and generalizable work on two-graphs (the set of unbalanced triangles of a signed complete graph) and on even and odd circles and paths in graphs and digraphs.

Nevertheless, it was not always easy to decide what belongs. I have employed the following principles:

Only works with mathematical content are entered, except for a few representative purely applied papers and surveys. I do try to include:

- Any (mathematical) work in which signed graphs are mentioned by name or signs are put on the edges of graphs, regardless of whether it makes essential use of signs. (However, due to lack of time and in order to maintain “balance” in the bibliography, I have included only a limited selection of items concerning binary clutters and postman theory, two-graphs, signed digraphs in qualitative matrix theory, and knot
theory. For clutters, see Cornuéjols (2001a); for postman theory, A. Frank (1996a). For two-graphs, see any of the review articles by Seidel. For qualitative matrix theory, see e.g. Maybee and Quirk (1969a) and Brualdi and Shader (1995a). For knot theory there are uncountable books and surveys.)

- Any work in which the notion of balance of a circle plays a role. Example: gain graphs. (Exception: purely topological papers concerning ordinary graph embedding.)

- Any work in which ideas of signed graph theory are anticipated, or generalized, or transferred to other domains. Examples: vertex-signed graphs; signed posets and matroids.

- Any mathematical structure that is an example, however disguised, of a signed or gain graph or generalization, and is treated in a way that seems in the spirit of signed graph theory. Examples: even-cycle and bicircular matroids; bidirected graphs; binary clutters (which are equivalent to signed binary matroids); some of the literature on two-graphs and double covering graphs.

- And some works that have suggested ideas of value for signed graph theory or that have promise of doing so in the future.

As for applications, besides works with appropriate mathematical content I include a few (not very carefully) selected representatives of less mathematical papers and surveys, either for their historical importance (e.g., Heider (1946a)) or as entrances to the applied literature (e.g., Taylor (1970a) and Wasserman and Faust (1994a) for psychosociology and Trinajstić (1983a) for chemistry). Particular difficulty is presented by spin glass theory in statistical physics—that is, Ising models and generalizations. Here one usually averages random signs and weights over a probability distribution; the problems and methods are rarely graph-theoretic, the topic is very specialized and hard to annotate properly, but it clearly is related to signed (and gain) graphs and suggests some interesting lines of graph-theoretic research. See Mézard, Parisi, and Virasoro (1987a) and citations in its annotation.

Plainly, judgment is required to apply these criteria. I have employed mine freely, taking account of suggestions from my colleagues. Still I know that the bibliography is far from complete, due to the quantity and even more the enormous range and dispersion of work in the relevant areas. I will continue to add both new and old works to future editions and I heartily welcome further suggestions.

There are certainly many errors, some of them egregious. For these I hand over responsibility to Sloth, Pride, Ambition, Envy, and Confusion. (Corrections, however, will be gratefully accepted by me.) And as Diedrich Knickerbocker says:

Should any reader find matter of offense in this [bibliography], I should heartily grieve, though I would on no account question his penetration by telling him he was mistaken, his good nature by telling him he was captious, or his pure conscience by telling him he was startled at a shadow. Surely when so ingenious in finding offense where none was intended, it were a thousand pities he should not be suffered to enjoy the benefit of his discovery.
Acknowledgement

I cannot name all the people who have contributed advice and criticism, but many of the annotations have benefited from suggestions by the authors or others and a number of items have been brought to my notice by helpful correspondents. I am very grateful to you all. Thanks also to the people who maintain the invaluable MR and Zbl indices (and a special thank-you for creating our own MSC classification: 05C22). However, I insist on my total responsibility for the final form of all entries, including such things as my restatement of results in signed or gain graphic language and, of course, all the praise and criticism (but not errors; see above) that they contain.

Bibliographical Data

Authors’ names are given usually in only one form, even should the name appear in different (but recognizably similar) forms on different publications. Journal abbreviations follow the style of Mathematical Reviews (MR) with minor ‘improvements’. Reviews and abstracts are cited from MR and its electronic form MathSciNet, from Zentralblatt für Mathematik (Zbl) and its electronic version (For early volumes, “Zbl VVV, PPP” denotes printed volume and page; the electronic item number is “(e VVV.PPPNN)”.), and occasionally from Chemical Abstracts (CA) or Computing Reviews (CR). A review marked (q.v.) has significance, possibly an insight, a criticism, or a viewpoint orthogonal to mine.

Some—not all—of the most fundamental works are marked with a ††; some almost as fundamental have a †. This is a personal selection.
Annotations

I try to describe the relevant content in a consistent terminology and notation, in the language of signed graphs despite occasional clumsiness (hoping that this will suggest generalizations), and sometimes with my [bracketed] editorial comments. I sometimes try also to explain idiosyncratic terminology, in order to make it easier to read the original item. Several of the annotations incorporate open problems (of widely varying degrees of importance and difficulty).

I use these standard symbols:

- $\Gamma$ is a graph $(V,E)$ of order $n = \#V$, undirected, possibly allowing loops and multiple edges. It is normally finite unless otherwise indicated.
- $\Gamma:X$ is the induced subgraph on $X \subseteq V$; also, $\Gamma:\pi := \bigcup_{X \in \pi} \Gamma: \pi$ and $\Gamma(\pi) := \Gamma \setminus E: \pi$, for $\pi$ a partition of $V$.
- $\Gamma^c$ is the complement of a simple graph.
- $\Sigma$ is a signed graph $(V,E,\sigma)$ of order $n$. $|\Sigma|$ is its underlying graph.
- $\sigma$ is the signature (sign function) of $\Sigma$.
- $E^+, E^-$ are the sets of positive and negative edges of $\Sigma$.
- $\Sigma^+, \Sigma^-$ are the corresponding spanning subgraphs (unsigned).
- $[\Sigma]$ is the switching class of $\Sigma$.
- $\tilde{\Sigma}$ is the double covering graph of $\Sigma$.
- $A(\ )$ is the adjacency matrix.
- $H(\ )$ (Eta) is the incidence matrix.
- $H[\ ]$ is the hyperplane arrangement of a graph (ordinary, signed, or gain).
- $L(\ )$ is the Laplacian matrix, $= H(\ )H(\ )^T$.
- $L(\ )$ is the intersection (semi)lattice of a hyperplane arrangement.
- $\lambda_1 = \lambda_{\max}$, the largest eigenvalue of a matrix.
- $\Phi$ is a gain graph $(V,E, \varphi)$ with gain function $\varphi$. $||\Phi||$ is its underlying graph.
- $\Phi$ is the gain function of a gain graph $\Phi$.
- $[\Phi]$ is the switching class of $\Phi$.
- $\sim$ means that two signed or gain graphs are switching equivalent (with the same underlying graph and no automorphism).
- $\simeq$ means that two signed or gain graphs are switching isomorphic (with isomorphic underlying graphs, which may be the same graph).
- $\cong$ denotes isomorphism.
- $\langle \rangle$ is the biased graph of $\Sigma$ or $\Phi$.
- $\Omega$ is a biased graph. $||\Omega||$ is its underlying graph.
- $l(\ )$ is the frustration index (line index of imbalance).
- $l_0(\ )$ is the frustration number (vertex frustration number, vertex elimination number).
- $G(\ )$ is the frame (formerly bias) matroid of a signed, gain, or biased graph.
- $L(\ ), L_0(\ )$ are the lift and extended lift matroids. [For line graphs see $\Lambda$. For the Laplacian matrix see $L$.]
- $\Lambda(\ )$ is a line graph. $\Lambda(\Gamma)$ is that of a graph. For a signed or gain graph, $\Lambda_{\BC}$ is that of Behzad–Chartrand (1969a); $\Lambda_\times$ is that of M. Acharya (2009a), $\Lambda_\bullet$ is that of M. Acharya (cf. B.D. Acharya (2010a)), $\Lambda$ is that of Zaslavsky (1979a), (1984c), (2010b), (20xxa).
Some standard terminology—much more will be found in the *Glossary* (Zaslavsky (1998c)):

**circle:** The graph of a simple closed path, or its edge set. (Also, “circuit”, “cycle”, “polygon”, “simple cycle”, etc.)

**cycle:** In a digraph, a coherently directed circle, i.e., “dicycle”. More generally: in an oriented signed, gain, or biased graph, a matroid circuit (usually, of the frame matroid) oriented to have no source or sink.
Subject Classification Codes

A code in *lower case* means the topic appears implicitly but not explicitly. A suffix *w* on *Sgnd, SG, SD, VS* denotes signs used as weights, i.e., treated as the numbers +1 and −1, added, and (usually) the sum compared to 0. A suffix *c* on *SG, SD, VS* denotes signs used as colors (often written as the numbers +1 and −1), usually permutable by the sign group. In a string of codes a colon precedes subtopics. A code may be refined through being suffixed in parentheses, as *Sgnd(M)* denoting signed matroids while *Sgnd: M* would indicate matroids of signed objects; thus *Sgnd(M): M* means matroids of signed matroids.

Adj Adjacency matrix.
Alg Algorithms.
Algeb Algebraic structures upon signed, gain, or biased graphs or digraphs.
Appl Applications other than (Chem), (Phys), (Biol), (PsS) (partial coverage).
Aut Automorphisms, symmetries, group actions.
Bal Balance (mathematical), cobalance; harmony, “consistency” of vertex signs.
Bic Bicircular matroids.
Biol Applications to biology (partial coverage).
Chem Applications to chemistry (partial coverage).
Circ Circles. (Cyc) for directed circles.
Clu Clusterability.
Col Vertex coloring.
Cov Covering graphs, double coverings.
Cyc Directed cycles. (Circ) for undirected.
D Duality (graphs, matroids, or matrices).
Dyn Dynamics in (di)graphs. Predicting aspects, e.g., an edge sign.
Eig Eigenvalues, eigenvectors, characteristic polynomial, energy.
Enum Enumeration of types of signed graphs, etc.
EC Even-cycle matroids.
ECol Edge coloring.
Exp Expository.
Exr Interesting exercises (in an expository work).
Fr Frustration (imbalance), esp. frustration index (line index of imbalance), other measures; minimum balancing set.
Geom Connections with geometry, e.g., linear programming, complex complement.
GD Digraphs with gains (or voltages).
Gen Generalization.
GG Gain, voltage, and biased graphs; includes Dowling lattices (with (M)).
GN Generalized or gain networks. (Multiplicative real gains.)
GH Hypergraphs with gains.
Hom Homomorphisms et al.
Incid Incidence matrix.
KG Signed complete graphs.
Knot Connections with knot theory (sparse coverage if signs are purely notational).
Lap Laplacian matrix $L(\ )$.
Lab Nonalgebraic labelling of signed, gain, or biased graphs, e.g., gracefulness.
M Matroids and geometric lattices, chain-groups (not signed matroids).
MtrdF Matroidal families.
Invar Numerical and algebraic invariants of signed, gain, biased graphs: polynomials, degree sequences, number of bases, etc.
Ori Orientations, bidirected graphs.
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>OG</td>
<td>Ordered gains.</td>
</tr>
<tr>
<td>Par</td>
<td>All-negative or antibalanced signed graphs, parity-biased graphs.</td>
</tr>
<tr>
<td>par</td>
<td>Includes results on even or odd paths or circles (partial coverage) that may generalize from antibalanced to all signed graphs.</td>
</tr>
<tr>
<td>Phys</td>
<td>Applications in physics (limited coverage).</td>
</tr>
<tr>
<td>PsS</td>
<td>Psychological, sociological, and anthropological applications (partial coverage).</td>
</tr>
<tr>
<td>QM</td>
<td>Qualitative (sign) matrices: sign patterns, sign stability, sign solvability, etc.: graphical methods.</td>
</tr>
<tr>
<td>Rand</td>
<td>Random signs or gains, signed or gain graphs.</td>
</tr>
<tr>
<td>Ref</td>
<td>Many references.</td>
</tr>
<tr>
<td>Sgnd</td>
<td>Signed objects other than graphs and hypergraphs: mathematical properties.</td>
</tr>
<tr>
<td>SD</td>
<td>Signed digraphs: mathematical properties.</td>
</tr>
<tr>
<td>SG</td>
<td>Signed graphs: mathematical properties. See (Par) for only all-negative, possibly implicit as with (par) for signless Laplacian.</td>
</tr>
<tr>
<td>SH</td>
<td>Signed hypergraphs: mathematical properties.</td>
</tr>
<tr>
<td>QSol</td>
<td>Sign solvability, sign nonsingularity (partial coverage).</td>
</tr>
<tr>
<td>QSta</td>
<td>Sign stability (partial coverage).</td>
</tr>
<tr>
<td>State</td>
<td>State space, state space landscape; ground state landscape (very partial coverage).</td>
</tr>
<tr>
<td>Str</td>
<td>Structure theory.</td>
</tr>
<tr>
<td>Sw</td>
<td>Switching of signs or gains.</td>
</tr>
<tr>
<td>Top</td>
<td>Topology applied to graphs; surface embeddings. (Not applications to topology.)</td>
</tr>
<tr>
<td>TG</td>
<td>Two-graphs, graph (Seidel) switching (partial coverage).</td>
</tr>
<tr>
<td>VS</td>
<td>Vertex-signed graphs (&quot;marked graphs&quot;); signed vertices and edges.</td>
</tr>
<tr>
<td>WD</td>
<td>Weighted digraphs.</td>
</tr>
<tr>
<td>WG</td>
<td>Weighted graphs.</td>
</tr>
<tr>
<td>WH</td>
<td>Weighted hypergraphs.</td>
</tr>
<tr>
<td>Xtreml</td>
<td>Extremal problems.</td>
</tr>
</tbody>
</table>
[Maria Abi Aad]
See M. Abi Aad (under ‘Ab’).

Ahmad Abdi & Bertrand Guenin

Extended abstract of (2018a). (SG)


Normalah S. Abdulcarim
See M.M. Mangontarum.

Takuro Abe

The arrangements are affino-signed-graphic arrangements. (sg, gg: Geom)


Takuro Abe, Koji Nuida, & Yasuhide Numata

Takuro Abe, Daisuke Suyama, & Shuhei Tsujie
Partial publication of (2017a).

Ish arrangements are supersolvable, and more. [Annot. 19 Dec 2020.]

Toshiki Abe
See Wang, Qian, and Abe (2019a).

Peter Abell
See also H.Z. Deng, B. Kujawski, and M. Ludwig.

Technical correction to probability estimates in (1968a). [Annot. 29 Aug 2013.]

Peter Abell & Robin Jenkins

Peter Abell & Mark Ludwig
Dynamics of signed graphs in a space of sign probabilities and tolerance of imbalance. There are three discernibly different domains of dynamical behavior. [Continued in Deng and Abell (2010a) and Kujawski, Ludwig, and Abell (2010a).] [Annot. 10 Sept, 9 Dec 2009.]
(SG, PsS: Bal, Fr, Dyn: Alg)

Robert P. Abelson
See also M.J. Rosenberg.

They introduce a modified adjacency matrix $R$, called the “structure matrix” [I call it the Abelson–Rosenberg adjacency matrix], with entries $a, p, n, a$ for, respectively, nonadjacency $[0$ in the usual adjacency matrix $A$], positive and negative adjacency $[+1, -1$ in $A]$ and simultaneous positive and negative adjacency $[0$ or indeterminate in $A]$. They define an algebra (i.e., associative, commutative, and distributive addition and multiplication) of these symbols (p. 8): $o$ acts as $0$, $p$ acts as $1$, $pn = n$, $n^2 = p$, $a = p + n$, $x + x = x$ and $ax = a$ for $x \neq 0$. In the algebra one can decide balance of $\Sigma$ via the permanent of $I + R$: $\Sigma$ is balanced if $\text{per}(I + R) = p$ and unbalanced if $\text{per}(I + R) = a$. (The “straightforward but space-consuming” proof is omitted [and the theorem is not completely correct]. They state that the permanent cannot equal $n$ or $o$ [but that is an error].) \[\text{See Harary–Norman–Cartwright (1965a) for more on this matrix, and Zaslavsky (2010b), Thm. 2.1, for a matrix with more precise counting properties.}\] They introduce a clumsy form of switching in terms of the Hadamard product of $R$ with a “passive $T$-matrix” [oversimplifying, that is a matrix obtained by switching the square all-$p$’s matrix; the actual definition involves operators $s$ and $c$ and is more interesting]. Thm. 11: Switching preserves balance.

They propose (p. 12) “complexity” \[= \text{frustration index } l(\Sigma)\] as a measure of imbalance. \[\text{Cf. Harary (1959b).}\] Thm. 12: Switching preserves frustration index. Thm. 14: $\max l(\Sigma)$, taken over all signed graphs $\Sigma$ of order $n$, equals $\lfloor (n - 1)^2/4 \rfloor$. (Proof omitted. \[\text{[Proved by Petersdorf (1966a) and Tomescu (1973a) for signed } K_n\text{'s and hence for all signed simple graphs of order } n.\text{]}\] (PsS)(SG: Adj, Bal, sw, Fr)
“Ribbongraphs” (Bollobás and Riordan (2002a)) are orientation-embedded signed graphs (cf. Zaslavsky (1992a)).

Lowell Abrams & Daniel Slilaty


M. Abreu, M.J. Funk, D. Labbate, & V. Napolitano

Nair Maria Maia de Abreu [Nair Abreu]
See also M.A.A. de Freitas, L.S. de Lima, A. Oliveira, and C.S. Oliveira.

Nair Abreu, Domingos M. Cardoso, Ivan Gutman, Enide A. Martins, & María Robbiano

Nair M.M. Abreu, Domingos M. Cardoso, Enide A. Martins, Maria Robbiano, & B. San Martín

Nair Maria Maia de Abreu & Vladimir Nikiforov


The largest eigenvalue of $L(\Gamma)$ is $\leq 2n(1-1/\omega)$ ($\omega =$ clique number), with equality if $\Gamma$ is complete $\omega$-partite and regular. [Annot. 20 Jan 2015.] (par: Lap: Eig)

Seyed Ebrahim Abtahi
See P. Esmailian.

B. Devadas Acharya [Belmannu Devadas Acharya]


A signed simple graph $\Sigma$ with positive edge weights $w$ is balanced iff $A(\Sigma, w)$ has the same spectrum as $A(|\Sigma|, w)$. A weighted, signed simple digraph $(\vec{\Gamma}, \sigma, w)$ is cycle balanced (every directed cycle is positive) iff $A(\vec{\Gamma}, \sigma, w)$ has the same spectrum as $A(\vec{\Gamma}, w)$. [Improved for connected, unweighted signed graphs in Stanić (2019c).]

Proposed measure of imbalance: the proportion of corresponding coefficients where the characteristic polynomials $p(A(\Sigma); \lambda)$ and $p(A(|\Sigma|); \lambda)$ differ. [See M.K. Gill (1981b).] [Annot. rev. 4 Apr 2012, 30 Nov 2014, 19 Dec 2020.]

(SD, SG: Bal, Adj)


[Annotation is very incomplete.] Let $\Sigma_1 \vee \Sigma_2$ be the join of underlying graphs, with edge signs $\{\pm 1\}$ as in $\Sigma_1 \cup \Sigma_2$ and with $\sigma(v_1 v_2) := \max(\mu_1(v_1), \mu_2(v_2))$, where $\mu(v) := \prod_{vw \in E} \sigma(vw)$. [Annot. 20 July 2009.]

(SG)


Begins an attack on the problem of characterizing by forbidden induced subgraphs the simple graphs that switch to forests. Among them are $K_5$ and $C_n$, $n \geq 7$. Problem. Find any others that may exist. [Solved by Hage and Harju (2004a). Forests that switch to forests were characterized by Hage and Harju (1998a).]

(TG: Sw)


Find the fewest colors to color the edges so that in each circle the number of edges of some color is even. [Possibly, inspired by §2 of Acharya and Acharya (1983a).]

(bal: Gen)


Converts a vertex-signed graph $(\Gamma, \mu)$ into a signed graph $\Sigma$ such that $(\Gamma, \mu)$ is consistent (as in Beineke and Harary (1978b)) iff every circle in $\Sigma$ is all negative or has an even number of all-negative components. [See Joglekar, Shah, and Diwan (2010a) for the definitive result on con-

Notably: nicely characterizes consistent vertex-signed graphs in which the subgraph induced by negative vertices is connected. [Subsumed by S.B. Rao (1984a).]


Includes an exposition of Sampathkumar and Nanjundaswamy (1973a).


Expounds the procedure of Katai and Iwai (1978a). Proposes a generalization to those $\Sigma$ that have a certain kind of circle basis. Construct a “dual” graph whose vertex set is a circle basis supplemented by the sum of basic circles. A “dual” vertex has sign as in $\Sigma$. Let $T = \text{set of negative “dual” vertices.}$ A $T$-join in the “dual”, if one exists, yields a negation set for $\Sigma$. A minimum $T$-join need not yield a minimum negation set. Indeed the procedure is unlikely to yield a minimum negation set (hence the frustration index $l(\Sigma)$) for all signed graphs, since it can be performed in polynomial time while $l(\Sigma)$ is NP-complete. *Questions.* To which signed graphs is the procedure applicable? For which ones does a minimum $T$-join yield a minimum negation set? Do the latter include all those that forbid an interesting subdivision or minor (cf. Gerards and Schrijver (1986a), Gerards (1988a), (1989a))?]


Signed hypergraph: hypergraph $H = (X, E)$ with $\sigma_H : E \rightarrow \{+, -\}$. Canonical marking $\mu_{\sigma_H}(x) := \prod_{e \ni x} \sigma_H(e)$ $(x \in X)$. Intersection edge sign $\sigma_{\Omega}(e) := \prod_{x \in e \cap f} \mu_{\sigma_H}(x)$. The signed intersection graph $\Omega(H, \sigma)$ is the intersection graph of $H$ with signature $\sigma_{\Omega}$. Main example: Maximal-clique hypergraph $\mathcal{K}($ of a signed graph $\mathcal{G}$ with $X = \{\text{maximal cliques of } \mathcal{G}\}$, signature $\sigma_{\mathcal{K}}(Q) := \prod_{e \in Q} \mu_{\sigma}(Q)$ for a max clique $Q$. Which signed graphs are $\Omega(\mathcal{K}(\mathcal{G}))$? Thm. 3.3: $\Sigma$ is a maxclique signed graph iff it has an edge clique cover with the Helly property, whose members
induce homogeneously signed subgraphs, an even number of which are all-negative.

On orbits of the operator $K$: Thm. 5.1: $K^m(\Sigma) = K^n(\Sigma)$ iff $K^m(|\Sigma|) = K^n(|\Sigma|)$, $\exists m < n$. However (§7), $m = 0$ ($\Sigma$ is “$K$-periodic) may hold for $|\Sigma|$ but not $\Sigma$. Problem 7.2. Characterize $K$-periodic signed graphs. [Annot. 28 Aug 2010.]

§8, “Signed line graphs”: Taking edges instead of max cliques defines a line graph $\Lambda(\Sigma)$ with signature $\sigma(e_f) := \mu(\sigma(e \cap f))$ (due to M. Acharya [M.K. Gill] (1982a), Acharya and Acharya (2015a)). [Annot. 28 Aug 2010.]

2010b Mathematical chemistry: Basic issues. In: Graph Theory Applied to Chemistry (Proc. Nat. Workshop, Pala, Kerala, India, 2010), Ch. 2.2, pp. 26–46.

§2.2.9, “Newer vistas”: Signed hypergraphs, signed semigraphs. [Annot. 31 Aug 2010.]


Many generalizations of graphs and digraphs. Mainly historical and expository. [Annot. 31 Jan 2012.]


Unlike with graphs, not every signed graph admits a minus domination function, i.e., $f \in \{0, \pm 1\}^V$ such that $(I + A(\Sigma))f \geq 1$. [Continued in Shreyas and Joseph (2020a). Cf. Walikar, Motammanavar, and Acharya (2015a) for signed domination.] [Annot. 18 May 2018.] thru (SG: Lab)


B. Devadas Acharya & Mukti Acharya [M.K. Gill]


The first half (most of §1) was improved and published as (1986a).

The second half (§§2–3) appears to be unpublished. Given: a graph $\Gamma$, a vertex signing $\mu$, and a covering $F$ of $E(\Gamma)$ by cliques of size $\leq 3$. Define a signed graph $S$ by $V(S) = F$ and $QQ' \in E(S)$ when at least half the elements of $Q$ or $Q'$ lie in $Q \cap Q'$; sign $QQ'$ negative iff there exist vertices $v \in Q' \setminus Q$, and $w \in Q \setminus Q'$ such that $\mu(v) \neq \mu(w)$. Suppose there is no edge $QQ'$ in which $|Q| = 3$, $|Q'| = 2$, and the two members of $Q \setminus Q'$ have differing sign. [This seems a very restrictive supposition.] Main result (Thm. 7): $S$ is balanced. The definitions, but not the theorem,
are generalized to multiple vertex signs $\mu$, general clique covers, and clique adjacency rules that differ slightly from that of the theorem.

(GG, VS, SG: Bal)


Four criteria for balance in an arbitrary gain graph. [See also Harary, Lindström, and Zetterström (1982a).] (GG: Bal, sw)


(SH: VS, SG: LG(Gen))

20xxa Consistent marked hypergraphs. Submitted. (SH(Gen): VS: Bal)

Belmannu Devadas Acharya, Mukti Acharya, & Deepa Sinha


Characterizes when $\Lambda_{BC}(\Sigma)$, the Behzad–Chartrand (1969a) line graph, with vertex signs $\sigma$ is harmonious. Dictionary: “cycle compatible” = harmonious (the product of all edge and vertex signs on each circle is positive). [Annot. 14 Oct 2009.] (SG, VS: LG: Bal)


Let $\Sigma$ be a signed simple graph. Thm. 2.1: The line graph $\Lambda(|\Sigma|)$, with vertex signs $\sigma$, is consistent (as in Beineke–Harary (1978b)) iff $\Sigma$ is balanced and, in $\Sigma$, a vertex of degree $\geq 4$ has only positive edges, while a trivalent vertex $v$ with negative edges has two such edges, which lie in every circle on $v$. [Slilaty and Zaslavsky (2015a) have a constructive approach. Sinha and Acharya (2016a) generalize to iterated line graphs.] [Cor. 1: A positive edge at a vertex with two negative edges is an isthmus. Cor. 2: Let $\Sigma$ be 2-connected. ($\Lambda(|\Sigma|), \sigma$) is consistent iff $\Sigma$ is balanced and every negative edge has endpoints of degree $\leq 2$. Problem. Find a structural characterization, by means of which all such $\Sigma$ can be constructed. Zaslavsky (2016a) has solutions.] [Annot. 2 Oct 2009, rev 15 Oct, 3 Nov 2013, 23 Jan 2014.] (SG, VS: LG: Bal)

B.D. Acharya, M.K. Gill, & G.A. Patwardhan


Continues M.K. Gill (1981a). A signed graph, or digraph, is “cycle-balanced” if every circle, or every cycle, is positive. Graphs, or digraphs, are “quasicospectral” if they have cospectral signings, “strictly quasicospectral” if they are quasicospectral but not cospectral, “strongly cospectral” if they are cospectral and have cospectral cycle-unbalanced signings. There exist arbitrarily large sets of strictly quasicospectral digraphs, which moreover can be assumed strongly connected, weakly but not strongly connected, etc. There exist pairs of unbalanced, strictly quasicospectral graphs; existence of larger sets is unsolved. There ex-
ist arbitrarily large sets of nonisomorphic, strongly cospectral connected graphs; also, of weakly connected digraphs, which moreover can be taken to be strongly connected, unilaterally connected, etc. Proofs, based on ideas of A.J. Schwenk, are sketched. (SD, SG: Eig)

Belmannu Devadas Acharya & Shalini Joshi


The complement of a signed digraph $D$ without loops or multiple signed arcs (a loopless, simply signed digraph, or “ambisidigraph”) is defined in the obvious way. Observation: If $D$ or $D^c$ contains a directed cycle of length $2k + 1$, then one of them contains a positive such cycle. (SD)


Popular exposition including ambisidigraphs (cf. (2003a)). [Annot. 7 Apr 2012.] (SD: Exp)


Sequel to Acharya and Joshi (2003a). For which loopless, simply signed digraphs $D$ do both $D$ and $D^c$ contain no positive 3-cycle? Thm.: If strongly connected, $D$ has order $< 6$. An attempt to use this to describe all loopless, simply signed digraphs that contain no positive 3-cycle. (SD: Str)

Mukti Acharya [Mukhtiar Kaur Gill]


See Gill and Patwardhan (1986a) for the $k$-path signed graph of $\Sigma$. The equation $\Sigma \simeq D_3(\Sigma)$ is solved. [Annot. 29 Apr 2009.] (SG, Sw)


$\Lambda_x : \Lambda_x(\Sigma) := (\Lambda(|\Sigma|), \sigma_x)$ where $\sigma_x(ef) := \sigma(e)\sigma(f)$. (Contrast with line
graphs of Behzad–Chartrand (1969a), or Zaslavsky (2010b), (2012c), (20xxa), or M. Acharya in B.D. Acharya (2010a).) [The definition originated in M.K. Gill (1982a). Publication of this article was delayed by many years.] [Annot. rev. 20 Dec 2010, 1 Sept 2012.] (SG: LG)


P. 119: Summary of \( k \)-square-sum signed graphs, where \( k \) edges classes are square-sum with the same vertex labels. \( k = 2 \) is signed graphs. [Annot. 30 Aug 2010.]


Graphs or digraphs are quasicospectral if they have cospectral signatures (signatures with the same adjacency spectrum). Properties and examples of quasicospectral graphs and digraphs that are not cospectral. Definitions and results from B.D. Acharya–Gill–Patwardhan (1984a) et al., as well as new results. [Annot. 4 Apr 2012.] (SG, SD: Eig: Exp)(SG, SD: Eig)


(VS: SG, SH: Lab)


(VS: SG: Lab)

20xxa Total C-cordial labeling in signed graphs – II. In preparation. (VS: SG: Lab)


Mukti Acharya, Rashmi Jain, & Sangita Kansal


“Lict” = line-cutpoint, an extension of the line graph. \( V(\Sigma) \) is signed by \( \mu_\Sigma \), the canonical vertex signature. \( L_c(S) \) or rather \( \Lambda_c(\Sigma) \) has vertex set \( E \cup C \) where \( C = \{ \text{cutpoints of } \Sigma \} \), edge \( uv \) iff \( u, v \) are adjacent or incident, and \( uv \) negative iff \( u \) and \( v \) are negative. \( \Lambda_c: E = \Lambda_{BC}(\Sigma) \), the Behzad–Chartrand (1969a) line graph. Characterized: (1) The signed \( K_n, C_n, \) and \( K_{n,s} \) that are \( \Lambda_{BC}(\Sigma) \) or \( \Lambda_c(\Sigma) \). (2) \( \Sigma \) such that \( \Lambda_{BC}(\Sigma) \) or \( \Lambda_c(\Sigma) \) is balanced. (3) \( \Sigma \) that are switching equivalent to \( \Lambda_{BC}(\Sigma) \) or \( \Lambda_c(\Sigma) \) or their negatives. [Annot. 8 Jan 2016.] (SG: LG(Gen): Bal, KG)

2017a Vertex equitable labeling of signed graphs. Int. Conf. Current Trends in Graph Theory and Computation (CTGTC-2016, New Delhi). Electronic Notes Dis-
Mukti Acharya & Joseph Varghese Kureethara

Parity labeling: \( f : V \leftrightarrow [n] \) such that \( \sigma(uv) = (-1)^{f(u)+f(v)} \). [I.e., \( E^- = \text{cut with nearly equal sides.} \] The *rnm* number := \( \min |E^-| \) over all parity signatures. Minor results. [Annot. 7 Oct 2019.] (SG: Lab)

Mukti Acharya, Joseph Varghese Kureethara, & Thomas Zaslavsky
20xxa Characterizations of some parity signed graphs. Submitted.


Mukti Acharya, Pranjali, Atul Gaur, & Amit Kumar

Mukti Acharya & Pranjali Sharma

Mukti Acharya & Tarkeshwar Singh

See (2004a). Here the graph is a circle and the second color class is a maximum matching. (SGc: Lab)


Announcement of (2010a). (SGc: Lab: Exp)


[Generalizing the definition in the article: Given: a graph with \( r \)-colored edges; integers \( k,d > 0 \). Required: a \( (k,d) \)-graceful labelling, i.e., an injection \( \lambda : V \rightarrow \{0,1,\ldots,k+(\#E-1)d\} \) so that, if \( f(uv) := |\lambda(v)-\lambda(w)| \), then \( f \) restricted to each color class is injective with range \( k,k+d,\ldots \].

The article concerns the case \( r = 2 \) with “results of our preliminary investigation”. *Conjecture*. Every 2-colored circle of length \( \geq 3 \) is \( (k,d) \)-graceful. (SGc: Lab)


\( \Sigma \) is a signed windmill with \( k > 1 \) blades. Only rim edges may be negative. Thm.: \( \Sigma \) is graceful \( \Rightarrow \) \( k \equiv 0 \mod 4 \) and \( \#E^- \) is even, or
The electronic journal of combinatorics #DS8

12


Proof of the conjecture of (2004a) for a circle of length \( k \not\equiv 1 \pmod{4} \) where the negative edge set is connected.


From Singh (2003a), Ch. III. See (2003b), Singh (2008a). “Skolem graceful” is the \((0,1)\)-gracefulness of (2004a). Thm.: A signed \( k \)-edge matching is Skolem graceful if \( k \equiv 0 \pmod{4} \) and \#\( E^- \) is even, or \( k \equiv 2 \pmod{4} \) and \#\( E^- \) is odd, or \( k \equiv 1 \pmod{4} \). Curiously complementary to the theorem of Singh (20xxa). [Annot. 20 July 2009.] (SGc: Lab)


20xxc Characterization of sigraphs whose negations are switching equivalent to their iterated line sigraphs.

The signed simple graphs \( \Sigma \) (which necessarily are signed circles) such that \( -\Sigma \) is switching isomorphic to any of its iterated Behzad–Chartrand (1969a) line graphs. [Annot. 20 July 2009.] (SG: Sw, LG)

20xxd Construction of certain infinite families of graceful sigraphs from a given graceful sigraph.

Let \( \lor \) denote the join of graphs or (defined in B.D. Acharya (1980c)) signed graphs. Thms.: If \( \Sigma \lor K^c_t \) is gracefully numbered, so are \( \Sigma \lor K^c_t \) and \((\Sigma \lor K^c_t \lor V) \lor K^c_t\). All \((K_2 \lor K^c_t, \sigma)\) are gracefully numbered. [Annot. 20 July 2009.] (SGc: Lab)

20xxe Graceful sigraphs: V. The case of union of signed cycles of length three with one vertex in common.

Mukti Acharya & Deepa Sinha

2002a A characterization of signed graphs that are switching equivalent to their jump signed graphs. Graph Theory Notes N.Y. 43 (2002), 7–8. MR 1960487 (no rev). (SG: LG)
2003a A characterization of sigraphs whose line sigraphs and jump sigraphs are switching equivalent. *Graph Theory Notes N.Y.* 44 (2003), 30–34. MR 2002894. (SG: LG)


Thm.: A signed simple graph $\Sigma$ is the Behzad–Chartrand (1969a) line graph of a signed graph iff the underlying graph is a line graph and $\Sigma$ is “sign compatible” (Sinha (2005a)). [Annot. 27 Apr 2009, 12 Oct 2010.]


The common-edge signed graph $C_E(\Sigma)$ is the second line graph $\Lambda^2(|\Sigma|)$ with signs $\sigma_{C_E}\{ef,fg\} = \sigma(f)$. Characterized in whole or part: When this is balanced (rarely), or isomorphic to $\Sigma$ (rarely), or switching isomorphic to the Behzad–Chartrand (1969a) line graph $\Lambda_{BC}(\Sigma)$ (rarely), or switching equivalent to $\Lambda_{2BC}(\Sigma)$. There are notions of consistency and compatibility of $C_E(\Sigma)$ with respect to a vertex signature of $\Sigma$, that seem ill defined. (SG: LG: Gen)


Sibel Adalı
See Y. Qian.

Gbemisola Adejumo, P. Robert Duimering, & Zhehui Zhong

See also R. Singh.

Bibhas Adhikari, Satyabrata Adhikari, & Subhashish Banerjee

A somewhat confusing attempt to model quantum states by a graph with weighted edges and vertices. There are several systems; in the most general, the edge weights may be complex, the vertex weights real. The adjacency matrix is Hermitian; for complex units that implies the edge weights are gains, but not so in general. [Annot. 13 Jan 2015.]

(par: Adj)(gg(Gen): Adj)

Bibhas Adhikari, Amrik Singh, & Sandeep Kumar Yadav

Satyabrata Adhikari
See B. Adhikari.
Chandrashekar Adiga, E. Sampathkumar, & M.A. Sriraj


(rgw: Adj: Eig)

Chandrashekar Adiga, E. Sampathkumar, M.A. Sriraj, & Shrikanth A.S.


Definitions at Sampathkumar and Sriraj (2013b). “Color matrix” := $A(\Sigma)$; “colorenergy” := energy of $A(\Sigma)$. Elementary results on characteristic polynomial, eigenvalues of $A(\Sigma)$. Thm. 2.4: Energy $\leq \sqrt{2n#E(\Sigma)}$.

“Complement” formed by complementing $\Sigma$ in $(K_n:\pi)c$. Examples: $K_n$, $K_{n,m}$, $C_n$, $CP(n)$, etc., and complements. [No mention of signed graphs.] [2.2, 2.4 for all signed (di)graphs are in Bhat (2017a).] Sequels: Adiga–Sampathkumar–Sriraj (2014a), Sampathkumar–Pushpalatha–Sriraj (2016a), et al. [For a strange “signed-graphic” generalization: Joshi–Joseph–Acharya (20xxa).] [Annot. 21 Dec 2018.]

L. Adler & S. Cosares


The class is that of the transshipment problem with gains. Along the way, a time bound on the uncapacitated, demands-only flows-with-gains
problem. (GN: Incid(D), Alg)

S.N. Afriat

See also Roy (1959a). (GG: OG, Sw, bal)


Amit Agarwal

A.A. Ageev, A.V. Kostochka, & Z. Szigeti

A Seymour graph satisfies with equality a general inequality between \( T \)-join size and \( T \)-cut packing. Thm.: A graph is not a Seymour graph iff it has a conservative \( \pm 1 \)-weighting such that there are two circles with total weight 0 whose union is an antibalanced subdivision of \( -K_n \) or \( -Pr_3 \) (the triangular prism). (SGw: Str, Bal, Par)


Virtually identical to (1995a). (SGw: Str, Bal, Par)

Charu Aggarwal

J.K. Aggarwal
See M. Malek-Zavarei.

Kalin Agrawal & William H. Batchelder

Priyanka Agrawal, Vikas K. Garg, & Ramasuri Narayanam

Boris Aguilar
See A. Veliz-Cuba.

F. Aguilera-Granja
See M.C. Salas-Solís.

Ron Aharoni, Rachel Manber, & Bronislaw Wajnryb
When do all perfect matchings in a signed bipartite graph have the same sign product? Solved. (sg: bal, Alg)(qm: QSol)

R. Aharoni, R. Meshulam, & B. Wajnryb

Given an edge weighting \( w : E \rightarrow R \) where \( R \) is a finite abelian group.
Main topic: perfect matchings \( M \) such that \( \sum_{e \in M} w(e) = 0 \) [I’ll call them 0-weight matchings]. (Also, in \( \S 2, = c \) where \( c \) is a constant.) Generalizes and extends Aharoni, Manber, and Wajnryb (1990a). [Continued by Kahn and Meshulam (1998a).] (GGw)

Prop. 4.1 concerns vertex-disjoint circles whose total sign product is + in certain signed digraphs. (SD: Circ)

Amnon Aharony

Physics of a random signed subgraph of \( \Gamma \): \( p, q, r \) = probabilities of +, −, or no edge. \( r = 0 \) is a randomly signed \( \Gamma \). \( p = 0 \) is a random subgraph \( -\Gamma \). Edges may have weights but the signs are most significant (pp. L461–2). Bipartite graphs (“simple systems, with two sublattices”) give easier results; e.g., switching exchanges \( p \) and \( q \), and transforms all-negative to all-positive. Analysis by the replica method: replicate the graph randomly \( n \) times. For temperature \( T \rightarrow 0 \): The case \( p = q \) has special properties. The limit \( r \rightarrow 0 \) gives all-positive (ferromagnetic) behavior because “only [constant states \( \zeta : V \rightarrow \{+1, -1\} \] contribute to the partition function.” \( T > 0 \): Special cases for equal weights, similarly to Houtappel (1950b), Newell (1950a). The replica method’s limitations include failure at \( T \rightarrow 0 \) when the signed subgraph is unbalanced (“frustrated”) (p. L463). [An interesting study. Problem. Interpret the replica method and results in terms of random signed graphs.] [Annot. 21 Jun 2012.] (Phys, SG, WG: Rand, Fr, sw)

Saeed Ahmadizadeh, Iman Shames, Samuel Martin, & Dragan Nešić

Luis von Ahn


Ravindra K. Ahuja, Thomas L. Magnanti, & James B. Orlin

§12.6: “Nonbipartite cardinality matching problem”. Nicely expounds theory of blossoms and flowers (Edmonds (1965a), etc.). Historical notes and references at end of chapter. (par: ori, Alg: Exp, Ref)

§5.5: “Detecting negative cycles” (i.e., sum < 0); §12.7, subsection
“Shortest paths in directed networks”. Weighted arcs with negative weights allowed. Techniques for detecting negative cycles and, if none exist, finding a shortest path. (OG: Alg: Exp)

Ch. 16: “Generalized flows”. §15.5: “Good augmented forests and linear programming bases”, Thm. 15.8, makes clear the connection between flows with gains and the frame matroid of the underlying gain graph. Some terminology: “breakeven cycle” = balanced circle; “good augmented forest” = basis of the frame matroid, assuming the gain graph is connected and unbalanced. (GN: M(Bases), Alg: Exp, Ref)

Martín Aigner

In § VII.1, pp. 333–334 and Exerc. 13–15 treat the Dowling lattices of $\text{GF}(q)^{\times}$ and higher-weight analogs. (GG, GG(Gen): M: Invar, Str)


Nir Ailon, Moses Charikar, & Alantha Newman


Saieed Akbari, Francesco Belardo, Ebrahim Dodongeh, & Mohammad Ali Nematollahi

$(C_n,\sigma)$ is spectrally unique for $A(\Sigma)$ iff $n$ is odd or $n = 4$; finds most cospectral signed graphs. It is spectrally nonunique for $L(\Sigma)$ iff even and balanced; finds all cospectral signed graphs. [Annot. 9 May 2018.]

(SG: Adj, Lap: Eig)

Saieed Akbari, Francesco Belardo, Farideh Heydari, Mohammad Maghasedi, & Mona Souri

§2, “Signed graphs perturbations”: For any $\Sigma$, effect on $\lambda_1 = \max \lambda_i$ of adding an edge. $\lambda_1$ and $\lambda_n$ for $\sigma \equiv 1, -1$ vs. arbitrary $\sigma$. §3, “Unbalanced unicyclic graphs with extremal index”: Those with max and min $\lambda_1$. [Annot. 23 Jan 2020.]

(SG: Adj: Eig)

S. Akbari, S.M. Cioabă, S. Goudarzi, A. Niaparast, & A. Tajdini

Nonsingular $A(K_n, \sigma)$ need not have a $\pm 1$-nullvector. [Annot. 6 Feb 2021.]

(SG: KG: Adj)

S. Akbari, A. Daemi, O. Hatami, A. Javanmard, & A. Mehrabian


Every signed Hamiltonian graph without a coloop has a nowhere-zero 12-flow: an improved result towards Bouchet’s (1983a) conjecture. The proofs are for unoriented flows on a graph (i.e., flows on an all-negative signed graph, which are equivalent to signed-graph flows). Better results if there is a negative Hamilton circle $C$. Thm. 3.2: An 8-flow if $\Sigma \setminus C$ is connected. Thm. 3.3: A 6-flow if $\Sigma \setminus C$ is unbalanced. [Annot. 5 Feb 2010.]

(SG: Flows, ori)

S. Akbari, M. Dalirrooyfard, K. Ehsani, & R. Sherkati


Follows C.-P. Wang (2013a). Thm. 2: $\beta^k_S \geq n - k - c(\Gamma)$. Thm. 3: If $\Gamma$ is connected and not Eulerian, $\beta^k_S = \max \#$ odd trails in a minimal trail decomposition. [Annot. 17 Dec 2020.]

(SG)

Saieed Akbari, Soudabeh Dalvandi, Farideh Heydari, & Mohammad Maghasedi


Thms.: Multiplicity of eigenvalue $\pm 1 \geq$ number of homogeneously positive (negative) vertices. Spectrum of regular $\Gamma$ is related to that of $K_\Gamma$. [Annot. 17 Dec 2020.]

(SG: KG: Adj: Eig)


Thm. 8: For $\Sigma^- = $ tree of $k$ edges, $\lambda_1$ is max iff $\Sigma^- = K_{1,k}$. Thm. 10 relates eigenvalues of $H \subseteq K_n$ to those of $K_H$. Thm. 16: For nonspanning $H \subseteq K_n$ and most eigenvalues $\lambda$ of $H$. $-1 - 2\lambda$ is an eigenvalue of $K_H$. [Annot. 17 Dec 2020.]

(SG: KG: Adj: Eig)

S. Akbari, M. Einollahzadeh, M.M. Karkhaneei, & M.A. Nematollahi


(SG: KG: Adj: Eig)

S. Akbari, K. Etemadi, P. Ezzati, & M. Ghadiri


(SG: Par: Circ)

S. Akbari, F.A.M. França, E. Ghasemian, M. Javarsineh, & L.S. de Lima

S. Akbari, A. Ghafari, K. Kazemian, & M. Nahvi

Given simple $\Gamma$, there is $\sigma$ such that $rk A(\Gamma, \sigma) = n$ iff $\Gamma$ has a $\{1, 2\}$-factor. [Annot. 10 Dec 2020.] (SG: Adj)

S. Akbari, A. Ghafari, M. Nahvi, & M.A. Nematollahi

The graphs are $\{\pm 1, \pm i\}$-gain graphs. (gg: Adj: Eig)

Saieed Akbari, Ebrahim Ghorbani, Jack [Jacobus] H. Koolen, & Mohammad Reza Oboudi

The sign-corrected coefficients of the characteristic polynomial of $L(-\Gamma)$ dominate those of $L(+\Gamma)$. [Problem 1. Prove they dominate those of $L(\Gamma, \sigma)$ for any $\sigma$. Problem 2. Generalize to any pair of signatures of $\Gamma$.] [Annot. 22 Nov 2010.] (Par: Eig, Incid)


S. Akbari, H.R. Maimani, & L. Parsaei Majd

Spectra of $K_n(-M)$ and $K_{p,q}(-M)$ for a matching $M$. A family of $(K_n, \sigma)$ with symmetric spectrum. [Cf. Etsuo Segawa & Yusuke Yoshie (20xxa).] [Annot. 17 Dec 2020.] (SG: Adj: Eig)

J. Akiyama, D. Avis, V. Chvátal, & H. Era
Bounds for $D(\Gamma)$, the largest frustration index $l(\Gamma, \sigma)$ over all signings of a fixed graph $\Gamma$ (not necessarily simple) of order $n$ and size $m = \#E$.

Main Thm.: $\frac{1}{2}m - \sqrt{mn} \leq D(\Gamma) \leq \frac{1}{2}m$. Thm. 4: $D(K_t,t) \leq \frac{1}{2}t^2 - c_0 t^{3/2}$, where $c_0$ can be taken as $\pi/480$. Probabilistic methods are used. Thus, Thm. 2: Given $\Gamma$, $\Prob(l(\Gamma, \sigma) > \frac{1}{2}m - \sqrt{mn}) \geq 1 - \left(\frac{2}{e}\right)^n$. Moreover, let $n_b(\Sigma)$ be the largest order of a balanced subgraph of $\Sigma$.

Thm. 5: $\Prob(n_b(K_n, \sigma) \geq k) \leq \binom{n}{k}/2^{(k^2)}$. (The problem of evaluating $n - n_b$ was raised by Harary (1959b).) Finally, Thm. 1: If $\Sigma$ has vertex-disjoint balanced induced subgraphs with $m'$ edges, then $l(\Sigma) \leq \frac{1}{2}(m - m')$.

[See Poljak and Turzík (1982a) for an upper bound on $D(\Gamma)$, Solé and Zaslavsky (1994a) for lower and (bipartite) upper bounds; Brown and Spencer (1971a), Gordon and Witsenhausen (1972a) for $D(K_t,t)$; Harary, Lindström, and Zetterström (1982a) for a result similar to Thm. 1.]

Tatsuya Akutsu, Sven Kosub, Avraham A. Melkman, & Takeyuki Tamura


Tatsuya Akutsu, Avraham A. Melkman, & Takeyuki Tamura


M.J. Alava, P.M. Duxbury, C.F. Moukarzel, & H. Rieger


Abdullah Alazemi, Milica Andelić, Francesco Belardo, Maurizio Brunetti, & Carlos M. da Fonseca

The gain group is \( \{ \pm 1, \pm i \} \). (GG: LG, LG(Gen), Eig, Incid)

Şahin Albayrak
See J. Kunegis.

István Albert
See A. Saadatpour.

J. James Albert
See Santhi. M.

Réka Albert
See A. Saadatpour.

Siavash Alemzadeh
See M.H. de Badyn.

John C. Alessio
Signed digraphs and graphs, with weights, used to describe a novel kind of “balance”, different from normal signed-graph balance, based on exchange between persons. [Annot. 24 Jan 2016.]
(PsS: SD, SG, WG: Bal(Gen))

S. Alexander & P. Pincus
Certain all-negative signed graphs where every edge is in a triangle: \( d = 2 \)-dimensional triangular lattice and \( d \geq 3 \)-dimensional face-centered cubic lattice. Phase phenomena depend on the parity of \( d \). Odd \( d \) implies interesting infinities of switchings with minimum \#E\(^{-}\). [Annot. 12 Aug 2012.] (Par: Phys)

Leonidas G. Alexopoulos
See I.N. Melas.

Artiom Alhazov, Ion Petre, & Vladimir Rogojin

Noga Alon
Lower bound on the largest bipartite subgraph of a simple graph with \( m \) edges. [I.e., upper bound on \( l(\Gamma) \). Problem. Generalize to \( l(\Sigma) \).] [Annot. 8 Mar 2011, 19 May 2012.] (sg: par: Fr)

Noga Alon & Yoshimi Egawa
Proves and improves a conjecture of B.D. Acharya (1983a). Thm.: The minimum number of colors for an “even edge coloring” = minimum
number of colors so each color class is bipartite = ⌈log₂χ(Γ)⌉. [Zaslavsky (1987b) generalizes the latter to Σ.](par: bal: Gen)

Noga Alon, Gregory Gutin, Eun Jung Kim, Stefan Szeider, & Anders Yeo

Extended abstract of (2011a). (SG: Alg)


Noga Alon, Konstantin Makarychev, Yury Makarychev, & Assaf Naor

Extended abstract of (2006a).


N. Alon & T.H. Marshall

(E.g., sign-colored graphs.) Thm.: For edge-colored graphs ∃ universal planar target. Bounds on target. [Relevant for signed-graph homomorphisms.] [Annot. 28 Dec 2019.] (sgc(Gen): Hom)

Stefanu Elias Aloysius, ed.


Yu. A. Al’pin

§2, “Harary’s theorem for digraphs over groups”: Thm. 3. A gain digraph (D, ϕ, ⋄), where D is strongly connected, is cycle balanced (every cycle has gain 1) ⇔ every closed walk has gain 1 ⇔ for each u, v all uv-paths have the same gain ⇔ ∃ potential for ϕ. This makes explicit properties implicit in previous work, e.g., Belitskiĭ and Lyubich (1984a). [This actually generalizes Harary-Norman-Cartwright (1965a), Thm. 13.11 for signed digraphs, not Harary (1953a) for undirected graphs.] Thm. 4. Two potentials θ, θ′ for ϕ differ by θ′ = hθ for some h ∈ ⋄. Ap-

[The important aspect: Strong connection suffices for gain-graph properties; the arcs need not be reversible as in a gain graph.] [Annot. 22 Feb 2021.] (GG: Bal, Appl)

Claudio Altafini
See also N. Ballber Torres, G. Facchetti, A. Fontan, G. Iacono, G.D. Shi, and N. Soranzo.

2012a Dynamics of opinion forming in structurally balanced social networks. PLoS ONE 7 (2012), no. 6, article 38135, 8 pp. + 6 supplements. URL https://doi.org/10.1371/journal.pone.0038135 (SD, sg: Bal, Dyn, PsS)


Claudio Altafini & Gabriele Lini

Dora Altbir
See E.E. Vogel.

Randolf Altmeyer
See D. Feng.

[Susan S. D’Amato]
See S.S. D’Amato (under ‘D’).

Meirav Amram, Robert Shwartz, & Mina Teicher

The structure of a quotient of a generalized Coxeter group depends on the presence of loops in the associated signed graph. [Annot. 17 Dec 2011.] (SG)

C. Amoruso & A.K. Hartmann

Xinhui An

Milica Andelić
See also A. Alazemi and S.K. Simić.
Milica Anđelić, Carlos M. da Fonseca, Slobodan K. Simić, & Dejan V. Tošić

Bounds on $\lambda_1(L(\Gamma))$ where $\Gamma$ is a nested split graph. (Cf. Cvetković, Rowlinson, and Simić (2007b), which shows nested split graphs maximize $\lambda_1(L(\Gamma))$.) [Annot. 2 Feb 2012.] (sg: par: Eig)


Milica Anđelić, Tamara Koledin, & Zoran Stanić


20xxb Bounds for Laplacian eigenvalues of signed graphs with given frustration number. Submitted. (SG: Lap: Eig: Fr)

Milica Anđelić & Slobodan K. Simić

Lars Døvling Andersen & Douglas D. Grant

Cf. Zelinka (1976b). If there are no coherent circles, no loops, and no parallel edges with the same orientation, then $\#E \leq 4n - 4$ (equality is characterized) and $\delta \leq 6$. Sufficient conditions for an antidirected Hamiltonian circle. Dictionary: “polar graph” = switching class of bidirected graphs, “homopolar circuit” = antidirected circle. [Later work, only on digraphs: e.g., cf. Diwan, Frye, Plantholt, and Tipnis (2011a).] [Annot. 27 Jul 2013.] (gg: Circ: Str)(sg: Ori: Circ)

P.W. Anderson
See S.F. Edwards.

Ascensión Andina-Díaz
See A. Parravano.

Kazutoshi Ando & Satoru Fujishige


Kazutoshi Ando, Satoru Fujishige, & Takeshi Naitoh

A balanced bisubmodular system corresponds to a bidirected graph that is balanced. The “flows” are arbitrary capacity-constrained functions,
not satisfying conservation at a vertex. (sg: Ori, Bal)

**Kazutoshi Ando, Satoru Fujishige, & Toshio Nemoto**


**Thomas Andreae**


**D. Angeli, M. Banaji, & C. Pantea**


**David Angeli, Patrick De Leenheer, & Eduardo Sontag**


Dictionary: “J-graph” = a signed graph of a Jacobian matrix. “Species-reaction graph” (“SR-graph”) = signed bipartite graph \((V_S, V_R, E, \sigma) =: \Sigma; \text{reaction graph” (“R-graph”) } =: -\Sigma^2; V_R; \text{“species graph” (“S-graph”) } = -\Sigma^2; V_S [\text{where } \Sigma^2 \text{ is the distance-2 signed graph: } V(\Sigma^2) := V_R \cup V_S, T_iT_j \in E^*(\Sigma^2) \iff \exists \text{ path } T_iT_kT_j \text{ with } \sigma(T_iT_kT_j) = \varepsilon]. \text{ Dictionary: “Simple loop” } \approx \text{ circle; “positive-loop property” } = \text{ balance. In a signed bipartite graph, “e-loop, o-loop” } = \text{ circle with } (-1)^{\#C/2}\sigma(C) = + \text{ or } -.

Prop. 4.5: \(\Sigma^2; V_R \text{ is antibalanced iff all circles in } \Sigma \text{ are e-loops and max } \deg(\Sigma; V_S) \leq 2. \text{ Thm. 1 (oversimplified): A certain differential system is monotone iff } \Sigma^2; V_R \text{ is antibalanced (the } R\text{-graph is balanced).} \] [Annot. 19 Feb 2010.] (SG: Bal, sw, Geom, Chem)

[A bipartite multiplicative gain graph \(\Phi := (V_S, V_R, E, \varphi) \text{ may be defined by } \varphi(S_iR_j) := (\text{a value from the stoichiometry matrix } \Gamma). \text{ Circle } C \text{ is “unitary” if } (-1)^{\#C/2}\varphi(C) = +1.] \Phi \text{ is implicated in the proof of geometrical Lemma 6.1.} \] [Annot. 19 Feb 2010.] (gg: Bal, Geom)

**David Angeli & Eduardo Sontag**


§IV, “Graphical conditions for strong monotonicity”: Mentions signed (di)graph balance and monotonicity. [Annot. 1 Jan 2012.]

(SD, SG: Bal, Chem: Exp)


§II, p. 167, mentions signed (di)graph balance and monotonicity. [Annot. 1 Jan 2012.]


J.C. Angles d’Auriac & R. Maynard

Signed square lattice graph: frustration index and ground states (minimum $\#E^-$ of switched $\Sigma$) via matching [cf. Katai and Iwai (1978a), Barahona (1981a), (1982a)]. Observed: natural clusters with relatively fixed spins (vertex signs) if the density of negative edges is in $(0.1, 0.2)$.
[Annot. 18 Aug 2012.]

Marcin Anholcer, Bartłomiej Bosek, & Jarosław Grytczuk

The “orientation” maps $I \rightarrow \mathbb{C}^*$, where $I$ is the set of vertex-hyperedge incidences.

Achu Aniyan & Sudev Naduvath

Induced signed graph $\Sigma_f$ of $\Gamma$: given $f : V \rightarrow \mathbb{Z}$, $\sigma(uv) := (-1)^{f(u)−f(v)}$.
§2: $f(u) = d(u)$, degree. Thm. 2.6: $\Sigma_d$ is balanced. Thm. 2.7: $−\Sigma_d$ is balanced iff $\Gamma$ is bipartite. §3: $f(u) = \varepsilon(u)$, eccentricity. Similar results. [Elementary. Balance of all $\Sigma_f$ by Harary (1953a). $−\Sigma_f$ property because $\Sigma_f$ is balanced.] §3: $f(u) = \varepsilon(u)$. Similar results. Dictionary: “switched signed graph” = negation $−\Sigma$, not a switching of $\Sigma$.
[Annot. 26 Sep, 13 Oct 2020.]


Cf. (2020a). $\sigma(uv) := (-1)^{d(u)d(v)}$. Simple results. [Thms. 2, 4, 5, 7 are wrong.] [In general, define $\sigma(uv) := (-1)^{f(u)f(v)}$, then $E^− = E:V_{odd}$]
Models for the evolution of a signed $K_n$ towards balance, with conclusions about the probable long-term behavior. A “state” of the graph is a signature. The unit of time $t$ is $\#E = \binom{n}{3}$ steps of the process. The density of edges is $\rho := \#E^+/\binom{n}{3}$. The number of triangles with $k$ negative edges (type $k$) is $N_k$; their density is $n_k := N_k/\binom{n}{3}$. The average density of type $k$ triangles on a positive edge is $n_k^+ = (3-k)N_k/(n-2)\#E^+ = (3-k)n_k/(3n_0+2n_1+n_2)$. Similarly, $n_k^- = kn_k/(3n_0+2n_1+n_2)$.

“Local triad dynamics”: At each step a random triangle $T$ is chosen. If it is all negative, a random edge in $T$ is chosen and negated. If it has one negative edge, a random edge in $T$ is chosen and negated with probability $p$ if it is negative and $1-p$ if positive. If it is balanced there is no change. The process is repeated ad infinitum. Finite $n$: For $p > 1/2$ the graph reaches all-positivity (“paradise”) in time $C \log t$ and for $p = 1/2$ in time $C/\sqrt{2t}$. For $p < 1/2$ the graph reaches a balanced state which is not all positive, in super-exponential time. (Time is in the units described.) “Infinite” $n$ [i.e., $n \to \infty$]: For $p < 1/2$ the density of negative edges approaches the stationary value $(1+\sqrt{3(1-2p)})^{-1}$. For $p > 1/2$ the network approaches all-positivity. Thus, at $p = 1/2$ there is a phase transition. Differential equations arise in the densities, with coefficients $\pi^+, \pi^-$ where $\pi^\varepsilon := \text{the probability that, in one step, the sign change is from } \varepsilon \text{ to } -\varepsilon$; thus $\pi^+ = (1-p)n_1$ and $\pi^- = pn_1 + n_3$. A stationary state has $\pi^+ = \pi^-$. For infinite $n$ the stationary states are in § III.B and temporal evolution of $\rho = \rho(t)$ is treated in § III.C. Finite $n$ is in § III.D.

“Constrained triad dynamics”: An edge is chosen randomly and is negated with probability 1 if the number of positive triangles increases, 0 if the number decreases, and 1/2 if the number remains the same. This corresponds to an Ising model with Hamiltonian $-\sum \sigma_i \sigma_j \sigma_k$, summed over all edge triples that form a triangle. This model approaches balance in time $C \log t$ with high probability if $n$ is large. The other alternatives are to reach an unbalanced absorbing state, where every edge is more positive than negative triangles (a “jammed state”), or a trajectory where every edge is in equally many triangles of each sign (a “blinker”). Blinkers were not observed in the simulations. The probability of a jammed state decreases quickly as $n \to \infty$. The “final” state, if balanced, has Harary bipartition $V = V_1 \bar{\cup} V_2$. For $\rho(0) \lesssim .4$, $\#V_1/\#V_2 \approx 1$. As $\rho(0) \to \beta \approx .65$, $\#V_1/\#V_2 \to \infty$, i.e., one set becomes dominant. When $\rho(0) > \beta$, $V_1 = V$ and all edges are positive. (§ IV.B) A jammed state can occur only when $n = 9$ or $n \geq 11$ (§ IV.C), e.g., certain 3-cluster states as in Davis (1967a). The number of jammed signatures $> 3^n \gg 2^{n-1} = \text{number of balanced ones}$, notwithstanding that the

Proposed research: Allow type 3 triangles (i.e., clustering). Allow incomplete graphs.

Dictionary: “network” = complete graph. [Annot. 27 Apr 2009.]

**SG: KG: Dyn: Bal**


Similar to (2005a), with some details omitted and some additional results. [Annot. 27 Apr 2009.]

**SG: KG: Dyn: Bal**

St. Antohe & E. Olaru


A “congruence” is an equivalence relation $R$ on $V(\Sigma)$ such that no negative edge is within an equivalence class. The quotient $\Sigma / R$ has the obvious simple underlying graph and signs $\sigma(\bar{x}y) = \sigma(xy)$ [which is ambiguous]. A signed-graph homomorphism is a function $f : V_1 \to V_2$ that is a sign-preserving homomorphism of underlying graphs. [This is inconsistent, since the sign of edge $f(x)f(y)$ can be ill defined. The defect might perhaps be remedied by allowing multiple edges with different signs or by passing entirely to multigraphs.] The canonical map $\Sigma \to \Sigma / R$ is such a homomorphism. Composition of homomorphisms is well defined and associative; hence one has a category $\text{Graph}^{\text{sign}}$. The categorial product is $\prod_{i \in I} \Sigma_i := \text{Cartesian product of the } |\Sigma_i| \text{ with the component-wise signature } \sigma((\ldots, u_i, \ldots)(\ldots, v_i, \ldots)) := \sigma_i(u_i,v_i)$. Some further elementary properties of signed-graph homomorphisms and congruences are proved. [The paper is hard to interpret due to mathematical ambiguity and grammatical and typographical errors.] (SGc: Hom)

Divya Antoney, Tabitha Agnes Mangam, & Mukti Acharya

20xxa Signed graphs from proper coloring of graphs. Submitted.

Parity signs (cf. Acharya and Kureethara (20xxa)) on $\Gamma$ derived from minimal proper colorings. [Annot. 12 Mar 2021.]

**SG: Lab**

Katsuaki Aoki

See M. Iri.

Mustapha Aouchiche & Pierre Hansen


Computer-generated conjectures. §4, “Signless Laplacian”: Several computer-generated conjectures about eigenvalues of $L(\Gamma)$; some are proved (mainly in Cvetković, Rowlinson, and Simić (2007b)) or disproved; some are difficult. [Question. How many generalize to all $\Sigma$, with or without proofs?] [Annot. 22 Jan 2012.]

(par: Lap: Eig)

§6, “Spectral invariants”: §6.3, “The eigenvalues of the signless Laplacian matrix”: Nordhaus–Gaddum-type relations imply theorems from Gutman, Kiani, Mirzakhah, and Zhou (2009a) about the eigenvalues, singular values, incidence energy of $L(\Gamma)$. Conjecture 6.19, generated by a computer—cf. (2010a): $\lambda_1(L(\Gamma)) + \lambda_1(L(\Gamma^c)) \leq 3n - 4$; $\lambda_1(L(\Gamma)) \cdot \lambda_1(L(\Gamma^c)) \leq 2n(n - 2)$; = iff $\Gamma$ is a star. [Annot. 22 Jan 2012.] (par: Lap: Eig)


The “distance signless Laplacian” is $D_L(\Gamma) := D + D(\Gamma)$, where $D$ = diagonal degree matrix, $D$ = distance matrix. Contrasts to the “distance Laplacian” $D_L(\Gamma) := D - D(\Gamma)$, in analogy to the Laplacian matrix $L(\Gamma) = D - A$ vs. signless Laplacian $L(\Gamma) = D + A$.) [Question. Is there a signed-graphic distance matrix $D(\Sigma)$ generalizing $D(\Gamma)$ and $-D(\Gamma)$, analogously to $A(\Sigma)$? E.g., is distance algebraically additive?] [Annot. 20 Mar 2016.] (sg: par: Eig)


**Mustapha Aouchiche, Pierre Hansen, & Claire Lucas**


(par: Lap: Eig)

**Gautam Appa**

See also L.S. Pitsoulis.

**Gautam Appa & Balázs Kotnyek**


2-regular matrices include binet matrices (2006a). A key property of $k$-regular matrices is that solutions of integral equations are $1/k$-integral. (sg: Incid: Ori)


Binet matrices are the network matrices of bidirected (or signed) graphs. Basic theory of binet matrices, generalizing that of network matrices, notably half-integrality theorems. [For a slight simplification see Bolker and Zaslavsky (2006a).] (sg: Incid: Ori)

**Gautam Appa, Balázs Kotnyek, Konstantinos Papalamprou, & Leonidas Pitsoulis**


**Julio Aracena**

See also J.-P. Comet, J. Demongeot, and M. Montalva.
A regulatory Boolean network $N$ is built on a signed digraph $D$. Thm. 6: If all (directed) cycles are positive then $N$ has at least 2 fixed points. Thm. 9: $N$ has at most $2^p$ fixed points, where $p :=$ minimum number of vertices that cover all positive cycles [unusually, not negative cycles!], and this is best possible. [Annot. 9 July 2009.]

J. Aracena, S. Ben Lamine, M.A. Mermet, O. Cohen, & J. Demongeot


(JD: Biol: Dyn: Fr(Gener))

Julio Aracena, Jacques Demongeot, & Eric Goles


Existence and upper bound on the number of fixed points of a “discrete neural network” $\mathcal{N}$, which consists of a real $n \times n$ matrix $W$, the associated signed digraph $D$ of order $n$, and a real vector $b$. A state is $x \in \{-1,+1\}^n$. A transition is $x \mapsto f(x) := \text{sgn}_+(Wx - b)$ where $\text{sgn}_+(t) := \text{sgn}(t)$ except $\text{sgn}_+(0) := +1$. Assume: $D$ is connected; no component of $f$ is constant, hence a cycle exists. Lemma 1: A cycle is positive iff it has a satisfied state. Thm. 1: If all cycles are positive, $f$ has a fixed point. Thm. 2: If all cycles are negative, $f$ has no fixed point. Thm. 3: $\#\{\text{fixed points}\} \leq 2^p$ where $p := \min(\text{size of vertex cover of positive cycles})$, and this is sharp. Dictionary: “positive feedback vertex set” = vertex cover of positive cycles = vertex set that covers all positive cycles; “circuit” = (directed) cycle; $1 = \text{sgn}_+$. [Annot. 20 July 2009.]

(JD: Biol: Dyn: Fr(Gener))

J. Aracena, E. Fanchon, M. Montalva, & M. Noual


Dictionary: “labeled digraph” = signed digraph. (JD: Dyn)

J. Aracena, E. Goles, A. Moreira, & L. Salinas


A “labelled digraph” is essentially a signed digraph. (JD: Dyn)(sd: Dyn, Biol)

Julio Aracena, Mauricio González, Alejandro Zuñiga, Marco A. Mendez, & Verónica Cambiazo


Figs. 2, 3 show particular proposed genetic regulatory networks based on signed digraphs. §3.2 describes how the mathematical model of Aracena, Ben Lamine, *et al.* (2003a) and Aracena, Demongeot, and Goles (2004a) applies to the situation of this paper. [Annot. 20 July 2009.]

(SD: Dyn: Appl(Biol))
Julio Aracena, Adrien Richard, & Lilian Salinas

Julián Aráoz, William H. Cunningham, Jack Edmonds, & Jan Green-Krótki

Marina Arav, Frank J. Hall, Zhongshan Li, & Hein van der Holst

Marina Arav, Hein van der Holst, & John Sinkovic

Marina Arav, Hein van der Holst, & John Sinkovic


Dan Archdeacon

The medial graph of $\Gamma \subset S$, a graph embedded in a surface, is a 4-regular graph $\overline{M} \subset S$ that encodes $\Gamma$ and its surface dual. Gains (“voltages”) on $\Gamma$ transfer to gains (“voltages”) on $\overline{M}$. Gains have both gains and signs; signs are determined by face orientations. [Question. Does this suggest a gain-graphic, surface-embedding theory of 4-regular gain graphs? It gives such a theory under certain conditions: faces are 2-colorable and black face boundaries must have balanced gains.] Dictionary: “straight, twisted edge” = positive, negative edge. [Annot. 16 Jan 2012, rev 20 Nov 2016.] (Top: GG, SG, D)

A compilation from various sources and contributors, updated every so often. “The genus sequence of a signed graph”, p. 10: A conjecture due to Širáň (?) on the demigenus range (here called “spectrum” [though unrelated to matrices]) for orientation embedding of Σ, namely, that the answer to Question 1 under Širáň (1991b) is affirmative. [The term “parity embedding” is mistakenly used for orientation embedding of any signed graph; parity embedding is of an unsigned graph.] (SG: Top)

MR 1411236 (98g:05044). Zbl 897.05026.

§2.5 describes orientation embedding (called “signed embedding” [although there are other kinds of signed embedding]) and switching (called “sequence of local switches of sense”) of signed graphs with rotation systems. §5.5, “Signed embeddings”, briefly mentions Širáň (1991b), Širáň and Škoviera (1991a), and Zaslavsky (1993a), (1996a). (SG: Top: Exp)


Dan Archdeacon & Marisa Debowsky

Similar to Archdeacon–Širáň (1998a), but for the projective plane. (SG, Sw: Top)

Dan Archdeacon, Joan Hutchinson, Atsuhiro Nakamoto, Seiya Negam [Seiya Negami], & Katsuhiro Ota


Dan Archdeacon & Jozef Širáň

A “claw” consists of a vertex and three incident half edges. Let C be the set of claws in Γ and T the set of theta subgraphs. Fix a rotation of each claw. Call t ∈ T an “edge” with endpoints c, c′ if t contains c and c′; sign it + or − according as t can or cannot be embedded in the plane so the rotations of its trivalent vertices equal the ones chosen for c and c′. This defines, independently (up to switching) of the choice of rotations, the “signed triple graph” T±(Γ). Theorem: Γ is planar iff T±(Γ) is balanced. (SG, Sw: Top)

Federico Ardila
2002a The Tutte polynomial of a hyperplane arrangement (extended abstract).


Ch. 2 is published as (2007a). [Annot. 4 Oct 2014.]


Applies the “finite field method” of Athansiadis (1996a) (in §5 this is actually the modular gains method for computing characteristic polynomials of integral gain graphs), cleverly extended, to compute Tutte polynomials (in the equivalent form of coboundary polynomials) of various gain graphs (in the equivalent form of affine hyperplane arrangements).

Thms. 4.2, 4.3: Coboundary polynomial of complete signed graph \(\pm K_n^*\) (“\(B_n\) arrangement”) and complete signed link graph \(\pm K_n\) (“\(D_n\) arrangement”) in terms of generating functions. Thm. 4.4: Balanced coboundary polynomial of a contrabalanced multigraph \((\Gamma, \emptyset)\) (coboundary polynomial of “\(A^\#\) arrangement” [equivalently, the bicircular lift matroid \(L(\Gamma, \emptyset)\)]). [Also computed, slightly generalized, in Zaslavsky (1995b), Ex. 3.4, \(q_{\{\emptyset, \Gamma\}}\).] Thm. 4.5: Coboundary polynomial of \(\mp K_n\) (“threshold arrangement” \(\mathcal{T}_n = \mathcal{H}[-K_n]\)).

§5, “Deformations of the braid arrangement”: Balanced subgraphs of \(A\)-expansions \(A \cdot K_n\), where \(A\) is a finite set of integers, are employed to compute the coboundary polynomials of integral deformations \(\mathcal{H}[A \cdot K_n]\) of the complete-graph (“braid”) arrangement \(\mathcal{H}[K_n]\). Prop. 5.8 treats the Catalan arrangement \(\mathcal{H}\{0, \mp 1\} \cdot K_n\) and its subarrangements. Thm. 5.14 treats the Catalan arrangement. Prop. 5.9 and Thm. 5.11 treat the Lineal arrangement \(\mathcal{H}\{1\} \cdot K_n\). Thm. 5.12 treats the Shi arrangement \(\mathcal{H}\{0, 1\} \cdot K_n\). Thm. 5.13 treats the semiorder arrangement \(\mathcal{H}\{\pm 1\} \cdot K_n\).

Dictionary: “finite field method” usually = modular gains method (Zaslavsky (2002a), §11.4 after (11.3); or see Berthomé et al. (2009a), Lemma 6.3); “type” = gain of edge; “planted graded graph with height function” = balanced integral gain graph with potential function; “planted graded \(A\)-graph” = balanced subgraph of \(A \cdot K_n\); \(\mathcal{E}_n = \mathcal{H}[A \cdot K_n]\); “threshold arrangement” = all-negative complete graph arrangement.

Federico Ardila, Federico Castillo, & Michael Henley


Extended abstract of (2015a).

(SG: M(Gen): Invar, Enum)(SG: Geom: Exp)

These invariants of signed graphs are obtained by a substitution in generating functions of the ordinary Tutte polynomials. G.f. computed for $A_{n-1}, B_n, C_n, D_n$ with respect to integer, root, and weight lattices. §4.1.2, “Enumeration of signed graphs”: Counted by several parameters, e.g., number of components without a negative cycle. §4.2, “From classical root systems to signed graphs”: Expository. $A_{n-1}, B_n, C_n, D_n \leftrightarrow \pm K_n, \pm K_n', \pm K_n, \pm K_n, ' \,$ denoting half edges [but $C_n$ should $\leftrightarrow \pm K_n$, with negative loops]. §4.3, “Computing the Tutte polynomials by signed graph enumeration”. §5, “Arithmetic characteristic polynomials”: Explicit, for some cases. Dictionary: “loop” = half edge (Lemma 4.9 depends on that). [Lemma 4.9: special case of Zaslavsky (1982a), Lemma 8A.2.]

*Question.* How to compute arithmetic Tutte polynomials and characteristic polynomials for other signed graphs? Preferably, directly. How does the lattice framework fit in? [Annot. 26 May 2018.]

(FG: M(Gen): Invar, Enum)(SG: Geom: Exp)

Federico Ardila & Alexander Postnikov

The polynomials $\tilde{Z}_A$ (Sokal’s (2005a) “multivariate Tutte polynomial”), slightly normalized, and $S_A$ are specializations of Traldi’s (1989a) “weighted dichromatic polynomial”, hence of Zaslavsky’s (1992b) “parametrized” dichromatic and corank-nullity polynomials. [Annot. 17 Oct 2017.]

(SGw(Gen): Invar, Geom)

Samin Aref


Samin Aref, Andrew J. Mason, & Mark C. Wilson

Samin Aref & Zachary Neal

Two-step algorithm for the NP-complete problem of frustration index \(l(\Sigma)\): first find upper and lower bounds by fast methods (by examples and LP relaxation), then use that to control the amount of computation. It also finds an optimal bipartition. [Annot. 15 Jul 2019.]

Samin Aref & Mark C. Wilson


Alex Arenas
See S. Gómez.

Srinivasa R. Arikati & Uri N. Peled

Given a graph with edges weighted from a group. The weight of a path is the product of its edge weights in order (not inverted, as with gains). Problem: to determined whether between two given vertices there is a chordless path of given weight. This is NP-complete in general but for chordal graphs there is a fast algorithm (linear in \((#E + #V) \cdot (\text{group order})\)). [Question. What if the edges have gains rather than weights?]


Nejat Arınık
Nejat Arinik, Rosa Figueiredo, & Vincent Labatut


20xxa Multiplicity and diversity: Analyzing the optimal solution space of the correlation clustering problem on complete signed graphs. J. Complex Networks (in
20xxb 2-Edge connected balanced subgraphs for correlation clustering problem. HAL hal-02428305.

Esther M. Arkin & Christos H. Papadimitriou

Reinterpreted: Treat a mixed graph as a bidirected graph B where a walk can only enter a vertex along an in-arrow (thus introverted edges are irrelevant). Given integral gains in $\mathbb{Z}^+$, finding negative cycles is NP-complete, though it is P if B is all positive or all negative. Also, “it is NP-complete to find shortest cycles . . ., even when the graph is known not to have negative cycles.” [Annot. 18 Sep 2018.] (SG: Clu: Alg) (ori: OG: Cyc)


As in (1985a), finding a feasible circulation is NP-complete for a general bidirected B but is P for all-negative (and all-positive) B. [Annot. 18 Sep 2018.] (ori: OG: Flows)

Drew Armstrong


Drew Armstrong & Brendon Rhoades


The integral gain graphs $0K_n \cup 1\vec{1}$ (the Shi gain graph, if $\Gamma = K_n$) and $0K_n \cup \{ie_{ij} : i \in [j−1]\}$ (the Ish gain graph, if $\Gamma = K_n$) have the same chromatic polynomial (but not Tutte polynomial) and the same numbers of acyclic orientations with specified properties. [$\vec{1}$ has $V = [n]$ and edges oriented upwards.] Proofs are [in effect] by counting proper integral colorations modulo a large prime. The results and proofs are cast in terms of the Shi and Ish hyperplane arrangements; cf. Athanasiadis
(1996a), whose method is used, called the “finite field method”. [Cf. Leven, Rhoades, and Wilson (2014a).] [Annot. 14 Mar 2013.]

S. Arockiaraj
See P. Jeyalakshmi.

A. Aromsawa and J. Poulter

Ashraf P K
See also K.A. Germina and N.K. Sudev.

P.K. Ashraf & K.A. Germina


Definition (from B.D. Acharya (2013a)): \( D \subseteq V(\Sigma) \) is “dominating” if for some vertex signature \( \mu \), \( u \notin D \) implies \( \exists v \in N(u) \cap D \), and \( \mu(v) = \sigma(uv)\mu(u), \forall v \in N(u) \cap D \). Characterizes minimal such sets \( D \) (dominating in \( |\Sigma| \) and three side conditions), and more. [Annot. 24 Mar 2017.]


Double dominating set \( D \) of \( \Sigma \): double dominating set for \( |\Sigma| \) such that the cut \( E(D, D^c) \) is balanced. \( \gamma_{\times 2}(\Sigma) := \min |D| \) can be 2 or \( n \). (SG)

P.K. Ashraf, K.A. Germina, & N.K. Sudev

Set-valuation: \( f : V \to P(X) \) (injective), \( X \) a set, \( f^\oplus(vw) := f(v) \oplus f(w) \); define \( \Sigma \) by \( \sigma(vw) := \sigma(f(v)|f(w)) \). Long proofs, results trivial or false except Thms. 2.9–10: \( \Sigma \) has a set-valuation iff balanced. [Quick proof: switch by \( \zeta(v) = (-1)^{|f(v)|} \).] [Annot. 3 Mar 2019.] (SG: Bal)


Ali Reza Ashrafi
See Z. Yarahmadi.

Jalal Askari, Ali Iranmanesh, & Kinkar Ch Das

Seidel–Estrada index := \( \sum e^\lambda \) over all eigenvalues of \( A(K_1) \). Bounds, and comparison with Seidel energy \( \sum |\lambda| \). [Annot. 9 Nov 2020.]

(sg: KG: Adj: Eig)
Fatihcan M. Atay

Fatihcan M. Atay & Bobo Hua

Fatihcan M. Atay & Shiping Liu

Fatihcan M. Atay & Hande Tunçel

Christos A. Athanasiadis

Treats the canonical lift representations (as affine hyperplane arrangements) of various gain graphs and signed gain graphs with additive gain group $\mathbb{Z}^+$. The article is largely a series of (sometimes brilliant) calculations of chromatic polynomials (mutatis mutandis, the characteristic polynomials of the representing arrangements) modulo a large integer $q$ using gain graph coloring, though disguised as applications of Crapo–Rota’s Critical Theorem. The fundamental principle is that, if $q$ is larger than the largest gain of a circle, then $\mathbb{Z}^+$ can be replaced as gain group by $\mathbb{Z}_q^+$ without changing the chromatic polynomial (a consequence of Zaslavsky (1995b), Thm. 4.2)—and the analog for signed gain graphs, whose theory needs to be developed. A non-graphical result of the general method is a unified proof (Thm. 2.4) of the theorem of Blass and Sagan (1998a).

§3, “The Shi arrangements”: these represent $\text{Lat}^b\{0,1\} K_n$ and signed-graph analogs. §4: “The Linial arrangement”: it represents $\text{Lat}^b\{1\} K_n$. §5, “Other interesting hyperplane arrangements”, treats: the arrangement representing $\text{Lat}^b AK_n$ where $A = \{-m,\ldots,m-1,m\}$ (which is the semilattice of $m$-composed partitions; see Zaslavsky (2002a), Ex. 10.5, also Edelman–Reiner (1996a)], and several generalizations, including to arbitrary sign-symmetric gain sets $L$ and to Weyl analogs; also, an antibalanced analog of the $A_n$ Shi arrangement (Thm. 5.4); and more. Most impressive result: Thm. 5.2: Let $A$ be a finite set of integers such that $0 \notin A = -A$ and let $A^0 = A \cup \{0\}$. For $\Phi = A^0 K_n$ and large integral $\lambda$, $\chi^*_{\Phi}(\lambda)/\lambda$ is the coefficient of $x^{\lambda-n}$ in $(1-x)^{-1} - f_A(x)/x$ where $f_A$ is the ordinary generating function for $A$. From this $\chi^*_{\Phi_{K_n}}(\lambda)/\lambda$ is derived.

The signed affinographic arrangements represent a kind of signed graph whose exact nature has not yet been penetrated by gain graph


The arrangement represents $\text{Lat}^b\{0,1\}K_n$. (gg: Geom, M, Invar)


The arrangements considered are the subarrangements of the projectivized Shi arrangements of type $A_{n-1}$ that contain $A_{n-1}$. Thms. 4.1 and 4.2 characterize those that are free or supersolvable. The extended Shi arrangements, representing $L_0([1-a,a]K_n)$ where $a \geq 1$, and a mild generalization, are of use in the proof. (gg: Geom, M, Invar)


§5, “Nonnesting partitions of fixed type”, has calculations like those in (1996a) for affinographic arrangements representing additional types of integral gain graph [of a kind that is not yet fully understood]. (gg: Geom, m, Invar)


The proof of Proposition 4.2 is essentially gain-graphic. (gg: m: Geom: Invar)


Christos A. Athanasiadis & Svante Linusson

**David Avis**  
See J. Akiyama.

**Remi C. Avohou, Joseph Ben Geloun, & Etera R. Livine**  

**F. Ayoobi, G.R. Omidi, & B. Tayfeh-Rezaie**  

**Ghodratollah Azadi**  
See V. Ghorbani.

**Habib Azanchiler**  
See V. Ghorbani.

**L. Babai & P.J. Cameron**  

Tournaments are treated as nowhere-zero GF(3)+-gain graphs based on $K_n$; “switching” is by negation in GF(3)+. [Cheng–Wells (1986a) treats all digraphs as $K_n$ with GF(3)+-gains. GF(3)+-gain switching differs from Babai–Cameron’s switching.] [Annot. rev. 7 Jan 2016, 4 Nov 2017.]

**Maxim A. Babenko**  


$O(mn^{2/3})$ algorithm for integral max flow, improving on Gabow (1983a), showing that max flow takes no longer on a bidirected graph than on a digraph. The time bound follows from an upper bound on the max flow value. Also, an acyclic flow of value $v$ is zero on all but $O(nv^{1/2})$ arcs. The technique involves transferring the flow to the double covering.
digraph. [Annot. 9 Sept 2010.] (sg: Ori: Flows, Alg, Cov)


The double covering graph of suitably oriented $-\Gamma$ [matching edges are introverted; nonmatching edges are extraverted] yields a proof that, if $\Gamma$ has a unique perfect matching $M$, then $M$ contains an isthmus. [Annot. 9 Sept 2010.]

Maxim A. Babenko & Alexander V. Karzanov


Optimization of integral odd-vertex flows on a bidirected graph, without or with capacities. [Annot. 9 Sept 2010.] (sg: Ori: Flows: Alg)


Minimizing the average weight in a cycle, or a closed trail, of an edge-weighted bidirected graph, in time $O(n^2 \min\{n^2, m \log n\})$. [Annot. 9 Sept 2010.] (sg: Ori: Alg)

[Jayapal Baskar Babujee]
See J. Baskar Babujee (under ‘Bas’).

Constantin P. Bachas


The frustration index decision problem on signed (3-dimensional) cubic lattice graphs is NP-complete. [Proof is incomplete; completed and improved by Green (1987a). Better result in Barahona (1982a).]

F. Bachmann
See B. Fierro.

Mathias Hudoba de Badyn, Siavash Alemzadeh, & Mehran Mesbahi


Y. Bagheri, A.R. Moghaddamfar, & F. Ramezani


The number of switching isomorphism classes of signed $\Gamma$. Thm. 3.2: For $\Gamma = GP(n, k)$ with prime $n > 5$ and $k \not\equiv \pm 1 \pmod{n}$, either $\text{Aut}[\Gamma, \sigma] = \text{Aut} GP(n, k)$ or $|\text{Aut}[\Gamma, \sigma]| \leq 2$. All 36 classes are shown for $GP(7, 2)$. [Annot. 29 Jul 2019.]

Chun-Hsiang Bai and Bang Ye Wu

Algorithm for frustration number $l_0(K_n, \sigma)$. [Annot. 5 Jun 2017.]

(G: KG: Fr: Alg)

G. David Bailey
20xxa Inductively factored signed-graphic arrangements of hyperplanes. Manuscript, under revision.


(SG: KG: Fr: Alg)

Keith Baker
See J.O. Morrissette.

V. Balachandran

(GN: M(bases))

V. Balachandran & G.L. Thompson

(GN: M)

R. Balakrishnan & K. Ranganathan

§10.6, “Application to social psychology”: Short introduction to balance in signed graphs. §10.7: Exercises on balance.

(SG: Bal: Exp)


§1.11, “Application to social psychology”: Short introduction to balance in signed graphs.

(SG: Bal: Exp)

R. Balakrishnan & N. Sudharsanam

$f : E(\Gamma) \rightarrow \mathbb{R}$ is “cycle-vanishing” if $f(C) := \sum_{e \in C} f(e) = 0$ for every circle. Thm. 3: $f$ is cycle-vanishing iff $f(S) = 0$ for every series class of non-isthmus edges. Thm. 4: dim{cycle-vanishing $f$} = $#E$ – number of series classes of non-isthmus edges. Thm. 5: Connected $\Gamma$ is 3-connected iff only $f = 0$ is cycle vanishing. [Specialized to a sign-weighted graph $\Sigma$, “cycle-vanishing” means $#E^+(C) = #E^-(C)$ for every circle. Thm. 3: $\sigma$ is cycle-vanishing iff every series class of non-isthmus edges has evenly many edges, half positive and half negative. Etc. [Cf. B.G. Xu (2009a), Vijayakumar (2011a) for generalization.] [Annot. 16 Oct 2011.]

(sgw: Gen)
P. Balamuralidhar
See also H.K. Rath.

P. Balamuralidhar & M.A. Rajan

Egon Balas


Linear (thus “fractional”, meaning half-integral) vs. integral programming solutions to maximum matching. The difference of their maxima = 1/2(max number of matching-separable vertex-disjoint odd circles). Also noted (p. 12): (max) fractional matchings in Γ correspond to (max) matchings in the double covering graph of −Γ. [Question. Does this lead to a definition of maximum matchings in signed graphs?] (par, ori: Incid, Geom, Alg, cov, Circ)

E. Balas & P.L. Ivanescu [P.L. Hammer]

A.R. Balasubramanian

The arrangements have hyperplanes of the form \(x_i + x_j = -l, -l + 1, \ldots, m - 1, m\) (integers) for \(i < j\). [Annot. 28 Jan 2020.] (gg: Geom, Invar)

K. Balasubramanian

Here a “signed graph” means, in effect, an acyclically oriented graph \(D\) along with the antisymmetric adjacency matrix \(A_\pm(D) = A(+D \cup -D^{-1}), D^{-1}\) being the converse digraph. [That is, \(A_\pm(D) = A(D) - A(D)^T\). The “signed graphs” are just acyclic digraphs with an antisymmetric adjacency matrix and, correspondingly, what we may call the ‘antisymmetric characteristic polynomial’. Proposes an algorithm for the polynomial. Observes in some examples a relationship between the characteristic polynomial of \(\Gamma\) and the antisymmetric characteristic polynomial of an acyclic orientation. (SD, wg: Eig: Invar: Alg, Chem)

Argues (heuristically) that a certain algorithm is superior to another, in particular for the antisymmetric polynomial defined in (1988a).

(SD: Eig: Invar: Alg)


Computed for graphs of six different cages of three different orders, in both ordinary and “signed” (see (1988a)) versions. Observes a property of the “signed graph” polynomials which is due to antisymmetry, as explained by P.W. Fowler (Comment on “Characteristic polynomials of fullerene cages”. *Chem. Phys. Letters* 203 (1993), 611–612).

(SD: Eig: Invar: Chem)


The “signed graphs” are as in (1988a). Simplified contents: It is shown by example that the antisymmetric characteristic polynomials of two nonisomorphic acyclic orientations of a graph (see (1988a)) may be equal or unequal. [Much smaller examples are provided by P.W. Fowler, Comment on “Characteristic polynomials of fullerene cages”. *Chem. Phys. Letters* 203 (1993), 611–612.]

[Question. Are there examples for which the underlying (di)graphs are nonisomorphic?] [For cospectrality of other kinds of signed graphs, see Acharya, Gill, and Patwardhan (1984a) (signed $K_n$’s).]

(SD: Eig: Invar)

R. Balian, J.M. Drouffe, & C. Itzykson


Gain group SO($n$) on a toroidal lattice graph (§ C, “Local invariance, gauge field, and minimal coupling), where SO(1) = {+1, −1} (developed in (1975a)). [Annot. 12 Aug 2012.]

(SG: Phys)


Dictionary: “Ising model” = signed hypercubical lattice, “gauge invariance” = switching invariance, “plaquette” = quadrilateral. The partition function depends on $p^+ − p^−$ where $p^\varepsilon = \#$ plaquettes with sign $\varepsilon$ and sometimes also $\#E^+ − \#E^-$. [Annot. 12 Aug 2012.]

(SG: Phys, Sw, Fr)

M.L. Balinski


Pp. 277–278 discuss integer programming problems on bidirected graphs in terms of the incidence matrix.

(ori: incid: par, Alg, Ref)

Igor Balla, Felix Dräxler, Peter Keevash, & Benny Sudakov


Núria Ballber Torres & Claudio Altafini

**Murad Banaji**

See also D. Angeli and N. Radde.


**Murad Banaji & Gheorghe Craciun**


**Murad Banaji & Carrie Rutherford**


**Afonso S. Bandeira, Amit Singer, & Daniel A. Spielman**


**Subhashish Banerjee**

See B. Adhikari.

**Jørgen Bang-Jensen, Stéphane Bessy, Bill Jackson, & Matthias Kriesell**


**Jørgen Bang-Jensen & Gregory Gutin**

1997a Alternating cycles and paths in edge-coloured multigraphs: A survey. *Discrete Math.* 165/166 (1997), 39–60. MR 1439259 (98d:05080). Zbl 876.05057. A rich source for problems on bidirected graphs. An edge 2-coloration of a graph becomes an all-negative bidirection by taking one color class to consist of introverted edges and the other to consist of extroverted edges. An alternating path becomes a coherent path; an alternating circle becomes a coherent circle. [General Problem. Generalize to bidirected graphs the results on edge 2-colored graphs mentioned in this paper.
Question. To what digraph properties do they specialize by taking the underlying signed graph to be all positive? [See e.g. Bárányi and Bárányi (1968a) (q.v.), Bang-Jensen and Gutin (1998a), Das and Rao (1983a), Grossman and Häggqvist (1983a), Mahadev and Peled (1995a), Saad (1996a).] (par: ori: Paths, Circ)


The longest coherent trail, having degrees bounded by a specified degree vector, in a bidirected all-negative complete multigraph that satisfies an extra hypothesis. Generalization of Das and Rao (1983a) and Saad (1996a), thus ultimately of Thm. 1 of Bárányi and Bárányi (1968a) (q.v.). Also, a polynomial-time algorithm. (par: ori: Paths, Alg)

M. Bárányi & Zs. Bárányi

Let B be a bidirected \(-K_{2n}\) which has a coherent 2-factor. (“Coherent” means that, at each vertex in the 2-factor, one edge is directed inward and the other outward.) Thm. 1: B has a coherent Hamiltonian circle iff, for every \(k \in \{2, 3, \ldots, n - 2\}\), \(s_k > k^2\), where \(s_k := \) the sum of the \(k\) smallest indegrees and the \(k\) smallest outdegrees. Thm. 2: The number of \(k\)’s for which \(s_k = k^2\) equals the smallest number \(p\) of circles in any coherent 2-factor of B. Moreover, the \(p\) values of \(k\) for which equality holds imply a partition of \(V\) into \(p\) vertex sets, each inducing \(B_i\) consisting of a bipartite [i.e., balanced] subgraph with a coherent Hamiltonian circle and in one color class only introverted edges, while in the other only extroverted edges. [Problem. Generalize these remarkable results to an arbitrary bidirected complete graph. The all-negative case will be these theorems; the all-positive case will give the smallest number of cycles in a covering by vertex-disjoint cycles of a tournament that has any such covering.] [See Bang-Jensen and Gutin (1997a) for further developments on alternating walks; also Busch, Jacobson, et al. (2013a), Busch, Mutar, and Slilaty (20xxa).] (par: ori: Paths, Alg)

Zs. Bárányi
See M. Bárányi.

C. Bankwitz

Introduces the sign-colored graph of a link diagram. [Further work by numerous writers, e.g., S. Kinoshita et al. and esp. Kauffman (1989a) and successors.] (Knot: SGc)

Nikhil Bansal, Avrim Blum, & Shuchi Chawla


Clusterability index $Q$ [minimum number of inconsistent edges; see Doreian and Mrvar (1996a) for notation] in signed complete graphs is NP-hard. Polynomial-time algorithms for approximate optimal clustering: up to a constant factor from $Q$ (§3); probably within $1 - \varepsilon$ of $\#E - Q$ for any $\varepsilon$ (i.e., maximizing consistent edges within $1 - \varepsilon$) (§4).

§3: A 2-clustering within $3Q_2$ (Thm. 2). A clustering within $cQ$ where $c \approx 20000$ (Thm. 13).

§4: A clustering within $\varepsilon n^2$ of $\#E - Q$ with high probability but slow in terms of $1/\varepsilon$ (Thm. 15). Asymptotically faster in terms of $1/\varepsilon$ (Thm. 22). The $1 - \varepsilon$ factor results from the fact that $\#E - Q = \binom{n}{2} - Q > \frac{1}{2}\binom{n}{2}$ [so is not strong].

§6: “Random noise”. §7: “Extensions”, considers edge weights in $[-1, 1]$ (thus allowing incomplete graphs). Thm. 23: An unweighted approximation algorithm will also approximate this case, assuming “linear cost”: $e$ costs $(1 - w(e))/2$ if within a cluster and $(1 + w(e))/2$ if between clusters. Thm. 24: The problem for clustering that minimizes the total weight of $+$ edges outside clusters and $-$ edges within clusters (“minimizing disagreements”) is APX-hard. [Improved in Charikar–Guruswami–Wirth (2003a), (2005a), Swamy (2004a). Generalized in Demaine et al. (2006a).]

Bo Bao, Rong Chen, & Genghua Fan


Assuming $G(\Sigma)$ has no coloop, $E$ can be covered by frame circuits so each edge is in exactly 6 circuits. [Annot. 28 Jan 2021.] (SG: flows)

R.B. Bapat

See also R. Singh.


§2.6, “0–1 Incidence matrix”. The rank and related properties of the the unoriented incidence matrix. [Cf. van Nuffelen (1973a).] [Annot. 25 Aug 2011.]

Ravindra B. Bapat, Jerrold W. Grossman, & Devadatta M. Kulkarni


Their “mixed graph” is a signed graph $\Sigma$: positive edges are called “directed” and negative edges “undirected”. The matrix-tree theorem is the unweighted case of Chaiken’s (1982a) all-minors theorem for signed graphs. The technical formalism differs somewhat. They point out that in case $U \cup W = V$, the minor is the sum of signed $UW$ matchings. Dictionary: “$k$-reduced substructure” $\cong$ independent set of rank $n - k$ in $G(\Sigma)$; “quasibipartite” = balanced. Successor to Grossman, Kulkarni, and Schochetman (1994a) [q.v. for more dictionary].

Successor to (1999a). Their “mixed tree” $T$ is a signed tree as in (1999a). Thm. 9 (simplified): The minor of $H^T H$ (H is the incidence matrix of $\Sigma$) obtained by deleting rows corresponding to $E \subseteq E(\Sigma)$ and columns corresponding to $F \subseteq E(\Sigma)$ has determinant equal, up to sign, to the number of common SDR’s of vertex sets of components of $T \setminus E$ and $T \setminus F$. [Interesting, but edge signs are irrelevant because any tree switches to all positive.] Dictionary: “substructure” = subgraph allowing retention of edges incident to deleted vertices [thus they become loose or half edges]. [See (1999a) for more dictionary.]

R.B. Bapat & E. Ghorbani

Generalizes to rings the inverses of $A(T)$ for a tree; cf. Godsil (1985a).

R.B. Bapat, D. Kalita, & S. Pati

They are complex unit gain graphs $\Phi$ with simple underlying graph. $L(\Phi)$ is obtained in the usual way from $H(\Phi)$. §2, “$D$-similarity and singularity in weighted directed graphs”: Thm. 8: $L(\Phi)$ is singular iff $\Phi \sim \|\Phi\|$ iff $\Phi$ (assumed connected) is balanced. [Cf. Zaslavsky (2003b), §2.1 esp. Thm. 2.1(a), noting that $\text{rk} L(\Phi) = \text{rk} H(\Phi) = \text{rk} G(\Phi)$.] §3, “Edge singularity of weighted directed graphs”: Elementary results on frustration index, appearing less elementary because treated indirectly, through eigenvalues, rather than directly, through the graph. Generalizing Y.Y. Tan and Fan (2008a) on signed graphs. §4, “3-Colored digraphs and their singularity”:

R.B. Bapat & Devadatta M. Kulkarni

Concerns a “mixed tree”, really an oriented signed tree without extroverted edges (see Bapat, Grossman, and Kulkarni (1999a)). The matrices are the incidence matrix $H$, the Laplacian matrix $HH^T$, and the “edge Laplacian” $H^TH$. Partly expository. New results concern Moore–Penrose inverses and their minor determinants. [Since a “mixed tree” is switching equivalent to an ordinary unsigned tree, their results should be identical to those for ordinary trees except for multiplication by a $V \times V$ diagonal matrix with signs on the diagonal.]

Given a 2-connected $\Sigma$ whose underlying graph is toroidal, polynomial-time algorithms are given for calculating the frustration index $l(\Sigma)$ and the generating function of switchings $\Sigma^{\mu}$ by $\#E^{-}(\Sigma^{\mu})$. The technique is to solve a Chinese postman ($T$-join) problem in the toroidal dual graph, $T$ corresponding to the frustrated face boundaries. Generalizes (1982a). [See (1990a), p. 4, for a partial description.] (SG: Fr, Alg)


The frustration-index problem, that is, minimization of $\#E^{-}(\Sigma^{\xi})$ over all switching functions $\zeta : V \rightarrow \{\pm 1\}$, for signed planar and toroidal graphs and subgraphs of 3-dimensional grids. Analyzed structurally, in terms of perfect matchings in a modified dual graph, and algorithmically. 3-dimensional is NP-hard, even when the grid has only 2 levels; the former are polynomial-time solvable even with weighted edges.

Also, the problem of minimizing $\#E^{-}(\Sigma^{\xi}) + \sum_{v} \zeta(v)$ for planar grids ("2-dimensional problem with external magnetic field"), which is NP-hard. [This corresponds to adding an extra vertex, positively adjacent to every vertex.] [See infinite analog in Istrail (2000a).] (SG: Phys, Fr, Fr(Gen): D, Alg)


Thm. 3.2: The real-weighted maximum cut problem is polynomial-time solvable for graphs not contractible to $K_{5}$. [Frustration index $l(-\Sigma)$ is the special case of weight $(e) = -\sigma(e) = \pm 1$.] [Annot. 19 Dec 2014.] (sg, WG: fr, Alg)


If the frustration number $l_{0}(-\Gamma) \leq 2$ (i.e., $-\Gamma \setminus \{u, v\}$ is balanced for some $u, v \in V$), then $\Gamma$ is weakly bipartite (cf. Guenin (2001a)) and $l(-\Gamma)$ is polynomial-time computable. [Problem. Characterize $\Sigma$ with $l_{0} \leq 2$. For $l_{0} \leq 1$ see Zaslavsky (1987c) with $\Omega = \Sigma$.] [Annot. 19 Dec 2014.] (Par, SG: fr, Sw)


§2: "Spin glasses."


Negative cutsets, where signs come from a network with real-valued capacities. Dual in the plane to negative circles. See §2.

Francisco Barahona & Adolfo Casari

Francisco Barahona & Michele Conforti

Francisco Barahona, Martin Grötschel, Michael Jünger, & Gerhard Reinelt

Francisco Barahona, Martin Grötschel, & Ali Ridha Mahjoub

Francisco Barahona & Enzo Maccioni

Francisco Barahona & Ali Ridha Mahjoub

Call $P_{BS}(\Sigma)$ the convex hull in $\mathbb{R}^E$ of characteristic vectors of negation sets (or “balancing [edge] sets”) in $\Sigma$. Finding a minimum-weight negation set in $\Sigma$ corresponds to a maximum cut problem, whence $P_{BS}(\Sigma)$ is a linear transform of the cut polytope $P_C(\Sigma)$, the convex hull of cuts.
Conclusions follow about facet-defining inequalities of $P_{BS}(\Sigma)$. See §5:
“Signed graphs”. (SG: Fr: Geom)


The “balanced induced subgraph polytope” $P_{BIS}(\Sigma)$ is the convex hull in $\mathbb{R}^V$ of incidence vectors of vertex sets that induce balanced subgraphs. Conditions are studied under which certain inequalities of form $\sum_{i \in Y} x_i \leq f(Y)$ define facets of this polytope: in particular, $f(Y) = \max$ size of balance-inducing subsets of $Y$, $f(Y) = 1$ or 2, $f(Y) = \#Y - 1$ when $Y = V(C)$ for a negative circle $C$, etc. (SG: Fr: Geom, Alg)


More on $P_{BIS}(\Sigma)$ (see (1989a)). A balance-inducing vertex set in $\pm \Gamma = a$ stable set in $\Gamma$. [See Zaslavsky (1982b) for a different correspondence.] Thm. 2.1 is an interesting preparatory result: If $\Sigma = \Sigma_1 \cup \Sigma_2$ where $\Sigma_1 \cap \Sigma_2 \cong \pm K_k$, then $P_{BIS}(\Sigma) = P_{BIS}(\Sigma_1) \cap P_{BIS}(\Sigma_2)$. The main result is Thm. 2.2: If $\Sigma$ has a 2-separation into $\Sigma_1$ and $\Sigma_2$, the polytope is the projection of the intersection of polytopes associated with modifications of $\Sigma_1$ and $\Sigma_2$. §5: “Compositions of facets”, derives the facets of $P_{BIS}(\Sigma)$. (SG: Geom, WG, Alg)

F. Barahona, R. Maynard, R. Rammal, & J.P. Uhry

Treats many important aspects of the quantity $l := \min_\zeta \#E^- (\Sigma^\zeta)$ [which equals the frustration index], over all switching functions $\zeta$ (“spin configurations $\sigma$” in the paper) of a signed graph, mainly a signed planar graph. ($\#E^- (\Sigma^\zeta)$ is the paper’s $\frac{1}{2}(\#E + H)$, $H :=$ Hamiltonian.) They maximize $-H = W^+ + W^- - W^{+-}$ where $W^+ + W^- := \#$ unswitched positive edges $- \#$ unswitched negative edges and $W^{+-} := \#$ switched positive edges $- \#$ switched negative edges. Thus, $-H = \#E^+ - \#E^- = \#E + 2\#E^-$ after switching. Maximizing it $\iff$ minimizing $\#E^-$ over all $\zeta$.

§2: “The frustration model as the Chinese postman’s problem”, describes how to find $l$ when $|\Sigma|$ is planar, by solving a Chinese postman ($T$-join) problem in the dual graph, $T$ corresponding to the frustrated (i.e., negative) face boundaries. The postman problem is solved by linear programming. [Solved independently by Katai and Iwai (1978a).] [Barahona (1981a) generalizes to signed toroidal graphs.]

§3: “Solution of the frustration problem by duality: rigidity”. An edge is “rigid” if it has the same sign in every $\Sigma^\zeta$ that minimizes $\#E$ (such an $\zeta$ is a “ground state”). The endpoints of a rigid edge are called “solidary”. Rigid edges are found via the dual linear program. The boundary contours of connected sets of frustrated faces play an important role.

§§4–5: “Numerical experimentation” and “Results”, for a randomly signed square lattice graph. The proportion $x$ of negative edges strongly affects the properties; esp., there is significant long-range order below
but not above \( x \approx 0.15 \). [See Deng and Abell (2010a) for numerical results on random signed graphs.]

More general problems discussed are (1) allowing positive edge weights (due to variable bond strengths); (2) minimizing \( \# E^- + c \sum_v \zeta(v) \), with \( c \neq 0 \) because of an external magnetic field. Then one cannot expect the LP to have a combinatorial optimum. [Annot. 20 Jan 2010.]

F. Barahona & J.P. Uhry

John S. Baras
See G.-D. Shi.

S. Barik, D. Kalita, S. Pati, & G. Sahoo

The signless Laplacian \( L(-\Gamma) \) is one of the matrices surveyed. No mention that it is signed-graphic. [Annot. 29 Dec 2020.]

J. Wesley Barnes
See P.A. Jensen.

Adriano Barra

Physical quantities of a random signed graph, with states (“configurations”) \( s \in \{\pm 1\}^n \), studied by replication. [Annot. 29 Dec 2012.]

Arun Kumar Baruah & Manoshi Kotoky

Confused; nothing of value. [Annot. 13 Oct 2019.]

Tamer Bašar
See W. Chen.

Nino Bašić, Patrick W. Fowler, Tomaž Pisanski, & Irene Sciriha

Jayapal Baskar Babujee & Shobana Loganathan

Does \( \Gamma \) admit a balanced edge signature \( \sigma = (+)^c \), such that \( \# \zeta^{-1}(+) \approx \# \zeta^{-1}(-) \) and \( \# E^+ \approx \# E^- \)? (\( \approx \) means difference \( \leq 1 \).) If so, \( \zeta \) is a “signed product cordial labeling”. Some constructions. [Equivalently, as it is a graph property without signs: Does \( \Gamma \) have a cut \( D = E(X,V \setminus X) \) such that \( \# X \approx \frac{1}{2} \# V \) and \( \# D \approx \frac{1}{2} \# E^- \)?] [Successors: Santhi and Albert (2015a), Rozario Raj and Manoharan (2016a), Shobana and Vasuki]
Lowell Bassett, John Maybee, & James Quirk

Lemma 3: A square matrix with every diagonal entry negative is nonsingular if and only if every cycle is negative in the associated signed digraph.
Thm. 4: A square matrix with negative diagonal is sign-invertible if and only if all cycles are negative and the sign of any (open) path is determined by its endpoints. And more.

Vladimir Batagelj
See also P. Doreian, N. Kejžar, and W. de Nooy.


§3, p. 6: Predicates to use for searching out balanced or clusterable partitions. [Annot. 10 Mar 2011.]

V. Batagelj & T. Pisanski

William H. Batchelder
See K. Agrawal.

Christian Bauckhage
See J. Kunegis.

Thierry-Pascal Baum
See J. Demongeot.

Jan Baumbach
See S. Böcker.

Andrei Bătutu & Elena Bătutu


§4, “Binary particle swarm optimization and Ising spin glasses”: The signed graph; spins and states; satisfied and frustrated edges; some history. In particle swarm optimization, each vertex acts as a cell in a cellular automaton, learning probabilistically, seeking a most satisfied spin $\zeta(v)$ in order to minimize $\#E^-(\Sigma_S)$. [It seems that this local minimization suffers from the same potential instability as Mitra’s (1962a) deterministic local minimization, hence is not accurate.] [Annot. 19 Aug 2012.]

Andrei Băutu, Elena Băutu, & Henri Luchian


Particle swarm optimization combined with hill-climbing to find $I(\Sigma)$ (ground state of Ising model); a hybrid method is said to be promising. [Annot. 19 Aug 2012.]


Bond-based representation means recording switched edge (“bond”) signs instead of vertex spins; cf. Pelikan and Hartmann (2007a), (2007b). Here, a state $s : V \rightarrow \{+1, -1\}$ is recorded as the signs of a spanning tree switched by $s$. This has ambiguity [2, obviously]. Negating one tree edge implies a chain of spin changes; this “may be considered a feature” [and its implications could be interesting]. Computational experiments tested the implied algorithm. [Annot. 19 Aug 2012.]

Andrei Băutu & Henri Luchian


Applies Băutu, Băutu, and Luchian (2008a). Shallower trees may produce better results due to the lesser effect of negating one tree edge. Computational comparisons of this and other algorithms for ground state (i.e., frustration index). [Annot. 19 Aug 2012.]

Elena Băutu

See A. Băutu.

Laurent Beaudou, Florent Foucaud, & Reza Naserasr

Matthias Beck & Mela Hardin

(SG: Hom, EC)

Matthias Beck, Erika Meza, Bryan Nevarez, Alana Shine, & Michael Young

Chromatic and zero-free chromatic polynomials of all six switching isomorphism types of signed Petersens (cf. Zaslavsky (2012b)) and all signed $K_n$'s for $n \leq 5$. Each switching isomorphism type has a different chromatic polynomial and each has a different zero-free polynomial. Computer code in SAGE. [Annot. 2 Nov 2013.] (SG: Col, Geom)

Matthias Beck & Thomas Zaslavsky

§5: “In which we color graphs and signed graphs.” A geometric interpretation of signed graph coloring by lattice points and hyperplane arrangements unifies the chromatic and zero-free chromatic polynomials and gives immediate proofs of theorems on the chromatic polynomials and acyclic orientations. (SG: Col: Geom, M, Invar, Bal)


The nowhere-zero flow polynomial of a signed graph, for flows in an odd abelian group, and the integral nowhere-zero flow quasipolynomial with period 2. [For even abelian groups see DeVos, Rollová, and Šámal (2019a).] (SG: Flows: Geom, M, Invar, Bal)


In magic labellings of a bidirected graph, the labels are distinct positive integers; at each vertex the sum over entering edge ends equals that over departing edge ends. Thms. (implicit): The number of magic labellings is a quasipolynomial function of the magic sum, if the magic sum is prescribed. It is also a quasipolynomial function of the upper bound on the labels, if an upper bound is prescribed. (ori: Geom, Enum)

§5: “Generalized exclusions.” Complementarity rules in magic squares, etc., can be expressed by signed-graphic hyperplanes. (sg: Lab: Geom, Enum)


§3: “Semimagic squares.” Counts magic labellings of the extraverted $-K_{3,3}$ by an explicit geometrical solution. Counted either by upper
bound on the values or by magic sum. 

Richard Behr

In Σ, does edge e, or vertex v, lie in exactly one negative circle? Exactly one positive circle? The structure of Σ and the corresponding edges or vertices are determined for each question. [Annot. 21 Dec 2017.]


Introduces edge coloring of signed graphs. Vizing’s Theorem generalizes to signed simple graphs: the number of proper edge colorings is \( \chi'(\Sigma) \leq \Delta(\Sigma) + 1 \). A proper edge coloring of Σ is the same as a proper vertex coloring (cf. Zaslavsky (1982b)) of \(-\Lambda(\Sigma)\), defined via bidirected graphs (cf. Zaslavsky (20xxa), (2010b)). An opposite definition (“antiproper coloring”) is equivalent to balanced decomposition of \(-\Sigma\) (cf. Zaslavsky (1987b)). Remarks on total coloring. [Independent partial proof by Zhang–Lu–Luo–Ye–Zhang (2020a) with results for signed planar graphs.]

[Sg: EC] (SG: LG: Ori, EC)

M. Behzad & G. Chartrand

\( \Lambda_{BC} \) Their line graph \( \Lambda_{BC}(\Sigma) \) of a signed simple graph Σ (not defined explicitly) is the line graph \( \Lambda(|\Sigma|) \) with an edge negative when its two endpoints are negative edges in Σ. They “color” as in Cartwright and Harary (1968a) [i.e., clustering]. Characterized: Σ with colorable line graphs. Found: the fewest colors for line graphs of signed trees, \( K_n \), and \( K_{r,s} \). [For a more sophisticated kind of line graph see Zaslavsky (1984c), (2010b), (20xxa). For another line graph, see M. Acharya (2009a).]

[Sg: LG: Ori, EC] (SG: EC)

Amos Beimel, Aner Ben-Efraim, Carles Padró, & Ilya Tyomkin

[Cf. Ben-Efraim (2016a).]

Lowell W. Beineke & Frank Harary
A “marked digraph” is a digraph $\vec{D}$ with signed vertices, $\vec{S} = \vec{D}, \mu$ where $\mu : V \to \{+, -\}$. It is “consistent” if all diwalks from $v$ to $w$ have the same sign $\mu(W)$. The sign of a walk is the vertex sign product.

Thm. 1. Assuming $\vec{D}$ is strongly connected, $\vec{S}$ is consistent iff every dicycle is positive. [An important difference from signed graphs, where no restriction is needed.] Thm. 2. $\vec{S}$ is consistent iff $V$ has a bipartition such that every arc with a positive tail lies within a set but no arc with a negative tail does so. Define $\sigma(\vec{uv}) := \mu(u)$. Thm. 3. Assuming $\vec{D}$ is strongly connected, this signed graph is balanced iff $\vec{S}$ is consistent.

Thm. 4. A vertex-signed tournament $\vec{S}$ is consistent iff: When strongly connected, [it is all positive or] it has exactly two negative vertices $u, v$ and, deleting $uv$, $u$ is a source and $v$ is a sink. When not strongly connected, it is consistent iff it is all positive, or it has one negative vertex which is a source or sink, or it has two negative vertices, one a source and the other a sink. Thm. 5. $\vec{D}$ has $\mu \not\equiv +$ such that $(\vec{D}, \mu)$ is consistent (“markable”) iff $\exists \emptyset \subset V_0 \subset V$ such that, $\forall v$, all out-arcs from $v$, or none, go to $V_0$. [Annot. 16 Sept 2010.] (VS)


A graph (not necessarily simple) with signed vertices is “consistent” if every circle has positive sign product. Thm. 2.2: $\Gamma$ with all negative vertices is consistent iff bipartite. Thm. 2.3: 3-connected vertices must have the same sign. Thm. 3.3: Contracting an edge with positive endpoints preserves consistency and inconsistency. Further partial results. Open problem: A full characterization of consistent vertex-signed graphs. [For a good solution see Hoede (1992a). For the best solution see Joglekar, Shah, and Diwan (2010a).] [Annot. rev. 11 Sept 2010.] (VS: Bal)
The signed digraph of a square matrix is “frustrated” if it has a negative cycle. Somewhat simplified: a negative cycle is necessary for there to be oscillation caused by intraneuronal processing delay. (SD: QM, Ref)

Francesco Belardo
See also S. Akbari, A. Alazemi, and J.F. Wang.


Francesco Belardo & Maurizio Brunetti

All connected Laplacian-cospectral mates of unbalanced “$\infty$-graphs” (tight handcuffs). They are certain bicyclic graphs. [Annot. 18 Jan 2019.] (SG: Lap: Eig) 20xxb Line graphs of complex unit gain graphs with least eigenvalue $-2$. In preparation.


Francesco Belardo, Maurizio Brunetti, Matteo Cavaleri, & Alfredo Donno

Francesco Belardo, Maurizio Brunetti, & Adriana Ciampella


2021a Godsil-McKay switching for mixed and gain graphs over the circle group. Linear Algebra Appl. 614 (2021), 256–269. MR 4209002. (sg, GG: Adj: Eig)


Francesco Belardo, Maurizio Brunetti, & Nathan Reff

Francesco Belardo, Sebastian M. Cioabă, Jack Koolen, & Jianfeng Wang


Eigenvalue questions, almost all for $A(\Sigma)$, with a review of relevant knowledge. [Annot. 13 Jun 2019.] (SG: Eig: Adj, KG)

Francesco Belardo, Enzo M. Li Marzi, & Slobodan K. Simić


Francesco Belardo, Enzo M. Li Marzi, Slobodan K. Simić, & Jianfeng Wang


The largest eigenvalue of $A(G)$ for $G = \text{chain or necklace of cliques}$, via $L(-\Gamma)$ where $G = \Lambda(\Gamma)$. [Annot. 16 Jan 2012.] (par: LG: Adj: Eig)


Francesco Belardo & Paweł Petecki


Francesco Belardo, Paweł Petecki, & Jianfeng Wang


Francesco Belardo, Tomaž Pisanski, & Slobodan K. Simić


Francesco Belardo, Irene Sciriha, & Slobodan K. Simić


Francesco Belardo & Slobodan K. Simić

§2: Adjacency eigenvalues of spectral line graph $\Lambda_{\text{Spec}}(\Sigma)$ compared to Laplacian eigenvalues of $\Sigma$ by standard method. §3: Coefficients of Laplacian characteristic polynomial. Dedò (1981a)-style matrix-tree theorem. §4, "Laplacian coefficients of signed unicyclic graphs". Dictionary: “oriented edge” = positive edge, “unoriented edge” = negative edge, “line graph” $L(\Sigma)_{\eta}$ = spectral line graph $\Lambda_{\text{Spec}}(\Sigma)$, i.e., reduced negated line graph $-\Lambda(\Sigma, \eta)$, “subdivision graph” $S(\Sigma, \eta)$ = signed vertex-edge incidence graph. [Annot. 5, 11 Jan 2020.]

Francesco Belardo, Zoran Stanić, & Thomas Zaslavsky

Francesco Belardo & Yue Zhou

Hacène Belbachir & Imad Eddine Bousbaa

G.R. Belitski˘ı & Yu.I. Lyubich

A. Bellacicco & V. Tulli

Signed (di)graphs (“spin graphs”) are defined. The main concepts are “dissimilarity”, “balance”, and “cluster” are defined and propositions are stated. Eigenvalues are mentioned. [This may be an announcement. There are no proofs. It is hard to be sure what is being said.] (SD: Eig)

Joachim von Below

Here a periodic graph [of dimension m] is defined as a connected graph $\Gamma = \tilde{\Psi}$ where $\Psi$ is a finite $\mathbb{Z}^m$-gain graph with gains contained in $\{0, b_i, b_i - b_j\}$. $(b_1, \ldots, b_m$ are the unit basis vectors of $\mathbb{Z}^m$.) Let us call such a $\Psi$ a small-gain base graph for $\Gamma$. Any $\tilde{\Phi}$, where $\Phi$ is a finite $\mathbb{Z}^m$-gain graph, has a small-gain base graph $\Psi$; thus this definition is equivalent to that of Collatz (1978a). The “index” $I(\Gamma)$, analogous to the largest eigenvalue of a finite graph, is the spectral radius of $A(\|\Psi\|)$ (here written $A(\Gamma, N)$) for any small-gain base graph of $\Gamma$. The paper contains basic theory and the lower bound $L_m = \inf \{I(\Gamma) : \Gamma$ is $m$-dimensional\}, where $1 = L_1, \sqrt{9/2} = L_2 \leq L_3 \leq \cdots$. (GG(Cov): Eig)

Jean Bénabou

Morphisms of signed graphs are employed in category-theoretic constructions. (SG)

Radel Ben-Av
See D. Kandel.

Edward A. Bender & E. Rodney Canfield

§3: “Self-dual signed graphs,” gives the number of $n$-vertex graphs that are signed, vertex-signed, or both; connected or not; self-isomorphic by reversing edge and/or vertex signs or not, for all $n \leq 12$. Some of this appeared in Harary, Palmer, Robinson, and Schwenk (1977a). (SG, VS: Enum)

Riccardo Benedetti
§8, “Spin manifolds”, hints at a use for decorated signed graphs in the structure theory of spin 3-manifolds. (sg: Appl: Exp)

Aner Ben-Efraim
See also A. Beimel.


Joseph Ben Geloun
See R.C. Avohou.

Samia Ben Lamine
See J. Aracena and J. Demongeot.

Curtis Bennett & Bruce E. Sagan

To illustrate the generalization, most of the article calculates the chromatic polynomial of $\pm K_n^{(k)}$ (called $DB_{n,k}$; this has half edges at $k$ vertices), builds an “atom decision tree” for $k = 0$, and describes and counts the bases of $G(\pm K_n^{(k)})$ (called $D_n$) that contain no broken circuits. (SG: M, Invar, col)

M.K. Bennett, Kenneth P. Bogart, & Joseph E. Bonin

Drawing an analogy between Desargues’ and Pappus’ theorems in projective spaces and similar incidence theorems in Dowling geometries. [The rigorous avoidance of gain graphs makes the results less obvious than they could be.] (gg: M, Geom)

Moussa Benoumhani

Cf. Dowling (1973b). Generating functions and identities for Whitney numbers of the first and second kinds, analogous to usual treatments of Stirling numbers. §2, “Whitney numbers of the second kind”: $W_m(n,k) := W_k(Q_n(\mathcal{S})) = W_k(G(\mathcal{S}K_n^{\bullet}))$ where $m = \#\mathcal{S}$. E.g., Thm. 1: $\sum_n W_m(n,k)z^n/n! = [(e^{mz} - 1)/m]^k e^z/k!$. Thm. 5: $\sum_n W_m(n,k)u^{n-k} = m^{k+1}/[(1-u)/mu]_{k+1}$. §3, “Whitney numbers of the first kind”: $w_m(n,k) := w_k(G(\mathcal{S}K_n))$. E.g., Thm. 10: $\sum_n w_m(n,k)z^n/n! = (1 + mz)^{-1/m} \cdot \ln^k(1 + mz)/k!m^k$. Thm. 12 is a reciprocity relation between $w_m(n,k)$ and $s(n,k)$. §4, “The integers maximizing $W_m(n,k)$ and $w_m(n,k)$”: Partial, complicated results. [Annot. 30 Apr 2012.] (gg: M: Invar)


Continuation of (1996a). §2, “Dowling polynomials”: $D_m(n,x) := \sum_k W_m(n,k)x^k$. Generating function, recurrence, infinite series expression. §3 similarly studies $F_m,1(x) := \sum_k k!m^k W_m(n,k)x^k$ and $F_m,1(x) := \sum_k k!W_m(n,k)x^k$. §4, “Log-concavity of $k!W_m(n,k)$”. Deduced from real
negativity of zeros. [Annot. 1 May 2012.] (gg: M: Invar)


Logarithmic concavity of Whitney numbers of the second kind is deduced by proving that their generating polynomial has only real zeros. [Cf. Stonesifer (1975a), Dür (1986a), and Damiani, D’Antona, and Regonati (1994a).] (gg: M: Invar)

Julien Bensmail


The max homomorphic chromatic number of grids \( \in \{8, 9, 10, 11\} \), improving \( \leq 12 \) from Nesetril–Raspaud (2000a). [Improved to \( \leq 9 \) in Dybizbański (20xxa).] Dictionary: “2-edge-coloured graph” = signed graph, “2-edge-coloured chromatic number” = min order of homomorphism target. [Annot. 3 Sep 2020.] (SG: Hom)

Julien Bensmail, Soumen Nandi, Mithun Roy, & Sagnik Sen

20xxa On homomorphisms of planar signed graphs and absolute cliques. Submitted. HAL hal-01919007. (SG: Hom)

Josh Bentley

See A.-M. Yang.

C. Benzaken

See also P.L. Hammer.

C. Benzaken, S.C. Boyd, P.L. Hammer, & B. Simeone


Cl. Benzaken, P.L. Hammer, & B. Simeone


C. Benzaken, P.L. Hammer, & D. de Werra


They are identical to “threshold signed graphs”. \( \Gamma \) is a threshold signed graph if \( \exists a : V \to \mathbb{R}, S, T \in \mathbb{R} \), such that \( v_i v_j \in E \) iff \( |a_i + a_j| \geq S \) or \( |a_i - a_j| \geq T \). [Proposed signed graph \( \Sigma \): \( -v_i v_j \in E \) iff \( |a_i + a_j| \geq S \), \( +v_i v_j \in E \) iff \( |a_i - a_j| \geq T \). Then \( \Gamma \) = simplification of \( |\Sigma| \). Question.
Is $\Sigma$ interesting? [Annot. 16 Jan 2012.] (VS, sg)

Michele Benzi
See E. Estrada.

C. Berge & J.-L. Fouquet

All-negative signed graphs in which the vertex frustration number equals the negative-circle vertex-packing number. This is called the “König property” [since it is a vertex König-type property for negative circles]. Example: the line graphs of cubic bipartite graphs. [Problems. Investigate for arbitrary signed and biased graphs.] (par: Fr, Circ)

Claude Berge & A. Ghouila-Houri


German edition(s) of (1962a). (GN: incid)

Claude Berge & Bruce Reed

If $-\Gamma$ is an all-negative signed graph in which the frustration index equals the negative-circle edge-packing number for every subgraph, then $\chi(\Gamma) \leq 3$. [Problem 1. Is it natural to state this bound in terms of the chromatic number of $-\Gamma$? Problem 2. Generalize to arbitrary signed graphs.] (par: Fr: Circ)


An upper bound on the frustration index in terms of the negative-circle edge-packing number. (par: Fr: Circ)

Joseph Berger, Bernard P. Cohen, J. Laurie Snell, & Morris Zelditch, Jr.

See Ch. 2: “Explicational models.” (PsS)(SG: Bal)(Ref)

Nantel Bergeron

A. Nihat Berker
See D. Blankschtein.

Abraham Berman & B. David Saunders

Olivier Bernardi & Guillaume Chapuy


Counting one-face cellular orientation embeddings of all graphs of order \( n \) in \( U_h \) (sphere with \( h \) crosscaps). Exact formulas if all degrees are 1 and 3 (Cors. 8, 9); asymptotic (as \( n \to \infty \)) in general (Thm. 11). Dictionary: “twist” = negative edge, “flip” of vertex = switching. [Cf. Širáň and Škoviera (1991a).] [Annot. 3 Nov 2017.] (sg: Top)

Gilles Bernot
See A. Richard.

Daniel Irving Bernstein

Pascal Berthomé, Raul Cordovil, David Forge, Véronique Ventos, & Thomas Zaslavsky

Calculating chromatic functions (which satisfy deletion-contraction for zero-gain edges and equal 0 if there is a balanced loop) by eliminating or adding identity-gain edges. Application to integral, modular, and zero-free chromatic polynomials of the Shi, Linial, Catalan, and intermediate hyperplane arrangements via their gain graphs [cf. Stanley (1999a)]. [See Ardila (2007a) for some of the corresponding coboundary and Tutte polynomials.]

(GG: Invar, Geom)

E.A. Bespalov

*Cf. D.S. Krotov (2010a).* Classifies \( \Gamma \) for which deleting single vertices is insufficient. [Annot. 31 Jul 2018.] (tg: Sw: Str)

E.A. Bespalov & D.S. Krotov


For \( g : E(K_n) \to \mathbb{F}_q^+ \), let \( E(\Gamma_g) := E(K_n) \setminus g^{-1}(0) \). For \( f : V \to \mathbb{F}_q \), let \( \delta f(vw) := f(v) + f(w) \), i.e., \( \delta f := \mathbb{H}(-\Gamma)f \). A “switching” of \( \Gamma_g \) (a nonstandard switching) = any \( \Gamma_g + \delta f \). “Separable” = nontrivially disconnected; cf. D.S. Krotov (2010a). “Switching separable” = switches to a separable graph. Thm. 1 classifies \( \Gamma \) that are switching inseparable but with all single-vertex deletions switching separable: \( \exists \) iff \( q \) even and odd \( n > 4 \); all are determined in Prop. 3. [Annot. 31 Jul 2018.]

(par: Sw(Gen), incid: Str)

Ouahiba Bessouf


(SG: Ori, Str)

Ouahiba Bessouf & Abdelkader Khelladi


Ouahiba Bessouf, Abdelkader Khelladi, & Thomas Zaslavsky


(SG: Ori: Str)


Stéphane Bessy

See J. Bang-Jensen.

Kenchappa S. Betageri

See also V.S. Shigehalli.


K.S. Betageri & G.H. Mokashi


Nadja Betzler

See F. Hüffner.

D. Bharathi

See S. Sajana.

Mushtaq A. Bhat

See also S. Pirzada and T. Shamsher.


Mushtaq A. Bhat & S. Pirzada


Mushtaq A. Bhat, U. Samee, & S. Pirzada


Pradeep G. Bhat & Sabitha D’Souza


The authors introduce a “label [adjacency] matrix” $A_{l}(\Gamma, \zeta)$ (see below) where $\zeta : V \to \{0, 1\}$ and $a, b, c$ are distinct real numbers. They investigate the characteristic polynomial $\varphi(\lambda)$ and energy $E_{l} := \sum |\lambda_{i}|$ where $\lambda_{i}$ are the eigenvalues. Thms. 2.1, 2.2: Top four coefficients of $\varphi$. Thms. 2.3, 2.4: Properties of $E_{l}$. Thm. 2.5: Eigenvalue upper bound.

§3, “Label energies of some families of graphs”: $K_{n}$, $K_{1,n}$, $K_{r,s}$, double star. Thm. 3.4: If $K_{n}$ has $m$ 0-labelled vertices, the eigenvalues are $-a$ (multiplicity $m - 1$), $-b$ (multiplicity $n - m - 1$), and the roots of a quadratic.

[Problem. Define a “generic adjacency matrix” $A_{l}(B)$ of a bidirected simple graph $B$ by $a_{ij} = a$ if $v_{i}v_{j}$ is extraverted, $b$ if $v_{i}v_{j}$ is introverted, $c$ if $v_{i}v_{j}$ is positive, and 0 otherwise, where $a, b, c$ are generic numbers, indeterminates, etc. (For a non-simple graph, sum the values of edges $v_{i}v_{j}$.)

Given a vertex-signed graph $(\Gamma, \zeta)$, bidirect it by $\tau(v_{i}, v_{j}) := \zeta(v_{i})$; that is, every vertex is a source (if $\zeta(v_{i}) = -$) or sink $\zeta(v_{i}) = +$). Note that $\{0, 1\} \cong \{+, -\}$. Does this matrix of a bidirected graph have interesting properties?] [Annot. 3 Oct 2013.] (VS: Eig)(sg: ori: Eig)


R.N. Bhatt & A.P. Young


Bikash Bhattacharjya
See D. Sehrawat.

Amitava Bhattacharya, Uri N. Peled, & Murali K. Srinivasan

The cone of Eulerian real-weighted subgraphs of a bidirected all-negative signed graph. (sg: par: Geom)


A “balanced subgraph” is an edge 2-colored graph where the red and blue degrees are equal at each vertex. [I.e., a signed graph whose net degree $d^\pm(v) = 0, \forall v$. Equivalent to an all-negative signed graph, oriented so that every vertex has equal in- and out-degree, which is the all-negative case of an Eulerian bidirected graph. P.D. Seymour, Sums of circuits, in *Graph Theory and Related Topics*, pp. 341–355, Academic Press, New York, 1979, treated the all-positive case.] The problem is to describe the facets of the convex cone generated by Eulerian subgraphs of an all-negative bidirected graph. [Problem. Solve for an arbitrary bidirected graph.] (sg: Par, Ori: Geom)

Anindya Bhattacharya & Rajat K. De

2010a Average correlation clustering algorithm (ACCA) for grouping of co-regulated genes with similar pattern of variation in their expression values. *J. Biomedical Informatics* 43 (2010), 560–568. (sg: Clu: Alg, Biol)

Gora Bhaumik
See P.A. Jensen.

V.N. Bhave
See E. Sampathkumar.

Mani Bhushan & Raghunathan Rengaswamy

Another application to fault diagnosis in chemical engineering, this one to location of sensors. (SD: Appl)

Ginestra Bianconi
See V. Ciotti.

Christin Bibby
Precursor of the marked Dowling posets of Bibby and Gadish (2018a).

**Christin Bibby & Nir Gadish**


Introduces marked Dowling (“S-Dowling”) posets. The posets can be viewed as consisting of decorated flats of the frame matroid $G(\mathcal{E}K_n^*)$. Dowling’s $Q_n(\mathcal{E}) = \text{Lat } G(\mathcal{E}K_n^*)$ is viewed as $\mathcal{F}^b := \{\text{closed, balanced subgraphs } B \text{ of } G(\mathcal{E}K_n^*), \text{ suitably ordered. A marking is } m : V(B)^c \to S\}$. The marked Dowling poset is $\{(N, m) : N \in \mathcal{F}^b\}$, suitably ordered. [More theory of marked Dowling posets in Delucchi–Girard–Paolini (2019a), Paolini (2020a).] [Annot. 1 Feb 2019.]

**I. Bieche, R. Maynard, R. Rammal, & J.P. Uhry**


The frustration index and ground states of a planar square grid graph can be found by matching in the dual graph. [Solved for all planar graphs by Katai and Iwai (1978a), Barahona (1982b).] [Annot. 29 Aug 2012.]

**Dan Bienstock**


Given a graph. Problem 1: Is there an odd hole on a particular vertex? Problem 2: Is there an odd induced path joining two specified vertices? Problem 3: Is every pair of vertices joined by an odd-length induced path? All three problems are NP-complete. [Obviously, one can replace the graph by a signed graph and “odd length” by “negative” and the problems remain NP-complete.]

**Norman Biggs**


Ch. 19: “The covering graph construction.” The covering graphs of gain graphs, with emphasis on automorphisms. Let $\Phi := (\Gamma, \varphi)$ with gain group $\mathbb{Z}_2E$ and $\varphi(e) = e$. Thm. 19.5: If $\Gamma$ is $t$-transitive ($t \geq 1$) [and connected], then $\Phi$ is vertex transitive [actually, $t$-transitive] and has $n - c(\Gamma)$ components (all isomorphic). [The number of components and the isomorphism of components of $\Phi$ require only connectedness of $\Phi$, because $\text{Aut } \Phi$ acts transitively on each vertex fiber.] 19A: “Double coverings.” The signed covering graph of $-\Gamma$. 19B: “The Desargues graph.” With $P :=$ Petersen graph, $-P$ is the Desargues graph. [Annot. 11 July 2009.]

[Tutte (1967a) implicitly develops the double covering of an oriented
\[ \Sigma; \text{it is a self-converse orientation of } \tilde{\Sigma}. \] (SG, GG: Cov, Aut, bal)


As in (1974a), but 19A, 19B have become Additional Results 19a, 19b. (SG, GG: Cov, Aut, bal)


A model of currency exchange rates in which no cyclic arbitrage is possible, hence the rates are given by a potential function. [That is, the exchange-rate gain graph is balanced, with the natural consequences.]

Assuming cash exchange without accumulation in any currency, exchange rates are determined. [See also Ellerman (1984a).] (GG, gn: Bal: Exp)

Yonatan Bilu & Nathan Linial


Conjecture 2 (based on (2006a)). Every \(d\)-regular Ramanujan graph can be signed so it has spectral radius \( \leq 2\sqrt{d-1} \). Conjecture 3. The same for every \(d\)-regular graph. Dictionary: “2-lift” = signed covering graph. [Annot. 2 Mar 2011.]


K. Binder & A.P. Young


§ III.F.2, “Frustration and gauge invariance”: A valuable summary of the state of knowledge and speculation at the time. Signed graphs with spin set \(\{+1, -1\}\) (Ising spins) and \(U(1)\) (“\(XY\) spins” = complex units). Frustration is treated via girth circles (“plaquettes”) in lattice graphs, where the girth is 3 or 4 (triangular or square planar lattice). Analytic solutions being too difficult, results are numerical, qualitative, or for “simpler limiting cases”. \(XY\) spins show quantization (cf. Villain (1977b)). For 3-dimensional lattices, plaquette duality leads to vector gains in a dual lattice, hence to closed paths of frustrated plaquettes.

In Ch. IV, “Mean-field theory”: Complete-graph (“infinite range”) models. § IV.A, “Sherrington-Kirkpatrick model and replica-symmetric solutions”: Ising models (\(G = \{+1, -1\}\)). § IV.H, “Non-Ising models”: Weighted edge signs are random variables. Spins may be normalized vectors (§1, “Isotropic vector spin glasses in zero field”) or other. §3, “Other models”: \(\mu\)-spin couplings” = \(\mu\)-uniform complete hypergraphs. Energy valleys and their shapes. Potts models (signed graphs, spins are multivalued).
Dictionary: “site” = vertex, “bond” = edge, “state” = function \( s : V \to \mathcal{G} \), “spin” = value \( s(v) \), “ferromagnetic” = positive, “antiferromagnetic” = negative, “quenched variable” = constant (instead of random variable), “gauge group” = gain group, “gauge transformation” = switching, “ground state” = state minimizing \( \#(E \setminus E^{\sigma}(\Phi)) \). [Annot. 17 Aug 2012.]

B.D. Bingham, D.D. Olesky, P. van den Driessche

“Even cycle pattern” means the signed digraph has negative cycles of every even length (remark before Thm. 4.1). [Annot. 13 Dec 2020.]

Robert E. Bixby

Türker Biyikoğlu & Josef Leydold

In a semiregular tree \( T \), all internal vertices have the same degree \( d \).

Thm. 2: Given \( n, d \geq 3 \), a semiregular \( T \) minimizes \( \lambda_1(L(T)) \) iff it is a caterpillar. The proof is via \( \text{Spec} L(-T) \), which = \( \text{Spec} L(T) \) since a signed tree is balanced. [Annot. 21 Jan 2012.] (sg: par: Eig)

Türker Biyikoğlu, Marc Hellmuth, & Josef Leydold

The \( p \)-Laplacian \( (1 < p < \infty) \) generalizes the Laplacian matrix acting on vertex functions. [Generalizing to signed graphs:] Define the \( p \)-Laplacian of \( \Sigma \) by \( \Delta_p(\Sigma)f(u) := \sum_{uv \in E} \text{sgn}[f(u) - \sigma(uv)f(v)]\cdot|f(u) - \sigma(uv)f(v)|^{p-1} \). Then \( p = 2 \) gives \( L(\Sigma) \).] The \( p \)-Laplacian of \( \Gamma \) is \( \Delta_p(+\Gamma) \) and its signless \( p \)-Laplacian is \( \Delta_p(-\Gamma) \). Prop. 3.3 et seq. concern \( \Delta_p(-\Gamma) \). [Unlike with the Laplacian \( L \), switching does not preserve properties, so signs matter in a tree.] [Problem. Generalize to signed graphs.] [Annot. 21 Jan 2012.] (sg: par: Eig: Gen)

Anders Björner & Bruce E. Sagan

Lattices \( \Pi_{n,k,h} \) (for \( 0 < h \leq k \leq n \)) consisting of all spanning subgraphs of \( \pm K^n_{n} \) that have at most one nontrivial component \( K \), for which either \( K \) is balanced and complete and \( \#V(K) \geq k \), or \( K \) is induced and \( \#V(K) \geq h \). (Also a generalization of this.) Characteristic polynomial, homotopy and homology of the order complex, cohomology of the real
complement. (SG: Geom, M(Gen): Invar, col)

Anders Björner, Michel Las Vergnas, Bernd Sturmfels, Neil White, & Günter M. Ziegler


The adjacency graph of bases of an oriented matroid is signed, using circuit signatures, to make the “signed basis graph”. See §3.5, “Basis orientations and chirotopes”, pp. 132–3. (M: SG)

Anders Björner & Michelle L. Wachs


§9, “Interpolating partition lattices”: Homology of $L(H[\pm K_n(T)])$ where $T$ is the set of vertices that have half edges. [Annot. 12 Aug 2014.]

J.A. Blackman

See also J.R. Gonçalves and J. Poulter.


The signed square lattice graph, including effect of density of negative edges (“antiferromagnetic bonds”). §III, “Frustration”: “Local-mode” eigenvectors of a Hermitian modification $D$ of a Pfaffian adjacency matrix correspond bijectively to negative squares (“frustrated plaquettes”). Weight $> 3$ of frustrated edge (compared to 1 of satisfied edges) alters the ground state [by switching; because the graph is 4-regular]. §IV, “Frustration at higher density”: Numerical studies of an $N^2$ grid with $N = 50$ [small by today’s standards] suggest a change of behavior at $p := l(\Sigma)/\#E \approx .045$ [a value that surely depends on 4-regularity]. Dictionary: “gauge transformation” = switching, “gauge invariance” = switching invariance, “wrong bond” = frustrated edge [in a ground state].

Continued in Blackman and Poulter (1991a) and in Gonçalves, Poulter, and Blackman (1997a) and (1998a). [Annot. 17 May 2013.]

J.A. Blackman, J.R. Gonçalves, & J. Poulter


J.A. Blackman & J. Poulter


Square lattice graph with a definite proportion of negative edges. Cf. Poulter and Blackman (2001a) for triangular lattice. [Annot. 16 Aug 2018.]

Franco Blanchini, Elisa Franco, & Giulia Giordano

Daniel Blankschtein, M. Ma, & A. Nihat Berker

Daniel Blankschtein, M. Ma, A. Nihat Berker, Gary S. Grest, & C.M. Soukoulis

Jarosław Błasiok
See C.E. Tsourakakis.

Andreas Blass

Andreas Blass & Frank Harary

The theorem that deletion index = negation index of a signed graph (Harary (1959b)) is shown to be a special case of a very general phenomenon involving hereditary classes of “partial choice functions”. Another special case: deletion index = alteration index of a gain graph [an immediate corollary of Harary–Lindström–Zetterström (1982a), Thm. 2]. (SG, GG: Bal, Fr)

Andreas Blass & Bruce Sagan

§3: “Non-crossing $B_n$ and $D_n$.” Lattices of noncrossing signed partial partitions. Atoms of the lattices are defined as edge fibers of the signed covering graph of $\pm K_n^o$, thus corresponding to edges of $\pm K_n^o$. [The “half edges” are perhaps best regarded as negative loops.] The lattices studied, called $NCB_n$, $NCD_n$, $NCBD_n(S)$, consist of the noncrossing members of the Dowling and near-Dowling lattices of the sign group, i.e., $\text{Lat}(\pm K_n^o(T))$ for $T = [n], \varnothing, [n] \setminus S$, respectively. (SG: Geom, M(Gen), Invar, cov)

Signed-graph chromatic polynomials are recast geometrically by observing that the number of $k$-colorings equals the number of points of $\{-k, -k+1, \ldots, k-1, k\}^n$ that lie in none of the edge hyperplanes of the signed graph. The interesting part is that this generalizes to subspace arrangements of signed graphs and, somewhat ad hoc, to the hyperplane arrangements of the exceptional root systems. [See also Athanasiadis (1996a), Zaslavsky (20xxi). For applications see articles of Sagan and P. Zhang.]

Matthias Bloss


Let $G$ denote any group. The algebra is $\mathbb{C}\text{Lat}^b G(\mathfrak{G}K_{2k}(U,W))$ where $\text{Lat}^b G(\mathfrak{G}K_{2k}(U,W))$ is the semilattice of balanced flats of the Dowling lattice $Q_{2k}(\mathfrak{G})$ on a set $V := U \uplus W$ of $2k$ vertices, $U := \{u_1, \ldots, u_k\}$, and $W := \{w_1, \ldots, w_k\}$.

The definition requires a multiplication on $\text{Lat}^b G(\mathfrak{G}K_{2k}(U,W))$ which involves an indeterminate $x$. For each balanced flat (equivalently, $\mathfrak{G}$-valued partition) $\alpha$ label its vertices $u_{\alpha i} := u_i$, $w_{\alpha i} := w_i$. Define $\gamma := \alpha \cdot \beta$ by identifying $w_{\alpha i}$ with $u_{\beta i}$ in $\alpha \uplus \beta$ (call the result $\gamma'$), taking the closure in $G(\mathfrak{G}K_{3k})$, multiplying by $x^m$ where $m := \#$ of components of $\gamma'$ contained completely within the identified vertices, and deleting the identified vertices $w_{\alpha i}$. Set $u_{\gamma i} := u_{\alpha i}$ and $w_{\gamma i} := w_{\beta i}$. [Annot. 20 Mar 2011.]

Avrim Blum

See N. Bansal.

Rafał Bocian, Mariusz Felisiak, & Daniel Simson


F.T. Boesch, X. Li [Xiao Ming Li], & J. Rodriguez


Two-graphs and switching are mentioned.

Irina E. Bocharova, Florian Hug, Rolf Johannesson, Boris D. Kudryashov, & Roman V. Satyukov


**Sebastian Böcker & Jan Baumbach**


Survey. Cluster editing is equivalent to finding the frustration index, cluster index, or $p$-cluster index of a signed complete graph $\Sigma = (K_n, \sigma)$, often with such restrictions as negating at most $k$ edges at each vertex, or minimizing the total negated weight if edges are weighted. Dictionary: “editing” = negating some edge signs, “$(p)$-cluster editing” = changing the fewest signs to make $\Sigma^+$ into a union of $(p)$ disjoint cliques; 2-cluster editing” = finding a minimum balancing edge set. [The field seems to be unaware of signed graphs, balance, and clustering and that this is the special case of Davis (1967a) with underlying complete graph.] [Annot. 4 Nov 2017.]

**Sebastian Böcker, Falk Hüffner, Anke Truss, & Magnus Wahlström**


The signed graph arises as a graph with edges labelled $= (\pm)$ or $\neq (\mp)$.

The “Balanced Subgraph” problem is to find a minimum balancing set. The algorithm of Hüffner, Betzler, and Niedermeier (2007a) is applied. [Annot. 6 Feb 2011.]

**Alexander Bockmayr**

See also H. Siebert.

**Alexander Bockmayr & Heike Siebert**


**Hans L. Bodlaender, Michael R. Fellows, Pinar Heggernes, Federico Mancini, Charis Papadopoulos, & Frances Rosamond**


The “fuzzy graphs” (not related to fuzzy graph theory) are, in essence, signed simple graphs; “fuzzy edges” are non-edges, which in a “normalization” become signed edges, resulting in a signed $K_n$.

(SG: Clu, KG: Alg)

Bernhard G. Bodmann & Vern I. Paulsen

Develops Strohmer and Heath (2003a) and Holmes and Paulsen (2004a). §4, “Two-uniform frames and graphs”: In Def. 4.1, the “signature matrix” $Q$ is $A(K_n, \varphi)$, $(K_n, \varphi)$ = real or complex unit gain graph and (Thm. 4.2) has exactly 2 distinct eigenvalues. Ex. 4.4, 4.5: $Q = A(K_\Gamma)$, $K_\Gamma$ = signed graph, from conference, skew-conference, and Hadamard matrices. Thm. 4.7: For real 2-uniform frames, $Q = A(K_\Gamma)$ where $\Gamma$ has 2 eigenvalues. Frame equivalence = graph switching equivalence. Hence, # of inequivalent real 2-uniform frames = # of switching classes = # regular two-graphs. §5, “Graphs and error bounds”. Def. 5.8: $G^{(m)}_n = \text{set of } (K_n, \sigma) \text{ with frustration index } s$. Induced complete bipartite subgraphs (up to switching) hinder error bounds and have other significance. Lemma 5.19: $E_3 = \#$ of non-triples of a regular two-graph. §6: Specific examples.


Bernhard G. Bodmann, Vern I. Paulsen, & Mark Tomforde

Adjacency matrices of cube-root-of-unity gain graphs on $K_n$. Dictionary: “Seidel matrix” = adjacency matrix of such a gain graph. [Annot. 27 Apr 2012.]

(gg: kg: TG: Adj: Geom, adj)

T.B. Boffey


Kenneth P. Bogart
See M.K. Bennett, J.E. Bonin, and J.R. Weeks.

Petre Boldescu

Generalized Ceva [strengthened via gain graphs in Zaslavsky (2003b) §2.6] and Menelaus theorems. [Problem. Formulate, explain, generalize Boldescu’s Menelaus generalization in terms of gain graphs.]

(gg: Geom)
Ethan D. Bolker


The elementary 1-cycles associated with circuits of $G(-\Gamma)$ (“bicycles”) are crucial. [Their first publication, I believe.]

**EC, sg: m**


**EC, SG: M, incid**

Ethan D. Bolker & Thomas Zaslavsky


An idea of Bolker’s (1979a), as developed by Bouchet (1983a), is turned into an algorithm slightly simpler than that of Appa and Kőnnyek (2006a).

**SG: Ori, Incid, Alg, Sw**

Béla Bollobás


A rich source of problems: find interesting generalizations to signed graphs of questions involving even or odd circles, or bipartite graphs or subgraphs.

**par: Xtreml**

§3.2, Thm. 2.2, is Lovász’s (1965a) characterization of the graphs having no two vertex-disjoint circles. [Problem. Generalize to biased graphs having no two vertex-disjoint unbalanced circles, Lovász’s theorem being the contrabalanced case.]

**GG: Circ**

§6.6, Problem 47, is the theorem on biparticity (all-negative vertex frustration number) from Bollobás, Erdős, Simonovits, & Szemerédi (1978a).

**par: Fr**


Sign-colored plane graphs in Ch. X, “The Tutte polynomial”, §6, “Polynomials of knots and links”, pp. 368–370. Little use is made of the signs.

**SGc: Knot**

B. Bollobás, P. Erdős, M. Simonovits, & E. Szemerédi


Thm. 9 asymptotically estimates upper bounds on frustration index and vertex frustration number for all-negative signed graphs with fixed negative girth. [Sharpened by Komlós (1997a).]

**par: Fr**

Béla Bollobás & András Gyárfás


**Bela Bollobás, Luke Pebody, & Oliver Riordan**


§4, “Coloured graphs”.

**Bela Bollobás & Oliver Riordan**


Discovers the fundamental relations for the commutative algebra underlying the parametrized Tutte polynomial of colored graphs. Cf. Zaslavsky (1992b).


The polynomial is a deletion-contraction invariant of signed graphs with rotation systems (called “ribbon graphs”).

**Erik G. Boman, Doron Chen, Ojas Parekh, & Sivan Toledo**


A real symmetric matrix $= H(\Phi)H(\Phi)^T$ for a real gain graph $\Phi$ with a link (called “factor width 2”). Thm. 9. $A$ has factor width 2 iff it is a symmetric $H$-matrix with diagonal $\geq 0$. [Annot. 8 Mar 2011.]

**Phillip Bonacich**


P. 214: The distribution of power depends in part on whether $H(–\Gamma)$ has full rank, i.e., $\Gamma$ is bipartite (cf. van Nuffelen (1973a)), where $\Gamma$ is the graph of potential exchanges. [Annot. 13 Aug 2012.]


§1.1.3, “Uses of $c(\beta)$ and $x$ in signed graphs”. [Annot. 12 Sept 2010.]

**Phillip Bonacich & Paulette Lloyd**


Compares the dominant-eigenvector measure of centrality in $\Sigma, \Sigma^+$, and dense induced subgraphs, in a standard example. [Annot. 22 Oct 2009.]

**Valerio Boncompagni, Irena Penev, & Kristina Vušković**


J.A. Bondy & L. Lovász

If $\Gamma$ is $k$-connected [and not bipartite], then any $k \lfloor k - 1 \rfloor$ vertices lie on an even [odd] circle. [Problem. Generalize to signed graphs, this being the all-negative case.]

J.A. Bondy & M. Simonovits

If a graph has enough edges, it has even circles of all moderately small lengths. [Problem 1. Generalize to positive circles in signed graphs, this being the antibalanced (all-negative) case. For instance, Problem 2. If an unbalanced signed simple graph has positive girth $\geq l$ (i.e., no balanced circle of length $< l$), what is its maximum size? Are the extremal examples antibalanced? Balanced?] (par: bal(Circ), Xtreml)

Joseph E. Bonin
See also M.K. Bennett.


A weight-$k$ higher Dowling geometry of rank $n$, $Q_{n,k}(GF(q)^{\times})$, is the union of all coordinate $k$-flats of $PG(n - 1, q)$: i.e., all flats spanned by $k$ elements of a fixed basis. If $k > 2$, the automorphism groups are those of $PG(n - 1, q)$ for $q > 2$ and are symmetric groups if $q = 2$. (gg: Gen: M, Aut)


See definition in (1993a). For $k > 2$ the only nontrivial modular flats are the projective coordinate $k$-flats and their subflats. This gives some information about the characteristic polynomials [which, however, are still only partially known]. [Kung (1996a), §6, has further results.] (gg: Gen: M: Invar)


The automorphisms of a Dowling geometry of a nontrivial group are the compositions of a coordinate permutation, switching, and a group automorphism. A similar result holds, with two exceptions, if some or all coordinate points are deleted. [A third exception is missed: the jointless
Dowling geometry $Q_3^0(\mathbb{Z}_3)$. [Cf. Schwartz (2002a).] (gg: M: Aut)


Problem 6.1. If a finite matroid embeds in the Dowling geometry of a group, does it embed in the Dowling geometry of some finite group? [No; see Brooksbank, Qin, Robertson, and Seress (2004a).] (gg: M: Aut)


Dowling geometries are used to prove Prop. 1.1. [Annot. 27 May 2018.]

§4 concerns Dowling lattices. (GG: M)

Joseph E. Bonin & Kenneth P. Bogart

Joseph E. Bonin & Joseph P.S. Kung


Prop. 4.6. The Dowling matroids $Q_r(\mathcal{E})$ are an example. [Annot. 10 Jan 2016.]

Joseph E. Bonin & William P. Miller

Dowling geometries are characterized amongst all simple matroids by numerical properties of large flats of ranks $\leq 7$ (Thm. 3.4); amongst all matroids by their Tutte polynomials. (gg: M)

Joseph E. Bonin & Hongxun Qin

Extremal matroid theory. The Dowling geometry $Q_3(GF(3)^\times) = G(\pm K^*_3)$ appears as an exceptional extremal matroid in Thm. 2.10. The extremal subset of $PG(n - 1, q)$ that does not contain the higher-weight Dowling geometry $Q_{m, m-1}(GF(q)^\times)$ (see Bonin (1993a)) is found in Thm. 2.14. (GG, Gen: M: Extrem, Invar)

C. Paul Bonnington & Charles H.C. Little

Signed-graph imbedding: see §2.3, §2.6 (esp. Thm. 2.4), pp. 44–48 (for the colorful 3-gem approach to crosscaps), §3.3, and Ch. 4 (esp. Thms.
Stefan Bornholdt
See J. Réichardt.

Bojana Borovičanin

E. Boros, Y. Crama, & P.L. Hammer


Endre Boros, Vladimir A. Gurvich, & Igor E. Zverovich

Oriented all-negative graphs in which every two vertices are joined by a unique coherent path. (The authors describe this as alternating paths in an edge 2-colored graph. The “two-graph” is the pair of monocolored graphs.) [Problem. Generalize to arbitrary bidirected graphs.] [Cf. Bánkfalvi and Bánkfalvi (1968a) and Bang-Jensen and Gutin (1997a) for alternating walks.] [Annot. 25 Oct 2012.]

Endre Boros & Peter L. Hammer

Includes finding a minimum-weight deletion set (as in Boros, Crama, and Hammer (1992a)).

Bartłomiej Bosek
See M. Anholcer.

J.-P. Bouchaud, F. Krzakala, & O.C. Martin

 Mostly, randomly weighted signed graphs (square and cubic lattices) with Gaussian signed weights. §VII, “Case of +/ − J couplings”: Calculation experiments suggest unweighted signed graphs behave very differently from weighted ones. “[T]he local environment of a spin has no disorder out to finite distances: any sign of the Jij can be gauged switched away ... .” [That seems to mean imbalance can be switched away, which is wrong and casts doubt on the conclusions.] [Annot. 28 Jan 2015.]

André Bouchet

Introduces nowhere-zero flows on signed graphs. A connected, coloop-free signed graph has a nowhere-zero integral flow with maximum weight \( \leq 216 \). The value 216 cannot be replaced by 5, but: *Conjecture* (Bouchet): it can be replaced by 6. [The bidirection is inessential; it is a device to keep track of the flow.] [For progress see Khelladi (1987a), Zýka (1987a), Xu and Zhang (2005a), Raspaud and Zhu (2011a), Akbari, Daemi, et al. (2015a), Wei, Tang, and Dan (2014a), Schubert and Steffen (2015a). See Jensen and Toft (1995a) for other contributions.]

A topological application is outlined. [Annot. ca. 1983.]

***Jean-Marie Bourjolly***


[See Sewell (1996a).]  

***J.-M. Bourjolly, P.L. Hammer, & B. Simeone***


***Jean-Marie Bourjolly & William R. Pulleyblank***


[It is hard to escape the feeling that we are dealing with all-negative signed graphs and that something here will generalize to other signed graphs. Especially see Thm. 5.1. Consult the references for related work.]
Maximum frustration is $l_{\text{max}}(\Gamma) := \max_{\sigma} l(\Gamma, \sigma)$. Thm. 27: $l_{\text{max}}(K_{l,r} = \frac{1}{2} lr \left(1 - 2^{-(l-1)} \left\lceil \frac{l-1}{2} \right\rceil \right)$). It is attained uniquely if $2^{l-1} | r$ and not at all otherwise.

Thm. 31: $l_{\text{max}}(K_{5,r}) = \left\lceil \frac{25}{16} r \right\rceil - \varepsilon_r$ where $\varepsilon_r \in \{0, 1\}$, $= 1$ iff $r \equiv 2, 4, 9, 13 \mod 16$. Thm. 33: $l_{\text{max}}(K_{6,r}) = \left\lceil \frac{69}{32} r \right\rceil - \varepsilon_r$ where $\varepsilon_r \in \{0, 1, 2\}$ and depends on $r \mod 32$ if $r > 6$. Thm. 33: $l_{\text{max}}(K_{7,r}) = \left\lceil \frac{154}{64} r \right\rceil - \varepsilon_r$ where $\varepsilon_r \in \{0, 1\}$ and depends on $r \mod 64$ if $r > 49$. Question. Is $\varepsilon_r$ for fixed $l$ bounded by a linear function of $l$?

Results in (2002a) on cheapest flow from source are incorrect. [Annot. 21 Mar 2011.] (GN: Alg)

Franz J. Brandenburg & Mao-Cheng Cai

See (2011a). (gg: incid: Alg, m)


Additive real gains. The lift matroid is implicit. Contrasts algorithmic complexity of additive with multiplicative gains. [Annot. 30 May 2012.] (gg: incid: Alg, m)

Benjamin Braun & Sarah Crown Rundell

A.J. Bray
See also G.J. Rodgers.

A.J. Bray, M.A. Moore, & P. Reed


Richard C. Brewster, Florent Foucaud, Pavol Hell, & Reza Naserasr

Richard C. Brewster & Timothy Graves

Richard C. Brewster & Mark Siggers

Matthew G. Brin
See G.S. Bowlin.

T. Britz, D.D. Olesky, & P. Van Den Driessche
2004a Matrix inversion and digraphs: the one factor case. Electronic J. Linear Algebra
Hajo Broersma

See D. Hu.

Jared C. Bronski & Lee DeVille


Bounds on the positive index of inertia, \( n_+ \), of a weighted graph, in terms of edge signs. [Annot. 20 Mar 2016.] (SG, WG: Adj: Eig)

Jared C. Bronski, Lee Deville, & Paulina Koutsaki


How the positive index of inertia of fixed \( \Sigma \) varies with edge weights. [Annot. 20 Mar 2016.] (SG, WG: Adj: Eig)

Peter Brooksbank, Hongxung Qin, Edmund Robertson, & Ákos Seress


Solution of Bonin (1996a). They produce a finite gain graph that has gains in no finite group. Dictionary: “Dowling geometry” = frame matroid of a gain graph [not an actual Dowling geometry, which would be impossible since a Dowling geometry determines its group; cf. Dowling (1973b)].

A.E. Brouwer, A.M. Cohen, & A. Neumaier


§1.5, “Taylor graphs and regular two-graphs”: Signed complete graphs appear in the form of double covers of the complete graph. §3.8, “Graph switching, equiangular lines, and representations of two-graphs”, §7.6C, “2-Transitive regular two-graphs”. (Tg: kg, Geom: Exp, Ref)

Andries E. Brouwer & Willem H. Haemers


§1.1, “Matrices associated to a graph”: “Laplace matrix” = Laplacian matrix \( L(\pm \Gamma) \), from the “directed [i.e., oriented] incidence matrix” \( H(\pm \Gamma) \). “Signless Laplace matrix” = Laplacian matrix \( L(-\Gamma) \), from the “(undirected) [unoriented] incidence matrix” \( H(-\Gamma) \) (with no \(-1\)s). Many results employ \( L(-\Gamma) \), but signed graphs are ignored; e.g., see §§1.4.5, 14.4.3, “Line graphs” [cf. G.R. Vijayakumar et al.]. §1.8.2, “Seidel switching”, defines the Seidel adjacency matrix \( A(K_\Gamma) \) and its switching. Ch. 10, “Regular two-graphs”.


Floor Brouwer & Peter Nijkamp
Edward M. Brown & Robert Messer

Their “signed graph” we might call a type of Eulerian partially bidi rected graph. That is, some edge ends are oriented (hence “partially bidirected”), and every vertex has even degree and at each vertex equally many edge ends point in and out (“Eulerian”). More specially, at each vertex all or none of the edge ends are oriented. (sg: ori: gen: Appl)

Gerald G. Brown & Richard D. McBride

Gerald G. Brown, Richard D. McBride, & R. Kevin Wood

Identifying largest embedded generalized network matrices (i.e., incidence matrices of real multiplicative gain graphs) in a matrix is NP-complete. Heuristic algorithms for finding such embedded matrices and using them to speed up linear programming. [Annot. 2 Oct 2009.] (GN: Incid: Alg)

John Brown, Chris Godsil, Devlin Mallory, Abigail Raz, & Christino Tamon

Kenneth S. Brown & Persi Diaconis

The real hyperplane arrangement representing $-K_n$ is studied in §3D. It leads to a random walk on threshold graphs. (par: Geom)

Thomas A. Brown
See also F.S. Roberts.

T.A. Brown, F.S. Roberts, & J. Spencer

Thomas A. Brown & Joel H. Spencer

Asymptotic estimates for the Gale–Berlekamp switching game, i.e., $l(K_{r,s})$, the maximum frustration index of signatures of $K_{r,s}$. [Improved by Gordon and Witsenhausen (1972a) and Bowlin (2009a), (2012a).] Also, exact values stated for $r \leq 4$ [extended by Solé and Zaslavsky (1994a) to $r = 5$, which was corrected and generalized by Bowlin (2009a), (2012a)]. [Cf. also Fishburn and Sloane (1989a), Carlson and Stolarski...
(2004a), and Roth and Viswanathan (2007a), (2008a) on Berlekamp’s game, where \( r = s \).

William G. Brown, ed.


See esp.: §208: “Signed graphs (+ or − on each edge), balance” (undirected and directed), Vol. 1, pp. 569–571. (SG, SD)

Richard A. Brualdi


§8.2, “Symmetric transportation polytopes”: The vertices of the polytope of symmetric, non-negative matrices with given line sums (Thm. 8.2.1, due to Brualdi (1976a), Converse and Katz (1975a), Lewin (1977a)) or bounded line sums (Thms. 8.2.6–8) correspond to the independent sets in the frame matroid \( G(\overline{K}_n) \). [Problem. Generalize to a polytope whose vertices are associated with independent sets in \( G(\pm K_n) \). Possibly, the matrices have prescribed entry signs determining a signed graph.] [Annot. 13 Oct 2012.] (sg: par: Adj)


§6.1, “Sign-nonsingular matrices”: Signed digraphs, called “weighted digraphs” of \( (0, \pm 1) \)-matrices such that every matrix with that sign pattern is nonsingular. Cf. esp. Maybee et al., van den Driessche et al.[Annot. 20 Nov 2011.] (QM: QSol: sd: Exp)


Richard A. Brualdi, Kathleen P. Kiernan, Seth A. Meyer, & Michael W. Schroeder


Richard A. Brualdi & Nancy Ann Neudauer


Minimal transversal presentations of \( G(\Gamma, \emptyset) \), given \( \Gamma \). (Bic)

Richard A. Brualdi & Herbert J. Ryser

See §7.5. (QM: QSol, SD, bal)(Exp, Ref)

Richard A. Brualdi & Bryan L. Shader


§1 reviews Seymour and Thomassen (1987a). Thm. 2.1: If two sign-nonsingular \((0,1,-1)\)-matrices have the same 0’s (and total support), their signed digraphs are switching equivalent. [Annot. 12 Jun 2012.]

(QM, SD: QSol: Exp)


Innumerable results and references on signed digraphs are contained herein. (QM, SD: QSol, QSta)(Exp, Ref, Alg)

Frank J. Bruggeman

See B.N. Kholodenko.

Jeroen Bruggeman

See V.A. Traag.

Michael Brundage


A concise expository survey. Ch. 1: “Even cycles in directed graphs”. Ch. 2: “\(L\)-matrices and sign-solvability”, esp. § “Signed digraphs”. Ch. 3: “Beyond”, esp. § “Balanced labellings” (vertices labelled from the set \(\{0, +1, -1\}\) so that from each vertex labelled \(\varepsilon \neq 0\) there is an arc to a vertex labelled \(-\varepsilon\) and § “Pfaffian orientations”.

(SD, Par: Circ, QSol, Alg, VS: Exp, Ref)

Maurizio Brunetti

See also A. Alazemi and F. Belardo.


Unicycles with pendants that are \(K_2\). For girth 3, extremal spectral radius and first and second Zagreb indices. [Annot. 15 Dec 2020.]

(SG: Lap: Eig)

Maurizio Brunetti, Matteo Cavaleri, & Alfredo Donno


I.e., composition \(\Sigma[A]\). Modified from Hameed and Germina’s (2012b) definition and said to respect switching (but see (2019b)). It has good eigenvalue properties. [Annot. 20 Apr, 5 Jul 2019.]

(SG: Bal, Sw, Eig: Adj, Lap)

Their definition does not respect switching. Problem: Find a definition that does. [Annot. 2 Sep 2019.]

(Michael Brusco, Patrick Doreian, Andrej Mrvar, & Douglas Steinley)


(Thomas H. Brylawski [Tom Brylawski])


Implicitly, switching in the bipartite gain graph of a matrix. (gg: sw)


§5, “q-Analogs from a Möbius identity”: §5.1, “Dowling lattices” (an example): A complicated identity is derived from the Möbius function of $Q_n(\mathbb{F}_q^\times)$. [Annot. 26 Dec 2015.]

(Thomas Brylawski & James Oxley)


(J.A. Brzozowski)

See C.J. Shi.

Changjiang Bu & Jiang Zhou


A subdivided star is determined, among all graphs, by Spec $L(-\Gamma)$. This completes work of Omidi (2009a) and of Omidi and Vatandoost (2010a). [Annot. 28 Nov 2012.]

(par: Lap: Eig)


Let $\Gamma = \Delta \vee K_1$, the join of $\Delta$ and a point. Spec $L(-\Gamma)$ determines $\Gamma$ if: $\Gamma$ or $\Gamma^c$ is a matching; 2-regular $\Gamma$ has $n \geq 11$; 2-regular $\Gamma^c$ is triangle-free. [This implies graphs with the same Spec $L(-\Gamma^c).$]

Also, Thm. 3.6: Spec $L([-K_{1,3} \vee \Gamma]) = \text{Spec} L(-[(C_3 \vee K_1) \vee \Gamma])$ for any $\Gamma$. [Annot. 28 Nov 2012.]

(par: Lap: Eig)

[Miguel Á. Valencia Bucio]

See M.A. Valencia Bucio (under ‘V’).
Fred Buckley, Lynne L. Doty, & Frank Harary

“Signed invertible graph” [i.e., sign-invertible graph] = graph \( \Gamma \) such that \( A(\Gamma)^{-1} = A(\Sigma) \) for some signed graph \( \Sigma \). Finds two classes of such graphs. Characterizes sign-invertible trees. [Cf. Godsil (1985a) and, for a different notion, Greenberg, Lundgren, and Maybee (1984b).]

(FG: Adj)

Fred Buckley & Frank Harary

Signed graphs and sign-invertible graphs (Buckley, Doty, and Harary (1988a)): pp. 120–122. (SG: Adj: Exp)

Yurii Burman, Andrey Ploskonosov, & Anastasia Trofimova

James R. Burns & Wayland H. Winstead

§ IV: “The computation of contradictory redundancy.” Summarized in modified notation: In a signed graph, define \( w_{ij}^+(r) = \) number of walks of length \( r \) and sign \( \varepsilon \) from \( v_i \) to \( v_j \). Define an adjacency matrix \( A \) by

\[
\begin{align*}
a_{ij} &= w_{ij}^+(1) + w_{ij}^-(1)\theta, \\
\text{where } \theta \text{ is an indeterminate whose square is 1.}
\end{align*}
\]

Then \( w_{ij}^+(r) + w_{ij}^-(r)\theta = (A^r)_{ij} \) for all \( r \geq 1 \). [We should regard this computation as taking place in the group ring of the sign group, where the sign group is treated as \(+1, \theta\). The generalization to arbitrary gain graphs and digraphs is obvious.] Other sections also discuss signed digraphs [but have little mathematical content]. (SD, gd: Adj, Paths)

Eugene Burnstein
See R.B. Zajonc.

Arthur H. Busch, Michael S. Jacobson, Timothy Morris, Michael J. Plantholt, & Shailesh K. Tipnis


Arthur Busch, Mohammed A. Mutar, & Daniel Slilaty
20xxa Hamilton cycles in bidirected complete graphs. Submitted.

A (coherent) cycle generalizes an alternating circle in an edge-2-colored graph and a directed cycle in a directed graph (cf. annotation on Bankfalvi and Bankfalvi (1968a)). Strong connection is defined for a bidirected graph. Thm. 3.3: An oriented \( \pm K_n \) \((n \geq 3)\) has a cycle of every length \( 3, \ldots, n \) iff it is strongly connected. Thm. 4.4: It has a Hamiltonian cycle iff it is strongly connected and has an alternating 2-factor.
F.C. Bussemaker, P.J. Cameron, J.J. Seidel, & S.V. Tsaranov

F.C. Bussemaker, D.M. Cvetković, & J.J. Seidel

The 187 simple graphs with eigenvalues $\geq -2$ that are not (negatives of) reduced line graphs of signed graphs are found, with computer aid. By Cameron, Goethals, Seidel, and Shult (1976a), all are represented by root systems $E_d$, $d = 6, 7, 8$. Most interesting is Thm. 2: each such graph is Seidel-switching equivalent to a line graph of a graph. [Problem. Explain this within signed graph theory.]

F.C. Bussemaker, R.A. Mathon, & J.J. Seidel

All $[K_n, \sigma]$, also cospectral pairs, and those that are regular, integral, or vertex-transitive, for $n \leq 9$, plus larger special types. [Annot. 29 Dec 2020.]

F.C. Bussemaker & A. Neumaier

They are the antibalanced signed graphs with largest eigenvalue $-2$. Also, largest eigenvalue around $-2$. Two-graphs and work of Vijayakumar et al. are mentioned. [Annot. 29 Apr 2012.]

Steve Butler

Generalizing D’Amato (1979a) and Bilu and Linial (2006a), the “signed graph” $G$ is vertex-signed; it is a branched double cover of a signed graph $H$ whose edge signs are incorporated into weights. The interesting new idea is the branching, wherein a vertex may be singly covered. [May the branches correspond to half edges?] Adjacency and normalized Laplacian spectra of $G$ are each obtained from those of $H$ and...
a modified $H$. [Annot. 9 Mar 2011.] (VS(\text{Gen: Eig})\langle SG: \text{cov, Eig} \rangle)

Steve Butler, Minerva Catral, H. Tracy Hall, Leslie Hogben, Xavier Martínez-Rivera, Bryan Shader, & Pauline van den Driessche


Example: Hermitian matrices of mixed graphs. Gain group \{±1, ±i\}: $\varphi(e) = 1$ for undirected, i for directed edges. (gg: Adj)

Jesper Makholm Byskov, Bolette Ammitzbøll Madsen, & Bjarke Skjernaa


Bounds on the number of maximal induced bipartite subgraphs. [Problem. Generalize to maximal induced balanced subgraphs, equivalently minimal balancing sets of vertices, especially in a signed graph.] (par: bal)

S. Cabasino, E. Marinari, P. Paolucci, & G. Parisi


He Cai

See Y. Jiang.

Leishen Cai & Baruch Schieber


By the negative-subdivision trick (subdividing each positive edge into two negative ones), the algorithm will find the intersection of all negative circles of a signed graph. (Par, sg: Fr, Circ: Alg)

Mao-cheng Cai

See F.J. Brandenburg.

Qing Cai, Maoguo Gong, Lijia Ma, Shanfeng Wang, Licheng Jiao, & Haifeng Du


“Balance problem” = clustering problem. Degree of inclusterability. As do many “social balance” articles, wrongly attributes Cartwright–Harary (1956a) balance theory to Heider and incorrectly says the former only treats complete graphs. [Annot. 27 Nov 2018.] (SG: Clu: Alg)

Qing Cai, Maoguo Gong, Bo Shen, Lijia Ma, & Licheng Jiao


Grant Cairns & Yuri Nikolayevsky


Thm. 2: \(\Gamma\), connected and not bipartite, has a generalized thrackle drawing in the orientable surface of genus \(g\) iff \(-\Gamma\) has an orientation
embedding in the nonorientable surface with demigenus $2g-1$. [Problem. Generalize to all signed graphs.]

T. Calamoneri & R. Petreschi


Tiziana Calamoneri, Angelo Monti, & Rossella Petreschi


“Threshold signed graph” [not a signed graph]: a graph such that $(\exists S, T \in \mathbb{R}_{>0})(\exists a : V \to \mathbb{R}) |a(v)| < \min(S, T)$ and $vw \in E \iff |a(v) + a(w)| \geq S$ or $|a(v) - a(w)| \geq T$. [Cf. Benzaken, Hammer, and de Werra (1985a).] (VS, sg)


Kyle David Calderhead


Ch. 6, “Type B analogs”, introduces threshold signed graphs and applies signed graphs to the slopes problem (the minimum number of slopes of $n$ points in the plane) for centrally symmetric points. A signed graph is threshold if its double cover is a threshold graph. (SG)

Laurence Calzone

See J.-P. Comet.

Verónica Cambiazo

See J. Aracena.

Peter J. Cameron

See also L. Babai and F.C. Bussemaker.


The first step towards (1977b), Thm. 3.1. (TG: Aut)


Introducing the cohomological theory of two-graphs. A two-graph $\tau$ is a 2-coboundary in the complex of GF(2)-cochains on $E(K_n)$. [The 1-cochains are the signed complete graphs, equivalently the graphs that are their negative subgraphs. Cf. D.E. Taylor (1977a).] Write $Z_i$, $Z^i$, $B^i$ for the $i$-cycle, $i$-cocycle, and $i$-coboundary spaces. Switching a signed complete graph means adding a 1-cocycle to it; a switching class of signed complete graphs is viewed as a coset of $Z^1$ and is equivalent to a two-graph.
Take a group $\mathcal{G}$ of automorphisms of $\tau$. Special cohomology elements $\gamma \in H^1(\mathcal{G}, B^1)$ and $\beta \in H^2(\mathcal{G}, B^0)$ (where $B^0 = \{0, V(K_n)\}$, the reduced 0-coboundary group) are defined. Thm. 3.1: $\gamma = 0$ iff $\mathcal{G}$ fixes a graph in $\tau$. Thm. 5.1: $\beta = 0$ iff $\mathcal{G}$ can be realized as an automorphism group of the canonical double covering graph of $\tau$ (viewing $\tau$ as a switching class of signed complete graphs). Conditions are explored for the vanishing of $\gamma$ (related to Harries and Liebeck (1978a)) and $\beta$.

$Z^1$ is the annihilator of $Z_1 = \text{the space of even-degree simple graphs}; the theorems of Mallows and Sloane (1975a) follow immediately. More generally: Lemma 8.2: $Z^i$ is the annihilator of $Z_i$. Thm. 8.3: The numbers of isomorphism types of $i$-cycles and $i$-cocycles are equal, for $i = 1, \ldots, n - 2$.

§8 concludes with discussion of possible generalizations, e.g., to oriented two-graphs (replacing $\text{GF}(2)$ by $\text{GF}(3)^r$) and double coverings of complete digraphs (Thms. 8.6, 8.7). [Cf. Moorhouse (1995a). A full ternary analog is developed in Cheng and Wells (1984a)]

(TG: Sw, Aut, Enum, Geom)


Exposition of parts of (1977b) with a simplified proof of the connection between $\beta$ and $\gamma$.

(TG: Aut, Enum, Geom, Exp)


[For generalized line graphs see Zaslavsky (1984c).] If two generalized line graphs are isomorphic, their underlying graphs and cocktail-party attachments are isomorphic, with small exceptions related to exceptional isomorphisms and automorphisms of root systems. The proof, along the lines of Cameron, Goethals, Seidel, and Shult (1976a), employs the canonical vector representation of the underlying signed graph.

(sg: LG: Aut, Geom)


§8, “Switching”: Graph switching, graph switching classes. Existence of a “representative”: a graph in a switching class that has the same automorphism group as the switching class. §9, “Digraphs”: Switching classes of tournaments on pp. 117–118. Switching a digraph means reversing all edges between $X \subseteq V$ and $X^c$.[Annot. 27 Dec 2010.]

(TG: Sw: Aut: Exp)


Let $T$ be a tree. Construction 1 (simplifying Seidel and Tsaranov (1990a)): Take all triples of edges such that none separates the other two. This defines a two-graph on $E(T)$ [whose underlying signed complete graph is described by Tsaranov (1992a)]. Construction 2: Choose $X \subseteq
the electronic journal of combinatorics #DS8

V(T). Take all triples of end vertices of T whose minimal connecting subtree has its trivalent vertex in X. The two-graphs (V, T) that arise from these constructions are characterized by forbidden substructures, namely, the two-graphs of (1) C_5 and C_6; (2) C_5. Also, trees that yield identical two-graphs are characterized.


Counting two-graphs of the types constructed in (1994a). (TG)


P.J. Cameron, J.M. Goethals, J.J. Seidel, & E.E. Shult


The essential idea is that graphs with least eigenvalue ≥ −2 are represented by the angles of root systems. It follows that line graphs are so represented. [Similarly, signed graphs with largest eigenvalue ≤ 2 are represented by the inner products of root systems, as in Vijayakumar et al. These include the line graphs of signed graphs as in Zaslavsky (1984c), since simply signed graphs are represented by B_n or C_n with a few exceptions. The representation of ordinary graphs by all-negative signed graphs is motivated in Zaslavsky (1984c).]

Peter J. Cameron, Bill Jackson, & Jason D. Rudd


The number of signatures of $K^*_n$, the complete graph with loops, under symmetry, switching, and negation (treated as totally nonzero symmetric sign pattern matrices) equals the number of switching isomorphism classes of signed complete graphs. (Cf. Mallows and Sloane (1975a) and Cameron (1979a).) [Annot. 12 Aug 2012, 16 Nov 2015.]

P.J. Cameron, J.J. Seidel, & S.V. Tsaranov


A generalized Coxeter group $Cox(\Sigma)$ and a Tsaranov group $Ts(\Sigma)$ are defined via Coxeter relations and an extra relation for each negative circle in $\Sigma$. They generalize Coxeter groups of tree Coxeter graphs and the Tsaranov groups of a two-graph ($|\Sigma| = K_n$; see Seidel and Tsaranov (1990a)). A new operation of “local switching” is introduced, which changes the edge set of $\Sigma$ but preserves the associated groups.
§2, “Signed graphs”, proves some well-known properties of switching and reviews interesting data from Bussemaker, Cameron, Seidel, and Tsaranov (1991a). §3, “Root lattices and Weyl groups”: The “intersection matrix” $2I + A(\Sigma)$ is a hyperbolic Gram matrix of a basis of $\mathbb{R}^n$ whose vectors form only angles $\pi/2, \pi/3, 2\pi/3$. To these vectors are associated the lattice $L(\Sigma)$ of their integral linear combinations and the Weyl group $W(\Sigma)$ generated by reflecting along the vectors. $W$ is finite iff $2I + A(\Sigma)$ is positive definite (Thm. 3.1). Problem 3.6. Determine which $\Sigma$ have this property. §4 introduces local switching to partially solve Problem 4.1: Which signed graphs generate the same lattice? Results and some experimental data are reported. All-negative signed graphs play a special role. Definition of local switching at $v$: (1) switch so the edges at $v$ are positive, (2) divide the components of the negative subgraph of the neighborhood of $v$ into two halves $J, K$, (3) add negative edges joining all vertices of $J$ to all those of $K$, (4) negate all edges from $v$ to $J$, (5) reverse the switching in step (1). [See Isihara (2007a) for more.] §6, “Coxeter groups”: The relationship between the Coxeter and Weyl groups of $\Sigma$. $\text{Cox}(\Sigma) = \text{Cox}(|\Sigma|)$ with additional relations for antinegative (i.e., negative in $-\Sigma$) induced circles. §7: “Signed complete graphs”. §8: “Tsaranov groups” of signed $K_n$’s §9: “Two-graphs arising from trees” (as in Seidel and Tsaranov (1990a)).

Dictionary: “$(\Gamma, f)$” = $\Sigma = (\Gamma, \sigma)$. “Fundamental signing” = all-negative signing, giving the antibalanced switching class. “The balance” of a cycle (i.e., circle) = its sign $\sigma(C)$; “the parity” = $\sigma(\overline{C})$ where $\overline{C} = C$ with all signs negated. “Even” = positive and “odd” = negative (referring to “parity”). “The balance” of $\Sigma$ = the partition of all circles into positive and negative classes $C^+$ and $C^-$; this is the bias on $|\Sigma|$ due to the signing and should not be confused with the customary meaning of “balance”, i.e., all circles are positive.

[A more natural definition of the intersection matrix would be $2I - A$. Then signs would be negative to those in the paper. The need for “parity” would be obviated, ordinary graphs would correspond to all-positive signings (and those would be “fundamental”), and the extra Coxeter relations would pertain to negative induced circles.]

(SG: Adj, Eig, Geom, Sw(Genera), lg

Peter J. Cameron & Sam Tarzi

The edges of $K_n$ are colored by $m$ colors. Thm.: For $m > 2$, the combined action of $\Sigma_n$ on vertices and $\Sigma_m$ on colors is transitive on $m$-edge-colored complete graphs for finite $n$ but not for infinite $n$.

(SGc: Gen: Sw)

P.J. Cameron & Albert L. Wells, Jr.

Federico Camia, Emilio De Santis, & Charles M. Newman
Paul Camion
Camion’s signing algorithm (implicitly) finds a set of sign reversals to balance a bipartite signed graph. (sg: Bal)


M. Campanino
Comparing the critical percolation values and the critical temperatures of a finite, positively edge-weighted, signed graph to those of the corresponding all-positive (“unfrustrated”, “ferromagnetic”) weighted graph. The graph is $\Lambda \subset \mathbb{Z}^d$ with a partition $\pi$ of the boundary $\partial \Lambda$ and all edges on each block of $\pi$. Dictionary: “frustrated path” = negative circle including a boundary edge; “frustrated configuration” = subgraph of $\Lambda$ having a negative circle including at least one boundary edge. [Annot. 2 Apr 2013. ] (Phys, SG: WG: Fr)

Sue Ann Campbell
See J. Bélair.

Manoel Campelo & Gérard Cornuéjols
The generalized stable set polyhedron of B is (equivalent to) $\text{conv}(\mathbb{Z}^n \cap \{0 \leq x \in \mathbb{R}^n : H(B)x \leq b\})$ where $b \in \mathbb{Z}^m$, $m = \#E$. Dictionary: “directed edge” = positive, “undirected edge” = negative; “odd cycle” = negative circle. [Annot. 9 Jun 2011.] (sg: ori, Incid, Geom)

Yves Candau
See N. Ramdani.

E. Rodney Canfield
See E.A. Bender.

Chun Zheng Cao
See X.X. Zhu.

D.S. Cao
See R. Simion.

Fayun Cao, Han Ren, & Hanlin Chen
Ming Cao
See A. Proskurnikov.

Andrea Capocci
See V. Ciotti.

Sergio Caracciolo
See also M Palassini.

S. Caracciolo, G. Parisi, & N. Sourlas

Approximation of physical quantities (free energy, critical temperature) on the triangular lattice with all-positive (“ferromagnetic”), all-negative (“antiferromagnetic”, “fully frustrated”), and arbitrary (“randomly frustrated”) signs, by moving edges (“bonds”, “couplings”), adjusting edge weights, and coarsening the lattice to get recursive formulas. Also, a tentative analog for the square lattice with possible diagonals. [Annot. 28 Mar 2013.]

Domingos M. Cardoso
See also N.M.M. de Abreu and I. Gutman.

Domingos M. Cardoso, Dragoš Cvetković, Peter Rowlinson, & Slobodan K. Simić

Thm.: \( \min_{\Gamma} \lambda_1(L(-\Gamma)) \), for connected, nonbipartite \( \Gamma \) with \( \#V = n \) is attained iff \( \Gamma \) is \( K_3 \) with an attached path. [Problem. Generalize to connected, unbalanced signed graphs.] [Annot. 4 Sept 2010.]

Charles Carlson, Karthekeyan Chandrasekaran, Hsien-Chih Chang, Naonori Kakimura, & Alexandra Kolla

Extended abstract of (2020a).

**Jordan Carlson & Daniel Stolarski**


The minimum frustration index of a signed $K_{n,n}$ for $n = 10, 11, 12$ and bounds up to 20. Corrects and extends Fishburn and Sloane (1989a).

Jaime Cartes


Dorwin Cartwright

See also T.C. Gleason, and Harary–Norman–Cartwright (1965a) *et al.*

Dorwin Cartwright & Terry C. Gleason


Pp. 194ff., “A generalization of the method”: Digraph edges have gains in a (commutative) semiring. A matrix method produces counts of paths and cycles of given lengths. Remarks on pp. 194, 199: The method can be applied to signed digraphs (unexplained) [the group ring $\mathbb{Z}[+, -]$ must be intended; matrix entry $= n_+ + n_-$] must be intended; matrix entry $= n_+ + n_-$. Remarks on pp. 194, 199: The method can be applied to signed digraphs (unexplained) [the group ring $\mathbb{Z}[+, -]$ must be intended; matrix entry $= n_+ + n_-$. Dictionary: “generalized addition, multiplication” = operations in the semiring; “value matrix” = adjacency matrix of the gain graph. [Annot. 28 Apr 2017.] (gd(Gen), sd: Adj, Paths, Circ)

Dorwin Cartwright & Frank Harary


Expounds Harary (1953a), (1955a) with sociological discussion. Proposes to measure imbalance by the proportion of balanced circles (the “degree of balance”) or balanced circles of length $\leq k$ (“degree of $k$-balance”). (PsS, SG: Bal, Fr)


“Coloring” is clustering as in Davis (1967a). Thm. 1 adds a bit to Davis (1967a) in Thm. 3: The clustering is unique $\iff$ all components of $\Sigma^+$ are adjacent. (SG: Clu)


Adolfo Casari
See F. Barahona.

Federico Castillo
See F. Ardila.

Paul A. Catlin

Thm. 2: If $\Gamma$ is 4-chromatic, $[\neg\Gamma]$ contains a subdivision of $[\neg K_4]$ (an “odd-$K_4$”). [Question. Can this possibly be a signed-graph theorem? For instance, should it be interpreted as concerning the chromatic number of $\neg \Gamma$?]

(par: col)

Minerva Catral
See also S. Butler.

M. Catral, D.D. Olesky, & P. van den Driessche

$D$ is the signed digraph of a square sign-pattern matrix $S$. Thm. 3.1: If the spectrum of $A$ with signs $S$ is arbitrary, $D$ has positive and negative disjoint cycle unions of all orders. Thm. 4.1: If the inertia is arbitrary, $D$ has a positive and a negative loop and a negative digon. [Annot. 4 Nov 2011.]

(SG: QM, Exp)

Bogdan Cautis
See C. Giatsidis.

Matteo Cavaleri
See also F. Belardo.

Matteo Cavaleri, Daniele D’Angeli, & Alfredo Donno


20xxc Characterizations of line graphs in signed and gain graphs. Submitted. (GG, SG: LG)

Michael S. Cavers


§4, “Characteristic polynomials of skew-adjacency matrices”: From $\Gamma$ form a signed digraph: each edge becomes a negative digon. An
orientation $\Gamma^\tau$ is a choice of one arc from each digon; thus, it is a signed digraph. A “skew adjacency matrix” of $\Gamma$ is a matrix $S := A(\Gamma^\tau)$. The characteristic polynomial $p_S(x)$ is odd/even for odd/even $n$. Eqs. (7), (8) give formulas for the coefficients in terms of matchings and cycle signs. A “generic skew-adjacency matrix” of $\Gamma$ has variables $x_{ij} = x_{ji}$ instead of 1’s in $S$. Thm. 4.2: Spec $S$ is unique iff $\Gamma$ has no even circles (an “odd-circle graph”). Dictionary: “cycle” = circle, “dicycle” = cycle, “$\sigma$” = $\tau$, “$\vec{G}(S)$” = $\Gamma^\tau$.

More results about odd-circle and bipartite (even-circle) graphs. Lemma 6.3 implicitly switches $\Gamma^\tau$ through $S$. [Problem. Generalize to bidirected graphs, thereby unifying symmetric and skew-symmetric adjacency matrices. A negative edge becomes a positive digon, all-positive if oriented extraverted and all-negative if introverted. The all-negative case is symmetric, the all-positive case is skew-symmetric.] [Annot. 4 Jan 2013.]

Michael S. Cavers & Shaun M. Fallat

Signed digraphs are generalized to edges labelled by $0, +, -, +_0 (\geq 0), -_0 (\leq 0), * (\neq 0), \# (\text{real})$. [Annot. 24 May 2013.] (QM: SD(Gen))

Michael S. Cavers & Kevin N. Vander Meulen

Lem. 5.1: An inertially arbitrary sign pattern contains a negative digon. [Annot. 5 Nov 2011.] (QM: sd, sw)

Nicolò Cesa-Bianchi, Claudio Gentile, Fabio Vitale, & Giovanni Zappella


Very short version of (2012c). (SG: Clu)


Less short version of (2012b) and (2012d).

See arXiv:1301.4767 for the full version (16 pp.). (SG: Clu)
Shortest version of (2012c).

Seth Chaiken

§4: “Extension to signed graphs”. Generalizing Zaslavsky (1982a), an all-minors matrix-tree theorem for weighted signed digraphs and a corollary for weighted signed graphs. Given: a signed graph on vertex set \([n]\). For a Laplace-type \(n \times n\) matrix \(L\) (A in the paper), \(L(\bar{U}, \bar{W})\) is \(L\) with the rows indexed by \(U\) and the columns indexed by \(W\) deleted. Take \(U, W \subseteq V\) with \(#U = #W = k \leq n\). Then \(\det L(\bar{U}, \bar{W})\) is a sum of terms, one for each independent set \(F\) of rank \(n - k\) in \(G(\Sigma)\) in which each tree component contains just one vertex from \(U\) and one from \(W\). Each term has a sign depending partly on the number of negative paths by which \(F\) links \(\bar{U}\) to \(\bar{W}\) and partly on the linking pattern, and with magnitude \(4^c\cdot(\text{weight product of } F)\), where \(c = \#\) of circles in \(F\). [The credit to Zaslavsky is overly generous: only the case \(U = W = \emptyset\) is his; the others are new.] The digraph version replaces 4 by 2 and imposes conditions on arc directions in the tree and nontree components of \(F\).

A brief remark describes a gain-graphic (“voltage-graphic”) generalization.

Connects a problem on common covectors of two subspaces of \(\mathbb{R}^m\), and more generally of a pair of oriented matroids, to the problem of sign-solvability of a matrix and the even-cycle problem for signed digraphs.

Possible generalizations to oriented matroids of sign-nonsingularity of a matrix.

Seth Chaiken, Christopher R.H. Hanusa, & Thomas Zaslavsky

Affinographic hyperplanes and rooted integral gain graphs, from Forge–Zaslavsky (2007a), imply the structure of formulas counting nonattacking arrangements of identical chess pieces in an \(m \times n\) strip, as a function of \(n\).

Hyperplane representation of sign-colored graphs with a new associated matroid, assisted by a related signed graph and its frame matroid. [Annot. 4 Apr 2011.]

(\text{SGc: Geom, M})(\text{SG: m})

Bikas K. Chakrabarti
See also S. Suzuki.

Bikas K. Chakrabarti, Amit Dutta, & Parongama Sen

Shows aspects of what physicists may ask about signed graphs. “Transverse” means external magnetic field(s), modellable as extra dominating vertices. Ch. 4, “ANNI model in transverse field”: Cf. Liebmann (1986a). Ch. 6, “Transverse Ising spin glass and random field systems”: The typical mountainous energy landscape. §6.1, “Classical Ising spin glasses: A summary”: Random signs. The $\pm J$ model is unweighted signed graphs. §6.2, “Quantum spin glasses”: Height of energy barriers between valleys may be less important than width due to quantum tunnelling.


(\text{Phys: SG, wg: Fr})

Nilanjan Chakraborty
See D. Li.

Sudip Chakravarty
See R.R.P. Singh.

AtMa P.O. Chan, Jeffrey C.Y. Teo, & Shinsei Ryu

A physics approach to embedding a signed graph on a surface, via orientable surfaces with parity defects. Also, in part, the same for gain graphs. [Annot. 2 Apr 2016.]

(\text{sg: Top})(\text{gg: Top})

Sarah Chand
See R. Farooq.

Karthekeyan Chandrasekaran
See C. Carlson.

Vijaya Chandru, Collette R. Coullard, & Donald K. Wagner

Determining whether a gain graph with real multiplicative gains has a balanced circle, i.e., is not contrabalanced, is NP-hard. So is determining whether a real matrix is projectively equivalent to the incidence matrix of a contrabalanced real gain graph. (\text{GN, Bic: Incid, Alg})

Chung-Chien Chang & Cheng-Ching Yu

Modifies the method of Iri, Aoki, O’Shima, and Matsuyama (1979a) of constructing the diagnostic signed digraph, e.g. by considering transient
and steady-state situations. (SD: Appl, Ref)

Gerard J. Chang
See J.H. Yan.

Hsien-Chih Chang
See C. Carlson.

Maw-Shang Chang
See L.-H. Chen.

Michael D. Chang
See M. Engquist.

Ting-Chung Chang [Ting-Jung Chang]
See T.J. Chang.

Ting-Jung Chang [Ting-Chung Chang]

Ting-Jung Chang & Bit-Shun Tam

Ting-Jung Chang (as Ting-Chung Chang) & Bit-Shun Tam

Ting-Jung Chang, Bit-Shun Tam, & Shu-Hui Wu

Yi Chang

Claudine Chaouiya
See G. Didier and A. Naldi.

Guillaume Chapuy
See O. Bernardi.

Pierre Charbit
See P. Aboulker.

Moses Charikar
See also N. Ailon.

Moses Charikar, Neha Gupta, & Roy Schwartz

Moses Charikar, Venkatesan Guruswami, & Anthony Wirth


Ankit Charls
See T. Sharma.

A. Charnes, M. Kirby, & W. Raike

A. Charnes & W.M. Raike

Clément Charpentier, Reza Naserasr, & ?Éric Sopena

Gary Chartrand
See also M. Behzad.


[Repr. (1985a).] (SG: Bal, Clu)


“Corrected reprint” of (1977a). (SG: Bal, Clu)

Gary Chartrand, Heather Gavlas, Frank Harary, & Michelle Schultz

Net degree sequences (i.e., \(d^\pm := d^+ - d^-\); called “signed degree sequences”) of signed simple graphs. A Havel–Hakimi-type reduction formula, but with an indeterminate length parameter; a determinant specialization to complete graphs. A necessary condition for a sequence to be a net degree sequence. Examples: paths, stars, double stars. [Continued in Yan, Lih, Kuo, and Chang (1997a). Solved in Michael (2002a).]

This is a special case of weighted degree sequences of \(K_n\) with integer edge weights chosen from a fixed interval of integers. Here the interval is \([-1, +1]\). The theory of such degree sequences is due to V. Chungphaisan, Conditions for sequences to be \(r\)-graphic, *Discrete Math.* 7 (1974), 31–39. MR 0351903 (50 #4391). Michael (2002a) characterizes net degree sequences by noticing this connection. (SGw: Invar)

[One can interpret net degrees as the net indegrees, \(d^\text{in} - d^\text{out}\), of certain bidirected graphs. Change the positive (negative) edges to extroverted...
Then we have the net indegree sequence of an oriented $-\Gamma$. Problem 1. Generalize to all bidirected (simple, or simply signed) graphs, especially $K_n$'s. Problem 2. Find an Erdős–Gallai-type characterization of net degree sequences of signed simple graphs. [Solved by Michael (2002a).] Problem 3. Characterize the separated signed degree sequences of signed simple graphs, where the separated signed degree is $(d^+(v), d^-(v))$. Problem 4. Generalize Problem 3 to edge $k$-colorings of $K_n$. 

(SG: ori: Invar)

Gary Chartrand, Frank Harary, Hector Hevia, & Kathleen A. McKeon


Bilal A. Chat

See H.A. Ganie.

Sourav Chatterjee


Sourav Chatterjee


Guy Chaty

Clarifies the structure of “free cyclic” digraphs and shows they include strong “upper” digraphs (see Harary, Lundgren, and Maybee (1985a)).

(SD: Str)

M. Chaves
See L. Tournier.

P.D. Chawathe & G.R. Vijayakumar

A list of the 49 switching classes of signed simple graphs that are the forbidden induced subgraphs for a signed simple graph to be a reduced line graph of a simply signed graph without loops or half edges. The graphs have orders 4, 5, and 6. [See several other works of Vijayakumar et al.]

(SG: adj, LG, Geom, incid)

Shuchi Chawla
See N. Bansal.

Charalampos Chelmis
See A. Srivastava.

Beifang Chen

Dual to Beifang Chen, Jue Wang, & Thomas Zaslavsky (2017a).

(SG: Flows(D))

20xxa Conformal decomposition of integral flows on signed graphs with outer-edges. Submitted.

(SG: Flows)

Beifang Chen & Shuchao Li

(SG)

Beifang Chen & Jue Wang

Introduces cuts, and directed circuits and cuts, of a signed graph; and the cycle (or circuit) and cut (or cocycle) spaces of a signed graph over a commutative, unital ring in which 2 is invertible. Definitions, basic theory, and graphical proofs. Orthogonal complementarity between real, or integral, circuit and cut spaces. Relationships between real and integral spaces. Interpretations in terms of flows and tensions.

A cut is an edge set $U := E(X, X^c) \cup U_X$ where $X \subseteq V$ and $U_X$ is a minimal balancing set of $E:X$. A minimal cut is a bond, i.e., a cocircuit in $G(\Sigma)$. A circuit or cut has two possible “directions”. A minimal directed cut need not be a directed bond. The indicator vectors of directed circuits generate the cycle (“circuit”) space; the indicator vectors of directed cuts generate the cocycle (“cut”) space.
The flow space or lattice is the real or integral null space of the incidence matrix. The tension space or lattice is the real or integral row space. The spaces equal lattices equal the real cycle and cut spaces and the lattices are their integral parts. Not every integral flow is in the integral span of circuit indicator vectors; but every integral tension is spanned by cut indicator vectors.

[Based upon and extending parts of J. Wang (2007a).]


[Based upon part of J. Wang (2007a).]


Beifang Chen, Jue Wang, & Thomas Zaslavsky


Irreducible integral flows include circuit flows as well as others of a complicated and unexpected nature. Resolved by lifting to the signed covering graph. [Based upon part of J. Wang (2007a) and also Chen and Wang (2011a), with a different method.]

Chao-Yang Chen

See B. Hu.

Chen Chen, Jin Huang, & Shuchao Li


For $\Phi$ with gain group $\{\pm 1, \pm i\}$ ($\varphi(e) = 1$ for undirected, $i$ for directed edges), $-2\xi(\|\Phi\|) \leq \text{rk } A(\Phi) - \mu(\|\Phi\|) \leq \xi(\|\Phi\|)$. Characterizes equalities. [A different matching formula in Tian–Chen–Chu (2018a).]

Chen Chen, Shuchao Li, & Minjie Zhang


$\Phi$ with gain group $\{\pm 1, \pm i\}$: $\varphi(e) = 1$ for undirected, $i$ for directed edges. $|\text{rk } A(\Phi) - \text{rk } A(\|\Phi\|)| \leq 2\xi(\|\Phi\|)$. Equality is characterized.

Doron Chen

See also E.G. Boman.

Doron Chen & Sivan Toledo


Certain matrices are related to gain graphs and others to signed graphs.
Gina Chen, Vivian Liu, Ellen Robinson, Lucas J. Rusnak, & Kyle Wang


Haiyan Chen
See Y.-N. Zhang.

Hanlin Chen
See F.-Y. Cao.

Jia-Fen Chen
See also B.Y. Wu.

Jia-Fen Chen & Bang Ye Wu

Minimum number of edge and vertex deletions and edge additions to arrive at balance, by two algorithms. [This thesis is presumably by Chen.] [Annot. 5 Jun 2017.] (SG: KG: Fr: Alg, Clu)

Jianer Chen & Jonathan L. Gross


Jianer Chen, Jonathan L. Gross, & Robert G. Rieper


Jianer Chen & Jie Meng

See (2012a). (sg: kg: Clu: Alg)

Equivalent: Is the clusterability index $l_{clu}(K_n, \sigma) \leq k$? Dictionary: “graph” = positive subgraph of $(K_n, \sigma)$; “editing” = edge sign changes.
[Definition: $l_{clu}$ = smallest number of sign changes that give a clusterable (cf. Davis (1967a)) signed $K_n$.] [Annot. 13 Jun, 1 July 2017.]
(sg: kg: Clu: Alg)

Jie Chen

Jing Chen & Genghua Fan


Li Chen  
See F.-L. Tian.

Li-Hsuan Chen  
See also B.Y. Wu.

Li-Hsuan Chen, Maw-Shang Chang, Chun-Chieh Wang, & Bang Ye Wu  
*Cf. (2013b), Chen and Wu (2017a).* (sg: kg: Fr: Alg)


Li-Hsuan Chen & Bang Ye Wu  
Equivalent to: Is $l(K_n,\sigma) \leq k$? Dictionary: “graph” = positive subgraph of $(K_n,\sigma)$; “editing” = edge sign changes; “2-clustering” = bipartition of $V$. [Annot. 13 Jun 2017.] (sg: kg: Fr: Alg)

Rong Chen  
See also B. Bao.


Rong Chen, Matt DeVos, Daryl Funk, & Irene Pivotto  

Rong Chen & Zifei Gao  

Rong Chen & Irene Pivotto  

Rong Chen & Geoff Whittle  

Rong Chen & Kai-nan Xiang  
A “spike-like” or “swirl-like” matroid is $L(\Gamma,\mathcal{B})$ or $G(\Gamma,\mathcal{B})$ where $\Gamma$ is a circle with all edges doubled or more. Thm. 1.1: A 3-connected vector
matroid with \( #E \geq 9 \) decomposes into spike-like, swirl-like, and “freely-placed-line” matroids and sequentially 4-connected matroids, assembled in a tree pattern along modular lines. [Annot. 18 Apr 2013.]

\[ (gg: \text{M: Str}) \]

Siyuan Chen
See Y.F. Huang.

Vinciane Chen, Angeline Rao, Lucas J. Rusnak, & Alex Yang

\[ (SH: \text{Bal}) \]

Xiaolin Chen, Xueliang Li, & Yingying Zhang

In defining the Hermitian adjacency matrix there seems to be a missing line \( h_{lk} = +i \) if \( l \to k \).

\[ (gg: \text{Adj: Eig}) \]

Wei Chen
See also Y.-H. Li.

Wei Chen, Dan Wang, Ji Liu, Tamer Başar, & Li Qiu

\[ (SG) \]

Weisheng Chen
See J.S. Wu.


\[ \S 3.4, \text{ “The Dowling polynomials”: } D_m(n;x) \text{ and } F_{m,1}(n;x) \text{ are from Benoumhani (1997a).} \text{ [Annot. 28 Jan 2015.]} \]

\[ (gg: \text{M: Invar}) \]

Y. Chen, X.L. Wang, B. Yuan, & B.Z. Tang

Ya-Hong Chen, Rong-Ying Pan, & Xiao-Dong Zhang

\[ (par: \text{Lap: Eig}) \]

Yanqing Chen & Ligong Wang

\[ (par: \text{Lap: Eig}) \]

Yu Qing Chen, Anthony B. Evans, Xiaoyu Liu, Daniel C. Slilaty, & Xiangqian Zhou
20xxa Representations of signed graphs. Submitted.

\[ (SG, GG: \text{Geom}) \]

Zhibin Chen & Wenan Zang

Zhi-Hong Chen, Ying-Qiang Kuang, & Hong-Jian Lai

The relationship between graph structure and the Tutte, vertical, and cyclic connectivities of the bicircular matroid. (Bic: Str)

Bo Cheng & Bolian Liu

The Abelson–Rosenberg (1958a) algebra is employed, with symbols 0, 1, −1, # for o, p, n, a. “Generalized sign pattern matrix”: # entries are allowed. “Generalized signed digraph”: #-arcs are allowed. (QM: SD)


Feng Cheng & Li Hua You

Jian Cheng

Jian Cheng, You Lu, Rong Luo, & Cun-Quan Zhang

Ying Cheng

This article studies what are described as $Z_4$-gain graphs $\Phi$ with underlying simple graph $\Gamma$. [However, see below.] They are regarded as digraphs $D$, the gains being determined by $D$ as follows: $\varphi(u, v) = 1$ or 2 if $(u, v)$ is an arc, 2 or 3 if $(v, u)$ is an arc. [N.B. $\Gamma$ is not uniquely determined by $D$.] Cheng's “switching” is gain-graph switching but only by switching functions $\eta : V \to \{0, 2\}$; I will call this “semiswitching”. His “isomorphisms” are vertex permutations that are automorphisms of $\Gamma$; I will call them “$\Gamma$-isomorphisms”. The objects of study are equivalence classes under semiswitching (semiswitching classes) or semiswitching and $\Gamma$-isomorphism (semiswitching $\Gamma$-isomorphism classes). Prop. 3.1 concerns adjacency of vertex orbits of a $\Gamma$-isomorphism that preserves a semiswitching class (call it a $\Gamma$-automorphism of the class). Thm. 4.3 gives the number of semiswitching $\Gamma$-isomorphism classes. Thm. 5.2 characterizes those $\Gamma$-automorphisms of a semiswitching class that fix an element of the class; Thm. 5.3 characterizes the $\Gamma$-isomorphisms $g$ that fix an element of every $g$-invariant semiswitching class.
[Likely the right viewpoint, as is hinted in §6, is that the edge labels are not $\mathbb{Z}_4$-gains but weights from the set $\{\pm 1, \pm 2, \ldots, \pm k\}$ with $k = 2$. Then semiswitching is ordinary signed switching, and so forth. However, I forbear to reinterpret everything here.]

In §6, $\mathbb{Z}_4$ is replaced by $\mathbb{Z}_{2k}$ [but this should be $\{\pm 1, \pm 2, \ldots, \pm k\}$]; semiswitching functions take values $0, k$ only. Generalizations of §§3, 4 are sketched and are applied to find the number of $H$-equivalent matrices of given size with entries $\pm 1, \pm 1, \ldots, \pm k$. ($H$- [or Hadamard] equivalence means permuting rows and columns and scaling them by $-1$.)

Ying Cheng & Albert L. Wells, Jr.


A two-digraph is a switching class of $\mathbb{Z}_3$-gain graphs based on $K_n$.


This exceptionally interesting paper treats a digraph as a ternary gain graph $\Phi$ (i.e., with gains in $\text{GF}(3)^+$) based on $K_n$. A theory of switching classes and triple covering graphs, analogous to that of signed complete graphs (and of two-graphs) is developed. The approach, analogous to that in Cameron (1977b), employs cohomology. The basic results are those of general gain-graph theory specialized to the ternary gain group and graph $K_n$.

The main results concern a switching class $[\Phi]$ of digraphs and an automorphism group $\mathfrak{A}$ of $[\Phi]$. §3, “The first invariant”: Thm. 3.2 characterizes, by a cohomological obstruction $\gamma$, the pairs $(\Phi, \mathfrak{A})$ such that some digraph in $[\Phi]$ is fixed. Thm. 3.5 is an [interestingly] more detailed result for cyclic $\mathfrak{A}$. §4: “Triple covers and the second invariant”. Di-graph triple covers of the complete digraph are considered. Those that correspond to gain covering graphs of ternary gain graphs $\Phi$ are characterized (“cyclic triple covers”, pp. 178–180). Automorphisms of $\Phi$ and its triple covering $\tilde{\Phi}$ are compared. Given $(\Phi, \mathfrak{A})$, Thm. 4.4 finds the cohomological obstruction $\beta$ to lifting $\mathfrak{A}$ to $\tilde{\Phi}$. Thm. 4.7 establishes an equivalence between $\gamma$ and $\beta$ in the case of cyclic $\mathfrak{A}$.

§5: “Enumeration”. Thm. 5.1 gives the number of isomorphism types of switching classes on $n$ vertices, based on the method of Wells (1984a) for signed graphs. §6: “The fixed signing property”. Thm. 6.1 characterizes the permutations of $V(K_n)$ that fix a gain graph in every invariant switching class, based on the method of Wells (1984a)

Dictionary: “Alternating function” on $X \times X = \text{GF}(3)^+$-valued gain function on $K_X$.

[See Babai and Cameron (2000a) for a treatment of tournaments as nowhere-zero ternary gain graphs based on $K_n$.]
Cheng Zhiyun & Gao Hongzhu
Which planar sign-colored graphs from link diagrams correspond to knots. [Annot. 26 Jul 2013.] (SGc: Knot)

Zhiyun Cheng, Ziyi Lei, Yitian Wang, & Yanguo Zhang
Cohomology theories for the chromatic polynomial and the balanced chromatic polynomial of \( \Sigma \) using a chain complex for each. Cohomology properties coordinate nicely with some polynomial properties. There are no categories. [Annot. 6 Jan 2021.] (SG: Col: Algeb, Invar)

Gi-Sang Cheon & Ji-Hwan Jung

T.C. Chern
See Kuo–Chern–Shih (1988a).

Yonah Cherniavsky, Avraham Goldstein, & Vadim E. Levit
(GG(Gen): Bal(Gen))


Yonah Cherniavsky, Avraham Goldstein, Vadim E. Levit, & Robert Shwartz
(GG(Gen): Bal(Gen))

William K. Cheung
See B. Yang.

Ming Chi
See B. Hu.

Kai-Yang Chiang
See also C.-J. Hsieh.

Kai-Yang Chiang, Cho-Jui Hsieh, Nagarajan Natarajan, Ambuj Tewari, & Inderjit S. Dhillon

Kai-Yang Chiang, Nagarajan Natarajan, Ambuj Tewari, & Inderjit S. Dhillon

Kai-Yang Chiang, Joyce Jiyoung Whang, & Inderjit S. Dhillon

Shuya Chiba, Shinya Fujita, Ken-ichi Kawarabayashi, & Tadashi Sakuma


K.P. Chithra
See N.K. Sudev.

Sergei Chmutov

Sign-colored graphs embedded in a surface (Chmutov and Pak (2007a)).
Duality with respect to an edge subset, applied to a sign-colored Bollobás–Riordan polynomial, gives a polynomial duality. [Further developments in Vignes-Tourneret (2009a) and Krushkal (2011a).] (SGc: Top, Invar)

Sergei Chmutov & Igor Pak

Sign-colored graphs embedded in a surface (orientable or not, independently of the edge signs. [The orientation properties of the ribbons make a signed graph, independent of the sign-colors.] (SGc, sg: Top, Invar)

Kwang-Hyun Cho

Hyeong-ah Choi, Kazuo Nakajima, & Chong S. Rim

Vertex biparticity [i.e., vertex frustration number $l_0(−Γ)$] is compared to edge biparticity [frustration index $l(−Γ)$] (for cubic graphs) and studied algorithmically. Proved: $l_0(−Γ) = l(−Γ)$ for cubic graphs; thus, cubic “$l_0 ≤ k$” is NP-complete because “$l ≤ k$” is and $l_0 = l$. [Equality is generalized in Sivaraman and Zaslavsky (20xxa).] (par: Fr: Alg)

Timothy Y. Chow

§4, “From graphs to symplectic matroids”: The matroid union of \( G(\Gamma, \sigma) \) over all signatures of a fixed graph yields a symplectic matroid.

(\text{SG: M})

**Deshabish Chowdhury**


Includes brief survey of how physicists look upon frustration. See esp. §1.3, “An elementary introduction to frustration”, where the signed square lattice graph illustrates balance vs. imbalance; Ch. 20, “Frustration, gauge invariance, defects and SG [spin glasses]”, discussing planar duality (see e.g. Barahona (1982a), “gauge theories”, where gains are in the orthogonal or unitary group (switching is called “gauge transformation” by physicists), and functions of interest to physicists; Addendum to Ch. 20, pp. 378–379, mentioning results on when the proportion of negative bonds is fixed, and on gauge theories.

(\text{Phys: SG, GG, VS, Fr: Exp, Ref})

**Nicholas A. Christakis**

See D. Feng.

**Dianhui Chu**

See D. Li.

**Rui Chu**

See F.-L. Tian.

**San Yan Chu**

See S.L. Lee.

**Ying Chu**

See R.M. Gao.

**Maria Chudnovsky**


**Maria Chudnovsky, William H. Cunningham, & Jim Geelen**


See Chudnovsky, Geelen, et al. (2006a). Structure theorem for optimal \( A \)-paths in terms of switching only vertices in \( A^c \); algorithm for finding such. Lemma 3.1 generalizes the basis result of Chudnovsky, Geelen, et al. (2006a). \[ \text{Question. B(II) is a subset of } V \times G. \text{ How is this related to the covering graph? Can one simplify their proofs? A “non-zero” path is like a level-changing path in } \tilde{\Phi} \text{ (covering graph). This suggests modelling their picture by } \Phi’ = \Phi \cup 1K_n, \text{ i.e., with distinguished identity-gain complete subgraph. Or, by } \Omega \subseteq M \cdot \Delta = \text{a biased expansion, with a distinguished maximal balanced subgraph.} \] (\text{GG: Paths: Str, Alg})

**Maria Chudnovsky, Jim Geelen, Bert Gerards, Luis Goddyn, Michael Lohman, & Paul Seymour**

In a gain graph $\Phi$, find the maximum number of vertex-disjoint paths with non-identity gain and with endpoints in $A \subseteq V$ (non-zero $A$-paths).

Thm.: If $\max k < k$, there is a set $X$ of up to $2k - 2$ vertices such that every $A$-path in $\Phi \setminus X$ has identity gain. This is not best possible.

They prove: \{\textrm{\textbf{$B(\Pi) : \Pi \in \mathcal{P}^*(G, A)$}}\} is the set of bases of a matroid.

Dictionary: “Group-labelled graph” = gain graph; $\Gamma$-labelled graph = $\Gamma$-gain graph (for a group $\Gamma$); “weight” = gain. “Shifting” = switching; “$A$-equivalent” = $A^c$-switching equivalent, i.e., obtained by switching vertices not in $A$.

**Maria Chudnovsky, Ken-ichi Kawarabayashi, & Paul Seymour**


Algorithm to detect positive holes (induced circles) in a signed graph. A polynomially equivalent problem is to decide whether a graph is negative-hole signable, i.e., has a signature in which every hole is negative.

**S.T. Chui**

See also B.W. Southern.

**S.T. Chui, G. Forgacs, & D.M. Hatch**


Physics of “fully frustrated” 3-dimensional cubic lattice, i.e., every square (“plaquette”) is negative. Each square has one negative edge. This is the unique fully frustrated signature up to switching [short proof: the squares generate the cycle space], but there are many nonisomorphic ground states ($\zeta : V \to \{+1, -1\}$ such that $\min \zeta \#(E^\zeta)^-$); they are said to form 12 mutually unreachable classes. App. A characterizes the ground states [and implies $l(\Sigma) = \frac{1}{4} \#V$ since each cube has one negative edge in each direction, neglecting boundary effects—or assuming toroidality]. The signed lattice is at times assumed to have a $2 \times 2$ fundamental domain; under that assumption there are 8 translational symmetry types of vertex, each forming a double-sized sublattice. Approximate clustering is discussed. [Annot. 18 Jun 2012.]

**Deborah Chun, Tyler Moss, Daniel Slilaty, & Xiangqian Zhou**


**Yang Chun**

See B. Jiao.
Fan Chung & Mark Kempton


Connection graph: A real-weighted $GL(\mathbb{R},d)$-gain graph.

Fan Chung, Wenbo Zhao, & Mark Kempton


Taeyoung Chung, Jack Koolen, Yoshio Sano, & Tetsuji Taniguchi


§2.2, “Generalized line graphs and generalized signless Laplace matrices”: The generalised signless Laplace matrix of $(\Gamma, f)$, where $f : V \to \mathbb{Z}_{\geq 0}$, is $L(-\Gamma) + 2D(f)$. The incidence matrix of $(\Gamma, f)$ is $H(\Sigma)$ where $\Sigma$ consists of $-\Gamma$ with $f(x)$ negative digons adjoined to $x \in V$. [See Zaslavsky (1984c), (2010b), (20xxa) for this construction, which is not stated here.] [Annot. 20 Dec 2011.]

S.M. Cioabă

See S. Akbari, F. Belardo, and M. Cavers.
Valerio Ciotti, Ginestra Bianconi, Andrea Capocci, Francesca Colaiori, & Pietro Panzarasa

Lane Clark

F.W. Clarke, A.D. Thomas, & D.A. Waller

Nancy E. Clarke, Samuel Fiorini, Gwenaël Joret, & Dirk Oliver Theis

A.M. Cohen
See A.E. Brouwer.

Bernard P. Cohen
See J. Berger.

Edith Cohen & Nimrod Megiddo


Looking for a closed walk (“cycle”) with gain 0 in a gain digraph with (additive) gains in $\mathbb{Q}^d$. [Cf. Kodialam and Orlin (1991a).]


Maximize the fraction of demand satisfied by a flow on a network with gains. Positive real gains in §3. Bidirected networks with positive gains in §4; these are more general than networks with arbitrary non-zero real gains.


(Lu): SG: Exp, Ref

Luke Collins
See I. Sciriha.

Barbara Coluzzi, Enzo Marinari, Giorgio Parisi, & Heiko Rieger

Ph. Combe & H. Nencka

Σ is balanced iff a fundamental system of circles is balanced [as is well known; see i.a. Popescu (1979a), Zaslavsky (1981b)]. An algorithm [incredibly complicated, compared to the obvious method of tracing a spanning tree] to determine all vertex signings of Σ that switch it to all positive. Has several physics references. (SG: Bal, Fr, Alg, Ref)


A signed-graphic model Σ of a neuron network. Obs.: A network is cooperative iff Σ has a non-frustrated state s : V → {+1, −1}, i.e., the Hamiltonian (“energy”) $H(s) := -\frac{1}{2} \sum_{uv \in E} \sigma(uv)s(u)s(v) = -\#E$. [Should be $-\frac{1}{2} \#E$.] H [i.e., Σ] is non-frustrated if some state is. Assertion: H is non-frustrated iff Σ is balanced. A proof idea (not a proof) is by setting up (real-valued) linear equations of positivity of generating circles; carried out for $K_n$. [See (1997b)]. [Easy proof: $H(s) = -\frac{1}{2} \#E + \#E^+(\Sigma^*)$, hence H is non-frustrated iff $\Sigma^*$ is all positive for some s iff Σ is balanced. See e.g. Zaslavsky (1982a), Cor. 3.3.] [Annot. 17 Jun, 17 Aug 2012.] (SG: Bal, Fr, Alg, Ref)


No proofs. Prop. 1: The signatures of Γ are a “GF(2)-vector space”. [Meaning: They are the points in $\{\pm 1\}^\#E \subset \mathbb{R}^\#E$.] Prop. 2: Nonfrustration corresponds to a large family of [real] linear systems. “Minimal” circles generalize plaquettes (girth circles) to arbitrary graphs. [“Minimal” = (?) minimum length, assuming such circles generate the cycle space. In general, choice of generating circles remains a good question.] “Fully frustrated”: all minimal circles are negative. Prop. 3: Full frustration corresponds to another family of [real] linear systems. [“Overblock- ing”: Fully frustrated and some nonminimal circles are negative.] Prop. 4: Linear system for overblocking in a fully frustrated signature. Cor. 5: $K_5$ is overblocking. $K_{3,2}$ cannot be fully frustrated. [Annot. 17 Jun
Jean-Paul Comet
See also A. Richard.
Jean-Paul Comet, Mathilde Noual, Adrien Richard, Julio Aracena, Laurence Calzone, Jacques Demongeot, Marcelle Kaufman, Aurélien Naldi, El Houssine Snoussi, & Denis Thieffry

F.G. Commoner

Michele Conforti
See also F. Barahona.
Michele Conforti & Gérard Cornuéjols

Bicolorability means every square submatrix contains the incidence matrix of a balanced signed graph. (SGw, sg: Bal(Gen))

Michele Conforti, Gérard Cornuéjols, Ajai Kapoor, & Kristina Vušković


The structure of graphs that are signable to be “without odd holes”: that is, so that each triangle is negative and each chordless circle of length greater than 3 is positive. Proof based on Truemper (1982a). (SG: Bal(Gen), Str)


Γ is “universally signable” if it can be signed so as to make every triangle negative and the holes independently positive or negative at will. Such graphs are characterized by a decomposition theorem which leads to a polynomial-time recognition algorithm. (SG: Bal, Str)


Recognition of graphs that are “strongly even-signable” (signable so triangles are − and longer circles with ≤ 1 chord are +) and “strongly odd-signable” (signable so quadrilaterals with a unique chord are + and all other circles with ≤ 1 chord are −). [Description adapted from Trotignon and Vušković (2010a).] [Annot. 19 Jan 2015.] (sg: Bal(Gen): Alg)

“Even hole” means a chordless circle, bigger than a triangle, that is positive in a given signing of the graph. The graphs of the title are characterized in several ways. Most of them have significant wheels. (SG: Bal, Str, Alg)


Michele Conforti, Gérard Cornuéjols, & M.R. Rao


Michele Conforti, Gérard Cornuéjols, & Klaus Truemper


The forbidden matrix type \(A’\) in Rem. 3, par. 2 is the transpose of a signed-graph incidence matrix. [Annot. 23 Aug 2014.] (sgw, sg: bal(gen))

Michele Conforti, Gérard Cornuéjols, & Kristina Vušković


Bipartite \(\Gamma\) is “balanceable” if it can be signed so each hole (chordless circle) is positive iff it is evenly even. [Truemper (1982a) implies a characterization by forbidden induced signed subgraphs.] “Strongly balancedable”: also no circle has a unique chord. [Annot. 19 Jan 2015.] (SGw, sg: Bal(Gen): Exp)

Michele Conforti & Bert Gerards

The problem is to find the most vertex-disjoint negative circles in a signed graph (thus, odd-length circles in an ordinary graph). It is NP-hard but it can be solved in polynomial time for the signed graphs that exclude the switching classes $[-K_5]$, $[K^{1,1}_{3,3}]$, $[K^{1,2}_{3,3}]$, $[K^2_{3,3}]$, which are defined as: $K^{1,1}_{3,3} = +K_{3,3}$ with edge $u_1v_1$ made negative and the additional negative edge $-u_2v_2$, $K^{1,2}_{3,3} = +K_{3,3}$ with $u_1v_1$ made negative and added edges $-u_1u_2$ and $-u_1u_3$, and $K^2_{3,3} = +K_{3,3}$ with edges $u_1v_1$ and $u_2v_2$ made negative.

Michele Conforti, Bert Gerards, & Ajai Kapoor


Michele Conforti & Ajai Kapoor

A new proof of Truemper’s (1982a) theorem on prescribed hole signs. Discussion of applications. (SG: Bal)

Antonio Coniglio

The “site-frustrated percolation model”: On a signed graph a vertex (“site”) may be occupied or not, but a frustrated circle cannot be fully occupied. Unoccupied vertices are considered “defects or holes”. Each possible configuration has a weight; the statistical properties are examined. The unoccupied vertices constitute a balancing vertex set (i.e., its deletion leaves a balanced subgraph). Maximally occupied configurations correspond to minimum balancing vertex sets. Question. What does the physics mean for such sets, and vice versa? [Annot. 22 Aug 2014.] (sg, Phys: Fr, State(Gen))


§2, “Frustrated lattice gas”: a signed-graph $(\pm J$ Ising) model “diluted with lattice gas variables”. §3, “Percolation in phase space”: see xrefd1994aAntonio Coniglio. (sg, Phys: Fr)


See esp. §3, “Clusters in the Ising spin-glass model”. (sg, Phys: State)

A. Coniglio, F. di Liberto, G. Monroy, & F. Peruggi

A cluster approach to the Ising spin glass model, i.e., vertex signs (“spins”) on a signed graph. Dictionary: “NN” = “nearest-neighbor” = interactions only between adjacent vertices. [Annot. 22 Aug 2014.]

Joseph G. Conlon

Main theorem: For 3-connected $G \neq K_4$, there is an even circle, deletion of whose vertices or edges leaves a 2-connected graph. [Problem. Generalize to signed graphs. And see Voss (1991a).] (par)

S. Contreras

George Converse & M. Katz


S.N. Coppersmith
See J.W. Landry.

Raul Cordovil
See P. Berthomé.

Denis Cornaz

Denis Cornaz & A. Ridha Mahjoub

Derek G. Corneil

Gérard Cornuéjols
See also M. Campelo and M. Conforti.


The topic is linear optimization over a clutter, esp. a “binary clutter”, which is the class of negative circuits of a signed binary matroid. The class $\mathcal{C}^{-}(\Sigma)$ is an important example (see Seymour (1977a)), as is its blocker $b(\mathcal{C}^{-}(\Sigma))$ [which is the class of minimal balancing edge sets;
Ch. 5: “Graphs without odd-$K_5$ minors”, i.e., signed graphs without $-K_5$ as a minor. Some esp. interesting results: Thm. 5.0.7 (special case of Seymour (1977a), Main Thm.): The clutter of negative circles of $\Sigma$ has the “Max-Flow Min-Cut Property” (Seymour’s “Mengerian” property) iff $\Sigma$ has no $-K_4$ minor. Conjecture 5.1.11 is Seymour’s (1981a) beautiful conjecture (his “weak MFMC” is here called “ideal”). §5.2 reports the partial result of Guenin (2001a). (See also §8.4.)

Def. 6.2.6 defines a signed graph “$G(A)$” of a $0, \pm 1$-matrix $A$, whose transposed incidence matrix is a submatrix of $A$. §6.3.3: “Perfect $0, \pm 1$-matrices, bidirected graphs and conjectures of Johnson and Padberg (1982a), associates a bidirected graph with a system of 2-variable pseudo-boolean inequalities; reports on Sewell (1996a) (q.v.).


(Sgnd(M), SG: M, Geom, Incid(Gen), Ori: Exp, Ref, Exr)

Gérard Cornuéjols & Bertrand Guenin

A partial proof of Seymour’s (1981a) conjecture. Main Thm.: A binary clutter is ideal if it has as a minor none of the circuit clutter of $F_7$, $C^-(\neg K_5)$ or its blocker, or $C^-(\neg K_4)$ or its blocker. Important are the lift and extended lift matroids, $L(M, \sigma)$ and $L_0(M, \sigma)$, defined as in signed graph theory. [See Cornuéjols (2001a), §8.4.]

(Sgnd(M), SG: M, Geom)

Sylvie Corteel, David Forge, & Véronique Ventos

S. Cosares
See L. Adler.

Collette R. Coullard
See also V. Chandru.

Collette R. Coullard, John G. del Greco, & Donald K. Wagner

§4: §4.1 describes 4 fairly simple types of “legitimate” graph operation that preserve the bicircular matroid. Thm. 4.11 is a converse: if $\Gamma_1$ and $\Gamma_2$ have the same connected bicircular matroid, then either they are related by a sequence of legitimate operations, or they belong to a small class of exceptions, all having order $\leq 4$, whose bicircular matroid isomorphisms are also described. This completes the isomorphism theorem of Wagner (1985a). §5: If finitely many graphs are related by a sequence of legitimate operations (so their bicircular matroids are isomorphic), then they have contrabalanced real gains whose incidence matrices are
row equivalent. These results are also found by a different approach in Shull, Orlin, et al. (1989a), Shull, Shuchat, et al. (1993a), (1997a).

(Bic: Str, Incid)


G. Coutinho, C. Godsil, H. Shirazi, & H. Zhan

Gheorghe Craciun
See also M. Banaji and M. Mincheva.

Gheorghe Craciun & Martin Feinberg


Gheorghe Craciun, Casian Pantea, & Eduardo D. Sontag

Yves Crama
See also E. Boros.


Balance and switching are used to study pseudo-Boolean functions. (§§2.2 and 4.) (SG: Bal, Sw)

Yves Crama & Peter L. Hammer

“Adjoint” = unoriented positive part of the line graph of a bidirected graph. “Quadratic graph” = graph that is an adjoint. Recognition of adjoints of bidirected simple graphs is NP-complete. (sg: Ori: LG: Alg)

Yves Crama, Peter L. Hammer, & Toshihide Ibaraki

§7: Signed hypergraphs, with a surprising generalization of balance. (SH: Bal)

Y. Crama, M. Loebl, & S. Poljak

R. Crowston, G. Gutin, M. Jones, & G. Muciaccia

An algorithm for \( l(\Sigma) \leq \frac{1}{2} \#E - \frac{1}{4}(n - 1) - \frac{1}{4}k \) with time linear in \( n \) and exponential in \( k \), assuming \( \Sigma \) is simple. (The upper bound \( l \leq \frac{1}{2} \#E - \frac{1}{4}(n - 1) \) is from Poljak and Turzík (1982a), (1986a).) [Annot. 2 Mar 2014.] (SG: Fr: Alg)

Anne Crumière & Paul Ruet

Regulatory graph: a signed digraph. (SD: Dyn, Biol)

Anne Crumière & Mathieu Sablik

Regulatory graph: a signed digraph. (Biol: SD: Dyn)

Lin Cui & Yi-Zheng Fan

Qing Cui
See W.-Z. Liu.

Shu-Yu Cui & Gui-Xian Tian

G.J. Culos, D.D. Olesky, & P. van den Driessche

William H. Cunningham
See J. Aráoz and M. Chudnovsky.

Dragoš M. Cvetković
See also R.A. Brualdi, F.C. Bussemaker, D.M. Cardoso, and M. Doob.


Pp. 128–130 discuss switching-equivalent graphs. Some of the theory is invariant, hence applicable to two-graphs. [Question. How can this be generalized to signed graphs and their switching classes?] (tg: Adj)


(par: Lap, LG: Eig)


(par: Lap: Eig)


Surveys the spectral theory of $L(-\Gamma)$. [Annot. 23 Nov 2014.]

(par: Lap: Eig: Exp)


Eigenvalues of $L(-\Gamma)$, with a brief history. $L(-\Gamma)$ is important because Spec $L(-\Gamma)$ “dominates” Spec $L(\Phi)$ for any signed or complex unit gain graph; cf. Reff (2012a). [Annot. 12 Dec 2020.] (sg: Par: Lap: Eig)

**Dragoš M. Cvetković & Michael Doob**


(sg: par: Geom, LG)

**Dragos M. Cvetković, Michael Doob, Ivan Gutman, & Aleksandar Torgašev**


**Dragoš M. Cvetković, Michael Doob, & Horst Sachs**


Update of (1980a).

 Appendices update (1982a), beyond the updating in Cvetković, Doob, Gutman, and Torgašev (1988a). App. B.3, p. 381 mentions work of Vijayakumar *(q.v.)*. P. 422: Pseudo-inverse graphs (when $A(\Gamma)^{-1} = A(\Sigma)$ for some balanced $\Sigma$, $|\Sigma|$ is the “pseudo-inverse” of $\Gamma$).

Dragoš Cvetković, Michael Doob, & Slobodan Simić

Abstract of (1981a). *(sg: LG, Eig(LG), Aut(LG))*


Dragoš Cvetković, Peter Rowlinson, & Slobodan K. Simić

Generalized line graphs are the fundamental example. Pp. 190–191 mention signed graphs representable in root systems as in papers of G.R. Vijayakumar *(q.v.)* [but not mentioning line graphs of signed graphs].


“Signless Laplacian” $Q(\Gamma) :=$ Laplacian matrix $L(\Gamma) = D(\Gamma) + A(\Gamma)$. Spectral properties; bounds for graph invariants; combinatorics of coefficients of characteristic polynomial of $L(\Gamma)$. *Problem.* Find all articles on “signless Laplacians”, herein called $L(\Gamma)$. Generalize to signed graphs, with nonbipartite graphs generalizing to unbalanced graphs.

[Annot. 14 Sept 2010.] *(sg: Par: Eig)*


See (2007a). Thm.: For connected $\Gamma$ with $\#V = n$ and $\#E = m$, $\lambda_1(L(\Gamma))$ is maximized when $\Gamma$ is a nested split graph. Also, many computer-generated conjectures *(cf. Aouchiche and Hansen (2010a)); some are proved (here or elsewhere) or disproved; some are difficult.


Dragoš M. Cvetković & Slobodan K. Simić


D. Cvetković, S.K. Simić, & Z. Stanić

Marek Cygan, Marcin Pilipczuk, Michał Pilipczuk, & Jakub Onufry Wojtaszczyk


V. Jude Annie Cynthia & E. Padmavathy

More, as in Baskar Babujee and Loganathan (2011a). (Lab: VS: SG, Bal)

[Ilda P.F. da Silva]
See I.P.F. da Silva (under ‘S’).

A. Daemi
See S. Akbari.

C. Dalf’o, M.A. Fiol, M. Miller, & J. Ryan [Joe Ryan]


Mina Dalirrooyfard
See S. Akbari.

Soudabeh Dalvandi
See S. Akbari.

E.R. van Dam & M.A. Fiol

Odd Girth Thm.: A graph with $d+1$ eigenvalues and odd girth $\geq 2d+1$ is a generalized odd graph. [Problem: Generalize to signed graphs, odd girth becoming negative girth and distance-regular and generalized odd graphs becoming one wants to know what. Knowing what would indicate what a distance-regular or generalized-odd signed graph should be.] [Annot. 17 Dec 2014.] (par: Eig: Str, Ref)

Edwin R. van Dam & Willem H. Haemers


New and old results on $L(\Gamma)$, the “signless Laplacian” of $\Gamma$. [Annot. 20 Dec 2011.] (par: Lap: Eig)(Par: Eig: Exp)

Susan S. D’Amato

Spectrum of signed covering graph. [See Butler (2010a).] [Annot. 9 Mar 2011.] (sg: cov: Eig)


Ternary gain graphs: spectrum of covering graph, as with signed graphs in (1979a). [Annot. 9 Mar 2011.] (gg: cov: Eig)

Jeffrey M. Dambacher, Richard Levins, & Philippe A. Rossignol

Jeffrey M. Dambacher, Hiram W. Li, & Philippe A. Rossignol
Feedback predictions from signed digraph \((D, \sigma)\) via “weighted predictions” \(W_{ij} := |C_{ij}(-A(D, \sigma))/P_{ij}(A(D))|\), where \(C_{ij}\) is the cofactor and \(P_{ij}\) is the permanental cofactor. \(W_{ij} = 1\) means perfect predictability, \(= 0\) means no predictability. Numerical tests. Dictionary: “Community matrix” = \(A(D, \sigma)\). [Annot. 9 Sept 2010.]

(SD: QM: QSta: Cyc, Ref)

E. Damiani, O. D’Antona, & F. Regonati

E.g., log concavity of Whitney numbers of the second kind of Dowling lattices. [Cf. Stonesifer (1975a) and Benoumhani (1999a).] [Annot. rev. 30 Apr 2012.]

A. Danielian

“Ground states”, i.e. \(\zeta : V \to \{+1, -1\}\) with smallest \#(\(E^\zeta\))^\(-\), of the all-negative (antiferromagnetic) \(R \times R \times R\) face-centered cubic lattice graph [assumed toroidal to avoid boundary effects?]. Frustration index \(l = 2\#V\); the number (“degeneracy”) of ground states is \(2^A \sqrt{\#V}\) where \(A > 0\); each ground state has 4-regular \(E^-\). See (1964a) for more structure.

[Problem. Determine the exact number and precise shape of all ground states \(\zeta\) in terms of the graph. Is there something interesting about \((\Sigma^\zeta)^-\), e.g., in its circle decomposition, symmetries, or transformations from one to another?] [Annot. 21 Jun 2012.]

(SG, Phys: Par: Fr, State(fr))


§ II, “The ground state”, continues (1961a) with more details on the structure of ground states \(\zeta\). The number of them is small compared to the all-negative triangular lattice [Question: and other all-negative, highly symmetric graphs?]. \(\zeta\) on each \(x\)-, \(y\)-, or \(z\)-layer has a form described in the paper. Low-weight distance-2 edges will fix the ground state (p. A1346). § III, “The partition function”, studies the effect of moving out of ground states. App. A derives a formula for the energy change from switching a cluster of vertices, in terms of frustrated and satisfied edges within and without the cluster. App. B estimates the effect of switching additional vertices. [Problem. Find rigorous treatments of such switchings; this means studying the energy landscape of state space \(\{\zeta\} = \{+1, -1\}^{(1,+1)^V}\}. Dictionary: “bond” = edge, “even/odd bond” = frustrated/satisfied edge = switches to + or −. [Annot. 21 Jun 2012.]

(Phys, SG: Par: Fr, State(fr))

Daniele D’Angeli
See M. Cavaleri.

O. D’Antona
See E. Damiani.

George B. Dantzig


F.A. Dar
See S. Pirzada.

Richard D’Ari
See R. Thomas.

Kinkar Ch. Das
See also J. Askari.


Assume $n \geq 4$; $\lambda_1 = \max$ eigenvalue. Thm. 3.2: $\lambda_1(-\Gamma) + \lambda_n(-\Gamma) \leq 3n - 2 - 2\alpha(\Gamma)$, where $\alpha :=$ independence number; $= \text{iff } \Gamma = K_{n-\alpha} \lor \overline{K_{\alpha}}$.

Thm. 3.3: $\lambda_1(-\Gamma) - \lambda_n(-\Gamma) \geq 2 + 2 \cos(\pi/n)$, with equality iff $\Gamma$ is a path or odd circle. [Annot. 21 Jan 2012.] (par: Lap: Eig)

Prabir Das & S.B. Rao

Given an all-negative bidirected $K_n$ and a positive integer $f_i = 2g_i$ for each vertex $v_i$. There is a connected subgraph having in-degree and out-degree $= g_i$ at $v_i$ iff there is a $g$-factor of introverted and one of extroverted edges and the degrees satisfy a complicated degree condition. Generalizes Thm. 1 of Bánkfalvi and Bánkfalvi (1968a). [See Bang-Jensen and Gutin (1997a) for how to convert an edge 2-coloring to an orientation of an all-negative graph and for further developments on alternating walks.] (par: ori)

Sandip Das, Prantar Ghosh, Swathy Prabhu [Swathyprabhu Mj], & Sagnik Sen
20xxa Relative clique number of planar signed graphs. Discrete Math. (in press). (SG)

Sandip Das, Soumen Nandi, Soumyajit Paul, & Sagnik Sen
20xxa Chromatic number of signed graphs with bounded maximum degree. arXiv:1603.09557. (SG: Col)

Bhaskar DasGupta, German Andres Enciso, Eduardo Sontag, & Yi Zhang
“...our problem amounts to finding ground states” $V \rightarrow \{+,-\}$ of a signed graph. Lemma 3: A dynamical system is monotone iff the associated signed graph is balanced. An algorithm to find $\#E(l(\Sigma))$ to within $7/8$. Dictionary: “sign-consistency” = balance, “consistent edge” = satisfied edge (in a state). [Annot. 1 Jan 2012.] (SD(sg): Dyn, Alg, Biol)

Brian Davis

James A. Davis
$\Sigma$ is “clusterable” if its vertices can be partitioned so that each positive edge is within a part and each negative edge joins different parts. Thm.: $\Sigma$ is clusterable $\iff$ no circle has exactly one negative edge. [Cf. Cartwright and Harary (1968a).] [Further developed in Doreian and Mrvar (1996a).] [Complete-graph clustering begins (?) in Zahn (1964a) and Moon (1966a), now called “cluster editing” and focussed on algorithms; cf., e.g., Böcker and Baumbach (2013a).] [Annot. rev. 18 Nov 2017, 11 Jan 2019.] (SG: Clu)
Survey of triad analysis in signed complete digraphs; cf. e.g. Davis and Leinhardt (1972a), Wasserman and Faust (1994a). [Annot. 28 Apr 2009.]

James A. Davis & Samuel Leinhardt

In “ranked clusterability” the vertices of a signed complete, symmetric digraph are divided into levels. The set of levels is totally ordered. A symmetric pair, \{+vw, +wv\} or \{-vw, -wv\}, should be within a level. For an asymmetric pair, \{+vw, -wv\}, \(w\) should be at a higher level than \(v\). Analysis in relation to both randomly generated and observational data. [Annot. 28 Apr 2009.]

A.C. Day, R.B. Mallion, & M.J. Rigby

A clumsy but intriguing way of representing some signed (or more generally, \(\mathbb{Z}_n\)-weighted) graphs: via 2-page (or, \(n\)-page) looseleaf book embedding (all vertices are on the spine and each edge is in a single page), with an edge in page \(k\) weighted by the “sheet parity index” \(\alpha_k = (-1)^k\) (or, \(e^{2\pi ik/n}\)). Described in the unnecessary terminology of an \(n\)-sheeted Riemann surface. [A \(\mathbb{Z}_n\)-weighted) graph has such a representation iff the subgraph of edges with each weight is outerplanar.]

A variation to get switching classes of signed circles: replace \(\alpha_k\) by the “connectivity parity index” \(\alpha^{\sigma_k}_k\) where \(\sigma_k\) = number of edges in page \(k\). [The variation is valid only for circles.]

Questions vaguely suggested by these procedures: Which signed graphs can be switched so that the edges of each sign form an outerplanar graph? Also, the same for gain graphs. And there are many similar questions: for instance, the same ones with “outerplanar” replaced by “planar.”

Rajat K. De
See A. Bhattacharya.

[Nair Maria Maia de Abreu]
See N.M.M. Abreu (under ‘A’).

[Mathias Hudoba de Badyn]
See M.H. de Badyn (under ‘B’).

Marisa Debowsky
See D. Archdeacon.

Ernesto Dedò
1981a Sulla ricostruibilità del polinomio caratteristico del commutato di un grafo. [The reconstructibility of the characteristic polynomial of the line-graph of a
C. De Dominicis
See G.J. Rodgers.

Matthias Dehmer
See G.H. Yu.

José F. De Jesús & Alexander Kelmans


Hidde de Jong

I.J. Dejter & V. Neumann-Lara

Thm. 1: Frustration number $l_0(-\Gamma)$ is unbounded for graphs with no disjoint odd circles. Their examples are projective-planar antibalanced signed graphs. Generalized to circles of length $L \mod N$ for many $L, N$.

[Annot. 1 May 2017.]


[Anne Delandtsheer]
See P. de la Harpe (under ‘H’).

Anne Delandtsheer


Patrick De Leenheer
See also D. Angeli and V.A. Traag.

Patrick De Leenheer, David Angeli, & Eduardo D. Sontag

**Gloria de Leon-Calio**
See G. de Leon-Calio (under ‘L’).

**John G. del Greco**
See also C.R. Coullard.


How to decide, given a matroid $M$ and a biased graph $\Omega$, whether $M = G(\Omega)$. (GG: M)

**Leonardo Silva de Lima**
See also S. Akbari, A. Oliveira, and C.S. Oliveira.

**Leonardo de Lima, Vladimir Nikiforov, & Carla Oliveira**


**Leonardo Silva de Lima, Carla Silva Oliveira, Nair Maria Maia de Abreu, & Vladimir Nikiforov**


**Alberto Del Pia & Giacomo Zambelli**


**Alberto Del Pia, Antoine Musitelli, & Giacomo Zambelli**


**Ernesto W. De Luca**
See J. Kunegis.

**Emanuele Delucchi**


Studies Dowling trees (cf. Hultman (2007a)). (gg: M: Invar)

**Emanuele Delucchi, Noriane Girard, & Giovanni Paolini**


Cf. Bibby and Gadish (2018a). (gg: M(Gen))

**Renata R. Del-Vecchio**
See M.A.A. de Freitas.
Erik D. Demaine, Dotan Emanuel, Amos Fiat, & Nicole Immorlica

Clustering in a weighted signed graph; cf. Bansal, Blum, and Chawla (2002a), (2004a). An $O(\log n)$-approximation algorithm based on linear-programming rounding and region growing. An $O(r^3)$-approximation algorithm for graphs without a $K_{r,r}$-minor [e.g., planar, if $r = 3$]. Equivalent to minimum multicut, hence hard to approximate better than $\Theta(\log n)$. [Annot. 13 Sept 2009.] (SG: WG: Clu: Alg)

Erik D. Demaine, MohammadTaghi Hajiaghayi, & Ken-ichi Kawarabayashi

Graphs excluding $-\Gamma$: structure and algorithms. Dictionary: “$\Gamma$ has odd minor $H$” $\iff$ $-\Gamma$ has minor $-H$. [Annot. 4 Feb 2021.] (sg: Par: Str, Alg)

Erik Demaine & Nicole Immorlica


Jacques Demongeot
See also J. Aracena, O. Cinquin, J.-P. Comet, and L. Forest.

Jacques Demongeot, Julio Aracena, Samia Ben Lamine, Sylvain Meignen, Arnaud Tonnelier, & Rene Thomas


Jacques Demongeot, Julio Aracena, Florence Thuderoz, Thierry-Pascal Baum, & Oliiver Cohen

Jacques Demongeot, Adrien Elena, Mathilde Noual, Sylvain Sené, & Florence Thuderoz
Jacques Demongeot, Marcelle Kaufman, & René Thomas  

Jacques Demongeot, Mathilde Noual, & Sylvain Sené  

The effect of cycles and intersecting cycles (“circuits”) in signed digraphs representing the action of a Boolean automaton. [“Automata” should be “automaton”.] Dictionary: “Boolean automata circuit” = digraph that is a signed cycle. “Double Boolean automata circuit” = digraph that is two signed cycles with one common vertex. [Annot. 16 Jan 2015.] (SD: Dyn)

Jacques Demongeot & René Thomas  

Jacques Demongeot, René Thomas, & Michel Thellier  


Hanyuan Deng & He Huang  

Hongzhong Deng & Peter Abell  

A random signed graph has edge \( uv \) with probability \( d \), which is positive with probability \( \alpha_0 \). Degree of balance is the proportion of triangles that are positive. A triangle of type \( T_i \) has \( i \) positive edges. They study the long-term proportions of triangle types in examples. §3, “Balance adjustment under a local rule”: A triangle \( \triangleuvw \) and edge \( uv \) are chosen at random; \( uv \) changes sign if \( \triangleuvw \) is negative. This “myopic adjustment rule” is iterated. For \( 0 < \alpha_0 < 1 \), the proportions approach 37% each of 1 or 2 and 13% each of 0 or 3 negative edges. This contradicts the Cartwright–Harary (1956a) balance hypothesis. Convergence behavior in examples depends interestingly on \( n \) and \( d \). §§4–7: The sign-change probability depends on the triangle type. Probabilities are suggested by models of Harary–Cartwright, Davis (1967a), and others, in which different sets of triangle types are “attractors”. Analytical and example results are reported.

Hongzhong Deng, Peter Abell, Ji Li, & Jun Wu

Hongzhong Deng, Peter Abell, Jun Wu, & Yuejin Tang

B. Derrida, Y. Pomeau, G. Toulouse, & J. Vannimenus
1979a Fully frustrated simple cubic lattices and the overblocking effect. J. Physique 40 (1979), 617–626.

Physics of the signed \( d \)-hypercube in which every plaquette is negative; specifically, [cleverly] construct \( \Sigma_d = (Q_d, \sigma_d) \), \( d > 0 \), as \( \Sigma_{d-1} \times (+Q_1) \) with the second copy of \( \Sigma_{d-1} \) negated. Invariants of physical interest are computed and compared to the balanced case. Dictionary: “plaquette” = square.


Madhav Desai & Vasant Rao

\[ \psi(\Gamma) := \min_{S}(l(-\Gamma:S) + \#E(S,S^c)/\#S), \text{ over } \emptyset \subset S \subseteq V, \text{ is a measure of nonbipartiteness of } \Gamma. \mu_1 := \text{smallest eigenvalue of } L(-\Gamma) \text{ satisfies } \psi(\Gamma)^2/4\Delta(\Gamma) \leq \mu_1 \leq 4\psi(\Gamma). \text{ Their } e_{\text{min}}(\Gamma) := l(-\Gamma). [\text{See Fan and Fallat (2012a) for another eigenvalue connection with } l(-\Gamma).] \]

Madhav Desai & Vasant Rao

C. De Simone, M. Diehl, M. Jünger, P. Müttel, G. Reinelt, & G Rinaldi

Improves the algorithm of Barahona, Grötschel, Jünger, and Reinelt (1988a) to find a switching with minimum \( \#E^- (= l(\Sigma)) \) for signed toroidal square lattice graphs with an extra vertex (exterior magnetic field) and a fixed proportion of negative edges. Applied to many signatures in order to find statistical properties. Continued in (1996a).

[Annot. 6 Dec 2009.] (SG: Bal)

[Wouter de Nooy]
See W. de Nooy (under ‘N’).

[Arnout van de Rijt]
See A. van de Rijt (under ‘V’).

L. de Sèze
See L. de Sèze (under ‘S’).

C. De Simone, M. Diehl, M. Jünger, P. Müttel, G. Reinelt, & G Rinaldi

[Annot. 19 Sept 2010, 29 Dec 2012.] (Par: Eig, Fr)

[L. de Sèze]
See L. de Sèze (under ‘S’).


A.H. Deutz, A. Ehrenfeucht, & G. Rozenberg

Lee DeVille
See J.C. Bronski.

Vincent Devloo, Pierre Hansen, & Martine Labbé

Matt DeVos
See also R. Chen.

Matt DeVos & Daryl Funk

Matt DeVos, Daryl Funk, & Irene Pivotto

Matt DeVos, Jiaao Li, You Lu, Rong Luo, Cun-Quan Zhang, & Zhang Zhang

Matt DeVos, Edita Rollová, & Robert Šámal
2019a A note on counting flows in signed graphs. *Electronic J. Combin.* 26 (2019), no. 2, article P2.38, 7 pp. arXiv:1701.07369. For each $k \geq 0$, the number of nowhere-zero flows in an abelian group of order $m$ that has $k$ factors $\mathbb{Z}_2$ is a polynomial function $f_k(m/2^k)$. (Generalizes $k = 0$ by Beck and Zaslavsky (2006b).) [Annot. 14 Aug 2017.] (SG: Flows: Invar)

M. Deza, V.P. Grishukhin, & M. Laurent

Switching (on coordinates) is an important symmetry of the cut polytope $P_n$ (of $K_n$); see p. 206. [See Deza and Laurent (1997a).] Thm. 2.6: $\text{Aut} P_n = \mathcal{D}_n$, the Weyl group $\supseteq \text{SwAut}(\pm K_n)$, the switching automorphism group. *Question* (p. 207): For the cut polytope $P_c(\Gamma)$, does $\text{Aut} P_c(\Gamma) = \text{SwAut}(\pm \Gamma)$? [Edge signs and SwAut are not stated as such.] [Annot. 12 Jun 2012.] (sg: par: KG: Geom, sw)

**Michel Marie Deza & Monique Laurent**


**Ayushi Dhama**

See also D. Sinha.


**Inderjit S. Dhillon**

See C.-J. Hsieh and K.-Y. Chiang.

[F. di Liberto]

See F. di Liberto (under “L”).

**Persi Diaconis**

See K.S. Brown.

**Yuanan Diao & Gábor Hetyei**


**Yuanan Diao, Gábor Hetyei, & Kenneth Hinson**


**Alicia Dickenstein & Mercedes Pérez Millán**
From a multiplicative gain digraph $\vec{\Phi} := (\vec{\Gamma}, \vec{\phi}, \mathcal{G})$ where $\vec{\Gamma}$ is a symmetric digraph, construct a gain graph $\Phi := (\Gamma, \phi, G)$ and $G = \mathbb{R}_{>0}^{\times}$, where $\Gamma$ has an edge $e_{ij}$ for each arc pair $(i, j), (j, i)$ and $\varphi_{ij}(e_{ij}) := \vec{\phi}(i, j)/\vec{\phi}(j, i)$. If $\Phi$ is balanced, $\vec{\Phi}$ is called "formally balanced". This property and related ones are studied. [The construction $\vec{\Phi} \mapsto \Phi$ is known in papers on chemical reaction graphs. This paper is more gain-graphic than most though the gain graph $\Phi$ is not explicit.]

A circle $C = e_{12} \cdots e_{l-1, l} e_{l1}$ is balanced iff $\vec{\varphi}(\vec{C}) = \vec{\varphi}(\vec{C}^{-1})$, where $\vec{C} = \vec{e}_{12} \cdots \vec{e}_{l-1, l} \vec{e}_{l1}$. Question. What is the general theory of gain graphs derived from $\vec{\Phi}$ of the above type with a general abelian gain group? In general define gains on a symmetric digraph: let $\vec{\Gamma}$ have arcs $\vec{e}_{ij}$, possibly with multiple arcs and loops, with a pairing $*: \vec{E} \leftrightarrow \vec{E}$ such that $\vec{e}_{ij}^* = \vec{e}_{ji}$. Example 1: For a gain graph, let $\vec{E} := \bigcup \{\vec{e}_{ij}, \vec{e}_{ji} = \vec{e}_{ij}^{-1} : \vec{e}_{ij} \in E\}$ and $\vec{\varphi}(\vec{e}_{ij}) := \varphi_{ij}(e_{ij})$; then each (nonloop) pair is a balanced digon. Example 2: Only one of each pair has identity gain; this seems inequivalent to Example 1 (Question: Is it?), so arbitrary gains on symmetric digraphs seem more general than such gains from gain graphs and more structured than general gain digraphs.]

Gilles Didier & Elisabeth Remy

Gilles Didier, Elisabeth Remy, & Claudine Chaouiya

M. Diehl
See C. De Simone.

Hung T. Diep
See also O. Nagai.

H.T. Diep, ed.


H.T. Diep & H. Giacomini

Frustration on signed graph with Ising ($\pm 1$) and vector spins (implicitly). §§1.1–1.2 introduce frustration and physics concerns, e.g., "degeneracy" = multiple ground states. Later, various periodic signed lattice
graphs (cf. Liebmann (1986a)) are solved and diagrammed for Ising and XY ($S^1$) spins, illustrating spin frustration. [Annot. 9 Aug 2018.]

(Hung T. Diep, P. Lallemand, & O. Nagai
Physics of fully frustrated 3-dimensional cubic lattice (cf. Chui, Forgacs, and Hatch (1982a)), but the negative edges are specifically chosen to form three orthogonal families of straight lines, alternating along each plane. As signed lattice has a $2 \times 2$ fundamental domain, there are 8 translational symmetry types of vertex, each forming a double-sized sublattice. The sublattices exhibit somewhat differentiated behavior. [Annot. 18 Jun 2012.]

Simulations on the the signed graph of (1985a). The 8 sublattices are equivalent in pairs. [Annot. 18 Jun 2012.]

V. Di Giorgio

Wil Dijkstra
Extends signed graphs to sign set $\{\pm 1, 0\}$ and extends the notion of (degree of) cycle balance. A circle $C$ is “balanced” if its sign product $\sigma(C) = +1$. Degree of balance = average sign product of all circles. Degree of local balance at $X \subseteq V$ is the average sign of all circles that contain $X$. Given a length weight function $1 \geq f(2) \geq f(3) \geq \cdots \geq 0$, the weighted degree of balance is the average value of $f(|C|)\sigma(C)$. [Cf. kinds of cycle balance in Cartwright and Harary (1956a), Morrissette (1958a), Norman and Roberts (1972a), (1972b).] (SG: Bal, Fr)

It is assumed [!] that answers have probability dependent on weighted degree of local balance at $\{p, y\}$ where $p =$ respondent and $y =$ answer. Speculation about choice of functions $f$ et al. One post-hoc application.

(SG: PsS)

Genhong Ding
See X.B. Ma.

Yvo M.I. Dirickx & M.R. Rao

Ajit A. Diwan
See also M. Joglekar.

Ajit A. Diwan, Josh B. Frye, Michael J. Plantholt, & Shailesh K. Tipnis

An antidirected circle is a balanced circle in the poise gains of a digraph. [For early bidirected work see Andersen and Grant (1981a). For connected 2-factor see Busch, Jacobson, Morris, Plantholt, and Tipnis (2013a). For Hamilton paths in $K_n$ see El Sahili and Abi Aad (2018a).]

[Question. How does this generalize to bidirected graphs?] [Annot. 20, 30 May 2018.]

Daniel B. Dix

Vlastimil Dlab

A valued graph is a simple symmetric digraph with $\mathbb{Z}>0$-gains, such that $\varphi(e_{ij})/\varphi(e_{ji}) = f(v_j)/f(v_i)$ for some $f: V \rightarrow \mathbb{Z}>0$ [equivalently, cycle gains invert by reversing direction]. $\varphi(e_{ij})$ represents the dimension of an $(F_i, F_j)$-bimodule corresponding to $e_{ij}$, where $F_i$ is an algebra associated to $v_i$. [There is no gain-graph theory.] [Based on 3 articles; see Zbl.]

Duong D. Doan & Patricia A. Evans

A pedigree is a kind of signed graph with $< n$ edges, with 3-colored vertices. Frustration index ("line index") $l =$ minimum number of necessary recombinations. Elementary relations among $l$, vertex cuts, and switching. Reduction rules, including the negative-subdivision trick, to test $l \leq k$. [Question. Does sparseness reduce the hardness of testing $l \leq k$?] [Annot. 29 Apr 2012.]

(Annot. 29 Apr 2012.)


See (2011a). This problem adds parity constraints. [Annot. 29 Apr 2012.]

(Biol: SG: Fr, Alg, sw)

R.L. Dobrushin & S.B. Shlosman

Partly expository. The problems are existence of nonperiodic ground states and stability for nonferromagnetic interactions, e.g., signed graphs that are not all positive. ("Ground state" means a state at temperature 0; a "state" is a probabilistic mixture of what are usually called states, here called "configurations"). The graphs are infinite hypercubic lattice...
graphs \( \mathbb{Z}^\nu, \nu \geq 1 \). The set of “configurations” \( \sigma : \mathbb{Z}^\nu \to S \), where \( S \) is a fixed set (usually finite; \( S = \{\pm 1\} \) for Ising model, etc.), is \( \Omega \). “State”: a probability measure on \( \Omega \) w.r.t. \( \mathcal{B} \), the finitely cylindrical \( \sigma \)-algebra on \( \mathbb{Z}^\nu \). “Interactions” between vertices have finite range, not necessarily only adjacent.

§2, “Ground states”: §2.8, “The symmetric ferromagnetic Ising model”, describes ground configurations in terms of the hypercubic lattice facets dual to frustrated edges of the configuration. 3-Dimensional examples. §2.9, “The antiferromagnetic Ising model” with external field strength \( h \). There are 1 and 2 ground configurations for \( |h| > 2\nu \) and \( < 2\nu \).

Some nonperiodic ground states are described. At \( h = \pm 2\nu \) ground states (not only configurations) exist, discussed in §§2.11–12. §2.11, “Random ground states—the antiferromagnetic case”. §2.12, “Nonperiodic random ground states”. App. I, “Ground states of the model of a one-dimensional lattice antiferromagnet”, by S.E. Burkov and Ya.G. Sinai. App. II, “On random ground states of one-dimensional antiferromagnetic model”, by A.A. Kerimov.

Dictionary: “ferromagnetic” = all-positive; “antiferromagnetic” = all-negative; “periodic” = toroidal; “external field” = extra vertex \( v_0 \), positive, with \( N(v_0) = V \) and arbitrary interaction strength. [Annot. 8 Mar, 23 May, 3 Jun 2015.]

(Phys, sg: bal: State(fr))(Phys, sg: par: State(fr))

Ebrahim Dodongeh
See S. Akbari.

Benjamin Doerr

The linear discrepancy of the transposed incidence matrix of a balanced signed graph.

(Phys, SG, WG: State(fr))

B.G.S. Doman & J.K. Williams

§2, “The random bond model at low temperatures”: A signed path with magnetic field \( B \) [interpretable as an extra all-positive dominating vertex with edge weights \( B \); cf. Barahona (1982a)]. §3, “Frustrated periodic bond model”: A path with edges signed \(+---\) periodically. Describes ground states (“allowed states”), which depend on \( B \). [Annot. 28 Aug 2012.]

(Phys, SG, WG: State(fr))

Eytan Domany
See G. Hed and D. Kandel.

Mirela Domijan & Elisabeth Pécou

(Biol, Chem: SD: Dyn)

Bing-can Dong
See R.L. Li.

Chun-Long Dong
See Y.-Z. Fan.
Fengming Dong
See S.J. He.

Jiu-Gang Dong & Lin Lin

Alfredo Donno
See F. Belardo and M. Cavaleri.

Michael Doob
See also D.M. Cvetković.


A readable, tutorial introduction to (1973a) (without matroids).
(ec: LG, Incid, Eig(LG))


Along with Simões-Pereira (1973a), introduces to the literature the even-cycle matroid $G(−Γ)$ [previously invented by Tutte, unpublished]. The multiplicity of $−2$ as an eigenvalue (in characteristic 0) equals the number of independent even circles $= n − rk G(−Γ)$. In characteristic $p$ there is a similar theorem, but the pertinent matroid is $G(Γ)$ if $p = 2$ and, when $p|n$, the matroid has rank 1 greater than otherwise [a fact that mystifies me].
(EC: LG, Incid, Eig(LG))


Thm. 3.2 is the theorem of van Nuffelen (1973a), supplemented by the observation that it remains true in any characteristic except 2.
(EC: Incid)


(ec: Incid)

Michael Doob & Dragoš Cvetković

Patrick Doreian
See also M. Brusco, N.P. Hummon and A. Mrvar.


(PsS, SD, sg: Bal, Fr, Clu, Adj: Exp)


(SG: Bal, PsS)


Reviews the development of balance and clustering theory for signed (di)graphs in social psychology, mainly Doreian and Mrvar (1996a), Doreian and Krackhardt (2001a), and especially Hummon and Doreian (2003a). The difference between Heider’s (1946a) and Cartwright and Harary’s (1956a) models, and the need to combine them. [Annot. 24 Apr 2009.]

(PsS: Exp: SD, Bal, Clu, Alg)


Signed graphs $\Sigma_1,\ldots$ (“multiple indicators”) may be approximations of a hidden signed graph $\Sigma$. Goals: detect whether $\Sigma$ exists, and find an optimal clustering of $\Sigma$. Methods: (1) Examine the $\Sigma_j$ for compatibility via statistical tests. (2) Estimate $\Sigma$ by $\sum_j \sigma_j$. (3) Applies the clusterability index and algorithm of Doreian and Mrvar (1996a). (2) implies using weighted signed graphs.) This article treats examples, with analysis of the methods’ success. [Annot. 27 Apr 2009.] (PsS, SD: sg: Clu)


Two books on and the philosophy of mathematics and sociology. [Annot. 27 Apr 2012.] (PsS: SG, SD)

Patrick Doreian, Vladimir Batagelj, & Anuška Ferligoj


Ch. 10: “Balance theory and blockmodeling signed networks”. Thm. (pp. 305–306; proof by Martin Everett): The sizes of the partitions of $V$ that minimize the clustering index (Doreian and Mrvar (1996a)) are consecutive integers.

(PsS, SD: sg: Clu, Bal)

Patrick Doreian, Roman Kapuscinski, David Krackhardt, & Janusz Szczy pulpia


§3.3: “Results for group balance”. Describes results from analysis of data on a small (social) group, in terms of frustration index \( l \) and a clusterability index \( \min_{k>2} 2P_{k,5} \) (slightly different from the index in Doreian and Mrvar (1996a)), finding both measures (but more so the latter) decreasing with time.

Patrick Doreian & David Krackhardt


In a signed digraph from empirical social-group data, a tendency to transitivity of signs on directed edges \( ij, ik, jk \) (i.e., \( \sigma(ij)\sigma(jk)\sigma(ik) = + \)) holds when \( \sigma(ij) = + \) and fails when \( \sigma(ij) = - \). This suggests that balance is not a primary tendency and Harary’s (1953a) and Davis’s (1967a) theorems on balance and clusterability have limited relevance to social groups. [Also, that undirected signed graphs have limited relevance. Digraph sign transitivity properties are more relevant.] [A thoughtful article.] [Annot. 13 Apr 2009.]

Patrick Doreian, Paulette Lloyd, & Andrej Mrvar


Patrick Doreian & Andrej Mrvar


They propose indices for clusterability (as in Davis (1967a)) that generalize the frustration index of \( \Sigma \). Fix \( k \geq 2 \) and \( \alpha \in [0,1] \). For a partition \( \pi \) of \( V \) into \( k \) “clusters”, they define \( P(\pi) := \alpha n_+ + (1 - \alpha)n_- \), where \( n_+ := \text{number of positive edges between clusters} \), \( n_- := \text{number of negative edges within clusters} \), and \( 0 \leq \alpha \leq 1 \) is fixed. The first proposed measure is \( P_k := \min \ P(\pi) \), minimized over \( k \)-partitions. A second suggestion is the “negation-minimal index of generalized imbalance” [i.e., of clusterability], the smallest number of edges whose negation equivalently, deletion] makes \( \Sigma \) clusterable. [Call it the ‘clusterability index’ \( Q(\Sigma) \)]. [Note that \( P(\pi) \) effectively generalizes the Potts Hamiltonian as given by Welsh (1993a).] Question. Does \( P(\pi) \) fit into an interesting generalized Potts model? [\( P(\pi) \) also resembles the Potts Hamiltonian in Fischer and Hertz (1991a) (q.v. for a related research question).] [The data in Doreian (2008a), and common sense, suggest that clusters should be allowed to overlap. This is an open research direction.]

They employ a local optimization algorithm to evaluate \( P_k(\alpha) \) and find an optimal partition: random descent from partition to neighboring partition, where \( \pi \) and \( \pi' \) are neighbors if they differ by transfer of one vertex or exchange of two vertices between two clusters. This was found
to work well if repeated many times. [A minimizing partition into at most $k$ clusters is equivalent to a ground state of the $k$-spin Potts model in the form given by Welsh (1993a), but not quite in that of Fischer and Hertz (1991a).]

Dictionary: $P(\pi)$ is the “criterion function”. [More explicitly, call $Q(\Sigma; \pi; \alpha) := 2P(\pi)$ the ‘$\alpha$-weighted clusterability index of $\pi$’, so the clusterability index is $Q(\Sigma) = \min_{\pi} Q(\Sigma; \pi; .5)$; and call $Q_k := 2P_k$ the ‘$k$-clustering index’ of $\Sigma$.]. Clusterability is “$k$-balance” or “generalized balance”. Clusters are “plus-sets”. Signed digraphs are employed in the notation but direction is ignored.

[Further developments in Doreian et al. (various), Hummon and Doreian (2003a), Bansal et al. (2004a), Demaine et al. (2006a), Mrvar and Doreian (2009a).]

[The data in Doreian (2008a), and common sense, suggest that clusters should be allowed to overlap. This is an unplumbed direction. [Annot. rev. 22 Sept 2009.]

(\text{SD: sg: Bal, Clu: Fr(Gen), Alg, PsS})


Similar to (1996a). Some lesser theoretical detail; some new examples. The $k$-clusterability index $P_k(\alpha)$ (1996a) is compared for different values of $k$, seeking the minimum. [But for which value(s) of $\alpha$ is not stated.]

Interesting observation: optimal values of $k$ were small. It is said that positive edges between parts are far more acceptable socially than negative edges within parts [thus, in the criterion function $\alpha$ should be rather near 1].

(\text{SD: sg: Bal, Clu: Fr(Gen), Alg, PsS})


Generalizes the ideas of (1996a) \textit{(q.v.)}. Given: A signed digraph $(\bar{\Gamma}, \sigma)$; a “criterion function” $P(\pi, \rho) := \alpha n^+ + (1 - \alpha) n^-$, where $\pi := \{B_1, \ldots, B_k\}$ partitions $V$ into “clusters”, $\rho : \pi \times \pi \rightarrow \{+,-\}, \ 0 \leq \alpha \leq 1$ is fixed, and $n^\varepsilon :=$ number of edges $\overline{B_iB_j}$ with sign $\varepsilon$ for which $\rho(B_i, B_j) = -\varepsilon$ (over all $i, j$). Objective: $(\pi, \rho)$, or simply $k := \# \pi$, that minimizes $P(\pi, \rho)$. What is new and most interesting is $\rho$; also new is using the edge directions.

Call $(\bar{\Gamma}, \sigma)$ “sign clusterable” if $\exists (\pi, \rho)$ with $P(\pi, \rho) = 0$. Clusterability is sign clusterability with $\rho = \rho_+$, where $\rho_+(B_i, B_j) := +$ if $i = j$, $-$ if $i \neq j$. Let $P(k) := \min \{P(\pi, \rho) : \# \pi = k\}$. Thm. 4: $P(k)$ is monotonically decreasing. [Thus, there is always an optimum $\pi$ with singleton clusters. Why this does not vitiate the model is not addressed.]

Thm. 5: If $(\bar{\Gamma}, \sigma)$ is sign clusterable, then $(\bar{\Gamma}, -\sigma)$ also is. If $(\bar{\Gamma}, \sigma)$ is clusterable, then $(\bar{\Gamma}, -\sigma)$ is not clusterable with the same $\pi$ [provided $E \neq \emptyset$]. If $(\bar{\Gamma}, \sigma)$ is sign clusterable with $\rho = -\rho_+$, then $(\bar{\Gamma}, -\sigma)$ is clusterable with the same $\pi$. “Relocation”: Shift one vertex, or exchange two vertices, between blocks so as to decrease $P$, as in (1996a). This is said (but not proved) to minimize $P$.

Refinements discussed: partially prespecified blocks; null blocks (with-
out outgoing edges); criterion functions with special treatment of null blocks.

Applications to standard test examples of social psychology.

Dictionary: “balanced” = clusterable; “relaxed balanced” = sign clusterable; “k-balanced” = clusterable with \#\pi = k; “relaxed structural balance blockmodel” = this whole system. [Annot. 7 Feb 2009.]

W. Dörfler


Tomislav Došlić

See also Z. Yarahmadi.

Tomislav Došlić & Damir Vukičević


Lynne L. Doty

See F. Buckley.

Peter Doubilet


Peter Doubilet, Gian-Carlo Rota, & Richard Stanley


§5.3: Brief treatment of Dowling lattices via symmetric gain digraphs (not quite the same as gain graphs). [Annot. rev 20 Jan 2021.] (gg: M)

Thomas A. Dowling


Pp. 221–223: The first intimations of Dowling lattices/geometries/matroids, as in (1973a), (1973b), and their higher-weight relatives (see Bonin (1993a)).

Linear-algebraic progenitor of (1973b). Treats the Dowling lattice of group $GF(q)^*$ as naturally embedded in $PG^{n-1}(q)$. Interesting is p. 105, Remark: One might generalize some results to any ambient (simple) matroid.


$Q_n(\mathfrak{G})$. Introduces the Dowling lattices $Q_n(\mathfrak{G})$ of a group, treated as lattices of group-labelled partial partitions. Equivalent to the frame matroid of complete $\mathfrak{G}$-gain graph $\mathfrak{G}K^*_n$. [The gain-graphic approach was known to Dowling (1973a), p. 109, but first published in Doubilet, Rota, and Stanley (1972a).] Isomorphism, vector representation, Whitney numbers and characteristic polynomial. [The first and still fundamental paper.]

[The Dowling Whitney numbers and polynomials have given rise to a little field of their own; cf. Mező (2010a) and references.]

Thomais Dowling & Hongxun Qin


Thm. 1.5: The Dowling geometry $Q_r(\mathbb{Z}_2)$ is the only matroid of rank $r \geq 4$ such that every contraction by a point is $Q_{r-1}(\mathbb{Z}_2)$. (sg: M)

Felix Dräxler

See I. Balla.

[Pauline van den Driessche]

See P. van den Driessche (under ‘V’).

François Dross, Florent Foucaud, Valia Mitsou, Pascal Ochem, & Théo Pierron


HAL hal-02990576. (SG: Hom: Alg)

J.M. Drouffe

See R. Balian.

K. Drühl & H. Wagner


Lúcia Drummond

See M. Levorato.

Natasha D’Souza

See T. Singh.
Sabitha D’Souza [S. D’Souza]
See P.G. Bhat.

Haifeng Du
See Q. Cai.

Hong Shan Du, Qing Jun Ren, Hou Chun Zhou, & Qing Yu Zheng

Mingjun Du, Baoli Ma, & Deyuan Meng


Wenxue Du
See also Y.-Z. Fan.

Wenxue Du, Xueliang Li, & Yiyang Li

Including a “tight bound” on signless Laplacian energy, of $L(-\Gamma)$, and “exact estimate” of incidence energy, of $H(-\Gamma)$. [Annot. 24 Jan 2012.] (Par: Eig, Incid)

Hangen Duan
See S.C. Gong.

M. Dub
See M. Doob.

P. Robert Duimering
See G. Adejumo.

Richard A. Duke, Paul Erdös, & Vojtěch Rödl

Results of the type in (1992a) for polygon lengths $\leq 8$. Thm. 2: Given $d > 0$, constant. Every $-\Gamma$ with $#V = n$ and $#E = dn^2$ has a subgraph $\Sigma'$ with $#E' \geq d^2n^2(1 - o(1))$ in which every two edges belong to a balanced polygon of length at most 8. (par: bal(Circ))


All graphs are simple. This is one of four related papers (including (1991a)) that prove extremal results concerning subgraphs of $-\Gamma$ within
which every two edges belong to a balanced circle of length at most $2k$; for all or particular $k$. Typical theorem: Let $F_l(n, m) =$ the largest number $m' = m'(n, m)$ such that every $-\Gamma$ with $\#V = n$ and $\#E \geq m$ has a subgraph $\Sigma'$ with $\#E' = m'$ in which every two edges belong to a balanced circle of length at most $l$. For $m = m(n) \geq n^{3/2}$, there is a constant $c_3 > 0$ such that $F_l(n, m) \leq c_3 m^2 n^{-2}$ for all $l$. (§2, (2).) [Problem. Extend these extremal results in an interesting way to arbitrary signed simple graphs, or to simply signed graphs (no repeated edges with the same sign). (Merely allowing positive edges in addition to negative ones just makes the problem easier. Something more is required.])

(par: bal(Circ): Xtrem)}

David M. Duncan, Thomas R. Hoffman, & James P. Solazzo


(ge: Geom, adj: kg)


Some measures to distinguish nonisomorphic two-graphs, i.e., switching-nonisomorphic signatures of $K_n$. [Annot. 25 Oct 2012.]

Yen Duong, Joel Foisy, Killian Meehan, Leanne Merrill, & Lynea Snyder

Arne Dür

Dowling lattices are an example of a categorial approach to incidence-algebra techniques in Ch. IV, §7. Computed are the characteristic polynomial and second kind of Whitney numbers. Binomial concavity, hence unimodality of the latter [cf. Stonesifer (1975a)] is proved by showing that a suitable generating polynomial has only distinct, negative zeros [cf. Benoumhani (1999a)].

(gg: M: Invar)

Amit Dutta
See B.K. Chakrabarti.

Luke Duttweiler & Nathan Reff

P.M. Duxbury
See M.J. Alava.

Zdeněk Dvořák & Luke Postle
A combinatorial variant of gains: partial injections between lists of a list coloring problem. [Short proof by Zając (20xxa).] [Annot. 29 Jul 2019.]

Janusz Dybizbański

20xxa 2-Edge-colored chromatic number of grids is at most 9. *Graphs Combin.* (to appear).


Janusz Dybizbański, Anna Nenca, & Andrzej Szepietowski


I.E. Dzyaloshinskii & G.E. Volovik


[Early attempt to apply frustration in physics.] §2: Heisenberg spins $V(\Sigma) \to$ unit sphere $S^2$, applying “frustration lines” according to Toulouse (1977a). “Local discrete invariance” = switching. §8: Briefly, partial antiferromagnet (= signed graph); phase diagram varies with the proportion $c$ of negative edges. [Annot. 6 Aug 2018.]

David Easley & Jon Kleinberg


Zbl 1205.91007.


§5.5B, “Approximately balanced networks”: Thm.: If the proportion of unbalanced triangles in a signed $K_n$ is $\leq \varepsilon < \frac{1}{3}$, and if $\delta := \sqrt{3}\varepsilon$, then there are $(1 - \delta)\#V$ vertices in which at most a fraction $\delta$ of the edges are negative, or there is a bipartition $V = X \cup Y$ such that at most a fraction $\delta$ of the edges in $X$ and also in $Y$ are negative and at most that fraction of the $XY$ edges are positive. [Annot. 22 March 2010.]
M. Ebrahimi
See A. Kargaran.

Paul H. Edelman & Victor Reiner

Characterizes all $\Sigma \supseteq +K_n$ whose frame matroid $G(\Sigma)$ is supersolvable, free, or inductively free. Essentially, iff the negative links form a threshold graph. [Continued in *Bailey* (20xxa). Generalized in part to arbitrary gain groups in *Zaslavsky* (2001a).] (sg: M, Geom, col)


Paul H. Edelman & Michael Saks

Given $\Gamma$ and abelian group $\mathfrak{A}$. Vertex and edge labelings $\lambda : V \to \mathfrak{A}$ and $\eta : E \to \mathfrak{A}$ are “compatible” if $\lambda(v) = \sum_e \eta(e)$ for every vertex $v$, the sum taken over all edges incident with $v$. $\lambda$ is “admissible” if it is compatible with some $\eta$. Admissible vertex labelings are characterized (differently for bipartite and nonbipartite graphs) and the number of edge labelings compatible with a given vertex labelling is computed. [Dual in a sense to *Gimbel* (1988a).] (WG, VS: Bal(D), Enum)

Herbert Edelsbrunner, Günter Rote, & Emo Welzl


§5.1, “Testing the feasibility of the linear program (2) or (1)”: The dual linear program (4) belongs to an oriented all-negative signed graph. Treated by expanding it to the graphic LP belonging to the canonical covering graph. (par: ori, Geom: Alg)

Jack Edmonds
See also J. Aráoz and E.L. Lawler (1976a).


Followed up by much work, e.g., Witzgall and Zahn (1965a); see Ahuja, Magnanti, and Orlin (1993a) for some references. (par: ori: incid, Alg)

Zbl 141.21802 (141, p. 218b).

Alludes to the polyhedron of Edmonds and Johnson (1970a).

(par: ori: Incid, Geom)

Jack Edmonds & Ellis L. Johnson


Introduces “bidirected graphs”. A “matching problem” is an integer linear program with nonnegative and possibly bounded variables and otherwise only equality constraints, whose coefficient matrix is the incidence matrix of a bidirected graph. No proofs. [See Aráoz, Cunningham, Edmonds, and Green-Krótki (1983a) for further work.]

(Par: Ori: Incid, Alg, Geom)


S.F. Edwards & P.W. Anderson


Spins $s_i$ are (unit) vectors; edge weights and signs are $J_{ij} \in \mathbb{R}$; the interaction between spins is $J_{ij}s_i \cdot s_j$ with the scalar product. [This seminal article leads to the appearance of signed graphs in physics. Later developments refine the definitions, e.g., to $J_{ij} = \pm 1$, $J_{ij}$ Gaussian random variables, $s_i \in \{+1, -1\}$, etc. Cf. esp. Sherrington and Kirkpatrick (1975a).]

(Phys: sg, wg)

Yoshimi Egawa

See N. Alon.

Richard Ehrenborg


(gg: col, Invar, m, Geom)


Calculates the characteristic (Cor. 2.5) and, implicitly, Whitney-number polynomials of $[-r+1, r]K_n$ in terms of its affinographic hyperplane representation, the extended Shi arrangement. The object is to count faces of the latter by dimension and dimension of the infinite part. [Zaslavsky
Richard Ehrenborg & Margaret A. Readdy


An abstract additive approach to the characteristic polynomial $p(\lambda)$, applied in particular (§3: “The divisor Dowling arrangement”) to “divisor Dowling” hyperplane arrangements $\mathcal{B}(m)$ and certain interpolating arrangements. [Let $\Phi = G_1 \cup \cdots \cup G_n K_n$, where $V(K_i) = \{v_1, \ldots, v_i\}$ and $\mathbb{Z}_m = G_1 \geq \cdots \geq G_n$. $\mathcal{B}(m)$ is the complex hyperplane representation of $\Phi^*$. Thus, $p_{g(m)}(\lambda) = \chi_{\Phi^*}(\lambda)$, the chromatic polynomial. This is computable via gain-graph coloring when $G_1$ is any finite group. The same is true for the other arrangements treated herein.] [Annot. 25 Apr 2009.]


Canonical complex hyperplane representation of the Dowling lattice of $\mathbb{Z}_k$. P. 395: an interesting EL-labelling of the Dowling lattice by a [disguised lexicographic] ordering of atoms. Thm. 4.9 is a recursive formula for its ab-index. Thm. 5.2: the c-2d-index of the face lattices in case $k = 1, 2$, i.e., those of the real root system arrangements $A_n^*$ and $B_n^*$. §6 presents a combinatorial description of the face lattice of $B_n^*$ [which it is interesting to compare with that in Zaslavsky (1991b)]. Dictionary: very confusingly, “region” = face. [Dictionary: “directed cycle” = circle (not directed).]


The group expansion of an ordinary graph is generalized to expansion of an $\mathbb{R}^\times_{>0}$-gain graph by a finite cyclic subgroup of $\mathbb{C}^\times$, with correspondingly generalized formulas for the chromatic polynomial. The computations are technically incorrect; they should be done by gain-graph coloring. [Dictionary: “directed cycle” = circle (not directed).]


A generalization of Stanley’s exponential structures, based on the partition lattice, to Dowling lattices. §2 defines Dowling lattices via partial partitions (“zero block” = set of non-partitioned elements). §3 defines Dowling exponential structures and gives compositional identities via generating functions. §4: generating-function identities for the Möbius invariant; structures with restricted block sizes—especially, block sizes divisible by $r$ with $K$ non-partitioned elements where $K \geq k$ and $K \equiv k \pmod{r}$. [Generalized in Koban (2004b).]

Andrzej Ehrenfeucht

See also A.H. Deutz.

Andrzej Ehrenfeucht, Jurriaan Hage, Tero Harju, & Grzegorz Rozenberg


TG: Sw: Alg


Every switching class of graphs except that of the edgeless graph contains a pancyclic graph. Thus Hamiltonicity is polynomial-time for graph switching classes.

(TG: Sw, Alg)


Andrzej Ehrenfeucht, Tero Harju, & Grzegorz Rozenberg


The “hierarchical structure” of a switching class of skew gain graphs based on $K_n$.

(GG: KG: Sw)


A tutorial (with some new proofs). The relevant sections, based on papers of Ehrenfeucht and Rozenberg with and without Harju, are those about dynamic labeled 2-structures, i.e., complete graphs with twisted gains. §6.7: “Dynamic labeled 2-structures”. §6.8: “Dynamic $\ell2$-structures with variable domains”. §6.9: “Quotients and plane trees”. §6.10: “Invariants”, concerns certain switching invariants called “free invariants” when the gains are not twisted. (gg: KG: Sw: Exp, Ref)


Given a gain graph $(K_n, \varphi, \mathfrak{G})$, a word $w$ in the oriented edges of $K_n$ has a gain $\varphi(w)$; call this $\psi_w(\varphi)$. A “free invariant” is a $\psi_w$ that is an invariant of switching classes. Thm.: There is a number $d = d(K_n, \mathfrak{G})$ such that the group of free invariants is generated by $\psi_w$ with $w = z_1^d \cdots z_k^d u_1 \cdots u_l$ where $w_i$ are triangular cycles (directed!) and $u_i$ are commutators. [The whole paper applies mutatis mutandis to arbitrary graphs, the triangular cycles being replaced by any set of cycles containing a fundamental system.] Dictionary: “Inversive 2-structure” = gain graph based on $K_n$.

(gg: KG: Sw, Invar)


(gg: KG: sw: Exp, Ref)
Let antilocal complementation at $v$ mean reversing the edges except within the neighborhood of $v$. Let strictly antilocal complementation mean reversing the edges except within the closed neighborhood of $v$. Every simple graph of order $n$ can be converted to every other one by antilocal complementations, and also by strictly antilocal complementations. 

Andrzej Ehrenfeucht & Grzegorz Rozenberg


Extended summary of (1994a). (GG(Gen): KG: Sw, Str)


They prove that a complicated definition of “reversible dynamic labeled 2-structure” $G$ amounts to a complete graph with a set, closed under switching, of twisted gains in a gain group $\Delta$. The twist is a gain-group automorphism $\alpha$ such that $\lambda(e; x, y) = [\alpha \lambda(e; y, x)]^{-1}$, $\lambda$ being the gain function. Dictionary: their “domain” $D =$ vertex set, “labeling function” $\lambda$ (or equivalently, $g$) = gain function, “alphabet” = gain group, “involution” $\delta = \alpha \circ \text{inversion}$, “$\delta$-selector” $\hat{S} =$ switching function, “transformation induced by $\hat{S}$” = switching by $\hat{S}$; a “single axiom” d.l. 2-structure consists of a single switching class.

Further, they investigate “clans” of $G$. Given $g$ (i.e., $\lambda$), deleting identity-gain edges leaves isolated vertices (“horizons”) and forms connected components, any union of which is a “clan” of $g$. A clan of $G$ is any clan of any $g \in G$.


Combinations and decompositions of complete graphs with twisted gains.

George C.M.A. Ehrhardt, Matteo Marsili, & Fernando Vega-Redondo


§ III, “The effect of negative links”: A random model where positive edges may appear, and may change to negative. Negative edges disap-
pear over time. [Annot. 12 Aug 2012. (SG: PsS: Rand, Phys)]

K. Ehsani
See S. Akbari.

M. Einollahzadeh
See S. Akbari.

Kurt Eisemann

Joyce Elam, Fred Glover, & Darwin Klingman

Adrien Elena
See J. Demongeot.

David P. Ellerman

M.N. Ellingham

Main theorem (§2) characterizes, given two signings of $K_n$ (where $n$ may be infinite) and a vertex set $S$, when switching $S$ makes the signings isomorphic. [Problem 1. Generalize to other underlying graphs. Problem 2. Prove an analog for bidirected $K_n$'s.] A corollary (§3) characterizes when vertices $u, v$ of $\Sigma = (K_n, \sigma)$ satisfy $\Sigma^{(u)} \cong \Sigma^{(v)}$ and discusses when in addition no automorphism of $\Sigma$ moves $u$ to $v$. All is done in terms of Seidel (graph) switching (here called “vertex-switching”) of unsigned simple graphs. (kg: sw, TG)


Deepens the folded-cube theory of Ellingham and Royle (1992a). Nicely generalizing Stanley (1985a), the number of subgraphs of a signed $K_n$ that are isomorphic to a fixed signed $K_m$ is reconstructible from the $s$-vertex switching deck if the Krawtchouk polynomial $K_s^n(x)$ has no even zeros between 0 and $m$. (Closely related to Krasikov and Roditty (1992a), Theorems 5 and 6.) Remark 4: balance equations (Krasikov and Roditty (1987a)) and Krawtchouk polynomials both reflect properties of folded cubes. All is done in terms of Seidel switching of unsigned simple graphs. [It seems clear that the folded cube appears because it corresponds to the effect of switchings on signatures of $K_n$ (or any connected graph), since switching by $X$ and $X^c$ have the same effect. For the bidirected case (Problem 2 under Stanley (1985a)), the unfolded cube should play a similar role. Question. When treating a general
underlying graph \( \Gamma \), will a polynomial influenced by \( \text{Aut} \Gamma \) replace the Krawtchouk polynomial?]

**M.N. Ellingham & Gordon F. Royle**


Reconstruction of induced subgraph numbers of a signed \( K_n \) from the \( s \)-vertex switching deck, dependent on linear transformation and thence Krawtchouk polynomials as in Stanley (1985a). The role of those polynomials is further developed. Done in terms of Seidel switching of unsigned simple graphs, with the advantage of reconstructing arbitrary subgraph numbers as well. A gap is noted in Krasikov and Roditty (1987a), proof of Lemma 2.5. [Methods and results are closely related to Krasikov (1988a) and Krasikov and Roditty (1987a), (1992a).]

**Joanna A. Ellis-Monaghan**

See also L. Abrams.

**Joanna A. Ellis-Monaghan & Iain Moffatt**


**Joanna A. Ellis-Monaghan & Irasema Sarmiento**


**Joanna Ellis-Monaghan & Lorenzo Traldi**


A variation on the multiplicative property of the parametrized Tutte polynomial.

**Abdelhakim El Maftouhi, Ararat Harutyunyan, and Yannis Manoussakis**

2014a (as Hakim El Maftouhi, Ararat Harutyunyan, and Yannis Manoussakis) Balance in random signed graphs. In: *Bordeaux Graph Workshop 2014*, pp. 43–
   (SG: Rand: Bal, Clu)

   (SG: Rand: Clu)

A. El Maftouhi, Y. Manoussakis, & O. Megalakaki
   (SG: Bal, Fr: Rand)

Amine El Sahili & Maria Abi Aad
   Antidirected means coherent in the poise gains of a digraph. [Cf. Diwan, Frye, Plantholt, and Tipnis (2011a).] [Question. How does this generalize to bidirected graphs?] [Annot. 30 May 2018.]
   (gg: KG: Str)(sg: KG: par: Ori)

D. Emanuel & A. Fiat
See also E. Demaine.

Frank Emmert-Streib
See G.H. Yu.

German Andres Enciso
See also B. DasGupta.

German Enciso & Eduardo D. Sontag
   (Dyn: SD)

   (SD: Dyn, Biol)

   (SD: Dyn, Biol)

Mechthild Enderle
See J.D. Noh.

Shin-ichi Endoh
See T. Nakamura.

Gernot M. Engel & Hans Schneider


Michael Engquist & Michael D. Chang


R.C. Entringer


See Erdős, Rubin, & Taylor (1980a). (par: bal)

H. Era

See J. Akiyama.

Paul Erdős [Paul Erdös, Paul Erdős, Pál Erdős]

See also B. Bollobás and R.A. Duke.


P. 119 mentions the theorem of Duke, Erdős, & Rödl (1991a) on even circles.

Pp. 120–121 mention (amongst similar problems) a theorem of Erdős and Hajnal (source not stated): Every all-negative signed graph with chromatic number \( \aleph_1 \) contains every finite bipartite graph [i.e., every finite, balanced, all-negative signed graph]. [Problem. Find generalizations to signed graphs. For instance: Conjecture. Every signed graph with chromatic number \( \aleph_1 \), that does not become antibalanced upon deletion of any finite vertex set, contains every finite, balanced signed graph up to switching equivalence.] [MR: “this is one of the best collections of problems that Erdos has published.”] (par: bal: Exp, Ref)

P. Erdős, R.J. Faudree, A. Gyárfás, & R.H. Schelp


A large, nonbipartite, 2-connected graph with large minimum degree contains a circle of given odd length or is one of a single type of excep-
tional graph. [Question. Can this be generalized to negative circles in unbalanced signed graphs?]

P. Erdős, E. Győri, & M. Simonovits

Assume $|\Sigma|$ simple of order $n$ and $\not\in$ a fixed graph $\Delta$. Results on frustration index $l$ of antibalanced $\Sigma$ if $\Delta$ is 3-chromatic, esp. $C_3$. Thm.: If $\#E > n^2/5 - o(n^2)$, then $l(\Sigma) < n^2/25 - o(n^2)$. Conjecture (Erdős): For $\Delta = C_3$ the hypothesis on $\#E$ is unnecessary. [Question 1(a)]. Is the answer different when $\Sigma$ need not be antibalanced? Question 2(a). Exclude a fixed signed graph whose signed chromatic number = 1. Question 3(a). In particular, exclude $-K_3$. Question 4(a). Exclude $-K_1$. Question 5(a). Exclude an unbalanced $C_l$. Questions 1–5(b). Even if $l(\Sigma)$ cannot be estimated, is there always an extremal graph that is antibalanced—as when no graph is excluded, by Petersdorf (1966a)?

P. Erdős & L. Pósa

An upper bound on $l_0$, the vertex frustration number, in terms of vertex packing of unbalanced circles, in the contrabalanced case. Problem. Find an analog for signed graphs and a generalization to biased graphs.

Paul Erdős, Arthur L. Rubin, & Herbert Taylor

Rubin’s block theorem (Thm. R, p. 136): a block graph, not complete or an odd circle, contains an induced even circle with at most one chord. [See also Entringer (1985a).] [Question. Does this generalize to signed graphs, Rubin’s block theorem being the antibalanced case? Rubin’s 2-choosability theorem, p. 132, is also tantalizingly reminiscent of antibalanced graphs, but in reverse.]

Carolyn Eschenbach
See also Z.-S. Li and J. Stuart.

Irreducible $A$ is sign-idempotent iff every entry is $. Necessary and sufficient conditions for reducible $A$ to be sign-idempotent; in particular, it need not have nonnegative entries, but $V$ must partition into $V_i$ inducing no arcs or an all-positive complete symmetric digraph with loops. [Counterexamples in Rong Huang, Sign idempotent sign patterns similar to nonnegative sign patterns, Linear Algebra Appl. 428 (2008), 2524–2535. MR 2416567 (2009c:15010). Zbl 1144.15014.]

Annot. 29
Carolyn A. Eschenbach, Frank J. Hall & Charles R. Johnson

Sign matrices whose sign patterns are self-inverse are essentially adjacency matrices of signed graphs and are very few. [Annot. 13 Apr 2009.]

Carolyn A. Eschenbach, Frank J. Hall, Charles R. Johnson, & Zhongshan Li

$A, B$ are nowhere-zero sign-pattern matrices. $(AB)_{ij}$ may be necessarily $+, −,$ or ambiguous [Abelson and Rosenberg’s (1958a) $p, n, a$]. Let $\mathcal{R} =$ set of rows, $\mathcal{C} =$ set of columns. The graph $G(A, B) \subseteq K_{\mathcal{R}(A), \mathcal{C}(B)}$ has an edge $ij$ for each unambiguous entry in $AB$. The digraph $D(A^2)$ has an arc $(i, j)$ for each unambiguous entry in $A^2$. Thm. 3.2: $\Gamma$ is a $G(A, B)$ iff it is the disjoint union of bicliques and isolated vertices. Characterizing $D(A^2)$ seems hard. Results on special cases. [$D$ and $G$ are signed by $p, n$. Thm. 5.11: $D(A^2)$, if a circle, is balanced iff the circle is positive. §6, “Characterization of permutation graphs in $D_n$”, i.e., $D(A^2)$ that are permutation graphs. [Problem. Investigate $D(A^2)$ and $G(A, B)$ signed by $p, n$. Problem. Generalize to allow 0 entries (thus working over Abelson–Rosenberg’s algebra $\{p, n, a, o\}$).] Dictionary: “signature similarity” of matrices = switching of digraph, “negative matching” = entry $n$ in $AB =$ negative edge in $G(A, B)$. [Annot. 4 Nov 2011.]

Carolyn A. Eschenbach, Frank J. Hall & Zhongshan Li

$S := \{\alpha + i\beta : \alpha, \beta = 0, \pm\},$ the set of complex signs. A complex sign pattern matrix has complex signs as entries. If it is square it has a digraph $D(A)$ with complex signs as gains. §3, “Cyclic nonnegativity”: Cycles with gain $\pm$. Switching (via matrices) by $\pm, \pm i$. Cor. 3.2: $D(A)$ is balanced iff it switches to all $+$. Thm. 3.3: iff $D$ is balanced and $D(A + A^*)$ switches to all $+$. §4, “Stability”: Lem. 4.1: If $A$ is sign stable, every digon has real or purely imaginary gain. Lem. 4.2: If $A$ is sign stable it is sign nonsingular. Thm. 4.4 (generalizing Quirk and Ruppert (1965a) and Maybee and Quirk (1969a)): Assume every vertex has a negative loop. Then $A$ is sign stable iff all digons are negative and no longer cycles exist. Thm. 5.2: Similar, for ray stability. Dictionary: “cyclically nonnegative” = all cycle gains are $+; “cyclically positive” =
cyclically nonnegative and no zeros. [Annot. 4 Nov 2011.]

(QM: Gen; gg, sw; QSta)

Carolyn A. Eschenbach & Charles R. Johnson

“The Perron property”: spectral radius is an eigenvalue. Thm. 1.1: Sign pattern matrix $A$ requires Perron property iff it is cyclically nonnegative.


(QM, SD: Bal(Cyc), sw)

Carolyn A. Eschenbach & Zhongshan Li

Matrices with tree digraph. Cycle sign = sign product, p. 82. 2-cycle signs in Thms. 5.3 (proof), 5.5, 5.7 (proof). [Annot. 5 Nov 2011.]

(QM: sd, sw)

Pouya Esmailian, Seyed Ebrahim Abtahi, & Mahdi Jalili

Pouya Esmailian & Mahdi Jalili

Ernesto Estrada

Axioms for measurement of imbalance. (SG: Bal, Fr: PsS)

Ernesto Estrada and Michele Benzi

For signed graph or digraph, $K(\Sigma) := \frac{\text{tr}(\exp A(\Sigma))}{\text{tr}(\exp A(\Sigma_n))}$, “degree of balance” based on walk signs. Weighting a closed $l$-walk by $w(W) = 1/l!$, degree of balance is $K = w(\Sigma)/w(\Sigma_n)$; then $0 \leq K \leq 1$; $K = 1$ iff $\Sigma$ is cycle balanced. Thm 1: For $\Sigma_n := (K_n, \sigma)$ with $\Sigma_n^+ = C_n$, $K \to 0$, interpreted as $\Sigma_n$ being “largely unbalanced”. Degree of balance of $v_i$ is $K_i := \frac{(\exp A(\Sigma))_{ii}}{(\exp A(\Sigma_n))_{ii}}$. §III: Replace $A$ by $\beta A$, $\beta > 0$ (inverse “temperature”), giving $K(\beta)$, interpreted as equilibrium constant of a graph with fluctuating signs. §§IV-VII: Compares walk-based measure $K$ to circle-based measures in some real examples including social networks. §VII, “Tuning balance in social networks”: Varying $\beta$ has interesting effects on $K$.


(SG, SD: Fr, Adj, PsS, Phys)

Ernesto Estrada & Naomichi Hatano

**Ernesto Estrada, Desmond J. Higham, & Naomichi Hatano**


**Ernesto Estrada & Juan A. Rodríguez-Velázquez**


**Khasheyar Etemadi**

See S. Akbari.

**Anthony B. Evans**

See Y.Q. Chen.

**Patricia A. Evans**

See D.D. Doan.

**Cloyd L. Ezell**


**Peyman Ezzati**

See S. Akbari.

**Giuseppe Facchetti, Giovanni Iacono, & Claudio Altafini**


**François Fages**

See also K. Sriram.

**François Fages & Sylvain Soliman**


**Ulrich Faigle & Rainer Schrader**


An example is threshold signed graphs (cf. Benzaken, Hammer, and de Werra (1985a)). [Annot. 16 Jan 2012.] (SG)

**M. Falcioni, E. Marinari, M.L. Paciello, G. Parisi, & B. Taglienti**


**Shaun Fallat**

See also M.S. Cavers and Y.-Z. Fan.
Shaun Fallat & Yi-Zheng Fan


“Bipartiteness” of $\Gamma$ [also known as biparticity] is $b(\Gamma)$. “Algebraic bipartiteness” is the smallest eigenvalue $\lambda_{\min}(L(\Gamma))$. Rephrased in terms of antibalanced signed graphs: Thm. 2.1. If $-\Gamma$ is unbalanced, $\lambda_{\min} \leq l_0(\Gamma)$, the vertex frustration number. Thm. 2.4. (1) Spec $L(\Gamma) = \text{Spec } L(\Gamma) \cup \text{Spec } L(-\Gamma)$. [A special case of Bilu and Linial (2006a), Lemma.] (2–4) Elementary properties of $\tilde{\Gamma}$ [found in Zaslavsky (1982a)]. (4) If $-\Gamma$ is connected and unbalanced, $\lambda_2(L(\tilde{\Gamma})) = \min\{\lambda_{\min}(\Gamma), \lambda_2(\Gamma)\} > 0$.

$\tilde{\psi}(\Gamma) := \min_S[2l(\Gamma : S) + \#E(S, S^c)]/\#S \ (S \neq \emptyset, V)$ (cf. Desai and Rao (1994a).) Thm. 2.6. If $\Gamma$ is connected, $\Delta := \text{max degree}$, $\lambda_{\min} \geq \Delta - \sqrt{\Delta^2 - \tilde{\psi}^2}$. Thm. 2.7. $\lambda_{\min} \leq 2\tilde{\psi} \leq 4l(\Gamma)/n$. (Strengthens Y.Y. Tan and Fan (2008a).) [Conjecture. The results must generalize to all $(\Gamma, \sigma)$.] [Annot. 20 Jan 2012.]

(Genghua Fan)

See B. Bao and J. Chen.

Yi-Zheng Fan

See also L. Cui, S. Fallat, S.C. Gong, B.S. Tam, Y.Y. Tan, Y. Wang, M.L. Ye, G.-D. Yu, and J. Zhou.


The signed graphs (not necessarily simple) for which adding an edge changes only one eigenvalue of the Laplacian matrix $L(\Sigma)$ and increases that by an integer. [Dictionary: “mixed graph” = bidirected graph $B$ where all negative edges are extraverted, in effect the signed graph $-\Sigma_B$; “quasibipartite” = balanced; “$e^\sigma$ = $e$ with reversed sign. The article’s sign $\text{sgn}(e)$ equals $-\sigma_B(e)$. The entire article is really about signed graphs $\Sigma$ and $A(\Sigma)$ and $L(\Sigma)$ and uses signed-graph matrices and methods.] Thm. 1: This eigenvalue property holds iff the column $x(e)$ of $e$ in $H(\Sigma)$ is an eigenvector of $L(\Sigma)$. Corollaries give other criteria and identify the change in the one eigenvalue. Lemma 5: $L$ is singular iff $\Sigma$ is balanced [special case of Zaslavsky (1982a), Theorem 8A.4]. [Annot. 13 Apr 2009, 10 Feb 2012.]

(Genghua Fan)


Yi-Zheng Fan


The “mixed graphs” are signed graphs with reversed signs; see (2003a). Graphs are simple. The eigenvalues are those of the Laplacian $L(\Sigma)$. Prop. 2.2: Laplacian spectrum of negative circle. [The first such proof. Equivalent to the adjacency spectrum because $C_n$ is regular.] Thm. 2.8:
The signed 1-trees with max and min $\lambda_{\min}(L(\Sigma))$. Thm. 2.9: Those with $\lambda_{\min} = n$. Thm. 2.10: Those with $\lambda_{\min} > n$. (N.B. Lem. 2.4: $\lambda_{\min} \leq n + 1$ from Hou, Li, and Pan (2003a), Thm. 3.5(1), or X.D. Zhang and Li (2002a).) [Annot. 10 Feb 2012.] (SG: incid, Eig)


The “mixed graphs” are signed graphs with reversed signs; see Y.-Z. Fan (2003a). Graphs are simple. The eigenvalue is that of $L(\Sigma)$. Eigenvector structure leads to results on minimum and maximum of the least eigenvalue, given order and girth. [Annot. 9 Jan 2013.] (sg: Eig)


The “mixed graphs” are signed graphs with reversed signs; see Y.-Z. Fan (2003a). Graphs are simple. The eigenquantities are those of the Laplacian $L(\Sigma)$. The eigenvector of the smallest eigenvalue is similar to that of the second smallest Laplacian eigenvalue of a graph. [Annot. 9 Jan 2013.] (sg: Eig)

Yi-Zheng Fan, Wen-Xue Du, & Chun-Long Dong


(SG: Eig)

Yi-Zheng Fan & Shaun Fallat


Cf. M.H. Liu and Liu (2010a), Oliveira, de Lima, de Abreu, and Kirkland (2010a). Weak relations between $l(\Sigma)$ and spread of $L(\Sigma)$. Min spread of $L(\Sigma)$ is $2 + 2\cos(\pi/n)$, attained only by a path and $-C_{odd}$. Next smallest spread = 4, attained only by $-K_4$, $-\Gamma$ consisting of two triangles joined by an edge, $K_{1,3}$, $C_{even}$. Proofs by cases: $l(\Sigma) \leq 1$ or $\geq 2$ (min spread is from $-K_4$). Dictionary: “edge bipartiteness” = frustration index $l(-\Gamma)$; “mixed graph” = (oriented) signed graph with reversed signs (oriented edges are called negative); Laplacian matrix of mixed graph $G = D(|\Sigma|) - A(\Sigma)$. [See Desai and Rao (1994a) for another eigenvalue connection with $l(\Sigma)$.] [Annot. 29 Dec 2012.] (Par: Eig, Fr, Cov)

Yi-Zheng Fan, Shi-Cai Gong, Yi Wang, & Yu-Bin Gao


The “mixed graphs” are signed graphs with reversed signs; see Fan (2003a). Graphs are simple. Eigenvalues are those of $L(\Sigma)$. (sg: Eig)

Yi-Zheng Fan, Shi-Cai Gong, Jun Zhou, Ying-Ying Tan, & Yi Wang

The “mixed graphs” are signed graphs with reversed signs; see Fan (2003a). Assume $\Sigma$ is connected. $m := $ number of eigenvalues $> 2$. Thm. 2.2: $d := $ longest path length, $\mu := $ matching number. (i) $m \geq \lfloor d/2 \rfloor$, (ii) $m \geq \mu$ if $n > 2\mu$, (iii) $m \geq \mu - 1$ if $n = 2\mu$. Now assume $\Sigma$ is unbalanced. Thm. 3.4. If $n \geq 7$, then $m = 2$ iff $|\Sigma|$ is one of two general types and $\Sigma$ has a certain negative triangle. Thm. 3.5. If $n \geq 6$, then $m = 1$ iff $\Sigma \sim -K_4$ or an unbalanced subgraph. Dictionary: See Bapat, Grossman, and Kulkarni (1999a).

Yi-Zheng Fan, Hai-Yan Hong, Shi-Cai Gong, & Yi Wang
Finds the unicyclic signed graphs with first, second, and third largest spectral radii. The “mixed graphs” are signed graphs with reversed signs; see Fan (2003a). Graphs are simple. The eigenvalues are those of $L(\Sigma)$.

Yi-Zheng Fan, Bit-Shun Tam, & Jun Zhou
The maximal graphs are $K_4 \setminus e$ with $n - 4$ pendant edges at one trivalent vertex. [Annot. 9 Sept 2010.]

Yi-Zheng Fan, Yue Wang, & Yi Wang
For $\text{rk} A(\Sigma) \geq 2$, the cases where $\text{rk} \leq 3$ are characterized. See also Y.-Z. Fan, W.-X. Du, and C.-L. Dong (2014a), X.-Z. Tan and B.-L. Liu (2006a). [Annot. 17 Dec 2011.]

Yi-Zheng Fan & Dan Yang

E. Fanchon
See J. Aracena.

Mohammad Reza Farahani
See M.R. Rajesh Kanna.

Thomas J. Fararo
See N.P. Hummon.

Luerbio Faria, Sulamita Klein, & Matěj Stehlík
If $\Gamma$ is a fullerene graph (cubic, plane, no isthmi, all faces are pentagons and hexagons), $l(-\Gamma) \leq \sqrt{12n/5}$. Dictionary: “odd cycle transversal” = balancing set of $-\Gamma$. [Annot. 1 Oct 2012.]

Arthur M. Farley & Andrzej Proskurowski

Calculating frustration index is NP-complete, since it is more general than max-cut. However, for signed outerplanar graphs with bounded size of bounded faces, it is solvable in linear time. [It is quickly solvable for signed planar graphs. See Katai and Iwai (1978a), Barahona (1981a), (1982a), and more.]

(RG: Fr)

Rashid Farooq
See also M. Khan.

Rashid Farooq, Mehtab Khan, & Sarah Chand
20xxa On iota energy of signed digraphs. Linear Multilinear Algebra (to appear).

(SD: Adj: Eig)

Rashid Farooq, Sarah Chand, & Mehtab Khan

(MF: Adj: Eig)

M. Farzan

A “double cover of a graph” means the double cover of a signing of a simple graph.

(GH: Cov, Aut)

G.H. Fath-Tabar
See E. Ghasemian.

R.J. Faudree
See also P. Erdős.

Ralph J. Faudree, Evelyne Flandrin, Michael S. Jacobson, Jenő Lehel, & Richard H. Schelp

(Par: Cyc)

Katherine Faust
See also S. Wasserman.


A documented warning that properties of triads (vertex triples) in a graph or digraph tend to be heavily dependent on monad (single-vertex) or dyad (vertex pair) properties such as the density of edges or the degree distribution and therefore must be evaluated in comparison to expected triad properties given the distribution of dyad types. The focus is on digraphs; pp. 9–10 mention structural balance, i.e., signed-graph models. [Problem 1. Carry out a similar analysis for signed graphs, and in particular, signed complete graphs (equivalent to graphs). Problem 2. The same, for switching classes of the preceding, in which the meaning
of a dyad census is unclear.] [Annot. 22 Aug 2014.]

Siamak Fayyaz Shahandashti, Mahmoud Salmasizadeh, & Javad Mohajeri

Edges have “signatures” for encryption. No edge signs! [Irresistible.] [Annot. 5 Mar 2011.]

N.T. Feather


Martin Feinberg
See G. Craciun and G. Shinar.

Anna Felikson
See M.D. Sikirić.

Mariusz Felisiak
See also R. Bocian.

Mariusz Felisiak & Daniel Simson

Michael R. Fellows
See H.L. Bodlaender.

Stefan Felsner & Kolja Knauer

“Generalized flows” are flows (conservative at each vertex, i.e., real 1-cycles) on a graph with positive real gains (“generalized network”). [Annot. 2 Apr 2013.]

Paul Fendley & Vyacheslav Krushkal

The Potts model treats a graph as all negative (“antiferromagnetic”; see the low-temperature expansion in §3). [Annot. 12 Jan 2012.]

Derek Feng, Randolf Altmeyer, Derek Stafford, Nicholas A. Christakis, and Harrison H. Zhou
A new, statistical model for measuring imbalance using triangles. Hidden assumption: triangles largely determine the graph. Examples may not satisfy this. [Annot. 19 Dec 2020.]  

**Gang Feng**
See Y.-Z. Wu.

**Lihua Feng**
See also G.H. Yu.

(par: Lap: Eig)

**Lihua Feng & Guihai Yu**

(par: Lap: Eig)

(par: Lap: Eig)

The graphs with maximum spectral radius. [Annot. 19 Nov 2011.]  
(par: Lap: Eig)

**Lihua Feng, Guihai Yu, & Aleksandar Ilić**

(par: Lap: Eig)

**Lihua Feng, Guihai Yu, Aleksandar Ilić, & Dragan Stevanović**

(par: Lap: Eig, Top)

**Lin Feng, Yan Hong Yao, Ji Ming Guo, & Shang Wang Tan**

(par: Lap: Eig)

**Shasha Feng, Li Wang, Yijia Li, Shiwen Sun, & Chengyi Xia**

(SG, WG: Alg)

**Zouhaier Ferchiou & Bertrand Guenin**

(sg: m)

**Anuška Ferligoj**
See P. Doreian.

**Lori Fern [Lori Koban]**
See also L. Koban.
Consider a subgroup $W$ of the hyperoctahedral group $Oc_n$ that is generated by reflections. Let $M(W)$ be the vector matroid of the vectors corresponding to reflections in $W$. The possible direct factors of any automorphism group of $M(W)$ are $S_k$, $Oc_k$, and $Oc_k^+$. The proof is strictly combinatorial, via signed graphs. (SG: M: Aut, Geom)

Rosário Fernandes


The “weights” are skew gains [cf. J. Hage (1999a) *et al.*] in $\mathbb{C}^\times$; the anti-involution is conjugation. Identities satisfied by the eigenvalues. [Annot. 11 Jan 2012.] (GG: Gen: Eig)

L.A. Fernández, V. Martin-Mayor, G. Parisi, & B. Seoane


Average behavior of random signed subhypercubes $(\Gamma, \sigma)$, with spanning $\Gamma \subseteq Q_D$, with random spins $\zeta : V \to \{+1, -1\}$. Each $(\Gamma, \sigma, \zeta)$ is a “sample”. To avoid irregularities $\Gamma$ is $z$-regular ("connectivity $z$") for a fixed $z$ (here, 6). [Annot. 19 Jun 2012.] (Phys, SG: State)

Daniela Ferrero


The product line graph $[= \Lambda_z(\Sigma)$ in M. Acharya (2009a)] is balanced. [Immediate from Harary’s (1953a) balance theorem or Sampathkumar’s (1972a), (1984a) similar theorem.] [Annot. 2008, 20 Dec 2010.] (SG: LG, Bal)

A. Fiat

See E. Demaine and D. Emanuel.

Miroslav Fiedler


If $V(\Sigma) = [n]$, the $k$-th additive compound graph $\Sigma[k]$ is defined for $k \in [n-1]$ via $A(\Sigma)$. It respects connection, also edge-disjoint union and induced subgraphs. Spec $\Sigma[k] =$ {sums of $k$ eigenvalues of $\Sigma$}. Spectral radius: $\rho(\Sigma) \leq \rho(\Sigma[k])$ (Thm. 2.17). For a path, $\rho(+P_n[k]) = -1 + \sin \frac{2k+1}{n+1} \pi / \sin \frac{1}{n+1} \pi$. $\Sigma[n-k]$ is $(-\Sigma)[k]$ where $\zeta$ switches odd-numbered vertices (Thm. 2.10, Rem. 2.11). $(+\Gamma)[k]$ may be unbalanced, e.g. (Thm. 2.15, proof) $(+C_n)[2]$ and $(+K_{1,3})[2]$; however, $(\tilde{C}_n)[2]$ is all-positive ($\tilde{C}_n$ = negative circle). Thm. 2.15: For connected $\Gamma$, $(+\Gamma)[2]$ is balanced iff $\Gamma = P_n$. [Annot. 26 Jul 2013.] (SG: LG(Gener): Adj, Sw)

Miroslav Fiedler & Vlastimil Ptak


B. Fierro, F. Bachmann, & E.E. Vogel

Physical parameters calculated on a signed square lattice ("Edwards–Anderson (1975a) model") with 1/32 of edges negative (p. 217). [Annot. 10 Jan 2015.]

**Tara Fife, Dillon Mayhew, James Oxley, & Charles Semple**

20xxa The unbreakable frame matroids. Submitted.

**Rosa M.V. Figueiredo**

See also N. Arıınık, M. Levorato, and I. Mendonça.

**Rosa Figueiredo & Yuri Frota**


Max order of balanced induced subgraph. [Annot. 30 Mar 2021.]

(SG: Clu: Alg)


(VS: Fr: Alg)

**Rosa Figueiredo, Yuri Frota, & Martine Labbé**


Preliminary version of (2019a).

(SG: Clu: Alg)


(SG: Clu: Alg)

**Rosa M.V. Figueiredo, Martine Labbé, & Cid C. de Souza**


**Rosa Figueiredo & Gisele Moura**


(SG: Clu: Alg, Appl)

**Joseph Fiksel**


**Miguel Angel Fiol**

See C. Dalf'o and E.R. van Dam.

**Samuel Fiorini**

See also N.E. Clarke.

**Samuel Fiorini, Nadia Hardy, Bruce Reed, & Adrian Vetta**

2005a Approximate min-max relations for odd cycles in planar graphs. In: M. Jünger and V. Kaibel, eds., *Integer Programming and Combinatorial Optimization*

See (2007a).


ν := maximum number of vertex-disjoint negative circles; ν′ := edge analog. ρ := minimum size of a transversal of negative face boundaries. Thm. 3 (Král and Voss (2004a)): frustration index l(Σ) ≤ 2ν′. (Here, a shorter proof.) Thm. 4: For an unbalanced signed plane graph, vertex frustration number l0(Σ) ≤ 7ν(Σ) + 3ρ(Σ) − 8. [Improved by Král’, Sereni, and Stacho (2012a).] Cor. 2: l(Σ) ≤ 10ν(Σ). Dictionary: “odd” = negative, “even” = positive. [Annot. 6 Feb 2011.] (SG: Fr, Circ)

Samuel Fiorini & Gwenaël Joret


E. Fischer, J.A. Makowsky, & E.V. Ravve

The incidence graph of clauses is a signed bipartite graph. [Annot. 16 Jan 2012.] (SG)

Ilse Fischer & C.H.C. Little

K.H. Fischer & J.A. Hertz

An excellent introduction to many aspects of physics (mainly theoretical) that often seem to be signed graph theory or to generalize it, e.g., by randomly weighting the edges. (Phys: sg: fr: Exp, Ref)

§2.5, “Frustration”, discusses the spin glass Ising model (essentially, signed graphs) in square and cubical lattices, including the “Matts model” (a switching of all positive signs), as well as a vector analog, the “XY” model (planar spins) and (p. 46) even a general gain-graph model with switching-invariant Hamiltonian. (Phys: SG: Fr, Sw: Exp, Ref)

Ch. 3 concerns the Ising and Potts models. In §3.7: “The Potts glass”, the Hamiltonian (without edge weights) is $H = -\frac{1}{2} \sum \sigma(e_{ij})(k\delta(s_i, s_j) - 1)$. [It is not clear that the authors intend to permit negative edges. If they are allowed, $H$ is rather like Doreian and Mrvar’s (1996a) $P(\pi)$. Question. Is there a worthwhile generalized signed and weighted Potts model with Hamiltonian that specializes both to this form of $H$ and to
\(P?\) [Also cf. Welsh (1993a) on the Ashkin–Teller–Potts model.]

(Phys: sg, clu: Exp)

Steven D. Fischer

§1.2: “Signed posets”. Definition of signed poset: a positively closed subset of the root system \(B_n\) whose intersection with its negative is empty. (Following Reiner (1990).) Equivalent to a partial ordering of \(\pm [n]\) in which negation is a self-duality and each dual pair of elements is comparable. [This is really a special type of signed poset. The latter restriction does not hold in general.]


[Partially summarized by Hanlon (1996a).]

(M: sg, ori, Geom, Invar)

M. Hamit Fişek, Robert Z. Norman, & Max Nelson-Kilger

The “strength” of a path depends on the number of edges of each sign.

[Annot. 10 Nov 2012.]

(PsS: SG)

P.C. Fishburn & N.J.A. Sloane

The maximum frustration index of a signed \(K_{t,t}\), which equals the covering radius of the Gale–Berlekamp code, is evaluated for \(t \leq 10\), thereby extending results of Brown and Spencer (1971a). See Table 1.

[Corrected and extended by Carlson and Stolarski (2004a).]

(Sg: Fr)

Susanna Fishel

Much geometry, no gain graphs [though a gain graph is implicit; cf. e.g. Zaslavsky (2003a)]. [Annot. 22 Jan 2020.]

(Geom: gg: Invar)

Michael E. Fisher & Rajiv R.P. Singh

Physics questions, e.g., phase transitions and high-temperature expansions, for signed lattice graphs (\(\pm J\) spins) and with random weights (Gaussian edge weights). [Annot. 24 Aug 2012.]

(Sg: Phys: Fr: Exp)

Claude Flament
Signed graphs are treated on pp. 126–129. *(SG: Bal, PsS: Exp)*


**Evelyne Flandrin**  
See R.J. Faudree.

**Erica Flapan**

Intrinsic chirality means the graph cannot be embedded in 3-space without a twist. *(Question. Can this be interpreted in terms of signed graphs?)* See also (1998a), Flapan and Weaver (1996a), Hu and Qiu (2009a). *(sg: Top: Chem)*


**Erica Flapan & Nikolai Weaver**


**T. Fleiner & G. Wiener**


**Herbert Fleischner**

Laura Florenc
See J. Spencer.

Rigoberto Flórez

Published as (2006a), (2009a), and Flórez and Forge (2007a), and in Flórez and Zaslavsky (2020a).


Lindström conjectured that a certain matroid $M(n)$ is algebraically nonrepresentable if $n$ is nonprime. Proved by showing that $M(n)$ extends by harmonic conjugation to $L_0(Z_nK_3)$, which in turn extends to a contradiction if $n$ is composite.


In a harmonic matroid $H$, harmonic conjugates exist and are unique. If $L_0(\mathfrak{S}K_3) \subseteq H$ and $\mathfrak{S} = \mathbb{Z}$ or $\mathbb{Z}_p$, then the closure of $L_0$ under harmonic conjugation is a projective plane over $\mathbb{Q}$ or $\mathbb{F}(p)$, as appropriate.

Rigoberto Flórez & David Forge

The minimal matroids are contained in lift matroids of $Z_nK_3$.

Rigoberto Flórez & Thomas Zaslavsky


Joel Foisy
See Y. Duong.

Wungkum Fong

A configuration consists of the vectors representing an acyclic orientation of a complete signed graph. The volume of the pyramid over the configuration with apex at the origin. [Ohsugi and Hibi (2003a) treats a similar problem. *Question*. Is there a connection with the chromatic polynomial?] [Annot. 11 Apr 2011.]

Carlos M. da Fonseca
See A. Alazemi, M. Andelić, and S.K. Simić.
Angela Fontan & Claudio Altafini

Loïc Forest, Nicolas Glade, & Jacques Demongeot

P. 101 and Fig. 5 describe the “regulon”, a signed digraph of order 2 with two stable states. [Annot. 23 Aug 2017.] (Biol: SD: Exp)

G. Forgacs
See also S.T. Chui and B.W. Southern.

Dictionary: “fully frustrated Ising model on a square lattice” = signed grid (square lattice) graph in which every quadrilateral is negative; “plaquette” = “square” = region boundary = quadrilateral. (Phys: sg)

G. Forgacs & E. Fradkin

David Forge
See also P. Berthomé, S. Corteel, and R. Flórez.

David Forge & Thomas Zaslavsky

The number of proper integral $m$-colorings of a rooted integral gain graph (root $v_0$ and a function $h: V \to \mathbb{Z}$ such that there are root edges $ge_{0i}$ for all $g \in (-\infty, h_i]$; the rest of the gain graph is finite).

(GG: Geom, Invar, M)


A weighted gain graph has lattice-ordered gain group and has vertex weights from an abelian semigroup acted upon by the gain group. The total dichromatic polynomial is a Tutte invariant (satisfying deletion-contraction and multiplicativity) with possibly uncountably many variables, but is not the universal one. Problem. Find the universal Tutte invariant. With integral gain group and integral weights, the integral chromatic function of (2007a) is an evaluation of the polynomial. Another special case is the polynomial of S.D. Noble and D.J.A. Welsh, A weighted graph polynomial from chromatic invariants of knots [Symposium à la Mémoire de François Jaeger (Grenoble, 1998). Ann. Inst.
Robin Forman

C.M. Fortuin & P.W. Kasteleyn

Most of the paper recasts classical physical and other models (percolation, ferromagnetic Ising, Potts, graph coloring, linear resistance) in a common form that is generalized in §7, ‘Random cluster model’. The “cluster (generating) polynomial” $Z(\Gamma; p, \kappa)$, where $p \in \mathbb{R}^E$ and $\kappa \in \mathbb{R}$, is a 1-variable specialization of the general parametrized dichromatic polynomial. In the notation of Zaslavsky (1992b) it equals $Q_\Gamma(q, p; \kappa, 1)$, where $q_e = 1 - p_e$. Thus it partially anticipates the general polynomials of Przytycka and Przytycki (1988a), Traldi (1989a), and Zaslavsky (1992b) that were based on Kauffman’s (1989a) sign-colored Tutte polynomial. A spanning-tree expansion is given only for the resistance model. A feature [that seems not to have been taken up by subsequent workers] is the differentiation relation (7.7) connecting $\partial \ln Z / \partial q_e$ with [I think!] the expectation that the endpoints of $e$ are disconnected in a subgraph. [Grimmett (1994a) summarizes subsequent work in the probabilistic direction.]

Florent Foucaud
See also L. Beaudou, R.C. Brewster, and F. Dross.

Florent Foucaud, Hervé Hocquard, Dimitri Lajou, Valia Mitsou, & Théo Pierron

Florent Foucaud & Reza Naserasr

Louiza Fouli & Susan Morey

J.-L. Fouquet
See C. Berge.

J.-C. Fournier

§3.12: “Matroïdes de Dowling” (p. 52). Definition by partial $\mathcal{G}$-
partitions and the linear representability theorem.  

Patrick W. Fowler
See also N. Basic.


§3, “Double covers and quotient surfaces”: A remarkable theorem: The spectrum of $A(\tilde{\Sigma})$ is $\text{Spec} A(\Sigma) \cup \text{Spec} A(|\Sigma|)$. Also, the eigenvectors of $A(\tilde{\Sigma})$ are $(x,x)$ and $(x,-x)$ for $x$ an eigenvector of $A(|\Sigma|)$ and of $A(\Sigma)$, respectively. A nice topological proof using orientation embedding of $\Sigma$ in a surface $S$ and the lift to an embedding of $\tilde{\Sigma}$ in the orientable double cover of $S$. [The graph theorem, not explicit, follows from the fact that every signed graph has an orientation embedding in some surface.] [Reproved independently, directly on the signed graph, by Bilu and Linial (2006a), Kalita and Pati (2012a), Gregory (2012a). Generalized to branched coverings in Butler (2010a).]

§4, “Chemical examples”: (i), “Monocyclic rings”: The eigenvalues and eigenvectors of a positive or negative circle, derived from the Frost–Musulin circle. [Reproved by Mathai and Zaslavsky (2012a) by a different method. Also proved by others.] An interesting discussion of the odd-order case. (ii), “In-plane π systems”, (iii), “Cyclic polyacenes”: Chemical applications. [Annot. 13 Jan 2015.]


Eduardo Fradkin
See G. Forgacs.

Eduardo Fradkin, B.A. Huberman, & Stephen H. Shenker

Properties of physical interest of switching classes (“gauge symmetric” properties) of signed graphs. [Annot. 11 Jan 2015.] (Phys, SG: Fr)

Aviezri S. Fraenkel & Peter L. Hammer

F.A.M. França
See S. Akbari.

Elisa Franco
See F. Blanchini.

András Frank
Pp. 89–91: Additively sign-weighted bipartite graphs. Thms. 8.1′, 8.5′: Criteria for negative circuit. [Questions. Is there a generalization to antibalanced signed graphs with additive sign-weights? Does the existence of minors help?] Thms. 8.1w, 8.1′w: Similar, for $\mathbb{Z}^+$-weighted or $\mathbb{Q}^+$-weighted graphs, not necessarily bipartite. Pp. 91–92 mention Gerards (1990a) and graphs with a bipartizing vertex. [Annot. 11 Jun 2012.]


A “conservative $\pm 1$-weighting” of $G$ is an edge labelling by $+1$’s and $-1$’s so that in every circle the sum of edge weights is nonnegative. It is a tool in several theorems. [Related: Ageev, Kostochka, and Szigeti (1995a), Sebő (1990a).]

Howard Frank & Ivan T. Frisch


Ove Frank & Frank Harary

An edge is present with probability $\alpha$ and positive with probability $p$. They compute the expected values of two kinds of measures of imbalance: the number of balanced triangles (whose variance is also given), and the number of induced subgraphs of order 3 having specified numbers of positive and negative edges. [Related: Škoviera (1992a), A.T. White (1994a).]

Giancarlo Franzese

The “fully frustrated” square lattice: alternate verticals are negative. Extending Kandel, Ben-Av, and Domany (1990a) by studying cluster properties in simulations, e.g., percolating clusters (that connect opposite sides of the lattice). Illuminating diagrams. [Annot. 18 Jun 2012.]

Maria Aguieiras A. de Freitas, Nair M.M. de Abreu, Renata R. Del-Vecchio, & Samuel Jurkiewicz

Maria Freitas, Renata Del-Vecchio, & Nair Abreu

The graph is $K_n \cup KK^j_n$ with $j$ additional edges. Spectral properties of $L(\Gamma)$. [Annot. 20 Jan 2015.]

Maria Aguieiras A. de Freitas, Renata R. Del-Vecchio, Nair M.M. de Abreu,
& Steve Kirkland

Maria Aguieiras A. de Freitas, Vladimir Nikiforov, & Laura Patuzzi

Thm. 1.1.: For $n \geq 4$, $C_4 \not\subseteq \Gamma \implies \lambda_1(L(-\Gamma)) \leq \lambda_1(F_n)$; $= \implies \Gamma = F_n$ ($F_n$ is the windmill of $(n-1)/2$ triangles for odd $n$ and is $F_{n-1}$ with a pendant edge on the center for even $n$). Thm. 1.3: For $n \geq 6$, $C_5 \not\subseteq \Gamma \implies \lambda_1(L(-\Gamma)) \leq \lambda_1(L(-K_2 \lor \bar{K}_{n-2}))$; $= \implies \Gamma = K_2 \lor \bar{K}_{n-2}$.

[Annot. 20 Jan 2015.] (par: Lap: Eig)

Christian Fremuth-Paeger & Dieter Jungnickel


Ivan T. Frisch
See H. Frank.
Tobias Fritz

A periodic graph is the (infinite) \( \mathbb{Z}^d \)-covering graph of a (finite) \( \mathbb{Z}^d \)-gain graph. (GG: Cov)

Yuri Frota
See R.M.V. Figueiredo and M. Levorato.

Josh B. Frye
See A.A. Diwan.

Toshio Fujisawa

Satoru Fujishige
See K. Ando.

See also Chiba.

Shinya Fujita & Ken-Ichi Kawarabayashi

If \( \Gamma \) is \( k \)-connected \( (k \geq 5) \), \( -\Gamma \) has a positive circle \( C \) such that \( \Gamma \setminus V(C) \) is \( (k - 4) \)-connected. If \( \Gamma \) has no triangles, we can say \( (k - 3) \)-connected. (Thomassen (2001a) conjectured the odd-circle analog.)

[Problem. Generalize to signed graphs such that \(|\Sigma| \) is \( k \)-connected.] [Annot. 26 Dec 2012.] (sg: Par: Circ)

Shinya Fujita & Colton Magnant


D.R. Fulkerson, A.J. Hoffman, & M.H. McAndrew

The “odd-cycle condition” is that any two odd circles without a common vertex are joined by an edge. Assuming it, certain conditions are necessary and sufficient for a degree sequence to be realized by a sub-multigraph of \( K_n \) with prescribed multiplicities. The incidence matrix of \( -K_n \) is employed in the geometrical proof. [Problem. Generalize to signed graphs.] [Annot. 30 May 2011.] (sg: Par: incid)

Edgar Fuller
See X.Q. Qi.

Atsushi Funato, Nan Li, & Akihiro Shikama

Daryl Funk
See also N. Bowler, R. Chen, and M. Devo.


Daryl Funk & Dillon Mayhew

Daryl Funk, Dillon Mayhew, & Mike Newman

Daryl Funk & Daniel Slilaty

Martin J. Funk
See M. Abreu.

H.N. Gabow

Nir Gadish
See C. Bibby.

Stephen M. Gagola

Giovanni Gaiffi

Anahí Gajardo
See M. Montalva.

David Gale
See also A.J. Hoffman.

David Gale & A.J. Hoffman

Joseph A. Gallian


§3.7, “Cordial labelings”; §3.8, “The friendly index–balance index”. From $f : V \rightarrow \mathbb{Z}_2$ obtain balanced edge gains $f^*(uv) = f(u) + f(v)$. $f$ is “friendly” if it has essentially equal numbers of each label, i.e., equal or differing by 1. $f$ is “cordial” if $f$ and $f^*$ have essentially equal numbers of each label. [Cf. Babujee and Loganathan (2011a).] A great many references. [(Γ, f) is like a balanced multiply signed graph but the questions are not gain-graphic.] [Annot. 9 Oct 2010.]

Vertex switching (switching one vertex) of some standard graph gives examples of many kinds of labellings. [Annot. 2 Jan 2015.]

Jean Gallier


§5, “Signed graphs”, uses Laplacian $L(Σ, w) := \text{diag}(d) - A(Σ, w)$, $w = \text{positive weight function}$, obtained from incidence matrix based on $\sqrt{w_{ij}}$ (= standard incidence matrix if $|w_{ij}| = 1$). Dictionary: “signed degree” := unsigned degree $d_i := \sum_j w_{ij}$. Cf. alternatives in Kunegis, Schmidt, *et al.* (2010a), Knyazev (2018a). [Annot. 8 Feb 2021.]

Anna Galluccio, Martin Loebl, & Jan Vondrák

See also J. Lukic.


Describes (2001a), emphasizing signed toroidal lattice graphs, i.e., toroidal lattice Ising models. [Annot. 18 Aug 2012.]


An algorithm for the generating function of weighted cuts (= partition function of Ising model), hence for $\sum_\zeta x^{\l(\zeta)}$ and frustration index $l(Σ)$, in polynomial time for graphs of bounded genus. [Annot. 18 Aug 2012.]

Huiling Gan and Ellis L. Johnson


Alberto Gandolfi

See also E. De Santis.
A. Gandolfi, C.M. Newman, & D.L. Stein

Robert Ganian, Petr Hliněný, & Jan Obdržálek

Hilal A. Ganie & Bilal A. Chat

Gao Hongzhu
See Cheng Z.Y.

Hua Gao, Zhijian Ji, & Ting Hou

Ruimei Gao & Ying Chu


Yu-Bin Gao
See also Y.Z. Fan, L.F. Huo, and Y.L. Shao.

Yubin Gao, Yihua Huang & Yanling Shao

Yubin Gao, Yanling Shao, & Jian Shen

Marianne L. Gardner [Marianne Lepp]
See R. Shull.

T. Garel & J.M. Maillard

Physics approach. Generalizes Southern, Chui, and Forgacs (1980a)’s square-lattice Ising model to four edge weights, symmetrically located, and reduces it to an all-positive graph with two weights. §3, “Application
to the Villain model”: All weights equal [hence a signed graph]; further results on Villain (1977a). [Annot. 16 Jun 2012.] (Phys: sg: wg)

Pravin Garg
See also D. Sinha.

Vikas K. Garg
See P. Agrawal.

Michael Gargano & Louis V. Quintas

Characterizes balance in abelian gain graphs. [See Harary, Lindström, and Zetterström (1982a).] Very simple results on existence, for a given graph, of balanced nowhere-zero gains from a given abelian group. [Elementary, if one notes that such gains exist iff the graph is $\#G$-colorable, $G$ being the gain group]. Comparison with the approach of Sampathkumar and Bhave (1973a). Dictionary: “Symmetric $G$-weighted digraph” = gain graph with gains in the (abelian) group $G$. “Weight” = gain. “Non-trivial” (of the gain function) = nowhere zero. (GG: Bal)

Michael L. Gargano, John W. Kennedy, & Louis V. Quintas

An abelian gain graph $\Phi$ is cobalanced (here called “cut-balanced”) if the sum of gains on the edges of each coherently oriented cutset is 0. [This generalizes Kabell (1985a).] Given $\Phi$ with $\|\Phi\|$ embedded in a surface, the surface dual graph is given gains by a right-rotation rule, thus forming a surface dual $\Phi^*$ of $\Phi$. [This appears to require that the surface be orientable. Note that cobalance generalizes to nonabelian gains on orientably embedded graphs, since the order of multiplication for the gain product on a cutset is given by the embedding.] Thm. 3.2: For a plane embedding of $\Phi$, $\Phi$ is cobalanced iff $\Phi^*$ is balanced. Thm. 3.4 restates as criteria for cobalance of $\Phi$ the standard criteria for balance of $\Phi^*$, as in Gargano and Quintas (1985a). More interesting are “well-balanced” graphs, which are both balanced and cobalanced. Problem. Characterize them. Dictionary (also see Gargano and Quintas (1985a)): Balance is called “cycle balance”. (GG: Bal(D))

Marcin Gąsiorek

Marcin Gąsiorek, Daniel Simson & Katarzyna Zajac


Gilles Gastou & Ellis L. Johnson

§10 introduces the co-postman and “odd circuit” problems, treated more thoroughly in Johnson and Mosterts (1987a) (q.v.). “Odd” edges and circuits are precisely negative edges and circles in an edge signing. The “odd circuit matrix” represents $L(\Sigma)$ (p. 30). The “odd circuit problem” is to find a shortest negative circle; a simple algorithm uses the signed covering graph (pp. 30–31). The “Fulkerson property” may be related to planarity and $K_5$ minors [which suggests comparison with Barahona (1990a), §5]. (SG: Fr(Gen), Circ, Incid, M(Bases), cov, Alg)

Heather Gavlas [Heather Jordon]

Premiysław Gawroński
See also F. Hassaniibesheli and K. Kułakowski.

P. Gawroński, P. Gronek, & K. Kułakowski


Przemysław Gawroński, Małgorzata J. Krawczyk, & Krzysztof Kułakowski


P. Gawroński & K. Kułakowski
See Gawroński, Krawczyk, and Kułakowski (2015a) [Annot. 21 Jan 2016.]


Jim [James F.] Geelen

See also M. Chudnovsky.


§6, “The basic classes”: Frame matroids $G(\Omega^*)$ of full biased graphs are called “framed Dowling matroids” and the submatroids $G(\Omega)$ without the extra unbalanced edges are called “Dowling matroids” [despite these generalizations of Dowling geometries $Q_n(\mathcal{G})$ having been introduced in Zaslavsky (1977a), (1987a), (1991a)]. Bicircular matroids $G(\Gamma, \emptyset)$ are mentioned as examples and are important in Conj. 6.1 due to Johnson, Robertson, and Seymour. Thm. 6.2 has as one important class $G(K_k, \emptyset)$. There follows a nice list of four basic questions about frame matroids:

Find the excluded minors. Can they be recognized in polynomial time?

Find the excluded minors for the frame matroids that have canonical representations (in the sense of Zaslavsky (2003b)) or frame representations (in the sense of papers of Geelen, Gerards, & Whittle) over $\mathbb{R}$ [the projective representation Thm. 7.1 of Zaslavsky (2003b) might be relevant?]. Can those matroids be recognized in polynomial time? [Annot. 26 Jan 2015.]

In Conj. 8.2 about the fewest flats that cover a matroid, one special class is the bicircular matroids. [Annot. 26 Jan 2015.]

Spikes $G(2C_n, \mathcal{B})$ are important (p. 630). [Annot. 29 Apr 2012.]


The lift matroid. (SG: M: Str)


Given finite, abelian $\mathfrak{G}$ and $\mathfrak{G}' \leq \mathfrak{G}$, and a $\mathfrak{G}$-gain graph $\Phi$ with a minor $\Psi \cong \mathfrak{G}' K_t$ where $t = 8n \# \mathfrak{G}^2$. Thm. 1.3: Either $\exists X \subseteq V$ with $\# X < t$ such that in $\Phi \setminus X$ the block containing most of $\Psi$ is $\mathfrak{G}'$-balanced, or $\Psi$ has a minor $\cong \mathfrak{G}'' K_n$ where $\mathfrak{G}' < \mathfrak{G}'' \leq \mathfrak{G}$. $t$ may not be best possible. Thm. 1.4: $\forall n, m$ such that $\| \Phi \|$ has a $K_{l(n)}$ minor $\implies \Phi$ has a $0K_n$ minor. Dictionary: “Group-labelled graph” = gain graph; $\Gamma$ means
\[ \mathfrak{G}; G \text{ means } \Phi; \bar{G} \text{ means } \|\Phi\|; \text{“shifting” means “switching”; } \mathfrak{G}'\text{-balanced means switchable so all gains are in } \mathfrak{G}'\].

(JG: Str)

**Jim Geelen, Bert Gerards, Bruce Reed, Paul Seymour, & Adrian Vetta**


(ng: par: Str)

**Jim Geelen, Bert Gerards, & Geoff Whittle**


Vector representation of spikes \( L(2C_n, \mathcal{B}) \) and tipped spikes \( L_0(2C_n, \mathcal{B}) \).

Thm.: Any representation of \( L(2C_n, \mathcal{B}) \) extends to one of \( L_0(2C_n, \mathcal{B}) \), if \( n \geq 4 \). [Greatly generalized in Zaslavsky (2003b), Thm. 7.1.] Thm.: All representations of \( L_0(2C_n, \mathcal{B}) \) are of a specific form, up to projective transformations. [Annot. 18 Apr 2013.]

(gg: M, Incid)


The full bicircular matroid \( G(\Gamma^*, \emptyset) \) appears on p. 589. (gg: bic)


See (2007a).

(gg: M: Exp)


Conjecture. A minor-closed proper subclass of all GF \( q \)-representable matroids is essentially constructible from frame matroids and their duals. Dictionary: “Dowling matroid” = simple frame matroid, i.e., submatroid of Dowling’s (1973a), (1973b) matroids \( G(K_n^*) \), for \( \mathcal{D} = \mathbb{F}_q^* \). [Annot. 25 May 2009.]

(gg: M: Exp)


(GG: M: Exp)


Perturbations of representations of frame matroids are important. But see Grace and van Zwam (2018a) for counterexamples. [Annot. 27 Feb 2017, rev 15 Jul 2019.]

(gg: M: Incid)(sg: M)


(GG: M)

2018b Retraction [of previous on-line version], *ibid.* 87 (2018), no. 2, 265.
The retraction is of a previous version. (GG: M)

James F. Geelen & Bertrand Guenin

Adds to Guenin’s theorem (2001a): Thm.: If \( \Sigma \) has no \(-K_5\) minor, then the dual linear program has a half-integral minimum (assuming \( f \) has nonnegative coefficients). Dictionary: “odd” = negative; the “Eulerian graph” is signed. (SG, D: Incid, Geom, Str)

Jim Geelen & Tony Huynh

Jim Geelen & Kasper Kabell

Bicircular matroid is an example of the property. §6.2, “Cleaning a nest”. Lemma 6.1: A sufficiently large “nest” without a large uniform minor has a frame matroid \( G(K_n^*, B) \) as a minor. §7, “Cliques”: A sufficiently large \( G(K_n^*, B) \) has \( G(K_n) \) or \( G(K_n, \emptyset) \) as a minor. Dictionary: “Dowling clique” = full frame matroid of biased \( K_n \), “Dowling representation” = the biased \( K_n \). [Annot. 12 Jul 2016.] (gg: M, Bic)

James Geelen, James Oxley, Dirk Vertigan, & Geoff Whittle

A rank-\( r \) swirl is \( G(2C_r, \emptyset) \). Free spikes and rank-\( r \) swirls, also the latter with one unbalanced loop, are important. Conjecture: The 3-connected, rank-\( k \) matroids, representable over GF(\( q \)) and having no \( L(2C_k, \emptyset) \) or \( G(2C_k, \emptyset) \) minor, have a bounded number of inequivalent GF(\( q \))-representations. [Annot. 25 May 2009, 29 Apr 2012.] (gg: M: Incid)


The “free swirl” \( \Delta_r \) is \( G(2C_k, \emptyset) \). The “free spike” \( \Lambda_r \) is \( L(2C_k, \emptyset) \). They play a main role re large totally free matroids (Thm. 3.4). [Annot. 8 Mar 2011.] (gg: M)

Jim Geelen & Cynthia Rodriguez

M.C. Geetha
See P.S.K. Reddy.

Laura Gellert & Raman Sanyal

Xianya Geng, Shuchao Li, & Slobodan K. Simić

§2 mentions \(L(−\Gamma)\). Quasi-\(k\)-cyclic means \(\exists v\) such that \(\Gamma \setminus v\) has cyclomatic number \(k\). For \(k \leq 2\), Thm. 3.2 describes all \(\Gamma\) maximizing the largest eigenvalue of \(L(−\Gamma)\). [Annot. 21 Jan 2012.] (par: Lap: Eig)

Claudio Gentile
See N. Cesa-Bianchi.

A.M.H. [Bert] Gerards
See also M. Chudnovsky, M. Conforti, and J. Geelen.


If an antibalanced, unbalanced signed graph has no homomorphism into its shortest negative circle, then it contains a subdivision of \(-K_4\) or of a loose \(\pm C_5\) (here called an “odd \(K_4\)” and an “odd \(K_5\)”). (A loose \(\pm C_n\) consists of \(n\) negative digons in circular order, each adjacent pair joined either at a common vertex or by a link.) [Question. Do the theorem and proof carry over to any unbalanced signed graph?] Other results about antibalanced signed graphs are corollaries. Several interesting results about signed graphs are lemmas. (Par, SG: Hom)


Let \(\Sigma\) be antibalanced and without isolated vertices and contain no subdivision of \(-K_4\). Then max. stable set size = min. cost of a cover by edges and negative circles. Also, min. vertex-cover size = max. profit of a packing of edges and negative circles. Also, weighted analogs. [Question. Do the theorem and proof extend to any \(\Sigma\)?] (par: sg: Circ)


The proof of Lemma 3 uses a signed graph. (SG: Bal)


[Very incomplete annotation.] Thm.: Given \(\Sigma\), the set \(\{x \in \mathbb{R}^n : d_1 \leq x \leq d_2, b_1 \leq H(\Sigma)^T x \leq b_2\}\) has Chvatal rank \(\leq 1\) for all integral vectors \(d_1, d_2, b_1, b_2\), iff \(\Sigma\) contains no subdivided \(-K_4\). (SG: Incid, Geom, Bal, Str)


Signed graphs used to prove Tutte’s theorem. The signed-graph matroid employed is the extended lift matroid $L_0(\Sigma)$ (“extended even cycle matroid”). The main theorem (Thm. 2): Let $\Sigma$ be a signed graph with no $-K_4$, $\pm K_3$, $-Pr_3$, or $\Sigma_4$ link minor; then $\Sigma$ can be converted by Whitney 2-isomorphism operations (“breaking” = splitting a component in two at a cut vertex, “glueing” = reverse, “switching” = twisting across a vertex 2-separation) to a signed graph that has a balancing vertex (“blocknode”). Here $\Sigma_4$ consists of $+K_4$ with a 2-edge matching doubled by negative edges and one other edge made negative.

More translation: His “$\Sigma$” is our $E^-$. “Even, odd” = positive, negative (for edges and circles). “Bipartite” = balanced; “almost bipartite” = has a balancing vertex. (SG: M, Str, Incid)


A.M.H. Gerards & M. Laurent


Thm. 5.1: The collection of negative circles of $\Sigma$ is box $\frac{1}{d}$-integral for some/any integer $d \geq 2$ iff it does not contain $-K_4$ as a link minor. (SG: Circ, Geom)

A.M.H. Gerards, L. Lovász, A. Schrijver, P.D. Seymour, C.-S. Shi, & K. Truemper

1990a Manuscript to be prepared as of 1990.

Extension of Gerards and Schrijver (1986b) [same comments apply. The proliferating authorship is preventing this major contribution from ever being published as such—though one hopes not! See Seymour (1995a) for description of two main theorems]. (SG: Str, M, Top)

A.M.H. Gerards & A. Schrijver

https://research.tilburguniversity.edu/en/publications/signed-graphs-regular-matroids-grafts

Essential, major theorems. The (extended) lift matroid of a signed graph is one of the objects studied. Some of this material is published in Gerards (1990a). This paper is in the process of becoming Gerards, Lovász, et al. (1990a) [was in process; that is unlikely to be written.]
A subsidiary result: If $-\Gamma$ contains no subdivided $-K_4$, then $\Gamma$ is $t$-perfect. (sg: Par: Geom, Str)

A.M.H. Gerards & F.B. Shepherd


1998b The graphs with all subgraphs $t$-perfect. *SIAM J. Discrete Math.* 11 (1998), 524–545. MR 1640924 (2000e:05074). Zbl 980.38493. Extension of Gerards (1989a). An “odd-$K_4$” is a graph whose all-negative signing is a subdivided $-K_4$. A “bad-$K_4$” is an odd-$K_4$ which does not consist of exactly two undivided $K_4$ edges that are nonadjacent while the other edges are replaced by even paths. Thm. 1: A graph that contains no bad-$K_4$ as a subgraph is $t$-perfect. Thm. 2 characterizes the graphs that are subdivisions of 3-connected graphs and contain an odd-$K_4$ but no bad-$K_4$. [The fact that ‘badness’ is not strictly a parity property weighs against the possibility that Gerards (1989a) extends well to signed graphs.] (par, sg: Str, Alg)

K.A. Germina


K.A. Germina & P.K. Ashraf


K.A. Germina & Shahul Hameed K (as K. Shahul Hameed)


Eigenvalues and energies of $A(\Sigma)$ and Laplacian matrices $L(\Sigma)$ of signed paths and circles; also recurrences for the characteristic polynomials. Energy of $A := \sum |\lambda_i(A)|$; energy of $L := \sum |\lambda_i(L) - \bar{d}|$ where $\bar{d} :=$ average degree. [Cf. Mathai and Zaslavsky (2012a).] [Annot. 14 Nov 2010.] (SG: Eig: Paths, Circ)

K.A. Germina, Shahul Hameed K, & Thomas Zaslavsky


Adjacency matrix $A$ and eigenvalues and energy for the general “Cvetko- vić product”, NEPS$(\Sigma_1, \ldots, \Sigma_k; B)$, and for a line graph $A(\Sigma)$ (as in Zaslavsky (2010b), (2012c), (20xxa)). Laplacian matrix $L(\Sigma)$; $L(+\Gamma) =$ Laplacian of graph $\Gamma$; $L(-\Gamma) =$ signless Laplacian; and its eigenvalues and energy for Cartesian product $\Sigma_1 \times \cdots \times \Sigma_r$. Also, $A(\Lambda(\Sigma))$. Thm.: The Cartesian product is balanced iff all $\Sigma_i$ are balanced. Examples: Planar, cylindrical, and toroidal grids with product signatures; line graphs of those grids and of $+K_n$ and $-K_n$. [Annot. 19 Oct 2010.]
K.A. Germina & Sahariya

Mehrdad Ghadiri
See S. Akbari.

A. Ghafari
See S. Akbari.

Joobin Gharibshah
See M. Shahriari.

E. Ghasemian
See also S. Akbari.

E. Ghasemian & G.H. Fath-Tabar
Classifies 3- and 4-regular, triangle-free $\Sigma$ with two eigenvalues. Cor. 2.5 says for 3-regular, the unique example is the cube $Q_3$ with all $C_4$’s negative. Thm. 2.7 states a 4-regular characterization [incomplete; cf. Hou, Tang, and Wang (2019a). Succeeded by Stanić (2020b).] [Annot. 29 May 2018.]

Anna Maria Ghirlanda
See L. Muracchini.

Ebrahim Ghorbani
See also R.B. Bapat and S. Akbari.

Ebrahim Ghorbani, Willem H. Haemers, Hamid Reza Maimani, & Leila Parsaei Majd
“Sign-symmetric” means $\Sigma \cong -\Sigma$. Nonbipartite examples are found. [Annot. 23 Nov 2020.]

Modjtaba Ghorbani
See also M. Hakimi-Nezhada.

Modjtaba Ghorbani, Mardjan Hakimi-Nezhada, & Bo Zhou

“Seidel matrix $S(\Gamma) := A(K_\Gamma)$. When does it have eigenvalue with multiplicity $\geq n - d$, $d \leq 3$? Partial solution. [Annot. 9 Nov 2020.] (sg: KG: sw, Adj: Eig)

Vahid Ghorbani, Ghodratollah Azadi, & Habib Azanchiler
Spike = extended lift matroid $L_0(2C_r, \mathcal{B})$ where $\mathcal{B} \subseteq \{C_r \subset 2C_r\}$. Construction of spikes by matroid operations from binary spikes (in which $\mathcal{B}$ is maximum). Question. Are these biased-graph operations? (gg: M)

Prantar Ghosh
See S. Das.

A. Ghouila-Houri
See C. Berge.

H. Giacomini
See H.T. Diep.

Christos Giatsidis
See also F.D. Malliaros.

Christos Giatsidis, Bogdan Cautis, Silviu Maniu, Dimitrios M. Thilikos, & Michalis Vazirgiannis

Rick Giles

In the author’s “mixed” graphs, the undirected edges are really extraverted bidirected edges. (sg: ori)

Mukhtiar Kaur Gill [Mukti Acharya]
See also B.D. Acharya and Mukti Acharya.


Introduces “quasicospectrality” of graphs or digraphs, i.e., they have cospectral signatures. See B.D. Acharya, Gill, and Pathwardhan (1984a) and M. Acharya (2012a). [Annot. 3 Feb 2012.] (SG, SD: Eig)


Assume $|\Sigma_1| = |\Sigma_2|$. If $\Sigma_1$ and $\Sigma_2$ have the same value of B.D. Acharya’s (1980a) measure of imbalance, $A(\Sigma_1)$ and $A(\Sigma_2)$ may have different spectra. [Not surprisingly.] (SG: Bal, Eig)


M.K. Gill & B.D. Acharya


M.K. Gill & G.A. Patwardhan

The line graph is that of Behzad–Chartrand (1969a). (SG: LG)


The line graph is that of Behzad–Chartrand (1969a). (SG: LG)


The \( k \)-path signed graph of \( \Sigma \) \( [I \text{ write } D_k(\Sigma)] \) is the distance-\( k \) graph on \( V \) with signs \( \sigma_k(\text{uv}) = -1 \) iff every length-\( k \) path is all negative. The equation \( \Sigma \cong D_2(\Sigma) \) is solved. [Annot. 29 Apr 2009.] (SG, Sw)

Robert Gill

(gg: m: Geom, Invar, Aut)


The semilattice is the intersection semilattice of an affinographic hyperplane arrangement representing \([-k, k]K_n\] and is therefore isomorphic to the geometric semilattice of all \( k \)-composed partitions of a set; see, e.g., Zaslavsky (2002a), Ex. 10.5]. The rank and the Whitney numbers of the first kind are calculated. See Kerr (1999a) for homology. (gg: m: Geom, Invar)


Ernst D. Gilles
See S. Klamt.

John Gimbel

The topic is “induced” edge labellings, that is, \( w(e_{uv}) = f(u)f(v) \) for some \( f : V \to \mathbb{A} \). The number of \( f \) that induce a given induced labelling, the number of induced labellings, and a characterization of
induced labellings. All involve the 2-torsion subgroup of \( \mathfrak{A} \), unless \( \Gamma \) is bipartite. The inspiration is dualizing magic graphs. [Somewhat dual to Edelman and Saks (1979a).]

(par: incid)(VS(Gen): Enum)

**Omer Giménez, Anna de Mier, & Marc Noy**


The number of bases is bounded above by \( C^n \cdot \text{(number of spanning trees)} \) in a simple graph but not in a multigraph. More precise results for \( K_n \) and \( K_{n,m} \). [See Neudauer, Meyers, and Stevens (2001a) and Neudauer and Stevens (2001a).]

(Bic: Incid)

**Omer Giménez & Marc Noy**


Known NP-hardness results for transversal matroids apply to their proper subclass, bicircular matroids, with a few possible exceptions.

(Bic: Incid: Alg)

**Giulia Giordano**

See F. Blanchini.

**Ioannis Giotis & Venkatesan Guruswami**


(SG: WG: Clu: Alg)


**Noriane Girard**

See E. Delucchi.

**Pierre-Louis Giscard, Paul Rochet, & Richard C. Wilson**


(SG: Fr, PsS)

**Nicolas Glade**

See L. Forest.

**Roland Glantz & Marcello Pelillo**


The polynomials arise from \( A(\Phi) \) where \( \Phi \) is an \( F^+ \)-gain graph, \( F \) a field.

(GG: Invar, Adj)

**Terry C. Gleason**

See also D. Cartwright.

**Terry C. Gleason & Dorwin Cartwright**

"Colorable" = clusterable. The adjacency matrices of $\Sigma^+$ and $\Sigma^-$ are employed separately. The arithmetic is mostly "Boolean", i.e., $1 + 1 = 0$.

A certain integral matrix $T$ shows whether or not $\Sigma$ is clusterable. [Annot. 11 Nov 2008.] (SG: Clu, Adj)

Fred Glover

See also J. Elam.

F. Glover, J. Hultz, D. Klingman, & J. Stutz


Fred Glover & D. Klingman


Fred Glover, Darwin Klingman, & Nancy V. Phillips


Textbook. See especially Ch. 5: “Generalized networks.” (GN: Alg: Exp)

F. Glover, D. Klingman, & J. Stutz


Luis Goddyn

See also M. Chudnovsky.

Luis Goddyn, Winfried Hochstättler, & Nancy Ann Neudauer


C.D. Godsil

See also J. Brown and G. Coutinho.


If $T$ is a tree with a perfect matching, then $A(T)^{-1} = A(\Sigma)$ where $\Sigma$ is balanced and $|\Sigma| \supseteq \Gamma$. *Question.* When does $|\Sigma| = \Gamma$? [Solved by Simion and Cao (1989a).] [Cf. Buckley, Doty, and Harary (1988a), Tifenbach & Kirkland (2009a), and for a generalization to rings Bapat and Ghorbani (2014a). For a different notion, Greenberg, Lundgren, and Maybee (1984b).] [Annot. < 1988 et seq.]

*Problem.* A bipartite graph with a unique perfect matching has an inverse signed multigraph: $A(\Gamma)^{-1} = A(\Sigma)$. When is $\Sigma$ balanced (i.e., switches to an unsigned graph)? Elegant solution by Yang and Ye (2018a). [Annot. 11 Dec 2018.] (sg: Adj, Bal, sw)

C.D. Godsil & I. Gutman


Chris Godsil & Gordon Royle


Ch. 11, “Two-graphs”: Equiangular lines (van Lint and Seidel (1966a), Lemmens and Seidel (1973a)), graph switching (van Lint and Seidel (1966a), Seidel (1976a)), regular two-graphs (Taylor (1977a)).

(TG: Adj, Eig, Geom, Sw)


(LG: sg: Eig, Geom, Sw)


(Sc, SGc: Adj, Incid, Top)

J.M. Goethals

See also P.J. Cameron.

J.M. Goethals & J.J. Seidel


A symmetric Hadamard matrix $H$ with constant diagonal can be put in the form $A(K_n, \sigma) \pm I$ for some signed $K_n$ that represents a regular two-graph [see D.E. Taylor (1977a)] of order $4s^2$ (Thm. 4.1). (tg: Adj)

Michael Goff


Andrew V. Goldberg & Alexander V. Karzanov


(sd: Flows, Cov)


Techniques for digraph flows are extended to bidirected flows, treated via the double covering digraph (cf. Tutte (1967a)). [Annot. 9 Sept 2010.] (sg: Ori: Flows, Cov)

**Felix Goldberg & Steve Kirkland**


The sign pattern of an eigenvector of the smallest eigenvalue of $L(-\Gamma)$ for a bipartite graph + some edges may be predictable. [Problem. Generalize to signed graphs.] [Annot. 23 Nov 2014.] (sg: par: Eig)

**Andrew V. Goldberg, Éva Tardos, & Robert E. Tarjan**


§1.5, “The generalized flow problem”: Max flow, conservative except at the source, in networks with (real, positive) gains; generalized augmenting paths. §1.6, “The restricted problem”: Flows with gains, conservative except at source and sink, whose residual flow has no gainy cycles that avoid the source. §1.7, “Decomposition theorems” for flows with or without gains. §6, “The generalized flow problem”: Combinatorial algorithms; connections between flow problems with and without gains [Annot. 11 Jun 2012.] (GN: Alg)

**Jay R. Goldman & Louis H. Kauffman**


The parametrized Tutte polynomial [as in Zaslavsky (1992b) et al.] of an $\mathbb{R}^\times$-weighted graph is used to define a two-terminal “conductance”. Interpreting weights as crossing signs (+1) in a planar link diagram with two blocked regions yields invariants of tunnel links. [Also see Kauffman (1997a).] (SGw: Gen: Invar, Knot, Phys)

**Avraham Goldstein**

See Y. Cherniavsky.

**Richard Z. Goldstein & Edward C. Turner**


**Eric Goles**

See J. Aracena.

**Harry F. Gollub**

**Martin Charles Golumbic**


Further results on chordal bipartite graphs. Their properties imply standard properties of ordinary chordal graphs. [See (1980a) for more.] (The “only if” portion of Thm. 4 is false, according to (1980a), p. 267.)


§12.3: “Perfect elimination bipartite graphs,” and §12.4: “Chordal bipartite graphs,” expound perfect elimination and chordality for bipartite graphs from Golumbic and Goss (1978a) and Golumbic (1979a). In particular, Cor. 12.11: A bipartite graph is chordal bipartite if every induced subgraph has perfect edge elimination scheme. [Problem. Guided by these results, find a signed-graph generalization of chordality that corresponds to supersolvability and perfect vertex elimination (cf. Zaslavsky (2001a)).] (sg: bal, cov)

**Martin Charles Golumbic & Clinton F. Goss**


A perfect edge elimination scheme is a bipartite analog of a perfect vertex elimination scheme. A chordal bipartite graph is a bipartite graph in which every circle longer than 4 edges has a chord. Analogs of properties of chordal graphs, e.g., Dirac’s separator theorem, are proved. In particular, a chordal bipartite graph has a perfect edge elimination scheme. [See Golumbic (1980a) for more.] (sg: bal)

**Sergio Gómez, Pablo Jensen, & Alex Arenas**


*Cf. Bansal, Blum, and Chawla (2004a).* (SG, WG: Clu)

**J.R. Gonçalves**

See also J.A. Blackman.

**J.R. Gonçalves, J. Poulter, & J.A. Blackman**


Signed square and triangular lattice graphs with all face circles (“plaquettes”) negative (“frustrated”). Entropy change due to changing an edge sign (creating a “bond defect”), or one or two vertex deletions (“site defects”). Eigenstates correspond to negative plaquettes, as in Blackman

(Phys, SG: Fr, Eig)

Maoguo Gong
See Q. Cai and J.S. Wu.

Shi-Cai Gong
See also Y. Wang.

2011a The unicyclic graphs with extremal signless Laplacian spectral spread. In: Proceedings of 2011 World Congress on Engineering and Technology (CET 2011), Vol. 1, pp. ?. [This may not have been published.] (par: Lap: Eig)

Shicai Gong, Hangen Duan, & Yizheng Fan

The “mixed graphs” are signed graphs. “[R]elations between the eigenvalues and matching number, diameter, and the number of quasi-pendant vertices of mixed graphs.” (From the abstract.) [Annot. 9 Jan 2013.]

Shi-Cai Gong & Yi-Zheng Fan

Characterizes such signed (“mixed”) graphs, for \( L(\Sigma) \), for \( n \geq 9 \). [Annot. 23 Mar 2009.]

Shi-Cai Gong & Guang-Hui Xu

A “weighted oriented graph” is an \( \mathbb{R}^+ \)-gain graph. The “skew adjacency matrix” is the gain-graphic adjacency matrix. [Annot. 7 Feb 2012.]

Mauricio González
See J. Aracena.

Andrew Goodall, Bart Litjens, Guus Regts, & Lluís Vena

Extended abstract of (2020a).


Early, shorter version of (2021a).


(SG: Top, Invar, M)

Gary Gordon

See also L. Fern.


An explicit bijection between the regions of the real hyperplane arrangement corresponding to $\pm K_{n}^\circ$ and the set of “good signed [complete] mixed graphs” $G_a$ of order $n$. The latter are a notational variant of the acyclic orientations $\tau$ of $\pm K_{n}^\circ$ [and therefore in bijective correspondence with the regions, by Zaslavsky (1991b), Thm. 4.4]. Dictionary: a directed edge in $G_a$ is an oriented positive edge in $\tau$, while a positive or negative undirected edge in $G_a$ is an introverted or extroverted negative edge of $\tau$. The main result, Thm. 1, is an interesting and significant explicit description of the acyclic orientations of $\pm K_{n}^\circ$. Namely, one orders the vertices and directs all positive edges upward; then one steps inward randomly from both ends of the ordered vertex set, one vertex at a time, at each new vertex orienting all previously unoriented negative edges to be introverted if the vertex was approached from below, extroverted if from above in the vertex ordering. [This clearly guarantees acyclicity.]

*(Problem). Generalize to arbitrary signed graphs.*

Lemma 2, “a standard exercise”, is that an orientation of $\pm K_{n}^\circ$ (with the loops replaced by half edges) is acyclic iff the magnitudes of its net degrees are a permutation of \{1, 3, ..., 2n − 1\}. [Similarly, an orientation of $\pm K_{n}^\circ$ is acyclic iff its net degree vector is a signed permutation of \{2, 4, ..., 2n\} (Zaslavsky (1991b), p. 369, but possibly known beforehand in other terminology). Both follow easily from Zaslavsky (1991b), Cor. 5.3: an acyclic orientation has a vertex that is a source or sink.]

*(SG: ori: incid, Geom)*


§5 presents the (signed-graph) question: an appealing presentation of material from (1997a). *(SG: ori, Incid, Geom, N: Exp)*

Gary Gordon, Jennifer McNulty, & Nancy Ann Neudauer


§4, “Cycle and bicircular matroids”: Thm. 4.3: If $\Gamma$ is 3-connected and $\# V \geq 5$, then $G(\Gamma)$ and $G(\Gamma, \emptyset)$ have the same fixing number. Thm. 4.4: If $\Gamma$ is 2-connected, $\# V \geq 5$, min degree $\geq 3$, then $\text{Aut } G(\Gamma, \emptyset) \cong \text{Aut } \Gamma$.

Cor. 4.6.2: Fixing number of $G(K_n, \emptyset)$ is 5 for $n = 4$, $\lfloor 2n/3 \rfloor$ if $n > 4$. [Annot. 8 Jan 2016.]

*(SG: bic: Aut)*

Y. Gordon & H.S. Witsenhausen

Asymptotic estimates of \( l(K_{r,s}) \), the maximum frustration index of signatures of \( K_{r,s} \), improving the bounds of Brown and Spencer (1971a).

Clinton F. Goss
See M.C. Golumbic.


eyric Gottlieb
See Gottlieb and Wachs (2000a).
The lattice is the subposet of \( \text{Lat} G(\mathfrak{s}K_n) \) consisting of the flats whose nontrivial balanced components have order \( \geq k \) and whose unbalanced component, if any, has order \( \geq h \). If \#\( \mathfrak{s} = 2 \) and \( h \leq k \) we have the lattice of Björner and Sagan (1996a).

Eric Gottlieb & Michelle L. Wachs
Two monomorphisms of the cohomology of the order complex of the lattice of flats of \( Q_n(\mathfrak{s}) \), upon which \( \mathfrak{s}_n \wr \mathfrak{s} \) acts as operators, into enveloping algebras of certain Lie algebras and Lie superalgebras.

Samira Goudarzi
See S. Akbari.

Ian P. Goulden, Jin Ho Kwak, & Jaeun Lee

Antoine Gournay

Jean-Luc Gouzé

Kevin Grace

Kevin Grace & Stefan H.M. van Zwam
Counterexamples to Geelen, Gerards, and Whittle (2015a). (GG: M)


Jarosław Grytczuk
See M. Anholcer.

R.L. Graham


R.L. Graham & N.J.A. Sloane

See Example b, p. 396 (the Gale-Berlekamp code). (sg: Fr)

M.J. Grannell & T.S. Griggs

The voltage graph (i.e., gain graph) construction is used to generate embeddings of combinatorial designs. [Annot. 12 Jun 2013.] (Top: GG, Cov: Exp)

Douglas D. Grant
See L.D. Andersen.

Ante Graovac, Ivan Gutman, & Nenad Trinajstić

§2.7. “Extension of graph-theoretical considerations to Mobius systems.” (SG: Adj, Eig, Chem)

A. Graovac & N. Trinajstić


The “Möbius graph” (i.e., signed graph of a suitably twisted ring hydrocarbon) is introduced with examples of the adjacency matrix and characteristic polynomial. (Chem: SG: Adj, Eig)

Timothy Graves
See Brewster and Graves (2009a).

Gary Greaves, Jack Koolen, Akihiro Munemasa, Yoshio Sano, & Tetsuji Tani-guchi
(SG: Adj: Eig: Str)  
Gary Greaves, Jacobus H. Koolen, Akihiro Munemasa, & Ferenc Szöllösi  
§3, “Some structural results on Seidel matrices”. Remarkable congruence results. Let \(\Sigma := (K_n, \sigma)\). Thm. 3.5: \(\det A(\Sigma) \equiv (-1)^n(1 - n) + 4n(\#E) \pmod{8}\). Cor. 3.6: \(\det A(\Sigma) \equiv 1 - n \pmod{4}\). Thm. 3.7 gives per \(A(\Sigma) \pmod{8}\). Prop. 3.8: \(\Sigma \not\equiv -\Sigma\) if \(n \equiv 3 \pmod{4}\).  
§4, “Small sets of equiangular lines”, finds all \(\Sigma\) for \(n \leq 12\), thus proving Haemers’ (2012a) conjecture for \(n \leq 12\). Application to equiangular lines. [Annot. 10 Dec 2020.]  
(SG: KG: Geom, sw, Adj: Eig)  
Gary Greaves, Bojan Mohar, & Suil O  
Gain group \(\{\pm 1, \pm i\} : \varphi(e) = 1\) for undirected, \(i\) for directed edges.  
(gg: Adj)  
[John G. del Greco]  
See J.G. del Greco (under ‘D’).  
F. Green  
Proves polynomial time for the reduction employed in Bachas (1984a) and improves the theorem to: The frustration-index decision problem on signed (3-dimensional) cubic lattice graphs with 9 layers is NP-complete. [2 layers, in Barahona (1982a).]  
(SG: Fr: Alg)  
Jan Green-Krótki  
See J. Aráoz.  
Harvey J. Greenberg, J. Richard Lundgren, & John S. Maybee  
From a matrix \(B\), with row set \(R\) and column set \(C\), form the “signed bipartite graph” \(BG^+\) with vertex set \(R \cup C\) and an edge \(r_i e_k\) signed \(\text{sgn} b_{ik} \) whenever \(b_{ik} \neq 0\). The “signed row graph” \(RG^+\) is the two-step signed graph of \(BG^+\) on vertex set \(R\); that is, \(r_i r_j\) is an edge if \(\text{dist}^{BG^+}(r_i, r_j) = 2\) and its sign is the sign of any shortest \(r_i r_j\)-path. If some edge has ill-defined sign, \(RG^+\) is undefined. The “signed column graph” \(CG^+\) is similar. The paper develops simple criteria for existence and balance of these graphs and the connection to matrix properties. It examines simple special forms of \(B\).  
(QM: SG, Bal, Appl)  
Application of (1983a), (1984b). “Netform” = incidence matrix of a positive real gain graph (neglecting a minor technicality). Thm. 1: $B$ is a netform iff $RG^+(B)$ exists and is all negative. (Then $CG^+(B)$ also exists.) Thm. 2: If the row set partitions so that all negative elements are in some rows and all positives are in the other rows, then $RG^+(B)$ is all negative and balanced. Thm. 3: If $\Sigma$ is all negative and balanced, then $B$ exists as in Thm. 2 with $RG^+(B) = \Sigma$. [Equivalent to theorem of Hoffman and Gale (1956a).] $B$ is an “inverse” of $\Sigma$. Thm. 4 concerns “inverting” $-\Gamma$ in a minimal way. Then $B$ will be (essentially) the incidence matrix of $+\Gamma$. (SG, gg: incid, Bal, VS, Exp, Appl)


See (1983a). “Inversion” means, given a signed graph $\Sigma_R$, or $\Sigma_R$ and $\Sigma_C$, finding a matrix $B$ such that $\Sigma_R = RG^+(B)$, or $\Sigma_R = RG^+(B)$ and $\Sigma_C = CG^+(B)$. The elementary solution is in terms of coverings of $\Sigma_R$ by balanced cliques. It may be desirable to minimize the size of the balanced clique cover; this difficult problem is not tackled. (QM: SG, VS, Bal)

Harvey J. Greenberg & John S. Maybee, eds.

Several articles relevant to signed (di)graphs. (QM)(SD, SG: Bal)

Curtis Greene & Thomas Zaslavsky

§9: “Acyclic orientations of signed graphs.” Continuation of Zaslavsky (1991b), counting acyclic orientations with specified unique source; also, with edge $e$ having specified orientation and with no termini except at the ends of $e$. The proof is geometric. (SG: M, Ori, Geom, Invar)

David A. Gregory
See also M. Cavers.


$\rho$ = spectral radius. Conjecture 2: Given simple $\Gamma$, for some $\sigma$ we have $\rho(A(\Gamma, \sigma)) \leq 2\sqrt{\Delta - 1}$. Thus, focus is on small $\rho$—via matching polynomial of $\Gamma$. Lemma 3: $\rho(\Sigma) \geq \sqrt{d(\Gamma)} = \text{iff } A(\Sigma)^2 = dI_n$. §6, “2-covers and Ramanujan families”: “2-cover” = $\Sigma$. Lemma 6 rediscovers Fowler’s (2002a) theorem (q.v.). [Annot. 13 Jun 2019.] (SG: Adj: Eig, sw, KG)

David A. Gregory, Kevin N. Vandermeulen, & Bryan L. Shader

For bipartite $\Sigma$, $\mathcal{M} := \text{class of matrices with weak sign pattern } \Sigma$. Every $A \in \mathcal{M}$ is the sum of $\text{rk} A$ rank-1 matrices in $\mathcal{M}$ iff (\text{(*)}) $\sigma(C) = \sigma(C') \text{ for every circle with } \#C + 1 \geq 6$. Thm. 3.2: $\Sigma$ has (\text{(*)}) for every
circle iff it is a spanning subgraph of a signed 4-cockade. Thm. 3.7. 
$\Sigma$ has (*) for circles with $\#C \geq 6$ iff, after switching, it is obtained by 
three constructions from a negative $C_4$, a subgraph of $+K_{3,n}$, or a signed 
graph $R_n$. [Annot. 6 Mar 2011.] (SG: QM, Circ)

Gary S. Grest
See also D. Blankschtein.
1985a Fully and partially frustrated simple cubic Ising models: a Monte Carlo study. 
Simulation of the cubic signed graph of Blankschtein, Ma, and Nihat 

T.S. Griggs
See M.J. Grannell.

Will Grilliette, Josephine Reynes, & Lucas J. Rusnak

Will Grilliette & Lucas J. Rusnak

G. Grimmett
Reviews Fortuin and Kasteleyn (1972a) and subsequent developments esp. in multidimensional lattices. The viewpoint is mainly probabilistic and asymptotic. §3.7, “Historical observations,” reports Kasteleyn’s account of the origin of the model. (sgc: Gen: Invar, Phys: Exp)

Ya.R. Grinberg & A.M. Rappoport
See (2011b). (SG: Fr, Str, Clu)

Thm. 1: The contrabalanced signed graphs are the cactus forests (Husimi forests) in which every circle is negative. Dictionary: “disbalance” = contrabalance, “junction” = cutpoint, “cyclically splittable” = every block is a circle, “$p$-groupable” = $p$-clusterable. [Annot. 9 Jun 2012, 22 Jan 2015.] (SG: Fr, Str, Clu)

Richard C. Grinold
Objective: to find the maximum output for given input. Basic solutions correspond to bases of $G(\Phi^\prime)$, $\Phi^\prime$ being the underlying gain graph $\Phi$ together with an unbalanced loop adjoined to the sink. Onaga (1967a)
also treats this problem. (GN: M(bases), Alg)

Heinz Gröflin & Thomas M. Liebling

Piotr Gronek
See P. Gawroński and K. Kulakowski.

Jonathan L. Gross
See also J. Chen.

§3. “Total embedding distributions”: “Twist” (= edge signature) is used to construct nonorientable embeddings, which increase the count of embeddings. [Annot. 12 Jun 2013.] (Top: sg: Exp)

Jonathan L. Gross & Thomas W. Tucker

Ch. 2: “Voltage graphs and covering spaces.” Ch. 4: “Imbedded voltage graphs and current graphs.” (GG: Top, Cov)
§3.2.2: “Orientability.” §3.2.3: “Rotation systems.” §4.4.5: “Nonorientable current graphs”, discusses how to deduce, from the signs on a current graph, the signs of the “derived” graph of the dual voltage graph. [The same rule gives the signs on the surface dual of any orientation-embedded signed graph.] (The sign group here is \(\mathbb{Z}_2\).) (GG: Top)


Jerrold W. Grossman
See also R.B. Bapat.

Jerrold W. Grossman & Roland Häggkvist

They prove the special case in which B is all negative of the following generalization, which is an immediate consequence of their result. *(Theorem)* If B is a bidirected graph such that for each vertex v there is a block of B in which v is neither a source nor a sink, then B contains a coherent circle. (“Coherent” means that at each vertex, one edge is directed inward and the other outward.)*

Jerrold W. Grossman, Devadatta M. Kulkarni, & Irwin E. Schochetman

Topics: The unoriented incidence matrix of Γ [i.e., the incidence matrix H(−Γ)], the Laplacian matrix L(−Γ), the even-cycle (“even circuit”) matroid G(−Γ), a partial all-minors matrix-tree theorem [completed in Bapat, Grossman, and Kulkarni (1999a)]. [This part is not new. See van Nuffelen (1973a) for rank(H(−Γ)); Zaslavsky (1982a), §8 for both matrices; Tutte (1981a), Doob (1973a), and Simões-Pereira (1973a) for the matroid; Chaiken (1982a) for the whole matrix-tree theorem.]

§§4, 5: Vector spaces associated with G(−Γ) and its dual, expressed both combinatorially in terms of vectors associated with matroid circuits and cocircuits (of two kinds) and as null and row spaces of H(−Γ) and H(−Γ)^T. E.g., in §5 is the all-negative case of: A basis for Nul H(Σ)^T consists of one switching function positivizing each balanced component of Σ. [The viewpoint, going from matroids to vector spaces over fields, usually with characteristic ≠ 2, contrasts sharply with that of Tutte (1981a), who starts with integral chain groups (Z-modules) and ends with chain-group properties and matroids. This is the only thorough development I know of vector spaces of a signed graph before Chen and Wang (2009a), despite some aspects’ having appeared e.g. in Bolker (1977a), (1979a), and Tutte (1981a). It will be still more valuable if it is extended to R^x-gain graphs and to F^x-gain graphs for any field F.]

Dictionary: M = H(−Γ); “k-reduced spanning substructure” ≈ independent set of rank n − k in G(−Γ); “quasi edge cut” = balancing set; “quasibond” = minimal balancing set; “even circuit” = positive closed walk; “bowtie” = contrabalanced handcuff; “marimba stick” = half edge.

EC, par: Incid, Bal, D


Rank of the unoriented incidence matrix of Γ (which equals H(−Γ)) [as in van Nuffelen (1973a)]. Finds all possible values of determinants of minors of H(−Γ) [repeating and refining Zaslavsky (1982a), §8A] and of maximal nonsingular minors. Consequences are the Smith normal form of H(−Γ) (§3) and the total integrality of some integer programs with
\( H(-\Gamma) \) as coefficient matrix. (par: Incid, ec, Geom)

Martin Grötschel
See also F. Barahona.

M. Grötschel, M. Jünger, & G. Reinelt

§2, “The spin glass model”: finding the weighted frustration index in a weighted signed graph \((\Sigma, w)\), or finding a ground state in the corresponding Ising model, is equivalent to the weighted max-cut problem in \((-\Sigma, w)\). This article concerns finding the exact weighted frustration index. §3, “Complexity”, describes previous results on NP-completeness and polynomial-time solvability. §4, “Exact methods”, discusses previous solution methods. §5, “Polyhedral combinatorics”, shows that finding weighted frustration index is a linear program on the cut polytope; also expounds related work. The remainder of the paper concerns a specific cutting-plane method suggested by the polyhedral combinatorics.

(sg: fr(gen), State: Alg, Geom, Ref)(Phys, Ref: Exp)

Martin Grötschel, László Lovász, & Alexander Schrijver

Ch. 8, “Combinatorial optimization: A tour d’horizon”: Topics mentioned include odd circles, maximum-gain flow, odd cuts.

(par, gg: Circ, Alg)


Essentially the same as (1988a). (par, gg: Circ, Alg)

M. Grötschel & W.R. Pulleyblank

Includes a polynomial-time algorithm, which they attribute to “Waterloo folklore”, for shortest (more generally, min-weight) even or odd path, hence (in an obvious way) odd or even circle. [Attributed by Thomassen (1985a) to Edmonds (unpublished). Adapts to signed graphs by the negative subdivision trick: Subdivide each positive edge of \(\Sigma\) into two negative edges, each with half the weight. The min-weight algorithm applied to the subdivision finds a min-weight (e.g., a shortest) negative circle of \(\Sigma\).] [This paper is very easy to understand. It is one of the best written I know.] [Weakly bipartite graphs are certain signed graphs. Further work: Barahona, Grötschel, and Mahjoub (1985a), Poljak and Tuza (1995a), and esp. Guenin (1998a), (2001a).]

(par: Alg, Geom, Paths, Circ)(sg: Geom)

D.A. Grundy, D.D. Olesky, & P. van den Driessche

Xiangbai Gu
See D. Peng.

Zhi-Hong Guan
See B. Hu.

Victor Guba & Mark Sapir

The “labelled oriented graph” (pp. 12–13) is a gain graph with a gain semigroup (instead of group) which is the semigroup generated by an alphabet and its inverse. (gg(Gen))

James E. Gubernatis
See N. Hatano.

Bertrand Guenin
See also A. Abdi, G. Cornuéjols, Z. Ferchiou, and J.F. Geelen.


Extended abstract of (2001a). (SG: Geom)


Σ is “weakly bipartite” (Grötschel and Pulleyblank (1981a)) if its clutter of negative circles is ideal (i.e., has the “weak MFMC” property of Seymour (1977a)). [This is a polyhedral property that can be equivalently stated: Define a “negative circle cover” to be an edge multiset that intersects every negative circle, and a “weighted negative circle cover” to be an edge weighting by nonnegative real numbers such that the total weight of each negative circle is at least 1. Weak biparticity means that, for every linear functional $f : E \to \mathbb{R}$, the minimum value over all weighted negative circle covers is attained by a negative circle cover.] Thm.: Σ is weakly bipartite iff it has no $-K_5$ minor. This proves part of Seymour’s (1981a) conjecture (see Cornuéjols (2001a)). [Short proof: Schrijver (2002a).] Dictionary: “odd” = negative, “even” = positive. (SG: Geom, Circ, Str)


See (2002a).

(sg: Par: M, Geom)

In $\Sigma$ distinguish a negative link $e_{st}$. An “unbalanced port” is $C \setminus e_{st}$ where $C$ is an unbalanced circuit of $L(\Sigma)$ that contains $e_{st}$. Replace "negative circle" by "negative port" in the definition of (2001a). Thm.: The minimum value over all weighted unbalanced port covers is attained by an unbalanced port cover, if $\Sigma$ has no $-K_5$ minor and $L(\Sigma)$ has no $F_7^*$ minor. [The latter can be replaced by: $\Sigma$ has no $(\pm C_4 \setminus \text{edge})$ minor, by Zaslavsky (1990a).] Dictionary: “even-cycle matroid” = lift matroid $L(\Sigma)$, not the even-cycle matroid $G(\Gamma)$ in W.T. Tutte (1981a), M. Doob (1973a); “odd $st$-walk” = unbalanced port. (SG: Geom, M, Str)


The excluded minor is $L(-K_4)$. (SG, Sgn(M): Flows)


**Bertrand Guenin, Irene Pivotto, & Paul Wollan**


Dictionary: “Even” = positive, “odd” = negative, “cycle” = binary cycle = even subgraph, “even cycle matroid” = lift matroid $L(\Sigma)$, not the even-cycle matroid defined in Doob (1973a) (cf. Tutte (1981a)). (sg: M: Str)

**Bertrand Guenin & Leanne Stuive**


Sylvain Guillemot


Basak Guler, Burak Varan, Kaya Tutuncuoglu, Mohamed Nafea, Ahmed A. Zewail, Aylin Yener, & Damien Octeau

Mahadevappa M. Gundloor
See H.S. Ramane.

Heng Guo & Mark Jerrum

Ji Ming Guo
See also L. Feng, J.X. Li, S.W. Tan, X.L. Wu, and J.M. Zhang.

Ji-Ming Guo, Jianxi Li, & Wai Chee Shiu

N. Gülpinar, G. Gutin, G. Mitra, & A. Zverovitch

Problem: Finding a largest embedded network matrix (up to “reflection” = row negation). Given a $0,\pm 1$-matrix $A$, let $\Sigma$ have for vertices the rows of $A$, with an edge $e_{ij}$ iff $\text{sgn}(a_{ik}a_{jk}) = -\varepsilon$ in the $k$-th column for some $k$. Let $\alpha := \text{maximum size of a stable set in a graph.}$ Thm.: The maximum height of a reflected network submatrix of $A$ equals $\max_X \alpha((\Sigma^X)^{-})$ over all switchings $\Sigma^X$. This implies a heuristic algorithm for finding a large embedded network matrix. [Annot. 30 Sept 2009.]
construct cospectral graphs. [Annot. 23 Nov 2014.]  

**Jiong Guo, Hannes Moser, & Rolf Niedermeier**


Iterative compression results in vast speed-up for, e.g., Graph Bipartization and Signed Graph Balancing (§3.1). *Cf.* Hüffner, Betzler, and Niedermeier (2007a). [Annot. 6 Feb 2011.]  

**Krystal Guo**


**Krystal Guo & Bojan Mohar**


“Digraph” = “mixed graph” = \{1, ±i\}-gain graph \(\Phi\). “Hermitian adjacency matrix” = \(A(\Phi)\). Studies eigenvalue properties in terms of gain-graph properties. §8, “Cospectrality”: Reversing arcs in a cut without undirected edges, replacing undirected edges in a cut without arcs by arcs in the same direction, and “four-way switching” retain the spectrum (because they are gain switching operations). So does reversing all arcs (it is conjugation). *Cf.* Liu and Li (2015a) and Mohar (2016a). [Annot. 4 Feb 2018, 23 Dec 2020.]  


**Qiumin Guo**

See also W.L. Guo.  

**Qiumin Guo, Weili Guo, Wentao Hu, & Guangfeng Jiang**


For the complex arrangement \(\mathcal{H}[\Sigma]\), the third quotient of the lower central series of \(\pi(C^n \setminus \bigcup \mathcal{H}[\Sigma])\) has a combinatorial interpretation in terms of \(\Sigma\). [Annot. 1 Nov 2014.]  

**Shuguang Guo**

See G.L. Yu.  

**Weili Guo**

See also Q.-M. Guo.  

**Weili Guo, Qiumin Guo, & Guangfeng Jiang**


**Weili Guo & Michele Torielli**


Yihao Guo
See M. Zhu.

Anika Gupta
See D. Li.

G. Gupta
See F. Harary.

Neha Gupta
See M. Charikar.

Razvan Gurau

Venkatesan Guruswami
See M. Charikar and I. Giotis.

Vladimir A. Gurvich
See E. Boros.

Gregory Gutin
See also J. Bang-Jensen, R. Crowston, and N. Gülpinar.

G. Gutin & D. Karapetyan

(SG: incid, Bal, Alg: Exp)

Gregory Gutin, Daniel Karapetyan, & Igor Razgon

Algorithmics of finding a balanced signed-graph incidence matrix (“reflected network matrix”) in a given matrix. [Annot. 26 Dec 2012.]

(GG: Bal: Sw, Alg)

Gregory Gutin, Benjamin Sudakov, & Anders Yeo

Existence of a coherent circle with alternating colors in a digraph with an edge 2-coloring is NP-complete. However, if the minimum in- and out-degrees of both colors are sufficiently large, such a cycle exists.
This problem generalizes the undirected, edge-2-colored alternating-circle problem, which is a special case of the existence of a bidirected coherent circle—see Bang-Jensen and Gutin (1997a). Question. Is this alternating cycle problem also signed-graphic?

(par: ori: Circ: Gen)

Gregory Gutin & Alexei Zverovitch

Ivan Gutman
See also N.M.M. de Abreu, D.M. Cvetković, A. Graovac, S.-L. Lee, and H.S. Ramane.


Points out an ambiguity in the definitions of Lee, Lucchese, and Chu (1987a) in the case of multiple eigenvalues. [See Lee and Gutman (1989a) for the repair.] (VS, SGw)

Ivan Gutman, Dariush Kiani, Maryam Mirzakhah, & Bo Zhou

Ivan Gutman, Shyi-Long Lee, Yeung-Long Luo, & Yeong-Nan Yeh

How to compute the balanced signing of $\Gamma$ that corresponds to eigenvalue $\lambda_i$ (see Lee, Lucchese, and Chu (1987a)), without computing the eigenvector $X_i$. Theorem: If $v_r, v_s \in E$, then $X_{ir}X_{is} = \sum_P f(P; \lambda_i)$, where $f(P; \lambda) := \varphi(G - V(P); \lambda)/\varphi'(G; \lambda)$, $\varphi(G; \lambda)$ is the characteristic polynomial, and the sum is over all paths connecting $v_r$ and $v_s$. Hence $\sigma_{ir}(v_r, v_s) = \text{sgn}(X_{ir}X_{is})$ is determined. [An interesting theorem. Questions. Does it generalize if one replaces $\Gamma$ by a signed graph, this being the balanced (all-positive) case? In such a generalization, if any, how will $\sigma$ enter in—by restricting the sum to positive paths, perhaps? What about graphs with real gains, or weights?]

(par: Incid Eig)

Ivan Gutman, Shyi-Long Lee, Jeng-Horng Sheu, & Chiuping Li

Points out some difficulties with the method of Lee and Li (1994a). “Net sign” of $\tau : V \to \{\pm 1\}$ means $\sum_v \tau(v) = \#V - 2(\text{cut size})$.

(VS, Chem)

Ivan Gutman, Shyi-Long Lee, & Yeong-Nan Yeh
A connected graph $\Gamma$ has $n$ eigenvalues and $n$ corresponding balanced signings (see Lee, Lucchese, and Chu (1987a)). Let $S_1 \geq S_2 \geq \ldots \geq S_n$ be the net signs of these signings and $m = \#E$. The net signs satisfy analogs of properties of eigenvalues. (A) If $\Delta \subset \Gamma$, then $S_1(\Delta) < S_1$. 
(B) $S_1 = m \geq S_2 + 2$. (C, D) For bipartite $\Gamma$, $S_n = -m$. Otherwise, $S_n \geq -m + 2$. From (B, C, D) we have $\#S_i \leq m - 2$ for all $i \neq 1$ and, if $\Gamma$ is bipartite, $i \neq n$. (E, F) If $\Gamma$ is bipartite, then $S_i = -S_{n+1-i}$ and at least $a - b$ net signs equal 0, where $a \geq b$ are the numbers of vertices in the two color classes. The analogy is imperfect, since $S_1 + S_2 + \cdots + S_n \geq 0$, while equality holds for eigenvalues. [Questions. Some of these conclusions require $\Gamma$ to be bipartite. Does that mean that they will generalize to an arbitrary balanced signed graph $\Sigma$ in place of the bipartite $\Gamma$, the eigenvectors being those of $\Sigma$? Will the other results generalize with $\Gamma$ replaced by any signed graph? How about real gains, or weights?] (VS, SGw)

Ivan Gutman, Maria Robbiano, Enide Andrade Martins, Domingos M. Cardoso, Luis Medina, & Oscar Rojo


Ivan Gutman & Oskar E. Polansky


Pavol Gvozdjak & Jozef Širáň


A. Gyárfás

See B. Bollobás and P. Erdős.

Eszter Gyimesi & Gábor Nyul


Combinatorial interpretation of the $r$-Dowling numbers, generalizing Belbachir and Bousbaa (2013a). $D_{n,m,r} = \#$ of ways to partition $[n+r]$ into $k$ blocks so $n+1, \ldots, n+r$ are in different blocks, then color non-minimal elements of each block $B \subseteq [n]$ with $m$ colors.

[Let group $\mathfrak{G}$ have order $m$; use color set $\mathfrak{G}$, least elements are colored
\[ \varepsilon; \text{ then } W_m(n,k,1) = W_k(Q_n(\mathfrak{G})) = \text{Whitney number of Dowling lattice (lattice of frame matroid } G(\mathfrak{G}K^n_j)). \text{ Proved by Dowling (1973b).} \]

**Improvements:**
1. \( W_m(n,k,r) = \# \text{ colorings by } \mathfrak{G} \text{ as above, but each minimal element has color } \varepsilon \text{ (a Dowling-like special case of Gyimesi–Nyul’s theorem).} \)
2. **Conjecture.** Replacing partitions by permutations and blocks by cycles, \( w_m(n,k,r) = \# \text{ colorings as above. See note at Belbachir and Bousbaa (2013a) for similar interpretations of } |w_m(n,k,0)| \text{ and } W_m(n,k,0). \]

[Annot. 28 May 2018.]

[Independently, Conjecture (2) is proved in (2019a).] [Annot. 22 Sep 2018.]

2019a New combinatorial interpretations of \( r \)-Whitney and \( r \)-Whitney-Lah numbers. 

[“Whitney” should be “Dowling” or “Dowling–Whitney”; Whitney numbers are more general.]

Definitions in terms of permutations (= partitions into circularly ordered blocks), \((r\text{-Dowling of first kind}), \text{ partitions (} r\text{-Dowling of second kind), and partitions into linearly ordered blocks (}\text{r-Dowling–Lah)}\text{, all with } r \text{ distinguished elements appearing in distinct blocks and an } m\text{-coloring rule for most elements of the other blocks. [Cf. (2018a) for Dowling-style reinterpretations.] Identities, formulas, with combinatorial proofs.} \]

[Cf. notes on (2018a). This } m\text{-coloring is equivalent to Dowling’s } (1973b) \text{ homogeneously } \mathbb{Z}_m\text{-labelled partitions (or coloring by any } \mathfrak{G} \text{ of order } m), \text{ applied also to circularly and linearly ordered blocks and generalized to multiple distinguished elements (Dowling being } r = 1). \text{ Each such type forms a lattice in the natural partial ordering of homogeneously } \mathfrak{G}\text{-colored partitions et al. (ignoring distinguished elements). Partitions generalize Dowling lattices (geometric lattice); permutations generalize regions of a hyperplane representation (Eulerian lattice; ). (3) Question. Is there similar geometry for } r > 1? \text{ (4) Question. With } r = 1, \text{ is there a geometry for linearly ordered blocks? (5) Problem. Determine the properties of these lattices and how they relate to each other. E.g., ranked, geometric, Eulerian? Effect of the natural homomorphisms?] [Annot. 22 Sep 2018.] \]

Ervin Győri
See also P. Erdős.

Ervin Győri, Alexandr V. Kostochka, & Tomasz Łuczak

Given \( \Sigma = -\Gamma \) and positive \( \rho \), suppose every negative circle has length \( \geq n/\rho \). Then \( \Sigma \) has frustration index \( \leq 200\rho^2(\ln(10\rho))^2 \) (best possible up to a constant factor) and vertex frustration number \( \leq 15\rho \ln(10\rho) \) (best possible up to a logarithmic factor). The proof is based on an interesting, refining lemma. [Problem. Generalize to arbitrary \( \Sigma \).] \text{ (sg: Par: Fr)}

M. Hachimori & M. Nakamura
Signed graph coloring is mentioned as an example. [Annot. 10 Mar 2011.] (SG: Invar: Exp)

Willem H. Haemers
See also S. Akbari, A.E. Brouwer, M. Cavers, E. Ghorbani, and E.R. van Dam.

Graphs whose energy is not altered by switching. §3, “Seidel matrix”: Energy $E(K_\Delta)$ of signed complete graphs $K_\Delta$ (“Seidel energy of $\Delta$”). Thm. 3.1: $E(K_\Delta) \leq n\sqrt{n-1}$, and = iff $\Delta$ is a conference graph. Thm. 3.3: $E(K_\Delta) > \sqrt{2n(n-1)}$ if $n > 2$. Fact: $E(K_n^+) = 2(n-1)$. Conjecture: $\min_{\sigma} E(K_n, \sigma) = 2(n-1)$. [Proved in Akbari, Einollahzadeh, et al. (2020a), previously for $n \leq 12$ in Greaves et al. (2016a).] [Annot. 13 Jan 2015, 10 Dec 2020.] (SG(KG): Sw, Adj: Eig)

W.H. Haemers & G.R. Omidi

Willem H. Haemers & Edward Spence

“Sign-less Laplacian” $Q(\Gamma) :=$ Laplacian matrix $L(-\Gamma) = D(\Gamma) + A(\Gamma)$. $L(-\Gamma)$ seems ($n \leq 11$) to allow fewer cospectral graphs than does $A(\Gamma)$ or $L(\Gamma)$. [Annot. Sept 2010.] (par: Lap: Eig)

Sumaira Hafeez & Mehtab Khan

Weights from $\mathbb{R}^\times$. Iota energy := $\sum |\text{Im} \lambda_i|$, $\lambda_i =$ eigenvalues. Results on cyclic and unicyclic weighted digraphs; sgn(weight product) affects the results. Thm. 3.5: General upper bound. §4, “Equienergetic weighted digraphs”: Examples. [Annot. 3 Jul 2018.] (SD: Adj: Eig)

Jurriaan Hage
See also A. Ehrenfeucht.

An algorithm to decide whether two skew gain graphs are switching equivalent. (GG(Gen): Sw, Alg)


Contains the material of the following papers, along with updates and improved results: Ehrenfeucht, Hage, Harju, and Rozenberg (2000a), (2000b), (2006a), Hage (1999a), and Hage and Harju (1998a), (2000a), (2004a).

Errata and a corrected version at URL (1/2002) http://www.cs.uu.nl/people/jur/2s.html (TG: Sw, Alg)(GG(Gen): Sw, Alg)

Skew gains reverse by an involutory antiautomorphism of the gain group (Hage and Harju (2000a)). Here switching is restricted by prescribing for each vertex a subgroup from which the switching value may be taken. Properties of ordinary switching generalize, or become more complicated, or become too difficult. Further research is needed. [Annot. 17 Dec, 5 Jan 2011–12] (GG: Gen: Sw: Gen)


Introducing “skew gain graphs”, which generalize gain graphs (see Zaslavsky (1989a)) to incorporate dynamic labelled 2-structures (see Ehrenfeucht and Rozenberg). Inversion is replaced by a gain-group antiautomorphism \( \delta \) of period at most 2. Thus \( \varphi(e^{-1}) = \delta(\varphi(e)) \), while in switching by \( \tau \), one defines \( \varphi^\tau(e;v,w) = \delta(\tau(v))\varphi(e;v,w)\tau(w) \). The authors find the size of a switching class \([\varphi] \) in terms of the centralizers and/or \( \delta \)-centralizers of various parts of the image of \( \varphi_T \), that is, \( \varphi \) switched to be the identity on a spanning tree \( T \). The exact formulas depend on whether \( \Gamma \) is complete, or bipartite, or general, and on the choice of \( T \) (the case where \( T \cong K_{1,n-1} \) being simplest). (GG(Gen): Sw)


Solves the problem raised by B.D. Acharya (1981a). (TG: Sw)


Partial results on the forbidden induced subgraphs for graph switching classes with no bipartite member. [Annot. 9 Sept 2010.] (TG: Sw)


Algebra related to the skewness, i.e., involutory anti-automorphisms,
of skew gains on a graph. [Annot. 12 Sep 2017.] (gg(Gen): Algeb)

Jurriaan Hage, Tero Harju, & Elmo Welzl

Polynomial-time algorithms for whether a graph switching class contains a triangle-free, or bipartite, or Eulerian, member. (TG: Sw)

Per Hage

Per Hage & Frank Harary
Signed graphs are treated in Ch. 3 and 6, marked (i.e., vertex-signed) graphs in Ch. 6. [Reviewed in Doreian (1985a).]
(SG, PsS: Bal: Exp)(VS: Exp)


Roland Häggkvist
See J.W. Grossman.

Mohammad Hassan Shirdareh Haghighi
See F. Motialah.

MohammadTaghi Hajiaghayi
See E.D. Demaine.

Mardjan Hakimi-Nezhaad
See also M. Ghorbani.

Mardjan Hakimi-Nezhaad & Modjtaba Ghorbani
(sg: KG: sw, Adj: Eig)

F.D.M. Haldane
See J. Vannimenus.
Frank J. Hall
See also M. Arav and C.A. Eschenbach.

Frank J. Hall & Zhongshan Li

H. Tracy Hall
See S. Butler.

Peter Hall
See B. Xiao.

Joshua Hallam, Jeremy L. Martin, & Bruce E. Sagan

§4, “ISFs in multigraphs”: The matroid is the Dowling geometry $G(C^* K_n)$ or a submatroid. Dictionary: “Labeled $n$-multigraph” = simple $C^*$-gain graph; “0-edge” = half edge; “ISF” (increasing spanning forest) = increasing basis of $G(C^* K_n)$; “perfectly labeled” = closed in $G(C^* K_n)$.

[Annot. 19 Oct 2019.] (GG: Geom, m, Invar)

Maureen Hallinan & David D. McFarland

§ “Signed directed graphs” (pp. 134–141): Tendency towards “transitivity” (balance or clusterability) in signed digraphs. The impetus for single-arc change (sign change, or deletion or introduction) cannot be determined by triangles alone (Props. 1.1–1.3). [Annot. 23 Nov 2012.]

(SD: bal, clu, PsS)

Mark D. Halsey

Dowling lattices are line closed; thus line closure does not imply vector representability. [Annot. 8 Apr 2016.] (gg: M: Str)

Shahul Hameed K
See also K.A. Germina, R.T. Roy, and T.V. Shijin.

Shahul Hameed K & K.A. Germina

Thm. 2.4: For a field $F$, an $F$-gain graph $\Phi$ is balanced iff $\text{Spec}_F A(\Phi) = \text{Spec}_F A(\|\Phi\|)$—generalizing B.D. Acharya (1980a). Thms. 2.2, 3.1: a formula and a recurrence (involving circles) for the characteristic polynomial. Corollaries for signed graphs. [Annot. 12 Jul 2019.]

(GG: Eig, Bal, SG)


Definitions of Cartesian, lexicographic, Cvetković (NEPS) products, with balance, adjacency matrices, eigenvalues. In part, generalizes to gain graphs portions of (2012b) and Germina, Hameed, and Zaslavsky (2011a). For balance of lex product see (2012b). Cvetković product NEPS($\Phi_1, \ldots, \Phi_\nu$) preserves balance. [Annot. 20 Apr 2019, rev 12 Jul 2019.]

20xxa On bounds for the eigenvalues and energy of signed graphs. Submitted. (SG: Eig)

Shahul Hameed K., Viji Paul, & K.A. Germina


Shahul Hameed K, Roshni T Roy, Soorya P, & Germina K A

20xxa On the characteristic polynomial of skew gain graphs. Submitted. (GG( Gen): Adj: Eig)

Shahul K. Hameed, T.V. Shijin, P. Soorya, K.A. Germina, & Thomas Zaslavsky


Set $\sigma_{\max}(u,v) := \max\{\sigma(P) : \text{shortest } uv\text{-path } P\}$, $\sigma_{\min}$ similarly. Signed distance: $d_{\max}(u,v) := \sigma_{\max}(u,v)d(u,v)$, $d_{\min}$. Signed distance matrices $D_{\max}$, $D_{\min}$. If $d_{\max} = d_{\min}$ (“distance compatibility”), $D_{\pm} := D_{\max}$. Thm. 3.5: $\Sigma$ balanced iff any $D(\Sigma)$ cospectral with $D(|\Sigma|)$.

$\Sigma$ is distance compatible if geodetic, balanced, or antibalanced. Thm. 5.1: Bipartite $\Sigma$ is distance-compatible iff balanced. Ex. 5.4: Distance compatible but not geodetic, balanced, or antibalanced. [Cf. Roy, Germina et al. (20xxa), Shijin, Soorya, et al. (20xxa).] (SG: WG, Adj, Bal, Eig, Sw)

Hasti Hamidzade & Dariush Kiani


Corrected proof of Y.P. Zhang, Liu, Zhang, and Yong (2009a), Thm. 3.3. [Annot. 16 Oct 2011.] (par: Lap: Eig)

Peter L. Hammer

See also E. Balas, C. Benzaken, E. Boros, J.-M. Bourjolly, Y. Crama, and A. Fraenkel.


P.L. Hammer, C. Benzaken, & B. Simeone

P.L. Hammer, T. Ibaraki, & U. Peled


See description of Thm. 8.5.2 in Mahadev and Peled (1995a). (par: ori)

P.L. Hammer & N.V.R. Mahadev


Peter L. Hammer, N.V.R. Mahadev, & Uri N. Peled


Peter L. Hammer & Sang Nguyen

J. Hammann
See E. Vincent.

P.R. Hampiholi, H.S. Ramane, Shailaja S. Shirkol, Meenal M. Kaliwal, & Saroja R. Hebbar

Semigraph := (V,E); edge e = (v_1, v_2, ..., v_k) = (v_k, ..., v_2, v_1), i.e., symmetrically ordered subset of V (due to E. Sampathkumar). Sign σ(e) = (−1)^k. Parity and sign are not distinguished. Balance results appear to be immediate corollaries of known signed-graph facts. Vertices may also be signed. [Annot. 11 May 2018.] (Sgnd: Bal, VS)

Miaomiao Han
See X.Y. Yuan.

Wei Han
See S.Y. Wang.

Phil Hanlon


Computes the Möbius functions of posets obtained from Lat G(±K_n^o) by discarding those flats with unbalanced vertex set in a given lower-hereditary list. Examples include Lat G(±K_n^{(k)}), the exponent denoting the addition of k negative loops. Generalized and superseded by Hanlon and Zaslavsky (1998a). (sg: M: Gen: Invar)


The lattices are based on a rank, n, a group, and a meet sublattice of the lattice of subgroups of the group. The Dowling lattices are a special case. (gg: M: Gen: Invar)


Phil Hanlon & Thomas Zaslavsky

Computes the characteristic polynomials (Thm. 4.1) and hence the Möbius functions (Cor. 4.4) of posets obtained from Lat G(Ω), Ω a biased graph, by discarding those flats with unbalanced vertex set in a given lower-hereditary list. Examples include Lat G(ØK_n^{(k)}) where Ø is...
a finite group, the exponent denoting the addition of \( k \) unbalanced loops. The interval structure, existence of a rank function, covering pairs, and other properties of these posets are investigated. There are many open problems. Simplifies, then generalizes, Hanlon (1988a).

"GG: M, Gen: Invar, Str, Col"

Pierre Hansen

See also M. Aouchiche, V. Devloo, and C.S. Oliveira.


§1: Algorithm 1 labels vertices of a signed graph to detect imbalance and a negative circle if one exists. [It is equivalent to switching a maximal forest to all positive and looking for negative edges. Independently discovered by Harary and Kabell (1980a).] §2: Algorithm 2 is the unweighted case of the algorithm of (1984a). Path balance in a signed digraph is discussed. §3: The frustration index of a signed graph is bounded below by the negative-circle packing number, which can be crudely bounded by Alg. 1. (SG, SD: Bal, Fr: Alg, sw)


Improves the characterization by Maybee (1981a) of sign-solvable digraphs with an eye to more effective algorithmic recognition. Thm. 2.2: A signed digraph \( D \) is sign solvable if its positive subdigraph is acyclic and each strongly connected component has a vertex that is the terminus of no negative, simple directed path. §3: “An algorithm for sign solvability” in time \( O(\#V \#E) \). (SD: QSol: Alg)


Algorithm to find shortest walks of each sign from vertex \( x_1 \) to each other vertex, in a signed digraph with positive integral(?) weights (i.e., lengths) on the edges. Applied to digraphs with signed vertices and edges; \( N \)-balance in signed graphs; sign solvability. The problem for (simple) paths is discussed [which is solvable by any min-weight parity path algorithm; see the notes on Grötschel and Pulleyblank (1981a)]. (SD, WD: Paths, VS, Bal, QSol: Alg)

Pierre Hansen & Claire Lucas

2009a An inequality for the signless Laplacian index of a graph using the chromatic number. *Graph Theory Notes N.Y.* 57 (2009), 39–42. MR 2666279 (2011c:05195). (par: Lap: Eig)

[See Liu and Lu (2014a) for solution of a conjecture.]

Pierre Hansen & Bruno Simeone


Three types of relatively easily maximizable pseudo-Boolean function (“unimodular” and two others) are defined. For quadratic pseudo-Boolean functions \( f \), the three types coincide; \( f \) is unimodular iff an associated signed graph is balanced (Thm. 3). Thus one can quickly recognize unimodular quadratic functions, although not unimodular functions in general. If the graph is a tree, the function can be maximized in linear time.

Christopher R.H. Hanusa

See Chaiken, Hanusa, and Zaslavsky (2019a).

Fei Hao, Stephen S. Yau, Geyong Min, & Laurence T. Yang


Dictionary: “\( k \)-balanced trusted clique” = all-positive \( k \)-clique in \( \Sigma \).

The problem: detect all such cliques. [Annot. 27 Nov 2018.]

Rong Xia Hao

See also S.-J. He.

Rong Xia Hao & Yan Pei Liu


\( \Sigma \) is projective planar iff an auxiliary graph is balanced. [The auxiliary graph may be a tree. It may have order linear in that of \( \Sigma \).] [Annot. 25 Apr 2012.]

Xiao Hui Hao & Bao Feng Li


(par: Lap: Eig)

Xiao Hui Hao & Li Jun Zhang


(par: Lap: Eig)

Masaaki Harada & Akihiro Munemasa


Signed bipartite graphs with \( B(\Sigma)B^T(\Sigma) = 5I \). [Annot. 14 Oct 2020.]
Frank Harary

See also L.W. Beineke, A. Blass, F. Buckley, D. Cartwright, G. Chartrand, O. Frank, and P. Hage.


The main theorem (Thm. 3) characterizes balanced signings as those for which there is a bipartition of the vertex set such that an edge is positive iff it lies within a part [I call this a Harary bipartition]. Thm. 2: A signing of a simple [or a loop-free] graph is balanced iff, for each pair of vertices, every path joining them has the same sign. The generating function for counting nonisomorphic signed simple graphs with \( n \) vertices by numbers of positive and negative edges is \( g_n(x + y) \) where \( g_n(x) \) is the g.f. of nonisomorphic simple graphs.

[The birth of signed graph theory. Although Thm. 3 was anticipated by König (1936a) (Thm. X.11, for finite and infinite graphs) without the terminology of signs, here is the first recognition of the crucial fact that labelling edges by elements of a group—specifically, the sign group—can lead to a general theory.] [Annot. ca. 1977. Rev. 20 Jan 2010.] [See also Whiteley (1991a).] [Annot. 12 Jun 2012.]


\( \Sigma \) is (locally) balanced at a vertex \( v \) if every circle on \( v \) is positive; then Thm. 3\': \( \Sigma \) is balanced at \( v \) iff every block containing \( v \) is balanced. \( \Sigma \) is \( N \)-balanced if every circle of length \( \leq N \) is positive; Thm. 2 concerns characterizing \( N \)-balance. Lemma 3: For each circle basis, \( \Sigma \) is balanced iff every circle in the basis is positive. [This strengthens König (1936a)] Thm. 13 for finite graphs.


“Antithetical duality” (pp. 260–261) introduces antibalence. Remarks on signed and vertex-signed graphs are scattered about the succeeding pages.


§6: “Balanced signed graphs”.


Proposes to measure imbalance by (i) \( \beta(\Sigma) \), the proportion of positive circles (“degree of balance”) from Cartwright and Harary (1956a), (ii) the
frustration index \( l(\Sigma) \) (here called “line index of balance”) \cite{AbelsonRosenberg1958}, i.e., the smallest number of edges whose deletion or equivalently (Thm. 7) negation results in balance, and (iii) the frustration number \( l_0(\Sigma) \) (“point index”). For \( \beta \) restricted to unbalanced blocks with cyclomatic number \( \xi \): Thm. 4: \( \min \beta \leq (\xi - 1)/(\xi - 1 + 2^{\xi - 1}) \).

Thm. 5: \( \max \beta \geq 1 - 2/(\xi + 1) \) (e.g., a ladder with \( \xi + 1 \) rungs and one rung negative). 

Cors.: \( \min \beta \to 0, \max \beta \to 1 \) as \( \xi \to \infty \). Conjecture. The bounds are best possible. 

[Annot. \( \leq 1980 \). Rev. 20 Jan 2010.]


First explicit appearance of the incidence matrix \( H(\Sigma) \), called \( J \). Thm. 2 (Heller and Tompkins (1956a), Hoffman and Gale (1956a)): \( \Sigma \) is balanced iff \( H(\Sigma) \) is totally unimodular. 

Cor.: The unoriented incidence matrix of \( \Gamma \) is totally unimodular iff \( \Gamma \) is bipartite. [Annot. 10 Nov 2008.]

Thm. 3: A necessary and sufficient condition that a subdeterminant is 0 in \( H(\Sigma) \), provided \( \Sigma \) is balanced. [Zaslavsky (1981a) §8A evaluates subdeterminants for any \( \Sigma \).] [Annot. 20 Jan 2010.] 


Signed, oriented half edges, applied to represent interpersonal relations. 


See remarks of Bixby (p. 111).

Historical remarks. E.g., it was Osgood and Tannenbaum (1955a) that inspired Harary to study vertex signings (Beineke and Harary (1978a), (1978b)).


Reconstruction from the multiset of vertex-deleted subgraphs. $\Sigma^+$ is reconstructible if $\Sigma$ is connected and balanced and not all positive or all negative.

F. Harary & G. Gupta


§3.9, “Signed graphs”, mentions that deletion index = negation index (Harary (1959b)).

Frank Harary & Jerald A. Kabell


Equivalent to switching so a spanning tree is all positive, then searching for a negative edge. [Independently discovered by Hansen (1978a)—and rather obvious by switching.]


Generating functions and partially explicit formulas for connected and all isomorphism types of vertex-signed, and balanced signed, graphs of order $n$. [Annot. 15 Aug 2017.]

Frank Harary & Helene J. Kommel


Frank Harary, Meng-Hiot Lim, Amit Agarwal, & Donald C. Wunsch


Thm. 1: The sizes of cuts in $K_n$. Thm. 2: A subgraph of a balanced
signed graph is balanced. [Annot. 12 Sept 2009.]

Frank Harary, Meng-Hiot Lim, & Donald C. Wunsch

“Assets” (vertices) have positive or negative correlation (edges of $K_n$). Balance is automatic. Switching is a means of hedging risk, which is highest with all positive edges. Imbalance indicates unpredictability; measured by the proportion of positive triangles. §5: “Balance analysis case study”. [Annot. 10 Sept 2009.]

Frank Harary & Bernt Lindström

Thm. 1: The number of balanced signings of matroid $M$ is $\leq 2^{rk(M)}$, with equality iff $M$ is binary. Thm. 3: Minimal deletion and negation sets coincide for all signings of $M$ iff $M$ is binary. Thm. 5: For connected binary $M$, a signing is balanced iff every circuit containing a fixed point is balanced.

Frank Harary, Bernt Lindström, & Hans-Olov Zetterström

Implicitly characterizes balance and balancing sets in a gain graph $\Phi$ by switching (proof of Thm. 1). [For balance, see also Acharya and Acharya (1986a), Zaslavsky (1977a) and ((1989a), Lemma 5.3. For abelian gains, see also Gargano and Quintas (1985a). In retrospect we can see that the characterization of balanced gains is as the 1-coboundaries with values in a group, which for abelian groups is essentially classical.] Thm. 1: The number of balanced gain functions. Thm. 2: Any minimal deletion set is an alteration set. Thm. 3: $l(\Phi) \leq m(1-\#\Phi^{-1})$. Thm. 4: $l(\Sigma) \leq \frac{1}{2}(m - \frac{n+1}{2})$, with strict inequality if not all degrees are even. [Compare with Akiyama, Avis, Chvátal, and Era (1981a), Thm. 1.]

Frank Harary, J. Richard Lundgren, & John S. Maybee

Which digraphs $D$ can be signed so that every cycle is negative? Three types of example. Type 1: The vertices can be numbered 1, 2, ..., $n$ so that the downward arcs are just (2, 1), (3, 2), ..., (n, $n-1$). (Strong “upper” digraphs; Thm. 2.) Type 2: No cycle is covered by the remaining cycles (“free cyclic” digraphs). This type includes arc-minimal strong digraphs. Type 3: A symmetric digraph, iff the underlying graph $\Gamma$ is bipartite and no two points on a common circle and in the same color class are joined by a path outside the cycle (Thm. 10; proved by signing $\Gamma$ via Zaslavsky (1981b)). [Further work in Chaty (1988a).]

Frank Harary, Robert Z. Norman, & Dorwin Cartwright

In Ch. 10, “Acyclic digraphs”: “Gradable digraphs”, pp. 275–280. That means a digraph whose vertices can be labelled by integers so that \( f(w) = f(v) + 1 \) for every arc \((v, w)\). [Equivalently, the Hasse diagram of a graded poset.] [Characterized by Topp and Ulatowski (1987a).]

(GD: bal, Exr)


“Limited balance”, pp. 352–355. Harary (1955a); also: Adjacency matrix \( A(D, \sigma) \) of a signed digraph: entries are 0, \( \pm 1 \). The “valency matrix” is the Abelson–Rosenberg (1958a) adjacency matrix \( R \). Thm. 13.8: Entries of \( (R - pI)^l \) show the existence of (undirected) walks of length \( l \) of each sign between pairs of vertices. [Strengthened in Zaslavsky (2010b), Thm. 2.1.]

“Cycle-balance and path-balance”, pp. 355–358: here directions of arcs are taken into account. E.g., Thm. 13.11: Every cycle is positive iff each strong component is balanced as an undirected graph.

(SG: Bal, Fr, Adj: Exp, Exr)(SD: Bal, Exr)


Frank Harary & Edgar M. Palmer


(SG: Bal: Enum)


Four exercises and a remark concern signed graphs, balanced signed graphs, and signed trees.

(SG: Enum, Bal)


Russian translation of (1973a).

(SG: Enum, Bal)

Frank Harary, Edgar M. Palmer, Robert W. Robinson, & Allen J. Schwenk


See Bender and Canfield (1983a).

(SG, VS: Enum)

Frank Harary & Michael Plantholt


\( L_{HP} \) A digraph \( D \) gives a signed line graph \( \Lambda_{HP}(D) \) with \( V_{HP} := E(D) \) and
edges \( +ef \) if \( e, f \) have the same head, \( -ef \) if \( e, f \) have the same tail.

[The negative part of \( \Lambda(+D) \) in Zaslavsky (2010b), (2012c), (20xxa) with extraverted edges made positive and introverted edges negative.]

[Annot. 4 Sept 2010, 17 Jan 2012.]  

Frank Harary & Geert Prins  

(SG: Enum)  

Frank Harary & Robert W. Robinson  

Counts signed trees. [Annot. 28 May 2017.]  

(SG, VS: Enum)  

Frank Harary & Bruce Sagan  

A signed poset is a (finite) partially ordered set \( P \) whose Möbius function takes on only values in \( \{0, \pm1\} \). \( S(P) \) is the signed graph with \( V = P \) and \( E_\varepsilon = \{xy : x \leq y \text{ and } \mu(x, y) = \varepsilon 1\} \) for \( \varepsilon = +, - \). Some examples are chains, tree posets, and any product of signed posets. Thm. 1 characterizes \( P \) such that \( \#S(P) \approx H(P) \), the Hasse diagram of \( P \). Thm. 3 characterizes posets for which \( S(P) \) is balanced. Thm. 4 gives a sufficient condition for clusterability of \( S(P) \). There are many unanswered questions, most basically Question 1. Which signed graphs have the form \( S(P) \)? [See Zelinka (1988a) for a partial answer.]  

(SG, Sgnd)  

Frank Harary & Marcello Truzzi  

(SG: Bal)  

Katsumi Harashima  
See H. Kosako.  

E. Harburg  
See K.O. Price.  

Mela Hardin  
See M. Beck.  

Nadia Hardy  
See S. Fiorini.  

Tero Harju  
See also A. Ehrenfeucht and J. Hage.  


A vertex-signed graph (called a “signed graph”) encodes the overlap of signed permutations (pp. 190ff.). [Annot. 6 Feb 2011.]

Tero Harju, Chang Li, & Ion Petre


See Harju, Li, Petre, and Rozenberg (2005a). The “parallel complexity” of a vertex-signed graph is the minimum number of operations required to reduce it to ∅. The value for an all-positive or all-negative tree is low (≤ 3 and 2, resp.). Conjecture. That of an all-negative graph is ≤ 3.


See (2008a). The parallel complexity of various examples, e.g., complete tripartite graphs with constant sign (complexity ≤ 3), and an all-positive circle with two negative leaves hanging off each circle vertex (complexity ≤ 4 or 5).

Tero Harju, Chang Li, Ion Petre, & Gregorz Rozenberg


The signs are on vertices. An operation is “local complementation” of a vertex v: in the neighborhood N(v), negate the vertices and complement the edges. Molecular operations formalized for vertex-signed graphs are: (1) deletion of an isolated negative vertex, (2) local complementation of a positive vertex, then deletion of the vertex, (3) a complementation in the neighborhood of two adjacent negative vertices v, w: complement in N(v) ∪ N(w), then complement in N(v) ∩ N(w). (The paper has a misprint.) The objective is to reduce the graph to ∅ by these operations, if possible. One consideration is when operations can be performed “in parallel”, i.e., independently of order of operations.

§7, “Fourth complexity measure: Parallelism”: A definition of parallelism in terms of applying rules (operations) to vertex-signed graphs. [Annot. 6 Feb 2011.] (VS: Alg)


See (2005a). (VS, Appl)

Tero Harju, Ion Petre, & Gregorz Rozenberg


See Harju, Li, Petre, and Rozenberg (2005a) et al. (VS, Appl: Exp)

Pierre de la Harpe


David Harries & Hans Liebeck


Given $\Sigma = (K_n, \sigma)$ and an automorphism group $G$ of the switching class $[\Sigma]$, is $G$ “exposable” on $[\Sigma]$? (does it fix a representative of $[\Sigma]$)? General techniques and a solution for the dihedral group. Done in terms of Seidel switching of unsigned simple graphs. (A further development from Mallows and Sloane (1975a). [Related work in M. Liebeck (1982a) and Cameron (1977a).]) (kg: sw, TG: Aut)

Matthew Hartley

See G.R. Walther.

Alexander K. Hartmann

See also C. Amoruso, G. Hed, O. Melchert, and M. Pelikan.


The distances of ground states in a signed cubic lattice with side $L$, measured by overlap, tested on many samples with $L \leq 14$ for the appearance of approaching ultrametricity in the infinite limit of $L$. There is such an appearance. $L$ is too small for quantitative statements. [Ultrametricity is a strong property that has been conjectured by Parisi et al. It is disavowed in Hed, Hartmann, Stauffer, and Domany (2001a).] Dictionary: cf. (2000a). [Annot. 11 Jan 2015.] (SG, Phys: Fr: State)


“Stiffness” = ground-state domain wall energy. The graph is a cubic lattice. Many ground states are generated and compared.


The ground states in examples of signed square, cubic, and tesseract (4-hypercubic) lattices are found to fall into relatively few clusters. An algorithmic method called “ballistic search” permits larger conclusions from smaller numbers of states. Dictionary: “ground state” = switching with fewest negative edges, “ground state energy” = $l(\Sigma)$, “ground state graph” has ground states $\zeta$ for vertices and an edge between ground states that differ by switching a vertex (necessarily having $d^+ = d^-$), “cluster” = component of ground-state graph, “overlap” of states $q(\zeta, \zeta') := n^{-1}[\#(\zeta \zeta'^{-1}(+) - \#(\zeta \zeta'^{-1}(-))] = (1/n) \cdot [\text{number of vertices of agreement} - \text{number of vertices of disagreement}].$ [Annot. 10 Jan 2015.]


Review of the ground state problem, methods, and conclusions.

**Alexander K. Hartmann & Federico Ricci-Tersenghi**


The state landscape appears to be more complex at small positive temperatures than at zero temperature.

**Alexander K. Hartmann & Heiko Rieger**


**Alexander K. Hartmann & Martin Weigt**


“Example: Ising spin glasses”: Frustration index of signed graphs on p. 6. §11.7, “Matchings and spin glasses”: Outlines the matching theory method (cf. Katai and Iwai (1978a) and Barahona (1982a)) for planar graphs, for calculating $l(\Sigma)$ and locating ground states (switchings with fewest negative edges). Also, locating interesting excited states (states
with more than the fewest unsatisfied edges), specifically, domain walls and droplets. A “domain” is generated by negating signs of a set of edges; the vertices whose spins remain the same form one domain and the complement is the other. The increased energy (the “domain wall energy”) has thermodynamic implications. [How to choose the negation set, and what can be the shapes of domain walls, are not obvious.] A “droplet” in a state $s$, vis-à-vis a ground state $s_0$, is a component of the subgraph induced by $(ss_0)^{-1}(-1)$. The sizes of droplets appear to have consequences for thermodynamics. [Annot. 24 Aug 2012.]

A.K. Hartmann & A.P. Young

For unweighted ($\pm J$) and randomly weighted (Gaussian) signed graphs, ground states are computed and compared. The lower critical dimension is different in the two types. [Annot. 22 Jan 2015.]

Nora Hartsfield & Gerhard Ringel

“Cascades”: see Youngs (1968a).

Forough Hassanibesheli, Leila Hedayatifar, Przemysław Gawroński, Maria Stojkow, Dorota Żuchowska-Skiba, & Krzysztof Kułakowski


Kurt Hässig


Ch. 5: “Verallgemeinerte Fluss- und Potentialdifferenzen-probleme.” The lift matroid arises from a side condition, i.e., extra row, added to the incidence matrix of the graph. [The side condition is expressed graphically by additive real gains.] (GN: Incid, M, Bal: Exp, Ref)

Refael Hassin


O. Hatami

See S. Akbari.

Naomichi Hatano

See also E. Estrada.

Naomichi Hatano & James E. Gubernatis


(Phys, sg: State(fr))


A Monte-Carlo investigation of infinite signed cubic lattice graphs at zero temperature, by means of large finite cubic lattices. Are there only two ground states, one the negative of the other, or are there many, unrelated ground states? The paper supports the former. See also Hatano and Gubernatis (2002b). [Having only one ground state (up to sign reversal) means there is only one switching that minimizes $\#E^-$. (Conjecture. That is not true of any signed graph, finite or infinite, except for very special, very regular graphs and signatures.) However, zero temperature may distort the normal behavior of a signed graph.] [Annot. 28 Mar 2013.]

(Phys, sg: State(fr))


D.M. Hatch
See S.T. Chui.

Emilie Haynsworth & A.J. Hoffman

Matrix $A(K_n, \sigma) + I$: properties as quadratic form. Thm.: It is copositive iff $(K_n, \sigma)$ is balanced. Cf. Hoffman and Pereira (1973a). [Annot. 28 May 2017.]

(Phys, kg: adj)

Bian He, Ya-Lei Jin, & Xiao-Dong Zhang

§4: The incidence energy (derived from $Q := L(-\Gamma)$) has a bound like that in Thm. 4.5. [Annot. 21 Jan 2012.]

(Par: Lap: Eig)

Chang-Xiang He & Min Zhou

(Par: Lap: Eig)

Jin-Ling He
See J.-Y. Shao.

Shengjie He & Rong-Xia Hao

Shengjie He, Rong-Xia Hao, & Fengming Dong

Bounds on $r(A(\Phi))$ in terms of matching number $\mu$ and cyclomatic number $\xi$. Thm. 1.10: $2\mu - 2\xi \leq r \leq 2\mu + \xi$. Thms. 1.11–12: Equality characterized. Thm. 1.13: Better bounds in terms of edge bipartitictity and acyclic induced subgraphs. [Annot. 2 Jan 2020.] (GG: Adj)

Shengjie He, Rong-Xia Hao, & Hong-Jian Lai

Bounds on rank: $2\mu - 2\xi \leq r(A(\Sigma)) \leq 2\mu + 2\xi$. Upper and lower bounds are attained by loose cacti with certain circle signs. [Cf. He, Hao, & Dong (2020a).] [Annot. 17 Jan 2019.] (SG: Adj)

Shengjie He, Rong-Xia Hao, & Aimei Yu


Shushan He & Shuchao Li

Patrick Headley

The characteristic polynomials of the Shi hyperplane arrangements $S(W)$ of type $W$ for each Weyl group $W$, evaluated computationally. $S(W)$ is obtained by splitting the reflection hyperplanes of $W$ in two in a certain way; thus $S(A_{n-1})$ splits the arrangement representing $Lat G(K_n)$—more precisely, it represents $Lat^b\{0, 1\}K_n$; that of type $B_n$ splits the arrangement representing $Lat G(\pm K^n)$, and so on. [See also Athanasiadis (1996a).] (gg: Geom, M, Invar)

Brian Healy & Arthur Stein

Describes balance (incorrectly) and clusterability of a signed graph; examines the relevance of, i.a., signed-graphic balance. [Annot. 9 Jun 2012.] (PsS; SG: Bal, Clu: Exp)

Robert W. Heath Jr.
See T. Strohmer.

Saroja R. Hebbar
See P.R. Hampiholi.
Guy Hed, Alexander K. Hartmann, Dietrich Stauffer, & Eytan Domany
Proposes an intermediate structure of ground states (switchings with smallest $\#E^-$) of a signed graph (“Ising spin glass” with edge weights $\pm 1$), not ultrametric (cf. Hartmann (1998a)) but “hierarchical”. [Annot. 11 Jan 2015.] (SG: State(fr), Phys)

Leila Hedayatifar
See F. Hassanibesheli.

Rajneesh Hegde
See A. van Zuylen.

Pinar Heggernes
See H.L. Bodlaender.

Fritz Heider
1946a Attitudes and cognitive organization. J. Psychology 21 (1946), 107–112.
No mathematics, but a formative article. Precursor and inspiration of Harary (1953a) and Cartwright and Harary (1956a) (q.v.). Signed digraphs (at most 4 vertices) and signed vertices are unsystematically present but there is no graph theory. Complicated by multiple types of vertices and edges [dispensed with by Cartwright–Harary]. [Annot. 15 Oct 2018.] (PsS, sd, vs: Bal)

How Heider came to his balance theory (1946a) and what it means. Remarks on the Cartwright–Harary (1956a) theory. [Annot. 1 Oct 2018.] (PsS)(SG: Bal)

E. Heilbronner

Matthias Hein
See P. Mercado.

Peter Christian Heinig

Richard V. Helgason
See J.L. Kennington.
Pavol Hell
See R.C. Brewster.

I. Heller

I. Heller & C.B. Tompkins

Thm.: The incidence matrix of a signed graph where all edges are links is totally unimodular iff the signed graph is balanced. (Not stated in terms of signed graphs.) See also Hoffman and Gale (1956a), Hoffman (1960a), and Reichmeider (1984a). (sg: Incid, Bal)

Marc Hellmuth
See T. Biyikoglu.

P.S. Hemavathi

J.L. van Hemmen

§2.3, “Frustration”: Physics of Ising models with edges (“bonds”) that are positive, negative, or of undetermined sign. [Annot. 16 Jun 2012.] (Phys: sg)

Robert L. Hemminger & Joseph B. Klerlein

An attempt, intrinsically unsuccessful, to represent the (signed) line graph of a digraph (see Zaslavsky (20xxa)) by a digraph. [Continued by Klerlein (1975a).] (sg: LG, ori)

Robert L. Hemminger & Bohdan Zelinka

Anthony Henderson

The subposets are $Q_n^{1 \mod d}(\emptyset)$ where $d > 1$, whose elements are the flats $A \in \text{Lat} G(\emptyset K_n)$ such that $d$ divides the order of the unbalanced part and the number of vertices every balanced component is $\equiv 1 \mod d$. (gg: M: Aut)

Michael Henley
See F. Ardila.
Cheolwon Heo
(SG: M, D: Alg)

Rainer Herges
A Möbius molecule has a half-twist in a ring structure [hence can be modeled by an unbalanced signed graph; cf. Heilbronner (1964a), Gutman (1978a) et al., Trinajstić (1983a), (1992a)]. A survey of specific types of Möbius molecules. §3.1.4, “Other Möbius systems": Rzepa (2005a) et al., Craig, and also Fowler (2002a) have proposed that certain annulenes are intrinsically twisted (i.e., unbalanced) due to the d-orbital or p-orbital structure. [Annot. 23 Nov 2012.]
(Chem: sg: bal: Exp, Ref)

Patricia Hersh & Ed Swartz
Remark 19: Chromatic polynomials of signed graphs vis-à-vis subarrangements of the root system arrangement $B_n$ in Thm. 18, which gives properties of an $h$-vector. [Annot. 1 Mar 2011.]
(SG: Invar)

Daniel Hershkowitz & Hans Schneider
Bipartite $\Sigma$ such that every matrix with sign pattern $\Sigma$ has the same rank, over each field $\neq \mathbb{F}_2$. [Annot. 6 Mar 2011.]
(SG: QM)

J.A. Hertz
See K.H. Fischer.

Gábor Hetyei
See Y. Diao.

Chris Heunen & Vaia Patta
§7.3, “Graphs”, mentions possible bicircular, frame, and lift functors.
[Annot. 2 Oct 2018.]
(gg:M, Bic)

Hector Hevia
See G. Chartrand.

Farideh Heydari
See S. Akbari and M. Souri.

Takayuki Hibi
See also T. Matsui and H. Ohsugi.

Takayuki Hibi, Nan Li, & Yan X Zhang
(sg: par: Geom)
Takayuki Hibi, Aki Mori, Hidefumi Ohsugi, & Akihiro Shikama

[This is the antibalanced case. *Problem.* Generalize to signed graphs, including balanced graphs.] (sg: Par: Geom)

Takayuki Hibi, Kenta Nishiyama, Hidefumi Ohsugi, & Akihiro Shikama

Antibalanced graph criteria. [*Problem.* Generalize to signed graphs.]
[Annot. 5 Oct 2014.] (sg: Par: Algeb)

Akihiro Higashitani
See T. Matsui.

Desmond J. Higham
See E. Estrada.

Yusuke Higuchi & Iwao Sato

“Balance” of \((D, \sigma) = \text{cycle balance (cf. B.D. Acharya (1980a)).} \) (SD: Bal)

Timo Hiller

Franziska Hinkelmann
See A. Veliz-Cuba.

K. Hinson
See Y. Diao.

André Hirschowitz
See M. Hirschowitz.

Michel Hirschowitz, André Hirschowitz, & Tom Hirschowitz

Tom Hirschowitz
See M. Hirschowitz.

Petr Hliněný
See R. Ganian.

Tuyen-Thanh-Thi Ho, Hung Thanh Vu, & Bac Hoai Le
Given $\vec{\Gamma}$ and signs on $E(\vec{\Gamma}) \setminus (u,v)$, $\sigma(u,v)$ is predicted by signed in-degrees of $v$ and signed out-degrees of $u$. Justified for social networks by appeal to psychological traits. [Annot. 22 Jan 2015.]

(DS: Pred: Alg: PsS)

Dorit S. Hochbaum


Integer programs with constraints of a generalized real gain-graphic form, $\alpha x - \beta y - \gamma \leq z$, the gain being $\beta/\alpha$. Slightly extends Hochbaum, Megiddo, Naor, and Tamir (1993a). (gn: Incid(D): Alg)


Integer programs as in (1998a) with “budget constraints”. (gn: Incid(D): Alg)


Integer programs as in (1998a). There is a polynomial-time solution via a minimum cut, or else a half-integral partial solution. (gn: Incid(D): Alg)


Constraints of a generalized positive-real gain-graphic form, $\alpha x - \beta y - \gamma \leq z$, the gain being $\beta/\alpha$, contrasting $\alpha, \beta \geq 0$ to the intrinsically hard case where a negative coefficient is allowed but a half-integral approximate solution is easy. (gn: Incid(D): Alg)

Dorit S. Hochbaum, Nimrod Megiddo, Joseph (Seffi) Naor, & Arie Tamir


Approximate solution of integer linear programs with real, dually gain-graphic coefficient matrix. [See Sewell (1996a).] (GN: Incid(D): Alg)

Dorit S. Hochbaum & Joseph (Seffi) Naor


Linear and integer programs with real, dually gain-graphic coefficient matrix: feasibility for linear programs, solution of integer programs when
the gains are positive (“monotone inequalities”), and identification of “fat” polytopes (that contain a sphere larger than a unit hypercube).

(GN: Incid(D): Alg, Ref)

Winfried Hochstättler
See also L. Goddyn.

Winfried Hochstättler, Robert Nickel, & Britta Peis

Incidence matrix used to find the circles in slow polynomial time. Use of graphic structure is explored.

(SG: Str: Circ: Alg, Incid)

Hervé Hocquard
See F. Foucaud.

Cornelis Hoede

(PsS: Gen)(SG, VS: Bal)


Teil 4: “Kognitive Konsistenz.” (PsS: Gen: Exp)


Characterizes when one can sign the vertices of a graph so every circle has positive sign product, solving the problem of Beineke and Harary (1978b). Given Γ, μ : V → {+, −}, and a spanning tree T: (Γ, μ) is consistent iff the fundamental circles with respect to T are positive and the endpoints of the intersection of two fundamental circles have the same sign. A polynomial-time algorithm ensues. [The definitive word until Joglekar, Shah, and Diwan (2010a). Does not include signed edges.]

[Annot. rev. 11 Sept 2010, 2 May 2012.]

(VS: Bal: Str)

Jan B. Hoek
See B.N. Kholodenko.

P. Hoever, W.F. Wolff, & J. Zittartz

Physics of Ising models on a planar square lattice. Exact solutions for partition function, free energy, ground state energy. The transition temperature depends only on the average edge sign, (#E⁺ − #E⁻)/#E. Switching is implicit (“substituting spins”). Model (a): all horizontal edges are + (attainable by switching); if horizontally periodic these are “random layered frustration” models. Model (b): Assumed switched to minimize #E⁻. Dictionary: “plaquette” = quadrilateral, “frustration index” = sign of plaquette.

They conjecture thermodynamic consequences if the ground states (s : V → {+1, −1} with l(Σ) frustrated edges) are connected in the
state graph \(\{+1, -1\}^V\). [Question. For which \(\Sigma\) are the ground states connected?]

(Phys: SG: sw)

Peter D. Hoff


(SG, PsS: Bal)

Alan J. Hoffman

See also D.R. Fulkerson, D. Gale, and E. Haynsworth.


(Par: Eig, Fr)


(LG)


(Par: Eig, Fr)


Eigens of signed complete graphs.

(Par: Eig, Fr)


(TG, Eig)


Abstract of (1977b). Also, bounding the least eigenvalue in terms of principal submatrices.

(SG: LG)


Introduces generalized line graphs. [They are the reduced line graphs of signed graphs of the form \(-\Gamma\) with any number of negative digons attached to each vertex; see Zaslavsky (2010b), Ex. 7.6; (20xxa)]. (LG)


\(\Sigma\) is a signed simple graph. Let \(\lambda\) be the least eigenvalue of \(A(\Sigma)\). Can (*) \(A(\Sigma) - \lambda I - KK^T\) be zero for some \(K\) with all entries 0, \(\pm 1\)? When \(\lambda = -2\), \(K\) exists [equivalently, \(\Sigma\) is a reduced line graph of a signed graph; *cf.* Zaslavsky (2010b), (20xxa)], with finitely many exceptions; the proof uses root systems; *cf.* Cameron, Goethals, Seidel, and Shult (1976a). In general, no \(K\) may give zero, but the minimum, over all \(K\), of the largest element of (*) is bounded by a function of \(\lambda\).

(SG: LG: Adj, Eig)
[A.J. Hoffman & D. Gale]

Alan J. Hoffman & Peter Joffe

Alan J. Hoffman & Francisco Pereira

Dean Hoffman & Heather Jordon
The net degree of a vertex in $\Sigma$ is $d^+(v) - d^-(v)$. [This is best viewed as degree in an all-negative bidirected graph; cf. p. 35.] Thms. 2.3 (for $\Sigma$) and 4.1 (for a bidirected graph $B$, called a “mixed signed graph”) are an interesting $f$-factor theorem in terms of net degrees. Thm. 4.1: Given $f : V \to \mathbb{Z}$, an “$f$-factor” is a subgraph whose net in-degree vector $= f$. For disjoint $S, T \subseteq V$ and a component $Q$ of $B \setminus (S \cup T)$, $J(Q, S, T)$ is computed in terms of $f$ and in-degrees and out-degrees of edges among $Q, S, T$. $q(S, T)$ is the number of $J$-odd components $Q$. An $f$-factor exists iff $q$ satisfies an inequality. Thm. 3.2: Fixing the maximum edge multiplicity, an Erdős–Gallai-type characterization of net degree sequences—simplifying the theorem of Michael (2002a). Thm. 4.2: Net in-degree sequences of bidirected simple graphs. [More in Jordon, McBride, and Tipnis (2009a).] [Annot. 14 Oct 2009.] (SG: ori: Invar)

Thomas R. Hoffman
See also D.M. Duncan.

Thomas R. Hoffman & James P. Solazzo
2012a Complex equiangular tight frames and erasures. Linear Algebra Appl. (gg: kg: Adj)

Karl Heinz Hoffmann
Expository, accessible. Main example is the Ising model: fixed weighted $\Sigma$ and variable $\zeta : V \to \{\pm 1\}$ with Hamming distance and energy func-
tion $e : \{\pm 1\}^V \to \mathbb{R}$ which is mountainous, i.e., many local minima (valleys) with low or high intermediate values $e(\zeta)$. Ground states (minimizing $e(\zeta)$) are therefore hard to find computationally. [Cf., e.g., Vogel et al.] Random “thermal variation” of $\zeta$ leads to slow “relaxation” from higher to lower local minima, in theory and practice. [Annot. 7 Aug 2018.] (sg, wg: VS: Str, Phys, Appl: Exp)

Franz Höfting & Egon Wanke

Given a finite gain digraph $\Phi$ (the “static graph”) with gains in $\mathbb{Z}^d$ and a rational cost for each edge, find a minimum-cost walk (“path”) in its canonical covering graph $\tilde{\Phi}$ with given initial and final vertices.


Take a gain digraph $\Phi$ (the “static graph”) with gains in $\mathbb{Z}_\alpha = \mathbb{Z}_{\alpha_1} \times \cdots \times \mathbb{Z}_{\alpha_d}$ (where $\alpha = (\alpha_1, \cdots, \alpha_d)$) and its canonical covering digraph $\tilde{\Phi}$ (the “toroidal periodic graph”). Treated algorithmically via integer linear programming and linear Diophantine equations: existence of directed paths (NP-complete, but polynomial-time if $\Phi$ is strongly connected) and number of strongly connected components of $\tilde{\Phi}$.


Full version of (1993a). The min-cost problem is expressed as an integer linear program. Various conditions under which the problem is NP-hard, even a very restricted version without costs (Thms. 3.3, 3.5), or polynomial-time solvable (e.g.: without costs, when $\Phi$ is an undirected gain graph: Thm. 3.4; with costs, when $d$ is fixed: Thm. 4.5).


Full version of (1994a).

Leslie Hogben
See also S. Butler.


Paul W. Holland & Samuel Leinhardt

A formulation of structure in terms of weak partial ordering, i.e., transitivity, hence triads (triples of elements). Refers to (1970b) for specific structures, e.g., structural balance of Cartwright–Harary (1956a) and clustering of Davis (1967a). The types of triples allowed determine what specific model applies. P. 495 states conditions for structural balance or clustering. [Annot. 26 Dec 2012.] (PsS: sg: Bal, Clu)


[Hein van der Holst] See H. van der Holst (under ‘V’).

Hai-Yan Hong See Y.-Z. Fan.

Sungpyo Hong See J.H. Kwak.

Yiguang Hong See D.-Y. Meng.


Thm. 2: If some neighbors of $v$ in $\Gamma$ are regrafted onto $u$, forming $\Gamma'$, and if $x_u \geq x_v$ in the Perron vector of $L(-\Gamma)$, then $\lambda_{\min}(L(-\Gamma)) < \lambda_{\min}(L(-\Gamma'))$. [Annot. 24 Jan 2012.] (par: Lap: Eig)


§6, “Spectrum and expansion in lifts of graphs”; covering graphs of permutation gain graphs, and from Bilu and Linial (2006a) of signed graphs. §6.1, “Covering maps and lifts”: Covering graphs of permutation gain graphs, presented as symmetric digraphs with invertible arc gains. §2.6,
“Eigenvalues - old and new”: Prop. 6.3. The covering graph’s eigenvalues include those of the (underlying) base graph \( \Gamma \) and its eigenvectors sum to 0 on fibers. Prop. 6.4. The signed covering graph’s eigenvalues are those of \( \Gamma \) and those of \((\Gamma, \sigma)\). §6.4, “Nearly-Ramanujan graphs by way of 2-lifts”: Conjectured and proven eigenvalue ranges when the base graph is a Ramanujan graph. Dictionary: “signing” of \( A(\Gamma) \) means \( A(\Gamma, \sigma) \) for any edge signature. “2-lift” = double covering graph. [Annot. 25 Aug 2011.] (sg: Cov, Eig: Exp)

John Hopcroft
See T. Joachims.

Tsuyoshi Horiguchi
See also O. Nagai.


On the square lattice, physical quantities for periodic signed graphs with up to four edge weights. A fairly general model of which several previous ones are special cases. [Annot. 22 Jan 2015.] (Phys: SG, WG)

A. Hosseiny
See A. Kargaran.

Jiangyou Hou
See D. Hu.

Ting Hou
See H. Gao and X.-Z. Liu.

Yaoping Hou
See also D.-J. Wang.


Yaoping Hou, Jiongsheng Li, & Yongliang Pan

Properties of (mainly) largest eigenvalue \( \lambda_{\text{max}}(\Sigma) \) of the Laplacian matrix \( L(\Sigma) \) of a signed simple graph. Thms. 2.5–2.6 repeat standard criteria for balance [with a sign error in (3) of each]. Main results:

Upper bounds, all in terms of underlying graph: Lemma 3.1: For connected \( \Gamma \), \( \lambda_{\text{max}}(\Gamma, \sigma) \leq \lambda_{\text{max}}(-\Gamma) \), = iff \( \sigma \) is antibalanced (e.g., \( -\Gamma \)). Thm. 3.4: \( \lambda_{\text{max}}(\Sigma) \leq 2(n-1) \), = iff \( \Sigma \sim -K_n \). Thm. 3.5: \( \lambda_{\text{max}}(\Sigma) \leq (1) \text{max edge degree} + 2, (2) \text{max(vertex degree + average neighbor degree)}, (3) \text{a combination of these degrees}; = iff \( \Sigma \) is antibalanced and \( |\Sigma| \) is semiregular bipartite.

Lower bounds: Cor. 3.8: \( \lambda_{\text{max}}(\Sigma^+) + \lambda_{\text{max}}(\Sigma^-) \geq \lambda_{\text{max}}(\Sigma) \geq \lambda_{\text{max}}(\Sigma^+) \). \( \lambda_{\text{max}}(\Sigma^-) \). Thm. 3.9: If \( \Sigma \) has a vertex of degree \( n-1 \), then \( \lambda_{\text{max}}(\Sigma) \geq \lambda_{\text{max}}(+|\Sigma|) \), with equality iff \( \Sigma \) is balanced. Thm. 3.10: \( \lambda_{\text{max}}(\Sigma) \geq 1 + \max_v d_{|\Sigma|}(v) \).
Interlacing: Lemma 3.7 (special case): \( \lambda_i(\Sigma) \geq \lambda_i(\Sigma \setminus e) \geq \lambda_{i+1}(\Sigma) \), where \( \lambda_1 \geq \lambda_2 \geq \cdots \).

Problems about existence of cospectral unbalanced signed graphs.

Yaoping Hou, Zikai Tang, & Dijian Wang


Classifies those with maximum degree \( \leq 4 \) (all are regular, by Ramezani (20xxa)). For 3-regular there are \( +K_4< -K_4 \), and \( Q_3 \) as in Ghasemian and Fath-Tabar (2017a). For 4-regular there are \( +K_5, -K_5 \), and triangle-free examples \( Q_4 \) with all \( C_4 \)'s negative, \( S_{14} \), and for \( n \geq 3 \), \( T_{2n} \) (with eigenvalues \( \pm 2 \)), contrary to Ghasemian and Fath-Tabar. For eigenvalues in \([-2, 2]\) they give an elementary proof of part of McKee and Smyth (2007a). [Annot. 29 May 2018, rev 3 Feb 2021.] (SG: Eig)


Classifies those with maximum degree \( \leq 4 \). The irregular ones are not obtained from (2019a). [Annot. 3 Feb 2021.] (SG: Adj: Eig)

Yao Ping Hou & Li Juan Wei


Combinatorial proof of an explicit formula for \( W_{k} \) [possibly the standard one?]. Studies “associated numbers” \( W_{k}^r \). Proved: \( W_{n-k} \leq W_{k} \) for \( k \leq 3 \) [this must be an error for \( W_{k} \leq W_{n-k} \) and must have some restriction on \( n \); well known for \( k = 1 \)].

C.H. Houghton


Cf. Rees (1940a), Graham (1968a). (gg: M: Invar)

R.M.F. Houtappel


Ising spins, i.e. \( \zeta : V \rightarrow \{+1, -1\} \), in the triangular and honeycomb (hexagonal) lattice graphs on a torus. Different edge weights (“bond strengths”) and signs are allowed in the three directions. The all-negative triangular signature (i.e., “antiferromagnetic” with equal weights) is an exceptional case. Switching the triangular lattice (p. 449, bottom) permits assuming that two chosen directions are positive. Exceptional weights are the antibalanced triangular lattice with equal smaller weights, e.g., all weights equal (p. 449, bottom). The honeycomb
cannot be exceptional [because it is balanced] (p. 451). [See also Newell (1950a), I. Syôzi (1950a), Wannier (1950a).] [Annot. 20 Jun 2012.]

(Phys, WG, sg: sw)

Cho-Jui Hsieh
See also K.-Y. Chiang.

Cho-Jui Hsieh, Kai-Yang Chiang, & Inderjit S. Dhillon

(SG: Bal, Clu: Alg)

L. Hsu
See E. Kaszkurwicz.

Bin Hu
See also C.-C. Huang.

Bin Hu, Zhi-Hong Guan, Xiao-Wei Jiang, Ming Chi, Rui-Quan Liao, & Chao-Yang Chen

(SG: Clu: Alg)

Dan Hu, Hajo Broersma, Jiangyou Hou, & Shenggui Zhang
2021a On the spectra of general random mixed graphs. Electronic J. Combin. 28 (2021), no. 1, article P1.3.

Random \{1,\pm 1\}-gain graphs. Probabilistic upper bound. Approximation of normalized Laplacian spectrum. [Annot. 8 Feb 2021.]

(gg: Rand: Adj, Lap: Eig)

Dan Hu, Xueliang Li, Xiaogang Liu, & Shenggui Zhang

A random \{1,\pm i\}-gain graph is \(\Phi_n\) with probabilities on each edge of \(p^2\) for gain 1, \(p(1-p)\) for each gain \(\pm i\), and \((1-p)^2\) for no edge, where \(p = p(n) \in (0,1)\). Thm. 2: The limiting spectral distribution of \(\Phi_n\) · 

\[
\frac{1}{\sqrt{np(2-p-p^3)}}
\]

as \(n \to \infty\) is the standard semicircle distribution. [Annot. 15 Dec 2020.]

(gg: Rand: Adj: Eig)

Guang Hu & Wen-Yuan Qiu


(sg: Top, Chem)

Jiangping Hu
See Y.-Z. Wu and Y.-L. Zhou.

Lili Hu & Xiangwen Li

(SG: Col)

**Wentao Hu**  
See Q.M. Guo.

**Bobo Hua**  
See F.M. Atay.

**Hongbo Hua**  
Fix $n \geq 13$. For connected unicyclic $\Gamma$ such that $-\Gamma$ is balanced, excluding circles and balloons (“tadpoles”, “lollipops”), the maximum energy occurs for a hexagon attached by an edge to the third vertex of a path. *Problem*. Replace “bipartite” by “signed”, i.e., allow unbalanced signed graphs.] [Annot. 24 Jan 2012.] (par: Lap: Eig)

**Chuanchao Huang, Bin Hu, Ruixian Yang, & Guangmei Wu**  

**Hao Huang**  
Lemma 2.2 has $A(Q_n,\sigma)$ in which every quadrilateral is negative. Lemma 2.3 is the [previously known] bound $\lambda_{\text{max}}(\Sigma) \leq \Delta(|\Sigma|)$. [Annot. 31 Jul 2019.] (sg: Adj, Eig)

**He Huang**  
See H.Y. Deng.

**Jin Huang**  
See C. Chen.

**Jing Huang, Shuchao Li, & Hua Wang**  
Skew adjacency matrix $= i^{-1}A(\Phi)$ where gain graph $\Phi$ has $\varphi(\overrightarrow{v_iv_j}) = i$. [Annot. 15 Jul 2019.] (gg: Adj)

**Qiongxiang Huang**  

**Rong Huang, Jianzhou Liu, & Li Zhu**  

**Ting-Zhu Huang**  

**Yihua Huang**  
See Y.-B. Gao.
Yufei Huang
See also C.H. Liang.

Yufei Huang, Bolian Liu, & Siyuan Chen

Yufei Huang, Bolian Liu & Yingluan Liu

The largest spectral radius and the extremal graphs. [Annot. 19 Nov 2011.] (par: Lap: Eig)

B.A. Huberman
See E. Fradkin.

Falk Hüffner
See also S. Böcker.

Falk Hüffner, Nadja Betzler, & Rolf Niedermeier

An improved algorithm for frustration index of $-\Sigma$. Dictionary: “balanced” = antibalanced, i.e., balance of $-\Sigma$; “2-coloring that minimizes inconsistencies with given edge labels” = switching function that minimizes $|E^-(-\Sigma)|$. [Annot. 10 Sept 2011.] (SG: Fr: Alg)


Dictionary: “balanced” = antibalanced, i.e., balance of $-\Sigma$. (SG: Fr: Alg)

Falk Hüffner, Christian Komusiewicz, & André Nichterlein

Florian Hug
See I.E. Bocharova.

JiSun Huh, Sangwook Kim, & Boram Park

Unifies toric ideals of graphs and digraphs (cf. Villarreal (1995a), Ohsugi and Hibi (1999a), Reyes, Tatakis, and Thoma (2012a), et al.) by generalization to bidirected graphs B (Beta). A positive closed walk $W = e_1 \cdots e_t$ is broken at incoherent vertices into segments $W_1, \ldots, W_k$ ($k \geq 1$) and gives binomial $B_W := \prod E(W_{\text{odd}}) - \prod E(W_{\text{even}})$, where $W_{\text{odd}} := W_1 \cup W_3 \cup \cdots$ and $W_{\text{even}} := W_2 \cup W_4 \cup \cdots$. The toric ideal $I_B \subseteq \mathbb{Z}[E]$ is generated by those binomials.
§3.1, “Primitive binomials”. Thm. 3.1 (restated): $B_W$ is primitive iff $W$ is an edge-simple walk that consists of a cactus whose outer circles are negative and inner circles are positive.

[See Reyes, Tataakis, and Thoma (2012a) for two proposed signed-graph problems.]

Dictionary: “signed graph” = bidirected graph $B$; “balanced vertex” in a walk = coherent vertex; “even-, odd-signed” walk = positive, negative walk; $A(B) = \text{incidence matrix } H$; $r(B) = \text{nul } H$.

Axel Hultman

§5, “A Dowling generalization”. (gg: M)


Introduces Dowling trees: “Natural Dowling analogues of the complex of phylogenetic trees”.

Axel Hultman & D. Klingman

John Hultz
See also F. Glover.

John Hultz & D. Klingman

Norman P. Hummon & Patrick Doreian

Presents a model for evolution of balance and clusterability (as in Davis (1967a)) of a signed digraph and explores it via computer simulations.

Definitions: Given a signed digraph $\Sigma$ and a partition $\pi$ of $V$, define the ‘clusterability’ $c(\Sigma, \pi) := (\# \text{ negative edges within blocks of } \pi) + (\# \text{ positive edges between blocks})$. Define $\pi(\Sigma) := \text{any } \pi \text{ that minimizes } c(\Sigma, \pi)$. Define $\Sigma(v_i) := \{v_i \bar{v}_j \in E(\Sigma)\}$ with signs. ($\Sigma$ models relations in a social group $V$. $\Sigma_i$ is the graph of relations perceived by $v_i$.)

Initial conditions: Fixed $\# V$, fixed “contentiousness” $p := \text{the probability that an initial edge is negative}$, a fixed “communication” rule, random $\Sigma^0$ and, for each $v_i \in V$, $\Sigma^0_i := \Sigma^0$. At time $t + 1$, $\Sigma^t_i(v_i)$ changes to $\Sigma^{t+1}_i(v_i)$ to minimize $d(d(\Sigma^{t+1}_i, \pi(\Sigma^t)))$. Then $\Sigma^{t+1}_j(i)$ changes to $\Sigma^{t+1}_i(v_i)$ for some $v_j$ (depending on $\Sigma_i$ and the communication rule).

Computer simulations examined the types of changes and emerging clusterability of $\Sigma'$ or $\Sigma'_t$ as $t$ increases, under four different communica-
tion rules, random initial conditions with various $p$, and $\#V = 3, 5, 7, 10$. The outcomes are highly suggestive (see §4; $p$ seems influential). [Problem. Predict the outcomes in terms of initial conditions through a mathematical analysis.] [Annot. 26 Apr 2009.]

(SD, sg: Bal, Clu: Alg)(PsS)

Norman P. Hummon & T.J. Fararo

David J. Hunter


(SG: Bal, Exp)

John E. Hunter

Bofeng Huo, Shengjin Ji, Xueliang Li, & Yongtang Shi

[Question. Do the results generalize to $A(\Sigma)$ for antibalanced signed bicyclic graphs?] [Annot. 8 Sep 2016.]

(par: Adj: Eig)

Bofeng Huo, Xueliang Li, & Yongtang Shi

[Question. Do the results generalize to $A(\Sigma)$ for antibalanced signed unicyclic graphs?] [Annot. 21 Mar 2011.]

(par: Adj: Eig)

Li Fang Huo & Yu Bin Gao

(SD: Adj)

C.A.J. Hurkens

Given: a bidirected graph $B$ (with no loose or half edges or positive loops) and an integer weight $b_e$ on each edge. Wanted: an integral vertex weighting $x$ such that $H(B)^T x \leq b$, where $H(B)$ is the incidence matrix. Such $x$ exists iff (i) every coherent circle or handcuff walk has nonnegative total weight and (ii) each doubly odd Korach walk (a generalization of a coherent handcuff that has a cutpoint dividing it into two parts, each with odd total weight) has positive total weight. This improves a theorem of Schrijver (1991a) and is best possible. Dictionary: “path”
(“cycle”) = coherent (closed) walk. (sg: Ori: Incid)

David A. Huse
See C.K. Thomas.

Joan Hutchinson
See D. Archdeacon.

Daniel Huttenlocher
See J. Leskovec.

Tony Huynh
See also J. Geelen.

Gains are in $GF(q)^\times$ or sometimes in a finite abelian group. Dictionary: “group-labelled graph” = gain graph, “Dowling matroid” = frame matroid (not Dowling geometry), “shifting” = switching. (GG: M)

Tony Huynh, Felix Joos, & Paul Wollan

Double gain graph: gains in $\mathcal{G}_1 \times \mathcal{G}_2$. Doubly nonneutral circle: both gains $\neq 1$. (GG: Circ: Str)

Tony Huynh, Andrew D. King, Sang-Il Oum, & Maryam Verdian-Rizi

\( \Gamma \) is “strongly even cycle decomposable” iff every signed graph \((\Gamma, \sigma)\) decomposes into positive circles (confusingly called “even cycles”). Dictionary: cf. Huynh, Oum, and Verdian-Rizi (2017a). [Annot. 26 Dec 2012.] (SG: Circ)

Tony Huynh, Sang-il Oum, & Maryam Verdian-Rizi

Despite the name, decomposability of \( \Sigma \) into (edge-disjoint) positive circles. Dictionary: “even” = positive, “odd” = negative (hence, unnecessary confusion), “even-length” = even, “re-signing” = switching, “parity” of vertex = parity of \( d^-(v) \). [Annot. 26 Dec 2012.] (SG: Circ)

Dawn Iacobucci


Giovanni Iacono
See also G. Facchetti and N. Soranzo.

Giovanni Iacono & Claudio Altafini
2010a Monotonicity, frustration, and ordered response: an analysis of the energy landscape of perturbed large-scale biological networks. BMC Systems Biol. 4 (2010),
G. Iacono, F. Ramezani, N. Soranzo, & C. Altafini

Toshihide Ibaraki
See also Y. Crama and P.L. Hammer.

T. Ibaraki & U.N. Peled

Takashi Iino
See T. Yoshikawa.

Takeo Ikai
See H. Kosako.

Yoshiko T. Ikebe & Akihisa Tamura

Rintaro Ikeshita & Shin-Ichi Tanigawa

Victor Il’ev, Svetlana Il’eva, & Alexander Kononov

Svetlana Il’eva
See V. Il’ev.

Aleksandar Ilić
See also L.H. Feng, G.H. Yu, and B. Zhou.

Denis Petrovich Ilyutko
See V.O. Manturov.

Nicole Immorlica
See E. Demaine.

Takehiro Inohara
(SD, PsS)

(SD, PsS)


Assumption: all $\sigma(i,i) = +$. Thm. 3: A signed complete digraph is clusterable iff $\sigma(i,j) = -$ or $\sigma(j,k) = \sigma(i,k)$ for every triple $\{i,j,k\}$ of vertices (not necessarily distinct). [The notation is unnecessarily complicated.]

(SD: KG: Bal, Clu, PsS)

(SD: Clu, PsS)

(SD: Clu, PsS)

(SD, PsS)

**Takehiro Inohara, Shingo Takahashi, & Bunpei Nakano**

(SD, PsS)

(SD, PsS)

**Jun-ichi Inoue**

See S. Suzuki.

**Yuri J. Ionin & Mohan S. Shrikhande**


§7.3, “Switching in strongly regular graphs”: Graph switching and two-graphs. 
(TG, Sw: Exp)

**Ali Iranmanesh**

See J. Askari.

**Masao Iri**

See also J. Shiozaki.

**Masao Iri & Katsuaki Aoki**

Masao Iri, Katsuaki Aoki, Eiji O’Shima, & Hisayoshi Matsuyama

1976a [A graphical approach to the problem of locating the system failure.] (In Japanese.) 76(135) (1976), 63–68. (SD, VS: Appl)


The process is modelled by a signed digraph with some nodes $v$ marked by $\mu(v) \in \{+, -, 0\}$. (Marks $+$, $-$ indicate a failure in the process.) Object: to locate the node which is origin of the failure. An oversimplified description of the algorithm: $\mu$ is extended arbitrarily to $V$. Arc $(u, v)$ is discarded if $0 \neq \mu(u)\mu(v) \neq \sigma(u, v)$. If the resulting digraph has a unique initial strongly connected component $S$, the nodes in it are possible origins. Otherwise, this extension provides no information. (I have overlooked: special marks on “controlled” nodes; speedup by stepwise extension and testing of $\mu$.) [Continued in Shiozaki, Matsuyama, O’Shima, and Iri (1985a).] [This article and (1976a) seem to be the origin of a whole literature. See e.g. Chang and Yu (1990a), Kramer and Palowitch (1987a)] (SD, VS: Appl, Alg)

Lucas Isenmann & Timothée Pecatte


Signed plane graph $\Sigma$ is orientation embedded via the plane vertex rotations in surface $S$. Thm.: All single-face embeddings are connected via two operations: negating certain edges, and simultaneously negating certain edge pairs. The edges involved depend on the 1-face walk. Dictionary: “Möbius edge” = negative edge; “painting walk” = face walk in $S$. “Möbius stanchion system (MSS)” = $\Sigma$ with single-face embedding in $S$ [cf. Širáň and Škoviera (1991a)]. [Annot. 2 Nov 2017.] (sg: Top)

Toru Ishihara


A new proof of Cameron (1994a). (TG)


A signed graph corresponding to a base of $A_n$ is a [signed] path of cliques and locally switches to a path. (For local switching see Cameron, Seidel, and Tsaranov (1994a).) (SG: Geom)(SG: Sw: Gen)


Which signed graphs locally switch to a tree? Examples only. (SG: Sw: Gen)

Converting an induced circle to a path by local switching.

(SG: Sw: Gen)


Local switching between trees. [Annot. 28 Dec 2011.]

(CSG)

Sorin Istrail


Extends Barahona (1982a) on finite signed lattice graphs to the computational complexity of (a) ground states (i.e., frustration index) and (more difficult) (b) partition function (generating function of frustrated edges over all states), for signed infinite lattice graphs. [An infinite lattice graph is (apparently) a graph drawn in $E^d$, crossings allowed, that has translational symmetry in $d$ independent directions.] General conclusion: For nonplanar ones they are NP-hard. Thm. 1: A lattice graph in $d = 2, 3$ is planar iff it does not contain a certain $d = 2$ lattice graph $K_0$, the “Basic Kuratowskian”. Lem. 2: Every 3-regular graph has a subdivision contained in $K_0$. §5, “Computational complexity of the 3D Ising models”: Lattice graphs with signs, subgraphs thereof, all-positive subgraphs, all-negative subgraphs. Thm. 2: For every subgraph of a signed non-planar infinite lattice graph, computing $l(\Sigma)$ for finite sublattice graphs $\Sigma$ is NP-hard. Thm. 3: For every subgraph of an all-negative non-planar infinite lattice graph, computing $l(\Sigma)$ for finite sublattice graphs $\Sigma$ is NP-hard. §5.3, “Ising models with $\{-J, +J\}$ interactions”: For every signed non-planar infinite lattice graph, computing $l(\Sigma)$ for finite sublattice graphs $\Sigma$ is NP-hard; the proof is postponed to “the full version of the paper” [which has not appeared]. [Annot. 21 Aug 2012.]

(SG, Phys: Fr)

Gabriel Istrate


Imbalance measured by triangles. Repeatedly change signs of edges of a fixed graph. Looks for recurrent states and time to become balanced. [Annot. 5 May 2010.]

(SG: Fr: Dyn)

C. Itzykson

See R. Balian.

P.L. Ivanescu [P.L. Hammer]

See E. Balas and P.L. Hammer.

Sousuke Iwai

See also O. Katai.

Sousuke Iwai & Osamu Katai

1978a Graph-theoretic models of social group structures and indices of group structures. (In Japanese.) *Systems and Control (Shisutemu to Seigyo)* 22 (1978),
Jaeger shows that the Kauffman polynomial, originally defined for link diagrams and here transformed to an invariant of signed plane graphs, depends only on the edge signs and the circle matroid. It can also be reformulated to be essentially independent of signs. Problem. Define a similar invariant for more general matroids.

(SGc, Sgnd(M): Invar, Knot)
Mahdi Jalili  
See P. Esmailian, A. Javari, and M. Shahriari.

Hye Jin Jang, Jack Koolen, Akihiro Munemasa, & Tetsuji Taniguchi  
2014a On fat Hoffman graphs with smallest eigenvalue at least $-3$. 
(SG: Eig)

Abdul Salam Jarrah  
See E. Sontag.

John J. Jarvis & Anthony M. Jezior  
1972a Maximal flow with gains through a special network. 
(GN: M(bases))

A. Javanmard  
See S. Akbari.

Amin Javari & Mahdi Jalili  
2014a Cluster-based collaborative filtering for sign prediction in social networks with positive and negative links. 
(SG: Clu, Pred: Alg)

M. Javarsineh  
See S. Akbari.

C. Jayaprakash  
See J. Vannimenus.

C. Jayasekaran  
See also V. Vilfred.

A self vertex switching is a Seidel (graph) switching $\Gamma^v \cong \Gamma$ for $v \in V$, or it is $v$ [better called a “self-switching vertex”, cf. MR for Vilfred and Jayasekaran (2009a)]. [ Cf. articles of J. Hage.] [Annot. 26 Sept 2012.]  
(tg: Sw)

2012a Self vertex switchings of unicyclic graphs. 
(tg: Sw)

2012b Self vertex switchings of connected unicyclic graphs. 
(tg: Sw)

20xxa Self vertex switchings of trees. Submitted.  
(tg: Sw)

Clark Jeffries  
1974a Qualitative stability and digraphs in model ecosystems. 
Sufficient (and necessary) conditions for sign stability in terms of negative cycles and a novel color test. Proofs are sketched or (for necessity)
absent. [Necessity is proved in Logofet and Ul’yanov (1982a), (1982b).]

(SD: QSta)


In a weighted symmetric digraph, a cycle is “balanced” if the product of its weights equals the weight product of the inverse cycle (p. 171). If all cycles of length \( \geq 3 \) are balanced, stability multipliers exist in an associated differential system (Thms. 9, 10). [For weights \( a_{ij} \), define gains \( \varphi(v_i, v_j) := a_{ij}/a_{ji} \). Then “balance” is balance in the gain graph.]

Question: What can be made of this? [Annot. 13 Apr 2009.] (gg: bal)

Clark Jeffries, Victor Klee, & Pauline van den Driessche


Clark Jeffries & P. van den Driessche


A is a real matrix whose bipartite graph is a forest. The signed digraph \( \tilde{\Sigma}(A) \) yields information about eigenvalues. Controllability of solutions of \( \dot{x}(t) = Ax(t) \) may be deduced from \( \tilde{\Sigma}(A) \). [Annot. 24 July 2010.] (QM: SD)


(QM: SD, QSol, QSta)

Eva Jelínková & Jan Kratochvíl


Characterizing graph switching classes that contain a graph with no \( H \) subgraph, for some particular graph \( H \). [An example of Kratochvíl, Nešetřil, & Žýka (1992a).] [Annot. 21 Mar 2011.] (TG: Sw)
generalized network.” Ch. 10: “Generalized minimum cost flow problems.”
§5.5: “Negative cycles.”

Russian translation of (1980a).

P.A. Jensen & Gora Bhaumik
1977a A flow augmentation approach to the network with gains minimum cost flow

T.R. Jensen & F.B. Shepherd
Proves Toft’s (1975a) conjecture for a 4-critical graph with a degree-3 vertex, whence for line graphs. [Annot. 2 Nov 2017.] (sg: par: Col)

Tommy R. Jensen & Bjarne Toft

See also H. Guo.

Mark Jerrum & Alistair Sinclair
§6, “Completeness results”: The problem ISING is to find the partition function \( \sum_{\Sigma} \sigma(\Sigma) \) of a signed simple graph Σ, where \( H(\Sigma) = \sum_{vw \in E} \sigma(vw) \). Thm. 14 suggests nonexistence of certain approximation algorithms. [Annot. 26 Jun 2011.] (sg: Fr, Phys)

R.H. Jeurissen
Involves the negative-circle edge-packing number of −Γ. (par: Fr)

The rank of the incidence matrix of a signed graph, in arbitrary characteristic, generalizing the all-negative results of Doob (1974a). Employs column operations on the incidence matrix. Application to magic labelings, where at each vertex a number (in a ring) is specified; the value of an edge is added if it enters the vertex and subtracted if it departs.


(\text{sg, ori: Incid, Eig(LG)})


The graphs, called “mixed”, are bidirected graphs without introverted edges. Dictionary: “‘bipartite’” = balanced (as a signed graph; the term “balanced” is herein used with another meaning). (sg, ori: incid)


Mostly, the graphs are all-negative signed graphs (oriented to be extroverted). §5, “Labelings of mixed graphs”, discusses bidirected graphs without introverted edges; as in the undirected problem, the (signed-graphically) balanced and unbalanced cases differ. (sg, ori: Incid)


Connected graphs with magic labellings are classified, separately for bipartite and nonbipartite graphs [as one might expect, due to the connection with the incidence matrix of $-\Gamma$; see Stewart (1966a)].

(par: incid)

William S. Jewell

P. Jeyalakshmi

$S \subseteq V$ dominates if every $v \notin S$ has more positive than negative neighbors in $S$. Thm. 2.3: Upper bound on $\#E^-$ if $\exists$ dominating set. [Independent of $\#E$; should be strengthened.] Thm. 3.1: $\#S \geq n/(1 + \Delta^+)$.

Thm. 3.4: Upper bound on $\text{min} \#S$. Characterizations for $\text{min} \#S \leq 4$.

Examples: paths and circles. [Annot. 27 Dec 2020.] (Lab: SG)

P. Jeyalakshmi, K. Karuppasamy, & S. Arockiaraj

Anthony M. Jezior
See J.J. Jarvis.

Samuel Ježný & Marián Trenkler

A weak characterization of magic graphs. [See Jeurissen (1988a) for a stronger characterization.] (par: Incid)

Shengjin Ji  
See B. Huo.

Zhijian Ji  
See H. Gao and X.-Z. Liu.

Guangfeng Jiang  
See also Q.M. Guo and W.L. Guo.

Guangfeng Jiang & Jianming Yu  

Characterizes supersolvability of $G(K_n, \sigma)$. [A special case of Zaslavsky (2001a).] (SG: Geom: m)

Guangfeng Jiang, Jianming Yu, & Jianghua Zhang  

The chromatic polynomial of $K_{K_3}$, i.e., $+K_3$ with a triangle made all negative, factors integrally except for a cubic factor. [See Zaslavsky (1982c), §7, for a graph-theoretic treatment of such examples. One expects a direct proof by adding positive vertices in sequence to $-K_3$. Problem. Evaluate $\chi_\Sigma(\lambda)$ where $\Sigma$ is $\Sigma_1$ with a new vertex positively adjacent to all vertices of $\Sigma_1$.] [Annot. 25 Feb 2012.] (SG: Geom, Invar)

Huidi Jiang & Hongwei Zhang  

Jing-Jing Jiang  
See S.W. Tan and X.L. Wu.

Jonathan Q. Jiang  

Xiao-Wei Jiang  
See B. Hu.

Ye Jiang, Hongwei Zhang, He Cai, & Jie Chen  

Ye Jiang, Hongwei Zhang, & Jie Chen


Yiting Jiang, Daphne Der-Fen Liu, Yeong-Nan Yeh, & Xuding Zhu

Zilin Jiang, Jonathan Tidor, Yuan Yao, Shengtong Zhang, & Yufei Zhao

Fix $-1 \leq \beta < 0 \leq \alpha < 1$. A two-distance set is $X \subseteq S^d$ such that all $\cos \angle(x,y) \in \{\alpha, \beta\}$. Thm. 1.11: $\max \#X \leq d + m$ where $m$ asymptotically $\to \infty = \max$ multiplicity of eigenvalue $(1 - \alpha)/(\alpha - \beta)$ in a certain family of signed graphs.

Dictionary: “Chromatic number $\chi^{\pm}(\Sigma)$” = cluster number, i.e., min $\#$ of clusters.

[Questions for clusterable $\Sigma$: How can it be switched to remain clusterable? How does the cluster number change?] [Annot. 7 Nov 2020.] (SG: Clu, Adj: Eig)

Bao Jiao, Yang Chun, & Tianyong Qiang (as Tianyongqiang)

Licheng Jiao
See Q. Cai and J.S. Wu.

Qiang Jiao
See X. Lin.

Yang Jiao
See J.-S. Wu.

Raúl D. Jiménez
See O. Rojo.

Chao Jin
See J.S. Wu.

Ligang Jin, Yingli Kang, & Eckhard Steffen

Ligang Jin, Tsai-Lien Wong, & Xuding Zhu

Dictionary: “weak $L$-coloring” of $\Gamma = L$-list coloring of $\pm \Gamma$; “generalized signed graph” = $\mathcal{S}_n$-gain graph. Cf. Jiang, Liu, Yeh, and Zhu (2019a),
X.D. Zhu (20xxb). (GG: Col)

Xian’an Jin & Fuji Zhang

They compute the Read–Whitehead chain polynomial of a sign-colored graph in which, for each divalent vertex, the two incident edges have the same color. This is applied to get the Kauffman bracket of small link diagrams. [Cf. W.L. Yang and Zhang (2007a).] (SGc: Invar, Knot)


Ya-Lei Jin
See B.A. He.

Thorsten Joachims & John Hopcroft

Herbert Jodlbauer
See G.H. Yu.

Peter Joffe
See A.J. Hoffman.

Manas Joglekar, Nisarg Shah, & Ajit A. Diwan


Γ has weights \( w : V \cup E \to \mathfrak{A} \) where \( \mathfrak{A} = \mathbb{Z}_2 \) is signs.) “Balance” = harmony: the sum around every circle = 0. Thm. 1: There are \( \#\mathfrak{A}^{\#V + t - c(\Gamma)} \) harmonious labellings, where \( t := \) number of edge 3-components of \( \Gamma \). Lemma 2. If \( (\Gamma, w) \) is balanced and \( u, v \) are edge 3-connected in \( \Gamma \), then \( 2w(P) = w(u) + w(v) \) for every \( uv \)-path. [Annot. 30 Aug 2010.] (GGw: Bal)

Thm. 3 is a construction for all edge 2-connected \( \Gamma \) such that \( \exists \) harmonious sign labelling, not all +. [The definitive characterization of consistent vertex signatures as in Beineke and Harary (1978b), improving on Hoede (1992a).] [Annot. 30 Aug 2010.] (SG, VS: Bal)

Rolf Johannesson
See I.E. Bocharova.

Karl H. Johansson
See G.-D. Shi.
Mikael Johansson
See G.-D. Shi.

David John

Polynomial-time algorithms to decide balance of a signed graph [this has long been known; see e.g. Hansen (1978a)] and allegedly to find the minimum number of negative edges whose deletion makes the graph balanced [call this the ‘negative frustration index’]. Contract the positive edges, leaving a graph consisting of the negative edges. To detect balance, look for bipartiteness of the contraction. [Inferior to the standard algorithm.] For negative frustration index, find a maximum cut of the contraction. [Something is wrong, since Max Cut is NP-complete and negative frustration index contains Max Cut. I believe the algorithm finds a nonmaximum cut.] (SG: Bal, Fr: Alg)

Eugene C. Johnsen

Signed complete digraphs \((K_n, \sigma)\). A list of permitted isomorphism types of triads (order-3 induced subgraphs) (a “microstructure”) implies a list of possible \(\sigma\)’s (a “macrostructure”). Ex. 2.1: Permitting symmetrically signed triads with 1 or 3 positive arc pairs gives balanced signed complete graphs as in Cartwright and Harary’s model (1956a). Ex. 2.2: Symmetric signs with 0, 1, or 3 positive arc pairs give clusterable signed complete graphs as in Davis’s model (1967a). Other examples are positive digraph models from the literature, with negative arcs inserted to complete the digraph. Throughout, empirical data are used to prune potential examples.

§3, “Submodels and their substantive interpretation: \(\sigma = (\sigma_t)_t\) evolves in [my simplification] discrete time \(t \in \mathbb{Z}\) according to some combination of four “processes”. The corresponding equilibrium signatures are the macrostructure and give the permitted triads. The processes:

1. \(\sigma_t(ab) = \sigma_t(bc) \implies \sigma_{t+1}(ab) = \sigma_{t+1}(ba) = +\).
2. \(\sigma_t(ab) = - \text{ or } \sigma_t(ba) = - \implies \sigma_{t+1}(ac) \neq \sigma_{t+1}(bc)\).
3. \(\sigma_t(ab) = + \text{ or } \sigma_t(ba) = + \implies \sigma_{t+1}(ac) = \sigma_{t+1}(bc)\).
4. \(\sigma_t(ab) = \sigma_t(ba) = + \implies \sigma_{t+1}(ac) = \sigma_{t+1}(bc)\).

§4, “Agreement–friendship processes related to affect”: The equilibrium triads for each process and some combinations (“microprocesses”) and the corresponding macrostructure. E.g., (1) with (3) gives the balance model. Some combinations do not yield macrostructures. Five combinations are “core”. Later §§: Further analysis of the core combinations.

An elaborate classificatory analysis of “triads” (signed complete directed graphs of 3 vertices) vis-à-vis “macrostructures” (signed complete directed graphs) with reference to structural interactions and implications of triadic numerical restrictions on “dyads” (s.c.d.g. of 2 vertices). Connections to certain models of affect in social psychology. “Impenetrability! That’s what I say!” “Would you tell me, please,” said Alice, “what that means?”

Eugene C. Johnsen & H. Gilman McCann

Balance and clustering analyzed via triples rather than edges. [Possible because the digraph is complete. A later analysis via triples is in Doreian and Krackhardt (2001a).] (SD: Bal, Clu)

Charles R. Johnson
See also P.J. Cameron and C.A. Eschenbach.

Charles R. Johnson, Frank Thomson Leighton, & Herbert A. Robinson

Charles R. Johnson & John Maybee

In square matrix $A$ let $A[S]$ be the principal submatrix with rows and columns indexed by $S$. Thm. 1: Assume $A[S]$ is sign-nonsingular in standard form and $i, j \notin S$. Then the $(i, j)$ entry of the Schur complement of $A[S]$ has sign determined by the sign pattern of $A$ iff, in the signed digraph of $A$, every path $i \to j$ via $S$ has the same sign. (QM: sd)

Charles R. Johnson, William D. McCuaig, & David P. Stanford

Charles R. Johnson, Michael Neumann, & Michael J. Tsatsomeros

Charles R. Johnson, D.D. Olesky, Michael Tsatsomeros, & P. van den Driessche
Suppose the signed digraph $D$ of an $n \times n$ matrix has longest cycle length $k$ and all cycles of $-D$ are negative. Theorem: If $k = n - 1$, the eigenvalues lie in a domain subtending angle $< 2\pi/k$. This is known for $k = 2$ but false for $k = n - 3$. (QM, SD)

**Charles R. Johnson, Frank Uhlig, & Dan Warner**


**David S. Johnson**


**Ellis L. Johnson**

See also J. Edmonds, H.L. Gan, and G. Gastou.


**Ellis L. Johnson & Sebastiano Mosterts**


Two of the problems: Given a signed graph (edges called “even” and “odd” rather than “positive” and “negative”). The co-postman problem is to find a minimum-cost deletion set (of edges). The “odd circuit” problem is to find a minimum-cost negative circle. The Chinese postman problem is described in a way that involves cobalance and “switching” around a circle. (SG: Fr(Gen), Incid)

**Ellis L. Johnson & Manfred W. Padberg**


Geometry of the bidirected stable set polytope $P(B)$ (which generalizes the stable set polytope to bidirected graphs), defined as the convex hull of $0, 1$ solutions of $x_i + x_j \leq 1$, $-x_i - x_j \leq -1$, $x_i \leq x_j$ for extroverted, introverted, and directed edges of $B$. (Thus, undirected graphs correspond to extroverted bidirected graphs.) It suffices to treat transitively closed bidirections of simple graphs ([unfortunately] called “bigraphs”). [Such a bidirected graph must be balanced.] A “biclique” $(S_+, S_-)$ is the Harary bipartition of a balanced complete subgraph $(S_+, S_-)$ are the source and sink sets of the subgraph). It is “strong” if no external vertex has an
edge directed out of every vertex of $S_+$ and an edge directed into every vertex of $S_-$. Strong bicliques generate facet inequalities of the polytope. Call $B$ perfect if these facets (and nonnegativity) determine $P(B)$. $\Gamma$ is “biperfect” if every transitively closed bidirection $B$ of $\Gamma$ is perfect. Conjectures: $\Gamma$ is biperfect iff it is perfect. $\Gamma$ is perfect iff some transitively closed bidirection is perfect. [Both proved by Sewell (1996a) and independently by Ikebe and Tamura (20xxa). See e.g. Tamura (1997a) for further work.]

Skyler J. Johnson
See L.J. Rusnak.

Will Johnson
Problem: Recover the conductances of branches of an electrical network, given boundary voltages and currents, when conductances can be nonlinear or negative. The “conductances” are gains. §11, “Applications of negative conductances”: §11.1, “Removing a mild failure of circular planarity”: A graphical transform of positive conductances can introduce negative conductances, using an idea from Schroeder (1995a) and Goff (2003a). §11.2, “Knot theory”: Treating the colors $\pm 1$ as conductances, a corollary on pseudoline arrangements and tangles. Suggests that the proof in Goff (2003a) is flawed (see fn. 4). [Annot. 26 Dec 2012.]

M. Jones
See R. Crowston.

[Hidde de Jong]
See H. de Jong (under ‘D’).

Felix Joos
See T. Huynh.

Mohammadreza Jooyandeh, Dariush Kiani, & Maryam Mirzakhah

Tibor Jordán, Viktória E. Kaszanitzky, & Shin-ichi Tanigawa

Heather Jordon [Heather Gavlas]
See also G. Chartrand and D. Hoffman.

Heather Jordon, Richard McBride, & Shailesh Tipnis
Consider signed simple graphs of order $n$. $P_n :=$ polytope determined by the inequalities from Hoffman and Jordon (2006a) that characterize net degree vectors. Thm. 2.7: $P_n = \text{conv(net degree vectors)}$. Thm. 2.9: Each vertex of $P_n \leftrightarrow$ a unique signed graph, which is a signed $K_n$. §3:
Comparison with net degree vectors of digraphs. [As in other papers on net degree sequences, the best viewpoint is that “signed” edges are oriented negative edges and “directed” edges are oriented positive edges.] [Annot. 1 Oct 2009.] (SG: ori: Invar: Geom)

Gwenaël Joret
See N.E. Clarke and S. Fiorini.

Leif Kjær Jørgensen

Let $\sigma_{\text{op}}(\Gamma)$, or $\sigma_{\text{odd}}(\Gamma)$, be the largest $s$ for which $-\Gamma$ contains a subdivision of $-K_s$ (an “odd-path-$K_s$”), or $[-\Gamma]$ contains an antibalanced subdivision of $K_s$ (an “odd-$K_s$”). Thm. 4: $\sigma_{\text{op}}(\Gamma), \sigma_{\text{odd}}(\Gamma) \approx \sqrt{n}$. Thms. 7, 8 (simplified): For $p = 4, 5$ and large enough $n = |V|, \sigma_{\text{odd}}(\Gamma) \geq p$ or $\Gamma$ is a specific exceptional graph. Conjecture 9. The same holds for all $p \geq 4$. [Problem. Generalize this to signed graphs.] (par: Xtreml)

J. Paulraj Joseph
See V. Vilfred.

Mayamma Joseph
See P.B. Joshi and S.R. Shreyas.

Prajakta Bharat Joshi & Mayamma Joseph


Prajakta Bharat Joshi, Mayamma Joseph, & Mukti Acharya
20xxa Color energy of signed graphs. Submitted. (SG: Adj: Eig)

Shalini Joshi
See B.D. Acharya.

Simon Joyce

Tadeusz Józefiak & Bruce Sagan


The hyperplane arrangements (over fields with characteristic $\neq 2$) corresponding to certain signed graphs are shown to be “free”. Explicit bases and the exponents are given. The signed graphs are: $+K_{n-1}$ \subseteq
\[\Sigma_1 \subseteq +K_n \text{ (known), } \pm K_n \subseteq \Sigma_2 \subseteq \pm K_n^\circ, \pm K_n \subseteq \Sigma_3 \subseteq \pm K_n^\circ; \text{ also,} \]

those obtained from \(+K_n\) or \(K_n^\circ\) by adding all negative links in the order of their larger vertex (assuming ordered vertices) (Thms. 4.1, 4.2) or smaller vertex (Thms. 4.4, 4.5); and those obtained from \(\pm K_{n-1}\) by adding positive edges ahead of negative ones (Thm. 4.3). [For further developments see Edelman and Reiner (1994a).] Similar theorems hold for complex arrangements when the sign group is replaced by the complex s-th roots of unity (§5). The Möbius functions of \(\Sigma_2\), known from Hanlon (1988a), are deduced in §6. (sg, gg: Geom, m, Invar)

**[Michael Juenger]**
See M. Jünger.

**Ji-Hwan Jung**
See G.-S. Cheon.

**Michael Jünger**
See F. Barahona, C. De Simone, M. Grötschel, and M. Palassini.

**Mark Jungerman & Gerhard Ringel**

“Cascades”: see Youngs (1968a).

**K. Jüngling**

[Annot. 16 Jun 2012.] (sg: Ori: Appl)

**K. Jüngling & G. Obermair**


**Dieter Jungnickel**
See C. Fremuth-Paeger.

**Samuel Jurkiewicz**
See M.A.A. de Freitas.

**James Justus**


**Jerald A. Kabell**
See also Harary and Kabell (1980a).

Cobalance means that every cutset has positive sign product. Thm.: $\Sigma$ is cobalanced iff every vertex star has evenly many negative edges. For planar graphs, corollaries of this criterion and Harary’s bipartition theorem result from duality. [The theorem follows easily by looking at the negative subgraph.]

(SG: Bal(D), Bal)


Kasper Kabell

Jeff Kahn


Jeff Kahn & Joseph P.S. Kung


Announcement of (1982a). (gg: M)


A “variety” is a class closed under deletion, contraction, and direct summation and having for each rank a “universal model”, a single member containing all others. There are two nontrivial types of variety of finite matroids: matroids representable over $GF(q)$, and gain-graphic matroids with gains in a finite group $G$. The universal models of the latter are the Dowling geometries $Q_n(G)$.

It is incidentally proved (§7, pp. 490–492) that Dowling geometries of non-group quasigroups cannot exist in rank $n \geq 4$. (gg: M)


Such a geometric lattice of rank $\geq 4$, if not a projective geometry with few points deleted, is a Dowling lattice. (gg: M)

Jeff Kahn & Roy Meshulam


Continues Aharoni, Meshulam, and Wajnryb (1995a) (q.v. for definitions), generalizing its Thm. 1.3 (the case $k = \#K = 2$ of the following). Let $m$ = number of 0-weight matchings, $\delta$ = minimum degree. Thm. 1.1: If $m > 0$ then $m \geq (\delta - k + 1)!$. *Conjecture* 1.2. $k$ can be reduced. (See
the paper for details.) [Question. Is there a generalization to weighted digraphs? One could have two kinds of arcs: some weighted from $\mathbb{R}$, and some weighted 0. The perfect matching might be replaced by an alternating Hamilton cycle or a spanning union of disjoint alternating cycles.]

Thm. 2.1: In a $\mathbb{R}$-weighted simple digraph with all outdegrees $> k$, there is a nonempty set of disjoint cycles whose total weight is 0. (WG)

**Tomáš Kaiser, Robert Lukoťka, Edita Máčajová, & Edita Rollová**


**Tomáš Kaiser, Robert Lukoťka, & Edita Rollová**


**Naonori Kakimura**

See also C. Carlson.


Symmetric matching theory of a bipartite graph with left-right symmetry, with a symmetric Mendelsohn–Dulmage theorem. [A symmetrically bipartite graph $\Gamma'$ is the signed covering graph of an all-negative signed graph $-\Gamma$, possibly with half edges. A symmetrical matching in $\Gamma'$ corresponds to a subgraph of $-\Gamma$ with maximum degree 1. Problem. Develop the symmetric matching theory of any graph with an involutory, fixed-point-free automorphism in terms of a matching theory of signed graphs with half edges.] [Annot. 29 Sept 2011.] (sg: cov: Str)

**Naonori Kakimura & Ken-ichi Kawarabayashi**


**Debajit Kalita**

See also R.B. Bapat and S. Barik.


“3-colored digraph” = gain graph with gains in $\{\pm 1, \pm i\}$ [not a digraph].


Debajit Kalita & Sukanta Pati


2014a A reciprocal eigenvalue property for unicyclic weighted directed graphs with weights from \(\{\pm 1, \pm i\}\}. Linear Algebra Appl. 449 (2014), 417–434. MR 3191876.

Meenal M. Kaliwal

See P.R. Hampiholi.

M. Kamaraj

See M. Parvathi.

Hidehiko Kamiya, Akimichi Takemura, & Hiroaki Terao


§5, “Example”: The affino-signed-graphic arrangement $\mathcal{B}_m^{[0,a]}$ from Athanasiadis (1999a). [Annot. 26 May 2018.] (gg: Geom, Invar)


§3.5, “Signed all-subset arrangement”. (SG: Geom, Invar)

[Axel von Kamp]
See A. von Kamp (under ‘V’).

Daniel Kandel, Radel Ben-Av, & Eytan Domany

A new probabilistic algorithm for clustering in a ground state (a function $s : V \to \{+1, -1\}$ such that $\#E^- = l(\Sigma)$) of an all-negative (“fully frustrated”) square lattice $\Sigma$. A “cluster” in $s$ is a partition of $V$ such that switching any part does not change $\#E^-$. The objective is to join vertices connected by satisfied edges but not those joined by frustrated edges; this cannot be solved uniquely for any unbalanced $\Sigma$, so previous methods (used for balanced $\Sigma$), e.g., nearest-neighbor moves in state space (“single spin flips”), are ineffective (see p. 942, col. 1; p. 943, col. 2). The algorithm depends on the square lattice structure since it works on squares (“plaquettes”); it succeeds because it works through plaquettes instead of edges (p. 943, col. 2). [Problem: Do state-space algorithms help to approximate signed-graph clustering in the sense of Davis (1967a)? Finding a ground state is NP-hard in general, though not for planar signed graphs (cf. Katai and Iawi (1978a), Barahona (1982a)).] [Annot. 18 Jun 2012.] (Phys, SG: Clu: Alg)

Yingli Kang
See also L.-G. Jin.


Yingli Kang & Eckhard Steffen


Mariusz Kaniecki, Justyna Kosakowska, Piotr Malicki, & Grzegorz Marczak

[M.R. Rajesh Kanna]
See M.R. Rajesh Kanna (under ‘R’) and .
M. Rajesh Kannan
See R. Mehatari and A. Samanta.

Sangita Kansal
See M. Acharya, R. Jain, and Payal.

Konstantinos Kaparis & Adam N. Letchford
(Par: Fr, Geom: Alg)

Vikram Singh Kapil
See R.P. Sharma.

Ajai Kapoor
See M. Conforti.

Roman Kapuscinski
See P. Doreian.

D. Karapetyan
See G. Gutin.

Mehran Kardar
See L. Saul.

František Kardoš & Jonathan Narboni
A signed planar graph may have chromatic number $> 4$, disproving a conjecture of Máčajová, Raspaud, and Škoviera (2016a). [Annot. 26 Jul 2019.]
(SG: Fr)

A. Kargaran, M. Ebrahimi, M. Riazi, A. Hosseiny, & G.R. Jafari
Measuring imbalance by triples and quadruples. [Annot. 18 Dec 2020.]
(SG: Fr)

M.M. Karkhaneei
See S. Akbari.

Richard M. Karp, Raymond E. Miller, & Shmuel Winograd
Implicitly, concerns the existence of nonpositive directed tours (closed trails) in a $Z^d$-gain graph (the “dependence graph” of a system of recurrences).
(gd: cov)

K. Karuppasamy
See P. Jeyalakshmi.

Alexander V. Karzanov
See M.A. Babenko and A.V. Goldberg.
Yasuhiro Kasai, Ayao Okiji, & Itiro Syozi


Grand partition function \( \sum_\theta \exp(\#E^- - \#E^+) \) over all edge signatures \( \theta \) and all switchings of a lattice graph, investigated for a physical phase via multiple replicates and analytic continuation. [The relevance to signed graphs is obscured by summing over all signatures.] [Annot. 17 Aug 2012.] (Phys: SG)


Yoshi Kashima

See G. Robins.

Stanisław Kasjan & Daniel Simson


P.W. Kasteleyn

See also C.M. Fortuin.


§V, “The Ising problem”: The ferromagnetic Ising model can be converted into a dimer-covering problem. The method has since been applied to signed graphs (the general Ising problem); cf. Thomas and Middleton (2009a), (2013a), and references therein. [Annot. 10 Jan 2015.] (Phys, Alg)

P.W. Kasteleyn & C.M. Fortuin


A specialization of the parametrized dichromatic polynomial of a graph: \( Q_\Gamma(q, p; x, 1) \) where \( q_e = 1 - p_e \). [Essentially, announcing Fortuin and
Kasteleyn (1972a).

Viktória E. Kaszanitzky
See T. Jordán.

E. Kaszkurwicz & L. Hsu

Osamu Katai
See also S. Iwai.

Osamu Katai & Sousuke Iwai

Balance and detecting balance are discussed at length. Finding the frustration index \( l(\Sigma) \) is solved for planar graphs by converting it into a matching problem in the dual graph with signed vertices. This applies also when edges are weighted by positive reals. [Barahona (1982a) and Barahona, Maynard, Rammal, and Uhry (1982a) have a similar, later, but independent solution for the planar frustration index. Barahona (1981a), (1990a) solves toroidal graphs.]

The nonplanar problem is treated via \( A(\Sigma) \), but amounts to finding \( \min_\zeta (\#E^+(\Sigma^\zeta) - \#E^-(\Sigma^\zeta)) \) [which is NP-hard]. This suggests an iterative procedure which consists of switching \( v \in V \) that minimizes \( d^\pm(v) \), and repeating; it may not attain the true minimum. [Mitra (1962a) also proposed this.] [Annot. 22 Jun 2012.]

(MG, WG, PsS: Bal, Fr, Alg, Adj, sw)


A shorter version of (1978a). Lem. 1 [restated]: \( \Sigma \) is balanced iff it switches to all positive. [Annot. 22 Jun 2012.]

(MG, VG, WG, PsS: Bal, Fr, Alg, Adj, sw)


Priya Kataria
See D. Sinha.

M. [Moshe] Katz
See also G. Converse.
The doubly stochastic case (all line sums = 1) of Thm. 8.2.1 in Brualdi (2006a). [Annot. 13 Oct 2012.]

Louis H. Kauffman

See also J.R. Goldman.


A leisurely development of Kauffman’s combinatorial bracket polynomial of a link diagram and the Jones and other knot polynomials, including the basics of (1989a).


The Tutte polynomial, also called “Kauffman’s bracket of a signed graph” and equivalent to his bracket of a link diagram, is of an edge-sign-colored graph. It is defined by a sum over spanning trees of terms that depend on the signs and activities of the edges and nonedges of the tree. The point is that the deletion-contraction recurrence over an edge has parameters dependent on the color of the edge; also, the parameters of the two colors are related. The purpose is to develop the bracket of a link diagram combinatorially. §3.2, “Link diagrams”: how link diagrams correspond to signed plane graphs. §4, “A polynomial for signed graphs”, defines the general sign-colored graph polynomial \( Q[\Sigma](A, B, d) \) by deletion-contraction, modified multiplication on components, and evaluation on graphs of loops and isthmi. §5, “A spanning tree expansion for \( Q[G] \)” \( [G \text{ means } \Sigma] \), proves \( Q[\Sigma] \) exists by producing a spanning-tree expansion, shown independent of the edge ordering by a direct argument. [No dichromatic form of \( Q[\Sigma] \) appears; but see successor articles.] §6, “Conclusion”, remarks that \( Q[\Sigma] \) is invariant under signed-graphic Reidemeister moves II and III. [This significant work, inspired by Thistlethwaite (1988a), led to independent but related generalizations by Przytycka and Przytycki (1988a), Schwärzler and Welsh (1993a), Traldi (1989a), and Zaslavsky (1992b) that were partially anticipated by Fortuin and Kasteleyn (1972a). Also see (1997a).]


§2, “A state summation for classical electrical networks”, uses a form of
the parametrized dichromatic polynomial $Q_{B,A;1,1}$ [as in Zaslavsky (1992b) et al.; cf.], where $A(e), B(e) \in \mathbb{C}$, to compute conductances as in Goldman and Kauffman (1993a). (sgc: Gen: Invar: Exp)

§3. “The bracket polynomial”, discusses the connections with sign-colored graphs and electricity. Problem: Is there a signed graph, not reducible by signed-graphic Reidemeister moves (see (1989a)) to a tree with loops, whose sign-colored dichromatic polynomial is trivial? If not, the Jones polynomial detects the unknot. (SGc: Invar: Exp)/(SGc: Invar)

Marcelle Kaufman
See also J.-P. Comet, J. Demongeot and R. Thomas.

M. Kaufman, C. Soulé, & R. Thomas

Marcelle Kaufman & René Thomas
2003a Emergence of complex behaviour from simple circuit structures. Émergence de comportements complexes à partir de structures de circuits simples. C.R. Biologies 326 (2003), 205–214. (SD: Dyn)

M. Kaufman, J. Urbain, & R. Thomas

Bableen Kaur
See D. Sinha.

Ken-ichi Kawarabayashi
See also Chiba, M. Chudnovsky, E.D. Demaine, S. Fujita, and N. Kakimura.


$\exists f(k)$ such that, if $-\Gamma$ has no $-K_k$ minor, then $V$ partitions into $496k$ parts of order $\le f(k)$. Based on Geelen, Gerards, Reed, Seymour, and Vetta (2009a). [Annot. 4 Feb 2021.] (Par: Col(Gen))


Short proof of Geelen, Gerards, Reed, Seymour, and Vetta (2009a): If $-\Gamma$ has no $-K_k$ minor, then $\Gamma$ is $O(k\sqrt{\log k})$-colorable. [Annot. 4 Feb 2021.] (Par: Col)


Ken-ichi Kawarabayashi & Yusuke Kobayashi
Conference version of (2016a). (Par: Str: Cyc)


Ken-ichi Kawarabayashi, Orlando Lee, & Bruce Reed


Ken-ichi Kawarabayashi, Zhentao Li, & Bruce Reed


Extended abstract. A totally odd subdivision of $\Gamma$ is a signed graph that is a (signed-graphic) subdivision of $-\Gamma$ and is itself all negative. Dictionary: “parity” = sign in $-\Gamma$; “parity path/circle” = path or circle of specified sign. (sg: par: Str, Alg)

Ken-Ichi Kawarabayashi & Bojan Mohar


Odd minor: specifically, a $-K_k$ minor of $-\Gamma$. [Annot. 4 Feb 2021.] (Par: Col: Alg)

Ken-Ichi Kawarabayashi & Atsuhiro Nakamoto


Ken-ichi Kawarabayashi & Kenta Ozeki


Thm. 1 states the characterization for internally 4-connected graphs. For the generalization to disjoint negative circles in signed graphs of any connectivity see the earlier paper by Slilaty (2007a). [Annot. 25 Jun 2013.] (sg: Par: Str)

Ken-Ichi Kawarabayashi & Bruce Reed


Thm.: If $-\Gamma$ has no $-K_p$ minor, then $\Gamma$ has a fractional $2p$-coloring. [Annot. 4 Feb 2021.] (Par: Col((Gen))


“Odd cycles transversal problem”: Is \( l_0(\Gamma) \leq k \)? An \( O(m\alpha(m,n)) \) algorithm for fixed \( k \), where \( \alpha = \) inverse Ackermann function; also solves edge form: Is \( l(\Gamma) \leq k \)? Also, an \( O(m\alpha(m,n) + n \log n) \) algorithm for 2-packing of \( k \) odd circles. Also, simplified proof of Bruce Reed (1999a).


\( \Gamma \) 3-connected, \( s, t \in V \), \( \forall \) 3-cut \( X \) each component of \( \Gamma \setminus X \) contains \( s \) or \( t \). Then (1) \( \exists \) induced st-path \( P \) such that \( \Gamma \setminus V(P) \) is unbalanced, or (2) \( \Gamma \) is planar such that every negative face contains one of \( s, t \); but not both (1) and (2). [Problem. Generalize to signed graphs.] [Annot. 4 Feb 2021.]


The graph minors algorithm for a \( \Gamma \) minor, specifically \( \Gamma \) (“odd complete minor of \( \Gamma \)” = \( \Gamma \) minor of \( \Gamma \)). Disjoint paths with parity conditions. Related problems. [Annot. 4 Feb 2021.]


Thm.: \( \exists f(k) \): If \( \Gamma \) is \( (496k + 13) \)-connected of order \( \geq f(k) \), then \( \Gamma \) has a \( \neg K_k \) minor or \( \Gamma \setminus 8k \) vertices is balanced. Dictionary: “odd complete minor of \( \Gamma \) = \( \neg K_k \) minor of \( \neg \Gamma \). [Problem. Generalize to signed graphs.] [Annot. 4 Feb 2021.]

Ken-ichi Kawarabayashi & David R. Wood

Yasushi Kawase, Yusuke Kobayashi, & Yutaro Yamaguchi


N. Kawashima and H. Rieger


Many subsections throughout on open problems about ±J models (signed graphs; ground state ↔ switching to fewest negative edges ↔ frustration index l(Σ)) and continuous models (stochastic edge signs and weights); esp., ground state computations, mostly on excessively small graphs, looking for phase transitions and critical points. For ±J, e.g: §§9.2.1, 9.2.4, 9.3.3, 9.6.3. §9.6: XY, Heisenberg, Potts spins (in S^2, S^3, [q]). §9.7, “Weak disorder”: “gauge invariance” (switching invariance) implies some properties. Ample references. [Annot. 15 Aug 2018.]

B. Kawecka-Magiera
See M.J. Krawczyk.

K. Kazemian
See S. Akbari.

Peter Keevash
See I. Balla.

Nataša Kejžar, Zoran Nikoloski, & Vladimir Batagelj

Alexander Kelman
See J.F. De Jesús.

Dzh. Kemeni & Dzh. Snell
See J.G. Kemeny and J.L. Snell.

John G. Kemeny
Model No. 1 expounds signed graphs in social psychology from Cartwright and Harary (1956a). [Annot. 27 Dec 2012.]

John G. Kemeny & J. Laurie Snell

Russian translation of (1962a).

B.K. Kempegowda
See M.R. Rajesh Kanna.

Mark Kempton
See F. Chung.

A. Joseph Kennedy
See also M. Parvathi.


John W. Kennedy
See M.L. Gargano.

Jeff L. Kennington & Richard V. Helgason
Ch. 5: “The simplex method for the generalized network problem.” (GN: M(Bases): Exp)

Richard Kenyon

Anne-Marie Kermarrec & Afshin Moin

Anne-Marie Kermarrec & Christopher Thraves

Can $(K_n, \sigma)$ be drawn in $\mathbb{R}^l$ so every positive neighbor is closer than every negative neighbor, for each vertex? Polynomial-time algorithm for $l = 1$. [Continued by Cygan, Pilipczuk, et al. (2012a).] [Annot. 26 Apr 2012.] (SG: KG: Bal, Alg, Clu)

Julie Kerr

The lattice is isomorphic to the semilattice of $k$-composed partitions of a set with a top element adjoined. (See R. Gill (1998b).) (gg: m: Geom, Top)

Mehtab Khan
See also R. Farooq and S. Hafeez.

Mehtab Khan & Rashid Farooq

Muhammad Ali Khan
See S. Pirzada.

H. Kharaghani

A “balanced generalized weighing matrix” is the group-ring adjacency matrix $\hat{A}$ of a gain digraph $\Phi$, with finite gain group $\mathfrak{G}$, such that $\hat{A}\hat{A}^* = kI + ls(J - I)$ where $s := \sum_{g \in \mathfrak{G}} g$. Constructs examples of $\hat{A}$ where $\mathfrak{G}$ is cyclic and $\Phi$ is symmetric with no loops. [The article does not mention gain digraphs.] (gg: Adj)

F. Kharari & É. Palmer [Frank Harary & Edgar M. Palmer]

Abdelkader Khelladi
See also O. Bessouf.


Introduces quasibalance (called “m-balance”): Two negative circles have at least 2 common vertices. [Annot. 10 Jan 2019.]

(OG: Ori: Str, Flows)


Improves the result of Bouchet (1983a) about nowhere-zero integral flows on a signed graph. $\Sigma$ has such an 18-flow if 4-connected, a 30-flow if 3-connected and without a positive triangle, and if quasibalanced (“m-balanced”; cf. (1985a)) a 6-flow (proving Bouchet’s conjecture in that case). [Annot. rev 10 Jan 2019.]

(OG: M, Flows, Ori)


Comments on the results of Bouchet (1983a) and Khelladi (1987a).

(BG: M, Flows, Ori)

Boris N. Kholodenko, Anatoly Kiyatkin, Frank J. Bruggeman, Eduardo Son-tag, Hans V. Westerhoff, & Jan B. Hoek


A matrix-based method to infer the signs (and magnitudes) of an interaction digraph from measurement of the interactions between modules of the digraph. [Annot. 25 Jan 2015.]

(SD: Alg: Biol)

Dariush Kiani

See I. Gutman, H. Hamidzade, M. Jooyandeh, and M. Mirzakhah.

Kathleen P. Kiernan

See R.A. Brualdi.

Dongseok Kim & Jaeun Lee


(OG: Cov: Col)

Eun Jung Kim

See N. Alon.

Jeong-Rae Kim, Yeoin Yoon, & Kwang-Hyun Cho


Collects the effects of the three types of coupled cycles (signed ++, +−, −−) in an interaction signed digraph in biological examples modelled by differential equations. Observes that ++ cycle pairs “enhance signal amplification and” bistability, −− enhance homeostasis, and +− “enable reliable decision-making” by the biological system. [Cf. Sriram, Soliman, and Fages (2009a).] [Annot. 16 Jan 2015.]

(BG: SD: Dyn)

Jong-Jae Kim

See O. Nagai.
Ringi Kim, Seog-Jin Kim, & Xuding Zhu  
20xxa Signed colouring and list colouring of $k$-chromatic graphs. Submitted.  
(\textit{SG: Col})

Sangwook Kim  
See J.-S. Huh.

Seog-Jin Kim  
See also R.-G. Kim.

Seog-Jin Kim & Kenta Ozeki  

Andrew D. King  
See T. Huynh.

Harunobu Kinoshita  
See T. Yamada.

Shin’ichi Kinoshita  
See also T. Yajima.

Shin’ichi Kinoshita & Hidetaka Terasaka  
Employs the sign-colored graph of a link diagram from Bankwitz (1930a) to form certain combinations of links. (\textit{SGc: Knot})

M. Kirby  
See A. Charnes.

Steve Kirkland  
See also M. Cavers, M.A.A. de Freitas, F. Goldberg, C.S. Oliveira, and J. Stuart.


Steve Kirkland, J.J. McDonald, & M.J. Tsatsomeros  

Steve Kirkland & Debdas Paul  
(\textit{Par: Eig, incid})

Scott Kirkpatrick  
See also D. Sherrington and J. Vannimenus.

Estimates the number of ground states of signed $d$-dimensional hypercubic lattices, $d \geq 2$. With random signs, of which the proportion $x$ is negative, the expected proportion of negative (“frustrated”) squares is computed to be $4x(1-x)[x^2 + (1-x)^2] \leq 0.5$, $\approx$ for $0.2 < x < 0.8$. [This assumes the squares’ signs are independent, which is only true when
Certain ice models are equivalent to signed graphs (p. 4632). §III, “Exact results”: In $d = 2$ the strings pairing negative squares in a ground state are short on average. In $d = 3$ ground states are more difficult to find [a conclusion essentially proved by Barahona (1982a)] but there are interesting remarks on strings pairing negative squares. §IV, “Monte Carlo results”: “carried out on fairly large samples” in $d = 2, 3$ for Ising spins ($\pm 1$) [with 1977 computing power]. Are there many ground states or only one (up to global spin reversal)? Evidence in $d = 2$ suggests signed graphs (“$\pm 1$”) are quite different from randomly weighted signed graphs (“Gaussian”). Signed-graph behavior differs for very low vs. middling density of negative edges; there seem to be more ground states at middling density. There seem to be fewer ground states in $d = 3$ than $d = 2$ (p. 4637). Discussion of expected behavior of low-frustration states; a remarkable planar example in Fig. 14. Dictionary: “bond” = edge; “state” = $\zeta : V \to \{\pm 1\}$; frustrated bond = $\sigma^\zeta(e) = -1$; frustration = $\#(\sigma^\zeta)^{-1}(-1)$; “ground state” = switching with min frustration $= l(\Sigma)$; “degeneracy” = multiple states with same amount of frustration.

Scott Kirkpatrick & David Sherrington

Random edge weights and signs on $K_n$ with vertex signs $\pm 1$. Most interesting: § VI, “Statics for $T \neq 0$”, where the “energy” (frustration index $l(\Sigma)$) landscape of random signs is described, based on computer experiments, as consisting of deep valleys, each having several local minima of $l$ separated by slightly higher ridges, and with high-$l$ barriers separating the valleys. [Presumably, the distance function is Hamming distance between reduced sign functions, i.e., those with $E^- = l$.] [A seminal successor to Edwards and Anderson (1975a). This picture, while convincing, has never been proved; it remains an object of intense curiosity. Cf. Marvel, Kleinberg, Kleinberg, and Strogatz (2011a), (2011b).] [Annot. 22 Aug 2012, 23 Jan 2015.] (Phys: SG, State(fr), Sw)

Nanao Kita

20xxb Bidirected graph II: Extension of basilica order. In preparation. (SG: Ori: Str)


Ouail Kitouni & Nathan Reff

Teeradej Kittipassorn & Gábor Mészáros

Thorough study of the number $c_3$ of negative triangles in a signed $K_n$. [Annot. 22 Jan 2015.] (Phys: SG, State(fr), Sw)
Two-thirds of the numbers from 0 to \( \binom{n}{3} \) cannot be \( c_3(K_n, \sigma) \). Some numbers that are, are

\[
0 = a_0 \leq b_0 \leq a_1 \leq \cdots \leq a_m \leq b_m \approx \frac{n^3}{2}
\]

where

\[
b_i = a_i + i(i - 1) \quad \text{and} \quad a_{i+1} = b_i + (n-2) - i(i+1).
\]

For \( i \leq m \), \( c_3(K_n, \sigma) \in [a_i, b_i] \) iff \( l(K_n, \sigma) = i \). \( \exists f(n) \) such that if \( n \gg 0 \), all \( j \in [f(n), \binom{n}{3} - f(n)] \) are \( c_3(K_n, \sigma) \)'s. Etc. [For other studies of negative circles cf. Tomescu (1976a), Popescu and Tomescu (1996a), Antal, Krapivsky, and Redner (2005a), Schaefer and Zaslavsky (2019a).] [Annot. 26 Sept 2015, 6 Jan 2017.]

Anatoly Kiyatkin
See B.N. Kholodenko.

Ralf Klamma
See M. Shahriari.

Steffen Klamt
See also I.N. Melas.

Steffen Klamt, Julio Saez-Rodriguez, Jonathan A. Lindquist, Luca Simeoni, & Ernst D. Gilles

(Biol, SD, SH: Alg)

Steffen Klamt & Axel von Kamp

Interaction graph: a signed digraph.

(SD Dyn: Alg(Paths, Cyc), Biol)

Victor Klee
See also C. Jeffries.


Along with Simões-Pereira (1972a), invents the bicircular matroid (here, for infinite graphs).

(Bic)


When are various forms of stability of a linear differential equation \( \dot{x} = Ax \) determined solely by the sign pattern of \( A \)? A survey of elegant combinatorial criteria. Signed digraphs [alas] play but a minor role.

(QSta, SD: Exp, Ref)


A question about sign solvability that generalizes “the infamous even
cycle problem.” [Annot. 13 Apr 2009.] (sd: QSol, QSta)

Victor Klee, Richard Ladner, & Rachel Manber

Victor Klee & Pauline van den Driessche

Sulamita Klein
See L. Faria.

Jon M. Kleinberg
See D. Easley, J. Leskovec, and S.A. Marvel.

Robert D. Kleinberg
See S.A. Marvel.

Peter Kleinschmidt & Shmuel Onn

In a graded partially ordered set with $0$ and $1$, assign a sign to each covering pair $(x, y)$ where $y$ is covered by $1$. This is an “exact signing” if in every upper interval there is just one $y$ whose coverings are all positive. Then the poset is “signable”. (Sgnd: Geom)


See (1995a) for definition. Signability is a generalization to posets of partitionability of a simplicial complex (Prop. 3.1). Shellable posets, and face lattices of spherical polytopes and oriented matroid polytopes, are signable. A stronger property of a simplicial complex, “total signability”, which applies for instance to simplicial oriented matroid polytopes (Thm. 5.12), implies the upper bound property (Thm. 4.4). Computational complexity of face counting and of deciding shellability and partitionability are discussed in §6. (Sgnd: Geom, Alg)

Joseph B. Klerlein
See also R.L. Hemminger.


Continues the topic of Hemminger and Kerlein (1979a). (sg: LG, ori)

Joerg Kliewer
See C.A. Kelley.

Darwin Klingman
See J. Elam, F. Glover, and J. Hultz.
Elizabeth Klipsch  
20xxa Some signed graphs that are forbidden link minors for orientation embedding. Manuscript in preparation.

For each $n \geq 5$, either $-K_n$ or its 1-edge deletion, but not both, is a forbidden link minor. Which one it is, is controlled by Euler’s polyhedral formula, provided $n \geq 7$. [A long version with excruciating detail is available.]  

(SG: Top, Par)

Ton Kloks, Haiko Müller, & Kristina Vušković  

A decomposition theorem for graphs without induced even circles and $K_4 \setminus e$’s. [Question. Does it make sense to generalize to signed graphs without chordless balanced circles (longer than 3?) or $[K_4 \setminus e]'s$?] [Annot. 10 Mar 2011.]  

(par: Str)

T. Klotz  
See also J.F. Valdés and E.E. Vogel.

T. Klotz & S. Kobe  

The energy (i.e., $\#E^-(\Sigma^c)$) landscape of switchings of a signed graph, the underlying graph being a cubic lattice. [Annot. 4 Jan 2015.]  

(SG: State(fr), Sw, Phys)

Kolja Knauer  
See S. Felsner.

Klaus Knorr  
See J.D. Noh.

Andrew Knyazev  

Spectral clustering: bipartition $V$ according to signs in an eigenvector of min eigenvalue of Laplacian. Argues that for $\Sigma$, $L(\Sigma) = \text{diag}(\text{deg}) - A(\Sigma)$ is better than “signed Laplacian” $D - A(\Sigma)$ where $D$ has large positive diagonal, cf. Kunegis, Schmidt, et al. (2010a). [Annot. 8 Feb 2021.]  

(SG: Clu: Lap)

Lori Koban [Lori Fern]  
See also L. Fern.


Correction to Thm. 2.1 and an improved (and corrected) proof of Thm. 2.2 of Zaslavsky (2001a).  

(GG: M)

2004b Two Generalizations of Biased Graph Theory: Circuit Signatures and Modular Triples of Matroids, and Biased Expansions of Biased Graphs. Doctoral disser-
Chapter 1: “Circuit signatures and modular triples.” When can gains be applied to matroids, as they are to graphs in Zaslavsky (1991a), to produce a linear class of circuits and hence a lift matroid? Theorem 1.4.1: When the group has exponent \( > 2 \), one needs a ternary circuit signature, thus a ternary matroid. Theorem 1.4.5: When the group has exponent 2 the matroid must be binary (no circuit signature is required).

Ch. 2: “Biased expansions of biased graphs.” Generalizes group and biased expansions of a graph and the chromatic (and bias-matroid characteristic) polynomial formulas (Zaslavsky (1995b), (20xxj)) to expansions of a biased graph. Ch. 3: “When are biased expansions actually group expansions?” Partial results about characterizing biased expansions of biased graphs that are group expansions; counterexamples to several plausible conjectures.


Four kinds of circuit signatures of a matroid can be characterized through modular triples of copoints or circuits. They are lift signatures as well as the previously known weak orientations, orientations, and ternary signatures. Lifting signatures are needed to define a matroid with gains and thereby a lift matroid determined by the gains.


Generalizes group and biased expansions of a graph (Zaslavsky (1995b), Ex. 3.8; Zaslavsky (2001a), Ex. 4.1; Zaslavsky (20xxj)) to biased expansions of a biased graph. The definition is similar but tricky. The chromatic polynomials follow similar formulas. [Annot. 20 Oct 2012.]

Yusuke Kobayashi
See K. Kawarabayashi and Y. Kawase.

S. Kobe

William Kocay & Douglas Stone

Balanced network = signed covering graph of \(-\Gamma\) with edges \(vw\) lifted to \(+v, -w\) and added source and sink. [Annot. 8 Mar 2011.] (sg: cov)


Muralidharan Kodialam & James B. Orlin
Linear programming methods to find the strongly connected components of a periodic digraph from the static graph: i.e., of the covering digraph of a gain digraph $\Phi$ with gains in $Q^d$ by looking at $\Phi$. Cf. Cohen and Megiddo (1993a), whose goals are similar but algorithms differ.

Vijay Kodiyalam, R. Srinivasan, & V.S. Sunder

Shungo Koichi

§§4 2–3: Signed partial partitions (treated as sign-symmetric partitions of $\pm [n] \cup \{0\}$). Two sets of signed partial partitions are equivalent if one is converted to the other by switching in $\pm K^*_n$. A “signed bipartition” is a signed partial partition with one block (that is, ignoring the 0 block). [Annot. 28 Jan 2015.]

Johan Kok
See N.K. Sudev.

Tamara Koledin
See also M. Andelić.

Tamara Koledin & Zoran Stanić
2017a Connected signed graphs of fixed order, size, and number of negative edges with maximal index. Linear Multilinear Algebra 65 (2017), no. 11, 2187–2198. MR 3740690.

Consider $\Sigma$ with connected simple $|\Sigma|$ and fixed values $|V| = n$, $|E| = m$, $|E^-| = k$. Thm. 4.1: $\lambda_{\text{max}}(A(\Sigma_0)) = \max\{\lambda_{\text{max}}(A(\Sigma)) : \forall \Sigma \}$ iff $\Sigma_0$ is balanced with Harary bipartition $\{U_1, U_2\}$, $\Sigma^-$ is a bipartite chain graph with that bipartition, and each $\Sigma^+: U_i$ is threshold; but there are exceptions. [Annot. 4 Apr 2021.]


Cf. Stanić (2019d). All SRSG are sorted into 5 classes; Class 3 is where $c = \frac{1}{2}(a + b) \neq 0$. Thms. 4.1–2: $\Sigma$ in Class 3 is net regular. $A(\Sigma)$ has exactly 3 eigenvalues. The net-regularity degree $d^\pm$ is a simple eigenvalue. For the other eigenvalues, $\lambda^2 + \frac{1}{2}(b - a)\lambda + \frac{1}{2}(a + b) = r$. Similarities to strongly regular graphs, e.g., Thm. 4.4: A connected, regular, net-regular, inhomogeneous signed graph with 3 eigenvalues, where $d^\pm$ is a simple eigenvalue, is strongly regular. Thm. 4.8: $\Sigma$ in Class 3 with $d^\pm = 0$ is complete with $a + b = -2$. Thm. 4.9: If $\Sigma$ is in Class 3 and the eigenvalues $\neq d^\pm$ have equal multiplicity, then $\Sigma$ is complete, $d^\pm = 0$, and $a = b = -1$. Thm. 5.2: $\Sigma$ in Class 3 with $r \leq 10$ is complete, with one possible exception. Two incomplete examples are constructed from 3-class Johnson schemes. [Annot. 4 Apr
Alexandra Kolla  
See C. Carlson.

János Komlós  

Sharp asymptotic upper bounds on frustration index and vertex frustration number for all-negative signed graphs with fixed negative girth. Improves Bollobás, Erdős, Simonovits, & Szemerédi (1978a).\[Problem.\] Generalize to arbitrary signed graphs or signed simple graphs.\[Par: Fr\]

Helene J. Kommel  

Christian Komusiewicz  
See F. Hüffner.

Dénes König  

§ X.3, “Komposition von Büschen”, contains Thms. 9–16 of Ch. X. I restate them in terms of a signature on the edge set; König says subgraph or \(p\)-subgraph (“\(p\)-Teilgraph”) to mean what we would call the negative edge set of a signature or a balanced signature. Instead of signed switching, König speaks of set summation (“composition”) with a vertex star (“Büschel”). His theorems apply to finite and infinite graphs except where stated otherwise. Thm. 9: The edgewise product of balanced signatures is balanced. Thm. 10: Every balanced signing of a finite graph is a switching of the all-positive signature. Thm. 11: A signature is balanced iff it has a Harary bipartition [see Harary (1953a)]. Thm. 12 (cor. of 11): A graph is bicolorable iff every circle has even length. [König makes this fundamental theorem a corollary of a signed-graph theorem!] Thm. 13: A signature is balanced if (not “only if”, but that is obvious) every circle of a fundamental system is positive. Thm. 14: A graph with \(n\) vertices (a finite number) and \(c\) components has \(2^{n-c}\) balanced signings. Thm. 16: The set of all vertex switchings, except for one in each finite component of \(\Gamma\), forms a basis for the space of all finitely generated switchings.

[Switching is born here but not recognized until reinvented. König’s one failing was not to see the role of edge and circle signs; thus, I regard Harary (1953a) as the true invention of signed graphs.] [Annot. rev. 26 Aug 2018.]


Reprint of (1936a) together with Euler’s paper (in Latin and German)
on the Königsberg bridges and supplementary material.

(\text{sg: Bal, sw, Enum})


English translation of (1936a). § X.3: “Composition of stars”. [“Kreis” (circle, meaning circle) is unfortunately translated as “cycle”—one of the innumerable meanings of “cycle”].

(\text{sg: Bal, sw, Enum})

Alexander Kononov
See V. Il’ev.

Jack [Jacobus] H. Koolen

Justin Koonin

An eigenspace poset is described in terms of “\textit{d}-divisible, \textit{k}-evenly colored Dowling lattices”, which are subposets of Dowling lattices. [Annot. 12 Jul 2016.]

(\text{gg: M, Geom, Gen})

Hideo Kosako
See also S.J. Moon.

Hideo Kosako, Suck Joong Moon, Katsumi Harashima, & Takeo Ikai

“Variable-signed graph” = signed simple (di)graph $\Sigma$ with switching function $p$ and switched graph $\Sigma^p$. Known basic properties of switching are established. More interesting: planar duality when $|\Sigma|$ is planar. The planar dual $|\Sigma|^*$ inherits the same edge signs; a dual vertex has sign of the surrounding primal face boundary. Property 9 is in effect the statements: (1) If a signed plane graph has $f$ negative face boundaries, then $l(\Sigma) \geq f/2$. (2) If the negative faces fall into two connected groups with oddly many faces in each, (1) can be improved to $\geq f/2 + 1$. Finally, incidence matrices are studied that are only superficially related to signs. [The paper is hard to interpret due to mathematical imprecision and language difficulty.]

(\text{SG: Sw, fr, D, Incid})

Justyna Kosakowska
See also M. Kaniecki.


George E. Kostakis
See K.C. Mondal.

Alexandr V. Kostochka
See A.A. Ageev and E. Györi.
Sven Kosub  
See T. Akutsu.

Balázs Kotnyek  
See also G. Appa and L.S. Pitsoulis.


Introducing binet matrices; cf. Appa and Kotnyek (2006a). A binet matrix is $A = HB^{-1}H(B)$ where $B$ is a bidirected graph (which may be assumed to have no balanced components) and $B$ is a basis for $G(\Sigma(B))$. 

**Problem:** To recognize a binet matrix. Thm.: If an $n \times m$ matrix $A$ is an indecomposable binet matrix, then at most one component of $B$ has no half edge (and the remaining component has a negative circle). [Further work in Appa and Kotnyek (2006a), Musitelli (2007a), (2007a).] [Annot. 15 January 2013.] (SG: Ori: Incid, Alg)

Manoshi Kotoky  
See A.K. Baruah.

A. Kotzig  

Paulina Koutsaki  
See J.C. Bronski.

István Kovács, Aleksander Malnič, Dragan Marušič, & Štefko Miklavič  

Uses gain graphs (“voltage graphs”) to construct graphs with certain kinds of automorphisms. [Annot. 28 Mar 2017.] (GG: Cov: Algeb)

Vladislav B. Kovchegov  


A “model of the institution with relations” consists of a loopless digraph $D = (V, A)$ with $V = \{1, 2, \ldots, n\}$, a group $\mathfrak{G}$, sets $X$ and $Y$, and functions $f : A \to Y$, $z_i : Y \to \mathcal{P}(\mathfrak{G}) \forall i \in V$. We consider $r : A \to \mathfrak{G}$
such that \( r(i, j) \in z_i(f(i, j)) \). That is, \((D, r)\) is a gain digraph with gain group \( \mathcal{G} \). The edges are colored by \( f \) and the gains are constrained by the list \( z_i(y) \) for each vertex \( i \) and color \( y \). [Annot. 24 Nov 2012.]

(SG: WG, Adj, Bal, Clu, Geom)

Robin Koytcheff
See E. Ziv.

David Krackhardt
See P. Doreian.

Thomas Krajewski, Iain Moffatt, & Adrian Tanasa


Surface-embedded signed graphs (“ribbon graphs”) are a main example. [Annot. 17 Apr 2019.] (sg: Top: Invar)

Thomas Krajewski, Vincent Rivasseau, & Fabien Vignes-Tourneret

Daniel Král’, Jean-Sébastien Sereni, & Ladislav Stacho

\( \nu := \max \) number of vertex-disjoint negative circles. The vertex frustration number \( l_0(\Sigma) \leq 6\nu(\Sigma) \) for planar \( |\Sigma| \), improving on Fiorini, Hardy, Reed, and Vetta (2005a), (2007a). Dictionary: “odd” = negative, “even” = positive, “transversal” = \( X \subseteq V \) such that \( \Sigma \setminus X \) is balanced. [Annot. 1 Oct 2012, rev 14 Jan 2017.]

(sg: par: Fr, Circ)

Daniel Král’ & Heinz-Jürgen Voss

Thm. 1: For a signed plane graph \( \Gamma \), the frustration index \( l(\Sigma) \leq 2\nu \), where \( \nu := \max \) number of edge-disjoint negative circles. Dictionary: “odd” = negative, “even” = positive. [Continued in Fiorini, Hardy, Reed, and Vetta (2007a), Thm. 3.] [Annot. 6 Feb 2011.]

(sg: Par: Fr, Circ)

Mark A. Kramer
See also O.O. Oyeleye.

M.A. Kramer & B.L. Palowitch, Jr.

Vertex signs indicate directions of change in vertex variables; signed directed edges describe relations among these directions.

Truth tables for a signed edge as a function of endpoint signs. Algorithms for deducing logical rules about states (assignments of vertex
signs) from the signed digraph. Has a useful discussion of previous literature, e.g., Iri, Aoki, O’Shima, and Matsuyama (1979a).

P.L. Krapivsky
See T. Antal.

I. Krasikov

Following up Stanley (1985a), a signed $K_n$ is reconstructible from its single-vertex switching deck if its negative subgraph is disconnected (therefore also if its positive subgraph is disconnected) or if the minimum degree of its positive or negative subgraph is sufficiently small. All done in terms of Seidel switching of unsigned simple graphs. 


Following up Krasikov and Roditty (1987a), $(K_n,\sigma)$ is reconstructible from its $s$-vertex switching deck if $s = \frac{1}{2}n - r$ where $r \in \{0,2\}$ and $r \equiv n \pmod{4}$, or $r = 1 \equiv n \pmod{2}$; also, if $s = 2$ and the minimum degree of the positive or negative subgraph is sufficiently small. Also, bounds on $\#E^-$ if $(K_n,\sigma)$ is not reconstructible. Negative-subgraph degree sequence: reconstructible when $s = 2$ and $n \geq 10$. Done in terms of Seidel switching of unsigned simple graphs. ([kg: sw, TG])


If the minimum degrees of its positive and negative subgraphs obey certain bounds, a signed $K_n$ is reconstructible from its $s$-switching deck. The main bound involves the least and greatest even zeros of the Krawtchouk polynomial $K_n^s(x)$. Done in terms of Seidel switching of unsigned simple graphs. [More details in Zbl.]

Ilia Krasikov & Simon Litsyn

Among the applications mentioned (pp. 72–73): 2. “Switching reconstruction problem”, i.e., graph-switching reconstruction as in Stanley (1985a) etc. 4. “Sign reconstruction problem”, i.e., reconstructing a signed graph from its $s$-edge negation deck, which is the multiset of signed graphs obtained by separately negating each subset of $s$ edges (here called “switching signs”, but it is not signed-graph switching); this is a new problem.

I. Krasikov & Y. Roditty

§2: “Reconstruction of graphs from vertex switching”. Corollary 2.3. If a signed $K_n$ is not reconstructible from its $s$-vertex switching deck, a certain linear Diophantine system (the “balance equations”) has a certain kind of solution. For $s = 1$ the balance equations are equivalent to
Stanley’s (1985a) theorem; for larger $s$ they may or may not be. All is done in terms of Seidel switching of unsigned simple graphs. [Ellingham and Royle (1992a) note a gap in the proof of Lemma 2.5.]


Main Theorem. Fix $s \geq 4$. If $n$ is large and (for odd $s$) not evenly even, every signed $K_n$ is reconstructible from its $s$-vertex switching deck. Different results hold for $s = 2, 3$. (This is based on and strengthens Stanley (1985a).) Theorems 5 and 6 concern reconstructing subgraph numbers. All done in terms of Seidel switching of unsigned simple graphs.

Jan Kratochvíl
See also E. Jelínková.


Results about properties as in Kratochvíl, Nešetřil, & Zýka (1992a). E.g., switchability to a regular graph is NP-complete. [Annot. 21 Mar 2011.]

Jan Kratochvíl, Jaroslav Nešetřil, & Ondřej Zýka

Is a given graph switching isomorphic to a graph with a specified property? (This is Seidel switching of simple graphs.) Depending on the property, this question may be in P or be NP-complete, whether the original property is in P or is NP-complete. Properties: containing a
Hamilton path; containing a Hamilton circle; no induced $P_3$; regularity; etc. Thm. 4.1: Switching isomorphism and graph isomorphism are polynomially equivalent.

(TG: Sw: Alg)

Stefan Kratsch & Magnus Wahlström


M.J. Krawczyk, K. Malarz, B. Kawecka-Magiera, A.Z. Maksymowicz, & K. Kulakowski


A. Krieger & B. O’Connor


Introduces trivariate Tutte polynomial. See Goodall, Litjens, Regts, and Vena (2017a) et seq. for further developments. (SG: Invar)

Matthias Kriesell

See J. Bang-Jensen.

D.S. Krotov

See also E. Bespalov.


Γ is “switching separable” if ∃ Seidel switching that is nontrivially disconnected. (Trivially: all vertices but one are connected.) Thm.: If all Γ \ v and Γ \ {u,v} are, then Γ is. Deleting only single vertices is insufficient, for odd $n > 4$. [Annot. 31 Jul 2018.] (tg: Sw: Str)

Uffe Krusenstjerna-Hastrøm & Bjarne Toft


Special case of Toft’s (1975a) conjecture. (sg: par: Col)

Vyacheslav Krushkal

See also P. Fendley.


§7, “A multivariate graph polynomial”: A partially parametrized rank-generating polynomial (“multivariate Tutte polynomial”) for graphs embedded in surfaces, with the somewhat awkward duality relation (7.3).

F. Krzakala
See J.-P. Bouchaud.

Ying-Qiang Kuang
See Z.H. Chen.

Boris D. Kudryashov
See I.E. Bocharova.

Lukas Kühne & Geva Yashfe

Bernard Kujawski, Mark Ludwig, & Peter Abell

Krzysztof Kułakowski
See also P. Gawroński, F. Hassanibesheli, A. Mańka-Krasoń, B. Tadić, and J. Tomkowicz.

Krzysztof Kułakowski, Premiysław Gawroński, & Piotr Gronek

Devadatta M. Kulkarni
See J.W. Grossman.

R. Pradeep Kumar
See M.R. Rajesh Kanna.

T.R. Vasanth Kumar
See P.S.K. Reddy.

[Vijaya Kumar]
See G.R. Vijayakumar.

[Anita Kumari Rao]
See A.K. Rao (under ‘R’).

Jérôme Kunegis
(SG, SD: Lap: Bal, Clu, Fr, Eig, WG, Pred, PsS)

§3, “Measuring structural balance: The signed clustering coefficient”: A new definition; the coefficient for $\Sigma$ is $3(\sum_{C_3 \subseteq \Sigma} \sigma(C_3))/\#\{(e,f) : e \sim f\}$. Also defined for signed digraphs. [Annot. 8 Jan 2016.]

§4, “Visualizing structural balance: Signed graph drawing”: Applies $L(\Sigma)$ to signed-graph drawing.

§5, “Capturing structural balance: The signed Laplacian”: I.e., $L(\Sigma)$.

§5.3, “Balanced graphs”: Then $\text{Spec} L(\Sigma) = \text{Spec} L(|\Sigma|)$ and the eigenvectors of $\Sigma$ are switched (componentwise) from those of $|\Sigma|$. §6, “Measuring structural balance 2: Algebraic conflict”: The smallest eigenvalue of $L(\Sigma)$ is dubbed “algebraic conflict” since it is $> 0$ iff $\Sigma$ is unbalanced. Cf. Hou (2005a). §7, ‘Maximizing structural balance: Signed spectral clustering”: Uses $L(\Sigma)$, alternatively $D^{-1}A(\Sigma)$ ($D =$ diagonal degree matrix). §8, “Predicting structural balance: Signed resistance distance”: A way to compute resistance distance for “signed resistances” = weighted signed edges, with an adapted Kirchhoff’s current law. Used for edge prediction.

Partly expository.

(SG, SD: Lap: Bal, Clu, Fr, Eig, WG, Pred, PsS)

Jérôme Kunegis, Andreas Lommatzsch, & Christian Bauckhage


(SG: WG: Clu: Alg)

Jérôme Kunegis, Julia Preusse, & Felix Schwagereit


A fairly large partial positive subgraph tends to predict negative edges but only by using a combination of centrality and proximity predictors. [Annot. 29 Dec 2020.] 

(PsS: SG, Pred: Alg)

Jérôme Kunegis & Stephan Schmidt


(SG, WG: Lap)

Jérôme Kunegis, Stephan Schmidt, Şahin Albayrak, Christian Bauckhage, & Martin Mehlitz


(SG: Adj, Alg)

Jérôme Kunegis, Stephan Schmidt, Andreas Lommatzsch, Jürgen Lerner, Ernesto W. De Luca, & Sahin Albayrak

2010a Spectral analysis of signed graphs for clustering, prediction and visualization. In: Srinivasan Parthasarathy et al., eds., Proceedings of the Tenth SIAM In-
ternational Conference on Data Mining (Columbus, Ohio, 2010), pp. 559–570. Soc. for Industrial and Appl. Math., 2010. (SG: Eig, Clu, Geom, Pred, Alg)

Joseph P.S. Kung
See also J.E. Bonin and J. Kahn.


Examples include Dowling geometries, Ex. (6.2), and the frame matroids of full group expansions of graphs in certain classes; see pp. 98–99. (GG: M)


P. 41: Exposition of Stanley (1985a) from the viewpoint of the finite Radon transform. (kg: sw, TG)


Conjecture: For every group \( G \), \( \exists k = k_G \) such that if \( M \) is a rank-\( n \) matroid \( (n > k) \) where every rank-\( k \) interval \( [x, \hat{1}] \cong Q_k(G) \), then \( M \subseteq Q_n(G) \). [This should be provable. \( k \) should be small.] [Annot. 9 Apr 1987.] (gg: M: Str)


The Dowling geometry over the sign group is the largest simple ternary matroid not containing the “Reid matroid”. (sg: M: Xtreml)


Dowling geometries used in the proof of Prop. (1.2). (gg: M)


Survey with new results; largely on size bounds and extremal matroids for certain minor-closed classes. §2.7: “Gain-graphic matroids,” i.e., frame matroids of gain graphs. P. 30, top and fn. 9 on extremal gain-graph theory. §4.3: “Varieties.” Conj. (4.9)(c) on growth rates. §4.5, “Framed gain-graphic matroids,” i.e., cones over (“framed”) frame matroids in projective space. §6.1: “Cones,” i.e., unions of long lines on a common point: p. 47. Thm. (6.15) is a quadratic bound on matroids whose minors exclude (approximately) \( q + 2 \)-point lines and non-frame planes. Conj. (7.1) on directions in \( \mathbb{C}^n \)-matroids proposes that cyclic Dowling matroids are extremal. §8: “Concluding remarks,” on a possible ternary analog of Seymour’s decomposition theorem. (GG: M: Xtreml, Str, Exp, Ref)

Dowling lattices are lower-half Sperner. The proof is given only for partition lattices. (gg: M)


§6.2: “Gain-graphic matroids,” i.e., frame matroids of gain graphs. (GG: M: Exp)


Delete from a Dowling geometry a subset S that contains no whole plane. Found: necessary and sufficient conditions for the characteristic polynomial to factor completely over the integers. When the geometry corresponds to a hyperplane arrangement, many more of the arrangements are not free than are free; however, if S contains no whole line, all are free (so the characteristic polynomial factors completely over Z) while many are not supersolvable. (gg: M: Invar)


Higher-weight Dowling geometries yield counterexamples to a conjecture. (gg: Gen: M: Invar)


§11, “Generic rank-generating polynomials”: The “Tugger polynomial” is a partially parametrized rank-generating polynomial (cf. Zaslavsky (1992b)). (Sc(M): Gen: Invar)

2002a Curious characterizations of projective and affine geometries. Special issue in
Dowling geometries $G(\mathcal{G}K_n^*)$ (if $\#\mathcal{G} > 2$) and jointless Dowling geometries $G(\mathcal{G}K_n)$ (if $\#\mathcal{G} > 4$) exemplify Lemma 3.4, which says that 5 numbers characterize the line sizes in a simple matroid with all lines of size 2, 3, or $l$.


§4, “Minimal blocks from graphs”: $GF(q)^\times \cdot \Gamma$ is a minimal $k$-block over $GF(q)$ if $\Gamma$ is minimally $j$-chromatic for a certain $j = f(k)$, and is a minimal 1-block if $\Gamma$ is an odd circle. [Annot. 20 Jun 2011.] (GG: M)

Joseph P.S. Kung & James G. Oxley


For $n \geq 4$, the Dowling geometry of rank $n$ over the sign group is the unique largest simple matroid of rank $n$ that is representable over $GF(3)$ and $GF(q)$.

H. Kunze & D. Siegel


David Kuo

See J.H. Yan.

Y.S. Kuo

See also W.-S. Shih.

Y.S. Kuo, T.C. Chern, & Wei-kuan Shih


Algorithm, by minimum perfect matching, for $l(\Sigma)$ for a weighted signed graph that is cubic and planar. See Kuo–Chern–Shih (1988a).

[Authors are unaware of Katai and Iwai (1978a) or Barahona (1982a) etc.] [Annot. 21 Dec 2014.] (WG, sg: fr: Alg)

Yueh-Er Kuo

See G. de Leon-Calio.

Ranan D. Kuperman

See Z. Maoz.

Joseph Varghese Kureethara

See also and M. Acharya.


Jin Ho Kwak
See also I.P. Goulden.

Jin Ho Kwak, Sungpyo Hong, Jaeun Lee, & Moo Young Sohn

Jin Ho Kwak & Jaeun Lee


J.H. Kwak, Jaeun Lee, & Young-hee Shin

The number of isomorphism types of regular balanced coverings of a signed graph. A covering is a sign-preserving covering projection from one signed graph to another. (SG: Top: Enum)

Yung-Keun Kwon & Kwang-Hyun Cho

Simulations suggest that more positive cycles lead to more fixed points and more negative cycles lead to more non-fixed-point attractors (with a fixed number of variables [or genes]). [Annot. 16 Jan 2015.] (SD: Dyn: Str)

Vincent Labatut
See N. Armik, R. Figueiredo, and I. Mendonça.

Domenico Labbate
See M. Abreu.
Martine Labbé
See V. Devloo and R.M.V. Figueiredo.

Nicholas Lacasse

Richard Ladner
See V. Klee.

George M. Lady, Thomas J. Lundy, & John Maybee

The signed digraph $S(A)$ of square matrix $A$. Thm. 1: $A$ is NSNS iff the rows can be permuted so that $S(A)$ has a negative loop at each vertex and no other negative cycles, and no vertex-disjoint positive cycles. [Annot. 12 Jun, 24 Nov 2012.]

George M. Lady & John S. Maybee

In terms of signed graphs, restates and completes the characterizations of sign-invertible matrices $A$ due to Bassett, Maybee, and Quirk (1968a) and George M. Lady (The structure of qualitatively determinate relationships. Econometrica 51 (1983), 197–218. MR 0694457 (85c:90019). Zbl 517.15004) and reveals the sign pattern of $A^{-1}$ in terms of path signs in the associated signed digraph.

J.C. Lagarias

Theorem F: Feasibility of integer linear programs with at most two variables per constraint is NP-complete.

Hong-Jian Lai
See Z.H. Chen, S.-J. He, and Y.T. Liang.

Dimitri Lajou
See also F. Foucaud.


Introduces achromatic numbers $\chi_a(\Sigma)$ for sign-colored simple graphs and $\chi_s([\Sigma]) := \max \chi_a(\Sigma')$ over $\Sigma' \in [\Sigma]$, for switching classes, leading to two variants for switching classes and four for unsigned graphs: max and min of $\chi_a(\Sigma')$ for $\Sigma' \in [\Sigma]$, max and min of $\chi_a(\Sigma)$ over $|\Sigma| = \Gamma$, max and min of $\chi_s([\Sigma])$ over $|\Sigma| = \Gamma$. Five of the eight, including $\chi_a$ and $\chi_s$, are shown NP-complete; the other three are open. A “$k$-coloring” of $\Sigma$ is $\alpha : V \to [k]$ such that $\forall\{i, j\}$ the set $\{uv \in E : \alpha(u) = i, \alpha(v) = j\}$ is homogeneously signed. A $k$-coloring of $[\Sigma]$ is a $k$-coloring of any $\Sigma' \in [\Sigma]$.

This coloring is unrelated to (signed) coloring defined in Zaslavsky (1982b). Is there an achromatic number for the latter?
Dictionary: “2-edge-colored graph” = sign-colored graph $\Sigma$, “re-signing” = switching, “signed graph” = switching class $[\Sigma]$, “2-edge-colored homomorphism” = edge-sign-preserving homomorphism, “signed homomorphism” = switching (circle-sign-preserving) homomorphism, “(un)balanced path/cycle” = (negative) positive path/circle. [Annot. 28 Dec 2019.]


Aparna Lakshmanan S
See J.J. Palathingal.

P. Lallemand
See H.T. Diep.

[S. Ben Lamine]
See S. Ben Lamine (under “B”).

Yanhua Lan
See K.C. Mondal.

Kelvin Lancaster

Comment on Maybee (1981a). (QM: QSol: SD)

J.W. Landry & S.N. Coppersmith


Similar to (2002a) with a “quantum term” added. The effect is that of an extra vertex $v_0$ added to $\Sigma$, positively adjacent to all of $V$ with an arbitrary strength. Quantum ground and low-energy states are linear combinations of ground states of $\Sigma$ in a single component of the ground=state graph. Dictionary: “low-energy” = relatively few negative edges, “ground state graph” has ground states $\zeta$ for vertices and an edge between ground states that differ by switching a vertex (necessarily having $d^+ = d^-$. [Annot. 10 Jan 2015.] (SG: State(fr): Alg, Phys)

Carsten Lange, Shuping Liu, Norbert Peyerimhoff, & Olaf Post
Jan-Hendrik Lange

Steven Landy

The theorem characterizes concurrence of lines drawn from each vertex of a rectilinear simplex to a point in the opposite side. [Problem. Reformulate, maybe generalize, in terms of gain graphs. Cf. Boldescu (1970a), Zaslavsky (2003b) §2.6.] (gg: Geom)

Andrea S. LaPaugh & Christos H. Papadimitriou

Fast algorithms for existence of even paths between two given vertices (or any two vertices) of a graph. The corresponding digraph problem is NP-complete. [Signed (di)graphs are similar, due to the standard reduction by negative subdivision.] [See also, e.g., works by Thomassen.] (Par: Paths: Alg)(sd: Par: Paths: Alg)

Michel Las Vergnas
See A. Björner.

Martin Lätsch & Britta Peis

A bidirected graph $(\Gamma, \tau)$ (where $\tau$ assigns + or − to each incidence) is “strongly connected” if there is a coherent walk from any vertex to any other vertex. The distance $\text{dist}_{(\Gamma, \tau)}(u, v) := \text{the minimum length of a coherent } \vec{w} \text{ walk.}$ The diameter $\text{diam}_{(\Gamma, \tau)} := \max_{(u, v) \in V^2} \text{dist}_{(\Gamma, \tau)}(u, v).$

In $\Gamma$ define $i := \text{number of isthmi, } \gamma := \text{domination number.}$ Thm. 5: $\Gamma$ has a strongly connected bidirection iff $\#V = 1$ or $\Gamma$ is connected and minimum degree $\geq 2.$ Thm. 10: If $\Gamma$ has strongly connected bidirections $\tau_j$ ($j = 1, \ldots, k$), then $\min_j \text{diam}_{(\Gamma, \tau_j)} \leq 2i + 2\min(i, 1) + 5\gamma - 1.$ When $i = 0,$ $\tau_j$ can be chosen so $\Sigma(\Gamma, \tau_j)$ is all positive. Conjecture. Also true when $i > 0.$ Thm. 11: If $\Gamma$ has a strongly connected bidirection, then $\min_j \text{diam}_{(\Gamma, \tau_j)} \leq 6\gamma + 3.$ By Fig. 8 this bound must be at least $6\gamma + 1$ if isthmi are allowed. The proofs are constructive, esp. by extending to $\Gamma$ a bidirection of a dominating subgraph. Dictionary: “path” = walk [not trail]. [Annot. 27 Apr 2007.] (sg: Ori: Invar)

Reinhard Laubenbacher
See E. Sontag and A. Veliz-Cuba.

Monique Laurent
See M.M. Deza and A.M.H. Gerards.

Eugene L. Lawler
Ch. 6: “Nonbipartite matching.” §3: Bidirected flows. (sg: Ori)

Ben Lawson
See C.E. Tsourakakis.

Bac Hoai Le
See T.T.T. Ho.

Jason Leasure
See L. Fern.

Walter Lebrecht
See also J.F. Valdés and E.E. Vogel.

W. Lebrecht, J.F. Valdés, & E.E. Vogel
Randomly signed Kagomé and five-point-star planar lattices with specified concentration \(x\) of positive edges: frustration index (“frustration length”) et al., with combinatorial and numerical results compared. Also, compared with results for homogeneous lattices like square and triangular to analyze effects of degree (“coordination number”), plaquette shape (degree of polygonal faces), et al. [Annot. 3 Jan 2015.]

Ground state energy \(l(\Sigma)\), et al., as functions of \(x := \#E^+ / \#E\). Analytical, probabilistic, and computational results are largely consistent. [Annot. 3 Jan 2015.]

W. Lebrecht & E.E. Vogel

In given \(\Gamma\), \(x := \#E^+ / \#E\) implies an expected number of frustrated (negative) plaquettes. \(\Gamma\) is triangular, square, hexagonal, Kagomé, etc., with periodic boundary conditions (i.e., toroidal) or is the graph of a regular or semiregular polyhedron. Dictionary: cf. Vogel, Cartes, Contreras, Lebrecht, and Villegas (1994a). [Annot. 2 Jan 2015.]

W. Lebrecht, E.E. Vogel, & J.F. Valdés
Probabilistic and computational analysis of average states on signed toroidal Kagomé and five-point-star lattices. Frustration, energy, et al. as functions of \(x := \#E^+ / \#E\). Comparison to honeycomb, square, and triangular lattices (cf. other papers of the authors). Dictionary: cf.

Toroidal (“periodic boundary conditions”) lattice (3, 4, 6, 4) (Grünbaum–Shephard classification) with random signs having proportion $x$ of positive edges. Distribution of frustrated (negative) plaquettes, proportion of satisfied edges, *et al.*, in ground states. Comparison to other lattices (*cf.* other papers of the authors). [Annot. 3 Jan 2015.]

Bruno Leclerc


J. Leclercq & R. Thomas


Gibaek Lee, Sang-Oak Song, & En Sup Yoon


Combines signed digraphs and partial least squares for fault analysis in chemical engineering. (*SD: Appl*)

Jaeun Lee


Jon Lee


Jon Lee & Matt Scobee


The results imply that a ternary matroid, such as the frame matroid of a signed graph, has at most three orientation classes. [Thanks to Stefan van Zwam.] [Annot. 2 Apr 2013.] (*sg: m*)

Orlando Lee

See K. Kawarabayashi.

Sang-Gu Lee & Jin-Woo Park


See §4, “Nonnegativity of sign idempotent sign pattern matrices”. (*SD, SG: sw, Bal*)

Sang-Gu Lee, Se-Won Park, & Han-Guk Seol

*Cf. Eschenbach and Johnson (1990a).* [Annot. 20 Oct 2020.]

(QM, sd: bal(Cyc), sw)

**Shyi-Long Lee**

See also I. Gutman and P.K. Sahu.


Response to Gutman (1988a). Proposes weighted net sign: divide by number of nonzero vertex signs. The goal is to have the ordering of net signs correlate more closely with that of eigenvalues. (VS, SGw, Chem)


Expounds principally Lee, Lucchese, and Chu (1987a) and Lee and Gutman (1989a). Examples include all connected, simple graphs of order \( \leq 4 \) and some aromatics. (VS, SGw, Chem: Exp)


See Lee, Lucchese, and Chu (1987a). More examples; again, eigenvalue and net-sign orderings are compared. (VS, SGw, Chem)

**Shyi-Long Lee & Ivan Gutman**


Supplements Lee, Lucchese, and Chu (1987a) to answer an objection by Gutman (1988a), by treating vertex signs corresponding to multidimensional eigenspaces. (VS, SGw, Chem)

**Shyi-Long Lee & Chiuping Li**


Varies Lee, Lucchese, and Chu (1987a) by taking net signs of all balanced signings, instead of only those obtained from eigenvectors, for small paths, circles, and circles with short tails. The distribution of net sign, over all signings of each graph, is more or less binomial. (VS, SGw, Chem)


Abbreviated presentation of (1994a). (VS, SGw: Exp)

**Shyi-Long Lee & Feng-Yin Li**


**Shyi-Long Lee, Feng-Yin Li, & Friday Lin**


**Shyi-Long Lee, Robert R. Lucchese, & San Yan Chu**


Introduces the net sign of a (balanced) signed graph. A graph has vertices signed according to the signs of an eigenvector $X_i$ of the adjacency matrix, $\mu(v_r) = \text{sgn}(X_{ir})$, and $\sigma(v_r,v_s) = \mu(v_r)\mu(v_s)$ [hence $\Sigma$ is balanced]. Note that a vertex can have 'sign' 0. Net sign of a [hydrocarbon] chemical graph is applied to prediction of properties of molecular orbitals.

**Shyi-Long Lee, Yeung-Long Luo, & Yeong-Nan Yeh**


See Lee, Lucchese, and Chu (1987a). Net signs for the Platonic polyhedra (Table I).

**Shyi-Long Lee & Yeong-Nan Yeh**


Follows up Lee, Lucchese, and Chu (1987a) and Lee and Gutman (1989a), calculating net signs of eigenspatially signed hypercube graphs of dimensions up to 6 by means of a general graph-product formula.

**Géraud Le Falher & Fabio Vitale**


**Hanno Lefmann**


Thm. 1.2 bounds the size of a family of lattice elements with prescribed meet ranks. Dowling lattices are an example of this and related results.
Jenő Lehel  
See R.J. Faudree.

Ziyi Lei  
See Z.-Y. Cheng.

Frank Thomson Leighton  
See C.R. Johnson.

Samuel Leinhardt  
See also J.A. Davis and P.W. Holland.

Samuel Leinhardt, ed.  
An anthology reprinting some basic papers in structural balance theory, including some elementary signed-graph theory. (PsS, SG: Bal, Clu)

P.W.H. Lemmens & J.J. Seidel  
Hints of graph switching; see van Lint and Seidel (1966a). (Geom, sw)

Gloria de Leon-Calio & Yueh-Er Kuo  
Simple properties of pulse properties, as in Roberts (1976a). Seems largely expository. [Annot. 5 Jun 2019.] (SDw: Exp)

Marianne Lepp [Marianne L. Gardner]  
See R. Shull.

Jürgen Lerner  
See J. Kunegis.

Jure Leskovec, Daniel Huttenlocher, & Jon Kleinberg  
(SD, SG: Bal, Clu)


Adam N. Letchford  
See K. Kaparis.

Emily Leven, Brendon Rhoades, & Andrew Timothy Wilson  

Richard Levins  
See also J.M. Dambacher and C.J. Puccia.


**Vadim E. Levit**
See Y. Cherniavsky.

**Mario Levorato, Rosa Figueiredo, Yuri Frota, & Lúcia Drummond**

**Mordechai Lewin**


**Torina Lewis, Jenny McNulty, Nancy Ann Neudauer, Talmage James Reid & Laura Sheppardson**

The connected bicircular matroids in which all circuits have the same size, i.e., which are duals of matroid designs, are certain uniform subdivisions of uniform matroids. [Annot. 9 Jun 2013.] (Bic)

**David W. Lewit**
See E.G. Shrader.

**Josef Leydold**
See T. Bıyıkoglu.

**Claire Lhuillier**
See G. Misguich.

**Bao Feng Li**
See X.H. Hao.

**Cai Heng Li & Jozef Širáň**

That is, signed expansion graphs $\pm\Gamma$, orientation embedded in a surface (Möbius), whose map automorphisms act transitively on flags (regularity). Properties of their automorphism groups. [Follows Wilson (1989a).] [Annot. rev. 31 Jul 2014.] (SG: Top: Aut)

**Chang Li**
See T. Harju.

**Chiuping Li**
See I. Gutman and S.L. Lee.
Dong Li, Cuihua Wang, Shengping Zhang, Guanglu Zhou, Dianhui Chu, & Chong Wu

Dong Li, Zhi-Ming Xu, Nilanjan Chakraborty, Anika Gupta, Katia Sycara, & Sheng Li

Feng-Hin Li
See S.L. Lee.

Guangbin Li

Guojun Li & Aimei Yu

Hao Li
See W.J. Ning.

Hiram W. Li
See J.M. Dambacher.

Hong-Hai Li
See also L. Su.

Hong-Hai Li & Jiong-Sheng Li


Hong-Hai Li, Bit-Shun Tam, & Li Su

Minimum and maximum magnitudes and associated graphs are found (for $n \geq 5$). Thms. 4.1, 5.1, 5.2, 5.3 on transforms of $\Gamma$ have two cases depending on whether (connected) $\Gamma$ is bipartite. [Conjecture. The results generalize to signed graphs with two (connected) cases: balanced or not.] [Annot. 20 Jan 2015.] (par: Lap: Eig)

Ji Li
See H.Z. Deng.
Jiaao Li
See M. DeVos.

Jianxi Li
See also J.M. Guo.

Jianxi Li & Ji-Ming Guo

Jing Li
See S.Y. Wang.

Ke Li
See also L.G. Wang.

Ke Li, Ligong Wang, & Guopeng Zhao


Jiong-Sheng Li
See Y.P. Hou, H.H. Li, and X.D. Zhang.

Lulu Li
See G.-H. Mu.

Nan Li
See A. Funato and T. Hibi.

Qian Li & Bolian Liu

Qian Li, Bolian Liu, & Jeffrey Stuart

Qingdu Li
See S.-D. Zhai.

Rao Li

Ruilin Li & Jinsong Shi

Rui-lin Li, Jin-song Shi, & Bing-can Dong

(par: Lap: Eig)

Selena Li
See L.J. Rusnak.

Sheng Li
See D. Li.

Shuang-Dong Li
See Y. Wang.

Shuchao Li
See also B. Chen, C. Chen, X.Y. Geng, S.S. He, J. Huang, G.-F. Wang, and M.J. Zhang.

Shuchao Li, Wanting Sun, & Baogen Xu
20xxa Relation between the adjacency rank of a complex unit gain graph and some classical parameters of its underlying graph. Submitted. (GG: Adj)

Shuchao Li & Yi Tian

Weight function $w : E \to \mathbb{R}_{>0}$. Since $\text{Spec } L(\Gamma, w) = \text{Spec } L(-\Gamma, w)$, $L(-\Gamma, w)$ is used to find $\lambda_1(L(\Gamma, w))$. [Annot. 21 Jan 2012.]

(par: Lap: Eig)


Laplacian matrix $L(-\Gamma)$. [Annot. 29 Jul 2019.]

(par: Lap: Eig)


Upper and lower bounds on the sum of largest eigenvalues of $L(-\Gamma)$ and $L(-\Gamma^c)$ for a simple graph. [See also Oliveira and de Lima (2016a).]

Eigenvalue and eigenvector bounds from $L(-\Gamma)$ on the clique and stability numbers. [Annot. 7 Jan 2015.]

(par: Lap: Eig)

Shuchao Li & Shujing Wang

(par: Lap: Eig)


The model: $\text{Prob}(+e_{ij}) = p$, $\text{Prob}(-e_{ij}) = q$, $\text{Prob}(\text{no } e_{ij}) = 1 - p - q$. An integral expression for energy $\mathcal{E}$ and Wigner matrices. Thm. 2.7: $\mathcal{E} = n^{3/2} \left( \frac{8}{3\pi} \sqrt{p + q - (p - q)^2} + o(1) \right)$. §3, “The energy of the random multipartite signed graph”: bounds. [Annot. 12 Jul 2019.]

(SG: Rand: Eig)

Shuchao Li & Li Zhang

Sharp upper and lower bounds for $\text{per}(L(-\Gamma))$ when $\Gamma$ is unicyclic or bipartite, with or without girth, and characterization of extremal graphs. (Authors' summary.) [Bipartite $\Gamma$ means they are doing $L(\Gamma)$; the truly signed part is for unicyclic graphs only.] [Annot. 19 Nov 2011.]

(par: Lap: Eig)


See Li and Zhang (2011a). Here, the second minimum of, and a lower bound for, $\text{per} L(-\Gamma)$. [Annot. 24 Jan 2012.]

(par: Lap)

**Shuchao Li & Minjie Zhang**


**Shuchao Li, Siqi Zhang, & Baogen Xu**

20xxa The relation between the $H$-rank of a mixed graph and the independence number of its underlying graph. *Linear Multilinear Algebra* (in press).

$\Phi$ with gain group $\{\pm 1, \pm i\}$: $\varphi(e) = 1$ for undirected, $i$ for directed edges. (gg: Adj)

**Xiangwen Li**

See L.L. Hu.

**Xiaodi Li**

See G.-H. Mu.

**Xiao Ming Li**

See F.T. Boesch and F.L. Tian.

**Xueliang Li**


**Xueliang Li & Wen Xia**


**Xueliang Li, Jianbin Zhang, & Lusheng Wang**


[Bipartite energy is the energy of $A(\Gamma)$ for bipartite $\Gamma$. Problem 1. Generalize to antibalanced signed graphs. Problem 2. Generalize to signed graphs.] [Annot. 24 Jan 2012.]

(par: bal: Lap: Eig)

**Yadong Li, Jing Liu, & Chenlong Liu**

Yang Li
See B. Yang.

Yanhua Li, Wei Chen, Yajun Wang, & Zhi-Li Zhang

Yijia Li
See S.-S. Feng.

Yiyang Li
See W.X. Du.

Yong Li
See J.-S. Wu.

Yuemeng Li
See also L.T. Wu.

Yuemeng Li, Xintao Wu, & Aidong Lu

Yuemeng Li, Shuhan Yuan, Xintao Wu, & Aidong Lu

Zhentao Li
See K. Kawarabayashi.

Zhongshan Li
See also M. Arav, C.A. Eschenbach, F.J. Hall and L. Zhang.

Zhongshan Li, Frank Hall, & Carolyn Eschenbach

Chaohua Liang, Bolian Liu, & Yufei Huang

Yanting Liang, Bolian Liu, & Hong-Jian Lai

Rui-Quan Liao
See B. Hu.
F. di Liberto
See A. Coniglio.

Hans Liebeck
See D. Harries.

Martin W. Liebeck
Examines the \( F\) \( \operatorname{Aut}(\Sigma)\)-module \( F^2V(\Sigma) \), where \( \Sigma \) is a signed complete graph and \( F \) is a field of characteristic 2.

Given an abstract group \( \mathfrak{A} \), which of its permutation representations are exposable on every invariant switching class of signed complete graphs [see Harries and H. Liebeck (1978a) for definitions]?

Thomas M. Liebling
See H. Gröflin.

Rainer Liebmann
Detailed and readable descriptions, often simplified and relatively combinatorial, of the state of knowledge about Ising systems in the form of signed graphs and weighted signed graphs. [Relatively accessible to combinatorists.] Dictionary: "model" = graph with signs and usually weights, “ferromagnetic” = positive edge, “antiferromagnetic” = negative edge, “fully frustrated” = all girth circles are negative, “state” = \( s: V \rightarrow \{+1, -1\} \), “ground state” = state with fewest frustrated edges, “ground state degeneracy” = number of ground states (1 being nondegenerate), “excited state” = non-ground state. §2.1.1, “Ground state degeneracy of the ANNNI-chain”, on chains of triangles with two bond signs and strengths, \( J_1 \) and \( J_2 \) (ANNI = Axial Next Nearest Neighbor Ising model.) The number and description of ground states are treated in detail, as well as less combinatorial physical quantities. §2.3.1, “Periodic frustrated chains”: All weights equal, so this is signed graphs. Restates Doman and Williams (1982a) in terms of a path with distance-2 edges, signed with period 4. The path edges have constant sign (either + or – by switching) and weight \( B \); the distance-2 edges are + − − with weight \( J \).

§3.1.2b, “Star-triangle transformation”: Edge signs and weights transform. The triangle-star transformation on a negative triangle gives imaginary signs. [Question. Does this indicate a use for complex unit gains?] §3.2, “Triangular lattice”: Based on Houtappel (1950a), (1950b) and Wannier (1950a). §3.3.1, “Union Jack lattice”: Square lattice, edges weighted \( J_1 \), with alternating diagonals in alternating squares weighted \( J_2 < 0 \). All triangles are negative. \( |J_2|/J_1 \) determines behavior. For ratio 1 (a signed graph), there are \( \approx C^{|V|} \) ground states for a finite sublattice, where \( C \geq \sqrt{17/8} \). §3.3.2, “Villain’s odd model”: Cf. Villain (1977a). §3.3.3, “Hexagon lattice”: Cf. Wolff and Zittartz (1982a),
§3.3.4, “Pentagon lattice”: Cf. Waldor, Wolff, and Zittartz (1985a). §3.3.5, “Kagomé lattice”: Various periodic sign patterns; references. §3.3.6, “Connection between GS [ground state] degeneracy and existence of a phase transition at $T_c = 0$”: The conjecture of Hoever, Wolff, and Zittartz (1981a). Also, a conjecture of Süto on the exact conditions under which the ground states are connected in the state graph. §3.4, “Frustrated Ising systems with crossing interactions”: Several more complicated extensions of previous models, usually by adding distance-2 edges (“nnn interactions”). See (2) below.

§4.1, “fcc antiferromagnet”: All-negative face-centered cubic lattice graph. Interesting remarks on how ground state and near-ground state structure might influence physical properties. §4.2, “Fully and partially frustrated simple cubic lattice”: The fully frustrated planar square lattice can be stacked in various ways to produce differently frustrated cubic lattices. §4.3, “AF pyrochlore model”: All-negative tetrahedra joined at corners. §4.4, “ANNNI-model”: All-positive cubic lattice with negative distance-2 vertical edges.

Two frequent remarks: (1) An external magnetic field reduces the number of ground states. (2) Slightly more complicated graphs give models that are not exactly solvable. [Combinatorial explanations: The magnetic field corresponds to an extra vertex, positively adjacent to all $V(\Sigma)$; see Barahona (1982a). The more complicated graphs are non-planar; Barahona (1982a) and Istrail (2000a) indicate that this is the obstacle to exact solution.] [Annot. 28 Aug 2012.]

Magnhild Lien & William Watkins


The Laplacian matrices of a signed plane graph and its dual have the same invariant factors. The proof is via the signed graphs of knot diagrams.

Frauke Liers

See M. Palassini and G. Pardella.

Ko-Wei Lih

See J.H. Yan.

Bart Litjens

See A. Goodall.

Chjan C. Lim


A family of bipartite signed wheels that prevent $A = (A^{-1})^T$. A family of bipartite signed graphs which allow it.[Annot. 6 Mar 2011.]

Ee-Peng Lim

See D. Lo.

Meng-Hiot Lim

See Harary, Lim, et al.
[Leonardo Silva de Lima]
See L.S. de Lima (under ‘D’).

Enzo M. Li Marzi

Friday Lin
See S.L. Lee.

Lin Lin
See J.-G. Dong.

Shangwei Lin
See S.Y. Wang.

Xue Lin, Qiang Jiao, & Long Wang

Jonathan A. Lindquist
See S. Klamt.

Bernt Lindström
See F. Harary.

Gabriele Lini
See C. Altafini.

Nathan Linial
See Y. Bilu and S. Hoory.

Sóstenes Lins

For Eulerian $\Sigma$ in projective plane, max. number of edge-disjoint negative circles = min. number of edges cut by a noncontractible closed curve that avoids the vertices. [Generalized by Schrijver (1989a).]

(SG: Top, fr, Circ, Alg)


See §4. [Cf. Zaslavsky (1992a) for related work.] (sg: Top: bal)


J.H. van Lint & J.J. Seidel

Introduces graph switching. (tg, Geom)

Svante Linusson
See C.A. Athanasiadis.

Marc J. Lipman & Richard D. Ringeisen
1978a Switching connectivity in graphs. In: F. Hoffman *et al.*, eds., *Proc. of the Ninth Southeastern Conf. on Combinatorics, Graph Theory and Computing* (Boca

A. Lipshtat
See A. Ma’ayan.

C.H.C. Little
See I. Fischer.

Simon Litsyn
See I. Krasikov.

Charles H.C. Little
See C.P. Bonnington.

Bolian Liu
See also B. Cheng, Y.F. Huang, C. Li, Q.A. Li, Y.T. Liang, J.P. Liu, M.H. Liu, and Z.F. You.

Bolian Liu, Muhuo Liu, & Zhifu You

For a degree sequence \( \pi \), define \( \mu_c(\pi) := \max_{\Gamma} \lambda_1(L(-\Gamma)) \) over connected \( \Gamma \) with degree sequence \( \pi \) and \( c \) circles. Let \( \pi \preceq \pi' \) in the majorization ordering. Thm. 2: Under certain assumptions on \( c \), \( \pi \), \( \pi' \), \( \mu(\pi) \leq \mu(\pi') \). For the special cases of unicyclic and bicyclic graphs: X.D. Zhang (2009a) and Huang, Liu, and Liu (2011a). For majorization also see Tam, Fan, and Zhou (2008a), M.H. Liu and Liu (2012a).

Chenlong Liu
See Y.D. Li.

Daphne Der-Fen Liu
See Y.-T. Jiang.

Fang Liu
See J.S. Wu.

Feng Liu
See X.-J. Tian.

Gui Zhen Liu & Qiang Wu

Describes some applications of and some results about balance in signed graphs. (SG: Bal, PsS: Exp, M)

Huan Liu

Henry Liu, Robert Morris, & Noah Prince


**Huiqing Liu**

See also Q. Wen.

**Huiqing Liu & Mei Lu**


Corrects and proves a conjecture of Hansen and Lucas (2010a) on max $\lambda_1(L(-\Gamma))D(\Gamma)$ for $\#V = n$, where $\lambda_1$ = the largest eigenvalue of $L(-\Gamma)$ and $D$ = diameter. [Annot. 23 Nov 2014.] (par: Lap: Eig)

**Ji Liu**

See W. Chen.

**Jia-Bao Liu & Shaohui Wang**


**Jianping Liu & Bolian Liu**


Bounds on the clique number $\omega(\Gamma)$ based on the least and greatest eigenvalues of $L(-\Gamma)$. A similar lower bound on the stability number $\alpha(\Gamma)$. [Annot. 23 Nov 2014.] (par: Lap: Eig)

**Jianxi Liu & Xueliang Li**


“Hermitian adjacency matrix” = $A(\Phi)$ with gain group \{±1, ±i\}: $\varphi(e) = 1$ for undirected, $i$ for directed edges. Thm. 2.7: Sachs formula for det $A(\Phi)$. Thm. 2.8: Coefficients of characteristic polynomial. Imaginary-gain circles contribute nothing to the formulas. Thm. 2.16: If $|\Phi_1| = |\Phi_2|$ and gains differ on some edge in a cut, then Spec $A(\Phi_1) = \text{Spec} A(\Phi_2) [\text{obviously false as stated}]. \ [Cf. Mohar (2016a).] \ S3, “Bounds of Hermitian energy”. §4, “Mixed graphs that share the same spectrum with their underlying graphs”. Thm. 4.1: Spec $A(\Phi) = \text{Spec} A(\|\Phi\|)$ iff $\Phi$ is balanced. Thm. 4.2: Cartesian product [as in] preserves balance. Dictionary: “value” = gain; “rank, corank” = those of matroid $G(\|\Phi\|)$; “positive” = balanced; “generalized orientation” = partial orientation. §5, “Oriented graphs”. §6, “Integral representation for Hermitian energy”. [Annot. 15 Dec 2020.] (gg: Adj, Sw: Eig)

**Jianzhou Liu**

See R. Huang.

**Jiming Liu**

See B. Yang.
Jing Liu
See Y.D. Li.

Lily L. Liu & Yi Wang
§3.5., “Compositions of multisets and Dowling lattices”.

Lu Liu
See Y.-Z. Wu.

Mu Huo Liu
See also B.L. Liu.

Muhuo Liu & Bolian Liu

Spread $S = \text{difference of largest and smallest eigenvalues}$, studied for $L(-\Gamma)$. Let $m(v) := \text{average degree in } N(v)$. Thm. 2.1: $\Delta - \delta + 1 \leq S \leq \max \{d(v) + m(v)\}$. Other lower bounds in terms of $\sum_v d(v)^2$, average degree of independent set of vertices. Thm. 2.5: Min spread of unicyclic graphs. [Cf. (2011a); Oliveira, de Lima, de Abreu, and Kirkland (2010a); Fan and Fallat (2012a).]

Also see Liu, Liu, and You (2013a).

Muhuo Liu, Bolian Liu, & Fuyi Wei


Also see Liu, Liu, and You (2013a).


The 2 or 4 largest eigenvalues and spreads of $L(-\Gamma)$. [Problem. Generalize to signed graphs, or complex unit gain graphs. Cf. Reff (2012a).]
[Annot. 19 May 2018.]

Muhuo Liu, Bolian Liu, & Xuezhong Tan

Ning Liu & William J. Stewart

Ruifang Liu
See M.Q. Zhai.

Shiping Liu
See also F.M. Atay and C. Lange.

Shiping Liu, Norbert Peyerimhoff, & Alina Vdovina
Gains in the group $T_3$ of cube roots of unity. Eigenvalues of the $T_3$-covering graph in terms of those of the gain graph $\Phi$ and underlying graph $\|\Phi\|$. [Annot. 30 Oct 2017.]

Vivian Liu
See G. Chen.

Wenzhong Liu, Huazheng You, & Qing Cui
Question of Markström (2012a) solved for oddness $\leq 4$. [Annot. 11 Nov 2020.]

Xianzhu Liu, Zhijian Ji, & Ting Hou

Xiaogang Liu
See also D. Hu and Y.P. Zhang.

Xiaogang Liu, Suijie Wang, Yuanping Zhang, & Xuerong Yong

Xiaoyu Liu
See Y.Q. Chen.

Xin Liu
See G.-H. Yu.

Xueyan Liu
See B. Yang.

Yan Pei Liu
See R.X. Hao.

Yingluan Liu
See Y.F. Huang.

Yu Liu & Lihua You

Yue Liu

See also X.Y. Yuan.

Yue Liu, Jia-Yu Shao, & Ling-Zhi Ren


Dictionary: “arc-weighted digraph of \( A \)” = complex gain digraph whose adjacency matrix is \( A \). (QM: gg: Adj)

Etera R. Livine

See R.C. Avohou.

Paulette Lloyd

See P. Bonacich and P. Doreian.

David Lo, Didi Surian, Philips Kokoh Prasetyo, Kuan Zhang, & Ee-Peng Lim


Martin Loebl

See also Y. Crama and A. Galluccio.

Martin Loebl & Iain Moffatt


[Shobana Loganathan]

See L. Shobana.

D.O. Logofet & N.B. Ul’yanov


Necessity of Jeffries’ (1974a) sufficient conditions. (QSta)


English trans. of (1982a). (QSta)

Michael Lohman

See M. Chudnovsky.

V. Lokesha

See also P.S.K. Reddy.

V. Lokesha, P.S. Hemavathi, & S. Vijay


V. Lokesha, P. Siva Kota Reddy, & S. Vijay

Definitions and notation as in Sampathkumar, Reddy, and Subramanya (2008a). Generalization of Subramanya and Reddy (2009a) to symmetric $n$-signed graphs, with similar definitions and results. [The results remain true without assuming symmetry.] [Annot. 10 Apr 2009.]

(SG(Gen), gg: Bal, LG(Gen), Sw)

Andreas Lommatzsch
See J. Kunegis.

Bo Long
See S.H. Yang.

M. Loréa


Discovers the “linearly bounded” (or “count”) matroids of graphs. [See White and Whiteley (1983a), Whiteley (1996a), Schmidt (1979a).]

(MtrdF: Bic, Gen)

Martin Lotz & Johann A. Makowsky


(SGw: Invar: Alg)

E. Loukakis


Another algorithm for detecting balance [cf. Hansen (1978a), Harary and Kabell (1980a)]. Also, once again proves that all-negative frustration index [obviously equivalent to Max Cut] is NP-complete.

(SG: Bal, Fr: Alg)

Janice R. Lourie


László Lovász
See also J.A. Bondy, Gerards, Lovász, et al. (1990a), and M. Grötschel.


Characterization of the graphs having no two vertex-disjoint circles. See Bollobás (1978a) for exposition in English. [Major Problem. Characterize the biased graphs having no two vertex-disjoint unbalanced circles. This theorem is the contrabalanced case. For the sign-biased case see Slilaty (2007a). McCuaig (1993a) might be relevant to the general problem.]

(GG: Circ: Str)


Prob. 7.21 finds $rkH(−Γ)$ [cf. van Nuffelen (1973a)]. Prob. 10.18: The vertex frustration number of a contrabalanced graph vs. the circle


It is hard to escape the feeling that we are dealing with all-negative signed graphs and their \( -K_4 \) and \( -K_2^2 \) minors. [And indeed, see Gerards and Schrijver (1986a) and Gerards, Lovász, et al. (1990a) and the notes on Seymour (1995a).]

(Par: Str)


L. Lovász & M.D. Plummer


Pp. 247–248: Shortest odd/even \( uv \)-path problem in \( \Gamma \). Lemma 6.6.9 reduces min length of odd path to a min-weight perfect matching problem in a modified graph. Exerc. 6.6.10–11 are similar for even paths and odd/even circles. [Problem. Generalize to negative/positive paths and circles in signed graphs.] §6.6, p. 252: \( l(\neg \Gamma) \) [i.e., max cut in \( \Gamma \)], \( l(\Sigma) \) for signed planar graphs. Cor. 6.19: For planar \( \Gamma \), \( l(\neg \Gamma) = \frac{1}{2} \) (max number of circles in a 2-packing of negative circles). [Question: How does this generalize to signed planar graphs?] Pp. 252–253: Odd-circle packing and 2-packing. [Annot. 10 Nov 2010.]


Reprint of (1986a) with errata and an appendix of updates. [Annot. 10 Nov 2010.]


L. Lovász, L. Pyber, D.J.A. Welsh, & G.M. Ziegler


§7: “Knots and the Tutte polynomial”, considers the signed graph of a
knot diagram (pp. 2076–77).  

Aidong Lu  
See Y.-M. Li and L.T. Wu.

Lingfei Lu  
See M. Zhu.

Mei Lu  
See H.Q. Liu and W.J. Ning.

Yong Lu  
See also L. Zhang.

Yong Lu, Ligong Wang, & Peng Xiao  
Rank of $A(\Phi)$.  

Yong Lu, Ligong Wang, & Qiannan Zhou  
Normalized adjacency matrix with gain group $\{\pm 1, \pm i\}$: $\varphi(e) = 1$ for undirected, $i$ for directed edges.  

Rank of $A(\Sigma)$ vs. $A(|\Sigma|)$.  

Rank of $A(\Phi)$ vs. $A(||\Phi||)$.  

Yong Lu & Jingwen Wu  
2021a No signed graph with the nullity $\eta(G, \sigma) = |V(G)| - 2m(G) + 2c(G) - 1$. *Linear Algebra Appl.* 615 (2021), 175–193. MR 4200821. arXiv:2006.00002.  
$\text{rk} A(\Sigma) + 2\xi - 2\mu = s$ is impossible for $s = 1$, where $\mu :=$ matching number, $\xi :=$ cyclomatic number. Fixing $\xi$, infinitely many $\Sigma$ exist for which $s \in \{0\} \cup [2, 3\xi]$. [Annot. 6 Feb 2021.]  

You Lu  
See also J.-A. Cheng, M. DeVos, and X. Wang.

You Lu, Rong Luo, Michael Schubert, Eckhard Steffen, & Cun-Quan Zhang  
In many ways they behave like flows on unsigned graphs.  

You Lu, Rong Luo, & Cun-Quan Zhang  
\(\Sigma\) without edge-disjoint negative circles has a nowhere-zero 6-flow, satisfying Bouchet’s (1983a) conjecture. [Annot. 26 Nov 2020.] (SG: Flows)

Claire Lucas  
See M. Aouchiche and P. Hansen.

Robert R. Lucchese  
See S.L. Lee.

Henri Luchian  
See A. Băutu.

Tomasz Łuczak  
See also E. Györi.

2016a Highly connected monochromatic subgraphs of two-colored complete graphs.  
Thm. If \(2 \leq k \leq (n + 3)/4\), then \(\Sigma = (K_n, \sigma)\) contains a \(k\)-connected homogeneously signed subgraph of order \(n - 2(k - 1)\) or \(k\)-connected all-positive and all-negative subgraphs of order \(n - 2(k - 1)\). Completes work of Bollobás and Gyárfás (2008a) (who conjectured most of this), Liu, Morris, and Prince (2009a), and Fujita and Magnant (2011a). [Annot. 24 Jan 2016.] (sg: Str)

Mark Ludwig  
See also P. Abell and B. Kujawski.

M. Ludwig & P. Abell  
2007a An evolutionary model of social networks.  
Signed edges are added to and deleted from a fixed set of nodes under a balancing rule. Imbalance measured by frustrated triangles impels evolution, which converges under some conditions. [Annot. 20 Jun 2011.] (SG: Bal, Fr: Dyn)

J. Lukic, A. Galluccio, E. Marinari, O.C. Martin, & G. Rinaldi  
2004a Critical thermodynamics of the two-dimensional \(\pm J\) Ising spin glass.  
Physical properties of a signed toroidal square lattice graph, from computation of the exact partition function (energy distribution) via Galluccio, Loebl, and Vondrák (2000a), (2001a). E.g., the approximate proportion of negative edges is important. [Annot. 18 Aug 2012.] (SG: Phys, Fr)

Robert Lukot’ka  
See T. Kaiser.

J. Richard Lundgren  
See H.J. Greenberg and F. Harary.

Thomas J. Lundy  
See also G.M. Lady.

Thomas J. Lundy, John Maybee, & James Van Buskirk  
1996a On maximal sign-nonsingular matrices.  
Constructions of such matrices. A matrix definition of $C_4$-cockades.  
[Annot. 6 Mar 2011.]  

(Rong Luo)  

(Yeung-Long Luo)  
See I. Gutman and S.L. Lee.

(Bob Lutz)  
“Dirichlet hyperplane arrangements” are a kind of gain-graphic arrangement; cf. (20xxb).  [Annot. 16 Nov 2018.]  

“Dirichlet matroids” are complete lift matroids of a kind of biased graph. Dirichlet arrangements (2019a) are hyperplane representations of them.  [Annot. 16 Nov 2018, 25 Oct 2019.]  


(Shengxiang Lv [Shengxiang Lyu])  
For $n \geq 3$, $D(K_{3,n})$ := max $d(K_{3,n}, \sigma) = 2[\frac{1}{4}(n - 2)] + 0$ if $n \equiv 3 \pmod{4}$, $+1$ otherwise ($d$ = demigenus, “Euler genus”). Most interesting feature: The maximum is attained (not uniquely) with only a single negative edge.  [Annot. 7 Nov 2017.]  

(Shengxiang Lyu & Zihan Yuan)  
Thm.: $D(K_{4,n}) := \max d(K_{4,n}, \sigma) = d(K_{4,n}) + 1$ for $n > 4$, $D(K_{4,4}) = 4$. As with $K_{2,n}$ and $K_{3,n}$ (cf. Lv (2015a)), the maximum is attained with only one negative edge, except for $K_{4,4}$, where the maximum requires a negative perfect matching. *Question.* Does this pattern continue for $K_{m,n}$, $m > 4$?  [Annot. 7 Nov 2017.]  

(S. X. Lv)  

(Yu.I. Lyubich)  
See G.R. Belitskii.

(Baoli Ma)  
See M.J. Du.

(Hongping Ma)  
See also L.Q. Wang.

Lijia Ma
See Q. Cai.

Hongping Ma & Zhengke Miao

M. Ma
See D. Blankschtein.

Xiaobin Ma, Genhong Ding, & Long Wang
20xxa On the nullity and the matching number of unicyclic signed graphs. Submitted. Further develops Y.Z. Fan, Wang, and Wang (2013a). Employs the matching number to express the adjacency nullity and to characterize adjacency rank 6 and 7 of a signed unicyclic graph. [Annot. 17 Dec 2011.] (SG: Eig)

Xiaobin Ma, Dein Wong, & Fenglei Tian

Φ := gain graph with gains ±i. Problem: rk A(Φ) [special case of adjacency rank for complex unit gain graphs; cf. Reff (2012a)]. Thm. 3.1(i): 2μ − 2ξ ≤ rk A(Φ) ≤ 2μ, where μ := matching number of ∥Φ∥, ξ := cyclomatic number. Thm. 4.1 characterizes 2μ − 2ξ = rk A(Φ). [Generalized in Fenglei Tian, Li Chen, & Rui Chu (2018a).] Dictionary: i = \sqrt{-1}, G := ∥Φ∥, \vec{G} := orientation making all gains i, skew adjacency matrix of \vec{G} := i^{-1} A(Φ), skew rank of \vec{G} = rk A(Φ), sgn(C) = \varphi(C)/i^{|C|}. [Annot. 10 May 2019.]

A. Ma’ayan, A. Lipshtat, R. Iyengar, & E.D. Sontag

See also T. Kaiser.

Edita Máčajová & Ján Mazák

Includes positive-circle decomposition (called “even cycle decomposition”) of a signed graph; cf. Markström (2012a). Thm. 2: An infinite class of 4-regular, 4-connected Σ without such a decomposition. Question 1: Does every signed line graph of a cubic graph without isthmus, with even #E−, have such a decomposition? [Annot. 4 Jun 2017.] (SG: Circ: Str)

Edita Máčajová, André Raspaud, Edita Rollová, & Martin Škoviera

Edita Máčajová, André Raspaud, & Martin Škoviera
χ(Σ) Coloring as in Zaslavsky (1982b). Chromatic number χ(Σ) := size of smallest color set [a better definition than Zaslavsky’s] = χ^0 + χ^∗ of Zaslavsky.
Main results: Thm. 6 (Brooks’ Theorem for signed simple graphs): χ(Σ) ≤ Δ(Σ) except for balanced (K_n, σ) and (C_{odd}, σ) and unbalanced (C_{even}, σ). [Fleiner and Wiener (2016a) have a short list-coloring proof. Zajac (20xxa) has a short, more general proof.] Prop. 4(i): Σ ∉ K_4 ⇒ χ(Σ) ≤ 3. Thm. 10: If |Σ| is planar, χ(Σ) ≤ 5, ≤ 4 if C_3-free, ≤ 3 if girth ≥ 5. Conjecture: Every planar signed graph is 4-colorable. [Annot. 14 Sept 2015, 7 May 2018, rev 9 Feb, 2 Jul 2020, 5 Jan 2021.] (SG: Col: Invar)

Edita Máčajová & Edita Rollová

Edita Máčajová & Martin Škoviera

Edita Máčajová & Eckhard Steffen
Enzo Maccioni  
See F. Barahona.

Gary MacGillivray  
See also K.H. Monfared.

Gary MacGillivray, Ben Tremblay, & Jacqueline M. Warren  
20xxa Colourings of \( m \)-edge-coloured graphs and switching. Submitted.  
Great generalization of Brewster and Graves (2009a). (gg(Gen), Cov)

Amila P. Macodi-Ringia  
See M.M. Mangontarum.

Bolette Ammitzbøll Madsen  
See J.M. Byskov.

K.V. Madhusudhan  
See also P.S.K. Reddy.

K.V. Madhusudhan & S. Vijay  
The detour radial graph of |\( \Sigma \)| is signed using the canonical vertex signature, by \( \sigma(uv) = \mu_\sigma(u)\mu_\sigma(v) \). [Annot. 12 Jul 2019.] (SG)

[A. El Maftouhi, Abdelakim El Maftouhi]  
See A. El Maftouhi (under ‘E’).

Mohammad Maghasedi  
See S. Akbari and M. Souri.

Colton Magnant  
See S. Fujita.

Thomas L. Magnanti  
See R.K. Ahuja.

N.V.R. Mahadev  
See also P.L. Hammer.

N.V.R. Mahadev & U.N. Peled  
§8.3: “Bithreshold graphs” (from Hammer and Mahadev (1985a)), and  
§8.4: “Strict 2-threshold graphs” (from Hammer, Mahadev, and Peled (1989a)), characterize two types of threshold-like graph. In each, a different signed graph \( H \) is defined on \( E(\Gamma) \) so that \( \Gamma \) is of the specified type iff \( H \) is balanced. (The negative part of \( H \) is the “conflict graph”, \( \Gamma^* \).) The reason is that one wants \( \Gamma \) to decompose into two subgraphs, and the subgraphs, if they exist, must be the two parts of the Harary bipartition of \( H \). [Thus one also gets a fast recognition algorithm, though not the fastest possible, for the desired type from the fast recognition of balance.]  
(SG: Bal: Appl)  
§8.5: “Recognizing threshold dimension 2.” Based on Raschle and Simon (1995a). Given: \( \Gamma \subseteq K_n \) such that \( \Gamma^* \) is bipartite. Orient \( \neg K_n \)
so that $\Gamma$-edges are introverted and the other edges are extroverted. Their “alternating cycle” is a coherent closed walk in this orientation. Let us call it “black” (in a given black-white proper coloring of $\Gamma^*$) if its $\Gamma$-edges are all black. Thm. 8.5.2 (Hammer, Ibaraki, and Peled (1981a)): If there is a black coherent closed walk in $E_0$, then there is a coherent tour (closed trail) of length 6 (which is a pair of joined triangles or a hexagon—their $AP_3$ and $AP_6$). Thm. 8.5.4: Given that there is no black coherent hexagon, one can recolor quickly so there is no black coherent 6-tour. Thm. 8.5.9: Given that there is no ‘double’ coherent hexagon (the book’s “double $AP_6$”), one can recolor quickly so there is no black coherent hexagon. Thm. 8.5.28: Any 2-coloring of $\Gamma^*$ can be quickly transformed into one with no ‘double’ coherent hexagon. [Question. Can any of this, especially Thm. 8.5.2, be generalized to arbitrary oriented all-negative graphs $B$? Presumably, this would require first defining a conflict graph on the introverted edges of $B$. More remotely, consider generalizing to bidirected complete or arbitrary graphs.]  

§9.2.1: “Threshold signed graphs.” See Benzaken, Hammer, and de Werra (1981a), (1985a). In this version it’s not clear where the signs are! (and their role is trivial). Real weights are assigned to the vertices and an edge receives the sign of the weight product of its endpoints.  


Uses the auxiliary signed graph of Hammer and Mahadev (1985a). 
[Annot. 22 Mar 2017.]  

John Maharry, Neil Robertson, Vaidy Sivaraman, & Daniel Slilaty  


§1: The flexibility is connected to duality of signed-graphic frame matroids by Slilaty (2005a). [Annot. 20 Dec 2011.]  

Ali Ridha Mahjoub  
See F. Barahona and D. Cornaz.

J.M. Maillard  
See T. Garel and J. Vannimenus.

Hamid Reza Maimani  
See S. Akbari and E. Ghorbani.

Leila Parsaei Majd  
See S. Akbari and E. Ghorbani.

Konstantin Makarychev  
See N. Alon.

Yury Makarychev  
See N. Alon.

J.A. Makowsky  
See also E. Fischer and M. Lotz.

2001a Colored Tutte polynomials and Kauffman brackets for graphs of bounded tree width. In: *Proceedings of the Twelfth Annual ACM-SIAM Symposium on Dis-


Polynomial-time computability for edge-colored graphs of bounded tree width. [Also see Traldi (2006a).] (SG: Gen: Invar: Alg, Knot)

A.Z. Maksymowicz
See M.J. Krawczyk.

Krzysztof Malarz
See M.J. Krawczyk and B. Tadić.

H.A. Malathi & H.C. Savithri

M. Malek-Zavarei & J.K. Aggarwal

Piotr Malicki
See M. Kaniecki.

Fragkiskos D. Malliaros, Christos Giatsidis, Apostolos N. Papadopoulos, & Michalis Vazirgiannis

§2.2.3, “Signed graphs”, defines “core” := maximum subgraph with $d^+_\text{in, out}, d^-_{\text{in, out}} \geq$ given bounds. [Annot. 2 Feb 2020.] (SD)

R.B. Mallion
See A.C. Day.

Devlin Mallory
See also J. Brown.

Devlin Mallory, Abigail Raz, Christino Tamon, & Thomas Zaslavsky

C.L. Mallows & N.J.A. Sloane

Thm. 1: For all $n$, the number of unlabelled two-graphs of order $n \ [i.e.,$ switching isomorphism classes of signed $K_n$'s$]$ equals the number of unlabelled even-degree simple graphs on $n$ vertices. The key to the proof is that a permutation fixing a switching class fixes a signing in the class. (Seidel (1974a) proved the odd case, where the fixing property is simple.) Thm. 2: The same for the labelled case. [More in Cameron (1977b), Cameron and Wells (1986a), Cheng and Wells (1984a), (1986a)]
To prove the fixing property they find the conditions under which a given permutation \( \pi \) of \( V(K_n) \) and switching set \( C \) fix some signed \( K_n \).

[More in Harries and Liebeck (1978a), Liebeck (1982a), and Cameron (1977b).]

(Aleksander Malnič)

See also I. Kovács.


Gain graphs (“voltage graphs”) and lifting automorphisms of their underlying graphs are a main example. [Annot. 11 Jun 2012.]

(Aleksander Malnič, Roman Nedela, & Martin Škoviera)


Automorphisms of gain graphs that lift to the covering graph. [Annot. 18 Apr 2012.]


§6, “Invariance of voltage assignments”, concerns automorphisms of a gain graph that preserve the gains, in connection with lifting automorphisms to the regular covering graph. The treatment is via maps as gain graphs with rotation systems. [Annot. 18 Apr 2012.]

(John W. Mamer)

See R.D. McBride.

(Rachel Manber)

See also R. Aharoni and V. Klee.


(Rachel Manber & Jia-Yu Shao)


(Federico Mancini)

See H.L. Bodlaender.

(Tabitha Agnes Mangam)

See D. Antoney.

(Mahid M. Mangontarum, Amila P. Macodi-Ringia, & Normalah S. Abdulcarim)


Introduces 0-Dowling polynomials \( D_{n,m,0}(x) := \sum_k W_{m,0}(n,k)x^k \); cf. Belbachir and Bousbaa (2013a) and 0-Dowling numbers \( D_{n,m,0}(1) \). Formulas, identities, integrals, etc., for 0-Whitney and 0-Dowling. Dictionary: “translated Dowling” = 0-Dowling | so named and coordinated with...
r-Whitney and r-Dowling numbers in Gyimesi and Nyul (2018a); polynomials and numbers $\tilde{D}_m(x), \tilde{D}_m(n) = D_{n,m,0}(x), D_{n,m,0}(1)$ [notation of Gyimesi–Nyul]. [Annot. 28 May 2018.]

Silviu Maniu
See C. Giatsidis.

Anna Mańka
See A. Mańka-Krasoń.

Anna Mańka-Krasoń & Krzysztof Kulakowski


Anna Mańka, Krzysztof Malarz, & Krzysztof Kulakowski

Computer experiments on physics aspects of all-negative signed graphs. [Annot. 14 Feb 2011.]

R. Lawrence Joseph Manoharan
See P.L. Rozario Raj.

Y. Manoussakis
See A. El Maftouhi.

Vassily Olegovich Manturov

Vassily Olegovich Manturov & Denis Petrovich Ilyutko

English trans. of Manturov (2010a). Ch. 9, “Theory of graph-links”: 0/1-labelled and sign-colored chords in chord diagrams. 0/1-labelled and sign-colored vertices. 0/1-labelled and sign-colored graphs. [Annot. 13 Sep 2013.]

Zeev Maoz, Lesley G. Terris, Ranan D. Kuperman, & Ilan Talmud

Dănuţ Marcu
[I cannot vouch for the authenticity of these articles. See MR 1324075 (97a: 05095) and Zbl 701.51004. Also see MR 1038400 (92a:51002), MR 1094344 (92b:51026), MR 1107637 (92h:11026), MR 1427830 (97k:05050); and Marcu (1981b).]

See Harary, Norman, and Cartwright (1965a) for the definition. *(GD: bal)*


See Harary, Norman, and Cartwright (1965a) for the definition. The tournaments of order 3 are [trivially] not gradable, whence the titular theorem. *(GD: bal)*


§1, “Preliminary considerations”, appears to be an edited, unacknowledged transcription of parts of Harary, Norman, and Cartwright (1965a) (or possibly (1968a)), pp. 341–345. Wording and notation have been modified, a trivial corollary has been added, and some errors have been introduced; but the mathematics is otherwise the same down to details of proofs. §2, “Results”, is largely a list of the corollaries resulting from setting all signs negative. The exception is Thm. 2.5, for which I am not aware of a source; however, it is simple and well known. *(sg(SD): Bal)*


Matroidal families of (multi)graphs (see Simões-Pereira (1973a)) correspond to functions on all isomorphism types of graphs that are similar to matroid rank functions, e.g., submodular. This provides insight into matroidal families, e.g., it immediately shows there are infinitely many. *(MtrdF: Bic, EC: Gen)*

Adam W. Marcus, Daniel A. Spielman, & Nikhil Srivastava


Preliminary version of (2015a); the latter has slight (and occasionally important) additions, deletions, and corrections. [Annot. 18 Oct 2015.]

*(SG: Cov, Adj: Eig)*


Dictionary: “2-lift” of $\Sigma = \text{signed covering graph} \tilde{\Sigma}$. “Double-cover” of $\Gamma = \text{that of } -\Gamma$. *(SG: Cov, Adj: Eig)*

Grzegorz Marczak
See also M. Kaniecki.

Grzegorz Marczak, Daniel Simson & Katarzyna Zajac

Enzo Marinari
See also S. Cabasino, B. Coluzzi, M. Falcioni, and J. Lukic.

Enzo Marinari, Giorgio Parisi, & Felix Ritort

Ising (spins, i.e., vertex values, $\in S^0 = \{+1, -1\}$) and XY (spins $\in S^1$, i.e., complex units) models behave differently on a fully frustrated signed hypercube graph $Q_D$ (all squares are negative). Numerical study of Ising spins of two such signatures: $\sigma_1(x, x + e_\mu) = (-1)^{x_1 + \cdots + x_\mu - 1}$, while $\sigma_2$ [“simplex” in the construction must mean hypercube] is from Derrida, Pomeau, Toulouse, and Vannimenus (1979a); “with identical results”.

Based on simulations with $D \leq 47$, Ising ground states seem to be few and hard to find. Near-ground states are easier to find but, apparently, tend to be far from ground states.

For positive temperature $T$, as $A(\Sigma)$ (“interaction matrix $J_{x,y}$”) is orthogonal [up to scaling], one can approximate by averaging over orthogonal adjacency matrices.

In simulations with XY spins the ground state is highly accessible. Dictionary: “ground state” = switching with minimum $\#E^-$. [Annot. 19 Jun 2012.]


Physics on hypercube $Q_D$ with complex unit gains and three types of spin, after Parisi (1994a), via simulations for $3 \leq D \leq 16$. [Annot. 19 Jun 2012.]


Fabrizio Marinelli & Angelo Parente

A.V. Markovski˘ı

Harry Markowitz
Klas Markström

Can $-\Gamma$ be decomposed into positive circles? Studied for 4-regular graphs and line graphs of cubic graphs. Cf. C.Q. Zhang (1994a), Máčajová and Mazák (2013a) (especially), and Liu–You–Cui (2020a). Dictionary: “oddness” of $\Gamma = \min \# \text{negative circles in a 2-factor of } -\Gamma$.


Clifford W. Marshall

“Consistency of choice” discusses signed graphs, pp. 262–266. (SG: Bal, Adj: Exp)

T.H. Marshall
See N. Alon.

Matteo Marsili
See G.C.M.A. Ehrhardt.

Florian Martin

O.C. Martin
See J.-P. Bouchaud and J. Lukic.

Jeremy L. Martin
See J. Hallam.

Samuel Martin
See S. Ahmadizadeh.

V. Martin-Mayor
See L.A. Fernández.

José Martínez-Bernal, Miguel A. Valencia-Bucio, & Rafael H. Villarreal

Xavier Martínez-Rivera
See S. Butler.

Enide Andrade Martins
See N.M.M. de Abreu and I. Gutman.

Dragan Marušič
See I. Kovács.

Seth A. Marvel, Jon Kleinberg, Robert D. Kleinberg, & Steven H. Strogatz

A differential equation model of balancing processes, based on Kułakowski, Gawroński, & Gronek (2005a). Conclusion: Final state is balance.

The mathematical support for (2011a). [Annot. 6 Feb 2011.]

Seth A. Marvel, Steven H. Strogatz, & Jon M. Kleinberg

Signed complete graphs under Antal, Krapivsky, and Redner’s (2005a) “constrained triad dynamics”: Imbalance measured by triangles; an edge is negated if it is in more negative than positive triangles. Paley graphs $P$ give $K_P$ with equally many positive and negative triangles on each edge (normalized “energy” = 0). Other such states exist. [Zyga (2009a) gives a popular exposition.] [Questions. Do unbalanced locally minimal regions with more than one point (graph) exist? How does the landscape look for switching classes?] [Annot. 5 May 2010, 26 Jan 2011.]

[Enzo M. Li Marzi]
See E.M. Li Marzi (under ‘L’).

Andrew J. Mason
See S. Aref.

J.H. Mason

§§2.5–2.6: “The lattice approach” and “Generalized coordinates”, pp. 172–174, propose a purely matroidal and more general formulation of Dowling’s (1973b) construction of his lattices. (gg(Gen): M)


Dowling matroids are an example in §1. (gg: M)

A.M. Mathai & Thomas Zaslavsky

Eigenvalues of $A(C_n, \sigma)$ (previously stated by Fowler (2002a); equivalent to Fan (2007a)’s Laplacian eigenvalues) by an elegant matrix method. [Cf. Germina and Hameed (2010a).] Some ways to partially or wholly distinguish different signatures of $C_n$ are compared. [Annot. 6 Sept 2010,
Anisha Jean Mathias, V. Sangeetha, & Mukti Acharya
Restrained dominating set $D: E \setminus E^c$ is balanced. [Annot. 3 Sep 2020.]

R.A. Mathon

Tetsushi Matsui, Akihiro Higashitani, Yuuki Nagazawa, Hidefumi Ohsugi, & Takayuki Hibi

Tatsuya Matsuoka & Shun Sato

Hisayoshi Matsuyama
See M. Iri and J. Shiozaki.

Amelia R.W. Mattern
Contains (20xxx) and (20xxb). [Annot. 28 May 2020.]

Laurence R. Matthews

Summary of (1977a).

(Bic)


Announcement of (1978c).

(gg: M)


Invents frame matroids of poise, modular poise, and antidirection bias on a digraph.

(gg: M)


(Bic: Gen)

Laurence R. Matthews & James G. Oxley

(Bic)

Alexey Matveev
See A. Proskurnikov.

Jean François Maurras

(GN: M)

Mano Ram Maurya, Raghunathan Rengaswamy, & Venkat Venkatasubramanian

(SD: QSta: Alg, Appl)


(SD: QSta: Alg, Appl)


(SD: Appl)


(SD: Appl)

John S. Maybee

Survey and simple proofs. (QM: sd, gg, QSta)(Exp)


For comments, see Lancaster (1981a). (QM: QSol: SD)


Signed (di)graphs play a role in characterizations. See e.g. §7. See also Roberts (1989a), §4. (QM, SD)

John S. Maybee & Stuart J. Maybee


A linear-time algorithm to determine balance or antibalance of the undirected signed graph of a signed digraph. The algorithm of Harary and Kabell (1980a) appears to be different. (SG: Bal, Par: Alg)

John Maybee & James Quirk


An important early survey with new results. (QM, SD: QSol, QSta, bal; Exp(in part), Ref)

John S. Maybee & Daniel J. Richman


Square matrix A is a GM-matrix if, for every positive and negative cycle P and N in its signed digraph, V(P) ⊇ V(N). Classification of irreducible GM-matrices; connections with the property that each p × p principal minor has sign (−1)^p; some conclusions about the inverse. (SD: QM)


John S. Maybee & Gerry M. Weiner


An L-function is a nonlinear generalization of a qualitative linear function. Signed digraphs play a small role. (QM, SD)

Stuart J. Maybee

See J.S. Maybee.

W. Mayeda & M.E. Van Valkenburg

**Dillon Mayhew**
See also T. Fife and D. Funk.


The number of inequivalent representations of a frame matroid over a fixed finite field is bounded, if the matroid does not have a free swirl $G(2C_n, \emptyset)$ as a minor. (GG: M)

**Dillon Mayhew, Geoff Whittle, & Stefan H.M. van Zwam**


**R. Maynard**
See J.C. Angles d’Auriac, F. Barahona, and I. Bieche.

**Ján Mazák**
See E. Máčajová.

**M.H. McAndrew**
See D.R. Fulkerson.

**Richard McBride**
See H. Jordon.

**Richard D. McBride**
See also G.G. Brown.


Introducing the algorithm “EMNET”, which employs embedded generalized-network matrices (i.e., incidence matrices of real multiplicative gain graphs) with side constraints (i.e., extra rows) to speed up linear programming. [Annot. 2 Oct 2009.] (GN: Incid: Alg)


**Richard D. McBride & John W. Mamer**


**Richard D. McBride & Daniel E. O’Leary**

H. Gilman McCann
See E.C. Johnsen.

William McCuaig
See also C.R. Johnson.


Characterizes the digraphs with no two disjoint cycles as well as those with no two arc-disjoint cycles. [Since cycles do not form a linear subclass of circles, this is not a biased-graphic theorem, but it might be of use in studying biased graphs that have no two disjoint balanced circles. See Lovász (1965a), Slilaty (2007a).] (Str)


Results needed for (2004a). (SD: par)


See the description of Robertson, Seymour, and Thomas (1999a), who independently prove the main theorem. (SD: par: Str)(SG)

20xxa When all dicycles have the same length. Manuscript.

Uses the main theorem of (2004a) and Robertson, Seymour, and Thomas (1999a) to prove: A digraph has an edge weighting in which all cycles have equal nonzero total weight iff it does not contain a “double dicycle”: a symmetric digraph whose underlying simple graph is a circle. There is also a structural description of such digraphs. (SD: par: Str)(Sw)

William McCuaig, Neil Robertson, P.D. Seymour, & Robin Thomas


Extended abstract of McCuaig (2004a) and Robertson, Seymour, and Thomas (1999a). (SD: par)

W.D. McCuaig & M. Rosenfeld


In a 3-connected graph, almost any two edges are in an even and an
odd circle. [By the negative-subdivision trick this generalizes to signed graphs.] (Par, sg: Bal)

Judith J. McDonald
See M. Cavers and S. Kirkland.

David D. McFarland
See M. Hallinan.

Sean McGuinness

Brendan D. McKay, Mirka Miller, & Jozef Širáň
Also see Šiagiová (2001a). (GG: Cov)

James McKee & Chris Smyth


2020a Symmetrizable integer matrices having all their eigenvalues in the interval \([-2, 2]\]. Algebraic Combin. 3 (2020), no. 3, 775–789. (Eig: sg, Sw)


Terry A. McKee
1984a Balance and duality in signed graphs. Proc. Fifteenth Southeastern Conf. on Combinatorics, Graph Theory and Computing (Baton Rouge, 1984). Con-
A chordally signed graph is a chordal graph signed so every positive circle $C$ of length at least 4 has a chord such that $C \cup e$ is balanced. Characterized in various ways. (SG)

P. 2312: An auxiliary graph can be treated as signed; chordal coloring is signed-graph clustering. [Annot. 11 Jul 2012] (SG: Clu)

Kathleen A. McKeon
See G. Chartrand.

Jennifer McNulty
See also G. Gordon, T. Lewis and N.A. Neudauer.

Jennifer McNulty & Nancy Ann Neudauer

Luis Medina
See I. Gutman.

Killian Meehan
See Y. Duong.

Yotsanan Meemark & Borworn Suntornpoch

O. Megalakaki
See A. El Maftouhi.

Nimrod Megiddo
See E. Cohen and D. Hochbaum.

Ranjit Mehatari, M. Rajesh Kannan, & Aniruddha Samanta

Ritu Rani Meherwal
See G.N. Purohit.

Kurt Mehlhorn & Dimitrios Michail
The “signed graph $G_i$” is a signed covering graph $\tilde{\Sigma}_i$. Used to find minimum cycle basis in a positively weighted graph $\Gamma$. $\Sigma_i$ has negative edge set $S_i$, the “witness set”. [Annot. 6 Feb 2011.]  


Martin Mehlitz  
See J. Kunegis.

A. Mehrabian  
See S. Akbari.

Sylvain Meignen  
See J. Demongeot.

Ioannis N. Melas., Regina Samaga., Leonidas G. Alexopoulos, & Steffen Klamt  

O. Melchert & A.K. Hartmann  
In other words, finding the frustration index of a signed planar graph. The titular method seems to be less efficient than others. [Annot. 9 Jan 2015.]  

Miguel A. Meléndez-Jiménez  
See A. Parravano.

Avraham A. Melkman  
See T. Akutsu.

C. Mendes Araújo & Juan R. Torregrosa  
The digraphs treated are acyclic or cycles. See Thm. 3.4 on signed cycles. Also, the “2-cycle property” means a negative 2-cycle. [Annot. 29 Sept 2012.]  

Almost identical to (2009a) with “$P_0$-matrices” replacing “$M$, $N$, $P$- or inverse $M$-matrices”. Treats directed circles as well as cycles and acyclic digraphs. [Annot. 29 Sept 2012.]  

J.F.F. Mendes  
See M. Ostilli.

Marco A. Mendez  
See J. Aracena.

Miguel A. Méndez & José L. Ramírez

**Israel Mendonça, Rosa Figueiredo, Vincent Labatut, & Philippe Michelon**


**Deyuan Meng**

See also M.J. Du.


**Deyuan Meng, Ziyang Meng, & Yiguang Hong**


**Jie Meng**

See J.-E. Chen.

**Ziyang Meng**

See D.-Y. Meng.

**Mircea Merca**


**Pedro Mercado, Francesco Tudisco, & Matthias Hein**


**Valeriu Mereacre**

See K.C. Mondal.
M.A. Mermet
See J. Aracena.

Leanne Merrill
See Y. Duong.

Russell Merris
Thm.: Spec \( L(\Gamma) = \text{Spec } L(-\Gamma) \) iff \( \Gamma \) is bipartite. [The antibalanced case of B.D. Acharya (1980a).] [Annot. 21 Jan 2012.] (Par: Eig, bal)


Mehran Mesbahi
See M.H. de Badyn.

Roy Meshulam
See R. Aharoni and J. Kahn.

Nacim Meslem
See N. Ramdani.

Robert Messer
See E.M. Brown.

Thomas Mestl
See E. Plahte.

Gábor Mészáros
See T. Kittipassorn.

Karola Mészáros
A signed simple graph generates a polytope \( P(\Sigma) \) whose volume is calculated. [Annot. 11 Sept 2010.] (SG: Geom)

[Positive edges are called “negative” and vice versa.] Flows on signed graphs give combinatorial interpretations of and identities for the partition functions. Negative (“positive”) edges are introverted, so flow goes in and disappears. [Annot. 25 Mar 2013.] (SG: Flows: Appl)

Karola Mészáros & Alejandro H. Morales

[The edge signs are the opposite of what they ought to be.]

(SG: Flows, Geom)

Frédéric Meunier & András Sebő


The negative circles of a signed graph and its minimal and minimum balancing sets are crucial. Dictionary: “signed graph” = $(\Sigma, E^-)$, “odd cycle” = negative circle, “odd cycle clutter” = $B_c(\Sigma)$, “transversal” = “uncut” = minimal balancing set, “BIP$(G,F)$” = MinBalSet$(\Sigma)$ (the problem of finding a minimum balancing set), “resigning” = switching.

[Annot. 22 Sept 2010.]

(SG: Fl)

---

David A. Meyer

See D.J. Song.

Seth A. Meyer

See R.A. Brualdi.

Hildegard Meyer-Ortmanns

See F. Radicchi.

Andrew M. Meyers

See N.A. Neudauer.

Erika Meza

See M. Beck.

Marc Mézard, Giorgio Parisi, & Miguel Angel Virasoro


Focuses on the Sherrington–Kirkpatrick model, i.e., underlying complete graph, emphasizing the Parisi-type model (see articles reprinted herein), which posits numerous metastable states, separated by energy barriers of greatly varying heights and subdividing as temperature decreases (cf. Kirkpatrick and Sherrington (1978a)). Essentially heuristic (as noted in MR): that is, the ideas awaited [and still largely await] mathematical justification.

Many original articles on Ising and vector models (both of which are based on weighted signed graphs) are reprinted herein, though few are of general signed-graphic interest.

[See also, i.a., Toulouse (1977a) et al., Chowdhury (1986a), Fischer and Hertz (1991a), Vincent, Hamman, and Ocio (1992a) for physics, Barahona (1982a), etc., Grötschel, Jünger, and Reinelt (1987a) for mathematics.]

Metastable states in the model appear to correspond to local minima of the state frustration function of the underlying weighted signed graph $(\Sigma, w)$, with ultrametric distance function $d(s, s') := \min_P \max_{s'' \in P} H(s'')$, where $P$ ranges over all paths $P : s \rightarrow s'$ in the graph of states and $H$
is the Hamiltonian of a state of $(\Sigma, w)$, equivalently the (weighted) frustration of the state. **Problem.** Study this metric on the state space of various signed $K_N$’s and other signed graphs. Possibly this will shed light on the physics; it will certainly be interesting for signed graphs.]

(Phys, SG: Fr, State: Exp, Ref)

Ch. 0, “Introduction”, briefly compares, in the obvious way, balance in social psychology [they neglect to mention the original paper, Cartwright and Harary (1956a)] with frustration in spin glasses.

(Phys, PsS: SG: Bal: Exp)

Pt. 1, “Spin glasses”, Ch. II, “The TAP approach”: pp. 19–20 describe 1-vertex switching of a weighted signed graph to reduce frustration, not however necessarily producing the frustration index (minimum frustration). **Question.** How does the valley (“basin of attraction”) of a ground state (minimum frustration) compare with the valley of a metastable state (locally minimum frustration); in particular can it be much smaller? [Annot. rev. 15 Aug 2018.]

(Phys: SG: Fr, Sw, Alg: Exp)

**István Mező**


Introduces the $r$-Whitney numbers [of Dowling lattices, not of geometric lattices in general]. $r = 1$ are the original numbers. [The numbers are popular. *Cf.* Cheon and Jung (2012a), Merca (2013a), Rahmani (2014a), Mező (2014a), Gyimesi and Nyul (2018a), etc.] [Annot. rev. 28 May 2018.]

(Phys: SG: Fr, Sw, Alg: Exp)


**Zhengke Miao**


**Isaac B. Michael & Mark R. Sepanski**


**T.S. Michael**


Characterizes net degree sequences of signed graphs with fixed maximum edge multiplicity. [See Chartrand, Gavlas, Harary, and Schultz (1994a) for explanation.]

(SGw: Invar, Alg)

**Dimitrios Michail**

See K. Mehlhorn.

**Philippe Michelon**

See I. Mendonça.

**Manuel Middendorf**

See E. Ziv.
A. Alan Middleton
See C.K. Thomas.

Anna de Mier
See O. Giménez.

S. Migowsky
See T. Wanschura.

Štefko Miklavič
See I. Kovács.

Alexander R. Miller


Mirka Miller
See C. Dalfo and B.D. McKay.

Raymond E. Miller
See R.M. Karp.

William P. Miller
See also J.E. Bonin.


Dowling geometries are reconstructible from their hyperplanes, their deletions, and their contractions. (gg: M)

Geyong Min
See F. Hao.

Maya Mincheva
See also G.R. Walther.

Maya Mincheva & Gheorghe Craciun

(SD: Chem, Biol: Dyn: Exp)

Edward Minieka


Maryam Mirzakhah
See also I. Gutman and M. Jooyandeh.

M. Mirzakhah & D. Kiani
(par: Lap: Eig)

(par: Lap: Eig)

Grégoire Misguich and Claire Lhuillier
I.a., details of ground-state spin alignments in XY and Heisenberg models ($S^2$ and $S^3$ spins) on simple periodic lattices. [Annot. 13 Aug 2018.]
(SG, Phys: Fr: Exp, Ref)

V. Mishra
The arrays are matrices.
⊗ Defines tensor product $\Sigma_1 \otimes \Sigma_2$ to have an edge $(v_1, v_2)(w_1, w_2)$ iff $v_1w_1$ and $v_2w_2$ are edges, with sign $\sigma((v_1, v_2)(w_1, w_2)) := \sigma_1(v_1w_1)\sigma_2(v_2w_2)$
(cf. Sinha and Garg (2014a)). [Annot. 23 Nov 2014.]
(SG)

U.K. Misra
See P.S.K. Reddy.

G. Mitra
See N. Gülpinar.

S. Mitra
Treats signed simple graphs via the Abelson–Rosenberg (1958a) structure matrix $R$. Observes that balance holds iff $R = rr^T$ for some vector $r \in \{p, n\}^V$; also, asserts that frustration index $l(\Sigma) =$ minimum number of negative edges over all switchings of $\Sigma$. [Proved in Barahona, Maynard, Rammal, and Uhry (1982a).] Asserts an algorithm for computing $l(\Sigma)$: switch vertices whose negative degree exceeds positive degree, one at a time, until no such vertices remain [incorrect: consider $K_6$, all positive except a negative $C_6$]. [Annot. corr. 20 Jan 2010.]
(sg: kg: Adj, sw, Fr)

Michael Mitzenmacher
See C.E. Tsourakakis.

Valia Mitsou
See F. Dross and F. Foucaud.

Seiji Miyashita
See O. Nagai.

Hirobumi Mizuno & Iwao Sato

Isomorphism types, under the action of a subgroup of $\text{Aut } \Gamma$, of coboundaries of 1-chains $f : V \to \mathbb{F}_q^+$ in $-\Gamma$. (In other words, the edge labels are $\delta f(uv) = f(u) + f(v).$) [Question. Does it generalize to signed graphs? The subgroup would be of $\text{Aut } \Sigma$, or one can count isomorphism types of switching classes under a subgroup of $\text{Aut } [\Sigma].$] [Annot. 16 Jan 2012.]

(par: incid)


[Swathyprabhu Mj]

See S. Das under ‘S’.

Iain Moffatt

See also J.A. Ellis-Monaghan, T. Krajewski, and M. Loebl.


A.R. Moghaddamfar

See Y. Bagheri.

Javad Mohajer

See S. Fayyaz Shahandashti.

Bojan Mohar


The “overlap matrix” of a signed graph with respect to a rotation system and a spanning tree provides a lower bound on the demigenus
that sometimes improves on that from Euler’s formula. (SG: Top)


Φ with gain group \{±1, ±i\}: \(\varphi(e) = 1\) for undirected, \(i\) for directed edges [as in Liu and Li (2015a)]. Switching. Adjacency cospectrality, including spectrally unique families. Cf. Guo and Mohar (2017a). [Annot. 15 Dec 2020.]

Bojan Mohar & Svatopluk Poljak


Bojan Mohar & Paul D. Seymour


[See also Nakamoto, Negami, and Ota (2002a), (2004a).] (sg: Top: sw)

Bojan Mohar & Carsten Thomassen


Afshin Moin

See A.-M. Kermarrec.

G.H. Mokashi

See K.S. Betageri.

K. Hassani Monfared, G. MacGillivray, D.D. Olesky, & P. van den Driessche


Dictionary: “flexibility” \(\tau(\Gamma, \sigma) = r(E^+) + r(e^-) - r(E)\), \(r = \text{rank in } G(\Gamma)\). [Hence related to connectivity of \(G(\Gamma)\) via sign partition of \(E\).]

G. Monroy

See A. Coniglio.
Kartik Chandra Mondal, Valeriu Mereacre, George E. Kostakis, Yanhua Lan, Christopher E. Anson, Ion Prisecaru, Oliver Waldmann, & Annie K. Powell


Novel arrangements of frustrated signs with $S^I$ spins in a ground state. Especially see Figs. 1, 4. [Annot. 20 Mar 2016.] (Phys: sg: Fr)

Marco Montalva
See also J. Aracena.

Marco Montalva, Julio Aracena, & Anahí Gajardo


Complexity of finding a minimum set of vertices, or arcs, that covers all positive, or negative, cycles in a signed digraph. All are NP-complete, by polynomial-time reduction to the existence problems Even Cycle and Odd Cycle in the positive and negative problems, respectively. [Directed frustration index and directed vertex frustration number are the negative-cycle cover problems, which are said to be easier than the positive-cycle cover problems.] [Annot. 20 July 2009.] (SD: Fr: Gen, Alg)

James D. Montgomery


Signed digraphs with possible multiple arcs of different sign, with two types of vertices (“actors” having positive and possibly negative loops, and “objects” having no loops), and with extra “awareness” arcs between actor vertices. Emphasis on directionality of arcs. “Boolean multiplication” [Boolean Hadamard product] of separate positive, negative, and awareness adjacency matrices to form mixed adjacency matrices. Assumption: Over time the signed digraph evolves towards sign-transitive closure constrained by the awareness arcs, whose absence impedes transitive closure. Four specific “mechanisms” are postulated for the evolution, of which two are essential (Lemma 1). Propositions present conclusions (no surprises) about intermediate and final (i.e., constrained sign-transitively closed) signed digraphs. Dictionary: “balance closure” = sign-transitive closure, i.e., arc-transitive closure with positive triple sign. [The idea of constrained closure is mathematically intriguing, though the notation is heavy.] [For more on sign-transitive closure in signed digraphs see Doreian and Krackhardt (2001a).] [Annot. 16 Apr 2009.] (SD, PsS: Bal)

Angelo Monti
See T. Calamoneri.

Elliott W. Montroll


J.W. Moon


Observe that $l_{	ext{clu}}(K_n, \sigma)/\binom{n}{2} \leq \frac{1}{2}$. Thm.: Let $\varepsilon > 0$. When $n \gg 0$,

$$l_{\text{clu}}(K_n, \sigma)/\binom{n}{2} \geq \frac{1}{2} - \varepsilon \text{ for almost all } \sigma.$$ Sequel to Zahn (1964a). [Annot. 10 Nov 2017.] (sg: Clu)

J.W. Moon & L. Moser


Studies the maximum frustration index of a signed $K_{r,s}$. (sg: Fr)

Suck Joong Moon

See also H. Kosako.

Suck-Joong Moon & Hideo Kosako


See Kosako, Moon, et al. (1993a) (SG, VS: Sw, fr, D, Incid)

M.A. Moore

See A.J. Bray.

G. Eric Moorhouse


Alejandro H. Morales

See K. Mészáros.

Katherine Tapia Morales

See M. Robbiano.

A. Moreira

See J. Aracena.

Susan Morey

See L. Fouli.
Aki Mori
See T. Hibi.

Michio Morishima

§4: “Alternative expression of the assumptions (1),” can be interpreted with hindsight as proving that, for a signed $K_n$, every triangle is positive iff the signature switches to all positive. (Everything is done with sign-symmetric matrices, not graphs, and switching is not mentioned in any form.)

Robert Morris
See H. Liu.

Timothy Morris
See A.H. Busch.

Julian O. Morrissette

Proposes that edges have strengths between $-1$ and $+1$ instead of pure signs. The Cartwright–Harary degree of balance (1956a), computed from circles, is modified to take account of strength. In addition, signed graphs are allowed to have edges of two types, say $U$ and $A$, and only short mixed-type circles enter into the degree of balance. This is said to be more consistent with the experimental data reported herein.

Julian O. Morrissette & John C. Jahnke

Reports an experiment; then discusses problems with and alternatives to the Cartwright–Harary (1956a) circle degree of balance. (PsS: Fr)

Julian O. Morrissette, John C. Jahnke, & Keith Baker

Proposes to measure degree of balance by $c^+(\Sigma)/c(K_n)$ instead of $c^-(\Sigma)/c(|\Sigma|)$ as in Cartwright and Harary (1956a), to overcome logical incompatibility between the latter measure, the principle of increasing balance, and an assumed tendency towards completeness in a (signed) graph of social relations; as well as for experimental reasons. [Annot. 3 Sep 2013.] (SG, PsS: Bal)

Hannes Moser
See J. Guo.

L. Moser
See J.W. Moon.

Tyler Moss
See D. Chun.

Sebastiano Mosterts
See E.L. Johnson.
Satish V. Motammanavar  
See H.B. Walikar.

Fatemeh Motialah & Mohammad Hassan Shirdareh Haghighi  
Sun = circle with one pendant edge at each vertex. Determined by Laplacian spectrum if unbalanced or has odd girth. [Annot. 15 Dec 2020.]  

(SG: Lap: Eig)

C.F. Moukarzel  
See M.J. Alava.

Gisele Moura  
See R. Figueiredo.

Nazanin Movarrae  
See P. Ochem.

[Eunice Gogo Mphako]  
See E. Mphako-Banda.

Eunice Mphako-Banda [Eunice Gogo Mphako]  
§5: Bracket polynomials of signed matroids, after Schwärzler and Welsh (1993a). [Annot. 21 May 2013.]  

(Them. 3.13: $p_L(v(M)) = (\lambda - 1)p_M(\lambda - \#H)$. §5, “$H$-lifts of tangential blocks”: $L_r(M^*)$ is a tangential block if $M^* := G(\Phi^*)$ is one. [This lift is not the lift matroid.] [Annot. 20 Oct 2020.]

(Them. 3.2 evaluates $#E(H \cdot \Delta^*)$. Thms. give the characteristic polynomials [they are special cases of Zaslavsky (1995b), §§4, 5]. [Annot. 20 Oct 2020.]

H is a finite group. “Complete $H$-tree”, “-path”, “-star”, “-cycle”, “fan”, “wheel” = full frame matroid $G(H \cdot \Delta^*)$ where $\Delta$ is a tree, path, etc. [The stated definition of $H$-tree is incomplete.] Spanning submatroids are called “$H$-trees”, etc. Props. 3.1, 5.1 list simple properties. Thm. 3.2 evaluates $#E(H \cdot \Delta^*)$. Thms. give the characteristic polynomials [they are special cases of Zaslavsky (1995b), §§4, 5]. [Annot. 20 Oct 2020.]

(Them. 3.2 evaluates $#E(H \cdot \Delta^*)$. Thms. give the characteristic polynomials [they are special cases of Zaslavsky (1995b), §§4, 5]. [Annot. 20 Oct 2020.]

Andrej Mrvar  
See also M. Brusco, P. Doreian and W. de Nooy.

Andrej Mrvar & Patrick Doreian  
§2, “Formalization of block-modeling signed two-mode data”: A signed two-mode network is a signed simple bipartite graph with color classes $V_1, V_2$. The objective is partitions $\pi_1, \pi_2$ of $V_1, V_2$ that minimize a “criterion function” $P := \alpha \bar{i}_- + (1 - \alpha)i_+$; usually $\alpha = .5$. $k_1 := \#\pi_1$ and $k_2 := \#\pi_2$, or other restrictions, may be specified. Definitions: $\pi_i := \{V_{i1}, \ldots, V_{ik_i}\}$. A “block” is a nonvoid set $E(V_{i1}, V_{i2})$. Its sign is the sign of the majority of edges, $+\bar{\epsilon}$ if $\epsilon$ is a draw. $e$ is “consistent” with $(\pi_1, \pi_2)$ if it is in a block of sign $\sigma(e)$. $i_\epsilon := \text{number of inconsistent edges of sign } \epsilon$. [Annot. 17 Aug 2009.] (SG: Clu, PsS)

Guohong Mu, Lulu Li, & Xiaodi Li
Balanced signed digraphs, not switched to all positive [which would be simpler]. [Annot. 19 Oct 2020.] (SD: Bal: Dyn)

Lili Mu & Richard P. Stanley
*Cf. Stanley (2015a).* [Annot. 16 Nov 2018.] (gg: Geom)

G. Muciaccia
See R. Crowston.

Haiko Müller
See T. Kloks.

Akihiro Munemasa
See also G. Greaves, M. Harada, and H.J. Jang.

Akihiro Munemasa, Yoshio Sano, & Tetsuji Taniguchi

Luigi Muracchini & Anna Maria Ghirlanda
A partially successful attempt to use unoriented signed graphs to define a line graph of a digraph. [See Zaslavsky (2010b), (20xxa), (2012c) for the correct signed-graph approach.] The Harary–Norman line digraph is also discussed. (SG: Bal, LG)

Kunio Murasugi
The signature of a sign-colored graph (see (1989a)) is an invariant of the sign-colored graphic matroid. (SGc: Incid, m)

Studies a dichromatic form, $P_\epsilon(x, y, z)$, of Kauffman’s (1989a) Tutte polynomial of a sign-colored graph. The deletion-contraction parameters are $a_\epsilon = 1$, $b_\epsilon = x^\epsilon$ for $\epsilon = \pm 1$; the initial values are such that
The polynomial is shown to be, in effect, an invariant of the sign-colored graphic matroid.

Much unusual graph theory is in here. A special focus is the degrees of the polynomial. First Main Thm. 3.1: Formulas for the maximum and minimum combined degrees of $P_{\Sigma}(x, y, z)$. §7, “Signature of a graph”, studies the signature ($\sigma$ in the paper, $s$ here) of the Laplacian matrix $L(\Sigma)$ ($B_\Sigma$ in the paper) obtained by changing the diagonal of $A(\Sigma)$ so the row sums are 0. Prop. 7.2 is a matrix-tree theorem [entirely different from that of Zaslavsky (1982a)]. The Second Main Thm. 8.1 bounds the signature: $#V - 2\beta_0(\Sigma^-) + 1 \leq s \leq #V - 2\beta_0(\Sigma^+) + 1$ ($\beta_0 =$ number of components), with equality characterized. The Laplacian matrix is further examined later on. §9, “Dual graphs”: Differing from most studies, here the dual of a sign-colored plane graph is the planar dual with same edge signs [however, negating all colors is a triviality]. §10, “Periodic graphs”: These graphs might be called branched covering graphs of signed gain graphs with finite cyclic gain group. [Thus they generalize the periodic graphs of Collatz (1978a) and others.] §§12–15 concern applications to knot theory.

\[
P_{\Sigma}(x, y, z) = y^{-1}Q_{\Sigma}(a, b; y, z)\]

of Zaslavsky (1992b). The polynomial is shown to be, in effect, an invariant of the sign-colored graphic matroid.


§§1–3 expound results from (1989a) on the dichromatic polynomial and the signature of a sign-colored graph and knot applications. §5 discusses the signed Seifert graph of a link diagram.


Kunio Murasugi & Jozef H. Przytycki


Ch. I, “Index of a graph”. The “index” is the largest number of “independent” edges, where “independent” has a complicated recursive definition (unrelated to matchings), one of whose requirements is that the edges be “singular” (= simple). The positive or negative index of a sign-colored graph is similar except that the independent edges must all be positive or negative. [The general notion is that of the index of a graph-subgraph pair. The signs pick out complementary subgraphs.] Thm. 2.4: Each of these indices is additive on blocks of a bipartite graph. The main interest, because of applications to knot theory, is in bipartite plane
graphs. Ch. II, “Link theory”: Pp. 26–27 define the sign-colored Seifert graph of an oriented link diagram and apply the graphical index theory. (SGc: Invar, D, Knot)

Tadao Murata

Antoine Musitelli
See also A. Del Pia.


A polynomial-time algorithm for recognizing binet matrices in time $O(n^6\#E)$. See (2010a). [Annot. 15 January 2013.]

(... SG: Ori: Incid, Alg)


*Cf. Kotnyek (2002a)*. Description of the algorithm of (2007a), which involves reduction to the cases of “cyclic” and “bicyclic” matrices. These are the incidence matrices of bidirected graphs with, respectively, one or two components that are without half edges. [Annot. 15 January 2013, rev 16 Oct 2017.]

(... SG: Ori: Incid, Alg)

Mohammed A. Mutar
See A.H. Busch.

A. Muthaiyan & A. Nesamathi
2016a Some new face and total face signed product cordial graphs. 3 (2016), no. 8, 399–407.

More examples as in *Rozario Raj and Manoharan (2016a)*. (Lab: VS: SG, Bal)

P. Mützel
See C. De Simone.

[Sudev Naduvath]
See N.K. Sudev (under ‘S’).

Mohamed Nafea
See B. Guler.

Ojiro Nagai
See also H.T. Diep.

Ojiro Nagai, Tsuyoshi Horiguchi, & Seiji Miyashita

Physics questions on various periodic signed lattice graphs, dim = 2, 3. 

*[Question. Do the phenomena treated here for periodic signed lattices suggest interesting mathematics, possibly for more general $\Sigma$?]*
§2.5, “Ising model with large $S$ on antiferromagnetic triangular lattice”: Spin $1/2$ generalizes to (integral) spin $S = \text{spin} \in \{-S, -(S-1), \ldots, 0, \ldots, S-1, S\}$ [as in coloring $\Sigma$ with $2S+1$ colors] on all-negative triangular lattice. Large $S$ gives new kinds of ground state (min energy).

[Question. Are these new signed-graph coloring problems?] §2.6, “Ising model with infinite-spin on antiferromagnetic triangular lattice”: “Infinite spin” means $S \to \infty$.

Dictionary: “local gauge transformation” = switching, “spin $1/2$” (often assumed in physics) = Ising spins $\pm 1$. [Annot. 9 Aug 2018.]

Ojiro Nagai, Koichi Nishino, Jong-Jae Kim, and Yuuzi Yamada


[Partly from previous works cited?] Cubic lattice with $E^- = \{(i,j,k)(i+1,j,k) : j+k \text{ even}\}$. Edge weights (“bond strengths”) $a, 1, 1$ in $x, y, z$ directions, $a > 0$. Ising ground states $\psi : V \to \{\pm 1\}$ (i.e., least weight of unsatisfied edges): For $a < 2$, each $yz$-plane is satisfied ($\psi$ is constant). For $a > 2$, each $x$ line is satisfied (constant $\psi$ on all-+ lines, alternating on all-- lines). For $a = 2$, both are ground states. Some results are for multivalued spins $-S, \ldots, S-1, S$. Consequence: At $a = 2$ there are “free” spins ($\psi(v) = \pm 1$ arbitrarily in ground states) at some vertices. [Question. Does this suggest interesting mathematics for more general $\Sigma$?] [Annot. 14 Aug 2018.]


Computer simulations on a cubic lattice with $E^- = \{(i,j,k)(i+1,j,k) : j \text{ even}\}$. In Ising ground states $\psi : V \to \{\pm 1\}$ (i.e., fewest unsatisfied edges), each $z$-line has constant $\psi$ along the line; in 1/4 of them the spin constant varies with time. [Question. Does this suggest interesting mathematics for more general $\Sigma$?] [Annot. 14 Aug 2018.]

O. Nagai, Y. Yamada, & H.T. Diep


Computer simulations on a cubic lattice with $E^- = \{(i,j,k)(i+1,j,k) : j \text{ even}\}$. In Ising ground states $\psi : V \to \{\pm 1\}$ (i.e., fewest unsatisfied edges), each $z$-line has constant $\psi$ along the line; in 1/4 of them the spin constant varies with time. [Question. Does this suggest interesting mathematics for more general $\Sigma$?] [Annot. 14 Aug 2018.]

(Phys: SG: Fr)

K.M. Nagaraja

See P.S.K. Reddy.

Yuuki Nagazawa

See T. Matsui.

P. Nageswari & P.B. Sarasija


Mina Nahvi

See S. Akbari.

T.A. Naikoo

See S. Pirzada.

Takeshi Naitoh

See K. Ando.
**Kazuo Nakajima**
See H. Choi.

**Atsuhiro Nakamoto**
See also D. Archdeacon.

**Atsuhiro Nakamoto, Seiya Negami, & Katsuhiro Ota**


A “cycle parity” on surface \( S = \text{homomorphism} \rho : \pi(S) \rightarrow \mathbb{Z}_2 \cong \{+,-\}, \) equivalently \( \rho : H_1(S; \mathbb{Z}_2 \rightarrow \mathbb{Z}_2 \cong \{+,-\}. \) \( \rho \) implies a signature (actually, a switching class) of any embedded graph \( \Gamma. \) There are one nontrivial type of cycle parity on an orientable surface and three on a nonorientable surface \( N_d, \) different for odd and even \( d, \) except two on \( N_2 \) and one on \( N_1. \) If \( \Gamma \hookrightarrow S \) so every face boundary is even (“even embedding”), then \( \rho(W) = \#W \mod 2 \) for closed walks is a cycle parity.

Thm. 9: For three of the six types on \( N_d's, \) there is a negative cut that opens \( N_d \) to an orientable surface. [See also Mohar and Seymour (2002a).] [Annot. 11 Jun 2012.] (sg: Top: sw)


**Daishin Nakamura & Akihisa Tamura**


The problem of the title is solvable in polynomial time. See Johnson and Padberg (1982a), Tamura (1997a) for definitions. They reduce to simple graphs, transitively bidirected with no sink or introverted edge (called “canonical” bidirected graphs). (sg: Ori: Geom, Sw, Alg)


**M. Nakamura**
See M. Hachimori.

**Tota Nakamura, Shin-ichi Endoh, & Takeo Yamamoto**

Physics of Ising, XY, and Heisenberg spin-glass models on a signed square lattice graph with 3-dimensional spin vectors. The Hamiltonian of state $S : V \to S^2$ (the sphere) is $\sum_{uv \in E} \sigma(uv) S_u \cdot S_v$. [Annot. 17 Jun 2012.] (Phys: SG)

Bunpei Nakano
See T. Inohara.

Norihiro Nakashima & Shuhei Tsujie

Preetum Nakkiran
See C.E. Tsourakakis.

Vasileios Nakos
See C.E. Tsourakakis.

Aurélien Naldi
See also J.-P. Comet.

Aurélien Naldi, Elisabeth Remy, Denis Thieffry, & Claudine Chaouiy

Soumen Nandi
See J. Bensmail and S. Das.

L. Nanjundaswamy
See E. Sampathkumar.

Assaf Naor
See N. Alon.

Joseph (Seffi) Naor
See D. Hochbaum.

Vito Napolitano
See M. Abreu.

Ramasuri Narayanam
See P. Agrawal.

Jonathan Narboni
See F. Kardoš.

Reza Naserasr
See also L. Beaudou, R.C. Brewster, C. Charpentier, and F. Foucaud.
20xxa Not $\{\pm 1, \pm 2\}$-colorable planar signed graphs. Submitted.
A simpler, smaller proof of Kardoš and Narboni (2021a), with remarks and questions on related homomorphisms. [Annot. 17 Nov 2019.] (SG: Col, Hom)

Reza Naserasr & Lan Anh Pham
20xxa Complex and homomorphism chromatic number of signed planar simple graphs. Submitted. HAL hal-03000542.
Color set $C_{k,l} = \{ \pm 1, \ldots, \pm k, \pm i_1, \ldots, \pm i_l \}, i = \sqrt{-1}$. Coloration $\kappa$ is proper when $\kappa(u)\kappa(v) \neq \sigma(uv)|\kappa(v)|^2$ for each edge $uv$ [more simply, $|\kappa(u)| \neq |\kappa(v)|$ or $\text{sgn} \kappa(u) \cdot \text{sgn} \kappa(v) \neq \sigma(uv)$, where $\text{sgn} z := z/|z|$].


Reza Naserasr, Lan Anh Pham, & Zhongxingn Wang
20xxa Density of $C_{-4}$-critical signed graphs. Submitted. HAL hal-03000545.

Reza Naserasr, Edita Rollová, & Éric Sopena


Reza Naserasr, Sagnik Sen, & Éric Sopena
20xxa The homomorphism order of signed graphs. Submitted. HAL hal-02969878.

Reza Naserasr, Sagnik Sen, & Qiang Sun

Reza Naserasr, Éric Sopena, & Thomas Zaslavsky

Several aspects of homomorphisms. Aims to set correct terminology, give proofs of fundamental properties, raise open questions. [Annot. 16 Sep 2019.]

Reza Naserasr & Zhongxingn Wang

Reza Naserasr, Zhongxingn Wang, & Xuding Zhu
20xxa Circular chromatic number of signed graphs. Submitted. HAL hal-02969872. (SG: Col)

C.St.J.A. Nash-Williams


Nagarajan Natarajan
See K.-Y. Chiang.

Nutan G. Nayak

Eigenvalues of $K_{-\Sigma_n}$. These lead to non-cospectral pairs of net-regular signed $K_n$’s with equal adjacency energy. [Annot. 20 Apr 2019.]

(SG: KG: Adj: Eig)


Elementary facts of net regularity. Eigenvalues of $K_{-\Sigma_n}$. [Annot. 20 Apr 2019.]

(SG: KG: Adj: Eig)


The net-Laplacian matrix has modified diagonal. (SG: Lap, Eig)


Zachary Neal
See S. Aref.

Roman Nedela
See also A. Malnič.

Roman Nedela & Martin Škoviera

By “canonical double covering” of $\Gamma$ they mean the signed covering graph $\Sigma$ of $\Sigma = -\Gamma$, but without reversing orientation at the negative covering vertex [as one would do in a signed covering graph (cf. e.g. Zaslavsky (1992a))], because orientable embeddings of $\Gamma$ are being lifted to orientable embeddings of $\Sigma$. [Thus these should be thought of not as signed graphs but rather as voltage (i.e., gain) graphs with 2-element gain group.] Instead of reversal they twist the negative-vertex rotations by taking a suitable power. In some cases this allows classifying the orientable, regular embeddings of $\Sigma$. (Par: Cov, Top, Aut)

Main topic: the theory of twisting of rotations as in (1996a).

(\textbf{GG: Cov, Top, Aut})

Portions concern double covering graphs of signed graphs. §7: “Antipodal and algebraically antipodal maps”. A map is “antipodal” if it is the orientable double covering of a nonorientable map; that is, as a graph it is the canonical double covering of an unbalanced signed graph. A partial algebraic criterion for a map to be antipodal. §9: “Regular embeddings of canonical double coverings of graphs”. See (1996a).

(\textbf{Par: Cov, Top, Aut})


Cases in which the classification of (1996a) is necessarily incomplete are studied by taking larger voltage (i.e., gain) groups and twisting the rotations at covering vertices by taking a power that depends on the position of the vertex in its fiber. Main result: the (very special) conditions on twisting under which a regular map lifts to a regular map.

(\textbf{GG: Cov, Top, Aut})

---

Seiya Negami

See D. Archdeacon and A. Nakamoto.

**Peter Nelson & Jorn van der Pol**


Almost all biased simple graphs are complete graphs with Hamiltonian bias. [Annot. 20 Aug 2019.]

(\textbf{GG: Enum})

Max Nelson-Kilger

See M.H. Fişek.

Mohammad Ali Nematollahi

See S. Akbari.

Toshio Nemoto

See K. Ando.

Anna Nenca

See J. Dybizbański.

Hanna Nencka [H. Nencka-Fisek]

See H. Nencka-Fisek.

H. Nencka-Fisek [H. Nencka]

See also Ph. Combe.


Signs are defined for arbitrary proper subhypercubes of the hypercube $Q_d$ [thus giving a signed hypergraph]. A "plaquette" (this is non-standard) is a $k-1$-dimensional band around a $k$-subhypercube; its sign is the product of signs of its $k-1$-faces. Overblocking means not all plaquettes can simultaneously be negative ("frustrated"). The interesting proof is by the adjacency graph of 2-faces of a 3-cube in $Q_d$. Identify opposite 2-faces to a single vertex whose sign is the product of 2-face signs; the faces of a 3-cube form a triangle whose vertices alternate in sign, if all plaquettes were negative. Conclusion: All 2-faces cannot be negative, if $d > 2$. [Presumably a similar argument should be applied to plaquettes of $k-1$-faces of a $k$-cube, $k > 3$, but it is not. There would be one plaquette per dimension. Question. Is there such a generalization?] [Annot. 19 Jun 2012.]


A higher-dimensional Ising model with a sign attached to each “plaquette” (see (1985a)). A plaquette is frustrated if the spin product (spin = ±1) of its sites (vertices) fails to match the attached sign. A necessary condition for frustration is said to be an “umbrella” (a topological construction, possibly a cap on the plaquette?). An example is a triangular lattice. The main example is $\mathbb{Z}^d$. The theorem implies that only $k < d$ can have a frustrated plaquette [obvious, if a plaquette lives in the boundary of a subcube]. [The article seems imprecise. The idea could be worth pursuing.] [Annot. 26 Dec 2014.]

A. Nesamathi
See A. Muthaiyan.

Jaroslav Nešetřil
See also J. Kratochvíl.

Jaroslav Nešetřil & André Raspaud

Nancy Ann Neudauer
See also R.A. Brualdi, L. Goddyn, G. Gordon, and J. McNulty.


The matroids are $G(\Gamma, \emptyset)$.


Survey of parts of Brualdi and Neudauer (1997a), Wagner (1985a), and Coullard, del Greco, and Wagner (1991a), with supplementary results on nice graphs whose bicircular matroid, $G(\Gamma, \emptyset)$, equals $M$.

Nancy Ann Neudauer, Andrew M. Meyers, & Brett Stevens
Counts bases and connected bases. Very complicated formulas. [The results count labelled simple 1-trees and 1-forests. A 1-tree is a tree with one extra edge forming a circle. A 1-forest is a disjoint union of 1-trees. A connected basis of the bicircular matroid \(G(K_n, \emptyset)\) for \(n \geq 3\) is a labelled simple 1-tree; a basis is a labelled simple 1-forest. Riddell (1951a) has a less complicated formula for 1-trees.]

**Nancy Ann Neudauer & Brett Stevens**


Bases are counted and their structure compared to the spanning trees of the graph. [A basis is a simple, labelled 1-forest (cf. Neudauer, Meyers, and Stevens (2001a)) whose circles are even.]

**Nancy Ann Neudauer & Daniel Slilaty**


---

**A. Neumaier**


In the signed graph \((K_n, \sigma)\) of a two-graph (see D.E. Taylor (1977a)), a “clique” is a vertex set that induces an antibalanced subgraph. A two-graph is “completely regular” if every clique of size \(i\) lies in the same number of cliques of size \(i + 1\), for all \(i\). Thm. 1.4 implies there is only a small finite number of completely regular two-graphs.

---

**Michael Neumann**

See C.R. Johnson.

**Víctor Neumann-Lara**

See I.J. Dejter.

**Bryan Nevarez**

See M. Beck.

**T.M. Newcomb**

See also K.O. Price.


**G.F. Newell**


The same physics conclusions as Houtappel’s (1950a), (1950b) for a signed, weighted triangular lattice. [See also I. Syôzi (1950a), Wannier (1950a).] [Annot. 20 Jun 2012.]

**Alantha Newman**

See N. Ailon.

**Charles M. Newman**

See also F. Camia and A. Gandolfi.

**Charles M. Newman & Daniel L. Stein**


(Phys: sg: State, fr)


Mike Newman
See D. Funk.


Sees especially §3, “The standard SK picture”. The Hamiltonian $H_{\sigma}(s) = -\sum_{vw \in E} \sigma(vw)s(v)s(w)$ is standard. Criticizes the typical physics application of randomly signed (and possibly weighted) $K_n$ (Sherrington–Kirkpatrick model) to $\mathbb{Z}^d$-lattice graphs by limits of finite (cubical) subgraphs. Raises the question of a “pure state” (cf. Mézard, Parisi, and Virasoro (1987a) et al.) of a signed $K_n$, where a state is $s : V \rightarrow \{+1, -1\}$ and a pure state is apparently a linear combination of or probability distribution on states, especially in the $\mathbb{Z}^d$ limit. A pure state is not well defined but is related to states of low frustration (and high probability).

[Question. Is there a graphical meaning of a pure state, based on the (ambiguous) physics definition? It should involve states with low frustration, because they dominate the partition function $Z(\sigma) = \sum_s e^{H_0(\sigma)}$, and on the qualities desired for computing quantities of physical interest, especially in terms of $H$ and $Z$.]

A “metastate” is a measure on states, essentially a linear combination with explicit coefficients. Pure states on $\mathbb{Z}^d$ should be metastates. See (1997a). [Question. Is there a graph-theory meaning to all this? Does it lead to a definition of frustration in an infinite signed (or gain) graph?]

[Annot. 26 Aug 2012.]

(Phys: sg: State, fr: Exp, Ref)


(Phys: sg: State, fr)

Sang Nguyen
See P.L. Hammer.

Aidin Niaparast
See S. Akbari.

André Nichterlein
See F. Hüffner.

Robert Nickel
See W. Hochstättler.

Rolf Niedermeier
See F. Hüffner.
F. Nieto
See A.J. Ramírez-Pastor and F. Romá.

Juhani Nieminen

Scisthe"signedclosure"ofasigneddigraphS.
Sis"weaklybalanced"if
inSall directed digons and all induced transitive triangles are positive.
Thm.: S is weakly balanced iff it is path- and cycle-balanced. Also, the degree of weak balance.

(SD: Bal)(SD: Fr: Alg)

Peter Nijkamp
See F. Brouwer.

Vladimir Nikiforov
See also N.M.M. de Abreu, M.A.A. de Freitas, and L.S. de Lima.


For order \( n \gg 0 \), if \(-\Gamma\) has least eigenvalue \( \sqrt{\lfloor n^2/4 \rfloor} \), then it has negative (i.e., odd) circles of all lengths \( \leq n/320 \). [Question. Does this property generalize to signed graphs as: eigenvalue bound \( \Rightarrow \) all negative circles of lengths \( \leq \) upper limit?] [Annot. 20 Sept 2015.]

(par: Lap: Eig)


Assume \( \Gamma \) is “\( k \)-degenerate”: every (induced) subgraph has a vertex of degree \( \leq k \). Thm. 1.2: \( \lambda_1(-\Gamma) \leq \lambda_1(-(K_k \vee \bar{K}_{n-k})) \), = iff \( \Gamma = K_k \vee \bar{K}_{n-k} \). (\( \lambda_1 \) = max eigenvalue of \( L(\Sigma) := \) Laplacian matrix.) Thm. 1.3: \( \lambda_1(-\Gamma) \leq \) function of \( n \), \( \#E \), \( \Delta \), and \( \delta \). [Annot. 20 Jan 2015.]

(par: Lap: Eig)


(par: Lap: Eig)

Vladimir Nikiforov & Xiying Yuan

Thm. 1.4: For large \( n \): (i) \( \Gamma \not\supset P_{2k+1} \) \( \Rightarrow \) \( \Gamma = K_k \vee \bar{K}_{n-k} \) or \( \lambda_1(-\Gamma) < \lambda_1(-(K_k \vee \bar{K}_{n-k})) \). (ii) \( \Gamma \not\supset P_{2k+2} \) \( \Rightarrow \) \( \Gamma = K_k \vee \bar{K}_{n-k} \cup e \) or \( \lambda_1(-\Gamma) < \lambda_1(-(K_k \vee \bar{K}_{n-k} \cup e)) \). (\( P_l \) = path of length \( l \). \( \lambda_1 \) = max eigenvalue of \( L(\Sigma) \)). [Annot. 20 Jan 2015.]

(par: Lap: Eig)

Yuri Nikolayevsky
See G. Cairns.

Zoran Nikoloski
See N. Kejžar.
Wenjie Ning, Hao Li, & Mei Lu

Koichi Nishino
See O. Nagai.

Kenta Nishiyama
See T. Hibi.

M. Nogala
See E.E. Vogel.

Kenta Noguchi


J.D. Noh, H. Rieger, M. Enderle, & K. Knorr


Rafidah MD Noor
See S.R. Shahriary.

Wouter de Nooy

Vertex ranking (a partial ordering) based on arc signs. Thm. 3 characterizes equality of rank. Thm. 6 characterizes strict inequality. [Annot. 11 Sept 2010.] *(SD: PsS, Bal, Clu)*

2008a Signs over time: statistical and visual analysis of a longitudinal signed network. *J. Social Structure* 9 (2008), article 1, 32 pp. *(SG: Fr, PsS: Dyn)*

Wouter de Nooy, Andrej Mrvar, & Vladimir Batagelj

Pajek is a computer package that analyzes networks, i.e., graphs, including signed graphs. Ch. 4: “Sentiments and friendship.” Computation of balance and clusterability of signed (di)graphs. §4.2: “Balance theory.” Introductory. §4.4: “Detecting structural balance and clusterability.” How to use Pajek to optimize clustering. §4.5: “Development in time.” Pajek can look for evolution towards balance or clusterability. §10.3: “Triadic analysis.” Types of balance and clusterability, with the triads (order-3 induced subgraphs) that do or do not occur in each. Table 16, p. 209, “Balance-theoretic models”, is a chart of 6 related models. §§10.7, 10.10: “Questions” and “Answers.” Some are on balance models. §10.9: “Further reading.” [Annot. 28 Apr 2009.] *(SG, SD, PsS: Bal, Clu, Alg: Exp)*
Robert Z. Norman
See also M.H. Fişek and F. Harary.

Robert Z. Norman & Fred S. Roberts

- Circle (“cycle”) indices of imbalance: the proportion of circles that are unbalanced, with circles weighted nonincreasingly according to length. (SG: Fr(Circ))


- Exposition and application of (1972a). (SG: Fr(Circ): Exp, PsS)

Mathilde Noual
See also J. Aracena, J.-P. Comet, and J. Demongeot.

Mathilde Noual, Damien Regnault, & Sylvain Sené

Beth Novick & András Sebő

- The clutter of negative circuits of a signed binary matroid \((M,\sigma)\).
- Important are the lift and extended lift matroids, \(L(M,\sigma)\) and \(L_0(M,\sigma)\), defined as in signed graph theory. An elementary result: the clutter is signed-graphic iff \(L_0(M,\sigma) / e_0\) is graphic (which is obvious). There are also more substantial but complicated results. [See Corméjols (2001a), §8.4.] (Sgnd(M), SG: M)


Marc Noy
See O. Giménez.

Cyriel van Nuffelen

- Theorem restated: the unoriented incidence matrix has rank \(\text{rk} G(\mathit{\Gamma})\).
- [Because the matrix represents \(G(\mathit{\Gamma})\): see Zaslavsky (1982a). In retrospect, partially implicit in Stewart (1966a) and completely so in Stanley (1973a).]

Summarizes (1973a). 

Koji Nuida
See also T. Abe.

Yasuhide Numata
See T. Abe.

Gábor Nyul
See E. Gyimesi.

Suil O
See G. Greaves.

Jan Obdržálek
See R. Ganian.

G. Obermaier
See K. Jüngling.

Mohammad Reza Oboudi
See also S. Akbari.

Cian O’Brien, Kevin Jennings, & Rachel Quinlan

Pascal Ochem
See also F. Dross.

Pascal Ochem & Nazanin Movarrae

Pascal Ochem, Alexandre Pinlou, & Sagnik Sen

M. Ocio
See E. Vincent.

B. O’Connor
See A. Krieger.

Damien Octeau
See B. Guler.

Hidefumi Ohsugi
See also T. Matsui.


Hidefumi Ohsugi & Takayuki Hibi


The odd-cycle condition of Fulkerson, Hoffman, and McAndrew (1965a) is employed in polynomial algebra. “Graph polytope” [later named “edge polytope”] $P_{-\Gamma} := \text{conv} x(E(\Gamma))$, where $x(E(-\Gamma)) = \{\text{columns of incidence matrix } H(-\Gamma)\}$. Dictionary: “$P_G = P_{-\Gamma}$.” [This is antibalanced. Problem. Generalize to signed graphs, including balanced graphs.] [Annot. 30 May 2011.] (sg: Par: Geom, Algeb)


§1, “Binomial ideals arising from finite graphs”: The edge ring of $-\Gamma$ (negative edges because the analysis is antibalanced—even and odd graph circles are different) is $K[x_{ij} : ij \in E(\Gamma)]$. Thm. 1.2: The toric ideal $I_{-\Gamma}$ is generated by quadratic binomials iff in $-\Gamma$, each positive circle has certain chords, each contrabalanced tight handcuff has an edge between its circles, and each two disjoint negative circles are joined by at least 2 edges [i.e., antibalanced criteria]. [Problem. Understand this via signed graphs.] §4: Edge polytope $P_{-\Gamma}$ properties such as minimal volume. §5, “Simple edge polytopes”: E.g., Cor. 5.4: $P_{-\Gamma}$ is simple iff $\Gamma = K_{p,q}$. [Problem. Generalize to signed graphs, including ordinary graphs $\Gamma$ (i.e., all positive). The edge ring would be bidirected: $K[x_{ij} : \vec{e}_{ij} \in E(B)]$ for a bidirection $B$ of $\Sigma$. E.g., one expects $P_{+\Gamma}$ to be simple iff $\Gamma = +K_n$.] [Annot. 5 Oct 2014, 3 Jun 2015.] (sg: Par: Algeb, Geom)

1999b A normal (0, 1)-polytope none of whose regular triangulations is unimodular. Discrete Comput. Geom. 21 (1999), 201–204.

The polytope is the edge polytope $\text{conv} H(-\Gamma)$ where $\Gamma = C_5$ with a triangle on each edge. [Annot. 18 Aug 2018.] (sg: Par: Algeb, Geom)


Edge polytope $P_{-\Gamma_{n_1,\ldots,n_k}}$ and edge ring. [This is antibalanced. Problem. Generalize to signed graphs, including balanced graphs.] [Annot. 5 Oct 2014.] (sg: Par: Geom, Algeb)

2003a Normalized volumes of configurations related with root systems and complete

A configuration consists of the vectors representing an acyclic orientation of a complete signed bipartite graph. The volume is that of the pyramid over the configuration with apex at the origin. (Successor to Fong (2000a).) **Question.** Is there a connection with the chromatic polynomial?**

Hidefumi Ohsugi & Kazuki Shibata


Ayao Okiji
See Y. Kasai.

E. Olaru
See St. Antohe.

Marián Olejár
See J. Širáň.

D.D. Olesky
See also B.D. Bingham, T. Britz, M. Catral, G.J. Culos, D.A. Grundy, C.R. Johnson, and K.H. Monfared.

D.D. Olesky, M.J. Tsatsomeros, & P. van den Driessche


Aroldo Oliveira, Leonardo Silva de Lima, & Nair Maria Maia de Abreu


For $L(−\Gamma)$, spread = (largest − smallest eigenvalue) $\leq \chi(\Gamma)$. [Annot. 20 Jan 2015.] (par: Lap: Eig)

Carla Silva Oliveira
See also L.S. de Lima.

Carla Oliveira & Leonardo de Lima


A degree bound for $L(−\Gamma)$.. See also Li and Tian (20xxa). [Annot. 8 Jan 2015.] (par: Lap: Eig)

Carla Silva Oliveira, Leonardo Silva de Lima, Nair Maria Maia de Abreu, &
Pierre Hansen
(par: Lap: Eig)

Carla Silva Oliveira, Leonardo Silva de Lima, Nair Maria Maia de Abreu, & Steve Kirkland
(par: Lap: Eig)

Stig W. Omholt
See E. Plahte.

G.R. Omidi
See also F. Ayoobi and W.H. Haemers.
A subdivided $K_{1,3}$ is determined by $\text{Spec}(L(-\Gamma))$. [Continued in Omidi and Vatandoost (2010a) and Bu and Zhou (2012a).] [Annot. 28 Nov 2012.]
(par: Lap: Eig)

Gholam R. Omidi & Ebrahim Vatandoost
A subdivided $K_{1,4}$ is determined by $\text{Spec}(L(-\Gamma))$. Continuation of Omidi (2009a). [Continued in Bu and Zhou (2012a).] [Annot. 28 Nov 2012.]
(par: Lap: Eig)

Kenji Onaga

Shmuel Onn
See also P. Kleinschmidt.
For “signability” see Kleinschmidt and Onn (1995a). A strong signing is an exact signing that satisfies a recursive condition on lower intervals.
(Sgd, Geom)

Rikio Onodera
The adjacency graph of trees of a graph is signed from a vertex signature and is shown to be balanced. [Trivial.] [Annot. 24 July 2010.]
(SG: Bal)
The Open University


Social sciences (pp. 21–23). Signed digraphs (pp. 50–52). [Published version: see Wilson and Watkins (1990a).] (SG, PsS, SD: Exp)

Peter Orlik & Louis Solomon


Thm. (4.8): The characteristic polynomials of the Dowling lattices and jointless Dowling lattices of $\mathbb{Z}_r$, computed via group theory as part of the general treatment of finite unitary reflection groups. (gg: m, Geom)


In the intersection lattice of reflection hyperplanes of a finite unitary reflection group, the characteristic polynomial of an upper interval has an integral factorization. The proofs involve detailed study of the group actions on $\mathbb{C}^l$. Dictionary: $\mathcal{A}_l(r)$ and $\mathcal{A}_k^l(r)$ are the arrangements corresponding to the rank-$l$ Dowling lattices and partially jointless Dowling lattices of $\mathbb{Z}_r$. Relevant results: §2: “Monomial groups”: Cor. (2.4) counts the flats, Prop. (2.5) and Cor. (2.7) gives the polynomials for $\mathcal{A}_l(r)$ [all known from Dowling (1973b)]. Cor. (2.10) counts the flats, Prop. (2.13) gives the polynomial of $\mathcal{A}_l^k(r)$, Prop. (2.14) notes that proper upper intervals are Dowling lattices [all fairly obvious via gain graphs and coloring (Zaslavsky (1995b))]. (gg: m, Geom, Invar)


James B. Orlin

See also R.K. Ahuja, M. Kodialam, and R. Shull.


Problems on 1-dimensional periodic graphs (i.e., covering (di)graphs of $\mathbb{Z}$-gain graphs $\Phi$) that can be solved in $\Phi$: connected components, strongly connected components, directed path from one vertex to another, Eulerian trail (directed or not), bicolorability, and spanning tree with minimum average cost.

(GG, GD: Cov: Paths, Circ, Col: Alg)


(GN: M(Bases): Alg)

Charles E. Osgood & Percy H. Tannenbaum

1955a The principle of congruity in the prediction of attitude change. Psychological
Eiji O’Shima
See M. Iri and J. Shiozaki.

M.A. Osorio
See E.E. Vogel.

Patric R.J. Östergård
See F. Szöllősi.

M. Ostilli & J.F.F. Mendes

Katsuhiro Ota
See D. Archdeacon and A. Nakamoto.

Sang-Il Oum
See T. Huynh.

James G. Oxley
See also T. Brylawski, T. Fife, J. Geelen, J.P.S. Kung, and L.R. Matthews.


See Exer. 3.20.


Thm. 6.6.3: proof from Brylawski’s (1975a).

§10.3: Exer. 3 concerns the Dowling lattices of $\Gamma(q)$. §12.2: Exer. 13 concerns $G(\Omega)$.


Spikes (with tips) and swirls, important in matroid structure theory, are the lift (extended lift) and frame matroids of biased $2C_n$’s. Spikes: pp. 40–42, 72–74, 111–112, 197–202, 545–548, 568, 662, *et al*. Swirls: pp. 552, 568, 664, *et al*. [The biased-graph representation could simplify some of the descriptions.] [Annot. 7 Feb 2013.]

James Oxley, Dirk Vertigan, & Geoff Whittle
§5: Free swirls, $G(2C_n, \emptyset)$ ($n \geq 4$), mentioning their relationship to Dowling lattices, and complete free spikes, $L_0(2C_n, \emptyset)$. (GG: M)

Olayiwola O. Oyeleye & Mark A. Kramer

Extends the signed digraph of Iri, Aoki, O’Shima, and Matsuyama (1979a) “to account for complex dynamics”. [Annot. 17 Feb 2013.] (SD, VS: Appl, Alg)

Kenta Ozeki
See K. Kawarabayashi and S.-J. Kim.

M.L. Paciello
See M. Falcioni.

Manfred W. Padberg
See E.L. Johnson.

E. Padmavathy
See V.J.A. Cynthia.

Carles Padró
See A. Beimel.

Steven R. Pagano

Ch. 1: “Separability”. Graphical characterization of bias-matroid $k$-separations of a biased graph. Also, some results on the possibility of $k$-separations in which one or both sides are connected subgraphs. (GG: M: Str)

Ch. 2: “Representability”. The frame matroid of every signed graph is representable over all fields with characteristic $\neq 2$. For which signed graphs is it representable in characteristic 2 (and therefore representable over $GF(4)$, by the theorem of Geoff Whittle, A characterization of the matroids representable over $GF(3)$ and the rationals. *J. Combin. Theory Ser. B* 65 (1995), 222–261. MR 1358987 (96m:05046). Zbl 835.05015)? Solved (for 3-connected signed graphs having vertex-disjoint negative circles and hence nonregular matroid). There are two essentially different types: (i) two balanced graphs joined by three independent unbalanced digons; (ii) a cylindrical signed graph, possibly with balanced graphs adjoined by 3-sums. [See notes on Seymour (1995a) for definition of (ii) and for Lovász–Slilaty’s structure theorem in the case without vertex-disjoint negative circles.]

Furthermore, the representations of these graphs in characteristic not 2 are all canonical signed-graphic, while any representations over $GF(4)$ are canonical $Z_3$-gain graphic. (SG: M: Incid, Str, Top)

Ch. 3: “Miscellaneous results”. (SG: M: Incid, Str)

Igor Pak
See S. Chmutov.

Matteo Palassini & Sergio Caracciolo

Physical quantities on the \( \pm J \) cubic lattice model, i.e., a signed cubic lattice graph. [Annot. 28 Mar 2013.]

Matteo Palassini, Frauke Liers, Michael Juenger, & A.P. Young

\( \S\S II–III: \) Take a ground state \( \zeta_0 \). Add \( (\epsilon/\#E)\zeta_0(v_i)\zeta_0(v_j) \) to \( J_{ij} \), raising energy of \( \zeta_0 \) by \( \epsilon > 0 \) and of any other state \( \zeta \) by less, the amount depending on the edges whose signs differ in \( \zeta_0 \) and \( \zeta \) [i.e., negative edges in \( \Sigma^{oc} \)]. This may change the relative energies of states. See if a near-ground state becomes a ground state. [This interesting approach makes sense only for weighted \( \Sigma \).]

\( \S\S IV–V: \) Algorithm for frustration index (equivalently, weighted frustration index) of a weighted signed graph, tested on small cubic lattice graphs. It uses a branch-and-cut method that requires solving many linear programs. [This part makes sense for unweighted \( \Sigma \).]

Dictionary: \( J_{ij} = \) signed weight of edge \( e_{ij} \); “state” = vertex signing \( \zeta \); “energy” of \( \zeta = \) weighted frustration of \( (\Sigma^c, \#J) := \) total unsigned weight of \( E^-(\Sigma^c) \); “ground state” = any \( \zeta \) that minimizes energy; minimum energy = energy of ground state = weighted frustration index \( l(\Sigma, \#J) \); “free” or “periodic” boundary conditions = nontoroidal ([path]³) or toroidal ([circle]³). [Annot. 22 Dec 2014.]

Jeepamol J. Palathingal

Ch. 4: Signed Gallai graph. Ch. 5: Signed anti-Gallai graph. Cf. Palathingal and Lakshmanan (2017a), (20xxa).

Jeepamol J. Palathingal & Aparna Lakshmanan S

Gallai graph \( \Gamma(\Sigma): V(\Gamma) := E(\Sigma), ef \in E(\Gamma) \) when \( e, f \notin \) triangle, and three different sign functions modelled on \( \Lambda_{BC}, \Lambda_x, \Lambda_s \). Thms.: All forbidden induced subgraphs for each type (but the type is not determined by forbidden induced subgraphs). [Annot. 17 Dec 2019.]

Jeepamol J. Palathingal

Ch. 4: Signed Gallai graph. Ch. 5: Signed anti-Gallai graph. Cf. Palathingal and Lakshmanan (2017a), (20xxa).

Jeepamol J. Palathingal & Aparna Lakshmanan S

Gallai graph \( \Gamma(\Sigma): V(\Gamma) := E(\Sigma), ef \in E(\Gamma) \) when \( e, f \notin \) triangle, and three different sign functions modelled on \( \Lambda_{BC}, \Lambda_x, \Lambda_s \). Thms.: All forbidden induced subgraphs for each type (but the type is not determined by forbidden induced subgraphs). [Annot. 17 Dec 2019.]

(\( SG: \) LG(Gen): Str)
20xxa Forbidden subgraph characterizations of extensions of anti-Gallai graph operator to signed graph. Preprint.

Anti-Gallai graph $\Delta(\Sigma) := E(\Sigma)$, $ef \in E(\Delta)$ when $e,f \in$ triangle, and the three sign functions in (2017a). Thms.: The same kind as in (2017a). [Annot. 17 Dec 2019.] (SG: LG(Gen): Str)

Edgar M. Palmer
See F. Harary and F. Kharari.

B.L. Palowitch, Jr.
See M.A. Kramer.

Rong-Ying Pan
See Y.H. Chen.

Yongliang Pan
See Y.P. Hou.

Casian Pantea
See D. Angeli and G. Craciun.

Pietro Panzarasa
See V. Ciotti.

Giovanni Paolini
See also E. Delucchi.


The marked Dowling posets of Bibby and Gadish (2018a) and Delucchi–Girard–Paolini (2019a). (gg: M(Gen))

P. Paolucci
See S. Cabasino.

Gyula Pap


See (2008a). (GG: Str, Paths, Alg)


Given: $\Phi$ with gain group $G(\Omega)$, the symmetric group of a set $\Omega$, $A \subseteq V$, and $\omega : A \to \Omega$. An $A$-path is a path $P$ with endpoints $v, w \in A$ and internally disjoint from $A$; it is “returning” if $\omega(v) \varphi(P) = \omega(w)$. Thm. The largest number of disjoint returning $A$-paths equals the minimum, over all satisfied edge subsets $F$, of the maximum number of disjoint $[A \cup V(F)]$-paths in $\|\Phi\|\setminus F$. [For “satisfied” edges see Zaslavsky (2009a).] Generalizes and simplifies Chudnovsky, Geelen, et al. (2006a), which is the case where the gains act regularly and $\omega = constant.$ (GG: Str, Paths)

**Christos H. Papadimitriou**
See also E.M. Arkin and A.S. LaPaugh.

**Christos H. Papadimitriou & Kenneth Steiglitz**
See Ch. 10, Problems 6–7, p. 244, for bidirected graphs and flows in relation to the matching problem. (sg: Ori: Flows)


Apostolos N. Papadopoulos
See F.D. Malliaros.

Charis Papadopoulos
See H.L. Bodlaender.

**Konstantinos Papalamprou**
See also G. Appa and L.S. Pitsoulis.


**Konstantinos Papalamprou & Leonidas Pitsoulis**

A binary matroid is signed-graphic iff, for some copoint $H$, all the bridges of $H$ (in the sense of Tutte) are graphic aside from one that is signed-graphic (and possibly graphic). [Annot. 5 Feb 2010.] (SG: M, Str)

Frame matroids $G(\Sigma)$ where all copoints are graphic matroids. (SG: M)

**Konstantinos Papalamprou, Leonidas S. Pitsoulis, & Eleni-Maria E. Vretta**
20xxa On characterizing the class of cographic signed-graphic matroids. Submitted.
(SG: M)

G. Pardella & F. Liers
(sg: Fr: Alg)

Eduardo G. Pardo, Mauricio Soto, & Christopher Thraves
(SG: Geom: Alg)

Ojas Parekh
See E.G. Boman.

Angelo Parente
See F. Marinelli.

Giorgio Parisi
See also S. Cabasino, S. Caracciolo, B. Coluzzi, M. Falcioni, L.A. fn dez, E. Marin nari, and M. Mézard.

Relatively (a careful word) simple explanation of physics of “Ising spin glasses” = spin states on signed graphs, and of the replica method for studying them. [Annot. 9 Aug 2018.]
(sg: Phys, Fr: Exp)

§5.3, “Spin glasses”: Friendly treatment of balance and frustration in signed graphs with a distinct physics slant, e.g., asymptotic behaviors. [Annot. 9 Aug 2018.]
(SG: Bal, Fr, Phys: Exp)

Physics on hypercube $Q_D$ with complex unit gains $\varphi$ ($\varphi$ is a “$U(1)$ gauge field”). Spins $\zeta(u)$ can be (i) complex units or (ii) Gaussian random complex numbers, or (iii) $\zeta$ can be a unit vector $\in \mathbb{C}^n$; mainly, (ii). Assumed: each square (“plaquette”) $C_{\alpha,\beta}$ (with vertices $x, x+e_{\alpha}, x+e_{\alpha}+e_{\beta}, x+e_{\beta}, x$ for any $x \in V(Q_D)$) has gain $e^{iB\sigma_{\alpha,\beta}}$ in a fixed orientation, where $\sigma_{\alpha,\beta} \in \{+1, -1\}$ determines which orientations have gains $e^{iB}$ and $e^{-iB}$. $B = 0$ gives balance; $B = \pi$ gives all plaquette gains $-1$ (full frustration). If $D \leq 3$, but not if $D > 3$, the choices of $\sigma$ are equivalent by switching in the gain group $\mathbb{C}^\times$. The statistics of random $\sigma$ are investigated. [Annot. 19 Jun 2012.]
(Phys, gg)

Complex unit gain graphs. The Hamiltonian is the quadratic form $\bar{z}A(\Phi)z$. [Annot. 12 Aug 2012.]

**Boram Park**

See J.S. Huh.

**Jin-Woo Park**

See also S.-G. Lee.

**Jin-Woo Park & Sung-Soo Pyo**


(QM: SD, bal, sw)

**Se-Won Park**

See S.-G. Lee.

**Antonio Parravano, Ascensión Andina-Díaz, & Miguel A. Meléndez-Jiménez**


(SG: Bal, Dyn)

**M. Parvathi**


They are the special case of Bloss (2003a) where $\mathcal{G} = \{+,-\}$. [Annot. 21 Mar 2011.]

**M. Parvathi & M. Kamaraj**


The algebra is generated by multiplying two-layer signed graphs (“Brauer graphs”). In the product $\Sigma_1\Sigma_2$ the bottom layer of $\Sigma_1$ cancels with the top layer of $\Sigma_2$ using edge-sign product. (Signs are represented by arrows $[\!\!]$.) [Annot. 5 Jun 2012.]


**M. Parvathi & D. Savithri**

M. Parvathi & C. Selvaraj


M. Parvathi & B. Sivakumar


M. Parvathi, B. Sivakumar, & A. Tamilselvi

M. Parvathi & A. Tamilselvi


Sukanta Pati
See R.B. Bapat, S. Barik, and D. Kalita.

Vaia Patta
See C. Heunen.

Philippa Pattison

Ch. 8, pp. 258–9: “The balance model. The complete clustering model.” They are embedded in a more general framework.

(SG, Sgnd: Adj, Bal, Clu: Exp)

Laura Patuzzi
See M.A.A. de Freitas.

G.A. Patwardhan
See B.D. Acharya and M.K. Gill.
Debdas Paul
See S. Kirkland.

Soumyajit Paul
See S. Das.

[Viji Paul]
See Viji Paul (under ‘V’).

Loïc Paulevé & Adrien Richard


Vern I. Paulsen
See B.G. Bodmann and R.B. Holmes.

Payal & Sangita Kansal
20xxa Analysis of signed Petri net. Submitted. (SG: Incid)


20xxd Structural matrices for signed Petri net. Submitted. (SG: Incid)

Charles Payan

See Benzaken, Hammer, and de Werra (1985a). (SGc)

Edmund R. Peay


Proposes an index of inclusterability for signed graphs and generalizes to edges weighted by a linearly ordered set. (SG, Gen: Clu: Fr(Gen))


Real-number edge weights; the value of a path is the minimum absolute weight. [Annot. 11 Sept 2010.] (WG)


See mainly §3: “Structural consistency.” (sd: Gen: Bal, Clu)

Luke Pebody
See B. Bollobás.

Timothée Pecatte
See L. Isenmann.
Elisabeth Pécou
See M. Domijan.

Britta Peis
See W. Hochstädtler and M. Lätsch.

David B. Peizer
See P.J. Runkel.

Uri N. Peled

Martin Pelikan & Alexander K. Hartmann


Marcello Pelillo
See R. Glantz.

R.A. Pendavingh & S.H.M. van Zwam

Dowling’s (1973b), (1973a) $Q_n(GF(q)^\times)$ is an example. [Annot. 1 Sept 2017.] (M: gg)


Irena Penev
See V. Boncompagni.

Di Peng, Xiangbai Gu, Yuan Xu, & Qunxiong Zhu

Francisco Pereira
See A.J. Hoffman.

Mercedes Pérez Millán
See A. Dickenstein.

Kavita S. Permi
See P.S.K. Reddy.

F. Peruggi
See A. Coniglio.
See F. Belardo.

M. Petersdorf


Treats signed $K_n$’s. Satz 1: \[ \max_{\sigma} l(K_n, \sigma) = \lfloor \frac{(n - 1)^2}{4} \rfloor \] with equality iff $(K_n, \sigma)$ is antibalanced. [From which follows easily the full Thm. 14 of Abelson and Rosenberg (1958a).] Also, some further discussion of antibalanced and unbalanced cases. [For extensions of this problem see notes on Erdős, Győri, & Simonovits (1992a).]

SG: Fr

Ion Petre

See A. Alhazov and T. Harju.

Rossella Petreschi

See also T. Calamoneri.

Rossella Petreschi & Andrea Sterbini


SG: Fr

[Rossella Petreschim & Andrea Sterbini]

Misprint for R. Petreschi & A. Sterbini.

Norbert Peyerimhoff

See C. Lange and S.-P. Liu.

Nathan Pflueger


SG: Fr

Lan Anh Pham

See R. Naserasr.

Geevarghese Philip, Ashutosh Rai, & Saket Saurabh


“Pseudoforest” [or 1-forest] = independent set in the bicircular matroid $G(\Gamma, \emptyset)$. Problem: Can $\Gamma \setminus (\leq k$ vertices) be a pseudoforest? Generally, “$l$-pseudoforest” = forest $+ l$ edges. Problem: Can $\Gamma \setminus (\leq k$ vertices) be an $l$-pseudoforest? [Annot. 22 Dec 2017.]

bic: Fr

J.L. Phillips


Proposes to measure imbalance of a signed (di)graph by largest eigenvalue of a matrix close to $I + A(\Sigma)$. (Cf. Abelson 1967a.) Possibly,
means to treat only graphs that are complete aside from isolated vertices.

Nancy V. Phillips
See F. Glover.

[Alberto Del Pia]
See A. Del Pia.

Jean-Claude Picard & H. Donald Ratliff
A minor application of signed switching to a weighted graph arising from an integer linear program. (sg: sw)

Théo Pierron
See F. Dross and F. Foucaud.

Marcin Pilipczuk
See M. Cygan.

Michał Pilipczuk
See M. Cygan.

P. Pincus
See S. Alexander.

Alexandre Pinlou
See P. Ochem.

Shariefuddin Pirzada
See also M.A. Bhat and T. Shamsher.

S. Pirzada & Mushtaq A. Bhat

S. Pirzada & F.A. Dar

S. Pirzada, Muhammad Ali Khan, & E. Sampathkumar
\[ \sigma \text{ bipartitions } V(\Gamma) \text{ as } V_{\text{odd}} \cup V_{\text{even}} \text{ by parity of } d^-(v). \] Coloring = properly coloring \( \Gamma_c := \Gamma:V_{\text{odd}} \cup \Gamma:V_{\text{even}}. \) [Why not an arbitrary bipartition?]
Signs seem superfluous. Results: chromatic polynomial [the usual one of $\Gamma_c$], examples like paths, circles. [Annot. 1 Apr 2019.] (SGc: Col)

S. Pirzada, T.A. Naikoo, & F.A. Dar

The set, as opposed to sequence, of net degrees [cf. Chartrand, Gavlas, Harary, and Schultz (1994a)] of a signed simple graph can be any finite set of integers. Also, the smallest order of a signed graph with given net degree set. (SG: ori: Invar)


Characterization of net degree sequences of signed, simple, bipartite graphs. [Annot. 15 Nov 2011.] (SG: ori: Invar)


Every finite set of integers is the signed degree set of some connected signed bipartite graph. [Annot. 10 Sept 2010.] (SG: ori: Invar)

S. Pirzada, Tahir Shamsher, & Mushtaq A. Bhat
2008b On ordering of minimal energies in bicyclic signed graphs. Submitted.
Among connected signed graphs with cyclomatic number 2 and order $n$, the 20 with least energy for $n \geq 30$ and the 16 for $n \geq 17$, via extensive computations. [Annot. 3 Feb 2021.] (SG: Adj: Eig)

Tomaž Pisanski
See also N. Basic, V. Batagelj, and F. Belardo.

Tomaž Pisanski & Primož Potočnik

Cryptic. Dictionary (my best guess): “signed edge” = oriented edge; “signed boundary walk” (of a face) = directed face boundary walk; “signature” = set of negative edges of an embedding; “switch” = negative (= orientation-reversing) edge of an embedding. (sg: Top)

Tomaž Pisanski & Jože Vrabec

Definition (see Pisanski, Shawe-Taylor, and Vrabec (1983a)), examples, superimposed structure, classification. (GG: Cov(Gen))

Tomaž Pisanski, John Shawe-Taylor, & Jože Vrabec

A graph bundle is, roughly, a covering graph with an arbitrary graph $F_v$ (the “fibre”) over each vertex $v$, so that the edges covering $e:vw$ induce
an isomorphism $F_v \rightarrow F_w$.  

Tomaž Pisanski & Arjana Žitnik


Leonidas S. Pitsoulis

See also G. Appa and K. Papalamprou.


Ch. 6, “Signed-graphic matroids”: Signed graphs; signed-graphic matroids; binary signed-graphic matroids; decomposition.  

Leonidas Pitsoulis, Konstantinos Papalamprou, Gautam Appa, & Balázs Kotnyek


Tour matrices of bidirected graphs are closed under 1-, 2-, and 3-sums. Possibly, every totally unimodular matrix is a tour matrix.  

Leonidas Pitsoulis & Eleni-Maria Vrette


Irene Pivotto

See also R. Chen, M. DeVos, and B. Guenin.


Erik Plahte, Thomas Mestl, & Stig W. Omholt


Michael Plancholt

See A.H. Busch, A.A. Diwan, and F. Harary. 

Andrey Ploskonosov

See Y. Burman. 

M.D. Plummer

See L. Lovász. 

Jorn van der Pol

See P. Nelson. 

Agnieszka Polak & Daniel Simson

2013a Algorithms computing $O(n, Z)$-orbits of P-critical edge-bipartite graphs and P-critical unit forms using Maple and C#. Algebra Discrete Math. 16 (2013),


Oskar E. Polansky
See I. Gutman.

Svatopluk Poljak
See also Y. Crama and B. Mohar.

Svatopluk Poljak & Daniel Turzík

Main Theorem: For a simple, connected signed graph of order \( n \) and size \( \#E = m \), the frustration index \( l(\Sigma) \leq \frac{1}{2}m - \frac{1}{4}(n - 1) \). The proof is algorithmic, by constructing a (relatively) small deletion set. Dictionary: \( \Sigma \) is an “edge-2-colored graph” \((G, c)\), \( E^+ \) and \( E^- \) are called \( E_1 \) and \( E_2 \), a balanced subgraph is “generalized bipartite”, and \( m - l(\Sigma) \) is what is calculated. [This gives an upper bound on \( D(\Gamma) := \max_{\sigma} l(\Gamma, \sigma) \) for a connected, simple graph, whereas Akiyama, Avis, Chvátal, and Era (1981a) has a lower bound on \( D. \)] (SG: Fr, Alg)


Generalizes (1982a), with application to signed graphs in Cor. 3. (SG: Fr, Alg)


The polytope \( P_B(\Sigma) \) (the authors write \( P_{BL} \)) is the convex hull in \( \mathbb{R}^E \) of characteristic vectors of balanced edge sets. It generalizes the bipartite subgraph polytope \( P_B(\Gamma) = P_B(-\Gamma) \) (see Barahona, Grötschel, and Mahjoub (1985a)), but is essentially equivalent to it according to Prop. 2: The negative-subdivision trick preserves facets of the polytope. Thm. 1 gives new facets, corresponding to certain circulant subgraphs. (They are certain unions of two Hamilton circles, each having constant sign.) (SG: Fr, Geom)

Further development of (1987a) for all-negative $\Sigma$. The import for general signed graphs is not discussed. [Developed more in Kaparis and Letchford (2018a).] (Par: Fr, Geom)

Svatopluk Poljak & Zsolt Tuza


Surveys max-cut and weighted max-cut [that is, max size balanced subgraph and max weight balanced subgraph in all-negative signed graphs]. See esp. §2.9: “Bipartite subgraph polytope and weakly bipartite graphs”. [The weakly bipartite classes announced by Gerards suggested that a signed-graph characterization of weakly bipartite graphs is called for. This is provided by Guenin (2001a).]

§1.2, “Lower bounds, expected size, and heuristics”, surveys results for all-negative signed graphs that are analogous to results in Akiyama, Avis, Chvátal, and Era (1981a) (q.v.), etc. [Problem. Generalize any of these results, that are not already generalized, to signed simple graphs and to simply signed graphs.] (par: Fr, tg(Sw): Exp, Ref)

Albert Pollatchek


Announcement of (1977a). (gg: aut, Algeb)


Y. Pomeau

See B. Derrida.

Dragos Popescu [Dragoş-Radu Popescu]

See D.-R. Popescu.

Dragoş-Radu Popescu [Dragos Popescu]


A signed $K_n$ is balanced or antibalanced or has a positive and a negative circle of every length $k = 3, \ldots, n$. For odd $n$, the signed $K_n$ if not balanced has at least $\frac{n-1}{2}$ negative Hamiltonian circles. For even $n$, $-K_n$ does not maximize the number of negative circles. A “circle basis” is a set of the smallest number of circles whose signs determine all circle signs. This is proved to have $\binom{n-1}{2}$ members. Furthermore, there is a basis consisting of $k$-circles for each $k = 3, \ldots, n$. [A circle basis in this sense is the same as a basis of circles for the binary cycle space. See

Ch. 1: “A-balance” (p. 91). Let $F$ be a spanning subgraph of $K_n$ and $A$ a signed $K_n$. The “product” of signed graphs is $\Sigma_1 \boxtimes \Sigma_2$ whose underlying graph is $|\Sigma_1| \cup |\Sigma_2|$, signed as in $\Sigma_i$ for an edge in only one $\Sigma_i$ but with sign $\sigma_1(e)\sigma_2(e)$ if in both. Let $G_F$ denote the group of all signings of $F$; let $G_F(A)$ be the group generated by the set of restrictions to $F$ of isomorphs of $A$. A member of $G_F(A)$ is “$A$-balanced”; other members of $G_F$ are $A$-unbalanced. We let $\hat{\Sigma}$ denote the coset class of $\Sigma$ and $\approx \Sigma$ the “isomorphism” of cosets induced by graph isomorphism, i.e., cosets are isomorphic if they have isomorphic members. Let $\hat{\Sigma}$ be the isomorphism class of $\Sigma$, $\hat{\Sigma}$ the isomorphism class of $\hat{\Sigma}$, and $\hat{\Sigma} : = \cup \hat{\Sigma}$. Now choose a system of representatives of the coset isomorphism classes, $R = \{\Sigma_1, \ldots, \Sigma_n\}$.

Prop. 1.4.1: Each $\hat{\Sigma}$ intersects exactly one $\Sigma_i$. Let $R_i = \{\Sigma_{i1}, \ldots, \Sigma_{in}\}$ be a system of representatives of $\hat{\Sigma}/\cong$, arranged so that $|E^-(\Sigma_{ij})|$ is a minimum when $j = 1$. This minimum value is the “line” index of $A$-imbalance” of each $\Sigma \in \hat{\Sigma}$ and is denoted by $\delta_A(\Sigma)$. (§2.1: Taking $A$ to be $K_n$ with one vertex star all negative makes this equal the frustration index $l(\Sigma)_i$.) Prop. 1.5.1: $\delta_A(\Sigma)$ is the least number of edges whose sign needs to be changed to make $\Sigma A$-balanced. Prop. 1.5.2: $\delta_A(\Sigma) = \#E^-(\Sigma)$ iff $\#(E^-(\Sigma) \cap E^-(F, \beta)) \leq \frac{1}{2}\#E^-(F, \beta)$ for every signing $\beta$ of $F$. Finally, for each $\Sigma \in G_F$ define the “$\Sigma$-relation” on coset isomorphism classes $\hat{\Sigma}_i$ to be the relation generated by negating in $\Sigma_1$ all the edges of $E^-(\Sigma)$, extended by isomorphism and transitivity. This is well defined (Prop. 1.6.1) and symmetric (Prop. 1.6.2) and is preserved under negation of coset isomorphism classes (Prop. 1.6.4, 1.6.5). Self-negative classes, such that $\hat{\Sigma} \approx -\hat{\Sigma}$, are the subject of Prop. 1.6.3.

Ch. 2: “Signed complete graphs” (p. 106). §2.5: “H-graphs”. If $H$ is a signed $K_n$, a “standard H-graph” $\Sigma$ is a signed $K_n$ such that $\Sigma \cong H^- \cup K_{n-h}$. Prop. 2.5.3. Assume certain hypotheses on $n$, $\#X_0$ for $X_0 \subseteq V(\Sigma)$, and a quantity $D^-(H)$ derived from negative degrees. Then $\#E^- = l(\Sigma)$ implies the induced subgraph $G,X_0$ is a standard $H$-graph with $\#E^-(\Sigma,X_0) = l(\Sigma,X_0)$. The cases $H^- = K_1, K_2$, and a 2-edge path are worked out. For the former, Prop. 2.5.3 reduces to Sozański’s (1976a) Thm. 3.

Ch. 3: “Frustration index” (p. 158). Some upper bounds.

Ch. 4: “Evaluations, divisibility properties” (p. 174). Similar to parts of (1996a) and Popescu and Tomescu (1996b).

Ch. 5: “Maximal properties” (p. 198). §5.1: “Minimum number and maximum number of negative stars, resp. 2-stars”. §5.2 is a special case of Popescu and Tomescu (1996a), Thm. 2. §5.3: “On the maximum number of negative cycles in some signed complete graphs”. Shows that Conjecture 1 is false for even $n \geq 6$. Some results on the odd case.

Conjecture 1 (Tomescu). A signed complete graph of odd order has...
the most negative circles iff it is antibalanced. (Partial results are in §5.3.) [This example maximizes $l(\Sigma)$. A somewhat related conjecture is in Zaslavsky (1997b).] Conjecture 2. See (1993a). Conjecture 3. Given $k$ and $m$, there is $n(k, m)$ so that for any $n \geq n(k, m)$, a signed $K_n$ with $m$ negative edges has (a) the most negative $k$-circles iff the negative edges are pairwise nonadjacent; (b) the fewest iff the negative edges form a star.

(SG: Bal(Gen), KG, Fr, Enum: Circ, Paths)


Conjecture. An unbalanced signed complete graph has the minimum number of negative circles iff its frustration index equals 1. [This has been proved.] (SG: Circ, Fr)


The numbers of negative subgraphs, especially circles and paths of length $k$, in an arbitrarily signed $K_n$. Complicated formulas; divisibility and congruence properties. Extends part of Popescu and Tomescu (1996a).

(SG: KG, Enum: Circ, Paths)


A generalization of signed-graph frustration index. Let $\mathcal{F} \subseteq \mathcal{P}(E)$; let $
abla_{\mathcal{F}}(S) := \min \{ |S \ominus F| : F \in \mathcal{F} \}$ and similarly $\nabla_{\mathcal{F}}(S) = l(\Sigma|S)$. [Annot. 3 Oct 2014.] (SG: Bal, Gen)


Similar to (1999a).

(SG: Fr)


Similar to (1999a) but [cf. MR] with improved results. (SG: Fr)

Dragoş-Radu Popescu & Ioan Tomescu


The number $c_p^-$ of negative circles of length $p$ in a signed $K_n$ with $s$ negative edges. Thm. 1: For $n$ sufficiently large compared to $p$ and $s$, $c_p^-$ is minimized if $E^-$ is a star (iff, when $s > 3$) and is maximized iff $E^-$ is a matching. Thm. 2: $c_p^-$ is divisible by $2^{p-2-\lfloor \log_2(p-1) \rfloor}$. Thm. 3: If $s \sim \lambda n$ and $p \sim \mu n$ and the negative-subgraph degrees are bounded (this is essential), then asymptotically the fraction of negative $p$-circles is $\frac{1}{2}(1 - e^{-4\lambda \mu})$. [Kittipassorn & Mészáros (2015a) performs a detailed study of the number of negative triangles.] (SG: KG: Fr, Enum: Circ)

A much earlier version of (1996a) with delayed publication. Contains part of (1996a): a version of Thm. 1 and a restricted form of Thm. 3.

(SG: KG: Fr, Enum: Circ)

L. Pósa
See P. Erdős.

Olaf Post
See C. Lange.

Luke Postle
See Z. Dvořák.

Alexander Postnikov
See also F. Ardila.


§4.2 mentions the lift matroid of \(\{1\}\vec{K}_n\), i.e., the integral poise gains of a transitively oriented complete graph, represented by the Linial arrangement. [See also Stanley (1996a).]

(Alexander Postnikov & Richard P. Stanley)


The arrangements are the canonical affine-hyperplane lift representations of certain additive real gain graphs. Characteristic polynomials of the former, equalling zero-free chromatic polynomials of the latter, are calculated. And much more.

(GG: Geom, M, Invar)

J. Poulter
See also A. Aromsawa, J.A. Blackman, and J.R. Goncalves.

J. Poulter & J.A. Blackman


(Phys: sg: Fr)


(Phys: sg: Fr, State)

Swathy Prabhu [Swathyprabhu Mj]
See Swathyprabhu.

U.M. Prajapati & K.K. Raval

More examples as in Baskar Babujee and Loganathan (2011a).
The Electronic Journal of Combinatorics #DS8

[Pranjali]
See P. Sharma.

Viktor K. Prasanna
See A. Srivastava.

Philips Kokoh Prasetyo
See D. Lo.

B. Prashanth
See P.S.K. Reddy.

Primož Potočnik
See also T. Pisanski.

Primož Potočnik & Mateja Šajna

Γ is “almost self-complementary” if Γ ≅ K_n \ M \ Γ where M is a perfect matching in K_{2n} \ Γ. Such graphs are double coverings (Δ, σ) that are ≅ (Δ^c, σ^c) for some σ^c. Dictionary: “Z_2-voltage assignment” = signature. [Annot. 22 Aug 2013.]


Annie K. Powell
See K.C. Mondal.

Julia Preusse
See J. Kunegis.

K.O. Price, E. Harburg & T.M. Newcomb
1966a Psychological balance in situations of negative interpersonal attitudes. J. Personality Social Psychol. 3 (1966), 265–270. (PsS)

Noah Prince
See H. Liu.

Geert Prins
See F. Harary.

Ion Prisecaru
See K.C. Mondal.

Sharon Pronchik
See L. Fern.

James Propp

Anton V. Proskurnikov and Ming Cao
Anton Proskurnikov, Alexey Matveev, & Ming Cao

Anton V. Proskurnikov & Roberto Tempo


(PsS: SD, WG: Bal, Dyn: Exp)

Andrzej Proskurowski
See A.M. Farley.

Alexandre Proutiere
See G.-D. Shi.

J. Scott Provan


§4: “Determinacy in a class of network models.” [Fig. 1 and Thm. 4.7 hint at a possible digraph version of the signed-graph or gain-graph frame matroid.]

(Teresa M. Przytycka & Józef H. Przytycki

Generalizing concepts from Kauffman (1989a). [See also Traldi (1989a) and Zaslavsky (1992b).]


A “chromatic graph” is a graph with edges weighted from the set \( Z \times \{d,l\} \), \( Z \) being [apparently] an arbitrary set of “colors”. A “dichromatic graph” has \( Z = \{+,−\} \). Such graphs have general dichromatic polynomials [see Przytycka and Przytycki (1988a), Traldi (1989a), and Zaslavsky (1992b)], as [partially] anticipated by Fortuin and Kasteleyn (1972a). I will not attempt to summarize this paper.

(Józef H. Przytycki
See K. Murasugi and T.M. Przytycka.)
Vlastimil Ptak
See M. Fiedler.

Charles J. Puccia & Richard Levins
(SD: QM: QSta: Cyc)

P. Simin Pulat
Decomposing an $R_{>0}$-gain graph. [Annot. 8 Jan 2016.]
(gg: Alg)

William R. Pulleyblank
See J.-M. Bourjolly and M. Grötschel.

G.N. Purohit & Ritu Rani Meherwal
(SG: WG: Eig, Alg)

L. Pushpalatha
See E. Sampathkumar.

L. Pyber
See L. Lovász.

Sung-Soo Pyo
See J.-W. Park.

Hao Qi
Abstract. Fix $\Gamma$ and $S \subseteq S_k$. $\Gamma$ is “$S$-colorable” if every $S$-permutation gain function $\varphi$ makes $(\Gamma, \varphi)$ [$k$]-colorable. “Critical” means $\forall S' \supseteq S$, $\exists \Gamma$ that is $S$- but not $S'$-colorable. Cf. Jin, Wong, and Zhu (20xxa). [Annot. 29 Jul 2019.]
(GG, SG: Col)

Hao Qi, Tsai-Lien Wong, & Xuding Zhu

Jian Qi
See S.W. Tan.

Xingqin Qi, Huimin Song, Jianliang Wu, Edgar Fuller, Rong Luo, & Cun-Quan Zhang
(SG: Clu: Alg)

Jianguo Qian

Yi Qian & Sibel Adalı

**Tianyong Qiang**  
See B. Jiao.

**Hongxun Qin**  
See also J.E. Bonin, P. Brooksbank, T. Dowling, and D.C. Slilaty.

These matroids are determined by their Tutte polynomials, except that only the order of the group can be determined. (gg: M: Incid)

**Hongxun Qin, Daniel C. Slilaty, & Xiangqian Zhou**  
The complete list of 31 forbidden minors that are regular matroids. [Annot. 10 Sept 2010.] (SG: M: Str)

**Li Qiu**  
See W. Chen.

**Lihong Qiu**  
See Wang, Qiu, Qian, and Wang (2020a).

**Wen-Yuan Qiu**  
See G. Hu.

**Hui Qu**  
See G.-H. Yu.

**Rachel Quinlan**  
See C. O’Brien.

**Louis V. Quintas**  
See M. Gargano.

**James P. Quirk**  
See also L. Bassett and J.S. Maybee.


Comments by W.M. Gorman (pp. 175–189) and Eli Hellerman (pp. 191–192). Discussion: see pp. 193–196. (QM: QSta: sd, bal: Exp)

**James Quirk & Richard Ruppert**

Nicole Radde


Nicole Radde, Nadav S. Bar, & Murad Banaji

Filippo Radicchi, Daniele Vilone, & Hildegard Meyer-Ortmanns

Filippo Radicchi, Daniele Vilone, Sooeyon Yoon, & Hildegard Meyer-Ortmanns

Marko Radovanović
See P. Aboulker.

Fahimeh Rahimi


Mourad Rahmani


Ashutosh Rai
See G. Philip.

W.M. Raike
See A. Charnes.

M.A. Rajan
See P. Balamuralidhar and H.K. Rath.

K.R. Rajanna
See P.S.K. Reddy.
R. Rajendra
See also P.S.K. Reddy.

R. Rajendra, P. Siva Kota Reddy, & V.M. Siddalingaswamy
Very elementary. [Annot. 19 Jan 2020.] (SG: Algeb)

M.R. Rajesh Kanna, R. Jagadeesh, & B.K. Kempegowda

M.R. Rajesh Kanna, R. Pradeep Kumar, & Mohammad Reza Farahani

M.R. Rajesh Kanna, R. Pradeep Kumar, & R. Jagadeesh
From Γ define Σ as in Adiga, Sampathkumar, et al. (2013a). Fix C a minimum vertex cover of Γ; let $I_C$ have 1 in $(i,i)$ for $v_i \in C$, otherwise 0. Minimum covering color energy $E^C := \text{energy of } A(\Sigma) + I_C$. [Annot. 22 Dec 2018.] (sg: Adj: Eig)

See also P.R. Hampiholi.

Harishchandra S. Ramane, Mahadevappa M. Gundloor, & Sunilkumar M. Hosamani

Harishchandra S. Ramane, Ivan Gutman, & Mahadevappa M. Gundloor

Nacim Ramdani, Nacim Meslem, & Yves Candau

Fahimeh Ramezani
See G. Iacono and N. Soranzo.

Farzaneh Ramezani
See also Y. Bagheri.


Farzaneh Ramezani, Peter Rowlinson, & Zoran Stanić

Eigenvalue \( \lambda \neq 0, \pm 1 \) of multiplicity \( m \) satisfies \( n \leq \binom{n-m+2}{3} \). [Annot. 30 May 2020.] (SG: Adj, Eig)


More on signed graphs with at most three eigenvalues. Submitted. (SG: Adj, Eig)

José L. Ramírez
See M.A. Méndez.

A.J. Ramírez-Pastor [Antonio José Ramírez Pastor]
See also F. Romá.

A.J. Ramírez-Pastor, F. Nieto, S. Contreras, & E.E. Vogel

A.J. Ramírez-Pastor, F. Nieto, & E.E. Vogel

Randomly signed square lattice with half positive and half negative edges plus randomly signed second-neighbor edges, added according to various schemes and calculated for random examples. Compares properties to 3-dimensional cubic and planar lattices, in particular to the pure square lattice in Vogel, Cartes, Contreras, Lebrecht, and Villegas (1994a) (q.v. for dictionary). [Annot. 3 Jan 2015.] (SG, Phys: Fr, Sw)

A.J. Ramirez-Pastor, F. Romá, F. Nieto, & E.E. Vogel

R. Rammal
See F. Barahona and I. Bieche.

K. Ranganathan
See R. Balakrishnan.

R. Rangarajan
See also P.S.K. Reddy.

R. Rangarajan & P. Siva Kota Reddy

\( S_n \) is a symmetric \( n \)-signed graph. Further definitions as in the notes to Sampathkumar, Reddy, and Subramanya (2008a), (2010c). §2, “Balance in an \( n \)-sigraph \( S_n = (G, \sigma) \).” Prop. 1 (generalizing Harary (1953a) for signed graphs): \( S_n \) is balanced iff for each pair \( u, v \in V \), every \( uv \)-path...
has the same gain. [The simple proof of \( \implies \), which depends on the fact that the gain group has exponent 2, is the best I have seen. The proof of \( \iff \) is incorrect.] Prop. 4: \( \Sigma_{S_n} \) is balanced iff \( V = V_1 \cup V_2 \) such that an edge has identity gain iff it lies within \( V_1 \) or \( V_2 \). Good proof via min as defined in the cited notes. §3, “Clustering in an n-sigraph \( S_n = (G, \sigma) \).” \( S_n \) is “clusterable” if \( V \) has a partition \( \pi \) such that an edge has identity gain iff it lies within a part of \( \pi \). Prop. 5 generalizes Davis (1967a) to \( n \)-signed graphs. §3.1: “Local balance (Local \( i \)-balance) in an \( n \)-sigraph \( S_n = (G, \sigma) \).” Prop. 6 generalizes Harary (1955a) on local balance [with a good proof]. Prop. 8: A complete \( S_n \) is balanced iff every triangle on one vertex is balanced. Prop. 9 [incorrect]: The same for imbalance. Prop. 10 gives the number of balanced \( S_n = (K_k, \sigma) \) [incorrect; the correct value is \( 2^\left\lceil k/2 \right\rceil (n-1) \)]. [The results are equally true, \emph{mutatis mutandis}, without assuming symmetry.] [Minor typos require correction.] [Annot. 9 July 2009.]
R. Rangarajan, P. Siva Kota Reddy, & M.S. Subramanya

Continuation of Reddy, Vijay, and Lokesha (2009a), (2010a). Definitions as at Sampathkumar, Reddy, and Subramanya (2008a). Prop. 4 characterizes $C_E(\Phi)$. Solved: $\Lambda_S(\Phi) \simeq \Phi$; $\Phi \simeq \Lambda_S(\Phi)$; $\Lambda_S(\Phi) \simeq C_E(\Phi)$; $J(\Phi) \simeq C_E(\Phi)$. [The results remain true without assuming symmetry.] [Annot. 2 Aug 2009.]

R. Rangarajan, M.S. Subramanya, & P. Siva Kota Reddy

$H$ is a connected graph of order $\geq 3$. $HL(\Sigma) \subseteq \Lambda_S(\Sigma)$ (defined at Sampathkumar, Reddy, and Subramanya (2010c)); $ef \in E(\Lambda_S(\Sigma))$ is in $HL(\Sigma)$ iff $e, f$ are in a copy of $H$ in $|\Sigma|$. $\Sigma'$ is an $HL(\Sigma)$ iff it is balanced and $|\Sigma'|$ is an $H$-line graph. Solved: $HL(\Sigma) \cong \Sigma$ for $H = C_k, P_k, K_r L(\Sigma) \simeq \Lambda_S(\Sigma)$. Connections with graphs derived from matrices. [Annot. 7 Jan 2011.]


Definitions as at Sampathkumar, Reddy, and Subramanya (2008a), (2010c). The neighborhood signed graph or 2-path graph $P_2(\Sigma)$ is $(V, E_2, \sigma^c)$ where $E_2 := \{vw : \exists vw$-path of length 2}. Thm. 5: $P_2(\Sigma)$ is balanced and the signature can be any balanced signature (by appropriate choice of $\sigma$). Solved: $\Sigma, P_2(\Sigma) \simeq \Sigma; P_2(\Sigma) \simeq \Sigma^c; P_2(\Sigma) \simeq \Lambda_S(\Sigma)$. For connected $\Sigma$: $P_2(\Sigma) \simeq \Lambda_S(\Sigma); P_2(\Sigma) \simeq J_S(\Sigma)$. Also, $P_2(\Sigma) \simeq \Lambda_S(\Sigma)$ when $|\Sigma|$ is unicyclic with circle length $l$ and $r, s < l/2$. [Annot. 2 Aug 2009.]

§5, “$(-1, 0, 1)$-Matrices and neighborhood signed graphs”: Given a $(-1, 0, 1)$-matrix $A$ with columns $a_1, \ldots, a_n$. Let $V_A := [n], E_A := \{ij : (\exists k) a_k a_{kj} \neq 0\}$, and $\sigma_A(ij) := \mu_i \mu_j$ where $\mu_i :=$ product of nonzero entries in $a_i$. Thm. 20: This signed graph of $A(\Sigma)$ is $P_2(\Sigma)$. [Annot. 10 Apr, 2 Aug 2009.]

Athira P Ranjith & Joseph Varghese Kureethara

“Sum signed graph” means $f(uv) = \text{sgn}(n + \frac{1}{2} - f(u) - f(v))$ for some bijection $f: [n] \rightarrow V$. Thms.: $\exists$ negative edge [if $n > 1$]. rna number (cf. Acharya and Kureethara (20xxa)) $\text{rna}(\Sigma) := \min_{f, \sigma} \#E^f = 1$ for path and star, 2 for circle, $\lfloor n^2/4 \rfloor$ for $K_n$. [Annot. 13 Dec 2020.

A.R. Rao
Angeline Rao
See V. Chen.

Anita Kumari Rao
See D. Sinha.

M.R. Rao
See Y.M.I. Dirickx.

S.B. Rao
See also B.D. Acharya, P. Das, and [G.R.] Vijaya Kumar.


A complicated solution, with a polynomial-time algorithm, to the problem of characterizing consistency in vertex-signed graphs (cf. Beineke and Harary (1978b)). Thm. 4.1 points out that graphs with signed vertices and edges can be easily converted to graphs with signed vertices only; thus harmony in graphs with signed vertices and edges is characterized as well. [This paper was independent of and approximately simultaneous with B.D. Acharya (1983b), (1984a).] [See Joglekar, Shah, and Diwan (2010a) for the last word.] (SG, VS: Bal, Alg)

S.B. Rao, B.D. Acharya, T. Singh, & Mukti Acharya

Extended abstract without proofs. “Graceful” means $(1,1)$-graceful, $r = 1$, as at M. Acharya and Singh (2004a). Thm. 1: $(K_n,\sigma)$ is graceful iff $n \leq 3$, $n = 4$ and $\#E^- \neq 3$, or $n = 5$ and $\#E^- \neq 5$ is odd and neither $\Sigma^+$ nor $\Sigma^-$ is $K_{1,3}$. The proof involves a recursive labelling procedure. [Annot. 21 July 2010.] (SGc: Lab)

S.B. Rao, N.M. Singhi, & K.S. Vijayan

These are the minimal forbidden induced subgraphs for an all-negative signed simple graph to be the reduced line graph of a signed graph. (sg: LG, par)

Vasant Rao
See M. Desai.

Anatol Rapoport

§3.3, “The theory of structural balance”. Nontechnical exposition. [An-
A.M. Rappoport
See Ya.R. Grinberg.

Thomas Raschle & Klaus Simon
Expounded by Mahadev and Peled (1995a), §8.5 (q.v.). (par: ori, Alg)

Andre Raspaud & Xuding Zhu
See also E. Mácajová and J. Nešetřil.
Thm. 1: \(\Sigma\) has a nowhere-zero integral and circular [i.e., real] 4-flow if it is edge 4-connected. It has a nowhere-zero circular \(r\)-flow with \(r < 4\) if it is edge 6-connected. A signed cut \(D\) [cf. Chen and Wang (2009a)] is described by a signed subset \(X = X^+ \cup X^-\) of \(V\). Lemma 3: \(\Sigma\) has a circular \(r\)-flow iff it has an orientation such that \(1/(r - 1) \leq \#\partial^+(X)/\#\partial^-(X) \leq r - 1\) for every \(X\). Here \(\partial^f(X)\) is the set of ends in \(X\), of \(e \in D\), that have a certain sign. [Annot. 23 March 2010.]
(SG: Ori, Flows)

Dieter Rautenbach & Bruce Reed
The smallest sufficient connectivity in Thomassen (2001a) is about 976\(k\). [For more, see Hochstättler, Nickel, and Peis (2006a).]
(par: Fr: Circ)

Hemant Kumar Rath, M. Rajan, & P. Balamuralidhar
(SD: Appl)

H. Donald Ratliff
See J.-Cl. Picard.

K.K. Raval
See U.M. Prajapati.

Bertram H. Raven
See B.E. Collins.

E.V. Ravve
See E. Fischer.

D.K. Ray-Chaudhuri, N.M. Singhi, & G.R. Vijayakumar
(SG: Eig, Geom: Exp)

Abigail Raz
See J. Brown and D. Mallory.
Igor Razgon
See G. Gutin.

Margaret A. Readdy
See also R. Ehrenborg.


S. Redner
See T. Antal.

A. Sashi Kanth Reddy

P. Siva Kota Reddy
See also V. Lokesha, R. Rajendra, R. Rangarajan, E. Sampathkumar, and M.S. Subramanya.


[Smarandache is irrelevant.] Cf. Sampathkumar, Reddy, and Subramanya (2010b). (SG: Sw, LG(Gen))

20xxa Switching invariant $t$-path sigraphs. Submitted. (SG: Sw, LG(Gen))

20xxb A note on characterization of jump signed graphs. Submitted. (SG: LG)

P. Siva Kota Reddy, M.C. Geetha, & K.R. Rajanna

See definitions at Sampathkumar, Reddy, and Subramanya (2008a), (2010c). The antipodal graph $A(\Phi)$ has $V(A) := V$, $E(A) := \{uv : d(u, v) = \max\}$, $\sigma_A(uv) = \mu_\varphi(u)\mu_\varphi(v)$ where $\mu_\varphi$ = canonical vertex labelling. Solved: $A(\Phi) \simeq \Phi$ [trivial], $A(\Phi) \simeq \Phi^c$, etc. [elementary]. [Cors. 3.2, 3.3 are wrongly stated.] [Annot. 13 Jul 2013.] (SG(Gen): Sw)


P. Siva Kota Reddy, P.S. Hemavathi, & R. Rajendra
Radial digraph of \((D, \sigma)\) with certain balanced signs. [Annot. 22 Jan 2020.]

P. Siva Kota Reddy & V. Lokesha

P. Siva Kota Reddy, V. Lokesha, B. Prashanth, & S. Vijay

P. Siva Kota Reddy, V. Lokesha, & Gurunath Rao Vaidya

P. Siva Kota Reddy & U.K. Misra

P. Siva Kota Reddy, U.K. Misra, & P.N. Samanta

P. Siva Kota Reddy, U.K. Misra, & P.N. Samanta
2013a The minimal equitable dominating signed graphs. *Scientia Magna* 9 (2013), no. 4, 64–70.

The intersection graph of minimal equitable dominating sets $P$ of $|\Sigma|$, with $\text{sgn}(PP') = \mu_\sigma(P)\mu_\sigma(P')$. [Results are trivial or (Prop. 3.1) wrong.]

Dictionary: “complement” of $\Sigma$ is $(|\Sigma|, \mu'_\sigma)$ where $\mu'_\sigma(uv) = \mu_\sigma(u)\mu_\sigma(v)$.

[Annot. 4 Oct 2019.]

P. Siva Kota Reddy, K.M. Nagaraja, & M.C. Geetha


P. Siva Kota Reddy, K.M. Nagaraja, & V.M. Siddalingaswamy


$E_k(\Sigma)$ has $V_k = E(\Gamma)$, $e,f \in E_k$ iff $e,f$ are adjacent or opposite in a $C_k$, $\sigma_k(e,f) = \sigma(e)\sigma(f)$. Generalizes $k = 3, 4$ Subramanya–Reddy (2009a), Rangarajan–Reddy (2010a). Thm. 7: $\Sigma \sim E_k(\Sigma)$ iff $\Sigma = \text{balanced } C_n, n \geq 5$. Minor graph equations solved. [Annot. 2 Jul 2019.]

P. Siva Kota Reddy & Kavita S. Permi

2014a Signed graph equations: $N(\Sigma) \sim CMD; CMD(\Sigma) \sim MD(\Sigma); MD(\Sigma) \sim L(\Sigma)$. *Bull. Int. Math. Virtual Institute* 4 (2014), 27–35.

$N = \text{neighborhood signed graph (Rangarajan, Subramanya, and Reddy (2012a)), (C)MD = (common) minimal dominating signed graph (Reddy and Prashanth (2012a), (2013a)).} [\text{Annot. 9 Apr 2020.}]

P. Siva Kota Reddy, Kavita S. Permi, & K.R. Rajanna


P. Siva Kota Reddy & B. Prashanth


Continuation of Rangarajan, Reddy, and Subramanya (2009a). Definitions as at Sampathkumar, Reddy, and Subramanya (2008a). Solved for an $n$-signed graph $\Phi$: $\Lambda_s(\Phi) \sim \Phi^c; \Lambda_S(\Phi) \sim \Phi^c$. [The results remain true without assuming symmetry.]


P. Siva Kota Reddy, B. Prashanth, & T.R. Vasanth Kumar

P. Siva Kota Reddy, B. Prashanth, & V. Lokesha

[The name Smarandache is used for no reason.] (SG)

P. Siva Kota Reddy, B. Prashanth, & V. Lokesha

[The name Smarandache is used for no reason.] (SG)

P. Siva Kota Reddy, B. Prashanth, & Kavita S. Permi

The antipodal graph $A$ of $|\Sigma|$ with $\sigma_A(uv) = \mu_\sigma(u)\mu_\sigma(v)$, hence balanced. Solved: $\Sigma \sim (A, \sigma_A)$, $-\Sigma \sim (A, \sigma_A)$. [The name Smarandache is used for no reason.]

(Annot. 3 Sep 2019.) (SG: LG(Gen))

P. Siva Kota Reddy, B. Prashanth, & M. Ruby Salestina

[The name Smarandache is used for no reason.] (SD)

P. Siva Kota Reddy, B. Prashanth, & Kavita S. Permi

P. Siva Kota Reddy, R. Rangarajan, & M.S. Subramanya

(SG, VS: LG)

P. Siva Kota Reddy, R. Rajendra, & M.C. Geetha

P. Siva Kota Reddy, E. Sampathkumar, & M.S. Subramanya

P. Siva Kota Reddy, R. Shivashankara, & K.V. Madhusudhan

Solved: $-\Sigma$, $\Lambda^k_x(\Sigma) \simeq A^2_x(\Sigma)$, based on existing solutions for unsigned isomorphism. (See M. Acharya (2009a) for $\Lambda_x$.) [Annot. 6 Feb 2011.]

(SG: LG, Sw)

P. Siva Kota Reddy & M.S. Subramanya
2479512 (2009m:05080). (SG( Gen): Sw, LG)


Definitions as at Sampathkumar, Reddy, and Subramanya (2008a).

Solved: $\Sigma \simeq \Lambda_2(S)$; $\Lambda_2^k(S) \simeq \Sigma$. (Continued in Reddy, Vijay, and Lokesha (2009a), (2010a).) [Annot. 3 Aug 2009.]


$V(P_k(S)) := \{\text{paths}\}, PP' \in E(P_k(S))$ iff $P \cup P'$ is a path of order $k + 1$ or a $C_k$, $\sigma(PP') = \sigma(P)\sigma(P')$. This is balanced. Solved: $\Sigma \simeq P_3(S), P_4(S)$. [Annot. 7 Jan 2011.] (SG: LG( Gen), Bal)

P. Siva Kota Reddy, M.S. Subramanya, & R. Rajendra


P. Siva Kota Reddy, Gurunath Rao Vaidya, & A. Sashi Kanth Reddy


P. Siva Kota Reddy & S. Vijay


The intersection graph $M_t$ of all total minimal dominating sets of $|\Sigma|$ is signed to be balanced using the canonical vertex signature of $\Sigma$. Such signed graphs are characterized. $M_t \simeq \Sigma, -\Sigma$ are solved, based on existing solutions for unsigned isomorphism. [Annot. 6 Feb 2011.]


$V(L_r(S)) := \mathcal{P}_r(E)$ with edge $PQ_{e,f} \in E(L_r(S))$ for each adjacent $e \in P, f \in Q$ and $\sigma_c(PQ_{e,f}) = \sigma(P)\sigma(Q)$. This is balanced. Solved: $\Sigma, \Lambda_2(S) \simeq \mathcal{L}_2(S), \Sigma \simeq \mathcal{L}_2(S), \eta_l et al.[Annot. 7 Jan 2011.]$ (SG: LG( Gen), Bal)

P. Siva Kota Reddy, S. Vijay, & V. Lokesha


$D_m$ The “$m^{th}$ power signed graph” $\Sigma^m$ [I will say “$\leq m$-distance signed graph” $D_m(S)$] is the graph of distance $\leq m$ in $|\Sigma|$ with signature
\[ \sigma^c \]. Prop. 5: \( \Sigma \) has the form \( D_m(\Sigma') \) iff it is balanced and \( |\Sigma| \) is a \( (\leq m) \)-distance graph. [Sufficiency is incorrect.] Solved [possibly incorrectly]: \( \Sigma^c \) or \( \Lambda_\times(\Sigma^c) \cong D_m(\Lambda_\times(\Sigma)) \); \( \Lambda_\times(\Sigma)^c \cong D_m(\Sigma)^c \); \( \Lambda_\times^2(\Sigma) \cong D_m(\Sigma), D_m(\Sigma)^c, D_m(\Sigma^c) \). [\( \Lambda_\times \) as in M. Acharya (2009a).] [Annot. 12 Apr 2009.] (SG: Bal, Sw, LG)


Continuing (2009a) with: \( \Lambda_\times(\Sigma) \cong D_m(\Sigma[c]); \Lambda_\times(\Sigma)^c \cong D_m(\Sigma) \). [Annot. 10 Apr 2009.] (SG: Bal, Sw, LG)

P. Siva Kota Reddy, S. Vijay, & Kavita S. Permi

P. Siva Kota Reddy, S. Vijay, & Kavita S Permi
Radial graph of \( \Sigma \) with certain balanced signs. [Annot. 22 Jan 2020.] (SG: Bal, Sw, VS)

P. Siva Kota Reddy, S. Vijay, & B. Prashanth

P. Siva Kota Reddy, S. Vijay, & H.C. Savithri

Bruce Reed
See also C. Berge, S. Fiorini, J. Geelen, K. Kawarabayashi, and D. Rautenbach.

Given \( k \), \( \exists f(k) \) such that \( \Gamma \) has a 2-packing of \( 2k \) odd circles, or \( l_0(-\Gamma) \leq f(k) \). [Simpler proof in Kawarabayashi and Reed (2010a).] [Annot. 29 Dec 2020.] (par: fr)

Bruce Reed, Kaleigh Smith, & Adrian Vetta


P. Reed
See A.J. Bray.

David Rees

A fundamental paper introducing bipartite gain graphs and their switching automorphisms (all not recognized as such) into the foundations of semigroup theory. Cf. Graham (1968a), Houghton (1977a), Pol-
Nathan Reff
See also F. Belardo, L. Duttweiler and O. Kitouni.


T Complex unit gain graphs have gain group $T := \{ z \in \mathbb{C} : |z| = 1 \}$. Bounds on largest and smallest eigenvalues of $A(\Phi)$ ($\S$3, “Eigenvalues of the adjacency matrix”) and $L(\Phi)$ ($\S$4, “Eigenvalues of the Laplacian matrix”). Most (except Thm. 4.9, where the edge gains affect the bounds) generalize known bounds for graphs, the signless Laplacian $L(-\Gamma)$, or signed graphs. Some generalizations are not obvious. Lemmas 3.1, 4.1: The spectrum of $A$ or $L$ depends only on the switching class. Lemmas 3.2, 4.2: If $\Phi$ is balanced, the spectra are the same as those of $||\Phi||$. (Problem. Generalize B.D. Acharya (1980a) by proving the converses.) Thm. 5.1: Exact eigenvalues for circle graphs. Thm. 5.4: Lemma 3.1 of Hou, Li, and Pan (2003a) generalized to complex unit gain graphs. [Annot. 30 Oct 2011, rev 20 Jan 2017.] (GG: Eig, Incid)


Nathan Reff & Lucas J. Rusnak

Walks, the adjacency matrix and its powers, and the incidence matrix and the Laplacian matrix $L$ of an oriented hypergraph have the same relationships as with graphs. [Annot. 19 Oct 2012.] (SH: Ori: Incid, Adj, Eig)

Nathan Reff & Howard Skogman
Damien Regnault
See M. Noual.

F. Regonati
See E. Damiani.

Guus Regts
See A. Goodall.

Jörg Reichardt & Stefan Bornholdt

Philip F. Reichmeider

Thm. 7.6, p. 107, attributed to Hoffman (1960a) (who credits Heller and Tompkins (1956a)): in effect the incidence matrix of a balanced signed graph is totally unimodular. König’s and Hall’s theorems are corollaries, per Hoffman. [Annot. 8 Nov 2015.] (sg: incid, bal: Exp)

Talmage James Reid
See also T. Lewis.

Talmage James Reid & Lee Inmon Virden

The Dowling geometry $Q_3(F_3^*)$ is one of two crucial matroids. [Annot. 9 Apr 2016.] (M: Str: gg)

Gerhard Reinelt
See F. Barahona, C. De Simone, and M. Grötschel.

Victor Reiner
See also P.H. Edelman.


They are equivalent to acyclic bidirected graphs. (Sgnd, sg: Ori: Str, geom)


“Noncrossing partitions [actually, partial partitions] of type $B$” are elements of the Dowling lattice $Q_n(\{+,−\})$ that, regarded as sign-symmetric partitions of $\pm[n]$, are noncrossing when drawn on the circular arrangement $[1,2,\ldots,n,−1,−2,\ldots,−n]$. [Annot. 28 Jan 2015.] (sg: M)

Victor Reiner & Dennis Tseng

J.B. Remmel & Michelle L. Wachs
Élisabeth Remy
See also G. Didier and A. Naldi.

Élisabeth Remy & Paul Ruet

2008a [From elementary signed circuits to the dynamics of Boolean regulatory networks.] Manuscript, 2008. (2008b) under a previous title; often cited as such. (SD: Dyn)

2008b From minimal signed circuits to the dynamics of Boolean regulatory networks, Bioinformatics 24 (2008), i220–i226. (SD: Dyn)

Élisabeth Remy, Paul Ruet, & Denis Thieffry


Han Ren
See F.-Y. Cao.

Ling-Zhi Ren
See Y. Liu.

Qing Jun Ren
See also H.S. Du.


Xiangyu Ren & Jianguo Qian

Raghunathan Rengaswamy
See M. Bhushan and M.R. Maurya.

Enrique Reyes, Christos Tatakis, & Apostolos Thoma

Cor. 3.3 (restated): A closed walk in $\Gamma$ is primitive iff its graph is a cactus tree in which each cutpoint is in exactly 2 blocks, the outer blocks are odd circles, and the inner blocks are even circles or isthmi. [Generalization: A closed walk in $\Sigma$ is primitive iff its graph is a cactus tree in which each cutpoint is in exactly 2 blocks, the outer blocks are negative circles, and the inner blocks are positive circles or isthmi.]

*Problem:* Prove it.

Thm. 4.13: Characterizes closed walks that give a minimal binomial of the ideal.

Thm. 4.16 (restated): A closed walk in $\Gamma$ is fundamental iff it is a minimal walk of an even circle $C$ with at most one chord, which creates two odd circles (i.e., $\Gamma : V(C)$ contains no other even circle, equivalently no other frame circuit), or of a handcuff circuit, of $G(\Gamma)$. [Generalization: A closed walk in $\Sigma$ is fundamental iff it is a minimal walk of an positive circle $C$ with at most one chord, which creates two negative circles (i.e., $\Sigma : V(C)$ contains no other positive circle, equivalently no other frame circuit), or of a handcuff circuit, of $G(\Sigma)$.]

*Problem:* Prove it.

Josephine Reynes
See W. Grilliette and L.J. Rusnak.

Brendon Rhoades
See D. Armstrong and E. Leven.

John Rhodes & Benjamin Steinberg


(RG: sw, aut: Algeb: Exp)

Ricardo Riaza


M. Riazi
See A. Kargaran.

Federico Ricci-Tersenghi
See A.K. Hartmann.

Adrien Richard
See also J. Aracena, J.-P. Comet, and L. Paulevé.


**Adrien Richard & Jean-Paul Comet**


**Adrien Richard, Jean-Paul Comet, & Gilles Bernot**

20xxa R. Thomas’ logical method. Manuscript. (SD: Dyn, Biol)


“Regulatory network” = signed digraph. (SD: Gen: Dyn, Biol)

**Adrien Richard & Paul Ruet**


**Jean Richelle**

See also D. Thieffry and R. Thomas.


The dynamics of a negative cycle differ under different models: continuous (differential equations), boolean (2-valued states), and stochastic, with or without the time delays found in biology. [Annot. 4 Aug 2018.] (sd: Dyn, Biol)

(84h:92020a).

Daniel J. Richman
See J.S. Maybee.

R.J. Riddell
Includes the number of labelled simple 1-trees of order n, i.e., bases of bicircular matroid $G(K_n, \emptyset)$. [Sequence A057500 in N.J.A. Sloane, The On-Line Encyclopedia of Integer Sequences, URL http://oeis.org/A057500. [Cf. Neudauer, Meyers, and Stevens (2001a).]

Heiko Rieger
See also M.J. Alava, B. Coluzzi, A.K. Hartmann, N. Kawashima, and J.D. Noh.

Robert G. Rieper
See J. Chen.

M.J. Rigby
See A.C. Day.

Simone Righi & Károly Takács
Extended abstract. See (2014b). [Annot. 8 Jan 2016.] (SG: Dyn, PsS)

Extended abstract. See (2014b). [Annot. 8 Jan 2016.] (SG: Dyn, PsS)

Random pairs of adjacent nodes of $\Sigma$ (= persons) play Prisoner’s Dilemma. Each node is a Cooperator, Defector, or Conditional player who cooperates iff the edge is positive. $\Sigma$ and (in some papers) the node strategies evolve depending on the outcome of each round, which is either a single play (dyadic) or a round robin in a triangle (triadic). Some conditions evolve into universal cooperation, some into universal defection; conditions for each outcome are explored in these articles. [Annot. 8 Jan 2016.] (SG: Dyn, PsS)
See (2014b). [Annot. 8 Jan 2016.] (SG: Dyn, PsS)

See (2014b). [Annot. 8 Jan 2016.] (SG: Dyn, PsS)

[Arnout van de Rijt] See A. van de Rijt (under ‘V’).

James E. Riley


G. Rinaldi See C. De Simone and J. Lukic.

R.D. Ringeisen See also M.J. Lipman.
Mathematical examination of a game based on switching vertices until a player creates an isolated vertex and wins. [Annot. 26 May 2017.] (tg: Sw)

Gerhard Ringel See also N. Hartsfield and M. Jungerman.
“Cascades” (§8.3): see Youngs (1968a).

signature. “Oriented” = all positive. (SG: Top, Sw)

Oliver Riordan
See B. Bollobás.

S. Risau-Gusman
See F. Romá.

F. Ritort
See E. Marinari.

Vincent Rivasseau
See T. Krajewski.

Nicolas Rivier

Engaging if physics-intensive exposition of continuous and discrete gauge transformations via fiber bundles. E.g., mixed Ising models (= signed graphs $\Sigma$) with discrete $\pm 1$ or continuous $S^3$ spins. “Odd ring” defects (p. 21) treated via all-negative graph $-\Gamma$; “odd lines” (pp. 22 ff.) = negative paths. $\mathbb{Z}_2$ is implicit in sign of gauge-invariant physical configuration (§3.3), explicit in magnetization (p. 38). §4, “Gauge invariance in discrete space”. §4.1, “Discrete gauge invariance in spin glasses”: Ising spin glass = general $\Sigma$. $\mathbb{Z}_2$ gauge transformation (switching) “cannot be meaningfully generalized to . . . XY or Heisenberg . . . spins [Question. Is that true?] because [signs $\pm 1$] are real numbers”. §4.2.1, “Potential valleys in configuration space”: “The configuration space is a direct product of . . . odd [= negative?] line[s].” “Tunnelling” between valleys (regions closer to ground state potential). §4.2.2, “Elasticity of random networks”: Fiber bundle = covering graph of permutation gain graph with $S_N$ action. Fig. 6: Signed graph on torus, cut along negative edges to become planar all-positive. §5.2, “Theory of surfaces”: ¶¶1–3 must be read. §7, “Conclusions”: Ising spin glass (i.e., $\Sigma$), p. 82. [Annot. 7 Aug 2018.] (SG, gg: VS: Phys, Bal, Fr: Exp)

Romeo Rizzi

[Annot. 13 Aug 2013.] (Par: Str: Circ)

María Robbiano
See also N.M.M. de Abreu and I. Gutman.

María Robbiano, Katherine Tapia Morales, & Bernardo San Martín

They find simple $\Gamma$ with “vertex bipartiteness number” $l_0(-\Gamma) \leq k$ that maximize spectral radii of $A(-\Gamma), L(-\Gamma)$. [Problem: Generalize to signed simple graphs.] [Corrected by Liu and Wang (20xxa).] [Annot. 8 May 2017.] (sg: Par: Fr, Eig)

Jakayla R. Robbins


The number of orientations of the free spike matroid $L(2C_n, \emptyset)$ is $2^{n-1}D_n$, $D_n :=$ Dedekind number. [Annot. 29 Sept 2011.]


In general, not all orientations of the free spike matroid $L(2C_n, \emptyset)$ have a real vector representation. Also, bounds on the number of representable orientations. [Annot. 29 Sept 2011.] (gg: M: Geom, Invar)

**Jakayla Robbins, Daniel Slilaty, & Xiangqian Zhou**


**Fred S. Roberts**

See also T.A. Brown and R.Z. Norman.


§3.1: “Signed graphs and the theory of structural balance.” Many topics are developed in the exercises. Exercise 4.2.7 (from Phillips (1967a)).

(SG, SD: Bal, Alg, Adj, Clu, Fr, PsS: Exp, Exr)

Ch. 4: “Weighted digraphs and pulse processes.” Signed digraphs here are treated as unit-weighted digraphs. Note esp.: §4.3: “The signed or weighted digraph as a tool for modelling complex systems.” Conclusions about models are drawn from very simple properties of their signed digraphs. §4.4: “Pulse processes.” §4.5: “Stability in pulse processes.” Stability is connected to eigenvalues of $A(\Sigma)$.

(SDw, SD, WD: Bal, Eig, PsS: Exp, Exr, Ref)


Ch. 9: “Balance theory and social inequalities.” Ch. 10: “Pulse processes and their applications.” Ch. 11: “Qualitative matrices.”

(SG, SD, SDw: Bal, PsS, QM: Exp, Ref)


§2: “Balance and clusterability.” Basics in brief. §7: “Signed and
weighted digraphs as decision-making models.” Cursory.


§4: “Qualitative stability.” A fine, concise basic survey.

§5: “Balanced signed graphs.” Another concise basic survey, and two open problems (p. 20).


Several characterizations of consistent vertex signatures of a graph. \( \Gamma \) is “markable” iff it has a consistent vertex signature that is not all +. Thm.: 3-connected \( \Gamma \) is markable iff it is bipartite. Thm.: A classification of markable 2-connected graphs with girth \( \leq 5 \). [See also Hoede (1992a).] [Annot. 27 Apr 2009.]


A survey of balance in signed graphs and consistency in vertex-signed graphs and their supposed applications in social psychology and elsewhere. Results from Xu (1998a) and Roberts and Xu (2003a). §4: “Connections among balance, consistency, and other graph-theoretical notions”. Lists some special and general equivalences, esp., with bipartiteness, or with all circle lengths divisible by 4. §5: “Coherent paths”. Characterizations of consistency or balance from Beineke and Harary (1978b), Roberts and Xu (2003a), Acharya (1983a), Rao (1984a). §6: “Fundamental cycles and cycle bases”. Hoede’s (1992a) characterization of consistency, a variant, and one from Roberts and Xu (2003a) in terms of a circle basis. §7: “Markable graphs”. \( \Gamma \) is “markable” iff it has a consistent vertex signature that is not all +. Thm. (Roberts). 3-connected \( \Gamma \) is markable iff it is bipartite. See Roberts (1995a) and S. Xu (1998a).

§8: “Open questions”. [Annot. 27 Apr 2009.]

Fred S. Roberts & Thomas A. Brown


Fred S. Roberts & Shaoji Xu

Several characterizations of consistent vertex-signed graphs, and algorithms to determine consistency, are surveyed or proved. Thm.: A vertex-signed graph is consistent iff every circle in some circle basis is positive and every two 3-connected vertices have the same sign. [Annot. 26 Apr 2009.]

Edmund Robertson
See P. Brooksbank.

Neil Robertson, P.D. Seymour, & Robin Thomas
See also W. McCuaig and J. Maharry.


Question 1. Does a given digraph \(D\) have an even cycle? Question 2. Can a given digraph \(D\) be signed so that every cycle is negative? (These problems are easily seen to be equivalent.) The main theorem (the “Even Dicycle Thm.”) is a structural characterization of digraphs that have a signing in which every cycle is negative. (These were previously characterized by forbidden minors in Seymour and Thomassen (1987a).)

The main theorem is proved also in McCuaig (2004a). See the joint announcement, McCuaig, Robertson, Seymour, and Thomas (1997a).

Garry Robins & Yoshi Kashima

Pp. 9–10: a critical review of signed-graph balance theory in social psychology. [See also that of R.C. Roistacher1974aRichard C. Roistacher.]

[Annot. 21 Aug 2014.]

Ellen Robinson
See L. Rusnak and G. Chen.

Herbert A. Robinson
See C.R. Johnson.

Robert W. Robinson
See also Harary, Palmer, Robinson, and Schwenk (1977a) and Harary and Robinson (1977a).

The “bilayered digraphs” of §7 are identical to simply signed, loop-free digraphs (where multiple arcs are allowed if they differ in sign or direction). Thm. 1: Their number $b_n = \text{number of self-complementary digraphs of order } 2n$. Cor. 1: Equality holds if the vertices are signed and $k$-colored. In §§, Cor. 2 concerns vertex-signed and 2-colored digraphs; Cor. 3 concerns vertex-signed tournaments. Assorted remarks on previous signed enumerations, mainly from Harary, Palmer, Robinson, and Schwenk (1977a), are scattered about the article. (SD, VS, SG: Enum)

Paul Rochet
See P.-L. Giscard.

G.J. Rodgers & A.J. Bray

Physical quantities via spectrum of random $A(\Sigma)$ with $n \to \infty$ assuming expected degree $d_{\Sigma}(v) \sim pn$ where $0 < p < 1$. [Annot. 29 Dec 2012.]

(Phys: sg: Rand: Eig)

G.J. Rodgers & C. De Dominicis

§3, “Solutions”: Random $A(\Sigma)$ where $\Sigma$ is sparse. [Annot. 29 Dec 2012.]

(Phys: sg: Rand: Eig)

Y. Roditty
See I. Krasikov.

Vojtěch Rödl

Jose Antonio Rodríguez
See R.T. Boesch.

Elisabeth Rodríguez-Heck

(Sh, SG: Bal, Geom)


Abstract of (2018a).

(Sh, SG: Bal, Geom)

Juan A. Rodríguez-Velázquez
See E. Estrada.

Vladimir Rogojin
See A. Alhazov.

Richard C. Roistacher

Pp. 20–22: a critical review of the use of signed graphs in social psychology. [See also that of Robins and Kashima (2008a).] [Annot. 21 Aug 2014.]

*(PsS: SG)*

**Oscar Rojo**

See also I. Gutman.


(*par: Adj: Eig*)


(*par: LG: Adj: Eig*)

**Oscar Rojo & Raúl D. Jiménez**


(*par: Adj: Eig, LG*)

**Oscar Rojo & Luis Medina**


(*par: Adj: Eig*)

**Edita Rollová**

See also M. DeVos, T. Kaiser, E. Máčajová, and R. Naserasr.

**Edita Rollová, Michael Schubert, & Eckhard Steffen**


*(SG: Flows)*

**F. Romá**

See also A.J. Ramírez-Pastor.

**F. Romá, F. Nieto, A.J. Ramirez-Pastor, & E.E. Vogel**


A signed toroidal square lattice with random signs. Analytical and numerical methods to study functions of $x := \#E^+/\#E$ such as proportion of frustrated (negative) plaquettes (face boundaries). (Cf. other papers of the authors.) [Annot. 3 Jan 2015.]

*(SG, Phys: State)*


Assuming (with little loss of generality) equally many edges of each sign, randomly distributed, in a square lattice. [Annot. 3 Jan 2015.]

*(Phys: SG: Rand: State, Sw)*
F. Romá, F. Nieto, E. Vogel, & A.J. Ramirez-Pastor

F. Romá, S. Risau-Gusman, A.J. Ramirez-Pastor, F. Nieto, & E.E. Vogel
2009a The ground state energy of the Edwards–Anderson spin glass model with a parallel tempering Monte Carlo algorithm. *Physica A* 388 (2009), 2821–2838. The efficiency of a Monte Carlo method for finding ground state energies et al., with either random pure signs (i.e., $\pm J$ with $J = 1$, half the edges having each sign) or Gaussian randomly weighted random signs. [Annot. 3 Jan 2015.] (SG: Fr, Sw, Phys: Alg)

S.V. Roopa
See E. Sampathkumar.

P. Lawrence Rozario Raj & R. Lawrence Joseph Manoharan
See Baskar Babujee and Shobana (2011a). For a face of plane $\Gamma$, $\zeta^*(F) :=$ product of boundary vertex signs. $\zeta$ is a “face signed product cordial labeling” if signed product cordial and $\#\zeta^*-1(+) \approx \#\zeta^*-1(-)$. $\zeta$ is a “total face signed product cordial labeling” if total $\#$ positive vertices, edges, and faces $\approx \#$ negative ones. $K_{1,1,n-2}$, $n \geq 5$, and another plane family are both. [Annot. 26 Dec 2020.] (Lab: VS: SG, Bal)

Frances Rosamond
See H.L. Bodlaender.

Milton J. Rosenberg
See also R.P. Abelson.

Milton J. Rosenberg & Robert P. Abelson
An attempt to test structural balance theory experimentally. The test involves, in effect, a signed $K_4$ [an unusually large graph for such an experiment]. Conclusion: there is a tendency to balance but it competes with other forces. (PsS: SG: kg)

Seymour Rosenberg

M. Rosenfeld
See W.D. McCuaig.

Elissa Ross


$\mathbb{Z}^d$-gain graphs. Dictionary: “net gain” = gain of closed walk, “$T$-gain procedure” = switching so $\varphi^t|_T \equiv 0$, “local gain group” = $\langle \text{Im} \varphi^t \rangle$, “derived graph” = gain covering graph. (GG: sw, Cov, Top, Geom)


Philippe A. Rossignol
See J.M. Dambacher.

Gian-Carlo Rota
See P. Doubilet.

Günter Rote
See H. Edelsbrunner.

Ron M. Roth & Krishnamurthy Viswanathan

See (2008a). (sg: fr, Alg)


Section III, “Relaxed problem”: The frustration index of a bipartite signed graph is NP-complete. Thm. 4.1: The frustration index of a signed $K_{n,n}$ (where $n$ is a variable) is NP-complete. The proofs use the bipartite adjacency matrix of the signed graph. The latter problem is polynomially reduced to the former by a construction using Kronecker product and a Hadamard matrix. The problems are interpreted as nearest-neighbor decoding of the Gale–Berlekamp code of order $n$.

Section V, “Decoding algorithm over the BSC”: A polynomial-time approximate decoding algorithm that is asymptotically reliable. [Annot. 2 Sept 2009.]

Uriel G. Rothblum & Hans Schneider


Jianling Rou
See F.L. Tian.

Peter Rowlinson
See also D.M. Cardoso, D.M. Cvetković, and F. Ramezani.

Peter Rowlinson & Zoran Stanić

**Bernard Roy**


Given real arc weights $a$ on a digraph, there exists $t : V \to \mathbb{R}$ such that $t(w) - t(v) \geq a(e)$ for every arc $e : (v, w)$ iff every cycle has nonpositive weight sum. [See also Afriat (1963a).] (WD: OG)


**Mithun Roy**

See J. Bensmail.

**Roshni T Roy**

See also S. Hameed.

**Roshni T Roy, K A Germina, K Shahul Hameed, and Thomas Zaslavsky**


*Cf. Hameed, Shijin, et al. (2021a).* Laplacian $L(\Sigma, w)$, $\det L(\Sigma, w)$ of positively weighted signed graph. Balance iff $\text{null } L(\Sigma, w) > 0$. Applied to $w := \min$, $\max$ signed distance. Balance iff $L(\Sigma, w)$ cospectral with $L(|\Sigma|, d)$, $d =$ distance in $|\Sigma|$. [Annot. 11 Oct 2020.]

(SG: WG, Incid, Lap, Bal, Eig, Sw)

**Gordon F. Royle**


**G. Rozenberg**


**Arthur L. Rubin**

See P. Erdős.

**Jason D. Rudd**

See P.J. Cameron.

**Paul Ruet**

See also A. Crumière and É. Remy, and A. Richard.


**Sarah Crown Rundell**

See B. Braun.

**Philip J. Runkel & David B. Peizer**
*(PsS: sg: Bal)*

Richard Ruppert  
See J. Quirk.

Lucas J. Rusnak  
See also G. Chen, V. Chen, W. Grilliette, and N. Reff.

MR 2941411 (no rev).  
Oriented hypergraphs generalize bidirected graphs: Each incidence gets a direction, or sign. Main interest: The linear dependencies of columns of a $(0, \pm 1)$-matrix, treated as the incidence matrix of an oriented hypergraph. Techniques are generalizations of those of signed graphic matroids (Zaslavsky (1982a)) but more complicated. The methods are most applicable to the matrices known as “balanceable”.

(SH: Incid, Str, SG, Ori)

Lucas J. Rusnak, Selena Li, Brian Xu, Eric Yan, & Shirley Zhu  
(SH: Ori, SG, Incid)

Lucas J. Rusnak, Josephine Reynes, Skyler J. Johnson, & Peter Ye  
(SG, SH, Ori: Lap, Flows(Gen))

Lucas J. Rusnak, Ellen Robinson, Martin Schmidt, & Piyush Shroff  
(Carrie Rutherford)  
See M. Banaji.

Joe Ryan  
See C. Dalf’o.

K. Rybnikov [K.A. Rybnikov, Jr.; Konstantin Rybnikov]  
See also S.S. Ryshkov.


§4, “Quality transfer”, concerns the existence of a satisfied state (called “quality translation” in Ryshkov and Rybnikov (1997a)) in a permutation gain graph $\Phi$, where $\mathcal{G}$ acts on a set $Q$. P. 487, top: A satisfied state exists iff $\Phi$ is balanced [Ryshkov and Rybnikov (1997a); but necessity is incorrect]. Lemma 4.1 appears to mean that a satisfied state exists iff it exists on each member of an arbitrary basis of the binary cycle space. [Not true, but interesting. The following special case, also invalid in general, was the author’s intention (as I was told, Oct. 2000): $\Phi$ is
balanced iff every member of a circle basis is balanced. The special case Lemma 4.2 is correct, because it is essentially homotopic. [See Rybnikov and Zaslavsky (2005a), (2006a).]

Konstantin Rybnikov & Thomas Zaslavsky


§§1–4: A condition on binary cycles that implies balance but does not depend on having a fundamental system of circles; it requires an abelian gain group. §5: Satisfied states and balance of a permutation gain graph.

(GG: Bal)

§6: The criterion is applied to calculate the dimension of the space of liftings of a piecewise-linear immersion of a \(d\)-cell complex in Euclidean \(d\)-space.

(GG: Bal, Geom)


The class of \(\Gamma\) such that the criterion of (2005a) works for any gains on \(\Gamma\) is minor-closed. Some forbidden minors are given. Which ones they are depends on the class of permitted gain groups in a way that is not understood.

(GG: Bal: Str)

S.S. Ryshkov & K.A. Rybnikov, Jr.


Announcement of results in (1997a). [“Generatrissa” corresponds to English “generatrix”.]

(gg: Geom)


Translation of (1996a).

(gg: Geom)


Let \(\Phi\) be a permutation gain graph, with gain group \(\mathfrak{G}\) acting on a set \(\mathfrak{Q}\), and with underlying graph the \(d\)-cell adjacency graph of a kind of simply connected polyhedral \(d\)-cell complex. A “quality translation” is a satisfied state: a mapping \(s : V \rightarrow \mathfrak{Q}\) such that \(s(w) = s(v)\varphi(e; v, w)\) for every edge. A “circuit” is a closed walk that is not trivially reducible. Call a “\(d−2\)-circle” any circle contained in the star of a \(d−2\)-cell. Assume \(\mathfrak{G}\) and \(\mathfrak{Q}\) fixed. Thms. 1–2 can be stated: In the free group on the edge set, the \(d−2\)-circles generate all circuits. Also, \(\Phi\) is balanced iff all \(d−2\)-circles have identity gain. Thm. 3: Identity gain of all \(d−2\)-circles is necessary and sufficient for the existence of a satisfied state. [Necessity is incorrect because the action may have nontrivial kernel.] The idea of quality transfer goes back to Voronoï in 1908. [See Rybnikov (1999a) and Rybnikov and Zaslavsky (2005a) for more.]

Sufficiency in Thm. 3 is applied to lifting of tilings of Euclidean and spherical space. Thm. 4 ((1996a), Thm. 9): Balance (“canonical definition”) of \(\Phi\) is sufficient for lifting a tiling of \(\mathbb{R}^d\). Here the qualities are
affine functions. Thms. 5–6 ((1996a), Thm. 10): Balance within each $d - 3$-cell star implies lifting. [See Rybnikov (1999a) and Rybnikov and Zaslavsky (2005a) for more.]

§8: “Applications to the colouring of tilings”. (gg: bal, Geom)

Herbert J. Ryser
See R.A. Brualdi.

Shinsei Ryu
See A.P.O. Chan.

Henry S. Rzepa

Rachid Saad

Thm.: In a bidirected all-negative complete graph with a suitable extra hypothesis, the maximum length of a coherent circle equals the maximum order of a coherent degree-2 subgraph. More or less generalizes part of Bánkfüf and Bánkfüf (1968a) (q.v.). [Generalized in Bang-Jensen and Gutin (1998a).] [Problem. Generalize to signed complete graphs or further.] (par: ori: Paths, Alg)

Assieh Saadatpour, István Albert, & Réka Albert

Mathieu Sablik
See A. Crumière.

Horst Sachs
See D.M. Cvetković.

Julio Saez-Rodriguez
See S. Klamt.

Bruce Sagan
See also C. Bennett, A. Björner, A. Blass, J. Hallam, F. Harary, and T. Józefiak.


In §4, coloring of a signed graph $\Sigma$, especially of $\pm K_n^*$ and $\pm K_n$, is used to calculate and factor the characteristic polynomial of $G(\Sigma)$. Presents the geometrical reinterpretation and generalization by Blass and Sagan (1998a). In §§5 and 6, other methods of calculation and factorization
are applied to some signed graphs (in their geometrical representation).

(SG, Gen: N: Col, G: Exp)

Sahariya
See K.A. Germina.

[Amine El Sahili]
See A. El Sahili (under ‘E’).

G. Sahoo
See S. Barik.

Prabhat K. Sahu & Shyi-Long Lee

The “net-sign identity information index” $I_s$ is expressed [obscurely] in terms of $\#E^+$ and $\#E^-$ in a molecular structure graph. The purpose is to correlate with chemical phenomena. $I_s$ and $\sqrt{I_s}$ are compared with other indices. [Annot. 6 Feb 2011.]

Shaik Sajana, D. Bharathi, & K.K. Srimitra

Balance and homogeneity for ring $\mathbb{Z}_n$. [Annot. 25 Apr 2019.]

Mateja Šajna
See P. Potočnik.

Michael Saks
See P.H. Edelman.

Tadashi Sakuma
See Chiba.

M.C. Salas-Solís, F. Aguilera-Granja, E.E. Vogel, & S. Contreras

Toroidal square lattice with fixed weights $f_xJ$ horizontally and $f_yJ$ vertically ($f_x, f_y, J > 0$), randomly signed with $\#E^+ = \frac{1}{2}\#E$ or with variable $x := \#E^+/\#E$. Studies dependence of “order parameters” on $f := f_x/f_y$ and $x$. [Annot. 3 Jan 2015.] (SG, WG, Phys: Fr)

Nicolau C. Saldanha

A generalized Kasteleyn matrix is the left-right adjacency matrix $B$ of a bipartite gain graph with the complex units as gain group. (A Kasteleyn matrix has for gain group the sign group.) The object is to interpret combinatorially the coefficients of the characteristic polynomial, or the eigenvalues, of $BB^*$. The approach is cohomological (cf. Cameron
M. Ruby Salestina
See P.S.K. Reddy.

Lillian Salinas
See J. Aracena.

Mahmoud Salmasizadeh
See S. Fayyaz Shahandashti.

Regina Samaga
See I.N. Melas.

Robert Šámal
See M. DeVos.

Aniruddha Samanta
See also R. Mehatari.

Aniruddha Samanta & M. Rajesh Kannan
(GG: Adj: Eig)

(GG: Adj: Eig)

20xxc Gain distance matrices for complex unit gain graphs. Submitted.
(GG: Str, Adj(Gen): Eig)

P.N. Samanta
See P.S.K. Reddy.

U. Samee
See M.A. Bhat.

E. Sampathkumar
See also C. Adiga and S. Pirzada.

See Graph Theory Newsletter 2 (Nov., 1972), no. 2, Abstract No. 7.
(SG, VS: Bal)

Consider a simple graph, an edge signature \( \sigma \), and a vertex signature \( \mu \). Define \( \mu_\sigma(v) := \prod_{e \text{ incident with } v} \sigma(e) \) [later dubbed “canonical marking”] and, for each component \( X \), \( \partial \mu(X) := \prod_{v \in X} \mu(v) \). \( \mu \) is “p-balanced” if \( \partial \mu \equiv + \). Thm. 1: \( \partial \mu \equiv + \) iff \( \mu = \mu_\sigma \) for some \( \sigma \). [An early homology theorem.] Thm. 2: If \( \sigma \) is balanced and \( \partial \mu_\sigma \equiv + \), then there exist all-negative, pairwise edge-disjoint paths connecting the \( \mu_\sigma \)-negative vertices in pairs. [Quick proof: \( \partial \mu \equiv + \) iff \( \mu \) has evenly many negative vertices in each component. Negative vertices of \( \mu_\sigma \) are odd-degree vertices of \( \Sigma^- \). Apply Listing’s Theorem (independently discovered in stronger form by Sampathkumar) to \( \Sigma^- \).] [It is interesting
to base homology 0-chains like $\partial \mu$ on the components. [Annot. rev. 27 Dec 2010.]

SG, VS: Bal


Within the class of simple graphs, what is a complement of a signed graph? An approach is to partition the edges of $K_n$ into 3 classes: $E^+$, $E^-$, and $E^c$ (the set of non-edges), and apply a specific permutation of these sets. Each permutation of order 2 implies a kind of complementation. Examination of self-complementarity. Generalizations of balance. Generalized to a graph $\Gamma$ with $k$ edge classes $R_i$ [i.e., $k$-edge-colored graphs].

§10, “Balanced graph structures”: “$R_i$-balance”: $(\exists X) R_i = E[X, X^c]$ (the cut between $X$ and $X^c$). “$R_1 \cdots R_r$-balance”: Similar for $R_1 \cup \cdots \cup R_r$. “Complete balance”: $R_i$ balance for all $i$. “Arbitrary balance”: $R_i \cdots R_i$-balance for every $i_1, \ldots, i_r \in [k]$. Problem 11: Characterize this property. “$r$-relation balance”: The same for fixed $r$. Problem 12: Characterize this property. Other, similar concepts based on partitioning $V$. [Annot. 4 Sept 2010.]

SG, SGc: Gen: Bal


E. Sampathkumar & V.N. Bhave


Group-weighted graphs, both in general and where the group has exponent 2 (so all $x^{-1} = x$). Analogs of elementary theorems of Harary and Flament. Here balance of a circle means that the weight product around the circle, taking for each edge either $w(e)$ or $w(e)^{-1}$ arbitrarily, equals 1 for some choice of where to invert. [Hence, the graphs are not gain graphs.]

E. Sampathkumar & L. Nanjundaswamy


Given a permutation of $\{1, 2, \ldots, n\}$, sign $K_n$ so edge $ij$ is negative if the permutation reverses the order of $i$ and $j$ and is positive otherwise. Kendall’s measure $\tau$ of correlation of rankings (i.e., permutations) $A$ and $B$ equals $(\#E^+ - \#E^-)/\#E$ in the signature due to $AB^{-1}$. (SG: KG)

E. Sampathkumar, L. Pushpalatha, & M.A. Sriraj


Definitions at Sampathkumar and Sriraj (2013b) and Cf. Adiga, Sampathkumar, et al. (2013a). The colors are unlabelled. $G$ colored by $\pi(\Sigma^-)$ is “color balanced” if $\Sigma$ is balanced. “Color balanced path num-
In the $n$-fold sign group $\{+, -\}^n$ an element is “symmetric” if it is its own reverse. A (symmetric) $n$-signed graph is a gain graph $\Phi = (\Gamma, \varphi)$ which has (symmetric) gains $\varphi(e) \in \{+, -\}^n$. [Equivalent to having arbitrary gains in $\{+, -\}^{\lceil n/2 \rceil}$.] Only symmetric $n$-signed graphs are treated.

$\Sigma_\Phi$ [The mapping $\min : \{+, -\}^n \to \{+, -\}^n$ by $\min(a_1, \ldots, a_n) = +$ if all $a_i = +$ and $= -$ otherwise gives a signed graph $\Sigma_\Phi$ with signs $\sigma_\Phi(e) := \min(\varphi(e))$.]

Def.: $\Phi_1 \simeq \Phi_2$ (“cycle isomorphism”) if there is an isomorphism $\|\Phi_1\| \cong \|\Phi_2\|$ that preserves circle gains. Prop. 3: Symmetric $n$-signed graphs are cycle isomorphic if and only if they are switching isomorphic—generalizing $n = 1$ due to Soząński (1980a), Zaslavsky (1981b). [The proof (omitted) applies iff the gain group has exponent 2.]

$\varphi_S$ Let $\varphi_S(ef) := \varphi(e)\varphi(f)$ for $e, f \in E$. [Generalizing $\sigma_x$ of M. Acharya (2009a).]

$J_S$ The jump graph is $J_S(\Phi) := (\Lambda(\Gamma)^t, \varphi_S)$. Solutions of $\Phi \simeq J_S(\Phi)$, $\Phi^t \simeq J_S(\Phi)$, $J_S(\Phi^t) \simeq J_S(\Phi)$, where $a^t := at$ for $a, t \in \{+, -\}^n$ and $t$ is one of three special $n$-signs. [The last solution extends to arbitrary $t \in \{+, -\}^n$.]

Dictionary: “identity balance”, “$i$-balance” = balance in $\Phi$; “balance” = balance in $\Sigma_\Phi$; $P(\vec{C}) := \varphi(C)$ in the indicated direction.

The gain group is the $n$-fold sign group $\{+, -\}^n$, with reversing automorphism $(a_1, \ldots, a_n)^r := (a_n, \ldots, a_1)$. The gains satisfy $\varphi(e^{-1}) = \varphi(e)^r$. For $t \in \{+, -\}^n$, the $t$-complement of $\Phi$ is $t\varphi(e)$. Elementary results on balance, $t$-complementation, switching, and isomorphism. Dictionary: “identity balance” = “$i$-balance” = balance in $\Phi$; “balance” = balance in $\Sigma_\Phi$ defined at (2008a); $P(C) := \varphi(C)$ in the indicated direction. [An interesting form of skew gain graph. The ideas should be pursued in directions suggested by Hage and Harju (2000a) and Hage (1999a).] [Annot. 10 Apr 2009.]

$\Lambda_S$ The line graph is $\Lambda_S(\Phi) := (\Lambda(\Gamma), \varphi_S)$ [generalizing $\Lambda_x$ of M. Acharya (2009a)]. For other definitions and notation see (2008a).

Line graphs and jump graphs in the sense of (2008a) are characterized, respectively, as balanced symmetric $n$-signings of (unsigned) line graphs and their complements. [The characterizations remain true for unsymmetric $n$-signatures.] There are remarks about the $t$-complement $t\varphi$ (2010b) for three $t \in \{+, -\}^n$,

$\mu_\varphi$, $\Phi^c$ The “complement” $\Phi^c$ is $(\Gamma^c, \varphi^c)$ defined by $\mu_\varphi(v) := \prod_{uv \in E} \varphi(uv)$ (“canonical marking”) (cf. Sampathkumar (1984a)) and $\varphi^c(uv) := \mu_\varphi(u) \cdot \mu_\varphi(v)$ [product of gains of all edges incident in $\Phi$ to $u$ or $v$ but not both]. [Gains $\varphi^c$ are clearly balanced.] Prop. 7: A symmetric $n$-signed graph is a line graph iff it is a balanced, symmetric $n$-signature of an unsigned line graph. [Because $\varphi_S$ is arbitrary balanced gains.] Prop. 9: $\Lambda_S(\Phi)^c \sim J_S(\Phi)$. [Because both are balanced and the underlying graphs are the same.] Prop. 10 solves $\Lambda_S(\Phi) \sim J_S(\Phi)$, generalizing M. Acharya and Sinha (2003a). [The solutions to such graph equations, here and in related papers of Rangarajan, Sampathkumar, Siva Kota Reddy, et al., are easy corollaries of the similar results for unsigned graphs.] [All results remain true without assuming symmetry.] [Annot. 10 Apr, 1 Aug 2009, 20 Dec 2010.]

$\Sigma_0$ is a common-edge signed graph iff it is balanced and $|\Sigma_0|$ is a common-edge graph. [Incorrect. $\Lambda_x^2(\Sigma)$ does not have arbitrary balanced signs. E.g., $|\Sigma| = C_4$.] Equations solved [possibly incorrectly]: $\Sigma \simeq C_E^0(\Sigma)$ and $\Sigma \simeq \Lambda_x^0(\Sigma)$ [this includes the preceding]. $\Lambda_x^k(\Sigma) \simeq C_E^k(\Sigma)$. The jump graph (2008a) $J_S(\Sigma) \simeq C_E(\Sigma)$. [Annot. 12 Apr 2009.]

“Smarandanchely $k$-signed/marked graphs” are defined as $k$-signed/-marked graphs [and not used]. Signed/marked graphs are the case $k = 2$ [correctly: $k = 1$]. [Smarandache has nothing to do with this.] [Annot.
E. Sampathkumar, S.V. Roopa, K.A. Vidya, & M.A. Sriraj
20xxa Partition energy of a graph. Submitted.
Definitions at Sampathkumar and Sriraj (2013b). Eigenvalues and
energy of A. Examples. [Annot. 4 Jan 2019.]
sink. [Annot. 29 Sept 2012.] (SG, Gen: Ori: Bal, Sw)

E. Sampathkumar, M.S. Subramanya, & P. Siva Kota Reddy

The line signed graph is $Λ_{x}(Σ)$ [see M. Acharya (2009a)]. Prop. 3: A signed graph is a line signed graph of this kind iff it is a line graph with balanced signs.

The line signed digraph is $Λ_{S}(Γ, σ):=$ the Harary–Norman line digraph of $Γ$, signed by $σ^c$ defined as $φ^c$ in Sampathkumar, Reddy, and Subramanya (2010c). Prop. 11: A signed digraph is a line signed digraph of this kind iff it is a Harary–Norman line digraph with (undirectedly) balanced signs. $(Γ, σ)$ is switching isomorphic to $Λ_{S}(Γ, σ)$ iff each component is a balanced directed cycle. [Annot. 4 Sep 2010.] (SG, SD: LG)

V. Sangeetha
See A.J. Mathias.

Bernardo San Martín
See N.M.M. Abreu and M. Robbiano.

Santhi. M & J. James Albert


Yoshio Sano
See T.Y Chung, G. Greaves, and A. Munemasa.

[Emilio De Santis]
See E. De Santis (under ‘D’).

Raman Sanyal
See L. Gellert.

Mark Sapir
See V. Guba.

S.V. Sapunov

Equivalence of signed graphs (?) that model languages. [Annot. 28 Dec 2011.] (SG?)

P.B. Sarasija
See P. Nageswari.

Irasema Sarmiento
See also J.A. Ellis-Monaghan.

They are 4-closed (determined by their flats of rank 4). They are characterized, among all matroids, by the statistics of flats of rank \( \leq 7 \) and therefore by their Tutte polynomials. There are exceptions in rank 3.

Iwao Sato

See also Y. Higuchi and H. Mizuno.


§3, “Weighted zeta functions of group covering of digraphs”: The covering graphs (“derived graphs”) of gain graphs (“voltage graphs”). (GG: Cov)

Shun Sato

See T. Matsuoka.

Roman V. Satyukov

See I.E. Bocharova.

Lawrence Saul & Mehran Kardar


Algorithm for the energy distributions (the partition function) of the states of a randomly signed square, toroidal lattice graph. Applied to find statistical properties of such a signed graph. [Annot. 17 Aug 2012.] (SG: Phys, Fr, state: Alg)

B. David Saunders

See also A. Berman.

B. David Saunders & Hans Schneider


James Saunderson

See T. Coleman.

Saket Saurabh

See G. Philip.
D. Savithri
See M. Parvathi.

H.C. Savithri

Alex Schaefer
Ch. 1, “Signed graphs: background and miscellaneous results”.
Ch. 2, “The dimension of the negative cycle vectors of a signed graph”: Same as Schaefer and Zaslavsky (2019a).
Ch. 3, “Graphs that contain multiply transitive matchings”: Same as Schaefer and Swartz (20xxa). [Annot. 15 Nov 2017.]

Alex Schaefer & Eric Swartz

Alex Schaefer & Thomas Zaslavsky
Negative cycle vector $c^-(\Sigma) := (c^-_3(\Sigma), \ldots, c^-_n(\Sigma))$, $c^-_l(\Sigma) := \#$ negative $l$-circles in $\Sigma$. NCV(\Gamma) := \{c^-(\Gamma,\sigma) : \sigma \text{ signs } \Gamma\}$. Trivially, \[ \dim \text{NCV} \leq \# \text{circle lengths in } \Gamma. \] Theorem: Equality for graphs with a strongly permutable matching, including $K_n$, $K_{r,s}$. [Annot. 30 Jul 2020.]

R.H. Schelp
See P. Erdős and R.J. Faudree.

Baruch Schieber
See L. Cai.

Frank Schmidt

Martin Schmidt
See L. Rusnak.

Rüdiger Schmidt
The “count” matroids of graphs (see Whiteley (1996a)) and an extensive further generalization of bicircular matroids that includes frame matroids of biased graphs. His “partly closed set” is a linear class of circuits in an arbitrary “count” matroid. (GG: MtrdF, Bic, EC: Gen)

Stephan Schmidt
See J. Kunegis.
**Hans Schneider**  

**Irwin E. Schochetman**  
See J.W. Grossman.

**Rainer Schrader**  
See U. Faigle.

**Alexander Schrijver**  
See also A.M.H. Gerards.


Remark 21.2 (p. 308) cites Truemper’s (1982a) definition of balance of a $0, \pm 1$-matrix. (sg: par: Incid: Exp)


Assume $\Sigma$ embedded in the Klein bottle. If $\Sigma$ is bipartite, negative girth = max. number of disjoint balancing edge sets. If $\Sigma$ is Eulerian, frustration index = max. number of edge-disjoint negative circles. Proved via polyhedral combinatorics. (SG: Top, Geom, Fr)


§3: “Edge-disjoint paths and multicommodity flows,” pp. 334 ff. [This work suggests there may be a signed-graph generalization with the theorems discussed corresponding to all-negative signatures.]

(par: Paths: Exp)


A streamlined proof of the theorem of Guenin (2001a).


Vol. A, Ch. 36, “Bidirected graphs”.

Vol. B, §68.6b, “Bidirected graphs”, includes incidence matrix and signed graph. §68.6c, “Characterizing odd-$K_4$-free graphs by mixing stable sets and vertex covers”: “Odd-$K_4^7$ = $-K_4$ minor of $-\Gamma$.


Dictionary: “positive, negative” edge = extraverted or introverted negative edge, “directed” edge = positive edge, “even, odd” = positive, negative (edge or circle), “bipartite” = balanced. [Annot. 9 Jun 2011, 31 Dec 2020.]


Konrad Schrøder


Michael W. Schroeder

See R.A. Brualdi.

Michael Schubert

See also Y. Lu, E. Rollová, and E. Steffen.

Michael Schubert & Eckhard Steffen


Michelle Schultz

See G. Chartrand.

Felix Schwagereit

See J. Kunegis.

Gary K. Schwartz


Aut $Q_n(\mathfrak{G})$ factors in a certain natural way if, but also only if, $\mathfrak{G}$ factors. [Succeeds Bonin (1995a). See also Sikirić, Felikson, and Tumarkin...
The electronic journal of combinatorics #DS8

(2011a) for (mostly) more restricted related results. [Annot. rev. 9 Apr 2016.]

Roy Schwartz
See M. Charikar.

W. Schwärzler & D.J.A. Welsh

Tutte and dichromatic polynomials of signed matroids, generalized from Kauffman (1989a); this is the 2-colored case of Zaslavsky’s (1992b) strong Tutte functions of colored matroids. [For terminology see Zaslavsky (1992b).] Applications to knot theory.

§2, “A matroid polynomial”, is foundational. Prop. 2.1 characterizes strong Tutte functions of signed matroids by two equations connecting their parameters and their values on signed coloops and loops. [If the function is 0 on positive coloops, the proof is incomplete and the functions = 0 except on $M = \emptyset$ are missed.] Prop. 2.2: The Tutte (basis-expansion) polynomial of a function $W$ of signed matroids is well defined iff $W$ is a strong Tutte function. Eq. (2.8) says $W =$ the rank generating polynomial $Q_{\Sigma}$ (here also called $W$) if certain variables are nonzero; (2.9) shows there are only 3 essential variables since, generically, only the ratio of parameters is essential [an observation that applies to general strong Tutte functions]. Prop. 2.5 computes $Q_{\Sigma}$ of a 2-sum.

§3 adapts $Q_{\Sigma}$ to Kauffman’s (1989a) and Murasugi’s (1989a) signed-graph polynomials and simplifies some of the latter’s results (esp. his chromatic degree). §4, “The anisotropic Ising model”, concerns the Hamiltonian of a state of a signed graph. The partition function is essentially an evaluation of $Q_{\Sigma}$. §5, “The bracket polynomial”, and §6, “The span of the bracket polynomial”: Certain substitutions reduce $Q_{\Sigma}$ to 1 variable; its properties are examined, esp. in light of knot-theoretic questions. Thm. 6.4 characterizes signed matroids with “full span” (a degree property). §7, “Adequate and semi-adequate link diagrams”, generalizes those notions to signed matroids. §8, “Zero span matroids”: when does span(bracket) = 0? Yes if $M = M(\Sigma)$ where $\Sigma$ reduces by Reidemeister moves to $K_1$, but the converse is open (and significant if true).

Allen J. Schwenk

Thomas Schweser & Michael Stiebitz

Irene Sciriha
See also N. Basic and F. Belardo.

Irene Sciriha & Luke Collins

Signed $K_n$’s with two eigenvalues. Dictionary: “NSSD” = non-singular
graph with a singular deck. [Annot. 4 Jun 2018.]

Matt Scobee
See J. Lee.

Alexander D. Scott & Alan D. Sokal


András Sebö
See also F. Meunier and B. Novick.


See A. Frank (1996a).

Etsuo Segawa & Yusuke Yoshie


Deepak Sehrawat & Bikash Bhattacharjya

Number of switching isomorphism types of \((P_{2n+1,1},\sigma)\). [Annot. 16 Apr 2019.]


Cf. Ashraf and Germina (2016a). Cubic: \(\bar{\gamma}_2(\Sigma) \geq n/2\). k-step generalized Petersen graph \(P_{m,k}\): upper bounds for \(k = 1\), \(k > 1\) and \(\gcd(m,k) = 1\) or \(> 1\), are \(\approx m + 2\), \(\frac{3}{2}m\), \(\frac{4}{3}m\). Similar bounds for I-graphs. “I-graph”: bipartite, union of 3 particular matchings. [Annot. 26 Dec 2020.]


\(D(P_{m,k}) \leq 1 + \lfloor m/2 \rfloor\) if \(d := \gcd(m,k) = 1\), \(\leq 1 + d(1 + \lfloor m/2d \rfloor)\) if \(d > 1\), thus improving Sivaraman (2012a) for these graphs. [Annot. 26 Dec 2020.]


[This is useful information.] Thm. 4.2: There are 16 switching isomorphism types. [Previously found as “two-graphs”, cf. Bussemaker, Mathon, and Seidel (1979a), (1981a).] New: Proof, unique minimal representatives, and (Thms. 5.1, 5.2) frustration indices \(l\) and frustration numbers \(l_0\). E.g., max \(l = 6\) and max \(l_0 = 4\), uniquely for \(E^- = C_3 \cup C_3\). [Annot. 16 Apr 2019, 28 Dec 2020.]

Book graph \( B(m,n) := \) union of \( n \) circles \( C_m \) at a single common edge.

\# switching isomorphism types = \( n + 1 \) (depending on number of negative \( C_m \)'s). Recursive formulas for chromatic and zero-free chromatic polynomials. [Annot. 16 Apr 2019.]

J.J. Seidel

See also F.C. Bussemaker, P.J. Cameron, P.W.H. Lemmens, and J.H. van Lint.


Cf. Mallows and Sloane (1975a).


Same as (1979a), with photograph. (TG: Adj, Eig)


Reprints many articles on two-graphs and related systems. (TG: Sw, Adj, Eig, Geom)

§4, “Signed graphs”: The “intersection matrix” $A + 2I$ of a signed simple graph is theGram matrix of a set of “root vectors” with respect to an “inner product” that may not be positive definite. Explains origin of local switching (cf. Cameron, Seidel, and Tsaranov (1994a) and Bussemaker, Cameron, Seidel, and Tsaranov (1991a)). For a signed complete graph, $A + 3I$ represents lines at angles $\cos^{-1} 1/3$; it is positive semidefinite only for few graphs, which are classified (implicit in Lemmens and Seidel (1973a)).

§3.2: “Equidistant sets in elliptic $(d - 1)$-space.” §3.3: “Regular two-graphs.”

J.J. Seidel & D.E. Taylor

J.J. Seidel & S.V. Tsaranov

A group $Ts(\Sigma)$ is defined from a signed complete graph $\Sigma$: its generators are the vertices and its relations are $(uv^{-\sigma(uv)})^2 = 1$ for each edge $uv$. It is invariant under switching, hence determined by the two-graph of $\Sigma$. A certain subgraph of a Coxeter group of a tree $T$ is isomorphic to $Ts(\Sigma_T)$ for suitable $\Sigma_T$ constructed from $T$. [Generalized in Cameron, Seidel, and Tsaranov (1994a). More on $\Sigma_T$ under Tsaranov (1992a). The construction of $\Sigma_T$ is simplified in Cameron (1994a).] (TG: Adj, Geom)

Chelliah Selvaraj
See also M. Parvathi.


See also T. Fife.

Charles Semple & Geoff Whittle

§7: “Dowling group geometries”. A Dowling geometry of a group $G$ has a partial-field representation iff $G$ is abelian and has at most one involution. [The condition is necessary but insufficient; see Vertigan
Parongama Sen
See B.K. Chakrabarti.

Sagnik Sen
See also J. Bensmail, S. Das, P. Ochem, and R. Naserasr.

Sylvain Sené
See J. Demongeot and M. Noual.

Masakazu Sengoku

A signed graph derived from trees and cotrees is balanced. [Annot. 24 July 2010.] (SG: Bal)

Seunghyun Seo

The characteristic polynomial of the Shi threshold arrangement \{\(x_i + x_j = 0, 1 : i < j\}\}, computed modulo a large prime (the “finite field method”, Athanasiadis (1996a)). [Annot. 14 Mar 2013, corr 22 Jan 2020.] (gg: Geom, Invar)


The characteristic polynomial of the Catalan threshold arrangement \{\(x_i + x_j = 0, \pm 1 : i \neq j\}\}, computed modulo a large prime. [Annot. 20 Feb 2020.] (gg: Geom, Invar)

B. Seoane
See L.A. Fernández.

Han-Guk Seol
See S.-G. Lee.

Mark R. Sepanski
See I.B. Michael.

Jean-Sebastien Sereni
See D. Král’.

Ákos Seress
See P. Brooksbank.

Anshu Sethi
See D. Sinha.

James P. Sethna

Textbook. P. 12, fn. 16: Frustration index (“spin-glass ground states”) is polynomially equivalent to graph coloring. §12.3.4, “Glassy systems:
random but frozen”, mentions frustration due to negative circles (“a loop with an odd number of antiferromagnetic couplings”). It is not yet known how many equilibrium [ground?] states exist. Fig. 12.17, “Frustration”: An all-negative triangle with Ising spins (±1). [Annot. 28 Aug 2012.] (Phys: SG: Fr, State(fr): Exp)

E.C. Sewell
See Johnson and Padberg (1982a) for definitions. §2, “Equivalence to stable set problem”: Optimization on the bidirected stable set polytope is reduced to optimization on a stable set polytope with no more variables. Results of Bourjolly (1988a) and Hochbaum, Megiddo, Naor, and Tamir (1993a) can thereby be explained. §3, “Perfect bigraphs”, proves the conjectures of Johnson and Padberg (1982a): a transitively closed bidirection of a simple graph is perfect iff its underlying graph is perfect. [Also proved by Ikebe and Tamura (20xxa).] Dictionary: “Bi-graph” = bidirected graph. “Stable” set in $B$ = vertex set inducing no introverted edge. (SG: Ori: Incid, Geom, sw)

“Even subdivision of $K_4$” = $|\Sigma|$ where $\Sigma$ is an all-negative subdivision of $-K_4$.

P.D. Seymour
The central example is $Q_6 = C^-(-K_4)$, the clutter of (edge sets of) negative circles in $-K_4$. P. 199: the extended lift matroid $L_0(-K_4) = F_7^*$, the dual Fano matroid. Result (3.4) readily generalizes (by the negative-subdivision trick) to: every $C^-(\Sigma)$ is a binary clutter, that is, a port of a binary matroid. [This is also immediate from the construction of $L_0(\Sigma)$.] P. 200, (i)–(iii): Amongst minor-minimal binary clutters without the “weak MFMC property” are the circuit clutter of $F_7^*$ and $C^-(-K_5)$ and its blocker.
Main Thm. (§5): A binary clutter is “Mengerian” (I omit the definition) iff it does not have $C^-(-K_4)$ as a minor. (See p. 200 for the antecedent theorem of Gallai.)
[See Cornuéjols (2001a), Guenin (2001a) for more.]

(\text{sg}, \text{Par}: \text{M, Geom})


The crucial matroid \( R_{10} = G(-K_5) \). [Annot. 3 Aug 2019.] (\text{sg: par: M})


Conjecture (based on (1977a)). A binary clutter has the weak MFMC property iff no minor is either the circuit clutter of \( F_7 \) or \( C^-(-K_5) \) or its blocker.


In Thm. 6.6, p. 546, interpreting \( G \) as a signed graph and an “odd-\( K_4 \)” as a subdivision of \( -K_4 \) gives the signed graph generalization, due to Gerards and Schrijver (1986a) [also Gerards (1990a), Thm. 3.2.3]. Let \( \Sigma \) be a signed simple, 3-connected graph in which no 3-separation has \( > 4 \) edges on both sides. Then \( \Sigma \) has no \( -K_4 \) minor iff either (i) deleting some vertex makes it balanced (the complete lift matroid of this type is graphic); or (ii) it is cylindrical: it can be drawn on a cylindrical surface that has a lengthwise red line so that an edge is negative iff it crosses the red line an odd number of times [Note: the extended lift matroid of this type is cographic, as observed by, I think, Gerards and Schrijver or by Lovász]. [See Pagano (1998a) for another use of cylindrical signed graphs.] [Problem. Find the forbidden topological subgraphs, link minors, and \( Y \Delta \) graphs for cylindrical signed graphs.] [Question. Embed a signed graph in the plane with \( k \) distinguished faces so that a circle’s sign is the parity of the number of distinguished faces it surrounds. Cylindrical embedding is \( k = 1 \). For each \( k \), which signed graphs are so embeddable?]

(\text{SG: Str, Top})

Thm. 6.7, pp. 546–547, generalizes to signed graphs, interpreting \( G \) as a signed graph and an “odd cycle” as a negative circle. Take a signed simple, 3-connected, internally 4-connected graph. It has no two vertex-disjoint negative circles iff it is one of four types: (i) deleting some vertex makes it balanced; (ii) deleting the edges of an unbalanced triangle makes it balanced; (iii) it has order \( \leq 5 \); (iv) it can be orientation-embedded in the projective plane. This is due to Lovász; see, if you can, Gerards, Lovász, \textit{et al.} (1990a). [A 2-connected \( \Sigma \) has no vertex-disjoint negative circles iff \( G(\Sigma) \) is binary iff \( G(\Sigma) \) is regular iff the lift matroid \( L(\Sigma) \) is regular. See Pagano (1998a) for classification of \( \Sigma \) with vertex-disjoint negative circles according to representability of the frame matroid.]

(\text{SG: Str, m, Top})

Paul Seymour & Carsten Thomassen


“Even” means every signing contains a positive cycle. A digraph is even
iff it contains a subdigraph that is obtained from a symmetric odd-circle digraph by subdivision and a vertex-splitting operation. [Cf. Thomassen (1985a).]

L. de Sèze
See J. Vannimenus.

Bryan L. Shader
See R.A. Brualdi, S. Butler, and D.A. Gregory.

Nisarg Shah
See M. Joglekar.

[Siamak Fayyaz Shahandashti]
See S. Fayyaz Shahandashti (under ‘F’).

Mohsen Shahriari
See also S.R. Shahriary.

Mohsen Shahriari & Ralf Klamma

Mohsen Shahriari, Omid Askari Sichani, Joobin Gharibshah, & Mahdi Jalili
2016a Sign prediction in social networks based on users reputation and optimism. Social Network Analysis Mining 6 (2016), article 91, 10 pp.

Ranking vertices by “reputation”, \( RR(v_i) := \frac{d^+(v_i) - d^-_i(v_i)}{d^+_i(v_i) + d^-_i(v_i)} \), and “optimism”, \( OP(v_i) := \frac{d^+_{out}(v_i) - d^-_{out}(v_i)}{d^+_{out}(v_i) + d^-_{out}(v_i)} \), produces better predictions of arc signs. [Annot. 22 Sep 2018.] (SD: Pred: Alg)

Saeed Reza Shahriary, Mohsen Shahriari, & Rafidah MD Noor

Naomi Shaked-Monderer

Tahir Shamsher
See also S. Pirzada.

Tahir Shamsher, Mushtaq A. Bhat, & S. Pirzada
20xxa Unicyclic signed graphs with first eleven minimal energies. Submitted. (SG: Adj: Eig)

Hai-Ying Shan
See also J.-Y. Shao and L. You.

Hai-Ying Shan & Jia-Yu Shao

Ronghua Shang
See J.S. Wu.
Jia-Yu Shao
See also Y. Liu, R. Manber, H.Y. Shan, and L. You.


Forbidden subgraphs are used to characterize the signed digraphs.[Annot. 6 Mar 2011.] (SD: QSol)

Jia-Yu Shao, Jin-Ling He, & Hai-Ying Shan


Yanling Shao
See also Y.-B. Gao.

Yanling Shao & Yubin Gao


Yanling Shao, Jian Shen, & Yubin Gao


2012a The $k$th upper and lower bases of primitive nonpowerful minimally strong signed digraphs. *Linear Multilinear Algebra* 60 (2012), no. 9, 1093–1113. MR 2966153. Zbl 1252.05077. (SD)

Deepakshi Sharma
See D. Sinha.

Pranjali Sharma [Pranjali]
See also M. Acharya.

Pranjali Sharma & Mukti Acharya


Pranjali [Pranjali Sharma], Mukti Acharya, & Atul Gaur


Ram Parkash Sharma & Vikram Singh Kapil


Tushar Sharma, Ankit Charls, & P.K. Singh
2011a Community mining in signed social networks – An automated approach. In: 
ipcsit.com/vol2/28-A217.pdf

John Shawe-Taylor
See T. Pisanski.

Bo Shen
See Q. Cai.

Jian Shen
See Y.-B. Gao and Y.L. Shao.

Jia Sheng & MiaoLin Ye
2010a The spectral radius of signless Laplacian of a connected graph with given in-
2765865 (no rev).

Stephen H. Shenker
See E. Fradkin.

F.B. Shepherd
See A.M.H. Gerards and T.R. Jensen.

Laura Sheppardson
See T. Lewis.

R. Sherkati
See S. Akbari.

Steven J. Sherman
See R.B. Zajonc.

David Sherrington & Scott Kirkpatrick
See also S. Kirkpatrick.
1796. Repr. in M. Mézard, G. Parisi, and M.A. Virasoro, Spin Glass Theory 

Introduces the Sherrington–Kirkpatrick spin-glass model, a randomly 
signed and (usually) weighted $K_n$. Announcement of part of Kirkpatrick 

Ronald G. Sherwin
1975a Structural balance and the sociomatrix: Finding triadic valence structures in 

A very simple [but not efficient] matrix algorithm for counting different 
types of circles in a signed (di)graph. [“Valence” means sign, unfortu-
nately.] (sg, SD: Bal, Circ: Invar: Alg)

Jeng-Horng Sheu
See I. Gutman.

Chuan-Jin Shi


C.-J. Shi & J.A. Brzozowski


A signed hypergraph $H = (V, E, \psi)$ is a hypergraph $(V, E)$ with an incidence signature $\psi: V \times E \to \{-1, 0, 1\}$. “Underlying graph” = bipartite incidence graph with edge signs $\psi$. Sign of a path [or walk] = product of incidence signs. Motivation: via minimization, i.e., minimize the number of connections between different planar layers of a two-layer circuit. [See Rusnak (2010a) for a different development of the same definitions. Path signs are different; the normal sign for signed graphs has an extra factor $-1$ for each edge.] $e$ is “balanced” by a bipartition $V = V_1 \cup V_2$ when incidences of $e$ are in the same $V_i$ iff they have the same sign. $H$ is “balanced” if some bipartition balances every edge. Thm. 3.1: $H$ is balanced iff every circle is positive. [I.e., antibalance, since walk signs are different from the norm.] Proof: Constructive [similar to but less exact than algorithms for signed graphs as in Harary and Kabell (1980a)], yielding Cor. 3.1: Testing balance takes linear time. Thm. 3.2: $H$ is balanced iff its incidence dual is balanced. “Maximum balance problem”: Minimize the number of unbalanced edges. Thm. 4.1: This is NP-complete, even for cubic graphs. [Known, as it contains the max-cut problem.] Thm. 4.2: NP-complete for planar signed hypergraphs with maximum degree $> 3$. (For max degree $\leq 3$, polynomial-time algorithms are given in Shi (1993b).) Problem: Minimum Covering: Find the minimum number of bipartitions of $V$ such that every edge is balanced by one of the bipartitions. Equivalently, decompose $H$ into the smallest number of balanced subhypergraphs. [See Zaslavsky (1987b) for signed graphs.] Thm. 5.1: NP-complete. Proof: Reduction to graph colorability via decomposability of a graph into bipartite subgraphs [special case of signed-graph decomposition as in Zaslavsky (1987b)].


§8, “Related notions: Signed graphs and $(0, \pm 1)$-matrices”. §8.1, “Harary’s signed graphs”, compares their work with Harary (1953a) [no mention of Harary and Kabell (1980a)]. §8.2, “Restricted unimodularity and balanced $(0, \pm 1)$ matrices”: The incidence matrix of $H(H)$ if $H$ is a graph
[H(−H) in the normal definition] is totally unimodular iff −H is balanced [essentially, Heller and Tompkins (1956a)].

[All problems and methods are equivalent to the similar problems for the signed graph derived by replacing each hyperedge by a balanced complete graph with Harary bipartition given by the sign bipartition of the hyperedge’s incidences.] [Annot. 4 Nov 2010.]

(SH: Incid, Bal, Alg, SG)

C.-J. Shi, A. Vannelli, & J. Vlach

(SH: sg: Bal)


Guodong Shi, Claudio Altafini, & John S. Baras

(SG: Dyn)

Guodong Shi, Alexandre Proutiere, Mikael Johansson, John S. Baras, & Karl H. Johansson


Jinsong Shi
See R.L. Li.

Lingsheng Shi & Zhang Zhang

Yongtang Shi
See B.F. Huo.

Kazuki Shibata
See H. Ohsugi.

V.S. Shigehalli & Kenchappa S. Betageri
Laplacian energy of the signed graph of Adiga, Sampathkumar, et al. (2013a).


*Cf. Adiga, Sampathkumar, et al. (2013a).*


*Cf. Adiga, Sampathkumar, et al. (2013a).*

Wei-Kuan Shih
See also Kuo–Chern–Shih (1988a).

Wei-Kuan Shih, Sun Wu, & Y.S. Kuo

Real edge weights make max and min cut equivalent. A faster algorithm than before. [This problem includes frustration index \(I(\Sigma)\).] Dictionary: “positive”, “negative” cut means total weight. [Annot. 19 Dec 2014.]

(WG, sg: fr: Alg)

Shijin T V [T.V. Shijin]
See also S. Hameed.

Shijin T V, Germina K A, & Shahul Hameed K

*Cf. Hameed et al. (2021a).* \(\Sigma^k_{\text{max}}\) has edge \(uv\) signed \(\sigma^k_{\text{max}}(u,v)\) when \(d(u,v) \leq k. \exists \Sigma^k\) if \(\Sigma^k_{\text{max}} = \Sigma^k_{\text{min}}.\) Theorems on balance and distance compatibility. [Annot. 27 Sep 2020.]

(SG: Bal, Adj(Gen))

Shijin T V, Soorya P, Shahul Hameed K, & Germina K A

*Cf. Hameed, Shijin, et al. (2021a).* Thm. 2.2 characterizes distance-compatible signed graphs, somewhat more simply than the definition. §3 characterizes distance compatibility for Cartesian and lexicographical products and partially for tensor product. §4: Formulas for signed distance matrices of the former two. §5: Examples. [Annot. 19 Mar 2021.]

(SG)(SG: Adj(Gen), Eig)

Akihiro Shikama
See A. Funato and T. Hibi.

Young-hee Shin
See J.H. Kwak.

Guy Shinar & Martin Feinberg

The “c-pair” (“complex pair”) edges act like a negative edge. [Annot. 21 Jan 2015.]

(sd, sg: Dyn, Chem)

Alana Shine
See M. Beck.

J. Shiozaki, H. Matsuyama, E. O’Sshima, & M. Iri
Continuation of Iri, Aoki, O’Shima, and Matsuyama (1979a).
(SD, VS: Appl, Alg)

H. Shirazi
See G. Coutinho.

Shailaja S. Shirkol
See P.R. Hampiholi.

Wai Chee Shiu
See J.M. Guo.

[Shivakumar Swamy C.S.]
See S. Swamy C.S.

K. Shivashankara
See P.S.K. Reddy.

S.B. Shlosman
See Dobrushin and Shlosman (1985a).

L. Shobana [Shobana Loganathan]
See also J. Baskar Babujee and B. Vasuki.

L. Shobana & B. Vasuki
More, as in Baskar Babujee and Loganathan (2011a).
(Lab: VS: SG, Bal)

Elizabeth G. Shrader & David W. Lewit
For \( \Gamma \subseteq K_n \) and signing \( \sigma \) of \( \Gamma \), “plausibility” = mean and “differentiability” = standard deviation of \( f(K_n, \sigma') \) over all extensions of \( \sigma \) to \( K_n \), where \( f \) is any function that measures degree of balance. Proposed: tendency toward balance is high when plausibility and differentiability are high. A specific \( f \), based on triangles and quite complicated, is studied for \( n = 4 \), with experiments.

(\( \text{sg, fr, PsS} \))

S.R. Shreyas & M. Joseph
Cf. B.D. Acharya (2012a). Characterizations are largely in terms of the lengths of positive and negative sections. [Annot. 25 Dec 2020.]
(SG: Lab)

A.S. Shrikanth
See C. Adiga.

Mohan S. Shrikhande
See Y.J. Ionin.

Shrikanth A.S.
See C. Adiga.
Piyush Shroff
See L. Rusnak.

Jinlong Shu
See G.L. Yu and M.Q. Zhai.

Alan Shuchat
See R. Shull.

Randy Shull, James B. Orlin, Alan Shuchat, & Marianne L. Gardner
[See Coullard, del Greco, and Wagner (1991a).] (Bic: Bases)

Randy Shull, Alan Shuchat, James B. Orlin, & Marianne Lepp

E.E. Shult
See P.J. Cameron.

Robert Shwartz
See M. Amram and Y. Cherniavsky.

Jana Šiagiová
See also J. Širáň.

Omid Askari Sichani
See M. Shahriari.

V.M. Siddalingaswamy

Heike Siebert
See also A. Bockmayr.

The local and global interaction graphs are signed digraphs (p. 111). (SD: Dyn, Biol)


Heike Siebert & Alexander Bockmayr


David Siegel
See H. Kunze.

Mark Siggers
See R.C. Brewster.

Mathieu Dutour Sikirić, Anna Felikson, & Pavel Tumarkin

Schwartz (2002a) is more general but excepts the exceptional root systems. [Annot. 9 Apr 2016.] (gg: M: Aut)

Ilda P.F. da Silva

B. Simeone

Luca Simeoni
See S. Klamt.
Slobodan K. Simić
See also M. Andelić, F. Belardo, D.M. Cardoso, D.M. Cvetković, and X.Y. Geng.


Slobodan K. Simić, Milica Andelić, Carlos M. da Fonseca, & Dejan Živković

Let $H_i'$ denote the bridges of a cutpoint $u$ in $\Gamma$ with each edge subdivided once. Order so $\varepsilon_i := \lambda_1(H_i' \setminus u)$ is decreasing. Cor. 3.4 (restated): $\varepsilon_2^2 \leq \lambda_2(\Gamma) \leq \varepsilon_2^2$, with $= \text{iff } \varepsilon_1 = \varepsilon_2$. [Annot. 20 Jan 2015.] (par: Lap: Eig)

Slobodan K. Simić & Zoran Stanić


§4, “Determination by the signless Laplacian spectrum”. Thm. 4.1: Among the forests whose trees are Smith trees (excluding a few), the three minimal graphs not determined amongst all graphs by Spec $L(\Gamma)$. [Annot. 20 Jan 2015.] (par: Lap: Eig)


Reconstructing the characteristic polynomial (of $A(\Sigma)$) from vertex-deleted subgraphs: solved for cyclomatic number 0 [known: same as for graphs] and 1. A trivial counterexample: positive and negative $C_n$. Question: Is there a counterexample with nonisomorphic $|\Sigma_1|, |\Sigma_2|$? [Annot. 18 Dec 2016.] (SG: Adj)


Rodica Simion

§6, “Dowling lattices”: They are an example, thus having an EL-labelling induced from $\Pi_n$. [Annot. 9 Apr 2016.] (gg: M, Invar)


“Type-B noncrossing partitions” are certain signed partial partitions of the ground set; i.e., certain elements of the Dowling lattice of $\{\pm\}$. (gg: M)
R. Simion & D.-S. Cao
1989a Solution to a problem of C. D. Godsil regarding bipartite graphs with unique
Zbl 688.05056.
Answering Godsil (1985a): \(|\Sigma| = \Gamma\) iff \(\Gamma\) consists of a bipartite graph
with a pendant edge attached to every vertex. [Surely there is a signed-
graphic generalization of Godsil’s and this theorem in which bipartite-
ness becomes balance or something like it.]

Aron Simis, Wolmer V. Vasconcelos, & Rafael H. Villarreal

J.M.S. Simões-Pereira
1972a On subgraphs as matroid cells. *Math. Z.* 127 (1972), 315–322. MR 0317973 (47

A family of (isomorphism types of) [simple] connected graphs is “ma-
troidal” if for any \(\Gamma\) the class of subgraphs of \(\Gamma\) that are in the family
constitute the circuits of a matroid on \(E(\Gamma)\). Bicircular and even-cycle
matroids are the two nicest examples. A referee contributes the even-
cycle matroid [cf. Tutte (1981a), Doob (1973a)]. Thm.: The circle and bicircular matroids [and free matroids] are the only
such matroids.

1975a On matroids on edge sets of graphs with connected subgraphs as circuits. *Proc.
Zbl 264.05126.

Partial results on describing matroidal families of simple, connected
graphs. Five basic types: free [omitted in the paper], cofree, circle,
bicircular, and even-cycle. If the family does not correspond to one of
these, then every member has \(\geq 3\) independent circles and minimum
degree \(\geq 3\). (MtrdF, Bic, EC: Gen)

1978a A comment on matroidal families. In: *Problèmes Combinatoires et Théorie des

Two small additions to (1973a), (1975a); one is that a matroidal family
not one of the five basic types must contain \(K_{p,q(p)}\) for each \(m \geq 3\), with
\(q(p) \geq p\). (MtrdF, Bic, EC: Gen)

“Count” matroids (see N. White (1986a)) in §4.3; Schmidt’s (1979a) remarkable generalization in §4.4.

(KG: MtrdF, Bic, EC: Gen: Exp, Exr, Ref)

Klaus Simon
See T. Raschle.

[C. De Simone]
See C. De Simone (under ‘D’).

M. Simonovits
See B. Bollobás, J.A. Bondy, and P. Erdős.

Daniel Simson
See also R. Bocian, M. Felisiak, M. Gąsiorek, S. Kasjan, G. Marczak, and A. Polak.


Daniel Simson & Katarzyna Zajac


Alistair Sinclair
See M. Jerrum.

Amit Singer
See A.S. Bandeira.

Amrik Singh
See B. Adhikari.

P.K. Singh
See T. Sharma.

Rajiv R.P. Singh
See also M.E. Fisher.

Rajiv R.P. Singh and Sudip Chakravarty

Ranveer Singh & Bibhas Adhikari

**Ranveer Singh & Ravindra B. Bapat**


**Tarkeshwar Singh**

See also M. Acharya and S.B. Rao.


[Cf. M. Acharya and Singh (2004a), (2003b). Generalizing the definition: Given: a graph with $r$-colored edges, $m_i$ of color $i$; a list $L$ of $n$ integers. Required: A bijection $\lambda : V \to L$ such that, if $f(vw) := |\lambda(v) - \lambda(w)|$, then $f$ restricted to color class $i$ is a bijection to $[m_i]$.] Signed graphs are the case $r = 2$. Skolem gracefulfulness is the case where $\lambda$ exists for $L = [n]$. Hooked Skolem gracefulfulness is the case where $\lambda$ exists for $L = [n + 1] \backslash \{n\}$. Results from M. Acharya and Singh (2010a) and Singh (20xxa), examples, some proofs. (SGc: Lab)


“Graceful” means $(1,1)$-graceful, $r = 1$, as at M. Acharya and Singh (2004a). $C_3^k$ is the windmill with $k$ blades. Let $\Sigma$ have $v$ negative rim edges, $1 \leq v \leq k/2$, and no other negative edges. Thm. 10: $\Sigma$ is graceful if $k \equiv 0 \text{mod} 4$ and $v$ is even. Thms. 11, 12: $\Sigma$ is graceful if $k \equiv 1,2 \text{mod} 4$. [Annot. 21 July 2010.] (SGc: Lab)


See (2008a). Thm.: A signed $k$-edge matching is hooked Skolem grace-
ful if $k \equiv 0 \pmod{4}$ and $\#E^-$ is odd, or $k \equiv 2 \pmod{4}$ and $\#E^-$ is even, or $k \equiv 3 \pmod{4}$. Curiously complementary to the theorem of M. Acharya and Singh (2010a).

Tarkeshwar Singh & Natasha D’Souza


Some graceful signed trees (see M. Acharya and Singh (2004a)). Every signed tree is an induced subgraph of a graceful signed tree. [Annot. 31 Aug 2010.]

N.M. Singhi


N.M. Singhi & G.R. Vijayakumar


A short proof that every such signed simple graph contains an induced subgraph with least eigenvalue $= -2$. [Their $M := 2I + A(\Sigma)$ is the Laplacian matrix of $-\Sigma$.]

Deepa Sinha

See also M. Acharya.


[Partial description] $\Sigma$ is “sign compatible” if $\exists X \subseteq V$ such that $E^- = E.X$. It is “canonically sign compatible” if $X = \mu_{-1}^e(-)$ (cf. Sampathkumar (1984a)). [Annot. 12 Oct 2010.]

Deepa Sinha & Mukti Acharya


Extensions to $\Lambda_{BC}(\Sigma)$ of M. Acharya and Sinha’s (2002a) characterization of balance in the Behzad–Chartrand (1969a) line graph $\Lambda_{BC}(\Sigma)$ and to $\Lambda_k(\Sigma)$ of Acharya, Acharya, and Sinha’s (2009a) criterion for consistency of $\Lambda(\Sigma)$.

Deepa Sinha & Ayushi Dhama


Cf. Sinha (2005a). (SG: Bal(Gen))


Characterizes balance, clusterability, and for some $n$ also canonical consistency and sign compatibility. [Annot. 11 Apr 2016.] (Algeb: SG: Bal, Clu, VS)

Thm. 2.2: Fairly elementary characterization of $\Sigma \simeq -\Sigma$. Thm. 2.3: Same for $\Sigma \cong -\Sigma$. [Problem. Find complete structural characterizations.] [Annot. 5 May 2014, 15 May 2018.] (SG: Sw, Aut(Gen): Str)

Define (semi)total signed graphs via the $\times$-line signed graph of M. Acharya (2009a). Characterizes those that are compatible (cf. Sinha (2005a)) with the canonical marking of the (semi)total signed graphs. [Annot. 9 Apr 2014.] (SG: LG(Gen): Bal(Gen))


20xxa On the unitary Cayley meet signed graphs $S_n^\wedge$. Submitted. (SG: Bal, Clu, Bal(Gen))

20xxb Negation-switching invariant $t$-path signed graphs, $t \leq 3$. Submitted. (SG: LG(Gen): Sw)

Deepa Sinha, Ayushi Dhama, & B.D. Acharya


Deepa Sinha & Pravin Garg


Consistency of the canonical vertex signature of certain graphs related to the line graph and total graph of $\Sigma$; see e.g. (2011f), (2015a), (2015b). [Annot. 31 Aug 2010.] (SG: LG(Gen): Bal(Gen))


Consistency of the canonical vertex signature of two kinds of line graph: (Thm. 2) $\Lambda_{BC}(\Sigma)$ of Behzad–Chartrand (1969a) and (Thm. 8) $\Lambda_{x}(\Sigma)$ of M. Acharya (2009a). [Annot. 25 Mar 2011.] (SG: LG: VS: Bal(Gen))


$T(\Sigma)$ characterizes balance and consistency of the total signed graph $T(\Sigma)$. The vertex signs are $\mu_1(v) := \sigma(E(v))$ ($E(v)$ := the vertex star), $\mu_1(e) = \sigma(e)$. The edge signs are $\sigma_T(uv) := \sigma(e_{uv})$, $\sigma_T(ue) := \sigma(e)\mu_1(u)$, and $\sigma_T(ef) := \sigma(e)\sigma(f)$ [thus $T(\Sigma) \supseteq \Lambda_{x}(\Sigma)$ of M. Acharya (2009a)]. [Annot. 13 Oct 2009, 20 Dec 2010.] (SG, VS: Bal, Bal(Gen))


$\Sigma$ is “regular” if $\Sigma^+$ and $\Sigma^-$ are regular. For the edge signs of line graphs and total graph see (2011b). Characterizes $\Sigma$ such that $\Lambda_{BC}$ or $\Lambda_{x}$ or $T(\Sigma)$ is regular. Dictionary: “signed-regular” = regular. [Annot. 25 July 2011.] (SG: LG, LG(Gen))


Thm. 2.3 characterizes semi-total signed graphs. Thm. 3.2 characterizes semi-total point signed graphs. Thm. 4.4 characterizes total signed graphs. Each result applies a pre-existing characterization of underlying graphs. [Annot. 23 Nov 2014.] (SG: LG, LG(Gen))


The unitary Cayley graph $X_n = \langle \mathbb{Z}_n, \{ab : \exists (b - a)^{-1}\} \rangle$. $S_n = \langle X_n, \sigma \rangle$ where $\sigma(ab) = -1$ iff $a^{-1} \not= b^{-1}$. Thm. 4: $S_n$ is balanced iff $n$ is even or a prime power. Cor. 5: $S_n$ is antibalanced iff $n$ is even. Cor. 7: $\Lambda_{BC}(S_n)$ is balanced iff $n$ is a prime power. Thm. 20: Let $n$ have at most 2 distinct odd prime factors. $S_n$ is canonically consistent iff $n$ is odd, evenly even, 2, or 6. [Annot. 16 Jan 2012.] (SG: Bal, LG, Bal(Gen): Algeb)

Similar to (2011b), but for $T_2(\Sigma) := T(\Sigma)$ without line-graph edges.  
[Annot. 13 Oct 2009.]


*Cf. Sinha and Garg (2011a).*


Tensor product defined by Mishra (1974a). For connected signed graphs: Thm. 2.6: $\Sigma_1 \otimes \Sigma_2$ is balanced iff $\Sigma_1$ and $\Sigma_2$ are both balanced or both antibalanced. Thm. 3.1: It is antibalanced iff one is balanced and the other is antibalanced. [Annot. 23 Nov 2014.]


*Cf. Sinha and Garg (2011a).*


*Cf. Sinha and Garg (2011a).*

Deepa Sinha, Pravin Garg, & H. Saraswat


Deepa Sinha & Anita Kumari Rao


Deepa Sinha, Anita Kumari Rao, & Ayushi Dhama


Main result is Thm. 3.4: $\text{Spec } A(\Sigma)$ is sign-symmetric iff $\Sigma \simeq -\Sigma$. [N.B. $\Sigma \simeq -\Sigma$ is unsolved.] ($\Sigma_i := a$ kind of signed $t$-path graph. Other results, e.g., Thm. 2.8: For signed $K_n$, $(\Sigma)_{2} \simeq -\Sigma$ iff $\Sigma$ is balanced or clusterable in a limited way. [Annot. 27 May 2018.]

Deepa Sinha, Anita Kumari Rao, & Pravin Garg

\(\Sigma\) is \((i,j)\)-regular if \(\Sigma^+\) is \(i\)-regular and \(\Sigma^-\) is \(j\)-regular. Embedding is as a subgraph. The aim is to minimize the order of the supergraph. [Annot. 13 Mar 2018.]

**Deepa Sinha & Anshu Sethi**


Definition: Sinha (2005a). The algorithm detects the forbidden subgraphs: a path with edges \(-,+,−\) and a triangle with edges \(-,+,−\). [Annot. 6 Jan 2016.] (SG: Alg)


20xxa An algorithmic characterization of line signed graph. Submitted.

For a signed simple graph \(\Sigma\), algorithms to construct (§3) the line graph \(\Lambda(\mid\Sigma\mid)\); (§4) \(\Gamma'\) such that \(|\Sigma\mid = \Lambda(\Gamma')\), if it exists; (§5) the Behzad–Chartrand (1969a) line graph \(\Lambda_{BC}(\Sigma)\); (§6) \(\Sigma'\) such that \(\Sigma = \Lambda_{BC}(\Sigma')\), if it exists. Cf. M. Acharya and Sinha (2005a). [Annot. 24 Dec 2014.] (SG: LG: Alg)

**Deepa Sinha & Deepakshi Sharma**

2014a Signed graphs whose 2-path signed graphs are isomorphic to their square signed graphs. Manuscript, 2014. Full version of (2014b). (SG: LG( Gen), Alg)


Extended abstract of (2014a). (SG: LG( Gen), Alg)

2016a On square and 2-path signed graph. *J. Interconnection Networks* 16 (2016), no. 1, article 1550011, 19 pp. (SG: LG( Gen), Alg)


2020a Characterization of 2-path signed network. *Complexity* 2020 (2020), article 1028941, 13 pp. (Σ)₂ := (V, E', σ'), where uv ∈ E' iff d(u, v) = 2, and σ'(uv) = − iff every 2-path uwv is all negative. Thm. 1 characterizes (Σ)₂’s. Thms. 2–4: balance, clusterability, sign-regularity of (Σ)₂. [Some seem too complicated. Some are not clear.] [Annot. 9 Nov 2020.] (SG: VS, Bal, Clu, Alg)

**Deepa Sinha, Deepakshi Sharma, & Bableen Kaur**


**Deepa Sinha, Somya Upadhyaya, & Priya Kataria**


For definition cf. M. Acharya and Sinha (2006a). Thm. 6: Σ is a common-edge signed graph iff |Σ| is a common-edge graph and its edges decompose into homogeneously signed complete graphs. §4, “Algorithm to output $C_E$-root sigraph of a given common-edge sigraph”. §5, “Complexity of COMMON-EDGE SIGRAPH”: It is $O(n^2\#E)$. [Annot. 23 Nov 2014.] (SG: LG(Gen))

**John Sinkovic**

See M. Arav.

**Jozef Širáň**

See also D. Archdeacon, P. Gvozdjak, C.H. Li, and B.D. McKay.

A signed graph orientation-embeds in only one surface iff any two circles are vertex disjoint. (SG: Top)


Richard A. Duke (The genus, regional number, and Betti number of a graph. *Canad. J. Math.*, 18 (1966), 817–822. MR 0196731 (33 #4917) proved that the (orientable) genus range of a graph forms a contiguous set of integers. Stahl (1978a) proved the analog for nonorientable embeddings. Širáň shows this need not be the case for the demigenus range of an unbalanced signed graph. However, any gaps consist of a single integer each. The main examples with gaps are vertex amalgamations of balanced and uniquely embeddable unbalanced signed graphs, but a 3-connected example is $+W_6$ together with the negative diameters of the rim. *Question* 1 (Širáň). Do all gaps occur at the bottom of the demigenus range? [*Question* 2. Can one in some way derive almost all signed graphs with gaps from balanced ones?] (SG: Top)

Jozef Širáň, Jana Šiagiová, & Marián Olejár


Connectivity and automorphisms of a covering graph of a gain graph (“voltage graph”). [Annot. 21 July 2010.] (GG: Cov: Aut, Exp)

Jozef Širáň & Martin Škoviera


The maximum demigenus $d_M(\Sigma) =$ the largest demigenus of a closed surface in which $\Sigma$ orientation embeds. Two formulas are proved for $d_M(\Sigma)$: one a minimum and the other a maximum of readily computable numbers. Thus $d_M(\Sigma)$ has a “good” (polynomial) characterization. Along the way, several results are proved about single-face embeddings. *Problem* (§11). Characterize those edge-2-connected $\Sigma$ such that $\Sigma$ and all $\Sigma \setminus e$ have single-face embeddings. [A complex and lovely paper.]

[Re single-face embeddings, cf. Bernardi and Chapuy (2011a) and Isenmann and Pecatte (2017a).] (SG: Top)

Jozef Širáň & Thomas W. Tucker


P. 203 mentions edge signs. P. 218 says the Petrie dual of a rotation system on $\Sigma$ is the same rotation system on $-\Sigma$. [Annot. 26 Aug 2018.] (sg: Top, sw: Exp)

[P. Siva Kota Reddy]

See P.S.K. Reddy (under ‘R’).
B. Sivakumar
See also M. Parvathi.

Vaidy Sivaraman
See also J. Maharry.

Thm.: $D(\Sigma) := \min_X l(\Sigma^X) \leq \frac{3}{8}n$ for cubic $\Sigma$. [Cf. Sehrawat and Bhattacharyya (2019b).]


The graphs for which $G(\Gamma, \emptyset)$ is a frame matroid of a signed graph: iff $G(\Gamma, \emptyset)$ is ternary, and other characterizations including forbidden subgraphs. Successor to Matthews (1977a), and implicitly Zaslavsky (2007a) for group $Z_2$. [Annot. 1 Oct 2017, rev 11 Jun 2019.]

Vaidy Sivaraman & Daniel Slilaty

$\Gamma$ characterized for $|\mathfrak{S}| \leq 6$ by structure, forbidden minors, and algorithmic recognition. Successor to Zaslavsky (2007a), Sivaraman (2014a), and Chun, Moss, Slilaty, and Zhou (2016a). [Annot. 11 Jun 2019.]


The 3-connected matroids that are bicircular and dual bicircular are the free swirls $G(2C_n, \emptyset)$ and their minors, with exceptions of rank and dual rank $\leq 5$. [Annot. 29 Dec 2020.]

Vaidy Sivaraman & Thomas Zaslavsky
20xxa The seven signed Heawood graphs. In preparation.

Successor to Zaslavsky (2012b), with several general theorems. There are 7 switching isomorphism classes of signatures of the Heawood graph $H$. $l_l, l_0, \chi, Q$ (inclusterability index) are computed. General thm.: For subcubic $|\Sigma|$, $l_0(\Sigma) = l(\Sigma)$ (for $-\Gamma$ see Choi, Nakajima, and Rim (1989a)). If $#E^-(\Sigma) = l(\Sigma)$, then $Q(\Sigma) = l(\Sigma)$. [Annot. 1 Oct 2017.]

A. Skhreîver [A. Schrijver]
See A. Schrijver.

D.B. Skillicorn
See Q. Zheng.

Bjarke Skjernaa
See J.M. Byskov.
Howard Skogman
See N. Reff.

Martin Škoviera
See also E. Máčajová, A. Malnič, R. Nedela, and J. Širáň.


Daniel C. Slilaty


(SG: Rand, Enum, Top)
When does the frame matroid \( G(\Omega) \) determine the biased graph \( \Omega \)? Given \( \Omega \) and \( \Omega_0 \), without isolated vertices, loose or half edges, or balanced loops. Assume \( \Omega \) is 3-connected and contains three vertex-disjoint unbalanced circles, at most one of which is a loop. Thm. 2: \( G(\Omega) \cong G(\Omega_0) \) if\( f(\Omega) = f(\Omega_0) \). [Annot. 14 Feb 2013.] (GG: M: Str, Circ)


Thm.: The signed graphs with no two vertex-disjoint negative circles are those with a balancing vertex, or obtained from a projective-planar signed graph (cf. Zaslavsky (1993a)) or from \([-K_5]\] by \( t \)-summation with balanced signed graphs for \( t = 1, 2, 3 \). (Previously announced in less general form by Lovász (see Seymour (1995a)) but the proof was incorrect.) [Major Problem. Characterize the biased graphs having no two vertex-disjoint unbalanced circles. Lovász (1965a), q.v., solved the contrabalanced case.] (GG: M: Str, Top, Circ)


Integral gains \( \phi : E \rightarrow \mathbb{Z} \) induce a cycle-space homomorphism \( \hat{\phi} : Z_1(\Gamma) \rightarrow \mathbb{Z} \). Let \( f : Z_1(\Gamma) \rightarrow \mathbb{Z} \). Thm. 3: \( f(W) \leq k\#W \) for every walk \( W \) iff \( f = \hat{\phi} \) for some \( \phi \) satisfying \( \max |\phi(e)| \leq k \). Thm. 2: For odd \( k \), if also \( f(W) \equiv \#W \mod 2 \), there is \( \phi \) which assumes only odd values; and conversely. [Annot. 5 Sept 2010.] (GG)

20xxa Connectivity in signed-graphic matroids. Submitted. (SG: M: Str)

Daniel C. Slilaty & Hongxun Qin


All frame matroids (of biased graphs) that are wheels and whirls, characterized topologically by embeddings in the projective plane (wheels) and the cylinder (whirls). (GG: M: Str)


Graphical biconnectivity of \( \Omega \) vs. matroid connectivity of \( G(\Omega) \), generalizing concepts developed by Wagner (1985a) for the bicircular matroid. (GG: M: Str)

Daniel C. Slilaty & Thomas Zaslavsky


Daniel Slilaty & Xiangqian Zhou

N.J.A. Sloane
See P.C. Fishburn, R.L. Graham, and C.L. Mallows.

Kaleigh Smith
See B. Reed.

Alex Smola
See S.H. Yang.

Chris Smyth
See J. McKee.

J. Laurie Snell
See J. Berger and J.G. Kemeny.

El Houssine Snoussi
See also J.-P. Comet and D. Thieffry.


El Houssine Snoussi & Rene Thomas

Lynea Snyder
See Y. Duong.

Moo Young Sohn
See J.H. Kwak.

Alan D. Sokal
See also A.D. Scott.


The parametrized dichromatic polynomial with parameters \( d_e = 1 \), called the “multivariate Tutte polynomial”. Partly expository, partly new. [See Zaslavsky (1992b).] (SGw: Gen: Invar)

James P. Solazzo
See D.M. Duncan and T.R. Hoffman.

Patrick Solé & Thomas Zaslavsky

Among other things, improves some results in Akiyama, Avis, Chvátal, and Era (1981a). Thm. 1: For a loopless graph with \( c \) components,
\[ D(\Gamma) \geq \frac{1}{2}m - \sqrt{\frac{1}{2} \ln 2 \sqrt{m(n - c)}}. \]

Thm. 2: For a simple, bipartite graph, \[ D(\Gamma) \leq \frac{1}{2}(m - \sqrt{m}). \]

**Conjecture.** The best general asymptotic lower bound is \[ D(\Gamma) \geq \frac{1}{2}m - c_1 \sqrt{mn} + o(\sqrt{mn}) \] where \( c_1 \) is some constant between \( \frac{1}{2} \ln 2 \) and \( \frac{1}{2} \pi \).

**Question.** What is \( c_1 \) for, e.g., \( k \)-connected graphs? Thm. 4 gives girth-based upper bounds on \( D(\Gamma) \). §5, “Embedded graphs”, has bounds for several examples obtained by surface duality. All proofs are via covering radius of the cutset code of \( \Gamma \). (SG: Fr, Top)

Extends to \( r = 5 \) the exact values of \( D(K_{r,s}) \) for \( r \leq 4 \) in Brown and Spencer (1971a). [But \( r = 5 \) has errors. Extended correctly to all \( r \) by Bowlin (2009a, 2012a).] [Annot. rev. 14 Feb 2011.] (SG: Fr)

Sylvain Soliman
See F. Fages and K. Sriram.

Louis Solomon
See P. Orlik.

N.D. Soner
See R. Rangarajan.

Dongjin Song & David A. Meyer


Huimin Song
See X.Q. Qi.

Joungmin Song


Regions of the hyperplane arrangement \( J_n := \{x_i + x_j = 1 \text{ and } x_i = 0, 1\} \) are counted via graph theory related to \( -K_n \). [The hyperplanes are translates of the hyperplanes in \( H[-K_n^\bullet] \). This calls for generalization via signed graphs.] [Annot. 14 Apr 2017.] (sg: par: Geom: Invar)


(SG: par: Geom: Invar)

**Sang-Oak Song**
See G. Lee.

**Song Yi-Zhe**
See B. Xiao.

**Zi-Xia Song**
See K. Kawarabayashi.

**Eduardo D. Sontag**


P. 13 describes how a signed digraph arises from differential equations, and that it is “monotone” [= isotope] if it has no negative cycles. [Annot. 25 Jan 2015.]

(Biol: Dyn: SD: Exp)


(SD, SG: Bal, Fr: Dyn, Biol)


Conference version of (2007b); almost the same. [Annot. 23 Jan 2015.]

(SD, SG: Bal, Fr: Dyn, Biol: Exp, Ref)


Dictionary: “graph” = signed signed digraph; “spin assignment” = state = function $\zeta : V \to \{+1, -1\}$; edge “consistent” with $\zeta = \text{satisfied}$ edge $(\sigma(e) = \zeta_i \zeta_j)$; “consistent spin assignment” $\Sigma$ = potential function $\zeta$ (edge directions are ignored); “monotone” = balanced (undirected); “consistency deficit” = frustration index (undirected).

(SD, SG: Bal, Fr, Dyn, Biol: Exp, Ref)

**Eduardo Sontag, Alan Veliz-Cuba, Reinhard Laubenbacher, & Abdul Salam Jarrah**

The directed frustration index $l(\vec{\Gamma}, \sigma)$ (called the “PF-distance”) of a signed digraph is the smallest number of edges whose signs should be changed to eliminate all negative cycles. This index is a measure of the number of independent negative cycles. Then $l_{\text{max}}(\vec{\Gamma}) := \max_{\sigma} l(\vec{\Gamma}, \sigma)$. P. 522: An algorithm “Distance to PF” for $l(\vec{\Gamma}, \sigma)$ with strongly connected $\vec{\Gamma}$ (sufficient, since $l$ is additive on strong components and a negative loop adds 1). Dictionary: “directed cycle” = cycle; “cycle” = circle; “odd parity” = negative sign; “positive feedback” (PF) = cycle balance (no negative cycles); “$\|\vec{\Gamma}\| = l_{\text{max}}(\vec{\Gamma})$.

A unate Boolean function $f : \mathbb{F}_2^n \to \mathbb{F}_2^n$ has a (signed) interaction digraph $\mathcal{D}(f)$. A computational experiment tests the connection between $l(\mathcal{D}(f))$ and the number and length of attractors (limit cycles) of $f$ in $\mathbb{F}_2^n$, which appear to be direct and inverse, respectively. Dictionary: “unate” = each component function is monotone (isotone or antitone); “monotone” = isotone (monotone weakly increasing); “signed dependency graph” = (signed) interaction digraph, “distance-to-positive-feedback”, “PF-distance” = frustration index. [Annot. 16 Jan 2015.]

(SD, SG: Fr, Alg)

P. Soorya
See S. Hameed and T.V. Shijin.

Éric Sopena
See C. Charpentier and R. Naserasr.

Nicola Soranzo
See also G. Iacono.

Nicola Soranzo, Fahimeh Ramezani, Giovanni Iacono, & Claudio Altafini
2012a Decompositions of large-scale biological systems based on dynamical properties. Bioinformatics 28 (2012), no. 1, 76–83. (SD)

Mauricio Soto
See E.G. Pardo.

C.M. Soukoulis
See D. Blankschtein.

Christophe Soulé
See also M. Kaufman.


Mona Souri
See also S. Akbari.

M. Souri, F. Heydari, & M. Maghasedi

N. Sourlas
See S. Caracciolo.
B.W. Southern, S.T. Chui, & G. Forgacs
Physics of signed square lattice graph, fully frustrated (all positive except for all-negative alternating vertical lines). Reduced to the “8-vertex” physics model by taking alternating sites (vertices) and observing they are 4-valent and all or half positive. [Cf. Garel and J.M. Maillard (1983a).] [Annot. 16 Jun 2012.] (Phys: sg)

Emilio De Santis
See also F. Camia.

E. De Santis & A. Gandolfi

Cid C. de Souza
See R.M.V. Figueiredo.

[Natasha D’Souza]
See N. D’Souza (under ‘D’).

Tadeusz Sozański
Σ denotes a signed $K_n$. The “level of balance” (“indice du niveau d’équilibre”) $\rho(\Sigma) :=$ maximum order of a balanced subgraph. [Complement of the vertex deletion number $l_0(\Sigma).$] Define distance $d(\Sigma_1, \Sigma_2) := \#(E_1+ \Delta E_2+)$. Say $\Sigma$ is $p$-clusterable if $\Sigma^+$ consists of $p$ disjoint cliques [its “clusters”]. Thm. 1 evaluates the frustration index of a $p$-clusterable $\Sigma$. Thm. 2 bounds $l(\Sigma)$ in terms of $n$ and $\rho(\Sigma)$. A negation set $U$ for $\Sigma$ “conserves” a balanced induced subgraph if they are edge-disjoint; it is “(strongly) conservative” if it conserves some (resp., every) maximum-order balanced induced subgraph. Thm. 3: Every minimum negation set conserves every balanced induced subgraph of order $> \frac{2}{3}n$. Thm. 4: A minimum negation set can be ordered so that, successively negating its edges one by one, $\rho$ never decreases. (SG: KG: Fr, Clu)

“Weak isomorphism” = switching isomorphism. Principal results: The number of switching nonisomorphic signed $K_n$’s. (Cf. Mallows and Sloane (1975a).) The number that are switching isomorphic to their negations. The number of nonisomorphic (not switching nonisomorphic!) balanced signings of a given graph. §2.3. “Space of signed graphs over a fixed graph”, implicitly contains the theorem that two signed graphs are switching isomorphic iff there is an isomorphism of underlying graphs that preserves circle signs [cf. Zaslavsky (1982a), Prop. 3.2; (1981b),}


Quico Spaen, Christopher Thraves Caro, & Mark Velednitsky

Valid distance drawing: $V \rightarrow \mathbb{R}^k$ such that $(\forall v)$ all positive neighbors of $v$ are closer than all negative neighbors. Problem: Find $L(n) := \min k$ with a valid drawing $\forall \Sigma$ such that $\#V = n$. Thm.: $[\log_5(n-3)] + 1 \leq L(n) \leq n - 2$. [Annot. 28 Dec 2019.] (SG: Geom)

Edward Spence
See W.H. Haemers.

Joel Spencer
See also T.A. Brown.

Daniel A. Spielman
See A.S. Bandeira and A.W. Marcus.

Joel Spencer with Laura Florescu

§6,8, “An exact formula for unicyclic graphs”: The number of bases of $L(K_n, \emptyset)$, the bicircular lift matroid of $K_n$. §6.4, “Counting unicyclic graphs in Asymptopia”: Asymptotics. [N.B. $L_0(K_n, \emptyset)$ has $n^{n-2}$ additional bases.] [Annot. 3 Oct 2014.] (bic: m: Invar)

K.K. Srimitra
See S. Sajana.

Aravind Srinivasan


Murali K. Srinivasan
See also A. Bhattacharya.


Decomposes the Dowling lattice $Q_n(\mathcal{S})$ into Boolean algebras, indexed in part by integer compositions, that are cover-preserving and centered above the middle rank. (GG: M)

R. Srinivasan
See V. Kodiyalam.

M.A. Sriraj
See C. Adiga and E. Sampathkumar.

K. Sriram, Sylvain Soliman, & François Fages


(Ajitesh Srivastava, Charalampos Chelmis, & Viktor K. Prasanna)


(SG: Dyn)

Nikhil Srivastava
See A.W. Marcus.

Ladislav Stacho
See D. Král’.

Derek Stafford
See D. Feng.

Saul Stahl

A generalized embedding scheme for a graph is identical to a rotation system for a signing of the graph. Thm. 2: Signed rotation systems describe all cellular embeddings of a graph. Thm. 4: Embeddings are homeomorphic iff their signed rotation systems are switching equivalent. Thm. 5: An embedding is orientable iff its signature is balanced. Compare Ringel (1977a). Dictionary: \( \lambda \) is the signature. “\( \lambda \)-trivial” means balanced.


§4.4, “Voltage graphs and their coverings”: Rotation system for surface embedding, embedded covering of embedded voltage graph, branched covering graph and embedding. [Annot. 25 Apr 2014.]

(GG: Top: Exp, Exr)

Saul Stahl & Catherine Stenson


(David P. Stanford
See C.R. Johnson.

Zoran Stanić
See also M. Andelić, F. Belardo, F. Ramezani, P. Rowlinson, and S.K. Simić.


(par: Lap: Eig)
I.e., Spec $L(\Gamma)$ is integral. [Annot. 19 Feb 2021.] (par: Lap: Eig)

§5: Spec $L(\Gamma)$ is mentioned. [Annot. 16 Jan 2012.]
(par: bal: Lap: Eig)

Perturbation by adding an edge or vertex or negating an edge. Effect on $\lambda_{\text{max}}(A(\Sigma))$. E.g., a case of Cor. 8: Negating $e$ in a nontrivial balanced block reduces $\lambda_1$. §4, “Representatives of small switching equivalent signed graphs”: Methodology for (2018b). Dictionary: “simple” = unsigned (treated as all positive). (SG: Adj: Eig)

Counts and lists of signed graphs for $n \leq 8$ and cospectral ones for $n \leq 7$. Data for (2018a). [Annot. 19 Feb 2021.] (SG: Sw, Adj, Eig)

Upper bounds such as $\lambda_{\text{max}} \leq \max_i \frac{1}{2}(\sqrt{5d_i^2 + 4(d_im_i - 4t_i^-)} - d_i)$, where $d_i =$ degree, $m_i =$ average 2-degree, $t_i^- =$ # negative triangles on $v_i$. [Annot. 15 Feb 2021.]
(SG: Adj: Eig)

Important result, “Lemma” 2.1: If $\Sigma$ is connected, then $\lambda_{\text{max}}(A(\Sigma)) = \lambda_{\text{max}}(A(|\Sigma|))$ iff $\Sigma$ is balanced. [This strengthens Acharya’s (1980a) spectral characterization of balance in signed graphs.]
Also, all net-regular $\Sigma$ that are 3-regular with integral Spec $A(\Sigma)$, and some 4-regular ones. [Annot. 19 Dec 2020, 21 Feb 2021.] (SG: Adj: Eig)

Introduces SRSG. The definition is combinatorial. Assume simple, $r$-regular $[\Sigma]$. Let $w^+_2(u,v) =$ # positive $- \#$ negative 2-edge $uv$-walks. Let $w^+_2(uv) =$ $a$, $b$, $c$ according as $uv \in E^+$, $e \in E^-$, or $\notin E$. $\Sigma$ is “strongly regular”, unless $E = \emptyset$ or $\Sigma = (K_n,+)$, $(K_n,-)$.
The balanced SRSG are the same as ordinary strongly regular graphs. The disconnected ones are characterized in Thm. 4.2. Zaslavsky’s (2010b) “very SRSG” are a special case.
The connection with matrices is not straightforward. Thm. 4.1: $\Sigma$ is strongly regular if $A(\Sigma)$ has two eigenvalues. Thm. 5.3: An unbal-
anced, net-regular \((K_{r,s}, \sigma)\) is an SRSG iff \(\Sigma^{+}\), equivalently \(\Sigma^{-}\), is the incidence graph of a symmetric block design. Thm. 5.4 combinatorially characterizes bipartite SRSG with 2, 3, and 4 eigenvalues.

*Cf. Koledin and Stanić (2020a).* [Annot. 4 Apr 2021.]

**SG: Adj: Eig**


\[
\lambda_{\min}(L(\Sigma)) > 4/c^{-n} and > \pi^{2} \frac{12n^{2} - \kappa^{2}}{12n^{2}} h^{-}, \text{ where } c^{-} = \text{negative circumference}, \ h^{-} = \text{packing number of negative Hamiltonian circles (if any exist). } [\text{Annot. 4 Apr 2021.}] 
\]

**SG: Adj: Eig**


*Cf. Ramezani (20xxa).* Completes list of 3-, 4-regular such \(\Sigma\), after Ghasemian and Fath-Tabar (2017a), Hou, Tang, and Wang (2019a). §4, “Relations to line systems”: Line systems at angles 90° and \(\theta\) (cf., e.g., Zaslavsky (2012c)). Includes reduced line graphs \(\Lambda(\Sigma)\). Thm. 4.3: Connected \(\Lambda(\Sigma)\) has 2 eigenvalues iff \(\Sigma \simeq +K_{n}, +K_{1,s}, \pm \Delta\) where \(\Delta\) is regular, or two other graphs. §5, “Computational results”: The 9 connected, incomplete \(\Sigma\) with \(n \leq 10\) that have 2 eigenvalues. §6, “Constructions” of some regular examples with \(n \leq 24\); related (non)existence results. [Annot. 4 Apr 2021.]

**SG: Adj: Eig, LG**


Cor 2.1 applies to signed graphs. [Annot. 19 Jan 2020.] **(Adj, Lap: SG)**


Determines the maximal signed graphs with \(\lambda_{\min}(A(\Sigma)) \geq -2\) that are not negated line graphs of signed graphs. [Annot. 4 Oct 2019, 4 Apr 2021.]

**SG: Adj: Eig, Geom**


Net Laplacian \(L^{\pm} := \text{diag}(d^{\pm}_{\Sigma}) - A(\Sigma)\). **(SG: Lap: Eig)**


Net Laplacian \(L^{\pm} := \text{diag}(d^{\pm}_{\Sigma}) - A(\Sigma)\). **(SG: Lap: Eig)**


The adjacency and Laplacian indices \(\lambda_{1}\) of \(\Sigma\) satisfy, for a certain vertex \(v\), \(\lambda_{1}(n^{+}_{r,v} + n^{-}_{r,v} + \lambda^{i}_{1}) \leq \text{complicated bounds in terms of } r\)-walks, where
\( n^r_{r,x} \) counts signed \( r \)-walks. [Annot. 12 Dec 2020.] (SG: Adj, Lap: Eig)


Net Laplacian \( L^\pm(\Sigma) := \text{diag}(d^\pm) - A(\Sigma) \). (SG: Lap: Eig)

20xxh Signed graphs with two eigenvalues and vertex degree five. Submitted.

Classifies 5-regular such graphs, based on weighing matrices as in *Harada and Munemasa (2012a)*. For \((\leq 4)\)-regular ones cf. e.g. (2020b). [Annot. 14 Oct 2020.]

Zoran Stanić & Ambat Vijayakumar


Richard P. Stanley

See also P. Doubilet, L.L. Mu, and A. Postnikov.


P. 630 restates Stewart (1966a), Cor. 2.4 in a clear way and observes that, if \( \Gamma \) is bipartite, then \( \dim V = \#E - n + 2 \). These two statements are equivalent to van Nuffelen (1973a). (par: incid, ec)


From the 1-vertex switching deck (the multiset of isomorphism types of signed graphs resulting by separately switching each vertex) of \( \Sigma = (K_n, \sigma) \), \( \Sigma \) can be reconstructed, provided that 4 \( \nmid \) \( n \). The same for \( i \)-vertex switchings, provided that the Krawtchouk polynomial \( K^i_n(x) \) has no even zeros from 0 to \( n \). When \( i = 1 \), the negative-subgraph degree sequence is always reconstructible. All done in terms of Seidel (graph) switching of unsigned simple graphs. [See Ellingham; Ellingham and Royle; Krasikov; Krasikov and Roditty for further developments.]

Problem 1. Generalize to signings of other highly symmetric graphs.

Problem 2. Prove a similar theorem for switching of a bidirected \( K_n \).


All-negative complete graphs (implicit in §3) and signed colorings (§4) are used to find the number of ordered degree sequences of \(n\)-vertex graphs and to study their convex hull. (SG: Geom, Col)


Deformed braid hyperplane arrangements, i.e., canonical affine hyperplanar lift representations of \(\text{Lat}^b \Phi\) where \(\|\Phi\| = K_n\) and edge \(ij\) has gain \(l_i \in \mathbb{Z}\) when \(i < j\). In particular (§4), all \(l_i = 1\). Also (§5), the Shi arrangement, which represents \(\text{Lat}^b \{0, 1\} \vec{K}_n\). (gg: Geom, M, Invar: Exp)


Additional exercises, some updating, some corrections to (1986a). (GG: M, Invar: Exr, Exp)


Exercise 5.50: The Shi arrangement [the affinographic hyperplane representation of \(\{0, 1\} \vec{K}_n\) with gain group \(\mathbb{Z}^+\)]. Exercise 5.41(h–i): The Linial arrangement and its characteristic polynomial \(\chi_{[1]}^\ell K_n(\lambda)\).

Exercise 6.19(lll) conceals the Catalan arrangement [which represents \(\{0, \pm 1\} \vec{K}_n\)]. Exercise 5.40(b): Counts two-graphs that \(\nsubseteq C_5\).

(\text{gg: Geom, m, Invar, TG: Exr, Exp})


Introduces “ψ-graphical” hyperplane arrangements, which are affine cross-sections of gain-graphic arrangements according to Lutz (2019a). Also see Mu–Stanley (2015a) and Suyama–Tsujie (2019a). [Annot. 16 Nov 2018.] (gg: Geom)

Dietrich Stauffer
See G. Hed.

Eckhard Steffen
See also L.-G. Jin, Y.-L. Kang, Y. Lu, E. Rollová, and M. Schubert.


Extended abstract. (SG: Flows)

Eckhard Steffen & Michael Schubert


Eckhard Steffen & Alexander Vogel

A survey of many concepts, from the direct generalization of ordinary graph coloring in Zaslavsky (1982b), to variants such as circular coloring as in Kang and Steffen (2017a), to homomorphic “coloring” as expounded in Naserasr, Sopena, and Zaslavsky (2021a). [Annot. 5 Jan 2021.] (SG: Col: Invar, Exp)

Matěj Stehlík
See L. Faria.

Kenneth Steiglitz
See C.H. Papadimitriou.

Arthur Stein
See B. Healy.

Daniel L. Stein
See also A. Gandolfi and C.M. Newman.


Informally describes frustration in spin glasses in terms of randomly ferromagnetic and antiferromagnetic interactions (see Toulouse (1977a)) and gives some history and applications. (Phys: sg: bal, Rand: Exp)

Benjamin Steinberg
See J. Rhodes.
Douglas Steinley  
See M. Brusco.

[R. Stenli (Richard P. Stanley)]  
See R.P. Stanley.

Catherine Stenson  
See S. Stahl.

Andrea Sterbini  
See R. Petreschi.

Dragan Stevanović  
See also L.H. Feng and G.H. Yu.


Two problems by Krzysztof Zwierzyński on the “signless Laplacian” matrix $L(-\Gamma)$ (see Cvetković, Rowlinson, and Simić (2007a) are: Problem AWGS.1, “The maximum clique and the signless Laplacian”. Compare the clique number with the min eigenvalue. Problem AWGS.2, “Integral graphs”. For which graphs are all eigenvalues (of $L(-\Gamma)$, in particular) integral? [Annot. 15 Sept 2010.] (par: Lap: Eig)

Brett Stevens  
See N.A. Neudauer.

B.M. Stewart  

In $\mathbb{R}^{1+E} = \mathbb{R} \times \mathbb{R}^E$ with $x_0$ the first coordinate, let $\sigma_v(x) = \sum \{x_e : e \text{ is incident to } v\}$, and let $V = \{x \in \mathbb{R}^E : \sigma_v(x) = x_0, \forall v \in V\}$. Cor. 2.4 (p. 1059): If $\Gamma$ is connected and contains an odd circle, then $\dim V = \#E - n + 1$. [Restated as in Stanley (1973a). Since $V \cap \{x_0 = 0\} = \text{null space of the incidence matrix } H(-\Gamma)$, this cryptically and partially anticipates the first calculation of $\text{rank}(H(-\Gamma))$, by van Nuffelen (1973a).] (par: incid, ec)

William J. Stewart  
See N. Liu.

Michael Stiebitz  
See T. Schweser.

Allen H. Stix  

Maria Stojkov  
See F. Hassanibesheli.

Daniel Stolarski  
See J. Carlson.

Douglas Stone  
See W. Kocay.

J. Randolph Stonesifer

The second kind of Whitney numbers of a Dowling lattice are binomially concave, hence strongly logarithmically concave, hence unimodal. [Cf. Damiani, D’Antona, and Regonati (1994a) and Benoumhani (1999a).]

[Famous Problem (Rota). Generalize this.] [Annot. rev. 30 Apr 2012.]

**Steven H. Strogatz**

See also S.A. Marvel.


**Thomas Strohmer & Robert W. Heath Jr.**


Notices connection with regular two-graphs via Seidel adjacency matrix (cf. Seidel (1976a)), because tight Grassmannian frames are equiangular. [Foundational, as explained in Bodmann and Paulsen (2005a), esp. §4.] [Annot. 6 Aug 2018.]

**Jeffrey Stuart**

See also Q.A. Li.

**Jeffrey Stuart, Carolyn Eschenbach, & Steve Kirkland**


**Leanne Stuive**

See B. Guenin.

**Bernd Sturmfels**

See A. Björner.

**J. Stutz**

See F. Glover.

**Li Su**

See also H.-H. Li.

**Li Su, Hong-Hai Li, & Jing Zhang**


[Questions. Does this apply to signed graphs, and what is the appropriate definition of a clique? Does it apply to complex unit gain graphs (cf. Reff (2012a))?] [Annot. 18 May 2018.]

**C.K. Subbarayya**

See C. Adiga.

**S.P. Subbiah**

M.S. Subramanya & P. Siva Kota Reddy


S.P. Subbiah & V. Swaminathan


M.S. Subramanya

See also R. Rangarajan, E. Sampathkumar, and P.S.K. Reddy.

M.S. Subramanya & P. Siva Kota Reddy


A “graph structure” (due to E. Sampathkumar in 2005) is $G := (V,R)$ where $S \subseteq R$ and $||S|| := \bigcup \{R : R \in S\}$. Define $||G|| := (V,||S||)$ and let $\Sigma(S)$ be the signed $||G||$ with negative edge set $||S||$. $\Sigma(S)$ is not defined but is implicit. $G$ is “$S$-balanced” if $\Sigma(S)$ is balanced, and “$S$-clusterable” if $\Sigma(S)$ is clusterable. Prop. 3 [hard to interpret] seems to be Harary’s (1953a) theorem for $\Sigma(S)$. Thm. 4: $G$ is $S$-balanced for all $S$ iff it is $\{R_i\}$-balanced for all $i$. Thm. 7 is Davis’s (1967a) characterization of clusterability applied to $S$-clusterability. Thm. 8 has three conditions equivalent to $S$-clusterability, assuming $\bigcup k R_i = P^2(V)$ and no $R_i = \emptyset$. [k = 2, #S = 1 is signed $K_n$.] Thm. 9: $G$ is $S$-clusterable for all $S$ iff it is $\{R_i\}$-clusterable [the paper says “balanced”] for all $i$. [Annot. 1 Aug 2009.]


Definitions as at Sampathkumar, Reddy, and Subramanya (2008a), (2010c). Let $T(\Gamma) := (E, E_T)$ where $E_T := \{ef : e, f \in C_3 \text{ in } \Gamma\}$. The triangular line signed graph is $T(\Sigma) := (T(|\Sigma|), \sigma^\Sigma)$. Solved: $\Lambda_\Sigma \simeq T(\Sigma)$, $T^k(\Sigma) \simeq T^2(\Sigma)$. [\Lambda_x \text{ as in M. Acharya (2009a).}] [Annot. 3 Aug 2010.]

(1)
2009.]

Benjamin Sudakov
See I. Balla and G. Gutin.

Naduvath K. Sudev
See also A. Aniyan, P.K. Ashraf, and K.A. Germina.

N.K. Sudev, P.K. Ashraf, & K.A. Germina

N.K. Sudev, K.P. Chithra, & K.A. Germina

Sudev Naduvath & Germina Augustine

§1.3, “Signed graphs”: expository. Fix $X \subseteq \mathbb{Z}$. §6.2, “Sumset labeled signed graphs”: A sumset labelling of $\Sigma$ is an injective $f : V \to \mathcal{P}(X) \setminus \{\emptyset\}$ such that $\sigma(uv) = (-1)^{#(f(u)+f(v))}$, $\forall uv \in E$. It is strong if $\forall uv \in E$, $f(u) + f(v)$ has no duplicate sums. Thm. 6.2.4 [6.2.5]: Let $f$ be strong and $(f(u) + f(v)) = k$, $\forall uv$. Then $\Sigma$ is balanced [clusterable] iff $|\Sigma|$ is bipartite or $\sqrt{k} \in \mathbb{Z}$ [$k$ is odd]. More results for strong and weak $f$. §6.3, “Arithmetic sumset labeled signed graphs”: Here $f(v)$ is an arithmetic progression. [Annot. 23 Oct 2019.] (SG: Lab: Bal, Clu)

Sudev Naduvath & Johan Kok

Definitions and simple properties of positive, negative, mixed, and net Zagreb, Schultz, Gutman indices, obtained from positive, negative, and net degrees. [Annot. 29 Dec 2020.] (SG: Invar)

N. Sudharsanam
See R. Balakrishnan.

Qiang Sun
See R. Naserasr.

Shiwen Sun
See S.-S. Feng.

Wanting Sun
See S.-C. Li.

Zhi Ren Sun
See X.X. Zhu.
Zhongyao Sun  
2015a *Analysis and Logical Modeling of Biological Signaling Transduction Networks.*  
Ch. 5, “Determining the attractors of a boolean network using an elementary signaling mode approach”, employs signed digraphs.  
(SD: Dyn)

V.S. Sunder  
See V. Kodiyalam.

Borworn Suntornpoch  
See Y. Meemark.

Didi Surian  
See D. Lo.

Daisuke Suyama  
See also T. Abe.

Daisuke Suyama, Michele Torielli, & Shuhei Tsujie  
(SG: Geom: Algeb)

Daisuke Suyama & Shuhei Tsujie  

Masuo Suzuki  

Sei Suzuki, Jun-ichi Inoue, & Bikas K. Chakrabarti  

V. Swaminathan  
See S.P. Subbiah.

Chaitanya Swamy  
(SG: WG: Clu: Alg)

Shivakumar Swamy C.S.  
See C. Adiga.
Ed Swartz
See P. Hersh.

Eric Swartz
See A. Schaefer.

Swathyprabhu Mj
See S. Das.

Robert H. Swendsen

Katia Sycara
See D. Li.

Itiro Syôzi
See also Y. Kasai.


Physics of the all-negative (“antiferromagnetic”) toroidal honeycomb (§7) and triangular (§9) lattices. The former is similar to all-positive (“ferromagnetic”) [because balanced] while the latter is not [because unbalanced]. [See also Houtappel (1950a), (1950b), Newell (1950a), Wannier (1950a).] [Annot. 21 Jun 2012.]

Edward Szczerbickl

A state (“pattern”) is $s : V \rightarrow \{+, -, 0\}$. An arc $uv$ is “consistent” if $s(v) = \sigma(uv)s(u)$. In a “solution” $\zeta$ all arcs are consistent. A state is propagated by $s'(v) = \sigma(uv)s(u)$. Rules for simplification to get a signed digraph with equivalent solutions. [The model is incomplete. The role of propagation is unclear.]

Janusz Szczypula
See P. Doreian.

Andrzej Szepietowski
See also J. Dybizbański.


Stefan Szeider
See N. Alon.

Endre Szemerédi
See B. Bollobás.

Zoltán Szigeti
See A.A. Ageev.

Ferenc Szöllősi
See also G. Greaves.

Ferenc Szöllősi & Patric R.J. Östergård
Seidel matrix = \( A(K_n, \sigma) \). Spectrum et al. for \( n \leq 13 \). Classification of those with 3 distinct eigenvalues for \( n \leq 23 \). Application to equiangular lines. [Annot. 22 Dec 2017.] (sg: KG: Adj, Geom)

Bosiljka Tadić, Krzysztof Malarz, & Krzysztof Kulakowski

B. Taglienti
See M. Falcioni.

Artin Tajdini
See S. Akbari.

Martin Takáč

Károly Takács
See S. Righi.

Shingo Takahashi
See T. Inohara.

Akimichi Takemura
See H. Kamiya.

Lynn Takeshita

Michel Talagrand

Irving Tallman

Ilan Talmud
See Z. Maoz.

Bit-Shun Tam, Yi-Zheng Fan, & Jun Zhou
See also T.-J. Chang [T.-C. Chang], Y.Z. Fan, and H.-H. Li.

The matrix is \( L(-\Gamma) \). “Maximizing” graphs are those whose degree sequences are maximal in the majorization ordering. [For majorization also see Liu, Liu, and You (2013a).] [Annot. 23 Mar 2009.] (Par: Lap)

Bit-Shun Tam & Shu-Hui Wu

**A. Tamilselvi**

See also M. Parvathi.


**Arie Tamir**

See also D. Hochbaum.


**Christino Tamon**

See J. Brown and D. Mallory.

**Akihisa Tamura**

See also Y.T. Ikebe and D. Nakamura.


Problem: maximize an integral weight function over the bidirected stable set polytope (cf. Johnson and Padberg (1982a)). §3 concerns the effect on perfection of deleting all incoming edges at a vertex. §4 reduces the “generalized stable set problem” for bidirected graphs to the maximum weighted stable set problem for ordinary graphs, whence the problem for perfect bidirected graphs is solvable in polynomial time.

**Takeyuki Tamura**

See T. Akutsu.


The stable set problem associated with bidirected graphs.

**Jinsong Tan**


**Shang Wang Tan**


The results on $L(\Gamma, w)$ with edge weights $w : E \to \mathbb{R}_{>0}$ are deduced from results on $L(\Gamma, w)$. [Problem. Show the same reasoning applies to all signatures of $\Gamma$.] [Annot. 20 Jan 2012.]
(par: WG: Eig)

Shang-wang Tan, Ji-ming Guo, & Jian Qi

Shang-Wang Tan & Jing-Jing Jiang
The “(signless) Laplacian” of a graph with positive edge weights, $(\Gamma, w)$ where $w : E \to \mathbb{R}_{>0}$, is $L(-\Gamma, w) := D(\Gamma, w) + A(\Gamma, w)$ (called $R$). The spectral radius is that of $L(-\Gamma, w)$. [Problem. Generalize to all weighted signed graphs.] [Annot. 11 Jan 2011, 21 Jan 2012.]
(par: WG, Eig)

Shang Wang Tan & Xing Ke Wang

Xuezhong Tan
See also M.H. Liu.

Xuezhong Tan & Bolian Liu
(Par: Eig, ec)

Ying-Ying Tan
See also Y.-Z. Fan.

Ying Ying Tan & Yi Zheng Fan
Relations between least Laplacian eigenvalue, its eigenvector, and $l(\Sigma)$. Properties of the eigenvector when $l(\Sigma) = 1$, e.g., $\lambda_{\min} \leq (4/n)l(\Sigma)$. Dictionary: “mixed graph” = signed graph, “edge singularity” = frustration index $l(\Sigma)$. [Generalized in Bapat, Kalita, and Pati (2012a).] [Annot. 28 Oct 2011, 20 Jan 2012.]
(sg: Fr, Eig)

Adrian Tanasa
See T. Krajewski.

B.Z. Tang
See Y. Chen.

Jiliang Tang, Yi Chang, Charu Aggarwal, & Huan Liu


Wen Tang
See E.L. Wei.

Wenliang Tang
See E.L. Wei

Zikai Tang
See Y.-P. Hou.

Shin-ichi Tanigawa
See also R. Ikeshita and T. Jordán.


Tetsuji Taniguchi
See T.Y. Chung, G. Greaves, Hye Jin Jang, and A. Munemasa.

Percy H. Tannenbaum
See C.E. Osgood.

Éva Tardos
See also A.V. Goldberg.

Éva Tardos & Kevin D. Wayne

Max flow in a network with positive rational gains. Multiple sources and sinks are allowed. “Relabeling” is switching the gains. Useful references to previous work. (GN: Sw, Alg, Ref)

Robert E. Tarjan
See A.V. Goldberg.

Christos Tatakis
See E. Reyes.

U. Tatt [W.T. Tutte]
See W.T. Tutte.

B. Tayfeh-Rezaie
See F. Ayoobi.

D.E. Taylor
See also J.J. Seidel.

Introducing two-graphs and regular two-graphs (defined by G. Higman, unpublished). [See Seidel (1976a) etc. for more.] A “two-graph” is the class \(\mathcal{C}_3^-\) of negative triangles of a signed complete graph \((K_n, \sigma)\). (See §2, p. 258, where the group is \(\mathbb{Z}_2 \cong \{+, -\}\) and the definition is in terms of the 2-coboundary operator.) Two-graphs and switching classes of signed complete graphs are equivalent concepts (stated in terms of Seidel switching in §2, p. 260). A two-graph is “regular” if every edge lies in the same number of negative triangles. Thm.: \(\mathcal{C}_3^-\) is regular iff \(A(K_n, \sigma)\) has at most two eigenvalues. Various parameters of regular two-graphs are calculated. (TG: Eig. Geom)

**Graeme Taylor**


See (2011a). (SG)


**Herbert Taylor**

See P. Erdős.

**Howard F. Taylor**


A thorough and pleasantly written survey of psychological theories of balance, including formalizations by signed graphs (Chs. 3 and 6), experimental tests and critical evaluation of the formalisms, and so forth. Ch. 2, “Substantive models of balance”, takes the perspective of social psychology. §2.2, “Varieties of balance theory”, reviews the theories of Heider (1946a) (the source of Harary’s (1953a) invention of signed graphs), Osgood and Tannenbaum (1955a), and others. §2.2e, “The Rosenberg-Abelson modifications”, discusses their introduction of the “cost” of change of relations, which led them (Abelson and Rosenberg (1958a)) to propose the frustration index as a measure of imbalance. (PsS: SG, WG: Exp, Ref)

Ch. 3, “Formal models of balance”, reviews various graph-theoretic models: signed and weighted signed, different ways to weigh imbalance, etc., the relationship to theories in social psychology being constantly kept in mind. §3.1, “Graph theory and balance theory”, presents the basics of balance, measures of degree of balance by circles (Cartwright and Harary (1956a)), circles with strengths of edges (Morissette (1958a)), local balance and \(N\)-balance (Harary (1955a)), edge deletion and negation (Abelson and Rosenberg (1958a), Harary (1959b)), vertex frustration number (Harary (1959b)). §3.2, “Evaluation of formalizations: strong points”, and §3.3, “Evaluation of formalizations: weak points”, judged from the applied standpoint. §3.3a, “Discrepancies between cycles or subsets of cycles”, suggests that differing degrees of imbalance among certain different subsets of the vertices may be significant [Is this reasonable?] and proposes measures, e.g., a variance measure (p. 71), of
this “discrepancy”. (PsS: SG, WG: Bal, Fr: Exp)

Ch. 6, “Issues involving formalization”, goes into more detail. §6.1, “Indices of balance”, compares five indices, in particular Phillips’ (1967a) eigenvalue index (also in Abelson (1967a)) with examples to show that the index differentiates among different balanced signings of the same graph. §6.2, “Extrabalance properties”, discusses Davis’s (1967a) clustering (§6.2b) and indices of clustering (§6.2c). §6.3, “The problem of cycle length and non-local cycles”. Are long circles less important? Do circles at a distance from an actor (that is, a vertex) have less effect on the actor in balancing processes? (PsS: SG: Fr, Adj: Exp)

Siamak Tazari


Mina Teicher
See M. Amram.

Roberto Tempo
See A.V. Proskurnikov.

Jeffrey C.Y. Teo
See A.P.O. Chan.

Hiroaki Terao
See H. Kamiya.

Hidetaka Terasaka
See S. Kinoshita.

Lesley G. Terris
See Z. Maoz.

Evimaria Terzi & Marco Winkler

$b := \text{fraction of balanced triangles, calculated via cubes of adjacency eigenvalues.}$ [Annot. 15 Jul 2019.] (SG: Fr: Adj)

Ambuj Tewari
See K.-Y. Chiang.

Dirk Oliver Theis
See N.E. Clarke.
Michel Thellier
See J. Demongeot.

Denis Thieffry
See also J.-P. Comet, A. Naldi, É. Remy, and R. Thomas.
Survey of positive and negative cycles in biological regulation. [Annot. 25 Jan 2015.]

D. Thieffry, E.H. Snoussi, J. Richelle, & R. Thomas

Dimitrios M. Thilikos
See C. Giatsidis.

Morwen B. Thistlethwaite

A 1-variable Tutte-style polynomial $\Gamma_{\Sigma}$ of a sign-colored graph. Fix an edge ordering. For each spanning tree $T$ and edge $e$, let $\mu_T(e) = -A^{3\tau_T(e)\sigma(e)}$ if $e$ is active with respect to $T$, $A^{\tau_T(e)\sigma(e)}$ if it is inactive, where $\tau_T(e) = +1$ if $e \in T$, $-1$ if $e \notin T$. Then $\Gamma_{\Sigma}(A) = \sum_T \prod_{e \in T} \mu_T(e)$.

[In the notation of Zaslavsky (1992a), $\Gamma_{\Sigma}(A) = Q_{\Sigma}$ with $a_\varepsilon = A^{-\varepsilon}, b_\varepsilon = A^\varepsilon$ for $\varepsilon = \pm 1$ and $u = v = -(A^2 + A^{-2})$.] §§3 and 4 show $\Gamma_{\Sigma}$ is independent of the ordering. Other sections derive consequences for knot theory. [This marks the invention of a Tutte-style polynomial of a colored, or parametrized or weighted, graph or matroid, developed in Kauffman (1989a) and successors.]

Apostolos Thoma
See E. Reyes.

A.D. Thomas
See F.W. Clarke.

Creighton K. Thomas, David A. Huse, & A. Alan Middleton

A droplet model of a signed square lattice shows long-range correlations (spin-glass behavior) in the ground state. [Annot. 3 Jan 2015.]

Creighton K. Thomas & A. Alan Middleton

Both pure signed graphs ($\pm J$ model) and randomly weighted ones (Gaussian model), using the Kastelyn and Temperley–Fisher decoration and Pfaffian method. [Annot. 10 Jan 2015.]

Both pure signed graphs (±J model) and randomly weighted ones (Gaussian model), using the Kastein and Temperley–Fisher decoration and Pfaffian method. [Annot. 10 Jan 2015.]

(SG, WG: State(fr), Phys, Alg)

René Thomas

See also J. Demongeot, M. Kaufman, J. Leclercq, E.H. Snoussi, and D. Thieffry.


A main progenitor of a large field of inquiry about biological and chemical regulatory systems with positive and negative feedback. [See, e.g., J. Aracena, É. Remy, A. Richard, H. Siebert, R. Thomas, and their many coauthors.] The diagrams show the embryonic appearance of signed digraphs. [Annot. 25 Apr 2014.]

(Biol: sd: Dyn)


(sd: Dyn, Biol)


Describes dynamics of very simple signed digraphs with up to two cycles. E.g.: One positive cycle leads to one of two steady states. One negative cycle implies cycling states. With two cycles having one common vertex, both positive are like one positive cycle. Both negative allow for multiple cyclic states. One of each sign allow both a steady state and cyclic states. [Annot. 4 Aug 2018.]

(SD: Dyn, Chem)


(SD: Dyn)


(SD: Dyn)


(sd: Dyn)


(SD: Dyn)

René Thomas & Richard D’Ari


(SD: Dyn, Biol)

René Thomas & Marcelle Kaufman

R. Thomas & J. Richelle


René Thomas, Denis Thieffry, & Marcelle Kaufman

Robin Thomas
See also W. McCuaig and N. Robertson.

Robin Thomas & Peter Whalen

An “odd $K_{3,3}$” is an all-negative subdivision of $-K_{3,3}$, treated as unsigned. (sg: Par: Str)

Andrew Thomason

The property is the existence of an Eulerian cut. The asymptotic probability is $.57$... [Problem. Generalize to gain graphs with finite gain group, esp. to signed graphs. The property is that of being switchable so that the identity-gain edges form an Eulerian subgraph. (This has various meanings.) Variation: The property is that of having a maximal balanced subgraph that is Eulerian. One expects the asymptotic probabilities to be the same for both problems and to depend only on the group’s order.] (par: Rand)

Carsten Thomassen
See also P.D. Seymour.


It is an NP-complete problem to decide whether a given signed digraph has a positive but not all-positive cycle, even if there are only 2 negative arcs. This follows from Lemma 3 of Steven Fortune, John Hopcroft, and James Wyllie, The directed subgraph homeomorphism problem (see *Theor. Computer Sci.* 10 (1980), 111–121. MR 0551599 (81e:68079). Zbl 419.05028.) by the simple argument in the proof of Prop. 2.1 here.

To decide whether a specified arc of a digraph lies in an even cycle, or in an odd cycle, are NP-complete problems (Prop. 2.1). To decide existence of an even cycle [hence, by the negative subdivision trick, of a positive cycle in a signed digraph] is difficult [but is solvable in polynomial time; see Robertson, Seymour, and Thomas (1999a)], although existence of an
odd cycle [resp., of a negative cycle] is easy, by a trick here attributed to Edmonds (unpublished). Prop. 2.2: Deciding existence of a positive cycle in a signed digraph is polynomial-time solvable if $\#E^-$ is bounded.

Thm. 3.2: If the outdegrees of a digraph are all $> \log_2 n$, then every signing has a positive cycle, and this bound is best possible; restricting to the all-negative signature, the lower bound might (it’s not known) go down by a factor of up to 2, but certainly (Thm. 3.1) a constant minimum on outdegree does not imply existence of an even cycle. [See (1992a) for the effect of connectivity.]


§8: “Even directed circuits and sign-nonsingular matrices.”

§§8–10 treat even cycles in digraphs.

[General Problem. Generalize even-cycle and odd-cycle results to positive and negative cycles in signed digraphs, the unsigned results corresponding to all-negative signatures.]


There is an algorithm for detecting a balanced circle in a $\mathbb{Z}_m$-gain graph. Balance of such a gain graph is characterized. (gg: Bal, Circ: Alg)


§5 describes the “fundamental cycle method”, a simple algorithm for a shortest unbalanced circle in a biased graph (Thm. 5.1). Thus the method finds a shortest noncontractible circle (Thm. 5.2). A noteworthy linear class: the surface-separating (“II-separating”) circles (p. 166). Dictionary: “3-path-condition” on a class $F$ of circles = property that $F^c$ is a linear class. “Möbius circle” = negative circle in the signature induced by a nonorientable embedding (only on p. 166). (gg, sg: Circ: Alg, Top)


A digraph that is strongly connected and has all in- and out-degrees $\geq 3$ contains an even cycle. (sd: par: Cyc)

A polynomial-time algorithm for deciding the existence of an even cycle in a planar digraph. (sd: par: Cyc: Alg)


P. 225 and Thm. 6.3: the “3-path-condition” and shortest unbalanced circle algorithm from (1990a). Examples mentioned (under other names) are parity bias (all-negative signs) [underlying the even-circle matroid of Tutte (1981a) and Doob (1973a) via Zaslavsky (1989a)], poise bias [underlying a matroid of Matthews (1978c)], and noncontractible or orientation-reversing embedded circles [for the latter see esp. Lins (1985a) and Zaslavsky (1992a)]. (gg, par: Exp)


Given \( k \), there exists \( K \) such that every sufficiently connected graph has \( k \) vertex-disjoint odd circles or \( K \) vertices whose deletion leaves a bipartite graph. [Problem. Given \( k \), there exists \( K \) such that every sufficiently connected signed graph has \( k \) vertex-disjoint negative circles or \( K \) vertices whose deletion leaves a balanced graph.] [Annot. rev. 26 Dec 2012. (par: Fr: Circ)


G.L. Thompson
See V. Balachandran.

Christopher Thraves Caro [Christopher Thraves]
See A.-M. Kermarrec, E.G. Pardo, and Q. Spaen.

Florence Thuderoz
See J. Demongeot.

Fenglei Tian
See also X.B. Ma and M. Zhu.

Fenglei Tian, Li Chen, & Rui Chu

Generalizes Ma–Wong–Tian (2016a) to \( 2\mu(\Gamma) - 2\xi(\Gamma) \leq \text{rk} A(\Phi) \leq 2\mu(\Gamma) + \xi(\Gamma) \) for gain graph \( \Phi \) with gains in \( \{\pm 1, \pm i\} \). Characterizes equalities. [A different matching formula is in Chen–Huang–Li (2018a).] [Annot. 10 May 2019, rev 16 Oct 2020. (gg: Adj)

Fenglei Tian, Xiaoming Li, & Jianling Rou

Fenglei Tian, Dengyin Wang, & Min Zhu
Characterized: All signed graphs with \( \text{rk}(A(\Sigma)) = 2 \), and signed planar graphs with \( \text{rk}(A(\Sigma)) = 4 \). [Annot. 22 Jan 2016.]

**Fenglei Tian & Dein Wong**

For \( \Phi \) with gain group \( \{ \pm 1, \pm i \} \) (\( \varphi(e) = 1 \) for undirected, \( i \) for directed edges), characterizes the spectrally unique ones with \( \text{rk}(A(\Phi)) = 3 \). [Annot. 15 Dec 2020.] (SG: Adj: Eig)

**Gui-Xian Tian**
See also S.-Y. Cui.

**Gui-Xian Tian, Ting-Zhu Huang, & Bo Zhou**

A lower bound on \( \sum_i \lambda_i((L(\Gamma))^\alpha) \), over nonzero eigenvalues, for bipartite \( \Gamma \) and \( \alpha \in \mathbb{R}^\times \). [Question. Is there a nonbipartite generalization involving \( L(-\Gamma) \)?] [Annot. 23 Jan 2012.]

**Xiao-Jun Tian, Xiao-Peng Zhang, Feng Liu, & Wei Wang**


**Yi Tian**
See S.C. Li.

**Jonathan Tidor**
See Z.-L. Jiang.

**R.M. Tifenbach**

Cf. Tifenbach and Kirkland (2009a). An \( h \)-graph \( \Gamma \) is “self-dual” if it has inverse \( \Sigma \) and \( \Gamma \cong |\Sigma| \), “strongly self-dual” if \( \Gamma = \Sigma \). Thm. 3.2 is Tifenbach and Kirkland (2009a) Thm. 2.5 with strong self-duality instead of duality. §4, “Constructions of strongly self-dual graphs”. §5, “Eigenvalues of self-dual \( h \)-graphs”: For an eigenvalue \( \lambda \) of a self-dual \( h \)-graph, \( -\lambda \) and \( \pm 1/\lambda \) are eigenvalues. \( \lambda = \pm 1 \) if rational. \( \pm 1 \) has multiplicity \( \equiv m(mod\, 2) \). Thm. 5.5: Let \( k^- := \# \) of vertices switched in changing \( \Sigma \) to \( \Gamma^+ \); then \( \lambda = \pm 1 \) has multiplicity \( \geq |m-2k^-| \). Examples. [Annot. 4 May 2017.] (SG: Adj, Eig)

**R.M. Tifenbach & S.J. Kirkland**

Inspired by Godsil (1985a) et al. Graphs are simple. An “\( h \)-graph” is bipartite with left set \( \{ u_1, \ldots, u_m \} \), right set \( \{ v_1, \ldots, v_m \} \), and a unique perfect matching \( M = \{ u_i v_i \} \). Thm. 1.1: \( \exists \) labelling so every edge \( u_i v_j \) has \( i < j \) (Simion and Cao (1989a)). Thus, \( \exists \) vertex labelling and partial order \( P_\Gamma \) on \( \{ v_1, \ldots, v_p \} \) so every edge \( u_i v_j \) has \( i < j \); this gives an
acyclic digraph. If $A(\Gamma)^{-1} = A(\Sigma)$ for some $\Sigma$, then $\Gamma^+ := |\Sigma|$ ("dual" of $\Gamma$) is an $h$-graph, $\Sigma$ is balanced, and every covering edge of $P_{\Gamma^+}$ is negative in $\Sigma$ (Thm. 2.3). Thm. 2.4: $P_{\Gamma} \cong P_{\Gamma^+}$. Thm. 2.5: Intervals of $P_{\Gamma}$ have duals; intervals respect duality. Thm. 2.6: $\Gamma^+$ exists iff $P_{\Gamma}$ has bipartite Hasse diagram and all intervals have duals. [The former is due to balance and the negative covering edges (i.e., antibalance).] §§3–4: Examples. [Annot. 4 May 2017.]

Problem. Generalize this and other $h$-graph research to bipartite signed graphs with unique perfect matching, so having a signed-graphic inverse is natural. What do balance and the negative covering edges (antibalance) of the inverse digraph become? [Annot. 4 May 2017.]

Shailesh K. Tipnis

R.L. Tobin

Bjarne Toft
See also T.R. Jensen and U. Krusenstjerna-Haastøm.


Sivan Toledo
See E.G. Boman and D. Chen.

Ioan Tomescu
See also D.R. Popescu.


Independent proof of Petersdorf’s (1966a) Satz 1. Also, treats similarly a variation on the frustration index. (SG: Fr)


The fewest sign changes needed to make $(K_n, \sigma)$ clusterable is $\leq \lfloor n/2 \rfloor \lfloor (n - 1)/2 \rfloor$. [Annot. 6 Jan 2017.]


Consider $(K_n, \sigma)$ with $\#E^- = p$. The parity of the number of negative triangles = that of $np$. The number of negative $t$-gons, for $t \geq 4$, is even [strengthened in Popescu (1991a), (1996a)]. [Kittipassorn & Mészáros
(2015a) performs a detailed study of the number of negative triangles. [SG: Bal]


Mark Tomforde
See B.G. Bodmann.

Joanna Tomkowicz & Krzysztof Kulakowski

C.B. Tompkins
See I. Heller.

Arnaud Tonnelier
See J. Demongeot.

J. Topp & W. Ulatowski

An additive real gain graph is balanced iff every circle in a circle basis is balanced, iff the gains are induced by a vertex labelling [in effect, switch to 0], iff every two paths with the same endpoints have the same gains. A digraph is gradable (Harary, Norman, and Cartwright (1965a); also see Marcu (1980a)) iff \( \varphi_1 \) is balanced, where for each arc \( e \), \( \varphi_1(e) = 1 \in \mathbb{Z} \) (Thm. 3). The Windy Postman Problem (Thms. 4, 5). [GG, GD: Bal]

Aleksandar Torgašev
See also D.M. Cvetković.


An infinite analog of Doob’s (1973a) characterization via the even-cycle matroid of when a line graph has \(-2\) as an eigenvalue. [Problem. Generalize to line graphs of infinite signed graphs.] [par: Eig(LG)]


An infinite graph is a generalized line graph iff its least “limit” eigenvalue \( \geq -2 \). [Problem. Generalize to line graphs of infinite signed graphs.] [par: Eig(LG)]

Michele Torielli
See also W.-L. Guo and D. Suyama.

Michele Torielli & Shuhei Tsujie
2020a Freeness of hyperplane arrangements between boolean arrangements and Weyl arrangements of type \( B_l \). Electronic J. Combin. 27 (2020), no. 3, article P3.10.
Juan R. Torregrosa
See C. Mendes Araújo.

[núria ballber torres]
See N. Ballber Torres (under ‘B’).

Dejan V. Tošić
See M. Andelić.

Gérard Toulouse
See also B. Derrida and J. Vannimenus.


Introduces the notion of imbalance (“frustration”) of a signed graph to account for inherent disorder in an Ising model (here synonymous with a signed graph, usually a lattice graph). (Positive and negative edges are called “ferromagnetic and antiferromagnetic bonds”.) Observes that switching the edge signs from all positive (the model of D.D. Mattis, Phys. Letters 56A (1976), 421–?) makes no essential difference. In a planar lattice [or any plane graph] frustration of face boundaries (“plaquettes”) can be thought of as curvature, i.e., failure of flatness. Proposes two kinds of asymptotic behavior of frustration as a circle encloses more plaquettes. The planar-duality approach for finding the states with minimum frustration (i.e., switchings with fewest negative edges); the number of such states is the “ground-state degeneracy” and is important. Ideas are sketched; no proofs.


Mainly for signed lattice graphs, with spins $s(v) \in S^{n-1}$ having symmetry group $\text{SO}(n)$; $n = 1$ (Ising model) gives $\text{SO}\{+1, -1\}$; $n = 2$ is planar spins; $n = 3$ is Heisenberg spins. Two symmetry groups: $\mathbb{Z}_2 \# V$ acts on $\Sigma$ (the “microscopic level”); $\text{SO}(n)$ or $\text{O}(n)$ acts on states $s$ (the “macroscopic level”). [An edge is satisfied if $s(w) = \sigma(vw)s(v)$, otherwise frustrated.] A “ground state” (where the most edges are satisfied) has a topology of frustrated plaquettes [negative girth circles], whose nature, depending on the lattice dimension, is described intuitively. Regions (“packets”) of relatively fixed spins can be identified. Topology of frustrated plaquettes leads to the homotopy groups of $\text{O}(n)$. The effect on thermodynamic phases is discussed. Dictionary: “Local transformation”

§3, “Frustration”: in signed graphs [after normalization to bond strength 1]. “Frustration function” of circles \(= \sigma(C)\) determines physical properties because they are “gauge [= switching] invariant”, if no external magnetic field. §3.i, “Periodic frustrated models” [= toroidally embedded graphs]. §3.ii, “Fully frustrated models”, where every “plaquette” [girth circle] is negative: overblocking effect, i.e., positive density of plaquettes with more than one negative edge. [A mathematically interesting concept, not understood today.] §3.iii, “Systems with finite residual entropy”: e.g., antiferromagnetic [all-negative] Potts models. §3.iv, “Approach to spin glasses, by dilution of periodic frustrated systems” [embedding an unbalanced toroidal graph in a larger balanced graph?]. §3.v, “Connections with gauge theories; topological defects and their hydrodynamics”: cf., e.g., (1979a). §3.vi, “Random frustration (edge weights \(\pm 1\)) models, in various space dimensions”: comparing random signs \(\pm 1\) with Gaussian random edge weights (centered at 0, hence with signs and magnitudes). For signed \(K_n\)’s (“Sherrington–Kirkpatrick model”), “in the thermodynamic limit [both] have the same physics.” [Annot. 20 Aug 2012.]

Gérard Toulouse & Jean Vannimenus

Popular exposition of the elements of frustration in relation to the Ising model [evidently based on Toulouse (1977a)]. Briefly mentions the social psychology application. (Phys: sg, Fr)(SG: PsS: Exp)


Annealed and quenched models on a square lattice are compared. Annealed: edge weights \(J_{ij}\) (“bond strengths”) are random variables; this is randomly weighted, randomly signed graphs. Quenched, edge weights = \(\pm J\); this is signed graphs. The annealed model “grossly underestimates frustration effects.” Proposed corrective: introduce Lagrange multipliers for the plaquettes. This leads to unexplored theory. App. (c), “The frustration model”: randomly signed graphs, especially regular graphs; compared to models with Gaussian random edge weights and signs. [Annot. 20 Aug 2012.] (Phys: sg, Fr)(Phys: sg, Fr: Exp)

L. Tournier & M. Chaves

V.A. Traag & Jeroen Bruggeman

Generalizes a Potts model for positive links to signed graphs. Method
is more general than the clustering model for signed graphs. [Applied in Yoshikawa, Iino, and Iyetomi (2012a).] 

Vincent Antonio Traag, Paul Van Dooren, & Patrick De Leenheer
2013a Dynamical models explaining social balance and evolution of cooperation. *PLOS One* 8 (2014), no. 4, article e60063, 7 pp. + 4 supplements.

Lorenzo Traldi
See also J. Ellis-Monaghan.

Generalizing Kauffman’s (1989a) Tutte polynomial of a sign-colored graph, Traldi’s “weighted dichromatic polynomial” \(Q(\Gamma; t, z)\) is Zaslavsky’s (1992b) \(Q_{\Gamma}(1, w; t, z)\), in which the deletion-contraction parameters \(a_e = 1\) and \(b_e = w(e)\), the weight of \(e\). Thm. 2 gives the Tutte-style spanning-tree expansion. Thm. 4: Kauffman’s Tutte polynomial \(Q[\Sigma](A, B, d) = d^{-1} A^{\#E^+} B^{\#E^-} Q_{\Sigma}(1, w; d, d)\) for connected \(\Sigma\), with \(w(e) = (AB^{-1})^{\sigma(e)}\). [See Kauffman (1989a) for other generalizations. Traldi gives perhaps too much credit to Fortuin and Kasteleyn (1972a).]

P. 284: Invariance under Reidemeister moves of type II constrains the weighted dichromatic polynomial to, in essence, equal Kauffman’s. Thus no generalization is evident in connection with general link diagrams. There is an interesting application to special link diagrams.


The corank-nullity expansion of the usual Tutte polynomial generalizes to colored Tutte polynomials in the universal sense of Bollobás and Riordan (1999a).


The Tutte polynomial of a parallel connection of colored graphs or matroids.


Polynomial-time computability for colored graphs of bounded tree width. [Also see Makowsky (2005a).] (SG: Gen: Invar, Alg, Knot)


Tan Nhat Tran
Tuan Tran & Günter M. Ziegler

Edge polytope $P_{\Gamma}$ (cf. Ohsugi and Hibi (1998a)). [This is the antibalanced case. Problem. Generalize to signed graphs, including balanced graphs.]

Ben Tremblay
See G. MacGillivray.

Marián Trenkler
See S. Jezný.

Vilmar Trevisan
See F. Belardo.

Nenad Trinajstić
See also A. Graovac.


Ch. 3, § V.B: “Möbius graphs.” Ch. 4, § I: “The adjacency matrix”; see pp. 42–43. Ch. 5: “The characteristic polynomial of a graph”, § II.B: “The extension of the Sachs formula to Möbius systems”; § III.D: “Möbius cycles”. Ch. 6, § VIII: “Eigenvalues of Möbius annulenes” (i.e., unbalanced circles); § IX: “A classification scheme for mon cyclic systems” (i.e., characteristic polynomials of circles). (SG: Eig, Chem)

Ch. 7: “Topological resonance energy,” § V.C: “Möbius annulenes”; § V.G: “Aromaticity in the lowest excited state of annulenes”. (Chem; sg: bal)

Anastasia Trofimova
See Y. Burman.

Nicolas Trotignon
See also P. Aboulker.

Nicolas Trotignon & Kristina Vušković

(SGw, sg: Bal(Gen): Exp)

L.E. Trotter, Jr.
See E.C. Sewell.

Klaus Truemper
See also Conforti, Cornuéjols, and Truemper (1994a) and Gerards, Lovász, et al. (1990a), Tseng and Truemper (1986a).


A 0,±1-matrix is called “balanced” if it contains no submatrix that is the incidence matrix of a negative circle. More generally, α-balance of a 0,±1-matrix corresponds to prescribing the signs of holes in a signed graph. Main theorem characterizes the sets of holes (chordless circles) in a graph that can be the balanced holes in some signing. [See Conforti and Kapoor (1998a) for a new proof and discussion of applications.]

(SGw, sg: Bal, Sw)


(SGw, sg: Bal, Sw)


According to Cornuéjols (2001a), this paper contains the following theorem: A bipartite graph is “balanceable” (has a ±1-weighting (mod 4) in which all polygons have sum 0 (mod 4)) iff it does not contain an induced subgraph that is a subdivided odd wheel or a theta graph with nodes in opposite color classes. [The weights are not gains because they are not oriented. However, this has major applications to signed hypergraphs; cf. Rusnak (2010a).] [Problem. Generalize to arbitrary graphs.]

In a bipartite graph the sum around a polygon has to be 0 or 2 (mod 4) and therefore belongs to a group $\cong \mathbb{Z}_2$ so can be considered a sign. However, it may not be possible to relabel the edges from $\mathbb{Z}_2$ so as to get the same polygon sums. I.e., the polygon signing may not be
derivable from a signed graph.

Théophile Trunck
See P. Aboulker.

Anke Truss
See S. Böcker.

Marcello Truzzi
See F. Harary.

S.V. Tsaranov
See also F.C. Bussemaker, P.J. Cameron, and J.J. Seidel.


A two-graph whose points are the edges of a tree $T$ and whose triples are the nonseparating triples of edges of $T$ (from Seidel and Tsaranov (1990a) via Cameron (1994a)). An associated signed complete graph $\Sigma_T$ on vertex set $E(T)$ is obtained by orienting $T$ arbitrarily, then taking $\sigma_T(ef) = +$ or $-$ depending on whether $e$ and $f$ are similarly or oppositely oriented in the path of $T$ that contains both. Reorienting edges corresponds to switching $\Sigma_T$. Thm.: Letting $n = \#V(T)$, the matrices $3I_n + A(\Sigma_T)$ and $2I_{n+1} - A(T)$ have the same numbers of zero and negative eigenvalues. ($\text{TG: Eig, Geom}$)


New proof of theorem on the group (Seidel and Tsaranov (1990a)) of the two-graph (Tsaranov (1992a)) of a tree. ($\text{TG: Eig, Geom}$)

Michael J. Tsatsomeros

Dennis Tseng
See V. Reiner.

F.T. Tseng & K. Truemper

A special case is decomposition of $L_0(\Sigma)$. [Annot. 18 Jan 2021.] (sg)

Charalampos E. Tsourakakis, Michael Mitzenmacher, Jarosław Błasiok, Ben Lawson, Preetum Nakkiran, & Vasileios Nakos

Shuhei Tsujie
D. Tsvetkovich, M. Dub, & Kh. Zakhs


Russian ed. of Cvetković, Doob, and Sachs (1980a).

(Par, TG: Sw, Adj, Eig, Geom: Exp, Exr, Ref)

Jianhua Tu
See G.-H. Yu.

Thomas W. Tucker
See also J.L. Gross and J. Širáň.


Francesco Tudisco
See P. Mercado.

Vanda Tulli
See A. Bellacicco.

Pavel Tumarkin
See M.D. Sikirić.

Hande Tunçel
See F.M. Atay.

Edward C. Turner
See R.Z. Goldstein.

Daniel Turzík
See S. Poljak.

W.T. Tutte

Integral \((u, u)\)-flows on a signed graph with edge capacities, presented in the language of integral \((\tilde{u}, \tilde{u}^\ast)\)-flows on a digraph with edge capacities, with an orientation-reversing, fixed-point free, capacity-preserving involution \(^*\). [Such a digraph is the double covering digraph of a bidirected graph, thus the capacities and flows are equivalent to \((u, u)\)-flows on a capacitated signed graph.] Analog of the Min-Flow Max-Cut Theorem (see 3.3). Structure of flows. Application to undirected graph factors. [Problem. Convert the entire paper to the language of signed graphs. Express the structure of \((u, u)\)-flows in terms of signed-graphic objects such as unbalanced unicyclic subgraphs. Extract the implicit matroid theory, including the structure of cocircuits (cf. Chen and Wang (2009a)].] [Annot. 9 Sept 2010, 12 Jan 2012.] (sg: ori, cov: Flows)

The chain-group approach to the dual even-cycle matroid, \( G(\Gamma)^* \). Developed entirely in terms of the group \( \Delta(\Gamma) \) [topologically, \( B^1(\Gamma, \mathbb{Z}) \)] of integral 1-coboundaries. Assuming \( \Gamma \) connected: “Dendroids of \( \Delta(\Gamma) \) = bases of \( G(\Gamma)^* \); Thms. 8.6–7 give their structure in the bipartite and nonbipartite cases. Support of an elementary coboundary = circuit of \( G(\Gamma)^* \); this is a bond of \( \Gamma \) if \( \Gamma \) is bipartite (Thm. 7.5) and a minimal balancing set otherwise (Thm. 7.6). Thm. 7.8: Any coboundary times some power of 2 is a sum of primitive coboundaries. [Problem. Explain how this is related to total dyadicity of the incidence matrix.] “Rank of \( \Delta(\Gamma) \) = \( \text{rk} \, G(\Gamma)^* \); its value is given at the end of §8. §9 develops a relationship between “homomorphisms” of \( \Delta(\Gamma) \) (linear functionals) and graph factors. §10: The dual chain group; characterization of circuits of \( \text{rk} \, G(\Gamma)^* \). It is amazing what can be done with nothing but integral 1-coboundaries. Problem 1. Extend Tutte’s theory of integral chain groups to all signed graphs. Grossman, Kulkarni, and Schochetman (1994a) have a development over a field but this is very different, even aside from their opposite viewpoint that goes from matroids to vector spaces. Problem 2. Extend to signed hypergraphs, where each hyperedge has a function \( \tau_e : V(e) \to \{+,-\} \) (not distinguished from \(-\tau_e\); as with bidirected graphs, choosing one of them corresponds to orienting \( e \)).

[Tutte knew and lectured on \( G(\Gamma)^* \) and/or \( G(\Gamma) \) before anyone (Doob (1973a), Simões-Pereira (1973a)) published it.—information from Neil Robertson.]


[Annot. 9 May 2014.]


Kaya Tutuncuoglu
See B. Guler.

Zsolt Tuza
See S. Poljak.

Ilya Tyomkin
See A. Beimel.

Frank Uhlig
See C.R. Johnson.

J.P. Uhry
See F. Barahona and I. Bieche.
Włodzimierz Ulatowski  
See also J. Topp.  
Examines injective, nowhere zero, balanced gains (called “graceful labellings”) from $Z_{m+1}$, $m = \# E$, on arbitrarily oriented circles and variously oriented paths. [Question. Does this work generalize to bidirected circles and paths?] (GD: bal: Circ, Paths)

[N.B. Ul’janov]  
See N.B. Ul’yanov.

N.B. Ul’yanov  
See D.O. Logofet.

Somya Upadhyaya  
See D. Sinha.

Gurunath Rao Vaidya  
See P.S.K. Reddy.

J.F. Valdés  
See also W. Lebrecht and E.E. Vogel.

J.F. Valdés, J. Cartes, & E.E. Vogel  
Physics and signed graph theory on a signed polyhedral graph, esp. properties of ground states as functions of $x := \# E^+ / \# E$. Effects of vertex and face degrees. [Annot. 17 Jun 2012, 9 Jan 2015.] (Phys, SG: State(fr))

J.F. Valdés, J. Cartes, E.E. Vogel, S. Kobe, & T. Klotz  

J.F. Valdés, W. Lebrecht, & E.E. Vogel  
Randomly signed dice lattice (planar, with rhombic faces) with specified $x := \# E^+ / \# E$: frustration index, distribution of frustrated plaquettes (rhombi), et al., as functions of $x$. This lattice is interesting because the average degree (“coordination number”) is not integral; cf. Lebrecht, Vogel, and Valdés (2004a) et al. [Annot. 3 Jan 2015.] (SG, Phys: Fr State, Alg)


Carlos E. Valencia & Rafael H. Villarreal

Miguel A. Valencia Bucio
See also J. Martínez-Bernal.

Miguel Ángel Valencia Bucio

James Van Buskirk
See T.J. Lundy.

[Edwin R. van Dam]
See E.R. van Dam (under ‘D’).

Pauline van den Driessche

Hein van der Holst
See M. Arav.

[Jorn van der Pol]
See J. van der Pol (under ‘P’).

Arnout van de Rijt

Kevin N. Vandermeulen
See M.S. Cavers and D.A. Gregory.

Paul Van Dooren
See V.A. Traag.

[J.L. van Hemmen]
See J.L. van Hemmen (under ‘H’).

Marc A.A. van Leeuwen

Elements of the hyperoctahedral group $D_d$ (signed permutations) of even degree $d = 2n$ permute $\pm [n]$ and of odd degree $d = 2n + 1$ permute $[-n, n]$ (pp. 22f. The natural involution is $\pi \mapsto -\bar{\pi}$, where $\bar{\pi}$ is the reverse of $\pi$ [reminiscent of signed graph coloring]. [Cf. Bloss (2003a) and Parvathi (2004a).] [Annot. 19 Mar 2011.] (sg: Algeb)

A. Vannelli
See C.J. Shi.

Jean Vannimenus
See also B. Derrida and G. Toulouse.
J. Vannimenus, S. Kirkpatrick, F.D.M. Haldane, & C. Jayaprakash

*XY* means signed graphs with complex-unit vertex spins. (Phys: sg)

J. Vannimenus, J.M. Maillard, & L. de Sèze

J. Vannimenus & G. Toulouse

[Cyriel van Nuffelen]
See C. v. Nuffelen (under ‘N’).

M.E. Van Valkenburg
See W. Mayeda.

Anke van Zuylen, Rajneesh Hegde, Kamal Jain, & David P. Williamson

[Stefan H.M. van Zwam]
See S.H.M. van Zwam (under “Z”).

Burak Varan
See B. Guler.

Patricio Vargas
See E.E. Vogel.

[T.R. Vasanth Kumar]
See P.S.K. Reddy.

Wolmer V. Vasconcelos
See A. Simis.

B. Vasuki
See also L. Shobana.

B. Vasuki & L. Shobana

More examples as in Baskar Babujee and Loganathan (2011a). (Lab: VS: SG, Bal)

Ebrahim Vatandoost
See G.R. Omidi.

Vijay V. Vazirani & Mihalis Yannakakis

Slightly abridged version of (1989a).


“Evenness” of a digraph (i.e., every signing contains a positive cycle) is polynomial-time equivalent to evaluability of a certain 0–1 permanent by a determinant and to parts of the existence and recognition problems for Pfaffian orientations of a graph. Briefly expounded in Brundage (1996a).

Michalis Vazirgiannis
See C. Giatsidis and F.D. Malliaros.

Alina Vdovina
See S.-P. Liu.

[Renata R. Del-Vecchio]
See R.R. Del-Vecchio (under ‘D’).

J.J.P. Veerman

Differential equation ˙X(t) = X(t)2 with X(0) = X0, an n×n invertible matrix, with λ := max real eigenvalue > 0 having right and left eigenvectors v and w. Thm. 2: ∃! X(t) for t ∈ [0, λ−1); and (λ−1 − 1)X(t) → vwT as t ↑ λ−1. Meaning (§4): For t ≈ λ−1, [n] develops up to 4 factions, two “cohesive” as in balance and (if X0 is not symmetric) two “dispersive” that have antisymmetric relations with the first two. [The paper’s Thm. 2 is unnecessarily constrained by a probabilistic statement that requires 2[n].] Cf. Marvel, Kleinberg, et al. (2011a) and Traag, Dooren, et al. (2013a). [Annot. 12 Jun 2019.] (SD, SG, WD: Bal, Dyn)

Fernando Vega-Redondo
See G.C.M.A. Ehrhardt.

Guido Veiner

Clustering some types of signed K_n. [Annot. 10 Nov 2017.] (sg: Clu)

Mark Velednitsky
See Q. Spaen.

Alan Veliz-Cuba
See also E.D. Sontag.


Alan Veliz-Cuba, Boris Aguilar, Franziska Hinkelmann, & Reinhard Laubenbacher
2014a Steady state analysis of Boolean molecular network models via model reduction and computational algebra. *BMC Bioinformatics* 21 (2014), article 221, 8 pp. (SD: Dyn)

**Alan Veliz-Cuba & Reinhard Laubenbacher**


**Lluís Vena**  
See A. Goodall.

**Venkat Venkatasubramanian**  
See M.R. Maurya.

**Véronique Ventos**  
See P. Berthomé, S. Corteel, and D. Forge.

**Maryam Verdian-Rizi**  
See T. Huynh.

**Dirk Vertigan**  
See also J. Geelen and J. Oxley.


Characterizes representability of the Dowling geometry $Q_n(\mathcal{G})$ over a ring, or equivalently a skew partial field, for any finite group $\mathcal{G}$, thereby solving Pendavingh and van Zwam (2013a), Problem 6.5. [Annot. 28 Jan 2015.] (gg: M)

**Adrian Vetta**  
See S. Fiorini, J. Geelen, and B. Reed.

**K.A. Vidya**  
See E. Sampathkumar.

**Fabien Vignes-Tourneret**  
See also T. Krajewski.


**S. Vijay**  

**Ambat Vijayakumar**  
See Z. Stanić.
[G.K. Vijayakumar]
See G.R. Vijayakumar.

G.R. Vijayakumar
See also P.D. Chawathe, D.K. Ray-Chaudhuri, and N.M. Singhi.


A connected, contrabalanced biased graph \((\Gamma, \emptyset)\) has a covering by \(\xi+1\) internally disjoint paths, where \(\xi = \text{cyclomatic number}\), iff every \((\Gamma \setminus v, \emptyset)\) has no balanced components. \(\text{[Question 1. Can this generalize to all connected biased graphs? Paths should become balanced subgraphs that are “path-like” (have at most two vertices of attachment). \(\xi\) should become a measure of the number of independent unbalanced circles. \text{[Question 2. Is there a recursive decomposition of a 2-connected biased graph into \(\xi\) path-like balanced subgraphs, generalizing the standard ear decomposition of a 2-connected, (contrabalanced biased) graph?]}}\) \(\text{[Annot. 8 Mar 2008.]}\)
G.R. Vijayakumar (as “Vijaya Kumar”), S.B. Rao, & N.M. Singhi

A minimal forbidden induced graph has order at most 10, which is best possible. [Annot. 2 Aug 2010.] (sg: adj, Geom, lg)

G.R. Vijayakumar & N.M. Singhi

K.S. Vijayan
See S.B. Rao.

Viji Paul
See also S. Hameed K.

Ch. 5, “Co-regular signed graphs”: $\Sigma$ is $(r,k)$-“co-regular” if $|\Sigma|$ is $r$-regular and $\Sigma^+$ is $\frac{1}{2}(r+k)$-regular. An $r$-regular $\Gamma$ has an $(r,k)$-co-regular signing iff $\Gamma$ has a $\frac{1}{2}(r+k)$-factor. Examples are treated. Thm. 5.4.2: Every $\Sigma$ is an induced subgraph of an $(r,k)$-co-regular signed graph for all $r,k$ satisfying $r \geq \Delta(\Sigma^+) + \Delta(\Sigma^-)$, $r \equiv k \mod 2$, and $2\Delta(\Sigma^+) - r \leq k \leq r - 2\Delta(\Sigma^-)$ ($\Delta =$ max degree). [Annot. 22 Nov 2012.] (SGc: Lab)

V. Vilfred & C. Jayasekaran

See Jayasekaran (2007a). Examines self-switching vertices that are interchanged by an automorphism of $\Gamma$ (“interchange similar”). [Annot. 26 Sept 2012.] (tg: Sw)

V. Vilfred, J. Paulraj Joseph, & C. Jayasekaran

See Jayasekaran (2007a). Examines when a cut vertex $x$ is self switching, assuming all self-switching vertices are interchanged by automorphisms. [Annot. 26 Sept 2012.] (tg: Sw)


Jacques Villain

Some physics spin models do, or may, or do not, have negative edges ($J < 0$). Precedes recognition of frustration and switching. [Annot. 11 Aug 2018.] *(Phys: sg)*


Partition function of “odd model” on signed lattice graph, i.e., all “elementary polygons” (“plaquettes” in *Toulouse* (1977a): small chordless circles: squares, triangles, or hexagons in different lattices) are unbalanced. Spins $S_i := S(v_i)$ may be Ising (in $\{\pm 1\} = S^0$), XY (in $S^1$), or Heisenberg (in $S^2$). §3, “The Ising version of the odd model on the two-dimensional, square lattice”. §4, “XY version of the odd model on the two-dimensional, square lattice”. Unique ground state up to a $\pm 1$ variable. §5, “The odd model on the diamond lattice”: Ising vs. higher-dimensional ground states. §6, “Magnetic susceptibility of the odd model on the diamond lattice (XY version)”. §7, “Modified odd models”: A few balanced plaquettes. Or, slightly varying spin magnitudes. §8, “The odd model on other types of lattice”. App. 1: The odd model switches to periodic form for the lattices treated herein. [Annot. 17 Jun 2012, 10 Aug 2018.] *(SG: Phys, Sw, VS(Gen))*


Rafael H. Villarreal

See also J. Martínez-Bernal, A. Simis, and C.E. Valencia.


The algebra for $\Gamma$ is closely related to the frame and lift matroids of $-\Gamma$. Cor. 3.2: The edge ideal $I(\Gamma)$ is of linear type iff $G(-\Gamma)$ is a free matroid. Lem. 3.1: The even cycle space has codimension $\text{rk } G(-\Gamma)$ [as found in van Nuffelen (1973a)]. Prop. 4.2: The elementary vectors of Nul $H(-\Gamma)$, i.e., the integral cycle space, are the indicator vectors of frame circuits [as in Doob (1973a)]. [Annot. 3 Jun 2015, rev 18 Aug 2018, 22 Oct 2020.] *(sg: Par: Incid Algeb, m)*


J. Villegas

See E.E. Vogel.

Daniele Vilone

See F. Radicchi.

Andrew Vince

See Theorem 6.1. (sg: bal: Top)

**E. Vincent, J. Hammann, & M. Ocio**


Surveys experiments with spin glass materials, especially aging behavior. Observations tend to support a landscape of graph signatures with numerous metastable states, subdividing as temperature decreases. [Presumably, the states correspond to clusters of low-frustration states, separated by high-frustration barriers, subdivided into smaller clusters by lower-frustration barriers, and so on. Mathematical examination is needed.] [Annot. 27 Aug 1998, 24 Aug 2012.] (Phys: sg: State: Dyn)

**Miguel Angel Virasoro**

See M. Mézard.

**Lee Inmon Virden**

See T.J. Reid.

**Krishnamurthy Viswanathan**

See R.M. Roth.

**Fabio Vitale**

See N. Cesa-Bianchi and G. Le Falher.

**J. Vlach**

See C.J. Shi.

**Alexander Vogel**

See E. Steffen.

**E.E. Vogel [Eugenio E. Vogel Matamala]**


**E.E. Vogel, J. Cartes, S. Contreras, W. Lebrecht, & J. Villegas**


Many small toroidal lattice examples are computed to estimate large-scale behavior of ground states: frustration index, ground state energy $E_g$, etc. The signs are random, half positive and half negative. Also near-ground states (energy $E_g + 4$), obtained either by switching one vertex of a ground state or as a local minimum in the state space.

Dictionary (for papers of Vogel *et al.*): “periodic boundary conditions” = toroidal; “plaquette” = face boundary circle; “curved” plaquette = frustrated = negative; “frustration length” = frustration index; “ground state” = minimizing switching function $\zeta$ of $\Sigma$ (i.e.; $#E^-(\Sigma^\zeta) = \min_\zeta = l$); “ground state degeneracy” $W = \text{number of ground states}; “magnetization per spin” (of ground state) = $([#\zeta^{-1}(+) - #\zeta^{-1}(-)]/nW$; “remnant entropy” = $(\ln W)/n$. [Annot. 4 Jan 2015.]

(Phys, SG: State(fr), Fr)

**Eugenio E. Vogel, Jaime Cartes, Patricio Vargas, & Dora Altbir**
(SG, Phys: State(fr), Fr)

**E.E. Vogel, J. Cartes, P. Vargas, D. Altbir, S. Kobe, T. Klotz, & M. Nogala**  
(Phys, SG: State(fr))

**E.E. Vogel, S. Contreras, J. Cartes, & M.A. Osorio**  

In a signed, toroidal square lattice graph keep the edges that are satisfied in every ground state. 500 $6 \times 6$ examples were computed. $h :=$ proportion of retained edges tends to $\approx 0.5$. The distribution of component sizes is quite different from that of random subgraphs of similar size. For large $h$ there is a boundary-linking giant component. For $h \approx 0.5$ there tends to be a definite proportion of small components of particular shapes. [Annot. 8 Feb 2015.]  
(SG: Fr: State, Phys)

**E.E. Vogel, S. Contreras, W. Lebrecht, & J. Cartes**  
(SG, Phys: Fr: State)

**E.E. Vogel, S. Contreras, F. Nieto, & A.J. Ramírez-Pastor**  
(SG, Phys: State(fr), Sw)

**E.E. Vogel, S. Contreras, M.A. Osorio, & J. Cartes**  
Square lattices on the torus. “Diluted” means removing every edge that is frustrated in one or more ground states. Dictionary: *cf. Vogel, Cartes, Contreras, Lebrecht, and Villegas (1994a).* [Annot. 3 Jan 2015.]  
(SG, Phys: State(fr), Sw)

**E.E. Vogel, S. Contreras, M.A. Osorio, A.J. Ramírez-Pastor, & F. Nieto**  
Square lattices on the torus. “Diluted” means removing every edge that is frustrated in one or more ground states. Dictionary: *cf. Vogel, Cartes, Contreras, Lebrecht, and Villegas (1994a).* [Annot. 3 Jan 2015.]  
(SG, Phys: State(fr), Sw)

**E.E. Vogel & W. Lebrecht**  
The expansions are for functions of signed toroidal triangular, square, and honeycomb lattices such as the “frustration length” $= \text{minimum length of a } T\text{-join in the dual graph connecting frustrated plaquettes in pairs, and the proportion of edges that are positive in } \Sigma$ for every ground
state \( \zeta \). Applies two theoretical methods and compares to computed examples. [Annot. 9 Jan 2015.] (SG, Phys: State, State(fr): Invar)

E.E. Vogel, A.J. Ramirez-Pastor, & F. Nieto

“All” signed square lattices studied for physical properties like order parameters and grouping of ground states (minimal switchings). Related numerical results in Lebrecht, Vogel, and Valdés (2002a). [Annot. 3 Jan 2015.] (SG, Phys: State(fr))

E.E. Vogel, J.F. Valdés, & W. Lebrecht

G.E. Volovik
See I.E. Dzyaloshinskii.

Jan Vondrák
See A. Galluccio.

Axel von Kamp
See S. Klamt.

Heinz-Jürgen Voss
See also D. Král’.


Jože Vrabek
See T. Pisanski.

Eleni-Maria E. Vrettta
See K. Papalamprou and L. Pitsoulis.

Hung Thanh Vu
See T.T.T. Ho.

Damir Vukičević
See T. Došlić.

Kristina Vušković
See also P. Aboulker, V. Boncompagni, M. Conforti, T. Kloks, and N. Trotignon.

Michelle L. Wachs
See also A. Björner, E. Gottlieb, and J.B. Remmel.


Donald K. Wagner
See also V. Chandru and C.R. Coullard.


Prop. 1 and Thm. 2 show that $n$-connectivity of the bicircular matroid $BG(\Gamma)$ is equivalent to “$n$-biconnectivity” of $\Gamma$.

When do two 3-biconnected graphs have isomorphic bicircular matroids? §5 proves that 3-biconnected graphs with $\leq 4$ vertices have isomorphic bicircular matroids iff one is obtained from the other by a sequence of operations called “edge rolling” and “3-star rotation”. This is the bicircular analog of Whitney’s circle-matroid isomorphism theorem, but it is complicated. [An important theorem, generalized to all bicircular matroids in Coullard, del Greco, and Wagner (1991a). *Major Research Problems*. Generalize to frame matroids of biased graphs. Find the analog for lift matroids.] (Bic: Str)


“Factor matroid” = even-cycle matroid $G(-\Gamma)$. Decides when $G(-\Gamma) \cong G(B)$ where $B$ is a given bipartite, 4-connected graph. (EC: Str)

H. Wagner
See K. Drühl.

Magnus Wahlström
See S. Böcker and S. Kratsch.

Bronislaw Wajnryb
See R. Aharoni.

Oliver Waldmann
See K.C. Mondal.

M.H. Waldor, W.F. Wolff, & J. Zittartz


Physics of all-positive and all-negative (“fully frustrated”: all face boundary circles are negative) signs as examples. § II, “Thermodynamics”, b) “Homogeneous case”: The all-negative signature has multiple
ground states that have energy $-J$ ($J =$ bond strength) per vertex because the frustrated edges form a perfect matching in a ground state. [I.e., frustration index $l(-\Gamma) = \#V$ for these pentagonal lattice graphs, assuming no boundary as, e.g., when the lattice is toroidal.] [Annot. 18 Jun 2012] (Phys: SG: Par: State(fr))

H.B. Walikar, Satish V. Motammanavar, & B.D. Acharya

Like B.D. Acharya (2012b), but a signed domination function is $f \in \{\pm 1\}^V$. NASC for a signed path, star, circle, or caterpillar to admit such a function. Every signed graph is an induced subgraph of a signed graph that admits one. [Annot. 18 May 2018, 29 Dec 2020]. (SG: Lab)

Derek A. Waller
See also F.W. Clarke.

Georg R. Walther, Matthew Hartley, & Maya Mincheva

O. Walther
See Q. Zheng.

Changping Wang

Fix $\Gamma$. Signed matching: $\sigma_m$ such that all $d^\pm(v) \leq 1$. Signed matching number $\beta'_1(\Gamma) := \max \sigma_m(\#E^+ - \#E^-)$. Finding a max $\sigma_m$ is strongly polynomial. Bounds on $\beta'_1$; exact values for $P_n, C_n, K_n, K_{p,q}$. Signed edge cover: $\sigma_e$ such that all $d^\pm(v) \geq 1$. Signed edge cover number $\rho'_1(\Gamma) := \min(\#E^+ - \#E^-)$. $\beta'_1(\Gamma) + \rho'_1(\Gamma)$ need not $= n$. [Annot. 17 Dec 2020.] (sg, Alg)


Generalizing (2008a), $b$- means $\geq b$ or $\leq b$. Bounds for strong product. Gallai theorem for $K_n$ and $K_{p,q}$. [Annot. 17 Dec 2020.] (sg)


Generalizing (2008a), $\sigma_S$ such that $d^\pm(v) \leq 1$ for $\geq k$ vertices. Sharp bounds on $\beta^k_S(\Gamma) := \max \sigma_S(\#E^+ - \#E^-)$ and some exact values. [More in S. Akbari, M. Dalirrooyfard, K. Ehsani, & R. Sherkati (2016a).] [Annot. 17 Dec 2020.] (sg)

Chong-Jun Wang
See Y. Wang.

Chun-Chieh Wang
See L.-H. Chen.
Cuihua Wang  
See D. Li.

Dan Wang  
See W. Chen.

Dengyin Wang  
See F.L. Tian.

Dijian Wang  
See also Y.-P. Hou.

Dijian Wang & Yaoping Hou  
(SG: Lap: Eig)

(SG: Adj: Eig)


There are one infinite family and 14 sporadic signed graphs with integral Spec $A(\Sigma)$. [Annot. 9 Dec 2020.]  
(SG: Adj: Eig)

(SG: Adj: Eig)

20xxb Laplacian integral subcubic signed graphs. Submitted.  
(SG: Lap: Eig)

Guangfu Wang, Shuchao Li, Wei Wei, & Siqi Zhang  

There are one infinite family and 14 sporadic signed graphs with integral Spec $A(\Sigma)$. [Annot. 9 Dec 2020.]  
(SG: Adj: Eig)

(SG: Adj: Eig)

Hai-Feng Wang  
See M.L. Ye.

Hua Wang  
See J. Huang.

Jianfeng Wang  
See F. Belardo.

Jianfeng Wang & Francesco Belardo  
(par: Lap: Eig)

(par: Lap: Eig)

Jianfeng Wang, Francesco Belardo, Wei Wang, & QiongXiang Huang

Jianfeng Wang & Qiongxiang Huang

The signless Laplacian \( Q = L(−\Gamma) \) is employed to derive results on the Laplacian \( L(\Gamma) \). Continued in Wang, Belardo, Huang, and Li Marzi (2013a). [Annot. 9 Feb 2013.]

Jian-Sheng Wang and Robert H. Swendsen

Jue Wang
See also B. Chen.


Kyle Wang
See G. Chen.
Larry X.W. Wang  
See W.Y.C. Chen.

Li Wang  
See S.-S. Feng.

Ligong Wang  
See also Y.Q. Chen, K. Li, and Y. Lu.

Ligong Wang, Guopeng Zhao, & Ke Li  

Long Wang  
See X. Lin and X.B. Ma.

Longqin Wang, Zhengke Miao, & Chao Yan  

Longqin Wang, Lihua You, & Hongping Ma  

Lusheng Wang  
See X.L. Li.

Qingwen Wang  
See G.H. Yu.

Shanfeng Wang  
See Q. Cai.

Shaohui Wang  
See J.-B. Liu.

Shilin Wang & Bo Zhou  

Shiyong Wang, Jing Li, Wei Han, & Shangwei Lin  

Shujing Wang  
See S.C. Li.

Suijie Wang  
See X.G. Liu.

Tianfei Wang  
See MR for the formulas, which apply to \( q_1(L(-\Gamma)) \) [hence to signed simple graphs]. The proofs use a normalized Laplacian. [Annot. 16 Jan 2012.] 

Wei Wang
See X.-J. Tian.

Wei Wang [mr36]
See also Wang, Qiu, Qian, and Wang (2020a).

Wei Wang & Jianguo Qian

Wei Wang, Jianguo Qian, & Toshiki Abe

Wei Wang, Zhidan Yan, & Jianguo Qian
2021a Eigenvalues and chromatic number of a signed graph. *Linear Algebra Appl.* 619 (2021), 137–145. Thm.: \( \chi(\Sigma) \geq 1 + \lambda_1/|\lambda_n|, \ n/(n - \lambda_1), \) where \( \lambda_1, \lambda_n = \max, \min \) adjacency eigenvalues. [Annot. 19 Mar 2021.] (SG: Col, Adj: Eig)

Wei Wang [mr21]
Wei Wang, Lihong Qiu, Jianguo Qian, & Wei Wang
2020a Generalized spectral characterization of mixed graphs. \( \Phi \) with gains in \{1, ±i\} as in Liu and Li (2015a) and Mohar (2016a). “Generalized spectrum” = (Spec \( A(\Phi) \), Spec(\( J - I - A(\Phi) \))). Characterization up to isomorphism, not switching isomorphism or conjugation (thus must be self-conjugate = digraph self-converse). |Conjugation preserves spectra. Switching preserves Spec \( A(\Phi) \) but not Spec(\( J - I - A(\Phi) \)).| [For gain graphs, this is unnatural; spectral characterization up to switching and conjugation is natural.] [Annot. 23 Dec 2020.] (gg: Adj: Eig)

X.L. Wang
See Y. Chen.

Xiao Wang, You Lu, Cun-Quan Zhang, & Shenggui Zhang

Xiao Wang, You Lu, & Shenggui Zhang

Xioafeng Wang
See E.L. Wei.
Xing Ke Wang
See S.W. Tan.

Yajun Wang
See Y.-H. Li.

Yi Wang
See also Y.Z. Fan, L.L. Liu, G.-D. Yu, B.-J. Yuan, and J. Zhou.

Yi Wang & Yi-Zheng Fan
(par: Lap: Eig)

Yi Wang, Shi-Cai Gong, & Yi-Zheng Fan

Yi Wang & Yeong-Nan Yeh

Yitian Wang
See Z.-Y. Cheng.

Yue Wang
See Y.Z. Fan.

Zhouningxin Wang
See R. Naserasr.

Egon Wanke
See also F. Höfting.


Broadly resembles Höfting and Wanke (1994a) but omits those edges of $\tilde{\Phi}$ that are affected by the modulus $\alpha$. (GD(Cov): Alg)

Ian M. Wanless

$\text{per}(B) := \sum_M \sigma(M)$, summed over all perfect matchings $M \subseteq \Sigma = (K_{n,n}, \sigma)$, where $A(\Sigma) = (\begin{bmatrix} B & 0 \\ 0 & \lambda \end{bmatrix})$. [Annot. 22 Aug 2012.] (sg: Adj)

G.H. Wannier

Physical consequences of frustration with Ising spins, i.e. $\zeta : V \to \{+1, -1\}$, in the all-negative triangular lattice graph. [See also Houtappel (1950a), (1950b), Newell (1950a), I. Syôzi (1950a).] [Annot. 16 Jun 2012.]

T. Wanschura, D.A. Coley, & S. Migowsky


Calculation by genetic algorithm for certain lattice graphs. [Annot. 3 Jan 2015.]

Dan Warner
See C.R. Johnson.

Jacqueline M. Warren
See G. MacGillivray.

Stanley Wasserman & Katherine Faust, eds.


Yusuke Watanabe


William C. Waterhouse


John J. Watkins
See R.J. Wilson.
William Watkins
See M. Lien.

Kevin D. Wayne
See É. Tardos.

Nikolai Weaver
See E. Flapan.

Jeffrey R. Weeks & Kenneth P. Bogart

Er Ling Wei, Wen Tang, & Dong Ye

Erling Wei, Wenliang Tang, & Xiaofeng Wang

Fuyi Wei
See F.-Y. Wei.

Fi-yi Wei
See also M.H. Liu.

Fi-yi Wei & Muhuo Liu

Li Juan Wei
See Y.P. Hou.

Po-Sun Wei and Bang Ye Wu

Wei Wei
See G.-F. Wang.

Martin Weigt
See A.K. Hartmann.
Gerry M. Weiner
See J.S. Maybee.

Volkmar Welker

The arrangement is the affine part (that is, where \( x_0 = 1 \)) of the projective representation of \( G(\Phi) \), where \( \Phi \) is the complex multiplicative gain graph \( \Phi = \{1\}K_{n+1} \cup \{re_0_i : 1 \leq i \leq n \text{ and } 2 \leq r \leq s\} \). Here the vertex set is \( \{0,1,\ldots,n\} \), \( s \) is any positive integer, and \( re_0_i(r) \) denotes an edge \( v_0v_i \) with gain \( r \). The topics of interest are those related to the complex complement. The study is based on the combinatorics of the intersection semilattice \( \text{Lat}^b \Phi \) of balanced flats, including the Poincaré polynomial of the arrangement [equivalent to the balanced chromatic polynomial of \( \Phi \) ].

Albert L. Wells, Jnr.
See also P.J. Cameron and Y. Cheng.


D.J.A. Welsh [Dominic Welsh]
See also L. Lovász and W. Schwärzler.


§11.4: “Partition matroids determined by finite groups”, sketches the most basic parts of Dowling (1973b). (GG: M: Exp)


The signed graph of a link diagram is employed to get an upper bound. (SGc: Enum)


Includes very brief treatments of some appearances of signed graphs.

§2.2, “Tait colourings”, defines the signed graph of a link diagram, mentioned again in observation (2.3.1) on alternating links and Prop (5.2.16) on “states models” (from Schwärzler and Welsh (1993a)). §5.6, “Thistlethwaite’s nontriviality criterion”: the criterion depends on the signed graph.
§2.5, “The braid index and the Seifert index of a link”, defines the Seifert graph, a signed graph based on splitting the link diagram.

(SGc, Knot)

§5.7, “Link invariants and statistical mechanics”, defines a relatively simple spin model for signed graphs, with an arbitrary finite number of possible spin values. The partition function is related to link diagrams.

§4.2, “The Ising model”, introduces the basic concepts in mathematical terms. §6.4, “The complexity of the Ising model”, “Computing ground states of spin systems”, pp. 105–107, discusses finding a ground state of the Ising model. This is described as the min-weight cut problem with weights the negatives [this is an error] of the Ising bond interaction values: that is, the weighted frustration index problem in the negative [erroneous] of the Ising graph. It is the max-cut problem when the Ising graph is balanced (ferromagnetic) [should be antibalanced (antiferromagnetic)]. For external magnetic field, follows Barahona (1982a).

(sg: State(fr), Fr, Phys)

§3.6, “Ice models”, counts “ice configurations” (certain graph orientations) via poise gains modulo 3, although the counting function is not gain-graphic.

(gg, Invar, Phys)

§4.4: “The Ashkin–Teller–Potts model”. This treatment of the Potts model has a different Hamiltonian from that of Fischer and Hertz (1991a). [It does not seem that Welsh intends to admit edge signs. If they are allowed then the Hamiltonian (without edge weights) is \(-\sum \sigma(e_{ij})\delta(s_i, s_j) - 1\). Up to halving and a constant term, this is Doreian and Mrvar’s (1996a) clusterability measure \(P(\pi)\), with \(\alpha = \frac{1}{2}\), of the vertex partition induced by the state.] [Also cf. Fischer and Hertz (1991a).] (clu, Phys)


Link diagrams ↔ dual pairs of sign-colored plane graphs: based on Yajima and Kinoshita (1957a). Unsolved algorithmic problems about knots based on link diagrams; in particular, triviality of diagrams is equivalent to Problem 4.2: A polynomial-time algorithm to decide whether the graphical Reidemeister moves can convert a given signed plane graph to one with edges all of one sign.

(SGc: D, Knot: Alg, Exp)


Pp. 258–259: Sign-colored graph of an alternating link diagram. [Annot. 27 Feb 2017.]

(Knot: SGc: Exp)

Mostly describes the signed graph of a link diagram and its relation to knot theory, including knot properties deducible directly from the signed graph, the Kauffman bracket and two-variable polynomials, etc. Similar to relevant parts of (1993a). (SGc: Knot: Invar: Exp)

Michael Welsh

Examples include: Def. 2.0.9: Dowling geometry $Q_n(GF(3)^t)$. §3.1.2, “Spikes”, i.e., $L_0(2C_n, B)$. [Annot. 27 Feb 2017.] (gg: M: Exp)

Emo Welzl
See H. Edelsbrunner and J. Hage.

Qin Wen, Qin Zhao, & Huiqing Liu

D. de Werra
See C. Benzaken.

Hans V. Westerhoff
See B.N. Kholodenko.

Peter Whalen
See R. Thomas.

Joyce Jiyoungh Whang
See K.-Y. Chiang.

Arthur T. White

Ch. 10: “Voltage graphs”. (GG: Top, Cov)


Take a graph $\Gamma$ with cyclomatic number $k$ and randomly sign it so that each edge is negative with probability $p$. The probability that $(\Gamma, \sigma)$ is balanced $= 2^{-k}$ if $p = \frac{1}{2}$ [obvious] and $\leq \left(\max(p, 1-p)\right)^k$ in general [not obvious] (this has an interesting asymptotic consequence due to Gimbel, given in this paper). [Related: Frank and Harary (1979a).] (SG: Rand, Bal)


§10-2, “Voltage graphs”: Voltage graphs and the covering graph. Thm. 10-8 is similar to Biggs (1974a), Thm. 19.5. Construction of surface
embeddings. §11-3, “Nonorientable voltage graph imbeddings”: Rotation schemes supplemented by edge signatures as in Ringel (1977a), Stahl (1978a), and Zaslavsky (1992a).


The voltage graph (i.e., gain graph) construction is used to generate embeddings of finite geometries. [Annot. 12 Jun 2013.]

Jacob A. White
20xxa Burnside chromatic polynomials of group-invariant graphs. Submitted.

Unlabelled coloring of a gain graph, in the form of its group covering graph, in the more general form of a graph with a group action. Incidentally, develops a remark in Zaslavsky (()009a)Thomas Zaslavsky. [Annot. 24 Nov 2020.]

Neil L. White
See also A. Björner.


Neil White & Walter Whiteley
† 1983a A class of matroids defined on graphs and hypergraphs by counting properties. Unpublished manuscript, 1983.

See Whiteley (1996a) for an exposition and extension. (Bic: Gen)

Walter Whiteley
See also N.L. White.


“Balance” used for circles with identity gain (in a gain graph with additive matrix gains), independently of Harary (1953a). §3, “Splines and matrices on graphs”: The matrix gains are $L_{hi}^{r+1}$ (p. 592) and the balance equation is ($\ast$) (p. 593). [Annot. 12 Jun 2012.] (gg: bal)


Appendix: “Matroids from counts on graphs and hypergraphs”, which expounds and extends Loréa (1979a), Schmidt (1979a), and especially White and Whiteley (1983a), describes matroids on the edge sets of graphs (and hypergraphs) that generalize the bicircular matroid. The
definition: given $m \geq 0$ and $k \in \mathbb{Z}$, $S$ is independent iff $\emptyset \subset S' \subseteq S$ implies $\#S' \leq m\#V(S') + k$. [Suggested name: “Linearly bounded matroids,” since they are defined by a linear bound on the rank.]  

(Bic: Gen)(Ref)

Geoff Whittle
See also R. Chen, J. Geelen, D. Mayhew, J. Oxley, and C. Semple.


A Dowling-lattice version of Crapo and Rota’s critical problem. Some minimal matroids whose critical exponent is $k$ (i.e., tangential $k$-blocks) are given, one being Dowling’s rank-$n$ matroid of $\{+, -, \} \setminus \emptyset$, $G(\pm K_n)$. [Annot. 25 May 2009.] (Bic: Gen: Invar)


Examples include bicircular and frame matroids. (GG: M, Bic)

2005a Recent work in matroid representation theory. Discrete Math. 302 (2005), 285–296. MR 2179649 (2006m:05053). Zbl 1076.05022. annot P. 288: The “free spike $\Phi_r$” is $L(2C_r, \emptyset)$. Pp. 290–291: Biased graphs and the bias [i.e., frame] matroid. Conjecture 5.2: With few exceptions, a highly connected matroid that is representable over more than one characteristic is a frame or dual frame matroid. P. 294: The “free swirl $\Psi_k$” is $G(2C_k, \emptyset)$. $U_{3,6} = L(2C_3, \emptyset) = G(2C_3, \emptyset)$ [the latter because there are no vertex-disjoint unbalanced circles]. [Annot. 25 May 2009.] (gg: M: Exp)

Gábor Wiener
See T. Fleiner.

Avi Wigderson
See S. Hoory.

Chris Wiggins
See E. Ziv.

J.K. Williams
See also B.G.S. Doman.


§2, “The random-bond Ising chain in a uniform field: $(T = 0)$”: A path with random edge signs, weighted $J$, magnetic field $B$ [interpretable as an extra all-positive vertex, as in ??]. Continued in Doman and Williams (1982a), §2. [Annot. 28 Aug 2012.] (Phys, SG, WG: State(fr))

David P. Williamson
See A. van Zuylen.

Andrew Timothy Wilson
See E. Leven.

Mark C. Wilson
See S. Aref.
Richard C. Wilson
   See P.-L. Giscard.

Robin J. Wilson
   See also L.W. Beineke.

Robin J. Wilson & John J. Watkins
      §3.2: “Social Sciences” (pp. 51–53) applies signed graphs. §5.1: “Signed digraphs” (pp. 96–98) discusses positive and negative feedback (i.e., positive and negative cycles) in applications. Based on Open University (1981a).

Steve Wilson
      Cantankerous map: signed expansion graph $\pm \Gamma$, orientation embedded in a surface, whose map automorphisms act transitively on flags. Rotary map: map with automorphisms that are cyclic permutations around a face and around a vertex on the face. Thm.: A rotary map is either cantankerous or a kind of branched covering. [See C.H. Li and Širáň (2007a) for more on cantankerous maps.] [Annot. rev. 31 Jul 2014.]

Marco Winkler
   See E. Terzi.

Shmuel Winograd
   See R.M. Karp.

Wayland H. Winstead
   See J.R. Burns.

Anthony Wirth
   See M. Charikar and T. Coleman.

H.S. Witsenhausen
   See Y. Gordon.

C. Witzgall & C.T. Zahn, Jr.

Jakub Onufry Wojtaszczyk
   See M. Cygan.

W.F. Wolff
   See also P. Hoever and M.H. Waldor.

W.F. Wolff & J. Zittartz
      § III, “The fully-frustrated square lattice model (FFS)”: Square lattice graph signed so every square (“plaquette”) is negative. § IV, “The chess-
board model": Square lattice graph with alternate squares negative and positive. [Annot. 28 Aug 2012.]


Paul Wollan
See B. Guenin, T. Huynh, and K. Kawarabayashi.

Dein Wong
See X.B. Ma and F.L. Tian.

Tsai-Lien Wong
See L.-G. Jin and H. Qi.

R. Kevin Wood
See G.G. Brown.

Bang Ye Wu


Bang Ye Wu & Li-Hsuan Chen

Equivalent: Find minimizing bipartition for \((K_n, \sigma)\). Objective: minimize \(\sum_v c(v)\) (thus finding \(l(K_n, \sigma)\)) or \(\sum_v c(v)^2\).

Dictionary: “graph” = positive subgraph of \((K_n, \sigma)\); “editing” = edge sign changes; “2-clustering” = bipartition \(V = X \cup Y\), “conflict number” \(c(x)\), for \(x \in X\) (say), = \#(edges +xx') + \#(edges −xy). [Annot. 13 Jun 2017.]

Chai Wah Wu

The graphs are weighted mixed graphs, i.e., bidirected graphs without introverted edges, and the matrices are digraph matrices, i.e., (weighted) outdegree matrices. The “Laplacian” is \(D - A\) where \(A\) is the adjacency matrix and \(D\) is the diagonal outdegree matrix. [Annot. 23 Mar 2009.]

(sg, sd: ori: incid, Eig)
Chong Wu
See D. Li.

Guangmei Wu
See C.-C. Huang.

Jianliang Wu
See X.Q. Qi.

Jianshe Wu, Licheng Jiao, Chao Jin, Fang Liu, Maoguo Gong, Ronghua Shang, & Weisheng Chen
Detects possibly overlapping clusters in $\Gamma$ via $(K_n, \sigma)$ with $E^+ = E(\Gamma)$.
Each vertex gets a randomly phased oscillator. Edge weights $w_+ > 0$, $w_- \geq 0$. Oscillators $\rightarrow$ in-phase on $+$ edges, out-of-phase on $-$ edges, revealing clusters. [Annot. 16 Jun 2018.] (sg: kg, WG: Clu: Dyn)

Jianshe Wu, Long Zhang, Yong Li, & Yang Jiao

Jingwen Wu
See Y. Lu.

Leting Wu, Xintao Wu, Aidong Lu, & Yuemeng Li
Extended abstract of (2017a). (SG: Clu: Adj: Eig, Geom)


Leting Wu, Xiaowei Ying, Xintao Wu, Aidong Lu, & Zhi-Hua Zhou


Qiang Wu
See G.Z. Liu.

Shu-Hui Wu
See B.S. Tam.

Sun Wu
See W.-S. Shih.

Xiao Li Wu, Jing Jing Jiang, Ji Ming Guo, & Shang Wang Tan

**Xintao Wu**
See Y.-M. Li and L.T. Wu.

**Yarong Wu**
See G.L. Yu.

**Yanzhi Wu, Lu Liu, Jiangping Hu, & Gang Feng**

Multilayer signed digraph. (SD, sg: Bal, Dyn)

**Yezhou Wu & Dong Ye**


**Yezhou Wu, Dong Ye, Wenan Zang, & Cun-Quan Zhang**

**Yuhan Wu**
See S.Y. Yi and L.H. You.

**Zhaoyang Wu**

A spike is \( L_0(\Omega) \) where \( \|\Omega\| = 2C_n \). (gg: M: Enum)

**Donald C. Wunsch**
See Harary, Lim, *et al.*

**Chengyi Xia**
See S.-S. Feng.

**Wen Xia**
See X.-L. Li.

**Kai-nan Xiang**
See R. Chen.

**Bai Xiao, Song Yi-Zhe, & Peter Hall**

Empirical tests of usefulness of the eigenvalues (the “feature vector”) of \( L(-\Gamma) \). [Annot. 24 Jan 2012.] (Par: Eig: Appl)

**Min Xiao**
See S.-D. Zhai.
Peng Xiao
See Y. Lu.

Mengmeng Xie & Chuixiang Zhou

Rundan Xing & Bo Zhou

Baogen Xu
See also S.-C. Li.


See Vijayakumar (2011a). [Annot. 10 Feb 2013.] (SGw)

Brian Xu
See L.J. Rusnak.

Guang-Hui Xu
See S.C. Gong.

Jing Xu
See B.-J. Yuan.

Meiling Xu
See G.L. Yu.

Rui Xu & Cun-Quan Zhang

Σ has a nowhere-zero 6-flow if it is coloop-free and edge 6-connected, thus solving Bouchet’s (1983a) conjecture in this case. [Annot. 5 Feb 2010.] (SG: Flows)

Shaoji Xu
See also F.S. Roberts.


Yuan Xu
See D. Peng.

Zhi-Ming Xu
See D. Li.

Sandeep Kumar Yadav
See B. Adhikari.

Takeshi Yajima & Shin’ichi Kinoshita

Examines the relationship between the two dual sign-colored graphs, $\Sigma$ and $\Sigma'$, of a link diagram (Bankwitz (1930a)), translating the Reidemeister moves into graph operations and showing that they will convert $\Sigma$ into $\Sigma'$.

**Takeo Yamada**


**Takeo Yamada & Harunobu Kinoshita**


In a real-weighted digraph, “negative” means the sum of weights is negative.

**Yuuzi Yamada**

See O. Nagai.

**Chiaki Yamaguchi**


**Yutaro Yamaguchi**

See Y. Kawase.

**Takeo Yamamoto**

See T. Nakamura.

**Chao Yan**


**Eric Yan**

See L.J. Rusnak.

**Jing-Ho Yan, Ko-Wei Lih, David Kuo, & Gerard J. Chang**


Net degree sequences of signed simple graphs. Thm. 2 improves the Havel–Hakimi-type theorem from Chartrand, Gavlas, Harary, and Schultz (1994a) by determining the length parameter. Thm. 7 characterizes the net degree sequences of signed trees. [There seems to be room to strengthen the characterization and generalize to weighted degree sequences: see notes on Chartrand, Gavlas, *et al.* (1994a).] (SGw: ori: Invar)

**Zhidan Yan**

See W. Wang.

**Aimei Yang & Josh Bentley**

Alex Yang
See V. Chen.

Arthur L.B. Yang
See W.Y.C. Chen.

Bo Yang, William K. Cheung, & Jiming Liu

An algorithm for an approximate clustering of a (weighted) signed (di)graph. Input: The graph and a length parameter $l$. Step 1: Construct transition probabilities $p_{ij} := [\sigma_{ij}w_{ij}]^+/d(v_i)$. Step 2: Apply the probabilities in a random walk of length $\leq l$ on positive edges; the matrix of $l$-step probabilities is $(p_{ij})^l$. Combine in a cluster the vertices that have high probabilities from a given starting point. “High” and $l$ are based on the network structure. Also, a cut algorithm for approximate clustering. A cluster is $X \subset V$ such that the total net degree $d^+(\Sigma:X) \geq d^+(X,X^c)$ and $d^-(X^c,X) \leq d^-((\Sigma:X^c)$.

Bo Yang, Xueyan Liu, Yang Li, & Xuehua Zhao

Bo Yang, Xuehua Zhao, & Xueyan Liu

Dan Yang
See Y.Z. Fan.

Fan Yang & Sanming Zhou

Withdrawn due to incomplete proof. [Can someone complete it?] [Annot. 20 Jul 2020.] (SG: Flows)

Laurence T. Yang
See F. Hao.

Ruixian Yang
See C.-C. Huang.

Shuang Hong Yang, Alex Smola, Bo Long, Hongyuan Zha, & Yi Chang
2012a Friend or frenemy? Predicting signed ties in social networks. In: *Proceedings of the 35th International ACM SIGIR Conference on Research and Development*
Weiling Yang & Fuji Zhang

The “chain polynomials” of sign-colored plane graphs with cyclomatic number ≤ 5 are obtained systematically. [Cf. Jin and Zhang (2005a), (2007a).] [Annot. 5 July 2009.] (SGc: Invar)

Yujun Yang & Dong Ye

Elegant solution to problem of Godsil (1985a): Iff Γ excludes an “odd flower”. Dictionary: “diagonally similar to a non-negative matrix” = balanced. *Question.* Is the answer the same if Γ is allowed to be signed graph? [Annot. 11 Dec 2018.] (sg: Adj, Bal, sw)

Mihalis Yannakakis
See also V.V. Vazirani.


“Restricted totally unimodular matrices”: Bipartite. Structural characterization by decomposition into incidence matrices (and transposes) of balanced bidirected graphs. (sg: Str)

Structure of “restricted balanceable graphs”, defined as bipartite and signable so a circle is positive iff it is evenly even. [Annot. 19 Jan 2015.] (SGw, sg: Str)

Milhalis Yannakakis [Mihalis Yannakakis]
See Mihalis Yannakakis.

Yan Hong Yao
See L. Feng.

Yuan Yao
See Z.-L. Jiang.

Zahra Yarahmadi

Dictionary: “bipartite edge frustration” of Γ = frustration index f(−Γ). (par: Fr)

Zahra Yarahmadi & Ali Reza Ashrafi

Dictionary: “bipartite vertex frustration” of $\Gamma = \text{frustration number } l_0(-\Gamma)$. (par: Fr)

**Z. Yarahmadi, T. Došlić, & A.R. Ashrafi**


**Geva Yashfe**

See L. Kühne.

**T. Yasuda**


**Stephen S. Yau**

See F. Hao.

**Dong Ye**

See E.L. Wei, Y.-J. Yang, Y.-Z. Wu, and L. Zhang.

**Miao-Lin Ye, Yi-Zheng Fan, & Hai-Feng Wang**

See also J. Sheng.


**Peter Ye**

See L.J. Rusnak.

**Yeong-Nan Yeh**


**Aylin Yener**

See B. Guler.

**Anders Yeo**

See N. Alon and G. Gutin.

**Shu Yong Yi & Li Hua You**

See also L.H. You.


**Shuyong Yi, Lihua You, & Yuhan Wu**


**Xiaowei Ying**

See L.T. Wu.

**Xuerong Yong**

En Sup Yoon
See G. Lee.

Yeoin Yoon

Young-Jin Yoon

An attempt to characterize supersolvability of $G(\Sigma)$ in terms of [bias-]simplicial vertices. [Fundamental conceptual and technical errors vitiate the entire paper; see Koban (2004a). For correct results see Zaslavsky (2001a) and Koban (2004a).]

Yusuke Yoshie
See E. Segawa.

Takeo Yoshikawa, Takashi Iino, & Hiroshi Iyetomi


Application of correlation clustering to the Tokyo stock market. The “frustration” of a clustering $\pi = \{B_1, \ldots, B_k\} \in \Pi_V$ in a weighted signed graph $(\Sigma, w)$ is $F(\pi) := -\sum_i \sum_{e \in E: B_i} w_e$ (cf. Traag and Bruggeman (2009a)). [Annot. 26 Jun 2012.] (SG, WG: Clu: Appl)

Huazheng You
See W.-Z. Liu.

Lihua You
See also F. Cheng, Y. Liu, L.Q. Wang, and S.Y. Yi.

Lihua You, Jiayu Shao, & Haiying Shan

Lihua You & Yuhan Wu

Lihua You & Shuyong Yi
Zhifu You
See also B.L. Liu.

Zhifu You & Bolian Liu

A.P. Young
See R.N. Bhatt, K. Binder and M. Palassini.

Michael Young
See M. Beck.

J.W.T. Youngs

Introducing “cascades”: current graphs with bidirected edges. A “cascade” is a bidirected graph, not all positive, that is provided with both a rotation system (hence it is orientation embedded in a surface) and a current (which is a special kind of bidirected flow). Dictionary: “broken” = negative edge. [Also see Ringel (1974a).] (sg: Ori: Appl, Flows)


“Cascades”: see (1968a). (sg: Ori: Appl)

Aimei Yu
See S.J. He G.J. Li, and W.J. Zhang.

Cheng-Ching Yu
See C.C. Chang.

Guanglong Yu, Shuguang Guo, & Meiling Xu

The graphs minimizing $\lambda_n(L(-\Gamma))$ given $n$ and the matching number $\mu$ or edge cover number. Thm. 3.5 finds the (few) connected, unbalanced, antibalanced signed graphs $-\Gamma$ with given $n, \mu$ that minimize $\lambda_n(L(-\Gamma))$.

[Problem. Generalize to connected, unbalanced signed graphs.] [Annot. 20 Jan 2015.] (par: Lap: Eig)

Guanglong Yu, Zhengke Miao, Chao Yan, & Jinlong Shu

Guanglong Yu, Yarong Wu, & Jinlong Shu

Gui-Dong Yu, Yi-Zheng Fan, & Yi Wang

Guihai Yu
See also L.H. Feng.

Guihai Yu, Matthias Dehmer, Frank Emmert-Streib, & Herbert Jodlbauer

It is a normalized Laplacian for digraph gains given by 1 if \(e_{ij}, e_{ji} \in E\), gain \(i\) if only \(e_{ij} \in E\). *[Cf. Liu and Li (2015a), Guo and Mohar (2017a).*]

Results about eigenvalues, e.g., symmetry, special values 0 and 2, edge interlacing, and expressions for coefficients of characteristic polynomial. [Annot. 15 Dec 2020.](gg: Lap: Eig)

Guihai Yu, Lihua Feng, Aleksandar Ilić, & Dragan Stevanović

Guihai Yu, Lihua Feng, & Hui Qu

Guihai Yu, Lihua Feng, Qingwen Wang, & Aleksandar Ilić
2014a The minimal positive index of inertia of signed unicyclic graphs. *Ars Combin.* (SG: Adj: Eig)

Guihai Yu, Xin Liu, & Hui Qu

\(\Phi\) with gain group \(\{\pm 1, \pm i\}\) \((\varphi(e) = 1\) for undirected, \(i\) for directed edges\). \(L(\Phi)\) and \(L(-\Phi)\) are positive semidefinite and singularity is characterized [cf. Guihai Yu & Hui Qu (2015a)]. Formula for determinant. [Annot. 15 Dec 2020.](gg: Lap)

Guihai Yu & Hui Qu

Based on Liu and Li (2015a), Guo and Mohar (2017a), and Harary (1953a). §2, “The positive of mixed graphs”. Thms. 2–3: For complete \(\|\Phi\|\) [an unnecessary restriction], \(\Phi\) is balanced iff \(V = V_1 \cup V_2\) so all edges in \(V_i\) are positive and all \(V_1V_2\) edges have gain \(i\) [note: \(\not\exists\) negative edges], iff all triangles are balanced. Thm. 4: \(\Phi\) is balanced iff all uv-paths have the same gain. §3, “Hermitian Laplacian matrix of mixed graphs”. Switching equivalence. Incidence matrix [special case of Zaslavsky (2003b), §2]. Laplacian matrix from incidence matrix, hence positive semidefinite (Thm. 9). Elementary eigenvalue properties and

Spectral symmetry around 1 and meaning of eigenvalue 2 of normalized Laplacian matrix.

Major error: Their incidence matrix \( S \) is that of \(-\Sigma\), i.e., \( H(-\Sigma) \) (Zaslavsky (1982a)); thus their Laplacian \( L(\Sigma) \) is really \( L(-\Sigma) \). Thm. 2: For connected \( \Sigma \), \( L(\Sigma) \) is singular iff all \( uv \)-walks have the same sign [i.e., \( \Sigma \) is balanced; known from Zaslavsky (1982a) in matroid form]. Wrong proof; the calculation reverses signs. Their Laplacian should be \( L(-\Sigma) \) throughout. Further results are known or wrong; e.g., Thm. 9 to the extent correct is Zaslavsky (1982a), Thm. 8A.4. [The new results can and should be corrected.] [Annot. 10 Nov 2018.] (SG: Lap: Eig)

Guihai Yu, Hui Qu, & Jianhua Tu


Jianming Yu
See G. Jiang.

B. Yuan
See Y. Chen.

Bo-Jun Yuan
See also Y. Wang.

Bo-Jun Yuan, Yi Wang, & Jing Xu

\( \Phi \) with gain group \( \{\pm 1, \pm i\} \): \( \varphi(e) = 1 \) for undirected, \( i \) for directed edges. Examples where spectrum determines \( \Phi \) [cf. Mohar2016aBojan Mohar]. [Annot. 15 Dec 2020.] (gg: Adj: Eig)

Shuhan Yuan
See Y.-M. Li.

Xiying Yuan
See also V. Nikiforov.


Xi-Ying Yuan, Yue Liu, & Miaomiao Han

\( \S 3: Q := L(-\Gamma) \) is used to prove results about trees. [Annot. 21 Jan
Zihan Yuan  
See S.-X. Lv.

Raphael Yuster & Uri Zwick  

Abbreviated version of (1997a).


For fixed even $k$, a very fast algorithm for finding a $k$-gon. Also, one for finding a shortest even circle. \[\text{Question.} Are these the all-negative cases of similarly fast algorithms to find positive $k$-gons, or shortest positive circles, in signed graphs?\] (par: Circ: Alg)

Sergey Yuzvinsky  

Prop. 3.3: A $k$-net in $\mathbb{C}P^2$ whose classes are pencils is the canonical representation of the jointless Dowling geometry $Q^1(\mathbb{Z}_m) = G(\mathbb{Z}_mK_3)$ of a finite cyclic group. If a $k$-net in $\mathbb{C}P^2$ represents $G(\mathcal{A}K_3)$ for a finite abelian group $\mathcal{A}$, then $\mathcal{A}$ is a subgroup of a 2-torus (Thm. 4.4) or has small invariant factors (Thm. 5.4); in particular it cannot be $\mathbb{Z}_3^2$ (Thm. 4.2). The author conjectures more definitive characterizations. (gg: Geom)

C.T. Zahn, Jr.  
See also C. Witzgall.


Let $l_{clu}(K_n, \sigma) := \min_{\sigma^*} \rho(\sigma, \sigma^*)$ over all clusterable $\sigma^*$, where $\rho(\sigma, \sigma^*) := \#(E^+(\sigma^*) \oplus E^+(\sigma))$. §3 finds a minimizing $\sigma^*$ when $\Sigma^+$ is a graph with a nontrivial clique attached to each vertex. §4 attaches cliques to the preceding clique vertices in certain cases. Dictionary: “symmetric relation” = graph $G = \Sigma^+$, “equivalence relation” = $\sigma^+$, “approximating equivalence relation” = “optimal partition” = $\sigma^*$ minimizing $\rho$. [Sequel: Moon (1966a).] [This appears to be the first paper that is implicitly on clustering a signed complete graph, before Davis (1967a).] [Annot. 10 Nov 2017.] (sg: Clu)


Katarzyna Zając  
See also M. Gąsiorek, G. Marczak, and D. Simson.


Mariusz Zając


Robert B. Zajonc


Robert B. Zajonc & Eugene Burnstein


Test of the relative importance of balance, reciprocity (= digon balance), and number of positive arcs on experimental subjects memorizing a [simple] signed digraph (represented by a sociological story). [The question raised is mathematically intriguing, but thus far undeveloped.] [Annot. 24 Nov 2012.] (PsS: SD: Bal)

Robert B. Zajonc & Steven J. Sherman


Experimental test of importance of balance in subjects’ attitudes towards signed graphs of order 3 suggests that balance is a weak criterion. Also, concise survey of several previous experiments. [Annot. 24 Nov 2012.] (PsS: SG: Bal) (PsS: SG, SD: Bal: Exp)

[Kh. Zakhs]

See H. Sachs.

Giacomo Zambelli

See also A. Del Pia.


Wenan Zang

See also Z.B. Chen and Y.-Z. Wu.

1998a Proof of Toft’s conjecture: Every graph containing no fully odd $K_4$ is 3-colorable. In: Wen-Lian Hsu and Ming-Yang Kao, eds., *Computing and Combi-

Proves Toft’s (1975a) conjecture: For every 4-chromatic graph $\Gamma$, $-\Gamma$ contains a subdivided $-K_4$. [Cf. Thomassen (2001b).] [Question. What is the signed-graph generalization?] [Annot. 29 Oct 2017.] (sg: par: Col)


An algorithm, based in part on Gerards (1994a), that, given $\Gamma$, finds a subdivided $[-K_4]$ in $\Gamma$ or a 3-coloring of $\Gamma$. [Question. Is there a generalization to all signed graphs?] [Annot. rev. 29 Oct 2017.]

Giovanni Zappella
See N. Cesa-Bianchi.

Thomas Zaslavsky


Proves Toft’s (1975a) conjecture: For every 4-chromatic graph $\Gamma$, $-\Gamma$ contains a subdivided $-K_4$. [Cf. Thomassen (2001b).] [Question. What is the signed-graph generalization?] [Annot. 29 Oct 2017.] (sg: par: Col)


An algorithm, based in part on Gerards (1994a), that, given $\Gamma$, finds a subdivided $[-K_4]$ in $\Gamma$ or a 3-coloring of $\Gamma$. [Question. Is there a generalization to all signed graphs?] [Annot. rev. 29 Oct 2017.]

Giovanni Zappella
See N. Cesa-Bianchi.

Thomas Zaslavsky


Published, greatly expanded, as (1989a), (1991a), (1995b) and more; as well as (but restricted to signed graphs) (1982a), (1982b).

(GG: Bal, M)


(SG: LG: Ori, Incid, Eig(LG). Sw)


(GG: M, Bic)


Signed graphs correspond to arrangements of hyperplanes in $\mathbb{R}^n$ of the forms $x_i = x_j$, $x_i = -x_j$, and $x_i = 0$. Consequently, one can compute the number of regions of the arrangement from graph theory, esp. for arrangements corresponding to “sign-symmetric” graphs, i.e., having both or none of each pair $x_i = \pm x_j$. Simplified account of parts of (1982a), (1982b), (1982c), emphasizing geometry.

(SG: M, Geom, Invar)


Notably, Thm. 6: A set $\mathcal{B}$ of circles in $\Gamma$ is the set of positive circles in some signing of $\Gamma$ iff every theta subgraph contains an even number of circles in $\mathcal{B}$. [Annot. rev. 22 Oct 2015.]

(SG: Bal)

See (1997a). (SG: Top, M)


$G(\Sigma)$ Basic results on: Switching (§3). Prop. 3.2: $\Sigma_1 \sim \Sigma_2$ iff $B(\Sigma_1) = B(\Sigma_2)$, i.e., signed graphs are switching equivalent iff they have the same circle signs. [Cf. Sozański (1980a).] Minors (§4). The frame matroid $G(\Sigma)$ in many cryptomorphisms (§5) (some erroneous: Thm. 5.1(f,g); partly corrected in the Erratum [and fully in (1991a)]), consistency of matroid with signed-graph minors; separators of $G(\Sigma)$. The signed covering graph $\tilde{\Sigma}$ (§6).

In §8A, the incidence and Laplacian matrices and matrix-tree theorem [different from that of Murasugi (1989a)] [generalized by Chaiken (1982a) to a weighted, all-minors version, both directed and undirected]. In §8B, vector representation of the matroid $G(\Sigma)$ by the incidence matrix [as multisubsets of root systems $B_n \cup C_n$].


Examples (§7) include: Sign-symmetric graphs and signed expansions $\pm \Gamma$. The all-negative graph $-\Gamma$, whose matroid (Cor. 7D.3; partly corrected in the Erratum) is the even-circle matroid (see Doob (1973a)) and whose incidence matrices include the unoriented incidence matrix of $\Gamma$. Signed complete graphs.

Generalizations to gain graphs (called “voltage graphs”) mentioned in §9. [Annot. rev. 22 Oct 2015.] (SG, GG: M, Bal, Sw, Cov, Incid, Geom; EC, KG)


$\chi_{\Sigma}(\lambda)$ A “proper $k$-coloring” of $\Sigma$ partitions $V$ into a special “zero” part, possibly void, that induces a stable subgraph, and up to $k$ other parts (labelled from a set of $k$ colors), each of which induces an antibalanced subgraph. A “zero-free proper $k$-coloring” is similar but without the “zero” part. [The suggestion is that a signed analog of a stable vertex set is one that induces an antibalanced subgraph. Problem. Use this insight to develop generalizations of stable-set notions, such as cliques and perfection. Example. Let $\alpha(\Sigma)$, the “antibalanced vertex set number”, be the largest size of an antibalance-inducing vertex set. Then $\alpha(\Gamma) = \alpha(+\Gamma \cup -K_n).$] §2, “Counting the coloring ways”: One gets two related chromatic polynomials. The chromatic polynomial, $\chi_{\Sigma}(2k + 1)$, counts all proper $k$-colorings; it is essentially the characteristic polyno-

[$k$-coloring and zero-free $k$-coloring may better be called $(2k + 1)$- and $2k$-coloring, as in Macajova–Raspaud–Škoviera (2014a).] [Annot. rev. 26 Aug 2018.]


The set of all forests in a graph forms a geometric lattice. The set of spanning forests forms a geometric semilattice. The characteristic polynomials count (spanning) forests. (GG: M, Bic, Geom, Invar)


The frame matroid of a gain graph. (GG: M, EC, Bic, Invar: Exp)


Studies zero-free [or balanced] chromatic number $\chi^* := \min(k : \exists$ proper coloring to $\{\pm 1, \ldots, \pm k\}$), and in particular that of a complete signed graph (which may have parallel edges). The signed graphs whose $\chi^*$ is largest or smallest.

[Updated notation: unbalanced chromatic number $\chi^0 :=$ similar for $\{0, \pm 1, \ldots, \pm k\}$, chromatic number (from Máčajová–Raspaud–Škoviera (2016a)) $\chi := \min(2\chi^0 + 1, 2\chi^*) = \min$ size of color set.] [Annot. rev 2

A modestly successful attempt to generalize two-graphs along the cohomological lines of Cameron and Wells (1986a) [Annot. 6 July 2011.]


\[\text{Λ(Σ)}\] 

The line graph of a switching class \([Σ]\) of signed graphs is a switching class of signed graphs; call it \([Λ'(Σ)]\). The reduced line graph \(Λ\) is formed from \(Λ'\) by deleting parallel pairs of oppositely signed edges. Then \(A(Λ) = A(Λ') = 2I - H^T H\), where \(H\) is an incidence matrix of \(Σ\).

Thm. 1: \(A(Λ)\) has all eigenvalues \(\leq 2\). Examples: For an ordinary graph \(Γ\), \(Λ(−Γ) = −Λ(Γ)\). Example: taking \(−Γ\) and attaching any number of pendant negative digons to each vertex yields (the negative of) Hoffman’s generalized line graph. Additional results are claimed but there are no proofs. [See also (20xxa).] [This work is intimately related to that of Vijayakumar *et al.*, which was then unknown to the author, and to Cameron (1980a) and Cameron, Goethals, Seidel, and Shult (1976a).]


For the frame (bias), lift, and extended lift matroids: forbidden-minor and structural characterizations. The latter for signed-graphic frame matroids is superseded by a result of Pagano (1998a).

[Error in Cor. 4.3: In the last statement, omit “\(G(Ω) = L(Ω)\)” That is true when \(Ω\) has no loops, but may not be if \(Ω\) has a loop \(e\) (because Theorem 3(3) applies with unbalanced block \(e\), but \((E \setminus e,e)\) is not a 2-separation).]


Decompose \(E(Σ)\) into the fewest balanced subsets (generalizing the decomposition biparticity of an unsigned graph), or the fewest balanced connected subsets. These minimum numbers are \(δ_0\) and \(δ_1\). Thm. 1: \(δ_0 = \lceil \log_2 χ^*(-Σ) \rceil + 1\), where \(χ^*\) is the zero-free chromatic number. Thm. 2: \(δ_0 = δ_1\) if \(Σ\) is complete. *Conjecture* 1. \(Σ\) partitions into \(δ_0\) balanced, connected, and spanning edge sets (whence \(δ_0 = δ_1\)) if it has \(δ_0\) edge-disjoint spanning trees. [Solved and generalized to basepointed matroids by D. Slilaty (unpublished).] *Conjecture* 2 is a formula for \(δ_1\) in terms of \(δ_0\) of subgraphs. [Thoroughly disproved by Slilaty (unpublished).]

Such a vertex $u$ (also, a “balancing vertex”) is a vertex of an unbalanced graph $\Omega$ whose removal leaves a balanced graph [i.e., frustration number $l_0 = 0$]. Some elementary results, e.g., $\Omega = \Omega'/e$ where $\Omega \setminus e$ is balanced and $e$ contracts to $u$. [Annot. rev. 19 Dec 2014.] 


An attempt to generalize two-graphs (here [alas?] called “unitogs”) in a way similar to that of Cameron and Wells (1986a) although largely independently. The notable new example is “Johnson togs”, based on the Johnson graph of $k$-subsets of a set. “Hamming togs” are based on a Hamming graph (that is, a Cartesian product of complete graphs) and generalize examples of Cameron and Wells. Other examples are as in (1984b).


A complete list of the biased graphs $\Omega$ such that $G(\Omega)$, $L_0(\Omega)$, or $L(\Omega)$ is one of the traditional special binary matroids, $G(K_5)$, $G(K_{3,3})$, $F_7$, their duals, and $G(K_m)$ (for $m \geq 4$) and $R_{10}$. [Unfortunately omitted are nonbinary matroids like the non-Fano plane and its dual.]

[Error: The graphs $\langle +K_n^c \rangle$ were overlooked in the last statement of Lemma 1H—due to an oversight in (1987a) Cor. 4.3—and thus in Props. 2A and 5A. A corrected last statement of Lemma 1H: “If $\Omega$ has no two vertex-disjoint negative circles, then $G(\Omega) = M \iff L(\Omega) = M$.” In Prop. 2A, add $\Omega = \langle +K_3^c \rangle$ to the list for $G(K_4)$. In Prop. 5A, add $\Omega = \langle +K_{m-1}^c \rangle$ to the list for $G(K_m)$. Thanks to Stefan van Zwam (25 July 2007).]


$G, L, L_0$ Basic theory of the bias [or better, “frame”] matroid $G$ (§2) and the lift and complete lift matroids, $L$ and $L_0$ (§3), of a gain graph or biased
graph. Infinite graphs. Matroids that are intermediate between the bias and lift matroids. Several questions and conjectures. (GG: M)


Oriented signed graph = bidirected graph. The oriented matroid of an oriented signed graph. A “cycle” in a bidirected graph is a bias circuit (a balanced circle, or a handcuff with both circles negative) oriented to have no source or sink. Cycles in $\Sigma$ are compared with those in its signed (i.e., derived) covering graph $\tilde{\Sigma}$. The correspondences among acyclic orientations of $\Sigma$ and regions of the hyperplane arrangements of $\Sigma$ and $\tilde{\Sigma}$, and dually the faces of the acyclotope of $\Sigma$.

Thm. 4.1: the net degree vector $d(\tau)$ of an orientation $\tau$ belongs to the face of the acyclotope that is determined by the union of all cycles. Cor. 5.3 (easy): a finite bidirected graph has a source or sink. (SG: Ori, M, Cov, Geom)(SGw: Invar)


Positive circles preserve orientation, negative ones reverse it. The minimal embedding surface of a one-point amalgamation of signed graphs. The formula is almost additive. Cf. Lins (1982a), (1985a) for related work in different formalism. (SG: Top)


Suppose that a function of matroids with labelled points is defined that is multiplicative on direct sums and satisfies a Tutte–Grothendieck recurrence with coefficients (the “parameters”) that depend on the element being deleted and contracted, but not on the particular minor from which it is deleted and contracted: specifically, $F(M) = a_e F(M \setminus e) + b_e F(M/e)$ if $e$ is not a loop or coloop in $M$. Thm. 2.1 completely characterizes such “strong Tutte functions” for each possible choice of parameters: there is one general type, defined by a rank generating polynomial $R_M(a; b; u, v)$ (the “parametrized rank generating polynomial”) involving the parameters $a = (a_e), b = (b_e)$ and the variables $u, v$, and there are a few special types that exist only for degenerate parameters. All have a Tutte-style basis expansion; indeed, a function has such an expansion iff it is a strong Tutte function (Thms. 7.1, 7.2). The Tutte expansion is a polynomial within each type. If the points are colored and the parameters of a point depend only on the color, one has a multicolored matroid generalization of Kauffman’s (1989a) Tutte polynomial of a sign-colored graph. Kauffman’s particular choices of parameters are shown to be related to matroid and color duality.

For a graph, “parametrized dichromatic polynomial” $Q_\Gamma = u^{\beta_0(\Gamma)} R_G(\Gamma)$, where $G$ = graphic matroid and $\beta_0 = \text{number of connected components}$. A “portable strong Tutte function” of graphs is multiplicative on disjoint unions, satisfies the parametrized Tutte–Grothendieck recurrence, and has value independent of the vertex set. Thm. 10.1: Such a function either equals $Q_\Gamma$ or is one of two degenerate exceptions. Prop. 11.1: Kauffman’s (1989a) polynomial of a sign-colored graph equals $R_G(\Sigma),(a; b; d, d)$ for connected $\Sigma$, where $a_+ = b_- = B$
and \( a_- = b_+ = A \). [Cf. Traldi (1989a).]

This paper differs from other generalizations of Kauffman’s polynomial, by Przytycka and Przytycki (1988a) and Traldi (1989a) (and partially anticipated by Fortuin and Kasteleyn (1972a)), who also develop the parametrized dichromatic polynomial of a graph, principally in that it characterizes all strong Tutte functions; also in generalizing to matroids and in having little to say about knots. Schwärzler and Welsh (1993a) generalize to signed matroids (and characterize their strong Tutte functions) but not to arbitrary colors. Bollabás and Riordan (1999a) initiate the study of the underlying commutative algebra.

(Sc(M), SGc: Gen: Invar, D, Knot)


\( \mathbb{P}^2 \) Characterized by six forbidden minors or eight forbidden topological subgraphs, all small. A close analog of Kuratowski’s theorem; the proof even has much of the spirit of the Dirac–Schuster proof of the latter, and all but one of the forbidden graphs are simply derived from the Kuratowski graphs. [Paul Seymour showed me an alternative proof from Kuratowski’s theorem that explains this; but it uses sophisticated results, as yet unpublished, of Robertson, Seymour, and Shih.] (SG: Top)

[Related: “projective outer-planarity” (POP): embeddable in the projective plane with all vertices on a common face. I have found most of the 40 or so forbidden topological subgraphs for POP of signed graphs (finding the rest will be routine); the proof is long and tedious and will probably not be published. Problem. Find a reasonable proof.]


A simple matroidal characterization of the frame (or “bias”) matroids of biased graphs. (GG: M)


Introducing the signed Heawood problem: what is the largest signed, or zero-free signed, chromatic number of any signed graph that orientation embeds in the sphere with \( h \) crosscaps? Solved for \( h = 1, 2 \).

(SG: Top, Col)


Polynomials of gain and biased graphs: the fundamental object is a four-variable polynomial, the “polychromial” (“polychromatic polynomial”), that specializes to the chromatic, dichromatic, and Whitney-number polynomials. The polynomials come in two flavors: unrestricted and balanced, depending on the edge sets that appear in their defining sums. (They can be defined in the even greater abstraction of “two-ideal graphs”, which clarifies the most basic properties.)

§4: “Gain-graph coloring”. In \( \Phi = (\Gamma, \varphi, \mathcal{G}) \), a “zero-free \( k \)-coloring”
is a mapping $f : V \to [k] \times G$; it is “proper” if, when $e : vw$ is a link or loop and $f(v) = (i, q)$, $f(w) = (i, h)$, then $h \neq q \varphi(e; v, w)$. A “$k$-coloring” is similar but the color set is enlarged by inclusion of a color $0$; propriety requires the additional restriction that $f(v)$ and $f(w)$ are not both $0$ (and $f(v) \neq 0$ if $v$ supports a half edge). In particular, a “group-coloring” of $\Phi$ is a zero-free $1$-coloring (ignoring the irrelevant numerical part of the color). A “partial group-coloring” is a group-coloring of an induced subgraph [which can only be proper if the uncolored vertices form a stable set]. The unrestricted and balanced chromatic polynomials count, respectively, unrestricted and zero-free proper $k$-colorings; the two Whitney-number polynomials count all colorings, proper and improper, by their improper edge sets.

§5: “The matroid connection”. The various polynomials are, in essence, frame matroid invariants and closely related to corresponding lift matroid and extended lift matroid invariants.

Almost infinitely many identities, some of them (esp., the balanced expansion formulas in §6) essential. Innumerable examples worked in detail. [The first half, to the middle of §6, is fundamental. The rest is more or less ornamental. Most of the results are, intentionally, generalizations of properties of ordinary graphs.]  


The smallest surface that holds $K_n$ with loops, if odd circles reverse orientation, even ones preserve it (this is parity embedding). I.e., the demigenus $d(-K^n_\circ)$.


Like (1996a), but without loops. Conjecture 1. The minimal surface for parity embedding $K_n$ is sufficient for orientation embedding of any signed $K_n$. Conjectures 3–4. The minimal surfaces of $\pm K^n_\circ$ and $\pm K^n$ are the smallest permitted by the lower bound obtained from Euler’s polyhedral formula.


Find an upper bound on $f(m) = \text{largest } r \text{ such that any group of order } \geq r \text{ has } m \text{ elements such that no product of any subset, possibly with inverted elements, equals the identity. Solution by Gagola (1999a).} [\text{The solution implies that } (*) \ f_1(m) \leq \lfloor 2^{n-1}(m - 1)! \sqrt{e} \rfloor, \text{ where } f_1(m) \text{ smallest } r \text{ such that every group of order } \geq r \text{ is a possible gain group for every contrabalanced gain graph of cyclomatic number } m. \text{ Problem 1. Find a good upper bound on } f_1. \text{ (*) is probably weak. Problem 2. Find a good lower bound. Problem 3. Estimate } f_1 \text{ asymptotically.}]$ (Par: KG: Top)

Basically, they are the antibalanced and bipartite signed graphs; but the exact description depends on the characterization one chooses for biparticity: whether it is evenness of circles, closed walks, face boundaries in surface embeddings, etc. Characterization by chromatic number leads to a slightly more different list of analogs. (SG: Str, Top)


Given an additive real gain graph Φ on n vertices and n reference
points \( Q_i \) in \( \mathbb{E}^d \), use \( \Phi \) to specify perpendicular hyperplanes to each of the lines \( Q_i Q_j \) by means of the “Pythagorean coordinate” along \( Q_i Q_j \). For generic points, the number of regions is computable based on the fact that the generic hyperplane intersection lattice is \( \text{Lat}^b \Phi \). Modifications of Pythagorean coordinates give intersection lattice \( \text{Lat}^b(\|\Phi\|, \emptyset) \) or a slightly more complex variant, still for generic reference points.

(GG: Geom, M, Invar)


§6, “Affinographic arrangements”: hyperplane arrangements that represent the extended lift matroid \( L_0(\Phi) \) where \( \Phi \) is an additive real gain graph. Examples: the weakly-composed-partition, extended Shi, and extended Linial arrangements. The faces are counted in terms of dimension and dimension of the infinite part. Ehrenborg (2019a) has more explicit formulas for Shi.

(GG: m, Geom, Invar)


§§2–4: Various ways in which to represent the bias and lift matroids of a gain or biased graph over a skew field \( F \). Bias matroid: canonical vector and hyperplanar representations (generalizing those of a graph) based on a gain group \( \subseteq F^\times \), Menelaean and Cevian representations (generalizations of theorems of Menelaus and Ceva), switching vs. change of ideal hyperplane, equational logic. Lift matroid: canonical vector and hyperplanar representations (the latter generalizing the Shi and Linial arrangements among others) based on a gain group \( \subseteq F^+ \), orthogonal representation (an affine variation on canonical representation), Pythagorean representation ((2002a)). Both: effect of switching, nonunique gain-group embedding. §5: Effect of Whitney operations, separating vertex. §6: Matroids characterized by restricted general position. §7, “Thick graphs”: A partial unique-representation theorem for biased graphs with sufficient edge multiplicity. §8: The 7 biased \( K_4 \)’s.

(GG: M, Geom, Invar)


Summary of (2012a).

(GG: Str)


Contrabalanced graphs, whose gains are called antivoltages. Emphasis on the existence of antivoltages in groups \( G = \mathbb{Z}_\mu \), \( \mathbb{Z} \), and \( \mathbb{Z}_p^k \) for application to canonical representation of the contrabalanced bias and lift matroids. The number of such antivoltages is a polynomial function of the group order or (for \( \mathbb{Z} \)) the bound on circle gains. Cf. Sivaraman (2014a), Chun, Moss, Slilaty, and Zhou (2016a), and especially Sivaraman–Slilaty (2019a).

[Annot. rev 11 Jun 2019.]

(GG: M, bic, Geom, Invar)

Given a set $Q$ of “spins”, a state is $s: V \to Q$. The gain group $\mathfrak{G}$ acts on the spin set. In a permutation gain graph $\Phi$ with gain group $\mathfrak{G}$, edge $e:vw$ is “satisfied” if $s(w) = s(v)\varphi(e)$, otherwise “frustrated”. A totally frustrated state (every edge is frustrated) generalizes a proper coloring. Enumerative theory, including deletion/contraction, a monodromy formula for the number of totally frustrated states, and a multivariate chromatic polynomial. An abstract partition function in the edge algebra.

(GG: Col: Gen: Invar, M)


Extended abstract of (2012b) [but not entirely correct]. There are six ways to sign the Petersen graph $P$ up to switching isomorphism. Their frustration indices, automorphism and switching automorphism groups, chromatic numbers, and clusterability indices. [Annot. 30 Aug, 26 Dec 2010.]

(SG: Fr, Aut, Col, Clu)


The adjacency, incidence, and Laplacian matrices, along with the adjacency matrices of line graphs. Balance, vertex degrees, eigenvalues, line graphs, strong regularity, etc. A survey, emphasizing work of Seidel, Vijayakumar, and Zaslavsky.

Abelson and Rosenberg’s (1958a) adjacency matrix is mentioned.

(SG: Adj, Lap, Eig, Incid, LG: Exp)


An $n$-ary quasigroup $(\mathcal{Q}, f)$ is essentially equivalent, up to isotopy, to a biased expansion $m \cdot C_{n+1}$. Factorizations of $f$ appear as chords in a maximal extension of $m \cdot C_{n+1}$. Thm.: A biased expansion of a 3-connected graph (of order $\geq 4$) is a group expansion. Cor.: If $n \geq 3$ and the factorization graph of $(\mathcal{Q}, f)$ is 3-connected, $(\mathcal{Q}, f)$ is isotopic to an iterated group. Thm.: For a biased expansion of a 2-connected graph of order $\geq 4$, if all minors of order 4 are group expansions, so is the whole expansion. Cor.: If in $(\mathcal{Q}, f)$ $(n \geq 3)$ all ternary residual quasigroups are iterated group isotopes, so is $(\mathcal{Q}, f)$. Cor.: $(\mathcal{Q}, f)$ is an iterated group isotope if $#\mathcal{Q} = 3$.

Other results: complete structural decomposition of nongroup biased expansions, or equivalently partially reducible multiary quasigroups, in terms of groups and either irreducible expansions or multiary quasigroups, respectively.

The matroids of maximal nongroup biased expansions are the nearest
generalization of Dowling’s (1973b) geometries. (GG: Str, M)

There are six ways to sign the Petersen graph $P$ up to switching isomorphism. The frustration indices, automorphism and switching automorphism groups (in extensive detail), chromatic numbers, and clusterability indices of them and their negatives. All but automorphisms and clusterability are switching invariant, thus are solved for all signed Petersens. [Annot. 26 Dec 2010.] (SG: Fr, Aut, Col, Clu)


Short version of (2018a). (SG: Circ, Bal, Fr)

A collection of mostly open questions, some with solutions, about circles of specified sign and their interactions: e.g., detection, uniqueness, intersection, packing, covering. [Annot. 15 Jan 2018.] (SG: Circ, Bal, Fr)


Λ Line graphs of signed graphs are, fundamentally, (bidirected) line graphs of bidirected graphs, $Λ(B)$. Then the line graph $Λ(Σ)$ of a signed graph is a polar graph, i.e., a switching class of bidirected graphs; the line graph of a polar graph is a signed graph; and the line graph of a sign-biased graph, i.e., of a switching class of signed graphs, is a sign-biased graph, $[ΛΣ]$. In particular, the line graph of an antibalanced switching class is an antibalanced switching class. (Partly for this reason, ordinary graphs should usually be regarded as antibalanced, i.e., all negative, in line graph theory.) Since a digraph is an oriented all-positive signed graph, its line graph is a bidirected graph whose positive part is the Harary–Norman line digraph. Among the line graphs of signed graphs, some reduce by cancellation of parallel but oppositely signed edges to all-negative graphs; these are precisely Hoffman’s generalized line graphs of ordinary graphs, a fact which explains their line-graph-like behavior.
Attempts at a completely descriptive line graph $\Lambda(\vec{\Gamma})$ of a digraph have been Muracchini and Ghirlanda (1965a) and Hemminger and Klerlein (1979a). [Annot. 1999.]

The geometry of line graphs and signed graphs has been developed by Vijayakumar et al. (q.v.). [Annot. 1999 et seq.]

The competition graph of a digraph $\vec{\Gamma}$ is the extraverted part of $\Lambda(\vec{\Gamma})$. [Annot. 16 Aug 2016.]

The naive approach to characteristic polynomials via lattice point counting (in characteristic 0) and Möbius inversion (as in Blass–Sagan (1998a)) can only work when one expects it to. (This is a theorem!)

Numerous types of examples of biased graphs, many having particular theory of their own, e.g., Hamiltonian bias. (GG: M, Geom)
See (1976a) for definitions. Railway tracks and switches modeled by edges and vertices of a polar graph. Forming its derived graph (see (1976d)), thence a digraph obtained therefrom by splitting vertices into two copies and adjusting arcs, the time for a train to go from one segment to another is found by a shortest path calculation in the digraph. A similar method is used to solve the problem for several trains.

(\text{sg: Ori, \textit{sw: LG: Appl}})


Basic definitions (Zítek (1972a)): “Polarized graph” \(B\) = bidirected graph (with no negative loops and no parallel edges sharing the same bidirection). “Polar graph” \(P\) \(\cong\) switching class of bidirected graphs (that is, we forget which direction at a vertex is in and which is out—here called “north” and “south” poles—but we remember that they are different).

Thms. 1–6. Elementary results about automorphisms, including finding the automorphism groups of the “complete polarized” and polar graphs. (The “complete polarized graph” has every possible bidirected link and positive loop, without repetition.) Thm. 7: With small exceptions, any (ordinary) graph can be made polar as, say, \(P\) so that \(\text{Aut} P\) is trivial.

Thms. 8–10. Analogs of Whitney’s theorem that the line graph almost always determines the graph. The “pole graph” \(B^*\) of \(B\) or \([B]\): Split each vertex into an “in” copy and an “out” copy and connect the edges appropriately. [Generalizes splitting a digraph into a bipartite graph. It appears to be a “twisted” signed double covering graph.] Thm. 8. The pole graph is determined, with two exceptions, by the edge relation \(e \sim_1 f\) if both enter or both leave a common vertex. (A trivial consequence of Whitney’s theorem.) Thm. 9. A polar graph \([B]\) with enough edges going in and out at each vertex is determined by the edge relation \(e \sim_2 f\) if one enters and the other exits a common vertex. (Examples show that too few edges going in and out leave \([B]\) undetermined.) Thm. 10. Knowing \(\sim_1, \sim_2,\) and which edges are parallel with the same sign, and if no component of the simplified underlying graph of \(B\) is one of twelve forbidden graphs, then \([B]\) is determined. [\text{Problem 1. Improve Thm. 10 to a complete characterization of the bidirected graphs that are reconstructible from their line graphs (which are to be taken as bidirected; see Zaslavsky (2010b), (20xxa)). In connection with this, see results on characterizing line graphs of bidirected (or signed) graphs by Vijayakumar (1987a). \text{Problem 2. It would be interesting to improve Thm. 9.}}]

(\text{sg: Ori, \text{sw: Aut, lg}})


See (1976a) for basic definitions. Here is the framework of the 8 theorems. Given a bidirected or polar graph, \(B\) or \(P\), vertices \(a\) and \(b\), and a type \(X\) of walk, let \(s_X [s_X'] = \) the fewest vertices [edges] whose deletion eliminates all \((a,b)\) walks of type \(X\), and let \(d_X [d_X'] = \) maximum number of suitably pairwise internally vertex-disjoint [or, suitably pairwise edge-disjoint] walks of type \(X\) from \(a\) to \(b\). [My notation.] By
"suitably" I mean that a common internal vertex or edge is allowed in $P$ (but not in $B$) if it is used oppositely by the two walks using it. (See the paper for details.) Thms. 1–4 (there are two Theorems 4) concern all-positive and all-introverted walks in a bidirected ("polarized") graph, and are simply the vertex and edge Menger theorems applied to the positive and introverted subgraphs. Thms. 4–7 concern polar graphs and have the form $s_X \leq d_X \leq 2s_X \lceil s'_X \leq d'_X \leq 2s'_X \rceil$, which is best possible. Thms. 4–5 concern type "heteropolar" (equivalently, directed walks in a bidirected graph). The proofs depend on Menger’s theorems in the double covering graph of the polar graph. [Since this has 2 vertices for each 1 in the polar graph, the range of $d_X$, $d'_X$ is explained.] Thms. 6–7 concern type "homopolar" (i.e., antidirected walks). The proofs employ the pole graph (see (1976a)).


See (1976a) for basic definitions. An Eulerian trail in a bidirected graph is a directed trail containing every edge. [Equivalently, a heteropolar trail that contains all the edges in the corresponding polar graph.] It is closed if the endpoints coincide and the trail enters at one end and departs at the other. The fewest directed trails needed to cover a connected bidirected graph is $\frac{1}{2}$ the total of the absolute differences between in-degrees and out-degrees at all vertices, or 1 if in-degree = out-degree everywhere.


See (1976a) for basic definitions. The "derived graph" of a bidirected graph [this is equivalent to the author’s terminology] is essentially the positive part of the bidirected line graph. The theorem can be restated, somewhat simplified: A finite connected bidirected graph $B$ is isomorphic to its derived graph iff $B$ is balanced and contains exactly one circle.


See (1976a) for basic definitions. A polar graph $PG(\mathfrak{G}, A)$ of a group and a subset $A$ is defined. [It is the Cayley digraph.] In bidirected language: a (bi)directed graph is “homogeneous” if it has automorphisms that are transitive on vertices, both preserving and reversing the orientations of edges, and that induce an arbitrary permutation of the incoming edges at any given vertex, and similarly for outgoing edges. It is shown that the Cayley digraph $PG(\mathfrak{G}, A)$, where $\mathfrak{G}$ is a group and $A$ is a set of generators, is homogeneous if $A$ is both arbitrarily permutable and invertible by $\text{Aut} \mathfrak{G}$. [Bidirection—i.e., the polarity—seems to play no part here.]


Is a simple graph $\Gamma$ a double cover of some signing of a simple graph? An elementary answer in terms of involutions of $\Gamma$. Further: if there are two
such involutions \( \alpha_0, \alpha_1 \) that commute, then \( \Gamma/\alpha_i \) has involution induced by \( \alpha_{1-i} \), so is a double cover of \( \Gamma/\langle \alpha_0, \alpha_1 \rangle \), which is not necessarily simple. [No properties of particular interest for signed covering are treated.]

(\text{sg: Cov})


The double covers here are those of all-negative simple graphs (hence are bipartite). Some properties of these double covers are proved, then connections with a certain lattice (the “logic”) of a graph. (\text{par: Cov: Aut})


The second half of (1983a).

(\text{par: Cov: Aut})


Harary and Sagan (1984a) asked: which signed graphs have the form \( S(P) \) for some poset \( P \)? Zelinka gives a rather complicated answer for all-negative signed graphs, which has interesting corollaries. For instance, Cor. 3: If \( S(P) \) is all negative, and \( P \) has \( \hat{0} \) or \( \hat{1} \), then \( S(P) \) is a tree.

(SG, Sgnd)

**Hans-Olov Zetterström**


**Ahmed A. Zewail**

See B. Guler.

**Hongyuan Zha**

See S.H. Yang.

**Mingqing Zhai, Ruifang Liu, & Jinlong Shu**


**Shidong Zhai**


**Shidong Zhai & Qingdu Li**


**Shidong Zhai, Min Xiao, & Qingdu Li**

H. Zhan
See G. Coutinho.

Bingyan Zhang
See Y.P. Zhang.

Cun-Quan Zhang


Conj. 12 is a sufficient condition for $-\Gamma$ to decompose into balanced circles. [Problem. Solve the obvious generalization to signed graphs. Is that easier because minors exist?] [Annot. 11 Jun 2012.] (sg: par: Str)


On decomposing $E(-\Gamma)$ into positive circles. Thm.: It is possible if $\Gamma$ is 2-connected and Eulerian and has no $K_5$ minor. [Problem. The same, for any signed graph. Is a $-K_5$ minor an obstruction? See also R. Rizzi (2001a) and K. Markström (2012a).] [Annot. 13 Aug 2013.] (Par: Str: Circ)

De Long Zhang & Shang Wang Tan

Fuji Zhang
See X.A. Jin and W. Yang.

Guang-Jun Zhang & Xiao-Dong Zhang


Hongwei Zhang
See also H.-D. Jiang and Y. Jiang.


Hongwei Zhang & Jie Chen


Jianbin Zhang
See X.L. Li.

Jianghua Zhang
See G. Jiang.

Jie Zhang & Xiao-Dong Zhang

Jing Zhang
See L. Su.

Jingming Zhang & Jiming Guo

Jing-Ming Zhang, Ting-Zhu Huang, & Ji-Ming Guo
(par: Lap: Eig)

Jing-Yue Zhang
See L. Zhang.

Kuan Zhang
See D. Lo.

Li Zhang
See also S.C. Li.

Li Zhang, You Lu, Rong Luo, Dong Ye, & Shenggui Zhang
(SG: EC: LG, Ori)

Li Jun Zhang
See X.H. Hao.

Ling Zhang, Ting-Zhu Huang, Zhongshan Li, & Jing-Yue Zhang

**Long Zhang**  
See J.-S. Wu.

**Minjie Zhang**  
See also C. Chen and S.C. Li.

**Minjie Zhang & Shuchao Li**  

**Ping Zhang**  

Blass and Sagan’s (1998a) geometrical form of signed-graph coloring is used to calculate (I) characteristic polynomials of several versions of \( k \)-equal subspace arrangements (these are the main results) and (II) [also in (2000a)] the chromatic polynomials (in geometrical guise) of ordinary graphs extending \( K_n \) by one vertex, signed graphs extending \( \pm K_n \) by one vertex, and \( \pm K_n \) with any number of negative loops adjoined.  

(\( \text{sg: Invar, Geom, col} \))


Uses signed-graph coloring (in geometrical guise) to evaluate the chromatic polynomials (in geometrical guise) of all signed graphs interpolating between (1) \( +K_n \) and \( +K_{n+1} \) [i.e., ordinary graphs extending a complete graph by one vertex]; (2) \( \pm K_{n-1}^\circ \) and \( \pm K_n^\circ \); (3) \( \pm K_n \) and \( \pm K_n^\circ \) [known already by several methods, including this one]; (4a) \( \pm K_{n-1} \) and \( \pm K_{n-1} \cup +K_n \); (4b) \( \pm K_n \cup +K_n \) and \( \pm K_n \); and certain signed graphs interpolating (by adding negative edges one vertex at a time, or working down and removing them one vertex at a time) between (5) \( +K_n \) and \( \pm K_n^\circ \); (6) \( +K_n \) and \( \pm K_n \). In cases (1)–(3) the chromatic polynomial depends only on how many edges are added [which is obvious from the coloring procedure, if it were not disguised by geometry].  

(\( \text{sg: Invar, col, Geom} \))

**Shenggui Zhang**  

**Shengping Zhang**  
See D. Li.

**Shengtong Zhang**  
See Z.-L. Jiang.

**Siqi Zhang**  

**W.J. Zhang & A.M. Yu**
Characterizes signed graphs with adjacency rank 4, separately for bipartite and non-bipartite graphs. [Annot. 27 Dec 2017.](SG, WG: Adj)

**Xiao-Dong Zhang**
See also Y.H. Chen, B.A. He, Y. Hong, G.J. Zhang, and J. Zhang.

§4, Remark 2: The main results extend to signed graphs (“mixed graphs”). [Annot. 23 Mar 2009.](sg: Eig)

*Problem.* Explain in terms of signed graphs, generalizing to $L(\Gamma)$. [Annot. bal: Lap: Eig]


**Xiao-Dong Zhang & Jiong-Sheng Li**
Spectrum and spectral radius of the Laplacian matrix of a signed simple graph. [For this topic, orientation is irrelevant so the results apply to all signed simple graphs, although they are stated for oriented signed graphs in the guise of mixed graphs.] Dictionary: “mixed graph” = bidirected graph where all negative edges are extraverted; “quasibipartite” = balanced; “line graph” = $-\Lambda(\Sigma)$ (the negative of the line graph of $\Sigma$). [Annot. 23 Mar 2009.] (sg: Eig, LG)

**Xiao-Dong Zhang & Rong Luo**
$\Sigma$ is a signed simple graph. $\lambda_{\text{max}}(L(\Sigma)) \leq \text{max edge degree} + 2$ (same as Hou, Li, and Pan (2003a), Thm. 3.5(1)). Also, other bounds on $\lambda_{\text{max}}$. Thm. 2.5: The second smallest eigenvalue is $\leq \kappa(|\Sigma|)$ if there exists a minimum separating vertex set $X$ such that $\Sigma \setminus X$ is balanced. Dictionary: See X.D. Zhang and Li (2002a). [Annot. 23 Mar 2009.](sg: Eig)

*Problem.* Generalize to connected, unbalanced signed graphs. [Annot. par: Lap: Eig]

**Xiao-Peng Zhang**
See X.-J. Tian.
Y. Zhang
See B. DasGupta.

Yan X Zhang
See T. Hibi.

Yanan Zhang & Haiyan Chen

Yanguo Zhang
See Z.-Y. Cheng.

Yingying Zhang
See X.-L. Chen.

Yuanping Zhang
See also X.G. Liu.

Yuan-ping Zhang & Xiao-gang Liu

Yuanping Zhang, Xiaogang Liu, Bingyan Zhang, & Xuerong Yong
[Correction by Hamidzade and Kiani (2010a).] (par: Lap: Eig)

Zhang Zhang
See M. DeVos and L.-S. Shi.

Zhi-Li Zhang
See Y.-H. Li.

Guopeng Zhao
See K. Li and L.G. Wang.

Qin Zhao
See Q. Wen.

Xuehua Zhao
See B. Yang.

Yufei Zhao
See Z.-L. Jiang.

Q. Zheng, D.B. Skillicorn, & O. Walther

Qing Yu Zheng
See also H.S. Du.

Qing Yu Zheng & Qing Jun Ren
2001a Quasi-Laplacian characteristic polynomials of graphs [or, The quasi-Laplacian characteristic polynomial]. (In Chinese.) *Qufu Shifan Daxue Xuebao Ziran*
Zhehui Zhong
See G. Adejumo.

Bo Zhou
See also M. Ghorbani, I. Gutman, G.X. Tian, S.L. Wang, and R.D. Xing.


Γ is bipartite. Subdivide every edge of Γ once. The eigenvalues are the square roots of the Laplacian eigenvalues of Γ, and 0. [Problem 1. Generalize to all graphs and the “signless Laplacian”.] [Annot. 28 Aug 2011.]


Bo Zhou & Aleksandar Ilić

See Tian, Huang, and Zhou (2009a). Lower and upper bounds in terms of sums of squared degrees. Thus, bounds on incidence energy et al.[Annot. 24 Jan 2012.]

Chuixiang Zhou
See M.-M. Xie.

Guanglu Zhou
See D. Li.

Haijun Zhou

“Long-range frustration” means correlation between spins (±1) of vertices at considerable distance, within the same “state” (a configuration domain separated by energy barriers). [This should be generalized to signed graphs.] [Annot. 12 Sept 2010.]

Harrison H. Zhou
See D. Feng.

Hou Chun Zhou
See H.S. Du.

Jiang Zhou
See C.J. Bu.

Jun Zhou, Yi-Zheng Fan & Yi Wang
See also Y.Z. Fan.

Sufficient condition for $\lambda_2(L(\Sigma)) \geq d_2$, the second largest eigenvalue and degree, respectively. Dictionary: See X.D. Zhang and Li (2002a).

Annot. 28 Oct 2011. (sg: Eig)

Min Zhou
See C.-X. He.

Qiannan Zhou
See Y. Lu.

Sanming Zhou
See F. Yang.

Xiangqian Zhou

Yue Zhou
See F. Belardo.

Yulong Zhou & Jiangping Hu

Zhi-Hua Zhou
See L.T. Wu.

Bao-Xuan Zhu


Jianming Zhu

Li Zhu
See R. Huang.

Min Zhu
See also F.L. Tian.

Min Zhu, Yihao Guo, Fenglei Tian, & Lingfei Lu

Qunxiong Zhu
See D. Peng.

Shirley Zhu
See L.J. Rusnak.
Xiao Xin Zhu, Zhi Ren Sun, & Chun Zheng Cao

Xuding Zhu
See also Y.-T. Jiang, L.-G. Jin, R.-G. Kim, H. Qi, and A. Raspaud.

Thm.: If every signed planar graph is 4-colorable (cf. Máčajová, Raspaud, and Škoviera (2016a)), then every planar graph is 2-list-bipartite-colorable. [Annot. 10 Nov 2017.] (SG: Col)

Zhongxun Zhu

Günter M. Ziegler
See A. Björner, L. Lovász, and T. Tran.

Howard E. Zimmerman

František Zítek
1972a Polarisované grafy. [Polarized graphs.] Lecture at the Czechoslovak Conf. on Graph Theory, Štířín, May, 1972.
For definitions see Zelinka (1976a). For work on these objects see many papers of Zelinka. (sg: Ori, sw)
Cf. Zelinka (1976a) for definitions. The number of labelled polar trees; the same with given degrees. [Annot. 27 Jul 2013.] (sg: Ori)

Arjana Žitnik
See T Pisanski.

J. Zittartz
See P. Hoever, M.H. Waldor, and W.F. Wolff.
Etay Ziv, Robin Koytcheff, Manuel Middendorf, & Chris Wiggins

Statistical analysis on a space of graphs. Mentions easy generalization to signed (and weighted) graphs. [Annot. 8 Sept 2010.] (SGw: Alg)

Dejan Živković
See S.K. Simić.

Dorota Żuchowska-Skiba
See F. Hassanibesheli.

Mark Zuckerberg

Example 3: The constraint matrix is the incidence matrix of a balanced signed graph. [Annot. 16 July 2016.] (SG: Bal: Incid)

Alejandro Zuñiga
See J. Aracena.

[Anke van Zuylen]
See A. van Zuylen (under “V”).

Alexei Zverovich
See G. Gutin.

Igor E. Zverovich
See also E. Boros.

The arc signed graph $Λ_Z(\vec{Γ})$ of a digraph $\vec{Γ}$ (simple $Γ$) is the line graph $Λ(Γ)$ with $σ_Z(\vec{uvw}) := +$ and $σ_Z(\vec{vuw}) := −$. [Thus, it is $−Λ(+Γ)$ where $+Γ$ has orientation $\vec{Γ}$; cf. Zaslavsky (2012c), (20xxa).]
Thm. 1: A Krausz-type characterization of $Λ_Z$. Cor. 1: $Λ_Z$ determines $\vec{Γ}$ up to isolated vertices and reversing the orientation. Thm. 2: Characterization by induced subgraphs: a finite list plus antibalanced circles of length $≥ 4$. Cor. 2: $Λ_Z(\vec{Γ})$ graphs can be recognized and $\vec{Γ}$ reconstructed in polynomial time. Dictionary: “(+)-complete” means $+K_n$; “bicomplete” means complete and balanced. [Antibalanced circles are forbidden due to having all-positive base graphs.] (SG: LG)

A. Zverovitch
See N. Gülpinar.

Stefan H.M. van Zwam
See also K. Grace, D. Mayhew and R.A. Pendavingh.
§3.2, “The Dowling lift of a partial field”. [Expanded in Pendavingh &
van Zwam (2013a).

Uri Zwick
See R. Yuster.

Krzysztof Zwierzyński
See D. Stevanović (2007a).

Lisa Zyga


Ondřej Zýka
See also J. Kratochvíl.


*Cf. Bouchet (1983a).* There is a nowhere-zero 30-flow [presumably, if the signed-graphic matroid has no coloop; there might be other restrictions]. [Annot. 15 Jan 2015.] (SG: Flows)