A Mathematical Bibliography of Signed and Gain Graphs and Allied Areas

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Colleagues:
HELP!
If you have any suggestions whatever for items to include in this bibliography, or for other changes, please let me hear from you. Thank you.

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> [I]t should be borne in mind that incompleteness is a necessary concomitant of every collection of whatever kind. Much less can completeness be expected in a first collection, made by a single individual, in his leisure hours, and in a field which is already boundless and is yet expanding day by day.
>
> —Robert Edouard Moritz, preface to *Memorabilia Mathematica: The Philomath's Quotation Book*, 1914.

Preface

A *signed graph* is a graph whose edges are labeled by signs. This is a bibliography of signed graphs and related mathematics.

Several kinds of labelled graph have been called "signed" yet are mathematically very different. I distinguish four types:

- *Group-signed graphs*: the edge labels are elements of a 2-element group and are multiplied around a circle (or along any walk). Among the natural generalizations are larger groups and vertex signs.
- *Sign-colored graphs*, in which the edges are labelled from a two-element set that is acted upon by the sign group: $-$ interchanges labels, $+$ leaves them unchanged. This is the kind of "signed graph" found in knot theory. The natural generalization is to more colors and more general groups—or no group.
- *Weighted graphs*, in which the edge labels are the elements $+1$ and -1 of the integers or another additive domain. Weights behave like numbers, not signs; thus I regard work on weighted graphs as outside the scope of the bibliography—except (to some extent) when the author calls the weights "signs".
- Labelled graphs where the labels have no structure or properties but are called "signs" for any or no reason.

Each of these categories has its own theory or theories, generally very different from the others, so in a logical sense the topic of this bibliography is an accident of terminology. However, narrow logic here leads us astray, for the study of true signed graphs, which arise in numerous areas of pure and applied mathematics, forms the great majority of the literature. Thus I regard as fundamental for the bibliography the notions of *balance* of a circle (sign product equals $+$, the sign group identity) and the vertex-edge incidence matrix (whose column for a negative edge has two $+1$'s or two -1's, for a positive edge one $+1$ and one -1, the rest being zero); this has led me to include work on *gain graphs* (where the edge labels are taken from any group) and "consistency" in *vertex-signed graphs*, and generalizable work on two-graphs (the set of unbalanced triangles of a signed complete graph) and on even and odd circles and paths in graphs and digraphs.

Nevertheless, it was not always easy to decide what belongs. I have employed the following principles:

Only works with mathematical content are entered, except for a few representative purely applied papers and surveys. I do try to include:

- •Any (mathematical) work in which signed graphs are mentioned by name or signs are put on the edges of graphs, regardless of whether it makes essential use of signs. (However, due to lack of time and in order to maintain "balance" in the bibliography, I have included only a limited selection of items concerning binary clutters and postman theory, two-graphs, signed digraphs in qualitative matrix theory, and knot theory. For clutters, see Cornuéjols (2001a); for postman theory, A. Frank (1996a). For two-graphs, see any of the review articles by Seidel. For qualitative matrix theory, see e.g. Maybee and Quirk (1969a) and Brualdi and Shader (1995a). For knot theory there are uncountable books and surveys.)
- •Any work in which the notion of balance of a circle plays a role. Example: gain graphs. (Exception: purely topological papers concerning ordinary graph embedding.)
- •Any work in which ideas of signed graph theory are anticipated, or generalized, or transferred to other domains. Examples: vertex-signed graphs; signed posets and matroids.
- •Any mathematical structure that is an example, however disguised, of a signed or gain graph or generalization, and is treated in a way that seems in the spirit of signed graph theory. Examples: even-cycle and bicircular matroids; bidirected graphs; binary clutters (which are equivalent to signed binary matroids); some of the literature on two-graphs and double covering graphs.
- •And some works that have suggested ideas of value for signed graph theory or that have promise of doing so in the future.

As for applications, besides works with appropriate mathematical content I include a few (not very carefully) selected representatives of less mathematical papers and surveys, either for their historical importance (e.g., Heider (1946a)) or as entrances to the applied literature (e.g., Taylor (1970a) and Wasserman and Faust (1993a) for psychosociology and Trinajstic (1983a) for chemistry). Particular difficulty is presented by spin glass theory in statistical physics—that is, Ising models and generalizations. Here one usually averages random signs and weights over a probability distribution; the problems and methods are rarely graph-theoretic, the topic is very specialized and hard to annotate properly, but it clearly is related to signed (and gain) graphs and suggests some interesting lines of graph-theoretic research. See Mézard, Parisi, and Virasoro (1987a) and citations in its annotation.

Plainly, judgment is required to apply these criteria. I have employed mine freely, taking account of suggestions from my colleagues. Still I know that the bibliography is far from complete, due to the quantity and even more the enormous range and dispersion of work in the relevant areas. I will continue to add both new and old works to future editions and I heartily welcome further suggestions.

There are certainly many errors, some of them egregious. For these I hand over responsibility to Sloth, Pride, Ambition, Envy, and Confusion. (Corrections, however, will be gratefully accepted by me.) And as Diedrich Knickerbocker says:

> Should any reader find matter of offense in this [bibliography], I should heartily grieve, though I would on no acount question his penetration by telling him he was mistaken, his good nature by telling him he was captious, or his pure conscience by telling him he was startled at a shadow. Surely when so ingenious in finding offense where none was intended, it were a thousand pities he should not be suffered to enjoy the benefit of his discovery.

Bibliographical Data. Authors' names are given usually in only one form, even should the name appear in different (but recognizably similar) forms on different publications. Journal abbreviations follow the style of *Mathematical Reviews* (MR) with minor 'improvements'. Reviews and abstracts are cited from MR and its electronic form MathSciNet, from *Zentralblatt für Mathematik* (Zbl) and its electronic version (For early volumes, "Zbl VVV, PPP" denotes printed volume and page; the electronic item number is "(e VVV.PPPNN)".), and occasionally from *Chemical Abstracts* (CA) or *Computing Reviews* (CR). A review marked (*q.v.*) has significance, possibly an insight, a criticism, or a viewpoint orthogonal to mine.

Some—not all—of the most fundamental works are marked with a ††; some almost as fundamental have a †. This is a personal selection.

Annotations. I try to describe the relevant content in a consistent terminology and notation, in the language of signed graphs despite occasional clumsiness (hoping that this will suggest generalizations), and sometimes with my [bracketed] editorial comments. I sometimes try also to explain idiosyncratic terminology, in order to make it easier to read the original item. Several of the annotations incorporate open problems (of widely varying degrees of importance and difficulty).

I use these standard symbols:

- Γ is a graph (V, E) of order $n = |V|$, undirected, possibly allowing loops and multiple edges. It is normally finite unless otherwise indicated.
- Σ is a signed graph (V, E, σ) of order n. $|\Sigma|$ is its underlying graph. E_+, E_- are the sets of positive and negative edges and Σ_+, Σ_- are the corresponding spanning subgraphs (unsigned).
- $[\Sigma]$ is the switching class of Σ.
- $A(\ )$ is the adjacency matrix.
- $\mathrm{H}(\ )$ is the incidence matrix.
- $K(\ )$ is the Kirchoff or Laplacian matrix, $\mathrm{H}(\ )\mathrm{H}(\ )^{\mathrm{T}}$.
- λ_1 is the largest eigenvalue of a matrix.
- Φ is a gain graph (V, E, φ). $\|\Phi\|$ is its underlying graph.
- $[\Phi]$ is the switching class of Φ.
- $\sim$ means that two signed or gain graphs are switching equivalent (with the same underlying graph).
- $\simeq$ means that two signed or gain graphs are switching isomorphic (with isomorphic underlying graphs).
- $\cong$ denotes isomorphism.
- $\langle\Sigma\rangle$ is the biased graph of Σ.
- $\langle\Phi\rangle$ is the biased graph of Φ.
- Ω is a biased graph. $\|\Omega\|$ is its underlying graph.
- $l(\ )$ is the frustration index (line index of imbalance).
- $l_0(\ )$ is the frustration number (vertex frustration number, vertex elimination number).

$G(\ )$ is the frame (bias) matroid of a signed, gain, or biased graph.
$L(\ ), L_0(\ )$ are the lift and extended lift matroids.
$\Lambda(\ )$ is a line graph. $\Lambda(\Gamma)$ is that of a graph. For a signed or gain graph, Λ_{BC} is that of Behzad and Chartrand (1969a); $\Lambda_\times$ is that of M. Acharya (2009a), Λ is that of Zaslavsky (1979a, 1984c, 2010b, 20xxa).

Some standard terminology—much more will be found in the *Glossary* (Zaslavsky (1998c)):

polygon, circle: The graph of a simple closed path, or its edge set.
cycle: In a digraph, a coherently directed circle, i.e., "dicycle". More generally: in an oriented signed, gain, or biased graph, a matroid circuit (usually, of the frame matroid) oriented to have no source or sink.

Acknowledgement. I cannot name all the people who have contributed advice and criticism, but many of the annotations have benefited from suggestions by the authors or others and a number of items have been brought to my notice by helpful correspondents. I am very grateful to you all. Thanks also to the people who maintain the invaluable MR and Zbl indices (and a special thank-you for creating our own MSC classification: 05C22). However, I insist on my total responsibility for the final form of all entries, including such things as my restatement of results in signed or gain graphic language and, of course, all the praise and criticism (but not errors; see above) that they contain.

Subject Classification Codes

A code in *lower case* means the topic appears implicitly but not explicitly. A suffix **w** on **Sgnd**, **SG**, **SD**, **VS** denotes signs used as weights, i.e., treated as the numbers $+1$ and -1, added, and (usually) the sum compared to 0. A suffix **c** on **SG**, **SD**, **VS** denotes signs used as colors (often written as the numbers $+1$ and -1), usually permuted by the sign group. In a string of codes a colon precedes subtopics. A code may be refined through being suffixed by a parenthesized code, as **Sgnd**(**M**) denoting signed matroids (while **Sgnd**: **M** would indicate matroids of signed objects; thus **Sgnd**(**M**): **M** means matroids of signed matroids).

Adj Adjacency matrix, Kirchhoff or Laplacian matrix; eigenvalues.
Alg Algorithms.
Algeb Algebraic structures upon signed, gain, or biased graphs or digraphs.
Appl Applications other than (**Chem**), (**Phys**), (**Biol**), (**PsS**) (partial coverage).
Aut Automorphisms, symmetries, group actions.
Bal Balance (mathematical), cobalance.
Bic Bicircular matroids.
Biol Applications to biology (partial coverage).
Chem Applications to chemistry (partial coverage).
Clu Clusterability.
Col Vertex coloring.
Cov Covering graphs, double coverings.
D Duality (graphs, matroids, or matrices).
Enum Enumeration of types of signed graphs, etc.
EC Even-cycle matroids.
ECol Edge coloring.

Exp Expository.
Exr Interesting exercises (in an expository work).
Fr Frustration (imbalance); esp. frustration index (line index of imbalance).
Geom Connections with geometry, including linear programming, toric varieties, complex complement, etc.
GD Digraphs with gains (or voltages).
Gen Generalization.
GG Gain graphs, voltage graphs, biased graphs; includes Dowling lattices.
GN Generalized or gain networks. (Multiplicative real gains.)
GH Hypergraphs with gains.
Incid Incidence matrix.
KG Signed complete graphs.
Knot Connections with knot theory (sparse coverage if signs are purely notational).
LG Line graphs.
M Matroids and geometric lattices, chain-groups (not signed matroids).
MtrdF Matroidal families.
Invar Numerical and algebraic invariants of signed, gain, biased graphs: polynomials, degree sequences, number of bases, etc.
Ori Orientations, bidirected graphs.
OG Ordered gains.
Par All-negative or antibalanced signed graphs; parity-biased graphs.
par Also, problems on even or odd length of paths or circles (partial coverage).
Phys Applications in physics (partial coverage).
PsS Psychological, sociological, and anthropological applications (partial coverage).
QM Qualitative (sign) matrices: sign patterns, sign stability, sign solvability, etc.: graphical methods.
Rand Random signs or gains, signed or gain graphs.
Ref Many references.
Sgnd Signed objects other than graphs and hypergraphs: mathematical properties.
SD Signed digraphs: mathematical properties.
SG Signed graphs: mathematical properties.
SH Signed hypergraphs: mathematical properties.
SM Signed matroids.
QSol Sign solvability, sign nonsingularity (partial coverage).
QSta Sign stability (partial coverage).
Str Structure theory.
Sw Switching of signs or gains.
Top Topology applied to graphs; surface embeddings. (Not applications to topology.)
TG Two-graphs, graph (Seidel) switching (partial coverage).
VS Vertex-signed graphs ("marked graphs"); signed vertices and edges.
WD Weighted digraphs.
WG Weighted graphs.
Xtreml Extremal problems.

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Takuro Abe

2009a The stability of the family of B_2-type arrangements. *Commun. Algebra* 37 (2009), no. 4, 1193–1215. MR 2510979 (2010d:32027). Zbl 1194.32014.

The arrangements are affino-signed-graphic arrangements.

(**sg, gg: Geom**)

Takuro Abe, Koji Nuida, and Yasuhide Numata

2009a An edge-signed generalization of chordal graphs, free multiplicities on braid arrangements, and their characterizations. In: Christian Krattenthaler, Volker Strehl, and Manuel Kauers, eds., *21st International Conference on Formal Power Series and Algebraic Combinatorics (FPSAC 2009)* (Hagenburg, Austria), pp. 1–12. Discrete Math. Theor. Computer Sci., Nancy, France, 2009. MR 2520381 (2010k:32039). (**SG: Str, Geom**)

2009b Signed-eliminable graphs and free multiplicities on the braid arrangement. *J. London Math. Soc.* (2) 80 (2009), no. 1, 121–134. MR 2520381 (2010k:32039). Zbl 1177.32017. (**SG: Str, Geom**)

Peter Abell

See also H. Deng, B. Kujawski, and M. Ludwig.

Peter Abell and Robin Jenkins

1967a Perception of the structural balance of part of the international system of nations. *J. Peace Res.* 4 (1967), no. 1, 76–82. (**PsS**)(**SG: Bal: Exp**)

Peter Abell and Mark Ludwig

2009a Structural balance: A dynamic perspective. *J. Math. Sociology* 33 (2009), no. 2, 129–155. Zbl 1169.91434.

Dynamics of signed graphs in a space of sign probabilities and tolerance of imbalance. There are three discernibly different domains of dynamical behavior. [Continued in Deng and Abell (2010a) and Kujawski, Ludwig, and Abell (20xxa).] [Annot. 10 Sept, 9 Dec 2009.]

(**SG, PsS: Bal, Fr: Alg**)

Robert P. Abelson

See also M.J. Rosenberg.

1967a Mathematical models in social psychology. In: Leonard Berkowitz, ed., *Advances in Experimental Social Psychology*, Vol. 3, pp. 1–54. Academic Press, New York, 1967.

§ II: "Mathematical models of social structure." Part B: "The balance principle." Reviews basic notions of balance and clusterability in signed (di)graphs and measures of degree of balance or clustering. Notes that signed K_n is balanced iff $I + A = vv^{\mathrm{T}}$, $v = \pm 1$-vector. Proposes: degree of balance $= \lambda_1/n$, where $\lambda_1 =$ largest eigenvalue of $I + A(\Sigma)$ and $n =$ order of the (di)graph. [*Cf.* Phillips (1967a).] Part C, 3: "Clusterability revisited." (**SG, SD: Bal, Clu, Fr, Adj**)

R.P. Abelson, E. Aronson, W.J. McGuire, T.M. Newcomb, M.J. Rosenberg, and P.H. Tannenbaum, eds.

1968a *Theories of Cognitive Consistency: A Sourcebook.* Rand-McNally, Chicago, Ill., 1968.

Robert P. Abelson and Milton J. Rosenberg

†1958a Symbolic psycho-logic: a model of attitudinal cognition. *Behavioral Sci.* 3 (1958), 1–13.

$R(\Sigma)$ They introduce a modified adjacency matrix R, called the "structure matrix" [I call it the Abelson–Rosenberg adjacency matrix], with entries o, p, n, a for, respectively, nonadjacency [0 in the usual adjacency matrix A], positive and negative adjacency [$+1$, -1 in A] and simultaneous positive and negative adjacency [0 or indeterminate in A]. They define an algebra (i.e., associative, commutative, and distributive addition and multiplication) of these symbols (p. 8): o acts as 0, p acts as 1, $pn = n$, $n^2 = p$, $a = p + n$, $x + x = x$ and $ax = a$ for $x \neq 0$. In the algebra one can decide balance of Σ via the permanent of $I + R$: Σ is balanced if $\operatorname{per}(I + R) = p$ and unbalanced if $\operatorname{per}(I + R) = a$. (The "straightforward but space-consuming" proof is omitted. They state that the permanent cannot equal n or o [but that is an error].) [See Harary, Norman, and Cartwright (1965a) for more on this matrix, and Zaslavsky (2010b), Thm. 2.1, for a matrix with more precise counting properties.] They introduce switching in terms of the Hadamard product of R with a "passive T-matrix" [oversimplifying, that is a matrix obtained by switching the square all-p's matrix; the actual definition involves operators s and c and is more interesting]. Thm. 11: Switching preserves balance.

They propose (p. 12) "complexity" [= frustration index $l(\Sigma)$] as a measure of imbalance. [*Cf.* Harary (1959b).] Thm. 12: Switching preserves frustration index. Thm. 14: $\max l(\Sigma)$, taken over all signed graphs Σ of order n, equals $\lfloor (n-1)^2/4 \rfloor$. (Proof omitted. [Proved by Petersdorf (1966a) and Tomescu (1973a) for signed K_n's and hence for all signed simple graphs of order n.]) (**PsS**)(**SG: Adj, Bal, sw, Fr**)

Marien Abreu

M. Abreu, M.J. Funk, D. Labbate, and V. Napolitano

20xxa On the ubiquity and utility of cyclic schemes. Submitted. arXiv:1111.3265. (**GG: Cov**)

Nair Maria Maia de Abreu

See also M.A.A. de Freitas, L.S. de Lima, and C.S. Oliveira.

Nair Abreu, Domingos M. Cardoso, Ivan Gutman, Enide A. Martins, and María Robbiano

2011a Bounds for the signless Laplacian energy. *Linear Algebra Appl.* 435 (2011), no. 10, 2365–2374. MR 2811121 (2012f:05164). Zbl 1222.05143. (**Par: Adj**)

B. Devadas Acharya [Belmannu Devadas Acharya]

See also M.K. Gill and S.B. Rao.

1973a On the product of p-balanced and l-balanced graphs. *Graph Theory Newsletter* 2 (Jan., 1973), no. 3, Results Announced No. 1. (**SG, VS: Bal**)

1979a New directions in the mathematical theory of balance in cognitive organizations. MRI Tech. Rep. No. HCS/DST/409/76/BDA (Dec., 1979). Mehta Research Institute of Math. and Math. Physics, Allahabad, 1979. (**SG, SD: Bal, Adj, Ref**)(**PsS: Exp, Ref**)

1979b A programme logic for listing of sigraphs, their characteristic polynomials, and their spectra. *Graph Theory Newsletter* 9 (1979), no. 2, 1.

Abstract of a plan for computation. (**SG: Adj: Alg**)

1980a Spectral criterion for cycle balance in networks. *J. Graph Theory* 4 (1980), 1–11. MR 81e:05097(*q.v.*). Zbl 445.05066.

A signed simple graph Σ is balanced iff $A(\Sigma)$ has the same spectrum as $A(|\Sigma|)$. A signed simple digraph $(\vec{\Gamma}, \sigma)$ is cycle balanced (every directed cycle is positive) iff $A(\vec{\Gamma}, \sigma)$ has the same spectrum as $A(\vec{\Gamma})$. [These are most interesting as criteria for cospectrality, since balance is easy to determine, although not cycle balance.]

Proposed measure of imbalance: the proportion of corresponding coefficients where the characteristic polynomials $p(A(\Sigma); \lambda)$ and $p(A(|\Sigma|); \lambda)$ differ. [See Gill (1981b).] [Annot. Rev. 4 Apr 2012.]

(**SD, SG: Bal, Adj**)

1980b An extension of the concept of clique graphs and the problem of K-convergence to signed graphs. *Nat. Acad. Sci. Letters (India)* 3 (1980), 239–242. Zbl 491.05052.

(**SG: LG, Clique graph**)

1980c Applications of sigraphs in behavioural sciences. M.R.I. Tech. Rep. No. DST/-HCS/409/79 (June, 1980). Mehta Research Institute of Math. and Math. Physics, Allahabad, 1979.

[Annotation is very incomplete.] Let $\Sigma_1 \vee \Sigma_2$ be the join of underlying graphs, with edge signs $\{\pm 1\}$ as in $\Sigma_1 \cup \Sigma_2$ and with $\sigma(v_1 v_2) := \max(\mu_1(v_1), \mu_2(v_2))$, where $\mu(v) := \prod_{vw \in E} \sigma(vw)$. [Annot. 20 July 2009.] (**SG**)

1981a On characterizing graphs switching equivalent to acyclic graphs. *Indian J. Pure Appl. Math.* 12 (1981), 1187-1191. MR 82k:05089. Zbl 476.05069.

Begins an attack on the problem of characterizing by forbidden induced subgraphs the simple graphs that switch to forests. Among them are K_5 and C_n, $n \geq 7$. *Problem.* Find any others that may exist. [Solved by Hage and Harju (2004a). Forests that switch to forests were characterized by Hage and Harju (1998a).] (**TG: Sw**)

1982a Connected graphs switching equivalent to their iterated line graphs. *Discrete Math.* 41 (1982), 115–122. MR 84b:05078. Zbl 497.05052. (**LG, TG**)

1982b Even edge colorings of a graph: II. A lower bound for maximum even edge-coloring index. *Nat. Acad. Sci. Letters (India)* 5 (1982), no. 3, 97–99.

(**bal: Gen**)

1983a Even edge colorings of a graph. *J. Combin. Theory Ser. B* 35 (1983), 78–79. MR 85a:05034. Zbl 505.05032, (515.05030).

Find the fewest colors to color the edges so that in each circle the number of edges of some color is even. [Possibly, inspired by §2 of Acharya and Acharya (1983a).] (**bal: Gen**)

1983b A characterization of consistent marked graphs. *Nat. Acad. Sci. Letters (India)* 6 (1983), no. 12, 433–440. MR 884837. Zbl 552.05052.

Converts a vertex-signed graph (Γ, μ) into a signed graph Σ such that (Γ, μ) is consistent (as in Beineke and Harary 1978b) iff every circle in Σ is all negative or has an even number of all-negative components. [See Joglekar, Shah, and Diwan (2010a) for the definitive result on consistency.] (**VS, SG: bal**)

1984a Some further properties of consistent marked graphs. *Indian J. Pure Appl. Math.* 15 (1984), 837–842. MR 86a:05101. Zbl 552.05053.

Notably: nicely characterizes consistent vertex-signed graphs in which the subgraph induced by negative vertices is connected. [Subsumed by S.B. Rao (1984a).] (**VS: bal**)

1984b Combinatorial aspects of a measure of rank correlation due to Kendall and its relation to social preference theory. In: B.D. Acharya, ed., *Proceedings of the National Symposium on Mathematical Modelling* (Allahabad, 1982). M.R.I. Lecture Notes in Appl. Math., 1. Mehta Research Institute of Math. and Math. Physics, Allahabad, India, 1984.

Includes an exposition of Sampathkumar and Nanjundaswamy (1973a). (**SG: KG: Exp**)

1985a *Signed Graphs With Applications in Behavioural Sciences.* M.R.I. Lect. Notes Appl. Math., No. 3. Mehta Research Institute of Math. and Math. Physics, Allahabad, 1985. (**SG: PsS**)

1986a An extension of Katai-Iwai procedure to derive balancing and minimum balancing sets of a social system. *Indian J. Pure Appl. Math.* 17 (1986), 875–882. MR 87k:92037. Zbl 612.92019.

Expounds the procedure of Katai and Iwai (1978a). Proposes a generalization to those Σ that have a certain kind of circle basis. Construct a "dual" graph whose vertex set is a circle basis supplemented by the sum of basic circles. A "dual" vertex has sign as in Σ. Let T = set of negative "dual" vertices. A T-join in the "dual", if one exists, yields a negation set for Σ. [A minimum T-join need not yield a minimum negation set. Indeed the procedure is unlikely to yield a minimum negation set (hence the frustration index $l(\Sigma)$) for all signed graphs, since it can be performed in polynomial time while $l(\Sigma)$ is NP-complete. *Questions.* To which signed graphs is the procedure applicable? For which ones does a minimum T-join yield a minimum negation set? Do the latter include all those that forbid an interesting subdivision or minor (*cf.* Gerards and Schrijver (1986a), Gerards (1988a, 1989a))?] (**SG: Fr: Alg**)

2009a Role of cognitive balance in some notions of graph labelings: Influence of Frank Harary, Fritz Heider, Gustav Kirchhoff and Leonhard Euler. *Bull. Allahabad Math. Soc.* 24 (2009), no. 2, 391–413. MR 2597634 (no rev). Zbl 1221.05278. (**SG, SD: Bal, sw**)

2010a Signed intersection graphs. *J. Discrete Math. Sci. Cryptography* 13 (2010), no. 6, 553–569. MR 2791608 (2011m:05129).

Signed hypergraph: hypergraph $H = (X, E)$ with $\sigma_H : E \to \{+, -\})$. Canonical marking $\mu_{\sigma_H}(x) := \prod_{e \ni x} \sigma_H(e)$ $(x \in X)$. Intersection edge sign $\sigma_\Omega(ef) := \prod_{x \in e \cap f} \mu_{\sigma_H}(x)$. The signed intersection graph $\Omega(H, \sigma)$ is the intersection graph of H with signature σ_Ω. Main example: Maximal-clique hypergraph $\mathcal{K}(\Xi)$ of a signed graph Ξ with $X = \{$maximal cliques of $|\Xi|\}$, signature $\sigma_\mathcal{K}(Q) := \prod_{v \in Q} \mu_\sigma(Q)$ for a max clique Q. Which signed graphs are $\Omega(\mathcal{K}(\Xi))$? Thm. 3.3: Σ is a maxclique signed graph iff it has an edge clique cover with the Helly property, whose members induce homogeneously signed subgraphs, an even number of which are all-negative.

On orbits of the operator $\mathcal{K}$: Thm. 5.1: $\mathcal{K}^m(\Sigma) = \mathcal{K}^n(\Sigma)$ iff $\mathcal{K}^m(|\Sigma|) =$

$\mathcal{K}^n(|\Sigma|)$, $\exists\ m < n$. However (§7), $m = 0$ (Σ is "$\mathcal{K}$-periodic) may hold for $|\Sigma|$ but not Σ. *Problem* 7.2. Characterize $\mathcal{K}$-periodic signed graphs. [Annot. 28 Aug 2010.] (**SH, SG: lg**)

§8, "Signed line graphs": Taking edges instead of max cliques defines a line graph $\Lambda(\bullet(\Sigma)$ with signature $\sigma_\bullet(ef) := \mu_\sigma(e \cap f)$ (due to M. Acharya). [Annot. 28 Aug 2010.] (**SG: LG**)

2010a Mathematical chemistry: Basic issues. In: *Graph Theory Applied to Chemistry* (Proc. Nat. Workshop, Pala, Kerala, India, 2010), Ch. 2.2, pp. 26–46.

§2.2.9, "Newer vistas": Signed hypergraphs, signed semigraphs. [Annot. 31 Aug 2010.] (**SG: Gen, SH: Exp**)

2011a On notions generalizing combinatorial graphs, with emphasis on linear symmetric dihypergraphs. *Bull. Allahabad Math. Soc.* 26 (2011), no. 2, 229–258.

Many generalizations of graphs and digraphs. Mainly historical and expository. [Annot. 31 Jan 2012.]
(**SG, SD, Gen: Exp, Ref**)(**SG, SD, Gen**)

20xxa Set-valuations of a signed digraph. Int. Workshop on Set-Valuations, Signed Graphs, Geometry and Their Appl. (IWSSG-2011, Mananthavady, Kerala, 2011). *J. Combin. Inform. Syst. Sci.*, to appear. (**SD, SG**)

20xxb Domination and absorbance in signed graphs and digraphs I. Foundations. Submitted. (**SG, SD**)

20xxc Minus domination in a signed graph. Int. Workshop on Set-Valuations, Signed Graphs, Geometry and Their Appl. (IWSSG-2011, Mananthavady, Kerala, 2011). *J. Combin. Inform. Syst. Sci.*, to appear. (**SG**)

B. Devadas Acharya and Mukti Acharya [M.K. Gill]

1983a A graph theoretical model for the analysis of intergroup stability in a social system. Manuscript, 1983.

The first half (most of §1) was improved and published as (1986a).

The second half (§§2–3) appears to be unpublished. Given; a graph Γ, a vertex signing μ, and a covering $\mathcal{F}$ of $E(\Gamma)$ by cliques of size ≤ 3. Define a signed graph S by; $V(S) = \mathcal{F}$ and $QQ' \in E(S)$ when at least half the elements of Q or Q' lie in $Q \cap Q'$; sign QQ' negative iff there exist vertices $v \in Q \backslash Q'$, and $w \in Q' \backslash Q$ such that $\mu(v) \neq \mu(w)$. Suppose there is no edge QQ' in which $|Q| = 3$, $|Q'| = 2$, and the two members of $Q \backslash Q'$ have differing sign. [This seems a very restrictive supposition.] Main result (Thm. 7): S is balanced. The definitions, but not the theorem, are generalized to multiple vertex signs μ, general clique covers, and clique adjacency rules that differ slightly from that of the theorem. (**GG, VS, SG: Bal**)

1986a New algebraic models of social systems. *Indian J. Pure Appl. Math.* 17 (1986), 150–168. MR 87h:92087. Zbl 591.92029.

Four criteria for balance in an arbitrary gain graph. [See also Harary, Lindstrom, and Zetterstrom (1982a).] (**GG: Bal, sw**)

Belmannu Devadas Acharya, Mukti Acharya, and Deepa Sinha

2008a Cycle-compatible signed line graphs. *Indian J. Math.* 50 (2008), no. 2, 407–414. MR 2517744 (2010h:05142). Zbl 1170.05032.

Characterizes when $\Lambda_{BC}(\Sigma)$ (Behzad and Chartrand 1969a) with vertex signs σ is harmonious. Dictionary: "cycle compatible" = harmonious (the product of all edge and vertex signs on each circle is positive). [Annot. 14 Oct 2009.] (**SG, VS: LG: Bal**)

2009a Characterization of a signed graph whose signed line graph is S-consistent. *Bull. Malaysian Math. Sci. Soc.* (2) 32 (2009), no. 3, 335–341. MR 2562172 (2010m:05135). Zbl 1176.05032.

Let Σ be a signed simple graph. Thm. 2.1: The line graph $\Lambda(|\Sigma|)$, with vertex signs σ, is consistent (as in Beineke and Harary 1978b) iff Σ is balanced and, in Σ, a vertex of degree ≥ 4 has only positive edges, while a trivalent vertex v with negative edges has two such edges, which lie in every circle on v. [Cor.: Let Σ be 2-connected. $(\Lambda(|\Sigma|), \sigma)$ is consistent iff Σ is balanced and every negative edge has endpoints of degree ≤ 2. *Problem.* Find a structural characterization, by means of which all such Σ can be constructed. Ear decomposition and Tutte's 3-decomposition should be the key.] [Annot. 2 Oct 2009.] (**SG, VS: LG: Bal**)

B.D. Acharya, M.K. Gill, and G.A. Patwardhan

1984a Quasicospectral graphs and digraphs. In: B.D. Acharya, ed., *Proceedings of the National Symposium on Mathematical Modelling* (Allahabad, 1982), pp. 133–144. M.R.I. Lect. Notes Appl. Math., No. 1. Mehta Research Institute of Math. and Math. Physics, Allahabad, 1984. MR 86c:05087. Zbl 556.05048.

Continues M. Acharya (1981a). A signed graph, or digraph, is "cycle-balanced" if every circle, or every cycle, is positive. Graphs, or digraphs, are "quasicospectral" if they have cospectral signings, "strictly quasicospectral" if they are quasicospectral but not cospectral, "strongly cospectral" if they are cospectral and have cospectral cycle-unbalanced signings. There exist arbitrarily large sets of strictly quasicospectral digraphs, which moreover can be assumed strongly connected, weakly but not strongly connected, etc. There exist pairs of unbalanced, strictly quasicospectral graphs; existence of larger sets is unsolved. There exist arbitrarily large sets of nonisomorphic, strongly cospectral connected graphs; also, of weakly connected digraphs, which moreover can be taken to be strongly connected, unilaterally connected, etc. Proofs, based on ideas of A.J. Schwenk, are sketched.

(**SD, SG: Adj**)

Belmannu Devadas Acharya and Shalini Joshi

2003a On the complement of an ambisidigraph. [Abstract.] Proc. R.C. Bose Centenary Sympos. Discrete Math. Appl. (Kolkata, 2002). *Electron. Notes Discrete Math.* 15 (2003), 5. MR 2159023. Zbl 1184.05100.

The complement of a signed digraph D without loops or multiple signed arcs (a loopless, simply signed digraph, or "ambisidigraph") is defined in the obvious way. Observation: If D or D^c contains a directed cycle of length $2k+1$, then one of them contains a positive such cycle. (**SD**)

2005a Mathematical modelling in social psychology–social networks. *Everyman's Science* 40 (2005), no. 2, 124–128.

Popular exposition including ambisidigraphs (*cf.* (2003a)). [Annot. 7 Apr 2012.] (**SD: Exp**)

B.D. Acharya, S. Joshi, and S.B. Rao

20xxa A Ramsey theorem for strongly connected ambisidigraphs. Submitted.

Sequel to Acharya and Joshi (2003a). For which loopless, simply signed digraphs D do both D and D^c contain no positive 3-cycle? Thm.: If strongly connected, D has order < 6. An attempt to use this to describe all loopless, simply signed digraphs that contain no positive 3-cycle. (**SD: Str**)

Mukti Acharya [Mukhtiar Kaur Gill]

See also B.D. Acharya, M.K. Gill, and S.B. Rao.

1988a Switching invariant three-path signed graphs. In: M.N. Gopalan and G.A. Patwardhan, eds., *Optimization, Design of Experiments and Graph Theory* (Proc. Sympos. in Honour of Prof. M.N. Vartak, Bombay, 1986), pp. 342–345. Indian Inst. of Technology, Bombay, 1988. MR 90b:05102. Zbl 744.05054.

See Gill and Patwardhan (1986a) for the k-path signed graph of Σ. The equation $\Sigma \simeq D_3(\Sigma)$ is solved. [Annot. 29 Apr 2009.] (**SG, Sw**)

2009a $\times$-line signed graphs. Int. Conf. Recent Developments Combin. Graph Theory (Krishnankoil, Tamil Nadu, India, 2007). *J. Combin. Math. Combin. Comput.* 69 (2009), 103–111. MR 2517311 (no rev). Zbl 1195.05031.

$\Lambda_\times(\Sigma)$ $\Lambda_\times(\Sigma) := (\Lambda(|\Sigma|), \sigma_\times)$ where $\sigma_\times(ef) := \sigma(e)\sigma(f)$. (Contrast with Behzad–Chartrand (1969a) and Zaslavsky (2010b, 20xxa, 20xxb) line graphs.) [The definition originated in Gill (1982a). Publication of this article was delayed by many years.] [Annot. Rev 20 Dec 2010.] (**SG: LG**)

2010a Square-sum graphs: Some new perspectives. In: *International Conference on Recent Trends in Graph Theory and Combinatorics (ICRTBC-2010)* (Cochin, 2010) [Summaries], pp. 114–119. Dept. of Mathematics, Cochin Univ. of Science and Technology, 2010.

P. 119: Summary of k-square-sum signed graphs, where k edges classes are square-sum with the same vertex labels. $k = 2$ is signed graphs. [Annot. 30 Aug 2010.] (**SGc**)

20xxa Quasicospectrality of graphs and digraphs: A creative review. Int. Workshop on Set-Valuations, Signed Graphs, Geometry and Their Appl. (IWSSG-2011, Mananthavady, Kerala, 2011). *J. Combin. Inform. Syst. Sci.*, to appear.

Graphs or digraphs are quasicospectral if they have cospectral signatures (signatures with the same adjacency spectrum). Properties and examples of quasicospectral graphs and digraphs that are not cospectral. Definitions and results from B.D. Acharya, Gill, and Patwardhan (1984a) (*q.v.*) et al., as well as new results. [Annot. 4 Apr 2012.] (**SG, SD: Adj: Exp**)(**SG, SD: Adj**)

20xxb Signed discrete structures. Submitted. (**SG, SH**)

Mukti Acharya and Tarkeshwar Singh

2003a Graceful signed graphs: III, The case of signed cycles in which the negative sections form a maximum matching. *Graph Theory Notes N.Y.* 45 (2003), 11–15. MR 2040207 (no rev).

See (2004a). Here the graph is a circle and the second color class is a maximum matching. (**SGc**)

2003b Skolem graceful signed graphs. Proc. R.C. Bose Centenary Sympos. Discrete Math. Appl. (Kolkata, 2002). *Electron. Notes Discrete Math.* 15 (2003), 10–11. MR 2159025 (no rev). Zbl 1184.05108.

Announcement of (2010a). (**SGc: Exp**)

2004a Graceful signed graphs. *Czechoslovak Math. J.* 54(129) (2004), no. 2, 291–302. MR 2005a:05193. Zbl 1080.05529.

[Generalizing the definition in the article: Given: a graph with r-colored edges; integers $k, d > 0$. Required: a (k, d)-*graceful labelling*, i.e., an injection $\lambda : V \to \{0, 1, \ldots, k + (|E| - 1)d\}$ so that, if $f(vw) := |\lambda(v) - \lambda(w)|$, then f restricted to each color class is injective with range $k, k + d, \ldots$.]

The article concerns the case $r = 2$ with "results of our preliminary investigation". *Conjecture.* Every 2-colored circle of length ≥ 3 is (k, d)-graceful. (**SGc**)

2004b Graceful signed graphs V. The case of union of signed cycles of the length three with one vertex in common. *Int. J. Management Syst.* 20 (2004), no. 3, 245–254.

Σ is a signed windmill with $k > 1$ blades. Only rim edges may be negative. Thm.: Σ is graceful $\implies$ $k \equiv 0 \bmod 4$ and $|E^-|$ is even, or $k \equiv 1 \bmod 4$, or $k \equiv 2 \bmod 4$ and $|E^-|$ is odd. Thm.: If $k \equiv 0, 1 \bmod 4$ and all rim edges are negative, then Σ is graceful. Thm.: If $k \equiv 2 \bmod 4$ and all rim edges but one are negative, then Σ is graceful. See also Singh (2009a). [Annot. 21 July 2010.] (**SGc**)

2004c A characterization of signed graphs whose negation is switching equivalent to its iterated line sigraphs. In: R.J. Wilson, R. Balakrishnan and G. Sethuraman, eds., *Proceedings of the Conference of Graph Theory and Applications* (CGTA-2001), pp. 15–24. Narosa Publishing House, New Delhi, 2004. (**SG: Sw, LG**)

2005a Graceful signed graphs: II. The case of signed cycles with connected negative sections. *Czechoslovak Math. J.* 55(130) (2005), no. 1, 25–40. MR 2005m:05192. Zbl 1081.05097.

Proof of the conjecture of (2004a) for a circle of length $\not\equiv 1 \pmod 4$ where the negative edge set is connected. (**SGc**)

2009a Skolem graceful signed stars. *J. Combin. Math. Combin. Comput.* 69 (2009), 113–124. MR 2517312 (2010e:05257). Zbl 1195.05065. (**SGc**)

2010a Skolem graceful signed graphs. *Utilitas Math.* 82 (2010), 97–109. MR 2663369 (2011h:05218). Zbl 1232.05198.

From Singh (2003a), Ch. III. See Acharya and Singh (2003b), Singh (2008a). "Skolem gracefulness" is the $(0, 1)$-gracefulness of (2004a). Thm.: A signed k-edge matching is Skolem graceful iff $k \equiv 0 \pmod 4$ and $|E^-|$ is even, or $k \equiv 2 \pmod 4$ and $|E^-|$ is odd, or $k \equiv 1 \pmod 4$. Curiously complementary to the theorem of Singh (20xxa). [Annot. 20 July 2009.] (**SGc**)

20xxb Embedding of signed graphs in graceful signed graphs. *Ars Combin.*, to appear.

See (2004a). Every signed graph whose vertices have distinct non-negative integral labels is an induced subgraph of a signed graph with $(1, 1)$-graceful labels. (**SGc**)

20xxc Characterization of sigraphs whose negations are switching equivalent to their iterated line sigraphs. Submitted.

The signed simple graphs Σ (which necessarily are signed circles) such that $-\Sigma$ is switching isomorphic to any of its iterated Behzad–Chartrand (1969a) line graphs. [Annot. 20 July 2009.] (**SG: Sw, LG**)

20xxd Construction of certain infinite families of graceful sigraphs from a given graceful sigraph. Submitted.

Let $\vee$ denote the join of graphs or (defined in B.D. Acharya (1980c)) signed graphs. Thms.: If Σ is gracefully numbered, so are $\Sigma \cup K_t^c$ and $\big(\Sigma \cup K^c_{|E|-|V|+1}\big) \vee K_t^c$. All $(K_2 \vee K_r^c, \sigma)$ are gracefully numbered. [Annot. 20 July 2009.] **(SGc)**

20xxe Graceful sigraphs: V. The case of union of signed cycles of length three with one vertex in common. Submitted. **(SGc)**

Mukti Acharya and Deepa Sinha

2002a A characterization of signed graphs that are switching equivalent to their jump signed graphs. *Graph Theory Notes N.Y.* 43 (2002), 7–8. MR 1960487 (no rev). **(SG: LG)**

2003a A characterization of sigraphs whose line sigraphs and jump sigraphs are switching equivalent. *Graph Theory Notes N.Y.* 44 (2003), 30–34. MR 2002894. **(SG: LG)**

2003b A characterization of line sigraphs. Proc. R.C. Bose Centenary Sympos. Discrete Math. Appl. (Kolkata, 2002). *Electron. Notes Discrete Math.* 15 (2003), 12. MR 2159026 (no rev).

Abstract of (2005a). **(SG: LG)**

2005a Characterizations of line sigraphs. *Nat. Acad. Sci. Letters (India)* 28 (2005), no. 1-2, 31–34. MR 2127289 (no rev).

Thm.: A signed simple graph Σ is the Behzad–Chartrand (1969a) line graph of a signed graph iff the underlying graph is a line graph and Σ is "sign compatible" (Sinha 2005a). [Annot. 27 Apr 2009, 12 Oct 2010.] **(SG: LG)**

2006a Common-edge sigraphs. *AKCE Int. J. Graphs Combin.* 3 (2006), no. 2, 115–130. MR 2285459 (2007k:05083). Zbl 1119.05053.

The common-edge signed graph $C_E(\Sigma)$ is the second line graph $L^2(|\Sigma|)$ with signs $\sigma_{C_E}\{ef, fg\} = \sigma(f)$. Characterized in whole or part: When this is balanced (rarely), or isomorphic to Σ (rarely), or switching isomorphic to the Behzad–Chartrand (1969a) line graph $\Lambda_{BC}(\Sigma)$ (rarely), or switching equivalent to $\Lambda^2_{BC}(\Sigma)$. There are notions of consistency and compatibility of $C_E(\Sigma)$ with respect to a vertex signature of Σ, that seem ill defined. **(SG: LG: Gen)**

Gbemisola Adejumo, P. Robert Duimering, and Zhehui Zhong

2008a A balance theory approach to group problem solving. *Social Networks* 30 (2008), 83–99. **(PsS, SG: Fr)**

L. Adler and S. Cosares

1991a A strongly polynomial algorithm for a special class of linear programs. *Operations Res.* 39 (1991), 955–960. MR 92k:90042. Zbl 749.90048.

The class is that of the transshipment problem with gains. Along the way, a time bound on the uncapacitated, demands-only flows-with-gains problem. **(GN: Incid(D), Alg)**

S.N. Afriat

1963a The system of inequalities $a_{rs} > X_r - X_s$. *Proc. Cambridge Philos. Soc.* 59 (1963), 125–133. MR 25 #5071. Zbl 118, 149 (e: 118.14901).

See also Roy (1959a). **(GG: OG, Sw, bal)**

1974a On sum-symmetric matrices. *Linear Algebra Appl.* 8 (1974), 129–140. MR 48 #11163. Zbl 281.15017. **(GG: Sw, bal)**

Amit Agarwal

See Harary, Lim, Agarwal, and Wunsch.

A.A. Ageev, A.V. Kostochka, and Z. Szigeti

1995a A characterization of Seymour graphs. In: Egon Balas and Jens Clausen, eds., *Integer Programming and Combinatorial Optimization* (4th Int. IPCO Conf., Copenhagen, 1995, Proc.), pp. 364–372. Lecture Notes in Computer Sci., Vol. 920. Springer, Berlin, 1995. MR 96h:05157.

A Seymour graph satisfies with equality a general inequality between T-join size and T-cut packing. Thm.: A graph is not a Seymour graph iff it has a conservative ± 1-weighting such that there are two circles with total weight 0 whose union is an antibalanced subdivision of $-K_n$ or $-Pr_3$ (the triangular prism). **(SGw: Str, Bal, Par)**

1997a A characterization of Seymour graphs. *J. Graph Theory* 24 (1997), 357–364. MR 97m:05217. Zbl 970.24507.

Virtually identical to (1995a). **(SGw: Str, Bal, Par)**

J.K. Aggarwal

See M. Malek-Zavarei.

Ron Aharoni, Rachel Manber, and Bronislaw Wajnryb

1990a Special parity of perfect matchings in bipartite graphs. *Discrete Math.* 79 (1990), 221–228. MR 91b:05140. Zbl 744.05036.

When do all perfect matchings in a signed bipartite graph have the same sign product? Solved. **(sg: bal, Alg)(qm: QSol)**

R. Aharoni, R. Meshulam, and B. Wajnryb

1995a Group weighted matchings in bipartite graphs. *J. Algebraic Combin.* 4 (1995), 165–171. MR 96a:05111. Zbl 950.25380.

Given an edge weighting $w : E \to K$ where K is a finite abelian group. Main topic: perfect matchings M such that $\sum_{e\in M} w(e) = 0$ [I'll call them 0-weight matchings]. (Also, in §2, $= c$ where c is a constant.) Generalizes and extends Aharoni, Manber, and Wajnryb (1990a). [Continued by Kahn and Meshulam (1998a).] **(GGw)**

Prop. 4.1 concerns vertex-disjoint circles whose total sign product is + in certain signed digraphs. **(SD)**

Amnon Aharony

1978a Low-temperature phase diagram and critical properties of a dilute spin glass. *J. Phys. C* 11 (1978), L457–L463.

Physics of a random signed subgraph of Γ: p, q, r = probabilities of +, −, or no edge. $r = 0$ is a randomly signed Γ. $p = 0$ is a random subgraph $-\Gamma_1$. Edges may have weights but the signs are most significant (pp. L461–2). Bipartite graphs ("simple systems, with two sublattices") give easier results; e.g., switching exchanges p and q, and transforms all-negative to all-positive. Analysis by the replica method: replicate the graph randomly n times. For temperature $T \to 0$: The case $p = q$ has special properties. The limit $r \to 0$ gives all-positive (ferromagnetic) behavior because "only [constant states $\zeta : V \to \{+1, -1\}$] contribute to the partition function." $T > 0$: Special cases for equal weights, similarly

to Houtappel (1950b), Newell (1950a). The replica method's limitations include failure at $T \to 0$ when the signed subgraph is unbalanced ("frustrated") (p. L463). [An interesting study. *Problem.* Interpret the replica method and results in terms of random signed graphs.] [Annot. 21 Jun 2012.] **(Phys, SG, WG: Rand, Fr, sw)**

Luis von Ahn

2008a Science of the Web: 15-396. Networks II: Structural Balance. Course slides. http://www.scienceoftheweb.org. Dept. of Computer Science, Carnegie Mellon University.

Triangle ("triad") balance and balance. **(SG: Bal: Exp)**

Ravindra K. Ahuja, Thomas L. Magnanti, and James B. Orlin

1993a *Network Flows: Theory, Algorithms, and Applications.* Prentice Hall, Englewood Cliffs, N.J., 1993. MR 94e:90035.

§12.6: "Nonbipartite cardinality matching problem". Nicely expounds theory of blossoms and flowers (Edmonds (1965a), etc.). Historical notes and references at end of chapter. **(par: ori, Alg: Exp, Ref)**

§5.5: "Detecting negative cycles"; §12.7, subsection "Shortest paths in directed networks". Weighted arcs with negative weights allowed. Techniques for detecting negative cycles and, if none exist, finding a shortest path. **(WD: OG, Alg: Exp)**

Ch. 16: "Generalized flows". §15.5: "Good augmented forests and linear programming bases", Thm. 15.8, makes clear the connection between flows with gains and the bias matroid of the underlying gain graph. Some terminology: "breakeven cycle" = balanced circle; "good augmented forest" = basis of the bias matroid, assuming the gain graph is connected and unbalanced. **(GN: M(Bases), Alg: Exp, Ref)**

Martin Aigner

1979a *Combinatorial Theory.* Grundl. math. Wiss., Vol. 234. Springer-Verlag, Berlin, 1979. Reprint: Classics in Mathematics. Springer-Verlag, Berlin, 1997. MR 80h:05002. Zbl 415.05001, 858.05001 (reprint).

In § VII.1, pp. 333–334 and Exerc. 13–15 treat the Dowling lattices of $\mathrm{GF}(q)^\times$ and higher-weight analogs. **(GG, GG(Gen): M: Invar, Str)**

M. Aĭgner [Martin Aigner]

1982a *Kombinatornaya teoriya.* "Mir", Moscow, 1982. MR 84b:05002.

Russian translation of (1979a). Transl. V.V. Ermakov and V.N. Lyamin. Ed. and preface by G.P. Gavrilov. **(GG, GG(Gen): M: Invar, Str)**

Nir Ailon, Moses Charikar, and Alantha Newman

2005a Aggregating inconsistent information: Ranking and clustering. In: *STOC'05: Proceedings of the 37th Annual ACM Symposium on the Theory of Computing* (Boston, 2005), pp. 684–693. Assoc. for Computing Machinery, New York, 2005. MR 2181673. Zbl 1192.90252.

Conference version of (2008a). **(SG: WG: Clu: Alg)**

2008a Aggregating inconsistent information: ranking and clustering. *J. ACM* 55 (2008), no. 5, Art. 23, 27 pp. MR 2456548 (2009k:68280). **(SG: WG: Clu: Alg)**

S. Akbari, A. Daemi, O. Hatami, A. Javanmard, and A. Mehrabian

20xxa Nowhere-zero unoriented flows in hamiltonian graphs. Submitted.

Every signed Hamiltonian graph without a coloop has a nowhere-zero 12-flow: an improved result towards Bouchet's (1983a) conjecture. The proofs are for unoriented flows on a graph (i.e., flows on an all-negative signed graph, which are equivalent to signed-graph flows). Better results if there is a negative Hamilton circle C. Thm. 3.2: An 8-flow if C^c is connected. Thm. 3.3: A 6-flow if C^c is unbalanced. (**SG: Flows, ori**)

Saieed Akbari, Ebrahim Ghorbani, Jack [Jacobus] H. Koolen, and Mohammad Reza Oboudi

2010a A relation between the Laplacian and signless Laplacian eigenvalues of a graph. *J. Algebraic Combin.* 32 (2010), no. 3, 459–464. MR 2721061 (2011i:05125).

The sign-corrected coefficients of the characteristic polynomial of $K(-\Gamma)$ dominate those of $K(+\Gamma)$. [*Problem.* Prove they dominate those of $K(\Gamma,\sigma)$ for any *sigma*.] [*Problem.* Generalize to all signatures of Γ.] [Annot. 22 Nov 2010.] (**Par: Adj, Incid**)

2010b On sum of powers of the Laplacian and signless Laplacian eigenvalues of graphs. *Electronic J. Combin.* 17 (2010), Article R115, 8 pp. MR 2679569 (2011j:05189). Zbl 1218.05086.

(**sg: Par: Adj**)

S. Akbari, E. Ghorbani, and M.R. Oboudi

2009a A conjecture on square roots of Laplacian and signless Laplacian eigenvalues of graphs. Unpublished manuscript. `arXiv:0905.2118`

Conjecture. The sum s of singular values is larger for $\mathrm{H}(-\Gamma)$ than for $\mathrm{H}(+\Gamma)$. Dictionary: "incidence matrix" = the unoriented incidence matrix $\mathrm{H}(-\Gamma)$; "directed incidence matrix" = oriented incidence matrix $\mathrm{H}(+\Gamma)$. [*Problem.* Generalize to other signatures of Γ? E.g., is $\max_\sigma s(\Gamma,\sigma) = s(-\Gamma)$?] [Annot. 8 Oct 2010.] (**Par: Adj, Incid**)

J. Akiyama, D. Avis, V. Chvátal, and H. Era

††1981a Balancing signed graphs. *Discrete Appl. Math.* 3 (1981), 227–233. MR 83k:05059. Zbl 468.05066.

Bounds for $D(\Gamma)$, the largest frustration index $l(\Gamma,\sigma)$ over all signings of a fixed graph Γ (not necessarily simple) of order n and size $m = |E|$. Main Thm.: $\frac{1}{2}m - \sqrt{mn} \leq D(\Gamma) \leq \frac{1}{2}m$. Thm. 4: $D(K_{t,t}) \leq \frac{1}{2}t^2 - c_0 t^{3/2}$, where c_0 can be taken $= \pi/480$. Probabilistic methods are used. Thus, Thm. 2: Given Γ, $\mathrm{Prob}(l(\Gamma,\sigma) > \frac{1}{2}m - \sqrt{mn}) \geq 1 - (\frac{2}{e})^n$. Moreover, let $n_\mathrm{b}(\Sigma)$ be the largest order of a balanced subgraph of Σ. Thm. 5: $\mathrm{Prob}(n_\mathrm{b}(K_n,\sigma) \geq k) \leq \binom{n}{k}/2^{\binom{k}{2}}$. (The problem of evaluating $n - n_\mathrm{b}$ was raised by Harary; see (1959b).) Finally, Thm. 1: If Σ has vertex-disjoint balanced induced subgraphs with m' edges, then $l(\Sigma) \leq \frac{1}{2}(m - m')$. [See Poljak and Turzík (1982a) for an upper bound on $D(\Gamma)$, Solé and Zaslavsky (1994a) for lower and (bipartite) upper bounds; Brown and Spencer (1971a), Gordon and Witsenhausen (1972a) for $D(K_{t,t})$; Harary, Lindström, and Zetterström (1982a) for a result similar to Thm. 1.]

(**SG: Fr, Rand**)

M.J. Alava, P.M. Duxbury, C.F. Moukarzel, and H. Rieger

2001a Exact combinatorial algorithms: Ground states of disordered systems. In: C. Domb and J.L. Lebowitz, eds., *Phase Transitions and Critical Phenomena*, Vol. 18. Academic Press, San Diego, 2001.

§7.1, "Random Ising magnets", (iv), "Frustrated magnets and spin glasses", introduces §7.4, "Ising spin glasses and Euclidean matching". §7.4.1, "Introduction and overview": Frustration index, in terms of Hamiltonian $H(s) := -\sum J_{ij}s_is_j$ where, mainly, $J_{ij} = \pm 1$ randomly (random signed graphs). Frustrated (negative) plaquettes (girth circles) in a lattice. §7.4.2, "Mapping to optimization problems": (i) "Mapping to a matching problem": Planar solution by dual matching as in Katai and Iwai (1978a) [not cited], Bieche, Maynard, Rammal, and Uhry (1980a), Barahona (1982b), *et al.*(ii) "Mapping to a cut problem": Equivalence to max cut. §7.4.3, "Ground-state calculation in two dimensions": Behavior of ground state (fewest frustrated edges) as function of negative-edge density. Remarks on external magnetic field, cubic grid graphs. [Annot. 29 Aug 2012.] (**Phys: SG: Fr: Rand: Exp, Ref**)

Şahin Albayrak

See J. Kunegis.

S. Alexander and P. Pincus

1980a Phase transitions of some fully frustrated models. *J. Phys. A: Math. Gen.* 13 (1980), no. 1, 263–273.

Certain all-negative signed graphs where every edge is in a triangle: $d = 2$-dimensional triangular lattice and $d \geq 3$-dimensional face-centered cubic lattice. Phase phenomena depend on the parity of d. Odd d implies interesting infinities of switchings with minimum $|E^-|$. [Annot. 12 Aug 2012.] (**Par: Phys**)

Artiom Alhazov, Ion Petre, and Vladimir Rogojin

2009a The parallel complexity of signed graphs: Decidability results and an improved algorithm. *Theor. Comput. Sci.* 410 (2009), no. 24–25, 2308–2315. MR 2522435 (2011a:68045). Zbl 1167.68022. (**SG: Alg**)

Noga Alon

1996a Bipartite subgraphs. *Combinatorica* 16 (1996), no. 3, 301–311.

Lower bound on the largest bipartite subgraph of a simple graph with m edges. [I.e., upper bound on $l(-\Gamma)$. *Problem.* Generalize to $l(\Sigma)$.] [Annot. 8 Mar 2011, 19 May 2012.] (**sg: par: Fr**)

Noga Alon and Yoshimi Egawa

1985a Even edge colorings of a graph. *J. Combin. Theory Ser. B* 38 (1985), no. 1, 93–94. MR 782628 (86f:05059). Zbl 556.05026.

Proves and improves a conjecture of B.D. Acharya (1983a). Thm.: The minimum number of colors for an even edge coloring = minimum number of colors so each color class is bipartite = $\lceil \log_2 \chi(\Gamma) \rceil$. (**bal: Gen**)

Noga Alon, Gregory Gutin, Eun Jung Kim, Stefan Szeider, and Anders Yeo

2010a Solving MAX-r-SAT above a tight lower bound. In: Moses Charikar, ed., *Proceedings of the Twenty-First Annual ACM-SIAM Symposium on Discrete Algorithms* (SODA 2010, Austin, Tex.), pp. 511–517. Soc. for Industrial and Appl. Math., Philadelphia, and Assoc. for Computing Machinery, New York, 2010. MR 2809695 (2012h:68266).

Extended abstract of (2011a). (**SG: Alg**)

2011a Solving MAX-r-SAT above a tight lower bound. *Algorithmica* 61 (2011), no. 3, 638–655. MR 2824999 (2012g:68101). Zbl 1242.68118. arXiv:0907.4573. (**SG: Alg**)

Susan S. D'Amato
See S.S. D'Amato (under D).

M. Amram, R. Shwartz, and M. Teicher
2010a Coxeter covers of the classical Coxeter groups. *Int. J. Algebra Comput.* 20 (2010), no. 8, 1041–1062. MR 2747415 (2012c:20104). arXiv:0803.3010.
The structure of a quotient of a generalized Coxeter group depends on the presence of loops in the associated signed graph. [Annot. 17 Dec 2011.] (**SG**)

Xinhui An
See J.F. Wang.

Milica Anđelić, Carlos M. da Fonseca, Slobodan K. Simić, and Dejan V. Tošić
2012a Connected graphs of fixed order and size with maximal Q-index: Some spectral bounds. *Discrete Appl. Math.* 160 (2012), no. 4-5, 448–459.
Bounds on $\lambda_1(K(-\Gamma))$ where Γ is a nested split graph. (*Cf.* Cvetković, Rowlinson, and Simić (2007b), which shows nested split graphs maximize $\lambda_1(K(-\Gamma))$.) [Annot. 2 Feb 2012.] (**sg: par: Adj**)

2012b Some further bounds for the Q-index of nested split graphs. *J. Math. Sci.* 182 (2012), no. 2, 193–199. (**sg: par: Adj**)

Milica Anđelić and Slobodan K. Simić
2010a Some notes on the threshold graphs. *Discrete Math.* 310 (2010), no. 17-18, 2241–2248. MR 2659175 (2011h:05145). Zbl 1220.05035. (**sg: par: Adj**)

Kazutoshi Ando and Satoru Fujishige
1996a On structures of bisubmodular polyhedra. *Math. Programming* 74 (1996), 293–317. MR 97g:90102. Zbl 855.68107. (**sg: Ori**)

Kazutoshi Ando, Satoru Fujishige, and Takeshi Naitoh
1997a Balanced bisubmodular systems and bidirected flows. *J. Operations Res. Soc. Japan* 40 (1997), 437–447. MR 98k:05073. Zbl 970.61830.
A balanced bisubmodular system corresponds to a bidirected graph that is balanced. The "flows" are arbitrary capacity-constrained functions, not satisfying conservation at a vertex. (**sg: Ori, Bal**)

Kazutoshi Ando, Satoru Fujishige, and Toshio Nemoto
1996a Decomposition of a bidirected graph into strongly connected components and its signed poset structure. *Discrete Appl. Math.* 68 (1996), 237–248. MR 97c:05096. Zbl 960.53208. (**sg: Ori**)

1996b The minimum-weight ideal problem for signed posets. *J. Operations Res. Soc. Japan* 39 (1996), 558-565. MR 98j:90084. Zbl 874.90188. (**sg: Ori**)

Thomas Andreae
1978a Matroidal families of finite connected nonhomeomorphic graphs exist. *J. Graph Theory* 2 (1978), 149–153. MR 80a:05160. Zbl 401.05070.
Partially anticipates the "count" matroids of graphs (see Whiteley (1996a)). (**Bic, EC: Gen**)

David Angeli, Patrick De Leenheer, and Eduardo Sontag
2010a Graph-theoretic characterizations of monotonicity of chemical networks in reaction coordinates. *J. Math. Biology* 61 (2010), no. 4, 581–616. MR 2672536 (2011d:92054). Zbl 1204.92038.

Dictionary: "J-graph" = a signed graph of a Jacobian matrix. "Species-reaction graph" ("SR-graph") = bipartite signed graph $\Sigma := (V_S, V_R, E, \sigma)$; "reaction graph" ("R-graph") = $-\Sigma^2{:}V_R$; "species graph" ("S-graph") = $-\Sigma^2{:}V_S$ [where Σ^2 is the distance-2 signed graph: $V(\Sigma^2) := V_R \cup V_S$, $T_iT_j \in E^\varepsilon(\Sigma^2) \iff \exists$ path $T_iU_kT_j$ with $\sigma(T_iU_kT_j) = \varepsilon$]. Dictionary: "Simple loop" $\approx$ circle; "positive-loop property" = balance. In a bipartite signed graph, "e-loop, o-loop" = circle with $(-1)^{|C|/2}\sigma(C) = +$ or $-$. Prop. 4.5: $\Sigma^2{:}V_R$ is antibalanced iff all circles in Σ are e-loops and max $\deg(\Sigma{:}V_S) \leq 2$. Thm. 1 (oversimplified): A certain differential system is monotone iff $\Sigma^2{:}V_R$ is antibalanced (the R-graph is balanced). [Annot. 19 Feb 2010.] (**SG: Bal, sw, Geom, Chem**)

[A bipartite multiplicative gain graph $\Phi := (V_S, V_R, E, \varphi)$ may be defined by $\varphi(S_iR_j) :=$(a value from the stoichiometry matrix Γ). Circle C is "unitary" if $(-1)^{|C|/2}\varphi(C) = +1$.] Φ is implicated in the proof of geometrical Lemma 6.1. [Annot. 19 Feb 2010.] (**gg: Bal, Geom**)

J.C. Angles d'Auriac and R. Maynard

1984a On the random antiphase state of the $\pm J$ spin glass model in two dimensions. *Solid State Commun.* 49 (1984), no. 8, 785–790.

Signed square lattice graph: frustration index and ground states (minimum $|E^-|$ of switched Σ) via matching [*cf.* Katai and Iwai (1978a), Barahona (1981a, 1982a)]. Observed: natural clusters with relatively fixed spins (vertex signs) when the density of negative edges is in $(0.1, 0.2)$. [Annot. 18 Aug 2012.] (**Phys, SG: Fr: Alg**)

T. Antal, P.L. Krapivsky, and S. Redner

††2005a Dynamics of social balance on networks. *Phys. Rev. E* 72 (2005), 036121. MR 2179924 (2006e:91124).

Models for the evolution of a signed K_n towards balance, with conclusions about the probable long-term behavior. A "state" of the graph is a signature. The unit of time t is $|E| = \binom{n}{2}$ steps of the process. The density of edges is $\rho := |E^+|/\binom{n}{2}$. The number of triangles with k negative edges (type k) is N_k; their density is $n_k := N_k/\binom{n}{3}$. The average density of type k triangles on a positive edge is $n_k^+ = (3-k)N_k/(n-2)|E^+| = (3-k)n_k/(3n_0 + 2n_1 + n_2)$. Similarly, $n_k^- = kn_k/(3n_0 + 2n_1 + n_2)$.

"Local triad dynamics": At each step a random triangle T is chosen. If it is all negative, a random edge in T is chosen and negated. If it has one negative edge, a random edge in T is chosen and negated with probability p if it is negative and $1-p$ if positive. If it is balanced there is no change. The process is repeated ad infinitum. Finite [i.e., fixed] n: For $p > 1/2$ the graph reaches all-positivity ("paradise") in time $C \log t$ and for $p = 1/2$ in time $C/\sqrt{2t}$. For $p < 1/2$ the graph reaches a balanced state which is not all positive, in superexponential time. (Time is in the units described.) "Infinite" n [i.e., $n \to \infty$]: For $p < 1/2$ the density of negative edges approaches the stationary value $(1 + \sqrt{(}3(1-2p))^{-1}$. For $p > 1/2$ the network approaches all-positivity. Thus, at $p = 1/2$ there is a phase transition. Differential equations arise in the densities, with coefficients π^+, π^- where $\pi^\varepsilon :=$ the probability that, in one step, the sign change is from ε to $-\varepsilon$; thus $\pi^+ = (1-p)n_1$

and $\pi^- = pn_1 + n_3$. A stationary state has $\pi^+ = \pi^-$. For infinite n the stationary states are in § III.B and temporal evolution of $\rho = \rho(t)$ is treated in § III.C. Finite n is in § III.D.

"Constrained triad dynamics": An edge is chosen randomly and is negated with probability 1 if the number of positive triangles increases, 0 if the number decreases, and 1/2 if the number remains the same. This corresponds to an Ising model with Hamiltonian $-\sum \sigma_i\sigma_j\sigma_k$, summed over all edge triples that form a triangle. This model approaches balance in time $C \log t$ with high probability if n is large. The other alternatives are to reach an unbalanced absorbing state, where every edge is more positive than negative triangles (a "jammed state"), or a trajectory where every edge is in equally many triangles of each sign (a "blinker"). Blinkers were not observed in the simulations. The probability of a jammed state decreases quickly as $n \to \infty$. The "final" state, if balanced, has Harary bipartition $V = V_1 \cup V_2$. For $\rho(0) \lesssim .4$, $|V_1|/|V_2| approx 1$. As $\rho(0) \to \beta \approx .65$, $|V_1|/|V_2| \to \infty$, i.e., one set becomes dominant. When $\rho(0) > \beta$, $V_1 = V$ and all edges are positive. (§ IV.B.) A jammed state can occur only when $n = 9$ or $n \geq 11$ (§ IV.C), e.g., certain 3-cluster states as in Davis (1967a). The number of jammed signatures $> 3^n \gg 2^{n-1}$ = number of balanced ones, notwithstanding that the probable long-term state is balanced (§ IV.C). [See Marvel, Strogatz, and Kleinberg (2009a), Abell and Ludwig (2009a), Kujawski, Ludwig, and Abell (20xxa), Deng and Abell (2010a).]

Proposed research: Allow type 3 triangles (i.e., clustering). Allow incomplete graphs.

Dictionary: "network" = complete graph. [Annot. 27 Apr 2009.]

(**SG: KG: Bal**)

2006a Social balance on networks: the dynamics of friendship and enmity. *Physica D* 224 (2006), no. 1-2, 130–136. MR 2301516 (2007k:91210). Zbl 1130.91041.

Similar to (2005a). [Annot. 27 Apr 2009.] (**SG: KG: Bal**)

St. Antohe and E. Olaru

1981a Singned graphs homomorphism [*sic*]. [Signed graph homomorphisms.] *Bul. Univ. Galati Fasc. II Mat. Fiz. Mec. Teoret.* 4 (1981), 35–43. MR 83m:05057.

A "congruence" is an equivalence relation R on $V(\Sigma)$ such that no negative edge is within an equivalence class. The quotient Σ/R has the obvious simple underlying graph and signs $\bar\sigma(\bar x\bar y) = \sigma(xy)$ [which is ambiguous]. A signed-graph homomorphism is a function $f : V_1 \to V_2$ that is a sign-preserving homomorphism of underlying graphs. [This is inconsistent, since the sign of edge $f(x)f(y)$ can be ill defined. The defect might perhaps be remedied by allowing multiple edges with different signs or by passing entirely to multigraphs.] The canonical map $\Sigma \to \Sigma/R$ is such a homomorphism. Composition of homomorphisms is well defined and associative; hence one has a category Graph$^{\text{sign}}$. The categorial product is $\prod_{i\in I} \Sigma_i :=$ Cartesian product of the $|\Sigma_i|$ with the component-wise signature $\sigma((\dots,u_i,\dots)(\dots,v_i,\dots)) := \sigma_i(u_iv_i)$. Some further elementary properties of signed-graph homomorphisms and congruences are proved. [The paper is hard to interpret due to mathematical ambiguity and grammatical and typographical errors.] (**SG**)

Katsuaki Aoki
See M. Iri.

Mustapha Aouchiche and Pierre Hansen

2010a A survey of automated conjectures in spectral graph theory. *Linear Algebra Appl.* 432 (2010), 2293–2322. MR 2599861 (2011b:05139).

Computer-generated conjectures. §4, "Signless Laplacian": Several computer-generated conjectures about eigenvalues of $K(-\Gamma)$; some are proved (mainly in Cvetković, Rowlinson, and Simić (2007b)) or disproved; some are difficult. [*Question.* How many generalize to all Σ, with or without proofs?] [Annot. 22 Jan 2012.] (**Par: Adj**)

20xxa A survey of Nordhaus–Gaddum type relations. *Discrete Appl. Math.*, in press.

§6, "Spectral invariants": §6.3, "The eigenvalues of the signless Laplacian matrix": Nordhaus–Gaddum-type relations imply theorems from Gutman, Kiani, Mirzakhah, and Zhou (2009a) about the eigenvalues, singular values, incidence energy of $K(-\Gamma)$. Conjecture 6.19, generated by a computer—*cf.* (2010a): $\lambda_1(K(-\Gamma)) + \lambda_1(K(-\Gamma^c)) \leq 3n - 4$; $\lambda_1(K(-\Gamma)) \cdot \lambda_1(K(-\Gamma^c)) \leq 2n(n-2)$; = iff Γ is a star. [Annot. 22 Jan 2012.] (**Par: Adj**)

Mustapha Aouchiche, Pierre Hansen, and Claire Lucas

2011a On the extremal values of the second largest Q-eigenvalue. *Linear Algebra Appl.* 435 (2011), no. 10, 2591–2606. MR 2811141 (2012h:05184). Zbl 1222.05146. (**Par: Adj**)

Gautam Appa
See also L. Pitsoulis.

Gautam Appa and Balázs Kotnyek

2004a Rational and integral k-regular matrices. *Discrete Math.* 275 (2004), 1–15. MR 2004m:05005. Zbl 1043.15011.

2-regular matrices include binet matrices (2006a). A key property of k-regular matrices is that solutions of integral equations are $1/k$-integral. (**sg: Incid: Ori**)

2006a A bidirected generalization of network matrices. *Networks* 47 (2006), no. 4, 185–198. MR 2008a:05157. Zbl 1097.05025.

Binet matrices are the network matrices of bidirected (or signed) graphs. Basic theory of binet matrices, generalizing that of network matrices, notably half-integrality theorems. [For a slight simplification see Bolker and Zaslavsky (2006a).] (**sg: Incid: Ori**)

Gautam Appa, Balázs Kotnyek, Konstantinos Papalamprou, and Leonidas Pitsoulis

2007a Optimization with binet matrices. *Operations Res. Letters* 35 (2007), 345–352. MR 2008a:90052. Zbl 1169.90407. (**Ori: Incid(Gen), m**)

Julio Aracena
See also M. Montalva.

2008a Maximum number of fixed points in regulatory Boolean networks. *Bull. Math. Biol.* 70 (2008), no. 5, 1398–1409. MR 2421503 (2009d:05088). Zbl 1144.92323.

A regulatory Boolean network N is built on a signed digraph D. Thm. 6: If all (directed) cycles are positive then N has at least 2 fixed points. Thm. 9: N has at most 2^p fixed points, where $p :=$ minimum number of vertices that cover all positive cycles [unusually, not negative cycles!], and this is best possible. [Annot. 9 July 2009.]

(**SD: Fr: Gen, Appl(Biol)**)

Julio Aracena, Jacques Demongeot, and Eric Goles

2004a Positive and negative circuits in discrete neural networks. *IEEE Trans. Neural Networks* 15 (2004), no. 1, 77–83.

Existence and upper bound on the number of fixed points of a "discrete neural network" $\mathcal{N}$, which consists of a real $n \times n$ matrix W, the associated signed digraph D of order n, and a real vector b. A state is $x \in \{-1, +1\}^n$. A transition is $x \mapsto f(x) := \mathrm{sgn}_+(Wx - b)$ where $\mathrm{sgn}_+(t) := \mathrm{sgn}(t)$ except $\mathrm{sgn}_+(0) := +1$. Assume: D is connected; no component of f is constant, hence a cycle exists. Lemma 1: A cycle is positive iff it has a satisfied state. Thm. 1: If all cycles are positive, f has a fixed point. Thm. 2: If all cycles are negative, f has no fixed point. Thm. 3: $\#\{\text{fixed points}\} \leq 2^p$ where $p := \min(\text{size of vertex cover of positive cycles})$, and this is sharp. Dictionary: "positive feedback vertex set" = vertex cover of positive cycles = vertex set that covers all positive cycles; "circuit" = (directed) cycle; $\mathbf{1} = \mathrm{sgn}_+$. [Annot. 20 July 2009.]

(**SD: Bal, Fr: Gen**)

Julio Aracena, Mauricio Gonzaĺez, Alejandro Zuñiga, Marco A. Mendez, and Verónica Cambiazo

2006a Regulatory network for cell shape changes during *Drosophila* ventral furrow formation. *J. Theoretical Biol.* 239 (2006), 49–62. MR 2224512.

Figs. 2, 3 show particular proposed genetic regulatory networks based on signed digraphs. §3.2 describes how the mathematical model of Aracena *et al.* (2002a, 2004a) applies to the situation of this paper. [Annot. 20 July 2009.] (**SD: Appl(Biol)**)

Julián Aráoz, William H. Cunningham, Jack Edmonds, and Jan Green-Krótki

1983a Reductions to 1-matching polyhedra. Proc. Sympos. on the Matching Problem: Theory, Algorithms, and Applications (Gaithersburg, Md., 1981). *Networks* 13 (1983), 455–473. MR 85d:90059. Zbl 525.90068.

The "minimum-cost capacitated b-matching problem in a bidirected graph B" is to minimize $\sum_e c_e x_e$ subject to $0 \leq x \leq u \in \{0, 1, \ldots, \infty\}^E$ and $\mathrm{H(B)}x = b \in \mathbb{Z}^V$. The paper proves, by reduction to the ordinary perfect matching problem, Edmonds and Johnson's (1970a) description of the convex hull of feasible solutions. Dictionary: "lobe" = half edge.

(**sg: Ori: Incid, Alg, Geom**)

Dan Archdeacon

1992a The medial graph and voltage-current duality. *Discrete Math.* 104 (1992), no. 2, 111–141. MR 1172842 (93i:05051). Zbl 757.05045.

The medial graph of $\Gamma \subset S$, a graph embedded in a surface, is a 4-regular graph $M \subset S$ that encodes Γ and its surface dual. Gains ("voltages") on Γ transfer to gains ("voltages") on M. [Question. Does this suggest a gain-graphic, surface-embedding theory of 4-regular gain graphs?] [Annot. 16 Jan 2012.] (**GG, Top**)

1995a Problems in topological graph theory. Manuscript, 1995. http://www.emba.uvm.edu/~archdeac/papers/papers.html

A compilation from various sources and contributors, updated every so often. "The genus sequence of a signed graph", p. 10: A conjecture due to Širáň (?) on the demigenus range (here called "spectrum" [though unrelated to matrices]) for orientation embedding of Σ, namely, that the answer to Question 1 under Širáň (1991b) is affirmative. (**SG: Top**)

1996a Topological graph theory: a survey. Surveys in Graph Theory (Proc., San Francisco, 1995). *Congressus Numer.* 115 (1996), 5–54. Updated version: (2/98) http://www.emba.uvm.edu/~archdeac/papers/papers.html
MR 98g:05044. Zbl 897.05026.

§2.5 describes orientation embedding (called "signed embedding" [although there are other kinds of signed embedding]) and switching (called "sequence of local switches of sense") of signed graphs with rotation systems. §5.5, "Signed embeddings", briefly mentions Širáň (1991b), Širáň and Škoviera (1991a), and Zaslavsky (1993a, 1996a). (**SG: Top: Exp**)

2005a Variations on a theme of Kuratowski. *Discrete Math.* 302 (2005), 22–31. MR 2179233 (2006g:05055). Zbl 1076.05027.

Mentions [and conflates] the theorems of Zaslavsky (1993a). [Annot. 20 June 2011.] (**Top: SG: Exp**)

Dan Archdeacon and Marisa Debowsky

2005a A characterization of projective-planar signed graphs. *Discrete Math.* 290 (2005), no. 2–3, 109–332. MR 2123383 (2005j:05041). Zbl 1060.05039.

Similar to Archdeacon and Širáň (1998a) but for the projective plane. (**SG, Sw: Top**)

Dan Archdeacon and Jozef Širáň

1998a Characterizing planarity using theta graphs. *J. Graph Theory* 27 (1998), 17–20. MR 98j:05055. Zbl 887.05016.

A "claw" consists of a vertex and three incident half edges. Let C be the set of claws in Γ and T the set of theta subgraphs. Fix a rotation of each claw. Call $t \in T$ an "edge" with endpoints c, c' if t contains c and c'; sign it $+$ or $-$ according as t can or cannot be embedded in the plane so the rotations of its trivalent vertices equal the ones chosen for c and c'. This defines, independently (up to switching) of the choice of rotations, the "signed triple graph" $T^{\pm}(\Gamma)$. Theorem: Γ is planar iff $T^{\pm}(\Gamma)$ is balanced. (**SG, Sw: Top**)

Alex Arenas

See S. Gómez.

Srinivasa R. Arikati and Uri N. Peled

1993a A linear algorithm for the group path problem on chordal graphs. *Discrete Appl. Math.* 44 (1993), 185–190. MR 94h:68084. Zbl 779.68067.

Given a graph with edges weighted from a group. The weight of a path is the product of its edge weights in order (not inverted, as with gains). Problem: to determined whether between two given vertices there is a chordless path of given weight. This is NP-complete in general but for chordal graphs there is a fast algorithm (linear in $(|E| + |V|) \cdot$ (group order)). [*Question.* What if the edges have gains rather than weights?] (**WG: par(Gen): Alg**)

1996a A polynomial algorithm for the parity path problem on perfectly orientable graphs. *Discrete Appl. Math.* 65 (1996), 5–20. MR 96m:05120. Zbl 854.68069.

Problem: Does a given graph contain an induced path of specified parity between two prescribed vertices? A polynomial-time algorithm for certain graphs. (no. Bienstock (1991a).) [*Problem.* Generalize to paths of specified sign in a signed graph.] **(par: Alg)(Ref)**

Esther M. Arkin and Christos H. Papadimitriou

1985a On negative cycles in mixed graphs. *Operations Res. Letters* 4 (1985), 113–116. MR 821170 (87h:68061). Zbl 585.05017. **(WG: OG)**

1986a On the complexity of circulations. *J. Algorithms* 7 (1986), 134–145. MR 834086 (88a:68033). Zbl 603.68039. **(sg: Flows)**

E.M. Arkin, C.H. Papadimitriou, and M. Yannakakis

1991a Modularity of cycles and paths in graphs. *J. Assoc. Comput. Mach.* 38 (1991), 255–274. MR 92h:68068. Zbl 799.68146.

Modular poise gains in digraphs (gain +1 on each oriented edge). **(gg: Bal)**

Ali Reza Ashrafi

See Z. Yarahmadi.

Christos A. Athanasiadis

†1996a Characteristic polynomials of subspace arrangements and finite fields. *Adv. Math.* 122 (1996), 193–233. MR 97k:52012. Zbl 872.52006.

Treats the canonical lift representations (as affine hyperplane arrangements) of various gain graphs and signed gain graphs with additive gain group $\mathbb{Z}^+$. The article is largely a series of (sometimes brilliant) calculations of chromatic polynomials (*mutatis mutandis*, the characteristic polynomials of the representing arrangements) modulo a large integer q using gain graph coloring, though disguised as applications of Crapo–Rota's Critical Theorem. The fundamental principle is that, if q is larger than the largest gain of a circle, then $\mathbb{Z}^+$ can be replaced as gain group by $\mathbb{Z}_q^+$ without changing the chromatic polynomial (a consequence of Zaslavsky (1995b), Thm. 4.2)—and the analog for signed gain graphs, whose theory needs to be developed. A non-graphical result of the general method is a unified proof (Thm. 2.4) of the theorem of Blass and Sagan (1998a).

§3: "The Shi arrangements": these represent $\mathrm{Lat}^{\mathrm{b}}\{0,1\}\vec{K}_n$ and signed-graph analogs. §4: "The Linial arrangement": this represents $\mathrm{Lat}^{\mathrm{b}}\{1\}\vec{K}_n$. §5: "Other interesting hyperplane arrangements", treats: the arrangement representing $\mathrm{Lat}^{\mathrm{b}}\,AK_n$ where $A = \{-m,\ldots,m-1,m\}$ [which is the semilattice of m-composed partitions; see Zaslavsky (2002a), Ex. 10.5, also Edelman and Reiner (1996a)], and several generalizations, including to arbitrary sign-symmetric gain sets L and to Weyl analogs; also, an antibalanced analog of the A_n Shi arrangement (Thm. 5.4); and more. Most impressive result: Thm. 5.2: Let A be a finite set of integers such that $0 \notin A = -A$ and let $A^0 = A \cup \{0\}$. For $\Phi = A^0K_n$ and large integral λ, $\chi^*_\Phi(\lambda)/\lambda$ is the coefficient of $x^{\lambda-n}$ in $(1-x)^{-1} - f_A(x)/x$ where f_A is the ordinary generating function for A. From this $\chi^*_{AK_n}(\lambda)/\lambda$ is derived.

[The signed affinographic arrangements represent a kind of signed gain graph whose exact nature has not yet been penetrated by gain graph theory.] **(sg, gg: Geom, M, Invar)**

1997a A class of labeled posets and the Shi arrangement of hyperplanes. *J. Combin. Theory Ser. A* 80 (1997), 158–162. MR 98d:05008. Zbl 970.66662.

The arrangement represents $\mathrm{Lat}^{\mathrm{b}}\{0,1\}\vec{K}_n$. **(gg: Geom, M, Invar)**

1998a On free deformations of the braid arrangement. *European J. Combin.* 19 (1998), 7–18. MR 99d:52008. Zbl 898.52008.

The arrangements considered are the subarrangements of the projectivized Shi arrangements of type A_{n-1} that contain A_{n-1}. Thms. 4.1 and 4.2 characterize those that are free or supersolvable. The extended Shi arrangements, representing $L_0([1-a,a]\vec{K}_n)$ where $a \geq 1$, and a mild generalization, are of use in the proof. **(gg: Geom, M, Invar)**

1998b On noncrossing and nonnesting partitions for classical reflection groups. *Electronic J. Combin.* 5 (1998), Research Paper R42, 16 pp. MR 1644234 (99i:05204). Zbl 898.05004.

§5, "Nonnesting partitions of fixed type", has calculations like those in (1996a) for affinographic arrangements representing additional types of gain graphs [of a kind that is not yet fully understood]. **(gg: Geom, m, Invar)**

1999a Extended Linial hyperplane arrangements for root systems and a conjecture of Postnikov and Stanley. *J. Algebraic Combin.* 10 (1999), 207–225. MR 2000i:52039. Zbl 948.52012. **(gg: Geom, m, Invar)**

1999b Piles of cubes, monotone path polytopes, and hyperplane arrangements. *Discrete Comput. Geom.* 21 (1999), no. 1, 117–130. MR 99j:52015. Zbl 979.52002.

The proof of Proposition 4.2 is essentially gain-graphic. **(gg: m: Geom: Invar)**

2000a Deformations of Coxeter hyperplane arrangements and their characteristic polynomials. In: Michael Falk and Hiroaki Terao, eds., *Arrangements—Tokyo, 1998*, pp. 1–26. Adv. Studies Pure Math., 27. Kinokuniya, for the Mathematical Soc. of Japan, Tokyo, 2000. MR 1796891 (2001i:52035). Zbl 976.32016. **(gg: Geom, m, Invar)**

2004a Generalized Catalan numbers, Weyl groups and arrangements of hyperplanes. *Bull. London Math. Soc.* 36 (2004), 294–302. MR 2005b:52055. Zbl 1068.20038. **(gg: Geom: Gen: Invar)**

2004b On a refinement of the generalized Catalan numbers for Weyl groups. *Trans. Amer. Math. Soc.* 357 (2004), no. 1, 179–196. MR 2005h:20091. Zbl 1079.20057. **(gg: Geom: Gen: Invar)**

Christos A. Athanasiadis and Svante Linusson

1999a A simple bijection for the regions of the Shi arrangement of hyperplanes. *Discrete Math.* 204 (1999), 27–39. MR 2000f:52031. Zbl 959.52019. **(gg: Geom)**

David Avis

See J. Akiyama.

F. Ayoobi, G.R. Omidi, and B. Tayfeh-Rezaie

2011a A note on graphs whose signless Laplacian has three distinct eigenvalues. *Linear Multilinear Algebra* 59 (2011), no. 6, 701–706. MR 2801363 (2012i:05158). Zbl 1223.05169. **(Par: Adj)**

L. Babai and P.J. Cameron

2000a Automorphisms and enumeration of switching classes of tournaments. *Electronic J. Combin.* 7 (2000), Research Paper R38, 25 pp. MR 1773295 (2001h:-05048). Zbl 956.05050.

Tournaments are treated as nowhere-zero $\mathrm{GF}(3)^+$-gain graphs based on K_n; "switching" is by negation in $\mathrm{GF}(3)^+$. (**gg: Sw, Aut, Enum**)

Maxim A. Babenko

2006a Acyclic bidirected and skew-symmetric graphs: algorithms and structure. In: Dima Grigoriev, John Harrison and Edward A. Hirsch, eds., *Computer Science—Theory and Applications* (Proc. 1st Int. Symp. Computer Sci. in Russia, CSR 2006, St. Petersburg, 2006), pp. 23–34. Lecture Notes in Comput. Sci., 3967. Springer, Berlin, 2006. MR 2260979 (2007f:05165). Zbl 1185.05133.

"Skew-symmetric graph" = double covering digraph of a bidirected $-\Gamma$. "Weak acyclicity": No positive dicycle. "Strong acyclicity": No positive closed diwalk. Algorithm to test for weak acyclicity. Construction of weakly acyclic graphs from strongly acyclic ones. [Annot. 9 Sept 2010.] (**sg: Ori: Str, Cov, Alg**)

2006b On flows in simple bidirected and skew-symmetric networks. (In Russian.) *Problemy Peredachi Informatsii* 42 (2006), no. 4, 104–120. English trans. *Probl. Inf. Transm.* 42 (2006), no. 4, 356–370. MR 2278815 (2008i:90013).

$O(mn^{2/3})$ algorithm for integral max flow (improving on Gabow 1983a), showing that max flow takes no longer on a bidirected graph than on a digraph. The time bound follows from an upper bound on the max flow value. Also, an acyclic flow of value v is zero on all but $O(nv^{1/2})$ arcs. The technique involves transferring the flow to the double covering digraph. [Annot. 9 Sept 2010.] (**sg: Ori: Flows, Alg, Cov**)

2007a On an application of the structural theory of acyclic skew-symmetric digraphs. (In Russian.) *Vestnik Moskov. Univ. Ser. I Mat. Mekh.* (2007), no. 2, 65–66, 80. English trans. *Moscow Univ. Math. Bull.* 62 (2007), no. 2, 85–86. MR 2357046 (2008i:05146). Zbl 1164.05056.

The double covering graph of suitably oriented $-\Gamma$ [matching edges are introverted; nonmatching edges are extraverted] yields a proof that, if Γ has a unique perfect matching M, then M contains an isthmus. [Annot. 9 Sept 2010.] (**par: Ori**)

Maxim A. Babenko and Alexander V. Karzanov

2007a Free multiflows in bidirected and skew-symmetric graphs. *Discrete Appl. Math.* 155 (2007), 1715–1730. MR 2348356 (2008j:90102). Zbl 1152.90574.

Optimization of integral odd-vertex flows on a bidirected graph, without or with capacities. [Annot. 9 Sept 2010.] (**sg: Ori: Flows: Alg**)

2009a Minimum mean cycle problem in bidirected and skew-symmetric graphs. *Discrete Optimization* 6 (2009), no. 1, 92–97. MR 2483322 (2010a:05105). Zbl 1161.05327.

Minimizing the average weight in a cycle, or a closed trail, of an edge-weighted bidirected graph, in time $O(n^2 \min\{n^2, m \log n\})$. [Annot. 9 Sept 2010.] (**sg: Ori: Alg**)

Constantin P. Bachas

1984a Computer-intractibility of the frustration model of a spin glass. *J. Phys. A* 17 (1984), L709–L712. MR 85j:82043.

The frustration index decision problem on signed (3-dimensional) cubic lattice graphs is NP-complete. [Proof is incomplete; completed and improved by Green (1987a). Better result in Barahona (1982a).]
(**SG: Fr: Alg**)

G. David Bailey

20xxa Inductively factored signed-graphic arrangements of hyperplanes. Submitted and under revision.
Continues Edelman and Reiner (1994a). (**SG: Geom, M**)

V. Balachandran

1976a An integer generalized transportation model for optimal job assignment in computer networks. *Operations Res.* 24 (1976), 742–759. MR 55 #12068. Zbl 356.90028. (**GN: M(bases)**)

V. Balachandran and G.L. Thompson

1975a An operator theory of parametric programming for the generalized transportation problem: I. Basic theory. II. Rim, cost and bound operators. III. Weight operators. IV. Global operators. *Naval Res. Logistics Quart.* 22 (1975), 79–100, 101–125, 297–315, 317–339. MR 52 ##2595, 2596, 2597, 2598. Zbl 331.90048, 90049, 90050, 90051. (**GN: M**)

R. Balakrishnan and K. Ranganathan

2000a *A Textbook of Graph Theory.* Springer, New York, 2000. MR 2000j:05001. Zbl 938.05001.
§10.6, "Application to social psychology": Short introduction to balance in signed graphs. §10.7: Exercises on balance. (**SG: Bal: Exp**)

R. Balakrishnan and N. Sudharsanam

1982a Cycle-vanishing edge valuations of a graph. *Indian J. Pure Appl. Math.* 13 (1982), no. 3, 313–316. MR 657670 (84d:05145). Zbl 485.05057.
$f : E(\Gamma) \to \mathbb{R}$ is "cycle-vanishing" if $f(C) := \sum_{e \in C} f(e) = 0$ for every circle. Thm. 3: f is cycle-vanishing iff $f(S) = 0$ for every series class of non-isthmus edges. Thm. 4: $\dim\{\text{cycle-vanishing } f\} = |E| -$ number of series classes of non-isthmus edges. Thm. 5: Connected Γ is 3-connected iff only $f = 0$ is cycle vanishing. [Specialized to a sign-weighted graph Σ, "cycle-vanishing" means $|E^+(C)| = |E^-(C)|$ for every circle. Thm. 3: σ is cycle-vanishing iff every series class of non-isthmus edges has evenly many edges, half positive and half negative. Etc. no. Vijayakumar (2011a).] [Annot. 16 Oct 2011.] (**sgw: Gen**)

Egon Balas

1966a The dual method for the generalized transportation problem. *Management Sci.* 12 (1966), no. 7 (March, 1966), 555–568. MR 32 #7232. Zbl 142, 166 (e: 142.16601). (**GN: M(bases)**)

1981a Integer and fractional matchings. In: P. Hansen, ed., *Studies on Graphs and Discrete Programming*, pp. 1–13. North-Holland Math. Stud., 59. Ann. Discrete Math., 11. North-Holland, Amsterdam, 1981. MR 84h:90084.
Linear (thus "fractional", meaning half-integral) vs. integral programming solutions to maximum matching. The difference of their maxima = $\frac{1}{2}$(max number of matching-separable vertex-disjoint odd circles). Also noted (p. 12): (max) fractional matchings in Γ correspond to (max) matchings in the double covering graph of $-\Gamma$. [*Question.* Does this lead to a definition of maximum matchings in signed graphs?]

(par, ori: Incid, Geom, Alg, cov)

E. Balas and P.L. Ivanescu [P.L. Hammer]

1965a On the generalized transportation problem. *Management Sci.* 11 (1965), no. 1 (Sept., 1964), 188–202. MR 30 #4599. Zbl 133, 425 (e: 133.42505).

(GN: M, Bal)

K. Balasubramanian

1988a Computer generation of characteristic polynomials of edge-weighted graphs, heterographs, and directed graphs. *J. Computational Chem.* 9 (1988), 204–211.

Here a "signed graph" means, in effect, an acyclically oriented graph D along with the antisymmetric adjacency matrix $A_{\pm}(D) = A(+D \cup -D^{-1})$, D^{-1} being the converse digraph. [That is, $A_{\pm}(D) = A(D) - A(D)^{\mathrm{T}}$. The "signed graphs" are just acyclic digraphs with an antisymmetric adjacency matrix and, correspondingly, what we may call the 'antisymmetric characteristic polynomial'.] Proposes an algorithm for the polynomial. Observes in some examples a relationship between the characteristic polynomial of Γ and the antisymmetric characteristic polynomial of an acyclic orientation.

(SD, wg: Adj: Invar: Alg, Chem)

1991a Comments on the characteristic polynomial of a graph. *J. Computational Chem.* 12 (1991), 248–253. MR 92b:92057.

Argues (heuristically) that a certain algorithm is superior to another, in particular for the antisymmetric polynomial defined in (1988a).

(SD: Adj: Invar: Alg)

1992a Characteristic polynomials of fullerene cages. *Chem. Phys. Letters* 198 (1992), 577–586.

Computed for graphs of six different cages of three different orders, in both ordinary and "signed" (see (1988a)) versions. Observes a property of the "signed graph" polynomials [which is due to antisymmetry, as explained by P.W. Fowler (Comment on "Characteristic polynomials of fullerene cages". *Chem. Phys. Letters* 203 (1993), 611–612)].

(SD: Adj: Invar: Chem)

1994a Are there signed cospectral graphs? *J. Chem. Information Computer Sci.* 34 (1994), 1103–1104.

The "signed graphs" are as in (1988a). Simplified contents: It is shown by example that the antisymmetric characteristic polynomials of two nonisomorphic acyclic orientations of a graph (see (1988a)) may be equal or unequal. [Much smaller examples are provided by P.W. Fowler, Comment on "Characteristic polynomials of fullerene cages". *Chem. Phys. Letters* 203 (1993), 611–612).] [*Question.* Are there examples for which the underlying (di)graphs are nonisomorphic?] [For cospectrality of other kinds of signed graphs, see Acharya, Gill, and Patwardhan (1984a) (signed K_n's).] **(SD: Adj: Invar)**

R. Balian, J.M. Drouffe, and C. Itzykson

1974a Gauge fields on a lattice. I. General model. *Phys. Rev. D* 10 (1974), no. 10, 3376-3395.

Gain group SO(n) on a toroidal lattice graph (§ C, "Local invariance, gauge field, and minimal coupling), where SO(1) = $\{+1, -1\}$ (developed in (1975a)). [Annot. 12 Aug 2012.] (**SG: Phys**)

1975a Gauge fields on a lattice. II. Gauge-invariant Ising model. *Phys. Rev. D* 11 (1975), no. 8, 2098–2103.

Dictionary: "Ising model" = signed hypercubical lattice, "gauge invariance" = switching invariance, "plaquette" = quadrilateral. The partition function depends on $p^{+} - p^{-}$ where $p^{\varepsilon} = \#$ plaquettes with sign ε and sometimes also $|E^{+}| - |E^{-}|$. [Annot. 12 Aug 2012.] (**SG: Phys, Sw, Fr**)

M.L. Balinski

1970a On recent developments in integer programming. *Proceedings of the Princeton Symposium on Mathematical Programming* (Princeton Univ., 1967), pp. 267–302. Princeton Univ. Press, Princeton, N.J., 1970. MR 55 #9957. Zbl 222.90036.

Pp. 277–278 discuss integer programming problems on bidirected graphs in terms of the incidence matrix. (**ori: incid: par, Alg, Ref**)

Murad Banaji

See also N. Radde.

2010a Graph-theoretic conditions for injectivity of functions on rectangular domains. *J. Math. Anal. Appl.* 370 (2010), 302–311. MR 2651147 (2011f:26012). Zbl 1227.26006. (**SD**)

Murad Banaji and Gheorghe Craciun

2009a Graph-theoretic approaches to injectivity and multiple equilibria in systems of interacting elements. *Comm. Math. Sci.* 7 (2009), no. 4, 867–900. MR 2604624 (2011i:05126). Zbl 1195.05038. (**SG, Chem**)

2010a Graph-theoretic criteria for injectivity and unique equilibria in general chemical reaction systems. *Adv. Appl. Math.* 44 (2010), 168–184. MR 2576846 (2010m:80010). Zbl 1228.05204.

Generalization of (2009a) to more general systems. [Annot. 26 Oct 2011.] (**SG, Chem**)

20xxa Graph theoretic approaches to injectivity in chemical reaction systems. Submitted. (**SG, Chem**)

Murad Banaji and Carrie Rutherford

2011a *P*-matrices and signed digraphs. *Discrete Math.* 311 (2011), no. 4, 295–301. Zbl 1222.05080. (**SD: QM**)

Jørgen Bang-Jensen and Gregory Gutin

1997a Alternating cycles and paths in edge-coloured multigraphs: A survey. *Discrete Math.* 165/166 (1997), 39–60. MR 98d:05080. Zbl 876.05057.

A rich source for problems on bidirected graphs. An edge 2-coloration of a graph becomes an all-negative bidirection by taking one color class to consist of introverted edges and the other to consist of extroverted edges. An alternating path becomes a coherent path; an alternating circle becomes a coherent circle. [*General Problem.* Generalize to bidirected graphs the results on edge 2-colored graphs mentioned in this paper. (See esp. §5.) *Question.* To what digraph properties do they specialize by taking the underlying signed graph to be all positive?] [See e.g. Bánkfalvi and Bánkfalvi (1968a) (*q.v.*), Bang-Jensen and Gutin (1998a), Das and Rao (1983a), Grossman and Häggqvist (1983a), Mahadev and Peled (1995a), Saad (1996a).] (**par: ori: Paths, Circles**)

1998a Alternating cycles and trails in 2-edge-colored complete multigraphs. *Discrete Math.* 188 (1998), 61–72. MR 99g:05072. Zbl 956.05040.

The longest coherent trail, having degrees bounded by a specified degree vector, in a bidirected all-negative complete multigraph that satisfies an extra hypothesis. Generalization of Das and Rao (1983a) and Saad (1996a), thus ultimately of Thm. 1 of Bánkfalvi and Bánkfalvi (1968a) (*q.v.*). Also, a polynomial-time algorithm. (**par: ori: Paths, Alg**)

M. Bánkfalvi and Zs. Bánkfalvi

1968a Alternating Hamiltonian circuit in two-coloured complete graphs. In: P. Erdős and G. Katona, eds., *Theory of Graphs* (Proc. Colloq., Tihany, 1966), pp. 11–18. Academic Press, New York, 1968. MR 38 #2052. Zbl 159, 542 (e: 159.54202).

Let B be a bidirected $-K_{2n}$ which has a coherent 2-factor. ("Coherent" means that, at each vertex in the 2-factor, one edge is directed inward and the other outward.) Thm. 1: B has a coherent Hamiltonian circle iff, for every $k \in \{2, 3, \dots, n-2\}$, $s_k > k^2$, where $s_k :=$ the sum of the k smallest indegrees and the k smallest outdegrees. Thm. 2: The number of k's for which $s_k = k^2$ equals the smallest number p of circles in any coherent 2-factor of B. Moreover, the p values of k for which equality holds imply a partition of V into p vertex sets, each inducing B_i consisting of a bipartite [i.e., balanced] subgraph with a coherent Hamiltonian circle and in one color class only introverted edges, while in the other only extroverted edges. [*Problem.* Generalize these remarkable results to an arbitrary bidirected complete graph. The all-negative case will be these theorems; the all-positive case will give the smallest number of cycles in a covering by vertex-disjoint cycles of a tournament that has any such covering.] [See Bang-Jensen and Gutin (1997a) for further developments on alternating walks.] (**par: ori: Circles**)

Zs. Bánkfalvi

See M. Bánkfalvi.

C. Bankwitz

1930a Über die Torsionszahlen der alternierenden Knotes. *Math. Ann.* 103 (1930), 145–161.

Introduces the sign-colored graph of a link diagram. [Further work by numerous writers, e.g., S. Kinoshita *et al.* and esp. Kauffman (1989a) and successors.] (**Knot: SGc**)

Nikhil Bansal, Avrim Blum, and Shuchi Chawla

2002a Correlation clustering. In: *Proc. 43rd Ann. IEEE Sympos. Foundations of Computer Science (FOCS '02)*, pp. 238–247. Zbl 1089.68085.

Preliminary version of (2004a). (**SG: KG: Clu: Alg**)

2004a Correlation clustering. Theoretical Advances in Data Clustering. *Machine Learning* 56 (2004), no. 1–3, 89–113. Zbl 1089.68085.

Clusterability index Q [minimum number of inconsistent edges; see Doreian and Mrvar (1996a) for notation] in signed complete graphs is NP-hard. Polynomial-time algorithms for approximate optimal clustering: up to a constant factor from Q (§3); probably within $1-\varepsilon$ of $|E|-Q$ for any ε (i.e., maximizing consistent edges within $1-\varepsilon$) (§4). §3: A 2-clustering within $3Q_2$ (Thm. 2). A clustering within cQ where $c \approx 20000$ (Thm. 13). §4: A clustering within εn^2 of $|E|-Q$ with high probability but slow in terms of $1/\varepsilon$ (Thm. 15). Asymptotically faster in terms of $1/\varepsilon$ (Thm. 22). The $1-\varepsilon$ factor results from the fact that $|E| - Q = \binom{n}{2} - Q > \frac{1}{2}\binom{n}{2}$ [so is not strong]. §6: "Random noise". §7: "Extensions", considers edge weights in $[-1, 1]$ (thus allowing incomplete graphs). Thm. 23: An unweighted approximation algorithm will also approximate this case, assuming "linear cost": e costs $(1-w(e))/2$ if within a cluster and $(1+w(e))/2$ if between clusters. Thm. 24: The problem for clustering that minimizes the total weight of $+$ edges outside clusters and $-$ edges within clusters ("minimizing disagreements") is APX-hard. [Improved in Charikar, Guruswami, and Wirth (2003a, 205a), Swamy (2004a). Generalized in Demaine *et al.* (2006a).] [Annot. 22 Sept 2009.] (**SG: KG: Clu: Alg**)

R.B. Bapat

2010a *Graphs and Matrices.* Hindustan Book Agency, New Delhi, and Springer, London, 2010.

§2.6, "$0-1$ Incidence matrix". The rank and related properties of the the unoriented incidence matrix. [*Cf.* van Nuffelen (1973a).] [Annot. 25 Aug 2011.] (**sg: Par: Incid: Exp**)

Ravindra B. Bapat, Jerrold W. Grossman, and Devadatta M. Kulkarni

1999a Generalized matrix tree theorem for mixed graphs. *Linear Multilinear Algebra* 46 (1999), 299–312. MR 2001c:05091. Zbl 940.05042.

Their "mixed graph" is a signed graph Σ: positive edges are called "directed" and negative edges "undirected". The matrix-tree theorem is the unweighted case of Chaiken's (1982a) all-minors theorem for signed graphs. The technical formalism differs somewhat. They point out that in case $U \cup W = V$, the minor is the sum of signed $\bar{U}\bar{W}$ matchings. Dictionary: "k-reduced substructure" $\cong$ independent set of rank $n-k$ in $G(\Sigma)$; "quasibipartite" = balanced. Successor to Grossman, Kulkarni, and Schochetman (1994a) [*q.v.* for more dictionary]. (**sg: Incid**)

2000a Edge version of the matrix tree theorem for trees. *Linear Multilinear Algebra* 47 (2000), 217–229. MR 1785029 (2001d:05112). Zbl 960.05067.

Successor to (1999a). Their "mixed tree" T is a signed tree as in (1999a). Thm. 9 (simplified): The minor of $\mathrm{H}^{\mathrm{T}}\mathrm{H}$ (H is the incidence matrix of Σ) obtained by deleting rows corresponding to $E \subseteq E(\Sigma)$ and columns corresponding to $F \subseteq E(\Sigma)$ has determinant equal, up to sign, to the number of common SDR's of vertex sets of components of $T \setminus E$ and $T \setminus F$. [Interesting, but edge signs are irrelevant because any tree switches to all positive.] Dictionary: "substructure" = subgraph allowing retention of edges incident to deleted vertices [thus they become loose or half edges]. [See (1999a) for more dictionary.] (**sg: Incid**)

R.B. Bapat, D. Kalita, and S. Pati

2012a On weighted directed graphs. *Linear Algebra Appl.* 436 (2012), no. 1, 99–111.

They are complex unit gain graphs Φ with simple underlying graph. $K(\Phi)$ is obtained in the usual way from $\mathrm{H}(\Phi)$. §2, "D-similarity and singularity in weighted directed graphs": Thm. 8: $K(\Phi)$ is singular iff $\Phi \sim \|\Phi\|$ iff Φ (assumed connected) is balanced. [*Cf.* Zaslavsky (2003b), §2.1 esp. Thm. 2.1(a), noting that $\operatorname{rk} K(\Phi) = \operatorname{rk} \mathrm{H}(\Phi) = \operatorname{rk} G(\Phi)$.] §3, "Edge singularity of weighted directed graphs": Elementary results on frustration index, appearing less elementary because treated indirectly, through eigenvalues, rather than directly, through the graph. Generalizing Tan and Fan (2008a) on signed graphs. §4, "3-Colored digraphs and their singularity": Gains restricted to $\pm 1, i$. Elementary results. Dictionary: "weighted directed graph" = complex unit gain graph; "mixed graph" = signed graph; D-similarity" [diagonal similarity] = switching equivalence, "edge singularity" = frustration index. [Annot. 28 Oct 2011.] (**gg: Adj, Incid, Bal**)

R.B. Bapat and Devadatta M. Kulkarni

2000a Minors of some matrices associated with a tree. In: *Algebra and Its Applications* (Athens, Ohio, 1999), pp. 45–66. Contemp. Math., Vol. 259. American Math. Soc., Providence, R.I., 2000. MR 2001h:05065. Zbl 979.05075.

Concerns a "mixed tree", really an oriented signed tree without extroverted edges (see Bapat, Grossman, and Kulkarni 1999a). The matrices are the incidence matrix H, the Laplacian (i.e., Kirchhoff) matrix $\mathrm{H}\,\mathrm{H}^{\mathrm{T}}$, and the "edge Laplacian" $\mathrm{H}^{\mathrm{T}}\mathrm{H}$. Partly expository. New results concern Moore–Penrose inverses and their minor determinants. [Since a "mixed tree" is switching equivalent to an ordinary unsigned tree, their results should be identical to those for ordinary trees except for multiplication by a $V \times V$ diagonal matrix with signs on the diagonal.] (**sg: Incid**)

Nadav S. Bar

See N. Radde.

Francisco Barahona

1981a Balancing signed toroidal graphs in polynomial-time. Unpublished manuscript, 1981.

Given a 2-connected Σ whose underlying graph is toroidal, polynomial-time algorithms are given for calculating the frustration index $l(\Sigma)$ and the generating function of switchings Σ^μ by $|E^-(\Sigma^\mu)|$. The technique is to solve a Chinese postman (T-join) problem in the toroidal dual graph, T corresponding to the frustrated face boundaries. Generalizes (1982a). [See (1990a), p. 4, for a partial description.] (**SG: Fr, Alg**)

1982a On the computational complexity of Ising spin glass models. *J. Phys. A: Math. Gen.* 15 (1982), 3241–3253. MR 84c:82022.

The frustration-index problem, that is, minimization of $|E^-(\Sigma^\zeta)|$ over all switching functions $\zeta : V \to \{\pm 1\}$, for signed planar and toroidal graphs and subgraphs of 3-dimensional grids. Analyzed structurally, in terms of perfect matchings in a modified dual graph, and algorithmically. The last is NP-hard, even when the grid has only 2 levels; the former are polynomial-time solvable even with weighted edges. Also, the problem of minimizing $|E^-(\Sigma^\zeta)| + \sum_v \zeta(v)$ for planar grids ("2-dimensional problem with external magnetic field"), which is NP-hard. (This corresponds to adding an extra vertex, positively adjacent to every vertex.) [See infinite analog in Istrail (2000a).] (**SG: Phys, Fr, Fr(Gen): D, Alg**)

1982b Two theorems in planar graphs. Unpublished manuscript, 1982. (**SG: Fr**)

1990a On some applications of the Chinese Postman Problem. In: B. Korte, L. Lovász, H.J. Prömel, and A. Schrijver, eds., *Paths, Flows and VLSI-Layout*, pp. 1–16. Algorithms and Combinatorics, Vol. 9. Springer-Verlag, Berlin, 1990. MR 92b:90139. Zbl 732.90086.

§2: "Spin glasses." (**SG: Phys, Fr: Exp**)

§5: "Max cut in graphs not contractible to K_5," pp. 12–13.
(sg: fr: Exp)

1990b Planar multicommodity flows, max cut, and the Chinese Postman Problem. In: William Cook and Paul D. Seymour, eds., *Polyhedral Combinatorics* (Proc. Workshop, 1989), pp. 189–202. DIMACS Ser. Discrete Math. Theor. Computer Sci., Vol. 1. Amer. Math. Soc. and Assoc. Comput. Mach., Providence, R.I., 1990. MR 92g:05165. Zbl 747.05067.

Negative cutsets, where signs come from a network with real-valued capacities. Dual in the plane to negative circles. See §2.
(SG: D: Bal, Alg)

Francisco Barahona and Adolfo Casari

1988a On the magnetisation of the ground states in two-dimensional Ising spin glasses. *Comput. Phys. Comm.* 49 (1988), 417–421. MR 89d:82004. Zbl 814.90132.
(SG: Fr: Alg)

Francisco Barahona, Martin Grötschel, Michael Jünger, and Gerhard Reinelt

1988a An application of combinatorial optimization to statistical physics and circuit layout design. *Oper. Res.* 36 (1988), no. 3, 493–513. Zbl 646.90084.

Frustration index of a weighted signed graph (Ising ground state; via minimum) is reduced to weighted max-cut. The algorithm uses cutting planes on the cut polytope of the underlying graph, specifically applied to toroidal grids with an extra vertex. Fractional solutions appear occasionally, especially for signed graphs. Possible use of negative-circle constraints is mentioned. [Annot. 18 Aug 2012.] **(sg: Fr: Alg)**

Francisco Barahona, Martin Grötschel, and Ali Ridha Mahjoub

1985a Facets of the bipartite subgraph polytope. *Math. Operations Res.* 10 (1985), 340–358. MR 87a:05123a. Zbl 578.05056.

The polytope $P_B(\Gamma)$ is the convex hull in $\mathbb{R}^E$ of characteristic vectors of bipartite edge sets. Various types of and techniques for generating facet-defining inequalities, thus partially extending the description of $P_B(\Gamma)$ from the weakly bipartite case (Grötschel and Pulleyblank (1981a)) in which all facets are due to edge and odd-circle constraints. [Some can be described best via signed graphs; see Poljak and Turzík (1987a).] [A brief expository treatment of the polytope appears in Poljak and Tuza (1995a).] **(sg: par: fr: Geom)**

Francisco Barahona and Enzo Maccioni

1982a On the exact ground states of three-dimensional Ising spin glasses. *J. Phys. A: Math. Gen.* 15 (1982), L611–L615. MR 83k:82044.

Discusses a 3-dimensional analog of Barahona, Maynard, Rammal, and Uhry (1982a). There may not always be a combinatorial LP optimum; hence LP may not completely solve the problem. **(SG: Phys, Fr, Alg)**

Francisco Barahona and Ali Ridha Mahjoub

1986a On the cut polytope. *Math. Programming* 36 (1986), 157–173. MR 866986 (88d:05049). Zbl 616.90058.

Call $P_{\mathrm{BS}}(\Sigma)$ the convex hull in $\mathbb{R}^E$ of characteristic vectors of negation sets (or "balancing [edge] sets") in Σ. Finding a minimum-weight negation set in Σ corresponds to a maximum cut problem, whence $P_{\mathrm{BS}}(\Sigma)$ is a linear transform of the cut polytope $P_{\mathrm{C}}(|\Sigma|)$, the convex hull of cuts.

Conclusions follow about facet-defining inequalities of $P_{\mathrm{BS}}(\Sigma)$. See §5: "Signed graphs". (**SG: Fr: Geom**)

1989a Facets of the balanced (acyclic) induced subgraph polytope. *Math. Programming Ser. B* 45 (1989), 21–33. MR 1017209 (91c:05178). Zbl 675.90071.

The "balanced induced subgraph polytope" $P_{\mathrm{BIS}}(\Sigma)$ is the convex hull in $\mathbb{R}^V$ of incidence vectors of vertex sets that induce balanced subgraphs. Conditions are studied under which certain inequalities of form $\sum_{i \in Y} x_i \leq f(Y)$ define facets of this polytope: in particular, $f(Y) =$ max. size of balance-inducing subets of Y, $f(Y) = 1$ or 2, $f(Y) = |Y| - 1$ when $Y = V(C)$ for a negative circle C, etc. (**SG: Fr: Geom, Alg**)

1994a Compositions of graphs and polyhedra. I: Balanced induced subgraphs and acyclic subgraphs. *SIAM J. Discrete Math.* 7 (1994), 344–358. MR 95i:90056. Zbl 802.05067.

More on $P_{\mathrm{BIS}}(\Sigma)$ (see (1989a)). A balance-inducing vertex set in $\pm\Gamma =$ a stable set in Γ. [See Zaslavsky (1982b) for a different correspondence.] Thm. 2.1 is an interesting preparatory result: If $\Sigma = \Sigma_1 \cup \Sigma_2$ where $\Sigma_1 \cap \Sigma_2 \cong \pm K_k$, then $P_{\mathrm{BIS}}(\Sigma) = P_{\mathrm{BIS}}(\Sigma_1) \cap P_{\mathrm{BIS}}(\Sigma_2)$. The main result is Thm. 2.2: If Σ has a 2-separation into Σ_1 and Σ_2, the polytope is the projection of the intersection of polytopes associated with modifications of Σ_1 and Σ_2. §5: "Compositions of facets", derives the facets of $P_{\mathrm{BIS}}(\Sigma)$. (**SG: Geom, WG, Alg**)

F. Barahona, R. Maynard, R. Rammal, and J.P. Uhry

†1982a Morphology of ground states of two-dimensional frustration model. *J. Phys. A: Math. Gen.* 15 (1982), 673–699. MR 83c:82045.

Treats many important aspects of the quantity $l := \min_\zeta |E^-(\Sigma^\zeta)|$ [which equals the frustration index], over all switching functions ζ ("spin configurations σ" in the paper) of a signed graph, mainly a signed planar graph. ($|E^-(\Sigma^\zeta)|$ is the paper's $\frac{1}{2}(|E| + H)$, $H :=$ Hamiltonian.) They maximize $-H = W^+ + W^- - W^{+-}$ where $W^+ + W^- :=$ # unswitched positive edges − # unswitched negative edges and $W^{+-} :=$ # switched positive edges − # switched negative edges. Thus, $-H = |E^+| - |E^-| = |E| - 2|E^-|$ after switching. Maximizing it $\iff$ minimizing $|E^-|$ over all ζ.

§2: "The frustration model as the Chinese postman's problem", describes how to find l when $|\Sigma|$ is planar, by solving a Chinese postman (T-join) problem in the dual graph, T corresponding to the frustrated (i.e., negative) face boundaries. The postman problem is solved by linear programming. [Solved independently by Katai and Iwai (1978a).] [Barahona (1981a) generalizes to signed toroidal graphs.]

§3: "Solution of the frustration problem by duality: rigidity". An edge is "rigid" if it has the same sign in every Σ^ζ that minimizes $|E|$ (such an ζ is a "ground state"). The endpoints of a rigid edge are called "solidary". Rigid edges are found via the dual linear program. The boundary contours of connected sets of frustrated faces play an important role.

§§4–5: "Numerical experimentation" and "Results", for a randomly signed square lattice graph. The proportion x of negative edges strongly affects the properties; esp., there is significant long-range order below but not above $x \approx .15$. [See Deng and Abell (2010a) for numerical results on random signed graphs.]

More general problems discussed are (1) allowing positive edge weights (due to variable bond strengths); (2) minimizing $|E^-(\Sigma^\zeta) + c\sum_V \zeta(v)$, with $c \neq 0$ because of an external magnetic field. Then one cannot expect the LP to have a combinatorial optimum. [Annot. 20 Jan 2010.]
(**SG: Phys, Fr, Fr(Gen), Alg**)

F. Barahona and J.P. Uhry

1981a An application of combinatorial optimization to physics. *Methods Operations Res.* 40 (1981), 221–224. Zbl 461.90080. (**SG: Phys, Fr: Exp**)

J. Wesley Barnes

See P.A. Jensen.

Lowell Bassett, John Maybee, and James Quirk

1968a Qualitative economics and the scope of the correspondence principle. *Econometrica* 36 (1968), 544–563. MR 38 #5456. Zbl (e: 217.26802).

Lemma 3: A square matrix with every diagonal entry negative is sign-nonsingular iff every cycle is negative in the associated signed digraph. Thm. 4: A square matrix with negative diagonal is sign-invertible iff all cycles are negative and the sign of any (open) path is determined by its endpoints. And more. (**QM: QSol, QSta: sd**)

Vladimir Batagelj

See also P. Doreian and W. de Nooy.

1990a [Closure of the graph value matrix.] (In Slovenian. English summary.) *Obzornik Mat. Fiz.* 37 (1990), 97–104. MR 91f:05058. Zbl 704.05035.
(**SG: Adj, Bal, Clu**)

1994a Semirings for social networks analysis. *J. Math. Sociology* 19 (1994), 53–68. Zbl 827.92029. (**SG: Adj, Bal, Clu**)

1997a Notes on blockmodeling. *Social Networks* 19 (1997), 143–155.

§3, p. 6: Predicates to use for searching out balanced or clusterable partitions. [Annot. 10 Mar 2011.] (**SG: PsS, Alg**)

V. Batagelj and T. Pisanski

1979a On partially directed Eulerian multigraphs. *Publ. Inst. Math. (Beograd) (N.S.)* 25(39) (1979), 16–24. MR 542818 (81a:05054). Zbl 418.05038. (**sg: Ori**)

Christian Bauckhage

See J. Kunegis.

Andrei Băutu and Elena Băutu

2007a Searching ground states of Ising spin glasses with particle swarms. *Rom. J. Phys.* 52 (2007), no. 3-4, 337–342.

Experimental results for $l(\Sigma)$ compared with known results. [Annot. 19 Aug 2012.] (**SG, Phys: Fr: Alg**)

2007b Searching ground states of Ising spin glasses with genetic algorithms and binary particle swarm optimization. In: Natalio Krasnogor *et al.*, eds., *Nature Inspired Cooperative Strategies for Optimization* (NICSO 2007, Int. Workshop, Acireale, Italy), pp. 85–94. Stud. Comput. Intelligence, Vol. 129. Springer, Berlin, 2008.

Compares the two algorithms for $l(\Sigma)$. [Annot. 19 Aug 2012.]
(**SG, Phys: Fr: Alg**)

2009a Particle swarms in statistical physics. In: Aleksandar Lazinica, ed., *Particle Swarm Optimization*, Ch. 4, pp. 77–88. InTech, Rijeka, Croatia, and Shanghai, 2009.

§4, “Binary particle swarm optimization and Ising spin glasses”: The signed graph; spins and states; satisfied and frustrated edges; some history. In particle swarm optimization, each vertex acts as a cell in a cellular automaton, learning probabilistically, seeking a most satisfied spin $\zeta(v)$ in order to minimize $|E_v^-(\Sigma^\zeta)|$. [It seems that this local minimization suffers from the same potential instability as Mitra’s (1962a) deterministic local minimization.] [Annot. 19 Aug 2012.]

(**SG, Phys: Fr: Alg: Exp**)

Andrei Băutu, Elena Băutu, and Henri Luchian

2007a Particle swarm optimization hybrids for searching ground states of Ising spin glasses. In: Viorel Negru *et al.*, eds., *SYNASC 2007: Ninth International Symposium on Symbolic and Numeric Algorithms for Scientific Computing, 2007* (Timisoara, Romania), pp. 415–418. IEEE Computer Soc., Los Alamitos, Cal., 2007.

Particle swarm optimization combined with hill-climbing to find $l(\Sigma)$ (ground state of Ising model); a hybrid method is promising. [Annot. 19 Aug 2012.] (**SG, Phys: Fr: Alg**)

2008a Searching ground states of Ising spin glasses with a tree bond-based representation. In: Viorel Negru *et al.*, eds., *SYNASC 2008: 10th International Symposium on Symbolic and Numeric Algorithms for Scientific Computing* (Timisoara, Romania), pp. 501–506. IEEE Computer Soc., Los Alamitos, Cal., 2008.

Bond-based representation means recording switched edge (“bond”) signs instead of vertex spins; *cf.* Pelikan and Hartmann (2007a, 2008a). Here, a state $s : V \to \{+1, -1\}$ is recorded as the signs of a spanning tree switched by s. This has ambiguity [2, obviously]. Negating one tree edge implies a chain of spin changes; this “may be considered a feature” [and its implications could be interesting]. Computational experiments tested the implied algorithm. [Annot. 19 Aug 2012.] (**SG, Phys: Fr: Alg**)

Andrei Băutu and Henri Luchian

2010a Particle swarm optimization with spanning tree representation for Ising spin glasses. In: *2010 IEEE Congress on Evolutionary Computation* (CEC 2010, Barcelona), pp. 1–6. doi 10.1109/CEC.2010.5586473. IEEE, 2010.

Applies Băutu, Băutu, and Luchian (2008a). Shallower trees may produce better results due to the lesser effect of negating one tree edge. Computational comparisons of this and other algorithms for ground state (i.e., frustration index). [Annot. 19 Aug 2012.] (**SG, Phys: Fr: Alg**)

Elena Băutu

See A. Băutu.

Matthias Beck and Mela Hardin

20xxa A bivariate chromatic polynomial for signed graphs. Submitted. arXiv:1204.2568.

(**SG: Col, Geom**)

Matthias Beck and Thomas Zaslavsky

2006a Inside-out polytopes. *Advances in Math.* 205 (2006), no. 1, 134–162. MR 2007e:52017. Zbl 1107.52009. arXiv:math/0309330.

§5: “In which we color graphs and signed graphs.” A geometric interpretation of signed graph coloring by lattice points and hyperplane

arrangements unifies the chromatic and zero-free chromatic polynomials and gives immediate proofs of theorems on the chromatic polynomials and acyclic orientations. (**SG: Col: Geom, M: Invar, Bal**)

2006b The number of nowhere-zero flows in graphs and signed graphs. *J. Combin. Theory Ser. B* 96 (2006), no. 6, 901–918. MR 2007k:05084. Zbl 1119.05105. arXiv:math/0309331.

The nowhere-zero flow polynomial of a signed graph, for flows in an odd abelian group, and the integral nowhere-zero flow quasipolynomial with period 2. (**SG: Flows: Geom: M: Invar, Bal**)

2006c An enumerative geometry for magic and magilatin labellings. *Ann. Combin.* 10 (2006), no. 4, 395–413. MR 2007m:05010. Zbl 1116.05071. arXiv:math/-0506315.

In magic labellings of a bidirected graph, the labels are distinct positive integers; at each vertex the sum over entering edge ends equals that over departing edge ends. Thms. (implicit): The number of magic labellings is a quasipolynomial function of the magic sum, if the magic sum is prescribed. It is also a quasipolynomial function of the upper bound on the labels, if an upper bound is prescribed. (**ori: Geom, Enum**)

§5: "Generalized exclusions." Complementarity rules in magic squares, etc., can be expressed by signed-graphic hyperplanes.

(**sg: Geom, Enum**)

2010a Six little squares and how their numbers grow. *J. Integer Sequences* 13 (2010), Article 10.6.2, 43 pp. MR 2659218 (2011j:05052). Zbl 1230.05062. arXiv:1004.-0282.

§3: "Semimagic squares." Counts magic labellings of the extraverted $-K_{3,3}$ by an explicit geometrical solution. Counted either by upper bound on the values or by magic sum. (**par: incid, Geom**)

M. Behzad and G. Chartrand

1969a Line-coloring of signed graphs. *Elem. Math.* 24 (1969), 49–52. MR 39 #5415. Zbl 175, 503 (e: 175.50302).

Λ_{BC} Their line graph $\Lambda_{BC}(\Sigma)$ of a signed simple graph Σ (not defined explicitly) is $\Lambda(|\Sigma|)$ with an edge negative when its two endpoints are negative edges in Σ. They "color" as in Cartwright and Harary (1968a) (i.e., clustering). Characterized: Σ with colorable line graphs. Found: the fewest colors for line graphs of signed trees, K_n, and $K_{r,s}$. [For a more sophisticated kind of line graph see Vijayakumar (various) and Zaslavsky (1984c, 2010b, 20xxa). For another line graph, see M. Acharya (2009a).]

(**SG: lg: Clu**)

Lowell W. Beineke and Frank Harary

1966a [As "W. Beineke and F. Harary"] Binary matrices with equal determinant and permanent. *Studia Sci. Math. Hungar.* 1 (1966), 179–183. MR 34 #7397. Zbl 145, 15e (e: 145.01505). (**SD**)

1978a Consistency in marked digraphs. *J. Math. Psychology* 18 (1978), 260–269. MR 522390 (80d:05026). Zbl 398.05040.

A "marked digraph" is a digraph $\vec{D}$ with signed vertices, $\vec{S} = \vec{D}, \mu)$ where $\mu : V \to \{+, -\}$. It is "consistent" if all diwalks from v to w have the same sign $\mu(W)$. The sign of a walk is the vertex sign product.

Thm. 1. Asssuming $\vec{D}$ is strongly connected, $\vec{S}$ is consistent iff every dicycle is positive. [An important difference from signed graphs, where no restriction is needed.] Thm. 2. $\vec{S}$ is consistent iff V has a bipartition such that every arc with a positive tail lies within a set but no arc with a negative tail does so. Define $\sigma(\vec{uv} := \mu(u)$. Thm. 3. Asssuming $\vec{D}$ is strongly connected, this signed graph is balanced iff $\vec{S}$ is consistent. Thm. 4. A vertex-signed tournament $\vec{S}$ is consistent iff: When strongly connected, [it is all positive or] it has exactly two negative vertices u, v and, deleting uv, u is a source and v is a sink. When not strongly connected, it is consistent iff it is all positive, or it has one negative vertex which is a source or sink, or it has two negative vertices, one a source and the other a sink. Thm. 5. $\vec{D}$ has $\mu \not\equiv +$ such that $(\vec{D}, \mu)$ is consistent ("markable") iff $\exists\ \varnothing \subset V_0 \subset V$ such that, $\forall\ v$, all out-arcs from v, or none, go to V_0. [Annot. 16 Sept 2010.] **(VS)**

1978b Consistent graphs with signed points. *Riv. Mat. Sci. Econom. Social.* 1 (1978), 81–88. MR 573718 (81h:05108). Zbl 493.05053.

A graph (not necessarily simple) with signed vertices is "consistent" if every circle has positive sign product. Thm. 2.2: Γ with all negative vertices is consistent iff bipartite. Thm. 2.3: 3-connected vertices must have the same sign. Thm. 3.3: Contracting an edge with positive endpoints preserves consistency and inconsistency. Further partial results. Open problem: A full characterization of consistent vertex-signed graphs. [For a good solution see Hoede (1992a). For the best solution see Joglekar, Shah, and Diwan (2010a).] [Annot. Rev. 11 Sept 2010.] **(VS: Bal)**

Jacques Bélair, Sue Ann Campbell, and P. van den Driessche

1996a Frustration, stability, and delay-induced oscillations in a neural network model. *SIAM J. Appl. Math.* 56 (1996), 245–255. MR 96j:92003. Zbl 840.92003.

The signed digraph of a square matrix is "frustrated" if it has a negative cycle. Somewhat simplified: frustration is necessary for there to be oscillation caused by intraneuronal processing delay. **(SD: QM, Ref)**

Francesco Belardo

See also J.F. Wang.

Francesco Belardo, Enzo M. Li Marzi, Slobodan K. Simić, and Jianfeng Wang

2010a On the index of necklaces. *Graphs Combin.* 26 (2010), no. 2, 163–172. MR 2606492 (2011g:05171).

The largest eigenvalue of $A(G)$ for G = chain or necklace of cliques, via $K(-\Gamma)$ where $G = \Lambda(\Gamma)$. [Annot. 16 Jan 2012.] **(Par: Adj)**

2011a Graphs whose signless Laplacian spectral radius does not exceed the Hoffman limit value. *Linear Algebra Appl.* 435 (2011), no. 11, 2913–2920. **(Par: Adj)**

A. Bellacicco and V. Tulli

1996a Cluster identification in a signed graph by eigenvalue analysis. In: *Matrices and Graphs: Theory and Applications to Economics* (full title *Proceedings of the Conferences on Matrices and Graphs: Theory and Applications to Economics*) (Brescia, 1993, 1995), pp. 233–242. World Scientific, Singapore, 1996. MR 99h:00029 (book). Zbl 914.65146.

Signed (di)graphs ("spin graphs") are defined. The main concepts are "dissimilarity", "balance", and "cluster" are defined and propositions are stated. Eigenvalues are mentioned. [This may be an announcement. There are no proofs. It is hard to be sure what is being said.] (**SD: Adj**)

Joachim von Below

1994a The index of a periodic graph. *Results Math.* 25 (1994), 198–223. MR 95e:05081. Zbl 802.05054.

Here a periodic graph [of dimension m] is defined as a connected graph $\Gamma = \tilde{\Psi}$ where Ψ is a finite $\mathbb{Z}^m$-gain graph with gains contained in $\{\mathbf{0}, \mathbf{b}_i, \mathbf{b}_i - \mathbf{b}_j\}$. ($\mathbf{b}_1, \ldots, \mathbf{b}_m$ are the unit basis vectors of $\mathbb{Z}^m$.) Let us call such a Ψ a small-gain base graph for Γ. Any $\tilde{\Phi}$, where Φ is a finite $\mathbb{Z}^m$-gain graph, has a small-gain base graph Ψ; thus this definition is equivalent to that of Collatz (1978a). The "index" $I(\Gamma)$, analogous to the largest eigenvalue of a finite graph, is the spectral radius of $A(||\Psi||)$ (here written $A(\Gamma, N)$) for any small-gain base graph of Γ. The paper contains basic theory and the lower bound $L_m = \inf\{I(\Gamma) : \Gamma \text{ is } m\text{-dimensional}\}$, where $1 = L_1$, $\sqrt{9/2} = L_2 \leq L_3 \leq \cdots$. (**GG(Cov): Adj**)

Jean Bénabou

1996a Some geometric aspects of the calculus of fractions. European Colloq. Category Theory (Tours, 1994). *Appl. Categ. Structures* 4 (1996), 139–165. MR 97g:18007. Zbl 874.18007.

Morphisms of signed graphs are employed in category-theoretic constructions. (**SG**)

Radel Ben-Av

See D. Kandel.

Edward A. Bender and E. Rodney Canfield

1983a Enumeration of connected invariant graphs. *J. Combin. Theory Ser. B* 34 (1983), 268–278. MR 85b:05099. Zbl 532.05036.

§3: "Self-dual signed graphs," gives the number of n-vertex graphs that are signed, vertex-signed, or both; connected or not; self-isomorphic by reversing edge and/or vertex signs or not, for all $n \leq 12$. Some of this appeared in Harary, Palmer, Robinson, and Schwenk (1977a). (**SG, VS: Enum**)

Riccardo Benedetti

1998a A combinatorial approach to combings and framings of 3-manifolds. In: A. Balog, G.O.H. Katona, A. Recski, and D. Sa'sz, eds., *European Congress of Mathematics* (Budapest, 1996), Vol. I, pp. 52–63. Progress in Math., Vol. 168. Birkhäuser, Basel, 1998. MR 1645797 (2000e:57033). Zbl 905.57018.

§8, "Spin manifolds", hints at a use for decorated signed graphs in the structure theory of spin 3-manifolds. (**sg: Appl: Exp**)

Curtis Bennett and Bruce E. Sagan

1995a A generalization of semimodular supersolvable lattices. *J. Combin. Theory Ser. A* 72 (1995), 209–231. MR 96i:05180. Zbl 831.06003.

To illustrate the generalization, most of the article calculates the chromatic polynomial of $\pm K_n^{(k)}$ (called $\mathcal{DB}_{n,k}$; this has half edges at k vertices), builds an "atom decision tree" for $k = 0$, and describes and counts the bases of $G(\pm K_n^{(k)})$ (called $\mathcal{D}_n$) that contain no broken circuits. (**SG: M, Invar, col**)

M.K. Bennett, Kenneth P. Bogart, and Joseph E. Bonin

1994a The geometry of Dowling lattices. *Adv. Math.* 103 (1994), 131–161. MR 95b:05050. Zbl 814.51003.

Drawing an analogy between Desargues' and Pappus' theorems in projective spaces and similar incidence theorems in Dowling geometries. [The rigorous avoidance of gain graphs makes the results less obvious than they could be.] **(gg: M, Geom)**

Moussa Benoumhani

1996a On Whitney numbers of Dowling lattices. *Discrete Math.* 159 (1996), 13–33. MR 98a:06005. Zbl 861.05004.

Cf. Dowling (1973b). Generating functions and identities for Whitney numbers of the first and second kinds, analogous to usual treatments of Stirling numbers. §2, "Whitney numbers of the second kind": $W_m(n,k) := W_k(Q_n(\mathfrak{G})) = W_k(G(\mathfrak{G}K_n^\bullet))$ where $m = |\mathfrak{G}|$. E.g., Thm. 1: $\sum_n W_m(n,k)z^n/n! = [(e^{mz}-1)/m]^k e^z/k!$. Thm. 5: $\sum_n W_m(n,k)u^{n-k} = m^{k+1}/([1-u]/mu)_{k+1}$. §3, "Whitney numbers of the first kind": $w_m(n,k) := w_k(G(\mathfrak{G}K_n^\circ))$. E.g., Thm. 10: $\sum_n w_m(n,k)z^n/n! = (1+mz)^{-1/m}\ln^k(1+mz)/k!m^k$. Thm. 12 is a reciprocity relation between $w_m(n,k)$ and $s(n,k)$. §4, "The integers maximizing $W_m(n,k)$ and $w_m(n,k)$": Partial, complicated results. [Annot. 30 Apr 2012.] **(gg: M: Invar)**

1997a On some numbers related to Whitney numbers of Dowling lattices. *Adv. Appl. Math.* 19 (1997), 106–116. MR 98f:05004. Zbl 876.05001.

Continuation of (1996a). §2, "Dowling polynomials": $D_m(n,x) := \sum_k W_m(n,k)x^k$. Generating function, recurrence, infinite series expression. §3 similarly studies $F_{m,1}(x) := \sum_k k!m^k W_m(n,k)x^k$ and $F_{m,1}(x) := \sum_k k!W_m(n,k)x^k$. §4, "Log-concavity of $k!W_m(n,k)$". Deduced from real negativity of zeros. [See (1999a) for $W_m(n,k)$.] [Annot. 1 May 2012.] **(gg: M: Invar)**

1999a Log-concavity of Whitney numbers of Dowling lattices. *Adv. Appl. Math.* 22 (1999), 186–189. MR 2000i:05008. Zbl 918.05003.

Logarithmic concavity of Whitney numbers of the second kind is deduced by proving that their generating polynomial has only real zeros. [*Cf.* Stonesifer (1975a), Dur (1986a), and Damiani, D'Antona, and Regonati (1994a).] **(gg: M: Invar)**

C. Benzaken

See also P.L. Hammer.

C. Benzaken, S.C. Boyd, P.L. Hammer, and B. Simeone

1983a Adjoints of pure bidirected graphs. Proc. Fourteenth Southeastern Conf. on Combinatorics, Graph Theory and Computing (Boca Raton, Fla., 1983). *Congressus Numer.* 39 (1983), 123–144. MR 85e:05077. Zbl 537.05024. **(sg: Ori: LG)**

Cl. Benzaken, P.L. Hammer, and B. Simeone

1980a Some remarks on conflict graphs of quadratic pseudo-boolean functions. In: L. Collatz, G. Meinardus, and W. Wetterling, eds., *Konstruktive Methoden der finiten nichtlinearen Optimierung* (Tagung, Oberwolfach, 1980), pp. 9–30. Int. Ser. Num. Math., 55. Birkhäuser, Basel, 1980. MR 83e:90096. Zbl 455.90063. **(par: fr)(sg: Ori: LG)**

C. Benzaken, P.L. Hammer, and D. de Werra

1981a Threshold signed graphs. Res. Rep. No. 237, IMAG, Univ. Grenoble, Grenoble, 1981.

See (1985a). **(SG: Bal)**

1985b Threshold characterization of graphs with Dilworth number two. *J. Graph Theory* 9 (1985), no. 2, 245–267. MR 797513 (87d:05135). Zbl 583.05048.

They are identical to "threshold signed graphs". Γ is a threshold signed graph if $\exists\ a : V \to \mathbb{R}$, $S, T \in \mathbb{R}$, such that $v_iv_j \in E$ iff $|a_i + a_j| \geq S$ or $|a_i - a_j| \geq T$. [Proposed signed graph Σ: $-v_iv_j \in E$ iff $|a_i + a_j| \geq S$, $+v_iv_j \in E$ iff $|a_i - a_j| \geq T$. Then Γ = simplification of $|\Sigma|$. *Question.* Is Σ interesting?] [Annot. 16 Jan 2012.] **(SG: Bal)**

C. Berge and J.-L. Fouquet

1997a On the optimal transversals of the odd cycles. *Discrete Math.* 169 (1997), 169–175. MR 1449714 (98c:05094). Zbl 883.05088.

All-negative signed graphs in which the vertex frustration number equals the negative-circle vertex-packing number. This is called the "König property" [since it is a vertex König-type property for negative circles]. Example: the line graphs of cubic bipartite graphs. [*Problems.* Investigate for arbitrary signed and biased graphs.] **(par: Fr)**

Claude Berge and A. Ghouila-Houri

1962a *Programmes, jeu et reseaux de transport.* Dunod, Paris, 1962. MR 33 #1137. Zbl (e: 111.17302).

2^e partie, Ch. IV, §2: "Les reseaux de transport avec multiplicateurs." Pp. 223–229. **(GN: incid)**

1965a *Programming, Games and Transportation Networks.* Methuen, London; Wiley, New York, 1965. MR 33 #7114.

English edition of (1962a).

Part II, 10.2: "The transportation network with multipliers." Pp. 221–227. **(GN: incid)**

1967a *Programme, Spiele, Transportnetze.* B.G. Teubner Verlagsgesellschaft, Leipzig, 1967, 1969. MR 36 #1195. Zbl (e: 183.23905, 194.19803).

German edition(s) of (1962a). **(GN: incid)**

Claude Berge and Bruce Reed

1999a Edge-disjoint odd cycles in graphs with small chromatic number. Symposium à la Mémoire de François Jaeger (Grenoble, 1998). *Ann. Inst. Fourier (Grenoble)* 49 (1999), 783–786. MR 1703423 (2000f:05051). Zbl 923.05034.

If $-\Gamma$ is an all-negative signed graph in which the frustration index equals the negative-circle edge-packing number for every subgraph, then $\chi(\Gamma) \leq 3$. [*Problem* 1. Is it natural to state this bound in terms of the signed chromatic number of $-\Gamma$? *Problem* 2. Generalize to arbitrary signed graphs.] **(par: Fr)**

2000a Optimal packings of edge-disjoint odd cycles. *Discrete Math.* 211 (2000), 197–202. MR 1735345 (2000h:05161). Zbl 945.05048.

An upper bound on the frustration index in terms of the negative-circle edge-packing number. **(par: Fr)**

Joseph Berger, Bernard P. Cohen, J. Laurie Snell, and Morris Zelditch, Jr.

1962a *Types of Formalization in Small Group Research.* Houghton Mifflin, Boston,

1962.

See Ch. 2: "Explicational models." **(PsS)(SG: Bal)(Ref)**

A. Nihat Berker

See D. Blankschtein.

Abraham Berman and B. David Saunders

1981a Matrices with zero line sums and maximal rank. *Linear Algebra Appl.* 40 (1981), 229–235. MR 82i:15029. Zbl 478.15013. **(QM, sd: ori)**

Abraham Berman and Miriam Farber

2011a A lower bound for the second largest Laplacian eigenvalue of weighted graphs. *Electronic J. Linear Algebra* 22 (2011), 1179-1184.

§4, "The signless Laplacian": Upper bounds on the second largest eigenvalue of $K(-\Gamma, w)$ for a positively edge-weighted graph. [Annot. 20 Jan 2012.] **(par: WG: Adj)**

Pascal Berthomé, Raul Cordovil, David Forge, Véronique Ventos, and Thomas Zaslavsky

2009a An elementary chromatic reduction for gain graphs and special hyperplane arrangements. *Electronic J. Combinatorics* 16 (1) (2009), Article R121, 31 pp. MR 2546324 (2010k:05253). Zbl 1188.05076.

Calculating chromatic functions (which satisfy deletion-contraction for zero-gain edges and equal 0 if there is a balanced loop) by eliminating or adding identity-gain edges. Application to integral, modular, and zero-free chromatic polynomials of the Shi, Linial, Catalan, and intermediate hyperplane arrangements via their gain graphs [*cf.* Stanley (1999a)]. **(GG: Invar, Geom)**

Nadja Betzler

See F. Hüffner.

Amitava Bhattacharya, Uri N. Peled, and Murali K. Srinivasan

2007a Cones of closed alternating walks and trails. *Linear Algebra Appl.* 423 (2007), no. 2-3, 351–365. MR 2312413 (2008j:05132). Zbl 1115.05067.

The cone of Eulerian real-weighted subgraphs of a bidirected all-negative signed graph. **(sg: Par: Geom)**

2009a The cone of balanced subgraphs. *Linear Algebra Appl.* 431 (2009), no. 1-2, 266–273. MR 2522574 (2010h:05226). Zbl 1169.05372.

A "balanced subgraph" is an edge 2-colored graph where the red and blue degrees are equal at each vertex. [Equivalent to an all-negative signed graph, oriented so that every vertex has equal in- and out-degree, which is the all-negative case of an Eulerian bidirected graph. P.D. Seymour, Sums of circuits, in *Graph Theory and Related Topics*, pp. 341–355, Academic Press, New York, 1979, treated the all-positive case.] The problem is to describe the facets of the convex cone generated by Eulerian subgraphs of an all-negative bidirected graph. [*Problem.* Solve for an arbitrary bidirected graph.] **(sg: Par: Geom)**

Gora Bhaumik

See P.A. Jensen.

V.N. Bhave

See E. Sampathkumar.

Mani Bhushan and Raghunathan Rengaswamy

2000a Design of sensor network based on the signed directed graph of the process for efficient fault diagnosis. *Ind. Eng. Chem. Res.* 39 (2000), 999–1019.

Another application to fault diagnosis in chemical engineering, this one to location of sensors. (**SD: Appl**)

I. Bieche, R. Maynard, R. Rammal, and J.P. Uhry

1980a On the ground states of the frustration model of a spin glass by a matching method of graph theory. *J. Phys. A: Math. Gen.* 13 (1980), 2553–2576. MR 81g:82037.

The frustration index and ground states of a planar square grid graph can be found by matching in the dual graph. [Solved for all planar graphs by Katai and Iwai (1978a), Barahona (1982b).] [Annot. 29 Aug 2012.] (**SG: Phys, Fr, Alg**)

Dan Bienstock

1991a On the complexity of testing for odd holes and induced odd paths. *Discrete Math.* 90 (1991), 85–92. MR 92m:68040a. Zbl 753.05046. Corrigendum. *ibid.* 102 (1992), 109. MR 92m.68040b. Zbl 760.05080.

Given a graph. Problem 1: Is there an odd hole on a particular vertex? Problem 2: Is there an odd induced path joining two specified vertices? Problem 3: Is every pair of vertices joined by an odd-length induced path? All three problems are NP-complete. [Obviously, one can replace the graph by a signed graph and "odd length" by "negative" and the problems remain NP-complete.] (**Par: Circles, Paths: Alg**)

Norman Biggs

1974a *Algebraic Graph Theory.* Cambridge Math. Tracts, No. 67. Cambridge Univ. Press, London, 1974. MR 50 #151. Zbl 284.05101.

Ch. 19: "The covering graph construction." The covering graphs of gain graphs, with emphasis on automorphisms. Let $\Phi := (\Gamma, \varphi)$ with gain group $\mathbb{Z}_2 E$ and $\varphi(e) = e$. Thm. 19.5: If Γ is t-transitive ($t \geq 1$) [and connected], then $\tilde{\Phi}$ is vertex transitive [actually, t-transitive] and has $n - c(\Gamma)$ components (all isomorphic). [The number of components and the isomorphism of components of $\tilde{\Phi}$ require only connectedness of Φ, because $\operatorname{Aut}\tilde{\Phi}$ acts transitively on each vertex fiber.] 19A: "Double coverings." The signed covering graph of $-\Gamma$. 19B: "The Desargues graph." With $P :=$ Petersen graph, $\widetilde{-P}$ is the Desargues graph. [Annot. 11 July 2009.]

[Tutte (1967a) implicitly develops the double covering of an oriented Σ; it is a self-converse orientation of $\tilde{\Sigma}$.] (**SG, GG: Cov, Aut, bal**)

1993a *Algebraic Graph Theory.* Second edn. Cambridge Math. Library, Cambridge Univ. Press, Cambridge, Eng., 1993. MR 95h:05105. Zbl 797.05032.

As in (1974a), but 19A, 19B have become Additional Results 19a, 19b. (**SG, GG: Cov, Aut, bal**)

1997a International finance. In: Lowell W. Beineke and Robin J. Wilson, eds., *Graph Connections: Relationships between Graph Theory and other Areas of Mathematics*, Ch. 17, pp. 261–279. The Clarendon Press, Oxford, 1997. MR 1634542 (99a:05001) (book). Zbl 876.90014.

A model of currency exchange rates in which no cyclic arbitrage is possible, hence the rates are given by a potential function. [That is, the exchange-rate gain graph is balanced, with the natural consequences.]

Assuming cash exchange without accumulation in any currency, exchange rates are determined. [See also Ellerman (1984a).]
(**GG, gn: Bal: Exp**)

Yonatan Bilu and Nathan Linial

2004a Ramanujan signing of regular graphs. *Combin. Probab. Comput.* 13 (2004), no. 6, 911–912. Zbl 1060.05040.

Conjecture 2 (based on (2006a)). Every d-regular Ramanujan graph can be signed so it has spectral radius $\leq 2\sqrt{d-1}$. *Conjecture* 3. The same for every d-regular graph. Dictionary: "2-lift" = signed covering graph. [Annot. 2 Mar 2011.] (**SG: Adj, Cov**)

2006a Lifts, discrepancy and nearly optimal spectral gap. *Combinatorica* 26 (2006), no. 5, 495–519. MR 2279667 (2008a:05160). Zbl 1121.05054.

A remarkable result: Lemma: The spectrum of $A(\tilde{\Sigma})$ is $\operatorname{Spec} A(\Sigma) \cup \operatorname{Spec} A(|\Sigma|)$. [Proved independently by Kalita and Pati (2012a). Generalized to branched coverings in Butler (2010a).] [Annot. 2 Mar 2011.]
(**SG: Adj, Cov**)

K. Binder and A.P. Young

1986a Spin glasses: Experimental facts, theoretical concepts, and open questions. *Rev. Modern Phys.* 58 (1986), no. 4, 801–976.

§ III.F.2, "Frustration and gauge invariance": A valuable summary of the state of knowledge and speculation. Signed graphs with spin set $\{+1, -1\}$ (Ising spins) and $U(1)$ ("XY spins" = complex units). Frustration is treated via girth circles ("plaquettes") in lattice graphs, where the girth is 3 or 4 (triangular or square planar lattice). Analytic solutions being too difficult, results are numerical, qualitative, or for "simpler limiting cases". XY spins show quantization (*cf.* Villain (1977b)). For 3-dimensional lattices, plaquette duality leads to vector gains in a dual lattice, thence to closed paths of frustrated plaquettes.

In Ch. IV, "Mean-field theory": Complete-graph ("infinite range") models. § IV.A, "Sherrington-Kirkpatrick model and replica-symmetric solutions": Ising models ($\mathfrak{G} = \{+1, -1\}$). § IV.H, "Non-Ising models": Weighted edge signs are random variables. Spins may be normalized vectors (§1, "Isotropic vector spin glasses in zero field") or other. §3, "Other models": "p-spin couplings" = p-uniform complete hypergraphs. Energy valleys and their shapes. Potts models (signed graphs, spins are multivalued).

Dictionary: "site" = vertex, "bond" = edge, "state" = function $s : V \to \mathfrak{G}$, "spin" = value $s(v)$, "ferromagnetic" = positive, "antiferromagnetic" = negative, "quenched variable" = constant (instead of random variable), "gauge group" = gain group, "gauge transformation" = switching, "ground state" = state minimizing $|E \setminus E^{1_{\mathfrak{G}}}(\Phi^s)|$. [Annot. 17 Aug 2012.] (**Phys: sg, gg: Fr, Sw, Exp, Ref**)

Robert E. Bixby

1981a Hidden structure in linear programs. In: Harvey J. Greenberg and John S. Maybee, eds., *Computer-Assisted Analysis and Model Simplification* (Proc. Sympos., Boulder, Col., 1980), pp. 327–360; discussion, pp. 397–404. Academic Press, New York, 1981. MR 82g:00016 (book). Zbl 495.93001 (book). (**GN**)

Türker Bıyıkoğlu and Josef Leydold

2010a Semiregular trees with minimal Laplacian spectral radius. *Linear Algebra Appl.* 432 (2010), 2335–2341. MR 2599863 (2011b:05114). Zbl 1225.05139.

In a semiregular tree T, all internal vertices have the same degree d. Thm. 2: Given $n, d \geq 3$, semiregular T minimizes $\lambda_1(K(T))$ iff it is a caterpillar. The proof is via $\operatorname{Spec} K(-T)$, which $= \operatorname{Spec} K(T)$ since a signed tree is balanced. [Annot. 21 Jan 2012.] (**Par: Adj**)

Türker Biyikoğlu, Marc Hellmuth, and Josef Leydold

†2009a Largest eigenvalues of the discrete p-Laplacian of trees with degree sequences. *Electronic J. Linear Algebra* 18 (2009), 202–210. MR 2491656 (2010d:05089). Zbl 1169.05335.

The p-Laplacian $(1 < p < \infty)$ generalizes the Laplacian or Kirchhoff matrix acting on vertex functions. [Generalizing to signed graphs, define the p-Laplacian of Σ by $\Delta_p(\Sigma)f(u) := \sum_{uv \in E} \operatorname{sgn}[f(u) - \sigma(uv)f(v)] \cdot |f(u) - \sigma(uv)f(v)|^{p-1}$. Then $p = 2$ gives $K(\Sigma)$.] The p-Laplacian of Γ is $\Delta_p(+\Gamma)$ and its signless p-Laplacian is $\Delta_p(-\Gamma)$. Prop. 3.3 *et seq.* concern $\Delta_p(-\Gamma)$. [Unlike with the Laplacian K, switching does not preserve properties, so signs matter in a tree.] [*Problem.* Generalize to signed graphs.] [Annot. 21 Jan 2012.] (**Par: Adj: Gen**)

Anders Björner and Bruce E. Sagan

1996a Subspace arrangements of type B_n and D_n. *J. Algebraic Combin.* 5 (1996), 291–314. MR 97g:52028. Zbl 864.57031.

They study lattices $\Pi_{n,k,h}$ (for $0 < h \leq k \leq n$) consisting of all spanning subgraphs of $\pm K_n^\circ$ that have at most one nontrivial component K, for which either K is balanced and complete and $|V(K)| \geq k$, or K is induced and $|V(K)| \geq h$. (They also study a generalization of this.) Characteristic polynomial, homotopy and homology of the order complex, cohomology of the real complement.

(**SG: Geom, M(Gen): Invar, col**)

Anders Björner, Michel Las Vergnas, Bernd Sturmfels, Neil White, and Günter M. Ziegler

1993a *Oriented Matroids.* Encyclop. Math. Appl., Vol. 46. Cambridge University Press, Cambridge, Eng., 1993. MR 95e:52023. Zbl 773.52001.

The adjacency graph of bases of an oriented matroid is signed, using circuit signatures, to make the "signed basis graph". See §3.5, "Basis orientations and chirotopes", pp. 132–3. (**M: SG**)

Daniel Blankschtein, M. Ma, and A. Nihat Berker

1984a Fully and partially frustrated simple-cubic Ising models: Landau-Ginzburg-Wilson theory. *Phys. Rev. B* 30 (1984), no. 3, 1362–1365. (**Phys, SG: Fr, sw**)

Daniel Blankschtein, M. Ma, A. Nihat Berker, Gary S. Grest, and C.M. Soukoulis

1984a Orderings of a stacked frustrated triangular system in three dimensions. *Phys. Rev. B* 29 (1984), no. 9, 5250–5252.

Physics of $(-L_3) \times (+P_m)$, consisting of m all-negative triangular lattice layers $-L_3$, stacked vertically with vertical positive edges forming paths P_m $(0 \ll m \leq \infty)$. The horizontal triangles are negative (the layers are "totally frustrated") while the vertical squares are positive. Ground states ($\zeta : V \to \{+1, -1\}$ such that $\min_\zeta |(E^\zeta)^-|$) are ground states of

$-L_3$ (*cf.* Wannier 1950a) repeated in every layer. [Annot. 18 Jun 2012.] (**Phys, SG: Fr, sw**)

Andreas Blass

1995a Quasi-varieties, congruences, and generalized Dowling lattices. *J. Algebraic Combin.* 4 (1995), 277–294. MR 96i:06012. Zbl 857.08002. Errata. *Ibid.* 5 (1996), 167. MR 96i:06012, 1382046. Zbl 857.08002.

Treats the generalized Dowling lattices of Hanlon (1991a) as congruence lattices of certain quasi-varieties, in order to calculate characteristic polynomials and generalizations. (**M(gg): Gen: Invar**)

Andreas Blass and Frank Harary

1982a Deletion versus alteration in finite structures. *J. Combin. Inform. System Sci.* 7 (1982), 139–142. MR 84d:05087. Zbl 506.05038.

The theorem that deletion index = negation index of a signed graph (Harary (1959b)) is shown to be a special case of a very general phenomenon involving hereditary classes of "partial choice functions". Another special case: deletion index = alteration index of a gain graph [an immediate corollary of Harary, Lindstöm, and Zetterström (1982a), Thm. 2]. (**SG, GG: Bal, Fr**)

Andreas Blass and Bruce Sagan

1997a Möbius functions of lattices. *Adv. Math.* 127 (1997), 94–123. MR 98c:06001. Zbl 970.32977.

§3: "Non-crossing B_n and D_n". Lattices of noncrossing signed partial partitions. Atoms of the lattices are defined as edge fibers of the signed covering graph of $\pm K_n^\circ$, thus corresponding to edges of $\pm K_n^\circ$. [The "half edges" are perhaps best regarded as negative loops.] The lattices studied, called $NCB_n, NCD_n, NCBD_n(S)$, consist of the noncrossing members of the Dowling and near-Dowling lattices of the sign group, i.e., $\operatorname{Lat} G(\pm K_n^{(T)})$ for $T = [n], \varnothing, [n]\backslash S$, respectively. (**SG: Geom, M(Gen), Invar, cov**)

1998a Characteristic and Ehrhart polynomials. *J. Algebraic Combin.* 7 (1998), 115–126. MR 99c:05204. Zbl 899.05003.

Signed-graph chromatic polynomials are recast geometrically by observing that the number of k-colorings equals the number of points of $\{-k, -k+1, \ldots, k-1, k\}^n$ that lie in none of the edge hyperplanes of the signed graph. The interesting part is that this generalizes to subspace arrangements of signed graphs and, somewhat *ad hoc*, to the hyperplane arrangements of the exceptional root systems. [See also Athanisiadis (1996a), Zaslavsky (20xxi). For applications see articles of Sagan and Zhang.] (**SG, Gen: M(Gen), Geom: col, Invar**)

Matthew Bloss

2003a G-colored partition algebras as centralizer algebras of wreath products. *J. Algebra* 265 (2003), no. 2, 690–710. MR 1987025 (2004e:20020). Zbl 1028.20007.

Let $\mathfrak{G}$ denote any group. The algebra is $\mathbb{C} \operatorname{Lat}^{\mathrm{b}} G(\mathfrak{G} K_{2k}(U, W))$ where $\operatorname{Lat}^{\mathrm{b}} G(\mathfrak{G} K_{2k}(U, W))$ = the semilattice of balanced flats of the Dowling lattice $Q_{2k}(\mathfrak{G})$ on a set $V := U \mathbin{\dot\cup} W$ of $2k$ vertices, $U := \{u_1, \ldots, u_k\}$, and $W := \{w_1, \ldots, w_k\}$.

The definition requires a multiplication on $\operatorname{Lat}^{\mathrm{b}} G(\mathfrak{G} K_{2k}(U, W))$ which involves an indeterminate x. For each balanced flat (equivalently, $\mathfrak{G}$-

valued partition) α label its vertices $u_{\alpha i} := u_i$, $w_{\alpha i} := w_i$. Define $\gamma := \alpha \cdot \beta$ by identifying $w_{\alpha i}$ with u_{β_i} in $\alpha \cup\!\!\!\cdot\, \beta$ (call the result γ'), taking the closure in $G(\mathfrak{G}K_{3k})$, multiplying by x^m where $m :=$ # of components of γ' contained completely within the identified vertices, and deleting the identified vertices $w_{\alpha i}$. Set $u_{\gamma i} := u_{\alpha i}$ and $w_{\gamma i} := w_{\beta i}$. [Annot. 20 Mar 2011.] (**gg: m: Algeb**)

Avrim Blum

See N. Bansal.

F.T. Boesch, X. Li [Xiao Ming Li], and J. Rodriguez

1995a Graphs with most number of three point induced connected subgraphs. *Discrete Appl. Math.* 59 (1995), no. 1, 1–10. MR 96b:05073 (*q.v.*). Zbl 835.05056.

Two-graphs and switching are mentioned. (**TG, Sw**)

Irina E. Bocharova, Florian Hug, Rolf Johannesson, Boris D. Kudryashov, and Roman V. Satyukov

2011a Some voltage graph-based LDPC tailbiting codes with large girth. *Information Theory (ISIT2011)* (Proc. 2011 IEEE Int. Sympos., St. Petersburg), pp. 732–736. IEEE, 2011. arXiv:1108.0840. (**GG: Cov**)

2011b Searching for voltage graph-based LDPC tailbiting codes with large girth. *IEEE Trans. Information Theory* 57 (2011), no. 12, to appear. arXiv:1108.0840. (**GG: Cov**)

Sebastian Böcker, Falk Hüffner, Anke Truss, and Magnus Wahlström

2009a A faster fixed-parameter approach to drawing binary tanglegrams. In: J. Chen and F.V. Fomin, eds., *Parameterized and Exact Computation* (4th Int. Workshop, IWPEC 2009, Copenhagen), pp. 38–49. Lect. Notes in Computer Sci., Vol. 5917. Springer, Berlin, 2009. MR 2773930 (no rev).

The signed graph arises as a graph with edges labelled $=$ $(+)$ or $\neq$ $(-)$. The "Balanced Subgraph" problem is to find a minimum balancing et. The algorithm of Hüffner, Betzler, and Niedermeier (2007a) is applied. [Annot. 6 Feb 2011.] (**sg: fr: Alg: Appl**)

Alexander Bockmayr

See H. Siebert.

Bernhard G. Bodmann, Vern I. Paulsen, and Mark Tomforde

2009a Equiangular tight frames from complex Seidel matrices containing cube roots of unity. *Linear Algebra Appl.* 430 (2009), 396–417. MR 2460526 (2010b:42040). Zbl 1165.42007.

Adjacency matrices of cube-root-of-unity gain graphs on K_n. Dictionary: "Seidel matrix" = adjacency matrix of such a gain graph. [Annot. 27 Apr 2012.] (**gg: Geom, adj: kg**)

T.B. Boffey

1982a *Graph Theory in Operations Research.* Macmillan, London, 1982. Zbl 509.90053.

Ch. 10: "Network flow: extensions." 10.1(g): "Flows with gains," pp. 224–226. 10.3: "The simplex method applied to network problems," subsection "Generalised networks," pp. 246–250. (**GN: m(bases): Exp**)

Kenneth P. Bogart

See M.K. Bennett, J.E. Bonin, and J.R. Weeks.

Petre Boldescu

1970a Les théorèmes de Menelaus et Ceva dans un éspace affine de dimension n. [The theorems of Menelaus and Ceva in an n-dimensional affine space.] (In

Romanian. French summary.) *An. Univ. Craiova Ser. a IV-a* 1 (1970), 101–106. MR 48 #12251. Zbl 275.50008.

Generalized Ceva [strengthened via gain graphs in Zaslavsky (2003b) §2.6] and Menelaus theorems. [*Problem.* Formulate, explain, generalize Boldescu's Menelaus generalization in terms of gain graphs.]

(**gg: Geom**)

Ethan D. Bolker

1977a Bracing grids of cubes. *Environment and Planning B* 4 (1977), 157–172.

The elementary 1-cycles associated with circuits of $G(-\Gamma)$ ("bicycles") are crucial. [Their first publication, I believe.] (**EC, sg: m**)

1979a Bracing rectangular frameworks. II. *SIAM J. Appl. Math.* 36 (1979), 491–503. MR 81j:73066b. Zbl 416.70010.

The elementary 1-cycles associated with circuits of $G(\Sigma)$ ("bicycles"), mostly for $\Sigma = -\Gamma$. General signed graphs appear at Thm. 7, p. 505. Dictionary: "Signed bicycle" = elementary 1-cycle (circulation) associated with a circuit. (**EC, SG: M, incid**)

Ethan D. Bolker and Thomas Zaslavsky

2006a A simple algorithm that proves half-integrality of bidirected network programming. *Networks* 48 (2006), no. 1, 36–38. MR 2007b:05098. Zbl 1100.05046.

An idea of Bolker's (1979a), as developed by Bouchet (1983a), is turned into an algorithm simpler than that of Appa and Kotnyek (2006a).

(**SG: Ori, Incid, Alg, Sw**)

Bela Bollobás

1978a *Extremal Graph Theory.* L.M.S. Monographs, Vol. 11. Academic Press, London, 1978. MR 80a:05120. Zbl 419.05031.

A rich source of problems: find interesting generalizations to signed graphs of questions involving even or odd circles, or bipartite graphs or subgraphs.

(**par: Xtreml**)

§3.2, Thm. 2.2, is Lovász's (1965a) characterization of the graphs having no two vertex-disjoint circles. [*Problem.* Generalize to biased graphs having no two vertex-disjoint unbalanced circles, Lovász's theorem being the contrabalanced case.] (**GG: Circles**)

§6.6, Problem 47, is the theorem on biparticity (all-negative vertex frustration number) from Bollobás, Erdős, Simonovits, and Szemerédi (1978a). (**par: Fr**)

1998a *Modern Graph Theory.* Springer, New York, 1998. MR 99h:05001. Zbl 902.-05016.

Sign-colored plane graphs in Ch. X, "The Tutte polynomial", §6, "Polynomials of knots and links", pp. 368-370. Little use is made of the signs.

(**SGc: Knot**)

B. Bollobás, P. Erdös, M. Simonovits, and E. Szemerédi

1978a Extremal graphs without large forbidden subgraphs. In: B. Bollobás, ed., *Advances in Graph Theory* (Proc. Cambridge Combin. Conf., 1977), pp. 29–41. Ann. Discrete Math., Vol. 3. North-Holland, Amsterdam, 1978. MR 80a:05119. Zbl 375.05034.

Thm. 9 asymptotically estimates upper bounds on frustration index and vertex frustration number for all-negative signed graphs with fixed

negative girth. [Sharpened by Komlós (1997a).] **(par: Fr)**

Bela Bollobás, Luke Pebody, and Oliver Riordan

2000a Contraction-deletion invariants for graphs. *J. Combin. Theory Ser. B* 80 (2000), 320–345. MR 2001j:05055. Zbl 1024.05028.

§4, "Coloured graphs". **(SGc: Gen: Invar)**

Bela Bollobás and Oliver Riordan

1999a A Tutte polynomial for coloured graphs. Recent Trends in Combinatorics (Mátraháza, 1995). *Combin. Probab. Comput.* 8 (1999), 45–93. MR 2000f:05033. Zbl 926.05017.

Discovers the fundamental relations for the commutative algebra underlying the parametrized Tutte polynomial of colored graphs. *Cf.* Zaslavsky (1992b). **(SGc: Gen: Invar, Knot)**

2002a A polynomial of graphs on surfaces. *Math. Ann.* 323 (2002), no. 1, 81–96. MR 2003b:05052. Zbl 1004.05021.

The polynomial is a deletion-contraction invariant of signed graphs with rotation systems (called "ribbon graphs"). **(sg: Top: Incid)**

Erik G. Boman, Doron Chen, Ojas Parekh, and Sivan Toledo

2005a On factor width and symmetric H-matrices. *Linear Algebra Appl.* 405 (2005), 239–248. MR 2148173 (2006e:15024). Zbl 1098.15014.

A real symmetric matrix $= \mathrm{H}(\Phi)\mathrm{H}(\Phi)^{\mathrm{T}}$ for a real gain graph Φ with a link (called "factor width 2"). Thm. 9. A has factor width 2 iff it is a symmetric H-matrix with diagonal ≥ 0. [Annot. 8 Mar 2011.] **(gg: Incid, Adj)**

Phillip Bonacich

1999a An algebraic theory of strong power in negatively connected exchange networks. *J. Math. Sociology* 23 (1999), no. 3, 203–224. Zbl 1083.91574.

P. 214: The distribution of power depends in part on whether $\mathrm{H}(-\Gamma)$ has full rank, i.e., Γ is bipartite (*cf.* van Nuffelen (1973a)), where Γ is the graph of potential exchanges. [Annot. 13 Aug 2012.] **(par: Incid, PsS)**

2007a Some unique properties of eigenvector centrality. *Social Networks* 29 (2007), 555–564.

§1.1.3, "Uses of $c(\beta)$ and x in signed graphs". [Annot. 12 Sept 2010.] **(SG, PsS: Adj)**

Phillip Bonacich and Paulette Lloyd

2004a Calculating status with negative relations. *Social Networks* 26 (2004), 331–338.

Compares the dominant-eigenvector measure of centrality in Σ, Σ^+, and dense induced subgraphs, in a standard example. [Annot. 22 Oct 2009.] **(SG: PsS: Adj)**

J.A. Bondy and L. Lovász

1981a Cycles through specified vertices of a graph. *Combinatorica* 1 (1981), 117–140. MR 82k:05073. Zbl 492.05049.

If Γ is k-connected [and not bipartite], then any k [$k-1$] vertices lie on an even [odd] circle. [*Problem.* Generalize to signed graphs, this being the all-negative case.] **(sg: bal)**

J.A. Bondy and M. Simonovits

1974a Cycles of even length in graphs. *J. Combin. Theory Ser. B* 16 (1974), 97–105. MR 49 #4851. Zbl 283.05108.

If a graph has enough edges, it has even circles of all moderately small lengths. [*Problem* 1. Generalize to positive circles in signed graphs, this being the antibalanced (all-negative) case. For instance, *Problem* 2. If an unbalanced signed simple graph has positive girth $\geq l$ (i.e., no balanced circle of length $< l$), what is its maximum size? Are the extremal examples antibalanced? Balanced?] (**par: bal(Circles), Xtreml**)

Joseph E. Bonin

See also M.K. Bennett.

1993a Automorphism groups of higher-weight Dowling geometries. *J. Combin. Theory Ser. B* 58 (1993), 161–173. MR 94k:51005. Zbl 733.05027, (789.05017).

A weight-k higher Dowling geometry of rank n, $Q_{n,k}(\mathrm{GF}(q)^\times)$, is the union of all coordinate k-flats of $\mathrm{PG}(n-1, q)$: i.e., all flats spanned by k elements of a fixed basis. If $k > 2$, the automorphism groups are those of $\mathrm{PG}(n-1, q)$ for $q > 2$ and are symmetric groups if $q = 2$. (**gg: Gen: M, Aut**)

1993b Modular elements of higher-weight Dowling lattices. *Discrete Math.* 119 (1993), 3–11. MR 94h:05018. Zbl 808.06012.

See definition in (1993a). For $k > 2$ the only nontrivial modular flats are the projective coordinate k-flats and their subflats. This gives some information about the characteristic polynomials [which, however, are still only partially known]. [Kung (1996a), §6, has further results.] (**gg: Gen: M: Invar**)

1995a Automorphisms of Dowling lattices and related geometries. *Combin. Probab. Comput.* 4 (1995), 1–9. MR 96e:05039. Zbl 950.37335.

The automorphisms of a Dowling geometry of a nontrivial group are the compositions of a coordinate permutation, switching, and a group automorphism. A similar result holds, with two exceptions, if some or all coordinate points are deleted. [A third exception is missed: the jointless Dowling geometry $Q_3^0(\mathbb{Z}_3)$.] [*Cf.* Schwartz (2002a).] (**gg: M: Aut**)

1996a Open problem 6. A problem on Dowling lattices. In: Joseph E. Bonin, James G. Oxley, and Brigitte Servatius, eds., *Matroid Theory* (Proc., Seattle, 1995), pp. 417–418. Contemp. Math., Vol. 197. Amer. Math. Soc., Providence, R.I., 1996.

Problem 6.1. If a finite matroid embeds in the Dowling geometry of a group, does it embed in the Dowling geometry of some finite group? [No; see Brooksbank, Qin, Robertson, and Seress (2004a).] (**gg: M**)

2006a Extending a matroid by a cocircuit. *Discrete Math.* 306 (2006), no. 8–9, 812–819. MR 2006m:05045. Zbl 1090.05008.

§4 concerns Dowling lattices. (**GG: M**)

Joseph E. Bonin and Kenneth P. Bogart

1991a A geometric characterization of Dowling lattices. *J. Combin. Theory Ser. A* 56 (1991), 195–202. MR 92b:05019. Zbl 723.05033. (**gg: M**)

Joseph E. Bonin and Joseph P.S. Kung

1994a Every group is the automorphism group of a rank-3 matroid. *Geom. Dedicata* 50 (1994), 243–246. MR 95m:20005. Zbl 808.05029. (**gg: M: Aut**)

Joseph E. Bonin and William P. Miller

1999a Characterizing combinatorial geometries by numerical invariants. *European J. Combin.* 20 (1999), 713–724. MR 2001a:51007. Zbl 946.05020.

Dowling geometries are characterized amongst all simple matroids by numerical properties of large flats of ranks ≤ 7 (Thm. 3.4); amongst all matroids by their Tutte polynomials. (**gg: M**)

Joseph E. Bonin and Hongxun Qin

2000a Size functions of subgeometry-closed classes of representable combinatorial geometries. *Discrete Math.* 224 (2000), 37–60. MR 2001g:05031. Zbl 968.52009.

Extremal matroid theory. The Dowling geometry $Q_3(\mathrm{GF}(3)^\times) = G(\pm K_3^\bullet)$ appears as an exceptional extremal matroid in Thm. 2.10. The extremal subset of $\mathrm{PG}(n-1,q)$ that does not contain the higher-weight Dowling geometry $Q_{m,m-1}(\mathrm{GF}(q)^\times)$ (see Bonin 1993a) is found in Thm. 2.14. (**GG, Gen: M: Xtreml, Invar**)

C. Paul Bonnington and Charles H.C. Little

1995a *The Foundations of Topological Graph Theory.* Springer, New York, 1995. MR 97e:05090. Zbl 950.48477.

Signed-graph imbedding: see §2.3, §2.6 (esp. Thm. 2.4), pp. 44–48 (for the colorful 3-gem approach to crosscaps), §3.3, and Ch. 4 (esp. Thms. 4.5, 4.6). (**sg: Top, bal**)

Stefan Bornholdt

See J. Reichardt.

Bojana Borovićanin

See J.F. Wang.

E. Boros, Y. Crama, and P.L. Hammer

1992a Chvàtal cuts and odd cycle inequalities in quadratic 0—1 optimization. *SIAM J. Discrete Math.* 5 (1992), 163–177. MR 93a:90043. Zbl 761.90069.

§4: "Odd cycles [i.e., negative circles] in signed graphs." Main problem: Find a minimum-weight deletion set in a signed graph with positively weighted edges. Related problems: A circle-covering formulation whose constraints correspond to negative circles. A dual circle-packing problem. (**SG: Fr, Geom, Alg**)

Endre Boros and Peter L. Hammer

1991a The max-cut problem and quadratic 0—1 optimization; polyhedral aspects, relaxations and bounds. *Ann. Operations Res.* 33 (1991), 151–180. MR 92j:90049. Zbl 741.90077.

Includes finding a minimum-weight deletion set (as in Boros, Crama, and Hammer (1991a)). (**SG, WG: Fr: Geom, Alg**)

André Bouchet

1982a Constructions of covering triangulations with folds. *J. Graph Theory* 6 (1982), 57–74. MR 83b:05057. Zbl 488.05032. (**sg: Ori, Appl(Top)**)

1983a Nowhere-zero integral flows on a bidirected graph. *J. Combin. Theory Ser. B* 34 (1983), 279–292. MR 85d:05109. Zbl 518.05058.

Introduces nowhere-zero flows on signed graphs. A connected, coloop-free signed graph has a nowhere-zero integral flow with maximum weight ≤ 216. The value 216 cannot be replaced by 5, but: *Conjecture*(Bouchet): it can be replaced by 6. [See Khelladi (1987a), Xu and Zhang (2005a), Raspaud and Zhu (2011a), and Akbari, Daemi, *et al.* (20xxa) for progress.

See Jensen and Toft (1995a) for other contributions.] A topological application is outlined. [The bidirection is inessential; it is a device to keep track of the flow.] [Annot. ca. 1983.]
(**SG: M, Ori, Flows, Appl(Top)**)

Jean-Marie Bourjolly

1988a An extension of the König–Egerváry property to node-weighted bidirected graphs. *Math. Programming* 41 (1988), 375–384. MR 90c:05161. Zbl 653.90083.
[See Sewell (1996a).] (**sg: Ori, GG: Alg**)

J.-M. Bourjolly, P.L. Hammer, and B. Simeone

1984a Node-weighted graphs having the König–Egerváry property. Mathematical Programming at Oberwolfach II (Oberwolfach, 1983). *Math. Programming Stud.* 22 (1984), 44–63. MR 86d:05099. Zbl 558.05054. (**par: ori**)

Jean-Marie Bourjolly and William R. Pulleyblank

1989a König–Egerváry graphs, 2-bicritical graphs and fractional matchings. *Discrete Appl. Math.* 24 (1989), 63–82. MR 90m:05069. Zbl 684.05036.
[It is hard to escape the feeling that we are dealing with all-negative signed graphs and that something here will generalize to other signed graphs. Especially see Thm. 5.1. Consult the references for related work.] (**Par; Ref**)

Garry Bowlin

2009a *Maximum Frustration of Bipartite Signed Graphs.* Doctoral dissertation, Binghamton Univ. (SUNY), 2009. MR 2713583 (no rev).
Strong results on structure, bounds, and asymptotics of the generalized Gale–Berlekamp switching game, i.e., maximum frustration of a signed $K_{r,s}$ (*cf.* Brown and Spencer (1971a)), by a linear programming method. Improves on Brown and Spencer (1971a) (*q.v.*), Gordon and Witsenhausen (1972a), Solé and Zaslavsky (1994a), [Annot. 9 Sept 2010, 30 Oct 2011.] (**SG: Fr: Geom**)

John Paul Boyd

1969a The algebra of group kinship. *J. Math. Psychology* 6 (1967), 139–167. Repr. in: Samuel Leinhardt, ed., *Social Networks: A Developing Paradigm*, pp. 319–346. Academic Press, New York, 1977. Zbl (e: 172.45501). Erratum. *J. Math. Psychology* 9 (1972), 339. Zbl 242.92010. (**SG: Bal**)

S.C. Boyd

See C. Benzaken.

John Bramsen

2002a Further algebraic results in the theory of balance. *J. Math. Sociology* 26 (2002), 309–319. Zbl 1014.05041.
Algorithmic ideas for estimating $l(\Sigma)$. Remarks on clusterability.
(**SH**) (**SG: Fr: Alg; Clu**)

Franz J. Brandenburg

2002a Cycles in generalized networks. In: Luděk Kučera, ed., *Graph-Theoretic Concepts in Computer Science* (28th Int. Workshop, WG 2002, Český Krumlov, Czech Rep., 2002), pp. 47–56. Lect. Notes in Computer Sci., Vol. 2573. Springer-Verlag, Berlin, 2002. MR 2062357. Zbl 1022.90035.
The effects of gainy and lossy cycles and negative cycles on cheapest flow from source or between two nodes. [Annot. 21 Mar 2011.] (**GN: Alg**)

2003a Erratum: "Cycles in generalized networks". In: Hans L. Bodlaender, ed., *Graph-Theoretic Concepts in Computer Science* (29th Int. Workshop, WG 2003, Elspeet, The Netherlands, 2003), p. 383. Lect. Notes in Computer Sci., Vol. 2880. Springer-Verlag, Berlin, 2003. MR 2080096.

Results in (2002a) on cheapest flow from source are incorrect. [Annot. 21 Mar 2011.] (**GN: Alg**)

Franz J. Brandenburg and Mao-Cheng Cai

2009a Shortest path and maximum flow problems in networks with additive losses and gains. In: X. Deng, J.E. Hopcroft, and J. Xue, eds., *Frontiers in Algorithmics: Third International Workshop* (FAW 2009, Hefei, China), pp. 4–15. Lect. Notes in Comput. Sci., Vol. 5598. Springer-Verlag, Berlin, 2009.

See (2011a). (**gg: incid: Alg, m**)

2011a Shortest path and maximum flow problems in networks with additive losses and gains. *Theor. Computer Sci.* 412 (2011), no. 4-5, 391–401. MR 2778472 (2011k:68052). Zbl 1230.90045.

Additive real gains. The lift matroid is implicit. Contrasts algorithmic complexity of additive with multiplicative gains. [Annot. 30 May 2012.] (**gg: incid: Alg, m**)

A.J. Bray, M.A. Moore, and P. Reed

1978a Vanishing of the Edwards-Anderson order parameter in two- and three-dimensional Ising spin glasses. *J. Phys. C: Solid State Phys.* 11 (1978), 1187–1202.

Random edge signs on a hypercubic lattice. [Annot. 12 Aug 2012.] (**Phys: SG: Rand, Fr**)

Peter Brooksbank, Hongxung Qin, Edmund Robertson, and Ákos Seress

2004a On Dowling geometries of infinite groups. *J. Combin. Theory Ser. A* 108 (2004), no. 1, 155–158. MR 2005e:51014. Zbl 1056.51011.

Solution of Bonin (1996a). They produce a finite gain graph that has gains in no finite group. (**gg: M**)

A.E. Brouwer, A.M. Cohen, and A. Neumaier

1989a *Distance-Regular Graphs.* Ergeb. Math., Third Ser., Vol. 18. Springer-Verlag, Berlin, 1989. MR 90e:05001. Zbl 747.05073.

§1.5, "Taylor graphs and regular two-graphs": Signed complete graphs appear in the form of double covers of the complete graph. §3.8, "Graph switching, equiangular lines, and representations of two-graphs". §7.6C, "2-Transitive regular two-graphs". (**TG: kg, Geom: Exp, Ref**)

Andries E. Brouwer and Willem H. Haemers

2012a *Spectra of Graphs.* Universitext. Springer-Verlag, Berlin, 2012.

§1.1, "Matrices associated to a graph": "Laplace matrix" = Kirchhoff matrix $K(+\Gamma)$, from the "directed [i.e., oriented] incidence matrix" H$(+\Gamma)$. "Signless Laplace matrix" = Kirchhoff matrix $K(-\Gamma)$, from the "(undirected) [unoriented] incidence matrix" H$(-\Gamma)$ (with no -1s). Many results employ $K(-\Gamma)$, but signed graphs are ignored; e.g., see §§1.4.5, 14.4.3, "Line graphs" [*cf.* G.R. Vijayakumar *et al.*]. §1.8.2, "Seidel switching", defines the Seidel adjacency matrix $A(K_\Gamma)$ and its switching. Ch. 10, "Regular two-graphs".

$K(-\Gamma)$ appears in: Ch. 3: "Eigenvalues and Eigenvectors of Graphs", §15.3: "Other matrices with at most three eigenvalues". §15.3.1: "Few

Seidel eigenvalues"; §15.3.3: "Three signless Laplace eigenvalues". [Annot. 19 Sept 2010, 23 Jan 2012.] (**sg: Par: Adj, incid, TG, sw**)

Floor Brouwer and Peter Nijkamp

1983a Qualitative structure analysis of complex systems. In: P. Nijkamp, H. Leitner, and N. Wrigley, eds., *Measuring the Unmeasurable*, pp. 509–530. Martinus Nijhoff, The Hague, 1983. (**QM, SD: QSol, QSta: Exp**)

Edward M. Brown and Robert Messer

1979a The classification of two-dimensional manifolds. *Trans. Amer. Math. Soc.* 255 (1979), 377–402. MR 80j:57007. Zbl 391.57010, (414.57003).

Their "signed graph" we might call a type of Eulerian partially bidirected graph. That is, some edge ends are oriented (hence "partially bidirected"), and every vertex has even degree and at each vertex equally many edge ends point in and out ("Eulerian"). More specially, at each vertex all or none of the edge ends are oriented. (**sg: ori: gen: Appl**)

Gerald G. Brown and Richard D. McBride

1984a Solving generalized networks. *Management Sci.* 30 (1984), 1497–1523. MR 0878883. Zbl 554.90032. (**GN: M(bases)**)

Gerald G. Brown, Richard D. McBride, and R. Kevin Wood

1985a Extracting embedded generalized networks from linear programming problems. *Math. Programming* 32 (1985), no. 1, 11–31. MR 0787741 (86f:90090). Zbl 574.90060.

Identifying largest embedded generalized network matrices (i.e., incidence matrices of real multiplicative gain graphs) in a matrix is NP-complete. Heuristic algorithms for finding such embedded matrices and using them to speed up linear programming. [Annot. 2 Oct 2009.]
(**GN: Incid: Alg**)

Kenneth S. Brown and Persi Diaconis

1998a Random walks and hyperplane arrangements. *Ann. Probab.* 26 (1998), 1813–1854. MR 1675083 (2000k:60138). Zbl 938.60064.

The real hyperplane arrangement representing $-K_n$ is studied in §3D. It leads to a random walk on threshold graphs. (**par: Geom**)

Thomas A. Brown

See also F.S. Roberts.

T.A. Brown, F.S. Roberts, and J. Spencer

1972a Pulse processes on signed digraphs: a tool for analyzing energy demand. Rep. R-926-NSF, Rand Corp., Santa Monica, Cal., March, 1972. (**SDw**)

Thomas A. Brown and Joel H. Spencer

1971a Minimization of ±1 matrices under line shifts. *Colloq. Math.* 23 (1971), 165–171. MR 46 #7059. Zbl 222.05016.

Asymptotic estimates for the Gale–Berlekamp switching game, i.e., $l(K_{r,s})$, the maximum frustration index of signatures of $K_{r,s}$. [Improved by Gordon and Witsenhausen (1972a) and Bowlin (2009a).] Also, exact values stated for $r \leq 4$ [extended by Solé and Zaslavsky (1994a) to $r = 5$, which was corrected and generalized by Bowlin (2009a)]. [*Cf.* Fishburn and Sloane (1989a), Carlson and Stolarski (2004a), and Roth and Viswanathan (2007a, 2008a) on Berlekamp's game, where $r = s$.]
(**sg: Fr**)

William G. Brown, ed.

1980a *Reviews in Graph Theory.* 4 vols. American Math. Soc., Providence, R.I., 1980. Zbl 538.05001.

See esp.: §208: "Signed graphs (+ or − on each edge), balance" (undirected and directed), Vol. 1, pp. 569–571. (**SG, SD**)

Richard A. Brualdi

2011a *The Mutually Beneficial Relationship of Graphs and Matrices.* CBMS Reg. Conf. Ser. Math., No. 115. American Math. Soc., Providence, R.I., 2011 MR 2808017 (2012i:05159). Zbl 1218.05002.

§6.1, "Sign-nonsingular matrices": Signed digraphs, called "weighted digraphs" of $(0, \pm 1)$-matrices such that every matrix with that sign pattern is nonsingular. *Cf.* esp. Maybee *et al.*, van den Driessche *et al.* [Annot. 20 Nov 2011.] (**QM: QSol: sd: Exp**)

§9.4, "ASM patterns": Signed graphs appear in the study of patterns in alternating sign matrices. *Cf.* Brualdi, Kiernan, *et al.* (20xxa). [Annot. 18 Nov 2011.] (**SG: Exp**)

Richard A. Brualdi, Kathleen P. Kiernan, Seth A. Meyer, and Michael W. Schroeder

20xxa Patterns of alternating sign matrices. *Linear Algebra Appl.*, to appear. arXiv:-1104.4075. (**SG**)

Richard A. Brualdi and Nancy Ann Neudauer

1997a The minimal presentations of a bicircular matroid. *Quart. J. Math. Oxford* (2) 48 (1997), 17–26. MR 97m:05065. Zbl 938.05023.

Minimal transversal presentations of $G(\Gamma, \varnothing)$, given Γ. (**Bic**)

Richard A. Brualdi and Herbert J. Ryser

1991a *Combinatorial Matrix Theory.* Encycl. Math. Appl., Vol. 39. Cambridge University Press, Cambridge, Eng., 1991. MR 93a:05087. Zbl 746.05002.

See §7.5. (**QM: QSol, SD, bal**)(**Exp, Ref**)

Richard A. Brualdi and Bryan L. Shader

1991a On sign-nonsingular matrices and the conversion of the permanent into the determinant. In: Peter Gritzman and Bernd Sturmfels, eds., *Applied Geometry and Discrete Mathematics: The Victor Klee Festschrift*, pp. 117–134. DIMACS Ser. Discrete Math. Theor. Comput. Sci., Vol. 4. American Math. Soc., Providence, R.I., 1991. MR 1116343 (92f:15003). Zbl 742.15001.

§1 reviews Seymour and Thomassen (1987a). Thm. 2.1: If two sign-nonsingular $(0, 1, -1)$-matrices have the same 0's (and total support), their signed digraphs are switching equivalent. [Annot. 12 Jun 2012.] (**QM, SD: QSol: Exp**)

1995a *Matrices of Sign-Solvable Linear Systems.* Cambridge Tracts in Math., Vol. 116. Cambridge University Press, Cambridge, Eng., 1995. MR 1358133 (97k:15001). Zbl 833.15002.

Innumerable results and references on signed digraphs are contained herein. (**QM, SD: QSol, QSta**)(**Exp, Ref, Alg**)

Jeroen Bruggeman

See V.A. Traag.

Michael Brundage

1996a From the even-cycle mystery to the L-matrix problem and beyond. M.S. thesis, Dept. of Mathematics, Univ. of Washington, Seattle, 1996.

http://www.math.washington.edu/~brundage/evcy/ (10/1997)

A concise expository survey. Ch. 1: "Even cycles in directed graphs". Ch. 2: "L-matrices and sign-solvability", esp. sect. "Signed digraphs". Ch. 3: "Beyond", esp. sect. "Balanced labellings" (vertices labelled from $\{0, +1, -1\}$ so that from each vertex labelled $\varepsilon \neq 0$ there is an arc to a vertex labelled $-\varepsilon$) and sect. "Pfaffian orientations".
(**SD, Par: Circles, QSol, Alg, VS: Exp, Ref**)

Tom Brylawski

1975a A note on Tutte's unimodular representation theorem. *Proc. Amer. Math. Soc.* 52 (1975), 499–502. MR 54 #7294. Zbl 328.05017.

Implicitly, switching in the bipartite gain graph of a matrix. (**gg: sw**)

2000a A Möbius identity arising from modularity in a matroid bilinear form. *J. Combin. Theory Ser. A* 91 (2000), 622–639. MR 2002a:05059. Zbl 966.05014.

Dowling lattices are an example in §5.1. (**gg: M, Invar**)

Thomas Brylawski and James Oxley

1992a The Tutte polynomial and its applications. In: Neil White, ed., *Matroid Applications*, Ch. 6, pp. 123–225. Encycl. Math. Appl., Vol. 40. Cambridge Univ. Press, Cambridge, Eng., 1992. MR 1165543 (93k:05060). Zbl 769.05026.

§6.4, "The critical problem", §6.4.B, "Minimal and tangential blocks", pp. 171–172: Tangential blocks in Dowling geometries $Q_n(\mathrm{GF}(q)^\times)$, after Whittle (1989a). [Annot. 16 Sept 2011.] (**gg: M**)

J.A. Brzozowski

See C.J. Shi.

Changjiang Bu and Jiang Zhou

2012a Starlike trees whose maximum degree exceed 4 are determined by their Q-spectra. *Linear Algebra Appl.* 436 (2012), 143–151. (**Par: Adj**)

2012b Signless Laplacian spectral characterization of the cones over some regular graphs. *Linear Algebra Appl.* 436 (2012), no. 9, 3634–3641. (**Par: Adj**)

Fred Buckley, Lynne L. Doty, and Frank Harary

1988a On graphs with signed inverses. *Networks* 18 (1988), 151–157. MR 89i:05222. Zbl 646.05061.

"Signed invertible graph" [i.e., sign-invertible graph] = graph Γ such that $A(\Gamma)^{-1} = A(\Sigma)$ for some signed graph Σ. Finds two classes of such graphs. Characterizes sign-invertible trees. [no. Godsil (1985a) and, for a different notion, Greenberg, Lundgren, and Maybee (1984b).]
(**SG: Adj**)

Fred Buckley and Frank Harary

1990a *Distance in Graphs.* Addison–Wesley, Redwood City, Cal., 1990. MR 90m:05002. Zbl 688.05017.

Signed graphs and sign-invertible graphs (Buckley, Doty, and Harary 1988a): pp. 120–122. (**SG: Adj: Exp**)

James R. Burns and Wayland H. Winstead

1982a Input and output redundancy. *IEEE Trans. Systems Man Cybernetics* SMC-12 (1982), no. 6, 785–793.

§ IV: "The computation of contradictory redundancy." Summarized in modified notation: In a signed graph, define $w_{ij}^{\varepsilon}(r)$ = number of walks of length r and sign ε from v_i to v_j. Define an adjacency matrix A by

$a_{ij} = w^+_{ij}(1) + w^-_{ij}(1)\theta$, where θ is an indeterminate whose square is 1. Then $w^+_{ij}(r) + w^-_{ij}(r)\theta = (A^r)_{ij}$ for all $r \geq 1$. [We should regard this computation as taking place in the group ring of the sign group, where the sign group is treated as $\{+1, \theta\}$. The generalization to arbitrary gain graphs and digraphs is obvious.] Other sections also discuss signed digraphs [but have little mathematical content]. (**SD, gd: Adj, Paths**)

F.C. Bussemaker, P.J. Cameron, J.J. Seidel, and S.V. Tsaranov

1991a Tables of signed graphs. EUT Report 91-WSK-01. Dept. of Math. and Computing Sci., Eindhoven Univ. of Technology, Eindhoven, 1991. MR 92g:05001. (**SG: Sw**)

F.C. Bussemaker, D.M. Cvetković, and J.J. Seidel

1976a Graphs related to exceptional root systems. T.H.-Report 76-WSK-05, 91 pp. Dept. of Math., Technological Univ. Eindhoven, Eindhoven, The Netherlands, 1976. Zbl 338.05116.

The 187 simple graphs with eigenvalues ≥ -2 that are not (negatives of) reduced line graphs of signed graphs are found, with computer aid. By Cameron, Goethals, Seidel, and Shult (1976a), all are represented by root systems E_d, $d = 6, 7, 8$. Most interesting is Thm. 2: each such graph is Seidel-switching equivalent to a line graph of a graph. [*Problem.* Explain this within signed graph theory.] (**LG: par: Adj**)

1978a Graphs related to exceptional root systems. In: A. Hajnal and Vera T. Sós, eds., *Combinatorics* (Proc. Fifth Hungar. Colloq., Keszthely, 1976), Vol. 1, pp. 185–191. Colloq. Math. Soc. János Bolyai, 18. North-Holland, Amsterdam, 1978. MR 80g:05049. Zbl 392.05055.

Announces the results of (1976a). (**LG: par: Adj**)

F.C. Bussemaker, R.A. Mathon, and J.J. Seidel

1979a Tables of two-graphs. TH-Report 79-WSK-05. Dept. of Math., Technological Univ. Eindhoven, Eindhoven, The Netherlands, 1979. Zbl 439.05032. (**TG**)

1981a Tables of two-graphs. In: S.B. Rao, ed., *Combinatorics and Graph Theory* (Proc. Sympos., Calcutta, 1980), pp. 70–112. Lecture Notes in Math., 885. Springer-Verlag, Berlin, 1981. MR 84b:05055. Zbl 482.05024.

"The most important tables from" (1979a). (**TG**)

F.C. Bussemaker and A. Neumaier

1992a Exceptional graphs with smallest eigenvalue −2 and related problems. *Math. Comput.* 59 (1992), 583–608. MR 1134718 (93a:05089). Zbl 770.05060.

They are the antibalanced signed graphs with largest eigenvalue −2. Also, largest eigenvalue around −2. Two-graphs and work of Vijayakumar *et al.* are mentioned. [Annot. 29 Apr 2012.] (**TG, LG, Adj**)

Steve Butler

2010a Eigenvalues of 2-edge coverings. *Linear Multilinear Algebra* 58 (2010), 413–423. MR 2663442 (2011g:05173). Zbl 1187.05047.

Generalizing D'Amato (1979a) and Bilu and Linial (2006a). The "signed graph" G is vertex-signed; it is a branched double cover of a signed graph H whose edge signs are incorporated into weights. The interesting new idea is the branching, wherein a vertex may be singly covered. [May the branches correspond to half edges?] Adjacency and normalized Laplacian spectra of G are each obtained from the those of H and a modified $|H|$. [Annot. 9 Mar 2011.] (**VS(Gen: Adj)**)(**SG: cov, Adj**)

Jesper Makholm Byskov, Bolette Ammitzbøll Madsen, and Bjarke Skjernaa

2005a On the number of maximal bipartite subgraphs of a graph. *J. Graph Theory* 48 (2005), no. 2, 127–132. MR 2005h:05099. Zbl 1059.05045.

Bounds on the number of maximal induced bipartite subgraphs. [*Problem*. Generalize to maximal induced balanced subgraphs, equivalently minimal balancing sets of vertices, especially in a signed graph.] (**par: bal**)

S. Cabasino, E. Marinari, P. Paolucci, and G. Parisi

1988a Eigenstates and limit cycles in the SK model. *J. Phys. A* 21 (1988), no. 22, 4201–4210. MR 983779 (89k:82070). (**Phys: SG**)

Leishen Cai and Baruch Schieber

1997a A linear-time algorithm for computing the intersection of all odd cycles in a graph, *Discrete Appl. Math.* 73 (1997), 27–34. MR 97g:05149. Zbl 867.05066.

By the negative-subdivision trick (subdividing each positive edge into two negative ones), the algorithm will find the intersection of all negative circles of a signed graph. (**Par, sg: Fr: Alg**)

Mao-cheng Cai

See F.J. Brandenburg.

Grant Cairns and Yuri Nikolayevsky

2009a Generalized thrackle drawings of non-bipartite graphs. *Discrete Comput. Geom.* 41 (2009), no. 1, 119–134. MR 2470073 (2010a:05059). Zbl 1191.05032.

Thm. 2: Γ, connected and not bipartite, has a generalized thrackle drawing in the orientable surface of genus g iff $-\Gamma$ has an orientation embedding in the nonorientable surface with demigenus $2g-1$. [*Problem*. Generalize to all signed graphs.] (**Par: Top**)

Kyle David Calderhead

2002a *Variations on the Slope Problem.* Doctoral dissertation, University of Minnesota, 2002.

Ch. 6, "Type B analogs", introduces threshold signed graphs and applies signed graphs to the slopes problem (the minimum number of slopes of n points in the plane) for centrally symmetric points. A signed graph is threshold if its double cover is a threshold graph. (**SG**)

Verónica Cambiazo

See J. Aracena.

Peter J. Cameron

See also L. Babai and F.C. Bussemaker.

1977a Automorphisms and cohomology of switching classes. *J. Combin. Theory Ser. B* 22 (1977), 297–298. MR 58 #16382. Zbl 331.05113, (344.05128).

The first step towards (1977b), Thm. 3.1. (**TG: Aut**)

†1977b Cohomological aspects of two-graphs. *Math. Z.* 157 (1977), 101–119. MR 58 #21779. Zbl 353.20004, (359.20004).

Introducing the cohomological theory of two-graphs. A two-graph τ is a 2-coboundary in the complex of GF(2)-cochains on $E(K_n)$. [The 1-cochains are the signed complete graphs, equivalently the graphs that are their negative subgraphs. *Cf.* D.E. Taylor (1977a).] Write Z_i, Z^i, B^i for the i-cycle, i-cocycle, and i-coboundary spaces. Switching a signed complete graph means adding a 1-cocycle to it; a switching class of signed complete graphs is viewed as a coset of Z^1 and is equivalent to a two-graph.

Take a group $\mathfrak{G}$ of automorphisms of τ. Special cohomology elements $\gamma \in H^1(\mathfrak{G}, B^1)$ and $\beta \in H^2(\mathfrak{G}, \tilde{B}^0)$ (where $\tilde{B}^0 = \{0, V(K_n)\}$, the reduced 0-coboundary group) are defined. Thm. 3.1: $\gamma = 0$ iff $\mathfrak{G}$ fixes a graph in τ. Thm. 5.1: $\beta = 0$ iff $\mathfrak{G}$ can be realized as an automorphism group of the canonical double covering graph of τ (viewing τ as a switching class of signed complete graphs). Conditions are explored for the vanishing of γ (related to Harries and Liebeck (1978a)) and β.

Z^1 is the annihilator of Z_1 = the space of even-degree simple graphs; the theorems of Mallows and Sloane (1975a) follow immediately. More generally: Lemma 8.2: Z^i is the annihilator of Z_i. Thm. 8.3: The numbers of isomorphism types of i-cycles and i-cocycles are equal, for $i = 1, \ldots, n-2$.

§8 concludes with discussion of possible generalizations, e.g., to oriented two-graphs (replacing GF(2) by GF(3)$^\times$) and double coverings of complete digraphs (Thms. 8.6, 8.7). [A full ternary analog is developed in Cheng and Wells (1986a).] (**TG: Sw, Aut, Enum, Geom**)

1979a Cohomological aspects of 2-graphs. II. In: C.T.C. Wall, ed., *Homological Group Theory* (Proc. Sympos., Durham, 1977), Ch. 11, pp. 241–244. London Math. Soc. Lect. Note Ser. 36. Cambridge Univ. Press, Cambridge, 1979. MR 81a:05061. Zbl 461.20001.

Exposition of parts of (1977b) with a simplified proof of the connection between β and γ. (**TG: Aut, Enum, Geom, Exp**)

1980a A note on generalized line graphs. *J. Graph Theory* 4 (1980), 243–245. MR 81j:05089. Zbl 403.05048, (427.05039).

[For generalized line graphs see Zaslavsky (1984c).] If two generalized line graphs are isomorphic, their underlying graphs and cocktail-party attachments are isomorphic, with small exceptions related to exceptional isomorphisms and automorphisms of root systems. The proof, along the lines of Cameron, Goethals, Seidel, and Shult (1976a), employs the canonical vector representation of the underlying signed graph. (**sg: LG: Aut, Geom**)

1983a Automorphism groups of graphs. Ch. 4 in: Lowell W. Beineke and Robin J. Wilson, eds., *Selected Topics in Graph Theory 2*, pp. 89–127. Academic Press, London, 1983. MR 797250 (86i:05079). Zbl 536.05037.

§8, "Switching": Graph switching, graph switching classes. Existence of a "representative": a graph in a switching class that has the same automorphism group as the switching class. §9, "Digraphs": Switching classes of tournaments on pp. 117–118. Switching a digraph means reversing all edges between $X \subseteq V$ and X^c. [Annot. 27 Dec 2010.] (**TG: Sw: Aut: Exp**)

1994a Two-graphs and trees. Graph Theory and Applications (Proc., Hakone, 1990). *Discrete Math.* 127 (1994), 63–74. MR 95f:05027. Zbl 802.05042.

Let T be a tree. Construction 1 (simplifying Seidel and Tsaranov (1990a)): Take all triples of edges such that none separates the other two. This defines a two-graph on $E(T)$ [whose underlying signed complete graph is described by Tsaranov (1992a)]. Construction 2: Choose $X \subseteq$

$V(T)$. Take all triples of end vertices of T whose minimal connecting subtree has its trivalent vertex in X. The two-graphs $(V, \mathcal{T})$ that arise from these constructions are characterized by forbidden substructures, namely, the two-graphs of (1) C_5 and C_6; (2) C_5. Also, trees that yield identical two-graphs are characterized. **(TG)**

2007a Orbit counting and the Tutte polynomial. In: Geoffrey Grimmett and Colin McDiarmid, eds., *Combinatorics, Complexity, and Chance: A Tribute to Dominic Welsh*, pp. 1–10. Oxford Lect. Ser. Math. Appl., Vol. 34. Oxford Univ. Press, Oxford, 2007. MR 2008a:05043. Zbl 1122.05022.

1995a Counting two-graphs related to trees. *Electronic J. Combin.* 2 (1995), Research Paper 4. MR 95j:05112. Zbl 810.05031.

Counting two-graphs of the types constructed in (1994a). **(TG: Enum)**

P.J. Cameron, J.M. Goethals, J.J. Seidel, and E.E. Shult

††1976a Line graphs, root systems, and elliptic geometry. *J. Algebra* 43 (1976), 305–327. MR 56 #182. Zbl 337.05142. Repr. in Seidel (1991a), pp. 208–230.

The essential idea is that graphs with least eigenvalue ≥ -2 are represented by the angles of root systems. It follows that line graphs are so represented. [Similarly, signed graphs with largest eigenvalue ≤ 2 are represented by the inner products of root systems, as in Vijayakumar *et al*. These include the line graphs of signed graphs as in Zaslavsky (1984c), since simply signed graphs are represented by B_n or C_n with a few exceptions. The representation of ordinary graphs by all-negative signed graphs is motivated in Zaslavsky (1984c).]

(LG: sg: Adj, Geom, Sw)

Peter J. Cameron, Bill Jackson, and Jason D. Rudd

2008a Orbit-counting polynomials for graphs and codes. *Discrete Math.* 308 (2008), 920–930. MR 2378927 (2009e:05140). Zbl 1133.05030. **(sg: Invar: Flows)**

Peter J. Cameron and Charles R. Johnson

2006a The number of equivalence classes of symmetric sign patterns. Int. Workshop Combin., Linear Algebra, Graph Coloring. *Discrete Math.* 306 (2006), no. 23, 3074–3077. MR 2273136 (2007j:05105). Zbl 1105.05034.

The number of signatures of K_n°, the complete graph with loops, under symmetry, switching, and negation. [Annot. 12 Aug 2012.]

(sg: Adj: Invar, sw, tg)

P.J. Cameron, J.J. Seidel, and S.V. Tsaranov

1994a Signed graphs, root lattices, and Coxeter groups. *J. Algebra* 164 (1994), 173–209. MR 95f:20063. Zbl 802.05043.

A generalized Coxeter group $\mathrm{Cox}(\Sigma)$ and a Tsaranov group $\mathrm{Ts}(\Sigma)$ are defined via Coxeter relations and an extra relation for each negative circle in Σ. They generalize Coxeter groups of tree Coxeter graphs and the Tsaranov groups of a two-graph ($|\Sigma| = K_n$; see Seidel and Tsaranov (1990a)). A new operation of "local switching" is introduced, which changes the edge set of Σ but preserves the associated groups.

§2, "Signed graphs", proves some well-known properties of switching and reviews interesting data from Bussemaker, Cameron, Seidel, and Tsaranov (1991a). §3, "Root lattices and Weyl groups": The "intersection matrix" $2I + A(\Sigma)$ is a hyperbolic Gram matrix of a basis of $\mathbb{R}^n$ whose vectors form only angles $\pi/2, \pi/3, 2\pi/3$. To these vectors are associated the lattice $L(\Sigma)$ of their integral linear combinations and the Weyl group $W(\Sigma)$ generated by reflecting along the vectors. W is finite iff $2I + A(\Sigma)$ is positive definite (Thm. 3.1). *Problem* 3.6. Determine which Σ have this property. §4 introduces local switching to partially solve *Problem* 4.1: Which signed graphs generate the same lattice? Results and some experimental data are reported. All-negative signed graphs play a special role. Definition of *local switching at* v: (1) switch so the edges at v are positive, (2) divide the components of the negative subgraph of the neighborhood of v into two halves J, K, (3) add negative edges joining all vertices of J to all those of K, (4) negate all edges from v to J, (5) reverse the switching in step (1). [See Isihara (2007a) for more.] §6, "Coxeter groups": The relationship between the Coxeter and Weyl groups of Σ. Cox(Σ) is Cox($|\Sigma|$) with additional relations for antinegative (i.e., negative in $-\Sigma$) induced circles. §7: "Signed complete graphs". §8: "Tsaranov groups" of signed K_n's §9: "Two-graphs arising from trees" (as in Seidel and Tsaranov (1990a)).

Dictionary: "(Γ, f)" = Σ = (Γ, σ). "Fundamental signing" = all-negative signing, giving the antibalanced switching class. "The balance" of a cycle (i.e., circle) = its sign $\sigma(C)$; "the parity" = $\sigma(-C)$ where $-C = C$ with all signs negated. "Even" = positive and "odd" = negative (referring to "parity"). "The balance" of Σ = the partition of all circles into positive and negative classes $\mathcal{C}^+$ and $\mathcal{C}^-$; this is the bias on $|\Sigma|$ due to the signing and should not be confused with the customary meaning of "balance", i.e., all circles are positive.

[A more natural definition of the intersection matrix would be $2I - A$. Then signs would be negative to those in the paper. The need for "parity" would be obviated, ordinary graphs would correspond to all-positive signings (and those would be "fundamental"), and the extra Coxeter relations would pertain to negative induced circles.]

(**SG: Adj, Geom, Sw(Gen), lg**)

Peter J. Cameron and Sam Tarzi

2004a Switching with more than two colours. *European J. Combin.* 25 (2004), no. 2, 169–177. MR 2005j:05059. Zbl 033.05038.

The edges of K_n are colored by m colors. Thm.: For $m > 2$, the combined action of $\mathfrak{S}_n$ on vertices and $\mathfrak{S}_m$ on colors is transitive on m-edge-colored complete graphs for finite n but not for infinite n.

(**SGc: Gen: Sw**)

P.J. Cameron and Albert L. Wells, Jr.

1986a Signatures and signed switching classes. *J. Combin. Theory Ser. B* 40 (1986), 344–361. MR 87m:05115. Zbl 591.05061. (**SG: TG: Gen**)

Paul Camion

1963a Caracterisation des matrices unimodulaires. *Cahiers Centre Études Recherche Opér.* 5 (1963), 181–190. MR 31 #3352. Zbl 124.00901.

Camion's signing algorithm (implicitly) finds a set of sign reversals to balance a bipartite signed graph.

1965a Characterization of totally unimodular matrices. *Proc. Amer. Math. Soc.* 16 (1965), 1068–1073. MR 31 #4802. Zbl 134.25201.

1968a Modules unimodulaires. *J. Combin. Theory* 4 (1968), 301–362. MR 48 #5918.

Zbl 174.29504.

2006a Unimodular modules. *Discrete Math.* 306 (2006), no. 19-20, 2355–2382. MR 2007e:05096. Zbl 1099.13021.

Sue Ann Campbell

See J. Bélair.

Manoel Campelo and Gérard Cornuéjols

2009a The Chvátal closure of generalized stable sets in bidirected graphs. LAGOS'09? Latin-American Algorithms, Graphs and Optimization Symposium. *Electronic Notes Discrete Math.* 35 (2009), 89–95. MR 2579413 (no rev).

The generalized stable set polyhedron of B is (equivalent to) conv($\mathbb{Z}^n \cap \{0 \leq x \in \mathbb{R}^n : \mathrm{H(B)}x \leq b\}$ where $b \in \mathbb{Z}^m$, $m = |E|$. Dictionary: "directed edge" = positive, "undirected edge" = negative; "odd cycle" = negative circle. [Annot. 9 June 2011.] (**sg: ori, Incid, Geom**)

E. Rodney Canfield

See E.A. Bender.

Chun Zheng Cao

See X.X. Zhu.

D.S. Cao

See R. Simion.

Domingos M. Cardoso

See also N.M.M. Abreu and I. Gutman.

Domingos M. Cardoso, Dragoš Cvetković, Peter Rowlinson, and Slobodan K. Simić

2008a A sharp lower bound for the least eigenvalue of the signless Laplacian of a non-bipartite graph. *Linear Algebra Appl.* 429 (2008), no. 11-12, 2770–2780. MR 2455532 (2009i:05145). Zbl 1148.05046.

See Cvetković, Rowlinson, and Simić (2007a). Thm.: $\min_\Gamma \lambda_1(K(-\Gamma))$, for connected, nonbipartite Γ with $|V| = n$ is attained iff Γ is K_3 with an attached path. [*Problem.* Generalize to connected, unbalanced signed graphs.] [Annot. 4 Sept 2010.] (**Par: Adj**)

Jordan Carlson and Daniel Stolarski

2004a The correct solution to Berlekamp's switching game. *Discrete Math.* 287 (2004), 145–150. MR 2005d:05005. Zbl 1054.94023.

The minimum frustration index of a signed $K_{n,n}$ for $n = 10, 11, 12$ and bounds up to 20. Corrects and extends Fishburn and Sloane (1989a). (**sg: fr**)

J. Cartes

See J.F. Valdés.

Dorwin Cartwright

See also T.C. Gleason; Harary, Norman, and Cartwright (1965a, etc.)

Dorwin Cartwright and Terry C. Gleason

1966a The number of paths and cycles in a digraph. *Psychometrika* 31 (1966), 179–199. MR 33 #5377. Zbl (e: 143.43702). (**SD: Adj, Paths**)

Dorwin Cartwright and Frank Harary

1956a Structural balance: a generalization of Heider's theory. *Psychological Rev.* 63 (1956), 277–293. Repr. in: Dorwin Cartwright and Alvin Zander, eds., *Group Dynamics: Research and Theory*, Second Edition, pp. 705–726. Harper and

Row, New York, 1960. Also reprinted in: Samuel Leinhardt, ed., *Social Networks: A Developing Paradigm*, pp. 9–25. Academic Press, New York, 1977.

Expounds Harary (1953a, 1955a) with sociological discussion. Proposes to measure imbalance by the proportion of balanced circles (the "degree of balance") or balanced circles of length $\leq k$ ("degree of k-balance"). (**PsS, SG: Bal, Fr**)

1968a On the coloring of signed graphs. *Elem. Math.* 23 (1968), 85–89. MR 38 #2053. Zbl 155, 317 (e: 155.31703).

"Coloring" is clustering as in Davis (1967a). Thm. 1 adds a bit to Davis (1967a). Thm. 3: The clustering is unique $\iff$ all components of Σ^+ are adjacent. (**SG: Clu**)

1970a Ambivalence and indifference in generalizations of structural balance. *Behavioral Sci.* 15 (1970), 497–513. (**SD, Bal**)

1977a A graph theoretic approach to the investigation of system-environment relationships. *J. Math. Sociology* 5 (1977), 87–111. MR 56 #2477. Zbl 336.92026. (**SD: Clu**)

1979a Balance and clusterability: an overview. In: Paul W. Holland and Samuel Leinhardt, eds., *Perspectives on Social Network Research* (Proc. Sympos., Dartmouth Coll., Hanover, N.H., 1975), Ch. 3, pp. 25–50. Academic Press, New York, 1979. (**SG, SD, VS: Bal, Fr, Clu, Adj: Exp**)

Adolfo Casari

See F. Barahona.

Paul A. Catlin

1979a Hajós' graph-coloring conjecture: variations and counterexamples. *J. Combin. Theory Ser. B* 26 (1979), 268–274. MR 81g:05057. Zbl 385.05033, 395.05033.

Thm. 2: If Γ is 4-chromatic, $[-\Gamma]$ contains a subdivision of $[-K_4]$ (an "odd-K_4"). [*Question.* Can this possibly be a signed-graph theorem? For instance, should it be interpreted as concerning the 0-free (signed) chromatic number of $-\Gamma$?] (**par: col**)

M. Catral, D.D. Olesky, and P. van den Driessche

2009a Allow problems concerning spectral properties of sign pattern matrices: A survey. *Linear Algebra Appl.* 430 (2009), no. 11-12, 3080–3094. MR 2517861 (2010i:15066). Zbl 1165.15009.

D is the signed digraph of a square sign-pattern matrix S. Thm. 3.1: If the spectrum of A with signs S is arbitrary, D has positive and negative disjoint cycle unions of all orders. Thm. 4.1: If the inertia is arbitrary, D has a positive and a negative loop and a negative digon. [Annot. 4 Nov 2011.] (**SG: QM, Exp**)

Michael S. Cavers

2010a On reducible matrix patterns. *Linear Multilinear Algebra* 58 (2010), no. 2, 257–267. MR 2641538 (2011b:15072). Zbl 1189.15010. (**SD: QM**)

Michael S. Cavers and Kevin N. Vander Meulen

2005a Spectrally and inertially arbitrary sign patterns. *Linear Algebra Appl.* 394 (2005), 53–72. MR 2100576 (2005f:15008). Zbl 1065.15009.

Lem. 5.1: An inertially arbitrary sign pattern contains a negative digon. [Annot. 5 Nov 2011.] (**QM: sd, sw**)

Seth Chaiken

1982a A combinatorial proof of the all minors matrix tree theorem. *SIAM J. Algebraic Discrete Methods* 3 (1982), 319–329. MR 83h:05062. Zbl 495.05018.

§4: "Extension to signed graphs". Generalizing Zaslavsky (1982a), an all-minors matrix-tree theorem for weighted signed digraphs and a corollary for weighted signed graphs. Given: a signed graph on vertex set $[n]$. For a Kirchoff (or "Laplace")-type $n \times n$ matrix K (A in the paper), $K(\bar{U}, \bar{W})$ is K with the rows indexed by U and the columns indexed by W deleted. Take $U, W \subseteq V$ with $|U| = |W| = k \leq n$. Then $\det K(\bar{U}, \bar{W})$ is a sum of terms, one for each independent set F of rank $n-k$ in $G(\Sigma)$ in which each tree component contains just one vertex from U and one from W. Each term has a sign depending partly on the number of negative paths by which F links U to W and partly on the linking pattern, and with magnitude 4^c·(weight product of F), where $c = \#$ of circles in F. [The credit to Zaslavsky is overly generous: only the case $U = W = \varnothing$ is his; the others are new.] The digraph version replaces 4 by 2 and imposes conditions on arc directions in the tree and nontree components of F.

A brief remark describes a gain-graphic ("voltage-graphic") generalization. (**SD, SG, GG: Adj, Incid, m**)

1996a Oriented matroid pairs, theory and an electrical application. In: Joseph E. Bonin, James G. Oxley, and Brigitte Servatius, eds., *Matroid Theory* (Proc., Seattle, 1995), pp. 313–331. Contemp. Math., Vol. 197. Amer. Math. Soc., Providence, R.I., 1996. MR 97e:05058.

Connects a problem on common covectors of two subspaces of $\mathbb{R}^m$, and more generally of a pair of oriented matroids, to the problem of sign-solvability of a matrix and the even-cycle problem for signed digraphs. (**QSol, sd: Par, Alg**)

1996b Open problem 5. A problem about common covectors and bases in oriented matroid pairs. In: Joseph E. Bonin, James G. Oxley, and Brigitte Servatius, eds., *Matroid Theory* (Proc., Seattle, 1995), pp. 415–417. Contemp. Math., Vol. 197. Amer. Math. Soc., Providence, R.I., 1996.

Possible generalizations to oriented matroids of sign-nonsingularity of a matrix. (**QSol, SD: Par**)

Seth Chaiken, Christopher R.H. Hanusa, and Thomas Zaslavsky

2010a Nonattacking queens in a rectangular strip. *Ann. Combin.* 14 (2010), 419–441. MR 2776757 (2012d:05034). Zbl 1233.05022.

Affinographic hyperplanes and rooted integral gain graphs, from Forge and Zaslavsky (2007a), imply the structure of formulas counting nonattacking arrangements of identical chess pieces in an $m \times n$ strip, as a function of n. (**GG: Geom, Invar**)

Vijaya Chandru, Collette R. Coullard, and Donald K. Wagner

1985a On the complexity of recognizing a class of generalized networks. *Operations Res. Letters* 4 (1985), 75–78. MR 87a:90144. Zbl 565.90078.

Determining whether a gain graph with real multiplicative gains has a balanced circle, i.e., is not contrabalanced, is NP-hard. So is determining whether a real matrix is projectively equivalent to the incidence matrix of a contrabalanced real gain graph. (**GN, Bic: Incid, Alg**)

Chung-Chien Chang and Cheng-Ching Yu

1990a On-line fault diagnosis using the signed directed graph. *Industrial and Engineering Chem. Res.* 29 (1990), 1290–1299.

Modifies the method of Iri, Aoki, O'Shima, and Matsuyama (1979a) of constructing the diagnostic signed digraph, e.g. by considering transient and steady-state situations. (**SD: Appl, Ref**)

Gerard J. Chang

See J.H. Yan.

Michael D. Chang

See M. Engquist.

Ting-Chung Chang [Ting-Jung Chang]

See T.J. Chang.

Ting-Jung Chang [Ting-Chung Chang]

Ting-Jung Chang and Bit-Shun Tam

2010a Graphs with maximal signless Laplacian spectral radius. *Linear Algebra Appl.* 432 (2010), no. 7, 1708–1733. MR 2592913 (2011e:15014).

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Ting-Jung Chang [as Ting-Chung Chang] and Bit-Shun Tam

2011a Connected graphs with maximal Q-index: The one-dominating-vertex case. *Linear Algebra Appl.* 435 (2011), no. 10, 2451–2461. MR 2811129 (2012d:05220). Zbl 1222.05029. (**Par: Adj**)

Ting-Jung Chang, Bit-Shun Tam, and Shu-Hui Wu

2011a Theorems on partitioned matrices revisited and their applications to graph spectra. *Linear Algebra Appl.* 434 (2011), 559–581. MR 2741241 (2012g:05131). Zbl 1225.05160. (**Par: Adj**)

Claudine Chaouiya

See A. Naldi.

Moses Charikar

See also N. Ailon.

Moses Charikar, Venkatesan Guruswami, and Anthony Wirth

2003a Clustering with qualitative information. In: *Proceedings of the 44th Annual IEEE Symposium on Foundations of Computer Science* (FOCS'03), pp. 524–533. IEEE, 2003.

Conference version of (2005a). (**SG: WG: Clu: Alg**)

2005a Clustering with qualitative information. Learning Theory 2003. *J. Comput. System Sci.* 71 (2005), no. 3, 360–383. MR 2168358 (2006f:68141). Zbl 1094.68075. (**SG: WG: Clu: Alg**)

A. Charnes, M. Kirby, and W. Raike

1966a Chance-constrained generalized networks. *Operations Res.* 14 (1966), 1113–1120. Zbl (e: 152.18302). (**GN**)

A. Charnes and W.M. Raike

1966a One-pass algorithms for some generalized network problems. *Operations Res.* 14 (1966), 914–924. Zbl (e: 149.38106). (**GN: Incid**)

Gary Chartrand

See also M. Behzad.

1977a *Graphs as Mathematical Models.* Prindle, Weber and Schmidt, Boston, 1977. MR 58 #9947. Zbl 384.05029.

[Repr. (1985a).] (**SG: Bal, Clu**)

1985a *Introductory Graph Theory.* Dover Publications, New York, 1985. MR 86c:05001. "Corrected reprint" of (1977a). (**SG: Bal, Clu**)

Gary Chartrand, Heather Gavlas, Frank Harary, and Michelle Schultz

1994a On signed degrees in signed graphs. *Czechoslovak Math. J.* 44(119) (1994), 677–690. MR 95g:05084. Zbl 837.05110.

Net degree sequences (i.e., $d^+ - d^-$; called "signed degree sequences") of signed simple graphs. A Havel–Hakimi-type reduction formula, but with an indeterminate length parameter; a determinate specialization to complete graphs. A necessary condition for a sequence to be a net degree sequence. Examples: paths, stars, double stars. [Continued in Yan, Lih, Kuo, and Chang (1997a). Solved in Michael (2002a).]

[This is a special case of weighted degree sequences of K_n with integer edge weights chosen from a fixed interval of integers. Here the interval is $[-1, +1]$. The theory of such degree sequences is due to V. Chungphaisan, Conditions for sequences to be r-graphic, *Discrete Math.* 7 (1974), 31–39. MR 50 #4391. Michael (2002a) characterizes net degree sequences by noticing this connection.] (**SGw: Invar**)

[One can interpret net degrees as the net indegrees ($d^{\text{in}} - d^{\text{out}}$) of certain bidirected graphs. Change the positive (negative) edges to extroverted (resp., introverted). Then we have the net indegree sequence of an oriented $-\Gamma$. *Problem* 1. Generalize to all bidirected (simple, or simply signed) graphs, especially K_n's. *Problem* 2. Find an Erdős–Gallai-type characterization of net degree sequences of signed simple graphs. [Solved by Michael (2002a).] *Problem* 3. Characterize the separated signed degree sequences of signed simple graphs, where the separated signed degree is $(d^+(v), d^-(v))$. *Problem* 4. Generalize Problem 3 to edge k-colorings of K_n.] (**SG: ori: Invar**)

Gary Chartrand, Frank Harary, Hector Hevia, and Kathleen A. McKeon

1992a On signed graphs with prescribed positive and negative graphs. *Vishwa Int. J. Graph Theory* 1 (1992), 9–18. MR 93m:05095.

What is the smallest order of an edge-disjoint union of two (isomorphism types of) simple graphs, Γ and Γ'? Bounds, constructions, and special cases. (The union is called a signed graph with Γ and Γ' as its positive and negative subgraphs.) Thm. 13: If Γ' is bipartite (i.e., the union is balanced) with color classes V_1' and V_2', the minimum order $= \min(|V_1'|, |V_2'|) + \max(|V|, |V_1'|, |V_2'|)$. (**wg**)(**SG: Bal**)

Guy Chaty

1988a On signed digraphs with all cycles negative. *Discrete Appl. Math.* 20 (1988), 83–85. MR 89d:05148. Zbl 647.05028.

Clarifies the structure of "free cyclic" digraphs and shows they include strong "upper" digraphs (see Harary, Lundgren, and Maybee (1985a)). (**SD: Str**)

P.D. Chawathe and G.R. Vijayakumar

1990a A characterization of signed graphs represented by root system D_∞. *European J. Combin.* 11 (1990), 523–533. MR 91k:05071. Zbl 764.05090.

A list of the 49 switching classes of signed simple graphs that are the forbidden induced subgraphs for a signed simple graph to be a reduced line graph of a simply signed graph without loops or half edges. The graphs have orders 4, 5, and 6. [See several other works of Vijayakumar et al.] (**SG: adj, LG, Geom, incid**)

Shuchi Chawla

See N. Bansal.

Beifang Chen and Shuchao Li

2011a The number of nowhere-zero tensions on graphs and signed graphs. *Ars Combin.* 102 (2011), 47–64. (**SG**)

Beifang Chen and Jue Wang

†2009a The flow and tension spaces and lattices of signed graphs. *European J. Combin.* 30 (2009), 263–279. MR 2460231 (2009i:05102). Zbl 1198.05085.

Introduces cuts, and directed circuits and cuts, of a signed graph; and the cycle (or circuit) and cut (or cocycle) spaces of a signed graph over a commutative, unital ring in which 2 is invertible. Definitions, basic theory, and graphical proofs. Orthogonal complementarity between real, or integral, circuit and cut spaces. Relationships between real and integral spaces. Interpretations in terms of flows and tensions.

A cut is an edge set $U := E\langle X, X^c\rangle \cup U_X$ where $X \subseteq V$ and U_X is a minimal balancing set of $E{:}X$. A minimal cut is a bond, i.e., a cocircuit in $G(\Sigma)$. A circuit or cut has two possible "directions". A minimal directed cut need not be a directed bond. The indicator vectors of directed circuits generate the cycle ("circuit") space; the indicator vectors of directed cuts generate the cocycle ("cut") space.

The flow space or lattice is the real or integral null space of the incidence matrix. The tension space or lattice is the real or integral row space. The spaces equal lattices equal the real cycle and cut spaces and the lattices are their integral parts. Not every integral flow is in the integral span of circuit indicator vectors; but every integral tension is spanned by cut indicator vectors.

[Based upon and extending parts of J. Wang (2007a).]

(**SG: Str, Ori, Incid**)

2010a Torsion formulas for signed graphs. *Discrete Appl. Math.* 158 (2010), 1148–1157. MR 2629892 (2011j:05131).

[Based upon part of J. Wang (2007a).] (**SG**)

20xxa Classification of indecomposable flows of signed graphs. Submitted. arXiv:-1112.0642. (**SG: Flows**)

Beifang Chen, Jue Wang, and Thomas Zaslavsky

20xxa Resolution of irreducible integral flows on a signed graph. In preparation.

Irreducible integral flows include circuit flows as well as others of a complicated and unexpected nature. Resolved by lifting to the signed covering graph. [Based on part of J. Wang (2007a).] (**SG: Incid, Str**)

Doron Chen

See also E.G. Boman.

Doron Chen and Sivan Toledo

2005a Combinatorial characterization of the null spaces of symmetric H-matrices. *Linear Algebra Appl.* 392 (2004), 71–90. MR 2005h:15016. Zbl 1061.65028.

Certain matrices are related to gain graphs and others to signed graphs. (**GG, SG**)

Jianer Chen, Jonathan L. Gross, and Robert G. Rieper

1994a Overlap matrices and total imbedding distributions. *Discrete Math.* 128 (1994), 73–94. MR 95f:05031. Zbl 798.05017. (**SG: Top, Sw**)

Ya-Hong Chen, Rong-Ying Pan, and Xiao-Dong Zhang

2011a Two sharp upper bounds for the signless Laplacian spectral radius of graphs. *Discrete Math. Algorithms Appl.* 3 (2011), no. 2, 185–191. MR 2822283 (2012f:-05173). Zbl 1222.05149.

(**Par: Adj**)

Yanqing Chen and Ligong Wang

2010a Sharp bounds for the largest eigenvalue of the signless Laplacian of a graph. *Linear Algebra Appl.* 433 (2010), no. 5, 908–913. MR 2658641 (2011h:05149). Zbl 1215.05100.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Zhi-Hong Chen, Ying-Qiang Kuang, and Hong-Jian Lai

1999a Connectivity of cycle matroids and bicircular matroids. *Ars Combin.* 52 (1999), 239–250. MR 2001d:05032. Zbl 977.05027.

The relationship between graph structure and the Tutte, verticial, and cyclic connectivities of the bicircular matroid. (**Bic: Str**)

Zhi-Hong Chen, Hong-Jian Lai, Xiankun Zhang, and Lei Xu

1998a Group coloring and group connectivity of graphs. Proc. Twenty-ninth Southeastern Int. Conf. on Combinatorics, Graph Theory and Computing (Boca Raton, Fla., 1998). *Congr. Numer.* 134 (1998), 123–130. MR 99j:05068. Zbl 952.05031.

§2 summarizes Lai and Zhang (2002a). §3 concerns the duality between [abelian] group colorability and [abelian] group connectivity. (**GG: Col**)

Bo Cheng and Bolian Liu

2008a The base sets of primitive zero-symmetric sign pattern matrices. *Linear Algebra Appl.* 428 (2008), 715–73. MR 2382083 (2009c:15028). Zbl 1135.15014.

The Abelson–Rosenberg (1958a) algebra is employed, with symbols $0, 1, -1, \#$ for o, p, n, a. "Generalized sign pattern matrix": # entries are allowed. "Generalized signed digraph": #-arcs are allowed. (**QM: SD**)

2010a Primitive zero-symmetric sign pattern matrices with the maximum base. *Linear Algebra Appl.* 433 (2010), no. 2, 365–379. MR 2645090 (2011e:15058). Zbl 1193.15029. (**QM: SD**)

Ying Cheng

1986a Switching classes of directed graphs and H-equivalent matrices. *Discrete Math.* 61 (1986), 27–40. MR 88a:05075. Zbl 609.05039.

This article studies what are described as $\mathbb{Z}_4$-gain graphs Φ with underlying simple graph Γ. [However, see below.] They are regarded as digraphs D, the gains being determined by D as follows: $\varphi(u, v) = 1$ or 2 if (u, v) is an arc, 2 or 3 if (v, u) is an arc. [N.B. Γ is not uniquely determined by D.] Cheng's "switching" is gain-graph switching but only by switching functions $\eta : V \to \{0, 2\}$; I will call this "semiswitching". His "isomorphisms" are vertex permutations that are automorphisms of Γ; I will call them "Γ-isomorphisms". The objects of study are equivalence classes under semiswitching (semiswitching classes) or semiswitching and Γ-isomorphism (semiswitching Γ-isomorphism classes). Prop. 3.1 concerns adjacency of vertex orbits of a Γ-isomorphism that preserves

a semiswitching class (call it a Γ-automorphism of the class). Thm. 4.3 gives the number of semiswitching Γ-isomorphism classes. Thm. 5.2 characterizes those Γ-automorphisms of a semiswitching class that fix an element of the class; Thm. 5.3 characterizes the Γ-isomorphisms g that fix an element of every g-invariant semiswitching class.

[Likely the right viewpoint, as is hinted in §6, is that the edge labels are not $\mathbb{Z}_4$-gains but weights from the set $\{\pm1, \pm2, \ldots, \pm k\}$ with $k = 2$. Then semiswitching is ordinary signed switching, and so forth. However, I forbear to reinterpret everything here.]

In §6, $\mathbb{Z}_4$ is replaced by $\mathbb{Z}_{2k}$ [but this should be $\{\pm1, \pm2, \ldots, \pm k\}$]; semiswitching functions take values $0, k$ only. Generalizations of §§3, 4 are sketched and are applied to find the number of H-equivalent matrices of given size with entries $\pm1, \pm1, \ldots, \pm k$. ($H$- [or Hadamard] equivalence means permuting rows and columns and scaling by -1.)

(**sg, wg, GG: Sw, Aut, Enum**)

Ying Cheng and Albert L. Wells, Jr.

1984a Automorphisms of two-digraphs. (Summary.) Proc. Fifteenth Southeastern Conf. on Combinatorics, Graph Theory and Computing (Baton Rouge, 1984). *Congressus Numer.* 45 (1984), 335–336. MR 86c:05004c (volume).

A two-digraph is a switching class of $\mathbb{Z}_3$-gain graphs based on K_n.

(**gg, SD: Sw, Aut**)

†1986a Switching classes of directed graphs. *J. Combin. Theory Ser. B* 40 (1986), 169–186. MR 87g:05104. Zbl 565.05034, (579.05027).

This exceptionally interesting paper treats a digraph as a ternary gain graph Φ (i.e., with gains in $\mathrm{GF}(3)^+$) based on K_n. A theory of switching classes and triple covering graphs, analogous to that of signed complete graphs (and of two-graphs) is developed. The approach, analogous to that in Cameron (1977b), employs cohomology. The basic results are those of general gain-graph theory specialized to the ternary gain group and graph K_n.

The main results concern a switching class $[\Phi]$ of digraphs and an automorphism group $\mathfrak{A}$ of $[\Phi]$. §3, "The first invariant": Thm. 3.2 characterizes, by a cohomological obstruction γ, the pairs $([\Phi], \mathfrak{A})$ such that some digraph in $[\Phi]$ is fixed. Thm. 3.5 is an [interestingly] more detailed result for cyclic $\mathfrak{A}$. §4: "Triple covers and the second invariant". Digraph triple covers of the complete digraph are considered. Those that correspond to gain covering graphs of ternary gain graphs Φ are characterized ("cyclic triple covers", pp. 178–180). Automorphisms of Φ and its triple covering $\tilde{\Phi}$ are compared. Given $([\Phi], \mathfrak{A})$, Thm. 4.4 finds the cohomological obstruction β to lifting $\mathfrak{A}$ to $\tilde{\Phi}$. Thm. 4.7 establishes an equivalence between γ and β in the case of cyclic $\mathfrak{A}$.

§5: "Enumeration". Thm. 5.1 gives the number of isomorphism types of switching classes on n vertices, based on the method of Wells (1984a) for signed graphs. §6: "The fixed signing property". Thm. 6.1 characterizes the permutations of $V(K_n)$ that fix a gain graph in every invariant switching class, based on the method of Wells (1984a).

Dictionary: "Alternating function" on $X \times X$ = $\mathrm{GF}(3)^+$-valued gain function on K_X.

[See Babai and Cameron (2000a) for a treatment of nowhere-zero ternary gain graphs based on K_n.] (**gg: Sw, Aut, Enum, Cov**)

William K. Cheung

See B. Yang.

Sergei Chmutov

2009a Generalized duality for graphs on surfaces and the signed Bollobás–Riordan polynomial. *J. Combin. Theory Ser. B* 99 (2009), no. 3, 617–638. MR 2507944 (2010f:05046). Zbl 1172.05015. arXiv:

Sign-colored graphs embedded in a surface (Chmutov and Pak 2007a). Duality with respect to an edge subset, applied to a sign-colored Bollobás–Riordan polynomial, gives a polynomial duality. [Further developments in Vignes-Tourneret (2009a) and Krushkal (2011a).]

(**SGc: Top, Invar**)

Sergei Chmutov and Igor Pak

2007a The Kauffman bracket of virtual links and the Bollobás-Riordan polynomial. *Moscow Math. J.* 7 (2007), no. 3, 409–418. MR 2343139 (2008h:57006). Zbl 1155.57004.

Sign-colored graphs embedded in a surface (orientable or not, independently of the edge signs. [The orientation properties of the ribbons make a signed graph, independent of the sign-colors.] (**SGc: Top, Invar**)

Hyeong-ah Choi, Kazuo Nakajima, and Chong S. Rim

1989a Graph bipartization and via minimization. *SIAM J. Discrete Math.* 2 (1989), 38–47. MR 89m:90132. Zbl 677.68036.

Vertex biparticity (the fewest vertices to delete to get a bipartite graph [i.e., vertex frustration number of $-\Gamma$]) is compared to edge biparticity [frustration index of $-\Gamma$] (for cubic graphs) and studied algorithmically. (**par: Fr**)

Timothy Y. Chow

2003a Symplectic matroids, independent sets, and signed graphs. *Discrete Math.* 263 (2003), 35–45. MR 2004a:05033. Zbl 1014.05017.

§4, "From graphs to symplectic matroids": The matroid union of $G(\Gamma, \sigma)$ over all signatures of a fixed graph yields a symplectic matroid.

(**SG: M**)

Debashish Chowdhury

1986a *Spin Glasses and Other Frustrated Systems.* Princeton Univ. Press, Princeton, and World Scientific, Singapore, 1986.

Includes brief survey of how physicists look upon frustration. See esp. §1.3, "An elementary introduction to frustration", where the signed square lattice graph illustrates balance vs. imbalance; Ch. 20, "Frustration, gauge invariance, defects and SG [spin glasses]", discussing planar duality (see e.g. Barahona (1982a), "gauge theories", where gains are in the orthogonal or unitary group (switching is called "gauge transformation" by physicists), and functions of interest to physicists; Addendum to Ch. 20, pp. 378–379, mentioning results on when the proportion of negative bonds is fixed, and on gauge theories.

(**Phys: SG, GG, VS, Fr: Exp, Ref**)

San Yan Chu

See S.L. Lee.

Maria Chudnovsky

2005a Even hole free graphs. *Graph Theory Notes N. Y.* 49 (2005), 22–24. MR 2006h:05185. (**SG: Cycles: Alg**)

Maria Chudnovsky, William H. Cunningham, and Jim Geelen

2008a An algorithm for packing non-zero A-paths in group-labelled graphs. *Combinatorica* 28 (2008), no. 2, 145–161. MR 2399016 (2009a:05103). Zbl 1164.05029.

See Chudnovsky, Geelen, *et al.* (2006a). Structure theorem for optimal A-paths in terms of switching only vertices in A^c; algorithm for finding such. Lemma 3.1 generalizes the basis result of (2006a). [*Question.* $B(\Pi)$ is a subset of $V \times \mathfrak{G}$. How is this related to the covering graph? Can one simplify their proofs? A "non-zero" path is like a level-changing path in $\tilde{\Phi}$ (covering graph). This suggests modelling their picture by $\Phi' = \Phi \cup 1K_n$, i.e., with distinguished identity-gain complete subgraph. Or, by $\Omega \subseteq M \cdot \Delta =$ a biased expansion, with a distinguished maximal balanced subgraph.] (**GG: Paths: Str, Alg**)

Maria Chudnovsky, Jim Geelen, Bert Gerards, Luis Goddyn, Michael Lohman, and Paul Seymour

2006a Packing non-zero A-paths in group-labelled graphs. *Combinatorica* 26 (2006), no. 5, 521–532. MR 2279668 (2007j:05184). Zbl 1127.05050.

In a gain graph Φ, find the maximum number of vertex-disjoint paths with non-identity gain and with endpoints in $A \subseteq V$ (non-zero A-paths). Thm.: If max $< k$, there is a set X of up to $2k-2$ vertices such that every A-path in $\Phi \setminus X$ has identity gain. This is not best possible.

They prove: $\{B(\Pi) : \Pi \in \mathcal{P}^*(G, A)\}$ is the set of bases of a matroid.

Dictionary: "Group-labelled graph" = gain graph; Γ-labelled graph = Γ-gain graph (for a group Γ); "weight" = gain. "Shifting" = switching; "A-equivalent" = A^c-switching equivalent, i.e., obtained by switching vertices not in A. (**GG: Str, Paths**)

Maria Chudnovsky, Ken-ichi Kawarabayashi, and Paul Seymour

2005a Detecting even holes. *J. Graph Theory* 48 (2005), no. 2, 85–111. MR 2006k:-05197. Zbl 1062.05135.

Algorithm to detect positive holes (induced circles) in a signed graph. A polynomially equivalent problem is to decide whether a graph is negative-hole signable, i.e., has a signature in which every hole is negative. (**SG: Cycles: Alg**)

S.T. Chui

See also B.W. Southern.

S.T. Chui, G. Forgacs, and D.M. Hatch

1982a Ground states and the nature of a phase transition in a simple cubic fully frustrated Ising model. *Phys. Rev. B* 25 (1982), no. 11, 6952–6958.

Physics of "fully frustrated" 3-dimensional cubic lattice, i.e., every square ("plaquette") is negative. Each square has one negative edge. This is the unique fully frustrated signature up to switching [short proof: the squares generate the cycle space], but there are many nonisomorphic ground states ($\zeta : V \to \{+1, -1\}$ such that $\min_\zeta |(E^\zeta)^-|$); they are said to form 12 mutually unreachable classes. App. A characterizes the ground states [and implies $l(\Sigma) = \frac{1}{4}|V|$ since each cube has one negative edge in each direction, neglecting boundary effects—or as-

suming toroidality]. The signed lattice is at times assumed to have a 2×2 fundamental domain; under that assumption there are 8 translational symmetry types of vertex, each forming a double-sized sublattice. Approximate clustering is discussed. [Annot. 18 Jun 2012.]

(**Phys, SG: Fr, sw, Clu**)

F.R.K. Chung, Wayne Goddard, and Daniel J. Kleitman

1994a Even cycles in directed graphs. *SIAM J. Discrete Math.* 7 (1994), 474–483. MR 1285584 (95e:05050). Zbl 809.05062.

A strongly connected digraph with $|E| \geq \lfloor (n+1)^2/4 \rfloor$ has an even cycle. This is best possible. [This equals Petersdorf's (1966a) bound for $l(K_n, \sigma)$. *Question.* Are they related?] [Annot. 12 Jun 2012.]

(**SD, Par: Bal**)

Taeyoung Chung, Jack Koolen, Yoshio Sano, and Tetsuji Taniguchi

2011a The non-bipartite integral graphs with spectral radius three. *Linear Algebra Appl.* 435 (2011), no. 10, 2544–2559. MR 2811137 (2012d:05224). Zbl 1222.05151.

§2.2, "Generalized line graphs and generalized signless Laplace matrices": The generalized signless Laplace matrix of (Γ, f), where $f : V \to \mathbb{Z}_{\geq 0}$, is $K(-\Gamma) + 2D(f)$. The incidence matrix of (Γ, f) is $\mathrm{H}(\Sigma)$ where Σ consists of $-\Gamma$ with $f(x)$ negative digons adjoined to $x \in V$. [See Zaslavsky (1984c, 2010b, 20xxa) for this construction, which is not stated here.] [Annot. 20 Dec 2011.] (**Par: Adj, Incid, LG**)

V. Chvátal

See J. Akiyama.

Olivier Cinquin and Jacques Demongeot

2002a Roles of positive and negative feedback in biological systems. *C. R. Biologies* 325 (2002), 1085–1095.

Stability of systems of nonlinear differential equations. Some mathematical treatment. (**SD: QSta, Appl**)

Lane Clark

2004a Limit theorems for associated Whitney numbers of Dowling lattices. *J. Combin. Math. Combin. Comput.* 50 (2004), 105–113. MR 2005b:06007. Zbl 1053.06003.

Aymptotics of numbers introduced by Benoumhani (1997a).

(**gg: M: Invar**)

F.W. Clarke, A.D. Thomas, and D.A. Waller

1980a Embeddings of covering projections of graphs. *J. Combin. Theory Ser. B* 28 (1980), 10–17. MR 81f:05066. Zbl 351.05126, (416.05069). (**gg: Top**)

A.M. Cohen

See A.E. Brouwer.

Bernard P. Cohen

See J. Berger.

Edith Cohen and Nimrod Megiddo

1989a Strongly polynomial-time and NC algorithms for detecting cycles in dynamic graphs. In: *Proceedings of the Twenty First Annual ACM Symposium on Theory of Computing* (Seattle, 1989), pp. 523–534.

Partial version of (1993a). (**GD: Bal: Alg**)

1991a Recognizing properties of periodic graphs. In: Peter Gritzmann and Bernd Sturmfels, eds., *Applied geometry and Discrete Mathematics: The Victor Klee*

Festschrift, pp. 135–146. DIMACS Ser. Discrete Math. Theor. Computer Sci., Vol. 4. Amer. Math. Soc., Providence, R.I., and Assoc. Computing Mach., 1991. MR 1116344 (92g:05166). Zbl 753.05047.

Given: a gain graph Φ with gains in $\mathbb{Z}^d$ (a "static graph"). Found: algorithms for (1) connected components and (2) bipartiteness of the covering graph $\tilde{\Phi}$ (the "periodic graph") and, (3) given costs on the edges of Φ, for a minimum-average-cost spanning tree in the covering graph. Many references to related work. (**GG: Cov: Alg, Ref**)

1992a New algorithms for generalized network flows. In: D. Dolev, Z. Galil, and M. Rodeh, eds., *Theory of Computing and Systems* (Proc., Haifa, 1992), pp. 103–114. Lect. Notes in Computer Sci., Vol. 601. Springer-Verlag, Berlin, 1992. MR 94b:68023 (book).

Preliminary version of (1994a), differing only slightly. (**GN: Alg**)(**sg: Ori: Alg**)

1993a Strongly polynomial-time and NC algorithms for detecting cycles in periodic graphs. *J. Assoc. Comput. Mach.* 40 (1993), 791–830. MR 96h:05182. Zbl 782.68053.

Looking for a closed walk ("cycle") with gain 0 in a gain digraph with (additive) gains in $\mathbb{Q}^d$. [*Cf.* Kodialam and Orlin (1991a).] (**GD: Bal: Alg**)

1994a New algorithms for generalized network flows. *Math. Programming* 64 (1994), 325–336. MR 95k:90111. Zbl 816.90057.

Maximize the fraction of demand satisfied by a flow on a network with gains. Positive real gains in §3. Bidirected networks with positive gains in §4; these are more general than networks with arbitrary non-zero real gains. (**GN: Alg**)(**sg: Ori: Alg**)

1994b Improved algorithms for linear inequalities with two variables per inequality. *SIAM J. Comput.* 23 (1994), 1313–1347. MR 95i:90040. Zbl 833.90094. (**GN: Incid: D: Alg**)

Charles J. Colbourn and Derek G. Corneil

1980a On deciding switching equivalence of graphs. *Discrete Appl. Math.* 2 (1980), 181–184. MR 81k:05090. Zbl 438.05054.

Deciding switching isomorphism of graphs is polynomial-time equivalent to graph isomorphism. (**TG: Alg**)

Tom Coleman, James Saunderson, and Anthony Wirth

2008a A local-search 2-approximation for 2-correlation-clustering. In: D. Halperin and K. Mehlhorn, eds., *Algorithms – ESA 2008* (16th Ann. Europ. Symp. Algorithms, Karlsruhe, 2008), pp. 308–319. Lect. Notes in Computer Sci., Vol. 5193. Springer, Berlin, 2008. Zbl 1158.68549. (**SG: Clu**)

L. Collatz

1978a Spektren periodischer Graphen. *Resultate Math.* 1 (1978), 42–53. MR 80b:-05042. Zbl 402.05054.

Introducing periodic graphs: these are connected canonical covering graphs $\Gamma = \tilde{\Phi}$ of finite $\mathbb{Z}^d$-gain graphs Φ. The "spectrum" of Γ is the set of all eigenvalues of $A(||\Phi||)$ for all possible Φ. The spectrum, while infinite, is contained in the interval $[-r, r]$ where r is the largest eigenvalue of each $A(||\Phi||)$ [the "index" of von Below (1994a)]. The inspiration is tilings. (**GG: Cov: Adj**)

Barry E. Collins and Bertram H. Raven

1968a Group structure: attraction, coalitions, communication, and power. In: Gardner Lindzey and Elliot Aronson, eds., *The Handbook of Social Psychology*, Second Edition, Vol. 4, Ch. 30, pp. 102–204. Addison-Wesley, Reading, Mass., 1968.

"Graph theory and structural balance," pp. 106–109.

(**PsS: SG: Exp, Ref**)

Barbara Coluzzi, Enzo Marinari, Giorgio Parisi, and Heiko Rieger

2000a On the energy minima of the Sherrington-Kirkpatrick model. *J. Phys. A* 33 (2000), no. 21, 3851–3862. (**Phys: SG**)

Ph. Combe and H. Nencka

1995a Non-frustrated signed graphs. In: J. Bertrand *et al.*, eds., *Modern Group Theoretical Methods in Physics* (Proc. Conf. in Honour of Guy Rideau, Paris, 1995), pp. 105–113. Math. Phys. Stud., Vol. 18. Kluwer, Dordrecht, 1995. MR 1361440 (96j:05105). Zbl 905.05071.

Σ is balanced iff a fundamental system of circles is balanced [as is well known; see *i.a.* Popescu (1979a), Zaslavsky (1981b)]. An algorithm [incredibly complicated, compared to the obvious method of tracing a spanning tree] to determine all vertex signings of Σ that switch it to all positive. Has several physics references. (**SG: Bal, Fr, Alg, Ref**)

1997a Cooperative networks and frustration on graphs. *Methods Funct. Anal. Topology* 3 (1997), 40–50. MR 2001e:91135. Zbl 933.92005.

A signed-graphic model Σ of a neuron network. Obs.: A network is cooperative iff Σ has a non-frustrated state $s : V \to \{+1, -1\}$, i.e., the Hamiltonian ("energy") $H(s) := -\frac{1}{2}\sum_{uv\in E}\sigma(uv)s(u)s(v) = -|E|$. [Should be $-\frac{1}{2}|E|$.] H [i.e., Σ] is non-frustrated if some state is. Assertion: H is non-frustrated iff Σ is balanced. A proof idea (not a proof) is by setting up (real-valued) linear equations of positivity of generating circles; carried out for K_n. [See (1997b).] [Easy proof: $H(s) = -\frac{1}{2}|E| + |E^-(\Sigma^s)|$, hence H is non-frustrated iff Σ^s is all positive for some s iff Σ is balanced. See e.g. Zaslavsky (1982a), Cor. 3.3.] [Annot. 17 Jun, 17 Aug 2012.] (**SG: Bal, sw, Fr, Biol**)

1997b Frustration and overblocking on graphs. *Math. Computer Modelling* 26 (1997), no. 8-10, 307–309. MR 1492513 (no rev). Zbl 1185.05147.

No proofs. Prop. 1: The signatures of Γ are a "GF(2)-vector space". [Meaning: They are the points in $\{\pm 1\}^{|E|} \subset \mathbb{R}^{|E|}$.] Prop. 2: Nonfrustration corresponds to a large family of [real] linear systems. "Minimal" circles generalize plaquettes (girth circles) to arbitrary graphs. ["Minimal" =(?) minimum length, assuming such circles generate the cycle space. In general, choice of generating circles remains a good question.] "Fully frustrated": all minimal circles are negative. Prop. 3: Full frustration corresponds to another family of [real] linear systems. ["Overblocking": Fully frustrated and some nonminimal circles are negative.] Prop. 4: Linear system for overblocking in a fully frustrated signature. Cor. 5: K_5 is overblocking. $K_{3,2}$ cannot be fully frustrated. [Annot. 17 Jun 2012.] (**SG: Bal, sw, Fr, Phys**)

Jean-Paul Comet

See A. Richard.

F.G. Commoner

1973a A sufficient condition for a matrix to be totally unimodular. *Networks* 3 (1973), 351–365. MR 49 #331. Zbl 352.05012. (**SD: Bal**)

Michele Conforti, Gérard Cornuéjols, Ajai Kapoor, and Kristina Vuškoviić

1994a Recognizing balanced $0, \pm 1$ matrices. In: *Proceedings of the 5th Annual ACM-SIAM Symposium on Discrete Algorithms* (Arlington, Va., 1994), pp. 103–111. Assoc. Computing Mach., New York, 1994. MR 95e:05022. Zbl 867.05014. (**SG: Bal**)

1995a A mickey-mouse decomposition theorem. In: Egon Balas and Jens Clausen, eds., *Integer Programming and Combinatorial Optimization* (4th Int. IPCO Conf., Copenhagen, 1995, Proc.), pp. 321–328. Lect. Notes in Computer Sci., Vol. 920. Springer, Berlin, 1995. MR 96i:05139. Zbl 875.90002 (book).

The structure of graphs that are signable to be "without odd holes": that is, so that each triangle is negative and each chordless circle of length greater than 3 is positive. Proof based on Truemper (1982a). (**SG: Bal, Str**)

1997a Universally signable graphs. *Combinatorica* 17 (1997), 67–77. MR 98g:05134. Zbl 980.00112.

Γ is "universally signable" if it can be signed so as to make every triangle negative and the holes independently positive or negative at will. Such graphs are characterized by a decomposition theorem which leads to a polynomial-time recognition algorithm. (**SG: Bal, Str**)

1999a Even and odd holes in cap-free graphs. *J. Graph Theory* 30 (1999), 289–308. MR 99m:05155. Zbl 920.05028. (**SG: Bal**)

2000a Triangle-free graphs that are signable without even holes. *J. Graph Theory* 34 (2000), 204–220. MR 2001b:05188. Zbl 953.05061.

"Even hole" means a chordless circle that is positive in a given signing of the graph. The graphs of the title are characterized in several ways. Most of them have significant wheels. (**SG: Bal, Str, Alg**)

2002a Even-hole-free graphs. Part I. Decomposition theorem. *Journal of Graph Theory* 39 (2002), 6–49. MR 2003c:05189. Zbl 1005.05034. (**SG: Bal**)

2002b Even-hole-free graphs. Part II. Recognition algorithm. *Journal of Graph Theory* 40 (2002), 238–266. MR 2004e:05182. Zbl 1003.05095. (**SG: Bal**)

Michele Conforti, Gérard Cornuéjols, and Kristina Vuškoviić

1999a Balanced cycles and holes in bipartite graphs. *Discrete Math.* 199 (1999), 27–33. MR 99j:05119. Zbl 939.05050. (**SGw, gg, sg: Bal**)

Michele Conforti and Bert Gerards

2007a Packing odd circuits. *SIAM J. Discrete Math.* 21 (2007), no. 2, 273–302. MR 2008g:05162. Zbl 1139.05323.

The problem is to find the most vertex-disjoint negative circles in a signed graph (thus, odd-length circles in an ordinary graph). It is NP-hard but it can be solved in polynomial time for the signed graphs that exclude the switching classes of the four signed graphs $-K_5$, $K_{3,3}^{1,1}$, $K_{3,3}^{1,2}$, $K_{3,3}^{2}$, which are defined as: $K_{3,3}^{1,1} = +K_{3,3}$ with edge u_1v_1 made negative and the additional negative edge $-u_2v_2$, $K_{3,3}^{1,2} = +K_{3,3}$ with u_1v_1 made

negative and added edges $-u_1u_2$ and $-u_1u_3$, and $K_{3,3}^2 = +K_{3,3}$ with edges u_1v_1 and u_2v_2 made negative. **(SG: Str)**

Michele Conforti, Bert Gerards, and Ajai Kapoor

2000a A theorem of Truemper. *Combinatorica* 20 (2000), no. 1, 15–26. MR 2001h:-05085. Zbl 949.05071.

Full version of Conforti and Kapoor (1998a). **(SG: Bal)**

Michele Conforti and Ajai Kapoor

1998a A theorem of Truemper. In: Robert E. Bixby, E. Andrew Boyd, and Roger Z. Ríos-Mercado, eds., *Integer Programming and Combinatorial Optimization* (6th Int. IPCO Conf., Houston, 1998, Proc.), pp. 53–68. Lect. Notes in Computer Sci., Vol. 1412. Springer, Berlin, 1998. MR 2000h:05184. Zbl 907.90269.

A new proof of Truemper's theorem on prescribed hole signs; discussion of applications. **(SG: Bal)**

Joseph G. Conlon

2004a Even cycles in graphs. *J. Graph Theory* 45 (2004), no. 3, 163–223. MR 2004m:05145. Zbl 1033.05062.

Main theorem: For 3-connected $G \neq K_4$, there is an even circle, deletion of whose vertices or edges leaves a 2-connected graph. [*Problem.* Generalize to signed graphs. And see Voss (1991a).] **(par)**

Raul Cordovil

See P. Berthomé.

Denis Cornaz

2006a On co-bicliques. *RAIRO Oper. Res.* 41 (2006), 295–304. MR 2348004 (2009f:-90053). Zbl 1227.90043. **(SG)**

Denis Cornaz and A. Ridha Mahjoub

2007a The maximum induced bipartite subgraph problem with edge weights. *SIAM J. Discrete Math.* 21 (2007), no. 3, 662–675. MR 2353996 (2008j:05331). Zbl 1141.05076. **(SD)**

Derek G. Corneil

See C.J. Colbourn and Seidel (1991a).

Gérard Cornuéjols

See also M. Campelo and M. Conforti.

2001a *Combinatorial Optimization: Packing and Covering.* CBMS-NSF Reg. Conf. Ser. Appl. Math., Vol. 74. Soc. Indust. Appl. Math., Philadelphia, 2001. MR 2002e:90004. Zbl 972.90059.

The topic is linear optimization over a clutter, esp. a "binary clutter", which is the class of negative circuits of a signed binary matroid. The class $\mathcal{C}_-(\Sigma)$ is an important example (see Seymour 1977a), as is its blocker $b\big(\mathcal{C}_-(\Sigma)\big)$ [which is the class of minimal balancing edge sets; hence the frustration index $l(\Sigma)$ = minimum size of a member of the blocker].

Ch. 5: "Graphs without odd-K_5 minors", i.e., signed graphs without $-K_5$ as a minor. Some esp. interesting results: Thm. 5.0.7 (special case of Seymour (1977a), Main Thm.): The clutter of negative circles of Σ has the "Max-Flow Min-Cut Property" (Seymour's "Mengerian" property) iff Σ has no $-K_4$ minor. Conjecture 5.1.11 is Seymour's (1981a) beautiful conjecture (his "weak MFMC" is here called "ideal"). §5.2 reports the partial result of Guenin (2001a). (See also §8.4.)

Def. 6.2.6 defines a signed graph "$G(A)$" of a $0, \pm 1$-matrix A, whose transposed incidence matrix is a submatrix of A. §6.3.3: "Perfect $0, \pm 1$-matrices, bidirected graphs and conjectures of Johnson and Padberg" (1982a), associates a bidirected graph with a system of 2-variable pseudoboolean inequalities; reports on Sewell (1997a) (*q.v.*).

§8.4: "On ideal binary clutters", reports on Cornuéjols and Guenin (2002a), Guenin (1998a), and Novick and Sebö (1995a) (*qq.v.*).

(**Sgnd(M), SG: M, Geom, Incid(Gen), Ori: Exp, Ref, Exr**)

Gérard Cornuéjols and Bertrand Guenin

2002a Ideal binary clutters, connectivity, and a conjecture of Seymour. *SIAM J. Discrete Math.* 15 (2002), no. 3, 329–352. MR 2003h:05057. Zbl 1035.90045.

A partial proof of Seymour's (1981a) conjecture. Main Thm.: A binary clutter is ideal if it has as a minor none of the circuit clutter of F_7, $\mathcal{C}_-(-K_5)$ or its blocker, or $\mathcal{C}_-(-K_4)$ or its blocker. Important are the lift and extended lift matroids, $L(M, \sigma)$ and $L_0(M, \sigma)$, defined as in signed graph theory. [See Cornuéjols (2001a), §8.4.]

(**Sgnd(M), SG: M, Geom**)

S. Cosares

See L. Adler.

Collette R. Coullard

See also V. Chandru.

Collette R. Coullard, John G. del Greco, and Donald K. Wagner

††1991a Representations of bicircular matroids. *Discrete Appl. Math.* 32 (1991), 223–240. MR 92i:05072. Zbl 755.05025.

§4: §4.1 describes 4 fairly simple types of "legitimate" graph operation that preserve the bicircular matroid. Thm. 4.11 is a converse: if Γ_1 and Γ_2 have the same connected bicircular matroid, then either they are related by a sequence of legitimate operations, or they belong to a small class of exceptions, all having order ≤ 4, whose bicircular matroid isomorphisms are also described. This completes the isomorphism theorem of Wagner (1985a). §5: If finitely many graphs are related by a sequence of legitimate operations (so their bicircular matroids are isomorphic), then they have contrabalanced real gains whose incidence matrices are row equivalent. These results are also found by a different approach in Shull *et al.* (1989a, 1993a, 1997a). (**Bic: Str, Incid**)

1993a Recognizing a class of bicircular matroids. *Discrete Appl. Math.* 43 (1993), 197–215. MR 94i:05021. Zbl 777.05036. (**Bic: Alg**)

1993b Uncovering generalized-network structure in matrices. *Discrete Appl. Math.* 46 (1993), 191–220. MR 95c:68179. Zbl 784.05044. (**GN: Bic: Incid, Alg**)

Gheorghe Craciun

See also M. Banaji and M. Mincheva.

Gheorghe Craciun and Martin Feinberg

2005a Multiple equilibria in complex chemical reaction networks: I. The injectivity property. *SIAM J. Appl. Math.* 65 (2005), 1526–1546. MR 2177713 (2006g:-92075). Zbl 1094.80005. (**SG, Chem**)

2006a Multiple equilibria in complex chemical reaction networks: II. The species-reaction graph. *SIAM J. Appl. Math.* 66, no. 4, 1321–1338. MR 2246058 (2007e:92027). Zbl 1136.80306. (**SG, Chem**)

2006b Multiple equilibria in complex chemical reaction networks: extensions to entrapped species models. *IEE Proc. Syst. Biol.* 153 (2006), no. 4, 179–186. (**SG, Chem**)

Gheorghe Craciun, Casian Pantea, and Eduardo D. Sontag

2011a Graph-theoretic analysis of multistability and monotonicity for biochemical reaction networks. In: Heinz Koeppl, Douglas Densmore, Gianluca Setti, and Mario di Bernardo, eds., *Design and Analysis of Biomolecular Circuits: Engineering Approaches to Systems and Synthetic Biology*, pp. 63–72. Springer, New York, 2011. (**SG, Chem: Exp**)

Yves Crama

See also E. Boros.

1989a Recognition problems for special classes of polynomials in 0–1 variables. *Math. Programming* A44 (1989), 139–155. MR 90f:90091. Zbl 674.90069.

Balance and switching are used to study pseudo-Boolean functions. (§§2.2 and 4.) (**SG: Bal, Sw**)

Yves Crama and Peter L. Hammer

1989a Recognition of quadratic graphs and adjoints of bidirected graphs. Combinatorial Math.: Proc. Third Int. Conf. *Ann. New York Acad. Sci.* 555 (1989), 140–149. MR 91d:05044. Zbl 744.05060.

"Adjoint" = unoriented positive part of the line graph of a bidirected graph. "Quadratic graph" = graph that is an adjoint. Recognition of adjoints of bidirected simple graphs is NP-complete. (**sg: Ori: LG: Alg**)

Yves Crama, Peter L. Hammer, and Toshihide Ibaraki

1986a Strong unimodularity for matrices and hypergraphs. *Discrete Appl. Math.* 15 (1986), 221–239. MR 88a:05105. Zbl 647.05042.

§7: Signed hypergraphs, with a surprising generalization of balance. (**SH: Bal**)

Y. Crama, M. Loebl, and S. Poljak

1992a A decomposition of strongly unimodular matrices into incidence matrices of digraphs. *Discrete Math.* 102 (1992), 143–147. MR 93g:05097. Zbl 776.05071. (**SG**)

Lin Cui and Yi-Zheng Fan

2010a The signless Laplacian spectral radius of graphs with given number of cut vertices. *Discuss. Math. Graph Theory* 30 (2010), no. 1, 85–93. MR 2676064 (2011j:05196). Zbl 1215.05101. (**Par: Adj**)

William H. Cunningham

See J. Aráoz and M. Chudnovsky.

Dragoš M. Cvetković

See also R.A. Brualdi, F.C. Bussemaker, D.M. Cardoso, and M. Doob.

1978a The main part of the spectrum, divisors and switching of graphs. *Publ. Inst. Math. (Beograd) (N.S.)* 23 (37) (1978), 31–38. MR 80h:05045. Zbl 423.05028.

1995a Star partitions and the graph isomorphism problem. *Linear Algebra Appl.* 39 (1995), 109–132. MR 97b:05105. Zbl 831.05043.

Pp. 128–130 discuss switching-equivalent graphs. Some of the theory is invariant, hence applicable to two-graphs. [*Question.* How can this be generalized to signed graphs and their switching classes?] (**TG: Adj**)

2005a Signless Laplacians and line graphs. *Bull. Cl. Sci. Math. Nat. Sci. Math.* No. 30 (2005), 86–92. MR 2213761 (2006m:05152). Zbl 1119.05066.
See Cvetković, Rowlinson, and Simić (2007a). **(Par: Adj)**

2008a New theorems for signless Laplacian eigenvalues. *Bull. Cl. Sci. Math. Nat. Sci. Math.* No. 33 (2008), 131–146. MR 2609604 (2011b:05145). Zbl 1199.05212.
See Cvetković, Rowlinson, and Simić (2007a). **(Par: Adj)**

Dragoš M. Cvetković and Michael Doob

1984a Root systems, forbidden subgraphs, and spectral characterizations of line graphs. In: *Graph Theory* (Novi Sad, 1983), pp. 69–99. Univ. Novi Sad, Novi Sad, 1984. MR 751442 (86a:05088). **(sg: par: Geom, LG)**

authorDragos M. Cvetković, Michael Doob, Ivan Gutman, and Aleksandar Torgašev

1988a *Recent Results in the Theory of Graph Spectra.* Ann. Discrete Math., 36. North-Holland, Amsterdam, 1988. MR 89d:05130. Zbl 634.05034.
Signed graphs mentioned: P. 40 cites Zaslavsky (1981a). Pp. 44–45 (with unusual terminology) describe B.D. Acharya (1980a) and Gill (1981b). P. 100 cites B.D. Acharya (1979b). All-negative signatures are implicated in the infinite-graph eigenvalue theorem of Torgašev (1982a), Thm. 6.29 of this book. Möbius molecules (with signed molecular graphs) mentioned on p. 149. **(SG, par: Adj: Exp, Appl, Ref)**

Dragoš M. Cvetković, Michael Doob, and Horst Sachs

1980a *Spectra of Graphs: Theory and Application.* VEB Deutscher Verlag der Wissenschaften, Berlin, 1980. Copublished as: Pure and Appl. Math., Vol. 87. Academic Press, New York-London, 1980. MR 81i:05054. Zbl 458.05042.
§4.6: Signed digraphs with multiple edges are employed to analyze the characteristic polynomial of a digraph. (Signed) switching, too. Pp. 187–188: Exercises involving Seidel switching and the Seidel adjacency matrix. Thm. 6.11 (Doob (1973a)): The even-cycle matroid determines the eigenvaluicity of -2. §7.3: "Equiangular lines and two-graphs." [Annot. $\leq$ 2000, rev. 20 Sept 2010.]
(SD, par, TG: Sw, Adj, Geom: Exp, Exr, Ref)

1982a *Spectra of Graphs: Theory and Application.* VEB Deutscher Verlag der Wissenschaften, Berlin, 1982. MR 84a:05046.
Update of (1980a). **(SD, par, TG: Sw, Adj, Geom: Exp, Exr, Ref)**

1995a *Spectra of Graphs: Theory and Applications.* Third ed. Johann Ambrosius Barth, Heidelberg, 1995. MR 96b:05108. Zbl 824.05046.
Appendices update (1982a), beyond the updating in Cvetković, Doob, Gutman, and Torgašev (1988a). App. B.3, p. 381 mentions work of Vijayakumar (*q.v.*). P. 422: Pseudo-inverse graphs (when $A(\Gamma)^{-1} = A(\Sigma)$ for some balanced Σ, $|\Sigma|$ is the "pseudo-inverse" of Γ).
(SD, par, TG: Adj, Sw, Geom, Bal: Exp, Exr, Ref)

Dragoš Cvetković, Michael Doob, and Slobodan Simić

1980a Some results on generalized line graphs. *C. R. Math. Rep. Acad. Sci. Canada* 2 (1980), 147–150. MR 81f:05136. Zbl 434.05057.
Abstract of (1981a). **(sg: LG, Adj(LG), Aut(LG))**

1981a Generalized line graphs. *J. Graph Theory* 5 (1981), 385–399. MR 82k:05091. Zbl 475.05061. **(sg: LG, Adj(LG), Aut(LG))**

Dragoš Cvetković, Peter Rowlinson, and Slobodan K. Simić

2004a *Spectral Generalizations of Line Graphs: On Graphs with Least Eigenvalue −2.* London Math. Soc. Lect. Note Ser., 314. Cambridge Univ. Press, Cambridge, Eng., 2004. MR 2120511 (2005m:05003). Zbl 1061.05057.

Generalized line graphs are the fundamental example. Pp. 190–191 mention signed graphs representable in root systems as in papers of G.R. Vijayakumar (*q.v.*) [but not mentioning line graphs of signed graphs]. [Annot. 13 Oct 2010.] (**LG: Gen, Geom, Adj**)(**SG: Geom: Exp**)

2007a Signless Laplacians of finite graphs. *Linear Algebra Appl.* 423 (2007), no. 1, 155–171. MR 2312332 (2008c:05105). Zbl 1113.05061.

"Signless Laplacian" $Q(\Gamma) := $ Kirchhoff matrix $K(-\Gamma) = D(\Gamma) + A(\Gamma)$. Spectral properties; bounds for graph invariants; combinatorics of coefficients of characteristic polynomial of $K(-\Gamma)$. [*Problem.* Find all articles on "signless Laplacians", herein called $K(-\Gamma)$. Generalize to signed graphs, with nonbipartite graphs generalizing to unbalanced graphs.] [Annot. 14 Sept 2010.] (**Par: Adj**)

2007b Eigenvalue bounds for the signless Laplacian. *Publ. Inst. Math. (Beograd) (N.S.)* 81(95) (2007), 11–27. MR 2401311 (2009e:05181). Zbl 1164.05038.

See (2007a). Thm.: For connected Γ with $|V| = n$ and $|E| = m$, $\lambda_1(K(-\Gamma))$ is maximized when Γ is a nested split graph. Also, many computer-generated conjectures (*cf.* Aouchiche and Hansen (2010a)); some are proved (here or elsewhere) or disproved; some are difficult. [Annot. 4 Sept 2010, 22 Jan 2012.] (**Par: Adj, LG**)

2010a *An Introduction to the Theory of Graph Spectra.* London Math. Soc. Student Texts, 75. Cambridge Univ. Press, Cambridge, Eng., 2010. MR 2571608 (2011g:05004). Zbl 1211.05002.

Graph switching in §1.1 Reduced line graphs of simply signed graphs are implicit in the construction of generalized line graphs in §1.2. [Annot. 14 Sept 2010.] (**tg: Sw: Exp**)(**sg: LG: Exp**)

§7.8, "The signless Laplacian". (**Par: Adj: Exp**)

Dragoš M. Cvetković and Slobodan K. Simić

1978a Graphs which are switching equivalent to their line graphs. *Publ. Inst. Math. (Beograd) (N.S.)* 23 (37) (1978), 39–51. MR 80c:05108. Zbl 423.05035. (**sw: LG**)

2009a Towards a spectral theory of graphs based on the signless Laplacian. I. *Publ. Inst. Math. (Beograd) (N.S.)* 85(99) (2009), 19–33. MR 2536686 (2010i:05203). Zbl 224.05293.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

2010a Towards a spectral theory of graphs based on the signless Laplacian. II. *Linear Algebra Appl.* 432 (2010), no. 9, 2257–2272. MR 2599858 (2011d:05217). Zbl 1218.05089.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

2010b Towards a spectral theory of graphs based on the signless Laplacian. III. *Appl. Anal. Discrete Math.* 4 (2010), no. 1, 156–166. MR 2654936 (2011m:05169).

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

2011a Graph spectra in Computer Science. *Linear Algebra Appl.* 434 (2011), no. 6, 1545–1562. MR 2775765 (2011m:05170). Zbl 1207.68230. (**Par: Adj: Exp**)

D. Cvetković, S.K. Simić, and Z. Stanić

2010a Spectral determination of graphs whose components are paths and cycles. *Computers Math. Appl.* 59 (2010), 3849–3857. MR 2651858 (2011j:05197). Zbl 1198.05110. **(Par: Adj)**

Marek Cygan, Marcin Pilipczuk, MichałPilipczuk, and Jakub Onufry Wojtaszczyk

20xxa Sitting closer to friends than enemies, revisited. Submitted. arXiv:1201.1869.

Sequel to Kermarrec and Thraves (2011a). [Annot. 26 Apr 2012.] **(SG: KG: Bal, Alg)**

A. Daemi

See S. Akbari.

Edwin R. van Dam and Willem H. Haemers

2003a Which graphs are determined by their spectrum? Special issue on the Combinatorial Matrix Theory Conference (Pohang, 2002). *Linear Algebra Appl.* 373 (2003), 241–272. MR 2022290 (2005a:05135). Zbl 1026.05079. **(Par: Adj)(Par: Adj: Exp)**

2009a Developments on spectral characterizations of graphs. Int. Workshop Design Theory, Graph Theory, Comput. Methods – IPM Combinatorics II. *Discrete Math.* 309 (2009), no. 3, 576–586. MR 2499010 (2010h:05178). Zbl 1205.05156.

New and old results on $K(-\Gamma)$, the "signless Laplacian" of Γ. [Annot. 20 Dec 2011.] **(Par: Adj)(Par: Adj: Exp)**

Susan S. D'Amato

1979a Eigenvalues of graphs with twofold symmetry. *Molecular Phys.* 37 (1979), 1363–1369. MR 535191 (80c:05098).

Spectrum of signed covering graph. [See Butler (2010a).] [Annot. 9 Mar 2011.] **(sg: cov: Adj)**

1979b Eigenvalues of graphs with threefold symmetry. *Theor. Chim. Acta* 53 (1979), 319–326.

Ternary gain graphs: spectrum of covering graph, as with signed graphs in (1979a). [Annot. 9 Mar 2011.] **(gg: cov: Adj)**

Jeffrey M. Dambacher, Richard Levins, and Philippe A. Rossignol

2005a Life expectancy change in perturbed communities: Derivation and qualitative analysis. *Math. Biosciences* 197 (2005), no. 1, 1–14. MR 2167483 (2006d:92058). Zbl 1074.92037. **(SD: QM: QSta: Cycles, Ref)**

Jeffrey M. Dambacher, Hiram W. Li, and Philippe A. Rossignol

2003a Qualitative predictions in model ecosystems. *Ecological Modelling* 161 (2003), no. 1, 79–93.

Feedback predictions from signed digraph (D, σ) via "weighted predictions" $W_{ij} := |C_{ij}(-A(D, \sigma))|/P_{ij}(A(D))$, where C_{ij} is the cofactor and P_{ij} is the permanental cofactor. $W_{ij} = 1$ means perfect predictability, $= 0$ means no predictability. Numerical tests. Dictionary: "Community matrix" $= A(D, \sigma)$. [Annot. 9 Sept 2010.] **(SD: QM: QSta: Cycles, Ref)**

E. Damiani, O. D'Antona, and F. Regonati

1994a Whitney numbers of some geometric lattices. *J. Combin. Theory Ser. A* 65 (1994), 11–25. MR 95e:06019. Zbl 793.05037.

E.g., log concavity of Whitney numbers of the second kind of Dowling lattices. [*Cf.* Stonesifer (1975a) and Benoumhani (1999a).] [Annot. Rev 30 Apr 2012.] (**gg: M: Invar**)

A. Danielian

1961a Ground state of an Ising face-centered cubic lattice. *Phys. Rev. Lett.* 6 (1961), 670–671.

"Ground states", i.e. $\zeta : V \to \{+1, -1\}$ with smallest $|(E^\zeta)^-|$, of the all-negative (antiferromagnetic) $R \times R \times$ face-centered cubic lattice graph [assumed toroidal to avoid boundary effects?]. Frustration index $l = 2|V|$; the number ("degeneracy") of ground states is $2^{A\sqrt{[3]|V|}}$ where $A > 0$; each ground state has 4-regular E^-. See (1964a) for more structure. [*Problem.* Determine the exact number and precise shape of all ground states ζ in terms of the graph. Is there something interesting about $(\Sigma^\zeta)^-$, e.g., in its circle decomposition, symmetries, or transformations from one to another?] [Annot. 21 Jun 2012.] (**SG, Phys: Par: Fr**)

1964a Low-temperature behavior of a face-centered cubic antiferromagnet. *Phys. Rev.* 133 (1964), no. 5A, A1344–A1349.

§ II, "The ground state", continues (1961a) with more details on the structure of ground states ζ. The number of them is small compared to the all-negative triangular lattice [*Question*: and other all-negative, highly symmetric graphs?]. ζ on each x-, y-, or z-layer has a form described in the paper. Low-weight distance-2 edges will fix the ground state (p. A1346). § III, "The partition function", studies the effect of moving out of ground states. App. A derives a formula for the energy change from switching a cluster of vertices, in terms of frustrated and satisfied edges within and without the cluster. App. B estimates the effect of switching additional vertices. [*Problem.* Find rigorous treatments of such switchings; this means studying the energy landscape of state space $\{\zeta\} = \{+1, -1\}^{\{+1,-1\}^V}$.] Dictionary: "bond" = edge, "even/odd bond" = frustrated/satisfied edge = switches to + or −. [Annot. 21 Jun 2012.] (**Phys, SG: Par: Fr**)

O. D'Antona

See E. Damiani.

George B. Dantzig

1963a *Linear Programming and Extensions.* Princeton Univ. Press, Princeton, N.J., 1963. MR 34 #1073. Zbl (e: 108.33103).

Chapter 21: "The weighted distribution problem." 21-2: "Linear graph structure of the basis." (**GN: M(Bases)**)

F.A. Dar

See S. Pirzada.

Kinkar Ch. Das

2010a On conjectures involving second largest signless Laplacian eigenvalue of graphs. *Linear Algebra Appl.* 432 (2010), no. 11, 3018–3029. MR 2639266 (2011h:05151). Zbl 1195.05040.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

2011a Proof of conjecture involving the second largest signless Laplacian eigenvalue and the index of graphs. *Linear Algebra Appl.* 435 (2011), no. 10, 2420–2424. Zbl 1223.05171. (**Par: Adj**)

2012a Proof of conjectures involving the largest and the smallest signless Laplacian eigenvalues of graphs. *Discrete Math.* 312 (2012), 992–998.

Assume $n \geq 4$; $\lambda_1 =$ max eigenvalue. Thm. 3.2: $\lambda_1(-\Gamma) + \lambda_n(-\Gamma) \leq 3n-2-2\alpha(\Gamma)$, where $\alpha :=$ independence number; = iff $\Gamma = K_{n-\alpha} \vee \bar{K}_\alpha$. Thm. 3.3: $\lambda_1(-\Gamma) - \lambda_n(-\Gamma) \geq 2 + 2\cos(/pi/n)$, with equality iff Γ is a path or odd circle. [Annot. 21 Jan 2012.] **(Par: Adj)**

Prabir Das and S.B. Rao

1983a Alternating eulerian trails with prescribed degrees in two edge-colored complete graphs. *Discrete Math.* 43 (1983), 9–20. MR 84k:05069. Zbl 494.05020.

Given an all-negative bidirected K_n and a positive integer $f_i = 2g_i$ for each vertex v_i. There is a connected subgraph having in-degree and out-degree = g_i at v_i iff there is a g-factor of introverted and one of extroverted edges and the degrees satisfy a complicated degree condition. Generalizes Thm. 1 of Bánkfalvi and Bánkfalvi (1968a). [See Bang-Jensen and Gutin (1997a) for how to convert an edge 2-coloring to an orientation of an all-negative graph and for further developments on alternating walks.] **(par: ori)**

B. Dasgupta, G.A. Enciso, E.D. Sontag, and Y. Zhang

2006a Algorithmic and complexity results for decompositions of biological networks into monotone subsystems. In: C. ?varez and M. Serna, eds., *Experimental Algorithms* (5th Int. Workshop, WEA 2006, Cala Galdana, Menorca, 2006), pp. 253–264. Lect. Notes in Comput. Sci., Vol. 4007. Springer-Verlag, Berlin, 2006. Zbl 196.92016.

James A. Davis

1963a Structural balance, mechanical solidarity, and interpersonal relations. *Amer. J. Sociology* 68 (1963), 444–463. Repr. with minor changes in: Joseph Berger, Morris Zelditch, Jr., and Bo Anderson, eds., *Sociological Theories in Progress, Vol. One*, Ch. 4, pp. 74–101. Houghton Mifflin, Boston, 1966. Also reprinted in: Samuel Leinhardt, ed., *Social Networks: A Developing Paradigm*, pp. 199–217. Academic Press, New York, 1977. **(PsS: SG, WG: Exp)**

1967a Clustering and structural balance in graphs. *Human Relations* 20 (1967), 181–187. Repr. in: Samuel Leinhardt, ed., *Social Networks: A Developing Paradigm*, pp. 27–33. Academic Press, New York, 1977.

Σ is "clusterable" if its vertices can be partioned so that each positive edge is within a part and each negative edge joins different parts. Thm.: Σ is clusterable $\iff$ no circle has exactly one negative edge. [See Doreian and Mrvar (1996a).] **(SG: Clu)**

1979a The Davis/Holland/Leinhardt studies: An overview. In: Paul W. Holland and Samuel Leinhardt, eds., *Perspectives on Social Network Research* (Proc. Sympos., Dartmouth Coll., Hanover, N.H., 1975), Academic Press, New York, 1979.

Survey of triad analysis in signed complete digraphs; *cf.* e.g. Davis and Leinhardt (1972a), Wasserman and Faust (1994a). [Annot. 28 Apr 2009.]

(PsS, SD: Clu(Gen): Exp)

James A. Davis and Samuel Leinhardt

1972a The structure of positive interpersonal relations in small groups. In: Joseph Berger, Morris Zelditch, Jr., and Bo Anderson, eds., *Sociological Theories in Progress, Vol. Two*, Ch. 10, pp. 218–251. Houghton Mifflin, Boston, 1972.

In "ranked clusterability" the vertices of a signed complete, symmetric digraph are divided into levels. The set of levels is totally ordered. A symmetric pair, $\{+vw, +wv\}$ or $\{-vw, -wv\}$, should be within a level. For an asymmetric pair, $\{+vw, -wv\}$, w should be at a higher level than v. Analysis in relation to both randomly generated and observational data. [Annot. 28 Apr 2009.] (**PsS, SD: Clu(Gen)**)

A.C. Day, R.B. Mallion, and M.J. Rigby

1983a On the use of Riemannian surfaces in the graph-theoretical representation of Möbius systems. In: R.B. King, ed., *Chemical Applications of Topology and Graph Theory* (Proc. Sympos., Athens, Ga., 1983), pp. 272–284. Stud. Phys. Theor. Chem., Vol. 28. Elsevier, Amsterdam, 1983. MR 85h:05039.

A clumsy but intriguing way of representing some signed (or more generally, $\mathbb{Z}_n$-weighted) graphs: via 2-page (or, n-page) looseleaf book embedding (all vertices are on the spine and each edge is in a single page), with an edge in page k weighted by the "sheet parity index" $\alpha_k = (-1)^k$ (or, $e^{2\pi ik/n}$). (Described in the [unnecessary] terminology of an n-sheeted Riemann surface.) [A $\mathbb{Z}_n$-weighted) graph has such a representation iff the subgraph of edges with each weight is outerplanar.]

A variation to get switching classes of signed circles: replace α_k by the "connectivity parity index" $\alpha_k^{\sigma_k}$ where $\sigma_k =$ number of edges in page k. [The variation is valid only for circles.] [*Questions* vaguely suggested by these procedures: Which signed graphs can be switched so that the edges of each sign form an outerplanar graph? Also, the same for gain graphs. And there are many similar questions: for instance, the same ones with "outerplanar" replaced by "planar."]

(**SG: sw, Adj, Top, Chem: Exp, Ref**)(**WG: Adj, Top: Exp, Ref**)

Nair Maria Maia de Abreu

See M.A.A. de Freitas and C.S. Oliveira.

Marisa Debowsky

See D. Archdeacon.

Pierre de la Harpe

See P. de la Harpe under "H".

Anne Delandtsheer

1995a Dimensional linear spaces. In: F. Buekenhout, ed., *Handbook of Incidence Geometry: Buildings and Foundations*, Ch. 6, pp. 193–294. North-Holland, Amsterdam,1995. MR 96k:51012. Zbl 950.23458.

"Dimensional linear space" (DLS) = simple matroid. §2.7: "Dowling lattices," from Dowling (1973b). §6.7: "Subgeometry-closed and hereditary classes of DLS's," from Kahn and Kung (1982a). In §2.6, the "Enough modular hyperplanes theorem" from Kahn and Kung (1986a). (**GG: M: Exp**)

Patrick De Leenheer

See D. Angeli.

John G. del Greco

See also C.R. Coullard.

1992a Characterizing bias matroids. *Discrete Math.* 103 (1992), 153–159. MR 93m:-05050. Zbl 753.05021.

How to decide, given a matroid M and a biased graph Ω, whether $M = G(\Omega)$. (**GG: M**)

Leonardo Silva de Lima

See also C.S. Oliveira.

Leonardo Silva de Lima, Carla Silva Oliveira, Nair Maria Maia de Abreu, and Vladimir Nikiforov

2011a The smallest eigenvalue of the signless Laplacian. *Linear Algebra Appl.* 435 (2011), no. 10, 2570–2584. MR 2811139 (2012g:05140). Zbl 1222.05180.

(**Par: Adj**)

Alberto Del Pia and Giacomo Zambelli

2009a Half-integral vertex covers on bipartite bidirected graphs: total dual integrality and cut-rank. *SIAM J. Discrete Math.* 23 (2009), no. 3, 1281–1296. MR 2538651 (2011b:05200). Zbl 1227.05209.

Dictionary: "Bipartite" = balanced. (**sg: Ori: Incid, Alg**)

Ernesto W. De Luca

See J. Kunegis.

Emanuele Delucchi

2007a Nested set complexes of Dowling lattices and complexes of Dowling trees. *J. Algebraic Combin.* 26 (2007), no. 4, 477–494. MR 2008i:05190. Zbl 1127.05107.

Studies Dowling trees (*cf.* Hultman 2007a). (**gg: M: Invar**)

Renata R. Del-Vecchio

See M.A.A. de Freitas.

Erik D. Demaine, Dotan Emanuel, Amos Fiat, and Nicole Immorlica

2006a Correlation clustering in general weighted graphs. Approximation and Online Algorithms. *Theoretical Computer Sci.* 361 (2006), no. 2–3, 172–187. MR 2252576 (2008e:68157). Zbl 1099.68074.

Weighted signed-graph clustering; *cf.* Bansal, Blum, and Chawla (2002a, 2004a). An $O(\log n)$-approximation algorithm for the weighted case based on linear-programming rounding and region growing. We also prove that this linear program has a gap of $\Omega(\log n)$, and therefore our approximation is tight under this approach. An $O(r^3)$-approximation algorithm for graphs without a $K_{r,r}$-minor [e.g., planar, if $r = 3$]. The problem is equivalent to minimum multicut, and therefore APX-hard and difficult to approximate better than $\Theta(\log n)$. [Annot. 13 Sept 2009.] (**SG: WG: Clu: Alg**)

Erik Demaine and Nicole Immorlica

2003a Correlation clustering with partial information. In: *Proceedings of the 6th International Workshop on Approximation Algorithms for Combinatorial Optimization Problems and 7th International Workshop on Randomization and Approximation Techniques in Computer Science (RANDOM-APPROX 2003)* (Princeton, N.J., 2003), pp. 1–13. Lect. Notes in Computer Sci., Vol. 2764. Springer, Berlin, 2003. MR 2080776 (2005c:68291). Zbl 1202.68479.

Conference version of Demaine, Emanuel, Fiat, and Immorlica (2006a). [Annot. 13 Sept 2009.] (**SG: WG: Clu: Alg**)

Jacques Demongeot

See J. Aracena and O. Cinquin.

Hongzhong Deng and Peter Abell

2010a A study of local sign change adjustment in balancing structures. *J. Math. Sociology* 34 (2010), no. 4, 253–282. Zbl 1201.91166.

A random signed graph has edge vw with probability d, which is positive with probability α_0. Degree of balance is the proportion of triangles that are positive. A triangle of type T_i has i positive edges. They study the long-term proportions of triangle types in examples. §3, "Balance adjustment under a local rule": A triangle $\triangle uvw$ and edge uv are chosen at random; uv changes sign iff $\triangle uvw$ is negative. This "myopic adjustment rule" is iterated. For $0 < \alpha_0 < 1$, the proportions approach 37% each of 1 or 2 and 13% each of 0 or 3 negative edges. This contradicts the Cartwright–Harary (1957a) balance hypothesis. Convergence behavior in examples depends interestingly on n and d. §§4–7: The sign-change probability depends on the triangle type. Probabilities are suggested by models of Harary–Cartwright, Davis (1967a), and others, in which different sets of triangle types are "attractors". Analytical and example results are reported.

Model based on Antal, Krapivsky, and Redner (2006a). Successor to Abell and Ludwig (2009a) and Kujawski, Abell, and Ludwig (20xxa). [See Barahona, Maynard, Rammal, and Uhry (1982a) for modelling of planar grid graphs.] [Annot. 6 Dec 2009.] (**SG: Bal**)

Hongzhong Deng, Peter Abell, Ji Li, and Jun Wu

2012a A study of sign adjustment in weighted signed networks. *Social Networks* 34 (2012), no. 2, 253–263. (**SG: WG, PsS**)

Hongzhong Deng, Peter Abell, Jun Wu, and Yuejin Tang

20xxa The influence of structural balance and homophily/heterophobia on the adjustment of random complete signed networks. Submitted. (**SG: Bal, PsS: KG**)

Wouter de Nooy

See W. de Nooy (under N).

Arnout van de Rijt

See A. van de Rijt (under V).

B. Derrida, Y. Pomeau, G. Toulouse, and J. Vannimenus

1979a Fully frustrated simple cubic lattices and the overblocking effect. *J. Physique* 40 (1979), 617–626.

Physics of the signed d-hypercube in which every plaquette is negative; specifically, [cleverly] construct $\Sigma_d = (Q_d, \sigma_d)$, $d > 0$, as $\Sigma_{d-1} \times (+Q_1)$ with the second copy of Σ_{d-1} negated. Invariants of physical interest are computed and compared to the balanced case. Dictionary: "plaquette" = square. (**Phys: SG**)

1980a Fully frustrated simple cubic lattices and phase transitions. *J. Physique* 41 (1980), 213–221. MR 566063 (80m:82020). (**Phys: SG**)

Madhav Desai and Vasant Rao

1994a A characterization of the smallest eigenvalue of a graph. *J. Graph Theory* 18 (1994), no. 2, 181–194. MR 1258251 (95c:05084). Zbl 792.05096.

$\psi(\Gamma) := \min_S(l(-\Gamma{:}S) + |E(S, S^c)|/|S|)$, over $\varnothing \subset S \subseteq V$, is a measure of nonbipartiteness of Γ. $\mu_1 :=$ smallest eigenvalue of $K(-\Gamma)$ satisfies $\psi(\Gamma)^2/4\Delta(\Gamma) \leq \mu_1 \leq 4\psi(\Gamma)$. Their $e_{\min}(\Gamma) := l(-\Gamma)$. [See Cvetković, Rowlinson, and Simić (2007a).] [Annot. 19 Sept 2010.] (**Par: Adj, Fr**)

L. de Sèze

See L. de Sèze (under S).

C. De Simone, M. Diehl, M. Jnger, P. Mützel, G. Reinelt, and G Rinaldi

1995a Exact ground states of Ising spin glasses: New experimental results with a branch and cut algorithm. *J. Stat. Phys.* 80 (1995), 487–496. Zbl 1106.82323.

Improves the algorithm of Barahona, Grötschel, Michael Jünger, and Gerhard Reinelt (1988a) to find a switching with minimum $|E^-|$ $(= l(\Sigma))$ for signed toroidal square lattice graphs with an extra vertex (exterior magnetic field) and a fixed proportion of negative edges. Applied to many signatures in order to find statistical properties. [Annot. 18 Aug 2012.] (**Phys, sg: Fr: Alg**)

1996a Exact ground states of two-dimensional $\pm J$ Ising spin glasses. *J. Stat. Phys.* 84 (1996), 1363–1371.

Continuation of (1995a). [Annot. 18 Aug 2012.] (**Phys, sg: Fr: Alg**)

A.H. Deutz, A. Ehrenfeucht, and G. Rozenberg

1994a Hyperedge channels are abelian. *Theor. Computer Sci.* 127 (1994), 367–393. MR 96b:68023. Zbl 824.68011. (**GH**)

M. DeVos

2004a Flows on bidirected graphs. Manuscript, 2004.

Corrected and extended in Raspaud and Zhu (2011a) (q.v.). [Annot. 23 March 2010.] (**SG: Ori, Flows**)

M. Deza, V.P. Grishukhin, and M. Laurent

1991a The symmetries of the cut polytope and of some relatives. In: Peter Gritzman and Bernd Sturmfels, eds., *Applied Geometry and Discrete Mathematics: The Victor Klee Festschrift*, pp. 205–220. DIMACS Ser. Discrete Math. Theor. Comput. Sci., Vol. 4. American Math. Soc., Providence, R.I., 1991. MR 1116350 (92e:52019). Zbl 748.05061.

Switching (on coordinates) is an important symmetry of the cut polytope P_n (of K_n); see p. 206. [See (1997a).] Thm. 2.6: $\operatorname{Aut} P_n = \mathfrak{D}_n$, the Weyl group [= $\operatorname{SwAut}(\pm K_n)$, the switching automorphism group]. *Question* (p. 207): For the cut polytope $P_c(\Gamma)$, does $\operatorname{Aut} P_c(\Gamma) = \operatorname{SwAut}(\pm\Gamma)$? [Edge signs and SwAut are not stated as such.] [Annot. 12 Jun 2012.] (**sg: par: KG: Geom, sw**)

Michel Marie Deza and Monique Laurent

1997a *Geometry of Cuts and Metrics.* Algorithms and Combin., Vol. 15. Springer, Berlin, 1997. MR 98g:52001. Zbl 885.52001.

A main object of interest is the cut polytope, which is the bipartite subgraph polytope (see Barahona, Grötschel, and Mahjoub (1985a)) of K_n, i.e., the balanced subgraph polytope (Poljak and Turzík (1987a)) of $-K_n$. §4.5, "An application to statistical physics", briefly discusses the spin glass application. §26.3, "The switching operation", discusses graph switching and its generalization to sets. §30.3, "Circulant inequalities", mentions Poljak and Turzík (1987a, 1992a). No explicit mention of signed graphs. (**sg: par: KG: fr, sw Geom: Exp**)

Persi Diaconis

See K.S. Brown.

Y. Diao, G. Hetyei, and K. Hinson

2009a Tutte polynomials of tensor products of signed graphs and their applications in knot theory. *J. Knot Theory Ramifications* 18 (2009), no. 5, 561–589. MR 2527677 (2010c:57010). Zbl 1185.05083. arXiv:math/0702328.

(**SGc: Invar, Knot**)

M. Diehl

See C. De Simone.

Hung T. Diep, P. Lallemand, and O. Nagai

1985a Simple cubic fully frustrated Ising crystal by Monte Carlo simulations. *J. Appl. Phys.* 57 (1985), 3309–3311.

Physics of fully frustrated 3-dimensional cubic lattice (*cf.* Chui, Forgacs, and Hatch (1982a), but the negative edges are specifically chosen to form three orthogonal families of straight lines, alternating along each plane. As signed lattice has a 2×2 fundamental domain, there are 8 translational symmetry types of vertex, each forming a double-sized sublattice. The sublattices exhibit somewhat differentiated behavior. [Annot. 18 Jun 2012.] (**Phys, SG: Fr, sw**)

1985b Critical properties of a simple cubic fully frustrated Ising lattice by Monte Carlo method. *J. Phys. C* 18 (1985), 1067–1078.

Simulations on the the signed graph of (1985a). The 8 sublattices are equivalent in pairs. [Annot. 18 Jun 2012.] (**Phys, SG: Fr**)

V. Di Giorgio

1974a 2-modules dans un graphe: equilibre et coequilibre d'un bigraphe—application taxonomique. *Bull. Math. Soc. Sci. Math. R. S. Roumanie (N.S.)* 18 (66) (1974), 81–102 (1975). MR 57 #16124. Zbl 324.05127. (**SG: Bal**)

Wil Dijkstra

1979a Response bias in the survey interview; an approach from balance theory. *Social Networks* 2(1979-1980), no. 3, 285–304.

Extends signed graphs to sign set $\{\pm1, 0\}$ and extends the notion of (degree of) cycle balance. A circle C is "balanced" if its sign product $\sigma(C) = +1$. Degree of balance = average sign product of all circles. Degree of local balance at $X \subseteq V$ is the average sign of all circles that contain X. Given a length weight function $1 \geq f(2) \geq f(3) \geq \cdots \geq 0$, the weighted degree of balance is the average value of $f(l(C))\sigma(C)$. [*Cf.* kinds of cycle balance in Cartwright and Harary (1956a), Morrissette (1958a), Norman and Roberts (1972a,b).] (**SG: Bal, Fr**)

It is assumed [!] that answers have probability dependent on weighted degree of local balance at $\{p, y\}$ where p = respondent and y = answer. Speculation about choice of functions f *et al.* One post-hoc application. (**SG: PsS**)

Genhong Ding

See X.B. Ma.

Yvo M.I. Dirickx and M.R. Rao

1974a Networks with gains in discrete dynamic programming. *Management Sci.* 20 (1974), No. 11 (July, 1974), 1428–1431. MR 50 #12279. Zbl 303.90052. (**GN: M(bases)**)

Ajit A. Diwan

See M. Joglekar.

Daniel B. Dix

2006a Polyspherical coordinate systems on orbit spaces with applications to biomolecular shape. *Acta Appl. Math.* 90 (2006), 247–306. MR 2248745 (2007k:92043). Zbl 1182.92028. (**GG: Appl**)

Duong D. Doan and Patricia A. Evans

2011a Haplotype inference in general pedigrees with two sites. 6th Int. Symp. Bioinformatics Res. Appl. (ISBRA10, Storrs, Conn., U.S.A., 2010). *BMC Proc.* 5

(2011), Suppl. 2, 56, 10 pp.

A pedigree is a kind of signed graph with $< n$ edges, with 3-colored vertices. Frustration index ("line index") l = minimum number of necessary recombinations. Elementary relations among l, vertex cuts, and switching. Reduction rules, inlcuding the negative-subdivision trick, to test $l \leq k$. [*Question.* Does sparseness reduce the hardness of testing $l \leq k$?] [Annot. 29 Apr 2012.] (**Biol: SG: Fr, Alg, sw**)

2011b An FPT haplotyping algorithm on pedigrees with a small number of sites. *Algorithms Molecular Biol.* 6 (2011), no. 8, 8 pp.

See (2011a). This problem adds parity constraints. [Annot. 29 Apr 2012.] (**Biol: SG: Fr, Alg**)

Benjamin Doerr

2000a Linear discrepancy of basic totally unimodular matrices. *Electronic J. Combin.* 7 (2000), Research Paper R48, 4 pp. MR 2001e:15017. Zbl 996.15012.

The linear discrepancy of the transposed incidence matrix of a balanced signed graph. (**sg: bal: Incid**)

B.G.S. Doman and J.K. Williams

1982a Low-temperature properties of frustrated Ising chains. *J. Phys. C* 15 (1982), 1693–1706.

§2, "The random bond model at low temperatures": A path with random edge signs, all weights J, magnetic field B [interpretable as an extra all-positive vertex with edge weights B; *cf.* Barahona (1982a)]. §3, "Frustrated periodic bond model": A path with edges signed $+ - --$ periodically, with weight J, and a magnetic field B. Describes allowed states [ground states], depending on B/J. [Annot. 28 Aug 2012.] (**Phys, SG, WG: fr**)

Eytan Domany

See D. Kandel.

Bing-can Dong

See R.L. Li.

Michael Doob

See also D.M. Cvetković.

1970a A geometric interpretation of the least eigenvalue of a line graph. In: *Proceedings of the Second Chapel Hill Conference on Combinatorial Mathematics and Its Applications* (1970), pp. 126–135. Univ. of North Carolina at Chapel Hill, Chapel Hill, N.C., 1970. MR 42 #2959. Zbl 209, 554 (e: 209.55403).

A readable, tutorial introduction to (1973a) (without matroids). (**ec: LG, Incid, Adj(LG)**)

1973a An interrelation between line graphs, eigenvalues, and matroids. *J. Combin. Theory Ser. B* 15 (1973), 40–50. MR 55 #12573. Zbl 245.05125, (257.05132).

Along with Simões-Pereira (1973a), introduces to the literature the even-cycle matroid $G(-\Gamma)$ [previously invented by Tutte, unpublished]. The multiplicity of -2 as an eigenvalue (in characteristic 0) equals the number of independent even circles $= n - \operatorname{rk} G(-\Gamma)$. In characteristic p there is a similar theorem, but the pertinent matroid is $G(\Gamma)$ if $p = 2$ and, when $p|n$, the matroid has rank 1 greater than otherwise [a fact that mystifies me]. (**EC: LG, Incid, Adj(LG)**)

1974a Generalizations of magic graphs. *J. Combin. Theory Ser. B* 17 (1974), 205–217. MR 51 #274. Zbl 271.05128, (287.05124).

Thm. 3.2 is the theorem of van Nuffelen (1973a), supplemented by the observation that it remains true in any characteristic except 2. (**EC: Incid**)

1974b On the construction of magic graphs. In: F. Hoffman *et al.*, eds., *Proceedings of the Fifth Southeastern Conference on Combinatorics, Graph Theory and Computing* (Boca Raton, 1974), pp. 361–374. Utilitas Math. Publ. Inc., Winnipeg, Man., 1974. MR 53 #13039. Zbl 325.05123. (**ec: Incid**)

1978a Characterizations of regular magic graphs. *J. Combin. Theory Ser. B* 25 (1978), 94–104. MR 58 #21840. Zbl 384.05054. (**ec: Incid**)

Michael Doob and Dragoš Cvetković

1979a On spectral characterizations and embeddings of graphs. *Linear Algebra Appl.* 27 (1979), 17–26. MR 81d:05050. Zbl 417.05025. (**sg: LG, Adj(LG)**)

Patrick Doreian

See also N.P. Hummon and A. Mrvar.

1970a Book review: *Balance in Small Groups* by Howard F. Taylor. *Sociological Rev.* 18 (1970), no. 3, 422–424.

Review of Taylor (1970a). [Annot. 27 Apr 2012.] (**PsS: SG, WG: Bal, Fr, Adj: Exp**)

1985a Book review: *Structural Models in Anthropology* by Per Hage and Frank Harary. *J. Math. Sociology* 11 (1985), 283–285.

Review of Hage and Harary (1983a). [Annot. 27 Apr 2012.] (**PsS: SG: Exp**)

2002a Event sequences as generators of social network evolution. *Social Networks* 24 (2002), 93–119. (**SG: Bal, PsS**)

2004a Evolution of signed human networks. *Metodoloŷski Zvezki* 1 (2004), 277–293.

Reviews the development of balance and clustering theory for signed (di)graphs in social psychology, mainly Doreian and Mrvar (1996a), Doreian and Krackhardt (2001a), and especially Hummon and Doreian (2003a). The difference between Heider's (1946a) and Cartwright and Harary's (1956a) models, and the need to combine them. [Annot. 24 Apr 2009.] (**PsS: Exp: SD, Bal, Clu, Alg**)

2006a Book review: W. de Nooy, A. Mrvar, and V. Batagelj, *Exploratory Social Network Analysis with Pajek*. *Social Networks* 28 (2006), 269–274.

Review of de Nooy, Mrvar, and Batagelj (2005a). (**PsS: SG, SD: Exp**)

2008a A multiple indicator approach to blockmodeling signed networks. *Social Networks* 30 (2008), 247–258.

Signed graphs $\Sigma_1, \ldots$ ("multiple indicators") may be approximations of a hidden signed graph Σ. Goals: detect whether Σ exists, and find an optimal clustering of Σ. Methods: (1) Examine the Σ_j for compatibility via statistical tests. (2) Estimate Σ by $\sum_j \sigma_j$. (3) Applies the clusterability index and algorithm of Doreian and Mrvar (1996a). ((2) implies using weighted signed graphs.) This article treats examples, with analysis of the methods' success. [Annot. 27 Apr 2009.] (**PsS, SD: sg: Clu**)

2008b Clashing paradigms and mathematics in the social sciences. *Contemp. Sociology* 37 (2008), no. 6, 542–545.

Two books on and the philosophy of mathematics and sociology. [Annot. 27 Apr 2012.] (**PsS: SG, SD**)

Patrick Doreian, Vladimir Batagelj, and Anuška Ferligoj

2005a *Generalized Blockmodeling.* Structural Analysis in the Social Sciences, No. 25. Cambridge Univ. Press, Cambridge, Eng., 2005.

Ch. 10: "Balance theory and blockmodeling signed networks". Thm. (pp. 305–306; proof by Martin Everett): The sizes of the partitions of V that minimize the clustering index (Doreian and Mrvar 1996a) are consecutive integers. (**PsS, SD: sg: Clu, Bal**)

Patrick Doreian, Roman Kapuscinski, David Krackhardt, and Janusz Szczypula

1996a A brief history of balance through time. *J. Math. Sociology* 21 (1996), 113–131. Repr. in Patrick Doreian and Frans N. Stokman, eds., *Evolution of Social Networks*, pp. 129–147. Gordon and Breach, Australia, Amsterdam, etc., 1997. Zbl 883.92034.

§2.3: "A method for group balance". Describes the negation-minimal index of clusterability (generalized imbalance) from Doreian and Mrvar (1996a). (**SG: Bal, Clu: Fr(Gen): Exp**)

§3.3: "Results for group balance". Describes results from analysis of data on a small (social) group, in terms of frustration index l and a clusterability index $\min_{k>2} 2P_{k,.5}$ (slightly different from the index in Doreian and Mrvar (1996a)), finding both measures (but more so the latter) decreasing with time. (**PsS: Bal, Clu: Fr(Gen)**)

Patrick Doreian and David Krackhardt

2001a Pre-transitive balance mechanisms for signed networks. *J. Math. Sociology* 25 (2001), no. 1, 43–67. Zbl 1017.91520.

In a signed digraph from empirical social-group data, a tendency to transitivity of signs on directed edges ij, ik, jk (i.e., $\sigma(ij)\sigma(jk)\sigma(ik) = +$) holds when $\sigma(ij) = +$ and fails when $\sigma(ij) = -$. This suggests that balance is not a primary tendency and Harary's (1953a) and Davis's (1967a) theorems on balance and clusterability have limited relevance to social groups. [Also, that undirected signed graphs have limited relevance. Digraph sign transitivity properties are more relevant.] [A thoughtful article.] [Annot. 13 Apr 2009.] (**PsS, sd**)

Patrick Doreian, Paulette Lloyd, and Andrej Mrvar

20xxa Partitioning large signed two-mode networks: Problems and prospects. *Social Networks*, to appear (**SG: Bal, Fr, PsS**)

Patrick Doreian and Andrej Mrvar

1996a A partitioning approach to structural balance. *Social Networks* 18 (1996), 149–168.

They propose indices for clusterability (as in Davis (1967a)) that generalize the frustration index of Σ. Fix $k \geq 2$ and $\alpha \in [0, 1]$. For a partition π of V into k "clusters", they define $P(\pi) := \alpha n_- + (1 - \alpha)n_+$, where $n_+ :=$ number of positive edges between clusters, $n_- :=$ number of negative edges within clusters, and $0 \leq \alpha \leq 1$ is fixed. The first proposed measure is $\min P(\pi)$, minimized over k-partitions. A second suggestion is the "negation-minimal index of generalized imbalance" [i.e., of clusterability], the smallest number of edges whose negation [equivalently, deletion] makes Σ clusterable. [Call it the 'clusterability index' $c(\Sigma)$.] [Note that $P(\pi)$ effectively generalizes the Potts Hamiltonian as given by Welsh (1993a). *Question.* Does $P(\pi)$ fit into an interesting generalized Potts model?] [$P(\pi)$ also resembles the Potts Hamiltonian in Fischer and Hertz (1991a) (*q.v.* for a related research question).] [The data in Doreian (2008a), and common sense, suggest that clusters should be allowed to overlap. This is an open research direction.]

They employ a local optimization algorithm to evaluate $P_{k,\alpha}$ and find an optimal partition: random descent from partition to neighboring partition, where π and π' are neighbors if they differ by transfer of one vertex or exchange of two vertices between two clusters. This was found to work well if repeated many times. [A minimizing partition into at most k clusters is equivalent to a ground state of the k-spin Potts model in the form given by Welsh (1993a), but not quite in that of Fischer and Hertz (1991a).]

Terminology: $P(\pi)$ is called the "criterion function" [more explicitly, one might call $c_\alpha(\Sigma, \pi) := 2P(\pi)$ the 'α-weighted clustering index of π', so the clusterability index $c(\Sigma) = \min_\pi c_{.5}(\Sigma, \pi)$]. Clusterability is "$k$-balance" or "generalized balance". The clusters are "plus-sets". Signed digraphs are employed in the notation but direction is ignored.

(**SD: sg: Bal, Clu: Fr(Gen), Alg, PsS**)

1996b Structural balance and partitioning signed graphs. In: A. Ferligoj and A. Kramberger, eds., *Developments in Data Analysis*, pp. 195–208. Metodološki zvezki, Vol. 12. FDV, Ljubljana, Slovenia, 1996.

Similar to (1996a). Some lesser theoretical detail; some new examples. The k-clusterability index $P_{k,\alpha}$ (1996a) is compared for different values of k, seeking the minimum. [But for which value(s) of α is not stated.] Interesting observation: optimal values of k were small. It is said that positive edges between parts are far more acceptable socially than negative edges within parts [thus, in the criterion function α should be rather near 1]. (**SD: sg: Bal, Clu: Fr(Gen), Alg, PsS**)

2009a Partitioning signed social networks. *Social Networks* 31 (2009), no. 1, 1–11.

Generalizes the ideas of (1996a) (*q.v.*). Given: A signed digraph $(\vec{\Gamma}, \sigma)$; a "criterion function" $P(\pi, \rho) := \alpha n^+ + (1-\alpha)n^-$, where $\pi := \{B_1, \ldots, B_k\}$ partitions V into "clusters", $\rho : \pi \times \pi \to \{+, -\}$, $0 \leq \alpha \leq 1$ is fixed, and $n^\varepsilon :=$ number of edges $\overrightarrow{B_i B_j}$ with sign ε for which $\rho(B_i, B_j) = -\varepsilon$ (over all i, j). Objective: (π, ρ), or simply $k := |\pi|$, that minimizes $P(\pi, \rho)$. What is new and most interesting is ρ; also new is using the edge directions.

Call $(\vec{\Gamma}, \sigma)$ "sign clusterable" if $\exists$ (π, ρ) with $P(\pi, \rho) = 0$. Clusterability is sign clusterability with $\rho = \rho_+$, where $\rho_+(B_i, B_j) := +$ if $i = j$, $-$ if $i \neq j$. Let $P(k) := \min\{P(\pi, \rho) : |\pi| = k\}$. Thm. 4: $P(k)$ is monotonically decreasing. [Thus, there is always an optimum π with singleton clusters. Why this does not vitiate the model is not addressed.] Thm. 5: If $(\vec{\Gamma}, \sigma)$ is sign clusterable, then $(\vec{\Gamma}, -\sigma)$ also is. If $(\vec{\Gamma}, \sigma)$ is clusterable, then $(\vec{\Gamma}, -\sigma)$ is not clusterable with the same π [provided $E \neq \varnothing$]. If $(\vec{\Gamma}, \sigma)$ is sign clusterable with $\rho = -\rho_+$, then $(\vec{\Gamma}, -\sigma)$ is clusterable with the same π. "Relocation": Shift one vertex, or exchange two vertices, between blocks so as to decrease P, as in (1996a). This is said (but not proved) to minimize P.

Refinements discussed: partially prespecified blocks; null blocks (without outgoing edges); criterion functions with special treatment of null blocks.

Applications to standard test examples of social psychology.

Dictionary: "balanced" = clusterable; "relaxed balanced" = sign clusterable; "k-balanced" = clusterable with $|\pi| = k$; "relaxed structural balance blockmodel" = this whole system. [Annot. 7 Feb 2009.]

(**SG: Bal, Clu, PsS**)

W. Dörfler

1977a Double covers of graphs and hypergraphs. In: *Beitrage zur Graphentheorie und deren Anwendungen* (Proc. Int. Colloq., Oberhof, D.D.R., 1977), pp. 67–79. Technische Hochschule, Ilmenau, 1977. MR 82c:05074. Zbl 405.05055.

(**SG: Cov, LG**)(**SD, SH: Cov**)

1978a Double covers of hypergraphs and their properties. *Ars Combinatoria* 6 (1978), 293–313. MR 82d:05085. Zbl 423.050532. (**SH: Cov, LG**)

Tomislav Došlić

See also Z. Yarahmadi.

Tomislav Došlić and Damir Vukičevic

2007a Computing the bipartite edge frustration of fullerene graphs. *Discrete Appl. Math.* 155 (2007), 1294–1301. MR 2332321 (2008b:05086). Zbl 1118.05092.

(**sg: Par: Fr**)

Lynne L. Doty

See F. Buckley.

Peter Doubilet

1971a Dowling lattices and their multiplicative functions. In: *Möbius Algebras* (Proc. Conf., Waterloo, Ont., 1971), pp. 187–192. Univ. of Waterloo, Ont., 1971, reprinted 1975. MR 50 #9605. Zbl 385.05008. (**GG: M**)

Peter Doubilet, Gian-Carlo Rota, and Richard Stanley

1972a On the foundations of combinatorial theory (VI): The idea of generating function. In: *Proceedings of the Sixth Berkeley Symposium on Mathematical Statistics and Probability* (Berkeley, Calif., 1970/71), Vol. II: *Probability Theory*, pp. 267–318. Univ. of California Press, Berkeley, Calif., 1972. MR 53 #7796. Zbl 267.05002. Repr. in: Gian-Carlo Rota, *Finite Operator Calculus*, pp. 83–134. Academic Press, New York, 1975. MR 52 #119. Zbl 328.05007. Repr. again in: Joseph P.S. Kung, ed., *Gian-Carlo Rota on Combinatorics: Introductory Papers and Commentaries*, pp. 148–199. Birkhäuser, Boston, 1995. MR 99b:01027. Zbl 841.01031.

§5.3: Brief treatment of Dowling lattices via symmetric gain digraphs.

(**GG: M**)

T.A. Dowling

1971a Codes, packings, and the critical problem. In: *Atti del Convegno di Geometria Combinatoria e sue Applicazioni (Perugia, 1970)*, pp. 209–224. Ist. Mat., Univ. di Perugia, Perugia, Italy, 1971. MR 49 #2438. Zbl 231.05029.

Pp. 221–223: The first intimations of Dowling lattices/geometries/matroids, as in (1973a, 1973b), and their higher-weight relatives (see Bonin 1993a). **(gg, Gen: M)**

1973a A q-analog of the partition lattice. Ch. 11 in: J.N. Srivastava *et al.*, eds., *A Survey of Combinatorial Theory* (Proc. Int. Sympos., Ft. Collins, Colo., 1971), pp. 101–115. North-Holland, Amsterdam, 1973. MR 51 #2954. Zbl 259.05023.

Linear-algebraic progenitor of (1973b). Treats the Dowling lattice of group $\mathrm{GF}(q)^\times$ as naturally embedded in $\mathrm{PG}^{n-1}(q)$. Interesting is p. 105, Remark: One might generalize some results to any ambient (simple) matroid. **(gg: Geom, M: Invar)**

††1973b A class of geometric lattices based on finite groups. *J. Combin. Theory Ser. B* 14 (1973), 61–86. MR 46 #7066. Zbl 247.05019. Erratum. *Ibid.* 15 (1973), 211. MR 47 #8369. Zbl 264.05022.

$Q_n(\mathfrak{G})$ Introduces the Dowling lattices $Q_n(\mathfrak{G})$ of a group, treated as lattices of group-labelled partial partitions. Equivalent to the bias matroid of complete $\mathfrak{G}$-gain graph $\mathfrak{G}K_n^\bullet$. [The gain-graphic approach was known to Dowling (1973a, p. 109) but first published in Doubilet, Rota, and Stanley (1972a).] Isomorphism, vector representation, Whitney numbers and characteristic polynomial. [The first and still fundamental paper.] **(gg: M: Invar)**

Thomas Dowling and Hongxun Qin

2005a Reconstructing ternary Dowling geometries. *Adv. Appl. Math.* 34 (2005), no. 2, 358–365. MR 2005j:05017. Zbl 1068.52017.

Thm. 1.5: The Dowling geometry $Q_r(\mathbb{Z}_2)$ is the only matroid of rank $r \geq 4$ such that every contraction by a point is $Q_{r-1}(\mathbb{Z}_2)$. **(sg: M)**

20xxa Excluded minors for classes of cographic matroids. Submitted. **(GG: M, Top, SG)**

Pauline van den Driessche

See van den Driessche (under 'V').

J.M. Drouffe

See R. Balian.

K. Drühl and H. Wagner

1982a Algebraic formulation of duality transformations for Abelian lattice models. *Ann. Phys.* 141 (1982), 225–253. MR 673981 (84h:82064). **(SG, GG: Gen: D, Fr, Phys)**

Natasha D'Souza

See T. Singh.

Hong Shan Du, Qing Jun Ren, Hou Chun Zhou, and Qing Yu Zheng

1998a The quasi-Laplacian permanental polynomial of a graph. (In Chinese.) *Qufu Shifan Daxue Xuebao Ziran Kexue Ban* [*J. Qufu Normal Univ., Nat. Sci.*] 24 (1998), no. 2, 59–62. MR 1655784 (no rev). **(Par: Adj)**

Wenxue Du, Xueliang Li, and Yiyang Li

2010a Various energies of random graphs. *MATCH Commun. Math. Comput. Chem.* 64 (2010), no. 1, 251–260. MR 2677586 (2011k:05133).

Including a "tight bound" on signless Laplacian energy, of $K(-\Gamma)$, and "exact estimate" of incidence energy, of $\mathrm{H}(-\Gamma)$. [Annot. 24 Jan 2012.] **(Par: Adj)**

Hangen Duan
See S.C. Gong.

P. Robert Duimering
See G. Adejumo.

Richard A. Duke, Paul Erdös, and Vojtěch Rödl
1992a Cycle-connected graphs. *Discrete Math.* 108 (1992), 261–278. MR 94a:05106. Zbl 776.05057.

All graphs are simple. This is one of four related papers that prove extremal results concerning subgraphs of $-\Gamma$ within which every two edges belong to a balanced circle of length at most $2k$, for all or particular k. Typical theorem: Let $F_l(n,m)$ = the largest number $m' = m'(n,m)$ such that every $-\Gamma$ with $|V| = n$ and $|E| \geq m$ has a subgraph Σ' with $|E'| = m'$ in which every two edges belong to a balanced circle of length at most l. For $m = m(n) \geq n^{3/2}$, there is a constant $c_3 > 0$ such that $F_l(n,m) \leq c_3 m^2 n^{-2}$ for all l. (§2, (2).) [*Problem.* Extend these extremal results in an interesting way to arbitrary signed simple graphs, or to simply signed graphs (no repeated edges with the same sign). (Merely allowing positive edges in addition to negative ones just makes the problem easier. Something more is required.)]
(**par: bal(Circles): Xtreml**)

David M. Duncan, Thomas R. Hoffman, and James P. Solazzo
2010a Equiangular tight frames and fourth root seidel matrices. *Linear Algebra Appl.* 432 (2010), 2816–2823. MR 2639246 (2011c:42081). Zbl 1223.05172.

Adjacency matrices of fourth-root-of-unity gain graphs on K_n. Dictionary: "Seidel matrix" = adjacency matrix of such a gain graph. [Annot. 20 June 2011.] (**gg: Geom, adj: kg**)

Yen Duong, Joel Foisy, Killian Meehan, Leanne Merrill, and Lynea Snyder
†2012a Intrinsically linked signed graphs in projective space. *Discrete Math.* 312 (2012), 2009–2022. Zbl 1243.05101. (**SG: Top**)

Arne Dür
1986a *Möbius Functions, Incidence Algebras and Power Series Representations.* Lect. Notes in Math., Vol. 1202. Springer-Verlag, Berlin, 1986. MR 88m:05005. Zbl 592.05006.

Dowling lattices are an example of a categorial approach to incidence-algebra techniques in Ch. IV, §7. Computed are the characteristic polynomial and second kind of Whitney numbers. Binomial concavity, hence unimodality of the latter [*cf.* Stonesifer (1975a)] is proved by showing that a suitable generating polynomial has only distinct, negative zeros [*cf.* Benoumhani (1999a)]. (**gg: M: Invar**)

P.M. Duxbury
See M.J. Alava.

David Easley and Jon Kleinberg
2010a *Networks, Crowds, and Markets: Reasoning About a Highly Connected World.* Cambridge Univ. Press, Cambridge, Eng., 2010. MR 2677125 (2011i:91001). Zbl 1205.91007.

Ch. 5, "Positive and negative relationships". §§5.1, "Structural balance", 5.2, "Balanced networks and the Cartwright-Harary Theorem": Balance by triangles in signed complete graphs. Proof of Harary's (1953a) bipartition theorem for complete graphs (§5.2). §5.3, "Applications of structural balance": Applications to history. Complete and incomplete graphs. Alternatives to structural balance. §5.4, "A weaker form of structural balance": Clusterability ("weak balance"). Proof of Davis's (1967a) clusterability theorem. §5.5, "Advanced material: Generalizing the definition of structural balance": Two parts. §5.5A, "Structural balance in arbitrary (non-complete) networks": Proof of Harary's bipartition theorem for general signed graphs by finding connected components of the positive subgraph, then applying breadth-first search to sign the components. [Annot. 22 March 2010.] **(SG: Bal: Exp, Exr)**

§5.5B, "Approximately balanced networks": Thm.: If the proportion of unbalanced triangles in a signed K_n is $\leq \varepsilon < \frac{1}{8}$, and if $\delta := \sqrt{[3]}\varepsilon$, then there are $(1-\delta)|V|$ vertices in which at most a fraction δ of the edges are negative, or there is a bipartition $V = X \cup Y$ such that at most a fraction δ of the edges in X and also in Y are negative and at most that fraction of the XY edges are positive. [Annot. 22 March 2010.] **(SG: Bal, fr)**

Paul H. Edelman and Victor Reiner

1994a Free hyperplane arrangements between A_{n-1} and B_n. *Math. Z.* 215 (1994), 347–365. MR 95b:52021. Zbl 793.05122.

Characterizes all $\Sigma \supseteq +K_n$ whose bias matroid $G(\Sigma)$ is supersolvable, free, or inductively free. Essentially, iff the negative links form a threshold graph. [Continued in Bailey (20xxa). Generalized in part to arbitrary gain groups in Zaslavsky (2001a).] **(sg: M, Geom, col)**

1996a Free arrangments and rhombic tilings. *Discrete Comput. Geom.* 15 (1996), 307–340. MR 97f:52019. Zbl 853.52013. Erratum. *Discrete Comput. Geom.* 17 (1997), 359. MR 97k:52013. Zbl 853.52013.

Paul H. Edelman and Michael Saks

1979a Group labelings of graphs. *J. Graph Theory* 3 (1979), 135–140. MR 80j:05071. Zbl 411.05059.

Given Γ and abelian group $\mathfrak{A}$. Vertex and edge labellings $\lambda : V \to \mathfrak{A}$ and $\eta : E \to \mathfrak{A}$ are "compatible" if $\lambda(v) = \sum_e \eta(e)$ for every vertex v, the sum taken over all edges incident with v. λ is "admissible" if it is compatible with some η. Admissible vertex labellings are characterized (differently for bipartite and nonbipartite graphs) and the number of edge labelings compatible with a given vertex labelling is computed. [Dual in a sense to Gimbel (1988a).] **(WG, VS: Bal(D), Enum)**

Herbert Edelsbrunner, Günter Rote, and Emo Welzl

1987a Testing the necklace condition for shortest tours and optimal factors in the plane. In: T. Ottmann, ed., *Automata, languages and programming* (Proc., Karlsruhe, 1987), pp. 364–375. Lect. Notes in Computer Sci., Vol. 267. Springer, Berlin, 1987. MR 88k:90065. Zbl 636.68042.

Summary of (1989a). **(par: ori, Geom: Alg)**

1989a Testing the necklace condition for shortest tours and optimal factors in the plane. *Theor. Computer Sci.* 66 (1989), 157–180. MR 90i:90042. Zbl 686.68035.

§5.1, "Testing the feasibility of the linear program (2) or (1)": The dual linear program (4) belongs to an oriented all-negative signed graph. Treated by expanding it to the graphic LP belonging to the canonical covering graph. **(par: ori, Geom: Alg)**

Jack Edmonds

See also J. Aráoz and E.L. Lawler (1976a).

1965a Paths, trees, and flowers. *Canad. J. Math.* 17 (1965), 449–467. MR 31 #2165. Zbl 132, 209 (e: 132.20903).

Followed up by much work, e.g., Witzgall and Zahn (1965a); see Ahuja, Magnanti, and Orlin (1993a) for some references.

(**par: ori: incid, Alg**)

1965b Maximum matching and a polyhedron with $0,1$-vertices. *J. Res. Nat. Bur. Standards (U.S.A.) Sect. B* 69B (1965), 125–130. MR 32 #1012. Zbl (e: 141.21802).

Alludes to the polyhedron of Edmonds and Johnson (1970a).

(**par: ori: Incid, Geom**)

Jack Edmonds and Ellis L. Johnson

††1970a Matching: a well-solved class of integral linear programs. In: Richard Guy *et al.*, eds., *Combinatorial Structures and Their Applications* (Proc. Calgary Int. Conf., Calgary, 1969), pp. 89–92. Gordon and Breach, New York, 1970. MR 42 #2799. Zbl 258.90032.

Introduces "bidirected graphs". A "matching problem" is an integer linear program with nonnegative and possibly bounded variables and otherwise only equality constraints, whose coefficient matrix is the incidence matrix of a bidirected graph. No proofs. [See Aráoz, Cunningham, Edmonds, and Green-Krótki (1983a) for further work.]

(**sg: Ori: Incid, Alg, Geom**)

2003a Matching: a well-solved class of integral linear programs. In: Michael Jünger, Gerhard Reinelt, and Giovanni Rinaldi, eds., *Combinatorial Optimization ?Eureka, You Shrink! Papers Dedicated to Jack Edmonds* (5th Int. Workshop, Aussois, France, 2001), pp. 27–30. Lect. Notes in Computer Sci., Vol. 2570, Springer-Verlag, Berlin, 2003. MR 2163946. Zbl 1024.90505.

Readably typeset reprint of (1970a). (**sg: Ori: Incid, Alg, Geom**)

Yoshimi Egawa

See N. Alon.

Richard Ehrenborg

2001a Counting faces in the extended Shi arrangement $\hat{\mathcal{A}}_n^r$. *Conference Proceedings of the 13th Int. Conference on Formal Power Series and Algebraic Combin.* (FPSAC, Tempe, Ariz., 2001), pp. 149–158.

Preliminary version of (20xxa). [Annot. 11 Mar 2011.]

(**gg: col, Invar, m, Geom**)

20xxa Counting faces in the extended Shi arrangement. Submitted.

Calculates the characteristic (Cor. 2.5) and, implicitly, Whitney-number polynomials of $[-r+1,r]\vec{K}_n$ in terms of its affinographic hyperplane representation, the extended Shi arrangement. The object is to count faces of the latter by dimension and dimension of the infinite part.

(**gg: col, Invar, m, Geom**)

Richard Ehrenborg and Margaret A. Readdy

1998a On valuations, the characteristic polynomial, and complex subspace arrangements. *Adv. Math.* 134 (1998), 32–42. MR 98m:52018. Zbl 906.52004.

An abstract additive approach to the characteristic polynomial $p(\lambda)$, applied in particular (§3: "The divisor Dowling arrangement") to "divisor Dowling" hyperplane arrangements" $\mathcal{B}(m)$ and certain interpolating arrangements. [Let $\Phi = \mathfrak{G}_1 K_1 \cup \cdots \cup \mathfrak{G}_n K_n$, where $V(K_i) = \{v_1, \ldots, v_i\}$ and $\mathbb{Z}_m = \mathfrak{G}_1 \geq \cdots \geq \mathfrak{G}_n$. $\mathcal{B}(m)$ is the complex hyperplane representation of $\Phi^\bullet$. Thus, $p_{\mathcal{B}(m)}(\lambda) = \chi_{\Phi^\bullet}(\lambda)$, the chromatic polynomial. This is computable via gain-graph coloring when $\mathfrak{G}_1$ is any finite group. The same is true for the other arrangements treated herein.] [Annot. 25 Apr 2009.] (**gg: m: Geom, Invar**)

1999a On flag vectors, the Dowling lattice, and braid arrangements. *Discrete Comput. Geom.* 21 (1999), 389–403. MR 2000a:52037. Zbl 941.52021.

Canonical complex hyperplane representation of the Dowling lattice of $\mathbb{Z}_k$. P. 395: an interesting EL-labelling of the Dowling lattice by a [disguised lexicographic] ordering of atoms. Thm. 4.9 is a recursive formula for its **ab**-index. Thm. 5.2: the **c**-2**d**-index of the face lattices in case $k = 1, 2$, i.e., those of the real root system arrangements A_n^* and B_n^*. §6 presents a combinatorial description of the face lattice of B_n^* [which it is interesting to compare with that in Zaslavsky (1991b)]. Dictionary: very confusingly, "region" = face. (**gg: Geom, Invar**)

2000a The Dowling transform of subspace arrangements. *J. Combin. Theory Ser. A* 91 (2000), 322–333. MR 2001k:52038. Zbl 962.05005.

The group expansion of an ordinary graph is generalized to expansion of an $\mathbb{R}^\times_{>0}$-gain graph by a finite cyclic subgroup of $\mathbb{C}^\times$, with correspondingly generalized formulas for the chromatic polynomial. The computations are technically incorrect; they should be done by gain-graph coloring. [Dictionary: "directed cycle" = circle (not directed).] [Generalized in Koban (2004b).] (**GG: Geom, Invar**)

2009a Exponential Dowling structures. *European J. Combin.* 30 (2009), 311–326. MR 2460236 (2010a:06007). Zbl 1157.05002.

A generalization of Stanley's exponential structures, based on the partition lattice, to Dowling lattices. §2 defines Dowling lattices via partial partitions ("zero block" = set of non-partitioned elements). §3 defines Dowling exponential structures and gives compositional identities via generating functions. §4: generating-function identities for the Möbius invariant; structures with restricted block sizes—especially, block sizes divisible by r with K non-partitioned elements where $K \geq k$ and $K \equiv k \pmod r$. (**gg: m: Invar, Enum, Exp**)

Andrzej Ehrenfeucht

See also A.H. Deutz.

Andrzej Ehrenfeucht, Jurriaan Hage, Tero Harju, and Grzegorz Rozenberg

2000a Complexity issues in switching classes of graphs. In: Hartmut Ehrig *et al.*, eds., *Theory and Applications of Graph Transformations (TAGT'98)* (Proc. 6th Int. Workshop, Paderborn, 1998), pp. 59–70. Lect. Notes in Computer Sci., Vol. 1764. Springer-Verlag, Berlin, 2000. MR 2001e:68013 (book). Zbl 958.68133. (**TG: Sw: Alg**)

2000b Pancyclicity in switching classes. *Inform. Proc. Letters* 73 (2000), 153–156. MR 2001c:05081.

Every switching class of graphs except that of the edgeless graph contains a pancyclic graph. Thus Hamiltonicity is polynomial-time for graph switching classes. (**TG: Sw, Alg**)

2006a The embedding problem for switching classes of graphs. Special issue on ICGT 2004. *Fund. Inform.* 74 (2006), no. 1, 115–134. MR 2282895 (2007h:68104). Zbl 1106.68053. (**GG: Sw**)

Andrzej Ehrenfeucht, Tero Harju, and Grzegorz Rozenberg

1996a Group based graph transformations and hierarchical representations of graphs. In: J. Cuny, H. Ehrig, G. Engels and G. Rozenberg, eds., *Graph Grammars and Their Application to Computer Science* (5th Int. Workshop, Williamsburg, Va., 1994), pp. 502–520. Lect. Notes in Computer Sci., Vol. 1073. Springer-Verlag, Berlin, 1996. MR 97h:68097.

The "heierarchical structure" of a switching class of skew gain graphs based on K_n. (**gg: KG: Sw**)

1997a 2-Structures—A framework for decomposition and transformation of graphs. In: Grzegorz Rozenberg, ed., *Handbook of Graph Grammars and Computing by Graph Transformation. Vol. 1: Foundations*, Ch. 6, pp. 401–478. World Scientific, Singapore, 1997. MR 99b:68006 (book). Zbl 908.68095 (book).

A tutorial (with some new proofs). The relevant sections, based on papers of Ehrenfeucht and Rozenberg with and without Harju, are those about dynamic labeled 2-structures, i.e., complete graphs with twisted gains. §6.7: "Dynamic labeled 2-structures". §6.8: "Dynamic ℓ2-structures with variable domains". §6.9: "Quotients and plane trees". §6.10: "Invariants", concerns certain switching invariants called "free invariants" when the gains are not twisted. (**gg: KG: sw: Exp, Ref**)

1997b Invariants of inversive 2-structures on groups of labels. *Math. Structures Computer Sci.* 7 (1997), 303–327. MR 98g:20089. Zbl 882.05119.

Given a gain graph $(K_n, \varphi, \mathfrak{G})$, a word w in the oriented edges of K_n has a gain $\varphi(w)$; call this $\psi_w(\varphi)$. A "free invariant" is a ψ_w that is an invariant of switching classes. Thm.: There is a number $d = d(K_n, \mathfrak{G})$ such that the group of free invariants is generated by ψ_w with $w = z_1^d \cdots z_k^d u_1 \cdots u_l$ where w_i are triangular cycles (directed!) and u_i are commutators. [The whole paper applies *mutatis mutandis* to arbitrary graphs, the triangular cycles being replaced by any set of cycles containing a fundamental system.] Dictionary: "Inversive 2-structure" = gain graph based on K_n. (**gg: KG: Sw, Invar**)

1999a *The Theory of 2-Structures: A Framework for Decomposition and Transformation of Graphs.* World Scientific, Singapore, 1999. MR 2001i:05001. Zbl 981.05002. (**gg: KG: sw: Exp, Ref**)

2004a Transitivity of local complementation and switching on graphs. *Discrete Math.* 278 (2004), 45–60. MR 2005d:05074. Zbl 1033.05052.

Let antilocal complementation at v mean reversing the edges except within the neighborhood of v. Let strictly antilocal complementation mean reversing the edges except within the closed neighborhood of v. Every simple graph of order n can be converted to every other one by antilocal complementations, and also by stricly antilocal complementations. (**TG**)

Andrzej Ehrenfeucht and Grzegorz Rozenberg

1993a An introduction to dynamic labeled 2-structures. In: Andrzej M. Borzyszkowski

and Stefan Sokołowski, eds., *Mathematical Foundations of Computer Science 1993* (Proc., 18th Int. Sympos., MFCS '93, Gdańsk, 1993), pp. 156–173. Lect. Notes in Computer Sci., Vol. 711. Springer-Verlag, Berlin, 1993. MR 95j:68126.

Extended summary of (1994a). (**GG(Gen): KG: Sw, Str**)

1994a Dynamic labeled 2-structures. *Math. Structures Comput. Sci.* 4 (1994), 433–455. MR 96j:68144. Zbl 829.68099.

They prove that a complicated definition of "reversible dynamic labeled 2-structure" G amounts to a complete graph with a set, closed under switching, of twisted gains in a gain group Δ. The twist is a gain-group automorphism α such that $\lambda(e;x,y) = [\alpha\lambda(e;y,x)]^{-1}$, λ being the gain function. Dictionary: their "domain" D = vertex set, "labeling function" λ (or equivalently, g) = gain function, "alphabet" = gain group, "involution" $\delta = \alpha\circ$ inversion, "δ-selector" $\hat{S}$ = switching function, "transformation induced by $\hat{S}$" = switching by $\hat{S}$; a "single axiom" d.l. 2-structure consists of a single switching class.

Further, they investigate "clans" of G. Given g (i.e., λ), deleting identity-gain edges leaves isolated vertices ("horizons") and forms connected components, any union of which is a "clan" of g. A clan of G is any clan of any $g \in G$. (**GG(Gen): KG: Sw, Str**)

1994b Dynamic labeled 2-structures with variable domains. In: J. Karhumäki, H. Maurer, and G. Rozenberg, eds., *Results and Trends in Theoretical Computer Science* (Proc. Colloq. in Honor of Arto Salomaa, Graz, 1994), pp. 97–123. Lect. Notes in Computer Sci., Vol. 812. Springer-Verlag, Berlin, 1994. MR 95m:68128.

Combinations and decompositions of complete graphs with twisted gains. (**GG(Gen): KG: Str, Sw**)

George C.M.A. Ehrhardt, Matteo Marsili, and Fernando Vega-Redondo

2005a On the rise and fall of networked societies. In: Joaquıın Marro, Pedro L. Garrido, and Miguel A. Muñoz, eds., *Modeling Cooperative Behavior in the Social Sciences* (Proc. 8th Granada Lect., Granada, Spain, 2005), pp. 96–103. AIP Conf. Proc., Vol. 779. Amer. Inst. Physics, Melville, N.Y., 2005. arxiv:physics/0505019.

§ III, "The effect of negative links": A random model where positive edges may appear, and may change to negative. Negative edges disappear over time. [Annot. 12 Aug 2012.] (**SG: PsS: Rand, Phys**)

Kurt Eisemann

1964a The generalized stepping stone method for the machine loading model. *Management Sci.* 11 (1964/65), No. 1 (Sept., 1964), 154–176. Zbl 136, 139 (e: 136.13901). (**GN: Incid, M(bases)**)

Joyce Elam, Fred Glover, and Darwin Klingman

1979a A strongly convergent primal simplex algorithm for generalized networks. *Math. Operations Res.* 4 (1979), 39–59. MR 81g:90049. Zbl 422.90081.

(**GN: M(bases), Incid**)

David P. Ellerman

1984a Arbitrage theory: A mathematical introduction. *SIAM Rev.* 26 (1984), 241–261. MR 85g:90024. Zbl 534.90014. (**GG: Bal, Incid, Flows: Appl, Ref**)

M.N. Ellingham

1991a Vertex-switching, isomorphism, and pseudosimilarity. *J. Graph Theory* 15

(1991), 563–572. MR 92g:05136. Zbl 802.05057.

Main theorem (§2) characterizes, given two signings of K_n (where n may be infinite) and a vertex set S, when switching S makes the signings isomorphic. [*Problem* 1. Generalize to other underlying graphs. *Problem* 2. Prove an analog for bidirected K_n's.] A corollary (§3) characterizes when vertices u, v of $\Sigma = (K_n, \sigma)$ satisfy $\Sigma^{\{u\}} \cong \Sigma^{\{v\}}$ and discusses when in addition no automorphism of Σ moves u to v. All is done in terms of Seidel (graph) switching (here called "vertex-switching") of unsigned simple graphs. **(kg: sw, TG)**

1996a Vertex-switching reconstruction and folded cubes. *J. Combin. Theory Ser. B* 66 (1966), 361–364. MR 96i:05120. Zbl 856.05071.

Deepens the folded-cube theory of Ellingham and Royle (1992a). Nicely generalizing Stanley (1985a), the number of subgraphs of a signed K_n that are isomorphic to a fixed signed K_m is reconstructible from the s-vertex switching deck if the Krawtchouk polynomial $K_s^n(x)$ has no even zeros between 0 and m. (Closely related to Krasikov and Roditty (1992a), Theorems 5 and 6.) Remark 4: balance equations (Krasikov and Roditty (1987a)) and Krawtchouk polynomials both reflect properties of folded cubes. All is done in terms of Seidel switching of unsigned simple graphs. [It seems clear that the folded cube appears because it corresponds to the effect of switchings on signatures of K_n (or any connected graph), since switching by X and X^c have the same effect. For the bidirected case (Problem 2 under Stanley (1985a)), the unfolded cube should play a similar role. *Question.* When treating a general underlying graph Γ, will a polynomial influenced by $\operatorname{Aut}\Gamma$ replace the Krawtchouk polynomial?] **(kg: sw, TG)**

M.N. Ellingham and Gordon F. Royle

1992a Vertex-switching reconstruction of subgraph numbers and triangle-free graphs. *J. Combin. Theory Ser. B* 54 (1992), 167–177. MR 93d:05112. Zbl 695.05053 (748.05071).

Reconstruction of induced subgraph numbers of a signed K_n from the s-vertex switching deck, dependent on linear transformation and thence Krawtchouk polynomials as in Stanley (1985a). The role of those polynomials is further developed. Done in terms of Seidel switching of unsigned simple graphs, with the advantage of reconstructing arbitrary subgraph numbers as well. A gap is noted in Krasikov and Roditty (1987a), proof of Lemma 2.5. [Methods and results are closely related to Krasikov (1988a) and Krasikov and Roditty (1987a, 1992a).] **(kg: sw, TG)**

Joanna A. Ellis-Monaghan and Iain Moffatt

20xxa Twisted duality for embedded graphs. Submitted. arXiv:0906.5557. **(sg: ori: Top, D)**

20xxb A Penrose polynomial for embedded graphs. Submitted. **(sg: Top, D)**

20xxc The Tutte–Potts connection in the presence of an external magnetic field. Submitted. **(sg: Top, D)**

Joanna A. Ellis-Monaghan and Irasema Sarmiento

2011a A recipe theorem for the topological Tutte polynomial of Bollobás and Riordan. *European J. Combin.* 32 (2011), no. 6, 782–794. Zbl 1223.05039. arXiv: **(sg: Top, D)**

Joanna Ellis-Monaghan and Lorenzo Traldi

2006a Parametrized Tutte polynomials of graphs and matroids. *Combin. Probab. Comput.*, 15 (2006), no. 6, 835–854. MR 2007j:05038. Zbl 1108.05024.

A variation on the multiplicative property of the parametrized Tutte polynomial. (**SGc: Gen: Invar**)

D. Emanuel and A. Fiat

See also E. Demaine.

2003a Correlation clustering—Minimizing disagreements on arbitrary weighted graphs. In: *Algorithms—ESA 2003* (Budapest, 2003), pp. 208–220. Lect. Notes in Computer Sci., Vol. 2832. Springer, Berlin, 2003. MR 2085454. Zbl pre05677126.

Conference version of Demaine, Emanuel, Fiat, and Immorlica (2006a). [Annot. 13 Sept 2009.] (**SG: WG: Clu: Alg**)

G.A. Enciso

See also B. Dasgupta.

G.A. Enciso and E.D. Sontag

2008a Monotone bifurcation graphs. *J. Biol. Dynamics* 2 (2008), no. 2, 121–139. MR 2427522 (2009e:34092). Zbl 1141.92005. (**SD, Biol**)

Shin-ichi Endoh

See T. Nakamura.

Gernot M. Engel and Hans Schneider

1973a Cyclic and diagonal products on a matrix. *Linear Algebra Appl.* 7 (1973), 301–335. MR 48 #2160. Zbl 289.15006. (**gg: Sw**)

1975a Diagonal similarity and equivalence for matrices over groups with 0. *Czechoslovak Math. J.* 25(100) (1975), 389–403. MR 53 #477. Zbl 329.15007. (**gg: Sw**)

1980a Matrices diagonally similar to a symmetric matrix. *Linear Algebra Appl.* 29 (1980), 131–138. MR 81k:15017. Zbl 432.15014. (**gg: Sw**)

Michael Engquist and Michael D. Chang

1985a New labeling procedures for the basis graph in generalized networks. *Operations Res. Letters* 4 (1985), no. 4, 151–155.

Generalizing pure-network procedures to get fast computations. [Annot. 4 Sept 2010.] (**GN: M**)

R.C. Entringer

1985a A short proof of Rubin's block theorem. In: B.R. Alspach and C.D. Godsil, eds., *Cycles in Graphs*, pp. 367–368. Ann. Discrete Math., Vol. 27. North-Holland Math. Stud., Vol. 115. North-Holland, Amsterdam, 1985. MR 87f:05144. Zbl 576.05037.

See Erdős, Rubin, and Taylor (1980a). (**par: bal**)

H. Era

See J. Akiyama.

Pál Erdős [Paul Erdös]

See also B. Bollobás and R.A. Duke.

1996a On some of my favourite theorems. In: D. Miklós, V.T. Sós and T. Szőnyi, eds., *Combinatorics, Paul Erdős is Eighty* (Papers from the Int. Conf. on Combinatorics, Keszthely, 1993), Vol. 2, pp. 97–132. Bolyai Soc. Math. Studies, 2. János Bolyai Mathematical Society, Budapest, 1996. MR 97g:00002. Zbl 837.00020 (book).

P. 119 mentions the theorem of Duke, Erdős, and Rödl (1991a) on even circles.

Pp. 120–121 mention (amongst similar problems) a theorem of Erdős and Hajnal (source not stated): Every all-negative signed graph with chromatic number $\aleph_1$ contains every finite bipartite graph [i.e., every finite, balanced, all-negative signed graph]. [*Problem.* Find generalizations to signed graphs. For instance: *Conjecture.* Every signed graph with chromatic number $\aleph_1$, that does not become antibalanced upon deletion of any finite vertex set, contains every finite, balanced signed graph up to switching equivalence.]

[The MR review: "this is one of the best collections of problems that Erdos has published."] (**par: bal: Exp, Ref**)

P. Erdös, R.J. Faudree, A. Gyárfás, and R.H. Schelp

1991a Odd cycles in graphs of given minimum degree. In: Y. Alavi, G. Chartrand, O.R. Oellermann, and A.J. Schwenk, eds., *Graph Theory, Combinatorics, and Applications* (Proc. Sixth Quadrennial Int. Conf. Theory Appl. Graphs, Kalamazoo, Mich., 1988), Vol. 1, pp. 407–418. Wiley, New York, 1991. MR 93d:05085. Zbl 840.05050.

A large, nonbipartite, 2-connected graph with large minimum degree contains a circle of given odd length or is one of a single type of exceptional graph. [*Question.* Can this be generalized to negative circles in unbalanced signed graphs?] (**par, sg: Circles, Xtreml**)

P. Erdős, E. Győri, and M. Simonovits

1992a How many edges should be deleted to make a triangle-free graph bipartite? In: G. Halász, L. Lovász, D. Miklós, and T. Szönyi, eds., *Sets, Graphs and Numbers* (Proc., Budapest, 1991), pp. 239–263. Colloq. Math. Soc. János Bolyai, Vol. 60. János Bolyai Math. Soc., Budapest, and North-Holland, Amsterdam, 1992. MR 94b:05104. Zbl 785.05052.

Assume $|\Sigma|$ simple of order n and $\not\supseteq$ a fixed graph Δ. Results on frustration index l of antibalanced Σ if Δ is 3-chromatic, esp. C_3. Thm.: If $|E| > n^2/5 - o(n^2)$, then $l(\Sigma) < n^2/25 - o(n^2)$. *Conjecture* (Erdős): For $\Delta = C_3$ the hypothesis on $|E|$ is unnecessary. [*Question* 1(a). Is the answer different when Σ need not be antibalanced? *Question* 2(a). Exclude a fixed signed graph whose signed chromatic number = 1. *Question* 3(a). In particular, exclude $-K_3$. *Question* 4(a). Exclude $-K_l$. *Question* 5(a). Exclude an unbalanced C_l. *Questions* 1–5(b). Even if $l(\Sigma)$ cannot be estimated, is there always an extremal graph that is antibalanced—as when no graph is excluded, by Petersdorf (1966a)?] (**par: Xtreml**)

P. Erdös and L. Pósa

1965a On independent circuits contained in a graph. *Canad. J. Math.* 17 (1965) 347–352. MR 31 #86. Zbl 129.39904.

An upper bound on l_0, the vertex frustration number, in terms of vertex packing of unbalanced circles, in the contrabalanced case. *Problem.* Find an analog for signed graphs and a generalization to biased graphs. (**gg: bal**)

Paul Erdös, Arthur L. Rubin, and Herbert Taylor

1980a Choosability in graphs. In: *Proceedings of the West Coast Conference on Combinatorics, Graph Theory and Computing* (Arcata, Calif., 1979), pp. 125–157. Congressus Numer., XXVI. Utilitas Math. Publ. Inc., Winnipeg, Man., 1980.

MR 82f:05038. Zbl 469.05032.

Rubin's block theorem (Thm. R, p. 136): a block graph, not complete or an odd circle, contains an induced even circle with at most one chord. [See also Entringer (1985a).] [*Question.* Does this generalize to signed graphs, Rubin's block theorem being the antibalanced case? Rubin's 2-choosability theorem, p. 132, is also tantalizingly reminiscent of antibalanced graphs, but in reverse.] **(par: Str, bal)**

Carolyn A. Eschenbach

See also Z. Li and J. Stuart.

Carolyn A. Eschenbach, Frank J. Hall and Charles R. Johnson

1993a Self-inverse sign patterns. In: Richard A. Brualdi, Shmuel Friedland, and Victor Klee, eds., *Combinatorial and Graph-Theoretical Problems in Linear Algebra* (IMA Workshop, Minneapolis, 1991), pp. 245–256. IMA Volumes in Mathematics and its Applications, 50. Springer-Verlag, New York, 1993. MR 1240969 (94e:15004). Zbl 792.15008.

Sign matrices whose sign patterns are self-inverse are essentially adjacency matrices of signed graphs and are very few. [Annot. 13 Apr 2009.] **(sg, QM)**

Carolyn A. Eschenbach, Frank J. Hall, Charles R. Johnson, and Zhongshan Li

1997a The graphs of the unambiguous entries in the product of two $(+, -)$-sign pattern matrices. *Linear Algebra Appl.* 260 (1997), 95–118. MR 1448352 (98e:05075). Zbl 881.05089.

A, B are nowhere-zero sign-pattern matrices. $(AB)_{ij}$ may be necessarily $+$, $-$, or ambiguous [Abelson and Rosenberg (1958a)'s p, n, a]. Let $\mathcal{R}$ = set of rows, $\mathcal{C}$ = set of columns. The graph $G(A, B) \subseteq K_{\mathcal{R}(A),\mathcal{C}(B)}$ has an edge ij for each unambigous entry in AB. The digraph $D(A^2)$ has an arc (i, j) for each unambiguous entry in A^2. Thm. 3.2: Γ is a $G(A, B)$ iff it is the disjoint union of bicliques and isolated vertices. Characterizing $D(A^2)$ seems hard. Results on special cases. [D and G are signed by p, n. Thm. 5.11: $D(A^2)$, if a circle, is balanced iff the circle is positive. §6, "Characterization of permutation graphs in D_n", i.e., $D(A^2)$ that are permutation graphs. [*Problem.* Investigate $D(A^2)$ and $G(A, B)$ signed by p, n. *Problem.* Generalize to allow 0 entries (thus working over Abelson–Rosenberg's algebra $\{p, n, a, o\}$.)] Dictionary: "signature similarity" of matrices = switching of digraph, "negative matching" = entry n in AB = negative edge in $G(A, B)$. [Annot. 4 Nov 2011.] **(QM: sd, sw)**

Carolyn A. Eschenbach, Frank J. Hall and Zhongshan Li

1998a From real to complex sign pattern matrices. *Bull. Austral. Math. Soc.* 5 (1998), 159–172. MR 1623848 (99d:15003). Zbl 951.15021.

$S := \{\alpha + i\beta : \alpha, \beta = 0, \pm\}$, the set of complex signs. A complex sign pattern matrix has complex signs as entries. If it is square it has a digraph $D(A)$ with complex signs as gains. §3, "Cyclic nonnegativity": Cycles with gain $\pm$. Switching (via matrices) by $\pm, \pm i$. Cor. 3.2: $D(A)$ is balanced iff it switches to all $+$. Thm. 3.3: iff D is balanced and $D(A + A^*)$ switches to all $+$. §4, "Stability": Lem. 4.1: If A is sign stable, every digon has real or purely imaginary gain. Lem. 4.2: If A is sign stable it is sign nonsingular. Thm. 4.4 (generalizing Quirk and

Ruppert (1965a) and Maybee and Quirk (1969a)): Assume every vertex has a negative loop. Then A is sign stable iff all digons are negative and no longer cycles exist. Thm. 5.2: Similar, for ray stability. Dictionary: "cyclically nonnegative" = all cycle gains are +; "cyclically positive" = cyclically nonnegative and no zeros. [Annot. 4 Nov 2011.]
(**QM: Gen; gg, sw; QSta**)

Carolyn A. Eschenbach and Zhongshan Li

1999a Potentially nilpotent sign pattern matrices. *Linear Algebra Appl.* 299 (1999). 81–99. MR 1723710 (2000i:15043). Zbl 941.15012.

Matrices with tree digraph. Cycle sign = sign product, p. 82. 2-cycle signs in Thms. 5.3 (proof), 5.5, 5.7 (proof). [Annot. 5 Nov 2011.]
(**QM: sd, sw**)

Ernesto Estrada and Naomichi Hatano

2008a Communicability in complex networks. *Phys. Rev. E* (3) 77 (2008), article 036111. MR 2495430 (2010i:91171). (**SG: KG**)

Ernesto Estrada, Desmond J. Higham, and Naomichi Hatano

2008a Communicability and multipartite structures in complex networks at negative absolute temperatures. *Phys. Rev. E* 78 (2008), article 026102. (**SG: KG: clu**)

Ernesto Estrada and Juan A. Rodríguez-Velázquez

2005a Spectral measures of bipartivity in complex networks. *Phys. Rev. E* (3) 72 (2005), no. 4, article 046105, 6 pp. MR 2202758 (2006i:94124). (**par: Fr, Adj**)

Patricia A. Evans

See D.D. Doan.

Cloyd L. Ezell

1979a Observations on the construction of covers using permutation voltage assignments. *Discrete Math.* 28 (1979), 7–20. MR 81a:05040. Zbl 413.05005.
(**GG: Top, Cov, sw**)

Ulrich Faigle and Rainer Schrader

1990a Orders and graphs. In: G. Tinhofer, E. Mayr, H. Noltemeier and M.M. Sysło, eds., *Computational Graph Theory.* Computing Supplementum, 7. Springer-Verlag, Vienna, 1990. MR 1059927 (91d:05085). Zbl 725.05045.

An example is threshold signed graphs (*cf.* Benzaken, Hammer, and de Werra 1985a). [Annot. 16 Jan 2012.] (**SG**)

M. Falcioni, E. Marinari, M.L. Paciello, G. Parisi, and B. Taglienti

1981a Phase transition analysis in Z_2 and U(1) lattice gauge theories. *Phys. Lett. B* 105 (1981), no. 1, 51–54. (**SG: Phys**)

Shaun Fallat and Yi-Zheng Fan

2012a Bipartiteness and the least eigenvalue of signless Laplacian of graphs. *Linear Algebra Appl.* 436 (2012), no. 9, 3254–3267.

"Bipartiteness" of Γ [also known as biparticity] is $b(-\Gamma)$. "Algebraic bipartiteness" is the smallest eigenvalue $\lambda_1(K(-\Gamma))$. Rephrased in terms of antibalanced signed graphs: Thm. 2.1. If $-\Gamma$ is unbalanced, $\lambda_1 \leq l_0(-\Gamma)$, the vertex frustration number. Thm. 2.4. (1) $\operatorname{Spec} K(\widetilde{-\Gamma}) = \operatorname{Spec} K(\Gamma) \cup \operatorname{Spec} K(-\Gamma)$. [A special case of Bilu and Linial (2006a), Lemma.] (2–4) Elementary properties of $\widetilde{-\Gamma}$ [found in Zaslavsky (1982a)]. (4) If $-\Gamma$ is connected and unbalanced, $\lambda_2(K(\widetilde{-\Gamma})) = \min\{\lambda_1, \lambda_2(\Gamma)\} > 0$.

$\bar\psi(-\Gamma) := \min_S[2l(-\Gamma : S) + |E(S, S^c)|]/|S|$ ($S \neq \varnothing, V$) (*cf.* Desai and Rao (1994a).) Thm. 2.6. If Γ is connected, $\Delta :=$ max degree, $\lambda_1 \geq \Delta - \sqrt{\Delta^2 - \bar\psi^2}$. Thm. 2.7. $\lambda_1 \leq 2\bar\psi \leq 4l(-\Gamma)/n$. (Strengthens Tan and Fan (2008a).) [*Conjecture.* The results must generalize to all (Γ, σ).] [Annot. 20 Jan 2012.] (**Par: Adj, Cov**)

Yi-Zheng Fan

See also L. Cui, S.C. Gong, B.S. Tam, Y.Y. Tan, Y. Wang, M.L. Ye, and J. Zhou.

2003a On spectral integral variations of mixed graphs. *Linear Algebra Appl.* 374 (2003), 307–316. MR 2008794 (2005h:05133). Zbl 1026.05076.

The signed graphs (not necessarily simple) for which adding an edge changes only one eigenvalue of the Laplacian (Kirchhoff) matrix $K(\Sigma)$ and increases that by an integer. [Dictionary: "mixed graph" = bidirected graph B where all negative edges are extraverted, in effect the signed graph $-\Sigma_{\mathrm{B}}$; "quasibipartite" = balanced; "e^c" = e with reversed sign. The article's sign sgn(e) equals $-\sigma_{\mathrm{B}}(e)$. The entire article is really about signed graphs Σ and uses signed-graph matrices and methods.] Thm. 1: This eigenvalue property holds iff the column $x(e)$ of e in $\mathrm{H}(\Sigma)$ is an eigenvector of K [i.e., $\mathrm{H}(\Sigma)x(e) = \lambda x(e)$]. Corollaries give other criteria and identify the change in the one eigenvalue. Lemma 5: K is singular iff Σ is balanced [special case of Zaslavsky (1982a), Theorem 8A.4]. [Annot. 13 Apr 2009, rev 10 Feb 2012.] (**SG: incid, Adj**)

2004a On structure of eigenvectors of mixed graphs. Sixth Int. Conf. Matrix Theory Appl. in China. *Heilongjiang Daxue Ziran Kexue Xuebao* (*J. Nat. Sci. Heilongjiang Univ.*) 21 (2004), no. 4, 50–54. MR 2129072 (no rev). Zbl 1077.05061.

The "mixed graphs" are signed graphs; see (2003a). Graphs are simple. The eigenvalues are those of the Laplacian, $K(\Sigma)$. (**sg: Adj**)

2004b Largest eigenvalue of a unicyclic mixed graph. *Appl. Math. J. Chinese Univ. Ser. B* 19 (2004), no. 2, 140–148. MR 2063313 (no rev).

The "mixed graphs" are signed graphs; see (2003a). Graphs are simple. The eigenvalues are those of the Laplacian $K(\Sigma)$. Prop. 2.2: Laplacian spectrum of negative circle. [The first such proof. Equivalent to the adjacency spectrum because C_n is regular.] Thm. 2.8: The signed 1-trees with max and min $\lambda_1(K(\Sigma))$. Thm. 2.9: Those with $\lambda_1 = n$. Thm. 2.10: Those with $\lambda_1 > n$. (N.B. Lem. 2.4: $\lambda_1 \leq n+1$ from Hou, Li, and Pan (2003a), Thm. 3.5(1), or X.D. Zhang and Li (2002a).) [Annot. 10 Feb 2012.] (**SG: incid, Adj**)

2005a On the least eigenvalue of a unicyclic mixed graph. *Linear Multilinear Algebra* 53 (2005), no. 2, 97–113. MR 2133313 (2005m:05145). Zbl 1062.05090.

The "mixed graphs" are signed graphs; see (2003a). Graphs are simple. The eigenvalues are those of $K(\Sigma)$. (**sg: Adj**)

2007a On eigenvectors of mixed graphs with exactly one nonsingular cycle. *Czechoslovak Math. J.* 57 (2007), no. 4, 1215–1222. MR 2357587 (2008i:05117). Zbl 1174.05075.

The "mixed graphs" are signed graphs; see (2003a). Graphs are simple. The eigenvalues are those of $K(\Sigma)$. (**sg: Adj**)

Yi-Zheng Fan, Shi-Cai Gong, Yi Wang, and Yu-Bin Gao

2009a First eigenvalue and first eigenvectors of a nonsingular unicyclic mixed graph. *Discrete Math.* 309 (2009), no. 8, 2479–2487. MR 2512565 (2010g:05212). Zbl 1182.05081.

The "mixed graphs" are signed graphs; see Fan (2003a). Graphs are simple. The eigenvalues are those of $K(\Sigma)$. (**sg: Adj**)

Yi-Zheng Fan, Shi-Cai Gong, Jun Zhou, Ying-Ying Tan, and Yi Wang

2007a Nonsingular mixed graphs with few eigenvalues greater than two. *European J. Combin.* 28 (2007), no. 6, 1694–1702. MR 2339495 (2008f:05115). Zbl 1122.05058.

The "mixed graphs" are signed graphs. Assume Σ is connected. $m :=$ number of eigenvalues > 2. Thm. 2.2: $d :=$ longest path length, $\mu :=$ matching number. (i) $m \geq \lfloor d/2 \rfloor$, (ii) $m \geq \mu$ if $n > 2\mu$, (iii) $m \geq \mu - 1$ if $n = 2\mu$. Now assume Σ is unbalanced. Thm. 3.4. If $n \geq 7$, then $m = 2$ iff $|\Sigma|$ is one of two general types and Σ has a certain negative triangle. Thm. 3.5. If $n \geq 6$, then $m = 1$ iff $\Sigma \sim -K_4$ or an unbalanced subgraph. Dictionary: See Bapat, Grossman, and Kulkarni (1999a). [Annot. 13 Jan 2012.] (**sg: Adj**)

Yi-Zheng Fan, Hai-Yan Hong, Shi-Cai Gong, and Yi Wang

2007a Order unicyclic mixed graphs by spectral radius. *Australas. J. Combin.* 37 (2007), 305–316. MR 2284395 (2007j:05137). Zbl 1122.05059. (**sg: Adj**)

Yi-Zheng Fan, Bit-Shun Tam, and Jun Zhou

2008a Maximizing spectral radius of unoriented Laplacian matrix over bicyclic graphs of a given order. *Linear Multilinear Algebra* 56 (2008), no. 4, 381–397. MR 2434109 (2009e:15070). Zbl 1146.05032.

The maximal graphs are $K_4 \backslash e$ with $n-4$ pendant edges at one trivalent vertex. [Annot. 9 Sept 2010.] (**par: Incid, Adj**)

Yi-Zheng Fan, Yue Wang, and Yi Wang

20xxa The nullity of unicyclic signed graphs. Submitted. arXiv:1107.0400.

The nullity ν is that of $A(\Sigma)$. $\nu \leq n - 2$. The cases where $\nu \geq n - 5$ are characterized. [Annot. 17 Dec 2011.] (**SG: Adj**)

Yi-Zheng Fan and Dan Yang

2009a The signless Laplacian spectral radius of graphs with given number of pendant vertices. *Graphs Combin.* 25 (2009), no. 3, 291–298. MR 2534887 (2010j:05233). Zbl 1194.05085.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Thomas J. Fararo

See N.P. Hummon.

Miriam Farber

See A. Berman.

Arthur M. Farley and Andrzey Proskurowski

1981a Computing the line index of balance of signed outerplanar graphs. Proc. Twelfth Southeastern Conf. on Combinatorics, Graph Theory and Computing (Baton Rouge, 1981), Vol. I. *Congressus Numer.* 32 (1981), 323–332. MR 83m:68119. Zbl 489.68065.

Calculating frustration index is NP-complete, since it is more general than max-cut. However, for signed outerplanar graphs with bounded size of bounded faces, it is solvable in linear time. [It is quickly solvable for signed planar graphs. See Katai and Iwai (1978a), Barahona (1981a, 1982a), and more.] (**SG: Fr**)

M. Farzan

1978a Automorphisms of double covers of a graph. In: *Problemes Combinatoires et Theorie des Graphes* (Colloq. Int., Orsay, 1976), pp. 137–138. Colloques Int. du CNRS, 260. Editions du C.N.R.S., Paris, 1978. MR 81a:05063. Zbl 413.05064.

A "double cover of a graph" means the double cover of a signing of a simple graph. (**sg: Cov, Aut**)

R.J. Faudree

See P. Erdős.

Katherine Faust

See S. Wasserman.

Siamak Fayyaz Shahandashti, Mahmoud Salmasizadeh, and Javad Mohajeri

2005a A provably secure short transitive signature scheme from bilinear group pairs. In: C. Blundo and S. Cimato, eds., *Security in Communication Networks* (4th Int. Conf., SCN 2004, Amalfi), pp. 60–76. Lect. Notes in Computer Sci., Vol. 3352. Springer-Verlag, Berlin, 2005. Zbl 1116.94320.

Edges have "signatures" for encryption. No edge signs! [Irresistible.] [Annot. 5 Mar 2011.] (**None**)

N.T. Feather

1971a Organization and discrepancy in cognitive structures. *Psychological Rev.* 78 (1971), 355–379.

A suggestion for defining balance in weighted digraphs: pp. 367–369. (**PsS: Bal: Exp**)(**WD: Bal**)

Martin Feinberg

See G. Craciun.

Paul Fendley and Vyacheslav Krushkal

2010a Link invariants, the chromatic polynomial and the Potts model. *Adv. Theor. Math. Phys.* 14 (2010), no. 2, 507–540. MR 2721654 (2011k:57015). Zbl 1207.82007. arXiv:0806.3484.

The Potts model treats a graph as all negative ("antiferromagnetic"; see the low-temperature expansion in §3). [Annot. 12 Jan 2012.] (**par: Invar**)

Lihua Feng

2010a The signless Laplacian spectral radius for bicyclic graphs with k pendant vertices. *Kyungpook Math. J.* 50 (2010), no. 1, 109–116. MR 2609079 (2011d:05221). Zbl 1205.05140.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Lihua Feng and Guihai Yu

2009a On three conjectures involving the signless Laplacian spectral radius of graphs. *Publ. Inst. Math. (Beograd) (N.S.)* 85(99) (2009), 35–38. MR 2536687 (2010i:-05204).

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

2009b The signless Laplacian spectral radius of unicyclic graphs with graph constraints. *Kyungpook Math. J.* 49 (2009), no. 1, 123–131. MR 2527378 (2011b:-05148). Zbl 1201.05056.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

2010a The signless Laplacian spectral radius of graphs with given diameter. *Utilitas Math.* 83 (2010), 265–276. MR 2742294 (2011i:05129).

The graphs with maximum spectral radius. [Annot. 19 Nov 2011.]
(**Par: Adj**)

Lihua Feng, Guihai Yu, and Aleksandar Ilić

2010a The Laplacian spectral radius for unicyclic graphs with given independence number. *Linear Algebra Appl.* 433 (2010), 934–944. MR 2658644 (2011f:05175). Zbl 1215.05102. (**Par: Adj**)

Lin Feng, Yan Hong Yao, Ji Ming Guo, and Shang Wang Tan

2011a The signless Laplacian spectral radius of unicyclic graphs with fixed girth. (In Chinese.) *Appl. Math. J. Chinese Univ. Ser. A* 26 (2011), no. 1, 121–126. MR 2807616 (no rev). (**Par: Adj**)

Anuška Ferligoj

See P. Doreian.

Lori Fern [Lori Koban]

See also L. Koban.

Lori Fern, Gary Gordon, Jason Leasure, and Sharon Pronchik

2000a Matroid automorphisms and symmetry groups. *Combin. Probab. Comput.* 9 (2000), 105–123. MR 2001g:05034. Zbl 960.05055.

Consider a subgroup W of the hyperoctahedral group Oc_n that is generated by reflections. Let $M(W)$ be the vector matroid of the vectors corresponding to reflections in W. The possible direct factors of any automorphism group of $M(W)$ are S_k, Oc_k, and Oc_k^+. The proof is stricly combinatorial, via signed graphs. (**SG: M: Aut, Geom**)

Rosário Fernandes

2010a Location of the eigenvalues of weighted graphs with a cut edge. *Linear Multilinear Algebra* 58 (2010), no. 3, 305–322. MR 2663432 (2011d:15014). Zbl 1203.05092.

The "weights" are skew gains [*cf.* J. Hage (1999a) *et al.*] in $\mathbb{C}^\times$; the anti-involution is conjugation. Identities satisfied by the eigenvalues. [Annot. 11 Jan 2012.] (**GG: Gen: Adj**)

L.A. Fernández, V. Martin-Mayor, G. Parisi, and B. Seoane

2010a Spin glasses on the hypercube. *Phys. Rev. B* 81 (2010), #134403, 14 pp.

Average behavior of random signed subhypercubes (Γ, σ), with spanning $\Gamma \subseteq Q_D$, with random spins $\zeta : V \to \{+1, -1\}$. Each (Γ, σ, ζ) is a "sample". To avoid irregularities Γ is z-regular ("connectivity z") for a fixed z (here, 6). [Annot. 19 Jun 2012.] (**Phys, SG: Fr**)

Daniela Ferrero

2008a Product line sigraphs. In: *The International Symposium on Parallel Architectures, Algorithms, and Networks* (i-span 2008), pp. 141–145. IEEE Computer Soc., 2008.

The product line graph [= $\Lambda_\times(\Sigma)$ in M. Acharya (2009a)] is balanced. [Immediate from Harary's (1953a) balance theorem or Sampathkumar's (1972a, 1984a) similar theorem.] [Annot. 2008, 20 Dec 2010.] (**SG: LG, Bal**)

A. Fiat

See E. Demaine and D. Emanuel.

Miroslav Fiedler

1957a Über qualitative Winkeleigenschaften der Simplexe. *Czechoslovak Math. J.* 7(82) (1957), 463–478. MR 20 #1252. Zbl 93, 336 (e: 093.33602). (**SG: Geom**)

1957b Einige Satze aus der metrischen Geometrie der Simplexe in euklidischen Raumen. *Schr. Forschungsinst. Math.* 1 (1957), 157. MR 19, 303. Zbl 89, 167 (e: 089.16706). (**SG: Geom**)

1961a Uber die qualitative Lage des Mittelpunktes der ungeschriebenen Hyperkugel im n-Simplex. *Comment. Math. Univ. Carolin.* 2, No. 1 (1961), 1–51. Zbl 101, 132 (e: 101.13205). (**SG: Geom**)

1964a Some applications of the theory of graphs in matrix theory and geometry. In: *Theory of Graphs and Its Applications* (Proc. Sympos., Smolenice, 1963), pp. 37–41. Publ. House Czechoslovak Acad. Sci., Prague, 1964. MR 30 #5294. Zbl (e: 163.45605). (**SG: Geom**)

1967a Graphs and linear algebra. In: *Theory of Graphs: International Symposium* (Rome, 1966), pp. 131–134. Gordon and Breach, New York; Dunod, Paris, 1967. MR 36 #6313. Zbl 263.05124. (**SG: Geom**)

1969a Signed distance graphs. *J. Combin. Theory* 7 (1969), 136–149. MR 39 #4034. Zbl 181, 260 (e: 181.26001). (**SG: Geom**)

1970a Poznámka o distancnich grafech [A remark on distance graphs] (in Czech). In: *Matematika (geometrie a teorie grafu)* [Mathematics (Geometry and Graph Theory)], pp. 85–88. Univ. Karlova, Prague, 1970. MR 43 #3143. Zbl 215.50203. (**SG: Geom**)

1975a Eigenvectors of acyclic matrices. *Czechoslovak Math. J.* 25(100) (1975), 607–618. MR 52 #8151. Zbl 325.15014. (**sg: Trees: Adj**)

1985a Signed bigraphs of monotone matrices. In: Horst Sachs, ed., *Graphs, Hypergraphs and Applications* (Proc. Int. Conf., Eyba, 1984), pp. 36–40. Teubner-Texte zur Math., B. 73. B.G. Teubner, Leipzig, 1985. MR 87m:05121. Zbl 626.05023. (**SG: Adj: Exp**)

1993a A geometric approach to the Laplacian matrix of a graph. In: Richard A. Brualdi, Shmuel Friedland, and Victor Klee, eds., *Combinatorial and Graph-Theoretical Problems in Linear Algebra*, pp. 73–98. IMA Vols. Math. Appl., 50. Springer-Verlag, New York, 1993. MR 1240957 (94g:05055). Zbl 791.05073.

The signed bipartite graph of a normalized Gram matrix (pp. 85–86). This is applied to study the types of angles in a geometric simplex. Dictionary: $\Gamma(A)$ = the signed bipartite graph of a symmetric real matrix. (**SG**)

1998a Additive compound graphs. *Discrete Math.* 187 (1998), 97–108. MR 99c:05131. Zbl 958.05091. (**SG**)

Miroslav Fiedler and Vlastimil Ptak

1967a Diagonally dominant matrices. *Czechoslovak Math. J.* 17(92) (1967), 420–433. MR 35 #6704. Zbl (e: 178.03402). (**GG: Sw, bal**)

1969a Cyclic products and an inequality for determinants. *Czechoslovak Math. J.* 19(94) (1969), 428–451. MR 40 #1409. Zbl 281.15014. (**gg: Sw**)

Rosa M.V. Figueiredo, Martine Labbé, and Cid C. de Souza

2011a An exact approach to the problem of extracting an embedded network matrix. *Computers Operations Res.* 38 (2011), no. 11, 1483–1492. MR 2781542 (2012f:90223) (*q.v.*). Zbl 1210.90038. (**SG: Incid**)

Joseph Fiksel

1980a Dynamic evolution in societal networks. *J. Math. Sociology* 7 (1980), 27–46. MR 81g:92023(*q.v.*). Zbl 434.92022. (**SG: Clu, VS**)

Samuel Fiorini, Nadia Hardy, Bruce Reed, and Adrian Vetta

2005a Approximate min-max relations for odd cycles in planar graphs. In: M. Jünger and V. Kaibel, eds., *Integer Programming and Combinatorial Optimization* (11th Int. IPCO Conf., IPCO 2005, Berlin), pp. 35–50. Lect. Notes in Computer Sci., Vol. 3509. Springer, Berlin, 2005. MR 2210011 (2006j:90108). Zbl 1119.90360.

See (2007a). (**SG: Fr**)

2007a Approximate min-max relations for odd cycles in planar graphs. *Math. Programming, Ser. B* 110 (2007), no. 1, 71–91. MR 2306131 (2008b:05087). Zbl 1113.05054.

$\tau :=$ minimum number of negative circles whose vertex deletion leaves a balanced signed graph. $\nu :=$ maximum number of vertex-disjoint negative circles. $\rho :=$ minimum size of a transversal of negative face boundaries. Let τ', ν' be the edge analogs. Thm. 3 (Král and Voss 2004a): $\tau' \leq 2\nu'$. (A shorter proof.) Thm. 4: For an unbalanced signed plane graph, $\tau \leq 7\nu + 3\rho - 8$. Cor. 2: $\tau \leq 10\nu$. Dictionary: "odd" = negative, "even" = positive. [Annot. 6 Feb 2011.] (**SG: Fr**)

E. Fischer, J.A. Makowsky, and E.V. Ravve

2008a Counting truth assignments of formulas of bounded tree-width or clique-width. *Discrete Appl. Math.* 156 (2008), no. 4, 511–529. MR 2379082 (2009k:68090). Zbl 1131.68093.

The incidence graph of clauses is a signed bipartite graph. [Annot. 16 Jan 2012.] (**SG**)

Ilse Fischer and C.H.C. Little

2004a Even circuits of prescribed clockwise parity. *Electronic J. Combin.* 10 (2003), Research Paper 45, 20 pp. MR 2004h:05071. Zbl 1031.05073. (**SG**)

K.H. Fischer and J.A. Hertz

1991a *Spin Glasses.* Cambridge Studies in Magnetism, Vol. 1. Cambridge Univ. Press, Cambridge, Eng., 1991. MR 93m:82019.

An excellent introduction to many aspects of physics (mainly theoretical) that often seem to be signed graph theory or to generalize it, e.g., by randomly weighting the edges. (**Phys: sg: fr: Exp, Ref**)

§2.5, "Frustration", discusses the spin glass Ising model (essentially, signed graphs) in square and cubical lattices, including the "Mattis model" (a switching of all positive signs), as well as a vector analog, the "XY" model (planar spins) and (p. 46) even a general gain-graph model with switching-invariant Hamiltonian. (**Phys: SG: Fr, Sw: Exp, Ref**)

Ch. 3 concerns the Ising and Potts models. In §3.7: "The Potts glass", the Hamiltonian (without edge weights) is $H = -\frac{1}{2}\sum \sigma(e_{ij})(k\delta(s_i, s_j) - 1)$. [It is not clear that the authors intend to permit negative edges. If they are allowed, H is rather like Doreian and Mrvar's (1996a) $P(\pi)$.

Question. Is there a worthwhile generalized signed and weighted Potts model with Hamiltonian that specializes both to this form of H and to P?] [Also *cf.* Welsh (1993a) on the Ashkin–Teller–Potts model.]
(**Phys: sg, clu: Exp**)

Steven D. Fischer
1993a *Signed Poset Homology and q-Analog Möbius Functions.* Ph.D. thesis, Univ. of Michigan, 1993.

§1.2: "Signed posets". Definition of signed poset: a positively closed subset of the root system B_n whose intersection with its negative is empty. (Following Reiner (1990).) Equivalent to a partial ordering of $\pm[n]$ in which negation is a self-duality and each dual pair of elements is comparable. [This is really a special type of signed poset. The latter restriction does not hold in general.]

Relevant contents: Ch. 2: "Cohen-Macaulay signed posets", §2.2: "EL-labelings of posets and signed posets", and shellability. Ch. 3: "Euler characteristics", and a fixed-point theorem. §5.1: "The homology of the signed posets S_Π" (a particular example). App. A: "Open problems", several concerning signed posets.

[Partially summarized by Hanlon (1996a).]
(**Sgnd: sg, ori, Geom, Invar**)

P.C. Fishburn and N.J.A. Sloane
1989a The solution to Berlekamp's switching game. *Discrete Math.* 74 (1989), 263–290. MR 90e:90151. Zbl 664.94024.

The maximum frustration index of a signed $K_{t,t}$, which equals the covering radius of the Gale–Berlekamp code, is evaluated for $t \leq 10$, thereby extending results of Brown and Spencer (1971a). See Table 1. [Corrected and extended by Carlson and Stolarski (2004a).] (**sg: Fr**)

Michael E. Fisher and Rajiv R.P. Singh
1990a Critical points, large-dimensionality expansions, and the Ising spin glass. In: G.R. Grimmett and D.J.A. Welsh, eds., *Disorder in Physical Systems: A Volume in Honour of John M. Hammersley on the Occasion of His 70th Birthday*, pp. 87–111. Clarendon Press, Oxford, 1990.

Physics questions, e.g., phase transitions and high-temperature expansions, for signed lattice graphs ($\pm J$ spins) and with random weights (Gaussian edge weights). [Annot. 24 Aug 2012.] (**sg: Phys: Fr: Exp**)

Claude Flament
1958a L'étude mathématique des structures psycho-sociales. *L'Année Psychologique* 58 (1958), 119–131.

Signed graphs are treated on pp. 126–129. (**SG: Bal, PsS: Exp**)

1963a *Applications of Graph Theory to Group Structure.* Prentice-Hall, Englewood Cliffs, N.J., 1963. MR 28 #1014. Zbl 141, 363 (e: 141.36301).

English edition of (1965a). Ch. 3: "Balancing processes."
(**SG: KG: Bal, Alg: Exp**)

1965a *Théorie des graphes et structures sociales.* Math. et sci. de l'homme, Vol. 2. Mouton and Gauthier-Villars, Paris, 1965. MR 36 #5018. Zbl 169, 266 (e: 169.26603).

Ch. III: "Processus d'équilibration." (**SG: KG: Bal, Alg: Exp**)

1970a Equilibre d'un graphe, quelques resultats algebriques. *Math. Sci. Humaines*, No. 30 (1970), 5–10. MR 43 #4704. Zbl 222.05124.

1979a Independent generalizations of balance. In: Paul W. Holland and Samuel Leinhardt, eds., *Perspectives on Social Network Research* (Proc. Sympos., Dartmouth Coll., Hanover, N.H., 1975), Ch. 10, pp. 187–200. Academic Press, New York, 1979. (**SG: Bal, PsS: Exp**)

Erica Flapan

1995a Intrinsic chirality. *Journal of Molecular Structure (Theochem)* 336 (1995), 157–164.

Intrinsic chirality means the graph cannot be embedded in 3-space without a twist. [*Question.* Can this be interpreted in terms of signed graphs?] See also Flapan (1998a), Flapan and Weaver (1996a), Hu and Qiu (2009a). [Annot. 4 Nov 2010.] (**sg: Top: Chem**)

1998a Knots and graphs in chemistry. *Chaos, Solitons & Fractals* 9 (415) (1998), 547–560. MR 1628741 (99c:57017). Zbl 933.57002.

A survey of chirality of 3-space embeddings. See Flapan (1995a). [Annot. 4 Nov 2010.] (**sg: Top: Chem: Exp**)

Erica Flapan and Nikolai Weaver

1996a Intrinsic chirality of 3-connected graphs. *J. Combinatorial Theory Ser. B* 68 (1996), 223–232. MR 1417798 (97k:05058). Zbl 861.05023.

See Flapan (1995a). [Annot. 4 Nov 2010.] (**sg: Top**)

Rigoberto Flórez

2005a *Four Studies in the Geometry of Biased Graphs.* Doctoral dissertation, State Univ. of New York at Binghamton, 2005. MR 2707450.

Published as (2006a, 2009a), Flórez and Forge (2007a), and (not yet) Flórez and Zaslavsky (20xxa). (**GG: M, Geom**)

2006a Lindström's conjecture on a class of algebraically non-representable matroids. *European J. Combin.* 27 (2006), no. 6, 896–905. MR 2006m:05048. Zbl 1090.-05010.

Lindström conjectured that a certain matroid $M(n)$ is algebraically nonrepresentable if n is nonprime. Proved by showing that $M(n)$ extends by harmonic conjugation to $L_0(\mathbb{Z}_n K_3)$, which in turn extends to a contradiction if n is composite. (**gg: M**)

2009a Harmonic conjugation in harmonic matroids. *Discrete Math.* 309 (2009), no. 8, 2365–2372. MR 2510362 (2010f:05038). Zbl 1207.05027.

In a harmonic matroid H, harmonic conjugates exist and are unique. If $L_0(\mathfrak{G} K_3) \subseteq H$ and $\mathfrak{G} = \mathbb{Z}$ or $\mathbb{Z}_p$, then the closure of L_0 under harmonic conjugation is a projective plane over $\mathbb{Q}$ or $\mathrm{GF}(p)$, as appropriate. (**gg: M**)

Rigoberto Flórez and David Forge

2007a Minimal non-orientable matroids in a projective plane. *J. Combin. Theory Ser. A* 114 (2007), no. 1, 175–183. MR 2007h:05031. Zbl 1120.52012.

The minimal matroids are contained in lift matroids of $\mathbb{Z}_n K_3$. (**gg: M**)

Rigoberto Flórez and Thomas Zaslavsky

20xxa Biased graphs. VI. Synthetic geometry. In preparation. (**GG: M, Geom**)

Joel Foisy

See Y. Duong.

Wungkum Fong

2000a *Triangulations and Combinatorial Properties of Convex Polytopes.* Doctoral dissertation, Massachusetts Inst. of Technology, 2000.

A configuration consists of the vectors representing an acyclic orientation of a complete signed graph. The volume of the pyramid over the configuration with apex at the origin. [See Ohsugi and Hibi (2003a). *Question.* Is there a connection with the chromatic polynomial?] [Annot. 11 Apr 2011.] (**sg: Geom: Invar**)

Carlos M. da Fonseca

See M. Anđelić.

G. Forgacs

See also S.T. Chui and B.W. Southern.

1980a Ground-state correlations and universality in two-dimensional fully frustrated systems. *Phys. Rev. B* (3) 22 (1980), no. 9, 4473–4480. MR 590596 (81i:82066).

Dictionary: "fully frustrated Ising model on a square lattice" = signed grid (square lattice) graph in which every quadrilateral is negative; "plaquette" = "square" = region boundary = quadrilateral. (**Phys: sg**)

G. Forgacs and E. Fradkin

1981a Anisotropy and marginality in the two-dimensional fully frustrated Ising model. *Phys. Rev. B* 23 (3) (1981), no. 7, 3442–3447. MR 607834 (82c:82094).

(**Phys: sg**)

David Forge

See also P. Berthomé and R. Flórez.

David Forge and Thomas Zaslavsky

2007a Lattice point counts for the Shi arrangement and other affinographic hyperplane arrangements. *J. Combin. Theory Ser. A* 114 (2007), no. 1, 97–109. MR 2007i:-52026. Zbl 1105.52014.

The number of proper integral m-colorings of a rooted integral gain graph (root v_0 and a function $h : V \to \mathbb{Z}$ such that there are root edges ge_{0i} for all $g \in (-\infty, h_i]$; otherwise the gain graph is finite).

(**GG: Geom, Invar, M**)

20xxb Colorations, orthotopes, and a huge polynomial Tutte invariant of weighted gain graphs. Submitted.

A weighted gain graph has lattice-ordered gain group and has vertex weights from an abelian semigroup acted upon by the gain group. The total dichromatic polynomial is a Tutte invariant (satisfying deletion-contraction and multiplicativity) with possibly uncountably many variables, but is not the universal one. *Problem.* Find the universal Tutte invariant. With integral gain group and integral weights, the integral chromatic function of (2007a) is an evaluation of the polynomial. Another special case is the polynomial of S.D. Noble and D.J.A. Welsh, A weighted graph polynomial from chromatic invariants of knots [Symposium à la Mémoire de François Jaeger (Grenoble, 1998). *Ann. Inst. Fourier (Grenoble)* 49 (1999), no. 3, 1057–1087]. (**GG: Invar, M**)

C.M. Fortuin and P.W. Kasteleyn

1972a On the random cluster model. I. Introduction and relation to other models. *Physica* 57 (1972), 536–564. MR 50 #12107.

Most of the paper recasts classical physical and other models (percolation, ferromagnetic Ising, Potts, graph coloring, linear resistance) in a common form that is generalized in §7, "Random cluster model". The "cluster (generating) polynomial" $Z(\Gamma; p, \kappa)$, where $p \in \mathbb{R}^E$ and $\kappa \in \mathbb{R}$, is a 1-variable specialization of the general parametrized dichromatic polynomial. In the notation of Zaslavsky (1992b) it equals $Q_\Gamma(q, p; \kappa, 1)$, where $q_e = 1 - p_e$. Thus it partially anticipates the general polynomials of Przytycka and Przytycki (1988a), Traldi (1989a), and Zaslavsky (1992b) that were based on Kauffman's (1989a) sign-colored Tutte polynomial. A spanning-tree expansion is given only for the resistance model. A feature [that seems not to have been taken up by subsequent workers] is the differentiation relation (7.7) connecting $\partial \ln Z / \partial q_e$ with [I think!] the expectation that the endpoints of e are disconnected in a subgraph. [Grimmett (1994a) summarizes subsequent work in the probabilistic direction.] (**sgc: Gen: Invar, Phys**)

J.-L. Fouquet

See C. Berge.

J.-C. Fournier

1979a *Introduction à la notion de matroïde (géométrie combinatoire)*. Publ. Math. d'Orsay, [No.] 79-03. Univ. Paris-Sud, Dép. Math., Orsay, 1979. MR 81a:05027. Zbl 424.05018.

[Ch.] 3.12: "Matroïdes de Dowling" (p. 52). Definition by partial $\mathfrak{G}$-partitions and the linear representability theorem. (**gg: M: Exp**)

E. Fradkin

See G. Forgacs.

Aviezri S. Fraenkel and Peter L. Hammer

1984a Pseudo-Boolean functions and their graphs. In: *Convexity and graph theory* (Jerusalem, 1981), pp. 137–146. North-Holland Math. Stud., 87. North-Holland, Amsterdam, 1984. MR 87b:90147. Zbl 557.94019. (**sh: lg**)

András Frank

1990a Packing paths, circuits, and cuts – a survey. In: B. Korte, L. Lovász, H.J. Prömel, and A. Schrijver, eds., *Paths, Flows, and VLSI-Layout*, pp. 47–100. Algorithms and Combinatorics, Vol. 9. Springer-Verlag, Berlin, 1990. MR 1083377 (91i:68116). Zbl 741.05042.

Pp. 89–91: Additively sign-weighted bipartite graphs. Thms. 8.1′, 8.5′: Criteria for negative circuit. [*Questions*. Is there a generalization to antibalanced signed graphs with additive sign-weights? Does the existence of minors help?] Thms. 8.1′w, 8.1″: Similar, for $\mathbb{Z}^+$-weighted or $\mathbb{Q}^+$-weighted graphs, not necessarily bipartite. Pp. 91–92 mention Gerards (1990a) and graphs with a bipartizing vertex. [Annot. 11 Jun 2012.] (**SGw, GGw: OG**)

1996a A survey on T-joins, T-cuts, and conservative weightings. In: D. Miklós, V.T. Sós, and T. Szőnyi, eds., *Combinatorics, Paul Erdős is Eighty*, Vol. 2, pp. 213–252. Bolyai Soc. Math. Stud., 2. János Bolyai Math. Soc., Budapest, 1996. MR 97c:05115. Zbl 846.05062.

A "conservative ±1-weighting" of G is an edge labelling by +1's and −1's so that in every circle the sum of edge weights is nonnegative. It is a tool in several theorems. [Related: Ageev, Kostochka, and Szigeti (1995a), Sebö (1990a).] (**SGw: Str, Alg: Exp, Ref**)

Howard Frank and Ivan T. Frisch

1971a *Communication, Transmission, and Transportation Networks.* Addison-Wesley, Reading, Mass., 1971. MR 49 #12063. Zbl 281.94012.

§6.12: "Graphs with gains," pp. 277–288. (**GN: Exp**)

Ove Frank and Frank Harary

1979a Balance in stochastic signed graphs. *Social Networks* 2 (1979/80), 155–163. MR 81e:05116.

An edge is present with probability α and positive with probability p. They compute the expected values of two kinds of measures of imbalance: the number of balanced triangles (whose variance is also given), and the number of induced subgraphs of order 3 having specified numbers of positive and negative edges. [Related: Škoviera (1992a), A.T. White (1994a).] (**SG: Rand, Fr**)

Giancarlo Franzese

1996a Cluster analysis for percolation on a two-dimensional fully frustrated system. *J. Phys. A* 29 (1996), 7367–7375. Zbl 904.60081.

The "fully frustrated" square lattice: alternate verticals are negative. Extending Kandel, Ben-Av, and Domany (1990a) by studying cluster properties in simulations, e.g., percolating clusters (that connect opposite sides of the lattice). Illuminating diagrams. [Annot. 18 Jun 2012.] (**Phys, SG: Clu**)

Maria Aguieiras A. de Freitas, Nair M.M. de Abreu, Renata R. Del-Vecchio, and Samuel Jurkiewicz

2010a Infinite families of Q-integral graphs. *Linear Algebra Appl.* 432 (2010), no. 9, 2352–2360. MR 2599865 (2011b:05150). Zbl 1219.05158.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Maria Aguieiras A. de Freitas, Renata R. Del-Vecchio, Nair M.M. de Abreu, and Steve Kirkland

2009a On Q-spectral integral variation. LAGOS'09—V Latin-Amer. Algor. Graphs Optim. Sympos. *Electron. Notes Discrete Math.* 35 (2009), 203–208. MR 2579431.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Christian Fremuth-Paeger and Dieter Jungnickel

1999a Balanced network flows. I: A unifying framework for design and analysis of matching algorithms. *Networks* 33 (1999), no. 1, 1–28. MR 1652254 (2000f:-90005). Zbl 999.90005. (**sg: par: Flows, cov**)

1999b Balanced network flows. II: Simple augmentation algorithms. *Networks* 33 (1999), no. 1, 29–41. MR 1652258 (2000g:90010). Zbl 999.90006. (**sg: par: Flows, cov**)

1999c Balanced network flows. III: Strongly polynomial augmentation algorithms. *Networks* 33 (1999), no. 1, 43–56. MR 1652262 (2000g:90011). Zbl 999.90007. (**sg: par: Flows, cov**)

2001a Balanced network flows. IV: Duality and structure theory. *Networks* 37 (2001), no. 4, 194–201. MR 1837197 (2002k:90010). Zbl 1038.90007. (**sg: par: Flows, cov**)

2001b Balanced network flows. V: Cycle-canceling algorithms. *Networks* 37 (2001), no. 4, 202–209. MR 1837198 (2002k:90011). Zbl 1038.90008. (**sg: par: Flows, cov**)

2001c Balanced network flows. VI: Polyhedral descriptions. *Networks* 37 (2001), no. 4, 210–218. MR 1837199 (2002k:90012). Zbl 1040.90002. (**sg: par: Flows, cov**)

2002a Balanced network flows. VII: Primal-dual algorithms. *Networks* 37 (2002), no. 1, 35–42. MR 1871705 (2003d:90007). Zbl 1040.90003. (**sg: par: Flows, cov**)

2002b An introduction to balanced network flows. In: K.T. Arasu and Á. Seress, eds., *Codes and Designs* (Columbus, Ohio, 2000), pp. 125–144. Ohio State Univ. Math. Res. Inst. Publ., 10. Walter de Gruyter, Berlin, 2002. MR 1948139 (2004b:05160). Zbl 1009.05113. (**sg: par: Flows, cov**)

2003a Balanced network flows. VIII: A revised theory of phase-ordered algorithms and the $O(\sqrt{n}m \log(n2/m)/ \log n)$ bound for the nonbipartite cardinality matching problem. *Networks* 37 (2003), no. 3, 137–142. MR 1970119 (2004f:90015). Zbl 1106.90013. (**sg: par: Flows, cov**)

Ivan T. Frisch

See H. Frank.

Toshio Fujisawa

1963a Maximal flow in a lossy network. In: J.B. Cruz, Jr., and John C. Hofer, eds., *Proceedings, First Annual Allerton Conference on Circuit and System Theory* (Monticello, Ill., 1963), pp. 385–393. Dept. of Electrical Eng. and Coordinated Sci. Lab., Univ. of Illinois, Urbana, Ill., [1963]. (**GN: M(bases)**)

Satoru Fujishige

See K. Ando.

D.R. Fulkerson, A.J. Hoffman, and M.H. McAndrew

1965a Some properties of graphs with multiple edges. *Canad. J. Math.* 17 (1965), 166–177. MR 177908 (31 #2166). Zbl 132.21002.

The "odd-cycle condition" is that any two odd circles without a common vertex are joined by an edge. Assuming it, certain conditions are necessary and sufficient for a degree sequence to be realized by a submultigraph of K_n with prescribed multiplicities. The incidence matrix of $-K_n$ is employed in the geometrical proof. [*Problem.* Generalize to signed graphs.] [Annot. 30 May 2011.] (**sg: Par: incid**)

Martin J. Funk

See M. Abreu.

H.N. Gabow

1983a An efficient reduction technique for degree-constrained subgraph and bidirected network flow problems. In: *Proceedings of the Fifteenth Annual ACM Symposium on Theory of Computing* (Boston, 1983), pp. 448–456. Assoc. for Computing Machinery, New York, 1983. MR 0842673 (87g:68004) (book).

$O(m^{3/2})$ algorithm for max integral flow. [See Babenko (2006b) for improved time.] [Annot. 9 Sept 2010.] (**sg: Ori: Alg**)

Stephen M. Gagola

1999a Solution to Problem 10606. *Amer. Math. Monthly* 106 (June–July, 1999), no. 6, 590–591.

Proposed by Zaslavsky (1997c), *q.v.* for statement of the problem and significance. (**gg**)

Anahí Gajardo

See M. Montalva.

David Gale

See also A.J. Hoffman.

David Gale and A.J. Hoffman

1982a Two remarks on the Mendelsohn–Dulmage theorem. In: Eric Mendelsohn, ed., *Algebraic and Geometric Combinatorics*, pp. 171–177. North-Holland Math. Stud., 65. Ann. Discrete Math., 15. North-Holland, Amsterdam, 1982. MR 85m:05054. Zbl 501.05049. (**sg: Incid, Bal**)

Joseph A. Gallian

2009a A dynamic survey of graph labeling. *Electronic J. Combin.* Dynamic Surveys in Combinatorics, # DS6.
http://www.combinatorics.org/ojs/index.php/eljc/article/view/ds6
MR 1668059 (99m:05141). Zbl 953.05067.

§3.7, "Cordial labelings"; §3.8, "The friendly index–balance index". From $f : V \to \mathbb{Z}_2$ obtain balanced edge gains $f^*(uv) = f(u) + f(v)$. f is "friendly" if it has essentially equal numbers of each label, i.e., equal or differing by 1. f is "cordial" if f and f^* have essentially equal numbers of each label. A great many references. [(Γ, f) is like a balanced multiply signed graph but the questions are not gain-graphic.] [Annot. 9 Oct 2010.] (**vs: Exp, Ref**)

Anna Galluccio, Martin Loebl, and Jan Vondrák

See also J. Lukic.

2000a New algorithm for the Ising problem: Partition function for finite lattice graphs. *Phys. Rev. Lett.* 84 (2000), no. 26, 5924–5927.

Describes (2001a), emphasizing signed toroidal lattice graphs, i.e., toroidal lattice Ising models. [Annot. 18 Aug 2012.]

(**SG, Phys: Fr: Alg**)

2001a Optimization via enumeration: a new algorithm for the Max Cut Problem. *Math. Programming Ser. A* 90 (2001), 273–290. MR 1824075 (2002b:90057). Zbl 989.90127.

An algorithm for the generating function of weighted cuts (= partition function of Ising model), hence for $\sum_\zeta x^{E^-(\Sigma^\zeta)}$ and frustration index $l(\Sigma)$, in polynomial time for graphs of bounded genus. [Annot. 18 Aug 2012.] (**SG: Fr: Alg, Phys**)

Yu-Bin Gao

See also Y.Z. Fan, L.F. Huo, and Y.L. Shao.

Yubin Gao, Yihua Huang and Yanling Shao

2009a Bases of primitive non-powerful signed symmetric digraphs with loops. *Ars Combin.* 90 (2009), 383–388. MR 2489540 (2010c:05049). Zbl 1224.05208.

(**SD, QM**)

Yubin Gao, Yanling Shao, and Jian Shen

2009a Bounds on the local bases of primitive nonpowerful nearly reducible sign patterns. *Linear Multilinear Algebra* 57 (2009), no. 2, 205–215. MR 2492103 (2010b:05103). Zbl 1166.15008. (**SD, QM**)

Marianne L. Gardner [Marianne Lepp]

See R. Shull.

T. Garel and J.M. Maillard

1983a Study of a two-dimensional fully frustrated model. *J. Phys. A* 16 (1983), 2257–2265. MR 713188 (85b:82069).

Physics approach. Generalizes Southern, Chui, and Forgacs's (1980a) square-lattice Ising model to four edge weights, symmetrically located, and reduces it to an all-positive graph with two weights. §3, "Application to the Villain model": All weights equal [hence a signed graph]; further results on Villain (1977a). [Annot. 16 Jun 2012.] (**Phys: sg: wg**)

Pravin Garg

See D. Sinha.

Michael Gargano and Louis V. Quintas

1985a A digraph generalization of balanced signed graphs. *Congressus Numerantium* 48 (1985), 133–143. MR 87m:05095. Zbl 622.05027.

Characterizes balance in abelian gain graphs. [See Harary, Lindström, and Zetterström (1982a).] Very simple results on existence, for a given graph, of balanced nowhere-zero gains from a given abelian group. [Elementary, if one notes that such gains exist iff the graph is $|G|$-colorable, G being the gain group]. Comparison with the approach of Sampathkumar and Bhave (1973a). Dictionary: "Symmetric G-weighted digraph" = gain graph with gains in the (abelian) group G. "Weight" = gain. "Non-trivial" (of the gain function) = nowhere zero. (**GG: Bal**)

Michael L. Gargano, John W. Kennedy, and Louis V. Quintas

1998a Group weighted balanced digraphs and their duals. Proc. Twenty-ninth Southeastern Int. Conf. on Combinatorics, Graph Theory and Computing (Boca Raton, Fla., 1998). *Congressus Numer.* 131 (1998), 161–167. MR 99j:05080. Zbl 951.05045.

An abelian gain graph Φ is cobalanced (here called "cut-balanced") if the sum of gains on the edges of each coherently oriented cutset is 0. [This generalizes Kabell (1985a).] Given Φ with $\|\Phi\|$ embedded in a surface, the surface dual graph is given gains by a right-rotation rule, thus forming a surface dual Φ^* of Φ. [This appears to require that the surface be orientable. Note that cobalance generalizes to nonabelian gains on orientably embedded graphs, since the order of multiplication for the gain product on a cutset is given by the embedding.] Thm. 3.2: For a plane embedding of Φ, Φ is cobalanced iff Φ^* is balanced. Thm. 3.4 restates as criteria for cobalance of Φ the standard criteria for balance of Φ^*, as in Gargano and Quintas (1985a). More interesting are "well-balanced" graphs, which are both balanced and cobalanced. *Problem.* Characterize them. Dictionary (also see Gargano and Quintas 1985a): Balance is called "cycle balance". (**GG: Bal(D)**)

Gilles Gastou and Ellis L. Johnson

1986a Binary group and Chinese postman polyhedra. *Math. Programming* 34 (1986), 1–33. MR 88e:90060. Zbl 589.52004.

§10 introduces the co-postman and "odd circuit" problems, treated more thoroughly in Johnson and Mosterts (1987a) (q.v). "Odd" edges and circuits are precisely negative edges and circles in an edge signing. The "odd circuit matrix" represents $L(\Sigma)$ (p. 30). The "odd circuit problem" is to find a shortest negative circle; a simple algorithm uses the signed covering graph (pp. 30–31). The "Fulkerson property" may be related to planarity and K_5 minors [which suggests comparison with Barahona (1990a), §5]. (**SG: Fr(Gen), Incid, M(Bases), cov, Alg**)

Heather Gavlas [Heather Jordon]
See G. Chartrand, D. Hoffman, and H. Jordon.

Premiysław Gawroński
See also K. Kułakowski.

P. Gawroński, P. Gronek, and K. Kułakowski

2005a The Heider balance and social distance. *Acta Phys. Polonica B* 36 (2005), no. 8, 2549–2558.

P. Gawroński and K. Kułakowski

2005a Heider balance in human networks. In: Joaquıın Marro, Pedro L. Garrido, and Miguel A. Muñoz, eds., *Modeling Cooperative Behavior in the Social Sciences* (Proc. 8th Granada Lect., Granada, Spain, 2005), pp. 93–95. AIP Conf. Proc., Vol. 779. Amer. Inst. Physics, Melville, N.Y., 2005.

2007a A numerical trip to social psychology: long-living states of cognitive dissonance. In: Y. Shi *et al.*, eds., *Computational Science – ICCS 2007* (7th Int. Conf., Beijing, 2007), Part IV, pp. 43–50. Lect. Notes in Computer Sci., Vol. 4490. Springer, Berlin, 2007.

Jim [James F.] Geelen
See also M. Chudnovsky.

2008a Some open problems on excluding a uniform matroid. *Adv. Appl. Math.* 41 (2008), 628–637. MR 2459453 (2009k:05050).

Spikes $G(2C_n, \mathcal{B})$ are important (see p. 630). [Annot. 29 Apr 2012.] **(gg: M)**

James F. [Jim] Geelen and A.M.H. [Bert] Gerards

2005a Regular matroid decomposition via signed-graphs. *J. Graph Theory* 48 (2005), no. 1, 74–84. MR 2005h:05037. Zbl 1055.05024.

The lift matroid. **(SG: M: Str)**

2009a Excluding a group-labelled graph. *J. Combin. Theory Ser. B* 99 (2009), 247–253. MR 2467829 (2009k:05169). Zbl 1226.05213.

Given finite, abelian $\mathfrak{G}$ and $\mathfrak{G}' \leq \mathfrak{G}$, and a $\mathfrak{G}$-gain graph Φ with a minor $\Psi \cong \mathfrak{G}'K_{4t}$ where $t = 8n|\mathfrak{G}|^2$. Thm. 1.3: Either $\exists\ X \subseteq V$ with $|X| < t$ such that in $\Phi \setminus X$ the block containing most of Ψ is $\mathfrak{G}'$-balanced, or Ψ has a minor $\cong \mathfrak{G}''K_n$ where $\mathfrak{G}' < \mathfrak{G}'' \leq \mathfrak{G}$. t may not be best possible. Thm. 1.4: $\forall\ n$, $\exists\ l(n)$ such that $\|\Phi\|$ has a $K_{l(n)}$ minor $\Longrightarrow \Phi$ has a $0K_n$ minor. Dictionary: "Group-labelled graph" = gain graph; Γ means $\mathfrak{G}$; G means Φ; $\tilde{G}$ means $\|\Phi\|$; "shifting" means "switching"; $\mathfrak{G}'$-balanced means switchable so all gains are in $\mathfrak{G}'$. **(GG: Str)**

Jim Geelen, Bert Gerards, Bruce Reed, Paul Seymour, and Adrian Vetta

2009a On the odd-minor variant of Hadwiger's conjecture. *J. Combin. Theory Ser. B* 99 (2009), no. 1, 20–29. MR 2467815 (2010f:05149). Zbl 1213.05079.

Jim Geelen, Bert Gerards, and Geoff Whittle

2006a Matroid T-connectivity. *SIAM J. Discrete Math.* 20 (2006), no. 3, 588–596. MR 2007j:05040. Zbl 1122.05023.

The full bicircular matroid $G(\Gamma^\bullet, \varnothing)$ appears on p. 589. **(gg: bic)**

2006b Towards a structure theory for matrices and matroids. In: Marta Sanz-Solé *et al.*, eds., *Proceedings of the International Congress of Mathematicians* (ICM, Madrid, 2006), Vol. III: *Invited Lectures*, pp. 827–842. European Mathematical Society, Zürich, 2006. MR 2275708 (2008a:05045). Zbl 1100.05016.

See (2007a). **(gg: M: Exp)**

2007a Towards a matroid-minor structure theory. In: Geoffrey Grimmett *et al.*, eds., *Combinatorics, Complexity, and Chance: A Tribute to Dominic Welsh*, pp. 72–82. Oxford Lect. Ser. Math. Appl., Vol. 34. Oxford Univ. Press, Oxford, 2007. MR 2008d:05037 (q.v.). Zbl 1130.05015.

Conjecture. A minor-closed proper subclass of all GF q-representable matroids is essentially constructible from frame matroids and their duals. Dictionary: "Dowling matroid" = simple frame matroid, i.e., submatroid of Dowling's (1973a, 1973b) matroids $G(\mathfrak{G}K_n^\bullet)$, for $\mathfrak{G} = \mathbb{H}_q^\times$. [Annot. 25 May 2009.] **(gg: M: Exp)**

James F. Geelen and Bertrand Guenin

2002a Packing odd circuits in Eulerian graphs. *J. Combin. Theory Ser. B* 86 (2002), no. 2, 280–295. MR 2004g:05129. Zbl 1023.05091.

Adds to Guenin's theorem (2001a): Thm.: If Σ has no $-K_5$ minor, then the dual linear program has a half-integral minimum (assuming f has nonnegative coefficients). **(SG: Geom, Str)**

Jim Geelen and Tony Huynh

2006a Colouring graphs with no odd-K_n minor. Manuscript, 2002, 2006. http://www.math.uwaterloo.ca/~jfgeelen/publications/colour.pdf **(SG, Col)**

James Geelen, James Oxley, Dirk Vertigan, and Geoff Whittle

2002a Totally free expansions of matroids. *J. Combin. Theory Ser. B* 84 (2002), no. 1, 130–179. MR 1877906 (2002j:05035). Zbl 1048.05020.

A rank-r swirl is $G(2C_r, \varnothing)$. Free spikes and rank-r swirls, also the latter with one unbalanced loop, are important. *Conjecture*: The 3-connected, rank-k matroids, representable over GF(q) and having no $L(2C_k, \varnothing)$ or $G(2C_k, \varnothing)$ minor, have a bounded number of inequivalent GF(q)-representations. [Annot. 25 May 2009, 29 Apr 2012.] **(gg: M)**

2004a A short proof of non-GF(5)-representability of matroids. *J. Combin. Theory Ser. B* 91 (2004), 105–121. MR 2047534 (2005b:05055). Zbl 1050.05024.

The "free swirl" Δ_r is $G(2C_k, \varnothing)$. The "free spike" Λ_r is $L(2C_k, \varnothing)$. They play a main role re large totally free matroids (Thm. 3.4). [Annot. 8 Mar 2011.] **(gg: M)**

M.C. Geetha

See P. Siva Kota Reddy.

Xianya Geng, Shuchao Li, and Slobodan K. Simić

2010a On the spectral radius of quasi-k-cyclic graphs. *Linear Algebra Appl.* 433 (2010), no. 8-10, 1561–1572. MR 2718221 (2011f:05178). Zbl 1211.05074.

§2 mentions $K(-\Gamma)$. Quasi-k-cyclic means $\exists\ v$ such that $\Gamma \setminus v$ has cyclomatic number k. For $k \leq 2$, Thm. 3.2 describes all $-\Gamma$ maximizing the largest eigenvalue of $K(-\Gamma)$. [Annot. 21 Jan 2012.] **(Par: Adj)**

A.M.H. Gerards

See also M. Chudnovsky, M. Conforti, and J. Geelen.

1988a Homomorphisms of graphs into odd cycles. *J. Graph Theory* 12 (1988), 73–83. MR 89h:05045. Zbl 691.05013.

If an antibalanced, unbalanced signed graph has no homomorphism into its shortest negative circle, then it contains a subdivision of $-K_4$ or of a loose $\pm C_3$ (here called an "odd K_4" and an "odd K_3^2"). (A loose $\pm C_n$ consists of n negative digons in circular order, each adjacent pair joined either at a common vertex or by a link.) [*Question.* Do the theorem and proof carry over to any unbalanced signed graph?] Other results about antibalanced signed graphs are corollaries. Several interesting results about signed graphs are lemmas. **(Par, SG: Str)**

1989a A min-max relation for stable sets in graphs with no odd-K_4. *J. Combin. Theory Ser. B* 47 (1989), 330–348. MR 91c:05143. Zbl 691.05021.

Let Σ be antibalanced and without isolated vertices and contain no subdivision of $-K_4$. Then max. stable set size = min. cost of a cover by edges and negative circles. Also, min. vertex-cover size = max. profit of a packing of edges and negative circles. Also, weighted analogs. [*Question.* Do the theorem and proof extend to any Σ?] **(par, sg: Str)**

1989b A short proof of Tutte's characterization of totally unimodular matrices. *Linear Algebra Appl.* 114/115 (1989), 207–212. MR 90b:05033. Zbl 676.05028.

The proof of Lemma 3 uses a signed graph. **(SG: Bal)**

††1990a *Graphs and polyhedra: Binary spaces and cutting planes.* CWI Tract, 73. Centrum voor Wiskunde en Informatica, Amsterdam, 1990. MR 1106635 (92f:-52027). Zbl 727.90044.

(Very incomplete annotation.) Thm.: Given Σ, the set $\{x \in \mathbb{R}^n : d_1 \leq x \leq d_2, b_1 \leq \mathrm{H}(\Sigma)^{\mathrm{T}} x \leq b_2\}$ has Chvatal rank ≤ 1 for all integral vectors d_1, d_2, b_1, b_2, iff Σ contains no subdivided $-K_4$. **(SG: Incid, Geom, Bal, Str)**

1992a On shortest T-joins and packing T-cuts. *J. Combin. Theory Ser. B* 55 (1992), 73–82. MR 93d:05093. Zbl 810.05056. **(SG: Str)**

1992b Odd paths and circuits in planar graphs with two odd faces. CWI Report BS-R9218, September 1992.

1994a An orientation theorem for graphs. *J. Combin. Theory Ser. B* 62 (1994), 199–212. MR 96d:05051. Zbl 807.05020. **(par, sg: M, Ori)**

1995a On Tutte's characterization of graphic matroids—a graphic proof. *J. Graph Theory* 20 (1995), 351–359. MR 96h:05038. Zbl 836.05017.

Signed graphs used to prove Tutte's theorem. The signed-graph matroid employed is the extended lift matroid $L_0(\Sigma)$ ("extended even cycle matroid"). The main theorem (Thm. 2): Let Σ be a signed graph with no $-K_4$, $\pm K_3$, $-Pr_3$, or Σ_4 link minor; then Σ can be converted by Whitney 2-isomorphism operations ("breaking" = splitting a component in two at a cut vertex, "glueing" = reverse, "switching" = twisting across a vertex 2-separation) to a signed graph that has a balancing vertex ("blocknode"). Here Σ_4 consists of $+K_4$ with a 2-edge matching doubled by negative edges and one other edge made negative.

More translation: His "Σ" is our E^-. "Even, odd" = positive, negative (for edges and circles). "Bipartite" = balanced; "almost bipartite" = has a balancing vertex. **(SG: M, Str, Incid)**

1995b Matching. In: M.O. Ball, T.L. Magnanti, C.L. Monma, and G.L. Nemhauser, eds., *Network Models*, Ch. 3, pp. 135–224. Handbooks Oper. Res. Management

Sci., Vol. 7. North-Holland, Amsterdam, 1995. MR 1420868. Zbl 839.90131.

§7.2.2, "Network flows and bidirected graphs". Generalized matchings in bidirected graphs. [Annot. 9 June 2011.] **(sg: Ori: Incid)**

A.M.H. Gerards and M. Laurent

1995a A characterization of box $\frac{1}{d}$-integral binary clutters. *J. Combin. Theory Ser. B* 65 (1995), 186–207. MR 96k:90052. Zbl 835.05017.

Thm. 5.1: The collection of negative circles of Σ is box $\frac{1}{d}$-integral for some/any integer $d \geq 2$ iff it does not contain $-K_4$ as a link minor. **(SG: Circles, Geom)**

A.M.H. Gerards, L. Lovász, A. Schrijver, P.D. Seymour, C.-S. Shi, and K. Truemper

†1990a Manuscript in preparation, 1990.

Extension of Gerards and Schrijver (1986b). [Same comments apply. The proliferating authorship may prevent this major contribution from ever being published—though one hopes not! See Seymour (1995a) for description of two main theorems.] **(SG: Str, M, Top)**

A.M.H. Gerards and A. Schrijver

1986b Signed graph – regular matroids – grafts. Research Memorandum, Faculteit der Economische Wetenschappen, Tilburg Univ., 1986.

Essential, major theorems. The (extended) lift matroid of a signed graph is one of the objects studied. Some of this material is published in Gerards (1990a). This paper is in the process of becoming Gerards, Lovász, *et al.* (1990a). **(SG: Str, M)**

1986a Matrices with the Edmonds–Johnson property. *Combinatorica* 6 (1986), 365–379. MR 879340 (88g:05087). Zbl (565.90048), 641.05039.

A subsidiary result: If $-\Gamma$ contains no subdivided $-K_4$, then Γ is t-perfect. **(sg: Par: Geom, Str)**

A.M.H. Gerards and F.B. Shepherd

1998a Strong orientations without even directed circuits. *Discrete Math.* 188 (1998), 111–125. MR 99i:05091. Zbl 957.05048.

1998b The graphs with all subgraphs t-perfect. *SIAM J. Discrete Math.* 11 (1998), 524–545. MR 2000e:05074. Zbl 980.38493.

Extension of Gerards (1989a). An "odd-K_4" is a graph whose all-negative signing is a subdivided $-K_4$. A "bad-K_4" is an odd-K_4 which does not consist of exactly two undivided K_4 edges that are nonadjacent while the other edges are replaced by even paths. Thm. 1: A graph that contains no bad-K_4 as a subgraph is t-perfect. Thm. 2 characterizes the graphs that are subdivisions of 3-connected graphs and contain an odd-K_4 but no bad-K_4. [The fact that 'badness' is not strictly a parity property weighs against the possibility that Gerards (1989a) extends well to signed graphs.] **(par, sg: Str, Alg)**

K.A. Germina

See also S. Hameed K.

K.A. Germina and Shahul Hameed K

2010a On signed paths, signed cycles and their energies. *Appl. Math. Sci. (Ruse)* 4 (2010), no. 70, 3455–3466. MR 2769200 (no rev).

Eigenvalues and energies of $A(\Sigma)$ and Laplacian (Kirchhoff) matrices $K(\Sigma)$ of signed paths and circles; also recurrences for the characteristic

polynomials. Energy of $A := \sum |\lambda_i(A)|$; energy of $K := \sum |\lambda_i(K) - \bar{d}$ where $\bar{d} :=$ average degree. [Annot. 14 Nov 2010.]
(**SG: Adj: Paths, Circles**)

K.A. Germina, Shahul Hameed K, and Thomas Zaslavsky

2011a On products and line graphs of signed graphs, their eigenvalues and energy. *Linear Algebra Appl.* 435 (2011), no. 10, 2432–2450. MR 2811128 (2012j:05254). Zbl 1222.05223. arXiv:1010.3884.

Adjacency matrix A and eigenvalues and energy for the general "Cvetković product" NEPS$(\Sigma_1, \ldots, \Sigma_k; \mathcal{B})$ and for a line graph $\Lambda(\Sigma)$ (as in Zaslavsky (2010b, 20xxa, 20xxb)). Kirchhoff ("Laplacian") matrix $K(\Sigma)$; $K(+\Gamma) =$ "Laplacian" of a graph Γ; $K(-\Gamma) =$ "signless Laplacian") and its eigenvalues and energy for Cartesian product $\Sigma_1 \times \cdots \times \Sigma_r$. Also, $A(\Lambda(\Sigma))$. Thm. The Cartesian product is balanced iff all Σ_i are balanced. Examples: Planar, cylindrical, and toroidal grids with product signatures; line graphs of those grids and of $+K_n$ and $-K_n$. [Annot. 19 Oct 2010.] (**SG: Bal, Adj, LG**)

Anna Maria Ghirlanda

See L. Muracchini.

Ebrahim Ghorbani

See S. Akbari.

A. Ghouila-Houri

See C. Berge.

Rick Giles

1982a Optimum matching forests. I: Special weights. II: General weights. III: Facets of matching forest polyhedra. *Math. Programming* 22 (1982), 1–11, 12–38, 39–51. MR 82m:05075a,b,c. Zbl 468.90053, 468.90054, 468.90055.

In the author's "mixed" graphs, the undirected edges are really extroverted bidirected edges. (**sg: ori**)

Mukhtiar Kaur Gill [Mukti Acharya]

See also B.D. Acharya.

1981a A graph theoretical recurrence formula for computing the characteristic polynomial of a matrix. In: S.B. Rao, ed., *Combinatorics and Graph Theory* (Proc. Sympos., Calcutta, 1980), pp. 261–265. Lect. Notes in Math., Vol. 885. Springer-Verlag, Berlin, 1981. MR 655622 (83f:05047). Zbl 479.05030.

Introduces "quasicospectrality" of graphs or digraphs, i.e., they have cospectral signatures. See B.D. Acharya, Gill, and Pathwardhan (1984a) and M. Acharya (20xxa). [Annot. 3 Feb 2012.] (**SG, SD: Adj**)

1981b A note concerning Acharya's conjecture on a spectral measure of structural balance in a social system. In: S.B. Rao, ed., *Combinatorics and Graph Theory* (Proc. Sympos., Calcutta, 1980), pp. 266–271. Lect. Notes in Math., Vol. 885. Springer-Verlag, Berlin, 1981. MR 655623 (84d:05121). Zbl 476.05073.

Assume $|\Sigma_1| = |\Sigma_2|$. If Σ_1 and Σ_2 have the same value of B.D. Acharya's (1980a) measure of imbalance, $A(\Sigma_1)$ and $A(\Sigma_2)$ may have different spectra. [Not surprisingly.] (**SG: Bal, Adj**)

1982a *Contributions to Some Topics in Graph Theory and Its Applications.* Ph.D. thesis, Dept. of Mathematics, Indian Institute of Technology, Bombay, 1982.

Most of the results herein have been published separately. See Gill (1981a, 1981b), Gill and Patwardhan (1981a, 1983a, 1986a), M. Acharya

(2009a). (**SG, SD: Bal, LG, Adj**)

M.K. Gill and B.D. Acharya

1980a A recurrence formula for computing the characteristic polynomial of a sigraph. *J. Combin. Inform. System Sci.* 5 (1980), 68–72. MR 586322 (81m:05097). Zbl 448.05048. (**SG: Adj**)

1980b A new property of two dimensional Sperner systems. *Bull. Calcutta Math. Soc.* 72 (1980), 165–168. MR 669580 (83m:05121). Zbl 531.05058. (**SG: Bal, Geom**)

M.K. Gill and G.A. Patwardhan

1981a A characterization of sigraphs which are switching equivalent to their line sigraphs. *J. Math. Phys. Sci.* 15 (1981), 567–571. MR 650430 (84h:05106). Zbl 488.05054.

The line graph is that of Behzad and Chartrand (1969a). (**SG: LG**)

1982a A characterization of sigraphs which are switching equivalent to their iterated line sigraphs. *J. Combin. Inform. System. Sci.* 7 (1982), 287–296. MR 724371 (86a:05103). Zbl 538.05060.

The line graph is that of Behzad and Chartrand (1969a). (**SG: LG**)

1986a Switching invariant two-path signed graphs. *Discrete Math.* 61 (1986), 189–196. MR 855324 (87j:05138). Zbl 594.05059.

The k-path signed graph of Σ [I write $D_k(\Sigma)$] is the distance-k graph on V with signs $\sigma_k(uv) = -$ iff every length-k path is all negative. The equation $\Sigma \simeq D_2(\Sigma)$ is solved. [Annot. 29 Apr 2009.] (**SG, Sw**)

Robert Gill

1998a The number of elements in a generalized partition semilattice. *Discrete Math.* 186 (1998), 125–134. MR 1623892 (99e:52014). Zbl 956.52009.

The semilattice is the intersection semilattice of a affinographic hyperplane arrangement representing $[-k, k]K_n$ [and is therefore isomorphic to the geometric semilattice of all k-composed partitions of a set; see, e.g., Zaslavsky (2002a), Ex. 10.5]. The rank and the Whitney numbers of the first kind are calculated. See Kerr (1999a) for homology. (**gg: m: Geom, Invar**)

2000a The action of the symmetric group on a generalized partition semilattice. *Electronic J. Combin.* 7 (2000), Research Paper 23, 20 pp. MR 1755612 (2001g:-05107). Zbl 947.06001.

See (1998a). (**gg: m: Geom, Invar, Aut**)

John Gimbel

1988a Abelian group labels on graphs. *Ars Combinatoria* 25 (1988), 87–92. MR 89k:05046. Zbl 655.05034.

The topic is "induced" edge labellings, that is, $w(e_{uv}) = f(u)f(v)$ for some $f : V \to \mathfrak{A}$. The number of f that induce a given induced labelling, the number of induced labellings, and a characterization of induced labellings. All involve the 2-torsion subgroup of $\mathfrak{A}$, unless Γ is bipartite. The inspiration is dualizing magic graphs. [Somewhat dual to Edelman and Saks (1979a).] (**par: incid**)(**VS(Gen): Enum**)

Omer Giménez, Anna de Mier, and Marc Noy

2005a On the number of bases of bicircular matroids. *Ann. Combin.* 9 (2005), no. 1, 35–45. MR 2005m:05049. Zbl 1059.05030.

The number of bases is bounded above by $C^n\cdot$(number of spanning trees) in a simple graph but not in a multigraph. More precise results for K_n and $K_{n,m}$. [See Neudauer, Meyers, and Stevens (2001a) and Neudauer and Stevens (2001a).] (**Bic: Incid**)

Omer Giménez and Marc Noy

2006a On the complexity of computing the Tutte polynomial of bicircular matroids. *Combin. Probab. Comput.* 15 (2006), no. 3, 385–395. MR 2007a:05029. Zbl 1094.05013.

Known NP-hardness results for transversal matroids apply to their proper subclass, bicircular matroids, with a few possible exceptions. (**Bic: Incid: Alg**)

Ioannis Giotis and Venkatesan Guruswami

2006a Correlation clustering with a fixed number of clusters. In: *Proceedings of the Seventeenth Annual ACM-SIAM Symposium on Discrete Algorithms*, pp. 1167–1176. ACM, New York, 2006. MR 2373844 (2009f:62098). Zbl 1194.62087. (**SG: WG: Clu: Alg**)

2006b Correlation clustering with a fixed number of clusters. *Theory Comput.* 2 (2006), 249–266. MR 2322880 (2009e:68118). (**SG: WG: Clu: Alg**)

Roland Glantz and Marcello Pelillo

2006a Graph polynomials from principal pivoting. *Discrete Math.* 306(2006), no. 24, 3253–3266. MR 2279060 (2008d:05112) (*q.v.*). Zbl 1125.05073. (**GG: Invar**)

Terry C. Gleason

See also D. Cartwright.

Terry C. Gleason and Dorwin Cartwright

1967a A note on a matrix criterion for unique colorability of a signed graph. *Psychometrika* 32 (1967), 291–296. MR 35 #989. Zbl 184, 492 (e: 184.49202).

"Colorable" = clusterable. The adjacency matrices of Σ^+ and Σ^- are employed separately. The arithmetic is mostly "Boolean", i.e., $1+1=0$. A certain integral matrix T shows whether or not Σ is clusterable. [Annot. 11 Nov 2008.] (**SG: Clu, Adj**)

Fred Glover

See also J. Elam.

F. Glover, J. Hultz, D. Klingman, and J. Stutz

1978a Generalized networks: A fundamental computer-based planning tool. *Management Sci.* 24 (1978), 1209–1220. (**GN: Alg, M(bases): Exp, Ref**)

Fred Glover and D. Klingman

1973a On the equivalence of some generalized network problems to pure network problems. *Math. Programming* 4 (1973), 269–278. MR 47 #6393. Zbl 259.90012. (**GN: Bal, Incid**)

1973b A note on computational simplifications in solving generalized transportation problems. *Transportation Sci.* 7 (1973), 351–361. MR 54 #6502. (**GN: M(bases), geom**)

Fred Glover, Darwin Klingman, and Nancy V. Phillips

1992a *Network Models in Optimization and Their Applications in Practice.* Wiley-Interscience, New York, 1992.

Textbook. See especially Ch. 5: "Generalized networks." (**GN: Alg: Exp**)

F. Glover, D. Klingman, and J. Stutz

1973a Extensions of the augmented predecessor index method to generalized network problems. *Transportation Sci.* 7 (1973), 377–384. **(GN: M(bases), m)**

Wayne Goddard

See F.R.K. Chung.

Luis Goddyn

See M. Chudnovsky.

C.D. Godsil

1985a Inverses of trees. *Combinatorica* 5 (1985), 33–39. MR 86k:05084. Zbl 578.05049.

If T is a tree with a perfect matching, then $A(T)^{-1} = A(\Sigma)$ where Σ is balanced and $|\Sigma| \supseteq \Gamma$. *Question.* When does $|\Sigma| = \Gamma$? [Solved by Simion and Cao (1989a).] [*Cf.* Buckley, Doty, and Harary (1984a) and, for a different notion, Greenberg, Lundgren, and Maybee (1984b).] **(sg: Adj, Bal)**

Chris Godsil and Gordon Royle

2001a *Algebraic Graph Theory.* Graduate Texts in Math., Vol. 207. Springer-Verlag, New York, 2001. MR 2002f:05002. Zbl 968.05002.

Ch. 11, "Two-graphs": Equiangular lines (van Lint and Seidel 1966a, Lemmens and Seidel 1973a), graph switching (van Lint and Seidel 1966a, Seidel 1976a), regular two-graphs (Taylor 1977a). **(TG: Adj, Geom, Sw)**

Ch. 12, "Line graphs and eigenvalues": Based on Cameron, Goethals, Seidel, and Shult (1976a). **(LG: sg: Adj, Geom, Sw)**

§15.3, "Signed matroids": Sign-colored matroids and graphs. Rank generating polynomial (see Kauffman 1989a). §16.3, "Signed plane graphs", §16.5, "Reidemeister invariants", §16.6, "The Kauffman bracket", §16.8, "Connectivity": Properties of Kauffman's (1989a) "signed-graph" (really sign-colored graph) Tutte polynomial. §16.7, "The Jones polynomial" of a knot. **(Sc, SGc: Adj, Incid, Top)**

J.M. Goethals

See also P.J. Cameron.

J.M. Goethals and J.J. Seidel

1970a Strongly regular graphs derived from combinatorial designs. *Canad. J. Math.* 22 (1970) 597–614. MR 44 #106. Zbl 198.29301.

A symmetric Hadamard matrix H with constant diagonal can be put in the form $A(K_n, \sigma) \pm I$ for some signed K_n that represents a regular two-graph [see D.E. Taylor (1977a)] of order $4s^2$ (Thm. 4.1). **(tg: Adj)**

Andrew V. Goldberg and Alexander V. Karzanov

1994a Path problems in skew-symmetric graphs. In: *Proceedings of the 5th annual ACM-SIAM symposium on discrete algorithms* (Arlington, Va., 1994), pp. 526–535. New York, Assoc. Comput. Machinery (ACM), 1994. MR 1285193 (95c:05074). Zbl 867.90118. **(sd: Flows, Cov)**

1996a Path problems in skew-symmetric graphs. *Combinatorica* 16 (1996), no. 3, 353–382. MR 1417346 (97h:05099). Zbl 867.05037. **(sd: Flows, Cov)**

2004a Maximum skew-symmetric flows and matchings. *Math. Program., Ser. A* 100 (2004), no. 3, 537–568. MR 2129927 (2005m:90142). Zbl 1070.90090.

Techniques for digraph flows are extended to bidirected flows, treated via the double covering digraph (*cf.* Tutte 1967a). [Annot. 9 Sept 2010.] **(sg: Ori: Flows, Cov)**

Andrew V. Goldberg, Éva Tardos, and Robert E. Tarjan

1990a Network flow algorithms. In: B. Korte, L. Lovász, H.J. Prömel, and A. Schrijver, eds., *Paths, Flows, and VLSI-Layout*, pp. 101–164. Algorithms and Combinatorics, Vol. 9. Springer-Verlag, Berlin, 1990. MR 1083378 (92i:90043). Zbl 728.90035.

§1.5, "The generalized flow problem": Max flow, conservative except at the source, in networks with (real, positive) gains; generalized augmenting paths. §1.6, "The restricted problem": Flows with gains, conservative except at source and sink, whose residual flow has no gainy cycles that avoid the source. §1.7, "Decomposition theorems" for flows with or without gains. §6, "The generalized flow problem": Combinatorial algorithms; connections between flow problems with and without gains [Annot. 11 Jun 2012.] **(GN: Alg)**

Jay R. Goldman and Louis H. Kauffman

1993a Knots, tangles, and electrical networks. *Adv. Appl. Math.* 14 (1993), 267–306. MR 94m:57013. Zbl 806.57002. Repr. in Louis H. Kauffman, *Knots and Physics*, 2nd edn., pp. 684–723. Ser. Knots Everything, Vol. 1. World Scientific, Singapore, 1993. MR 95i:57010. Zbl 868.57001.

The parametrized Tutte polynomial [as in Zaslavsky (1992b) *et al.*] of an $\mathbb{R}^{\times}$-weighted graph is used to define a two-terminal "conductance". Interpreting weights as crossing signs (± 1) in a planar link diagram with two blocked regions yields invariants of tunnel links. [Also see Kauffman (1997a).] **(SGw: Gen: Invar, Knot, Phys)**

Richard Z. Goldstein and Edward C. Turner

1979a Applications of topological graph theory to group theory. *Math. Z.* 165 (1979), 1–10. MR 80g:20050. Zbl 377.20027, (387.20034). **(SG: Top)**

Eric Goles

See J. Aracena.

Harry F. Gollub

1974a The subject-verb-object approach to social cognition. *Psychological Rev.* 81 (1974), 286–321. **(PsS: vs)**

Martin Charles Golumbic

1979a A generalization of Dirac's theorem on triangulated graphs. In: Allan Gewirtz and Louis V. Quintas, eds., Second Int. Conf. on Combinatorial Mathematics (New York, 1978). *Ann. New York Acad. Sci.* 319 (1979), 242–246. MR 81c:05077. Zbl 479.05055.

Further results on chordal bipartite graphs. Their properties imply standard properties of ordinary chordal graphs. [See (1980a) for more.] (The "only if" portion of Thm. 4 is false, according to (1980a), p. 267.) **(sg: bal, cov)**

1980a *Algorithmic Graph Theory and Perfect Graphs.* Academic Press, New York, 1980. MR 81e:68081. Zbl 541.05054.

§12.3: "Perfect elimination bipartite graphs," and §12.4: "Chordal bipartite graphs," expound perfect elimination and chordality for bipartite graphs from Golumbic and Goss (1978a) and Golumbic (1979a). In particular, Cor. 12.11: A bipartite graph is chordal bipartite iff every induced subgraph has perfect edge elimination scheme. [*Problem.* Guided by these results, find a signed-graph generalization of chordality that corresponds to supersolvability and perfect vertex elimination (*cf.* Zaslavsky (2001a)).] (**sg: bal, cov**)

Martin Charles Golumbic and Clinton F. Goss

1978a Perfect elimination and chordal bipartite graphs. *J. Graph Theory* 2 (1978), 155–163. MR 80d:05037. Zbl 411.05060.

A perfect edge elimination scheme is a bipartite analog of a perfect vertex elimination scheme. A chordal bipartite graph is a bipartite graph in which every circle longer than 4 edges has a chord. Analogs of properties of chordal graphs, e.g., Dirac's separator theorem, are proved. In particular, a chordal bipartite graph has a perfect edge elimination scheme. [See Golumbic (1980a) for more.] (**sg: bal**)

Sergio Gómez, Pablo Jensen, and Alex Arenas

2009a Analysis of community structure in networks of correlated data. *Phys. Rev. E* 80 (2009), no. 1, 016114. arXiv:0812.3030.

Cf. Bansal, Blum, and Chawla (2004a), *et al.* (**SG, WG: Clu**)

Shicai Gong, Hangen Duan, and Yizheng Fan

2006a On eigenvalues distribution of mixed graphs. *J. Math. Study* 39 (2006), no. 2, 124–128. MR 2248100 (2007b:05136). Zbl 1104.05045. (**sg: Adj**)

Shi-Cai Gong and Yi-Zheng Fan

2007a Nonsingular unicyclic mixed graphs with at most three eigenvalues greater than two. *Discuss. Math. Graph Theory* 27 (2007), no. 1, 69–82. MR 2321423 (2008b:05102). Zbl 1139.05033.

Dictionary: See X.-D. Zhang and Li (2002a). [Annot. 23 Mar 2009.] (**sg: incid, Adj**)

Shi-Cai Gong and Guang-Hui Xu

2012a The characteristic polynomial and the matchings polynomial of a weighted oriented graph. *Linear Algebra Appl.* 436 (2012), no. 9, 3597–3607.

A "weighted oriented graph" is an $\mathbb{R}^+$-gain graph. The "skew adjacency matrix" is the gain-graphic adjacency matrix. [Annot. 7 Feb 2012.] (**gg: CAdj**)

Mauricio Gonzaĺez

See J. Aracena.

Gary Gordon

See also L. Fern.

1997a Hyperplane arrangements, hypercubes and mixed graphs. Proc. Twenty-eighth Southeastern Int. Conf. on Combinatorics, Graph Theory and Computing (Boca Raton, Fla., 1997). *Congressus Numer.* 126 (1997), 65–72. MR 98j:05038. Zbl 901.05055.

An explicit bijection between the regions of the real hyperplane arrangement corresponding to $\pm K_n^\circ$ and the set of "good signed [complete] mixed graphs" $G_{\mathbf{a}}$ of order n. The latter are a notational variant of the acyclic orientations τ of $\pm K_n^\circ$ [and are therefore in bijective correspondence with the regions, by Zaslavsky (1991b), Thm. 4.4]; the dictionary is: a directed edge in $G_{\mathbf{a}}$ is an oriented positive edge in τ, while a positive or negative undirected edge in $G_{\mathbf{a}}$ is an introverted or extroverted negative edge of τ. The main result, Thm. 1, is an interesting and significant

explicit description of the acyclic orientations of $\pm K_n^\circ$. Namely, one orders the vertices and directs all positive edges upward; then one steps inward randomly from both ends of the ordered vertex set, one vertex at a time, at each new vertex orienting all previously unoriented negative edges to be introverted if the vertex was approached from below, extroverted if from above in the vertex ordering. [This clearly guarantees acyclicity.] [*Problem.* Generalize to arbitrary signed graphs.]

Lemma 2, "a standard exercise", is that an orientation of $\pm K_n^\circ$ (with the loops replaced by half edges) is acyclic iff the magnitudes of its net degrees are a permutation of $\{1, 3, \ldots, 2n-1]\}$. [Similarly, an orientation of $\pm K_n^\circ$ is acyclic iff its net degree vector is a signed permutation of $\{2, 4, \ldots, 2n\}$ (Zaslavsky (1991b), p. 369, but possibly known beforehand in other terminology). Both follow easily from Zaslavsky (1991b), Cor. 5.3: an acyclic orientation has a vertex that is a source or sink.] (**SG: ori: incid, Geom**)

1999a The answer is $2^n \cdot n!$ What's the question? *Amer. Math. Monthly* 106 (Aug.–Sept., 1999), no. 7, 636–645. MR 2000j:05050. Zbl 982.05052.

§5 presents the signed-graph question: an appealing presentation of material from (1997a). (**SG: ori, Incid, Geom, N: Exp**)

Y. Gordon and H.S. Witsenhausen

1972a On extensions of the Gale–Berlekamp switching problem and constants of l_p-spaces. *Israel J. Math.* 11 (1972), 216–229. MR 46 #3213. Zbl 238.46009.

Asymptotic estimates of $l(K_{r,s})$, the maximum frustration index of signatures of $K_{r,s}$, improving the bounds of Brown and Spencer (1971a). (**sg: Fr**)

Clinton F. Goss

See M.C. Golumbic.

Eric Gottlieb

2003a On the homology of the h, k-equal Dowling lattice. *SIAM J. Discrete Math.* 17 (2003), no. 1, 50–71. MR 2004k:05209. Zbl 1033.05098.

The lattice is the subposet of $\mathrm{Lat}\, G(\mathfrak{G}K_n)$ consisting of the flats whose nontrivial balanced components have order $\geq k$ and whose unbalanced component, if any, has order $\geq h$. If $|\mathfrak{G}| = 2$ and $h \leq k$ we have the lattice of Björner and Sagan (1996a). (**gg: M: Invar**)

Eric Gottlieb and Michelle L. Wachs

2000a Cohomology of Dowling lattices and Lie (super)algebras. *Adv. in Appl. Math.* 24 (2000), no. 4, 301–336. MR 2001i:05161. Zbl 1026.05104.

Two monomorphisms of the cohomology of the order complex of the lattice of flats of $Q_n(\mathfrak{G})$, upon which $\mathfrak{S}_n \wr \mathfrak{G}$ acts as operators, into enveloping algebras of certain Lie algebras and Lie superalgebras. (**gg: M: Invar**)

Ian P. Goulden, Jin Ho Kwak, and Jaeun Lee

2005a Enumerating branched orientable surface coverings over a non-orientable surface. *Discrete Math.* 303 (2005), 42–55. MR 2181041 (2006i:05089). Zbl 1079.05025. (**SG: Cov, Top, gg**)

R.L. Graham and N.J.A. Sloane

1985a On the covering radius of codes. *IEEE Trans. Inform. Theory* IT-31 (1985), 385–401. MR 87c:94048. Zbl 585.94012.

See Example b, p. 396 (the Gale–Berlekamp code). (**sg: Fr**)

Ante Graovac, Ivan Gutman, and Nenad Trinajstić

1977a *Topological Approach to the Chemistry of Conjugated Molecules*. Lect. Notes in Chem., Vol. 4. Springer-Verlag, Berlin, 1977. Zbl 385.05032.

§2.7. "Extension of graph-theoretical considerations to Mobius systems." (**SG: Adj, Chem**)

A. Graovac and N. Trinajstić

1975a Mobius molecules and graphs. *Croatica Chemica Acta (Zagreb)* 47 (1975), 95–104. (**SG: Adj, Chem**)

1976a Graphical description of Möbius molecules. *J. Molecular Structure* 30 (1976), 416–420.

The "Möbius graph" (i.e., signed graph of a suitably twisted ring hydrocarbon) is introduced with examples of the adjacency matrix and characteristic polynomial. (**Chem: SG: Adj**)

John G. del Greco

See del Greco (under 'D').

F. Green

1987a More about NP-completeness in the frustration model. *OR Spektrum* 9 (1987), 161–165. MR 88m:90053. Zbl 625.90070.

Proves polynomial time for the reduction employed in Bachas (1984a) and improves the theorem to: The frustration-index decision problem on signed (3-dimensional) cubic lattice graphs with 9 layers is NP-complete. [2 layers, in Barahona (1982a).] (**SG: Fr: Alg**)

Jan Green-Krótki

See J. Aráoz.

Harvey J. Greenberg, J. Richard Lundgren, and John S. Maybee

1983a Rectangular matrices and signed graphs. *SIAM J. Algebraic Discrete Methods* 4 (1983), 50–61. MR 84m:05052. Zbl 525.05045.

From a matrix B, with row set R and column set C, form the "signed bipartite graph" BG^+ with vertex set $R \cup C$ and an edge $r_i c_k$ signed $\operatorname{sgn} b_{ik}$ whenever $b_{ik} \neq 0$. The "signed row graph" RG^+ is the two-step signed graph of BG^+ on vertex set R: that is, $r_i r_j$ is an edge if $\operatorname{dist}^{BG^+}(r_i, r_j) = 2$ and its sign is the sign of any shortest $r_i r_j$-path. If some edge has ill-defined sign, RG^+ is undefined. The "signed column graph" CG^+ is similar. The paper develops simple criteria for existence and balance of these graphs and the connection to matrix properties. It examines simple special forms of B. (**QM: SG, Bal, Appl**)

1984a Signed graphs of netforms. Proc. Fifteenth Southeastern Conf. on Combinatorics, Graph Theory and Computing. *Congressus Numer.* 44 (1984), 105–115. MR 87c:05085. Zbl 557.05048.

Application of (1983a, 1984b). "Netform" = incidence matrix of a positive real gain graph (neglecting a minor technicality). Thm. 1: B is a netform iff $RG^+(B)$ exists and is all negative. (Then $CG^+(B)$ also exists.) Thm. 2: If the row set partitions so that all negative elements are in some rows and all positives are in the other rows, then $RG^+(B)$ is all negative and balanced. Thm. 3: If Σ is all negative and balanced, then B exists as in Thm. 2 with $RG^+(B) = \Sigma$. [Equivalent to theorem of Hoffman and Gale (1956a).] B is an "inverse" of Σ. Thm. 4 concerns

"inverting" $-\Gamma$ in a minimal way. Then B will be (essentially) the incidence matrix of $+\Gamma$. (**SG, gg: incid, Bal, VS, Exp, Appl**)

1984b Inverting signed graphs. *SIAM J. Algebraic Discrete Methods* 5 (1984), 216–223. MR 86d:05085. Zbl 581.05052.

See (1983a). "Inversion" means, given a signed graph Σ_R, or Σ_R and Σ_C, finding a matrix B such that $\Sigma_R = RG^+(B)$, or $\Sigma_R = RG^+(B)$ and $\Sigma_C = CG^+(B)$. The elementary solution is in terms of coverings of Σ_R by balanced cliques. It may be desirable to minimize the size of the balanced clique cover; this difficult problem is not tackled. (**QM: SG, VS, Bal**)

Harvey J. Greenberg and John S. Maybee, eds.

1981a *Computer-Assisted Analysis and Model Simplification* (Proc. First Sympos., Univ. of Colorado, Boulder, Col., 1980). Academic Press, New York, 1981. MR 82g:00016. Zbl 495.93001.

Several articles relevant to signed (di)graphs. (**QM**)(**SD, SG: Bal**)

Curtis Greene and Thomas Zaslavsky

1983a On the interpretation of Whitney numbers through arrangements of hyperplanes, zonotopes, non-Radon partitions, and orientations of graphs. *Trans. Amer. Math. Soc.* 280 (1983), 97–126. MR 84k:05032. Zbl 539.05024.

§9: "Acyclic orientations of signed graphs." Continuation of Zaslavsky (1991b), counting acyclic orientations with specified unique source; also, with edge e having specified orientation and with no termini except at the ends of e. The proof is geometric. (**SG: M, Ori, Geom, Invar**)

David A. Gregory, Kevin N. Vandermeulen, and Bryan L. Shader

1996a Rank decompositions and signed bigraphs. *Linear Multilinear Algebra* 40 (1996), 283–301. MR 1384648 (97a:05147). Zbl 866.05042.

For bipartite Σ, $\mathcal{M} :=$ class of matrices with weak sign pattern Σ. Every $A \in \mathcal{M}$ is the sum of $\operatorname{rk} A$ rank-1 matrices in $\mathcal{M}$ iff (*) $\sigma(C) = -(-1)^{|C|/2}$ for every circle with $|C| \geq 6$. Thm. 3.2: Σ has (*) for every circle iff it is a spanning subgraph of a signed 4-cockade. Thm. 3.7. Σ has (*) for circles with $|C| \geq 6$ iff, after switching, it is obtained by three constructions from a negative C_4, a subgraph of $+K_{3,n}$, or a signed graph R_n. [Annot. 6 Mar 2011.] (**SG: QM, Circles**)

Gary S. Grest

See also D. Blankschtein.

1985a Fully and partially frustrated simple cubic Ising models: a Monte Carlo study. *J. Phys. C* 18 (1985), 6239–6246.

Simulation of the cubic signed graph of Blankschtein, M. Ma, and A. Nihat Berker (1984a). [Annot. 18 Jun 2012.] (**Phys, SG: Fr**)

G. Grimmett

1994a The random-cluster model. In: F.P. Kelly, ed., *Probability, Statistics and Optimisation*, Ch. 3, pp. 49–63. Wiley, Chichester, 1994. MR 96d:60154. Zbl 858.60093.

Reviews Fortuin and Kasteleyn (1972a) and subsequent developments esp. in multidimensional lattices. The viewpoint is mainly probabilistic and asymptotic. §3.7, "Historical observations," reports Kasteleyn's account of the origin of the model. (**sgc: Gen: Invar, Phys: Exp**)

Ya.R. Grinberg and A.M. Rappoport

2011a Configuration and minimal coloring of disbalanced graphs. (In Russian.) *Dok-*

lady Akad. Nauk 439 (2011), no. 6, 743–745. Zbl 1238.05118.
See (2011b). **(SG: Fr, Clu)**

2011b Configuration and minimal coloring of disbalanced graphs. *Doklady Math.* 84 (2011), no. 1, 579–581. Zbl 1238.05118.

Contrabalanced signed graphs are characterized. [Easy.] Dictionary: "disbalance" = contrabalance, "junction" = cutpoint, "cyclically splittable" = every block is a circle, "p-groupable" = p-clusterable. [Annot. 9 Jun 2012.] **(SG: Fr, Clu)**

Richard C. Grinold

1973a Calculating maximal flows in a network with positive gains. *Operations Res.* 21 (1973), 528–541. MR 50 #3900. Zbl 304.90043.

Objective: to find the maximum output for given input. Basic solutions correspond to bases of $G(\Phi')$, Φ' being the underlying gain graph Φ together with an unbalanced loop adjoined to the sink. Onaga (1967a) also treats this problem. **(GN: M(bases), Alg)**

Heinz Gröflin and Thomas M. Liebling

1981a Connected and alternating vectors: polyhedra and algorithms. *Math. Programming* 20 (1981), 233–244. MR 83k:90061. Zbl 448.90035. **(sg, Geom)**

Piotr Gronek

See P. Gawroński and K. Kułakowski.

Jonathan L. Gross

See also J. Chen.

1974a Voltage graphs. *Discrete Math.* 9 (1974), 239–246. MR 50 #153. Zbl 286.05106. **(GG: Top, Cov)**

Jonathan L. Gross and Thomas W. Tucker

1977a Generating all graph coverings by permutation voltage assignments. *Discrete Math.* 18 (1977), 273–283. MR 57 #5803. Zbl 375.55001. **(GG: Top, Cov)**

1979a Fast computations in voltage graph theory. In: Allan Gewirtz and Louis V. Quintas, eds., Second Int. Conf. on Combinatorial Mathematics (New York, 1978). *Ann. New York Acad. Sci.* 319 (1979), 247–253. MR 80m:94111. Zbl 486.05027.
(GG: Top, Cov, Sw)

1987a *Topological Graph Theory.* Wiley, New York, 1987. MR 88h:05034. Zbl 621.05013. Repr. with minor additions: Dover Publications, Mineola, N.Y., 2001. MR 1855951. Zbl 991.05001.

Ch. 2: "Voltage graphs and covering spaces." Ch. 4: "Imbedded voltage graphs and current graphs." **(GG: Top, Cov)**

§3.2.2: "Orientability." §3.2.3: "Rotation systems." §4.4.5: "Nonorientable current graphs", discusses how to deduce, from the signs on a current graph, the signs of the "derived" graph of the dual voltage graph. [The same rule gives the signs on the surface dual of any orientation-embedded signed graph.] (The sign group here is $\mathbb{Z}_2$.) **(SG: Top)**

Jerrold W. Grossman

See also R.B. Bapat.

Jerrold W. Grossman and Roland Häggkvist

1983a Alternating cycles in edge-partitioned graphs. *J. Combin. Theory Ser. B* 34 (1983), 77–81. MR 84h:05044. Zbl 491.05039, (506.05040).

They prove the special case in which B is all negative of the following generalization, which is an immediate consequence of their result. [*Theorem.* If B is a bidirected graph such that for each vertex v there is a block of B in which v is neither a source nor a sink, then B contains a coherent circle. ("Coherent" means that at each vertex, one edge is directed inward and the other outward.)] (**par: ori**)

Jerrold W. Grossman, Devadatta M. Kulkarni, and Irwin E. Schochetman

1994a Algebraic graph theory without orientation. *Linear Algebra Appl.* 212/213 (1994), 289–307. MR 96b:05111. Zbl 817.05047.

Topics: The unoriented incidence matrix of Γ [which equals the incidence matrix $\mathrm{H}(-\Gamma)$], the Kirchoff or "Laplacian" matrix of $-\Gamma$, the even-cycle ("even circuit") matroid $G(-\Gamma)$, a partial all-minors matrix-tree theorem [completed in Bapat, Grossman, and Kulkarni (1999a)]. [This part is not new. See van Nuffelen (1973a) for rank($\mathrm{H}(-\Gamma)$); Zaslavsky (1982a), §8 for both matrices; Tutte (1981a), Doob (1973a), and Simões-Pereira (1973a) for the matroid; Chaiken (1982a) for the whole matrix-tree theorem.]

§§4, 5: Vector spaces associated with $G(-\Gamma)$ and its dual, expressed both combinatorially in terms of vectors associated with matroid circuits and cocircuits (of two kinds) and as null and row spaces of $\mathrm{H}(-\Gamma)$ and $\mathrm{H}(-\Gamma)^{\mathrm{T}}$. E.g., in §5 is the all-negative case of: A basis for $\mathrm{Nul}\,\mathrm{H}(\Sigma)^{\mathrm{T}}$ consists of one switching function positivizing each balanced component of Σ. [The viewpoint, going from matroids to vector spaces over fields, usually with characteristic $\neq 2$, contrasts sharply with that of Tutte (1981a), who starts with integral chain groups ($\mathbb{Z}$-modules) and ends with chain-group properties and matroids. This is the only thorough development I know of vector spaces of a signed graph before Chen and Wang (2009a), despite some aspects' having appeared e.g. in Bolker (1977a, 1979a) and Tutte (1981a). It will be still more valuable if it is extended to $\mathbb{R}^{\times}$-gain graphs and to $F^{\times}$-gain graphs for any field F.]

Dictionary: $M = \mathrm{H}(-\Gamma)$; "k-reduced spanning substructure" $\cong$ independent set of rank $n-k$ in $G(-\Gamma)$; "quasi edge cut" = balancing set; "quasibond" = minimal balancing set; "even circuit" = positive closed walk; "bowtie" = contrabalanced handcuff; "marimba stick" = half edge. (**EC, par: Incid, Bal, D**)

1995a On the minors of an incidence matrix and its Smith normal form. *Linear Algebra Appl.* 218 (1995), 213–224. MR 95m:15020. Zbl 819.05043.

Rank of the unoriented incidence matrix of Γ (which equals $\mathrm{H}(-\Gamma)$) [as in van Nuffelen (1973a)]. Finds all possible values of determinants of minors of $\mathrm{H}(-\Gamma)$ [repeating and refining Zaslavsky (1982a), §8A] and of maximal nonsingular minors. Consequences are the Smith normal form of $\mathrm{H}(-\Gamma)$ (§3) and the total integrality of some integer programs with $\mathrm{H}(-\Gamma)$ as coefficient matrix. (**par: Incid, ec, Geom**)

Martin Grötschel

See also F. Barahona.

M. Grötschel, M. Jünger, and G. Reinelt

1987a Calculating exact ground states of spin glasses: a polyhedral approach. In: J.L. van Hemmen and I. Morgenstern, eds., *Heidelberg Colloquium on Glassy*

Dynamics (Proc., 1986), pp. 325–353. Lect. Notes in Physics, Vol. 275. Springer-Verlag, Berlin, 1987. MR 88g:82002 (book).

§2, "The spin glass model": finding the weighted frustration index in a weighted signed graph (Σ, w), or finding a ground state in the corresponding Ising model, is equivalent to the weighted max-cut problem in $(-\Sigma, w)$. This article concerns finding the exact weighted frustration index. §3, "Complexity", describes previous results on NP-completeness and polynomial-time solvability. §4, "Exact methods", discusses previous solution methods. §5, "Polyhedral combinatorics", shows that finding weighted frustration index is a linear program on the cut polytope; also expounds related work. The remainder of the paper concerns a specific cutting-plane method suggested by the polyhedral combinatorics.

(**sg: fr(gen): Alg, G, Ref**)(**Phys, Ref: Exp**)

Martin Grötschel, László Lovász, and Alexander Schrijver

1988a *Geometric Algorithms and Combinatorial Optimization.* Algorithms and Combin. Vol. 2. Springer-Verlag, Berlin, 1988. MR 89m:90135. Zbl 634.05001.

Ch. 8, "Combinatorial optimization: A tour d'horizon": Topics mentioned include odd cycles, maximum-gain flow, odd cuts.

(**par, gg: Cycles, Alg**)

1993a *Geometric Algorithms and Combinatorial Optimization.* Second corrected ed. Algorithms and Combin., Vol. 2. Springer-Verlag, Berlin, 1993. MR 95e:90001. Zbl 837.05001.

Essentially the same as (1988a). (**par, gg: Cycles, Alg**)

M. Grötschel and W.R. Pulleyblank

1981a Weakly bipartite graphs and the max-cut problem. *Operations Res. Letters* 1 (1981/82), 23–27. MR 83e:05048. Zbl 478.05039, 494.90078.

Includes a polynomial-time algorithm, which they attribute to "Waterloo folklore", for shortest (more generally, min-weight) even or odd path, hence (in an obvious way) odd or even circle. [Attributed by Thomassen (1985a) to Edmonds (unpublished). Adapts to signed graphs by the negative subdivision trick: Subdivide each positive edge of Σ into two negative edges, each with half the weight. The min-weight algorithm applied to the subdivision finds a min-weight (e.g., a shortest) negative circle of Σ.] [This paper is very easy to understand. It is one of the best written I know.] [Weakly bipartite graphs are certain signed graphs. Further work: Barahona, Grötschel, and Mahjoub (1985a), Polyak and Tuza (1995a), and esp. Guenin (1998a, 2001a).]

(**par: Alg, Geom, Paths, Circles**)(**sg: Geom**)

Victor Guba and Mark Sapir

1997a *Diagram Groups.* Mem. Amer. Math. Soc., vol. 130 (1997), no. 620. MR 98f:20013. Zbl 930.20033.

The "labelled oriented graph" (pp. 12–13) is a gain graph with a gain semigroup (instead of group) which is the semigroup generated by an alphabet and its inverse. (**gg: Gen**)

Bertrand Guenin

See also G. Cornuéjols and J.F. Geelen.

1998a *On Packing and Covering Polyhedra.* Ph.D. dissertation, Grad. Sch. Industrial Engin., Carnegie–Mellon Univ., 1998. (**SG: Geom**)(**Sgnd(M): Geom**)

1998b A characterization of weakly bipartite graphs. In: Robert E. Bixby, E. Andrew Boyd, and Roger Z. Ríos-Mercado, eds., *Integer Programming and Combinatorial Optimization* (6th Int. IPCO Conf., Houston, 1998, Proc.), pp. 9–22. Lect. Notes in Computer Sci., Vol. 1412. Springer, Berlin, 1998. MR 2000i:05158. Zbl 909.90264.

Outline of (2001a). (**SG: Geom**)

†2001a A characterization of weakly bipartite graphs. *J. Combin. Theory Ser. B* 83 (2001), 112–168. MR 2002h:05145. Zbl 1030.05103.

Σ is "weakly bipartite" (Grötschel and Pulleyblank 1981a) if its clutter of negative circles is ideal (i.e., has the "weak MFMC" property of Seymour 1977a). [This is a polyhedral property that can be equivalently stated: Define a "negative circle cover" to be an edge multiset that intersects every negative circle, and a "weighted negative circle cover" to be an edge weighting by nonnegative real numbers such that the total weight of each negative circle is at least 1. Weak biparticity means that, for every linear functional $f : E \to \mathbb{R}$, the minimum value over all weighted negative circle covers is attained by a negative circle cover.] Thm.: Σ is weakly bipartite iff it has no $-K_5$ minor. This proves part of Seymour's conjecture (1981a) (see Cornuéjols 2001a). [Short proof: Schrijver (2002a).] Dictionary: "odd" = negative, "even" = positive.

(**SG: Geom, Str**)

2001b Integral polyhedra related to even cycle and even cut matroids. In: Karen Aardal, ed., *Integer Programming and Combinatorial Optimization* (8th Int. IPCO Conf., Utrecht, 2001). Lect. Notes in Computer Sci., Vol. 2081, 196–209. Springer, Berlin, 2001. MR 1939172 (2003j:90090). Zbl 1010.90088.

(**sg: Par: M, Geom**)

2002a Integral polyhedra related to even-cycle and even-cut matroids. *Math. Operations Res.* 27 (2002), no. 4, 693–710. MR 2003j:90090. Zbl 1082.90584.

In Σ distinguish a negative link e_{st}. An "unbalanced port" is $C \setminus e_{st}$ where C is an unbalanced circuit of $L(\Sigma)$ that contains e_{st}. Replace "negative circle" by "negative port" in the definition of (2001a). Thm.: The minimum value over all weighted unbalanced port covers is attained by an unbalanced port cover, iff Σ has no $-K_5$ minor and $L(\Sigma)$ has no F_7^* minor. [The latter can be replaced by: Σ has no $(\pm C_4 \setminus \text{edge})$ minor, by Zaslavsky (1990a).] Dictionary: "odd *st*-walk" = unbalanced port.

(**SG: Geom, Str**)

Bertrand Guenin, Irene Pivotto, and Paul Wollan

20xxa Isomorphism for even cycle matroids – I. Submitted. arXiv:1109.2978.

(**sg: Par: M**)

N. Gülpinar, G. Gutin, G. Mitra, and A. Zverovitch

2004a Extracting pure network submatrices in linear programs using signed graphs. *Discrete Appl. Math.* 137 (2004), no. 3, 359–372. MR 2004k:90145. Zbl 1095.-90112.

Problem: Finding a largest embedded network matrix (up to "reflection" = row negation). Given a $0, \pm 1$-matrix AS, let Σ have for vertices the rows of A, with an edge εe_{ij} iff $\operatorname{sgn}(a_{ik}a_{jk}) = -\varepsilon$ in the k-th column for some k. Let $\alpha :=$ maximum size of a stable set in a graph. Thm.: The maximum height of a reflected network submatrix of

A equals $\max_\eta \alpha((\Sigma^X)_-)$ over all switchings of Σ. This implies a heuristic algorithm for finding a large embedded network matrix. [Annot. 30 Sept 2009.] (**SG: incid: Bal, Alg**)f

Ji Ming Guo

See L. Feng, S.W. Tan, and X.L. Wu.

Jiong Guo, Hannes Moser, and Rolf Niedermeier

2009a Iterative compression for exactly solving NP-hard minimization problems. In: J. Lerner, D. Wagner, and K.A. Zweig, eds., *Algorithmics of Large and Complex Networks: Design, Analysis, and Simulation*, pp. 65–80. Lect. Notes in Computer Sci., Vol. 5515. Springer, Berlin, 2009.

Iterative compression results in vast speed-up for, e.g., Graph Bipartization and Signed Graph Balancing (§3.1). *Cf.* Hüffner, Betzler, and Niedermeier (2007a). [Annot. 6 Feb 2011.] (**SG: Fr: Alg**)

G. Gupta

See F. Harary.

Venkatesan Guruswami

See M. Charikar and I. Giotis.

Gregory Gutin

See also J. Bang-Jensen and N. Gülpinar.

G. Gutin and D. Karapetyan

2009a A selection of useful theoretical tools for the design and analysis of optimization heuristics. *Memetic Computing* 1 (2009), 25–34.

§2.1, "Preprocessing in linear programming": Exposition of Gülpinar, Gutin, Mitra, and Zverovitch (2004a). [Annot. 30 Sept 2009.] (**SG: incid, Bal, Alg: Exp**)

Gregory Gutin, Daniel Karapetyan, and Igor Razgon

2009a Fixed-parameter algorithms in analysis of heuristics for extracting networks in linear programs. In: J. Chen and F.V. Fomin, eds., *Parameterized and Exact Computation* (4th Int. Workshop, IWPEC 2009, Copenhagen), pp. 222–233. Lect. Notes in Computer Sci., Vol. 5917. Springer, Berlin, 2009. MR 2773945. (**SG: Fr, Sw, Alg**)

Gregory Gutin, Benjamin Sudakov, and Anders Yeo

1998a Note on alternating directed cycles. *Discrete Math.* 191 (1998), 101–107. MR 99d:05050. Zbl 956.05060.

Existence of a coherent circle with alternating colors in a digraph with an edge 2-coloring is NP-complete. However, if the minimum in- and out-degrees of both colors are sufficiently large, such a cycle exists. [This problem generalizes the undirected, edge-2-colored alternating-circle problem, which is a special case of the existence of a bidirected coherent circle—see Bang-Jensen and Gutin (1997a). *Question.* Is this alternating cycle problem also signed-graphic?] (**par: ori: Circles: Gen**)

Ivan Gutman

See also N.M.M. Abreu, D.M. Cvetković, A. Graovac, and S.-L. Lee.

1978a Electronic properties of Möbius systems. *Z. Naturforsch.* 33a (1978), 214–216. MR 58 #8800. (**SG: Adj, Chem**)

1988a Topological analysis of eigenvalues of the adjacency matrices in graph theory: A difficulty with the concept of internal connectivity. *Chem. Phys. Letters* 148

(1988), 93–94.

Points out an ambiguity in the definitions of Lee, Lucchese, and Chu (1987a) in the case of multiple eigenvalues. [See Lee and Gutman (1989a) for the repair.] (**VS, SGw**)

Ivan Gutman, Dariush Kiani, Maryam Mirzakhah, and Bo Zhou

2009a On incidence energy of a graph. *Linear Algebra Appl.* 431 (2009), no. 8, 1223–1233. MR 2547906 (2010k:05174). Zbl 1175.05084.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Ivan Gutman, Shyi-Long Lee, Yeung-Long Luo, and Yeong-Nan Yeh

1994a Net signs of molecular graphs: dependence of molecular structure. *Int. J. Quantum Chem.* 49 (1994), 87–95.

How to compute the balanced signing of Γ that corresponds to eigenvalue λ_i (see Lee, Lucchese, and Chu (1987a)), without computing the eigenvector X_i. Theorem: If $v_r v_s \in E$, then $X_{ir} X_{is} = \sum_P f(P;\lambda_i)$, where $f(P;\lambda) := \varphi(G - V(P);\lambda)/\varphi'(G;\lambda)$, $\varphi(G;\lambda)$ is the characteristic polynomial, and the sum is over all paths connecting v_r and v_s. Hence $\sigma_i(v_r v_s) = \operatorname{sgn}(X_{ir} X_{is})$ is determined. [An interesting theorem. *Questions.* Does it generalize if one replaces Γ by a signed graph, this being the balanced (all-positive) case? In such a generalization. if any, how will σ enter in—by restricting the sum to positive paths, perhaps? What about graphs with real gains, or weights?] (**VS, SGw**)

Ivan Gutman, Shyi-Long Lee, Jeng-Horng Sheu, and Chiuping Li

1995a Predicting the nodal properties of molecular orbitals by means of signed graphs. *Bull. Inst. Chem., Academica Sinica* No. 42 (1995), 25–31.

Points out some difficulties with the method of Lee and Li (1994a). (**VS, SGw, Chem**)

Ivan Gutman, Shyi-Long Lee, and Yeong-Nan Yeh

1992a Net signs and eigenvalues of molecular graphs: some analogies. *Chem. Phys. Letters* 191 (1992), 87–91.

A connected graph Γ has n eigenvalues and n corresponding balanced signings (see Lee, Lucchese, and Chu (1987a)). Let $S_1 \geq S_2 \geq \cdots \geq S_n$ be the net signs of these signings and $m = |E|$. The net signs satisfy analogs of properties of eigenvalues. (A) If $\Delta \subset \Gamma$, then $S_1(\Delta) < S_1$. (B) $S_1 = m \geq S_2 + 2$. (C, D) For bipartite Γ, $S_n = -m$. Otherwise, $S_n \geq -m + 2$. From (B, C, D) we have $|S_i| \leq m - 2$ for all $i \neq 1$ and, if Γ is bipartite, $i \neq n$. (E, F) If Γ is bipartite, then $S_i = -S_{n+1-i}$ and at least $a - b$ net signs equal 0, where $a \geq b$ are the numbers of vertices in the two color classes. The analogy is imperfect, since $S_1 + S_2 + \cdots + S_n \geq 0$, while equality holds for eigenvalues. [*Questions.* Some of these conclusions require Γ to be bipartite. Does that mean that they will generalize to an arbitrary balanced signed graph Σ in place of the bipartite Γ, the eigenvectors being those of Σ? Will the other results generalize with Γ replaced by any signed graph? How about real gains, or weights?] (**VS, SGw**)

Ivan Gutman, María Robbiano, Enide Andrade Martins, Domingos M. Cardoso, Luis Medina, and Oscar Rojo

2010a Energy of line graphs. *Linear Algebra Appl.* 433 (2010), no. 7, 1312–1323. MR 2680258 (2012a:05188). Zbl 1194.05137.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj, Incid, LG**)

Ivan Gutman and Oskar E. Polansky

1986a *Mathematical Concepts in Organic Chemistry.* Springer-Verlag, Berlin, 1986. MR 861119 (87m:92102). Zbl 657.92024.

See pp. 54–55 for eigenvalues of adjacency matrices of positive and negative circles. [Annot. 4 Nov 2010.] (**Chem: Exp: SG: Adj**)

Pavol Gvozdjak and Jozef Širáň

1993a Regular maps from voltage assignments. In: Neil Robertson and Paul Seymour, eds., *Graph Structure Theory* (Proc., Seattle, 1991), pp. 441–454. Contemp. Math., Vol. 147. Amer. Math. Soc., Providence, R.I., 1993. MR 1224722 (94j:05047).

§3, "Voltage assignments and derived maps", defines gain graph and covering graph (and map). §4, "Lifting map automorphisms": A map automorphism lifts iff it preserves the class of identity-gain walks. [Initiates method developed in Nedela and Škoviera (1997b), Malnič, Nedela, and Škoviera (2000a, 2002a), *et al.*] Dictionary: "voltage" = gain, "derived graph" = gain covering graph, "map" = combinatorial definition of embedded graph, "local group" (at a vertex) = fundamental group (at the vertex). [Annot. 11 Jun 2012.] (**GG: Aut, Cov, Top**)

A. Gyárfás

See P. Erdős.

Ervin Győri

See also P. Erdős.

Ervin Györi, Alexandr V. Kostochka, and Tomasz Łuczak

1997a Graphs without short odd cycles are nearly bipartite. *Discrete Math.* 163 (1997), 279–284. MR 97g:05203. Zbl 871.05040.

Given all-negative Σ and positive ρ, suppose every odd circle has length $\geq n/\rho$. Then Σ has frustration index $\leq 200\rho^2(\ln(10\rho))^2$ (best possible up to a constant factor) and vertex deletion number $\leq 15\rho\ln(10\rho)$ (best possible up to a logarithmic factor). The proof is based on an interesting, refining lemma. [*Problem.* Generalize to arbitrary Σ.] (**sg: Par: Fr**)

M. Hachimori and M. Nakamura

2007a A factorization theorem of characteristic polynomials of convex geometries. *Ann. Combin.* 11 (2007), 39–46. MR 2311929 (2008b:52001). Zbl 1110.06006 .

Signed graph coloring is mentioned as an example. [Annot. 10 Mar 2011.] (**SG: Invar: Exp**)

Willem H. Haemers

See also A.E. Brouwer and E.R. van Dam.

W.H. Haemers and G.R. Omidi

2011a Universal adjacency matrices with two eigenvalues. *Linear Algebra Appl.* 435 (2011), no. 10, 2520–2529. MR 2811135 (2012e:05230). Zbl 1221.05233.

(**sg: Par: Adj**)

Willem H. Haemers and Edward Spence

2004a Enumeration of cospectral graphs. *European J. Combin.* 25 (2004), 199–211. MR 2070541 (2005d:05102). Zbl 1033.05070.

"Sign-less Laplacian" $Q(\Gamma) :=$ Kirchhoff matrix $K(-\Gamma) = D(\Gamma)+A(\Gamma)$. $K(-\Gamma)$ seems ($n \leq 11$) to allow fewer cospectral graphs than do $A(\Gamma)$

or $K(\Gamma)$. [Annot. Sept 2010.] **(sg: Par: Adj)**

Jurriaan Hage

See also A. Ehrenfeucht.

1999a The membership problem for switching classes with skew gains. *Fund. Inform.* 39 (1999), 375–387. MR 1823982 (2002b:05071). Zbl 944.68144.

An algorithm to decide whether two skew gain graphs are switching equivalent. **(GG(Gen): Sw, Alg)**

2001a *Structural Aspects of Switching Classes*. Doctoral dissertation, Universiteit Leiden, 2001. IPA Dissertation Ser., UL.2001-8. [Instituut voor Programmatuurkunde en Algoritmiek, 2001.]

Contains the material of the following papers, along with updates and improved results: Ehrenfeucht, Hage, Harju, and Rozenberg (2000a,b, 2006a), Hage (1999a), Hage and Harju (1998a, 2000a, 2004a).

Errata and a downloadable corrected version at http://www.cs.uu.nl/people/jur/2s.html (1/2002).

(TG: Sw, Alg)(GG(Gen): Sw, Alg)

20xxa Subgroup switching of skew gain graphs. *Fund. Inform.* 116 (2012), 111–122.

Skew gains reverse by an involutory antiautomorphism of the gain group (Hage and Harju 2000a). Here switching is restricted by prescribing for each vertex a subgroup from which the switching value may be taken. Properties of ordinary switching generalize, or become more complicated, or become too difficult. Further research is needed. [Annot. 17 Dec, 5 Jan 2011–12.] **(GG: Gen: Sw: Gen)**

Jurriaan Hage and Tero Harju

1998a Acyclicity of switching classes. *European J. Combin.* 19 (1998), 321–327. MR 99d:05051. Zbl 905.05057.

Classifies the switching-equivalent pairs of forests. Thm. 2.2: In a Seidel switching class of graphs there is at most one isomorphism type of tree; and there is at most one tree, with exceptions that are completely classified. Thms. 3.1 and 4.1: In a switching class that contains a disconnected forest there are at most 3 forests (not necessarily isomorphic); the cases in which there are 2 or 3 forests are completely classified. (Almost all are trees plus isolated vertices.) [*Question*. Regarding these results as concerning the negative subgraphs of switchings of signed complete graphs, to what extent do they generalize to switchings of arbitrary signed simple graphs?] [B.D. Acharya (1981a) asked which simple graphs switch to forests, with partial results.] **(TG: Sw)**

2000a The size of switching classes with skew gains. *Discrete Math.* 215 (2000), 81–92. MR 1746450 (2001d:05074). Zbl 949.05039.

Introducing "skew gain graphs", which generalize gain graphs (see Zaslavsky (1989a)) to incorporate dynamic labelled 2-structures (see Ehrenfeucht and Rozenberg). Inversion is replaced by a gain-group antiautomorphism δ of period at most 2. Thus $\varphi(e^{-1}) = \delta(\varphi(e))$, while in switching by τ, one defines $\varphi^{\tau}(e; v, w) = \delta(\tau(v))\varphi(e; v, w)\tau(w)$. The authors find the size of a switching class $[\varphi]$ in terms of the centralizers and/or δ-centralizers of various parts of the image of φ_T, that is, φ switched to be the identity on a spanning tree T. The exact formulas depend on whether Γ is complete, or bipartite, or general, and on the choice of T (the case where $T \cong K_{1,n-1}$ being simplest). (**GG(Gen): Sw**)

2004a A characterization of acyclic switching classes of graphs using forbidden subgraphs. *SIAM J. Discrete Math.* 18 (2004), no. 1, 159–176. MR 2005k:05205. Zbl 1071.05063.

Solves the problem raised by B.D. Acharya (1981a). (**TG: Sw**)

2007a Towards a characterization of bipartite switching classes by means of forbidden subgraphs. *Discuss. Math. Graph Theory* 27 (2007), no. 3, 471–483. MR 2412359 (2009b:05126). Zbl 1142.05042.

Partial results on the forbidden induced subgraphs for graph switching classes with no bipartite member. [Annot. 9 Sept 2010.] (**TG: Sw**)

Jurriaan Hage, Tero Harju, and Elmo Welzl

2002a Euler graphs, triangle-free graphs and bipartite graphs in switching classes. In: *Graph Transformation* (Proc. First Int. Conf., Rome, 2002), pp. 148–160. Lect. Notes in Computer Sci., vol. 2505. Springer-Verlag, London, 2002. MR 2049362. Zbl 1028.68101.

Preliminary version of (2003a). [Annot. 9 Sept 2010.] (**TG: Sw**)

2003a Euler graphs, triangle-free graphs and bipartite graphs in switching classes. Special issue on ICGT 2002. *Fund. Inform.* 58 (2003), no. 1, 23–37. MR 2056589 (2005b:05206). Zbl 1054.05092.

Polynomial-time algorithms for whether a graph switching class contains a triangle-free, or bipartite, or Eulerian, member. (**TG: Sw**)

Per Hage

1979a Graph theory as a structural model in cultural anthropology. *Annual Rev. Anthropology* 8 (1979), 115–136.

"Signed graphs", pp. 120–124. "Structural duality", pp. 132–133. Other examples. [Annot. 2 Aug 2010.] (**SG, PsS: Bal, Fr, Clu: Exp**)

Per Hage and Frank Harary

1983a *Structural Models in Anthropology.* Cambridge Univ. Press, Cambridge, Eng., 1983. MR 86e:92002.

Signed graphs are treated in Ch. 3 and 6, marked graphs in Ch. 6. [Reviewed in Doreian (1985a).] (**SG, PsS: Bal: Exp**)(**VS: Exp**)

1986a Some genuine graph models in anthropology. *J. Graph Theory* 10 (1986), no. 3, 353–361. MR 856121 (87i:92061). Zbl 605.05042. [Annot. 9 Sept 2010.] (**PsS, SG: Exp**)

1987a *Exchange in Oceania.* Routledge and Kegan Paul, London, 1987. (**PsS**)

Roland Häggkvist

See J.W. Grossman.

F.D.M. Haldane

See J. Vannimenus.

Frank J. Hall

See also C.A. Eschenbach.

Frank J. Hall and Zhongshan Li

2007a Sign pattern matrices. In: Leslie Hogben, ed., *Handbook of Linear Algebra*, pp. 33-1–33-21. Discrete Math. Appl. Chapman & Hall/CRC Press, Boca Raton, 2007. MR 2279160 (2007j:15001) (book). Zbl 1122.15001. (**QM: sd**)

Peter Hall

See B. Xiao.

Shahul Hameed K
See also K.A. Germina.

Shahul Hameed K and K.A. Germina
2012a Balance in gain graphs – A spectral analysis. *Linear Algebra Appl.* 436 (2012), no. 5, 1114–1121. Zbl 1236.05096. (**GG: Adj, Bal**)

2012b On composition of signed graphs. *Discuss. Math. Graph Theory* 32 (2012), no. 3, 507–516. (**SG: Adj**)

20xxc Balance in certain gain graph products. Int. Workshop on Set-Valuations, Signed Graphs, Geometry and Their Appl. (IWSSG-2011, Mananthavady, Kerala, 2011). *J. Combin. Inform. Syst. Sci.*, to appear. (**GG, Bal**)

Hasti Hamidzade and Dariush Kiani
2010a Erratum to "The lollipop graph is determined by its Q-spectrum" . *Discrete Math.* 310 (2010), no. 10-11, 1649. MR 2601277 (2011c:05194). Zbl 1210.05080.
Corrected proof of Y.P. Zhang, Liu, Zhang, and Yong (2009a), Thm. 3.3. [Annot. 16 Oct 2011.] (**Par: Adj**)

Peter L. Hammer
See also E. Balas, C. Benzaken, E. Boros, J.-M. Bourjolly, Y. Crama, and A. Fraenkel.

1974a Boolean procedures for bivalent programming. In: P.L. Hammer and G. Zoutendijk, eds., *Mathematical Programming in Theory and Practice* (Proc. NATO Adv. Study Inst., Figueira da Foz, Portugal, 1972), pp. 311–363. North-Holland, Amsterdam, and American Elsevier, New York, 1974. MR 57 #18817. Zbl 335.90034 (book).

1977a Pseudo-Boolean remarks on balanced graphs. In: L. Collatz, G. Meinardus, and W. Wetterling, eds., *Numerische Methoden bei Optimierungsaufgaben, Band 3: Optimierung bei graphentheoretischen und ganzzahligen Problemen* (Tagung, Oberwolfach, 1976), pp. 69–78. Int. Ser. Numer. Math., Vol. 36. Birkhäuser, Basel, 1977. MR 57 #5833. Zbl 405.05054. (**SG: Bal**)

P.L. Hammer, C. Benzaken, and B. Simeone
1980a Graphes de conflit des fonctions pseudo-booleennes quadratiques. In: P. Hansen and D. de Werra, eds., *Regards sur la Theorie des Graphes* (Actes du Colloq., Cerisy, 1980), pp. 165–170. Presses Polytechniques Romandes, Lausanne, Switz., 1980. MR 82d:05054 (book).

P.L. Hammer, T. Ibaraki, and U. Peled
1980a Threshold numbers and threshold completions. In: M. Deza and I.G. Rosenberg, eds., *Combinatorics 79* (Proc. Colloq., Montreal, 1979), Part II. *Ann. Discrete Math.* 9 (1980), 103–106. MR 81k:05092. Zbl 443.05064. (**par: ori**)

1981a Threshold numbers and threshold completions. In: Pierre Hansen, ed., *Studies on Graphs and Discrete Programming* (Proc. Workshop, Brussels, 1979), pp. 125–145. North-Holland Math. Studies, 59. Ann. Discrete Math., 11. North-Holland, Amsterdam, 1981. MR 83m:90062. Zbl 465.00007 (book).
See description of Thm. 8.5.2 in Mahadev and Peled (1995a). (**par: ori**)

P.L. Hammer and N.V.R. Mahadev
1985a Bithreshold graphs. *SIAM J. Algebraic Discrete Methods* 6 (1985), 497–506. MR 86h:05093. Zbl 579.05052.
See description of §8.3 of Mahadev and Peled (1995a). (**SG: Bal: Appl**)

P.L. Hammer, N.V.R. Mahadev, and U.N. Peled

1989a Some properties of 2-threshold graphs. *Networks* 19 (1989), 17–23. MR 89m:-05096. Zbl 671.05059.

A restricted line graph with signed edges is a proof tool. (**SG, LG**)

Peter L. Hammer and Sang Nguyen

1979a A partial order in the solution space of bivalent programs. In: Nicos Christofides, Aristide Mingozzi, Paolo Toth, and Claudio Sandi, eds., *Combinatorial Optimization*, Ch. 4, pp. 93–106. Wiley, Chichester, 1979. MR 82a:90099 (book). Zbl 414.90063. (**sg: ori**)

J. Hammann

See E. Vincent.

Miaomiao Han

See X.Y. Yuan.

Wei Han

See S.Y. Wang.

Phil Hanlon

1984a The characters of the wreath product group acting on the homology groups of the Dowling lattices. *J. Algebra* 91 (1984), 430–463. MR 86j:05046. Zbl 557.20009. (**gg: M: Aut**)

1988a A combinatorial construction of posets that intertwine the independence matroids of B_n and D_n. Manuscript, 1988.

Computes the Möbius functions of posets obtained from Lat $G(\pm K_n^\circ)$ by discarding those flats with unbalanced vertex set in a given lower-hereditary list. Examples include Lat $G(\pm K_n^{(k)})$, the exponent denoting the addition of k negative loops. Generalized and superseded by Hanlon and Zaslavsky (1998a). (**sg: M: Gen: Invar**)

1991a The generalized Dowling lattices. *Trans. Amer. Math. Soc.* 325 (1991), 1–37. MR 91h:06011. Zbl 748.05043.

The lattices are based on a rank, n, a group, and a meet sublattice of the lattice of subgroups of the group. The Dowling lattices are a special case. (**gg: M: Gen: Invar**)

1996a A note on the homology of signed posets. *J. Algebraic Combin.* 5 (1996), 245–250. MR 97f:05194. Zbl 854.06004.

Partial summary of Fischer (1993a). (**Sgnd**)

Phil Hanlon and Thomas Zaslavsky

1998a Tractable partially ordered sets derived from root systems and biased graphs. *Order* 14 (1997–98), 229–257. MR 2000a:06016. Zbl 990.03811.

Computes the characteristic polynomials (Thm. 4.1) and hence the Möbius functions (Cor. 4.4) of posets obtained from Lat $G(\Omega)$, Ω a biased graph, by discarding those flats with unbalanced vertex set in a given lower-hereditary list. Examples include Lat $G(\mathfrak{G}K_n^{(k)})$ where $\mathfrak{G}$ is a finite group, the exponent denoting the addition of k unbalanced loops. The interval structure, existence of a rank function, covering pairs, and other properties of these posets are investigated. There are many open problems. (**GG: M, Gen: Invar, Str, Col**)

Pierre Hansen

See also M. Aouchiche and C.S. Oliveira.

1978a Labelling algorithms for balance in signed graphs. In: *Problèmes Combinatoires et Theorie des Graphes* (Colloq. Int., Orsay, 1976), pp. 215–217. Colloques Int. du CNRS, 260. Editions du C.N.R.S., Paris, 1978. MR 80m:68057. Zbl 413.05060.

§1: Algorithm 1 labels vertices of a signed graph to detect imbalance and a negative circle if one exists. [It is equivalent to switching a maximal forest to all positive and looking for negative edges. Independently discovered by Harary and Kabell (1980a).] §2: Algorithm 2 is the unweighted case of the algorithm of (1984a). Path balance in a signed digraph is discussed. §3: The frustration index of a signed graph is bounded below by the negative-circle packing number, which can be crudely bounded by Alg. 1. (**SG, SD: Bal, Fr: Alg, sw**)

1979a Methods of nonlinear 0–1 programming. In: P.L. Hammer, E.L. Johnson, and B.H. Korte, eds., *Discrete Optimization II* (Proc., Banff and Vancouver, 1977), pp. 53–70. Ann. Discrete Math., Vol. 5. North-Holland, Amsterdam, 1979. MR 84h:90034 (book). Zbl 426.90063.

See pp. 58–59. (**SG: Bal: Exp**)

1983a Recognizing sign solvable graphs. *Discrete Appl. Math.* 6 (1983), 237–241. MR 84i:68112. Zbl 524.05048.

Improves the characterization by Maybee (1981a) of sign-solvable digraphs with an eye to more effective algorithmic recognition. Thm. 2.2: A signed digraph D is sign solvable iff its positive subdigraph is acyclic and each strongly connected component has a vertex that is the terminus of no negative, simple directed path. §3: "An algorithm for sign solvability" in time $O(|V|\,|E|)$. (**SD: QSol: Alg**)

1984a Shortest paths in signed graphs. In: A. Burkard *et al.*, eds., *Algebraic Methods in Operations Research*, pp. 201–214. North-Holland Math. Stud., 95. Ann. of Discrete Math., 19. North-Holland, Amsterdam, 1984. MR 86i:05086. Zbl 567.05032.

Algorithm to find shortest walks of each sign from vertex x_1 to each other vertex, in a signed digraph with positive integral(?) weights (i.e., lengths) on the edges. Applied to digraphs with signed vertices and edges; N-balance in signed graphs; sign solvability. The problem for (simple) paths is discussed [which is solvable by any min-weight parity path algorithm; see the notes on Grötschel and Pulleyblank (1981a)].

(**SD, WD: Paths, VS, Bal, QSol: Alg**)

Pierre Hansen and Claire Lucas

2009a An inequality for the signless Laplacian index of a graph using the chromatic number. *Graph Theory Notes N.Y.* 57 (2009), 39–42. MR 2666279 (2011c:05195).

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

2010a Bounds and conjectures for the signless Laplacian index of graphs. *Linear Algebra Appl.* 432 (2010), no. 12, 3319–3336. MR 2639286 (2011m:05173). Zbl 1214.05079.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Pierre Hansen and Bruno Simeone

1986a Unimodular functions. *Discrete Appl. Math.* 14 (1986), 269–281. MR 88a:90138. Zbl 597.90058.

Three types of relatively easily maximizable pseudo-Boolean function ("unimodular" and two others) are defined. For quadratic pseudo-Boolean functions f, the three types coincide; f is unimodular iff an

associated signed graph is balanced (Thm. 3). Thus one can quickly recognize unimodular quadratic functions, although not unimodular functions in general. If the graph is a tree, the function can be maximized in linear time. (**SG: Bal, Alg**)

Christopher R.H. Hanusa

See S. Chaiken.

Rong Xia Hao and Yan Pei Liu

2010a Auxiliary graphs of projective planar signed graphs. (In Chinese.) *J. Systems Sci. Math. Sci.* 30 (2010), no. 9, 1251–1258. MR 2785248 (2012c:05093).

Σ is projective planar iff an auxiliary graph is balanced. [The auxiliary graph may be a tree. It may have order linear in that of Σ.] [Annot. 25 Apr 2012.] (**SG: Top**)

Xiao Hui Hao and Bao Feng Li

2008a The quasi-Laplacian spectral radius of a graph. (In Chinese.) *Math. Practice Theory* 38 (2008), no. 4, 158–160. MR 2435555 (no rev). Zbl 1174.05438. (**Par: Adj**)

Xiao Hui Hao and Li Jun Zhang

2009a The largest eigenvalue of the quasi-Laplacian matrix of a connected graph. (In Chinese.) *Math. Pract. Theory* 39 (2009), no. 7, 178–181. MR 2553871 (no rev). Zbl 1212.05154. (**Par: Adj**)

Frank Harary

See also L.W. Beineke, A. Blass, F. Buckley, D. Cartwright, G. Chartrand, O. Frank, and P. Hage.

††1953a On the notion of balance of a signed graph. *Michigan Math. J.* 2 (1953–1954), 143–146 and addendum preceding p. 1. MR 16, 733. Zbl 56, 421c (e: 056.42103).

Σ The main theorem (Thm. 3) characterizes balanced signings as those for which there is a bipartition of the vertex set such that an edge is positive iff it lies within a part [I call this a Harary bipartition]. Thm. 2: A signing of a simple [or a loop-free] graph is balanced iff, for each pair of vertices, every path joining them has the same sign. The generating function for counting nonisomorphic signed simple graphs with n vertices by numbers of positive and negative edges is $g_n(x+y)$ where $g_n(x)$ is the g.f. of nonisomorphic simple graphs. [The birth of signed graph theory. Although Thm. 3 was anticipated by König (1936a) (Thm. X.11, for finite and infinite graphs) without the terminology of signs, here is the first recognition of the crucial fact that labelling edges by elements of a group—specifically, the sign group—can lead to a general theory.] [Annot. ca. 1977. Rev. 20 Jan 2010.] [See also Whiteley (1991a).] [Annot. 12 Jun 2012.] (**SG: Bal, Enum**)

1955a On local balance and N-balance in signed graphs. *Michigan Math. J.* 3 (1955–1956), 37–41. MR 17, 394. Zbl 70, 185 (e: 070.18502).

Σ is (locally) balanced at a vertex v if every circle on v is positive; then Thm. 3′: Σ is balanced at v iff every block containing v is balanced. Σ is N-balanced if every circle of length $\leq N$ is positive; Thm. 2 concerns characterizing N-balance. Lemma 3: For each circle basis, Σ is balanced iff every circle in the basis is positive. [For finite graphs this strengthens

König (1936a) Thm. 13.] (**SG: Bal**)

1957a Structural duality. *Behavioral Sci.* 2 (1957), 255–265. MR 24B #B851.

"Antithetical duality" (pp. 260–261) introduces antibalance. Remarks on signed and vertex-signed graphs are scattered about the succeeding pages. (**SG: Bal, Par**)

1958a On the number of bi-colored graphs. *Pacific J. Math.* 8 (1958), 743–755. MR 21 #2598. Zbl 84, 194 (e: 084.19402).

§6: "Balanced signed graphs". (**SG: Bal: Enum**)

1959a Graph theoretic methods in the management sciences. *Management Sci.* 5 (1959), 387–403. MR 21 #7103. Repr. in: Samuel Leinhardt, ed., *Social Networks: A Developing Paradigm*, pp. 371–387. Academic Press, New York, 1977.

Pp. 400–401: List of characterizations of balance. (**SG: Bal: Exp**)

1959b On the measurement of structural balance. *Behavioral Sci.* 4 (1959), 316–323. MR 22 #3696.

Proposes to measure imbalance by (i) $\beta(\Sigma)$, the proportion of positive circles ("degree of balance") from Cartwright and Harary (1956a), (ii) the frustration index $l(\Sigma)$ ("line index") [*cf.* Abelson and Rosenberg (1958a)], i.e., the smallest number of edges whose deletion or equivalently (Thm. 7) negation results in balance, and (iii) the vertex frustration number $l_0(\Sigma)$ ("point index"). For β of unbalanced blocks with cyclomatic number ξ: Thm. 4: $\min\beta \leq (\xi-1)/(\xi-1+2^{\xi-1})$. Thm. 5: $\max\beta \geq 1-2/(\xi+1)$ (e.g., a ladder with $\xi+1$ rungs and one rung negative). Cors.: $\min\beta \to 0$, $\max\beta \to 1$ as $\xi \to \infty$. *Conjecture*. The bounds are best possible. [I know of no work on this.] Thm. 6 (contributed by J. Riordan): Asymptotically, $\beta(-K_n) - \frac{1}{2} \sim \frac{1}{2}(-1/e)^n$. [Annot. ≤1980. Rev. 20 Jan 2010.] (**SG: Fr**)

1960a A matrix criterion for structural balance. *Naval Res. Logistics Quarterly* 7, No. 2 (June, 1960), 195–199. Zbl 91, 159 (e: 091.15904).

First explicit appearance of the incidence matrix $\mathrm{H}(\Sigma)$, called J. Thm. 2 (Heller and Tompkins 1956a, Gale and Hoffman 1956a): Σ is balanced iff $\mathrm{H}(\Sigma)$ is totally unimodular. Cor.: The unoriented incidence matrix of Γ is totally unimodular iff Γ is bipartite. [Annot. 10 Nov 2008.] (**SG: Bal, Incid: Exp**)

Thm. 3: A necessary and sufficient condition that a subdeterminant is 0 in $\mathrm{H}(\Sigma)$, provided Σ is balanced. [Zaslavsky (1981a) §8A evaluates subdeterminants for any Σ.] [Annot. 20 Jan 2010.] (**SG: Bal, Incid**)

1970a Graph theory as a structural model in the social sciences. In: Bernard Harris, ed., *Graph Theory and Its Applications*, pp. 1–16. Academic Press, New York, 1970. MR 41 #8277. Zbl 224.05129.

1971a Demiarcs: An atomistic approach to relational systems and group dynamics. *J. Math. Sociology* 1 (1971), 195–205. MR 371738 (51 #7955).

Signed, oriented half edges, applied to represent interpersonal relations. (**PsS: SD**)

1979a Independent discoveries in graph theory. In: Frank Harary, ed., Topics in Graph Theory (Proc. Conf., New York, 1977). *Ann. New York Acad. Sci.* 328 (1979), 1–4. MR 81a:05001. Zbl 465.05026.

1980a Some theorems about graphs from social sciences. In: *Proceedings of the West Coast Conference on Combinatorics, Graph Theory and Computing* (Arcata, Calif., 1979), pp. 41–47. Congressus Numerantium, XXVI. Utilitas Math. Publ. Inc., Winnipeg, Man., 1980. MR 81m:05118. Zbl 442.92027.
(**SG: Bal: History, Exp**)

1981a Structural models and graph theory. In: Harvey J. Greenberg and John S. Maybee, eds., *Computer-Assisted Analysis and Model Simplification* (Proc. Sympos., Boulder, Col., 1980), pp. 31–58. Discussion, pp. 103–111. Academic Press, New York, 1981. MR 82g:00016 (book). Zbl 495.93001 (book).
See remarks of Bixby (p. 111). (**SG, VS, SD: Bal, Alg: Exp**)

1983a Consistency theory is alive and well. *Personality and Social Psychology Bull.* 9 (1983), 60–64.
Historical remarks. E.g., it was Osgood and Tannenbaum (1955a) that inspired Harary to study vertex signings (Beineke and Harary 1978a,b).
(**PsS, SG: Exp**)

1985a The reconstruction conjecture for balanced signed graphs. In: B.R. Alspach and C.D. Godsil, eds., *Cycles in Graphs*, pp. 439–442. Ann. Discrete Math., Vol. 27. North-Holland Math. Stud., Vol. 115. North-Holland, Amsterdam, 1985. MR 87d:05122. Zbl 572.05048.
Reconstruction from the multiset of vertex-deleted subgraphs. Σ^+ is reconstructible if Σ is connected and balanced and not all positive or all negative. (**SG: Bal**)

F. Harary and G. Gupta

1997a Dynamic graph models. *Math. Computer Modelling* 25 (1997), no. 7, 79–87. MR 98b:05092. Zbl 879.68085.
§3.9, "Signed graphs", mentions that deletion index = frustration index (Harary (1959b)). (**SG: Fr: Exp**)

Frank Harary and Jerald A. Kabell

1980a A simple algorithm to detect balance in signed graphs. *Math. Social Sci.* 1 (1980/81), 131–136. MR 81j:05098. Zbl 497.05056.
Equivalent to switching so a spanning tree is all positive, then searching for a negative edge. [Independently discovered by Hansen (1978a).]
(**SG: Bal, Alg**)

1981a Counting balanced signed graphs using marked graphs. *Proc. Edinburgh Math. Soc.* (2) 24 (1981), 99–104. MR 83a:05072. Zbl 476.05043. (**SG, VS: Enum**)

Frank Harary and Helene J. Kommel

1978a Matrix measures for transitivity and balance. *J. Math. Sociology* 6 (1978/79), 199–210. MR 81a:05056. Zbl 408.05028.
§2, "Balance measures in signed graphs": Imbalance in Σ is measured by the proportion of negative triangles, or quadrilaterals, as computed from small powers of $A(\Sigma)$. §3, "Balance in signed digraphs": Similar measures for a signed digraph using digons or triangles. [Annot. 10 Nov 2008.] (**SG: Fr, Adj**)

1979a The graphs with only self-dual signings. *Discrete Math.* 26 (1979), 235–241. MR 80h:05047. Zbl 408.05045. (**SG, VS: Aut**)

Frank Harary, Meng-Hiot Lim, Amit Agarwal, and Donald C. Wunsch

2004a Algorithms for derivation of structurally stable Hamiltonian signed graphs. *Int.*

J. Computer Math. 81 (2004), no. 11, 1349–1356. MR 2172923 (no rev). Zbl 1065.05049.

Thm. 1: The sizes of cuts in K_n. Thm. 2: A subgraph of a balanced signed graph is balanced. [Annot. 12 Sept 2009.] (**SG: Bal**)

Frank Harary, Meng-Hiot Lim, and Donald C. Wunsch

2002a Signed graphs for portfolio analysis in risk management. *IMA J. Management Math.* 13 (2002), no. 3, 201–210. Zbl 1065.91025.

"Assets" (vertices) have positive or negative correlation (edges of K_n). Balance is automatic. Switching is a means of hedging risk, which is highest with all positive edges. Imbalance indicates unpredictability; measured by the proportion of positive triangles. §5: "Balance analysis case study". [Annot. 10 Sept 2009.] (**SG: KG: Appl: Bal, Sw, Exp**)

Frank Harary and Bernt Lindström

1981a On balance in signed matroids. *J. Combin. Inform. System. Sci.* 6 (1981), 123–128. MR 83i:05024. Zbl 474.05021.

Thm. 1: The number of balanced signings of matroid M is $\leq 2^{\mathrm{rk}(M)}$, with equality iff M is binary. Thm. 3: Minimal deletion and negation sets coincide for all signings of M iff M is binary. Thm. 5: For connected binary M, a signing is balanced iff every circuit containing a fixed point is balanced. (**Sgnd: M: Bal, Fr**)

Frank Harary, Bernt Lindström, and Hans-Olov Zetterström

1982a On balance in group graphs. *Networks* 12 (1982), 317–321. MR 84a:05055. Zbl 496.05052.

Implicitly characterizes balance and balancing sets in a gain graph Φ by switching (proof of Thm. 1). [For balance, see also Acharya and Acharya (1986a), Zaslavsky (1977a) and (1989a), Lemma 5.3. For abelian gains, see also Gargano and Quintas (1985a). In retrospect we can see that the characterization of balanced gains is as the 1-coboundaries with values in a group, which for abelian groups is essentially classical.] Thm. 1: The number of balanced gain functions. Thm. 2: Any minimal deletion set is an alteration set. Thm. 3: $l(\Phi) \leq m(1 - |\mathfrak{G}|^{-1})$. Thm. 4: $l(\Sigma) \leq \frac{1}{2}(m - \frac{n-1}{2})$, with strict inequality if not all degrees are even. [Compare with Akiyama, Avis, Chvátal, and Era (1981a), Thm. 1.]

(**GG, SG: sw(Bal), Enum(Bal), Fr**)

Frank Harary, J. Richard Lundgren, and John S. Maybee

1985a On signed digraphs with all cycles negative. *Discrete Appl. Math.* 12 (1985), 155–164. MR 87g:05108. Zbl 586.05019.

Which digraphs D can be signed so that every cycle is negative? Three types of example. Type 1: The vertices can be numbered $1, 2, \ldots, n$ so that the downward arcs are just $(2,1), (3,2), \ldots, (n, n-1)$. (Strong "upper" digraphs; Thm. 2.) Type 2: No cycle is covered by the remaining cycles ("free cyclic" digraphs). This type includes arc-minimal strong digraphs. Type 3: A symmetric digraph, iff the underlying graph Γ is bipartite and no two points on a common circle and in the same color class are joined by a path outside the cycle (Thm. 10; proved by signing Γ via Zaslavsky (1981b)). [Further work in Chaty (1988a).] (**SD: Bal, SG**)

Frank Harary, Robert Z. Norman, and Dorwin Cartwright

1965a *Structural Models: An Introduction to the Theory of Directed Graphs.* Wiley, New York, 1965. MR 32 #2345. Zbl 139, 415 (e: 139.41503).

In Ch. 10, "Acyclic digraphs": "Gradable digraphs", pp. 275–280. That means a digraph whose vertices can be labelled by integers so that $f(w) = f(v) + 1$ for every arc (v, w). [Equivalently, the Hasse diagram of a graded poset.] [Characterized by Topp and Ulatowski (1987a).] (**GD: bal, Exr**)

Ch. 13: "Balance in structures". "Criteria for balance", pp. 340–346 (*cf.* Harary (1953a)); local balance (Harary (1955a)). "Measures of structural balance", pp. 346–352: "degree of balance" (proportion of balanced circles; Cartwright and Harary (1956a)); "line-index for balance" [frustration index] (Abelson and Rosenberg (1958a), Harary (1959b)).

"Limited balance", pp. 352–355. Harary (1955a); also: Adjacency matrix $A(D, \sigma)$ of a signed digraph: entries are $0, \pm 1$. The "valency matrix" is the Abelson–Rosenberg (1958a) adjacency matrix R. Thm. 13.8: Entries of $(R - pI)^l$ show the existence of (undirected) walks of length l of each sign between pairs of vertices. [Strengthened in Zaslavsky (2010b), Thm. 2.1.]

"Cycle-balance and path-balance", pp. 355–358: here directions of arcs are taken into account. E.g., Thm. 13.11: Every cycle is positive iff each strong component is balanced as an undirected graph.
(**SG: Bal, Fr, Adj: Exp, Exr**)(**SD: Bal, Exr**)

1968a *Introduction a la théorie des graphes orientés. Modèles structuraux.* Dunod, Paris, 1968. Zbl 176, 225 (e: 176.22501).

French edition of (1965a). (**GD: bal, Exr**)
(**SG: Bal, Fr, Adj: Exp, Exr**)(**SD: Bal, Exr**)

Frank Harary and Edgar M. Palmer

1967a On the number of balanced signed graphs. *Bull. Math. Biophysics* 29 (1967), 759–765. Zbl 161, 209 (e: 161.20904). (**SG: Bal: Enum**)

1973a *Graphical Enumeration.* Academic Press, New York, 1973. MR 50 #9682. Zbl 266.05108.

Four exercises and a remark concern signed graphs, balanced signed graphs, and signed trees. Russian transl.: Kharari and Palmer (1977a). (**SG: Enum, Bal**)

1977a (As "F. Kharari and È. Palmer") *Perechislenie grafov.* "Mir", Moscow, 1977. MR 56 #5353.

Russian translation of (1973a). (**SG: Enum, Bal**)

Frank Harary, Edgar M. Palmer, Robert W. Robinson, and Allen J. Schwenk

1977a Enumeration of graphs with signed points and lines. *J. Graph Theory* 1 (1977), 295–308. MR 57 #5818. Zbl 379.05035.

See Bender and Canfield (1983a). (**SG, VS: Enum**)

Frank Harary and Michael Plantholt MR 782975 (86h:05056). Zbl 525.05030.

A digraph D gives a signed graph $L_{HP}(D)$ with $V_{HP} := E(D)$ and edges $+ef$ if e, f have the same head, $-ef$ if e, f have the same tail. [The negative part of $\Lambda(+D)$ in Zaslavsky (2010b, 20xxa, 20xxb) with extraverted edges made positive and introverted edges negative.] [Annot. 4 Sept 2010, 17 Jan 2012.] (**SG: LG, Bal**)

Frank Harary and Geert Prins

1959a The number of homeomorphically irreducible trees, and other species. *Acta Math.* 101 (1959), 141–162. MR 21 #653. Zbl 84, 193 (e: 084.19304). (**SG: Enum**)

Frank Harary and Robert W. Robinson

1977a Exposition of the enumeration of point-line-signed graphs enjoying various dualities. In: R.C. Read and C.C. Cadogan, eds., *Proceedings of the Second Carribean Conference in Combinatorics and Computing* (Cave Hill, Barbados, 1977), pp. 19–33. Dept. of Math., Univ. of the West Indies, Cave Hill, Barbados, 1977. (**SG, VS: Enum**)

Frank Harary and Bruce Sagan

1984a Signed posets. In: *Calcutta Mathematical Society Diamond-cum-Platinum Jubilee Commemoration Volume (1908–1983)*, Part I, pp. 3–10. Calcutta Math. Soc., Calcutta, 1984. MR 87k:06003. Zbl 588.05048.

A signed poset is a (finite) partially ordered set P whose Möbius function takes on only values in $\{0, \pm 1\}$. $S(P)$ is the signed graph with $V = P$ and $E_\varepsilon = \{xy : x \leq y \ and \mu(x, y) = \varepsilon 1\}$ for $\varepsilon = +, -$. Some examples are chains, tree posets, and any product of signed posets. Thm. 1 characterizes P such that $|S(P)| \cong H(P)$, the Hasse diagram of P. Thm. 3 characterizes posets for which $S(P)$ is balanced. Thm. 4 gives a sufficient condition for clusterability of $S(P)$. There are many unanswered questions, most basically *Question* 1. Which signed graphs have the form $S(P)$? [See Zelinka (1988a) for a partial answer.] (**SG, Sgnd**)

Frank Harary and Marcello Truzzi

1979a The graph of the zodiac: On the persistence of the quasi-scientific paradigm of astrology. *J. Combin. Inform. System Sci.* 4 (1979), 147–160. MR 82e:00004 (*q.v.*). (**SG: Bal**)

Katsumi Harashima

See H. Kosako.

E. Harburg

See K.O. Price.

Mela Hardin

See M. Beck.

Nadia Hardy

See S. Fiorini.

Tero Harju

See also A. Ehrenfeucht and J. Hage.

2005a Combinatorial models of gene assembly. In: S.B. Cooper, B. Löwe, and L. Torenvliet, eds., *New Computational Paradigms* (First Conf. Computability in Europe, CiE 2005, Amsterdam, 2005), pp. 188–195. Lect. Notes in Computer Sci., Vol. 3526. Springer, Berlin, 2005. Zbl 1113.68400.

A vertex-signed graph (called a "signed graph") encodes the overlap of signed permutations (pp. 190ff.). [Annot. 6 Feb 2011.] (**VS: Alg, Appl**)

2004a Tutorial on DNA computing and graph transformation. In: H. Ehrig *et al.*, eds., *ICGT 2004*, pp. 434–436. Lect. Notes in Computer Sci., Vol. 3256. Springer, Berlin, 2004.

[Vertex-]signed overlap graphs mentioned on p. 436. [Annot. 6 Feb 2011.] (**VS: Alg, Appl**)

Tero Harju, Chang Li, and Ion Petre

2008a Graph theoretic approach to parallel gene assembly. *Discrete Appl. Math.* 156 (2008), no. 18, 3416–3429. MR 2467313 (2010c:92075). Zbl 1200.05238.

See Harju, Li, Petre, and Rozenberg (2005a). The "parallel complexity" of a vertex-signed graph is the minimum number of operations required to reduce it to $\varnothing$. The value for an all-positive or all-negative tree is low (≤ 3 and 2, resp.). *Conjecture.* That of an all-negative graph is ≤ 3. (**VS, Appl**)

2008b Parallel complexity of signed graphs for gene assembly in ciliates. *Soft Computing* 12 (2008), 731–737. Zbl 1137.92305.

See (2008a). The parallel complexity of various examples, e.g., complete tripartite graphs with constant sign (complexity ≤ 3), and an all-positive circle with two negative leaves hanging off each circle vertex (complexity ≤ 4 or 5). (**VS, Appl**)

Tero Harju, Chang Li, Ion Petre, and Gregorz Rozenberg

2005a Parallelism in gene assembly. In: C. Ferretti, G. Mauri, and C. Zandron, eds., *DNA Computing* (Proc. 10th Int. Workshop on DNA Computing, DNA10, Milan, 2004), pp. 138–148. Lect. Notes in Computer Sci., Vol. 3384. Springer, Berlin, 2005. MR 2179032 (no rev). Zbl 1116.68454.

The signs are on vertices. An operation is "local complementation" of a vertex v: in the neighborhood $N(v)$, negate the vertices and complement the edges. Molecular operations formalized for vertex-signed graphs are: (1) deletion of an isolated negative vertex, (2) local complementation of a positive vertex, then deletion of the vertex, (3) a complementation in the neighborhood of two adjacent negative vertices v, w: complement in $N(v) \cup N(w)$, then complement in $N(v) \cap N(w)$. (The paper has a misprint.) The objective is to reduce the graph to $\varnothing$ by these operations, if possible. One consideration is when operations can be performed "in parallel", i.e., independently of order of operations. (**VS, Appl**)

2007a Complexity measures for gene assembly. In: K. Tuyls *et al.*, eds., *Knowledge Discovery and Emergent Complexity in Bioinformatics* (First Int. Workshop, KDECB 2006, Ghent, 2006), pp. 42–60. Lect. Notes in Bioinformatics, Vol. 4366. Springer, Berlin, 2007.

§7, "Fourth complexity measure: Parallelism": A definition of parallelism in terms of applying rules (operations) to vertex-signed graphs. [Annot. 6 Feb 2011.] (**VS: Alg**)

2006a Parallelism in gene assembly. *Nat. Computing* 5 (2006), no. 2, 203–223. MR 2259034 (2007h:68043). Zbl 1114.68043.

See (2005a). (**VS, Appl**)

Tero Harju, Ion Petre, and Gregorz Rozenberg

2004a Tutorial on DNA computing and graph transformation. In: H. Ehrig *et al.*, eds., *ICGT 2004*, pp. 434-436. Lect. Notes in Computer Sci., Vol. 3256. Springer, Berlin, 2004.

See Harju, Li, Petre, and Rozenberg (2005a) *et al.* (**VS, Appl: Exp**)

Pierre de la Harpe

1994a Spin models for link polynomials, strongly regular graphs and Jaeger's Higman-Sims model. *Pacific J. Math.* 162 (1994), no. 1, 57–96. MR 1247144 (94m:-57014). Zbl 795.57002. (**SGc: Knot, Invar**)

David Harries and Hans Liebeck

1978a Isomorphisms in switching classes of graphs. *J. Austral. Math. Soc. (A)* 26 (1978), 475–486. MR 80a:05109. Zbl 411.05044.

Given $\Sigma = (K_n, \sigma)$ and an automorphism group $\mathfrak{A}$ of the switching class $[\Sigma]$, is $\mathfrak{A}$ "exposable" on $[\Sigma]$ (does it fix a representative of $[\Sigma]$)? General techniques and a solution for the dihedral group. Done in terms of Seidel switching of unsigned simple graphs. (A further development from Mallows and Sloane (1975a). [Related work in M. Liebeck (1982a) and Cameron (1977a).]) (**kg: sw, TG: Aut**)

Alexander K. Hartmann

See also M. Pelikan.

Alexander K. Hartmann and Martin Weigt

2005a *Phase Transitions in Combinatorial Optimization Problems: Basics, Algorithms and Statistical Mechanics.* Wiley-VCH, Weinheim, Germany, 2005. MR 2293999 (2009b:82028). Zbl 1094.82002.

"Example: Ising spin glasses": Frustration index of signed graphs on p. 6. §11.7, "Matchings and spin glasses": Outlines the matching theory method (cf. Katai and Iwai (1978a) and Barahona (1982a)) for planar graphs, for calculating $l(\Sigma)$ and locating ground states (fewest unsatisfied edges). Also, locating interesting excited states (states with more than the fewest unsatisfied edges), specifically, domain walls and droplets. A "domain" is generated by negating signs of a set of edges; the vertices whose spins remain the same form one domain and the complement is the other. The increased energy (the "domain wall energy") has thermodynamic implications. [How to choose the negation set, and what can be the shapes of domain walls, are not obvious.] A "droplet" in a state s, vis-á-vis a ground state s_0, is a component of the subgraph induced by $(ss_0)^{-1}(-1)$. The sizes of droplets appear to have consequences for thermodynamics. [Annot. 24 Aug 2012.]

(**SG: WG, Fr: Phys, Alg: ExpRef**)

Nora Hartsfield and Gerhard Ringel

1989a Minimal quadrangulations of nonorientable surfaces. *J. Combin. Theory Ser. A* 50 (1989), 186-195. MR 90j:57003. Zbl 665.51007.

"Cascades": see Youngs (1968b). (**sg: Ori: Appl**)

Kurt Hässig

1975a Theorie verallgemeinerter Flüsse und Potentiale. In: *Siebente Oberwolfach-Tagung uber Operations Research* (1974), pp. 85–98. Operations Research Verfahren, Band XXI. A. Hain, Meisenheim am Glan, 1975. MR 56 #8434. Zbl 358.90070. (**GN: Incid**)

1979a *Graphentheoretische Methoden des Operations Research.* Leitfaden der angew. Math. und Mechanik, 42. B.G. Teubner, Stuttgart, 1979. MR 80f:90002. Zbl 397.90061.

Ch. 5: "Verallgemeinerte Fluss- und Potentialdifferenzen-probleme." The lift matroid arises from a side condition, i.e., extra row, added to the incidence matrix of the graph. [The side condition is expressed graphically by additive real gains.] (**GN: Incid, M, Bal: Exp, Ref**)

Refael Hassin

1981a Generalizations of Hoffman's existence theorem for circulations. *Networks* 11 (1981), 243–254. MR 83c:90055. Zbl 459.90026. (**GN**)

O. Hatami

See S. Akbari.

Naomichi Hatano

See E. Estrada.

D.M. Hatch

See S.T. Chui.

Bian He, Ya-Lei Jin, and Xiao-Dong Zhang

2011a Sharp bounds for the signless Laplacian spectral radius in terms of clique number. *Linear Algebra Appl.* 434 (2011), no. 3, 683–687.

§4: The incidence energy (derived from $Q := K(-\Gamma)$) has a bound like that in Thm. 4.5. [Annot. 21 Jan 2012.] (**Par: Adj**)

Jin-Ling He

See J.-Y. Shao.

Shushan He and Shuchao Li

2012a On the signless Laplacian index of unicyclic graphs with fixed diameter. *Linear Algebra Appl.* 436 (2012), no. 1, 252–261. MR 2859926 (2012j:05257). Zbl 1229.05201. (**sg: par: Adj**)

Patrick Headley

1997a On a family of hyperplane arrangements related to the affine Weyl groups. *J. Algebraic Combin.* 6 (1997), 331–338. MR 98e:52010. Zbl 911.52009.

The characteristic polynomials of the Shi hyperplane arrangements $\mathcal{S}(W)$ of type W for each Weyl group W, evaluated computationally. $\mathcal{S}(W)$ is obtained by splitting the reflection hyperplanes of W in two in a certain way; thus $\mathcal{S}(A_{n-1})$ splits the arrangement representing $\operatorname{Lat} G(K_n)$—more precisely, it represents $\operatorname{Lat}^{\mathrm{b}}\{0,1\}\vec{K}_n$; that of type B_n splits the arrangement representing $\operatorname{Lat} G(\pm K_n^\bullet)$, and so on. [See also Athanasiadis (1996a).] (**gg: Geom, M, Invar**)

Brian Healy and Arthur Stein

20xxa The balance of power in international history.

Describes balance (incorrectly) and clusterability of a signed graph; examines the relevance of, *i.a.*, signed-graphic balance. [Annot. 9 Jun 2012.] (**PsS; SG: Bal, Clu: Exp**)

Fritz Heider

1946a Attitudes and cognitive organization. *J. Psychology* 21 (1946), 107–112.

No mathematics, but a formative article. [See Cartwright and Harary (1956a).] (**PsS**)

1979a On balance and attribution. In: Paul W. Holland and Samuel Leinhardt, eds., *Perspectives on Social Network Research* (Proc. Sympos., Dartmouth Coll., Hanover, N.H., 1975), Ch. 2, pp. 11–23. Academic Press, New York, 1979. (**PsS**)(**SG: Bal**)

E. Heilbronner

1964a Hückel molecular orbitals of Möbius-type conformations of annulenes. *Tetrahedron Letters* 5 (1964), no. 29, 1923–1928.

Eigenvalues of adjacency matrices of negative circles. [Annot. 4 Nov 2010.] (**SG: Adj, Chem**)

Peter Christian Heinig

20xxa Chio condensation and random sign matrices. Submitted. arXiv:1103.2717. (**SG**)

Richard V. Helgason

See J.L. Kennington.

I. Heller

1957a On linear systems with integral valued solutions. *Pacific J. Math.* 7 (1957), 1351–1364. MR 20 #899. Zbl 79, 19 (e: 079.01903). (**sg: Incid: bal**)

I. Heller and C.B. Tompkins

1956a An extension of a theorem of Dantzig's. In: H.W. Kuhn and A.W. Tucker, eds., *Linear Inequalities and Related Systems*, pp. 247–252. Annals of Math. Studies, No. 38. Princeton Univ. Press, Princeton, N.J., 1956. MR 18, 459. Zbl 72, 378 (e: 072.37804).

Thm.: The incidence matrix of a signed graph where all edges are links is totally unimodular iff the signed graph is balanced. (Not stated in terms of signed graphs.) See also Hoffman and Gale (1956a). (**sg: Incid, Bal**)

Marc Hellmuth

See T. Biyikoğlu.

J.L. van Hemmen

1983a Equilibrium theory of spin glasses: mean-field theory and beyond. In: J.L. van Hemmen and I. Morgenstern, eds., *Heidelberg Colloquium on Spin Glasses* (Proc., Heidelberg, 1983), pp. 203–233. Lect. Notes in Physics Vol. 192. Springer-Verlag, Berlin, 1983. MR 733800 (85d:82086).

§2.3, "Frustration": Physics of Ising models with edges ("bonds") that are positive, negative, or of undetermined sign. [Annot. 16 Jun 2012.] (**Phys: sg**)

Robert L. Hemminger and Joseph B. Klerlein

1979a Line pseudodigraphs. *J. Graph Theory* 1 (1977), 365–377. MR 57 #5812. Zbl 379.05032.

An attempt, intrinsically unsuccessful, to represent the (signed) line graph of a digraph (see Zaslavsky 20xxa) by a digraph. [Continued by Klerlein (1975a).] (**sg: LG, ori**)

Robert L. Hemminger and Bohdan Zelinka

1973a Line isomorphisms on dipseudographs. *J. Combin. Theory Ser. B* 14 (1973), 105–121. MR 47 #3230. Zbl 263.05107. (**sg: LG, ori**)

Anthony Henderson

2006a Plethysm for wreath products and homology of sub-posets of Dowling lattices. *Electronic J. Combin.* 13 (2006), no. 1, Research article R87, 25 pp. MR 2007f:05187. Zbl 1113.05101.

The subposets are $Q_n^{1 \bmod d}(\mathfrak{G})$ where $d > 1$, whose elements are the flats $A \subseteq E(\mathfrak{G}K_n^\bullet)$ such that d divides the order of the unbalanced part and the number of vertices every balanced component is $\equiv 1 \bmod d$. (**gg: M: Aut**)

Patricia Hersh and Ed Swartz

2008a Coloring complexes and arrangements. *J. Algebraic Combin.* 27 (2008), 205–214. MR 2375492 (2008m:05109). Zbl 1154.05315.

Remark 19: Chromatic polynomials of signed graphs vis-á-vis subarrangements of the root system arrangement $\mathcal{B}_n$ in Thm. 18, which gives properties of an h-vector. [Annot. 1 Mar 2011.] (**SG: Invar**)

Daniel Hershkowitz and Hans Schneider

1993a Ranks of zero patterns and sign patterns. *Linear Multilinear Algebra* 34 (1993), no. 1, 3–19. MR 1334927 (96g:15004). Zbl 793.05027.

Bipartite Σ such that every matrix with sign pattern Σ has the same rank, over each field $\neq \mathbb{H}_2$. [Annot. 6 Mar 2011.] (**SG: QM**)

J.A. Hertz

See K.H. Fischer.

G. Hetyei

See Y. Diao.

Hector Hevia

See G. Chartrand.

Takayuki Hibi

See H. Ohsugi.

Desmond J. Higham

See E. Estrada.

K. Hinson

See Y. Diao.

André Hirschowitz

See M. Hirschowitz.

Michel Hirschowitz, André Hirschowitz, and Tom Hirschowitz

2007a A theory for game theories. In: V. Arvind and S. Prasad, eds., (FSTTCS 2007: Foundations of Software Technology and Theoretical Computer Science), pp. 192–203. Lect. Notes in Computer Sci., Vol. 4855. Springer-Verlag, Berlin, 2007. MR 2480201 (2010h:91057). Zbl 1136.68035. (**SD: Appl**)

Tom Hirschowitz

See M. Hirschowitz.

Dorit S. Hochbaum

1998a Instant recognition of half integrality and 2-approximations. In: Klaus Jansen and José Rolim, eds., *Approximation Algorithms for Combinatorial Optimization* (Aalborg, 1998), pp. 99–110. Lect. Notes in Computer Sci., Vol. 1444. Springer, Berlin, 1998. MR 1677400. Zbl 911.90261.

Integer programs with constraints of a generalized real gain-graphic form, $\alpha x - \beta y - \gamma \leq z$, the gain being β/α. Slightly extends Hochbaum, Megiddo, Naor, and Tamir (1993a). (**gn: Incid(D): Alg**)

1998b The t-vertex cover problem: extending the half integrality framework with budget constraints. In: Klaus Jansen and José Rolim, eds., *Approximation Algorithms for Combinatorial Optimization* (Aalborg, 1998), pp. 111–122. Lect. Notes in Computer Sci., Vol. 1444. Springer, Berlin, 1998. MR 2000b:90032. Zbl 908.90213.

Integer programs as in (1998a) with "budget constraints".

(**gn: Incid(D): Alg**)

2000a Instant recognition of polynomial time solvability, half integrality and 2-approximations. In: Klaus Jansen and Samir Khuller, eds., *Approximation Algorithms* *for Combinatorial Optimization* (Saarbrücken, 2000), pp. 2–14. Lect. Notes in Computer Sci., Vol. 1913. Springer, Berlin, 2000. MR 1850069. Zbl 976.90123.

Integer programs as in (1998a). There is a polynomial-time solution via a minimum cut, or else a half-integral partial solution.

(gn: Incid(D): Alg)

2002a Solving integer programs over monotone inequalities in three variables: a framework for half integrality and good approximations. O.R. for a United Europe (Budapest, 2000). *European J. Operational Res.* 140 (2002), no. 2, 291–321. MR 2003e:90052. Zbl 1001.90050.

Constraints of a generalized positive-real gain-graphic form, $\alpha x - \beta y - \gamma \leq z$, the gain being β/α, contrasting $\alpha, \beta \geq 0$ to the intrinsically hard case where a negative coefficient is allowed but a half-integral approximate solution is easy. **(gn: Incid(D): Alg)**

Dorit S. Hochbaum, Nimrod Megiddo, Joseph (Seffi) Naor, and Arie Tamir

1993a Tight bounds and 2-approximation algorithms for integer programs with two variables per inequality. *Math. Programming Ser. B* 62 (1993), 69–83. MR 94k:90050. Zbl 802.90080.

Approximate solution of integer linear programs with real, dually gain-graphic coefficient matrix. [See Sewell (1996a).] **(GN: Incid(D): Alg)**

Dorit S. Hochbaum and Joseph (Seffi) Naor

1994a Simple and fast algorithms for linear and integer programs with two variables per inequality. *SIAM J. Computing* 23 (1994), 1179–1192. MR 95h:90066. Zbl 831.90089.

Linear and integer programs with real, dually gain-graphic coefficient matrix: feasibility for linear programs, solution of integer programs when the gains are positive ("monotone inequalities"), and identification of "fat" polytopes (that contain a sphere larger than a unit hypercube).

(GN: Incid(D): Alg, Ref)

Winfried Hochstättler, Robert Nickel, and Britta Peis

2006a Two disjoint negative cycles in a signed graph. CTW2006 – Cologne-Twente Workshop on Graphs and Combinatorial Optimization. *Electronic Notes Discrete Math.* 50 (2006), 107–111. MR2307287 (no rev). Zbl 1134.05319.

Incidence matrix used to find the circles in slow polynomial time. Use of graphic structure is explored. **(SG: Str: Circles: Alg, Incid)**

Cornelis Hoede

1981a The integration of cognitive consistency theories. Memorandum nr. 353, Dept. of Appl. Math., Twente Univ. of Tech., Enschede, The Netherlands, Oct., 1981.

(PsS: Gen)(SG, VS: Bal)

1982a Anwendungen von Graphentheoretischen Methoden und Konzepten in den Socialwissenschaften. Memorandum nr. 390, Dept. of Appl. Math., Twente Univ. of Tech., Enschede, the Netherlands, May, 1982.

Teil 4: "Kognitive Konsistenz." **(PsS: Gen: Exp)**

†1992a A characterization of consistent marked graphs. *J. Graph Theory* 16 (1992), 17–23. MR 93b:05141. Zbl 748.05081.

Characterizes when one can sign the vertices of a graph so every circle has positive sign product, solving the problem of Beineke and Harary (1978b). (Γ, μ), where $\mu : V \to \{+, -\}$, is consistent iff, with respect to some spanning tree, the fundamental circles are positive and the endpoints of the intersection of two fundamental circles have the same sign.

A polynomial-time algorithm ensues. [The definitive word until Joglekar, Shah, and Diwan (2010a). Does not treat signed vertices and edges.] [Annot. Rev. 11 Sept 2010.] **(VS: Bal: Str)**

P. Hoever, W.F. Wolff, and J. Zittartz

1981a Random layered frustration models. *Z. Phys. B* 41 (1981), 43–53. MR 600279 (81m:82027).

Physics of Ising models on a planar square lattice. Exact solutions for partition function, free energy, ground state energy. The transition temperature depends only on the average edge sign, $(|E^+| - |E^-|)/|E|$. Switching is implicit ("substituting spins"). Model (a): all horizontal edges are + (attainable by switching); if horizontally periodic these are "random layered frustration" models. Model (b): Assumed switched to minimize $|E^-|$. Dictionary: "plaquette" = quadrilateral, "frustration index" = sign of plaquette.

They conjecture thermodynamic consequences if the ground states ($s : V \to \{+1, -1\}$ with $l(\Sigma)$ frustrated edges) are connected in the state graph $\{+1, -1\}^V$. [*Question.* For which Σ are the ground states connected?] [Annot. 16 Jun, 28 Aug 2012.] **(Phys: SG: sw)**

Alan J. Hoffman

See also D.R. Fulkerson and D. Gale.

1970a $-1 - \sqrt{2}$? In: Richard Guy *et al.*, eds., *Combinatorial Structures and Their Applications* (Proc. Calgary Int. Conf., 1969), pp. 173–176. Gordon and Breach, New York, 1970. Zbl 262.05133. **(LG)**

1972a Eigenvalues and partitionings of the edges of a graph. *Linear Algebra Appl.* 5 (1972), 137–146. MR 46 #97. Zbl 247.05125. **(Par: Adj, Fr)**

1974a On eigenvalues of symmetric $(+1, -1)$ matrices. *Israel J. Math.* 17 (1974), 69–75. MR 50 #2202. Zbl 281.15003.

Eigenvalues of signed complete graphs. **(sg: kg: Adj)**

1975a Spectral functions of graphs. In: *Proceedings of the International Congress of Mathematicians* (Vancouver, 1974), Vol. 2, pp. 461–463. Canad. Math. Congress, Montreal, 1975. MR 55 #7850. Zbl 344.05164. **(TG, Adj)**

1976a On spectrally bounded signed graphs. (Abstract.) In: *Trans. Twenty-First Conference of Army Mathematicians* (White Sands, N.M., 1975), pp. 1–5. ARO Rep. 76-1. U.S. Army Research Office, Research Triangle Park, N.C., 1976. MR 58 #27648.

Abstract of (1977b). Also, bounding the least eigenvalue in terms of principal submatrices. **(SG: LG)**

1977a On graphs whose least eigenvalue exceeds $-1 - \sqrt{2}$. *Linear Algebra Appl.* 16 (1977), 153–165. MR 57 #9607. Zbl 354.05048.

Introduces generalized line graphs. [They are the reduced line graphs of signed graphs of the form $-\Gamma$ with any number of negative digons attached to each vertex; see Zaslavsky (20xxb), Ex. 7.6, (20xxa)]. **(LG)**

1977b On signed graphs and gramians. *Geometriae Dedicata* 6 (1977), 455–470. MR 57 #3167. Zbl 407.05064.

Σ is a signed simple graph. Let λ be the least eigenvalue of $A(\Sigma)$. Can (*) $A(\Sigma) - \lambda I - KK^{\mathrm{T}}$ be zero for some K with all entries $0, \pm 1$?

When $\lambda = -2$, K exists [equivalently, Σ is a reduced line graph of a signed graph; *cf.* Zaslavsky (2010b, 20xxa)], with finitely many exceptions; the proof uses root systems; *cf.* Cameron, Goethals, Seidel and Shult (1976a). In general, no K may give zero, but the minimum, over all K, of the largest element of (*) is bounded by a function of λ.
(**SG: LG: Adj**)

[**A.J. Hoffman and D. Gale**]

1956a Appendix [to the paper of Heller and Tompkins (1956a)]. In: H.W. Kuhn and A.W. Tucker, eds., *Linear Inequalities and Related Systems*, pp. 252–254. Annals of Math. Studies., No. 38. Princeton Univ. Press, Princeton, N.J., 1956.
(**sg: Incid: bal**)

Alan J. Hoffman and Peter Joffe

1978a Nearest $\mathcal{S}$-matrices of given rank and the Ramsey problem for eigenvalues of bipartite $\mathcal{S}$-graphs. In: *Problèmes Combinatoires et Théorie des Graphes* (Colloq. Int., Orsay, 1976), pp. 237–240. Colloques Int. du CNRS, 260. Editions du C.N.R.S., Paris, 1978. MR 81b:05080. Zbl 413.05031. (**SG: Adj**)

Alan J. Hoffman and Francisco Pereira

1973a On copositive matrices with −1, 0, 1 entries. *J. Combinatorial Theory Ser. A* 14 (1973), 302–309. MR 47 #5029. Zbl 273.15019.

Dean Hoffman and Heather Jordon

2006a Signed graph factors and degree sequences. *J. Graph Theory* 52 (2006), no. 1, 27–36. MR 2006k:05174. Zbl 1117.05089.

The net degree of a vertex in Σ is $d^+(v) - d^-(v)$. [This is best viewed as degree in an all-negative bidirected graph; *cf.* p. 35.] Thms. 2.3 (for Σ) and 4.1 (for a bidirected graph B, called a "mixed signed graph") are an interesting f-factor theorem in terms of net degrees. Thm. 4.1: Given $f : V \to \mathbb{Z}$, an "f-factor" is a subgraph whose net in-degree vector $= f$. For disjoint $S, T \subseteq V$ and a component Q of $\mathrm{B} \setminus (S \cup T)$, $J(Q, S, T)$ is computed in terms of f and in-degrees and out-degrees of edges among Q, S, T. $q(S, T)$ is the number of J-odd components Q. An f-factor exists iff q satisfies an inequality. Thm. 3.2: Fixing the maximum edge multiplicity, an Erdős–Gallai-type characterization of net degree sequences—simplifying the theorem of Michael (2002a). Thm. 4.2: Net in-degree sequences of bidirected simple graphs. [More in Jordon, McBride, and Tipnis (2009a).] [Annot. 14 Oct 2009.] (**SG: ori: Invar**)

Thomas R. Hoffman

See D.M. Duncan.

Franz Höfting and Egon Wanke

1993a Polynomial algorithms for minimum cost paths in periodic graphs. In: Vijaya Ramachandran *et al.*, eds., *Proceedings of the Fourth Annual ACM-SIAM Symposium on Discrete Algorithms* (Austin, Tex., 1993), pp. 493–499. Assoc. for Computing Machinery, New York, and Soc. for Industrial and Appl. Math., Philadelphia, 1993. MR 93m:05184. Zbl 801.68133.

Given a finite gain digraph Φ (the "static graph") with gains in $\mathbb{Z}^d$ and a rational cost for each edge, find a minimum-cost walk ("path") in its canonical covering graph $\tilde{\Phi}$ with given initial and final vertices.
(**GD(Cov): Alg**)

1994a Polynomial time analysis of toroidal periodic graphs. In: Serge Abiteboul and

Eli Shamir, eds., *Automata, Languages and Programming* (Proc. 21st Int. Colloq., ICALP 94, Jerusalem, 1994), pp. 544–555. Lect. Notes in Computer Sci., Vol. 820. Springer-Verlag, Berlin, 1994. MR 96c:05164.

Take a gain digraph Φ (the "static graph") with gains in $\mathbb{Z}_\alpha = \mathbb{Z}_{\alpha_1} \times \cdots \times \mathbb{Z}_{\alpha_d}$ (where $\alpha = (\alpha_1, \cdots, \alpha_d)$) and its canonical covering digraph $\tilde{\Phi}$ (the "toroidal periodic graph"). Treated algorithmically via integer linear programming and linear Diophantine equations: existence of directed paths (NP-complete, but polynomial-time if Φ is strongly connected) and number of strongly connected components of $\tilde{\Phi}$.

(**GD(Cov): Alg, Geom**)

1995a Minimum cost paths in periodic graphs. *SIAM J. Computing* 24 (1995), 1051–1067. MR 96d:05061. Zbl 839.05063.

Full version of (1993a). The min-cost problem is expressed as an integer linear program. Various conditions under which the problem is NP-hard, even a very restricted version without costs (Thms. 3.3, 3.5), or polynomial-time solvable (e.g.: without costs, when Φ is an undirected gain graph: Thm. 3.4; with costs, when d is fixed: Thm. 4.5). (**GD, GG(Cov): Alg, Geom, Ref**)

2000a Polynomial-time analysis of toroidal periodic graphs. *J. Algorithms* 34 (2000), no. 1, 14–39. MR 1732196 (2001k:68111). Zbl 958.68129.

Full version of (1994a). (**GD(Cov): Alg, Geom**)

Leslie Hogben

2005a Spectral graph theory and the inverse eigenvalue problem of a graph. *Electronic J. Linear Algebra* 14 (2005), 12–31. MR 2202430 (2006k:05133). Zbl 1162.05333. (**Par: Adj**)

Paul W. Holland and Samuel Leinhardt

1971a Transitivity in structural models of small groups. *Comparative Group Studies* 2 (1971), 107–124. (**PsS: SG: Bal**)

Paul W. Holland and Samuel Leinhardt, eds.

1979a *Perspectives on Social Network Research* (Proc. Math. Soc. Sci. Board Adv. Res. Sympos. on Social Networks held at Dartmouth College, Hanover, N.H., September 18–21, 1975). Academic Press, New York, 1979. (**PsS, SG**)

Roderick B. Holmes and Vern I. Paulsen

2004a Optimal frames for erasures. *Linear Algebra Appl.* 377 (2004), 31–51. MR 2021601 (2004j:42028). Zbl 1042.46009.

Adjacency matrices of cube-root-of-unity gain graphs. [Annot. 20 June 2011.] (**gg: adj**)

Hai-Yan Hong

See Y.-Z. Fan.

Sungpyo Hong

See J. H. Kwak.

Yuan Hong and Xiao-Dong Zhang

2005a Sharp upper and lower bounds for largest eigenvalue of the Laplacian matrices of trees. *Discrete Math.* 296 (2005), no. 2-3, 187–197. MR 2154712 (2006g:05127). Zbl 1068.05044 .

Thm. 2: If some neighbors of v in Γ are regrafted onto u, forming Γ', and if $x_u \geq x_v$ in the Perron vector of $K(-\Gamma)$, then $\lambda_1(K(-\Gamma)) <$

$\lambda_1(K(-\Gamma'))$. [Annot. 24 Jan 2012.] **(Par: Adj)**

Shlomo Hoory, Nathan Linial, and Avi Wigderson

2006a Expander graphs and their applications. *Bull. Amer. Math. Soc. (N.S.)* 43 (2006), no. 4, 439–561. MR 2247919 (2007h:68055). Zbl 1147.68608.

§6, "Spectrum and expansion in lifts of graphs": covering graphs of permutation gain graphs, and from Bilu and linial (2006a) of signed graphs. §6.1, "Covering maps and lifts": Covering graphs of permutation gain graphs, presented as symmetric digraphs with invertible arc gains. §2.6, "Eigenvalues - old and new": Prop. 6.3. The covering graph's eigenvalues include those of the (underlying) base graph Γ and its eigenvectors sum to 0 on fibers. Prop. 6.4. The signed covering graph's eigenvalues are those of Γ and those of (Γ, σ). §6.4, "Nearly-Ramanujan graphs by way of 2-lifts": Conjectured and proven eigenvalue ranges when the base graph is a Ramanujan graph. Dictionary: "signing" of $A(\Gamma)$ means $A(\Gamma, \sigma)$ for any edge signature. "2-lift" = double covering graph. [Annot. 25 Aug 2011.] **(sg: Cov, Adj: Exp)**

Tsuyoshi Horiguchi

1986a Fully frustrated Ising model on a square lattice. *Progress Theor. Phys. Suppl.* No. 87 (1986), 33–42. MR 884854 (88g:82063). **(Phys: SG: Fr)**

Yaoping Hou

2005a Bounds for the least Laplacian eigenvalue of a signed graph. *Acta Math. Sinica (Engl. Ser.)* 21 (2005), no. 4, 955–960. MR 2156977 (2006d:05120). Zbl 1080.05060. **(SG: Adj, Bal)**

Yaoping Hou, Jiongsheng Li, and Yongliang Pan

2003a On the Laplacian eigenvalues of signed graphs. *Linear Multilinear Algebra* 51 (2003), 21–30. MR 2003j:05084. Zbl 1020.05044.

Properties of (mainly) largest eigenvalue $\lambda_1(\Sigma)$ of the Laplacian (Kirchhoff) matrix $K(\Sigma)$ of a signed simple graph. Thms. 2.5–2.6 repeat standard criteria for balance [with a sign error in (3) of each]. Main results:

Upper bounds, all in terms of underlying graph: Lemma 3.1: For connected Γ, $\lambda_1(\Gamma, \sigma) \leq \lambda_1(-\Gamma)$, equal iff σ is antibalanced (e.g., all negative). Thm. 3.4: $\lambda_1(\Sigma) \leq 2(n-1)$, equal iff $\Sigma \sim -K_n$. Thm. 3.5: $\lambda_1(\Sigma) \leq$ (1) max edge degree + 2, (2) max (vertex degree + average neighbor degree), (3) a combination of these degrees; equal iff Σ is antibalanced and $|\Sigma|$ is semiregular bipartite.

Lower bounds: Cor. 3.8: $\lambda_1(\Sigma^+) + \lambda_1(\Sigma^-) \geq \lambda_1(\Sigma) \geq \lambda_1(\Sigma^+), \lambda_1(\Sigma^-)$. Thm. 3.9: If Σ has a vertex of degree $n-1$, then $\lambda_1(\Sigma) \geq \lambda_1(+|\Sigma|)$, with equality iff Σ is balanced. Thm. 3.10: $\lambda_1(\Sigma) \geq 1 + \max_v d_{|\Sigma|}(v)$.

Interlacing: Lemma 3.7 (special case): $\lambda_i(\Sigma) \geq \lambda_i(\Sigma \setminus e) \geq \lambda_{i+1}(\Sigma)$, where $\lambda_1(\Sigma) \geq \lambda_2(\Sigma) \geq \cdots$.

Problems about existence of cospectral unbalanced signed graphs. **(SG: Adj)**

Yao Ping Hou and Li Juan Wei

1999a Whitney numbers of the second kind for Dowling lattices. (In Chinese.) *Acta Sci. Natur. Univ. Norm. Hunan.* 22 (1999), No. 3, 6–10. MR 2000k:05017. Zbl 948.05004.

Combinatorial proof of an explicit formula for W_k [possibly the standard one?]. Studies "associated numbers" W_k^r. Proved: $W_{n-k} \leq W_k$ for $k \leq 3$ [this must be an error for $W_k \leq W_{n-k}$ and must have some restriction on n; well known for $k = 1$]. [*Cf.* Benoumhani (1996a).] (**gg: M: Invar**)

R.M.F. Houtappel

1950a Statistics of two-dimensional hexagonal ferromagnetics with "Ising"- interaction between nearest neighbours only. *Physica* 16 (1950), 391–392. Zbl 38, 419c (e: 038.41903).

Announcement of (1950b). (**Phys, WG, sg: Fr**)

1950b Order-disorder in hexagonal lattices. *Physica* 16 (1950), 425–455. MR 039632 (12, 576j). Zbl 38, 139c (e: 038.13903).

Ising spins, i.e. $\zeta : V \rightarrow \{+1, -1\}$, in the triangular and honeycomb (hexagonal) lattice graphs on a torus. Different edge weights ("bond strengths") and signs are allowed in the three directions. The all-negative triangular signature (i.e., "antiferromagnetic" with equal weights) is an exceptional case. Switching the triangular lattice (p. 449, bottom) permits assuming that two chosen directions are positive. Exceptional weights are the antibalanced triangular lattice with equal smaller weights, e.g., all weights equal (p. 449, bottom). The honeycomb cannot be exceptional [because it is balanced] (p. 451). [See also G.F. Newell (1950b), I. Syôzi (1950a), G.H. Wannier (1950a).] [Annot. 20 Jun 2012.] (**Phys, WG, sg: Fr. sw**)

Guang Hu and Wen-Yuan Qiu

2009a Extended Goldberg polyhedral links with odd tangles. *MATCH Commun. Math. Comput. Chem.* 61 (2009), no. 3, 753–766. Zbl 1189.92027.

See Flapan (1995a). [Annot. 4 Nov 2010.] (**sg: Top, Chem**)

Hongbo Hua

2007a Bipartite unicyclic graphs with large energy. *MATCH Commun. Math. Comput. Chem.* 58 (2007), no. 1, 57–73. MR 2335478 (2008d:05101). Zbl 1224.05301.

Fix $n \geq 13$. For connected unicyclic Γ such that $-\Gamma$ is balanced, excluding circles and balloons ("tadpoles", "lollipops"), the maximum energy occurs for a hexagon attached by an edge to the third vertex of a path. [*Problem.* Replace "bipartite" by "signed", i.e., allow unbalanced signed graphs.] [Annot. 24 Jan 2012.] (**par: Adj**)

Qiongxiang Huang

See J.F. Wang.

Rong Huang, Jianzhou Liu, and Li Zhu

2011a A structural characterization of real k-potent matrices. *Linear Multilinear Algebra* 59 (2011), no. 4, 433–439. MR 2802524 (2012d:15023). (**QM: SD**)

Ting-Zhu Huang

See G.X. Tian.

Yihua Huang

See Y.-B. Gao.

Yufei Huang

See also C.H. Liang.

Yufei Huang, Bolian Liu and Yingluan Liu

2011a The signless Laplacian spectral radius of bicyclic graphs with prescribed degree sequences. *Discrete Math.* 311 (2011), no. 6, 504–511. MR 2799902

(2012a:05189). Zbl 1222.05130.

The largest spectral radius and the extremal graphs. [Annot. 19 Nov 2011.] (**Par: Adj**)

Falk Hüffner

See also S. Böcker.

Falk Hüffner, Nadja Betzler, and Rolf Niedermeier

2007a Optimal edge deletions for signed graph balancing. In: Camil Demetrescu, ed., *Experimental Algorithms* (6th Int. Workshop, WEA 2007, Rome, 2007), pp. 297–310. Lect. Notes in Computer Sci., Vol. 4525. Springer-Verlag, Berlin, 2007. Zbl 1203.68125.

An improved algorithm for frustration index. Dictionary: "2-coloring that minimizes inconsistencies with given edge labels" = switching function that minimizes number of negative edges. [Annot. 10 Sept 2011.] (**SG: Fr: Alg**)

2010a Separator-based data reduction for signed graph balancing. *J. Combin. Optim.* 20 (2010), no. 4, 335–360. MR 2734305 (2011j:05325). Zbl 1206.90201. (**SG: Fr: Alg**)

Florian Hug

See I.E. Bocharova.

Axel Hultman

2007a The topology of spaces of phylogenetic trees with symmetry. *Discrete Math.* 307 (2007), no. 14, 1825–1832. MR 2008a:05055. Zbl 1109.92031.

Introduces Dowling trees: "Natural Dowling analogues of the complex of phylogenetic trees". (**gg: M: Invar**)

2007b Link complexes of subspace arrangements. *European J. Combin.* 28 (2007), no. 3, 781–790. MR 2007m:52029. Zbl 1113.52038. arXiv:math/0507314.

Interprets chromatic polynomials of signed graphs in terms of Hilbert polynomials. (**SG: Invar**)

John Hultz

See also F. Glover.

John Hultz and D. Klingman

1979a Solving singularly constrained generalized network problems. *Appl. Math. Optim.* 4 (1978), 103–119. MR 57 #15414. Zbl 373.90075. (**GN: M(bases)**)

Norman P. Hummon and Patrick Doreian

†2003a Some dynamics of social balance processes: bringing Heider back into balance theory. *Social Networks* 25 (2003), 17–49.

Presents a model for evolution of balance and clusterability (as in Davis 1967a) of a signed digraph and explores it via computer simulations.

Definitions: Given a signed digraph $\vec{\Sigma}$ and a partition π of V, define the 'clusterability' $c(\vec{\Sigma}, \pi) :=$ (# negative edges within blocks of π) + (# positive edges between blocks). Define $\pi(\vec{\Sigma}) :=$ any π that minimizes $c(\vec{\Sigma}, \pi)$. Define $\vec{\Sigma}(v_i) := \{v_i\vec{v}_j \in \vec{E}(\vec{\Sigma})\}$ with signs. ($\vec{\Sigma}$ models relations in a social group V. $\vec{\Sigma}_i$ is the graph of relations perceived by v_i.)

Initial conditions: Fixed $|V|$, fixed "contentiousness" $p :=$ the probability that an initial edge is negative, a fixed "communication" rule, random $\vec{\Sigma}^0$ and, for each $v_i \in V$, $\vec{\Sigma}_i^0 := \vec{\Sigma}^0$. At time $t+1$, $\vec{\Sigma}_i^t(v_i)$

changes to $\vec{\Sigma}_i^{t+1}(v_i)$ to minimize $d(d(\vec{\Sigma}_i^{t+1}, \pi(\vec{\Sigma}^t))$. Then $\vec{\Sigma}_j^{t+1}(i)$ changes to $\vec{\Sigma}_i^{t+1}(v_i)$ for some v_j (depending on $\vec{\Sigma}_i$ and the communication rule).

Computer simulations examined the types of changes and emerging clusterability of $\vec{\Sigma}^t$ or $\vec{\Sigma}_i^t$ as t increases, under four different communication rules, random initial conditions with various p, and $|V| = 3, 5, 7, 10$. The outcomes are highly suggestive (see §4; p seems influential). [*Problem.* Predict the outcomes in terms of initial conditions through a mathematical analysis.] [Annot. 26 Apr 2009.]

(**SD, sg: Bal, Clu: Alg**)(**PsS**)

Norman P. Hummon and T.J. Fararo

1995a Assessing hierarchy and balance in dynamic network models. *J. Math. Sociology* 20 (1995), 145–159. Zbl 858.92032.

John E. Hunter

1978a Dynamic sociometry. *J. Math. Sociology* 6 (1978), 87–138. MR 58 #20631.

(**SG: Bal, Clu**)

Bofeng Huo, Shengjin Ji, Xueliang Li, and Yongtang Shi

2011a Solution to a conjecture on the maximal energy of bipartite bicyclic graphs. *Linear Algebra Appl.* 435 (2011), no. 4, 804–810. MR 2807234 (2012e:05232). Zbl 1220.05073.

[*Question.* Does this naturally generalize to antibalanced signed bicyclic graphs?] [Annot. 21 Mar 2011.] (**sg: Par: Adj**)

Bofeng Huo, Xueliang Li, and Yongtang Shi

2011a Complete solution to a problem on the maximal energy of unicyclic bipartite graphs. *Linear Algebra Appl.* 434 (2011), no. 5, 1370–1377. MR 2763594 (2011m-05176). Zbl 1205.05146.

[*Question.* Do the results generalize to antibalanced signed unicyclic graphs?] [Annot. 21 Mar 2011.] (**sg: Par: Adj**)

Li Fang Huo and Yu Bin Gao

2010a Local bases of two class of primitive nonpowerful signed digraphs with girth 2. *Math. Pract. Theory* 40 (2010), no. 10, 235–239. MR 2730313 (no rev).

(**SD: Adj**)

C.A.J. Hurkens

1989a On the existence of an integral potential in a weighted bidirected graph. *Linear Algebra Appl.* 114/115 (1989), 541–553. MR 90c:05142. Zbl 726.05050.

Given: a bidirected graph B (with no loose or half edges or positive loops) and an integer weight b_e on each edge. Wanted: an integral vertex weighting x such that $\mathrm{H}(B)^{\mathrm{T}} x \le b$, where $\mathrm{H}(B)$ is the incidence matrix. Such x exists iff (i) every coherent circle or handcuff walk has nonnegative total weight and (ii) each doubly odd Korach walk (a generalization of a coherent handcuff that has a cutpoint dividing it into two parts, each with odd total weight) has positive total weight. This improves a theorem of Schrijver (1991a) and is best possible. Dictionary: "path" ("cycle") = coherent (closed) walk. (**sg: Ori: Incid**)

Daniel Huttenlocher

See J. Leskovec.

Tony Chi Thong Huynh

See also J. Geelen.

2009a *The Linkage Problem for Group-labelled Graphs.* Doctoral thesis, Univ. of Waterloo, 2009.

Gains are in GF$(q)^\times$ or sometimes in a finite abelian group. Dictionary: "group-labelled graph" = gain graph, "Dowling matroid" = frame matroid (not Dowling geometry), "shifting" = switching. (**GG: M**)

T. Ibaraki

See also Y. Crama and P.L. Hammer.

T. Ibaraki and U.N. Peled

1981a Sufficient conditions for graphs to have threshold number 2. In: Pierre Hansen, ed., *Studies on Graphs and Discrete Programming* (Proc. Workshop, Brussels, 1979), pp. 241–268. North-Holland Math. Studies, 59. Ann. Discrete Math., 11. North-Holland, Amsterdam, 1981. MR 84f:05056. Zbl 479.05058. (**par: ori**)

Takashi Iino

See T. Yoshikawa.

Takeo Ikai

See H. Kosako.

Yoshiko T. Ikebe and Akihisa Tamura

20xxa Perfect bidirected graphs. Submitted.

A transitively closed bidirection of a simple graph is perfect iff its underlying graph is perfect. (See Johnson and Padberg (1982a) for definitions.) [Also proved by Sewell (1996a).] (**sg: Ori: Incid, Geom**)

Aleksandar Ilić

See also L.H. Feng and B. Zhou.

2010a Trees with minimal Laplacian coefficients. *Computers Math. Appl.* 59 (2010), 2776–2783. MR 2607982 (2010m:05069). Zbl 1193.05060. (**Par: Adj**)

Nicole Immorlica

See E. Demaine.

Takehiro Inohara

1999a On conditions for a meeting not to reach a recurrent argument. *Appl. Math. Comput.* 101 (1999), 281–298. MR 1677966 (99k:90010). Zbl 942.91019. (**SD, PsS**)

2000a Meetings in deadlock and decision makers with interperception. *Appl. Math. Comput.* 109 (2000), 121–133. MR 1738208 (2000m:91035). Zbl 1042.91010. (**SD, PsS**)

2002a Characterization of clusterability of signed graph in terms of Newcomb's balance of sentiments. *Appl. Math. Comput.* 133 (2002), no. 1, 93–104. MR 1923185 (2003i:05064). Zbl 1023.05072.

Assumption: all $\sigma(i,i) = +$. Thm. 3: A signed complete digraph is clusterable iff $\sigma(i,j) = -$ or $\sigma(j,k) = \sigma(i,k)$ for every triple $\{i,j,k\}$ of vertices (not necessarily distinct). [The notation is unnecessarily complicated.] (**SD: Clu, PsS**)

2003a Clusterability of groups and information exchange in group decision making with approval voting system. *Appl. Math. Comput.* 136 (2003), no. 1, 1–15. MR 2004b:91059. Zbl 1042.91086. (**SD: KG: Bal, Clu, PsS**)

2004a Quasi-clusterability of signed graphs with negative self evaluation. *Appl. Math. Comput.* 158 (2004), no. 1, 201–215. MR 2091243 (2005f:05072). Zbl 1055.05074. (**SD: Clu, PsS**)

2004b Signed graphs with negative self evaluation and clusterability of graphs. *Appl. Math. Comput.* 158 (2004), no. 2, 477–487. MR 2094633 (2005f:05073). Zbl 1054.05048. **(SD: Clu, PsS)**

2007a Relational dominant strategy equilibrium as a generalization of dominant strategy equilibrium in terms of a social psychological aspect of decision making. *Europ. J. Oper. Res.* 182 (2007), 856–866. Zbl 1121.90355. **(SD, PsS)**

Takehiro Inohara, Shingo Takahashi, and Bunpei Nakano

1998a On conditions for a meeting not to reach a deadlock. *Appl. Math. Comput.* 90 (1998), 1–9. MR 1485601. Zbl 907.90014. **(SD, PsS)**

2000a Credibility of information in 'soft' games with interperception of emotions. *Appl. Math. Comput.* 115 (2000), 23–41. MR 1779380 (2001e:91037). Zbl 1046.91004. **(SD, PsS)**

Yuri J. Ionin and Mohan S. Shrikhande

2006a *Combinatorics of Symmetric Designs*. Cambridge Univ. Press, Cambridge, Eng., 2006. MR 2234039 (2008a:05001). Zbl 1114.05001.

§7.3, "Switching in strongly regular graphs": Graph switching and two-graphs. **(TG, Sw: Exp)**

Masao Iri and Katsuaki Aoki

1980a A graphical approach to the problem of locating the origin of the system failure. *J. Operations Res. Soc. Japan* 23 (1980), 295–312. MR 82c:90041. Zbl 447.90036.

(SD, VS: Appl)

Masao Iri, Katsuaki Aoki, Eiji O'Shima, and Hisayoshi Matsuyama

1976a [A graphical approach to the problem of locating the system failure.] (In Japanese.) [???] 76(135) (1976), 63–68. **(SD, VS: Appl)**

1979a An algorithm for diagnosis of system failures in the chemical process. *Computers and Chem. Eng.* 3 (1979), 489–493 (1981).

The process is modelled by a signed digraph with some nodes v marked by $\mu(v) \in \{+, -, 0\}$. (Marks $+, -$ indicate a failure in the process.) Object: to locate the node which is origin of the failure. An oversimplified description of the algorithm: μ is extended arbitrarily to V. Arc (u, v) is discarded if $0 \neq \mu(u)\mu(v) \neq \sigma(u, v)$. If the resulting digraph has a unique initial strongly connected component S, the nodes in it are possible origins. Otherwise, this extension provides no information. (I have overlooked: special marks on "controlled" nodes; speedup by stepwise extension and testing of μ.) [This article and/or (1976a) seems to be the origin of a whole literature. See e.g. Chang and Yu (1990a), Kramer and Palowitch (1987a).] **(SD, VS: Appl, Alg)**

Toru Ishihara

2000a Cameron's construction of two-graphs. *Discrete Math.* 215 (2000), 283–291. MR 2000k:05090. Zbl 959.05099.

A new proof of Cameron (1994a). **(TG)**

2002a Signed graphs associated with the lattice A_n. *J. Math. Univ. Tokushima* 36 (2002), 1–6 (2003). MR 1974060 (2004c:05086). Zbl 1032.05061.

A signed graph corresponding to a base of A_n is a [signed] path of cliques and locally switches to a path. (For local switching see Cameron, Seidel, and Tsaranov (1994a).) **(SG: Geom)(SG: Sw: Gen)**

2004a Local switching of some signed graphs. *J. Math. Univ. Tokushima* 38 (2004), 1–7. MR 2123167 (2005m:05110). Zbl 1067.05032.

Which signed graphs locally switch to a tree? Examples only.
(**SG: Sw: Gen**)

2005a Local switching of signed induced cycles. *J. Math. Univ. Tokushima* 39 (2005), 1–5. MR 2194305 (2006i:05077).

Converting an induced circle to a path by local switching.
(**SG: Sw: Gen**)

2007a Signed graphs and Hushimi trees. *J. Math. Univ. Tokushima* 41 (2007), 13–23. MR 2380208 (no rev). Zbl 1138.05316.

Local switching between trees. [Annot. 28 Dec 2011.] (**CSG**)

Sorin Istrail

2000a Statistical mechanics, three-dimensionality and NP-completeness. I. Universality of intractability for the partition function of the Ising model across non-planar lattices. In: *Proceedings of the Thirty-Second Annual ACM Symposium on Theory of Computing* (STOC, Portland, Ore., 2000), pp. 87–96. ACM, New York, 2000. MR 2114521 (no rev).

Extends Barahona (1982a) on finite signed lattice graphs to the computational complexity of (a) ground states (i.e., frustration index) and (more difficult) (b) partition function (generating function of frustrated edges over all states), for signed infinite lattice graphs. [An infinite lattice graph is (apparently) a graph drawn in $\mathbb{E}^d$, crossings allowed, that has translational symmetry in d independent directions.] General conclusion: For nonplanar ones they are NP-hard. Thm. 1: A lattice graph in $d = 2, 3$ is planar iff it does not contain a certain $d = 2$ lattice graph K_0, the "Basic Kuratowskian". Lem. 2: Every 3-regular graph has a subdivision contained in K_0. §5, "Computational complexity of the 3D Ising models": Lattice graphs with signs, subgraphs thereof, all-positive subgraphs, all-negative subgraphs. Thm. 2: For every subgraph of a signed non-planar infinite lattice graph, computing $l(\Sigma)$ for finite sublattice graphs Σ is NP-hard. Thm. 3: For every subgraph of an all-negative non-planar infinite lattice graph, computing $l(\Sigma)$ for finite sublattice graphs Σ is NP-hard. §5.3, "Ising models with $\{-J, +J\}$ interactions": For every signed non-planar infinite lattice graph, computing $l(\Sigma)$ for finite sublattice graphs Σ is NP-hard; the proof is postponed to "the full version of the paper" [which has not appeared]. [Annot. 21 Aug 2012.] (**SG, Phys: Fr**)

Gabriel Istrate

2009a On the dynamics of social balance on general networks (with an application to XOR-SAT). Machines, Computations and Universality, Part II. *Fund. Inform.* 91 (2009), no. 2, 341–356. MR 2516378 (2010f:68140). Zbl 1181.91282.

Imbalance measured by triangles. Repeatedly change signs of edges of a fixed graph. Looks for recurrent states and time to become balanced. [Annot. 5 May 2010.] (**SG: Fr**)

C. Itzykson

See R. Balian.

P.L. Ivanescu [P.L. Hammer]

See E. Balas and P.L. Hammer.

Sousuke Iwai
See also O. Katai.

Osamu Katai and Sousuke Iwai
1978a Graph-theoretic models of social group structures and indices of group structures. (In Japanese.) *Systems and Control* (*Shisutemu to Seigyo*) 22 (1978), 713–722. MR 80d:92038 (no rev). (**CPsS: Exp**)

Hiroshi Iyetomi
See T. Yoshikawa.

Bill Jackson
See P.J. Cameron.

François Jaeger
1992a On the Kauffman polynomial of planar matroids. In: Jaroslav Nešetřil and Miroslav Fiedler, eds., *Fourth Czechoslovak Symposium on Combinatorics, Graphs and Complexity* (Prachatice, 1990), pp. 117–127. Ann. Discrete Math., Vol. 51. North-Holland, Amsterdam, 1992. MR 94d:57016. Zbl 763.05021.

(This is not the colored Tutte polynomial of Kauffman (1989a).) Jaeger shows that the Kauffman polynomial, originally defined for link diagrams and here transformed to an invariant of signed plane graphs, depends only on the edge signs and the circle matroid. It can also be reformulated to be essentially independent of signs. *Problem.* Define a similar invariant for more general matroids. (**SGc, Sgnd(M): Invar, Knot**)

François Jaeger, Nathan Linial, Charles Payan, and Michael Tarsi
1992a Group connectivity of graphs—a nonhomogeneous analogue of nowhere-zero flow properties. *J. Combin. Theory Ser. B* 56 (1992), 165–182. MR 93h:05088. Zbl 824.05043.

Let $\mathfrak{A}$ be an abelian group. Γ is "$\mathfrak{A}$-colorable" if every $\mathfrak{A}$-gain graph on Γ has a proper group-coloring (as in Zaslavsky 1991a). Prop. 4.2: Every simple planar graph is $\mathfrak{A}$-colorable for every abelian group of order ≥ 6. (For the same reason as the classical 6-Color Theorem.) [Improved by Lai and Zhang (2002b).] (**GG: Col**)

John C. Jahnke
See J.O. Morrissette.

John J. Jarvis and Anthony M. Jezior
1972a Maximal flow with gains through a special network. *Operations Res.* 20 (1972), 678–688. MR 47 #6286. Zbl 241.90021. (**GN: M(bases)**)

A. Javanmard
See S. Akbari.

C. Jayaprakash
See J. Vannimenus.

Clark Jeffries
1974a Qualitative stability and digraphs in model ecosystems. *Ecology* 55 (1974), 1415–1419.

Sufficient (and necessary) conditions for sign stability in terms of negative cycles and a novel color test. Proofs are sketched or (for necessity) absent. (**SD: QSta**)

1993a Some matrix patterns arising in queuing theory. In: Richard A. Brualdi, Shmuel Friedland, and Victor Klee, eds., *Combinatorial and Graph-Theoretical Problems in Linear Algebra*, pp. 165–174. IMA Vols. Math. Appl., 50. Springer-Verlag, New York, 1993. MR 1240961 (94e:15056). Zbl 789.60069.

In a weighted symmetric digraph, a cycle is "balanced" if the product of its weights equals the weight product of the inverse cycle (p. 171). If all cycles of length ≥ 3 are balanced, stability multipliers exist in an associated differential system (Thms. 9, 10). [For weights a_{ij}, define gains $\varphi(v_i, v_j) := a_{ij}/a_{ji}$. Then "balance" is balance in the gain graph. *Question*: What can be made of this?] [Annot. 13 Apr 2009.] (**gg: bal**)

Clark Jeffries, Victor Klee, and Pauline van den Driessche

1977a When is a matrix sign stable? *Canad. J. Math.* 29 (1977), 315–326. MR 56 #5603. Zbl 383.15005. (**SD: QSta**)

Clark Jeffries and P. van den Driessche

1988a Eigenvalues of matrices with tree graphs. *Linear Algebra Appl.* 101 (1988), 109–120. MR 941299 (89i:05198). Zbl 686.05037.

A is a real matrix whose bipartite graph is a forest. The signed digraph $\vec{\Sigma}(A)$ yields information about eigenvalues. Controllability of solutions of $\dot{x}(t) = Ax(t)$ may be deduced from $\vec{\Sigma}(A)$. [Annot. 24 July 2010.] (**QM: SD**)

Eva Jelínková and Jan Kratochvíl

2008a On switching to H-free graphs. In: Hartmut Ehrig *et al.*, eds., *Graph Transformations* (4th Int. Conf., ICGT 2008, Leicester, U.K., 2008), pp. 379–395. Lect. Notes in Computer Sci., Vol. 5214. Springer-Verlag, Berlin, 2008. Zbl 1175.68298.

Characterizing graph switching classes that contain a graph with no H subgraph, for some particular H. [An example of Kratochvíl, Nešetřil, and Zýka (1992a).] [Annot. 21 Mar 2011.] (**TG: Sw**)

Robin Jenkins

See P. Abell.

Pablo Jensen

See S. Gómez.

Paul A. Jensen and J. Wesley Barnes

1980a *Network Flow Programming.* Wiley, New York, 1980. MR 82f:90096. Zbl 502.90057. Repr.: Robert E. Krieger, Melbourne, Fla., 1987. MR 89a:90152.

§1.4: "The network-with-gains model." §2.8: "Networks with gains—-example applications." Ch. 9: "Network manipulation algorithms for the generalized network." Ch. 10: "Generalized minimum cost flow problems." (**GN: M(bases)**)

§5.5: "Negative cycles." (**OG: M(bases)**)

1984a *Potokovoe programmirovanie.* Radio i Svyaz, Moskva, 1984. Zbl 598.90035.

Russian translation of (1980a). (**GN: M(bases)**) (**OG: M(bases)**)

P.A. Jensen and Gora Bhaumik

1977a A flow augmentation approach to the network with gains minimum cost flow problem. *Management Sci.* 28 (1976/77), no. 6 (Feb., 1977), 631–643. MR 55 #14163. Zbl 352.90024. (**GN**)

Tommy R. Jensen and Bjarne Toft

1995a *Graph Coloring Problems.* Wiley, New York, 1995. MR 95h:05067. Zbl 950.45277.

§8.14: "t-perfect graphs." Related to all-negative Σ with no subgraph homeomorphic to $-K_4$ (no "odd-K_4"). See Gerards and Schrijver (1986a), Gerards and Shepherd (1998b). (**sg: Par: Geom, Str**)

§13.4: "Bouchet's 6-flow conjecture" (for signed graphs). See Bouchet (1983a), Khelladi (1987a). **(SG: Flows)**

§15.9: "Square hypergraphs." Related to nonexistence of even cycles in a digraph and to sign nonsingularity. See Seymour (1974a) and Thomassen (1985a, 1986a, 1992a). **(sd: Par: bal, QSol: Exp)**

Mark Jerrum and Alistair Sinclair

1990a Polynomial-time approximation algorithms for the Ising model (extended abstract). In: Michael S. Paterson, ed., *Automata, Languages and Programming* (Proc. 17th Int. Colloq., Warwick, 1990), pp. 462–475. Lect. Notes in Computer Sci., Vol. 443. Springer-Verlag, Berlin, 1990. MR 1076810 (91e:68004) (book). Zbl 764.65091.

Extended abstract of (1993a). [Annot. 26 June 2011.] **(sg: Fr, Phys)**

1993a Polynomial-time approximation algorithms for the Ising model. *SIAM J. Comput.* 22 (1993), no. 5, 1087–1116. MR 1237164 (94g:82007). Zbl 782.05076.

§6, "Completeness results": The problem Ising is to find the partition function $\sum_{\zeta:V\to\{+1,-1\}} 2^{-\beta H(\Sigma^\zeta)}$ of a signed simple graph Σ, where $H(\Sigma^\zeta) = \sum_{vw\in E} \sigma^\zeta(vw)$. Thm. 14 suggests nonexistence of certain approximation algorithms. [Annot. 26 June 2011.] **(sg: Fr, Phys)**

R.H. Jeurissen

1975a Covers, matchings and odd cycles of a graph. *Discrete Math.* 13 (1975), 251–260. MR 54 #168. Zbl 311.05129.

Involves the negative-circle edge-packing number of $-\Gamma$. **(par: Fr)**

1981a The incidence matrix and labellings of a graph. *J. Combin. Theory Ser. B* 30 (1981), 290–301. MR 83f:05048. Zbl 409.05042, (457.05047).

The rank of the incidence matrix of a signed graph, in arbitrary characteristic, generalizing the all-negative results of Doob (1974a). Employs column operations on the incidence matrix. Application to magic labellings, where at each vertex a number (in a ring) is specified; the value of an edge is added if it enters the vertex and subtracted if it departs. §5, "Generalizations": "Mixed" graphs, really signed graphs. §6: A new proof of Doob's (1973a) theorem on the multiplicity of -2 as a line-graph eigenvalue in arbitrary characteristic. **(sg, ori: Incid, Adj(LG))**

1983a Disconnected graphs with magic labellings. *Discrete Math.* 43 (1983), 47–53. MR 84c:05064. Zbl 499.05053.

The graphs, called "mixed", are bidirected graphs without introverted edges. Dictionary: " 'bipartite' " = balanced (as a signed graph; the term "balanced" is herein used with another meaning). **(sg, ori: incid)**

1983b Pseudo-magic graphs. *Discrete Math.* 43 (1983), 207–214. MR 84g:05122. Zbl 514.05054.

Mostly, the graphs are all-negative signed graphs (oriented to be extroverted). §5, "Labelings of mixed graphs", discusses bidirected graphs without introverted edges; as in the undirected problem, the (signed-graphically) balanced and unbalanced cases differ. **(sg, ori: Incid)**

1988a Magic graphs, a characterization. *European J. Combin.* 9 (1988), 363–368. MR 89f:05138. Zbl 657.05065.

Connected graphs with magic labellings are classified, separately for bipartite and nonbipartite graphs [as one might expect, due to the connection with the incidence matrix of $-\Gamma$; see Stewart (1966a)]. **(par: incid)**

William S. Jewell

1962a Optimal flow through networks with gains. *Operations Res.* 10 (1962), 476–499. MR 26 #2325. Zbl (e: 109.38203). **(GN)**

Anthony M. Jezior

See J.J. Jarvis.

Samuel Jezný and Marián Trenkler

1983a Characterization of magic graphs. *Czechoslovak Math. J.* 33(108) (1983), 435–438. MR 85c:05030. Zbl 571.05030.

A weak characterization of magic graphs. [See Jeurissen (1988a) for a stronger characterization.] **(par: Incid)**

Shengjin Ji

See B. Huo.

Guangfeng Jiang and Jianming Yu

2004a Supersolvability of complementary signed-graphic hyperplane arrangements. *Australasian J. Combin.* 30 (2004), 261–276. MR 2080474 (2005j:05042). Zbl 1054.05049.

Characterizes supersolvability of $G(K_n, \sigma)$. [A special case of Zaslavsky (2001a).] **(SG: Geom: m)**

Guangfeng Jiang, Jianming Yu, and Jianghua Zhang

2008a Poincaré polynomial of a class of signed complete graphic arrangements. In: Konno, Kazuhiro *et al.*, eds., *Algebraic Geometry in East Asia—Hanoi 2005* (Proc. 2nd Int. Conf. Algebraic Geometry in East Asia, Hanoi, 2005), pp. 289–297. Adv. Stud. Pure Math., Vol. 50. Mathematical Society of Japan, Tokyo, 2008. MR 2409562 (2009j:52024). Zbl 1144.52025.

The chromatic polynomial of K_{K_3}, i.e., $+K_n$ with a triangle changed to negative edges. It factors integrally except for a cubic factor. [See Zaslavsky (1982c), §7, for a graph-theoretic treatment of such examples. One hopes for a direct proof by adding positive vertices in sequence to $-K_3$. *Problem.* Evaluate $\chi_\Sigma(\lambda)$ where Σ is Σ_1 with a new vertex positively adjacent to all vertices of Σ_1.] [Annot. 25 Feb 2012.] **(SG: Geom, Invar)**

Jing-Jing Jiang

See S.W. Tan and X.L. Wu.

Raúl D. Jiménez

See O. Rojo.

Xian'an Jin and Fuji Zhang

2005a The Kauffman brackets for equivalence classes of links. *Adv. Appl. Math.* 34 (2005), no. 1, 47–64. MR 2102274 (2005j:57009). Zbl 1060.05041.

They compute the Read–Whitehead chain polynomial of a sign-colored graph in which, for each divalent vertex, the two incident edges have the same color. This is applied to get the Kauffman bracket of small link diagrams. [*Cf.* Yang and Zhang (2007a).] **(SGc: Invar, Knot)**

2007a The replacements of signed graphs and Kauffman brackets of link families. *Adv. Appl. Math.* 39 (2007), no. 2, 155–172. MR 2333646 (2009b:57005). Zbl 1129.57004. arXiv:math/0511326. **(SGc: Invar, Knot)**

Ya-Lei Jin

See B.A. He.

Peter Joffe

See A.J. Hoffman.

Manas Joglekar, Nisarg Shah, and Ajit A. Diwan

2010a Balanced group labeled graphs. In: *International Conference on Recent Trends in Graph Theory and Combinatorics (ICRTBC-2010)* (Cochin, 2010) [Summaries], pp. 120–121. Dept. of Mathematics, Cochin Univ. of Science and Technology, 2010.

Extended abstract of (20xxa). [Annot. 13 Jan 2012.] (**SG, VS: Bal**)

††2012a Balanced group labeled graphs. Recent Trends in Graph Theory and Combinatorics (Cochin, 2010). *Discrete Math.* 312 (2012), no. 9, 1542–1549.

Γ has weights $w : V \cup E \to \mathfrak{A}$ where $\mathfrak{A}$ is an abelian group. ($\mathfrak{A} = \mathbb{Z}_2$ is signs.) "Balance" = harmony: the sum around every circle = 0. Thm. 1: There are $|fA|^{|V|+t-c(\Gamma)}$ harmonious labellings, where $t :=$ number of edge 3-components of Γ. Lemma 2. If (Γ, w) is balanced and u, v are edge 3-connected in Γ, then $2w(P) = w(u) + w(v)$ for every uv-path. [Annot. 30 Aug 2010.] (**GGw: Bal**)

Thm. 3 is a construction for all edge 2-connected Γ such that $\exists$ harmonious sign labelling, not all $+$. [The best characterization of consistent vertex signatures as in Beineke and Harary (1978b), improving on Hoede (1992a).] [Annot. 30 Aug 2010.] (**SG, VS: Bal**)

Rolf Johannesson

See I.E. Bocharova.

David John

1998a Minimal edge cuts to induce balanced signed graphs. Proc. Twenty-ninth Southeastern Int. Conf. Combinatorics, Graph Theory and Computing (Boca Raton, Fla., 1998). *Congr. Numer.* 132 (1998), 5–8. MR 99j:05178. Zbl 991.53332.

Polynomial-time algorithms to decide balance of a signed graph [this has long been known; see e.g. Hansen (1978a)] and allegedly to find the minimum number of negative edges whose deletion makes the graph balanced [call this the 'negative frustration index']. Contract the positive edges, leaving a graph consisting of the negative edges. To detect balance, look for bipartiteness of the contraction. [Inferior to the standard algorithm.] For negative frustration index, find a maximum cut of the contraction. [Something is wrong, since Max Cut is NP-complete and negative frustration index contains Max Cut. I believe the algorithm finds a nonmaximum cut.] (**SG: Bal, Fr: Alg**)

Eugene C. Johnsen

1989a The micro-macro connection: Exact structure and process. In: Fred Roberts, ed., *Applications of Combinatorics and Graph Theory to the Biological and Social Sciences*, pp. 169–201. IMA Vols. Math. Appl., Vol. 17. Springer-Verlag, New York, 1989. MR 90g:92089. Zbl 725.92026 (*q.v.*).

An elaborate classificatory analysis of "triads" (signed complete directed graphs of 3 vertices) vis-á-vis "macrostructures" (signed complete directed graphs) with reference to structural interactions and implications of triadic numerical restrictions on "dyads" (s.c.d.g. of 2 vertices).

Connections to certain models of affect in social psychology. ["Impenetrability! That's what *I* say!" "Would you tell me, please," said Alice, "what that means?"] (**KG, SD, SG: Bal, PsS: Exp**)

Eugene C. Johnsen and H. Gilman McCann

1982a Acyclic triplets and social structure in complete signed digraphs. *Social Networks* 3 (1982), 251–272.

Balance and clustering analyzed via triples rather than edges. [Possible because the digraph is complete. A later analysis via triples is in Doreian and Krackhardt (2001a).] (**SD: Bal, Clu**)

Charles R. Johnson

See also P.J. Cameron and C.A. Eschenbach.

1983a Sign patterns of inverse nonnegative matrices. *Linear Algebra Appl.* 55 (1983), 69–80. MR 719863 (86i:15001). Zbl 519.15008. (**SG: QSol**)

Charles R. Johnson, Frank Thomson Leighton, and Herbert A. Robinson

1979a Sign patterns of inverse-positive matrices. *Linear Algebra Appl.* 24 (1979), 75–83. (**SG: QSol**)

Charles R. Johnson and John Maybee

1991a Qualitative analysis of Schur complements. In: *Applied Geometry and Discrete Mathematics: The Victor Klee Festschrift*, pp. 359–365. DiMACS Ser. Discrete Math. Theor. Computer Sci., Vol. 4. Amer. Math. Soc., Providence, 1991. MR 92h:15004. Zbl 742.15009.

In square matrix A let $A[S]$ be the principal submatrix with rows and columns indexed by S. Thm. 1: Assume $A[S]$ is sign-nonsingular in standard form and $i, j \notin S$. Then the (i, j) entry of the Schur complement of $A[S]$ has sign determined by the sign pattern of A iff, in the signed digraph of A, every path $i \to j$ via S has the same sign. (**QM: sd**)

Charles R. Johnson, William D. McCuaig, and David P. Stanford

1995a Sign patterns that allow minimal semipositivity. *Linear Algebra Appl.* 223–224 (1995), 363–373. MR 1340701 (96g:15021). Zbl 829.15017. (**SG: QM: QSol**)

Charles R. Johnson, Michael Neumann, and Michael J. Tsatsomeros

1996a Conditions for the positivity of determinants. *Linear Multilinear Algebra* 40 (1996), 241–148. MR 1382081 (97a:15014). Zbl 866.15001. (**SD: QM**)

Charles R. Johnson, D.D. Olesky, Michael Tsatsomeros, and P. van den Driessche

1993a Spectra with positive elementary symmetric functions. *Linear Algebra Appl.* 180 (1993), 247–261. MR 94a:15028. Zbl 778.15006.

Suppose the signed digraph D of an $n \times n$ matrix has longest cycle length k and all cycles of $-D$ are negative. Theorem: If $k = n - 1$, the eigenvalues lie in a domain subtending angle $< 2\pi/k$. This is known for $k = 2$ but false for $k = n - 3$. (**QM, SD**)

Charles R. Johnson, Frank Uhlig, and Dan Warner

1982a Sign patterns, nonsingularity, and the solvability of $Ax = b$. *Linear Algebra Appl.* 47 (1982), 1–9. MR 672727 (84h:15005). Zbl 488.15002. (**SG: QSol**)

David S. Johnson

1983a The NP-completeness column: An ongoing guide. *J. Algorithms* 4 (1983), 87–100.

§4, Problem 3, "Ground state of a spin glass": Is the "ground state spin energy" of a weighted signed graph $\leq K$? NP-complete for weights ± 1

on a two-layer cubic lattice; *cf.* Barahona (1982a). Related problems. [Annot. 18 Jun 2012.] (**SG: wg, Fr, Alg**)

Ellis L. Johnson

See also J. Edmonds and G. Gastou.

1965a Programming in networks and graphs. Report ORC 65-1, Operations Research Center, Univ. of California, Berkeley, Calif., Jan. 1965.

§7: "Flows with gains." §8: "Linear programming in an undirected graph." §9: "Integer programming in an undirected graph."
(**GN: Incid, M(bases)**)(**ec: Incid, M(bases), Alg**)

1966a Networks and basic solutions. *Operations Res.* 14 (1966), 619–623. (**GN**)

Ellis L. Johnson and Sebastiano Mosterts

1987a On four problems in graph theory. *SIAM J. Algebraic Discrete Methods* 8 (1987), 163–185. MR 88d:05097. Zbl 614.05036.

Two of the problems: Given a signed graph (edges called "even" and "odd" rather than "positive" and "negative"). The co-postman problem is to find a minimum-cost deletion set (of edges). The "odd circuit" problem is to find a minimum-cost negative circle. The Chinese postman problem is described in a way that involves cobalance and "switching" around a circle. (**SG: Fr(Gen), Incid**)

Ellis L. Johnson and Manfred W. Padberg

1982a Degree-two inequalities, clique facets, and biperfect graphs. In: Achim Bachem, Martin Grotschel, and Bernhard Korte, eds., *Bonn Workshop on Combinatorial Optimization* (Fourth, 1980), pp. 169–187. North-Holland Math. Studies, 66. Ann. Discrete Math., 16. North-Holland, Amsterdam, 1982. MR 84j:05085. Zbl 523.52009.

Geometry of the bidirected stable set polytope $P(\mathrm{B})$ (which generalizes the stable set polytope to bidirected graphs), defined as the convex hull of $0,1$ solutions of $x_i + x_j \leq 1$, $-x_i - x_j \leq -1$, $x_i \leq x_j$ for extroverted, introverted, and directed edges of B. (Thus, undirected graphs correspond to extroverted bidirected graphs.) It suffices to treat transitively closed bidirections of simple graphs ([unfortunately] called "bigraphs"). [Such a bidirected graph must be balanced.] A "biclique" (S_+, S_-) is the Harary bipartition of a balanced complete subgraph (S_+, S_- are the source and sink sets of the subgraph). It is "strong" if no external vertex has an edge directed out of every vertex of S_+ and an edge directed into every vertex of S_-. Strong bicliques generate facet inequalities of the polytope. Call B perfect if these facets (and nonnegativity) determine $P(\mathrm{B})$. Γ is "biperfect" if every transitively closed bidirection B of Γ is perfect. Conjectures: Γ is biperfect iff it is perfect. Γ is perfect iff some transitively closed bidirection is perfect. [Both proved by Sewell (1996a) and independently by Ikebe and Tamura (20xxa). See e.g. Tamura (1997a), Conforti (20xxa) for further work.]
(**sg: Ori: Incid, Geom, sw**)

Mohammadreza Jooyandeh, Dariush Kiani, and Maryam Mirzakhah

2009a Incidence energy of a graph. *MATCH Commun. Math. Comput. Chem.* 62 (2009), no. 3, 561–572. MR 2568740 (2010j:05238). (**par: Incid**)

Heather Jordon [Heather Gavlas]

See also G. Chartrand and D. Hoffman.

Heather Jordon, Richard McBride, and Shailesh Tipnis

2009a The convex hull of degree sequences of signed graphs. *Discrete Math.* 309 (2009), no. 19, 5841–5848. MR 2551962 (2010k:05120). Zbl 1208.05043.

Consider signed simple graphs of order n. $P_n :=$ polytope determined by the inequalities from Hoffman and Jordon (2006a) that characterize net degree vectors. Thm. 2.7: $P_n =$conv(net degree vectors). Thm. 2.9: Each vertex of $P_n \leftrightarrow$ a unique signed graph, which is a signed K_n. §3: Comparison with net degree vectors of digraphs. [As in other papers on net degree sequences, the best viewpoint is that "signed" edges are oriented negative edges and "directed" edges are oriented positive edges.] [Annot. 1 Oct 2009.] (**SG: ori: Invar: Geom**)

Leif Kjær Jørgensen

1989a Some probabilistic and extremal results on subdivisions and odd subdivisions of graphs. *J. Graph Theory* 13 (1989), 75–85. MR 90d:05186. Zbl 672.05070.

Let $\sigma_{\mathrm{op}}(\Gamma)$, or $\sigma_{\mathrm{odd}}(\Gamma)$, be the largest s for which $-\Gamma$ contains a subdivision of $-K_s$ (an "odd-path-K_sS"), or $[-\Gamma]$ contains an antibalanced subdivision of K_s (an "odd-K_sS"). Thm. 4: $\sigma_{\mathrm{op}}(\Gamma), \sigma_{\mathrm{odd}}(\Gamma) \approx \sqrt{n}$. Thms. 7, 8 (simplified): For $p = 4, 5$ and large enough $n = |V|$, $\sigma_{\mathrm{odd}}(\Gamma) \geq p$ or Γ is a specific exceptional graph. *Conjecture* 9. The same holds for all $p \geq 4$. [*Problem.* Generalize this to signed graphs.] (**par: Xtreml**)

Shalini Joshi

See B.D. Acharya.

Tadeusz Józefiak and Bruce Sagan

1992a Free hyperplane arrangements interpolating between rootsystem arrangements. In: *Séries formelles et combinatoirealgébrique* (Actes du colloque, Montréal, 1992), pp. 265–270.Publ. Lab. Combin. Inform. Math., Vol. 11. Dép. de math. et d'informatique, Univ. de Québec à Montréal, 1992.

Summarizes the freeness results in (1993a).

(**sg, gg: Geom, m, Invar**)

1993a Basic derivations for subarrangements of Coxeter arrangements. *J. Algebraic Combin.* 2 (1993), 291–320. MR 94j:52023. Zbl 798.05069.

The hyperplane arrangements (over fields with characteristic $\neq 2$) corresponding to certain signed graphs are shown to be "free". Explicit bases and the exponents are given. The signed graphs are: $+K_{n-1} \subseteq \Sigma_1 \subseteq +K_n$ (known), $\pm K_n \subseteq \Sigma_2 \subseteq \pm K_n^\circ$, $\pm K_n \subseteq \Sigma_3 \subseteq \pm K_n^\circ$; also, those obtained from $+K_n$ or K_n° by adding all negative links in the order of their larger vertex (assuming ordered vertices) (Thms. 4.1, 4.2) or smaller vertex (Thms. 4.4, 4.5); and those obtained from $\pm K_{n-1}$ by adding positive edges ahead of negative ones (Thm. 4.3). [For further developments see Edelman and Reiner (1994a).] Similar theorems hold for complex arrangements when the sign group is replaced by the complex s-th roots of unity (§5). The Möbius functions of Σ_2, known from Hanlon (1988a), are deduced in §6. (**sg, gg: Geom, m, Invar**)

Michael Jünger

See F. Barahona, C. De Simone, and M. Grötschel.

Mark Jungerman and Gerhard Ringel

1978a The genus of the n-octahedron: Regular cases. *J. Graph Theory* 2 (1978), 69-75. MR 58 #5315. Zbl 384.05037.

"Cascades": see Youngs (1968b). (**sg: Ori: Appl**)

Dieter Jungnickel

See C. Fremuth-Paeger.

Samuel Jurkiewicz

See M.A.A. de Freitas.

James Justus

2005a Qualitative scientific modeling and loop analysis. *Philosophy of Science* 72 (2005), 1272–1286. MR 2295282 (2007j:00008).

Philosophical discussion of qualitative differential equations with emphasis on Levins (1975a). [Annot. 9 Sept 2010.] (**SD: QM: QSta: Exp**)

Jerald A. Kabell

See also F. Harary.

1985a Co-balance in signed graphs. *J. Combin. Inform. System Sci.* 10 (1985), 5–8. MR 89i:05232. Zbl 635.05028.

Cobalance means that every cutset has positive sign product. Thm.: Σ is cobalanced iff every vertex star has evenly many negative edges. For planar graphs, corollaries of this criterion and Harary's bipartition theorem result from duality. [The theorem follows easily by looking at the negative subgraph.] (**SG: Bal(D), Bal**)

1988a An algorithmic look at cycles in signed graphs. 250th Anniversary Conf. on Graph Theory (Fort Wayne, Ind., 1986). *Congressus Numerantium* 63 (1988), 229–230. MR 90d:05143. Zbl 666.05046. (**SG, SD: Bal: Alg**)

Jeff Kahn and Joseph P.S. Kung

1980a Varieties and universal models in the theory of combinatorial geometries. *Bull. Amer. Math. Soc. (N.S.)* 3 (1980), 857–858. MR 81i:05051. Zbl 473.05025.

Announcement of (1982a). (**gg: M**)

††1982a Varieties of combinatorial geometries. *Trans. Amer. Math. Soc.* 271 (1982), 485–499. MR 84j:05043. Zbl 503.05010. Repr. in: Joseph P.S. Kung, *A Source Book in Matroid Theory*, pp. 395–409, with commentary, pp. 335–338. Birkhäuser, Boston, 1986. MR 88e:05028. Zbl 597.05019.

A "variety" is a class closed under deletion, contraction, and direct summation and having for each rank a "universal model", a single member containing all others. There are two nontrivial types of variety of finite matroids: matroids representable over GF(q), and gain-graphic matroids with gains in a finite group $\mathfrak{G}$. The universal models of the latter are the Dowling geometries $Q_n(\mathfrak{G})$.

It is incidentally proved (§7, pp. 490–492) that Dowling geometries of non-group quasigroups cannot exist in rank $n \geq 4$. (**gg: M**)

1986a A classification of modularly complemented geometric lattices. *European J. Combin.* 7 (1986), 243–248. MR 87i:06026. Zbl 614.05018.

A geometric lattice of rank ≥ 4, if not a projective geometry with a few points deleted, is a Dowling lattice. (**gg: M**)

Jeff Kahn and Roy Meshulam

1998a On the number of group-weighted matchings. *J. Algebraic Combin.* 7 (1998), 285–290. MR 99b:05113. Zbl 899.05042.

Continues Aharoni, Meshulam, and Wajnryb (1995a) (*q.v.*, for definitions), generalizing its Thm. 1.3 (the case $|K| = 2$ of the following).

Let m = number of 0-weight matchings, δ = minimum degree. Thm. 1.1: If $m > 0$ then $m \geq (\delta - k + 1)!$ where $k = |K|$. *Conjecture* 1.2. k can be reduced. (See the paper for details.) [*Question.* Is there a generalization to weighted digraphs? One could have two kinds of arcs: some weighted from K, and some weighted 0. The perfect matching might be replaced by an alternating Hamilton cycle or a spanning union of disjoint alternating cycles.] (**WG**)

Thm. 2.1: Let D be a simple digraph with weights in an abelian group K. If all outdegrees are $> k$, where $k = |K|$, then there is a nonempty set of disjoint cycles whose total weight is 0. (**WD**)

Naonori Kakimura

2010a Matching structure of symmetric bipartite graphs and a generalization of Pólyas problem. *J. Combin. Theory Ser. B* 100 (2010), 650–670. MR 2718684 (2011j:-05265). Zbl 1208.05112.

Symmetric matching theory of a bipartite graph with left-right symmetry, with a symmetric Mendelsohn–Dulmage theorem. [A symmetrically bipartite graph Γ' is the signed covering graph of an all-negative signed graph $-\Gamma$, possibly with half edges. A symmetrical matching in Γ' corresponds to a subgraph of $-\Gamma$ with maximum degree 1. *Problem.* Develop the symmetric matching theory of any graph with an involutory, fixed-point-free automorphism in terms of a matching theory of signed graphs with half edges.] [Annot. 29 Sept 2011.] (**sg: cov: Str**)

D. Kalita

See also R.B. Bapat.

D. Kalita and S. Pati

2012a On the spectrum of 3-colored digraphs. *Linear and Multilinear Algebra* 60 (2012), no. 6, 743–756.

Theorem: $\operatorname{Spec} A(\tilde{\Sigma}) = \operatorname{Spec} A(\Sigma) \cup \operatorname{Spec} A(|\Sigma|)$. [Also in Bilu and Linial (2006a).] [Annot. 13 Jan 2012.] (**SG: Cov, Adj**)

M. Kamaraj

See M. Parvathi.

Daniel Kandel, Radel Ben-Av, and Eytan Domany

†1990a Cluster dynamics for fully frustrated systems. *Phys. Rev. Lett.* 65 (1990), no. 8, 941–944.

A new probabilistic algorithm for clustering in a ground state (a function $s : V \to \{+1, -1\}$ such that $|E^-| = l(\Sigma)$) of an all-negative ("fully frustrated") square lattice Σ. A "cluster" in s is a partition of V such that switching any part does not change $|E^-|$. The objective is to join vertices connected by satisfied edges but not those joined by frustrated edges; this cannot be solved uniquely for any unbalanced Σ, so previous methods (used for balanced Σ), e.g., nearest-neighbor moves in state space ("single spin flips"), are ineffective (see p. 942, col. 1; p. 943, col. 2). The algorithm depends on the square lattice structure since it works on squares ("plaquettes"); it succeeds because it works through plaquettes instead of edges (p. 943, col. 2). [*Problem*: Do state-space algorithms help to approximate signed-graph clustering in the sense of Davis (1967a)? Finding a ground state is NP-hard in general, though not for planar signed graphs (*cf.* Katai and Iawi (1978a), Barahona (1982a)).] [Annot. 18 Jun 2012.] (**Phys, SG: Clu: Alg**)

Vikram Singh Kapil
See R.P. Sharma.

Ajai Kapoor
See M. Conforti.

Roman Kapuscinski
See P. Doreian.

D. Karapetyan
See G. Gutin.

Mehran Kardar
See L. Saul.

Richard M. Karp, Raymond E. Miller, and Shmuel Winograd

1967a The organization of computations for uniform recurrence equations. *J. Assoc. Computing Machinery* 14 (1967), 563–590. MR 38 #2920. Zbl (e 171.38305).

Implicitly, concerns the existence of nonpositive directed tours (closed trails) in a $\mathbb{Z}^d$-gain graph (the "dependence graph" of a system of recurrences). (**gd: cov**)

Alexander V. Karzanov
See M.A. Babenko and A.V. Goldberg.

Yasuhiro Kasai, Ayao Okiji, and Itiro Syozi

1981a The ground state of a replicated Ising system. *Progress Theor. Phys.* 65 (1981), no. 4, 1439–1442. MR 620472 (82h:82030).

Grand partition function := $\sum_\theta \exp(|E^-| - |E^+|)$ over all edge signatures θ and all switchings of a lattice graph, investigated for a physical phase via multiple replicates and analytic continuation. [The relevance to signed graphs is obscured by summing over all signatures.] [Annot. 17 Aug 2012.] (**Phys: SG, Fr**)

1981b Ising replicated system of $\pm J$ model. *Progress Theor. Phys.* 66 (1981), no. 5, 1561–1573. MR 642957 (83b:82081).

Similar to (1981a), without analytic continuation. §2 recapitulates (1981a). §3 does calculations for the path graph. §4, "The ground state". [Annot. 17 Aug 2012.] (**Phys: SG, Fr**)

P.W. Kasteleyn
See also C.M. Fortuin.

P.W. Kasteleyn and C.M. Fortuin

1969a Phase transitions in lattice systems with random local properties. In: *International Conference on Statistical Mechanics* (Proc., Kyoto, 1968), pp. 11–14. Supplement to *J. Physical Soc. Japan*, Vol. 26, 1969. Physical Society of Japan, [Tokyo?], 1969.

A specialization of the parametrized dichromatic polynomial of a graph: $Q_\Gamma(q, p; x, 1)$ where $q_e = 1 - p_e$. [Essentially, announcing Fortuin and Kasteleyn (1972a).] (**sgc: Gen: Invar, Phys**)

Osamu Katai
See also S. Iwai.

1979a Studies on aggregation of group structures and group attributes through quantification methods. D.Eng. dissertation, Kyoto Univ., 1979.

Osamu Katai and Sousuke Iwai

1978a Studies on the balancing, the minimal balancing, and the minimum balancing processes for social groups with planar and nonplanar graph structures. *J.*

Math. Psychology 18 (1978), 140–176. MR 83m:92072. Zbl 394.92027.

Balance and detecting balance are discussed at length. Finding the frustration index $l(\Sigma)$ is solved for planar graphs by converting it into a matching problem in the dual graph with signed vertices. This applies also when edges are weighted by positive reals. [Barahona (1982a) and Barahona, Maynard, Rammal, and Uhry (1982a) have a similar, later, but independent solution for the planar frustration index. Barahona (1981a, 1990a) solves toroidal graphs.]

The nonplanar problem is treated via $A(\Sigma)$, but amounts to finding $\min_\zeta(|E^+(\Sigma^\zeta)|-|E^-(\Sigma^\zeta)|)$ [which is NP-hard]. This suggests an iterative procedure which consists of switching $v \in V$ that minimizes $d^\pm(v)$, and repeating; it may not attain the true minimum. [Mitra (1962a) also proposed this.] [Annot. 22 Jun 2012.]

(**SG, CVS, WG, CPsS: Bal, Fr, Alg, Adj, sw**)

1978c On the characterization of balancing processes of social systems and the derivation of the minimal balancing processes. *IEEE Trans. Systems Man Cybernetics* SMC-8 (1978), 337–348. MR 57 #18886 (*q.v.*). Zbl 383.92025.

A shorter version of (1978a). Lem. 1 [restated]: Σ is balanced iff it switches to all positive. [Annot. 22 Jun 2012.]

(**SG, CVS, WG, CPsS: Bal, Fr, Alg, Adj, sw**)

1978d Characterization of social balance by statistical and finite-state systems theoretical analysis. In: *Proceedings of the International Conference on Cybernetics and Society* (Tokyo, 1978). IEEE, 1978. (**SG, WG, CPsS: Bal, Fr**)

Louis H. Kauffman

See also J.R. Goldman.

1986a Signed graphs. Abstract 828-57-12, *Abstracts Amer. Math. Soc.* 7 (1986), no. 5, p. 307.

Announcement of (1989a). (**SGc: Knot: Invar**)

1988a New invariants in the theory of knots. *Amer. Math. Monthly* 95 (1988), 195–242. MR 89d:57005. Zbl 657.57001.

A leisurely development of Kauffman's combinatorial bracket polynomial of a link diagram and the Jones and other knot polynomials, including the basics of (1989a). (**Knot, SGc: Invar: Exp**)

†1989a A Tutte polynomial for signed graphs. *Discrete Appl. Math.* 25 (1989), 105–127. MR 91c:05082. Zbl 698.05026.

The Tutte polynomial, also called "Kauffman's bracket of a signed graph" and equivalent to his bracket of a link diagram, is defined by a sum over spanning trees of terms that depend on the signs and activities of the edges and nonedges of the tree. The point is that the deletion-contraction recurrence over an edge has parameters dependent on the color of the edge; also, the parameters of the two colors are related. The purpose is to develop the bracket of a link diagram combinatorially. §3.2, "Link diagrams": how link diagrams correspond to signed plane graphs. §4, "A polynomial for signed graphs", defines the general sign-colored graph polynomial $Q[\Sigma](A,B,d)$ by deletion-contraction, modified multiplication on components, and evaluation on graphs of loops and isthmi. §5, "A spanning tree expansion for $Q[G]$" [G means Σ], proves $Q[\Sigma]$ exists by producing a spanning-tree expansion, shown independent of the edge ordering by a direct argument. [No dichromatic form of $Q[\Sigma]$ appears; but see successor articles.] §6, "Conclusion", remarks that $Q[\Sigma]$ is invariant under signed-graphic Reidemeister moves II and III. [This significant work, inspired by Thistlethwaite (1988a), led to independent but related generalizations by Przytycka and Przytycki (1988a), Schwärzler and Welsh (1993a), Traldi (1989a), and Zaslavsky (1992b) that were partially anticipated by Fortuin and Kasteleyn (1972a). Also see (1997a).] (**SGc: Invar, Knot**)

1997a Knots and electricity. In: S. Suzuki, ed., *Knots '96* (Proc. Fifth Int. Research Inst., Math. Soc. Japan, Tokyo, 1996), pp. 213–230. World Scientific, Singapore, 1997. MR 99m:57006. Zbl 967.57007.

§2, "A state summation for classical electrical networks", uses a form of the parametrized dichromatic polynomial $Q_\Gamma(B, A; 1, 1)$ [as in Zaslavsky (1992b) *et al.*], where $A(e), B(e) \in \mathbb{C}^\times$, to compute conductances as in Goldman and Kauffman (1993a). (**sgc: Gen: Invar: Exp**)

§3: "The bracket polynomial", discusses the connections with signed graphs and electricity. *Problem*: Is there a signed graph, not reducible by signed-graphic Reidemeister moves (see (1989a)) to a tree with loops, whose sign-colored dichromatic polynomial is trivial? If not, the Jones polynomial detects the unknot. (**SGc: Invar: Exp**)(**SGc: Invar**)

Ken-ichi Kawarabayashi

See also M. Chudnovsky.

Ken-Ichi Kawarabayashi and Atsuhiro Nakamoto

2007a The Erdős–Pósa property for vertex- and edge-disjoint odd cycles in graphs on orientable surfaces. *Discrete Math.* 307 (2007), no. 6, 764–768. MR 2291454 (2007h:05084). Zbl 1112.05056. (**sg: Par: Circles, Top**)

Ken-Ichi Kawarabayashi and Bruce Reed

2009a Highly parity linked graphs. *Combinatorica* 29 (2009), no. 2, 215–225. MR 2520281 (2010k:05157). Zbl 1212.05143. (**sg: Par: Str**)

B. Kawecka-Magiera

See M.J. Krawczyk.

Christine A. Kelley and Joerg Kliewer

20xxa Algebraic constructions of graph-based nested codes from protographs. Submitted. arXiv:1006.2977. (**GG**)

John G. Kemeny and J. Laurie Snell

1962a *Mathematical Models in the Social Sciences*. Blaisdell, Waltham, Mass., 1962. Repr.: MIT Press, Cambridge, Mass., 1972. MR 25 #3797. Zbl (256.92003).

Chapter VIII: "Organization theory: Applications of graph theory." See pp. 97–101 and 105–107. (**SG: Bal: Exp**)

A. Joseph Kennedy

See also M. Parvathi.

2007a Class partition algebras as centralizer algebras of wreath products. *Comm. Algebra* 35 (2007), no. 1, 145–170. MR 2287557 (2008j:16072). Zbl 1151.20006. (**gg: m: Algeb**)

John W. Kennedy

See M.L. Gargano.

Jeff L. Kennington and Richard V. Helgason

1980a *Algorithms for Network Programming.* Wiley, New York, 1980. MR 82a:9013. Zbl 502.90056.

Ch. 5: "The simplex method for the generalized network problem." **(GN: M(Bases): Exp)**

Anne-Marie Kermarrec and Christopher Thraves

2011a Can everybody sit closer to their friends than their enemies? In: Filip Murlak and Piotr Sankowski, eds., *Mathematical Foundations of Computer Science 2011* (36th Int. Symp., Warsaw), pp. 388–399. Lecture Notes in Computer Science, Vol. 6907. Springer, Heidelberg, 2011.

Can (K_n, σ) be drawn in $\mathbb{R}^l$ so every positive neighbor is closer than every negative neighbor, for each vertex? Polynomial-time algorithm for $l = 1$. [Continued by Cygan, Pilipczuk, *et al.* (20xxa).] [Annot. 26 Apr 2012.] **(SG: KG: Bal, Alg, Clu)**

Julie Kerr

1999a A basis for the top homology of a generalized partition lattice. *J. Algebraic Combin.* 9 (1999), 47–60. MR 2000k:05265. Zbl 921.05063.

The lattice is isomorphic to the semilattice of k-composed partitions of a set with a top element adjoined. (See R. Gill (1998a).) **(gg: m: Geom, Top)**

H. Kharaghani

2003a On a class of symmetric balanced generalized weighing matrices. *Designs Codes Cryptogr.* 30 (2003), no. 2, 139–149. MR 2004j:05027. Zbl 1036.05016.

A "balanced generalized weighing matrix" is the group-ring adjacency matrix $\hat{A}$ of a gain digraph $\vec{\Phi}$, with finite gain group $\mathfrak{G}$, such that $\hat{A}\hat{A}^* = kI + ls(J - I)$ where $s := \sum_{g \in \mathfrak{G}} g$. Constructs examples of $\hat{A}$ where $\mathfrak{G}$ is cyclic and $\vec{\Phi}$ is symmetric with no loops. [The article does not mention gain digraphs.] **(gg: Adj)**

F. Kharari and È. Palmer [Frank Harary and Edgar M. Palmer]

See F. Harary and E.M. Palmer (1977a).

A. Khelladi

1987a Nowhere-zero integral chains and flows in bidirected graphs. *J. Combin. Theory Ser. B* 43 (1987), 95–115. MR 88h:05045. Zbl 617.90026.

Improves the result of Bouchet (1983a) about nowhere-zero integral flows on a signed graph. Σ has such an 18-flow if 4-connected, a 30-flow if 3-connected and without a positive triangle, and in some cases a 6-flow (proving Bouchet's conjecture in those cases). **(SG: M: Flows)**

1999a Colorations généralisées, graphes biorientés et deux ou trois choses sur François. Symposium à la Mémoire de François Jaeger (Grenoble, 1998). *Ann. Inst. Fourier (Grenoble)* 49 (1999), 955–971. MR 2000h:05083. Zbl 917.05026.

Comments on the results of Bouchet (1983a) and Khelladi (1987a). **(SG: M, Flows)**

Dariush Kiani

See I. Gutman, H. Hamidzade, M. Jooyandeh, and M. Mirzakhah.

Kathleen P. Kiernan

See R.A. Brualdi.

Dongseok Kim and Jaeun Lee

2008a The chromatic numbers of double coverings of a graph. *Discrete Math.* 308 (2008), no. 22, 5078–5086. MR 2450445 (2009k:05082). Zbl 1158.05026. (**SG: Col, Cov**)

Eun Jung Kim

See N. Alon.

Harunobu Kinoshita

See T. Yamada.

Shin'ichi Kinoshita

See also T. Yajima.

Shin'ichi Kinoshita and Hidetaka Terasaka

1957a On unions of knots. *Osaka Math J.* 9 (1957), 131–153. MR 20 #4846. Zbl 080.17001.

Employs the sign-colored graph of a link diagram from Bankwitz (1930a) to form certain combinations of links. (**SGc: Knot**)

M. Kirby

See A. Charnes.

Steve Kirkland

See also M.A.A. de Freitas, C.S. Oliveira, and J. Stuart.

2011a Sign patterns for eigenmatrices of nonnegative matrices. *Linear Multilinear Algebra* 59 (2011), no. 9, 999–1018. MR 2826068 (2012j:15049). (**QM, SD**)

Steve Kirkland, J.J. McDonald, and M.J. Tsatsomeros

1996a Sign-patterns which require a positive eigenvalue. *Linear Multilinear Algebra* 41 (1996), no. 3, 199–210. MR 1430028 (97j:15009). (**QM, SD**)

Steve Kirkland and Debdas Paul

2011a Bipartite subgraphs and the signless Laplacian matrix. *Appl. AnalḊiscrete Math.* 5 (2011), no. 1, 1–13. MR 2809028 (2012c:05191). (**Par: Adj: Adj, incid**)

Scott Kirkpatrick

See also D. Sherrington and J. Vannimenus.

1977a Frustration and ground-state degeneracy in spin glasses. *Phys. Rev. B* 16 (1977), no. 10, 4630–4641. (**Phys: SG, Fr, Sw**)

Scott Kirkpatrick and David Sherrington

1978a Infinite-ranged models of spin-glasses. *Phys. Rev. B* 17 (1978), no. 11, 4384–4403. Repr. in M. Mézard, G. Parisi, and M.A. Virasoro, *Spin Glass Theory and Beyond*, pp. 109–128. World Scientific Lect. Notes in Physics, Vol. 9. World Scientific, Singapore, 1987.

Random edge weights and signs on K_n. Most interesting: § VI, "Statics for $T \neq 0$", where the "energy" (frustration-index $l(\Sigma)$) landscape of random signs is described, based on computer experiments, as consisting of deep valleys, each having several local minima of l separated by slightly higher ridges, and with high-l barriers separating the valleys. [Presumably, the distance function is Hamming distance between reduced sign functions, i.e., those with $E^- = l$.] [This picture, while convincing, has never been proved; it remains an object of intense curiosity. *Cf.* Marvel, Kleinberg, Kleinberg, and Strogatz (2011a,b).] [Annot. 22 Aug 2012.] (**Phys: sg: Fr**)

Victor Klee

See also C. Jeffries.

1971a The greedy algorithm for finitary and cofinitary matroids. In: Theodore S. Motzkin, ed., *Combinatorics*, pp. 137–152. Proc. Sympos. Pure Math., Vol. 19. Amer. Math. Soc., Providence, R.I., 1971. MR 48 #10865. Zbl 229.05031.

Along with Simões-Pereira (1972a), invents the bicircular matroid (here, for infinite graphs). (**Bic**)

1989a Sign-patterns and stability. In: Fred Roberts, ed., *Applications of Combinatorics and Graph Theory to the Biological and Social Sciences*, pp. 203–219. IMA Vols. Math. Appl., Vol. 17. Springer-Verlag, New York, 1989. MR 90h:34081. Zbl 747.05057.

When are various forms of stability of a linear differential equation $\dot{x} = Ax$ determined solely by the sign pattern of A? A survey of elegant combinatorial criteria. Signed digraphs [alas] play but a minor role. (**QSta, SD: Exp, Ref**)

1993a Open Problem 2. In: Richard A. Brualdi, Shmuel Friedland, and Victor Klee, eds., *Combinatorial and Graph-Theoretical Problems in Linear Algebra*, p. 257. IMA Vols. Math. Appl., 50. Springer-Verlag, New York, 1993. MR 1240954 (94d:00012) (book). Zbl 780.00017 (book).

A question about sign solvability that generalizes "the infamous even cycle problem." [Annot. 13 Apr 2009.] (**sd: QSol, QSta**)

Victor Klee, Richard Ladner, and Rachel Manber

1984a Signsolvability revisited. *Linear Algebra Appl.* 59 (1984), 131–157. MR 86a:-15004. Zbl 543.15016. (**SD, QM: QSol, Alg**)

Victor Klee and Pauline van den Driessche

1977a Linear algorithms for testing the sign stability of a matrix and for finding Z-maximum matchings in acyclic graphs. *Numer. Math* 28 (1977), 273–285. Zbl 348.65032, (352.65020). (**SD: QM, QSta, Alg**)

Jon M. Kleinberg

See D. Easley, J. Leskovec, and S.A. Marvel.

Robert D. Kleinberg

See S.A. Marvel.

Peter Kleinschmidt and Shmuel Onn

1995a Oriented matroid polytopes and polyhedral fans are signable. In: Egon Balas and Jens Clausen, eds., *Integer Programming and Combinatorial Optimization* (4th Int. IPCO Conf., Copenhagen, 1995, Proc.), pp. 198–211. Lect. Notes in Computer Sci., Vol. 920. Springer, Berlin, 1995. MR 97b:05040.

In a graded partially ordered set with 0 and 1, assign a sign to each covering pair (x, y) where y is covered by 1. This is an "exact signing" if in every upper interval there is just one y whose coverings are all positive. Then the poset is "signable". (**Sgnd: Geom**)

1996a Signable posets and partitionable simplicial complexes. *Discrete Comput. Geom.* 15 (1996), 443–466. MR 97a:52014. Zbl 853.52010.

See (1995a) for definition. Signability is a generalization to posets of partitionability of a simplicial complex (Prop. 3.1). Shellable posets, and face lattices of spherical polytopes and oriented matroid polytopes, are signable. A stronger property of a simplicial complex, "total signability", which applies for instance to simplicial oriented matroid polytopes

(Thm. 5.12), implies the upper bound property (Thm. 4.4). Computational complexity of face counting and of deciding shellability and partitionability are discussed in §6. (**Sgnd: Geom, Alg**)

Daniel J. Kleitman

See F.R.K. Chung.

Joseph B. Klerlein

See also R.L. Hemminger.

1975a Characterizing line dipseudographs. In: F. Hoffman *et al.*, eds., *Proceedings of the Sixth Southeastern Conference on Combinatorics, Graph Theory and Computing* (Boca Raton, 1975), pp. 429–442. Congressus Numerantium, XIV. Utilitas Math. Publ. Inc., Winnipeg, Man., 1975. MR 53 #190. Zbl 325.05106.

Continues the topic of Hemminger and Kerlein (1977a). (**sg: LG, ori**)

Joerg Kliewer

See C.A. Kelley.

Darwin Klingman

See J. Elam, F. Glover, and J. Hultz.

Elizabeth Klipsch

20xxa Some signed graphs that are forbidden link minors for orientation embedding. Manuscript in preparation.

For each $n \geq 5$, either $-K_n$ or its 1-edge deletion, but not both, is a forbidden link minor. Which one it is, is controlled by Euler's polyhedral formula, provided $n \geq 7$. [A long version with excruciating detail is available.] (**SG: Top, Par**)

Ton Kloks, Haiko Müller, and Kristina Vušković

2009a Even-hole-free graphs that do not contain diamonds: A structure theorem and its consequences. *J. Combin. Theory Ser. B* 99 (2009), 733–800. MR 2522592 (2010j:05345). Zbl 1218.05160.

A decomposition theorem for graphs without induced even circles and $K_4 \setminus e$'s. [*Question.* Does it make sense to generalize to signed graphs without chordless balanced circles (longer than 3?) or $[K_4 \setminus e]$'s?] [Annot. 10 Mar 2011.] (**par: Str**)

Lori Koban [Lori Fern]

See also L. Fern.

2004a Comments on "Supersolvable frame-matroid and graphic-lift lattices" by T. Zaslavsky. *European J. Combin.* 25 (2004), 141–144. MR 2004k:05054. Zbl 1031.05032.

Correction to Thm. 2.1 and an improved (and corrected) proof of Thm. 2.2 of Zaslavsky (2001a). (**GG: M**)

2004b *Two Generalizations of Biased Graph Theory: Circuit Signatures and Modular Triples of Matroids, and Biased Expansions of Biased Graphs.* Doctoral dissertation, State Univ. of New York at Binghamton, 2004. MR 2706325 (no rev).

Chapter 1: "Circuit signatures and modular triples." When can gains be applied to matroids, as they are to graphs in Zaslavsky (1991a), to produce a linear class of circuits and hence a lift matroid? Theorem 1.4.1: When the group has exponent > 2, one needs a ternary circuit signature, thus a ternary matroid. Theorem 1.4.5: When the group has exponent 2 the matroid must be binary (no circuit signature is required).

(**M: GG: Gen**)

Ch. 2: "Biased expansions of biased graphs." Generalizes group and biased expansions of a graph and the chromatic (and bias-matroid characteristic) polynomial formulas (Zaslavsky 1995b, 20xxj) to expansions of a biased graph. Ch. 3: "When are biased expansions actually group expansions?" Partial results about characterizing biased expansions of biased graphs that are group expansions; counterexamples to several plausible conjectures. (**GG: M, Invar, Geom**)

2008a A modular triple characterization of circuit signatures. *European J. Combin.* 29 (2008), no. 1, 159–170. MR 2368623 (2008k:05040). Zbl 1127.05021.

Four kinds of circuit signatures of a matroid can be characterized through modular triples of copoints or circuits. They are lift signatures as well as the previously known weak orientations, orientations, and ternary signatures. Lifting signatures are needed to define a matroid with gains and thereby a lift matroid determined by the gains.

(**GG: Gen, M**)

William Kocay and Douglas Stone

1993a Balanced network flows. *Bull. Inst. Combin. Appl.* 7 (1993), 17–32. MR 1206759 (93j:05148). Zbl 804.05057.

Balanced network = signed covering graph of $-\Gamma$ with edges vw lifted to $\overrightarrow{+v, -w}$ and added source and sink. [Annot. 8 Mar 2011.] (**sg: cov**)

1995a An algorithm for balanced flows. *J. Combin. Math. Combin. Comput.* 19 (1995), 3–31. MR 1358494 (96j:90087). Zbl 841.68098.

Continuation of (1993a). [Annot. 8 Mar 2011.] (**sg: cov: Alg**)

Muralidharan Kodialam and James B. Orlin

1991a Recognizing strong connectivity in (dynamic) periodic graphs and its relation to integer programming. In: *Proceedings of the Second Annual ACM-SIAM Symposium on Discrete Algorithms* (San Francisco, 1991), pp. 131–135. Assoc. for Computing Machinery, New York, 1991. Zbl 800.68639.

Linear programming methods to find the strongly connected components of a periodic digraph from the static graph: i.e., of the covering digraph of a gain digraph Φ with gains in $\mathbb{Q}^d$ by looking at Φ. *Cf.* Cohen and Megiddo (1993a), whose goals are similar but algorithms differ.

(**GD(Cov): Bal, Circles: Alg**)

Vijay Kodiyalam, R. Srinivasan, and V.S. Sunder

2000a The algebra of G-relations. *Proc. Indian Acad. Sci., Math. Sci.* 110 (2000), no. 3, 263–292. MR 1781906 (2001k:16019) (*q.v.*). Zbl 992.16015. (**gg: Algeb, m**)

János Komlós

1997a Covering odd cycles. *Combinatorica* 17 (1997), 393–400. MR 99b:05114. Zbl 902.05036.

Sharp asymptotic upper bounds on frustration index and vertex frustration number for all-negative signed graphs with fixed negative girth. Improves Bollobás, Erdős, Simonovits, and Szemerédi (1978a). [*Problem.* Generalize to arbitrary signed graphs or signed simple graphs.]

(**Par: Fr**)

Helene J. Kommel

See F. Harary.

Dénes König

1936a *Theorie der endlichen und unendlichen Graphen: Kombinatorische Topologie der Streckenkomplexe.* Mathematik und ihre Anwendungen, Band 16. Akademische Verlagsges., Leipzig, 1936. Repr.: Chelsea, New York, 1950. MR 12, 195. Zbl 13, 228 (e: 013.22803).

§ X.3, "Komposition von Büsheln", contains Thms. 9–16 of Ch. X. I restate them in terms of a signature on the edge set; König says subgraph or p-subgraph ("p-Teilgraph") to mean what we would call the negative edge set of a signature or a balanced signature. Instead of signed switching, König speaks of set summation ("composition") with a vertex star ("Büschel"). His theorems apply to finite and infinite graphs except where stated otherwise. Thm. 9: The edgewise product of balanced signatures is balanced. Thm. 10: Every balanced signing of a finite graph is a switching of the all-positive signature. Thm. 11: A signature is balanced iff it has a Harary bipartition [see Harary (1953a)]. Thm. 12 (cor. of 11): A graph is bicolorable iff every circle has even length. [König makes this fundamental theorem a corollary of a signed-graph theorem!] Thm. 13: A signature is balanced if (not only if) every circle of a fundamental system is positive. Thm. 14: A graph with n vertices (a finite number) and c components has 2^{n-c} balanced signings. Thm. 16: The set of all vertex switchings except for one in each finite component of Γ forms a basis for the space of all finitely generated switchings.

(**sg: Bal, sw, Enum**)

1986a *Theorie der endlichen und unendlichen Graphen. Mit einer Abhandlung von L. Euler* Ed. and introd. by H. Sachs, introd. by P. Erdos, biographical essay by T. Gallai [in English]. Teubner-Archiv zur Math., 6. BSB B. G. Teubner, Leipzig, 1986. MR 88i:01168. Zbl 608.05002.

Reprint of (1936a) together with Euler's paper (in Latin and German) on the Königsberg bridges and supplementary material.

(**sg: Bal, sw, Enum**)

1990a *Theory of Finite and Infinite Graphs.* Trans. Richard McCoart, commentary by W.T. Tutte, biographical sketch by T. Gallai. Birkhäuser, Boston, 1990. MR 91f:01026. Zbl 695.05015.

English translation of (1936a). § X.3: "Composition of stars". [The term "Kreis" (circle, meaning circle) is unfortunately translated as "cycle"—one of the innumerable meanings of "cycle".]

(**sg: Bal, sw, Enum**)

Jack H. Koolen

See T.Y. Chung and S. Akbari.

Hideo Kosako, Suck Joong Moon, Katsumi Harashima, and Takeo Ikai

1993a Variable-signed graph. *Bull. Univ. Osaka Pref. Ser. A* 42 (1993), 37–49. MR 96e:05167. Zbl 798.05070.

"Variable-signed graph" = signed simple (di)graph Σ with switching function p and switched graph Σ^p. Known basic properties of switching are established. More interesting: planar duality when $|\Sigma|$ is planar. The planar dual $|\Sigma|*$ inherits the same edge signs; a dual vertex has sign of the surrounding primal face boundary. Property 9 is in effect the statements: (1) If a signed plane graph has f negative face boundaries, then $l(\Sigma) \geq f/2$. (2) If the negative faces fall into two connected groups with oddly many faces in each, (1) can be improved to $\geq f/2+1$. Finally, incidence matrices are studied that are only superficially related to signs. [The paper is hard to interpret due to mathematical imprecision and language difficulty.] (**SG: Sw, fr, D, Incid**)

Alexandr V. Kostochka
See A.A. Ageev and E. Györi.

Balázs Kotnyek
See G. Appa and L. Pitsoulis.

A. Kotzig
1968a Moves without forbidden transitions in a graph. *Mat. Časopis* 18 (1968), 76–80. MR 39 #4038. Zbl (e: 155.31901). (**par: ori**)

Robin Koytcheff
See E. Ziv.

David Krackhardt
See P. Doreian.

Daniel Král', Ondřej Pangrác, and Heinz-Jürgen Voss
2005a A note on group colorings. *J. Graph Theory* 50 (2005), no. 2, 123–129. MR 2006d:05072. Zbl 1077.05044.

The group chromatic number (Lai and Zhang 2002a) $\chi_1(\Gamma) \geq \delta/2 \ln \delta$, where δ = minimum degree. A planar graph may have $\chi_1(\Gamma) = 5$, the maximum allowed by Lai and Zhang (2002b). Etc. (**gg: Col**)

Daniel Král' and Heinz-Jürgen Voss
2004a Edge-disjoint odd cycles in planar graphs. *J. Combin. Theory Ser. B* 90 (2004), 107–120. MR 2041320 (2005d:05089). Zbl 1033.05064.

Thm. 1: For a plane graph Γ, the frustration index $l(-\Gamma) \leq 2\nu'$, where $\nu' :=$ maximum number of edge-disjoint odd circles. [See Fiorini, Hardy, Reed, and Vetta (2007a), Thm. 3.] [Annot. 6 Feb 2011.] (**sg: Par: Fr**)

M.A. Kramer and B.L. Palowitch, Jr.
1987a A rule-based approach to fault diagnosis using the signed directed graph. *AIChE J.* 33 (1987), 1067–1078. MR 88j:94060.

Vertex signs indicate directions of change in vertex variables; signed directed edges describe relations among these directions.

Truth tables for a signed edge as a function of endpoint signs. Algorithms for deducing logical rules about states (assignments of vertex signs) from the signed digraph. Has a useful discussion of previous literature, e.g., Iri, Aoki, O'Shima, and Matsuyama (1979a). (**SD, VS: Appl, Alg, Ref**)

P.L. Krapivsky
See T. Antal.

I. Krasikov
1988a A note on the vertex-switching reconstruction. *Int. J. Math. Math. Sci.* 11 (1988), 825–827. MR 89i:05204. Zbl 663.05046.

Following up Stanley (1985a), a signed K_n is reconstructible from its single-vertex switching deck if its negative subgraph is disconnected [therefore also if its positive subgraph is disconnected] or if the minimum degree of its positive or negative subgraph is sufficiently small. All done in terms of Seidel switching of unsigned simple graphs. (**kg: sw, TG**)

1994a Applications of balance equations to vertex switching reconstruction. *J. Graph Theory* 18 (1994), 217–225. MR 95d:05091. Zbl 798.05039.

Following up Krasikov and Roditty (1987a), (K_n, σ) is reconstructible from its s-vertex switching deck if $s = \frac{1}{2}n - r$ where $r \in \{0, 2\}$ and $r \equiv n \pmod 4$, or $r = 1 \equiv n \pmod 2$; also, if $s = 2$ and the minimum degree of the positive or negative subgraph is sufficiently small. Also, bounds on $|E^-|$ if (K_n, σ) is not reconstructible. Negative-subgraph degree sequence: reconstructible when $s = 2$ and $n \geq 10$. Done in terms of Seidel switching of unsigned simple graphs. **(kg: sw, TG)**

1996a Degree conditions for vertex switching reconstruction. *Discrete Math.* 160 (1996), 273–278. MR 97f:04137. Zbl 863.05056.

If the minimum degrees of its positive and negative subgraphs obey certain bounds, a signed K_n is reconstructible from its s-switching deck. The main bound involves the least and greatest even zeros of the Krawtchouk polynomial $K_s^n(x)$. Done in terms of Seidel switching of unsigned simple graphs. [More details in Zbl.] **(kg: sw, TG)**

Ilia Krasikov and Simon Litsyn

1996a On integral zeros of Krawtchouk polynomials. *J. Combin. Theory Ser. A* 74 (1996), 71–99. MR 97i:33005. Zbl 853.33008.

Among the applications mentioned (pp. 72–73): 2. "Switching reconstruction problem", i.e., graph-switching reconstruction as in Stanley (1985a) etc. 4. "Sign reconstruction problem", i.e., reconstructing a signed graph from its s-edge negation deck, which is the multiset of signed graphs obtained by separately negating each subset of s edges (here called "switching signs", but it is not signed-graph switching); this is a new problem. **(kg: sw, TG)(SG)**

I. Krasikov and Y. Roditty

1987a Balance equations for reconstruction problems. *Arch. Math. (Basel)* 48 (1987), 458–464. MR 88g:050996. Zbl 594.05049.

§2: "Reconstruction of graphs from vertex switching". Corollary 2.3. If a signed K_n is not reconstructible from its s-vertex switching deck, a certain linear Diophantine system (the "balance equations") has a certain kind of solution. For $s = 1$ the balance equations are equivalent to Stanley's (1985a) theorem; for larger s they may or may not be. All is done in terms of Seidel switching of unsigned simple graphs. [Ellingham and Royle (1992a) note a gap in the proof of Lemma 2.5.] **(kg: sw, TG)**

1992a Switching reconstruction and Diophantine equations. *J. Combin. Theory Ser. B* 54 (1992), 189–195. MR 93e:05072. Zbl 702.05062 (749.05047).

Main Theorem. Fix $s \geq 4$. If n is large and (for odd s) not evenly even, every signed K_n is reconstructible from its s-vertex switching deck. Different results hold for $s = 2, 3$. (This is based on and strengthens Stanley (1985a).) Theorems 5 and 6 concern reconstructing subgraph numbers. All done in terms of Seidel switching of unsigned simple graphs. **(kg: sw, TG)**

1994a More on vertex-switching reconstruction. *J. Combin. Theory Ser. B* 60 (1994), 40–55. MR 94j:05090. Zbl 794.05092.

Based on (1987a) and strengthening Stanley (1985a): Theorem 7. A signed K_n is reconstructible if the Krawtchouk polynomial $K_s^n(x)$

"has one or two even roots [lying] far from $n/2$" (the precise statement is complicated). Numerous other partial results, e.g., a signed K_n is reconstructible if $s = \frac{1}{2}(n-r)$ where $r = 0, 1, 3$, or $2, 4, 5, 6$ with side conditions. All is done in terms of Seidel switching of unsigned simple graphs. **(kg: sw, TG)**

Jan Kratochvíl

See also E. Jelínková.

1989a Perfect codes and two-graphs. *Comment. Math. Univ. Carolin.* 30 (1989), no. 4, 755–760. MR 1045906 (91a:05080). Zbl 693.05060.

A two-graph $\mathcal{T}$ has a perfect code if every graph in its switching class has a 1-perfect vertex code (a perfect dominating set). Thm. $\mathcal{T}$ has a perfect code iff one of its graphs is the union of up to 3 disjoint cliques iff $\mathcal{T}$ has no sub-pentagons and no sub-4-cocliques. [Annot. 21 Mar 2011.] **(TG: Sw)**

2003a Complexity of hypergraph coloring and Seidel's switching. In: Hans L. Bodlaender, ed., *Graph-Theoretic Concepts in Computer Science* (29th Int. Workshop, WG 2003, Elspeet, Neth., 2003), pp. 297–308. Lect. Notes in Computer Sci., Vol. 2880. Springer-Verlag, Berlin, 2003. MR 2080089.

Results about properties as in Kratochvíl, Nešetřil, and Zýka (1992a). E.g., switching to a regular graph is NP-complete. [Annot. 21 Mar 2011.] **(TG: Sw)**

Jan Kratochvíl, Jaroslav Nešetřil, and Ondřej Zýka

1992a On the computational complexity of Seidel's switching. In: Jaroslav Nešetřil and Miroslav Fiedler, eds., *Fourth Czechoslovak Symposium on Combinatorics, Graphs and Complexity* (Prachatice, 1990), pp. 161–166. Ann. Discrete Math., Vol. 51. North-Holland, Amsterdam, 1992. MR 93j:05156. Zbl 768.68047.

Is a given graph switching isomorphic to a graph with a specified property? (This is Seidel switching of simple graphs.) Depending on the property, this question may be in P or be NP-complete, whether the original property is in P or is NP-complete. Properties: containing a Hamilton path; containing a Hamilton circle; no induced P_2; regularity; etc. Thm. 4.1: Switching isomorphism and graph isomorphism are polynomially equivalent. **(TG: Sw: Alg)**

M.J. Krawczyk, K. Malarz, B. Kawecka-Magiera, A.Z. Maksymowicz, and K. Kułakowski

2005a Spin-glass properties of an Ising antiferromagnet on the Archimedean $(3, 12^2)$ lattice. *Phys. Rev. B* 72 (2005), article 24445. **(par: Fr)**

Vyacheslav Krushkal

See also P. Fendley.

2011a Graphs, links, and duality on surfaces. *Combin. Prob. Computing* 20 (2011), 267–287. MR 2769192 (2012d:05190). Zbl 1211.05029. arXiv:.

§7, "A multivariate graph polynomial": A partially parametrized rank-generating polynomial ("multivariate Tutte polynomial") for graphs embedded in surfaces, with the somewhat awkward duality relation (7.3). *Cf.* Chmutov and Pak (2007a) and Chmutov (2009a). [Annot. 12 Jan 2012.] **(GGw: Invar)**

Ying-Qiang Kuang

See Z.H. Chen.

Boris D. Kudryashov
See I.E. Bocharova.

Bernard Kujawski, Mark Ludwig, and Peter Abell
20xxa Structural balance dynamics and group formation: An exploratory study. Submitted. (**SG: Bal**)

Krzysztof Kułakowski
See also P. Gawroński, A. Mańka-Krasoń, B. Tadić, and J. Tomkowicz.

2007a Some recent attempts to simulate the Heider balance problem. *Computing in Science and Engineering* 9 (July/Aug. 2007), no. 4, 86–91.

Krzysztof Kułakowski, Premiysław Gawroński, and Piotr Gronek
2005a The Heider balance: a continuous approach. *Int. J. Mod. Phys. C* 16 (2005), no. 5, 707–716. Zbl 1103.91405.

Devadatta M. Kulkarni
See J.W. Grossman.

T.R. Vasanth Kumar
See P. Siva Kota Reddy.

Vijaya Kumar [G.R. Vijayakumar]
See G.R. Vijayakumar.

Jérôme Kunegis, Andreas Lommatzsch, and Christian Bauckhage
2009a The slashdot zoo: mining a social network with negative edges. In: *Proceedings of the 18th International Conference on the World Wide Web* (Madrid, 2009), pp. 741–750. Assoc. for Computing Machinery, New York, 2009. (**SG: WG: Clu: Alg**)

Jérôme Kunegis, Stephan Schmidt, Şahin Albayrak, Christian Bauckhage, and Martin Mehlitz
2008a Modeling collaborative similarity with the signed resistance distance kernel. In: Malik Ghallab *et al.*, eds., *ECAI 2008 – 18th European Conference on Artificial Intelligence*, pp. 261–265. Frontiers in Artificial Intelligence and Applications, Vol. 178. IOS Press, Amsterdam, 2008. (**SG: Adj, Alg**)

Jérôme Kunegis, Stephan Schmidt, Andreas Lommatzsch, Jürgen Lerner, Ernesto W. De Luca, and Sahin Albayrak
2010a Spectral analysis of signed graphs for clustering, prediction and visualization. In: Srinivasan Parthasarathy *et al.*, eds., *Proceedings of the Tenth SIAM International Conference on Data Mining* (Columbus, Ohio, 2010), pp. 559–570. Soc. for Industrial and Appl. Math., 2010. (**SG: Adj, Clu, Geom, Alg**)

Joseph P.S. Kung
See also J.E. Bonin and J. Kahn.

1986a Numerically regular hereditary classes of combinatorial geometries. *Geom. Dedicata* 21 (1986), 85–105. MR 87m:05056. Zbl 591.05019.

Examples include Dowling geometries, Ex. (6.2), and the bias matroids of full group expansions of graphs in certain classes; see pp. 98–99. (**GG: M**)

1986b Radon transforms in combinatorics and lattice theory. In: Ivan Rival, ed., *Combinatorics and Ordered Sets* (Proc., Arcata, Calif., 1985), pp. 33–74. Contemp. Math., Vol. 57. Amer. Math. Soc., Providence, R.I., 1986. MR 88d:05024. Zbl 595.05006.

P. 41: Exposition of Stanley (1985a) from the viewpoint of the finite Radon transform. (**kg: sw, TG**)

1990a Combinatorial geometries representable over GF(3) and GF(q). I. The number of points. *Discrete Comput. Geom.* 5 (1990), 83–95. MR 90i:05028. Zbl 697.51007.

The Dowling geometry over the sign group is the largest simple ternary matroid not containing the "Reid matroid". (**sg: M: Xtreml**)

1990b The long-line graph of a combinatorial geometry. II. Geometries representable over two fields of different characteristic. *J. Combin. Theory Ser. B* 50 (1990), 41–53. MR 91m:51007. Zbl 645.05026.

Dowling geometries used in the proof of Prop. (1.2). (**gg: M**)

1993a Extremal matroid theory. In: Neil Robertson and Paul Seymour, eds., *Graph Structure Theory* (Proc., Seattle, 1991), pp. 21–61. Contemp. Math., Vol. 147. Amer. Math. Soc., Providence, R.I., 1993. MR 94i:05022. Zbl 791.05018.

Survey with new results; largely on size bounds and extremal matroids for certain minor-closed classes. §2.7: "Gain-graphic matroids," i.e., frame matroids of gain graphs. P. 30, top and fn. 9 on extremal gain-graph theory. §4.3: "Varieties." Conj. (4.9)(c) on growth rates. §4.5. "Framed gain-graphic matroids," i.e., cones over ("framed") frame matroids in projective space. §6.1: "Cones," i.e., unions of long lines on a common point: p. 47. Thm. (6.15) is a quadratic bound on matroids whose minors exclude (approximately) $q+2$-point lines and non-frame planes. Conj. (7.1) on directions in $\mathbb{C}^n$-matroids proposes that cyclic Dowling matroids are extremal. §8: "Concluding remarks," on a possible ternary analog of Seymour's decomposition theorem.
(**GG: M: Xtreml, Str, Exp, Ref**)

1993b The Radon transforms of a combinatorial geometry. II. Partition lattices. *Adv. Math.* 101 (1993), 114–132. MR 95b:05051. Zbl 786.05018.

Dowling lattices are lower-half Sperner. The proof is given only for partition lattices. (**gg: M**)

1996a Matroids. In: M. Hazewinkel, ed., *Handbook of Algebra*, Vol. 1, pp. 157–184. North-Holland (Elsevier), Amsterdam, 1996. MR 98c:05040. Zbl 856.05001.

§6.2: "Gain-graphic matroids," i.e., frame matroids of gain graphs.
(**GG: M: Exp**)

1996b Critical problems. In: Joseph E. Bonin, James G. Oxley, and Brigitte Servatius, eds., *Matroid Theory* (Proc., Seattle, 1995), pp. 1–127. Contemp. Math., Vol. 197. Amer. Math. Soc., Providence, R.I., 1996. MR 97k:05049. Zbl 862.05019.

A remarkable more-than-survey with numerous new results and open problems. §4.5: "Abstract linear functionals in Dowling group geometries". §6: "Dowling geometries and linear codes", concentrates on higher-weight Dowling geometries, extending Bonin (1993b). §7.4: "Critical exponents of classes of gain-graphic geometries". §7.5: "Growth rates of classes of gain-graphic geometries". §8.5: "Jointless Dowling group geometries". Cor. 8.30. §8.11: "Tangential blocks in $\mathcal{Z}(A)$". Also see pp. 56, 61, 88, 92, 114. Dictionary: "Gain-graphic matroids" = frame matroids of gain graphs. (**GG, Gen: M**)

1998a A geometric condition for a hyperplane arrangement to be free. *Adv. Math.* 135 (1998), 303–329. MR 2000f:05023. Zbl 905.05017.

Delete from a Dowling geometry a subset S that contains no whole plane. Found: necessary and sufficient conditions for the characteristic polynomial to factor completely over the integers. When the geometry

corresponds to a hyperplane arrangement, many more of the arrangements are not free than are free; however, if S contains no whole line, all are free (so the characteristic polynomial factors completely over $\mathbb{Z}$) while many are not supersolvable. (**gg: M: Invar**)

2000a Critical exponents, colines, and projective geometries. *Combin. Probab. Comput.* 9 (2000), 355–362. MR 2002f:05048. Zbl 974.51008.

Higher-weight Dowling geometries yield counterexamples to a conjecture. (**gg: Gen: M: Invar**)

2001a Twelve views of matroid theory. In: Sungpyo Hong *et al.*, eds., *Combinatorial & Computational Mathematics* (Proc., Pohang, 2000), pp. 56–96. World Scientific, Singapore, 2001. MR 2002i:05028. Zbl 1001.05038.

§5: "Graph theory and lean linear algebra". "Lean" means at most 2 nonzero coordinates, hence gain graphs. §6, "Varieties of finite matroids", summarizes Kahn and Kung (1982a). §7, "Secret-sharing matroids": *Question.* Is the Dowling matroid $Q_n(\mathfrak{G})$ a secret-sharing matroid? (**GG: M**)

§11, "Generic rank-generating polynomials": The "Tugger polynomial" is a partially parametrized rank-generating polynomial (*cf.* Zaslavsky 1992b). (**Sc(M): Gen: Invar**)

2002a Curious characterizations of projective and affine geometries. Special issue in memory of Rodica Simion. *Adv. Appl. Math.* 28 (2002), 523–543. MR 2003c:51008. Zbl 1007.51001.

Dowling geometries $G(\mathfrak{G}K_n^\bullet)$ (if $|\mathfrak{G}| > 2$) and jointless Dowling geometries $G(\mathfrak{G}K_n)$ (if $|\mathfrak{G}| > 4$) exemplify Lemma 3.4, which says that 5 numbers characterize the line sizes in a simple matroid with all lines of size 2, 3, or l. (**gg: M: Invar**)

2006a Minimal blocks of binary even-weight vectors. *Linear Algebra Appl.* 416 (2006), 288–297. MR 2242730 (2008d:05038). Zbl 1115.05012.

§4, "Minimal blocks from graphs": $\mathrm{GF}(q)^\times \cdot \Gamma$ is a minimal k-block over $\mathrm{GF}(q)$ if Γ is minimally j-chromatic for a certain $j = f(k)$, and is a minimal 1-block if Γ is an odd circle. [Annot. 20 June 2011.] (**GG: M**)

Joseph P.S. Kung and James G. Oxley

1988a Combinatorial geometries representable over GF(3) and GF(q). II. Dowling geometries. *Graphs Combin.* 4 (1988), 323–332. MR 90i:05029. Zbl 702.51004.

For $n \geq 4$, the Dowling geometry of rank n over the sign group is the unique largest simple matroid of rank n that is representable over GF(3) and GF(q). (**sg: M: Xtreml**)

David Kuo

See J.H. Yan.

Ranan D. Kuperman

See Z. Maoz.

Jin Ho Kwak

See also I.P. Goulden.

Jin Ho Kwak, Sungpyo Hong, Jaeun Lee, and Moo Young Sohn

2000a Isoperimetric numbers and bisection widths of double coverings of a complete graph. *Ars Combin.* 57 (2000), 49–64. MR 2001h:05083. Zbl 1064.05076.

(**sg: KG: Cov**)

J.H. Kwak and Jaeun Lee

2001a Enumeration of graph coverings, surface branched coverings and related group theory. In: Sungpyo Hong *et al.*, eds., *Combinatorial & Computational Mathematics* (Proc., Pohang, 2000), pp. 97–161. World Scientific, Singapore, 2001. MR 2003b:05083. Zbl 1001.05092.

Voltage graphs (i.e., gain graphs) and their covering graphs ("derived graphs") defined in §1; emphasis on groups and counting group covering graphs of a graph. (**gg: Cov, Top**)

J.H. Kwak, Jaeun Lee, and Young-hee Shin

2004a Balanced regular coverings of a signed graph and regular branched orientable surface coverings over a non-orientable surface. *Discrete Math.* 275 (2004), 177–193. MR 2004i:05036. Zbl 1030.05034.

The number of isomorphism types of regular balanced coverings of a signed graph. A covering is a sign-preserving covering projection from one signed graph to another. (**SG: Top: Enum**)

Domenico Labbate

See M. Abreu.

Martine Labbé

See R.M.V. Figueiredo.

Richard Ladner

See V. Klee.

George M. Lady, Thomas J. Lundy, and John Maybee*

1995a Nearly sign-nonsingular matrices. *Linear Algebra Appl.* 220 (1995), 229–248. MR 1334579 (96e:15007).

The signed digraph $S(A)$ of square matrix A. [Annot. 12 Jun 2012.] (**SD: QM**)

George M. Lady and John S. Maybee

1983a Qualitatively invertible matrices. *Math. Social Sci.* 6 (1983), 397–407. MR 85f:15005. Zbl 547.15002.

In terms of signed graphs, restates and completes the characterizations of sign-invertible matrices A due to Bassett, Maybee, and Quirk (1968a) and George M. Lady (The structure of qualitatively determinate relationships. *Econometrica* 51 (1983), 197–218. MR 85c:90019. Zbl 517.15004) and reveals the sign pattern of A^{-1} in terms of path signs in the associated signed digraph. (**QM: QSol: SD**)

J.C. Lagarias

1985a The computational complexity of simultaneous diophantine approximation problems. *SIAM J. Computing* 14 (1985), 196–209. MR 86m:11048. Zbl 563.10025.

Theorem F: Feasibility of integer linear programs with at most two variables per constraint is NP-complete. (**GN(Incid): D: Alg**)

Hong-Jian Lai

See also Z.H. Chen and Y.T. Liang.

2000a Group connectivity of 3-edge-connected chordal graphs. *Graphs Combin.* 16 (2000), 165–176. MR 2001f:05074. Zbl 966.05041.

Hong-Jian Lai and Xiangwen Li

2006a Group chromatic number of planar graphs of girth at least 4. *J. Graph Theory* 52 (2006), no. 1, 51–72. MR 2006m:05088. Zbl 1088.05032.

Definition: see Lai and Zhang (2002a). Thm. 1.1: If Γ is a plane graph with girth ≥ 4, every 4-circle is a face boundary, and no 4-circle meets another 4-circle or a 5-circle, then $\chi_1(\Gamma) \leq 3$. Thm. 1.2: If Γ has no $K_{3,3}$ minor and has girth ≥ 5, then $\chi_1(\Gamma) \leq 3$. The proofs are by producing $\mathbb{Z}_3$-colorings. (**gg: Col**)

Hong-Jian Lai and Xiankun Zhang

2002a Group colorability of graphs. *Ars Combin.* 62 (2002), 299–317. MR 2003c:05109.

Simple graphs only are considered. The [abelian] "group chromatic number" $\chi_1(\Gamma) = \min m$ such that, for every abelian group $\mathfrak{A}$ of order $\geq m$, every $\mathfrak{A}$-gain graph on Γ is $\mathfrak{A}$-colorable (see Jaeger, Linial, Payan, and Tarsi 1992a). Various results, e.g., Γ is $\mathbb{Z}_2$-colorable iff it is a forest; analog of Brooks' Theorem (a strengthening because $\chi_1 \geq \chi$); analog of Nordhaus–Gaddum Theorem involving the complementary graph. [Thus $\chi_1(\Gamma)$ seems to resemble ordinary chromatic number more than it does gain-graph coloring.] (**gg: Col**)

2002b Group chromatic number of graphs without K_5-minors. *Graphs Combin.* 18 (2002), no. 1, 147–154. MR 2002m:05089. Zbl 993.05073.

Continues (2002a). Thm.: If Γ is simple and has no K_5 minor, then $\chi_1(\Gamma) \leq 5$, improving on Jaeger, Linial, Payan, and Tarsi (1992a). [See Král', Pangrác, and Voss (2005a).] (**gg: Col**)

P. Lallemand

See H.T. Diep.

Kelvin Lancaster

1981a Maybee's "Sign solvability". In: Harvey J. Greenberg and John S. Maybee, eds., *Computer-Assisted Analysis and Model Simplification* (Proc. Sympos., Boulder, Col., 1980), pp. 259–270. Academic Press, New York, 1981. MR 82g:00016 (book). Zbl 495.93001 (book).

Comment on Maybee (1981a). (**QM: QSol: SD**)

Steven Landy

1988a A generalization of Ceva's theorem to higher dimensions. *Amer. Math. Monthly* 95 (Dec., 1988), no. 10, 936–939. MR 90c:51020. Zbl 663.51011.

The theorem characterizes concurrence of lines drawn from each vertex of a rectilinear simplex to a point in the opposite side. [*Problem.* Reformulate, maybe generalize, in terms of gain graphs. *Cf.* Boldescu (1970a), Zaslavsky (2003b) §2.6.] (**gg: Geom**)

Andrea S. LaPaugh and Christos H. Papadimitriou

1984a The even-path problem for graphs and digraphs. *Networks* 14 (1984), 507–513. MR 86g:05057. Zbl 552.68059.

Fast algorithms for existence of even paths between two given vertices (or any two vertices) of a graph. The corresponding digraph problem is NP-complete. [Signed (di)graphs are similar, due to the standard reduction by negative subdivision.] [See also, e.g., works by Thomassen.] (**Par: Paths: Alg**)(**sd: Par: Paths: Alg**)

Michel Las Vergnas

See A. Björner.

Martin Lätsch and Britta Peis

2008a On a relation between the domination number and a strongly connected bidirection of an undirected graph. *Discrete Appl. Math.* 156 (2008), 3194–3202. MR 2468789 (2010a:05139). Zbl 1176.05058.

A bidirected graph (Γ, τ) (where τ assigns $+$ or $-$ to each incidence) is "strongly connected" if there is a coherent walk from any vertex to any other vertex. The distance $\mathrm{dist}_{(\Gamma,\tau)}(u, v) :=$ the minimum length of a coherent $\vec{uv}$ walk. The diameter $\mathrm{diam}(\Gamma, \tau) := \max_{(u,v)\in V^2} \mathrm{dist}_{(\Gamma,\tau)}(u, v)$. In Γ define $i :=$ number of isthmi, $\gamma :=$ domination number. Thm. 5: Γ has a strongly connected bidirection iff $|V| = 1$ or Γ is connected and minimum degree ≥ 2. Thm. 10: If Γ has strongly connected bidirections τ_j $(j = 1, \ldots, k)$, then $\min_i \mathrm{diam}(\Gamma, \tau_j) \leq 2i + 2\min(i, 1) + 5\gamma - 1$. When $i = 0$, τ_j can be chosen so $\Sigma(\Gamma, \tau_j)$ is all positive. *Conjecture.* Also true when $i > 0$. Thm. 11: If Γ has a strongly connected bidirection, then $\min_j \mathrm{diam}(\Gamma, \tau_j) \leq 6\gamma + 3$. By Fig. 8 this bound must be at least $6\gamma + 1$ if isthmi are allowed. The proofs are constructive, esp. by extending to Γ a bidirection of a dominating subgraph. Dictionary: "path" = walk [not trail]. [Annot. 27 Apr 2007.] (**sg: Ori: Invar**)

Monique Laurent

See M.M. Deza and A.M.H. Gerards.

Eugene L. Lawler

1976a *Combinatorial Optimization: Networks and Matroids.* Holt, Rinehart and Winston, New York, 1976. MR 55 #12005. Zbl 413.90040.

Ch. 6: "Nonbipartite matching." §3: Bidirected flows. (**sg: Ori**)

Ch. 4: "Network flows." §8: "Networks with losses and gains." §12: "Integrality of flows and the unimodular property." (**GN**)(**sg: Incid, Bal**)

Jason Leasure

See L. Fern.

Bruno Leclerc

1981a Description combinatoire des ultramétriques. *Math. Sci. Humaines* No. 73 (1981), 5–37. MR 82m:05083. Zbl 476.05079. (**SG: Bal**)

Gibaek Lee, Sang-Oak Song, and En Sup Yoon

2003a Multiple-fault diagnosis based on system decomposition and dynamic PLS. *Indust. Engin. Chem. Res.* 42 (2003), 6145–6154.

Combines signed digraphs and partial least squares for fault analysis in chemical engineering. (**SD: Appl**)

Jaeun Lee

See I.P. Goulden, D. Kim, and J.H. Kwak.

Jon Lee

1989a Subspaces with well-scaled frames. *Linear Algebra Appl.* 114/115 (1989), 21–56. MR 90k:90111. Zbl 675.90061.

See §9. (**sg: Ori: Incid, Flows, Alg**)

Shyi-Long Lee

See also I. Gutman and P.K. Sahu.

1989a Comment on 'Topological analysis of the eigenvalues of the adjacency matrices in graph theory: A difficulty with the concept of internal connectivity'. *J. Chinese Chem. Soc.* 36 (1989), 63–65.

Response to Gutman (1988a). Proposes weighted net sign: divide by number of nonzero vertex signs. The goal is to have the ordering of net signs correlate more closely with that of eigenvalues. (**VS, SGw, Chem**)

1989b Net sign analysis of eigenvectors and eigenvalues of the adjacency matrices in graph theory. *Bull. Inst. Chem., Academica Sinica* No. 36 (1989), 93–104.

Expounds principally Lee, Lucchese, and Chu (1987a) and Lee and Gutman (1989a). Examples include all connected, simple graphs of order ≤ 4 and some aromatics. (**VS, SGw, Exp, Chem**)

1992a Topological analysis of five-vertex clusters of group IVa elements. *Theoretica Chimica Acta* 81 (1992), 185–199.

See Lee, Lucchese, and Chu (1987a). More examples; again, eigenvalue and net-sign orderings are compared. (**VS, SGw, Chem**)

Shyi-Long Lee and Ivan Gutman

1989a Topological analysis of the eigenvectors of the adjacency matrices in graph theory: Degenerate case. *Chem. Phys. Letters* 157 (1989), 229–232.

Supplements Lee, Lucchese, and Chu (1987a) to answer an objection by Gutman (1988a), by treating vertex signs corresponding to multidimensional eigenspaces. (**VS, SGw, Chem**)

Shyi-Long Lee and Chiuping Li

1994a Chemical signed graph theory. *Int. J. Quantum Chem.* 49 (1994), 639–648.

Varies Lee, Lucchese, and Chu (1987a) by taking net signs of all balanced signings, instead of only those obtained from eigenvectors, for small paths, circles, and circles with short tails. The distribution of net sign, over all signings of each graph, is more or less binomial. (**VS, SGw, Chem**)

1994b On generating molecular orbital graphs: the first step in signed graph theory. *Bull. Inst. Chem., Academica Sinica* No. 41 (1994), 69–75.

Abbreviated presentation of (1994a). (**VS, SGw: Exp**)

Shyi-Long Lee and Feng-Yin Li

1990a Net sign approach in graph spectral theory. *J. Molecular Structure (Theochem)* 207 (1990), 301–317.

Similar topics to S.L. Lee (1989a, 1989b). Several examples of order 6. (**VS, SGw, Exp, Chem**)

1990b Net sign analysis of five-vertex chemical graphs. *Bull. Inst. Chem., Academica Sinica* No. 37 (1990), 83–97.

See Lee, Lucchese, and Chu (1987a). Treats all connected, simple graphs of order 5. (**VS, SGw, Chem**)

Shyi-Long Lee, Feng-Yin Li, and Friday Lin

1991a Topological analysis of eigenvalues of particle [*sic*] in one- and two-dimensional simple quantal systems: Net sign approach. *Int. J. Quantum Chem.* 39 (1991), 59–70.

See Lee, Lucchese, and Chu (1987a). § II: Net signs calculated for paths. §§ III, IV: Planar graphs with two different types of potential, yielding complicated results. (**VS, SG, Chem**)

Shyi-Long Lee, Robert R. Lucchese, and San Yan Chu

1987a Topological analysis of eigenvectors of the adjacency matrices in graph theory: The concept of internal connectivity. *Chem. Phys. Letters* 137 (1987), 279–284. MR 88i:05130. Zbl none.

Introduces the net sign of a (balanced) signed graph. A graph has vertices signed according to the signs of an eigenvector X_i of the adja-

cency matrix, $\mu(v_r) = \mathrm{sgn}(X_{ir})$, and $\sigma(v_r v_s) = \mu(v_r)\mu(v_s)$ [hence Σ is balanced]. Note that a vertex can have 'sign' 0. Net sign of a [hydrocarbon] chemical graph is applied to prediction of properties of molecular orbitals. (**VS, SGw, Chem**)

Shyi-Long Lee, Yeung-Long Luo, and Yeong-Nan Yeh

1991a Topological analysis of some special graphs. III. Regular polyhedra. *J. Cluster Sci.* 2 (1991), 105–116.

See Lee, Lucchese, and Chu (1987a). Net signs for the Platonic polyhedra (Table I). (**VS, SGw, Chem**)

Shyi-Long Lee and Yeong-Nan Yeh

1990a Topological analysis of some special classes of graphs. Hypercubes. *Chem. Phys. Letters* 171 (1990), 385–388.

Follows up Lee, Lucchese, and Chu (1987a) and Lee and Gutman (1989a), calculating net signs of eigenspatially signed hypercube graphs of dimensions up to 6 by means of a general graph-product formula. (**VS, SGw, Chem**)

1993a Topological analysis of some special classes of graphs. II. Steps, ladders, cylinders. *J. Math. Chem.* 14 (1993), 231–241. MR 95f:05079.

See Lee, Lucchese, and Chu (1987a). Net signs and eigenvalues are compared. (**VS, SGw, Chem**)

Frank Thomson Leighton

See C.R. Johnson.

Samuel Leinhardt

See also J.A. Davis and P.W. Holland.

Samuel Leinhardt, ed.

1977a *Social Networks: A Developing Paradigm.* Academic Press, New York, 1977.

An anthology reprinting some basic papers in structural balance theory. (**PsS, SG: Bal, Clu**)

P.W.H. Lemmens and J.J. Seidel

1973a Equiangular lines. *J. Algebra* 24 (1973), 494–512. MR 46 #7084. Zbl 255.50005. Repr. in Seidel (1991a), pp. 127–145.

Hints of graph switching; see van Lint and Seidel (1966a). (**Geom, sw**)

Marianne Lepp [Marianne L. Gardner]

See R. Shull.

Jürgen Lerner

See J. Kunegis.

Jure Leskovec, Daniel Huttenlocher, and Jon Kleinberg

2010a Signed networks in social media. In: *CHI '10: Proceedings of the 28th ACM Conference on Human Factors in Computing Systems* (Atlanta, 2010). Assoc. for Computing Machinery, New York, 2010. (**SD, SG: Bal, Clu**)

2010b Predicting positive and negative links in online social networks. In: *WWW '10: Proceedings of the 19th International Conference on World Wide Web* (Raleigh, N.C., 2010). Assoc. for Computing Machinery, New York, 2010. (**SD: Bal**)

Richard Levins

See also J.M. Dambacher and C.J. Puccia.

1974a The qualitative analysis of partially specified systems. *Ann. N.Y. Acad. Sci.* 231 (1974), 123–138. Zbl 285.93028. (**SD: QM: QSta: Cycles**)

1975a Evolution in communities near equilibrium. In: M. Cody and J.M. Diamond, eds., *Ecology and Evolution of Communities*, pp. 16–50. Harvard Univ. Press, Cambridge, Mass., 1975. (**SD: QM: QSta: Cycles**)

David W. Lewit
See E.G. Shrader.

Josef Leydold
See T. Bıyıkoğlu.

Bao Feng Li
See X.H. Hao.

Bin Li
See T.F. Wang.

Cai Heng Li and Jozef Širáň

2007a Möbius regular maps. *J. Combin. Theory Ser. B* 97 (2007), no. 1, 57–73. MR 2007h:05043. Zbl 1106.05033.

That is, graphs in a surface, that are signed so every edge belongs to a negative digon (Möbius), and whose map automorphisms act transitively on flags (regularity). Properties of their automorphism groups. [Follows Wilson (1989a).] (**SG: Top: Aut**)

Chang Li
See T. Harju.

Chiuping Li
See I. Gutman and S.L. Lee.

Feng-Hin Li
See S.L. Lee.

Hiram W. Li
See J.M. Dambacher.

Hong-Hai Li and Jiong-Sheng Li

2008a An upper bound on the Laplacian spectral radius of the signed graphs. *Discuss. Math. Graph Theory* 28 (2008), no. 2, 345–359. MR 2477235 (2010a:05115). Zbl 1156.05035.

Dictionary: See X.D. Zhang and Li (2002a). [Annot. 23 Mar 2009.] (**SG: incid, Adj**)

2009a Note on the normalized Laplacian eigenvalues of signed graphs. *Australasian J. Combin.* 44 (2009), 153–162. MR 2527006 (2010i:05210). Zbl 1177.05050. (**SG: Adj**)

Ji Li
See H.Z. Deng.

Jing Li
See S.Y. Wang.

Ke Li, Ligong Wang, and Guopeng Zhao

2011a The signless Laplacian spectral radius of tricyclic graphs and trees with k pendant vertices. *Linear Algebra Appl.* 435 (2011), no. 4, 811–822. MR 2807235 (2012f:05179). Zbl 1220.05075. (**Par: Adj**)

2011b The signless Laplacian spectral radius of unicyclic and bicyclic graphs with a given girth. *Electronic J. Combin.* 18 (2011), no. 1, Paper 183, 10 pp. MR 2836818 (2012g:05138). Zbl 1230.05200. (**Par: Adj**)

Jiong-Sheng Li
See Y. Hou, H.H. Li and X.D. Zhang.

Qian Li and Bolian Liu
2008a Bounds on the kth multi-g base index of nearly reducible sign pattern matrices. *Discrete Math.* 308 (2008), 4846–4860. MR 2446095 (2010a:05037). Zbl 1167.15013. (**QM: SD**)

Qian Li, Bolian Liu, and Jeffrey Stuart
2010a Bounds on the k-th generalized base of a primitive sign pattern matrix. *Linear Multilinear Algebra* 58 (2010), no. 3, 355–366. MR 2663436 (2011c:15090). Zbl 1196.15030. (**SD: QM**)

Rao Li
2010a Inequalities on vertex degrees, eigenvalues and (signless) Laplacian eigenvalues of graphs. *Int. Math. Forum* 5 (2010), no. 37-40, 1855–1860. MR 2672449 (no rev). Zbl 1219.05088. (**Par: Adj**)

Ruilin Li and Jinsong Shi
2010a The minimum signless Laplacian spectral radius of graphs with given independence number. *Linear Algebra Appl.* 433 (2010), no. 8-10, 1614–1622. MR 2718223 (2011m:05181). Zbl 1211.05075.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Rui-lin Li, Jin-song Shi, and Bing-can Dong
2011a Maximal signless Laplacian spectral radius of bicyclic graphs with given independence number. (In Chinese?) *J. East China Norm. Univ. Natur. Sci. Ed.* 2011 (2011), no. 3, 73–84, 99. MR 2867304 (no rev). (**Par: Adj**)

Shuchao Li
See also B. Chen, X.Y. Geng, and S.S. He.

Shuchao Li and Yi Tian
2011a On the (Laplacian) spectral radius of weighted trees with fixed matching number q and a positive weight set. *Linear Algebra Appl.* 435 (2011), no. 6, 1202–1212. MR 2807144 (2012f:05180). Zbl 1222.05165.

Weight function $w : E \to \mathbb{R}_{>0}$. Since $\operatorname{Spec} K(\Gamma, w) = \operatorname{Spec} K(-\Gamma, w)$, $K(-\Gamma, w)$ is used to find $\lambda_1(K(\Gamma, w))$. [Annot. 21 Jan 2012.] (**Par: Adj**)

2012a Some bounds on the largest eigenvalues of graphs. *Appl. Math. Lett.* 25 (2012), 326–332. (**Par: Adj**)

Shuchao Li and Shujing Wang
2012a The least eigenvalue of the signless Laplacian of the complements of trees. *Linear Algebra Appl.* 436 (2012), no. 7, 2398–2405. (**Par: Adj**)

Shuchao Li and Li Zhang
2011a Permanental bounds for the signless Laplacian matrix of bipartite graphs and unicyclic graphs. *Linear Multilinear Algebra* 59 (2011), no. 2, 145–158. MR 2773647 (2012a:05194).

Sharp upper and lower bounds for $\operatorname{per}(K(-\Gamma))$ when Γ is unicyclic or bipartite, with or without girth, and characterization of extremal graphs. (Authors' summary.) [Bipartite Γ means they are doing $K(\Gamma)$; the truly signed part is for unicyclic graphs only.] [Annot. 19 Nov 2011.] (**Par: Adj**)

20xxa Permanental bounds for the signless Laplacian matrix of a unicyclic graph with diameter d. *Graphs Combin.*, in press.

See Li and Zhang (2011a). Here, the second minimum of, and a lower bound for, per $K(-\Gamma)$. [Annot. 24 Jan 2012.] (**Par: Adj**)

Shuchao Li and Minjie Zhang

2012a On the signless Laplacian index of cacti with a given number of pendant vertices. *Linear Algebra Appl.* 436 (2012), no. 12, 4400–4411. (**Par: Adj**)

Xiangwen Li

See H.J. Lai.

Xiao Ming Li

See F.T. Boesch.

Xueliang Li

See also W.X. Du and B.F. Hou.

Xueliang Li, Jianbin Zhang, and Lusheng Wang

2009a On bipartite graphs with minimal energy. *Discrete Appl. Math.* 157 (2009), no. 4, 869–873. MR 2499503 (2010f:05116). Zbl 1226.05161.

[Bipartite energy is the energy of $A(\Gamma)$ for bipartite Γ. *Problem* 1. Generalize to antibalanced signed graphs. *Problem* 2. Generalize to signed graphs.] [Annot. 24 Jan 2012.] (**Par: Adj**)

Yiyang Li

See W.X. Du.

Zhongshan Li

See also C.A. Eschenbach and F.J. Hall.

Zhongshan Li, Frank Hall, and Carolyn Eschenbach

1994a On the period and base of a sign pattern matrix. *Linear Algebra Appl.* 212-213 (1994), 101–120. MR 1306974 (95m:15026). Zbl 821.15017.

Chaohua Liang, Bolian Liu, and Yufei Huang

2010a The kth lower bases of primitive non-powerful signed digraphs. *Linear Algebra Appl.* 432 (2010), no. 7, 1680–1690. MR 2592910 (2011b:15076). Zbl 1221.05190. (**SD**)

Yanting Liang, Bolian Liu, and Hong-Jian Lai

2009a Multi-g base index of primitive anti-symmetric sign pattern matrices. *Linear Multilinear Algebra* 57 (2009), no. 6, 535–546. MR 2543715 (2010i:05151. Zbl 1221.15019. (**QM: SD**)

Hans Liebeck

See D. Harries.

Martin W. Liebeck

1980a Lie algebras, 2-graphs and permutation groups. *Bull. London Math. Soc.* 33 (1982), 76–85. MR 81f:05095. Zbl 499.05031.

Examines the F Aut$([\Sigma])$-module $F\,V(\Sigma)$, where Σ is a signed complete graph and F is a field of characteristic 2. (**TG: Aut**)

1982a Groups fixing graphs in switching classes. *J. Austral. Math. Soc. (A)* 33 (1982), 76–85. MR 83h:05048. Zbl 499.05031.

Given an abstract group $\mathfrak{A}$, which of its permutation representations are exposable on every invariant switching class of signed complete graphs [see Harries and H. Liebeck (1978a) for definitions]? (**kg: sw, TG: Aut**)

Thomas M. Liebling

See H. Gröflin.

Rainer Liebmann

†1986a *Statistical Mechanics of Periodic Frustrated Ising Systems.* Lect. Notes in Phys., Vol. 251. Springer-Verlag, Berlin, 1986. MR 850837 (87k:82004).

Detailed and readable descriptions, often simplified and relatively combinatorial, of the state of knowledge about Ising systems in the form of signed graphs and weighted signed graphs. [Relatively accessible to combinatorists.] Dictionary: "model" = graph with signs and usually weights, "ferromagnetic" = positive edge, "antiferromagnetic" = negative edge, "fully frustrated" = all girth circles are negative, "state" = $s : V \to \{+1, -1\}$, "ground state" = state with fewest frustrated edges, "ground state degeneracy" = number of ground states (1 being nondegenerate), "excited state" = non-ground state. §2.1.1, "Ground state degeneracy of the ANNNI-chain", on chains of triangles with two bond signs and strengths, J_1 and J_2. The number and description of ground states are treated in detail, as well as less combinatorial physical quantities. §2.3.1, "Periodic frustrated chains": All weights equal, so this is signed graphs. Restates Doman and Williams (1982a) in terms of a path with distance-2 edges, signed with period 4. The path edges have constant sign (either + or − by switching) and weight B; the distance-2 edges are $+ - --$ with weight J.

§3.1.2b, "Star-triangle transformation": Edge signs and weights transform. The triangle-star transformation on a negative triangle gives imaginary signs. [*Question.* Does this indicate a use for complex unit gains?] §3.2, "Triangular lattice": Based on Houtappel (1950a,b) and Wannier (1950a). §3.3.1, "Union Jack lattice": Square lattice, edges weighted J_1, with alternating diagonals in alternating squares weighted $J_2 < 0$. All triangles are negative. $|J_2|/J_1$ determines behavior. For ratio 1 (a signed graph), there are $\approx C^{|V|}$ ground states for a finite sublattice, where $C \geq \sqrt{(17/8)}$. §3.3.2, "Villain's odd model": *Cf.* Villain (1977a). §3.3.3, "Hexagon lattice": *Cf.* Wolff and Zittartz (1982a, 1983a). §3.3.4, "Pentagon lattice": *Cf.* Waldor, Wolff, and Zittartz (1985a). §3.3.5, "Kagomé lattice": Various periodic sign patterns; references. §3.3.6, "Connection between GS [ground state] degeneracy and existence of a phase transition at $T_c = 0$": The conjecture of Hoever, Wolff, and Zittartz (1981a). Also, a conjecture of Süto on the exact conditions under which the ground states are connected in the state graph. §3.4, "Frustrated Ising systems with crossing interactions": Several more complicated extensions of previous models, usually by adding distance-2 edges ("nnn interactions"). See (2) below.

§4.1, "fcc antiferromagnet": All-negative face-centered cubic lattice graph. Interesting remarks on how ground state and near-ground state structure might influence physical properties. §4.2, "Fully and partially frustrated simple cubic lattice": The fully frustrated planar square lattice can be stacked in various ways to produce differently frustrated cubic lattices. §4.3, "AF pyrochlore model": All-negative tetrahedra joined at corners. §4.4, "ANNNI-model": All-positive cubic lattice with negative distance-2 vertical edges.

Two frequent remarks: (1) An external magnetic field reduces the number of ground states. (2) Slightly more complicated graphs give models that are not exactly solvable. [Combinatorial explanations: The magnetic field corresponds to an extra vertex, positively adjacent to all $V(\Sigma)$; see Barahona (1982a). The more complicated graphs are non-planar; Barahona (1982a) and Istrail (2000a) indicate that this is the obstacle to exact solution.] [Annot. 28 Aug 2012.]

(**Phys, SG: WG: Fr: Exp, Ref**)

Magnhild Lien and William Watkins

2000a Dual graphs and knot invariants. *Linear Algebra Appl.* 306 (2000) 123–130. MR 2000k:05187. Zbl 946.05061.

The Kirchhoff ("Laplacian") matrices of a signed plane graph and its dual have the same invariant factors. The proof is via the signed graphs of knot diagrams. (**SGc: D, Adj, Knot**)

Ko-Wei Lih

See J.H. Yan.

Chjan C. Lim

1993a Nonsingular sign patterns and the orthogonal group. *Linear Algebra Appl.* 184 (1993), 1–12. MR 1209379 (94c:15036). Zbl 782.68098.

A family of bipartite signed wheels that prevent $A = (A^{-1})^{\mathrm{T}}$. A family of bipartite signed graphs which allow it. [Annot. 6 Mar 2011.]

(**SG: QM**)

Meng-Hiot Lim

See Harary, Lim, *et al.*

Leonardo Silva de Lima

See L.S. de Lima (under D).

Enzo M. Li Marzi

See F. Belardo and J.F. Wang.

Friday Lin

See S.L. Lee.

Shangwei Lin

See S.Y. Wang.

Bernt Lindström

See F. Harary.

Nathan Linial

See Y. Bilu, S. Hoory, and F. Jaeger.

Sóstenes Lins

1981a A minimax theorem on circuits in projective graphs. *J. Combin. Theory Ser. B* 30 (1981), 253–262. MR 82j:05074. Zbl 457.05057.

For Eulerian Σ in projective plane, max. number of edge-disjoint negative circles = min. number of edges cut by a noncontractible closed curve that avoids the vertices. [Generalized by Schrijver (1989a).]

(**SG: Top, fr, Alg**)

1982a Graph-encoded maps. *J. Combin. Theory Ser. B* 32 (1982), 171–181. MR 83e:05049. Zbl 465.05031, (478.05040).

See §4. (**sg: Top: bal**)

1985a Combinatorics of orientation reversing circles. *Aequationes Math.* 29 (1985), 123–131. MR 87c:05051. Zbl 592.05019. (**sg, par: Top, Bal, Fr**)

J.H. van Lint and J.J. Seidel

1966a Equilateral point sets in elliptic geometry. *Proc. Koninkl. Ned. Akad. Wetenschap. Ser. A* 69 (= *Indag. Math.* 28) (1966), 335–348. MR 34 #685. Zbl 138, 417 (e: 138.41702). Repr. in Seidel (1991a), pp. 3–16.

Introduces graph switching. (**tg, Geom**)

Svante Linusson

See C.A. Athanasiadis.

Marc J. Lipman and Richard D. Ringeisen

1978a Switching connectivity in graphs. In: F. Hoffman *et al.*, eds., *Proc. of the Ninth Southeastern Conf. on Combinatorics, Graph Theory and Computing* (Boca Raton, 1978), pp. 471–478. Congressus Numerantium, XXI. Utilitas Math. Publ. Inc., Winnipeg, Man., 1978. MR 80k:05073. Zbl 446.05033. (**TG**)

C.H.C. Little

See I. Fischer.

Simon Litsyn

See I. Krasikov.

Charles H.C. Little

See C.P. Bonnington.

Bolian Liu

See also B. Cheng, Y.F. Huang, C. Li, Q.A. Li, Y.T. Liang, J.P. Liu, M.H. Liu, and Z.F. You.

2007a The period and base of a reducible sign pattern matrix. *Discrete Math.* 307 (2007), 3031–3039. MR 2371074 (2009i:15043). Zbl 1127.15018. (**QM: SD**)

Bolian Liu, Muhuo Liu, and Zhifu You

20xxa The majorization theorem for signless Laplacian spectral radii of connected graphs. *Graphs Combin.*, in press.

For a degree sequence π, define $\mu_c(\pi) := \max_\Gamma \lambda_1(K(-\Gamma))$ over connected Γ with degree sequence π and c circles. Let $\pi \preccurlyeq \pi'$ in the majorization ordering. Thm. 2: Under certain assumptions on c, π, π', $\mu(\pi) \leq \mu(\pi')$. For the special cases of unicyclic and bicyclic graphs: X.D. Zhang (2009a) and Huang, Liu, and Liu (2011a). [Also see B.L. Liu and Liu (2012a).] [Annot. 24 Jan 2012.] (**Par: Adj**)

Feng Liu

See X.-J. Tian.

Gui Zhen Liu and Qiang Wu

Applications of graph theory to social science. (In Chinese. English summary.) *Shandong Daxue Xuebao Ziran Kexue Ban* 30 (1995), no. 4, 361–366. MR 97c:05149. Zbl 882.05116.

Describes some applications of and some results about balance in signed graphs. (**SG: Bal, PsS: Exp, M**)

Jianping Liu and Bolian Liu

2008a The maximum clique and the signless Laplacian eigenvalues. *Czechoslovak Math. J.* 58(133) (2008), no. 4, 1233–1240. MR 2471179 (2010a:05116). Zbl 1174.05079.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Jianzhou Liu

See R. Huang.

Jiming Liu

See B. Yang.

Mu Huo Liu
See also B.L. Liu and F.Y. Wei.

Muhuo Liu and Bolian Liu
2010a The signless Laplacian spread. *Linear Algebra Appl.* 432 (2010), no. 2-3, 505–514. MR 2577696 (2011d:05226). Zbl 1206.05064.
See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

2011a On the spectral radii and the signless Laplacian spectral radii of c-cyclic graphs with fixed maximum degree. *Linear Algebra Appl.* 435 (2011), no. 12, 3045–3055. MR 2831596 (2012h:05197). Zbl 1226.05138. (**Par: Adj**)

2012a New method and new results on the order of spectral radius. *Computers Math. Appl.* 63 (2012), no. 3, 679–686. MR 2871667 (2012i:05171).
Also see B.L. Liu, Liu, and You (20xxa). (**Par: Adj**)

20xxa On the signless Laplacian spectral radii of bicyclic and tricyclic graphs. Submitted.
See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Muhuo Liu, Bolian Liu, and Fuyi Wei
2011a Graphs determined by their (signless) Laplacian spectra. *Electron. J. Linear Algebra* 22 (2011), 112–124. MR 2781040 (2012g:05141). Zbl 1227.05185. (**Par: Adj**)

Muhuo Liu, Xuezhong Tan, and Bolian Liu
2010a The (signless) Laplacian spectral radius of unicyclic and bicyclic graphs with n vertices and k pendant vertices. *Czechoslovak Math. J.* 60 (2010), no. 3, 849–867. MR 2672419 (2011f:05185). Zbl 1224.05311.
See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

2011a The largest signless Laplacian spectral radius of connected bicyclic and tricyclic graphs with n vertices and k pendant vertices. (In Chinese.) *Appl. Math. J. Chinese Univ. Ser. A* 26 (2011), no. 2, 215–222. MR 2838952 (2012e:05238). (**Par: Adj**)

20xxa On the ordering of the signless Laplacian spectral radii of unicyclic graphs. *Appl. Math. J. Chinese Univ. Ser. B*, to appear.
See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Ning Liu and William J. Stewart
2011a Markov chains and spectral clustering. In: *Performance Evaluation of Computer and Communication Systems: Milestones and Future Challenges*, pp. 87–98. Lect. Notes in Comput. Sci. Vol. 6821. Springer-Verlag, Berlin, 2011. (**Par: Adj: Appl**)

Ruifang Liu
See M.Q. Zhai.

Xiaogang Liu
See also Y.P. Zhang.

Xiaogang Liu, Suijie Wang, Yuanping Zhang, and Xuerong Yong
2011a On the spectral characterization of some unicyclic graphs. *Discrete Math.* 311 (2011), 2317–2336. (**Par: Adj**)

Yan Pei Liu
See R.X. Hao.

Yingluan Liu
See Y.F. Huang.

Yue Liu
See X.Y. Yuan.

Paulette Lloyd
See P. Bonacich and P. Doreian.

Martin Loebl
See also Y. Crama and A. Galluccio.

Martin Loebl and Iain Moffatt
2008a The chromatic polynomial of fatgraphs and its categorification. *Adv. Math.* 217 (2008), no. 4, 1558–1587. MR 2382735 (2008j:05114). Zbl 1131.05036.

D.O. Logofet and N.B. Ul'yanov
1982a Necessary and sufficient conditions for the sign stability of matrices. (In Russian.) *Dokl. Akad. Nauk SSSR* 264 (1982), 542–546. MR 84j:15018. Zbl 509.15008.
Necessity of Jeffries' (1974a) sufficient conditions. (**QSta**)

D.O. Logofet and N.B. Ul'janov [N.B. Ul'yanov]
1982b Necessary and sufficient conditions for the sign stability of matrices. *Soviet Math. Dokl.* 25 (1982), 676–680. MR 84j:15018. Zbl 509.15008.
English translation of (1982a). (**QSta**)

Michael Lohman
See M. Chudnovsky.

V. Lokesha
See also P. Siva Kota Reddy.

V. Lokesha, P. Siva Kota Reddy, and S. Vijay
2009a The triangular line n-sigraph of a symmetric n-sigraph. *Adv. Stud. Contemp. Math. (Kyungshang)* 19 (2009), no. 1, 123–129. MR 2542128 (2010k:05121). Zbl 1213.05120.
Definitions and notation as in Sampathkumar, Siva Kota Reddy, and Subramanya (2008a). Generalization of Subramanya and Siva Kota Reddy (2009a) to symmetric n-signed graphs, with similar definitions and results. [The results remain true without assuming symmetry.] [Annot. 10 Apr 2009.] (**SG(Gen), gg: Bal, LG(Gen), Sw**)

Andreas Lommatzsch
See J. Kunegis.

M. Loréa
1979a On matroidal families. *Discrete Math.* 28 (1979), 103–106. MR 81a:05029. Zbl 409.05050.
Discovers the "linearly bounded" (or "count") matroids of graphs. [See White and Whiteley (1983a), Whiteley (1996a), Schmidt (1979a).]
(**MtrdF: Bic, Gen**)

E. Loukakis
2003a A dynamic programming algorithm to test a signed graph for balance. *Int. J. Computer Math.* 80 (2003), no. 4, 499–507. MR 1983308. Zbl 1024.05034.
Another algorithm for detecting balance [*cf.* Hansen (1978a), Harary and Kabell (1980a)]. Also, once again proves that all-negative frustration index [obviously equivalent to Max Cut] is NP-complete.
(**SG: Bal, Fr: Alg**)

Janice R. Lourie
1964a Topology and computation of the generalized transportation problem. *Management Sci.* 11 (1965) (Sept., 1964), no. 1, 177–187. (**GN: M(bases)**)

László Lovász

See also J.A. Bondy, Gerards *et al.* (1990a), and M. Grötschel.

1965a On graphs not containing independent circuits. (In Hungarian.) *Mat. Lapok* 16 (1965), 289–299. MR 35 #2777. Zbl 151, 334c (e 151.33403).

Characterization of the graphs having no two vertex-disjoint circles. See Bollobás (1978a) for exposition in English. [*Major Problem.* Characterize the biased graphs having no two vertex-disjoint unbalanced circles. This theorem is the contrabalanced case. For the sign-biased case see Slilaty (2007a). McQuaig (1993a) might be relevant to the general problem.] (**GG: Circles**)

1979a *Combinatorial Problems and Exercises.* North-Holland, Amsterdam, and Akadémiai Kiadó, Budapest, 1979. MR 537284 (80m:05001). Zbl 439.05001.

Prob. 7.21 finds $\mathrm{rk}\,\mathrm{H}(-\Gamma)$ [*cf.* van Nuffelen (1973a)]. Prob. 10.18: The vertex frustration number of a contrabalanced graph vs. the circle edge-packing number. [Annot. 16 Jun 2012.] (**sg: par: Incid**)(**gg: fr**)

1983a Ear-decompositions of matching-covered graphs. *Combinatorica* 3 (1983), 105–117. MR 85b:05143. Zbl 516.05047.

It is hard to escape the feeling that we are dealing with all-negative signed graphs and their $-K_4$ and $-K_2^\circ$ minors. [And indeed, see Gerards and Schrijver (1986a) and Gerards *et al.* (1990a) and the notes on Seymour (1995a).] (**Par: Str**)

1993a *Combinatorial Problems and Exercises*, Second Ed. Elsevier, Amsterdam, and Akadémiai Kiadó, Budapest, 1993. MR 1265492 (94m:05001). Zbl 785.05001.

See (1979a). [Annot. 16 Jun 2012.] (**sg: par: Incid**)(**gg: fr**)

2007a *Combinatorial Problems and Exercises*, Second Ed., corr. reprint. AMS Chelsea Publ., American Mathematical Soc., Providence, R.I., 2007. MR 2321240 (no rev). Zbl 439.05001.

See (1979a). [Annot. 16 Jun 2012.] (**sg: par: Incid**)(**gg: fr**)

2011a Subgraph densities in signed graphons and the local Simonovits–Sidorenko conjecture. *Electronic J. Combin.* 18 (2011), #P127. MR 2811096 (2012f:05158). Zbl 1219.05084. arXiv:1004.3026. (**SG**)

L. Lovász and M.D. Plummer

1986a *Matching Theory.* North-Holland Math. Stud., Vol. 121. Ann. Discrete Math., Vol. 29. Akadémiai Kiadó, Budapest, and North-Holland, Amsterdam, 1986. MR 88b:90087. Zbl 618.05001.

Pp. 247–248: Shortest odd/even uv-path problem in Γ. Lemma 6.6.9 reduces min length of odd path to a min-weight perfect matching problem in a modified graph. Exerc. 6.6.10–11 are similar for even paths and odd/even circles. [*Problem.* Generalize to negative/positive paths and circles in signed graphs.] §6.6, p. 252: $l(-\Gamma)$ [i.e., max cut in Γ], $l(\Sigma)$ for signed planar graphs. Cor. 6.19: For planar Γ, $l(-\Gamma) = \frac{1}{2}$(max number of circles in a 2-packing of negative circles). [*Question*: How does this generalize to signed planar graphs?] Pp. 252–253: Odd-circle packing and 2-packing. [Annot. 10 Nov 2010.] (**sg, par: fr, Paths, Circles: Exp**)

§8.7, pp. 353–354: Weighted non-ferromagnetic Ising model. [Annot. 10 Nov 2010.] (**SG, WG: Phys, fr: Exp**)

2009a *Matching Theory.* AMS Chelsea Publ. (Amer. Math. Soc.), Providence, R.I., 2009. MR 88b:90087. Zbl 618.05001.

Reprint of (1986a) with errata and an appendix of updates. [Annot. 10 Nov 2010.]

(sg: par: Circles, Paths, fr: Exp)(sg, WG: Phys, fr: Exp)

L. Lovász, L. Pyber, D.J.A. Welsh, and G.M. Ziegler

1995a Combinatorics in pure mathematics. In: R.L. Graham, M. Grötschel, and L. Lovász, eds., *Handbook of Combinatorics*, Vol. II, Ch. 41, pp. 2039–2082. North-Holland (Elsevier), Amsterdam, and MIT Press, Cambridge, Mass., 1995. MR 97f:00003. Zbl 851.52017.

§7: "Knots and the Tutte polynomial", considers the signed graph of a knot diagram (pp. 2076–77). **(SGc: Knot)**

Aidong Lu

See L.T. Wu.

Claire Lucas

See M. Aouchiche and P. Hansen.

Robert R. Lucchese

See S.L. Lee.

Henri Luchian

See A. Băutu.

Tomasz Łuczak

See E. Györi.

Mark Ludwig

See also P. Abell and B. Kujawski.

M. Ludwig and P. Abell

2007a An evolutionary model of social networks. *Europ. Phys. J. B* 58 (2007), 97–105.

Signed edges are added to and deleted from a fixed set of nodes under a balancing rule. Imbalance measured by frustrated triangles impels evolution, which converges under some conditions. [Annot. 20 June 2011.] **(SG: Bal, Fr)**

J. Lukic, A. Galluccio, E. Marinari, O.C. Martin, and G. Rinaldi

2004a Critical thermodynamics of the two-dimensional $\pm J$ Ising spin glass. *Phys. Rev. Lett.* 92 (2004), no. 11, #117202.

Physical properties of a signed toroidal square lattice graph, from computation of the exact partition function (energy distribution) via Galluccio, Loebl, and Vondrák (2000a, 2001a). E.g., the approximate proportion of negative edges is important. [Annot. 18 Aug 2012.]

(SG: Phys, Fr)

J. Richard Lundgren

See H.J. Greenberg and F. Harary.

Thomas J. Lundy

See also G.M. Lady.

Thomas J. Lundy, John Maybee, and James Van Buskirk

1996a On maximal sign-nonsingular matrices. *Linear Algebra Appl.* 247 (1996), 55–81. MR 1412740 (97k:15020) (*q.v.*). Zbl 862.15019.

Constructions of such matrices. A matrix definition of C_4-cockades. [Annot. 6 Mar 2011.] **(SG: QSol)**

Rong Luo

See X.D. Zhang.

Yeung-Long Luo

See I. Gutman and S.L. Lee.

Hongping Ma

See also L.Q. Wang.

2009a Bounds on the local bases of primitive, non-powerful, minimally strong signed digraphs. *Linear Algebra Appl.* 430 (2009), no. 2-3, 718–731. MR 2473178 (2009i:05100). Zbl 1151.05020. (**SD: Adj**)

Hongping Ma and Zhengke Miao

2011a Imprimitive non-powerful sign pattern matrices with maximum base. *Linear Multilinear Algebra* 59 (2011), no. 4, 371–390. Zbl 1221.15042. arXiv:. (**SD: Adj**)

M. Ma

See D. Blankschtein.

Xiaobin Ma, Genhong Ding, and Long Wang

20xxa On the nullity and the matching number of unicyclic signed graphs. Submitted.

Further develops Y.'Z. Fan, Wang, and Wang (20xxa). Employs the matching number to express the nullity and to characterize nullity $n-6, n-7$ of a signed unicyclic graph. [Annot. 17 Dec 2011.] (**SG: Adj**)

Enzo Maccioni

See F. Barahona.

Bolette Ammitzbøll Madsen

See J.M. Byskov.

K.V. Madhusudhan

See P. Siva Kota Reddy.

Thomas L. Magnanti

See R.K. Ahuja.

N.V.R. Mahadev

See also P.L. Hammer.

N.V.R. Mahadev and U.N. Peled

1995a *Threshold Graphs and Related Topics.* Ann. Discrete Math., Vol. 56. North-Holland, Amsterdam, 1995. MR 97h:05001. Zbl 950.36502.

§8.3: "Bithreshold graphs" (from Hammer and Mahadev (1985a)), and §8.4: "Strict 2-threshold graphs" (from Hammer, Mahadev, and Peled 1989a), characterize two types of threshold-like graph. In each, a different signed graph H is defined on $E(\Gamma)$ so that Γ is of the specified type iff H is balanced. (The negative part of H is the "conflict graph", Γ^*.) The reason is that one wants Γ to decompose into two subgraphs, and the subgraphs, if they exist, must be the two parts of the Harary bipartition of H. [Thus one also gets a fast recognition algorithm, though not the fastest possible, for the desired type from the fast recognition of balance.] (**SG: Bal: Appl**)

§8.5: "Recognizing threshold dimension 2." Based on Raschle and Simon (1995a). Given: $\Gamma \subseteq K_n$ such that Γ^* is bipartite. Orient $-K_n$ so that Γ-edges are introverted and the other edges are extroverted. Their "alternating cycle" is a coherent closed walk in this orientation. Let us call it "black" (in a given black-white proper coloring of Γ^*) if its Γ-edges are all black. Thm. 8.5.2 (Hammer, Ibaraki, and Peled 1981a): If there is a black coherent closed walk in E_0, then there is a coherent tour (closed trail) of length 6 (which is a pair of joined triangles or a hexagon—their AP_5 and AP_6). Thm. 8.5.4: Given that there is no black coherent hexagon, one can recolor quickly so there is no black coherent 6-tour. Thm. 8.5.9: Given that there is no 'double' coherent hexagon (the book's "double AP_6"), one can recolor quickly so there is no black coherent hexagon. Thm. 8.5.28: Any 2-coloring of Γ^* can be quickly transformed into one with no 'double' coherent hexagon. [*Question.* Can any of this, especially Thm. 8.5.2, be generalized to arbitrary oriented all-negative graphs B? Presumably, this would require first defining a conflict graph on the introverted edges of B. More remotely, consider generalizing to bidirected complete or arbitrary graphs.] (**par: ori, Alg**)

§9.2.1: "Threshold signed graphs." See Benzaken, Hammer, and de Werra (1981a, 1985a). In this version it's not clear where the signs are! (and their role is trivial). Real weights are assigned to the vertices and an edge receives the sign of the weight product of its endpoints. (**sg: bal**)

John Maharry, Neil Robertson, Vaidy Sivaraman, and Daniel Slilaty

20xxa Flexibility of projective-planar embeddings. Submitted.

§1: Flexibility is connected to duality of signed-graphic frame matroids by Slilaty (2005a). [Annot. 20 Dec 2011.] (**SG: Top**)

Ali Ridha Mahjoub

See F. Barahona and D. Cornaz.

J.M. Maillard

See T. Garel and J. Vannimenus.

J.A. Makowsky

See also E. Fischer.

2001a Colored Tutte polynomials and Kauffman brackets for graphs of bounded tree width. In: *Proceedings of the Twelfth Annual ACM-SIAM Symposium on Discrete Algorithms* (Washington, D.C., 2001), pp. 487–495. Soc. for Industrial and Appl. Math., Philadelphia, Pa., 2001. MR 1958441 (no rev). Zbl 988.05087.

See (2005a). (**SGc: Invar: Alg**)

2005a Coloured Tutte polynomials and Kauffman brackets for graphs of bounded tree width. *Discrete Appl. Math.* 145 (2005), no. 2, 276–290. MR 2005m:05214. Zbl 1084.05505.

Polynomial-time computability for edge-colored graphs of bounded tree width. [Also see Traldi (2006a).] (**SGc: Gen: Invar: Alg, Knot**)

A.Z. Maksymowicz

See M.J. Krawczyk.

Krzysztof Malarz

See M.J. Krawczyk and B. Tadić.

H.A. Malathi and H.C. Savithri

2010a A note on jump symmetric n-sigraph. *Int. J. Math. Combin.* 2010 (2010), vol. 2, 65–67. Zbl 1216.05051. (**SG: LG**)

M. Malek-Zavarei and J.K. Aggarwal

1971a Optimal flow in networks with gains and costs. *Networks* 1 (1971), 355–365. MR 45 #4896. Zbl 236.90026. (**GN: bal**)

R.B. Mallion
See A.C. Day.

C.L. Mallows and N.J.A. Sloane
1975a Two-graphs, switching classes and Euler graphs are equal in number. *SIAM J. Appl. Math.* 28 (1975), 876–880. MR 55 #164. Zbl 275.05125, (297.05129).

Thm. 1: For all n, the number of unlabelled two-graphs of order n [i.e., switching isomorphism classes of signed K_n's] equals the number of unlabelled even-degree simple graphs on n vertices. The key to the proof is that a permutation fixing a switching class fixes a signing in the class. (Seidel (1974a) proved the odd case, where the fixing property is simple.) Thm. 2: The same for the labelled case. [More in Cameron (1977b), Cameron and Wells (1986a), Cheng and Wells (1984a, 1986a).]

To prove the fixing property they find the conditions under which a given permutation π of $V(K_n)$ and switching set C fix some signed K_n. [More in Harries and Liebeck (1978a), M. Liebeck (1982a), and Cameron (1977b).] (**TG: Aut, Enum**)

Aleksander Malnič
2002a Action graphs and coverings. *Discrete Math.* 244 (2002), 299–322. MR 1844040 (2003b:05081). Zbl 996.05067.

Gain graphs ("voltage graphs") and lifting automorphisms of their underlying graphs are a main example. [Annot. 11 Jun 2012.] (**GG: Aut, Cov**)

Aleksander Malnič, Roman Nedela, and Martin Škoviera
2000a Lifting graph automorphisms by voltage assignments. *Europ. J. Combin.* 21 (2000), 927–947. MR 1787907 (2001i:05086).

Automorphisms of gain graphs that lift to the covering graph. [Annot. 18 Apr 2012.] (**GG: Cov, Aut**)

2002a Regular homomorphisms and regular maps. *Europ. J. Combin.* 23 (2002), 449–461. MR 1914482 (2003g:05045).

§6, "Invariance of voltage assignments", concerns automorphisms of a gain graph that preserve the gains, in connection with lifting automorphisms to the regular covering graph. The treatment is via maps as gain graphs with rotation systems. [Annot. 18 Apr 2012.] (**GG: Cov, Aut**)

John W. Mamer
See R.D. McBride.

Rachel Manber
See also R. Aharoni and V. Klee.

1982a Graph-theoretical approach to qualitative solvability of linear systems. *Linear Algebra Appl.* 48 (1982), 457–470. MR 84g:68054. Zbl 511.15008. (**SD, QM: QSol**)

Rachel Manber and Jia-Yu Shao
1986a On digraphs with the odd cycle property. *J. Graph Theory* 10 (1986), 155–165. MR 88i:05090. Zbl 593.05032. (**SD, SG: Par**)

Anna Mańka
See A. Mańka-Krasoń.

Anna Mańka-Krasoń and Krzysztof Kułakowski
2009a Magnetism of frustrated regular networks. *Acta Phys. Polonica B* 40 (2009), no. 5, 1455–1461.

20xxa Frustration and collectivity in spatial networks. arXiv:0904.4002. (**sg: par: Fr**)

Anna Mańka, Krzysztof Malarz, and Krzysztof Kułakowski

2007a Clusterization, frustration and collectivity in random networks. *Int. J. Modern Phys. C* 18 (2007), no. 11, 1765–1773. Zbl 1170.82371.

Computer experiments on physics aspects of all-negative signed graphs. [Annot. 14 Feb 2011.] (**sg: Par**)

Zeev Maoz, Lesley G. Terris, Ranan D. Kuperman, and Ilan Talmud

2007a What is the enemy of my enemy? Causes and consequences of imbalanced international relations, 1816–2001. *J. Politics* 69 (2007), no. 1, 100–115. (**PsS, SG: Bal**)

Dănuţ Marcu

I cannot vouch for the authenticity of these articles. See MR 97a:05095 and Zbl 701.51004. Also see MR 92a:51002, 92b:51026, 92h:11026, 97k:05050; and Marcu (1981b).

1980a On the gradable digraphs. *An. Ştiinţ. Univ. "Al. I. Cuza" Iaşi Secţ. I a Mat. (N.S.)* 26 (1980), 185–187. MR 82k:05056 (*q.v.*). Zbl 438.05032.

See Harary, Norman, and Cartwright (1965a) for the definition. (**GD: bal**)

1981a No tournament is gradable. *An. Univ. Bucureşti Mat.* 30 (1981), 27–28. MR 83c:05069. Zbl 468.05028.

See Harary, Norman, and Cartwright (1965a) for the definition. The tournaments of order 3 are [trivially] not gradable, whence the titular theorem. (**GD: bal**)

1981b Some results concerning the even cycles of a connected digraph. *Studia Univ. Babeş-Bolyai Math.* 26 (1981), 24–28. MR 83e:05058. Zbl 479.05032.

§1, "Preliminary considerations", appears to be an edited, unacknowledged transcription of portions of Harary, Norman, and Cartwright (1965a) (or possibly (1968a)), pp. 341–345. Wording and notation have been modified, a trivial corollary has been added, and some errors have been introduced; but the mathematics is otherwise the same down to details of proofs. §2, "Results", is largely a list of the corollaries resulting from setting all signs negative. The exception is Thm. 2.5, for which I am not aware of a source; however, it is simple and well known. (**sg(SD): Bal**)

1987a Note on the matroidal families. *Riv. Math. Univ. Parma* (4) 13 (1987), 407–412. MR 89k:05025.

Matroidal families of (multi)graphs (see Simões-Pereira (1973a)) correspond to functions on all isomorphism types of graphs that are similar to matroid rank functions, e.g., submodular. This provides insight into matroidal families, e.g., it immediately shows there are infinitely many. (**MtrdF: Bic, EC: Gen**)

Enzo Marinari

See also S. Cabasino, B. Coluzzi, M. Falcioni, and J. Lukic.

Enzo Marinari, Giorgio Parisi, and Felix Ritort

1995a The fully frustrated hypercubic model is glassy and aging at large *D*. *J. Phys. A* 28 (1995), 327–334.

Ising (spins, i.e., vertex values, $\in \mathbb{S}^0 = \{+1, -1\}$) and XY (spins $\in \mathbb{S}^1$, i.e., complex units) models behave differently on a totally frustrated signed hypercube graph Q_D (all squares are negative). Numerical study of Ising spins of two such signatures: $\sigma_1(x, x+e_\mu) = (-1)^{x_1+\cdots+x_{\mu-1}}$, while σ_2 ["simplex" in the construction must mean hypercube] is from Derrida, Pomeau, Toulouse, and Vannimenus (1979a); "with identical results". [Reason: $\Sigma_1 \cong \Sigma_2$ under the coordinate transformation $i \leftrightarrow D+1-i$.] Based on simulations with $D \leq 47$, Ising ground states seem to be few and hard to find. Near-ground states are easier to find but, apparently, tend to be far from ground states.

For positive temperature T, as $A(\Sigma)$ ("interaction matrix $J_{x,y}$") is orthogonal [up to scaling], one can approximate by averaging over orthogonal adjacency matrices.

In simulations with XY spins the ground state is highly accessible.

Dictionary: "ground state" = switching with minimum $|E^-|$. [Annot. 19 Jun 2012.] (**Phys, SG**)

1995b Replica theory and large-D Josephson junction hypercubic models. *J. Phys. A* 28 (1995), 4481–4503. MR 1352169 (96g:82032). Zbl 925.82088.

Physics on hypercube Q_D with complex unit gains and three types of spin, after Parisi (1994a), via simulations for $3 \leq D \leq 16$. [Annot. 19 Jun 2012.] (**Phys, gg**)

2000a On the 3D Ising spin glass. *J. Phys. A* 27 (1994), no. 8, 2687–2708. (**Phys: SG**)

A.V. Markovskiĭ

1997a Analysis of the structure of signed directed graphs. (In Russian.) *Izv. Akad. Nauk Teor. Sist. Upr.* 1997 (1997), no. 5, 144–149. Eng. trans., *J. Comput. Syst. Sci. Int.* 36 (1997), no. 5, 788–793. MR 1679025 (2000a:05099). Zbl 898.05078. (**SD: WG**)

Harry Markowitz

1955a Concepts and computing procedures for certain X_{ij} programming problems. In: H.A. Antosiewicz, ed., *Proceedings of the Second Symposium in Linear Programming* (Washington, D.C., 1955), Vol. II, pp. 509–565. Nat. Bur. Standards of U.S. Dept. of Commerce, and Directorate of Management Analysis, DCS Comptroller, HQ, U.S. Air Force, 1955. Sponsored by Office of Scientific Res., Air Res. and Develop. Command. MR 17, 789.

Also see RAND Corporation Paper P-602, 1954. (**GN: m(bases)**)

Klas Markström

2012a Even cycle decompositions of 4-regular graphs and line graphs. *Discrete Math.* 312 (2012), no. 17, 2676–2681.

Can a graph be decomposed into even circles? Studied for 4-regular and line graphs. [*Question.* Can a signed graph be decomposed into positive circles?] [Annot. 12 Jan 2012.] (**par: Str**)

Clifford W. Marshall

1971a *Applied Graph Theory.* Wiley-Interscience, New York, 1971. MR 48 #1951. Zbl 226.05101.

"Consistency of choice" discusses signed graphs, pp. 262–266.
(**SG: Bal, Adj: Exp**)

Matteo Marsili

See G.C.M.A. Ehrhardt.

O.C. Martin

See J. Lukic.

V. Martin-Mayor
See L.A. Fernández.

Enide Andrade Martins
See N.M.M. Abreu and I. Gutman.

Seth A. Marvel, Jon Kleinberg, Robert D. Kleinberg, and Steven H. Strogatz

2011a Continuous-time model of structural balance. *Proc. Nat. Acad. Sci. (U.S.A.)* 108 (2011), no. 5, 1771–1776.

A differential equation model of balancing processes, based on Kułakowski, Premiysław Gawroński, and Piotr Gronek (2005a). See (2011b) for the mathematics. [See commentary, Srinivasan (2011a).] [Annot. 6 Feb 2011.] (**SG: KG: Fr**)

2011b Supporting information. *Proc. Nat. Acad. Sci. (U.S.A.)* 108 (2011), http://www.pnas.org/cgi/doi/10.1073/pnas.1013213108.

The mathematical support for (2011a). [Annot. 6 Feb 2011.] (**SG: KG: Fr**)

Seth A. Marvel, Steven H. Strogatz, and Jon M. Kleinberg

2009a Energy landscape of social balance. *Phys. Rev. Letters* 103 (2009), article 198701.

Signed complete graphs under Antal, Krapivsky, and Redner's (2005a) "constrained triad dynamics": Imbalance measured by triangles; an edge negated if it is in more negative than positive triangles. Paley graphs P give K_P with equally many positive and negative triangles on each edge (normalized "energy" = 0). Other such states exist. [See also Zyga (2009a).] [*Questions.* Do unbalanced locally minimal regions with more than one point (graph) exist? How does the landscape look for switching classes?] [Annot. 5 May 2010, rev 26 Jan 2011.] (**SG: KG: Fr**)

Enzo M. Li Marzi
See E.M. Li Marzi (under L).

J.H. Mason

1977a Matroids as the study of geometrical configurations. In: *Higher Combinatorics* (Proc. NATO Adv. Study Inst., Berlin, 1976), pp. 133–176. NATO Adv. Study Inst. Ser., Ser. C: Math. Phys. Sci., Vol. 31. Reidel, Dordrecht, 1977. MR 80k:05037. Zbl 358.05017.

§§2.5–2.6: "The lattice approach" and "Generalized coordinates", pp. 172–174, propose a purely matroidal and more general formulation of Dowling's (1973b) construction of his lattices. (**gg(Gen): M**)

1981a Glueing matroids together: A study of Dilworth truncations and matroid analogues of exterior and symmetric powers. In: *Algebraic Methods in Graph Theory* (Proc., Szeged, 1978), Vol. II, pp. 519–561. Colloq. Math. Soc. János Bolyai, 25. North-Holland, Amsterdam, 1981. MR 84i:05041. Zbl 477.05022.

Dowling matroids are an example in §1. (**gg: M**)

A.M. Mathai

20xxa On adjacency matrices and descriptors of signed cycle graphs. Int. Workshop on Set-Valuations, Signed Graphs, Geometry and Their Appl. (IWSSG-2011, Mananthavady, Kerala, 2011). *J. Combin. Inform. Syst. Sci.*, to appear.

Eigenvalues of $A(C_n, \sigma)$ (equivalent to Fan (2007)'s Laplacian eigenvalues) by an elegant matrix method. Some ways to partially or wholly distinguish different signatures of C_n are compared. [Annot. 6 Sept 2010.] (**SG: Adj**)

R.A. Mathon

See F.C. Bussemaker and Seidel (1991a).

Hisayoshi Matsuyama

See M. Iri.

Laurence R. Matthews

1977a Bicircular matroids. *Quart. J. Math. Oxford* (2) 28 (1977), 213–227. MR 58 #21732. Zbl 386.05022.

Thorough study of bicircular matroids, introduced by Klee (1971a) and Simões-Pereira (1972a). (**Bic**)

1978a Properties of bicircular matroids. In: *Problèmes Combinatoires et Théorie des Graphes* (Colloq. Int., Orsay, 1976), pp. 289–290. Colloques Int. du CNRS, 260. Editions du C.N.R.S., Paris, 1978. MR 81a:05030. Zbl 427.05021. (**Bic**)

1978b Matroids on the edge sets of directed graphs. In: *Optimization and Operations Research* (Proc. Workshop, Bonn, 1977), pp. 193–199. Lect. Notes in Economics and Math. Systems, 157. Springer-Verlag, Berlin, 1978. MR 80a:05103. Zbl 401.05031.

Announcement of (1978c). (**gg: M**)

1978c Matroids from directed graphs. *Discrete Math.* 24 (1978), 47–61. MR 81e:05055. Zbl 388.05005.

Invents poise, modular poise, and antidirection matroids of a digraph. (**gg: M**)

1979a Infinite subgraphs as matroid circuits. *J. Combin. Theory Ser. B* 27 (1979), 260–273. MR 81e:05056. Zbl 433.05018. (**Bic: Gen**)

Laurence R. Matthews and James G. Oxley

1977a Infinite graphs and bicircular matroids. *Discrete Math.* 19 (1977), 61–65. MR 58 #16348. Zbl 386.05021. (**Bic**)

Jean François Maurras

1972a Optimization of the flow through networks with gains. *Math. Programming* 3 (1972), 135–144. MR 47 #2993. Zbl 243.90048. (**GN: M**)

Mano Ram Maurya, Raghunathan Rengaswamy, and Venkat Venkatasubramanian

2003a A systematic framework for the development and analysis of signed digraphs for chemical processes. 1. Algorithms and analysis. *Ind. Eng. Chem. Res.* 42 (2003), 4789–4810. (**SD: QSta: Alg, Appl**)

2003b A systematic framework for the development and analysis of signed digraphs for chemical processes. 2. Control loops and flowsheet analysis. *Ind. Eng. Chem. Res.* 42 (2003), 4811–4827. (**SD: QSta: Alg, Appl**)

John S. Maybee

See also L. Bassett, H.J. Greenberg, F. Harary, C.R. Johnson, G.M. Lady, and T.J. Lundy.

1974a Combinatorially symmetric matrices. *Linear Algebra Appl.* 8 (1974), 529–537. MR 56 #11845. Zbl (438.15021).

Survey and simple proofs. (**QM: sd, gg, QSta**)(**Exp**)

1980a Sign solvable graphs. *Discrete Appl. Math.* 2 (1980), 57–63. MR 81g:05063. Zbl 439.05024. (**SD: QM: QSol**)

1981a Sign solvability. In: Harvey J. Greenberg and John S. Maybee eds., *Computer-Assisted Analysis and Model Simplification* (Proc. Sympos., Boulder, Col., 1980), pp. 201–257. Discussion, p. 321. Academic Press, New York, 1981. MR 82g:00016 (book). Zbl 495.93001 (book).

For comments, see Lancaster (1981a). (**QM: QSol: SD**)

1989a Qualitatively stable matrices and convergent matrices. In: Fred Roberts, ed., *Applications of Combinatorics and Graph Theory to the Biological and Social Sciences*, pp. 245–258. IMA Vols. Math. Appl., Vol. 17. Springer-Verlag, New York, 1989. MR 90h:34082. Zbl 708.15007.

Signed (di)graphs play a role in characterizations. See e.g. §7. See also Roberts (1989a), §4. (**QM, SD**)

John S. Maybee and Stuart J. Maybee

1983a An algorithm for identifying Morishima and anti-Morishima matrices and balanced digraphs. *Math. Social Sci.* 6 (1983), 99–103. MR 85f:05084. Zbl 567.05038.

A linear-time algorithm to determine balance or antibalance of the undirected signed graph of a signed digraph. The algorithm of Harary and Kabell (1980a) appears to be different. (**SG: Bal, Par: Alg**)

John Maybee and James Quirk

1969a Qualitative problems in matrix theory. *SIAM Rev.* 11 (1969), 30–51. MR 40 #1127. Zbl 186, 335 (e: 186.33503).

An important early survey with new results.
(**QM, SD: QSol, QSta, bal; Exp(in part), Ref**)

John S. Maybee and Daniel J. Richman

1988a Some properties of GM-matrices and their inverses. *Linear Algebra Appl.* 107 (1988), 219–236. MR 89k:15039. Zbl 659.15021.

Square matrix A is a GM-matrix if, for every positive and negative cycle P and N in its signed digraph, $V(P) \supseteq V(N)$. Classification of irreducible GM-matrices; connections with the property that each $p \times p$ principal minor has sign $(-1)^p$; some conclusions about the inverse.
(**SD: QM**)

John S. Maybee and Gerry M. Weiner

1987a *L*-functions and their inverses. *SIAM J. Algebraic Discrete Methods* 8 (1987), 67–76. MR 88a:26021. Zbl 613.15005.

An L-function is a nonlinear generalization of a qualitative linear function. Signed digraphs play a small role. (**QM, SD**)

Stuart J. Maybee

See J.S. Maybee.

W. Mayeda and M.E. Van Valkenburg

1965a Properties of lossy communication nets. *IEEE Trans. Circuit Theory* CT-12 (1965), 334–338. (**GN**)

Dillon Mayhew

2005a Inequivalent representations of bias matroids. *Combin. Probab. Comput.* 14 (2005), 567–583. MR 2006j:05040. Zbl 1081.05021.

The number of inequivalent representations of a frame matroid over a fixed finite field is bounded, if the matroid does not have a free swirl $G(2C_n, \varnothing)$ as a minor. (**GG: M**)

R. Maynard

See J.C. Angles d'Auriac, F. Barahona, and I. Bieche.

M.H. McAndrew

See D.R. Fulkerson.

Richard McBride

See H. Jordon.

Richard D. McBride

See also G.G. Brown.

1985a Solving embedded generalized network problems. *European J. Operational Res.* 21 (1985), 82–92. Zbl 565.90038.

Introducing the algorithm "EMNET", which employs embedded generalized-network matrices (i.e., incidence matrices of real multiplicative gain graphs) with side constraints (i.e., extra rows) to speed up linear programming. [Annot. 2 Oct 2009.] (**GN: Incid: Alg**)

1998a Progress made in solving the multicommodity flow problem. *SIAM J. Optim.* 8 (1998), no. 4, 947–955. MR 1641274 (99i:90110). Zbl912.90128.

Employing embedded generalized-network matrices to speed up linear programming. [Annot. 2 Oct 2009.] (**GN: Incid: Alg: Exp**)

Richard D. McBride and John W. Mamer

1997a Solving multicommodity flow problems with a primal embedded network simplex algorithm. *INFORMS J. Comput.* 9 (1997), no. 2, 154–163. MR 1477311. Zbl 885.90040. (**GN: Incid: Alg**)

2004a Implementing an LU factorization for the embedded network simplex algorithm. *INFORMS J. Comput.* 16 (2004), no. 2, 109–119. MR 2063190.

Matrix factorization to speed up the method of McBride (1985a). [Annot. 2 Oct 2009.] (**GN: Incid: Alg**)

Richard D. McBride and Daniel E. O'Leary

1997a An intelligent modeling system for generalized network flow problems: With application to planning for multinational firms. *Ann. Operations Res.* 75 (1997), 355–372. Zbl 894.90060. (**GN: Incid: Alg**)

H. Gilman McCann

See E.C. Johnsen.

William McCuaig

See also C.R. Johnson.

1993a Intercyclic digraphs. In: Neil Robertson and Paul Seymour, eds., *Graph Structure Theory* (Proc., Seattle, 1991), pp. 203–245. Contemp. Math., Vol. 147. Amer. Math. Soc., Providence, R.I., 1993. MR 94f:05062. Zbl 789.05042.

Characterizes the digraphs with no two disjoint cycles as well as those with no two arc-disjoint cycles. [Since cycles do not form a linear subclass of circles, this is not a biased-graphic theorem, but it might be of use in studying biased graphs that have no two disjoint balanced circles. See Lovász (1965a), Slilaty (2007a).] (**Str**)

2000a Even dicycles. *J. Graph Theory* 35 (2000), no. 1, 46–68. MR 2001f:05087. Zbl 958.05070. (**SD: par: Str**)

2001a Brace generation. *J. Graph Theory* 38 (2001), no. 3, 124–169. MR 2002h:05136. Zbl 991.05086.

Results needed for (2004a). (**SD: par**)

†2004a Pólya's permanent problem. *Electron. J. Combin.* 11 (2004), Research Paper 79, 83 pp. MR 2005i:05004. Zbl 1062.05066.

See the description of Robertson, Seymour, and Thomas (1999a), who independently prove the main theorem. (**SD: par: Str**)(**SG**)

20xxa When all dicycles have the same length. Manuscript.

Uses the main theorem of (2004a) and Robertson, Seymour, and Thomas (1999a) to prove: A digraph has an edge weighting in which all cycles have equal nonzero total weight iff it does not contain a "double dicycle": a symmetric digraph whose underlying simple graph is a circle. There is also a structural description of such digraphs. (**SD: par: Str**)(**Sw**)

William McCuaig, Neil Robertson, P.D. Seymour, and Robin Thomas

1997a Permanents, Pfaffian orientations, and even directed circuits. Extended abstract. In: *Proceedings of the Twenty-Ninth Annual ACM Symposium on Theory of Computing* (STOC 97, El Paso, Tex., 1997), pp. 402–405. ACM Press, New York, 1997. Zbl 963.68153.

Extended abstract of McCuaig (2004a) and Robertson, Seymour, and Thomas (1999a). (**SD: par**)

W.D. McCuaig and M. Rosenfeld

1985a Parity of cycles containing specified edges. In: B.R. Alspach and C.D. Godsil, eds., *Cycles in Graphs*, pp. 419–431. Ann. Discrete Math., Vol. 27. North-Holland Math. Stud., Vol. 115. North-Holland, Amserdam, 1985. MR 87g:05139. Zbl 583.05037.

In a 3-connected graph, almost any two edges are in an even and an odd circle. [By the negative-subdivision trick this generalizes to signed graphs.] (**Par, sg: Bal**)

J.J. McDonald

See S. Kirkland.

James McKee and Chris Smyth

2007a Integer symmetric matrices having all their eigenvalues in the interval $[-2, 2]$. *J. Algebra* 317 (2007), 260–290. MR 2008j:15038. Zbl 1140.15007. arXiv :0705.3599.

The matrices (except those of orders 1, 2) are signed-graph "adjacency" matrices A with diagonal entries $0, 1, -1$. There are 3 infinite families and a few sporadic examples of maximal such signed graphs; all of which satisfy $A^2 = 4I$. The proof uses "charged signed graphs", i.e., a signed graph with 0, +1, or −1 attached to each vertex (and appearing on the diagonal of the adjacency matrix). Switching a vertex negates the charge. Dictionary: "strongly equivalent" = switching isomorphic; "bipartite" = switching isomorphic to its negation. [The charged signed graphs are really oriented all-negative graphs with half edges. The adjacency matrix is not $A(\Sigma)$ but an oriented adjacency matrix $\vec{A}$ defined by $\vec{a}_{ij}$ = net in-degree of v_iv_j edges at v_i. [Annot. 27 June 2008.]]

(**SG: Adj**)

Terry A. McKee

1984a Balance and duality in signed graphs. Proc. Fifteenth Southeastern Conf. on Combinatorics, Graph Theory and Computing (Baton Rouge, 1984). *Congressus Numer.* 44 (1984), 11–18. MR 87b:05124. Zbl 557.05046. (**SG: Bal: D**)

1987a A local analogy between directed and signed graphs. *Utilitas Math.* 32 (1987), 175–180. MR 89a:05075. Zbl 642.05023. (**SG: D, Clu, Bal**)

2002a Chordally signed graphs. *Discrete Appl. Math.* 119 (2002), 273–280. MR 1906865 (2003d:05101). Zbl 1003.05051.

A chordally signed graph is a chordal graph signed so every positive circle C of length at least 4 has a chord such that $C \cup e$ is balanced. Characterized in various ways. (**SG**)

2007a Chordal multipartite graphs and chordal colorings. *Discrete Math.* 307 (2007), 2309–2314. MR 2340631 (2008f:05063).

P. 2312: An auxiliary graph can be treated as signed; chordal coloring is signed-graph clustering. [Annot. 11 Jul 2012.] (**SG: Clu**)

Kathleen A. McKeon
See G. Chartrand.

Luis Medina
See I. Gutman.

Killian Meehan
See Y. Duong.

Nimrod Megiddo
See E. Cohen and D. Hochbaum.

Kurt Mehlhorn and Dimitrios Michail

2005a Implementing minimum cycle basis algorithms. In: S.E. Nikoletseas, ed., *Experimental and Efficient Algorithms* (4th Int. Workshop, WEA 2005, Santorini Island, 2005), pp. 32–43. Lect. Notes in Computer Sci., Vol. 3503. Springer, Berlin, 2005. Zbl 1121.05314.

The "signed graph G_i" is a signed covering graph $\tilde{\Sigma}_i$. Used to find minimum cycle basis in a positively weighted graph Γ. Σ_i has negative edge set S_i, the "witness set". [Annot. 6 Feb 2011.] (**SG: Alg, Cov**)

2006a Implementing minimum cycle basis algorithms. *ACM J. Exper. Algorithmics* 11 (2006), 14 pp. MR 2306622 (2007m:05139). Zbl 1143.05310.

See (2005a). (**SG: Alg, Cov**)

Martin Mehlitz
See J. Kunegis.

A. Mehrabian
See S. Akbari.

Marco A. Mendez
See J. Aracena.

Leanne Merrill
See Y. Duong.

Russell Merris

1994a Laplacian matrices of graphs: a survey. *Linear Algebra Appl.* 197/198 (1994), 143–176. MR 1275613 (95e:05084). Zbl 802.05053.

Thm.: $\operatorname{Spec} K(\Gamma) = \operatorname{Spec} K(-\Gamma)$ iff Γ is bipartite. [The antibalanced case of B.D. Acharya (1980a).] [Annot. 21 Jan 2012.] (**Par: Adj, bal**)

1995a A survey of graph Laplacians. *Linear Multilinear Algebra* 39 (1995), no. 1–2, 19–31. MR 1374468 (97c:05104). Zbl 832.05081. (**Par: Adj**)

Roy Meshulam
See R. Aharoni and J. Kahn.

Robert Messer
See E.M. Brown.

Karola Mészáros

2011a Root polytopes, triangulations, and the subdivision algebra, II. *Trans. Amer. Math. Soc.* 363 (2011), no. 11, 6111–6141. MR 2817421 (2012g:52021). Zbl 1233.05216. arXiv:0904.3339.

A signed simple graph generates a polytope $P(\Sigma)$ whose volume is calculated. [Annot. 11 Sept 2010.] (**SG: Geom**)

Frédéric Meunier and András Sebő

2009a Paintshop, odd cycles and necklace splitting. *Discrete Appl. Math.* 157 (2009), 780–793. MR 2499492 (2010e:90102). Zbl 1163.90774.

Dictionary: "signed graph" = $(|\Sigma|, E^-)$, "odd cycle" = negative circle, "odd cycle clutter" = $\mathcal{B}^c(\Sigma)$, "uncut" = minimal balancing set, "BIP(G, F)" = MinBalSet(Σ) (the problem of finding a minimum balancing set), "resigning" = switching. [Annot. 22 Sept 2010.] (**SG: Fr**)

Seth A. Meyer

See R.A. Brualdi.

Hildegard Meyer-Ortmanns

See F. Radicchi.

Andrew MṀeyers

See N.A. Neudauer.

Marc Mézard, Giorgio Parisi, and Miguel Angel Virasoro

1987a *Spin Glass Theory and Beyond.* World Scientific Lect. Notes in Physics, Vol. 9. World Scientific, Singapore, 1987. MR 91k:82066.

Focuses on the Sherrington–Kirkpatrick model, i.e., underlying complete graph, emphasizing the Parisi-type model (see articles reprinted herein), which posits numerous metastable states, separated by energy barriers of greatly varying heights and subdividing as temperature decreases (*cf.* Kirkpatrick and Sherrington (1978a)). Essentially heuristic (as noted in MR): that is, the ideas awaited [and still largely await] mathematical justification.

Many original articles on Ising and vector models (both of which are based on weighted signed graphs) are reprinted herein, though few are of general signed-graphic interest.

[See also, i.a., Toulouse (1977a, etc.), Chowdhury (1986a), Stern (1989a), Fischer and Hertz (1991a), Vincent, Hammann, and Ocio (1992a) for physics, Barahona (1982a, etc.), Grötschel, Jünger, and Reinelt (1987a) for mathematics.] (**Phys, SG: Fr: Exp, Ref**)

Ch. 0, "Introduction", briefly compares, in the obvious way, balance in social psychology [they neglect to mention the original paper, Cartwright and Harary (1956a)] with frustration in spin glasses. (**Phys, PsS: SG: Bal: Exp**)

Pt. 1, "Spin glasses", Ch. II, "The TAP approach": pp. 19–20 describe 1-vertex switching of a weighted signed graph to reduce frustration, not however necessarily producing the frustration index (minimum frustration). (**Phys: SG: Fr, Sw, Alg: Exp**)

Zhengke Miao

See H.P. Ma and L.Q. Wang.

T.S. Michael

2002a Signed degree sequences and multigraphs. *J. Graph Theory* 41 (2002), 101–105.

MR 2003g:05042. Zbl 1012.05052.

Characterizes net degree sequences of signed graphs with fixed maximum edge multiplicity. [See Chartrand, Gavlas, Harary, and Schultz (1994a) for explanation.] (**SGw: Invar, Alg**)

Dimitrios Michail
See K. Mehlhorn.

Manuel Middendorf
See E. Ziv.

Anna de Mier
See O. Giménez.

Raymond E. Miller
See R.M. Karp.

William P. Miller
See also J.E. Bonin.

1997a Techniques in matroid reconstruction. *Discrete Math.* 170 (1997), 173–183. MR 98f:05039. Zbl 878.05020.

Dowling matroids are reconstructible from their hyperplanes, their deletions, and their contractions. (**gg: M**)

Maya Mincheva and Gheorghe Craciun

2008a Multigraph conditions for multistability, oscillations and pattern formation in biochemical reaction networks. *Proc. IEEE* 96 (2008), no. 8, 1281–1291. (**SD: Chem, Biol: Exp**)

Edward Minieka

1972a Optimal flow in a network with gains. *INFOR* 10 (1972), 171–178. Zbl 234.90012. (**GN: M(indep), Bal**)

1978a *Optimization Algorithms for Networks and Graphs.* Marcel Dekker, New York and Basel, 1978. MR 80a:90066. Zbl 427.90058.

§4.6: "Flows with gains," pp. 151–174. Also see pp. 80–81. (**GN: Bal, Sw, m(indep): Exp**)

1981a *Algoritmy Optimizatsii na Setyakh i Grafakh.* Transl. M.B. Katsnel'son and M.I. Rubinshteĭn; ed. E.K. Maslovskiĭ. Mir, Moskva, 1981. MR 83f:90118. Zbl 523.90058.

Russian translation of (1978a). (**GN: Bal, Sw, m(indep): Exp**)

Maryam Mirzakhah
See also I. Gutman and M. Jooyandeh.

M. Mirzakhah and D. Kiani

2010a The Sun graph is determined by its signless Laplacian spectrum. *Electronic J. Linear Algebra* 20 (2010), 610–620. MR 2735977 (2011j:05209). Zbl 1205.05149. (**Par: Adj**)

G. Mitra
See N. Gülpinar.

S. Mitra

1962a Letter to the editors. *Behavioral Sci.* 7 (1962), 107.

Treats signed simple graphs via the Abelson–Rosenberg (1958a) structure matrix R. Observes that balance holds iff $R = rr^{\mathrm{T}}$ for some vector $r \in \{p, n\}^V$; also, asserts that frustration index $l(\Sigma)$ = minimum number of negative edges over all switchings of Σ. [Proved in Barahona, Maynard, Rammal, and Uhry (1982a).] Asserts an algorithm for computing

$l(\Sigma)$: switch vertices whose negative degree exceeds positive degree, one at a time, until no such vertices remain [incorrect: consider K_6, all positive except a negative C_6]. [Annot. Corr. 20 Jan 2010.]

(sg: kg: Adj, sw, Fr)

V. Mishra

1974a *Graphs Associated With* $(0, +1, -1)$ *Arrays.* Doctoral thesis, Indian Institute of Technology, Bombay, 1974.

The arrays are matrices. **(SG)**

Hirobumi Mizuno and Iwao Sato

1997a Enumeration of finite field labels on graphs. *Discrete Math.* 176 (1997), 197–202. MR 1477289 (98e:05059). Zbl 893.05015.

Isomorphism types, under the action of a subgroup of $\operatorname{Aut}\Gamma$, of coboundaries of 1-chains $f : V \to \mathbb{H}_q^+$ in $-\Gamma$. (In other words, the edge labels are $\delta f(uv) = f(u) + f(v)$.) [*Question.* Does it generalize to signed graphs? The subgroup would be of $\operatorname{Aut}\Sigma$, or one can count isomorphism types of switching classes under a subgroup of $\operatorname{Aut}[\Sigma]$.] [Annot. 16 Jan 2012.]

(par: incid)

2010a Weighted scattering matrices of regular coverings of graphs. *Linear Multilinear Algebra* 58 (2010), no. 7, 927–940. MR 2742326 (2011k:05145). Zbl 1231.05170.

(GGw: Cov: Invar, Adj: Gen)

Iain Moffatt

See also J.A. Ellis-Monaghan and M. Loebl.

2011a Unsigned state models for the Jones polynomial. *Ann. Combin.* 15 (2011), no. 1, 126–146. MR 2785760 (2012b:05087).

Vertex models (graphs with vertices labelled ± 1) and Potts (edges labelled ± 1) models can be replaced by unsigned models by converting an edge-labelled graph into an orientable ribbon graph. A limited parametrized rank-corank polynomial appears (in the standard way) as the Potts partition function. [Annot. 23 Apr 2009.] **(SGc: Invar)**

Javad Mohajeri

See S. Fayyaz Shahandashti.

A. Mohammadian and B. Tayfeh-Rezaie

2011a Graphs with four distinct Laplacian eigenvalues. *J. Algebraic Combin.* 34 (2011), no. 4, 671–682. MR 2842915 (2012j:05265). Zbl 1242.05166.

$K(-\Gamma)$ is used to prove Thm. 6, characterizing bipartite Γ with four Laplacian eigenvalues. [Annot. 24 Jan 2012.] **(Par: Adj)**

Bojan Mohar

1989a An obstruction to embedding graphs in surfaces. *Discrete Math.* 78 (1989), 135–142. MR 90h:05046. Zbl 686.05019.

The "overlap matrix" of a signed graph with respect to a rotation system and a spanning tree provides a lower bound on the demigenus that sometimes improves on that from Euler's formula. **(SG: Top)**

Bojan Mohar and Svatopluk Poljak

1993a Eigenvalues in combinatorial optimization. In: Richard A. Brualdi, Shmuel Friedland, and Victor Klee, eds., *Combinatorial and Graph-Theoretical Problems in Linear Algebra*, pp. 107–151. IMA Vols. Math. Appl., 50. Springer-Verlag, New York, 1993. MR 1240959 (95e:90003). Zbl 806.90104.

Switching of a weight function on an unsigned graph (p. 119), from C. Delorme and S. Poljak, Combinatorial properties and the complexity of a max-cut approximation, Tech. Rep. 91687, Inst. Diskrete Math., Univ. Bonn, 1991. [Annot. 13 Apr 2009.] (**Sw**)

Bojan Mohar and Carsten Thomassen

2001a *Graphs on Surfaces*. Johns Hopkins Stud. Math. Sci. Johns Hopkins Univ. Press, Baltimore, 2001. MR 1844449 (2002e:05050). Zbl 979.05002.

§3.3, "Embedding schemes", surveys rotation systems and edge signatures for embedding in nonorientable surfaces. *Cf.* Ringel (1977a) and Stahl (1978a). §4.1, "Embeddings combinatorially": Detailed treatment of embeddings from rotation systems and optionally an edge signature. [Lins (1982a), Širáň and Škoviera (1991a), Zaslavsky (1992a, 1993a), *et al.* are regrettably never mentioned in this valuable book.] (**sg: Top**)

Marco Montalva, Julio Aracena, and Anahí Gajardo

2008a On the complexity of feedback set problems in signed digraphs. IV Latin-American Algorithms, Graphs, and Optimization Sympos. (Puerto Varas, Chile, 2007). *Electron. Notes Discrete Math.* 30 (2008), 249–254. MR 2570648.

Complexity of finding a minimum set of vertices, or arcs, that covers all positive, or negative, cycles in a signed digraph. All are NP-complete, by polynomial-time reduction to the existence problems Even Cycle and Odd Cycle in the positive and negative problems, respectively. [Directed frustration index and directed vertex frustration number are the negative-cycle cover problems, which are said to be easier than the positive-cycle cover problems.] [Annot. 20 July 2009.]
(**SD: Fr: Gen, Alg**)

James D. Montgomery

2009a Balance theory with incomplete awareness. *J. Math. Sociology* 33 (2009), 69–96. Zbl 1169.91438.

Signed digraphs with possible multiple arcs of different sign, with two types of vertices ("actors" having positive and possibly negative loops, and "objects" having no loops), and with extra "awareness" arcs between actor vertices. Emphasis on directionality of arcs. "Boolean multiplication" [Boolean Hadamard product] of separate positive, negative, and awareness adjacency matrices to form mixed adjacency matrices. Assumption: Over time the signed digraph evolves towards sign-transitive closure constrained by the awareness arcs, whose absence impedes transitive closure. Four specific "mechanisms" are postulated for the evolution, of which two are essential (Lemma 1). Propositions present conclusions (no surprises) about intermediate and final (i.e., constrained sign-transitively closed) signed digraphs. Dictionary: "balance closure" = sign-transitive closure, i.e., arc-transitive closure with positive triple sign. [The idea of constrained closure is mathematically intriguing, though the notation is heavy.] [For more on sign-transitive closure in signed digraphs see Doreian and Krackhardt (2001a).] [Annot. 16 Apr 2009.] (**SD, PsS: Bal**)

Elliott W. Montroll

1964a Lattice statistics. In: Edwin F. Beckenbach, ed., *Applied Combinatorial Mathematics*, Ch. 4, pp. 96–143. Wiley, New York, 1964. MR 30 #4687 (book). Zbl 141, 155 (e: 141.15503).

§4.4: "The Pfaffian and the dimer problem". Exemplified by the square lattice, expounds Kasteleyn's method of signing edges to make the Pfaffian term signs all positive. Partial proofs. §4.7, "The Ising problem", pp. 127–129, explains application to the Ising model. Exceptionally readable. [Further development in, e.g., Vazirani and Yannakakis (1988a, 1989a).] (**SG, Phys: Exp**)

J.W. Moon and L. Moser

1966a An extremal problem in matrix theory. *Mat. Vesnik* N.S. 3(18) (1966), 209–211. MR 34 #7385. Zbl 146, 14a (e: 146.01401).

Studies the maximum frustration index of a signed $K_{r,s}$. (**sg: Fr**)

Suck Jung Moon

See H. Kosako.

M.A. Moore

See A.J. Bray.

G. Eric Moorhouse

1995a Two-graphs and skew two-graphs in finite geometries. *Linear Algebra Appl.* 226/228 (1995), 529–551. MR 96f:51012. Zbl 839.05024.

[Skew two-graphs are likely related to the two-digraphs of Cheng and Wells (1984a).] (**gg, sd: sw: Invar**)

Michio Morishima

1952a On the laws of change of the price-system in an economy which contains complementary commodities. *Osaka Economic Papers* 1 (1952), 101–113.

§4: "Alternative expression of the assumptions (1)," can be interpreted with hindsight as proving that, for a signed K_n, every triangle is positive iff the signature switches to all positive. (Everything is done with sign-symmetric matrices, not graphs, and switching is not mentioned in any form.) (**sg: bal, sw**)

Julian O. Morrissette

1958a An experimental study of the theory of structural balance. *Human Relations* 11 (1958), 239–254.

Proposes that edges have strengths between -1 and $+1$ instead of pure signs. The Cartwright–Harary degree of balance (1956a), computed from circles, is modified to take account of strength. In addition, signed graphs are allowed to have edges of two types, say U and A, and only short mixed-type circles enter into the degree of balance. This is said to be more consistent with the experimental data reported herein. (**PsS, SG, Gen: Fr**)

Julian O. Morrissette and John C. Jahnke

1967a No relations and relations of strength zero in the theory of structural balance. *Human Relations* 20 (1967), 189–195.

Reports an experiment; then discusses problems with and alternatives to the Cartwright–Harary (1956a) circle degree of balance. (**PsS: Fr**)

Hannes Moser

See J. Guo.

L. Moser

See J.W. Moon.

Sebastiano Mosterts

See E.L. Johnson.

C.F. Moukarzel
See M.J. Alava.

Andrej Mrvar
See also P. Doreian and W. de Nooy.

Andrej Mrvar and Patrick Doreian

2009a Partitioning signed two-mode networks. *J. Math. Sociology* 33 (2009), no. 3, 196–221. Zbl 1168.91511.

§2, "Formalization of block-modeling signed two-mode data": A signed two-mode network is a bipartite signed simple graph with color classes V_1, V_2. The objective is partitions π_1, π_2 of V_1, V_2 that minimize a "criterion function" $P := \alpha i_- + (1-\alpha) i_+$; usually $\alpha = .5$. $k_1 := |\pi_1|$ and $k_2 := |\pi_2|$, or other restrictions, may be specified. Definitions: $\pi_i := \{V_{i1}, \dots, V_{ik_i}\}$. A "block" is a nonvoid set $E(V_{1i}, V_{2j})$. Its sign is the sign of the majority of edges, + if a draw. e is "consistent" with (π_1, π_2) if it is in a block of sign $\sigma(e)$. $i_\varepsilon :=$ number of inconsistent edges of sign ε. [Annot. 17 Aug 2009.] (**SG: Clu, PsS**)

Haiko Müller
See T. Kloks.

Luigi Muracchini and Anna Maria Ghirlanda

1965a Sui grafi segnati ed i grafi commutati. *Statistica* (Bologna) 25 (1965), 677–680. MR 33 #7272.

A partially successful attempt to use unoriented signed graphs to define a line graph of a digraph. [See Zaslavsky (2010b, 20xxa, 20xxb) for the correct signed-graph approach.] The Harary–Norman line digraph is also discussed. (**SG: Bal, LG**)

Kunio Murasugi

1988a On the signature of a graph. *C.R. Math. Rep. Acad. Sci. Canada* 10 (1988), 107–111. MR 89h:05056.

The signature of a sign-colored graph (see 1989a) is an invariant of the sign-colored graphic matroid. (**SGc: Incid, m**)

1989a On invariants of graphs with applications to knot theory. *Trans. Amer. Math. Soc.* 314 (1989), 1–49. MR 89k:57016. Zbl 726.05051.

Studies a dichromatic form, $P_\Sigma(x, y, z)$, of Kauffman's (1989a) Tutte polynomial of a sign-colored graph. The deletion-contraction parameters are $a_\varepsilon = 1$, $b_\varepsilon = x^\varepsilon$ for $\varepsilon = \pm 1$; the initial values are such that $P_\Sigma(x, y, z) = y^{-1} Q_\Sigma(a, b; y, z)$ of Zaslavsky (1992b). The polynomial is shown to be, in effect, an invariant of the sign-colored graphic matroid.

Much unusual graph theory is in here. A special focus is the degrees of the polynomial. First Main Thm. 3.1: Formulas for the maximum and minimum combined degrees of $P_\Sigma(x, y, z)$. §7, "Signature of a graph", studies the signature (σ in the paper, s here) of the Kirchhoff matrix $K(\Sigma)$ (B_Σ in the paper) obtained by changing the diagonal of $A(\Sigma)$ so the row sums are 0. Prop. 7.2 is a matrix-tree theorem [entirely different from that of Zaslavsky (1982a)]. The Second Main Thm. 8.1 bounds the signature: $|V| - 2\beta_0(\Sigma^-) + 1 \leq s \leq |V| - 2\beta_0(\Sigma^+) + 1$ ($\beta_0 =$ number of components), with equality characterized. The Kirchhoff matrix is further examined later on. §9, "Dual graphs": Differing from most studies, here the dual of a sign-colored plane graph is the planar dual with same edge signs [however, negating all colors is a triviality]. §10, "Periodic graphs": These graphs might be called branched covering graphs of signed gain graphs with finite cyclic gain group. [Thus they generalize the periodic graphs of Collatz (1978a) and others.] §§12–15 concern applications to knot theory.

(**SGc: Invar, Incid, GG(Cov), D, Knot**)

1991a Invariants of graphs and their applications to knot theory. In: S. Jackowski, B. Oliver, and K. Pawałowski, eds., *Algebraic topology Poznań 1989* (Proc., Poznań, 1989), pp. 83–97. Lect. Notes in Math., Vol. 1474. Springer-Verlag, Berlin, 1991. MR 92m:57015. Zbl 751.57007.

§§1–3 expound results from (1989a) on the dichromatic polynomial and the signature of a sign-colored graph and knot applications. §5 discusses the signed Seifert graph of a link diagram.

(**SGc: Invar, Incid, Knot: Exp**)

1993a *Musubime riron to sono ōnō*. [Knot Theory and Its Applications.] (In Japanese.) 1993.

See (1996a). (**SGc: Knot**)

1996a *Knot Theory and Its Applications*. Birkhäuser, Boston, 1996. MR 97g:57011. Zbl 864.57001.

Updated translation of (1993a) by Bohdan Kurpita. Pp. 36–37: Construction of signed plane graph from link diagram, and conversely.

(**SGc: Knot**)

Kunio Murasugi and Jozef H. Przytycki

1993a *An Index of a Graph with Applications to Knot Theory*. Mem. Amer. Math. Soc., Vol. 106, No. 158. Amer. Math. Soc., Providence, R.I., 1993. MR 94d:57025. Zbl 792.05047.

Ch. I, "Index of a graph". The "index" is the largest number of "independent" edges, where "independent" has a complicated recursive definition (unrelated to matchings), one of whose requirements is that the edges be "singular" (simple, i.e., nonmultiple links). The positive or negative index of a sign-colored graph is similar except that the independent edges must all be positive or negative. [The general notion is that of the index of a graph-subgraph pair. The signs pick out complementary subgraphs.] Thm. 2.4: Each of these indices is additive on blocks of a bipartite graph. The main interest, because of applications to knot theory, is in bipartite plane graphs. Ch. II, "Link theory": Pp. 26–27 define the sign-colored Seifert graph of an oriented link diagram and apply the graphical index theory. (**SGc: Invar, D, Knot**)

Tadao Murata

1965a Analysis of lossy communication nets by modified incidence matrices. In: M.E. Van Valkenburg, ed., *Proceedings, Third Annual Allerton Conference on Circuit and System Theory* (Monticello, Ill., 1965), pp. 751–761. Dept. of Electrical Eng. and Coordinated Sci. Lab., Univ. of Illinois, Urbana, Ill.; and Circuit Theory Group, Inst. of Electrical and Electronics Engineers, [1965]. (**GN: Incid**)

Antoine Musitelli

2010a Recognizing binet matrices. *Math. Programming* 124 (2010), no. 1-2, 349–381. MR 2679995 (2011g:68121). Zbl 1206.68149. (**SG: Ori: Incid**)

P. Mützel

See C. De Simone.

O. Nagai
See H.T. Diep.

K.M. Nagaraja
See P. Siva Kota Reddy.

T.A. Naikoo
See S. Pirzada.

Takeshi Naitoh
See K. Ando.

Kazuo Nakajima
See H. Choi.

Atsuhiro Nakamoto, Seiya Negami, and Katsuhiro Ota

2002a Chromatic numbers and cycle parities of quadrangulations on nonorientable closed surfaces. Ninth Quadrennial Int. Conf. Graph Theory, Combinatorics, Algorithms Appl. *Electronic Notes Discrete Math.* 11 (2002), 509–518. MR 2155788 (no rev).

Cycle parity on surface S = homomorphism $\rho : \pi(S) \to \mathbb{Z}_2 \cong \{+,-\}$, equivalently $\rho : H_1(S;\mathbb{Z}_2 \to \mathbb{Z}_2 \cong \{+,-\}$. ρ implies a signature (actually, a switching class) of any embedded graph Γ. There are one nontrivial type of cycle parity on an orientable surface and three on a nonorientable surface N_d, different for odd and even d, except two on N_2 and one on N_1. If $\Gamma \hookrightarrow S$ so every face boundary is even ("even embedding"), $\rho(W) = |W| \bmod 2$ for closed walks is a cycle parity. Thm. 9: For three of the six types on N_d's, there is a negative cut that opens N_d to an orientable surface. [Annot. 11 Jun 2012.] (**sg: Top: sw**)

Daishin Nakamura and Akihisa Tamura

1998a The generalized stable set problem for claw-free bidirected graphs. In: Robert E. Bixby, E. Andrew Boyd, and Roger Z. Ríos-Mercado, eds., *Integer Programming and Combinatorial Optimization* (6th Int. IPCO Conf., Houston, 1998, Proc.), pp. 69–83. Lect. Notes in Computer Sci., Vol. 1412. Springer, Berlin, 1998. MR 2000h:05209. Zbl 907.90272.

The problem of the title is solvable in polynomial time. See Johnson and Padberg (1982a), Tamura (1997a) for definitions. They reduce to simple graphs, transitively bidirected with no sink or introverted edge (called "canonical" bidirected graphs). (**sg: Ori: Geom, Sw, Alg**)

1998b Generalized stable set problems for claw-free bi-directed graphs. (In Japanese.) Theory and Applications of Mathematical Optimization (Kyoto, 1998). *Sūrikaisekikenkyūsho Kōkyūroku* No. 1068 (1998), 100–109. Zbl (939.05506). (**sg: Ori: Geom, Sw, Alg**)

2000a A linear time algorithm for the generalized stable set problem on triangulated bidirected graphs. New Trends in Mathematical Programming (Kyoto, 1998). *J. Operations Res. Soc. Japan* 43 (2000), 162–175. MR 2001c:90093. Zbl 1138.90494. (**sg: Ori: Geom: Alg**)

M. Nakamura
See M. Hachimori.

Tota Nakamura, Shin-ichi Endoh, and Takeo Yamamoto

2003a Weak universality of spin-glass transitions in three-dimensional $\pm J$ models. *J. Phys. A* 36 (2003), 10895–10906. MR 2025232 (no rev). Zbl 1075.82508.

Physics of Ising, XY, and Heisenberg spin-glass models on a signed square lattice graph with 3-dimensional spin vectors. The Hamiltonian of state $S : V \to \mathbb{S}^2$ (the sphere) is $\sum_{uv \in E} \sigma(uv) S_u \cdot S_v$. [Annot. 17 Jun 2012.] (**Phys: SG**)

Bunpei Nakano

See T. Inohara.

Aurélien Naldi, Elisabeth Remy, Denis Thieffry, and Claudine Chaouiya

2011a Dynamically consistent reduction of logical regulatory graphs. *Theor. Computer Sci.* 412 (2011), no. 21, 2207–2218, MR 2809505 (2012a:92077). Zbl 1211.92024. (**SD, Biol**)

L. Nanjundaswamy

See E. Sampathkumar.

Joseph (Seffi) Naor

See D. Hochbaum.

Vito Napolitano

See M. Abreu.

C.St.J.A. Nash-Williams

1960a On orientations, connectivity, and odd-vertex-pairings in finite graphs. *Canad. J. Math.* 12 (1960), 555–567. MR 22 #9455. Zbl 96, 380 (e: 096.38002).

1969a Well-balanced orientations of finite graphs and unobtrusive odd-vertex-pairings. In: W.T. Tutte, ed., *Recent Progress in Combinatorics* (Proc. Third Waterloo Conf., 1968), pp. 133–149. Academic Press, New York, 1969. MR 40 #7146. Zbl 209, 557 (e: 209.55701).

Roman Nedela

See also A. Malnič.

Roman Nedela and Martin Škoviera

1996a Regular embeddings of canonical double coverings of graphs. *J. Combin. Theory Ser. B* 67 (1996), 249–277. MR 1399678 (97e:05078). Zbl 856.05029.

By "canonical double covering" of Γ they mean the signed covering graph $\tilde{\Sigma}$ of $\Sigma = -\Gamma$, but without reversing orientation at the negative covering vertex [as one would do in a signed covering graph (*cf.* e.g. Zaslavsky 1992a)], because orientable embeddings of Γ are being lifted to orientable embeddings of $\tilde{\Sigma}$. [Thus these should be thought of not as signed graphs but rather as voltage (i.e., gain) graphs with 2-element gain group.] Instead of reversal they twist the negative-vertex rotations by taking a suitable power. In some cases this allows classifying the orientable, regular embeddings of $\tilde{\Sigma}$. (**Par: Cov, Top, Aut**)

1997a Exponents of orientable maps. *Proc. London Math. Soc.* (3) 75 (1997), 1–31. MR 1444311 (98i:05059). Zbl 877.05012.

Main topic: the theory of twisting of rotations as in (1996a). (**GG: Cov, Top, Aut**)

Portions concern double covering graphs of signed graphs. §7: "Antipodal and algebraically antipodal maps". A map is "antipodal" if it is the orientable double covering of a nonorientable map; that is, as a graph it is the canonical double covering of an unbalanced signed graph. A partial algebraic criterion for a map to be antipodal. §9: "Regular embeddings of canonical double coverings of graphs". See (1996a). (**sg, Par: Cov, Top, Aut**)

1997b Regular maps from voltage assignments and exponent groups. *European J. Combin.* 18 (1997), 807–823. MR 1478826 (98j:05061). Zbl 908.05036.

Cases in which the classification of (1996a) is necessarily incomplete are studied by taking larger voltage (i.e., gain) groups and twisting the rotations at covering vertices by taking a power that depends on the position of the vertex in its fiber. Main result: the (very special) conditions on twisting under which a regular map lifts to a regular map.

(**GG: Cov, Top, Aut**)

Seiya Negami

See A. Nakamoto.

Toshio Nemoto

See K. Ando.

H. Nencka [H. Nencka-Ficek]

See Ph. Combe and H. Nencka-Fisek.

H. Nencka-Fisek [H. Nencka]

See also Ph. Combe.

1984a Necessary and sufficient conditions for the overblocking effect. In: A. Pękalski and J. Sznajd, eds., *Static Critical Phenomena in Inhomogeneous Systems* (Proc. XX Karpacz Winter School Theor. Phys., Karpacz, Poland, 1984), pp. 337–343. Lect. Notes in Physics Vol. 206. Springer-Verlag, Berlin, 1984. MR 839663 (87i:82096).

Signs are defined for arbitrary proper subhypercubes of the hypercube Q_d [thus giving a signed hypergraph]. A "plaquette" (this is nonstandard) is a $k-1$-dimensional band around a k-subhypercube; its sign is the product of signs of its $k-1$-faces. Overblocking means not all plaquettes can simultaneously be negative ("frustrated"). The interesting proof is by the adjacency graph of 2-faces of a 3-cube in Q_d. Identify opposite 2-faces to a single vertex whose sign is the product of 2-face signs; the faces of a 3-cube form a triangle whose vertices alternate in sign, if all plaquettes were negative. Conclusion: All 2-faces cannot be negative, if $d > 2$. [Presumably a similar argument should be applied to plaquettes of $k-1$-faces of a k-cube, $k > 3$, but it is not. Is it valid? There would be one plaquette per dimension.] [Annot. 19 Jun 2012.]

(**SH, Phys: Fr**)

Jaroslav Nešetřil

See J. Kratochvíl.

Nancy Ann Neudauer

See also R.A. Brualdi.

2002a Graph representations of a bicircular matroid. *Discrete Appl. Math.* 118 (2002), 249–262. MR 2003b:05047. Zbl 990.05025.

Survey of parts of Brualdi and Neudauer (1997a), Wagner (1985a), and Coullard, del Greco, and Wagner (1991a), with supplementary results on nice graphs whose bicircular matroid, $G(\Gamma, \varnothing)$, equals M. (**Bic**)

Nancy Ann Neudauer, Andrew MṀeyers, and Brett Stevens

2001a Enumeration of the bases of the bicircular matroid on a complete graph. Proc. Thirty-second Southeastern Int. Conf. Combinatorics, Graph Theory and Computing (Baton Rouge, La., 2001). *Congr. Numer.* 149 (2001), 109–127. MR 2002m:05054. Zbl 1003.05031.

Counts bases and connected bases. Very complicated formulas. [The results count labelled simple 1-trees and 1-forests. A 1-*tree* is a tree with one extra edge forming a circle. A 1-*forest* is a disjoint union of 1-trees. A connected basis of the bicircular matroid $G(K_n, \varnothing)$ for $n \geq 3$ is a labelled simple 1-tree; a basis is a labelled simple 1-forest. Riddell (1951a) has a less complicated formula for 1-trees.] (**Bic: Invar(Bases)**)

Nancy Ann Neudauer and Brett Stevens

2001a Enumeration of the bases of the bicircular matroid on a complete bipartite graph. *Ars Combin.* 66 (2003), 165–178. MR 2004a:05034. Zbl 1075.05510.

Bases are counted and their structure compared to the spanning trees of the graph. [A basis is a simple, labelled 1-forest (*cf.* Neudauer, Meyer, and Stevens 2001a) whose circles are even.] (**Bic: Invar(Bases)**)

A. Neumaier

1982a Completely regular twographs. *Arch. Math. (Basel)* 38 (1982), 378–384. MR 83g:05066. Zbl 475.05045.

In the signed graph (K_n, σ) of a two-graph (see D.E. Taylor 1977a), a "clique" is a vertex set that induces an antibalanced subgraph. A two-graph is "completely regular" if every clique of size i lies in the same number of cliques of size $i + 1$, for all i. Thm. 1.4 implies there is only a small finite number of completely regular two-graphs. (**TG**)

Michael Neumann

See C.R. Johnson.

T.M. Newcomb

See also K.O. Price.

1968a Interpersonal balance. In: R.P. Abelson *et al.*, eds., *Theories of Cognitive Consistency: A Sourcebook*. Rand-McNally, Chicago, Ill., 1968. (**PsS**)

G.F. Newell

1950b Crystal statistics of a two-dimensional triangular Ising lattice. *Phys. Rev.* (2) 79 (1950), 876–882. MR 039631 (12, 576i). Zbl 38, 139b (e: 038.13902).

The same physics conclusions as R.M.F. Houtappel's (1950a,b) for a signed, weighted triangular lattice. [See also I. Syôzi (1950a), G.H. Wannier (1950a).] [Annot. 20 Jun 2012.] (**Phys, WG, sg: Fr**)

Alantha Newman

See N. Ailon.

Charles M. Newman and Daniel L. Stein

1997a Metastate approach to thermodynamic chaos. *Phys. Rev. E* (3) 55 (1997), no. 5, part A, 5194–5211. MR 1448389 (98k:82098).

A technical paper supporting (1998a). [Annot. 26 Aug 2012.] (**Phys: sg, fr**)

1998a Thermodynamic chaos and the structure of short-range spin glasses. In: Anton Bovier and Pierre Picco, eds., *Mathematical Aspects of Spin Glasses and Neural Networks*, pp. 243–287. Progress in Prob., Vol. 41. Birkhäuser, Boston, 1998. MR 1601751 (99b:82056). Zbl 896.60078.

See especially §3, "The standard SK picture". The Hamiltonian $H_\sigma(s) = -\sum_{vw\in E} \sigma(vw)s(v)s(w)$ is standard. Criticizes the typical physics application of randomly signed (and possibly weighted) K_n (Sherrington–Kirkpatrick model) to $\mathbb{Z}^d$-lattice graphs by limits of finite (cubical) subgraphs. Raises the question of a "pure state" (*cf.* Mézard, Parisi, and Virasoro (1987a) *et al.*) of a signed K_n, where a state is $s : V \to \{+1, -1\}$ and a pure state is apparently a linear combination of or probability distribution on states, especially in the $\mathbb{Z}^d$ limit. A pure state is not well defined but is related to states of low frustration (and high probability). [*Question.* Is there a graphical meaning of a pure state, based on the (ambiguous) physics definition? It should involve states with low frustration, because they dominate the partition function $Z(\sigma) = \sum_s e^{H_\sigma(s)}$, and on the qualities desired for computing quantities of physical interest, especially in terms of H and Z.]

A "metastate" is a measure on states, essentially a linear combination with explicit coefficients. Pure states on $\mathbb{Z}^d$ should be metastates. See (1997a). [*Question.* Is there a graph-theory meaning to all this? Does it lead to a definition of frustration in an infinite signed (or gain) graph?] [Annot. 26 Aug 2012.] (**Phys: sg, fr: Exp, Ref**)

2010a Distribution of pure states in short-range spin glasses. *Int. J. Modern Phys. B* 24 (2010), no. 14, 2091–2106. MR 2659908 (2011g:82055).

Further development of (1997a); *cf.* (1998a). [Annot. 26 Aug 2012.] (**Phys: sg, fr**)

Sang Nguyen
See P.L. Hammer.

Robert Nickel
See W. Hochstättler.

Rolf Niedermeier
See F. Hüffner.

Juhani Nieminen

1976a Weak balance: A combination of Heider's theory and cycle and path-balance. *Control Cybernet.* 5 (1976), 69–73. MR 55 #2639.

S^c denotes the "signed closure" of a signed digraph S. S is "weakly balanced" if in S^c all directed digons and all induced transitive triangles are positive. Thm.: S is weakly balanced iff it is path- and cycle-balanced. (**SD: Bal**)

Peter Nijkamp
See F. Brouwer.

Vladimir Nikiforov
See L.S. de Lima.

Yuri Nikolayevsky
See G. Cairns.

Wouter de Nooy

1999a The sign of affection: Balance-theoretic models and incomplete signed digraphs. *Social Networks* 21 (1999), 269–286.

Vertex ranking (a partial ordering) based on arc signs. Thm. 3 characterizes equality of rank. Thm. 6 characterizes strict inequality. [Annot. 11 Sept 2010.] (**SD: PsS, Bal, Clu**)

2008a Signs over time: statistical and visual analysis of a longitudinal signed network. *J. Social Structure* 9 (2008), Article 1, 32 pp. (**SG: Fr, PsS**)

Wouter de Nooy, Andrej Mrvar, and Vladimir Batagelj

2005a *Exploratory Social Network Analysis with Pajek.* Structural Anal. Soc. Sci., No. 27. Cambridge Univ. Press, Cambridge, Eng., 2005.

Pajek is a computer package that analyzes networks, i.e., graphs, including signed graphs. Ch. 4: "Sentiments and friendship." Computation of balance and clusterability of signed (di)graphs. §4.2: "Balance theory." Introductory. §4.4: "Detecting structural balance and clusterability." How to use Pajek to optimize clustering. §4.5: "Development in time." Pajek can look for evolution towards balance or clusterability.

§10.3: "Triadic analysis." Types of balance and clusterability, with the triads (order-3 induced subgraphs) that do or do not occur in each. Table 16, p. 209, "Balance-theoretic models", is a chart of 6 related models. §§10.7, 10.10: "Questions" and "Answers." Some are on balance models. §10.9: "Further reading." [Annot. 28 Apr 2009.]

(**SG, SD, PsS: Bal, Clu, Alg: Exp**)

Robert Z. Norman

See also F. Harary.

Robert Z. Norman and Fred S. Roberts

1972a A derivation of a measure of relative balance for social structures and a characterization of extensive ratio systems. *J. Math. Psychology* 9 (1972), 66–91. MR 45 # 2121. Zbl 233.92006.

Circle ("cycle") indices of imbalance: the proportion of circles that are unbalanced, with circles weighted nonincreasingly according to length.

(**SG: Fr(Circles)**)

1972b A measure of relative balance for social structures. In: Joseph Berger, Morris Zelditch, Jr., and Bo Anderson, eds., *Sociological Theories in Progress*, Ch. 14, pp. 358–391. Houghton Mifflin, Boston, 1972.

Exposition and application of (1972a). (**SG: Fr(Circles): Exp, PsS**)

Beth Novick and András Sebö

1995a On combinatorial properties of binary spaces. In: Egon Balas and Jens Clausen, eds., *Integer Programming and Combinatorial Optimization* (4th Int. IPCO Conf., Copenhagen, 1995, Proc.), pp. 212–227. Lect. Notes in Computer Sci., Vol. 920. Springer-Verlag, Berlin, 1995. MR 96h:0503.

The clutter of negative circuits of a signed binary matroid (M, σ). Important are the lift and extended lift matroids, $L(M, \sigma)$ and $L_0(M, \sigma)$, defined as in signed graph theory. An elementary result: the clutter is signed-graphic iff $L_0(M, \sigma)/e_0$ is graphic (which is obvious). There are also more substantial but complicated results. [See Cornuéjols (2001a), §8.4.] (**SM, SG: M**)

1996a On ideal clutters, metrics and multiflows. In: William H. Cunningham, S. Thomas McCormick, and Maurice Queyrann, eds., *Integer Programming and Combinatorial Optimization* (5th Int. IPCO Conf., Vancouver, 1996, Proc.), pp. 275–287. Lect. Notes in Computer Sci., Vol. 1084. Springer-Verlag, Berlin, 1996. MR 98i:90075. (**SM: M**)

Marc Noy

See O. Giménez.

Cyriel van Nuffelen

1973a On the rank of the incidence matrix of a graph. Colloque sur la Theorie des Graphes (Bruxelles, 1973). *Cahiers Centre Etudes Rech. Oper.* 15 (1973), 363–365. MR 50 #162. Zbl 269.05116.

The theorem restated: unoriented incidence matrix has rank $\mathrm{rk}\, G(-\Gamma)$. [Because the matrix represents $G(-\Gamma)$: see Zaslavsky (1982a). In retrospect, partially implicit in Stewart (1966a) and completely so in Stanley (1973a).] **(par: Incid, ec)**

1976a On the incidence matrix of a graph. *IEEE Trans. Circuits Systems* CAS-23 (1976), 572. MR 56 #186.

Summarizes (1973a). **(par: Incid, ec)**

Koji Nuida

See also T. Abe.

2010a A characterization of signed graphs with generalized perfect elimination orderings. *Discrete Math.* 310 (2010), no. 4, 819–831. MR 2574831 (2011a:05140. Zbl 1209.05119. arXiv:0712.4118. **(SG: Str, Geom)**

Yasuhide Numata

See T. Abe.

Mohammad Reza Oboudi

See S. Akbari.

M. Ocio

See E. Vincent.

Hidefumi Ohsugi and Takayuki Hibi

1998a Normal polytopes arising from finite graphs. *J. Algebra* 207 (1998), 409–426. MR 1644250 (2000a:13010). Zbl 926.52017.

The odd-cycle condition of Fulkerson, Hoffman, and McAndrew (1965a) is employed in polynomial algebra. "Graph polytope" = conv(columns of H($-\Gamma$)). [*Problem.* Generalize to signed graphs.] [Annot. 30 May 2011.] **(sg: Par: Geom)**

2003a Normalized volumes of configurations related with root systems and complete bipartite graphs. *Discrete Math.* 268 (2003), 217–242. MR 1983280 (2004m:-52018). Zbl 1080.14059.

A configuration consists of the vectors representing an acyclic orientation of a complete bipartite signed graph. The volume is that of the pyramid over the configuration with apex at the origin. (Successor to Fong (2000a).) [*Question.* Is there a connection with the chromatic polynomial?] **(sg: Geom: Invar)**

Ayao Okiji

See Y. Kasai.

E. Olaru

See St. Antohe.

Marián Olejár

See J. Širáň.

D.D. Olesky

See M. Catral and C.R. Johnson.

Carla Silva Oliveira

See also L.S. de Lima.

Carla Silva Oliveira, Leonardo Silva de Lima, Nair Maria Maia de Abreu, and Pierre Hansen

2010a Bounds on the index of the signless Laplacian of a graph. *Discrete Appl. Math.* 158 (2010), no. 4, 355–360. MR 2588119 (2011d:05228). Zbl 1225.05174.

See Cvetković, Rowlinson, and Simić (2007a). **(Par: Adj)**

Carla Silva Oliveira, Leonardo Silva de Lima, Nair Maria Maia de Abreu, and Steve Kirkland

2010a Bounds on the Q-spread of a graph. *Linear Algebra Appl.* 432 (2010), no. 9, 2342–2351. MR 2599864 (2011k:05146). Zbl 1214.05082.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

G.R. Omidi

See also F. Ayoobi and W.H. Haemers.

2009a On a signless Laplacian spectral characterization of T-shape trees. *Linear Algebra Appl.* 431 (2009), no. 9, 1607–1615. MR 2555062 (2010m:05181). Zbl 1169.05351.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Gholam R. Omidi and Ebrahim Vatandoost

2010a Starlike trees with maximum degree 4 are determined by their signless Laplacian spectra. *Electron. J. Linear Algebra* 20 (2010), 274–290. MR 2653539 (2011c:05205). Zbl 1205.05151.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Kenji Onaga

1966a Dynamic programming of optimum flows in lossy communication nets. *IEEE Trans. Circuit Theory* CT-13 (1966), 282–287. (**GN**)

1967a Optimal flows in general communication networks. *J. Franklin Inst.* 283 (1967), 308–327. MR 36 #1189. Zbl (e: 203.22402). (**GN**)

Shmuel Onn

See also P. Kleinschmidt.

1997a Strongly signable and partitionable posets. *European J. Combin.* 18 (1997), 921–938. MR 99d:06007. Zbl 887.06003.

For "signability" see Kleinschmidt and Onn (1995a). A strong signing is an exact signing that satisfies a recursive condition on lower intervals. (**Sgnd, Geom**)

Rikio Onodera

1968a On signed tree-graphs and cotree-graphs. *RAAG Res. Notes* (3) No. 133 (1968), ii + 29 pp. MR 38 #5671. Zbl 182, 582a (e: 182.58201).

The adjacency graph of trees of a graph is signed from a vertex signature and is shown to be balanced. [Trivial.] [Annot. 24 July 2010.] (**SG: Bal**)

The Open University

1981a Graphs and Digraphs. Unit 2 in Course TM361: Graphs, Networks and Design. The Open University Press, Walton Hall, Milton Keynes, England, 1981. MR none. Zbl none.

Social sciences (pp. 21–23). Signed digraphs (pp. 50–52). [Published version: see Wilson and Watkins (1990a).] (**SG, PsS, SD: Exp**)

Peter Orlik and Louis Solomon

1980a Unitary reflection groups and cohomology. *Invent. Math.* 59 (1980), 77–94. MR 81f:32017. Zbl 452.20050.

Thm. (4.8): The characteristic polynomials of the Dowling lattices and jointless Dowling lattices of $\mathbb{Z}_r$, computed via group theory as part of the general treatment of finite unitary reflection groups. (**gg: m, Geom**)

1982a Arrangements defined by unitary reflection groups. *Math. Ann.* 261 (1982), 339–357. MR 84h:14006. Zbl 491.51018.

In the intersection lattice of reflection hyperplanes of a finite unitary reflection group, the characteristic polynomial of an upper interval has an integral factorization. The proofs involve detailed study of the group actions on $\mathbb{C}^l$. Dictionary: $\mathcal{A}_l(r)$ and $\mathcal{A}_l^k(r)$ are the arrangements corresponding to the rank-l Dowling lattices and partially jointless Dowling lattices of $\mathbb{Z}_r$. Relevant results: §2: "Monomial groups": Cor. (2.4) counts the flats, Prop. (2.5) and Cor. (2.7) gives the polynomials for $\mathcal{A}_l(r)$ [all known from Dowling (1973b)]. Cor. (2.10) counts the flats, Prop. (2.13) gives the polynomial of $\mathcal{A}_l^k(r)$, Prop. (2.14) notes that proper upper intervals are Dowling lattices [all fairly obvious via gain graphs and coloring (Zaslavsky 1995b)]. (**gg: m, Geom, Invar**)

1983a Coxeter arrangements. In: Peter Orlik, ed., *Singularities* (Arcata, Calif., 1981), Part 2, pp. 269–291. Proc. Sympos. Pure Math., Vol. 40. Amer. Math. Soc., Providence, R.I., 1983. MR 85b:32016. (**gg: m, Geom, Invar**)

James B. Orlin

See also R.K. Ahuja, M. Kodialam, and R. Shull.

1984a Some problems on dynamic/periodic graphs. In: *Progress in combinatorial optimization* (Proc. Conf., Waterloo, Ont., 1982), pp. 273–293. Academic Press, Toronto, 1984. MR 86m:90058. Zbl 547.05060.

Problems on 1-dimensional periodic graphs (i.e., covering (di)graphs of $\mathbb{Z}$-gain graphs Φ) that can be solved in Φ: connected components, strongly connected components, directed path from one vertex to another, Eulerian trail (directed or not), bicolorability, and spanning tree with minimum average cost. (**GG, GD: Cov: Paths, Circles, Col: Alg**)

1985a On the simplex algorithm for networks and generalized networks. *Math. Programming Study* 24 (1985), 166–178. MR 87k:90102. Zbl 592.90031. (**GN: M(Bases): Alg**)

Charles E. Osgood and Percy H. Tannenbaum

1955a The principle of congruity in the prediction of attitude change. *Psychological Rev.* 62 (1955), 42–55. (**VS: PsS**)

Eiji O'Shima

See M. Iri.

Katsuhiro Ota

See A. Nakamoto.

James G. Oxley

See also T. Brylawski, J. Geelen, J.P.S. Kung, and L.R. Matthews.

1992a Infinite matroids. In: Neil White, ed., *Matroid Applications*, Ch. 3, pp. 73–90. Encycl. Math. Appl., Vol. 40. Cambridge Univ. Press, Cambridge, Eng., 1992. MR 93f:05027. Zbl 766.05016.

See Exer. 3.20. (**Bic: Exp**)

1992b *Matroid Theory.* Oxford Univ. Press, Oxford, 1992. MR 1207587 (94d:05033). Zbl 784.05002.

Thm. 6.6.3: proof from Brylawski (1975a). (**gg: sw: Exp**)

§10.3: Exer. 3 concerns the Dowling lattices of GF$(q)^\times$. §12.2: Exer. 13 concerns $G(\Omega)$. (**gg: M: Exp**)

2011a *Matroid Theory*, 2nd ed. Oxford Grad. Texts Math., 21. Oxford Univ. Press, Oxford, 2011.

§6.10, "Dowling geometries": Frame (i.e., bias) matroid theory of biased graphs. Examples: gain and signed graphs, Dowling (1973a,b) geometries, bicircular, even-circle (even-cycle, factor), and poise and antidirection matroids. Representability of Dowling geometries. Kahn and Kung's (1982a) varieties. Other mentions of Dowling geometries in Prop. 14.10.22, §15.3 p. 590, §15.9 p. 605, and Appendix, "Some interesting matroids", p. 663; of bicircular matroids in Exer. 10.4.12, Prop. 11.1.6, Exer. 11.1.7, Conj. 14.3.12, Thm. 14.10.19. Spikes and swirls (the lift and frame matroids of biased $2C_n$'s) are important in matroid structure theory. [Annot. 21 Mar 2011.] (**GG: M, Bic, EC: Exp, Exr**)

James Oxley, Dirk Vertigan, and Geoff Whittle

1996a On inequivalent representations of matroids over finite fields. *J. Combin. Theory Ser. B* 67 (1996), 325–343. MR 97d:05052. Zbl 856.05021.

§5: Free swirls, $G(2C_n, \varnothing)$ $(n \geq 4)$, mentioning their relationship to Dowling lattices, and complete free spikes, $L_0(2C_n, \varnothing)$. (**GG: M**)

M.L. Paciello

See M. Falcioni.

Manfred W. Padberg

See E.L. Johnson

Steven R. Pagano

†1998a *Separability and Representability of Bias Matroids of Signed Graphs.* Doctoral thesis, State Univ. of New York at Binghamton, 1998. MR 2697393 (no rev).

Ch. 1: "Separability". Graphical characterization of bias-matroid k-separations of a biased graph. Also, some results on the possibility of k-separations in which one or both sides are connected subgraphs. (**GG: M: Str**)

Ch. 2: "Representability". The bias matroid of every signed graph is representable over all fields with characteristic $\neq 2$. For which signed graphs is it representable in characteristic 2 (and therefore representable over GF(4), by the theorem of Geoff Whittle, A characterization of the matroids representable over GF(3) and the rationals. *J. Combin. Theory Ser. B* 65 (1995), 222–261. MR 96m:05046. Zbl 835.05015.)? Solved (for 3-connected signed graphs having vertex-disjoint negative circles and hence nonregular matroid). There are two essentially different types: (i) two balanced graphs joined by three independent unbalanced digons; (ii) a cylindrical signed graph, possibly with balanced graphs adjoined by 3-sums. [See notes on Seymour (1995a) for definition of (ii) and for Lovász's structure theorem in the case without vertex-disjoint negative circles.]

Furthermore, the representations of these graphs in characteristic not 2 are all canonical signed-graphic, while any representations over GF(4) are canonical $\mathbb{Z}_3$-gain graphic. (**SG: M: Incid, Str, Top**)

Ch. 3: "Miscellaneous results". (**SG: M: Incid, Str**)

1999a Binary signed graphs. Manuscript, ca. 1999. (**SG: M: Incid, Str**)

1999b Signed graphic GF(4) forbidden minors. Manuscript, ca. 1999. (**SG: M**)

1999c GF(4)-representations of bias matroids of signed graphs: The 3-connected case. Manuscript, ca. 1999. (**SG: M: Incid, Str, Top**)

Igor Pak
See S. Chmutov.

Edgar M. Palmer
See F. Harary and F. Kharari.

B.L. Palowitch, Jr.
See M.A. Kramer.

Rong-Ying Pan
See Y.H. Chen.

Yongliang Pan
See Y. Hou.

Ondřej Pangrác
See D. Král'.

Casian Pantea
See G. Craciun.

P. Paolucci
See S. Cabasino.

Gyula Pap

2005a Packing non-returning A-paths algorithmically. In: Stefan Felsner, ed., *2005 European Conference on Combinatorics, Graph Theory and Applications (EuroComb '05)* (Berlin, 2005), pp. 139–144, electronic. Discrete Math. & Theor. Computer Sci. Proceedings AE, 2005. Zbl 1192.05123.
http://www.dmtcs.org/dmtcs-ojs/index.php/proceedings/issue/view/77
(**GG: Str, Paths, Alg**)

2007a Packing non-returning A-paths. *Combinatorica* 27 (2007), no. 2, 247–251. MR 2008c:05148. Zbl 1136.05060.
Given: Φ with gain group $\mathfrak{S}(\Omega)$, the symmetric group of a set Ω, $A \subseteq V$, and $\omega : A \to \Omega$. An A-path is a path P with endpoints $v, w \in A$ and internally disjoint from A; it is "returning" if $\omega(v)\varphi(P) = \omega(w)$. Thm. The largest number of disjoint returning A-paths equals the minimum, over all satisfied edge subsets F, of the maximum number of disjoint $[A \cup V(F)]$-paths in $\|\Phi\| \setminus F$. [For "satisfied" edges see Zaslavsky (2009a).] Generalizes and simplifies Chudnovsky *et al.* (2006a), which is the case where the gains act regularly and $\omega =$ constant. (**GG: Str, Paths**)

2008a Packing non-returning A-paths algorithmically. *Discrete Math.* 308 (2008), no. 8, 1472–1488. MR 2392063 (2009e:05160). Zbl 1135.05060.
(**GG: Str, Paths, Alg**)

Christos H. Papadimitriou
See also E.M. Arkin and A.S. LaPaugh.

Christos H. Papadimitriou and Kenneth Steiglitz

1982a *Combinatorial Optimization: Algorithms and Complexity*. Prentice-Hall, Englewood Cliffs, N.J., 1982. MR 84k:90036. Zbl 503.90060.
See Ch. 10, Problems 6–7, p. 244, for bidirected graphs and flows in relation to the matching problem. (**sg: Ori: Flows**)

1985a *Kombinatornaya optimiztsiya. Algoritmy i Slozhnost'*. Transl. V.B. Alekseev. Mir, Moskva, 1985. MR 86i:90067. Zbl 598.90067.
Russian translation of (1982a). (**sg: Ori: Flows**)

Konstantinos Papalamprou

See G. Appa and L. Pitsoulis.

Ojas Parekh

See E.G. Boman.

Giorgio Parisi

See also S. Cabasino, B. Coluzzi, M. Falcioni, L.A. Fernández, E. Marinari, and M. Mézard.

1994a *D*-dimensional arrays of Josephson junctions, spin glasses and *q*-deformed harmonic oscillators. *J. Phys. A* 27 (1994), 7555–7568. MR 1312271 (95m:82070). Zbl 844.60095.

Physics on hypercube Q_D with complex unit gains φ (φ is a "$U(1)$ gauge field"). Spins $\zeta(v)$ can be (i) complex units or (ii) Gaussian random complex numbers, or (iii) ζ can be a unit vector $\in \mathbb{C}^n$; mainly, (ii). Assumed: each square ("plaquette") $C_{\alpha,\beta}$ (with vertices $x, x+e_\alpha, x+e_\alpha+e_\beta, x+e_\beta, x$ for any $x \in V(Q_D)$) has gain $e^{iB\sigma_{\alpha,\beta}}$ in a fixed orientation, where $\sigma_{\alpha,\beta} \in \{+1,-1\}$ determines which orientations have gains e^{iB} and e^{-iB}. $B=0$ gives balance; $B=\pi$ gives all plaquette gains -1 (full frustration). If $D \leq 3$, but not if $D > 3$, the choices of σ are equivalent by switching in the gain group $\mathbb{C}^\times$. The statistics of random σ are investigated. [Annot. 19 Jun 2012.] **(Phys, gg)**

1996a A mean field theory for arrays of Josephson junctions. *J. Math. Phys.* 37 (1996), no. 10, 5158–5170. MR 1411624 (97i:82029). Zbl 872.60038.

Complex unit gain graphs. The Hamiltonian is the quadratic form $\bar{z}A(\Phi)z$. [Annot. 12 Aug 2012.] **(GG: Phys)**

M. Parvathi

2004a Signed partition algebras. *Comm. Algebra* 32 (2004), no. 5, 1865–1880. MR 2099708 (2005g:16060). Zbl 1081.20008.

They are the special case of Bloss (2003a) where $\mathfrak{G} = \{+,-\}$. [Annot. 21 Mar 2011.] **(gg: Algeb, m)**

M. Parvathi and M. Kamaraj

1998a Signed Brauer's algebras. *Comm. Algebra* 26 (1998), no. 3, 839–855. MR 1606174 (99c:16028). Zbl 944.16015.

The algebra is generated by multiplying two-layer signed graphs ("Brauer graphs"). In the product $\Sigma_1\Sigma_2$ the bottom layer of Σ_1 cancels with the top layer of Σ_2 using edge-sign product. (Signs are represented by arrows [!].) [Annot. 5 Jun 2012.] **(gg: Algeb, m)**

2002a Matrix units for signed Brauer's algebras. *Southeast Asian Bull. Math.* 26 (2002), no. 2, 279–297. MR 2047807 (2005b:16055). Zbl 1066.16014. **(gg: Algeb, m)**

M. Parvathi and A. Joseph Kennedy

2004a *G*-vertex colored partition algebras as centralizer algebras of direct products. *Comm. Algebra* 32 (2004), no. 11, 4337–4361. MR 2102453 (2005i:16068). Zbl 1081.20009. **(gg: Algeb, m)**

2004b Representations of vertex colored partition algebras. *Southeast Asian Bull. Math.* 28 (2004), no. 3, 493–518. MR 2084740 (2006c:16051). Zbl 1081.20010. **(gg: Algeb, m)**

2005a Extended G-vertex colored partition algebras as centralizer algebras of symmetric groups. *Algebra Discrete Math.* 2005, no. 2, 58–79. MR 2238218 (2007b:-16068). Zbl 1091.20005. **(gg: Algeb, m)**

M. Parvathi and D. Savithri

2002a Representations of G-Brauer algebras. *Southeast Asian Bull. Math.* 26 (2002), no. 3, 453–468. MR 2047837 (2005b:16056). Zbl 1065.20017. **(gg: Algeb, m)**

M. Parvathi and C. Selvararj

1999a Signed Brauer's algebras as centralizer algebras. *Comm. Algebra* 27 (1999), no. 12, 5985–5998. MR 1726289 (2000j:16051). Zbl 944.16016. **(gg: Algeb, m)**

2004a Note on signed Brauer's algebras. *Southeast Asian Bull. Math.* 27 (2004), no. 5, 883–898. MR 2175793 (2006i:16047). Zbl 1071.16010. **(gg: Algeb, m)**

2006a Characters of signed Brauer's algebras. *Southeast Asian Bull. Math.* 30 (2006), no. 3, 495–514. MR 2243691 (2007d:16068). Zbl 1150.16303. **(gg: Algeb, m)**

M. Parvathi and B. Sivakumar

2008a The Klein-4 diagram algebras. *J. Algebra Appl.* 7 (2008), no. 2, 231–262. MR 2417044 (2009b:16032). Zbl 1167.16012. **(gg: Algeb, m)**

2008b R-S correspondence for $(Z_2 \times Z_2) \wr S_n$ and Klein-4 diagram algebras. *Electronic J. Combin.* 15 (2008), no. 1, Research Paper R98, 28 pp. MR 2426161 (2009i:05233). Zbl 1163.05300. **(gg: Algeb, m)**

M. Parvathi, B. Sivakumar, and A. Tamilselvi

2007a R-S correspondence for the hyper-octahedral group of type B_n—a different approach. *Algebra Discrete Math.* 2007 (2007), no. 1, 86–107. MR 2367517 (2008k:05203). Zbl 1164.05465. **(gg: Algeb, m)**

M. Parvathi and A. Tamilselvi

2007a Robinson-Schensted correspondence for the signed Brauer algebras. *Electronic J. Combin.* 14 (2007), no. 1, Research Paper 49, 26 pp. MR 2336326 (2008e:-05143). Zbl 1163.05336. **(gg: Algeb, m)**

2008a Robinson-Schensted correspondence for the G-Brauer algebras. In: S.K. Jain and S. Parvathi, eds., *Noncommutative rings, group rings, diagram algebras and their applications* (Proc. Int. Conf., Chennai, 2006), pp. 137–150. Contemp. Math., Vol. 456. Amer. Math. Soc., Providence, R.I., 2008. MR 2416147 (2009m:16060). Zbl 1187.05085. **(gg: Algeb, m)**

S. Pati

See R.B. Bapat and S. Kalita.

Philippa Pattison

1993a *Algebraic Models for Social Networks.* Structural Analysis in the Social Sciences, 7. Cambridge Univ. Press, Cambridge, 1993.

Ch. 8, pp. 258–9: "The balance model. The complete clustering model." Embedded in a more general framework.

(SG, Sgnd: Adj, Bal, Clu: Exp)

G.A. Patwardhan

See B.D. Acharya and M.K. Gill.

Debdas Paul

See S. Kirkland.

Vern I. Paulsen

See B.G. Bodmann and R.B. Holmes.

Charles Payan

See also F. Jaeger.

1983a Perfectness and Dilworth number. *Discrete Math.* 44 (1983), no. 2, 229–230. MR 689816 (84e:05090). Zbl 518.05053.

See Benzaken, Hammer, and de Werra (1985a). (**SGc**)

Edmund R. Peay

1977a Matrix operations and the properties of networks and directed graphs. *J. Math. Psychology* 15 (1977), 89–101. MR 58 #20631. (**SD, WD: Adj: Gen**)

1977b Indices for consistency in qualitative and quantitative structures. *Human Relations* 30 (1977), 343–361.

Proposes an index of nonclusterability for signed graphs and generalizes to edges weighted by a linearly ordered set. (**SG, Gen: Clu: Fr(Gen)**)

1980a Connectedness in a general model for valued networks. *Social Networks* 2 (1980), 385–410. MR 602317 (82h:92053) (*q.v.*).

Real-number edge weights; the value of a path is the minimum absolute weight. [Annot. 11 Sept 2010.] (**WG**)

1982a Structural models with qualitative values. *J. Math. Sociology* 8 (1982), 161–192. MR 83d:92107. Zbl 486.05060.

See mainly §3: "Structural consistency." (**sd: Gen: Bal, Clu**)

Luke Pebody

See B. Bollobás.

Britta Peis

See W. Hochstättler and M. Lätsch.

Uri N. Peled

See S.R. Arikati, A. Bhattacharya, P.L. Hammer, T. Ibaraki, and N.V.R. Mahadev.

Martin Pelikan and Alexander K. Hartmann

2007a Obtaining ground states of Ising spin glasses via optimizing bonds instead of spins. (Extended abstract.) In: *GECCO '07: Genetic and Evolutionary Computation Conference* (GECCO 2007, London), p. 628. ACM, New York, 2007.

Announcement of (2007b). (**SG, Phys: Fr: Alg**)

2007b Obtaining ground states of Ising spin glasses via optimizing bonds instead of spins. Report, Missouri Estimation of Distribution Algorithms Laboratory, Dept. of Mathematics and Computer Science, University of Missouri–St. Louis, 2007. http://medal-cs.umsl.edu/ (**SG, Phys: Fr: Alg**)

Marcello Pelillo

See R. Glantz.

Francisco Pereira

See A.J. Hoffman.

Kavita S. Permi

See P. Siva Kota Reddy.

M. Petersdorf

1966a Einige Bemerkungen über vollständige Bigraphen. *Wiss. Z. Techn. Hochsch. Ilmenau* 12 (1966), 257–260. MR 37 #1275. Zbl (e: 156.44302).

Treats signed K_n's. Satz 1: $\max_\sigma l(K_n, \sigma) = \lfloor (n-1)^2/4 \rfloor$ with equality iff (K_n, σ) is antibalanced. [From which follows easily the full Thm. 14 of Abelson and Rosenberg (1958a).] Also, some further discussion of antibalanced and unbalanced cases. [For extensions of this problem see notes on Erdős, Győri, and Simonovits (1992a).] (**SG: Fr**)

Ion Petre
See A. Alhazov and T. Harju.

J.L. Phillips
1967a A model for cognitive balance. *Psychological Rev.* 74 (1967), 481–495.

Proposes to measure imbalance of a signed (di)graph by largest eigenvalue of a matrix close to $I + A(\Sigma)$. (*Cf.* Abelson 1967a.) Possibly, means to treat only graphs that are complete aside from isolated vertices. [Somewhat imprecise.] Summary of Ph.D. thesis. (**SG: Bal, Fr, Adj**)

Nancy V. Phillips
See F. Glover.

Alberto Del Pia
See A. Del Pia.

Jean-Claude Picard and H. Donald Ratliff
1973a A graph-theoretic equivalence for integer programs. *Operations Res.* 21 (1973), 261–269. MR 50 #12240. Zbl 263.90021.

A minor application of signed switching to a weighted graph arising from an integer linear program. (**sg: sw**)

Marcin Pilipczuk
See M. Cygan.

MichałPilipczuk
See M. Cygan.

P. Pincus
See S. Alexander.

S. Pirzada
20xxa Signed degree sequences in signed graphs. Int. Workshop on Set-Valuations, Signed Graphs, Geometry and Their Appl. (IWSSG-2011, Mananthavady, Kerala, 2011). *J. Combin. Inform. Syst. Sci.*, to appear.
(**SG: ori: Invar: Exp**)(**SG: ori: Invar**)

S. Pirzada and F.A. Dar
2007a Signed degree sets in signed 3-partite graphs. *Mat. Vesnik* 59 (2007), no. 3, 121–124. MR 2361920 (2008k:05095). Zbl 1224.05222. (**SG: ori: Invar**)

2007b Signed degree sequences in signed 3-partite graphs. *J. Korean Soc. Ind. Appl. Math.* 11 (2007), no. 1, 9–14. (**SG: ori: Invar**)

S. Pirzada, T.A. Naikoo, and F.A. Dar
2007a Signed degree sets in signed graphs. *Czech. Math. J.* 57 (2007), no. 3, 843–848. MR 2008g:05088. Zbl 1174.05059. arXiv:math/0609121.

The set, as opposed to sequence, of net degrees [*cf.* Chartrand, Gavlas, Harary, and Schultz (1994a)] of a signed simple graph can be any finite set of integers. Also, the smallest order of a signed graph with given net degree set. (**SG: ori: Invar**)

2007b Signed degree sequences in signed bipartite graphs. *AKCE Int. J. Graphs Combin.* 4 (2007), no. 3, 301–312. MR 2384886 (no rev). Zbl 1143.05307.

Characterization of net degree sequences of signed, simple, bipartite graphs. [Annot. 15 Nov 2011.] (**SG: ori: Invar**)

2008a A note on signed degree sets in signed bipartite graphs. *Appl. Anal. Discrete Math.* 2 (2008), no. 1, 114–117. MR 2396733 (2009a:05092). Zbl 1199.05159.

Every finite set of integers is the signed degree set of some connected signed bipartite graph. [Annot. 10 Sept 2010.] (**SG: ori: Invar**)

Tomaž Pisanski

See also V. Batagelj.

Tomaž Pisanski and Primož Potočnik

2004a Graphs on surfaces. In: Jonathan L. Gross and Jay Yellen, eds., *Handbook of Graph Theory*, pp. 611–624. Discrete Math. Appl. (Boca Raton). CRC Press, Boca Raton, Fla., 2004. MR 2004j:05001 (book). Zbl 1036.05001 (book).

Cryptic. Dictionary (my best guess): “signed edge” = oriented edge; “signed boundary walk” (of a face) = directed face boundary walk; “signature” = set of negative edges of an embedding; “switch” = negative (= orientation-reversing) edge of an embedding. (**sg: Top**)

Tomaž Pisanski and Jože Vrabec

1982a Graph bundles. Preprint Ser., Dept. Math., Univ. Ljubljana, 1982.

Definition (see Pisanski, Shawe-Taylor, and Vrabec (1983a)), examples, superimposed structure, classification. (**GG: Cov(Gen)**)

Tomaž Pisanski, John Shawe-Taylor, and Jože Vrabec

1983a Edge-colorability of graph bundles. *J. Combin. Theory Ser. B* 35 (1983), 12–19. MR 85b:05086. Zbl 505.05034, (515.05031).

A graph bundle is, roughly, a covering graph with an arbitrary graph F_v (the “fibre”) over each vertex v, so that the edges covering $e : vw$ induce an isomorphism $F_v \to F_w$. (**GG: Cov(Gen): ECol**)

Leonidas Pitsoulis

See also G. Appa.

Leonidas Pitsoulis and Konstantinos Papalamprou

20xxa Decomposition of binary signed-graphic matroids. Submitted. arXiv:1011.6497.

A binary matroid is signed-graphic iff, for some copoint H, all the bridges of H (in the sense of Tutte) are graphic aside from one that is signed-graphic (and possibly graphic). (**SG: M, Str**)

Leonidas Pitsoulis, Konstantinos Papalamprou, Gautam Appa, and Balázs Kotnyek

2009a On the representability of totally unimodular matrices on bidirected graphs. *Discrete Math.* 309 (2009), no. 16, 5024–5042. MR 2548904 (2010m:05182). Zbl 1182.05120.

Tour matrices of bidirected graphs are closed under 1-, 2-, and 3-sums. Possibly, every totally unimodular matrix is a tour matrix. (**Ori: Incid(Gen)**)

Irene Pivotto

See B. Guenin.

Michael Plantholt

See F. Harary.

M.D. Plummer

See L. Lovász.

Oskar E. Polansky

See I. Gutman.

Svatopluk Poljak

See also Y. Crama and B. Mohar.

Svatopluk Poljak and Daniel Turzík

1982a A polynomial algorithm for constructing a large bipartite subgraph, with an application to a satisfiability problem. *Canad. J. Math.* 34 (1982), 519–524. MR 83j:05048. Zbl 471.68041, (487.68058).

Main Theorem: For a simple, connected signed graph of order n and size $|E| = m$, the frustration index $l(\Sigma) \leq \frac{1}{2}[m - \frac{1}{2}(n-1)]$. The proof is algorithmic, by constructing a (relatively) small deletion set. Dictionary: Σ is an "edge-2-colored graph" (G, c), E^+ and E^- are called E_1 and E_2, a balanced subgraph is "generalized bipartite", and $m - l(\Sigma)$ is what is calculated. [This gives an upper bound on $D(\Gamma) := \max_\sigma l(\Gamma, \sigma)$ for a connected, simple graph, whereas Akiyama, Avis, Chvátal, and Era (1981a) has a lower bound on D.] (**SG: Fr, Alg**)

1986a A polynomial time heuristic for certain subgraph optimization problems with guaranteed worst case bound. *Discrete Math.* 58 (1986), 99–104. MR 87h:68131. Zbl 585.05032.

Generalizes (1982a), with application to signed graphs in Cor. 3. (**SG: Fr, Alg**)

1987a On a facet of the balanced subgraph polytope. *Časopis Pěst. Mat.* 112 (1987), 373–380. MR 89g:57009. Zbl 643.05059.

The polytope $P_B(\Sigma)$ (the authors write P_{BL}) is the convex hull in $\mathbb{R}^E$ of characteristic vectors of balanced edge sets. It generalizes the bipartite subgraph polytope $P_B(\Gamma) = P_B(-\Gamma)$ (see Barahona, Grötschel, and Mahjoub (1985a)), but is essentially equivalent to it according to Prop. 2: The negative-subdivision trick preserves facets of the polytope. Thm. 1 gives new facets, corresponding to certain circulant subgraphs. (They are certain unions of two Hamilton circles, each having constant sign.) (**SG: Fr, Geom**)

1992a Max-cut in circulant graphs. *Discrete Math.* 108 (1992), 379–392. MR 93k:05101.

Further development of (1987a) for all-negative Σ. The import for general signed graphs is not discussed. (**Par: Fr, Geom**)

Svatopluk Poljak and Zsolt Tuza

1995a Maximum cuts and large bipartite subgraphs. In: W. Cook, L. Lovaśz, and P. Seymour, eds., *Combinatorial Optimization* (Papers from the DIMACS Special Year), pp. 181–244. DIMACS Ser. Discrete Math. Theor. Computer Sci., Vol. 20. Amer. Math. Soc., Providence, R.I., 1995. MR 95m:90008. Zbl 819.00048.

Surveys max-cut and weighted max-cut [that is, max size balanced subgraph and max weight balanced subgraph in all-negative signed graphs]. See esp. §2.9: "Bipartite subgraph polytope and weakly bipartite graphs". [The weakly bipartite classes announced by Gerards suggested that a signed-graph characterization of weakly bipartite graphs is called for. This is provided by Guenin (2001a).]

§1.2, "Lower bounds, expected size, and heuristics", surveys results for all-negative signed graphs that are analogous to results in Akiyama, Avis, Chvátal, and Era (1981a) (*q.v.*), etc. [*Problem.* Generalize any of these results, that are not already generalized, to signed simple graphs and to simply signed graphs.] (**par: Fr, tg(Sw): Exp, Ref**)

Y. Pomeau

See B. Derrida.

Dragos Popescu [Dragoş-Radu Popescu]

See Dragoş-Radu Popescu.

Dragoş-Radu Popescu [Dragos Popescu]

1979a Proprietati ale grafurilor semnate. [Properties of signed graphs.] (In Romanian. French summary.) *Stud. Cerc. Mat.* 31 (1979), 433–452. MR 82b:05111. Zbl 426.05048.

A signed K_n is balanced or antibalanced or has a positive and a negative circle of every length $k = 3, \ldots, n$. For odd n, the signed K_n if not balanced has at least $\frac{n-1}{2}$ negative Hamiltonian circles. For even n, $-K_n$ does not maximize the number of negative circles. A "circle basis" is a set of the smallest number of circles whose signs determine all circle signs. This is proved to have $\binom{n-1}{2}$ members. Furthermore, there is a basis consisting of k-circles for each $k = 3, \ldots, n$. [A circle basis in this sense is the same as a basis of circles for the binary cycle space. See Zaslavsky (1981b), Topp and Ulatowski (1987a).] (**SG: Fr**)

1991a Cicluri în grafuri semnate. [Cycles in signed graphs.] (In Romanian; French summary.) *Stud. Cercet. Mat.* 43 (1991), no. 3/4, 85–219. MR 92j:05114. Zbl 751.05060.

Ch. 1: "A-balance" (p. 91). Let F be a spanning subgraph of K_n and A a signed K_n. The "product" of signed graphs is $\Sigma_1 * \Sigma_2$ whose underlying graph is $|\Sigma_1| \cup |\Sigma_2|$, signed as in Σ_i for an edge in only one Σ_i but with sign $\sigma_1(e)\sigma_2(e)$ if in both. Let $\mathcal{G}_F$ denote the group of all signings of F; let $\mathcal{G}_F(A)$ be the group generated by the set of restrictions to F of isomorphs of A. A member of $\mathcal{G}_F(A)$ is "A-balanced"; other members of $\mathcal{G}_F$ are A-unbalanced. We let $\hat{\Sigma}$ denote the coset of Σ and $\approx$ the "isomorphism" of cosets induced by graph isomorphism, i.e., cosets are isomorphic if they have isomorphic members. Let $\dot{\Sigma}$ be the isomorphism class of Σ, $\dot{\hat{\Sigma}}$ the isomorphism class of $\hat{\Sigma}$, and $\overset{\circ}{\to} \Sigma := \bigcup \dot{\hat{\Sigma}}$. Now choose a system of representatives of the coset isomorphism classes, $R = \{\Sigma_1, \ldots, \Sigma_l\}$. Prop. 1.4.1. Each $\dot{\Sigma}$ intersects exactly one $\hat{\Sigma}_i$. Let $R_i = \{\Sigma_{i1}, \ldots, \Sigma_{ia_i}\}$ be a system of representatives of $\hat{\Sigma}_i / \cong$, arranged so that $|E^-(\Sigma_{ij})|$ is a minimum when $j = 1$. This minimum value is the "[line] index of A-imbalance" of each $\Sigma \in \overset{\circ}{\to} \Sigma_i$ and is denoted by $\delta_A(\Sigma)$. (§2.1: Taking A to be K_n with one vertex star all negative makes this equal the frustration index $l(\Sigma)$.) Prop. 1.5.1. $\delta_A(\Sigma)$ is the least number of edges whose sign needs to be changed to make Σ A-balanced. Prop. 1.5.2. $\delta_A(\Sigma) = |E^-(\Sigma)|$ iff $|E^-(\Sigma) \cap E^-(F, \beta)| \leq \frac{1}{2}|E^-(F, \beta)|$ for every signing $\beta)$ of F. Finally, for each $\Sigma \in \mathcal{G}_F$ define the "Σ-relation" on coset isomorphism classes $\dot{\hat{\Sigma}}_i$ to be the relation generated by negating in Σ_1 all the edges of $E^-(\Sigma)$, extended by isomorphism and transitivity. This is well defined (Prop. 1.6.1) and symmetric (Prop. 1.6.2) and is preserved under negation of coset isomorphism classes (Prop. 1.6.4, 1.6.5). Self-negative classes, such that $\dot{\hat{\Sigma}} \approx -\dot{\hat{\Sigma}}$, are the subject of Prop. 1.6.3.

Ch. 2: "Signed complete graphs" (p. 106). §2.5: "H-graphs". If

H is a signed K_h, a "standard H-graph" Σ is a signed K_n such that $\Sigma^- \cong H^- \cup K^c_{n-h}$. Prop. 2.5.3. Assume certain hypotheses on n, $|X_0|$ for $X_0 \subseteq V(\Sigma)$, and a quantity $D^-(H)$ derived from negative degrees. Then $|E^-| = l(\Sigma) \Rightarrow$ the induced subgraph $G{:}X_0$ is a standard H-graph with $|E^-(\Sigma{:}X_0)| = l(\Sigma{:}X_0)$. The cases $H^- = K_1$, K_2, and a 2-edge path are worked out. For the former, Prop. 2.5.3 reduces to Sozański's (1976a) Thm. 3.

Ch. 3: "Frustration index" (p. 158). Some upper bounds.

Ch. 4: "Evaluations, divisibility properties" (p. 174). Similar to parts of (1996a) and Popescu and Tomescu (1996b).

Ch. 5: "Maximal properties" (p. 198). §5.1: "Minimum number and maximum number of negative stars, resp. 2-stars". §5.2 is a special case of Popescu and Tomescu (1996a), Thm. 2. §5.3: "On the maximum number of negative cycles in some signed complete graphs". Shows that Conjecture 1 is false for even $n \geq 6$. Some results on the odd case.

Conjecture 1 (Tomescu). A signed complete graph of odd order has the most negative circles iff it is antibalanced. (Partial results are in §5.3.) [This example maximizes $l(\Sigma)$. A somewhat related conjecture is in Zaslavsky (1997b).] *Conjecture* 2. See (1993a). *Conjecture* 3. Given k and m, there is $n(k,m)$ so that for any $n \geq n(k,m)$, a signed K_n with m negative edges has (a) the most negative k-circles iff the negative edges are pairwise nonadjacent; (b) the fewest iff the negative edges form a star. (**SG: Bal(Gen), KG, Fr, Enum: Circles, Paths**)

1993a Problem 17. *Research Problems* at the Int. Conf. on Combinatorics (Keszthely, 1993). Unpublished manuscript. János Bolyai Math. Soc., Budapest, 1993.

Conjecture. An unbalanced signed complete graph has the minimum number of negative circles iff its frustration index equals 1. (**SG: Fr**)

1996a Une méthode d'énumération des cycles négatifs d'un graphe signé. *Discrete Math.* 150 (1996), 337–345. MR 97c:05077. Zbl 960.39919.

The numbers of negative subgraphs, especially circles and paths of length k, in an arbitrarily signed K_n. Formulas and divisibility and congruence properties. Extends part of Popescu and Tomescu (1996a). (**SG: KG, Enum: Circles, Paths**)

1999a Balance in systems of finite sets. Proc. Annual Meeting Fac. Math. (Bucharest, 1999). Proc. Annual Meeting Faculty Math. (Bucharest, 1999). *An. Univ. Bucureşti Mat. Inform.* 48 (1999), no. 2, 29–40. MR 1829295 (2002c:05082). (**SG: Bal, Gen**)

2001a An inequality on the maximum number of negative cycles in complete signed graphs. *Math. Rep. (Bucur.)* 3(53) (2001), no. 1, 53–60. MR 2002m:05193. Zbl 1017.05099. (**SG: Fr**)

Dragoş-Radu Popescu and Ioan Tomescu

1996a Negative cycles in complete signed graphs. *Discrete Appl. Math.* 68 (1996), 145–152. MR 98f:05098. Zbl 960.35935.

The number c_p of negative circles of length p in a signed K_n with s negative edges. Thm. 1: For n sufficiently large compared to p and s, c_p is minimized if E^- is a star (iff, when $s > 3$) and is maximized iff E^- is a matching. Thm. 2: c_p is divisible by $2^{p-2-\lfloor \log_2(p-1) \rfloor}$. Thm. 3:

If $s \sim \lambda n$ and $p \sim \mu n$ and the negative-subgraph degrees are bounded (this is essential), then asymptotically the fraction of negative p-circles is $\frac{1}{2}(1 - e^{-4\lambda\mu})$. (**SG: KG: Fr, Enum: Circles**)

1996b Bonferroni inequalities and negative cycles in large complete signed graphs. *European J. Combin.* 17 (1996), 479–483. MR 97d:05177. Zbl 861.05036.

A much earlier version of (1996a) with delayed publication. Contains part of (1996a): a version of Thm. 1 and a restricted form of Thm. 3. (**SG: KG: Fr, Enum: Circles**)

L. Pósa

See P. Erdős.

Alexander Postnikov

1997a Intransitive trees. *J. Combin. Theory Ser. A* 79 (1997), 360–366. MR 98b:05036. Zbl 876.05042.

§4.2 mentions the lift matroid of $\{1\}\vec{K}_n$, i.e., the integral poise gains of a transitively oriented complete graph, represented by the Linial arrangement. [See also Stanley (1996a).] (**GG: M, Geom**)

Alexander Postnikov and Richard P. Stanley

2000a Deformations of Coxeter hyperplane arrangements. *J. Combin. Theory Ser. A* 91 (2000), 544–597. MR 2002g:52032. Zbl 962.05004.

The arrangements are the canonical affine-hyperplane lift representations of certain additive real gain graphs. Characteristic polynomials of the former, equalling zero-free chromatic polynomials of the latter, are calculated. And much more. (**gg: Geom, M, Invar**)

B. Prashanth

See P. Siva Kota Reddy.

Primož Potočnik

See T. Pisanski.

K.O. Price, E. Harburg and T.M. Newcomb

1966a Psychological balance in situations of negative interpersonal attitudes. *J. Personality Social Psychol.* 3 (1966), pp. 265–270. (**PsS**)

Geert Prins

See F. Harary.

Sharon Pronchik

See L. Fern.

Andrzey Proskurowski

See A.M. Farley.

J. Scott Provan

1983a Determinacy in linear systems and networks. *SIAM J. Algebraic Discrete Methods* 4 (1983), 262–278. MR 84g:90061. Zbl 558.93018. (**QSol, GN**)

1987a Substitutes and complements in constrained linear models. *SIAM J. Algebraic Discrete Methods* 8 (1987), 585–603. MR 89c:90072. Zbl 645.90049.

§4: "Determinacy in a class of network models." [Fig. 1 and Thm. 4.7 hint at a possible digraph version of the signed-graph or gain-graph frame matroid.] (**sg?, gg: m(bases?): gen**)

Teresa M. Przytycka and Józef H. Przytycki

1988a Invariants of chromatic graphs. Tech. Rep. No. 88-22, Univ. of British Columbia, Vancouver, B.C., 1988.

Generalizing concepts from Kauffman (1989a). [See also Traldi (1989a) and Zaslavsky (1992b).] (**SGc: Gen: Invar, Knot**)

1993a Subexponentially computable truncations of Jones-type polynomials. In: Neil Robertson and Paul Seymour, eds., *Graph Structure Theory* (Proc., Seattle, 1991), pp. 63–108. Contemp. Math., Vol. 147. Amer. Math. Soc., Providence, R.I., 1993. MR 95c:57016. Zbl 812.57010.

A "chromatic graph" is a graph with edges weighted from the set $Z \times \{d, l\}$, Z being [apparently] an arbitrary set of "colors". A "dichromatic graph" has $Z = \{+, -\}$. Such graphs have general dichromatic polynomials [see Przytycka and Przytycki (1988a), Traldi (1989a), and Zaslavsky (1992b)], as [partially] anticipated by Fortuin and Kasteleyn (1972a). I will not attempt to summarize this paper. (**SGc: Invar, Knot, Ref**)

Jozef H. Przytycki

See K. Murasugi and T.M. Przytycka.

Vlastimil Ptak

See M. Fiedler.

Charles J. Puccia and Richard Levins

1986a *Qualitative Modeling of Complex Systems: An Introduction to Loop Analysis and Time Averaging.* Harvard Univ. Press, Cambridge, Mass., 1986. (**SD: QM: QSta: Cycles**)

William R. Pulleyblank

See J.-M. Bourjolly and M. Grötschel.

L. Pyber

See L. Lovász.

Jian Qi

See S.W. Tan.

Hongxun Qin

See also J.E. Bonin, P. Brooksbank, T. Dowling, and D.C. Slilaty.

2004a Complete principal truncations of Dowling lattices. *Adv. Appl. Math.* 32 (2004), no. 1-2, 364–379. MR 2005e:06003. Zbl 1041.05019.

These matroids are determined by their Tutte polynomials, except that only the order of the group can be determined. (**gg: M: Incid**)

Hongxun Qin, Daniel C. Slilaty, and Xiangqian Zhou

2009a The regular excluded minors for signed-graphic matroids. *Combin. Prob. Computing* 18 (2009), 953–978. MR 2550378 (2010m:05062). Zbl 1231.05063.

The complete list of 31 forbidden minors that are regular matroids. [Annot. 10 Sept 2010.] (**SG: M: Str**)

Wen-Yuan Qiu

See G. Hu.

Louis V. Quintas

See M. Gargano.

James P. Quirk

See also L. Bassett and J.S. Maybee.

1974a A class of generalized Metzlerian matrices. In: George Horwich and Paul A. Samuelson, eds., *Trade, Stability, and Macroeconomics: Essays in Honor of Lloyd A. Metzler*, pp. 203–220. Academic Press, New York, 1974. (**QM: QSta: sd**)

1981a Qualitative stability of matrices and economic theory: a survey article. In: Harvey J. Greenberg and John S. Maybee, eds., *Computer-Assisted Analysis and Model Simplification* (Proc. Sympos., Boulder, Col., 1980), pp. 113–164. Discussion, pp. 193–199. Academic Press, New York, 1981. MR 82g:00016 (book). Zbl 495.93001 (book).

Comments by W.M. Gorman (pp. 175–189) and Eli Hellerman (pp. 191–192). Discussion: see pp. 193–196. (**QM: QSta: sd, bal: Exp**)

James Quirk and Richard Ruppert

1965a Qualitative economics and the stability of equilibrium. *Rev. Economic Stud.* 32 (1965), 311–326. (**QM: QSta: sd**)

Nicole Radde, Nadav S. Bar, and Murad Banaji

2010a Graphical methods for analysing feedback in biological networks – A survey. *Int. J. Systems Sci.* 41 (2010), no. 1, 35–46. MR 2599706 (no rev). (**SD, Biol: Exp**)

Filippo Radicchi, Daniele Vilone, and Hildegard Meyer-Ortmanns

2007a Universality class of triad dynamics on a triangular lattice. *Phys. Rev. E* 75 (2007), 021118. (**SG: Bal**)

Filippo Radicchi, Daniele Vilone, Sooeyon Yoon, and Hildegard Meyer-Ortmanns

2007a Social balance as a satisfiability problem of computer science. *Phys. Rev. E (3)* 75 (2007), no. 2, 026106, 17 pp. MR 2354025 (2008g:91190).

Antal, Krapivsky, and Redner (2005a) is generalized to k-cycle dynamics. [Annot. 20 June 2011.] (**SG: Bal: Alg**)

W.M. Raike

See A. Charnes.

K.R. Rajanna

See P. Siva Kota Reddy.

R. Rammal

See F. Barahona and I. Bieche.

K. Ranganathan

See R. Balakrishnan.

R. Rangarajan

See also P. Siva Kota Reddy.

R. Rangarajan and P. Siva Kota Reddy

2008a Notions of balance in symmetric n-sigraphs. *Proc. Jangjeon Math. Soc.* 11 (2008), no. 2, 145–151. MR 2482598 (2010h:05143). Zbl 1205.05102.

S_n is a symmetric n-signed graph. Further definitions as in the notes to Sampathkumar, Siva Kota Reddy, and Subramanya (2008a, 2010c). §2, "Balance in an n-sigraph $S_n = (G, \sigma)$." Prop. 1 (generalizing Harary (1953a) for signed graphs): S_n is balanced iff for each pair $u, v \in V$, every uv-path has the same gain. [The simple proof of $\implies$, which depends on the fact that the gain group has exponent 2, is the best I have seen. The proof of $\Leftarrow$ is incorrect.] Prop. 4: Σ_{S_n} is balanced iff $V = V_1 \cup V_2$ such that an edge has identity gain iff it lies within V_1 or V_2. Good proof via min as defined in the cited notes. §3, "Clustering in an n-sigraph $S_n = (G, \sigma)$." S_n is "clusterable" if V has a partition π such that an edge has identity gain iff it lies within a part of π. Prop. 5 generalizes Davis (1967a) to n-signed graphs. §3.1: "Local balance (Local i-balance) in

an n-sigraph $S_n = (G, \sigma)$." Prop. 6 generalizes Harary (1955a) on local balance [with a good proof]. Prop. 8: A complete S_n is balanced iff every triangle on one vertex is balanced. Prop. 9 [incorrect]: The same for imbalance. Prop. 10 gives the number of balanced $S_n = (K_k, \sigma)$ [incorrect; the correct value is $2^{\lceil k/2 \rceil (n-1)}$]. [The results are equally true, *mutatis mutandis*, without assuming symmetry.] [Minor typos require correction.] [Annot. 9 July 2009.] (**SG(Gen), gg: Bal**)

2009a Notions of balance and consistency on symmetric n-marked graphs. *Bull. Pure Appl. Math.* 3 (2009), no. 1, 1–8. MR 2537685 (2010i:05156). Zbl 1200.05097. (**VS(Gen), SG(Gen), gg: Bal**)

2010a The edge C_4 signed graph of a signed graph. *Southeast Asian Bull. Math.* 34 (2010), 1066–1082. MR 2746741 (2011k:05100). Zbl 1240.05141.

Definitions as at Sampathkumar, Siva Kota Reddy, and Subramanya (2008a, 2010c). The edge C_4 signed graph $E_4(\Sigma) := (V', E', \sigma_S)$ where $V' := E$ and $E' := \{ef : \exists\ C_4 \ni e, f \text{ in } |\Sigma|\}$. Prop. 2.1: $E_4(\Sigma)$ is balanced. Cor. 2.5: $E_4(\Sigma) = E_4(-\Sigma)$. Prop. 2.3: $\Sigma \simeq E_4(\Sigma)$ iff Σ is a balanced signing of C_n, $n \geq 5$. Prop. 3.1: Σ' is an $E_4(\Sigma)$ iff it is balanced and $|\Sigma'|$ is an $E_4(\Gamma)$. [Annot. 2 Aug 2009, rev 20 Dec 2010.] (**SG: Bal, Sw, LG(Gen)**)

R. Rangarajan, P. Siva Kota Reddy, and N.D. Soner

2009a Switching equivalence in symmetric n-sigraphs. II. *J. Orissa Math. Soc.* 28 (2009), no. 1–2, 1–12. MR 2664129 (2011k:05099).

Continuation of Rangarajan, Siva Kota Reddy, and Subramanya (2009a) and Siva Kota Reddy and B. Prashanth (2009a). Definitions as at Sampathkumar, Siva Kota Reddy, and Subramanya (2008a, 2010c). Φ is a symmetric n-signed graph. Prop. 4: Φ is the $(\leq m)$-distance graph $D_m(\Phi')$ of some Φ' iff it is balanced and $\|\Phi\|$ is a $(\leq m)$-distance graph. [Sufficiency is incorrect.] Solved [possibly incorrectly]: Φ^c or $\Lambda_S(\Phi^c) \simeq D_m(\Lambda_S(\Phi))$; $\Lambda_S(\Phi)$ or $\Lambda^2_S(\Phi) \simeq D_m(\Phi^{[c]})^{[c]}$ (except $\Lambda^2_S(\Phi) \simeq D_m(\Phi^c)^c$). [The results are equally true without requiring symmetry.] [Annot. 3 Aug 2009.] (**SG(Gen), gg: Sw, LG**)

R. Rangarajan, P. Siva Kota Reddy, and M.S. Subramanya

2009a Switching equivalence in symmetric n-sigraphs. *Adv. Stud. Contemp. Math. (Kyungshang)* 18 (2009), no. 1, 79–85. MR 2479750 (2011a:05141). Zbl 1183.-05033.

Continuation of Siva Kota Reddy, Vijay, and Lokesha (2009a, 2010a). Definitions as at Sampathkumar, Siva Kota Reddy, and Subramanya (2008a). Prop. 4 characterizes $C_E(\Phi)$. Solved: $\Lambda_S(\Phi) \simeq \Phi$; $\Phi \simeq \Lambda_S(\Phi)$; $\Lambda_S(\Phi) \simeq C_E(\Phi)$; $J(\Phi) \simeq C_E(\Phi)$. [The results remain true without assuming symmetry.] [Annot. 2 Aug 2009.] (**SG(Gen), gg: Sw**)

R. Rangarajan, M.S. Subramanya, and P. Siva Kota Reddy

2010a The H-line signed graph of a signed graph. *Int. J. Math. Combin.* 2 (2010), 37–43. Zbl 1216.05052.

H is a connected graph of order ≥ 3. $HL(\Sigma) \subseteq \Lambda_S(\Sigma)$ (defined at Sampathkumar, Siva Kota Reddy, and Subramanya (2010c)); $ef \in E(\Lambda_S(\Sigma))$ is in $HL(\Sigma)$ iff e, f are in a copy of H in $|\Sigma|$. Σ' is an $HL(\Sigma)$ iff it is balanced and $|\Sigma'|$ is an H-line graph. Solved: $HL(\Sigma) \simeq \Sigma$ for

$H = C_k, P_k, K_r L(\Sigma) \simeq \Lambda_S(\Sigma)$. Connections with graphs derived from matrices. [Annot. 7 Jan 2011.] (**SG: LG(Gen), Bal, Adj**)

20xxa Neighborhood signed graphs. *Southeast Asian Bull. Math.*, to appear.

Definitions as at Sampathkumar, Siva Kota Reddy, and Subramanya (2008a, 2010c). The neighborhood signed graph or 2-path graph $P_2(\Sigma)$ is (V, E_2, σ^c) where $E_2 := \{vw : \exists\ vw\text{-path of length } 2\}$. Thm. 5: $P_2(\Sigma)$ is balanced and the signature can be any balanced signature (by appropriate choice of σ). Solved: Σ, $P_2(\Sigma) \simeq \Sigma$; $P_2(\Sigma) \simeq \Sigma^c$; $P_2(\Sigma) \simeq \Lambda_S(\Sigma)$. For connected Σ: $P_2^r(\Sigma) \simeq \Lambda_S(\Sigma)$; $P_2(\Sigma) \simeq J_S(\Sigma)$. Also, $P_2^r(\Sigma) \simeq \Lambda_S^s(\Sigma)$ when $|\Sigma|$ is unicyclic with circle length l and $r, s < l/2$. [Annot. 2 Aug 2009.] (**SG: Bal, Sw, LG(Gen)**)

§5, "$(-1,0,1)$-Matrices and neighborhood signed graphs": Given a $(-1,0,1)$-matrix A with columns $a_1, \ldots, a_n$. Let $V_A := [n]$, $E_A := \{ij : (\exists\ k)\ a_{ki}a_{kj} \neq 0\}$, and $\sigma_A(ij) := \mu_i\mu_j$ where $\mu_i :=$ product of nonzero entries in a_i. Thm. 20: This signed graph of $A(\Sigma)$ is $P_2(\Sigma)$. [Annot. 10 Apr, 2 Aug 2009.] (**SG: Adj: Bal**)

M.R. Rao

See Y.M.I. Dirickx.

S.B. Rao

See also B.D. Acharya, P. Das, and [G.R.] Vijaya Kumar.

1984a Characterizations of harmonious marked graphs and consistent nets. *J. Combin. Inform. System Sci.* 9 (1984), 97–112. MR 89h:05048. Zbl 625.05049.

A complicated solution, with a polynomial-time algorithm, to the problem of characterizing consistency in vertex-signed graphs (*cf.* Beineke and Harary 1978b). Thm. 4.1 points out that graphs with signed vertices and edges can be easily converted to graphs with signed vertices only; thus harmony in graphs with signed vertices and edges is characterized as well. [This paper was independent of and approximately simultaneous with B.D. Acharya (1983b, 1984a).] [See Joglekar, Shah, and Diwan (2010a) for the last word.] (**SG, VS: Bal, Alg**)

S.B. Rao, B.D. Acharya, T. Singh, and Mukti Acharya

2005a Graceful complete signed graphs. In: S. Arumugam, B.D. Acharya, and S.B. Rao, eds., *Graphs, Combinatorics, Algorithms and Applications* (Proc. Nat. Conf., Anand Nagar, Krishnankul, India, 2004), pp. 123–124. Narosa Publishing House, New Delhi, 2005.

Extended abstract without proofs. "Graceful" means $(1,1)$-graceful, $r = 1$, as at M. Acharya and Singh (2004a). Thm. 1: (K_n, σ) is graceful iff $n \leq 3$, $n = 4$ and $|E^-| \neq 3$, or $n = 5$ and $|E^-| \neq 5$ is odd and neither Σ^+ nor Σ^- is $K_{1,3}$. The proof involves a recursive labelling procedure. [Annot. 21 July 2010.] (**SG**)

S.B. Rao, N.M. Singhi, and K.S. Vijayan

1981a The minimal forbidden subgraphs for generalized line graphs. In: S.B. Rao, ed., *Combinatorics and Graph Theory* (Proc. Sympos., Calcutta, 1980), pp. 459–472. Lect. Notes in Math., 885. Springer-Verlag, Berlin, 1981. MR 83i:05062. Zbl 494.05053.

These are the minimal forbidden induced subgraphs for an all-negative signed simple graph to be the reduced line graph of a signed graph. (**sg: LG, par**)

Vasant Rao
See M. Desai.

A.M. Rappoport
See Ya.R. Grinberg.

Thomas Raschle and Klaus Simon
1995a Recognition of graphs with threshold dimension two. In: *Proceedings of the Twenty-Seventh Annual ACM Symposium on the Theory of Computing* (Las Vegas, 1995), pp. 650–661.
Expounded by Mahadev and Peled (1995a), §8.5 (*q.v.*). (**par: ori, Alg**)

Andre Raspaud and Xuding Zhu
2011a Circular flow on signed graphs. *J. Combin. Theory Ser. B* 101 (2011), 464–479. MR 2832812 (2012j:05191).
Thm. 1: Σ has a nowhere-zero integral and circular [i.e., real] 4-flow if it is edge 4-connected. It has a nowhere-zero circular r-flow with $r < 4$ if it is edge 6-connected. A signed cut D [*cf.* Chen and Wang (2009a)] is described by a signed subset $X = X^+ \cup X^-$ of V. Lemma 3: Σ has a circular r-flow iff it has an orientation such that $1/(r-1) \leq |\partial^+(X)|/|\partial^-(X)| \leq r-1$ for every X. Here $\partial^\varepsilon(X)$ is the set of ends in X, of $e \in D$, that have a certain sign. [Annot. 23 March 2010.]
(**SG: Ori, Flows**)

Dieter Rautenbach and Bruce Reed
2001a The Erdös–Pósa property for odd cycles in highly connected graphs. Paul Erdös and His Mathematics (Budapest, 1999). *Combinatorica* 21 (2001), no. 2, 267–278. MR 2002i:05073. Zbl 981.05066.
The smallest sufficient connectivity in Thomassen (2001a) is about $976k$. [For more, see Hochstättler, Nickel, and Peis (2006a).]
(**par: Fr: Circles**)

H. Donald Ratliff
See J.-Cl. Picard.

Bertram H. Raven
See B.E. Collins.

E.V. Ravve
See E. Fischer.

D.K. Ray-Chaudhuri, N.M. Singhi, and G.R. Vijayakumar
1992a Signed graphs having least eigenvalue around -2. *J. Combin. Inform. System Sci.* 17 (1992), 148–165. MR 94g:05056. (**SG: Adj, Geom: Exp**)

Igor Razgon
See G. Gutin.

Margaret A. Readdy
See also R. Ehrenborg.

2001a The Yuri Manin ring and its $\mathcal{B}_n$-analogue. *Adv. Appl. Math.* 26 (2001), 154–167. MR 2001k:13034. Zbl 989.13016.
P. 164: Lattice of signed compositions ("ordered signed partitions"), from Ehrenborg and Readdy (1999a), §6. Pp. 164–165: Signed permutahedron [equivalent to acyclotope of $\pm K_n^\bullet$]. (**Sgnd**)(**sg: kg: Geom**)

S. Redner
See T. Antal.

P. Siva Kota Reddy

See Siva Kota Reddy (under 'S').

Bruce Reed

See C. Berge, S. Fiorini, J. Geelen, K. Kawarabayashi, and D. Rautenbach.

P. Reed

See A.J. Bray.

Nathan Reff

2012a Spectral properties of complex unit gain graphs. *Linear Algebra Appl.* 436 (2012), no. 9, 3165–3176.

$\mathbb{T}$ Complex unit gain graphs have gain group $\mathbb{T} := \{z \in \mathbb{C} : |z| = 1\}$. Bounds on largest and smallest eigenvalues of $A(\Phi)$ (§3, "Eigenvalues of the adjacency matrix") and $K(\Phi)$ (§4, "Eigenvalues of the Laplacian matrix"). Most (except Thm. 4.9, where the edge gains affect the bounds) generalize known bounds for graphs, the "signless Laplacian" ($K(-\Gamma)$), or signed graphs. Some generalizations are not obvious. Lemmas 3.1, 4.1: The spectrum of A or L depends only on the switching class. Lemmas 3.2, 4.2: If Φ is balanced, the spectra are the same as those of $\|\Phi\|$. [*Problem.* Generalize B.D. Acharya (1980a) by proving the converse.] Thm. 5.1: Exact eigenvalues for circle graphs. [Annot. 30 Oct 2011.]

(**GG: Adj, Incid**)

2012b *Gain Graphs, Group-Oriented Hypergraphs, and Matrices.* Doctoral dissertation, Binghamton University (SUNY), 2012. (**SG, GG, GH: Adj, Incid**)

20xxa New bounds for the Laplacian spectral radius of a signed graph. In preparation. arXiv:1103.4629. (**SG: Adj**)

F. Regonati

See E. Damiani.

Jörg Reichardt and Stefan Bornholdt

2006a Statistical mechanics of community detection. *Phys. Rev. E* 74 (2006), 016110, 14 pp. MR 2276596 (2007h:82089). (**sg: kg: Phys, Clu**)

Gerhard Reinelt

See F. Barahona, C. De Simone, and M. Grötschel.

Victor Reiner

See also P. Edelman.

1993a Signed posets. *J. Combin. Theory Ser. A* 62 (1993), 324–360. MR 94d:06011. Zbl 773.06008.

They are equivalent to acyclic bidirected graphs.

(**Sgnd, sg: Ori: Str, geom**)

Élisabeth Remy

See also A. Naldi.

Élisabeth Remy and Paul Ruet

2007a On differentiation and homeostatic behaviours of Boolean dynamical systems. In: Corrado Priami, ed., *Transactions on Computational Systems Biology VIII*, pp. 92–101. Lect. Notes in Bioinformatics. Lect. Notes in Computer Sci., Vol. 4780. Springer, Berlin, 2007. MR 2303765. Zbl 1141.92330. (**SD**)

Élisabeth Remy, Paul Ruet, and Denis Thieffry

2006a Positive or negative regulatory circuit inference from multilevel dynamics. In: Christian Commault *et al.*, eds., *Positive Systems* (Proceedings of the second multidisciplinary international symposium on positive systems: Theory and

applications, POSTA 06, Grenoble, 2006), pp. 263–270. Lect. Notes in Control and Inform. Sci., Vol. 341. Springer, Berlin, 2006. MR 2250264 (2007e:92032). Zbl 1132.93305. (**SD**)

2008a Graphic requirements for multistability and attractive cycles in a Boolean dynamical framework. *Adv. Appl. Math.* 41 (2008), no. 3, 335–350. MR 2449595 (2010j:37018). Zbl 1169.05333. (**SD**)

Qing Jun Ren

See also H.S. Du.

2001a A note on the quasi-Laplacian spectra of graphs. (In Chinese.) *J. Nanjing Normal Univ. Nat. Sci. Ed.* 24 (2001), no. 2, 23–25. MR 1849157 (no rev). Zbl 984.05059. (**Par: Adj**)

Raghunathan Rengaswamy

See M. Bhushan and M.R. Maurya.

Adrien Richard

2009a Positive circuits and maximal number of fixed points in discrete dynamical systems. *Discrete Appl. Math.* 157 (2009), no. 15, 3281–3288. MR 2554797 (2010m:05132). Zbl 1193.05085. (**SG**)

2010a Negative circuits and sustained oscillations in asynchronous automata networks. *Adv. Appl. Math.* 44 (2010), 378–392. MR 2600786 (2011a:92007). Zbl 1201.37117. (**SD**)

2011a Local negative circuits and fixed points in non-expansive Boolean networks. *Discrete Appl. Math.* 159 (2011), no. 11, 1085–1093. (**SD**)

Adrien Richard and Jean-Paul Comet

2007a Necessary conditions for multistationarity in discrete dynamical systems. *Discrete Appl. Math.* 155 (2007), no. 18, 2403–2413. MR 2365052 (2008m:37032). Zbl 1125.37062. (**SG**)

J. Richelle

See R. Thomas.

Daniel J. Richman

See J.S. Maybee.

R.J. Riddell

1951a *Contributions to the Theory of Condensation.* Dissertation, University of Michigan, Ann Arbor, 1951.

Includes the number of labelled simple 1-trees of order n. [Sequence A057500 in N.J.A. Sloane, *The On-Line Encyclopedia of Integer Sequences*, http://oeis.org/.]

[*Cf.* Neudauer, Meyers, and Stevens (2001a).] (**bic: Invar(bases)**)

Heiko Rieger

See M.J. Alava and B. Coluzzi.

Robert G. Rieper

See J. Chen.

M.J. Rigby

See A.C. Day.

Arnout van de Rijt

See A. van de Rijt (under V).

James E. Riley

1969a An application of graph theory to social psychology. In: G. Chartrand and S.F.

Kapoor, eds., *The Many Facets of Graph Theory* (Proc., Kalamazoo, 1968), pp. 275–280. Lecture Notes in Mathematics, Volume 110. MR 252266 (40 #5487). (**PsS: SG: Exp**)

Chong S. Rim
See H. Choi.

G. Rinaldi
See C. De Simone and J. Lukic.

R.D. Ringeisen
See also M.J. Lipman.

1974a Isolation, a game on a graph. *Math. Mag.* 47 (May, 1974), no. 3, 132–138. MR 51 #9842. (**TG**)

Gerhard Ringel
See also N. Hartsfield and M. Jungerman.

1974a *Map Color Theorem.* Grundl. math. Wiss., B. 209. Springer-Verlag, Berlin, 1974. MR 50 #2860. Zbl 287.05102.
"Cascades": see Youngs (1968b). (**sg: Ori: Appl**)

1974a *Teorema o Raskraske Kart.* Transl. V.B. Alekseev. Ed. G.P. Gavrilov. "Mir", Moskva, 1977. MR 57 #5809. Zbl 439.05019.
Russian translation of (1974a). (**sg: Ori: Appl**)

1977a The combinatorial map color theorem. *J. Graph Theory* 1 (1977), 141–155. MR 56 #2860. Zbl 386.05030.
Signed rotation systems for graphs. Thm. 12: Signed rotation systems describe all cellular embeddings of a graph; an embedding is orientable iff its signature is balanced. Compare Stahl (1978a). Dictionary: "Triple" means graph with signed rotation system. "Orientable" triple means balanced signature. "Oriented" means all positive. (**SG: Top, Sw**)

Oliver Riordan
See B. Bollobás.

F. Ritort
See E. Marinari.

María Robbiano
See N.M.M. Abreu and I. Gutman.

Jakayla R. Robbins

2003a *On Orientations of the Free Spikes.* Doctoral dissertation, Univ. of Kentucky, 2003. MR 2704379 (no rev). (**gg: M: Invar**)

2007a Enumerating orientations of the free spikes. *European J. Combin.* 28 (2007), 868–875. MR 2300767 (2007k:05047). Zbl 1112.05022.
The number of orientations of the free spike matroid $L(2C_n, \varnothing)$ is $2^{n-1}D_n$, $D_n :=$ Dedekind number. [Annot. 29 Sept 2011.]
(**gg: M: Invar**)

2008a Representable orientations of the free spikes. *Discrete Math.* 308 (2008), no. 22, 5174–5183. MR 2450452 (2009h:05052). Zbl 1157.05020.
In general, not all orientations of the free spike matroid $L(2C_n, \varnothing)$ have a real vector representation. Also, bounds on the number of representable orientations. [Annot. 29 Sept 2011.] (**gg: M: Geom, Invar**)

authorFred S. Roberts
See also T.A. Brown and R.Z. Norman.

1974a Structural characterizations of stability of signed digraphs under pulse processes. In: Ruth A. Bari and Frank Harary, eds., *Graphs and Combinatorics* (Proc. Capital Conf., Geo. Washington Univ., 1973), pp. 330–338. Lect. Notes in Math., 406. Springer-Verlag, Berlin, 1974. MR 50 #12792. Zbl 302.05107. (**SDw**)

1976b *Discrete Mathematical Models, With Applications to Social, Biological, and Environmental Problems.* Prentice-Hall, Englewood Cliffs, N.J., 1976. Zbl 363.90002.

§3.1: "Signed graphs and the theory of structural balance." Many topics are developed in the exercises. Exercise 4.2.7 (from Phillips (1967a)). (**SG, SD: Bal, Alg, Adj, Clu, Fr, PsS: Exp, Exr**)

Ch. 4: "Weighted digraphs and pulse processes." Signed digraphs here are treated as unit-weighted digraphs. Note esp.: §4.3: "The signed or weighted digraph as a tool for modelling complex systems." Conclusions about models are drawn from very simple properties of their signed digraphs. §4.4: "Pulse processes." §4.5: "Stability in pulse processes." Stability is connected to eigenvalues of $A(\Sigma)$. (**SDw, SD, WD: Bal, Adj, PsS: Exp, Exr, Ref**)

1978a *Graph Theory and Its Applications to Problems of Society.* CBMS-NSF Regional Conf. Ser. in Appl. Math., 29. Soc. Indust. Appl. Math., Philadelphia, 1978. MR 80g:90036. Zbl 452.05001.

Ch. 9: "Balance theory and social inequalities." Ch. 10: "Pulse processes and their applications." Ch. 11: "Qualitative matrices." (**SG, SD, SDw: Bal, PsS, QM: Exp, Ref**)

1979a Graph theory and the social sciences. In: Robin J. Wilson and Lowell W. Beineke, eds., *Applications of Graph Theory*, Ch. 9, pp. 255–291. Academic Press, London, 1979. MR 81h:05050 (book). Zbl 444.92018.

§2: "Balance and clusterability." Basics in brief. §7: "Signed and weighted digraphs as decision-making models." Cursory. (**SG, PsS, SD, SDw: Bal, Clu, KG: Exp, Ref**)

1986a *Diskretnye matematicheskie modeli s prilozheniyami k sotsialnym, biologicheskim i ekologicheskim zadacham.* Transl. A.M. Rappoport and S.I. Travkin. Ed. and preface by A.I. Teĭman. Teoriya i Metody Sistemnogo Analiza [Theory and Methods of Systems Analysis]. "Nauka", Moscow, 1986. MR 88e:00020. Zbl 662.90002.

Russian edition of (1976b).
(**SG, SD: Bal, Alg, Adj, Clu, Fr, PsS: Exp, Exr**)
(**SDw, SD, WD: Bal, Adj, PsS: Exp, Exr, Ref**)

1989a Applications of combinatorics and graph theory to the biological and social sciences: Seven fundamental ideas. In: Fred Roberts, ed., *Applications of Combinatorics and Graph Theory to the Biological and Social Sciences*, pp. 1–37. IMA Vols. Math. Appl., Vol. 17. Springer-Verlag, New York, 1989. MR 91c:92001.

§4: "Qualitative stability." A fine, concise basic survey. (**QM: SD: Exp, Ref**)

§5: "Balanced signed graphs." Another concise basic survey, and two open problems (p. 20). (**SG: Bal: Exp, Ref**)

1995a On the problem of consistent marking of a graph. *Linear Algebra Appl.* 217 (1995), 255–263. MR 95k:05157. Zbl 830.05059.

Several characterizations of consistent vertex signatures of a graph. Γ is "markable" iff it has a consistent vertex signature that is not all +. Thm.: 3-connected Γ is markable iff it is bipartite. Thm.: A classification of markable 2-connected graphs with girth ≤ 5. [See also Hoede (1992a).] [Annot. 27 Apr 2009.] (**VS: Bal**)

1999a On balanced signed graphs and consistent marked graphs. Ordinal and Symbolic Data Analysis (OSDA '98, Univ. of Mass., Amherst, Mass., 1998). *Electron. Notes Discrete Math.* 2 (1999), 12 pp. MR 1990307 (2004e:05088). Zbl 971.68115.

A survey of balance in signed graphs and consistency in vertex-signed graphs and their supposed applications in social psychology and elsewhere. Results from Xu (1998a) and Roberts and Xu (2003a). §4: "Connections among balance, consistency, and other graph-theoretical notions". Lists some special and general equivalences, esp., with bipartiteness, or with all circle lengths being divisible by 4. §5: "Coherent paths". Characterizations of consistency or balance from Beineke and Harary (7), Roberts and Xu (2003a), Acharya (3), Rao(34). §6: "Fundamental cycles and cycle bases". Hoede's (1992a) characterization of consistency, a variant, and one from Roberts and Xu (2003a) in terms of a circle basis. §7: "Markable graphs". Γ is "markable" iff it has a consistent vertex signature that is not all +. Thm. (Roberts). 3-connected Γ is markable iff it is bipartite. See Roberts (1995a) and Xu (1998a). §8: "Open questions". [Annot. 27 Apr 2009.]

(**SG, VS: Bal, PsS, Appl: Exp**)

2001a Discrete mathematics. In: Neil J. Smelser and Paul B. Baltes, eds., *International Encyclopedia of the Social & Behavioral Sciences*, pp. 3743–3746. Pergamon (Elsevier), 2001.

§5: "Signed and marked graphs". (**SG, VS, PsS: Exp**)

Fred S. Roberts and Thomas A. Brown

1975a Signed digraphs and the energy crisis. *Amer. Math. Monthly* 82 (June–July, 1975), no. 6, 577–594. MR 51 #5195. Zbl 357.90070. (**SD, SDw**)

1977a [Reply to Waterhouse (1977a)]. *Amer. Math. Monthly* 84 (1977), 27.

Fred S. Roberts and Shaoji Xu

2003a Characterizations of consistent marked graphs. 1998 Conf. Ordinal and Symbolic Data Analysis (OSDA '98) (Amherst, Mass.). *Discrete Appl. Math.* 127 (2003), 357–371. MR 1984094 (2004b:05097). Zbl 1026.05054.

Several characterizations of consistent vertex-signed graphs, and algorithms to determine consistency, are surveyed or proved. Thm.: A vertex-signed graph is consistent iff every circle in some circle basis is positive and every two 3-connected vertices have the same sign. [Annot. 26 Apr 2009.] (**SG, VS: Bal, Alg**)

Edmund Robertson

See P. Brooksbank.

Neil Robertson, P.D. Seymour, and Robin Thomas

See also W. McCuaig and J. Maharry.

†1999a Permanents, Pfaffian orientations, and even directed circuits. *Ann. of Math.* (2) 150 (1999), no. 3, 929–975. MR 2001b:15013. Zbl 947.05066.

Question 1. Does a given digraph D have an even cycle? Question

2. Can a given digraph D be signed so that every cycle is negative? (These problems are easily seen to be equivalent.) The main theorem (the "Even Dicycle Thm.") is a structural characterization of digraphs that have a signing in which every cycle is negative. (These were previously characterized by forbidden minors in Seymour and Thomassen (1987a).)

The main theorem is proved also in McCuaig (2004a). See the joint announcement, McCuaig, Robertson, Seymour, and Thomas (1997a). (**SD: par: Str**)

Herbert A. Robinson
See C.R. Johnson.

Robert W. Robinson
See also Harary, Palmer, Robinson, and Schwenk (1977a) and Harary and Robinson (1977a).

1981a Counting graphs with a duality property. In: H.N.V. Temperley, ed., *Combinatorics* (Proc. Eighth British Combinatorial Conf., Swansea, 1981), pp. 156–186. London Math. Soc. Lect. Note Ser., 52. Cambridge Univ. Press, Cambridge, England, 1981. MR 83c:05071. Zbl 462.05035.

The "bilayered digraphs" of §7 are identical to simply signed, loop-free digraphs (where multiple arcs are allowed if they differ in sign or direction). Thm. 1: Their number b_n = number of self-complementary digraphs of order $2n$. Cor. 1: Equality holds if the vertices are signed and k-colored. In §8, Cor. 2 concerns vertex-signed and 2-colored digraphs; Cor. 3 concerns vertex-signed tournaments. Assorted remarks on previous signed enumerations, mainly from Harary, Palmer, Robinson, and Schwenk (1977a), are scattered about the article. (**SD, VS, SG: Enum**)

Y. Roditty
See I. Krasikov.

Vojtěch Rödl
See R.A. Duke.

Jose Antonio Rodriguez
See R.T. Boesch.

Juan A. Rodríguez-Velázquez
See E. Estrada.

Vladimir Rogojin
See A. Alhazov.

Oscar Rojo
See also I. Gutman.

2009a Spectra of copies of a generalized Bethe tree attached to any graph. *Linear Algebra Appl.* 431 (2009), 863–882. MR 2535558 (2011c:05209). Zbl 1168.05328. (**Par: Adj**)

2011a Line graph eigenvalues and line energy of caterpillars. *Linear Algebra Appl.* 435 (2011), 2077–2086. MR 2810648 (2012e:05243). Zbl 1222.05177. (**Par: Adj, LG**)

Oscar Rojo and Raúl D. Jiménez

2011a Line graph of combinations of generalized Bethe trees: Eigenvalues and energy. *Linear Algebra Appl.* 435 (2011), no. 10, 2402–2419. MR 2811125 (2012f:05185). Zbl 1222.05178. (**Par: Adj, LG**)

Oscar Rojo and Luis Medina

2010a Spectral characterization of some weighted rooted graphs with cliques. *Linear Algebra Appl.* 433 (2010), no. 7, 1388–1409. MR 2680266 (2011h:05163). Zbl 1194.05095.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Milton J. Rosenberg

See also R.P. Abelson.

Milton J. Rosenberg and Robert P. Abelson

1960a An analyisis of cognitive balancing. In: Milton J. Rosenberg *et al.*, eds., *Attitude Organization and Change: An Analysis of Consistency Among Attitude Components*, Ch. 4, pp. 112–163. Yale Univ. Press, New Haven, 1960.

An attempt to test structural balance theory experimentally. The test involves, in effect, a signed K_4 [an unusually large graph for such an experiment]. Conclusion: there is a tendency to balance but it competes with other forces. (**PsS**)

Seymour Rosenberg

1968a Mathematical models of social behavior. In: Gardner Lindzey and Elliot Aronson, eds., *The Handbook of Social Psychology*, Second Edn., Vol. 1, Ch. 3, pp. 179–244. Addison-Wesley, Reading, Mass., 1968.

"Balance model," pp. 196–199. "Congruity model," pp. 199–203. (**PsS, SG: Bal: Exp, Ref**)

M. Rosenfeld

See W.D. McCuaig.

Elissa Ross

2011a *Geometric and Combinatorial Rigidity of Periodic Frameworks as Graphs on the Torus.* Doctoral dissertation, York University, 2011. MR 2941979 (no rev). (**GG: Cov, Top, Geom**)

Philippe A. Rossignol

See J.M. Dambacher.

Gian-Carlo Rota

See P. Doubilet.

Günter Rote

See H. Edelsbrunner.

Ron M. Roth and Krishnamurthy Viswanathan

2007a On the hardness of decoding the Gale–Berlekamp code. In: *Information Theory, 2007* (ISIT 2007. IEEE Int. Symp., Nice, 2007), pp. 1356–1360. IEEE Press, 2007.

See (2008a). (**sg: fr, Alg**)

2008a On the hardness of decoding the Gale–Berlekamp code. *IEEE Trans. Inform. Theory* 54 (2008), no. 3, 1050–1060. MR 2445050 (2010h:94267).

Section III, "Relaxed problem": The frustration index of a bipartite signed graph is NP-complete. Thm. 4.1: The frustration index of a signed $K_{n,n}$ (where n is a variable) is NP-complete. The proofs use the bipartite adjacency matrix of the signed graph. The latter problem is polynomially reduced to the former by a construction using Kronecker product and a Hadamard matrix. The problems are interpreted as nearest-neighbor decoding of the Gale–Berlekamp code of order n.

Section V, "Decoding algorithm over the BSC": A polynomial-time approximate decoding algorithm that is asymptotically reliable. [Annot. 2 Sept 2009.] (**sg: fr, Alg**)

Uriel G. Rothblum and Hans Schneider

1980a Characterizations of optimal scalings of matrices. *Math. Programming* 19 (1980), 121–136. MR 81j:65064. Zbl 437.65038. (**gg: m, Sw**)

1982a Characterizations of extreme normalized circulations satisfying linear constraints. *Linear Algebra Appl.* 46 (1982), 61–72. MR 84d:90047. Zbl 503.05032. (**gg: m**)

Peter Rowlinson

See D.M. Cardoso and D.M. Cvetković.

Bernard Roy

1959a Contribution de la théorie des graphes à l'étude de certains problèmes linéaires. *C.R. Acad. Sci. Paris* 248 (1959), 2437–2439. MR 22 #2493. (**WD: OG**)

1970a *Algèbre moderne et théorie des graphes, orientées vers les sciences économiques et sociales. Tome II: Applications et problèmes spécifiques.* Dunod, Paris, 1970. MR 41 #5039. Zbl 238.90073.

§ IX.B.3.b: "Flots multiplicatifs et non conditionnels, ou k-flots." § IX.E.1.b: "Extension du problème central aux k-flots." § IX.E.2.c: "Quelques utilisations concretes des k-flots." (**GN: m(circuit): Exp**)

Gordon F. Royle

See M.N. Ellingham and C.D. Godsil.

G. Rozenberg

See A.H. Deutz, A. Ehrenfeucht, and T. Harju.

Arthur L. Rubin

See P. Erdős.

Jason D. Rudd

See P.J. Cameron.

Paul Ruet

See É. Remy.

Richard Ruppert

See J. Quirk.

Lucas Rusnak

††2010a *Oriented Hypergraphs.* Doctoral dissertation, Binghamton Univ. (SUNY), 2010. MR 2941411 (no rev).

Oriented hypergraphs generalize bidirected graphs: Each incidence gets a direction, or sign. Main interest: The linear dependencies of columns of a $(0, \pm 1)$-matrix, treated as the incidence matrix of an oriented hypergraph. Techniques are generalizations of those of signed graphic matroids (Zaslavsky 1982a) but more complicated. The methods are most applicable to the matrices known as "balanceable".

(**SH: Incid, Str, SG, Ori**)

Carrie Rutherford

See M. Banaji.

K. Rybnikov [K.A. Rybnikov, Jr.; Konstantin Rybnikov]

See also S.S. Ryshkov.

1999a Stresses and liftings of cell-complexes. *Discrete Comput. Geom.* 21 (1999), 481–517. MR 2001a:52016. Zbl 941.52008.

§4, "Quality transfer", concerns the existence of a satisfied state (called "quality translation" in Ryshkov and Rybnikov 1997a) in a permutation

gain graph Φ, where $\mathfrak{G}$ acts on a set Q. P. 487, top: A satisfied state exists iff Φ is balanced [Ryshkov and Rybnikov (1997a); necessity is incorrect]. Lemma 4.1 appears to mean that a satisfied state exists iff it exists on each member of an arbitrary basis of the binary cycle space. [Not true, but interesting. The following special case, also invalid in general, was the author's intention (as I was told, Oct. 2000): Φ is balanced iff every member of a circle basis is balanced. The special case Lemma 4.2 is correct, because it is essentially homotopic.] [See Rybnikov and Zaslavsky (2005a, 2006a).] (**gg: bal, Cov**)

Konstantin Rybnikov and Thomas Zaslavsky

2005a Criteria for balance in abelian gain graphs, with an application to piecewise-linear geometry. *Discrete Comput. Geom.* 34 (2005), no. 2, 251–268. MR 2006f:05086. Zbl 1074.05047.

§§1–4: A condition on binary cycles that implies balance but does not depend on having a fundmental system of circles; it requires an abelian gain group. §5: Satisfied states and balance of a permutation gain graph. (**GG: Bal**)

§6: The criterion is applied to calculate the dimension of the space of liftings of a piecewise-linear immersion of a d-cell complex in Euclidean d-space. (**(GG: Bal, Geom)**)

2006a Cycle and circle tests of balance in gain graphs: Forbidden minors and their groups. *J. Graph Theory* 51 (2006), no. 1, 1–21. MR 2006i:05078. Zbl 1085.05033.

The class of Γ such that the criterion of (2005a) works for any gains on Γ is minor-closed. Some forbidden minors are given. Which ones they are depends on the class of permitted gain groups in a way that is not understood. (**GG: Bal: Str**)

S.S. Ryshkov and K.A. Rybnikov, Jr.

1996a Generatrissa. The Maxwell and Voronoĭ problems. (In Russian.) *Dokl. Akad. Nauk* 349 (1996), no. 6, 743–746. MR 98b:52027. Zbl 906.51007.

Announcement of results in (1997a). (**gg: Geom**)

1996b Generatrissa: The problems due to Maxwell and Voronoi. *Dokl. Math.* 54 (1996), no. 1, 614–617. Zbl 906.51007.

Translation of (1996a). (**gg: Geom**)

1997a The theory of quality translations with applications to tilings. *European J. Combin.* 18 (1997), 431–444. MR 98d:52031. Zbl 881.52015.

Let Φ be a permutation gain graph, with gain group $\mathfrak{G}$ acting on a set $\mathfrak{Q}$, and with underlying graph the d-cell adjacency graph of a kind of simply connected polyhedral d-cell complex. A "quality translation" is a satisfied state: a mapping $s : V \to \mathfrak{Q}$ such that $s(w) = s(v)\varphi(e; v, w)$ for every edge. A "circuit" is a closed walk that is not trivially reducible. Call a "$d-2$-circle" any circle contained in the star of a $d-2$-cell. Assume $\mathfrak{G}$ and $\mathfrak{Q}$ fixed. Thms. 1–2 can be stated: In the free group on the edge set, the $d-2$-circles generate all circuits. Also, Φ is balanced iff all $d-2$-circles have identity gain. Thm. 3: Identity gain of all $d-2$-circles is necessary and sufficient for the existence of a satisfied state. [Necessity is incorrect because the action may have nontrivial kernel.] The idea of quality transfer goes back to Voronoĭ in 1908. [See Rybnikov (1999a) and Rybnikov and Zaslavsky (2005a) for more.]

Sufficiency in Thm. 3 is applied to lifting of tilings of Euclidean and spherical space. Thm. 4 (1996a, Thm. 9): Balance ("canonical definition") of Φ is sufficient for lifting a tiling of $\mathbb{R}^d$. Here the qualities are affine functions. Thms. 5–6 (1996a, Thm. 10): Balance within each $d-3$-cell star implies lifting. [See Rybnikov (1999a) and Rybnikov and Zaslavsky (2005a) for more.]

§8: "Applications to the colouring of tilings". (**gg: bal, Geom**)

Herbert J. Ryser

See R.A. Brualdi.

Rachid Saad

1996a Finding a longest alternating cycle in a 2-edge-coloured complete graph is in RP. *Combin. Probab. Computing* 5 (1996), 297–306. MR 97g:05156. Zbl 865.05054.

Thm.: In a bidirected all-negative complete graph with a suitable extra hypothesis, the maximum length of a coherent circle equals the maximum order of a coherent degree-2 subgraph. More or less generalizes part of Bánkfalvi and Bánkfalvi (1968a) (*q.v.*). [Generalized in Bang-Jensen and Gutin (1998a).] [*Problem.* Generalize to signed complete graphs or further.] (**par: ori: Paths, Alg**)

Horst Sachs

See D.M. Cvetković.

Bruce Sagan

See also C. Bennett, A. Björner, A. Blass, F. Harary, and T. Józefiak.

1995a Why the characteristic polynomial factors. *Sém. Lotharingien Combin.* 35 (1995) [1998], Article B35a, iii + 20 pp. MR 98a:06006. Zbl 855.05012. arXiv:math/9812136.

A shorter predecessor of (1999a). (**SG, Gen: N: Col, G: Exp**)

1999a Why the characteristic polynomial factors. *Bull. Amer. Math. Soc. (N.S.)* 36 (1999), 113–133. MR 2000a:06021. Zbl 921.06001.

In §4, coloring of a signed graph Σ, especially of $\pm K_n^\bullet$ and $\pm K_n$, is used to calculate and factor the characteristic polynomial of $G(\Sigma)$. Presents the geometrical reinterpretation and generalization by Blass and Sagan (1998a). In §§5 and 6, other methods of calculation and factorization are applied to some signed graphs (in their geometrical representation). (**SG, Gen: N: Col, G: Exp**)

Prabhat K. Sahu and Shyi-Long Lee

2008a Net-sign identity information index: A novel approach towards numerical characterization of chemical signed graph theory. *Chem. Phys. Letters* 454 (2008), 133–138.

The "net-sign identity information index" I_k is expressed [obscurely] in terms of $|E^+|$ and $|E^-|$ in the molecular structure graph. The purpose is to correlate with chemical phenomena. I_k and $\sqrt{I_k}$ are compared with other indices. [Annot. 6 Feb 2011.] (**SG: Chem**)

Michael Saks

See P.H. Edelman.

Nicolau C. Saldanha

2002a Singular polynomials of generalized Kasteleyn matrices. *J. Algebraic Combin.* 16 (2002), no. 2, 195–207. MR 2004c:05051. Zbl 1017.05077.

A generalized Kasteleyn matrix is the left-right adjacency matrix B of a bipartite gain graph with the complex units as gain group. (A Kasteleyn matrix has for gain group the sign group.) The object is to interpret combinatorially the coefficients or eigenvalues of BB^{T}. The approach is cohomological (*cf.* Cameron 1977b). (**GG, SG: Adj, Sw**)

Mahmoud Salmasizadeh

See S. Fayyaz Shahandashti.

E. Sampathkumar

1972a Point-signed and line-signed graphs. Karnatak Univ. Graph Theory Res. Rep. 1, 1972.

See *Graph Theory Newsletter* 2 (Nov., 1972), no. 2, Abstract No. 7. (**SG, VS: Bal**)

1984a Point signed and line signed graphs. *Nat. Acad. Sci. Letters (India)* 7 (1984), no. 3, 91–93. Zbl 552.05051.

$\partial\sigma = \mu_\sigma$, $\partial\mu$ Consider a simple graph, an edge signature σ, and a vertex signature μ. Define $\partial\sigma(v) := \prod\{\sigma(e) : e$ incident with $v\}$ [later dubbed "canonical marking"] and, for each component X, $\partial\mu(X) := \prod_{v\in X}\mu(v)$. μ is "p-balanced" if $\partial\mu \equiv +$. Thm. 1: $\partial\mu \equiv +$ iff $\mu = \partial\sigma$ for some σ. [An early homology theorem.] Thm. 2: If σ is balanced and $\partial^2\sigma \equiv +$, then there exist all-negative, pairwise edge-disjoint paths connecting the $\partial\sigma$-negative vertices in pairs. [Quick proof: $\partial\mu \equiv +$ iff μ has evenly many negative vertices in each component. Negative vertices of $\partial\sigma$ are odd-degree vertices of Σ^-. Apply Listing's Theorem (independently discovered in stronger form by Sampathkumar) to Σ^-.] [It is interesting to base homology 0-chains like $\partial\mu$ on the components.] [Annot. Rev 27 Dec 2010.] (**SG, VS: Bal**)

2006a Generalized graph structures. *Bull. Kerala Math. Assoc.* 3, no. 2 (Dec., 2006), 67–123. MR 2290946 (no rev).

Within the class of simple graphs, what is a complement of a signed graph? An approach is to partition the edges of K_n into 3 classes: E^+, E^-, and E^c (the set of non-edges), and apply a specific permutation of these sets. Each permutation of order 2 implies a kind of complementation. Examination of self-complementarity. Generalizations of balance. Generalized to a graph Γ with k edge classes R_i [i.e., k-edge-colored graphs].

§10, "Balanced graph structures": "R_i-balance": $(\exists X)\ R_i = E[X, X^c]$ (the cut between X and X^c). "$R_1 \cdots R_r$-balance": Similar for $R_1 \cup \cdots \cup R_r$. "Complete balance": R_i balance for all i. "Arbitrary balance": $R_{i_1} \cdots R_{i_r}$-balance for every $i_1, \ldots, i_r \in [k]$. *Problem* 11: Characterize this property. "r-relation balance": The same for fixed r. *Problem* 12: Characterize this property. Other, similar concepts based on partitioning V. [Annot. 4 Sept 2010.] (**SG, SGc: Gen: Bal**)

2006b 4-Sigraphs. In: *International Conference on Discrete Mathematics, ICDM 2006* (Lecture Notes, Bangalore, 2006), p. 288. (**SG: Gen: GG**)

2011a Two new characterizations of consistent marked graph. *Adv. Stud. Contemp. Math. (Kyungshang)* 21 (2011), no. 4, 437–439. MR 2885007 (2012j:05192). (**VS: Bal, SG**)

E. Sampathkumar and V.N. Bhave

1973a Group valued graphs. *J. Karnatak Univ. Sci.* 18 (1973), 325–328. MR 50 #177. Zbl 284.05113.

Group-weighted graphs, both in general and where the group has exponent 2 (so all $x^{-1} = x$). Analogs of elementary theorems of Harary and Flament. Here balance of a circle means that the weight product around the circle, taking for each edge either $w(e)$ or $w(e)^{-1}$ arbitrarily, equals 1 for some choice of where to invert. (**WG, GG: Bal**)

E. Sampathkumar and L. Nanjundaswamy

1973a Complete signed graphs and a measure of rank correlation. *J. Karnatak Univ. Sci.* 18 (1973), 308–311. MR 54 #11649 (*q.v.*). Zbl 291.62066.

Given a permutation of $\{1, 2, \ldots, n\}$, sign K_n so edge ij is negative if the permutation reverses the order of i and j and is positive otherwise. Kendall's measure τ of correlation of rankings (i.e., permutations) A and B equals $(|E^+| - |E^-|)/|E|$ in the signature due to AB^{-1}. (**SG: KG**)

E. Sampathkumar, P. Siva Kota Reddy, and M.S. Subramanya

2008a Jump symmetric n-sigraph. *Proc. Jangjeon Math. Soc.* 11 (2008), no. 1, 89–95. MR 2429334 (2009j:05107). Zbl 1172.05028.

In the n-fold sign group $\{+, -\}^n$ an element is "symmetric" if it is its own reverse. A (symmetric) n-signed graph is a gain graph $\Phi = (\Gamma, \varphi)$ which has (symmetric) gains $\varphi(e) \in \{+, -\}^n$. [Equivalent to having arbitrary gains in $\{+, -\}^{\lceil n/2 \rceil}$.] Only symmetric n-signed graphs are treated.

Σ_Φ [The mapping min : $\{+, -\}^n \to \{+, -\}$ by $\min(a_1, \ldots, a_n) = +$ if all $a_i = +$ and $= -$ otherwise gives a signed graph Σ_Φ with signs $\sigma_\Phi(e) := \min(\varphi(e))$.]

Def.: $\Phi_1 \simeq \Phi_2$ ("cycle isomorphism") if there is an isomorphism $\|\Phi_1\| \cong \|\Phi_2\|$ that preserves circle gains. Prop. 3: Symmetric n-signed graphs are cycle isomorphic iff they are switching isomorphic—generalizing $n = 1$ due to Sozański (1980a), Zaslavsky (1981b). [The proof (omitted) requires that the gain group have exponent 2.]

φ_S Let $\varphi_S(ef) := \varphi(e)\varphi(f)$ for $e, f \in E$. [Generalizing $\sigma_\times$ of M. Acharya (2009a).]

J_S The jump graph is $J_S(\Phi) := (\Lambda(\Gamma)^c, \varphi_S)$. Solutions of $\Phi \simeq J_S(\Phi)$, $\Phi^t \simeq J_S(\Phi)$, $J_S(\Phi^t) \simeq J_S(\Phi)$, where $a^t := at$ for $a, t \in \{+, -\}^n$ and t is one of three special n-signs. [The last solution extends to arbitrary $t \in \{+, -\}^n$.]

Dictionary: "identity balance", "i-balance" = balance in Φ; "balance" = balance in Σ_Φ; $P(\vec{C}) := \varphi(C)$ in the indicated direction.

[The results remain true without assuming symmetry.] [Continued in (2010c, 2010d), Sampathkumar, Subramanya, and Siva Kota Reddy (2011a), and papers of Siva Kota Reddy.] [Annot. 2 Aug 2009, 20 Dec 2010.] (**SG(Gen), gg: LG, Sw, Bal**)

2008b $(3, d)$-Sigraph and its applications. *Adv. Stud. Contemp. Math. (Kyungshang)* 17 (2008), no. 1, 57–67. MR 2428537 (2009g:05073).

The $n = 3$ case of (2010b). [Annot. 10 Apr 2009.]

(**SG(Gen), gg: Bal, Sw**)

2009a Directionally n-signed graphs. II. *Int. J. Math. Combin.* 2009 (2009), vol. 4, 89–98 (2010). MR 2598676 (no rev). **(GG: Gen: Bal)**

2010a $(4, d)$-Sigraph and its applications. *Adv. Stud. Contemp. Math. (Kyungshang)* 20 (2010), no. 1, 115–124. MR 2597997 (2011i:05089). Zbl 1192.05067.

The $n = 4$ case of (2010b). [Annot. 9 Sept 2010.]

(SG(Gen), gg: Bal, Sw)

2010b Directionally n-signed graphs. In: B.D. Acharya, G.O.H. Katona, and J. Nesetril, eds., *Advances in Discrete Mathematics and Applications: Mysore, 2008* (Proc. Int. Conf. Discrete Math., ICDM-2008, Mysore, India, 2008), pp. 153–160. Ramanujan Math. Soc. Lect. Notes Ser., No. 13. Ramanujan Mathematical Soc., Mysore, India, 2010. MR 2766915 (2012g:05097). Zbl 1231.05119.

The gain group is the n-fold sign group $\{+,-\}^n$, with reversing automorphism $(a_1, \ldots, a_n)^r := (a_n, \ldots, a_1)$. The gains satisfy $\varphi(e^{-1}) = \varphi(e)^r$. For $t \in \{+,-\}^n$, the t-complement of Φ is $\|\Phi\|$ with gains $\varphi^t(e) := t\varphi(e)$. Elementary results on balance, t-complementation, switching, and isomorphism. Dictionary: "identity balance" = "i-balance" = balance in Φ; "balance" = balance in Σ_Φ defined at (2008a); $P(\vec{C}) := \varphi(C)$ in the indicated direction. [An interesting form of skew gain graph. The ideas should be pursued in directions suggested by Hage and Harju (2000a) and Hage (1999a).] [Annot. 10 Apr 2009.] **(SG(Gen), gg: Bal, Sw)**

2010c The line n-sigraph of a symmetric n-sigraph. *Southeast Asian Bull. Math.* 34 (2010), no. 5, 953–958. MR 2746762 (2012a:05142).

Λ_S The line graph is $\Lambda_S(\Phi) := (\Lambda(\Gamma), \varphi_S)$ [generalizing $\Lambda_\times$ of M. Acharya (2009a)]. For other definitions and notation see (2008a).

Line graphs and jump graphs in the sense of (2008a) are characterized, respectively, as balanced symmetric n-signings of (unsigned) line graphs and their complements. [The characterizations remain true for unsymmetric n-signatures.] There are remarks about the t-complement $t\varphi$ (2010b) for three $t \in \{+,-\}^n$.

$\mu_\varphi = \partial\varphi, \varphi^c$ The "complement" Φ^c is (Γ^c, φ^c) defined by $\mu_\varphi(v) := \prod_{uv\in E} \varphi(uv)$ ("canonical marking") (*cf.* Sampathkumar 1984a) and $\varphi^c(uv) := \mu_\varphi(u) \cdot \mu_\varphi(v)$ [= product of gains of all edges incident in Φ to u or v but not both]. [Gains φ^c are clearly balanced.] Prop. 7: A symmetric n-signed graph is a line graph iff it is a balanced, symmetric n-signature of an unsigned line graph. [Because φ_S is arbitrary balanced gains.] Prop. 9: $\Lambda_S(\Phi)^c \sim J_S(\Phi)$. [Because both are balanced and the underlying graphs are the same.] Prop. 10 solves $\Lambda_S(\Phi) \simeq J_S(\Phi)$, generalizing M. Acharya and Sinha (2003a). [The solutions to such graph equations, here and in related papers of Rangarajan, Sampathkumar, Siva Kota Reddy, *et al.*, are easy corollaries of the similar results for unsigned graphs.] [All results remain true without assuming symmetry.] Dictionary: Their μ_σ is my $\partial\sigma$. [Annot. 10 Apr, 1 Aug 2009, 20 Dec 2010.]

(SG, gg: LG, Sw, Bal)

2010d Common-edge signed graph of a signed graph. *J. Indones. Math. Soc.* 16 (2010), no. 2, 105–112. MR 2752773 (no rev). Zbl 1236.05098.

C_E See (2008a), Sampathkumar, Subramanya, and Siva Kota Reddy (2011a)

for definitions. The common-edge signed graph $C_E(\Sigma)$ is $\Lambda^2_\times(\Sigma)$. Prop. 4: Σ_0 is a common-edge signed graph iff it is balanced and $|\Sigma_0|$ is a common-edge graph. [Incorrect. $\Lambda^2_\times(\Sigma)$ does not have arbitrary balanced signs. E.g., $|\Sigma| = C_4$.] Equations solved [possibly incorrectly]: $\Sigma \simeq C^k_E(\Sigma)$ and $\Sigma \simeq \Lambda^k_\times(\Sigma)$ [this includes the preceding]. $\Lambda^k_\times(\Sigma) \simeq C^r_E(\Sigma)$. The jump graph (2008a) $J_S(\Sigma) \simeq C_E(\Sigma)$. [$\Lambda_\times$ as in M. Acharya (2009a).] [Annot. 12 Apr 2009.]

"Smarandanchely k-signed/marked graphs" are defined as k-signed/-marked graphs [and not used]. Signed/marked graphs are the case $k = 2$ [correctly: $k = 1$].

[Smarandanche has absolutely nothing to do with this.] [Annot. 7 Jan 2011.] (**SG: Bal, Sw, LG**)

E. Sampathkumar and M.A. Sriraj

20xxa Vertex labeled/colored graphs, matrices and signed graphs. Int. Workshop on Set-Valuations, Signed Graphs, Geometry and Their Appl. (IWSSG-2011, Mananthavady, Kerala, 2011). *J. Combin. Inform. Syst. Sci.*, to appear. (**SG**)

E. Sampathkumar, M.S. Subramanya, and P. Siva Kota Reddy

2011a Characterization of line sidigraphs. *Southeast Asian Bull. Math.* 35 (2011), no. 2, 297–304. MR 2866547 (2012j:05193). Zbl 1240.05143.

The line signed graph is $\Lambda_\times(\Sigma)$ [see M. Acharya (2009a)]. Prop. 3: A signed graph is a line signed graph of this kind iff it is a line graph with balanced signs.

The line signed digraph is $\Lambda_S(\vec{\Gamma}, \sigma) :=$ the Harary–Norman line digraph of $\vec{\Gamma}$, signed by σ^c defined as φ^c in Sampathkumar, Siva Kota Reddy, and Subramanya (2010c). Prop. 11: A signed digraph is a line signed digraph of this kind iff it is a Harary–Norman line digraph with (undirectedly) balanced signs. $(\vec{\Gamma}, \sigma)$ is switching isomorphic to $\Lambda_S(\vec{\Gamma}, \sigma)$ iff each component is a balanced directed cycle. [Annot. 4 Sep 2010.]

(**SG, SD: LG**)

Yoshio Sano

See T.Y Chung.

Mark Sapir

See Victor Guba.

S.V. Sapunov

2002a Equivalence of marked graphs. [Or: Equivalence of labeled graphs.] (In Russian.) *Proceedings of the Institute of Applied Mathematics and Mechanics* [*Tr. Inst. Prikl. Mat. Mekh.*], Vol. 7, pp. 162–167. Nats. Akad. Nauk Ukrainy Inst. Prikl. Mat. Mekh., Donetsk, 2002. MR 2141811 (2006c:05070). Zbl 1081.68074.

Equivalence of signed graphs that model languages. [Annot. 28 Dec 2011.] (**SG?**)

Irasema Sarmiento

See also J.A. Ellis-Monaghan.

1999a A characterisation of jointless Dowling geometries. 16th British Combinatorial Conf. (London, 1997). *Discrete Math.* 197/198 (1999), 713–731. MR 99m:-51020. Zbl 929.05016.

They are 4-closed (determined by their flats of rank 4). They are characterized, among all matroids, by the statistics of flats of rank ≤ 7 and therefore by their Tutte polynomials. There are exceptions in rank 3. **(GG: M: Invar)**

Iwao Sato

See also H. Mizuno.

2008a The stochastic weighted complexity of a group covering of a digraph. *Linear Algebra Appl.* 429 (2008), 1905–1914. MR 2446628 (2009h:05137) . Zbl 1144.05322 .

§3, "Weighted zeta functions of group covering of digraphs": The covering graphs ("derived graphs") of gain graphs ("voltage graphs"). **(GG)**

Roman V. Satyukov

See I.E. Bocharova.

Lawrence Saul and Mehran Kardar

1993a Exact integer algorithm for the two-dimensional $\pm J$ Ising spin glass. *Phys. Rev. E* 48 (1993), no. 5, R3221–R3224.

Announcement of (1994a) with some details, observations, and conclusions. [Annot. 18 Aug 2012.] **(SG: Phys, Fr: Alg)**

1994a The 2D $\pm J$ Ising spin glass: exact partition functions in polynomial time. *Nuclear Phys. B* 432 [FS] (1994), 641–667.

Algorithm for the energy distributions (the partition function) of the states of a randomly signed square, toroidal lattice graph. Applied to find statistical properties of such a signed graph. [Annot. 17 Aug 2012.] **(SG: Phys, Fr: Alg)**

B. David Saunders

See also A. Berman.

B. David Saunders and Hans Schneider

1978a Flows on graphs applied to diagonal similarity and diagonal equivalence for matrices. *Discrete Math.* 24 (1978), 205–220. MR 80e:15008. Zbl 393.94046. **(gg: Sw)**

1979a Cones, graphs and optimal scalings of matrices. *Linear Multilinear Algebra* 8 (1979), 121–135. MR 80k:15036. Zbl 433.15005. **(gg: Sw)(Ref)**

James Saunderson

See T. Coleman.

D. Savithri

See M. Parvathi.

H.C. Savithri

See H.A. Malathi and P. Siva Kota Reddy.

R.H. Schelp

See P. Erdős.

Baruch Schieber

See L. Cai.

Rüdiger Schmidt

1979a On the existence of uncountably many matroidal families. *Discrete Math.* 27 (1979), 93–97. MR 80i:05029. Zbl 427.05024.

The "count" matroids of graphs (see Whiteley (1996a)) and an extensive further generalization of bicircular matroids that includes bias matroids of biased graphs. His "partly closed set" is a linear class of circuits in an arbitrary "count" matroid. (**GG: MtrdF, Bic, EC: Gen**)

Stephan Schmidt
See J. Kunegis.

Hans Schneider
See G.M. Engel, D. Hershkowitz, U.G. Rothblum, and B.D. Saunders.

Irwin E. Schochetman
See J.W. Grossman.

Rainer Schrader
See U. Faigle.

Alexander Schrijver
See also A.M.H. Gerards.

1986a *Theory of Linear and Integer Programming.* Wiley, Chichester, 1986. MR 88m:90090. Zbl 665.90063.

Remark 21.2 (p. 308) cites Truemper's (1982a) definition of balance of a $0, \pm 1$-matrix. (**sg: par: Incid: Exp**)

1989a The Klein bottle and multicommodity flows. *Combinatorica* 9 (1989), 375–384. MR 92b:90083. Zbl 708.05019.

Assume Σ embedded in the Klein bottle. If Σ is bipartite, negative girth = max. number of disjoint balancing edge sets. If Σ is Eulerian, frustration index = max. number of edge-disjoint negative circles. Proved via polyhedral combinatorics. (**SG: Top, Geom, Fr**)

1990a Applications of polyhedral combinatorics to multicommodity flows and compact surfaces. In: William Cook and P.D. Seymour, eds., *Polyhedral Combinatorics*, pp. 119–137. DIMACS Ser. in Discrete Math. and Theor. Comp. Sci., Vol. 1. Amer. Math. Soc. and Soc. Indust. Appl. Math., Providence, R.I., 1990. MR 92d:05057. Zbl 727.90025.

§2: "The Klein bottle," surveys (1989a). (**SG: Top, Geom, Fr: Exp**)

1990b Homotopic routing methods. In: B. Korte, L. Lovász, H.J. Prömel, and A. Schrijver, eds., *Paths, Flows, and VLSI-Layout*, pp. 329–371. Algorithms and Combinatorics, Vol. 9. Springer-Verlag, Berlin, 1990. MR 92f:68139. Zbl 732.90087.

§4: "Edge-disjoint paths in planar graphs," pp. 342–345, "The projective plane and the Klein bottle," surveys (1989a).
(**SG: Top, Geom, Fr: Exp**)

§3: "Edge-disjoint paths and multicommodity flows," pp. 334 ff. [This work suggests there may be a signed-graph generalization with the theorems discussed corresponding to all-negative signatures.]
(**par: Paths: Exp**)

1991a Disjoint circuits of prescribed homotopies in a graph on a compact surface. *J. Combin. Theory Ser. B* 51 (1991), 127–159. MR 92a:05048. Zbl 723.05050.

§2: "An auxiliary theorem on linear inequalities," concerns feasibility of inequalities with coefficient matrix containing incidence matrix of $-\Gamma$. [See Hurkens (1988a).] (**ec: Incid**)

1991b (As "A. Skhreĭver") *Teoriya lineĭnogo i tselochislennogo programmirovaniya*, Vols. 1 and 2. Mir, Moscow, 1991. MR 94c:90003, 94g:90005.

Russian transl. of (1986a). (**sg: par: Incid: Exp**)

2002a A short proof of Guenin's characterization of weakly bipartite graphs. *J. Combin. Theory Ser. B* 85 (2002), 255–260. MR 2003e:05119. Zbl 1024.05079.

A streamlined proof of the theorem of Guenin (2001a).

(**SG: Geom, Str**)

2003a *Combinatorial Optimization: Polyhedra and Efficiency.* Vol. A, *Paths, Flows, Matchings.* Vol. B, *Matroids, Trees, Stable Sets.* Vol. C, *Disjoint Paths, Hypergraphs.* Algor. Combin., Vol. 24 A, B, C. Springer, Berlin, 2003. MR 1956924 (2004b:90004a), 1956925 (2004b:90004b), 1956926 (2004b:90004c). Zbl 1041.90001, 1072.90030.

Vol. A, Ch. 36, "Bidirected graphs".

Vol. C, Ch. 75, "Cuts, odd circuits, and multiflows". Signed graphs, weakly and strongly balanced signed graphs. Ch. 78, "Ideal hypergraphs". §80.4, "On characterizing binary ideal hypergraphs". Dictionary: "Odd" = negative (edge or circle). "Bipartite" = balanced. [Annot. 9 June 2011.] (**sg: Ori: Incid, Geom**)

Vol. C, Ch. 76, "Homotopy and graphs on surfaces". [Annot. 9 June 2011.] (**gg**)

Michael W. Schroeder

See R.A. Brualdi.

Michelle Schultz

See G. Chartrand.

Gary K. Schwartz

2002a On the automorphism groups of Dowling geometries. *Combin. Probab. Comput.* 11 (2002), no. 3, 311–321. MR 2004c:20005. Zbl 1008.06007.

Aut $Q_n(\mathfrak{G})$ factors in a certain natural way if, but also only if, $\mathfrak{G}$ factors. [Succeeds Bonin (1995a).] (**gg: M: Aut**)

W. Schwärzler and D.J.A. Welsh

1993a Knots, matroids and the Ising model. *Math. Proc. Cambridge Philos. Soc.* 13 (1993), 107–139. MR 94c:57019. Zbl 797.57002.

Tutte and dichromatic polynomials of signed matroids, generalized from Kauffman (1989a); this is the 2-colored case of Zaslavsky's (1992b) strong Tutte functions of colored matroids. [For terminology see Zaslavsky 1992b.] Applications to knot theory.

§2, "A matroid polynomial", is foundational. Prop. 2.1 characterizes strong Tutte functions of signed matroids by two equations connecting their parameters and their values on signed coloops and loops. [If the function is 0 on positive coloops, the proof is incomplete and the functions = 0 except on $M = \varnothing$ are missed.] Prop. 2.2: The Tutte (basis-expansion) polynomial of a function W of signed matroids is well defined iff W is a strong Tutte function. Eq. (2.8) says $W =$ the rank generating polynomial Q_Σ (here also called W) if certain variables are nonzero; (2.9) shows there are only 3 essential variables since, generically, only the ratio of parameters is essential [an observation that applies to general strong Tutte functions]. Prop. 2.5 computes Q_Σ of a 2-sum.

§3 adapts Q_Σ to Kauffman's and Murasugi's (1989a) signed-graph polynomials and simplifies some of the latter's results (esp. his chromatic degree). §4, "The anisotropic Ising model", concerns the Hamiltonian of a state of a signed graph. The partition function is essentially an

evaluation of Q_Σ. §5, "The bracket polynomial", and §6, "The span of the bracket polynomial": Certain substitutions reduce Q_Σ to 1 variable; its properties are examined, esp. in light of knot-theoretic questions. Thm. 6.4 characterizes signed matroids with "full span" (a degree property). §7, "Adequate and semi-adequate link diagrams", generalizes those notions to signed matroids. §8, "Zero span matroids": when does span(bracket) = 0? Yes if $M = M(\Sigma)$ where Σ reduces by Reidemeister moves to K_1, but the converse is open (and significant if true).
(**Sc(M), SGc: Invar, Knot, Phys**)

Allen J. Schwenk

See Harary, Palmer, Robinson, and Schwenk (1977a).

András Sebö

See also F. Meunier and B. Novick.

1990a Undirected distances and the postman-structure of graphs. *J. Combin. Theory Ser. B* 49 (1990), 10–39. MR 91h:05049. Zbl 638.05032.
See A. Frank (1996a). (**SGw: Str**)

J.J. Seidel

See also F.C. Bussemaker, P.J. Cameron, P.W.H. Lemmens, and J.H. van Lint.

1968a Strongly regular graphs with $(-1, 1, 0)$ adjacency matrix having eigenvalue 3. *Linear Algebra Appl.* 1 (1968), 281–298. MR 38 #3175. Zbl 159, 254 (e: 159.25403). Reprinted in Seidel (1991a), pp. 26–43. (**tg**)

1969a Strongly regular graphs. In: W.T. Tutte, ed., *Recent Progress in Combinatorics* (Proc. Third Waterloo Conf. on Combinatorics, 1968), pp. 185–198. Academic Press, New York, 1969. MR 54 #10047. Zbl 191, 552 (e: 191.55202). (**TG**)

1974a Graphs and two-graphs. In: F. Hoffman *et al.*, eds., *Proceedings of the Fifth Southeastern Conference on Combinatorics, Graph Theory, and Computing* (Boca Raton, 1974), pp. 125–143. Congressus Numerantium X. Utilitas Math. Publ. Inc., Winnipeg, Man., 1974. MR 51 #283. Zbl 308.05120. (**TG**)

†1976a A survey of two-graphs. In: *Colloquio Internazionale sulle Teorie Combinatorie* (Roma, 1973), Tomo I, pp. 481–511. Atti dei Convegni Lincei, No. 17. Accad. Naz. Lincei, Rome, 1976. MR 58 #27659. Zbl 352.05016. Reprinted in Seidel (1991a), pp. 146–176. (**TG: Adj, Cov, Aut**)

1978a Eutactic stars. In: A. Hajnal and Vera T. Sós, eds., *Combinatorics* (Proc. Fifth Hungar. Colloq., Keszthely, 1976), Vol. 2, pp. 983–999. Colloq. Math. Soc. János Bolyai, 18. North-Holland, Amsterdam, 1978. MR 80d:05016. Zbl 391.05050.

1979a The pentagon. In: Allan Gewirtz and Louis V. Quintas, eds., Second Int. Conf. on Combinatorial Mathematics (New York, 1978). *Ann. New York Acad. Sci.* 319 (1979), 497–507. MR 81e:05004. Zbl 417.51005. (**TG: Adj**)

1979b The pentagon. In: P.C. Baayen *et al.*, eds., *Proceedings, Bicentennial Congress, Wiskundig Genootschaap* (Amsterdam, 1978), Part I, pp. 80–96. Mathematical Center Tracts, 100. Mathematisch Centrum, Amsterdam, 1979. MR 80f:51008. Zbl 417.51005.
Same as (1979a), with photograph. (**TG: Adj**)

1991a *Geometry and Combinatorics: Selected Works of J.J. Seidel.* D.G. Corneil and R. Mathon, eds. Academic Press, Boston, 1991. MR 92m:01098. Zbl 770.05001.
Reprints many articles on two-graphs and related systems.

(**TG: Sw, Adj, Geom**)

1992a More about two-graphs. In: Jaroslav Nešetřil and Miroslav Fiedler, eds., *Fourth Czechoslovakian Symposium on Combinatorics, Graphs and Complexity* (Prachatice, 1990), pp. 297–308. Ann. Discrete Math., Vol. 51. North-Holland, Amsterdam, 1992. MR 94h:05040. Zbl 764.05036. (**TG: Exp, Ref**)

1995a Geometric representations of graphs. *Linear Multilinear Algebra* 39 (1995), 45–57. MR 97e:05149a. Zbl 832.05079. Errata. *Linear Multilinear Algebra* 39 (1995), 405. MR 97e:05149b. Zbl 843.05078.

§4, "Signed graphs": The "intersection matrix" $A + 2I$ of a signed simple graph is the Gram matrix of a set of "root vectors" with respect to an "inner product" that may not be positive definite. Explains origin of local switching (*cf.* Cameron, Seidel, and Tsaranov 1994a and Bussemaker, Cameron, Seidel, and Tsaranov 1991a). For a signed complete graph, $A+3I$ represents lines at angles $\cos^{-1} 1/3$; it is positive semidefinite only for few graphs, which are classified (implicit in Lemmens and Seidel 1973a). (**SG: Adj, Geom: Exp**)

1995b Discrete non-Euclidean geometry. In: F. Buekenhout, ed., *Handbook of Incidence Geometry: Buildings and Foundations*, Ch. 15, pp. 843–920. North-Holland (Elsevier), Amsterdam, 1995. MR 96m:52001. Zbl 826.51012.

§3.2: "Equidistant sets in elliptic $(d-1)$-space." §3.3: "Regular two-graphs." (**TG: Adj, Geom: Exp**)

J.J. Seidel and D.E. Taylor

1981a Two-graphs, a second survey. In: L. Lovász and Vera T. Sós, eds., *Algebraic Methods in Graph Theory* (Proc. Int. Colloq., Szeged, 1978), Vol. II, pp. 689–711. Colloq. Math. Soc. János Bolyai, 25. János Bolyai Math. Soc., Budapest, and North-Holland, Amsterdam, 1981. MR 83f:05070. Zbl 475.05073. Reprinted in Seidel (1991a), pp. 231–254. (**TG**)

J.J. Seidel and S.V. Tsaranov

1990a Two-graphs, related groups, and root systems. Algebra, Groups and Geometry. *Bull. Soc. Math. Belg. Ser. A* 42 (1990), 695–711. MR 95m:20046. Zbl 736.05048.

A group $\mathrm{Ts}(\Sigma)$ is defined from a signed complete graph Σ: its generators are the vertices and its relations are $(uv^{-\sigma(uv)})^2 = 1$ for each edge uv. It is invariant under switching, hence determined by the two-graph of Σ. A certain subgraph of a Coxeter group of a tree T is isomorphic to $\mathrm{Ts}(\Sigma)$ for suitable Σ_T constructed from T. [Generalized in Cameron, Seidel, and Tsaranov (1994a). More on Σ_T under Tsaranov (1992a). The construction of Σ_T is simplified in Cameron (1994a).] (**TG: Adj, Geom**)

Chelliah Selvaraj

See also M. Parvathi.

2007a Factor algebras of signed Brauer's algebras. *Kyungpook Math. J.* 47 (2007), no. 4, 549–568. MR 2397479 (2009b:16076). Zbl 1187.16013. (**gg: Algeb, m**)

Charles Semple and Geoff Whittle

1996a Partial fields and matroid representation. *Adv. Appl. Math.* 17 (1996), 184–208. MR 97g:05046. Zbl 859.05035.

§7: "Dowling group geometries". A Dowling geometry of a group $\mathfrak{G}$ has a partial-field representation iff G is abelian and has at most one involution. (**gg: M: Incid**)

Masakazu Sengoku

1974a On hybrid tree graphs. *Electron. Commun. Japan* 57 (1974), no. 5, 18–23. MR 56 #15210.

A signed graph derived from trees and cotrees is balanced. [Annot. 24 July 2010.] (**SG: Bal**)

B. Seoane

See L.A. Fernández.

Ákos Seress

See P. Brooksbank.

James P. Sethna

2006a *Statistical Mechanics: Entropy, Order Parameters, and Complexity.* Oxford Master Ser. in Physics, Vol. 14. Oxford Univ. Press, Oxford, 2006.

Textbook. P. 12, fn. 16: Frustration index ("spin-glass ground states") is polynomially equivalent to graph coloring. §12.3.4, "Glassy systems: random but frozen", mentions frustration due to negative circles ("a loop with an odd number of antiferromagnetic couplings"). It is not yet known how many equilibrium states exist. Fig. 12.17, "Frustration": An all-negative triangle with Ising spins (± 1). [Annot. 28 Aug 2012.] (**Phys: SG: Fr: Exp**)

E.C. Sewell

1996a Binary integer programs with two variables per inequality. *Math. Programming* 75 (1996), Ser. A, 467–476. MR 97m:90059. Zbl 874.90138.

See Johnson and Padberg (1982a) for definitions. §2, "Equivalence to stable set problem": Optimization on the bidirected stable set polytope is reduced to optimization on a stable set polytope with no more variables. Results of Bourjolly (1988a) and Hochbaum, Megiddo, Naor, and Tamir (1993a) can thereby be explained. §3, "Perfect bigraphs", proves the conjectures of Johnson and Padberg (1982a): a transitively closed bidirection of a simple graph is perfect iff its underlying graph is perfect. [Also proved by Ikebe and Tamura (20xxa).] Dictionary: "Bigraph" = bidirected graph B. "Stable" set in B = vertex set inducing no introverted edge. (**SG: Ori: Incid, Geom, sw**)

P.D. Seymour

See also M. Chudnovsky; J. Geelen; Gerards, Lovász, *et al.* (1990a); W. McCuaig; and N. Robertson.

1974a On the two-colouring of hypergraphs. *Quart. J. Math. Oxford* (2) 25 (1974), 303–312. MR 51 #7927. Zbl 299.05122. (**sd: Par: bal**)

1977a The matroids with the max-flow min-cut property. *J. Combin. Theory Ser. B* 23 (1977), 189–222. MR 57 #2960. Zbl 375.05022.

The central example is $Q_6 = \mathcal{C}_-(-K_4)$, the clutter of (edge sets of) negative circles in $-K_4$. P. 199: the extended lift matroid $L_0(-K_4) = F_7^*$, the dual Fano matroid. Result (3.4) readily generalizes (by the negative-subdivision trick) to: every $\mathcal{C}_-(\Sigma)$ is a binary clutter, that is, a port of a binary matroid. [This is also immediate from the construction of $L_0(\Sigma)$.]

P. 200, (i)–(iii): Amongst minor-minimal binary clutters without the "weak MFMC property" are the circuit clutter of F_7^* and $\mathcal{C}_-(-K_5)$ and its blocker.

Main Thm. (§5): A binary clutter is "Mengerian" (I omit the definition) iff it does not have $\mathcal{C}_-(-K_4)$ as a minor. (See p. 200 for the antecedent theorem of Gallai.)

[See Cornuéjols (2001a), Guenin (2001a) for more.]

(sg, Par: M, Geom)

1981a Matroids and multicommodity flows. *European J. Combin.* 2 (1981), 257–290. MR 82m:05030. Zbl 479.05023.

Conjecture (based on (1977a)). A binary clutter has the weak MFMC property iff no minor is either the circuit clutter of F_7 or $\mathcal{C}_-(-K_5)$ or its blocker. **(sm, sg: M)**

†1995a Matroid minors. In: R.L. Graham, M. Grötschel, and L. Lovász, eds., *Handbook of Combinatorics*, Vol. I, Ch. 10, pp. 527–550. North-Holland (Elsevier), Amsterdam, and MIT Press, Cambridge, Mass., 1995. MR 97a:05055. Zbl 960.24825.

In Thm. 6.6, p. 546, interpreting G as a signed graph and an "odd-K_4" as a subdivision of $-K_4$ gives the signed graph generalization, due to Gerards and Schrijver (1986a) [also Gerards (1990a), Thm. 3.2.3]. Let Σ be a signed simple, 3-connected graph in which no 3-separation has > 4 edges on both sides. Then Σ has no $-K_4$ minor iff either (i) deleting some vertex makes it balanced (the complete lift matroid of this type is graphic); or (ii) it is cylindrical: it can be drawn on a cylindrical surface that has a lengthwise red line so that an edge is negative iff it crosses the red line an odd number of times [Note: the extended lift matroid of this type is cographic, as observed by, I think, Gerards and Schrijver or by Lovász]. [See Pagano (1998a) for another use of cylindrical signed graphs.] [*Problem.* Find the forbidden topological subgraphs, link minors, and $Y\Delta$ graphs for cylindrical signed graphs.] [*Question.* Embed a signed graph in the plane with k distinguished faces so that a circle's sign is the parity of the number of distinguished faces it surrounds. Cylindrical embedding is $k = 1$. For each k, which signed graphs are so embeddable?] **(SG: Str, Top)**

Thm. 6.7, pp. 546–547, generalizes to signed graphs, interpreting G as a signed graph and an "odd cycle" as a negative circle. Take a signed simple, 3-connected, internally 4-connected graph. It has no two vertex-disjoint negative circles iff it is one of four types: (i) deleting some vertex makes it balanced; (ii) deleting the edges of an unbalanced triangle makes it balanced; (iii) it has order ≤ 5; (iv) it can be orientation-embedded in the projective plane. This is due to Lovász; see, if you can, Gerards *et al.* (1990a). [A 2-connected Σ has no vertex-disjoint negative circles iff $G(\Sigma)$ is binary iff $G(\Sigma)$ is regular iff the lift matroid $L(\Sigma)$ is regular. See Pagano (1998a) for classification of Σ with vertex-disjoint negative circles according to representability of the bias matroid.] **(SG: Str, m, Top)**

Paul Seymour and Carsten Thomassen

1987a Characterization of even directed graphs. *J. Combin. Theory Ser. B* 42 (1987), 36–45. MR 88c:05089. Zbl 607.05037.

"Even" means every signing contains a positive cycle. A digraph is even iff it contains a subdigraph that is obtained from a symmetric odd-circle digraph by subdivision and a vertex-splitting operation. [*Cf.* Thomassen (1985a).] **(sd: par: Str)**

L. de Sèze
See J. Vannimenus.

Bryan L. Shader
See R.A. Brualdi and D.A. Gregory.

Nisarg Shah
See M. Joglekar.

Siamak Fayyaz Shahandashti
See S. Fayyaz Shahandashti (under F).

Hai-Ying Shan
See J.-Y. Shao and L. You.

Jia-Yu Shao
See also R. Manber and L. You.

1998a On digraphs and forbidden configurations of strong sign nonsingular matrices. *Linear Algebra Appl.* 282 (1998), 221–232. MR 1648336 (99h:05086). Zbl 940.05044. (**SD: QSol**)

2000a On the digraphs of sign solvable linear systems. *Linear Algebra Appl.* 313 (2000), 115–126. MR 1770361 (2001e:05083). Zbl 958.15003.
Forbidden subgraphs used to characterize the signed digraphs. [Annot. 6 Mar 2011.] (**SD: QSol**)

Jia-Yu Shao, Jin-Ling He, and Hai-Ying Shan

2003a Number of nonzero entries of S^2NS matrices and matrices with signed generalized inverses. *Linear Algebra Appl.* 373 (2003), 223–239. MR 1648336 (99h:05086). Zbl 1036.15005. (**SG: QSol**)

Yanling Shao
See also Y.-B. Gao.

Yanling Shao and Yubin Gao

2009a The local bases of primitive non-powerful signed symmetric digraphs with loops. *Ars Combin.* 90 (2009), 357–369. MR 2489538 (2010c:05054). Zbl 1224.05223. (**SD, sg**)

Yanling Shao, Jian Shen, and Yubin Gao

2009a The kth upper bases of primitive non-powerful signed digraphs. *Discrete Math.* 309 (2009), no. 9, 2682–2686. MR 2523775 (2010h:05144). Zbl 1207.05073. (**SD**)

Ram Parkash Sharma and Vikram Singh Kapil

2011a Irreducible $\overrightarrow{S_n}$-modules and a cellular structure of the signed Brauer algebras. *Southeast Asian Bull. Math.* 35 (2011), no. 3, 497–522, MR 2856396 (2012h:16003). (**gg: Algeb, m**)

John Shawe-Taylor
See T. Pisanski.

Jia Sheng and MiaoLin Ye

2010a The spectral radius of signless Laplacian of a connected graph with given independence number. *Math. Appl. (Wuhan)* 23 (2010), no. 4, 709–712. MR 2765865 (no rev).

Jian Shen
See Y.-B. Gao and Y.L. Shao.

F.B. Shepherd
See A.M.H. Gerards.

David Sherrington and Scott Kirkpatrick

See also S. Kirkpatrick.

1975a Solvable model of a spin-glass. *Phys. Rev. Lett.* 35 (1975), no. 26, 1792–1796. Repr. in M. Mézard, G. Parisi, and M.A. Virasoro, *Spin Glass Theory and Beyond*, pp. 104–108. World Scientific Lect. Notes in Physics, Vol. 9. World Scientific, Singapore, 1987.

Announcement of part of Kirkpatrick and Sherrington (1978a). Introduces the Sherrington–Kirkpatrick spin-glass model, a randomly signed and (usually) weighted K_n. [Annot. 22 Aug 2012.] (**Phys: sg: Fr**)

Ronald G. Sherwin

1975a Structural balance and the sociomatrix: Finding triadic valence structures in signed adjacency matrices. *Human Relations* 28 (1975), 175–189.

A very simple [but not efficient] matrix algorithm for counting different types of circles in a signed (di)graph. ["Valence" means sign, unfortunately.] (**sg, SD: Bal: Alg**)

Jeng-Horng Sheu

See I. Gutman.

Chuan-Jin Shi

1992a A signed hypergraph model of constrained via minimization. In: *VLSI, 1992. Proceedings of the Second Great Lakes Symposium on VLSI* (Kalamazoo, Mich., 1992), pp. 159–166. IEEE, 1992. (**SH: Appl**)

1992b A signed hypergraph model of constrained via minimization. *Microelectronics J.* 23 (1992), no. 7, 533–542. (**SH: Appl**)

1993a Constrained via minimization and signed hypergraph partitioning. In: D.T. Lee and M. Sarrafzadeh, eds., *Algorithmic Aspects of VLSI Layouts*, pp. 337–356. World Scientific, Singapore, 1993. (**SH: Appl: Exp**)

1993b *Optimum Logic Encoding and Layout Wiring for VLSI Design: A Graph-Theoretic Approach*. Ph.D. thesis, Univ. of Waterloo, 1993.
(**SH: Incid, Bal, Alg, SG, Appl**)

C.-J. Shi and J.A. Brzozowski

1999a A characterization of signed hypergraphs and its applications to VLSI via minimization and logic synthesis. *Discrete Appl. Math.* 90 (1999), no. 1-3, 223–243. MR 1666019 (99m:68155). Zbl 913.68104.

A signed hypergraph $H = (V, E, \psi)$ is a hypergraph (V, E) with an incidence signature $\psi : V \times E \to \{-1, 0, 1\}$. "Underlying graph" = bipartite incidence graph with edge signs ψ. Sign of a path [or walk] = product of incidence signs. Motivation: via minimization, i.e., minimize the number of connections between different planar layers of a two-layer circuit. [See Rusnak (2009a) for a different development of the same definitions. Path signs are different; the normal sign for signed graphs has an extra factor -1 for each edge.] e is "balanced" by a bipartition $V = V_1 \uplus V_2$ when incidences of e are in the same V_i iff they have the same sign. H is "balanced" if some bipartition balances every edge. Thm. 3.1: H is balanced iff every circle is positive. [I.e., antibalance, since walk signs are different from the norm.] Proof: Constructive [similar to but less exact than algorithms for signed graphs as in Harary and Kabell (1980a)], yielding Cor. 3.1: Testing balance takes linear time. Thm. 3.2: H is balanced iff its incidence dual is balanced. "Maximum balance problem": Minimize the number of unbalanced edges. Thm. 4.1: This is NP-complete, even for cubic graphs. [Known, as it contains the max-cut problem.] Thm. 4.2: NP-complete for planar signed hypergraphs with maximum degree > 3. (For max degree ≤ 3, polynomial-time algorithms are given in Shi 1993b.) *Problem*: Minimum Covering: Find the minimum number of bipartitions of V such that every edge is balanced by one of the bipartitions. Equivalently, decompose H into the smallest number of balanced subhypergraphs. [See Zaslavsky (1987b) for signed graphs.] Thm. 5.1: NP-complete. Proof: Reduction to graph colorability via decomposability of a graph into bipartite subgraphs [special case of signed-graph decomposition as in Zaslavsky (1987b)].

§6, "Constrained via minimization", summarizes connection with signed hypergraphs, based on Shi (1992a,b). §7, "Constrained logic encoding".

§8, "Related notions: Signed graphs and $(0, \pm 1)$-matrices". §8.1, "Harary's signed graphs", compares their work with Harary (1953a) [no mention of Harary and Kabell (1980a)]. §8.2, "Restricted unimodularity and balanced $(0, \pm 1)$ matrices": The incidence matrix of $\mathrm{H}(H)$ if H is a graph [$\mathrm{H}(-H)$ in the normal definition] is totally unimodular iff $-H$ is balanced [essentially, Heller and Tompkins (1956a)].

[All problems and methods are equivalent to the similar problems for the signed graph derived by replacing each hyperedge by a balanced complete graph with Harary bipartition given by the sign bipartition of the hyperedge's incidences.] [Annot. 4 Nov 2010.]

(**SH: Incid, Bal, Alg, SG**)

C.-J. Shi, A. Vannelli, and J. Vlach

1990a A hypergraph partitioning approach to the via minimization problem. In: *Proceedings of the Canadian Conference on VLSI* (1990), pp. 2.7.1-2.7.8.

(**SH: sg: Bal**)

1997a Performance-driven layer assignment by integer linear programming and path-constrained hypergraph partitioning. *J. Heuristics* 3 (1997), no. 3, 225–243. Zbl 1071.90584. (**SH: sg: Bal, Alg: Appl**)

Jinsong Shi

See R.L. Li.

Yongtang Shi

See B.F. Hou.

Young-hee Shin

See J.H. Kwak.

K. Shivashankara

See P. Siva Kota Reddy.

Elizabeth G. Shrader and David W. Lewit

1962a Structural factors in cognitive balancing behavior. *Human Relations* 15 (1962), 265–276.

For $\Gamma \subset K_n$ and signing σ of Γ, "plausibility" = mean and "differentiability" = standard deviation of $f(K_n, \sigma')$ over all extensions of σ to K_n, where f is any function that measures degree of balance. Proposed: tendency toward balance is high when plausibility and differentiability are high. A specific f, based on triangles and quite complicated, is studied for $n = 4$, with experiments. (**sg, fr, PsS**)

Mohan S. Shrikhande
See Y.J. Ionin.

Jinlong Shu
See G.L. Yu and M.Q. Zhai.

Alan Shuchat
See R. Shull.

Randy Shull, James B. Orlin, Alan Shuchat, and Marianne L. Gardner

1989a The structure of bases in bicircular matroids. *Discrete Appl. Math.* 23 (1989), 267–283. MR 90h:05040. Zbl 698.05022.
[See Coullard, del Greco, and Wagner (1991a).] (**Bic(Bases)**)

Randy Shull, Alan Shuchat, James B. Orlin, and Marianne Lepp

1993a Recognizing hidden bicircular networks. *Discrete Appl. Math.* 41 (1993), 13–53. MR 94e:90122. Zbl 781.90089. (**GN: Bic: Incid, Alg**)

1997a Arc weighting in hidden bicircular networks. Proc. Twenty-eighth Southeastern Int. Conf. on Combinatorics, Graph Theory and Computing (Boca Raton, Fla., 1997). *Congressus Numer.* 125 (1997), 161–171. MR 98m:05181. Zbl 902.90157. (**GN: Bic: Incid, Alg**)

E.E. Shult
See P.J. Cameron.

R. Shwartz
See M. Amram.

Jana Šiagiová
See J. Širáň.

Heike Siebert

2008a Local structure and behavior of boolean bioregulatory networks. In: Katsuhisa Horimoto *et al.*, eds., *Algebraic Biology* (Third Int. Conf., AB 2008, Castle of Hagenberg, Austria, 2008), pp. 185–199. Lect. Notes in Computer Sci., Vol. 5147. Springer, Berlin, 2008. Zbl 1171.92303. (**SD**)

2009a Deriving behavior of Boolean bioregulatory networks from subnetwork dynamics. *Math. Computer Sci.* 2 (2009), no. 3, 421–442. MR 2507427 (2010g:92009). Zbl 1205.37097. (**SD**)

2011a Analysis of discrete bioregulatory networks using symbolic steady states. *Bull. Math. Biology* 73 (2011), no. 4, 873–898. MR 2785148 (2012c:92006). Zbl 1214.92033. (**SD**)

Heike Siebert and Alexander Bockmayr

2006a Incorporating time delays into the logical analysis of gene regulatory networks. In: Corrado Priami, ed.,*Computational Methods in Systems Biology* (Proc. Int. Conf. CMSB 2006, Trento, Italy), pp. 169–183. Lect. Notes in Computer Sci., Vol. 4210. Springer, Berlin, 2006. MR 2288350 (2007k:92067). (**SD**)

2007a Context sensitivity in logical modeling with time delays. In: Muffy Calder and Stephen Gilmore, eds., *Computational Methods in Systems Biology* (Proc. Int. Conf. CMSB 2007, Edinburgh, 2007), pp. 64–79. Lect. Notes in Computer Sci., Vol. 4695. Springer, Berlin, 2007. (**SD**)

2008a Temporal constraints in the logical analysis of regulatory networks. *Theoretical Computer Sci.* 391 (2008), no. 3, 258–275. MR 2386791 (2008k:92035). Zbl 1133.68041. (**SD**)

2008b Relating attractors and singular steady states in the logical analysis of bioregulatory networks. In: Hirokazu Anai *et al.*, eds., *Algebraic Biology* (Second Int. Conf., AB 2007, Castle of Hagenberg, Austria, 2007), pp. 36–50. Lect. Notes in Computer Sci., Vol. 4545. Springer, Berlin, 2008. Zbl 1127.92002. (**SD**)

B. Simeone

See C. Benzaken, J.-M. Bourjolly, P.L. Hammer, and P. Hansen.

Slobodan K. Simić

See also M. Anđelić, F. Belardo, D.M. Cardoso, D.M. Cvetković, and X.Y. Geng.

1980a Graphs which are switching equivalent to their complementary line graphs I. *Publ. Inst. Math. (Beograd) (N.S.)* 27(41) (1980), 229–235. MR 82m:05077. Zbl 531.05050. (**TG: LG**)

1982a Graphs which are switching equivalent to their complementary line graphs II. *Publ. Inst. Math. (Beograd) (N.S.)* 31(45) (1982), 183–194. MR 85d:05207. Zbl 531.05051. (**TG: LG**)

Slobodan K. Simić and Zoran Stanić

2008a *Q*-integral graphs with edge-degrees at most five. *Discrete Math.* 308 (2008), 4625–4634. MR 2438168 (2010g:05229). Zbl 1156.05037. (**Par: Adj**)

2009a On some forests determined by their Laplacian or signless Laplacian spectrum. *Comput. Math. Appl.* 58 (2009), no. 1, 171–178. MR 2535979 (2010j:05252). Zbl 1189.05106 .

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Rodica Simion

2000a Combinatorial statistics on type-B analogues of noncrossing partitions and restricted permutations *Electronic J. Combin.* 7 (2000), Research Paper R9, 27 pp. MR 2000k:05013. Zbl 938.05003.

"Type-B noncrossing partitions" are certain signed partial partitions of the ground set; i.e., certain elements of the Dowling lattice of $\{\pm\}$. (**gg: M**)

R. Simion and D.-S. Cao

1989a Solution to a problem of C. D. Godsil regarding bipartite graphs with unique perfect matching. *Combinatorica* 9 (1989), 85–89. MR 90f:05113. Zbl 688.05056.

Answering Godsil (1985a): $|\Sigma| = \Gamma$ iff Γ consists of a bipartite graph with a pendant edge attached to every vertex. [Surely there is a signed-graphic generalization of Godsil's and this theorem in which bipartiteness becomes balance or something like it.] (**sg: Adj, bal**)

J.M.S. Simões-Pereira

1972a On subgraphs as matroid cells. *Math. Z.* 127 (1972), 315–322. MR 47 #6522. Zbl 226.05016, (243.05022).

"Cell" = circuit. Along with Klee (1971a), invents the bicircular matroid (here, for finite graphs) (Thm. 1). Suppose we have matroids on the edge sets of all [simple] graphs, such that the class of circuits is a [nonempty] union of homeomorphism classes of connected graphs. Thm. 2: The circle and bicircular matroids [and free matroids] are the only such matroids. (**MtrdF, Bic**)

1973a On matroids on edge sets of graphs with connected subgraphs as circuits. *Proc. Amer. Math. Soc.* 38 (1973), 503–506. MR 47 #3214. Zbl 241.05114, 264.05126.

A family of (isomorphism types of) [simple] connected graphs is "matroidal" if for any Γ the class of subgraphs of Γ that are in the family constitute the circuits of a matroid on $E(\Gamma)$. Bicircular and even-cycle matroids are the two nicest examples. A referee contributes the even-cycle matroid [*cf.* Tutte (1981a), Doob (1973a)]. Thm.: The family cannot be finite [unless it is void or consists of K_2]. [See Marcu (1987a) for a valuable new viewpoint.] (**MtrdF, Bic, EC, Gen**)

1975a On matroids on edge sets of graphs with connected subgraphs as circuits II. *Discrete Math.* 12 (1975), 55–78. MR 54 #7298. Zbl 307.05129.

Partial results on describing matroidal families of simple, connected graphs. Five basic types: free [omitted in the paper], cofree, circle, bicircular, and even-cycle. If the family does not correspond to one of these, then every member has ≥ 3 independent circles and minimum degree ≥ 3. (**MtrdF, Bic, EC: Gen**)

1978a A comment on matroidal families. In: *Problèmes Combinatoires et Théorie des Graphes* (Colloq. Int., Orsay, 1976), pp. 385–387. Colloques Int. du CNRS, 260. Editions du C.N.R.S., Paris, 1978. MR 81b:05031. Zbl 412.05023.

Two small additions to (1973a, 1975a); one is that a matroidal family not one of the five basic types must contain $K_{p,q(p)}$ for each $m \geq 3$, with $q(p) \geq p$. (**MtrdF, Bic, EC: Gen**)

1992a Matroidal families of graphs. In: Neil White, ed., *Matroid Applications*, Ch. 4, pp. 91–105. Encycl. Math. Appl., Vol. 40. Cambridge Univ. Press, Cambridge, Eng., 1992. MR 93c:05036. Zbl 768.05024.

"Count" matroids (see N. White (1996a)) in §4.3; Schmidt's (1979a) remarkable generalization in §4.4.
(**GG: MtrdF, Bic, EC: Gen: Exp, Exr, Ref**)

Klaus Simon
See T. Raschle.

C. De Simone
See C. De Simone under D.

M. Simonovits
See B. Bollobás, J.A. Bondy, and P. Erdős.

Alistair Sinclair
See M. Jerrum.

Rajiv R.P. Singh
See M.E. Fisher.

Tarkeshwar Singh
See also M. Acharya and S.B. Rao.

2003a *Advances in the Theory of Signed Graphs.* Doctoral dissertation, University of Delhi, India.

Fairly complete accounts of Acharya and Singh (various) and Singh (20xxa), supplemented with background, appendix, etc. Ch. II, "Graceful signed graphs", is in Acharya and Singh (2003a, 2004a, 2005a, 20xxd, 20xxe). Ch. III, "Skolem graceful sigraphs": Announced in Acharya and Singh (2003b). Thm. 3.12: See Acharya and Singh (2010a). Also: Thm. 3.13: A necessary condition for Skolem-gracefulness of signed multiple stars. Thm. 3.14: A sufficient condition for two signed stars. Ch. IV, "Negation-switching invariant sigraphs": See Acharya and Singh

(20xxc). Also: A binary encoding of signed circles. App., "A catalog of assorted labelled sigraphs". [Annot. 20 July 2009.]
(**SGc**)(**SG: Sw, LG**)

2008a Skolem and hooked Skolem graceful sigraphs. In: B.D. Acharya, S. Arumugam, and Alexander Rosa, eds., *Labelings of Discrete Structures and Applications* (Mananthavady, Kerala, 2006), pp. 155–164. Narosa, New Delhi, 2008. MR 2391786 (2009e:05281) (book). Zbl 1161.05340.

[*Cf.* Acharya and Singh (2004a, 2003b). Generalizing the definition: Given: a graph with r-colored edges, m_i of color i; a list L of n integers. Required: A bijection $\lambda : V \to L$ such that, if $f(vw) := |\lambda(v) - \lambda(w)|$, then f restricted to color class i is a bijection to $[m_i]$.] Signed graphs are the case $r = 2$. Skolem gracefulness is the case where λ exists for $L = [n]$. Hooked Skolem gracefulness is the case where λ exists for $L = [n+1] \setminus \{n\}$. Results from Acharya and Singh (2010a) and Singh (20xxa), examples, some proofs. (**SGc: Exp**)

2009a Graceful signed graphs on C_3^k. Fifth Int. Workshop on Graph Labelings (IWOGL 2009) (Krishnankoil, 2009). *AKCE Int. J. Graphs Combin.* 6 (2009), no. 1, 201–208. MR 2533200 (2010g:05330). Zbl 1210.05155.

"Graceful" means $(1,1)$-graceful, $r = 1$, as at M. Acharya and Singh (2004a). C_3^k is the windmill with k blades. Let Σ have ν negative rim edges, $1 \leq \nu \leq k/2$, and no other negative edges. Thm. 10: Σ is graceful if $k \equiv 0 \bmod 4$ and ν is even. Thms. 11, 12: Σ is graceful if $k \equiv 1, 2 \bmod 4$. [Annot. 21 July 2010.] (**SGc**)

20xxa A note on hooked Skolem graceful sigraphs and its application. Submitted.

See (2008a). Thm.: A signed k-edge matching is hooked Skolem graceful iff $k \equiv 0 \pmod 4$ and $|E^-|$ is odd, or $k \equiv 2 \pmod 4$ and $|E^-|$ is even, or $k \equiv 3 \pmod 4$. Curiously complementary to the theorem of Acharya and Singh (2010a). (**SGc**)

Tarkeshwar Singh and Natasha D'Souza

2010a Some results in graceful signed tree. In: *International Conference on Recent Trends in Graph Theory and Combinatorics (ICRTBC-2010)* (Cochin, 2010) [Summaries], p. 169. Dept. of Mathematics, Cochin Univ. of Science and Technology, 2010.

Abstract. Some graceful signed trees (see M. Acharya and Singh 2004a). Every signed tree is an induced subgraph of a graceful signed tree. [Annot. 31 Aug 2010.] (**SGc**)

N.M. Singhi

See also S.B. Rao, D.K. Ray-Chaudhuri, and G.R. Vijayakumar.

N.M. Singhi and G.R. Vijayakumar

1992a Signed graphs with least eigenvalue < -2. *European J. Combin.* 13 (1992), 219–220. MR 93e:05069. Zbl 769.05065.

A short proof that every such signed simple graph contains an induced subgraph with least eigenvalue $= -2$. [Their $M := 2I + A(\Sigma)$ is the Kirchhoff matrix of $-\Sigma$.] (**SG: adj**)

Deepa Sinha

See also M. Acharya.

2005a *New Frontiers in the Theory of Signed Graphs.* Doctoral dissertation, University of Delhi, 2005.

[Partial description] Σ is "sign compatible" if $\exists\ X \subseteq V$ such that $E^{-} = E{:}X$. [Annot. 12 Oct 2010.] (**SG**)

Deepa Sinha and Ayushi Dhama

20xxa Sign-compatibility of common-edge sigraphs and 2-path sigraphs. Submitted. (**SG: LG: Gen**)

Deepa Sinha and Pravin Garg

2010a Consistency of semi-total signed graphs. In: *International Conference on Recent Trends in Graph Theory and Combinatorics (ICRTBC-2010)* (Cochin, 2010) [Summaries], p. 153. Dept. of Mathematics, Cochin Univ. of Science and Technology, 2010.

Abstract. Consistency of the canonical vertex signature of certain graphs related to the line graph and total graph of Σ; see e.g. (2011f). [Annot. 31 Aug 2010.] (**SG: VS: Bal**)

2011a Canonical consistency of signed line structures. *Graph Theory Notes N. Y.* 59 (2011), 22–27. MR 2849400 (2012g:05098).

Thm. 2: Consistency of the canonical vertex signature of two kinds of line graph: (Thm. 2) $\Lambda_{BC}(\Sigma)$ (Behzad–Chartrand 1969a) and (Thm. 8) $\Lambda_{\times}(\Sigma)$ (M. Acharya 2009a). [Annot. 25 Mar 2011.] (**SG: LG: VS: Bal**)

2011b Balance and consistency of total signed graphs. *Indian J. Math.* 53 (2011), no. 1, 71–81. MR 2809572 (2012d:05174).

$T(\Sigma)$ Characterizes balance and consistency of the total graph $T(\Sigma)$. The vertex signs are $\mu_1(v) := \sigma(E(v))$ ($E(v) :=$ the vertex star), $\mu_1(e) = \sigma(e)$. The edge signs are $\sigma_T(uv) := \sigma(e_{uv})$, $\sigma_T(ue) := \sigma(e)\mu_1(u)$, $\sigma_T(ef) := \sigma(e)\sigma(f)$ [thus $T(\Sigma) \supseteq \Lambda_{\times}(\Sigma)$ of M. Acharya (2009a)]. [Annot. 13 Oct 2009, 20 Dec 2010.] (**SG, VS: Bal**)

2011c On the regularity of some signed graph structures. *AKCE Int. J. Graphs Combin.* 8 (2011), no. 1, 63–74. MR 2839176 (2012f:05126).

$T(\Sigma)$ Σ is regular if Σ^{+} and Σ^{-} are regular graphs. For the edge signs of line graphs and total graph see (2011b). Characterizes Σ such that Λ_{BC} or $\Lambda_{\times}$ or T is regular. Dictionary: "signed-regular" = regular. [Annot. 25 July 2011.] (**SG: LG**)

2011d Characterization of total signed graph and semi-total signed graphs. *Int. J. Contemp. Math. Sci.* 6 (2011), no. 5-8, 221–228. MR 2797063 (no rev). Zbl 1235.05058. (**SG: LG: Gen**)

2011e On the unitary Cayley signed graphs. *Electron. J. Combin.* (2011), Article P229, 13 pp.

The unitary Cayley graph $X_n = (\mathbb{Z}_n, \{ab : \exists\ (b-a)^{-1}\})$. $S_n = (X_n, \sigma)$ where $\sigma(ab) = -$ iff $\nexists\ a^{-1}, b^{-1}$. Thm. 4: S_n is balanced iff n is even or a prime power. Cor. 5: S_n is antibalanced iff n is even. Cor. 7: $\Lambda_{BC}(S_n)$ is balanced iff n is a prime power. Thm. 20: Let n have at most 2 distinct odd prime factors. S_n is canonically consistent iff n is odd, evenly even, 2, or 6. [Annot. 16 Jan 2012.] (**SG: Bal**)

2011f Some results on semi-total signed graphs. *Discuss. Math. Graph Theory* 31 (2011), no. 4, 625–638.

Similar to (2011b), but for $T_2(\Sigma) := T(\Sigma)$ without line-graph edges. [Annot. 13 Oct 2009.] (**SG, VS: LG: Gen: Bal**)

20xxd Canonical consistency of semi-total signed graphs. Submitted. (**SG, VS: LG: Gen**)

20xxe A characterization of canonically consistent total signed graphs. Submitted. (**SG, VS: LG: Gen**)

Jozef Širáň

See also D. Archdeacon, P. Gvozdjak, and C.H. Li.

1991a Characterization of signed graphs which are cellularly embeddable in no more than one surface. *Discrete Math.* 94 (1991), 39–44. MR 92i:05086. Zbl 742.05035.

A signed graph orientation-embeds in only one surface iff any two circles are vertex disjoint. (**SG: Top**)

1991b Duke's theorem does not extend to signed graph embeddings. *Discrete Math.* 94 (1991), 233–238. MR 92j:05065. Zbl 742.05036.

Richard A. Duke (The genus, regional number, and Betti number of a graph. *Canad. J. Math.* 18 (1966), 817–822. MR 33 #4917.) proved that the (orientable) genus range of a graph forms a contiguous set of integers. Stahl (1978a) proved the analog for nonorientable embeddings. Širáň shows this need not be the case for the demigenus range of an unbalanced signed graph. However, any gaps consist of a single integer each. The main examples with gaps are vertex amalgamations of balanced and uniquely embeddable unbalanced signed graphs, but a 3-connected example is $+W_6$ together with the negative diameters of the rim. *Question* 1 (Širáň). Do all gaps occur at the bottom of the demigenus range? [*Question* 2. Can one in some way derive almost all signed graphs with gaps from balanced ones?] (**SG: Top**)

Jozef Širáň, Jana Šiagiová, and Marián Olejár

2009a Graph coverings and graph labellings. Special Issue on Graph Labelings. Fifth Int. Workshop on Graph Labelings (IWOGL 2009) (Krishnankoil, 2009). *AKCE Int. J. Graphs Combin.* 6 (2009), no. 1, 127–133. MR 2533240 (2010g:05331). Zbl 1210.05129.

Connectivity and automorphisms of a covering graph of a gain graph ("voltage graph"). [Annot. 21 July 2010.] (**GG: Cov: Aut, Exp**)

Jozef Širáň and Martin Škoviera

††1991a Characterization of the maximum genus of a signed graph. *J. Combin. Theory Ser. B* 52 (1991), 124–146. MR 92b:05033. Zbl 742.05037.

The maximum demigenus $d_M(\Sigma)$ = the largest demigenus of a closed surface in which Σ orientation embeds. Two formulas are proved for $d_M(\Sigma)$: one a minimum and the other a maximum of readily computable numbers. Thus $d_M(\Sigma)$ has a "good" (polynomial) characterization. Along the way, several results are proved about single-face embeddings. *Problem* (§11). Characterize those edge-2-connected Σ such that Σ and all $\Sigma \backslash e$ have single-face embeddings. [A complex and lovely paper.] (**SG: Top**)

P. Siva Kota Reddy

See also V. Lokesha, R. Rangarajan, E. Sampathkumar, and M.S. Subramanya.

2010a t-path sigraphs. *Tamsui Oxford J. Math. Sci.* 26 (2010), no. 4, 433–441. MR 2840769 (2012g:05096). **(SG)**

20xxa Switching invariant t-path sigraphs. Submitted.

In the t-path signed graph $(\Sigma)_t$, u, v are adjacent when joined by a path of length t, with signature σ^c (see Sampathkumar, Siva Kota Reddy, and Subramanya (2010c)). (The signature differs from that of Gill and Patwardhan (1986a) and M. Acharya (1988a).) Solved: $\Sigma \simeq (\Sigma)_2$, $(\Sigma)_3$. [Annot. 10 Apr 2009.] **(SG: Sw, LG(Gen))**

20xxb A note on characterization of jump signed graphs. Submitted. **(SG: LG)**

P. Siva Kota Reddy, V. Lokesha, and Gurunath Rao Vaidya

2010a The line sigraph of a symmetric n-sigraph. II. *Proc. Jangjeon Math. Soc.* 13 (2010), no. 3, 305–312. Zbl 1223.05109.

Definitions as at Sampathkumar, Siva Kota Reddy, and Subramanya (2008a). **(GG: LG)**

2010b The line n-sigraph of a symmetric n-sigraph—III. *Int. J. Open Problems Computer Sci. Math.* 3 (2010), no. 5, 172–178.

Definitions as at Sampathkumar, Siva Kota Reddy, and Subramanya (2008a). **(GG: LG)**

2011a Switching equivalence in symmetric n-sigraphs—III. *Int. J. Math. Sci. Engineering Appl.* 5 (2011), no. 1, 95–101. MR 2791536 (2012a:05141).

Definitions as at Sampathkumar, Siva Kota Reddy, and Subramanya (2008a). **(GG: Sw)**

P. Siva Kota Reddy, K.M. Nagaraja, and M.C. Geetha

2012a The line n-sigraph of a symmetric n-sigraph–IV. *Int. J. Math. Combin.* 2012 (2012), vol. 1, 106–112. **(SG: Gen: LG)**

P. Siva Kota Reddy, Kavita S. Permi, and K.R. Rajanna

2012a Combinatorial aspects of a measure of rank correlation due to Kendall and its relation to complete signed digraphs. *Int. J. Math. Combin.* 2012 (2012), vol. 1, 74–77. **(SD)**

P. Siva Kota Reddy and B. Prashanth

2009a Switching equivalence in symmetric n-sigraphs. I. *Adv. Appl. Discrete Math.* 4 (2009), no. 1, 25–32. MR 2555623 (2010k:05122). Zbl 1176.05034.

Continuation of Rangarajan, Siva Kota Reddy, and Subramanya (2009a). Definitions as at Sampathkumar, Siva Kota Reddy, and Subramanya (2008a). Solved for an n-signed graph Φ: $\Lambda_S(\Phi) \simeq \Phi^c$; $\Lambda^k_S(\Phi) \simeq \Phi^c$. [The results remain true without assuming symmetry.] **(SG, gg: Sw, LG)**

P. Siva Kota Reddy, B. Prashanth, and T.R. Vasanth Kumar

2011a Antipodal signed digraphs. *Adv. Stud. Contemp. Math. (Kyungshang)* 21 (2011), no. 4, 355–360. **(SD)**

P. Siva Kota Reddy, B. Prashanth, and V. Lokesha

20xxa A note on switching in symmetric n-sigraphs. Submitted.

Switching multiple signs $\sigma(e) \in \{+,-\}^k$ by signs $\mu(v) \in \{+,-\}$. [Equivalent to restricted switching, where $\mu(v) \in \{\pm(+,\dots,+)\}$.] Characterized by cutset negation. [Annot. 7 Jan 2011.] **(SG(Gen): Sw)**

P. Siva Kota Reddy, B. Prashanth, and Kavita S. Permi

2011a A note on antipodal signed graphs. *Int. J. Math. Combin.* 2011 (2011), vol. 1,

107–112. MR 2829740 (2012d:05173). (**SG**)

P. Siva Kota Reddy, R. Rangarajan, and M.S. Subramanya

2011a Switching invariant neighborhood signed graphs. *Proc. Jangjeon Math. Soc.* 14 (2011), no. 2, 249–258. MR 2829740 (2012d:05173). Zbl 1238.05121. (**SG, VS: LG**)

P. Siva Kota Reddy, E. Sampathkumar, and M.S. Subramanya

2010a Common-edge signed graph of a signed graph. *J. Indonesian Math. Soc.* 16 (2010), no. 2, 105–113. MR 2752773 (no rev). (**SG**)

P. Siva Kota Reddy, K. Shivashankara, and K.V. Madhusudhan

2010a Negation switching equivalence in signed graphs. *Int. J. Math. Combin.* 3 (2010), 85–90. Zbl 1238.05122.

Solved: $-\Sigma$, $\Lambda^k_\times(\Sigma) \simeq \Lambda^2_\times(\Sigma)$, based on existing solutions for unsigned isomorphism. (See M. Acharya (2009a) for $\Lambda_\times$.) [Annot. 6 Feb 2011.] (**SG: LG, Sw**)

P. Siva Kota Reddy and M.S. Subramanya

2007a A characterization of symmetric 3-sigraphs whose line symmetric 3-sigraphs are switching equivalent. *J. Appl. Math. Anal. Appl.* 3 (2007), no. 1, 23–31. MR 2479512 (2009m:05080).

2009b Signed graph equation $L^K(S) \sim \bar{S}$. *Int. J. Math. Combin.* 4 (2009), 84–88 (2010). MR 2598675 (no rev).

Definitions as at Sampathkumar, Siva Kota Reddy, and Subramanya (2008a). Solved: $\Sigma^c \simeq \Lambda^2_\times(\Sigma)$; $\Lambda^k_\times(\Sigma) \simeq \Sigma^c$. [$\Lambda_\times$ as in M. Acharya (2009a).] [Continued in Siva Kota Reddy, Vijay, and Lokesha (2009a, 2010a)]. [Annot. 3 Aug 2009.] (**SG: Bal, Sw, LG**)

2009c Note on path signed graphs. *Notes Number Theory Discrete Math.* 15 (2009), no. 4, 1–6.

$V(P_k(\Sigma)) := \{\text{paths}\}$, $PP' \in E(P_k(\Sigma))$ iff $P \cup P'$ is a path of order $k+1$ or a C_k, $\sigma(PP') = \sigma(P)\sigma(P')$. This is balanced. Solved: $\Sigma \simeq P_3(\Sigma), P_4(\Sigma)$. [Annot. 7 Jan 2011.] (**SG: LG(Gen), Bal**)

P. Siva Kota Reddy and S. Vijay

2010a Total minimal dominating signed graph. *Int. J. Math. Combin.* 3 (2010), 11–16. Zbl 1238.05124.

The intersection graph M_t of all total minimal dominating sets of $|\Sigma|$ is signed to be balanced using the canonical vertex signature of Σ. Such signed graphs are characterized. $M_t \simeq \Sigma, -\Sigma$ are solved, based on existing solutions for unsigned isomorphism. [Annot. 6 Feb 2011.] (**SG**)

20xxa The super line signed graph $\mathcal{L}_r(S)$ of a signed graph. *Southeast Asian Bull. Math.*, to appear.

$V(\mathcal{L}_r(\Sigma)) := \mathcal{P}_r(E)$ with edge $PQ_{e,f} \in E(\mathcal{L}_r(\Sigma))$ for each adjacent $e \in P$, $f \in Q$ and $\sigma_{\mathcal{L}}(PQ_{e,f}) = \sigma(P)\sigma(Q)$. This is balanced. Solved: $\Sigma, \Lambda_\times(\Sigma) \simeq \mathcal{L}_2(\Sigma)$, $\Sigma \cong \mathcal{L}_2(\Sigma)$, et al. [Annot. 7 Jan 2011.] (**SG: LG(Gen), Bal**)

P. Siva Kota Reddy, S. Vijay, and V. Lokesha

2009a n^{th} power signed graphs. *Proc. Jangjeon Math. Soc.* 12 (2009), no. 3, 307–313. MR 2582796 (2011e:05109). Zbl 1213.05121.

Definitions and notation as in Sampathkumar, Siva Kota Reddy, and Subramanya (2008a, 2010c).

D_m The "mth power signed graph" Σ^m [I will say "$\leq m$-distance signed graph" $D_m(\Sigma)$] is the graph of distance $\leq m$ in $|\Sigma|$ with signature σ^c. Prop. 5: Σ has the form $D_m(\Sigma')$ iff it is balanced and $|\Sigma|$ is a $(\leq m)$-distance graph. [Sufficiency is incorrect.] Solved [possibly incorrectly]: Σ^c or $\Lambda_\times(\Sigma^c) \simeq D_m(\Lambda_\times(\Sigma))$; $\Lambda_\times(\Sigma)^c \simeq D_m(\Sigma^c)$; $\Lambda^2_\times(\Sigma) \simeq D_m(\Sigma)$, $D_m(\Sigma)^c$, $D_m(\Sigma^c)$. [$\Lambda_\times$ as in M. Acharya (2009a).] [Annot. 12 Apr 2009.] (**SG: Bal, Sw, LG**)

2010a The n^{th} power signed graphs. II. *Int. J. Math. Combin.* 1 (2010), 74–79. MR 2662418 (no rev). Zbl 1207.05074.

Continuing (2009a) with: $\Lambda_\times(\Sigma) \simeq D_m(\Sigma^{[c]})$; $\Lambda_\times(\Sigma)^c \simeq D_m(\Sigma)$. [Annot. 10 Apr 2009.] (**SG: Bal, Sw, LG**)

P. Siva Kota Reddy, S. Vijay, and B. Prashanth

2009a The edge C_4 n-sigraph of a symmetric n-sigraph. *Int. J. Math. Sci. Eng. Appl.* 3 (2009), no. 2, 21–27. (**SG(Gen), gg: LG(Gen)**)

P. Siva Kota Reddy, S. Vijay, and H.C. Savithri

2010a A note on path signed digraphs. *Int. J. Math. Combin.* 2010 (2010), vol. 1, 42–46. MR 2662415 (no rev). Zbl 1203.05065. (**SD: LG: Gen**)

B. Sivakumar

See also M. Parvathi.

2009a Matrix units for the group algebra $kG_f = k((Z_2 \times Z_2) \wr S_f)$. *Asian-Eur. J. Math.* 2 (2009), no. 2, 255–277. MR 2532703 (2010g:16043). Zbl 1198.20013. (**gg: m: Algeb**)

Vaidy Sivaraman

See J. Maharry.

A. Skhreĭver [A. Schrijver]

See A. Schrijver.

Bjarke Skjernaa

See J.M. Byskov.

Martin Škoviera

See also A. Malnič, R. Nedela, and J. Širáň.

1983a Equivalence and regularity of coverings generated by voltage graphs. In: Miroslav Fiedler, ed., *Graphs and Other Combinatorial Topics* (Proc. Third Czechoslovak Sympos. on Graph Theory, Prague, 1982), pp. 269–272. Teubner-Texte Math., 59. Teubner, Leipzig, 1983. MR 85e:05064. Zbl 536.05019. (**GG: Top, Cov, Sw**)

1986a A contribution to the theory of voltage graphs. *Discrete Math.* 61 (1986), 281–292. MR 88a:05060. Zbl 594.05029.

Automorphisms of covering projections of canonical covering graphs of gain graphs. (**GG: Top, Cov, Aut, Sw**)

1992a Random signed graphs with an application to topological graph theory. In: Alan Frieze and Tomasz Luczak, eds., *Random Graphs, Vol. 2* (Proc., Poznań, 1989), Ch. 17, pp. 237–246. Wiley, New York, 1992. MR 93g:05126. Zbl 817.05059.

The model: each edge is selected with probability p, positive with probability s. Under mild hypotheses on p and s, Σ is almost surely unbalanced and almost surely has a 1-face orientation embedding. [Related: Frank and Harary (1979a).] (**SG: Rand, Enum, Top**)

Daniel Slilaty

See also H. Qin.

2000a *Orientations of Biased Graphs and Their Matroids.* Doctoral dissertation, State University of New York at Binghamton, 2000. MR 2701091 (no rev).

Introducing orientation of biased graphs and biased signed graphs by means of proper circle orientations and their generalization, "graphical orientation schemes". The definition is chosen so as to produce orientations of the bias and complete lift matroids and (though not in the thesis) to model the orientation of the bias or complete lift matroid of, respectively, an $\mathbb{R}^\times$- or $\mathbb{R}^+$-gain graph induced by its canonical bias or lift representation (Zaslavsky 2003b). Characterizations of equivalence of different orientation schemes. The completeness question: when do graphical orientation schemes yield all orientations of the bias matroid? Always, for additively biased (i.e., signed) graphs and for some other kinds of biased graphs. (**GG: Ori, M, OG, SG**)

2002a Matroid duality from topological duality in surfaces of nonnegative euler characteristic. *Combin. Probab. Computing* 11 (2002), no. 5, 515–528. MR 1930356 (2003i:05034). Zbl 1009.05036.

Duality of matroids of biased graphs, obtained by defining gains through embedding in a surface and dualizing the graph in the surface. (**GG, SG: M, D, Top**)

2005a On cographic matroids and signed-graphic matroids. *Discrete Math.* 301 (2005), no. 12, 207–217. MR 2171313 (2007c:05049). Zbl 1078.05017. (**SG: M, Top**)

2006a Bias matroids with unique graphical representations. *Discrete Math.* 306 (2006), no. 12, 1253–1256. MR 2245651 (2007b:05044). Zbl 1093.05015. (**GG: M: Str**)

†2007a Projective-planar signed graphs and tangled signed graphs. *J. Combin. Theory Ser. B* 97 (2007), no. 5, 693–717. MR 2344133 (2008j:05161). Zbl 1123.05046.

Thm.: The signed graphs with no two vertex-disjoint negative circles are those with a balancing vertex, or obtained from a projective-planar signed graph (*cf.* Zaslavsky 1993a) or from $[-K_5]$ by t-summation with balanced signed graphs for $t = 1, 2, 3$. (Previously announced in less general form by Lovász (see Seymour 1995a) but the proof was incorrect.) [*Major Problem.* Characterize the biased graphs having no two vertex-disjoint unbalanced circles. Lovász (1965a, *q.v.*) solved the contrabalanced case.] (**SG: Top, Str**)

2010a Integer functions on the edges and cycle space of a graph. *Graphs Combin.* 20 (2010), no. 2, 293–299. MR 2606501 (2011b:05092). Zbl 1230.05142.

Integral gains $\varphi : E \to \mathbb{Z}$ induce a cycle-space homomorphism $\hat{\varphi} : Z_1(\Gamma) \to \mathbb{Z}$. Let $f : Z_1(\Gamma) \to \mathbb{Z}$. Thm. 3: $f(W) \leq k|W|$ for every walk W iff $f = \hat{\varphi}$ for some φ satisfying $\max|\varphi(e)| \leq k$. Thm. 2: For odd k, if also $f(W) \equiv |W| \bmod 2$, there is φ which assumes only odd values; and conversely. [Annot. 5 Sept 2010.] (**GG**)

20xxa Connectivity in signed-graphic matroids. Submitted. (**SG: M: Str**)

Daniel C. Slilaty and Hongxun Qin

2007a Decompositions of signed-graphic matroids. *Discrete Math.* 307 (2007), no. 17-18, 2187–2199. MR 2340600 (2008f:05032). Zbl 1121.05055. (**SG: M: Str**)

2008a The signed-graphic representations of wheels and whirls. *Discrete Math.* 308 (2008), no. 10, 1816–1825. MR 2394450 (2009c:05043). Zbl 1173.05311.

All frame matroids (of biased graphs) that are wheels and whirls, characterized topologically by embeddings in the projective plane (wheels) and the cylinder (whirls). (**GG: M: Str**)

2008b Connectivity in frame matroids. *Discrete Math.* 308 (2008), no. 10, 1994–2001. MR 2394467 (2009e:05139). Zbl 1170.05323.

Graphical biconnectivity of Ω vs. matroid connectivity of $G(\Omega)$, generalizing concepts developed by Wagner (1985a) for the bicircular matroid. (**GG: M: Str**)

Daniel C. Slilaty and Thomas Zaslavsky

20xxa Construction of line-consistent signed graphs. Submitted.

A constructive proof of Acharya, Acharya, and Sinha's (2009a) criterion for consistency of $\Lambda(\Sigma)$. [Annot. 14 Oct 2009.] (**SG, VS: LG: Bal**)

N.J.A. Sloane

See P.C. Fishburn, R.L. Graham, and C.L. Mallows.

Chris Smyth

See J. McKee.

J. Laurie Snell

See J. Berger and J.G. Kemeny.

Lynea Snyder

See Y. Duong.

Moo Young Sohn

See J. H. Kwak.

Alan D. Sokal

2005a The multivariate Tutte polynomial (alias Potts model) for graphs and matroids. In: Bridget S. Webb, ed., *Surveys in Combinatorics 2005*, pp. 173–226. Cambridge University Press, Cambridge, Eng., 2005. MR 2006k:05052. Zbl 1110.05020.

The parametrized dichromatic polynomial with parameters $d_e = 1$, called the "multivariate Tutte polynomial". Partly expository, partly new. [See Zaslavsky (1992b).] (**SGw: Gen: Invar, Exp**)

James P. Solazzo

See D.M. Duncan.

Patrick Solé and Thomas Zaslavsky

1994a A coding approach to signed graphs. *SIAM J. Discrete Math.* 7 (1994), 544–553. MR 95k:94041. Zbl 811.05034.

Among other things, improves some results in Akiyama, Avis, Chvátal, and Era (1981a). Thm. 1: For a loopless graph with c components, $D(\Gamma) \geq \frac{1}{2}m - \sqrt{\frac{1}{2}\ln 2}\sqrt{m(n-c)}$. Thm. 2: For a simple, bipartite graph, $D(\Gamma) \leq \frac{1}{2}(m - \sqrt{m})$. *Conjecture.* The best general asymptotic lower bound is $D(\Gamma) \geq \frac{1}{2}m - c_1\sqrt{mn} + o(\sqrt{mn})$ where c_1 is some constant between $\sqrt{\frac{1}{2}\ln 2}$ and $\frac{1}{2}\pi$. *Question.* What is c_1 for, e.g., k-

connected graphs? Thm. 4 gives girth-based upper bounds on $D(\Gamma)$. §5, "Embedded graphs", has bounds for several examples obtained by surface duality. All proofs are via covering radius of the cutset code of Γ. (**SG: Fr, Top**)

Extends to $r = 5$ the exact values of $D(K_{r,s})$ for $r \leq 4$ in Brown and Spencer (1971a). [But $r = 5$ has errors. Extended correctly to all r by Bowlin (2009).] [Annot. Rev. 14 Feb 2011.] (**SG: Fr**)

Louis Solomon
See P. Orlik.

N.D. Soner
See R. Rangarajan.

Sang-Oak Song
See G. Lee.

Song Yi-Zhe
See B. Xiao.

Eduardo Sontag
See also D. Angeli, C. Craciun, B. Dasgupta and G.A. Enciso.

2007a Monotone and near-monotone systems. In: I. Queinnec, S. Tarbouriech, G. Garcia, and S.-I. Niculescu, eds., *Biology and Control Theory: Current Challenges*, pp. 79–122. Lect. Notes in Control and Inform. Sci., Vol. 357. Springer-Verlag, Berlin, 2007. MR 2352229 (2008k:92021). (**sg: Biol**)

C.M. Soukoulis
See D. Blankschtein.

B.W. Southern, S.T. Chui, and G. Forgacs

1980a Non-universality for two-dimensional frustrated lattices? *J. Phys. C* 13 (1980), L827–L830.

Physics of signed square lattice graph, fully frustrated (all positive except for all-negative alternating vertical lines). Reduced to the "8-vertex" physics model by taking alternating sites (vertices) and observing they are 4-valent and all or half positive. [*Cf.* Garel and J.M. Maillard (1983a).] [Annot. 16 June 2012.] (**Phys: sg**)

Cid C. de Souza
See R.M.V. Figueiredo.

Natasha D'Souza
See T. Singh.

Tadeusz Sozański

1976a Processus d'équilibration et sous-graphes équilibrés d'un graphe signé complet. *Math. Sci. Humaines*, No. 55 (1976), 25–36, 83. MR 58 #27613.

Σ denotes a signed K_n. The "level of balance" ("indice du niveau d'équilibre") $\rho(\Sigma)$:= maximum order of a balanced subgraph. [Complement of the vertex deletion number.] Define distance $d(\Sigma_1, \Sigma_2) := |E_{1+} \triangle E_{2+}|$. Say Σ is p-clusterable if Σ^+ consists of p disjoint cliques [its "clusters"]. Thm. 1 evaluates the frustration index of a p-clusterable Σ. Thm. 2 bounds $l(\Sigma)$ in terms of n and $\rho(\Sigma)$. A negation set U for Σ "conserves" a balanced induced subgraph if they are edge-disjoint; it is "(strongly) conservative" if it conserves some (resp., every) maximum-order balanced induced subgraph. Thm. 3: Every minimum negation set conserves every balanced induced subgraph of order $> \frac{2}{3}n$. Thm. 4:

A minimum negation set can be ordered so that, successively negating its edges one by one, ρ never decreases. (**SG: KG: Fr, Clu**)

1980a Enumeration of weak isomorphism classes of signed graphs. *J. Graph Theory* 4 (1980), 127–144. MR 81g:05070. Zbl 434.05059.

"Weak isomorphism" = switching isomorphism. Principal results: The number of switching nonisomorphic signed K_n's. (*Cf.*Mallows and Sloane (1975a).) The number that are switching isomorphic to their negations. The number of nonisomorphic (not switching nonisomorphic!) balanced signings of a given graph. (**SG, KG: Sw: Enum**)

1982a Model rownowagi strukturalnej. Teoria grafow oznakowanych i jej zastosowania w naukach spotecznych. [The structural balance model. The theory of signed graphs and its applications in the social sciences.] (In Polish.) Ph.D. thesis, Jagellonian Univ., Krakow, 1982. (**SG, PsS: Bal, Fr, Clu, Aut, Adj, Ref**)

Edward Spence

See W.H. Haemers.

Joel Spencer

See T.A. Brown.

Aravind Srinivasan

2011a Local balancing influences global structure in social networks. *Proc. Nat. Acad. Sci. (U.S.A.)* 108 (2011), no. 5, 1751–1752.

Summary and commentary on Marvel, Kleinberg, Kleinberg, and Strogatz (2011a). [Annot. 7 Feb 2011.] (**SG: KG: Fr**)

Murali K. Srinivasan

See also A. Bhattacharya.

1998a Boolean packings in Dowling geometries. *European J. Combin.* 19 (1998), 727–731. MR 99i:05059. Zbl 990.10387.

Decomposes the Dowling lattice $Q_n(\mathfrak{G})$ into Boolean algebras, indexed in part by integer compositions, that are cover-preserving and centered above the middle rank. (**GG: M**)

R. Srinivasan

See V. Kodiyalam.

Saul Stahl

1978a Generalized embedding schemes. *J. Graph Theory* 2 (1978), 41–52. MR 58 #5318. Zbl 396.05013.

A generalized embedding scheme for a graph is identical to a rotation system for a signing of the graph. Thm. 2: Signed rotation systems describe all cellular embeddings of a graph. Thm. 4: Embeddings are homeomorphic iff their signed rotation systems are switching equivalent. Thm. 5: An embedding is orientable iff its signature is balanced. Compare Ringel (1977a). Dictionary: λ is the signature. "λ-trivial" means balanced. (**sg: Top, Sw**)

1978b The embeddings of a graph—a survey. *J. Graph Theory* 2 (1978), 275–298. MR 80a:05085. Zbl 406.05027. (**sg: Top**)

David P. Stanford

See C.R. Johnson.

Zoran Stanić

See also S.K. Simić.

2007a *Some Reconstructions in Spectral Graph Theory and Q-Integral Graphs.* (In Serbian.) Doctoral Thesis, Faculty of Math., Belgrade, 2007. **(Par: Adj)**

2007b There are exactly 172 connected Q-integral graphs up to 10 vertices. *Novi Sad J. Math.* 37 (2) (2007), 193–205. MR 2401613 (no rev). Zbl 1164.05046. **(Par: Adj)**

2009a On determination of caterpillars with four terminal vertices by their Laplacian spectrum. *Linear Algebra Appl.* 431 (2009), 2035–2048. MR 2567810 (2010j:05253). Zbl 226.05165.

§5: $\operatorname{Spec} K(-\Gamma)$ is mentioned. [Annot. 16 Jan 2012.] **(Par: Adj)**

Richard P. Stanley

See also P. Doubilet and A. Postnikov.

1973a Linear homogeneous diophantine equations and magic labelings of graphs. *Duke Math. J.* 40 (1973), 607–632. MR 47 #6519. Zbl 269.05109.

P. 630 restates Stewart (1966a), Cor. 2.4 in a clear way and observes that, if Γ is bipartite, then $\dim V = |E| - n + 2$. These two statements are equivalent to van Nuffelen (1973a). **(par: incid, ec)**

1985a Reconstruction from vertex-switching. *J. Combin. Theory Ser. B* 38 (1985), 132–138. MR 86f:05096. Zbl 572.05046.

From the 1-vertex switching deck (the multiset of isomorphism types of signed graphs resulting by separately switching each vertex) of $\Sigma = (K_n, \sigma)$, Σ can be reconstructed, provided that $4 \nmid n$. The same for i-vertex switchings, provided that the Krawtchouk polynomial $K_i^n(x)$ has no even zeros from 0 to n. When $i = 1$, the negative-subgraph degree sequence is always reconstructible. All done in terms of Seidel (graph) switching of unsigned simple graphs. [See Ellingham; Ellingham and Royle; Krasikov; Krasikov and Roditty for further developments. *Problem* 1. Generalize to signings of other highly symmetric graphs. *Problem* 2. Prove a similar theorem for switching of a bidirected K_n.] **(kg: sw, TG)**

1986a *Enumerative Combinatorics, Volume I.* Wadsworth and Brooks/Cole, Monterey, Cal., 1986. MR 87j:05003. Zbl 608.05001.

Ch. 3, "Partially ordered sets": Exercise 51, pp. 165 and 191, concerns the Dowling (1973a,b) lattices of a group and mentions Zaslavsky's generalizations [signed and biased graphs]. **(GG: M, Invar: Exr, Exp)**

1990a (As "R. Stenli") *Perechislitel'naya kombinatorika.* "Mir", Moscow, 1990. MR 91m:05002.

Russian translation of Stanley (1986). **(GG: M, Invar: Exr, Exp)**

1991a A zonotope associated with graphical degree sequences. In: Peter Gritzmann and Bernd Sturmfels, eds., *Applied Geometry and Discrete Mathematics: The Victor Klee Festschrift*, pp. 555–570. DIMACS Ser. Discrete Math. and Theor. Computer Sci., Vol. 4. American Mathematical Soc. and Assoc. for Computing Machinery, Providence and Baltimore, 1991. MR 92k:52020. Zbl 737.05057.

All-negative complete graphs (implicit in §3) and signed colorings (§4) are used to find the number of ordered degree sequences of n-vertex graphs and to study their convex hull. **(SG: Geom, Col)**

1996a Hyperplane arrangements, interval orders, and trees. *Proc. Nat. Acad. Sci. USA* 93 (1996), 2620–2625. MR 97i:52013. Zbl 848.05005.

Deformed braid hyperplane arrangements, i.e., canonical affine hyperplanar lift representations of $\mathrm{Lat}^{\mathrm{b}}\,\Phi$ where $\|\Phi\| = K_n$ and edge ij has gain $l_i \in \mathbb{Z}$ when $i < j$. In particular (§4), all $l_i = 1$. Also (§5), the Shi arrangement, which represents $\mathrm{Lat}^{\mathrm{b}}\{0,1\}\vec{K}_n$.

(**gg: Geom, M, Invar: Exp**)

1997a *Enumerative Combinatorics, Volume 1.* Corrected reprint. Cambridge Stud. Adv. Math., Vol. 49. Cambridge University Press, Cambridge, Eng., 1997. MR 98a:05001. Zbl 970.29805, 945.05006.

Additional exercises, some updating, some corrections to (1986a).

(**GG: M, Invar: Exr, Exp**)

1998a Hyperplane arrangements, parking functions and tree inversions. In: B.E. Sagan and R. Stanley, eds., *Mathematical Essays in Honor of Gian-Carlo Rota*, Progress in Math., Vol. 161, pp. 359–375. Birkhäuser, Boston, 1998. MR 99f:05006. Zbl 980.39546. (**gg: Geom, M, Invar: Exp**)

1999a *Enumerative Combinatorics, Volume 2.* Cambridge Stud. Adv. Math., Vol. 62. Cambridge University Press, Cambridge, Eng., 1999. MR 2000k:05026. Zbl 928.05001.

Exercise 5.50: The Shi arrangement [the affinographic hyperplane representation of $\{0,1\}\vec{K}_n$ with gain group $\mathbb{Z}^+$]. Exercise 5.41(h–i): The Linial arrangement and its characteristic polynomial [$= \chi^*_{\{1\}\vec{K}_n}(\lambda)$]. Exercise 6.19(lll) conceals the Catalan arrangement [representing $\{0,\pm1\}\vec{K}_n$]. Exercise 5.40(b): Counts two-graphs that $\not\supseteq [C_5]$.

(**gg: Geom, m, Invar, TG: Exr, Exp**)

2012a *Enumerative Combinatorics, Volume 1.* Second edition. Cambridge Stud. Adv. Math., Vol. 49. Cambridge University Press, Cambridge, Eng., 2012.

Vastly enlarged from (1986a, 1997a). Ch. 3, "Partially ordered sets": Exercise 115b, solution, p. 434, mentions Zaslavsky (1981a). Exercise 117, solution, p. 435, mentions Zaslavsky (2002a). Exercise 131, pp. 385 and 439–440, concerns the Dowling (1973a,b) lattices of a group and mentions Zaslavsky's generalizations to signed and gain [and biased] graphs. [Annot. 14 Jun 2012.] (**GG: M, Invar: Exr, Exp**)

Kenneth Steiglitz

See C.H. Papadimitriou.

Arthur Stein

See B. Healy.

Daniel L. Stein

See also C.M. Newman.

1989a Spin glasses. *Scientific American* 261 (July, 1989), no. 1, 52–59.

Informally describes frustration in spin glasses in terms of randomly ferromagnetic and antiferromagnetic interactions (see Toulouse (1977a)) and gives some history and applications. (**Phys: sg: bal, Rand: Exp**)

R. Stenli [Richard P. Stanley]

See R.P. Stanley.

Dragan Stevanović

2007a Research problems from the Aveiro Workshop on Graph Spectra. *Linear Algebra Appl.* 423 (2007), no. 1, 172–181. MR 2312333.

Two problems by Krzysztof Zwierzyński on the "signless Laplacian" matrix $K(-\Gamma)$ (see Cvetković, Rowlinson, and Simić 2007a) are: Problem AWGS.1, "The maximum clique and the signless Laplacian". Compare the clique number with the min eigenvalue. Problem AWGS.2, "Integral graphs". For which graphs are all eigenvalues (of $K(-\Gamma)$, in particular) integral? [Annot. 15 Sept 2010.] (**Par: Adj**)

Brett Stevens
See N.A. Neudauer.

B.M. Stewart
1966a Magic graphs. *Canad. J. Math.* 18 (1966), 1031–1059. MR 33 #5523. Zbl 149, 214 (e: 149.21401).

In $\mathbb{R}^{1+E} = \mathbb{R} \times \mathbb{R}^E$ with x_0 the first coordinate, let $\sigma_v(x) = \sum\{x_e : e \text{ is incident to } v\}$, and let $V = \{x \in \mathbb{R}^E : \sigma_v(x) = x_0, \forall v \in V\}$. Cor. 2.4 (p. 1059): If Γ is connected and contains an odd circle, then $\dim V = |E| - n + 1$. [Restated as in Stanley (1973a). Since $V \cap \{x_0 = 0\}$ = null space of the incidence matrix $\mathrm{H}(-\Gamma)$, this cryptically and partially anticipates the first calculation of rank($\mathrm{H}(-\Gamma)$), by van Nuffelen (1973a).] (**par: incid, ec**)

William J. Stewart
See N. Liu.

Allen H. Stix
1974a An improved measure of structural balance. *Human Relations* 27 (1974), 439–455. (**SG: Fr**)

Daniel Stolarski
See J. Carlson.

Douglas Stone
See W. Kocay.

J. Randolph Stonesifer
1975a Logarithmic concavity for a class of geometric lattices. *J. Combin. Theory Ser. A* 18 (1975), 216–218. MR 50 #9637. Zbl 312.05019.

The second kind of Whitney numbers of a Dowling lattice are binomially concave, hence strongly logarithmically concave, hence unimodal. [*Cf.* Damiani, D'Antona, and Regonati (1994a) and Benoumhani (1999a).] [*Famous Problem* (Rota). Generalize this.] [Annot. Rev 30 Apr 2012.] (**gg: M: Invar**)

Steven H. Strogatz
See also S.A. Marvel.

2010a The enemy of my enemy. *New York Times*, online edition, February 14, 2010, the Opinionator blog. http://opinionator.blogs.nytimes.com/2010/02/14/the-enemy-of-my-enemy/

A gentle explanation of negatives and negation, with special reference to balance in signed graphs. [Annot. 21 March 2010.] (**SG: Bal: Exp**)

Jeffrey Stuart
See also Q.A. Li.

Jeffrey Stuart, Carolyn Eschenbach, and Steve Kirkland
1999a Irreducible sign k-potent sign pattern matrices. *Linear Algebra Appl.* 294 (1999), 85–92. MR 1693935 (2000f:15017). Zbl 935.15008. (**QM: SD**)

Bernd Sturmfels
See A. Björner.

J. Stutz
See F. Glover.

S.P. Subbiah
2008a *A Study of Graph Theory: Topology, Steiner Domination and Semigraph Concepts*. Ph.D. thesis, Madurai Kamaraj Univ., 2008.
Contains material summarized in Subbiah and Swaminathan (2009a). [Annot. 2 Aug 2010.] (**SG**)

S.P. Subbiah and V. Swaminathan
2009a Properties of topological spaces associated with sigraphs. In: K. Somasundaram, ed., *Graph Theory and its Applications* (Proc.), pp. 233–241. Macmillan Publishers India, Delhi, 2009. MR 2574613.
Topologies τ_+, τ_- on $V \longleftrightarrow \Sigma^\varepsilon, \varepsilon = +, -$ for a signed graph Σ [not necessarily simple or finite]. $\Sigma \mapsto (\tau_\pm)$: $\tau_\varepsilon = \{$unions of subsets of $\pi(\Sigma^\varepsilon)\}$, $\pi(\Gamma) :=$ connected-component partition of V in Γ. "Exclusive property": If $u, v \in$ same component of Σ^ε, they are not in the same component of $\Sigma^{-\varepsilon}$, for $\varepsilon = \pm$. "Transitivity": Every component of $\Sigma^\pm$ is a clique. Thm. 1: Bijection between topology pairs (τ_+, τ_-) and transitive signed graphs on a set V (Subbiah 2008a). Further results [made elementary by observing that topology pairs are equivalent to partitions π_+, π_- of V. Exclusivity is $\pi_+ \wedge \pi_- = 0_V$ and is equivalent to simplicity of $|\Sigma|$. Topology is an epiphenomenon]. [It is not always clear when $|\Sigma|$ is meant to be simple.] [Annot. 2 Aug 2010.] (**SG**)

2009b Properties of topological spaces associated with sigraphs. Int. Conf. Graph Theory Appl. (Coimbatore, 2008). *Electron. Notes Discrete Math.* 33 (2009), 59–66. MR 2574613.
Shorter version of (2009a). [Annot. 2 Aug 2010.] (**SG**)

M.S. Subramanya
See also R. Rangarajan, E. Sampathkumar, and P. Siva Kota Reddy.

M.S. Subramanya and P. Siva Kota Reddy
2008a On balance and clusters in graph structures. *Int. J. Phys. Sci.* 20(1) (2008), 159-162.
A "graph structure" (due to E. Sampathkumar in 2005) is $G := (V, \mathcal{R})$ where $\mathcal{R} = \{R_1, \ldots, R_k)$, $k \geq 2$, $R_i \subseteq \mathcal{P}^{(2)}(V)$, and the R_i are disjoint. Let $\mathcal{S} \subseteq \mathcal{R}$ and $\|\mathcal{S}\| := \bigcup\{R : R \in \mathcal{S}\}$. Define $\|G\| := (V, \|\mathcal{S}\|)$ and let $\Sigma(\mathcal{S})$ be the signed $\|G\|$ with negative edge set $\|\mathcal{S}\|$. [$\Sigma(\mathcal{S})$ is not defined but is implicit.] G is "$\mathcal{S}$-balanced" if $\Sigma(\mathcal{S})$ is balanced, and "$\mathcal{S}$-clusterable" if $\Sigma(\mathcal{S})$ is clusterable. Prop. 3 [hard to interpret] seems to be Harary's (1953a) theorem for $\Sigma(\mathcal{S})$. Thm. 4: G is $\mathcal{S}$-balanced for all $\mathcal{S}$ iff it is $\{R_i\}$-balanced for all i. Thm. 7 is Davis's (1967a) characterization of clusterability applied to $\mathcal{S}$-clusterability. Thm. 8 has three conditions equivalent to $\mathcal{S}$-clusterability, assuming $\bigcup_1^k R_i = \mathcal{P}^{(2)}(V)$ and no $R_i = \varnothing$. [$k = 2, |\mathcal{S}| = 1$ is signed K_n.] Thm. 9: G is $\mathcal{S}$-clusterable for all $\mathcal{S}$ iff it is $\{R_i\}$-clusterable [the paper says "balanced"] for all i. [Annot. 1 Aug 2009.] (**sg, SG(Gen), gg: Bal, Sw, Clu**)

2009a Triangular line signed graph of a signed graph. *Adv. Appl. Discrete Math.* 4 (2009), no. 1, 17–23. MR 2555622 (2010m:05136). Zbl 1176.05036.

Definitions as at Sampathkumar, Siva Kota Reddy, and Subramanya (2008a, 2010c). Let $T(\Gamma) := (E, E_T)$ where $E_T := \{ef : e, f \in C_3 \text{ in } \Gamma\}$. The triangular line signed graph is $T(\Sigma) := (T(|\Sigma|), \sigma^c)$. Solved: $T(\Sigma) \simeq \Lambda_\times(\Sigma)$, $T^k(\Sigma) \simeq T^2(\Sigma)$. [$\Lambda_\times$ as in M. Acharya (2009a).] [Annot. 3 Aug 2009.] (**SG: Bal, Sw, LG(Gen), LG**)

Benjamin Sudakov
See G. Gutin.

N. Sudharsanam
See R. Balakrishnan.

Zhi Ren Sun
See X.X. Zhu.

V.S. Sunder
See V. Kodiyalam.

Masuo Suzuki

1991a Lee-Yang complex-field systems and frustrated Ising models. *J. Phys. Soc. Japan* 60 (1991), no. 2, 441–449. MR 1104390 (92h:82034) (*q.v.*).

§2, "Equivalence of Villain's frustrated system to Lee-Yang's complex-field systems": (2.3) summarizes Villain's (1977a) "fully frustrated" signed-graphic Ising model. [Annot. 17 Jun 2012.] (**Phys: SG**)

V. Swaminathan
See S.P. Subbiah.

Chaitanya Swamy

2004a Correlation clustering: Maximizing agreements via semidefinite programming. In: *Proceedings of the Fifteenth Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)* (New Orleans, 2004), pp. 526–527. Assoc. for Computing Machinery, New York, and SIAM, Philadelphia, 2004. MR 2291092. (**SG: WG: Clu: Alg**)

Ed Swartz
See P. Hersh.

Itiro Syôzi
See also Y. Kasai.

1950a The statistics of honeycomb and triangular lattice. II. *Progress Theor. Phys.* 5 (1950), 341–351. MR 039629 (12, 576g).

Physics of the all-negative ("antiferromagnetic") toroidal honeycomb (§7) and triangular (§9) lattices. The former is similar to all-positive ("ferromagnetic") [because balanced] while the latter is not [because unbalanced]. [See also R.M.F. Houtappel (1950a,b), G.F. Newell (1950b), G.H. Wannier (1950a).] [Annot. 21 Jun 2012.] (**Phys, sg: Fr**)

Edward Szczerbickl

1996a Signed directed graphs and reasoning for agents and multi-agent systems. *Int. J. Syst. Sci.* 27 (1996), no. 10, 1009–1015. Zbl 860.90071. (**SD: Appl**)

Janusz Szczypula
See P. Doreian.

Stefan Szeider
See N. Alon.

E. Szemerédi
See B. Bollobás.

Z. Szigeti
See A.A. Ageev.

Bosiljka Tadić, Krzysztof Malarz, and Krzysztof Kułakowski

2005a Magnetization reversal in spin patterns with complex geometry. *Phys. Rev. Letters* 94 (2005), article 137204. (**sg: par: Fr**)

B. Taglienti

See M. Falcioni.

Shingo Takahashi

See T. Inohara.

Michel Talagrand

1998a Huge random structures and mean field models for spin glasses. Proc. Int. Congress of Mathematicians, Vol. I (Berlin, 1998). *Documenta Math.*, Extra Vol. ICM 1998 (1998), Vol. I, pp. 507–536. MR 2000c:60164. Zbl 902.60089. (**sg: Gen: fr**)

Irving Tallman

1967a The balance principle and normative discrepancy. *Human Relations* 20 (1967), 341–355. (**PsS: ECol**)

Ilan Talmud

See Z. Maoz.

Bit-Shun Tam, Yi-Zheng Fan, and Jun Zhou

See also T.-J. Chang [T.-C. Chang] and Y.Z. Fan.

2008a Unoriented Laplacian maximizing graphs are degree maximal. *Linear Algebra Appl.* 429 (2008), 735–758. MR 2428127 (2009c:05143). Zbl 1149.05034.

The matrix is the Kirchhoff matrix of $-\Gamma$. "Maximizing" graphs are those whose degree sequences are maximal in the majorization ordering. [Annot. 23 Mar 2009.] (**Par: Adj**)

Bit-Shun Tam and Shu-Hui Wu

2010a On the reduced signless Laplacian spectrum of a degree maximal graph. *Linear Algebra Appl.* 432 (2010), no. 7, 1734–1756. MR 2592914 (2011c:15041). Zbl 1230.05202.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

A. Tamilselvi

See also M. Parvathi.

2010a Robinson-Schensted correspondence for the G-vertex colored partition algebra. *Asian-Eur. J. Math.* 3 (2010), no. 2, 369–385. MR 2669040 (2011j:16057). Zbl 1230.05010. (**gg: Algeb, m**)

Arie Tamir

See also D. Hochbaum.

1976a On totally unimodular matrices. *Networks* 6 (1976), 373–382. MR 57 #12553. Zbl 356.15020. (**SD: Bal**)

Akihisa Tamura

See also Y.T. Ikebe and D. Nakamura.

1997a The generalized stable set problem for perfect bidirected graphs. *J. Operations Res. Soc. Japan* 40 (1997), 401–414. MR 99e:05063. Zbl 894.90156.

Problem: maximize an integral weight function over the bidirected stable set polytope (*cf.* Johnson and Padberg (1982a)). §3 concerns the effect on perfection of deleting all incoming edges at a vertex. §4 reduces the "generalized stable set problem" for bidirected graphs to the maximum weighted stable set problem for ordinary graphs, whence the problem for perfect bidirected graphs is solvable in polynomial time.

(sg: Ori: Incid, Geom, Sw, Alg)

2000a Perfect $(0, \pm 1)$-matrices and perfect bidirected graphs. Combinatorics and Optimization (Okinawa, 1996). *Theor. Comput. Sci.* 235 (2000), no. 2, 339–356. MR 2001i:15019. Zbl 938.68061.

The stable set problem associated with bidirected graphs.

(sg: Ori: Geom, Alg)

Shang Wang Tan

See also L. Feng, X.L. Wu, and D.L. Zhang.

2010a On the Laplacian spectral radius of weighted trees with a positive weight set. *Discrete Math.* 310 (2010), no. 5, 1026–1036. MR 2575820 (2011e:05156).

The results on $K(\Gamma, w)$ with edge weights $w : E \to \mathbb{R}_{>0}$ are deduced from results on $K(-\Gamma, w)$. [*Problem.* Show the same reasoning applies to all signatures of Γ.] [Annot. 20 Jan 2012.] **(par: WG: Adj)**

2010b On the weighted trees with given degree sequence and positive weight set. *Linear Algebra Appl.* 433 (2010), no. 2, 380–389. MR 2645091 (2011e:05157). Zbl 1209.05054.

Similar to (2010a). [Annot. 20 Jan 2012.] **(par: WG: Adj)**

Shang-wang Tan, Ji-ming Guo, and Jian Qi

2003a The spectral radius of Laplacian matrices and quasi-Laplacian matrices of graphs. *Gongcheng Shuxue Xuebao* [*Chinese J. Engineering Math.*] 20 (2003), no. 6, 69–74. MR 2031534 (2004k:05137). **(Par: Adj)**

Shang-Wang Tan and Jing-Jing Jiang

2011a On the Laplacian spectral radius of weighted trees with fixed diameter and weight set. *Linear Multilinear Algebra* 59 (2011), no. 2, 173–192. MR 2773649 (2012a:05202). Zbl 1226.05169.

The "(signless) Laplacian" of a graph with positive edge weights, (Γ, w) where $w : E \to \mathbb{R}_{>0}$, is $K(-\Gamma, w) := D(\Gamma, w) + A(\Gamma, w)$ (but called R). The spectral radius is that of $K(-\Gamma, w)$. [*Problem.* Generalize to all weighted signed graphs.] [Annot. 11 Jan 2011, 21 Jan 2012.]

(par: WG, Adj)

Shang Wang Tan and Xing Ke Wang

2009a On the largest eigenvalue of signless Laplacian matrix of a graph. *J. Math. Res. Exposition* 29 (2009), no. 3, 381–390. MR 2510212 (2010h:05183). Zbl 1212.05164.

See Cvetković, Rowlinson, and Simić (2007a). **(Par: Adj)**

Xuezhong Tan

See also M.H. Liu.

Xuezhong Tan and Bolian Liu

2006a On the spectrum of the quasi-Laplacian matrix of a graph. *Australasian J. Combin.* 34 (2006), 49–55. MR 2195309 (2006i:05106). Zbl 1102.05039.

(Par: Adj, ec)

Ying-Ying Tan

See also Y.-Z. Fan.

Ying Ying Tan and Yi Zheng Fan

2008a On edge singularity and eigenvectors of mixed graphs. *Acta Math. Sinica (English Ser.)* 24 (2008), no. 1, 139–146. MR 2384238 (2008k:05134). Zbl 1143.05058.

Relations between least Laplacian eigenvalue, its eigenvector, and $l(\Sigma)$. Properties of the eigenvector when $l = 1$, e.g., $\lambda_1 \leq (4/n)l$. Dictionary: "mixed graph" = signed graph, "edge singularity" = frustration index $l(\Sigma)$. [Generalized in Bapat, Kalita, and Pati (2012a).] [Annot. 28 Oct 2011, 20 Jan 2012.] (**sg: Fr, Adj**)

Wenliang Tang

See E.L. Wei.

Shin-ichi Tanigawa

20xxa Matroids of gain graphs in applied discrete geometry. Submitted. arXiv:1207.-3601. (**GG: M: Gen**)

Tetsuji Taniguchi

See T.Y. Chung.

Percy H. Tannenbaum

See C.E. Osgood.

Éva Tardos

See also A.V. Goldberg.

Èva Tardos and Kevin D. Wayne

1998a Simple generalized maximum flow algorithms. In: Robert E. Bixby, E. Andrew Boyd, and Roger Z. Ríos-Mercado, eds., *Integer Programming and Combinatorial Optimization* (6th Int. IPCO Conf., Houston, 1998, Proc.), pp. 310–324. Lect. Notes in Computer Sci., Vol. 1412. Springer, Berlin, 1998. MR 2000i:90111. Zbl 911.90156.

Max. flow in a network with positive rational gains. Multiple sources and sinks are allowed. "Relabeling" is switching the gains. Useful references to previous work. (**GN: Sw, Alg, Ref**)

Robert E. Tarjan

See A.V. Goldberg.

Michael Tarsi

See F. Jaeger.

B. Tayfeh-Rezaie

See F. Ayoobi and A. Mohammadian.

D.E. Taylor

See also J.J. Seidel.

1977a Regular 2-graphs. *Proc. Lond. Math. Soc.* (3) 35 (1977), 257–274. MR 57 #16147. Zbl 362.05065.

Introducing two-graphs and regular two-graphs (defined by G. Higman, unpublished). [See Seidel (1976a) etc. for more.] A "two-graph" is the class $\mathcal{C}_{3-}$ of negative triangles of a signed complete graph (K_n, σ). (See §2. p. 258, where the group is $\mathbb{Z}_2 \cong \{+,-\}$ and the definition is in terms of the 2-coboundary operator.) Two-graphs and switching classes of signed complete graphs are equivalent concepts. (Stated in terms of Seidel switching in §2, p. 260.) A two-graph is "regular" if every edge lies in the same number of negative triangles. Thm.: $\mathcal{C}_{3-}$ is regular iff $A(K_n, \sigma)$ has at most two eigenvalues. Various parameters of regular two-graphs are calculated. (**TG: Adj. Geom**)

Graeme Taylor

2011a Cyclotomic matrices and graphs over the ring of integers of some imaginary quadratic fields. *J. Algebra* 331 (2011), no. 1, 523–545. MR 2774674 (2012b:-15058). Zbl 1238.05166. arXiv:1011.2737. (**SG**)

Herbert Taylor

See P. Erdős.

Howard F. Taylor

1970a *Balance in Small Groups.* Van Nostrand Reinhold, New York, 1970.

A thorough and pleasantly written survey of psychological theories of balance, including formalizations by signed graphs (Chs. 3 and 6), experimental tests and critical evaluation of the formalisms, and so forth. Ch. 2: "Substantive models of balance", takes the perspective of social psychology. §2.2: "Varieties of balance theory", reviews the theories of Heider (1946a) (the source of Harary's (1953a) invention of signed graphs), Osgood and Tannenbaum (1955a), and others. §2.2e: "The Rosenberg-Abelson modifications", discusses their introduction of the "cost" of change of relations, which led them (Abelson and Rosenberg 1958a) to propose the frustration index as a measure of imbalance. (**PsS: SG, WG: Exp, Ref**)

Ch. 3: "Formal models of balance", reviews various graph-theoretic models: signed and weighted signed, different ways to weigh imbalance, etc., the relationship to theories in social psychology being constantly kept in mind. §3.1: "Graph theory and balance theory", presents the basics of balance, measures of degree of balance by circles (Cartwright and Harary (1956a)), circles with strengths of edges (Morrissette (1958a)), local balance and N-balance (Harary (1955a)), edge deletion and negation (Abelson and Rosenberg (1958a), Harary (1959b)), vertex frustration number (Harary (1959b)). §3.2: "Evaluation of formalizations: strong points", and §3.3: "Evaluation of formalizations: weak points", judged from the applied standpoint. §3.3a: "Discrepancies between cycles or subsets of cycles", suggests that differing degrees of imbalance among certain different subsets of the vertices may be significant [Is this reasonable?] and proposes measures, e.g., a variance measure (p. 71), of this "discrepancy". (**PsS: SG, WG: Bal, Fr: Exp**)

Ch. 6: "Issues involving formalization", goes into more detail. §6.1: "Indices of balance", compares five indices, in particular Phillips' (1967a) eigenvalue index (also in Abelson (1967a)) with examples to show that the index differentiates among different balanced signings of the same graph. §6.2: "Extrabalance properties", discusses Davis's (1967a) clustering (§6.2b) and indices of clustering (§6.2c). §6.3: "The problem of cycle length and non-local cycles". Are long circles less important? Do circles at a distance from an actor (that is, a vertex) have less effect on the actor in balancing processes?

[Reviewed in Doreian (1970a).] (**PsS: SG: Fr, Adj: Exp**)

M. Teicher

See M. Amram.

Hidetaka Terasaka

See S. Kinoshita.

Lesley G. Terris

See Z. Maoz.

Denis Thieffry

See A. Naldi and É. Remy.

Morwen B. Thistlethwaite

1988a On the Kauffman polynomial of an adequate link. *Invent. Math.* 93 (1988), 285–296. MR 89g:57009. Zbl 645.57007.

A 1-variable Tutte-style polynomial Γ_Σ of a sign-colored graph. Fix an edge ordering. For each spanning tree T and edge e, let $\mu_T(e) = -A^{3\tau_T(e)\sigma(e)}$ if e is active with respect to T, $A^{\tau_T(e)\sigma(e)}$ if it is inactive, where $\tau_T(e) = +1$ if $e \in T$, -1 if $e \notin T$. Then $\Gamma_\Sigma(A) = \sum_T \prod_{e\in T} \mu_T(e)$. [In the notation of Zaslavsky (1992a), $\Gamma_\Sigma(A) = Q_\Sigma$ with $a_\varepsilon = A^{-\varepsilon}, b_\varepsilon = A^\varepsilon$ for $\varepsilon = \pm 1$ and $u = v = -(A^2 + A^{-2})$.] §§3 and 4 show Γ_Σ is independent of the ordering. Other sections derive consequences for knot theory. [This marks the invention of a Tutte-style polynomial of a colored, or parametrized or weighted, graph or matroid, developed in Kauffman (1989a) and successors.] (**SGc: Knot: Invar**)

A.D. Thomas

See F.W. Clarke.

René Thomas

R. Thomas and J. Richelle

1988a Positive feedback loops and multistationarity. *Discrete Appl. Math.* 19 (1988), 381–386. MR 936224 (89g:92007). Zbl 639.92003. (**sd: bal**)

Robin Thomas

See W. McCuaig and N. Robertson.

Andrew Thomason

1988a A graph property not satisfying a "zero-one law". *European J. Combin.* 9 (1988), 517–521. MR 90e:05051. Zbl 675.05057.

The property is the existence of an Eulerian cut. The asymptotic probability is .57 . . . . [*Problem.* Generalize to gain graphs with finite gain group, esp. to signed graphs. The property is that of being switchable so that the identity-gain edges form an Eulerian subgraph. (This has various meanings.) Variation: The property is that of having a maximal balanced subgraph that is Eulerian. One expects the asymptotic probabilities to be the same for both problems and to depend only on the group's order.] (**par: Rand**)

Carsten Thomassen

See also P.D. Seymour.

1985a Even cycles in directed graphs. *European J. Combin.* 6 (1985), 85–89. MR 86i:05098. Zbl 606.05039.

It is an NP-complete problem to decide whether a given signed digraph has a positive but not all-positive cycle, even if there are only 2 negative arcs. This follows from Lemma 3 of Steven Fortune, John Hopcroft, and James Wyllie, The directed subgraph homeomorphism problem (*Theor. Computer Sci.* 10 (1980), 111–121. MR 81e:68079. Zbl 419.05028.) by the simple argument in the proof of Prop. 2.1 here.

To decide whether a specified arc of a digraph lies in an even cycle, or in an odd cycle, are NP-complete problems (Prop. 2.1). To decide existence of an even cycle [hence, by the negative subdivision trick, of a positive cycle in a signed digraph] is difficult [but is solvable in polynomial time; see Robertson, Seymour, and Thomas (1999a)], although existence of an

odd cycle [resp., of a negative cycle] is easy, by a trick here attributed to Edmonds (unpublished). Prop. 2.2: Deciding existence of a positive cycle in a signed digraph is polynomial-time solvable if $|E^-|$ is bounded. Thm. 3.2: If the outdegrees of a digraph are all $> \log_2 n$, then *every signing* has a positive cycle, and this bound is best possible; restricting to the all-negative signature, the lower bound might (it's not known) go down by a factor of up to 2, but certainly (Thm. 3.1) a constant minimum on outdegree does not imply existence of an even cycle. [See (1992a) for the effect of connectivity.] (**SD, Par: Bal, Alg**)

1986a Sign-nonsingular matrices and even cycles in directed graphs. *Linear Algebra Appl.* 75 (1986), 27–41. MR 87k:05120. Zbl 589.05050. Erratum. *Linear Algebra Appl.* 240 (1996), 238. MR 1387301 (no rev).
(**QM, sd: par: QSol, bal, Alg**)

1988a Paths, circuits and subdivisions. In: Lowell W. Beineke and Robin J. Wilson, eds., *Selected Topics in Graph Theory 3*, Ch. 5, pp. 97–131. Academic Press, London, 1988. MR 93h:05003 (book). Zbl 659.05062.

§8: "Even directed circuits and sign-nonsingular matrices."
(**SD, QM: Bal, QSol: Exp**)

§§8–10 treat even cycles in digraphs. (**SD: Bal: Exp**)

[*General Problem.* Generalize even-cycle and odd-cycle results to positive and negative cycles in signed digraphs, the unsigned results corresponding to all-negative signatures.]

1988b On the presence of disjoint subgraphs of a specified type. *J. Graph Theory* 12 (1988), 101–111. MR 89e:05174. Zbl 662.05032.

There is an algorithm for detecting a balanced circle in a $\mathbb{Z}_m$-gain graph. Balance of such a gain graph is characterized. (**gg: Bal, Circles: Alg**)

1989a When the sign pattern of a square matrix determines uniquely the sign pattern of its inverse. *Linear Algebra Appl.* 119 (1989), 27–34. MR 90f:05099. Zbl 673.05067. (**QM, SD: QSol, Adj**)

1990a Embeddings of graphs with no short noncontractible cycles. *J. Combin. Theory Ser. B* 48 (1990), 155–177. MR 91b:05069. Zbl 704.05011.

§5 describes the "fundamental cycle method", a simple algorithm for a shortest unbalanced circle in a biased graph (Thm. 5.1). Thus the method finds a shortest noncontractible circle (Thm. 5.2). A noteworthy linear class: the surface-separating ("Π-separating") circles (p. 166). Dictionary: "3-path-condition" on a class F of circles = property that F^c is a linear class. "Möbius cycle" = negative circle in the signature induced by a nonorientable embedding. (**gg, sg: Alg, Top**)

1992a The even cycle problem for directed graphs. *J. Amer. Math. Soc.* 5 (1992), 217–229. MR 93b:05064. Zbl 760.05051.

A digraph that is strongly connected and has all in- and out-degrees ≥ 3 contains an even cycle. (**sd: par: bal**)

1993a The even cycle problem for planar digraphs. *J. Algorithms* 15 (1993), 61–75. MR 94d:05077. Zbl 784.68045.

A polynomial-time algorithm for deciding the existence of an even cycle in a planar digraph. (**sd: par: bal: Alg**)

1994a Embeddings of graphs. Graphs and Combinatorics (Qawra, 1990). *Discrete Math.* 124 (1994),217–228. MR 95f:05035. Zbl 797.05035.

P. 225 and Thm. 6.3: the "3-path-condition" and shortest unbalanced circle algorithm from (1990a). Examples mentioned (under other names) are parity bias (all-negative signs), poise bias, and noncontractible or orientation-reversing embedded circles. (**gg, par: Exp**)

2001a The Erdös–Pósa property for odd cycles in graphs of large connectivity. Paul Erdös and his mathematics (Budapest, 1999). *Combinatorica* 21 (2001), no. 2, 321–333. MR 2002c:05108. Zbl 989.05062.

Given k there exists K such that every sufficiently connected graph has k vertex-disjoint odd circles or K vertices whose deletion leaves a bipartite graph. [There follows by the negative-subdivision trick the corollary: Given k there exists K (the same K) such that every sufficiently connected signed graph has k vertex-disjoint negative circles or K vertices whose deletion leaves a balanced graph.] (**par: Fr: Circles**)

2001b Totally odd K_4-subdivisions in 4-chromatic graphs. *Combinatorica* 21 (2001), no. 3, 417–443. MR 2002e:05058. Zbl 1012.05064.

Proves Zang (1998a): Every 4-chromatic graph, when signed all negative, contains a subdivided $-K_4$. (**par: Col**)

G.L. Thompson

See V. Balachandran.

Christopher Thraves

See A.-M. Kermarrec.

Gui-Xian Tian, Ting-Zhu Huang, and Bo Zhou

2009a A note on sum of powers of the Laplacian eigenvalues of bipartite graphs. *Linear Algebra Appl.* 430 (2009), no. 8-9, 2503–2510. MR 2508309 (2010e:05191). Zbl 1165.05020.

A lower bound on $\sum_i \lambda_i(K(\Gamma))^\alpha$, over nonzero eigenvalues, for bipartite Γ and $\alpha \in \mathbb{R}^\times$. [*Question.* Is there a nonbipartite generalization involving $K(-\Gamma)$?] [Annot. 23 Jan 2012.] (**par: Adj**)

Xiao-Jun Tian, Xiao-Peng Zhang, Feng Liu, and Wei Wang

2009a Interlinking positive and negative feedback loops creates a tunable motif in gene regulatory networks. *Phys. Rev. E* 80 (2009), no. 1, 011926. (**SD: Biol**)

Yi Tian

See S.C. Li.

Shailesh Tipnis

See H. Jordon.

R.L. Tobin

1975a Minimal complete matchings and negative cycles. *Networks* 5 (1975), 371–387. MR 52 #16578. Zbl 348.90151.

Bjarne Toft

See T.R. Jensen.

Sivan Toledo

See E.G. Boman and D. Chen.

Ioan Tomescu

See also D.R. Popescu.

1973a Note sur une caracterisation des graphes dont le degré de deséquilibre est maximal. *Math. Sci. Humaines*, No. 42 (1973), 37–40. MR 51 #3003. Zbl 266.05115.

Independent proof of Petersdorf's (1966a) Satz 1. Also, treats similarly a variation on the frustration index. (**SG: Fr**)

1974a La réduction minimale d'un graphe à une réunion de cliques. *Discrete Math.* 10 (1974), 173–179. MR 51 #247. Zbl 288.05127. (**SG: Bal, Clu**)

1976a Sur le nombre des cycles négatifs d'un graphe complet signé. *Math. Sci. Humaines*, No. 53 (1976), 63–67. MR 56 #15493. Zbl 327.05119.

The parity of the number of negative triangles = that of $n|E^-|$. The number of negative t-gons is even when $n, t \geq 4$ [strengthened in Popescu (1991a), (1996a)]. (**SG: Bal**)

1978a Problem 2. In: A. Hajnal and Vera T. Sós, eds., *Combinatorics* (Proc. Fifth Hungarian Colloq., Keszthely, 1976), Vol. 2, p. 1217. Colloq. Math. Soc. János Bolyai, 18. János Bolyai Math. Soc., Budapest, and North-Holland, Amsterdam, 1978. MR 80a:05002 (book). Zbl 378.00007. (**SG: Bal**)

Mark Tomforde

See B.G. Bodmann.

Joanna Tomkowicz and Krzysztof Kułakowski

20xxa Scaling of spin avalanches in growing networks. Preprint. arXiv:0904.2697. (**par: Fr**)

C.B. Tompkins

See I. Heller.

J. Topp and W. Ulatowski

1987a On functions which sum to zero on semicycles. *Zastosowanie Mat.* (*Applicationes Math.*) 19 (1987), 611–617. MR 89i:05138. Zbl 719.05044.

An additive real gain graph is balanced iff every circle in a circle basis is balanced, iff the gains are induced by a vertex labelling [in effect, switch to 0], iff every two paths with the same endpoints have the same gains. A digraph is gradable (Harary, Norman, and Cartwright (1965a); also see Marcu (1980a)) iff φ_1 is balanced, where for each arc e, $\varphi_1(e) = 1 \in \mathbb{Z}$ (Thm. 3). The Windy Postman Problem (Thms. 4, 5). (**GG, GD: Bal**)

Aleksandar Torgašev

See also D.M. Cvetković.

1982a The spectrum of line graphs of some infinite graphs. *Publ. Inst. Math. (Beograd) (N.S.)* 31(45) (1982), 209–222. MR 85d:05175. Zbl 526.05039.

An infinite analog of Doob's (1973a) characterization via the even-cycle matroid of when a line graph has -2 as an eigenvalue. [*Problem.* Generalize to line graphs of infinite signed graphs.] (**par: Adj(LG)**)

1983a A note on infinite generalized line graphs. In: D. Cvetković *et al.*, eds., *Graph Theory* (Proc. Fourth Yugoslav Seminar, Novi Sad, 1983), pp. 291–297. Univ. Novom Sadu, Inst. Mat., Novi Sad, 1984. MR 85i:05168. Zbl 541.05042.

An infinite graph is a generalized line graph iff its least "limit" eigenvalue ≥ -2. [*Problem.* Generalize to line graphs of infinite signed graphs.] (**par: Adj(LG)**)

Dejan V. Tošić

See M. Anđelić.

Gérard Toulouse

See also B. Derrida and J. Vannimenus.

1977a Theory of the frustration effect in spin glasses: I. *Commun. Phys.* 2 (1977), 115–119. Repr. in M. Mézard, G. Parisi, and M.A. Virasoro, *Spin Glass Theory and Beyond*, pp. 99–103. World Scientific Lect. Notes in Physics, Vol. 9. World Scientific, Singapore, 1987.

Introduces the notion of imbalance ("frustration") of a signed graph to account for inherent disorder in an Ising model (here synonymous with a signed graph, usually a lattice graph). (Positive and negative edges are called "ferromagnetic and antiferromagnetic bonds".) Observes that switching the edge signs from all positive (the model of D.D. Mattis, *Phys. Letters* 56A (1976), 421–?) makes no essential difference. In a planar lattice [or any plane graph] frustration of face boundaries ("plaquettes") can be thought of as curvature, i.e., failure of flatness. Proposes two kinds of asymptotic behavior of frustration as a circle encloses more plaquettes. The planar-duality approach for finding the states with minimum frustration (i.e., switchings with fewest negative edges); the number of such states is the "ground-state degeneracy" and is important. Ideas are sketched; no proofs. [A foundational paper. See Wannier (1950a) and, e.g., Villain *et al.* (all), Hoever, Wolff, and Zittartz (1981a), Barahona, Maynard, Rammal, and Uhry (1982a), van Hemmen (1983a), Wolff and Zittartz (1983a), Mézard, Parisi, and Virasoro (1987a), Fischer and Hertz (1991a), Schwärzler and Welsh (1993a).]

(**SG: Phys, Sw, Bal**)

1979a Symmetry and topology concepts for spin glasses and other glasses. Nonperturbative Aspects in Quantum Field Theory (Proc. Les Houches Winter Adv. Study Inst., 1978). *Phys. Rep.* 49 (1979), no. 2, 267–272. MR 518399 (82j:82063).

Mainly for signed lattice graphs, with spins $s(v) \in S^{n-1}$ having symmetry group SO(n); $n = 1$ (Ising model) gives SO$\{+1, -1\}$; $n = 2$ is planar spins; $n = 3$ is Heisenberg spins. Two symmetry groups: $\mathbb{Z}_2^{|V|}$ acts on Σ (the "microscopic level"); SO(n) or O(n) acts on states s (the "macroscopic level"). [An edge is satisfied if $s(w) = \sigma(vw)s(v)$, otherwise frustrated.] A "ground state" (where the most edges are satisfied) has a topology of frustrated plaquettes [negative girth circles], whose nature, depending on the lattice dimension, is described intuitively. Regions ("packets") of relatively fixed spins can be identified. Topology of frustrated plaquettes leads to the homotopy groups of O(n). The effect on thermodynamic phases is discussed. Dictionary: "Local transformation" = switching. [Annot. 20 Aug 2012.]

(**Phys, CSG: CFr, CSw: CExp, CRef**)

1981a Spin glasses with special emphasis on frustration effects. In: Claudio Castellani *et al.*, eds., *Disordered Systems and Localization* (Rome, 1981), pp. 166–173. Lect. Notes in Phys., Vol. 149. Springer, Berlin, 1981.

§3, "Frustration": in signed graphs [after normalization to bond strength 1]. "Frustration function" of circles [$= \sigma(C)$] determines physical properties because they are "gauge [= switching] invariant", if no external magnetic field. §3.i, "Periodic frustrated models" [= toroidally embedded graphs]. §3.ii, "Fully frustrated models", where every "plaquette" [girth circle] is negative: overblocking effect, i.e., positive density of plaquettes with more than one negative edge. [A mathematically interesting concept, not understood today.] §3.iii, "Systems with finite residual entropy": e.g., antiferromagnetic [all-negative] Potts models. §3.iv, "Approach to spin glasses, by dilution of periodic frustrated systems" [embedding an unbalanced toroidal graph in a larger balanced graph?]. §3.v, "Connections with gauge theories; topological defects and their hydrodynamics": *cf.*, e.g., (1979a). §3.vi, "Random frustration ($J = \pm 1$) models, in various space dimensions": comparing random signs ± 1 with Gaussian random edge weights (centered at 0, hence with signs and magnitudes). For signed K_n's ("Sherrington–Kirkpatrick model"), "in the thermodynamic limit [both] have the same physics." [Annot. 20 Aug 2012.] (**Phys: Csg, CFr, CSw: CExp, CRef**)

Gérard Toulouse and Jean Vannimenus

1977a La frustration: un monde semé de contradictions. *La Recherche*, No. 83, Vol. 8 (Nov., 1977), 980–981.

Popular exposition of the elements of frustration in relation to the Ising model [evidently based on Toulouse (1977a)]. Briefly mentions the social psychology application. [See also Stern (1989a).]
(**Phys: SG, Bal: Exp**)(**SG: PsS: Exp**)

1980a On the connection between spin glasses and gauge field theories. *Phys. Rep.* 67 (1980), no. 1, 47–54.

Annealed and quenched models on a square lattice are compared. Annealed: edge weights J_{ij} ("bond strengths") are random variables; this is randomly weighted, randomly signed graphs. Quenched, edge weights = $\pm J$; this is signed graphs. The annealed model "grossly underestimates frustration effects." Proposed corrective: introduce Lagrange multipliers for the plaquettes. This leads to unexplored theory. App. (c), "The frustration model": randomly signed graphs, especially regular graphs; compared to models with Gaussian random edge weights and signs. [Annot. 20 Aug 2012.] (**Phys: Csg, CFr**)(**Phys: Csg, CFr: Exp**)

V.A. Traag and Jeroen Bruggeman

2009a Community detection in networks with positive and negative links. *Phys. Rev. E* 80 (2009), article 036115. arXiv:0811.2329.

Generalizes a Potts model for positive links to signed graphs. Method is more general than the clustering model for signed graphs. [Applied in Yoshikawa, Iino, and Iyetomi (2012a).] (**SG: Clu, PsS**)

Lorenzo Traldi

See also J. Ellis-Monaghan.

1989a A dichromatic polynomial for weighted graphs and link polynomials. *Proc. Amer. Math. Soc.* 106 (1989), 279–286. MR 90a:57013. Zbl 713.57003.

Generalizing Kauffman's (1989a) Tutte polynomial of a sign-colored graph, Traldi's "weighted dichromatic polynomial" $Q(\Gamma; t, z)$ is Zaslavsky's (1992b) $Q_\Gamma(1, w; t, z)$, in which the deletion-contraction parameters $a_e = 1$ and $b_e = w(e)$, the weight of e. Thm. 2 gives the Tutte-style spanning-tree expansion. Thm. 4: Kauffman's Tutte polynomial $Q[\Sigma](A, B, d) = d^{-1}A^{|E^+|}B^{|E^-|}Q_{|\Sigma|}(1, w; d, d)$ for connected Σ, with $w(e) = (AB^{-1})^{\sigma(e)}$. [See Kauffman (1989a) for other generalizations. Traldi gives perhaps too much credit to Fortuin and Kasteleyn (1972a).]

P. 284: Invariance under Reidemeister moves of type II constrains the weighted dichromatic polynomial to, in essence, equal Kauffman's. Thus no generalization is evident in connection with general link diagrams. There is an interesting application to special link diagrams. (**SGc: Gen: Invar, Knot**)

2004a A subset expansion of the coloured Tutte polynomial. *Combin. Probab. Comput.* 13 (2004), no. 2, 269–275.

The corank-nullity expansion of the usual Tutte polynomial generalizes to colored Tutte polynomials in the universal sense of Bollobás and Riordan (1999a). (**SGc: Gen: M: Invar**)

2005a Parallel connections and coloured Tutte polynomials. *Discrete Math.* 290 (2005), no. 2–3, 291–299. MR 2005j:05033. Zbl 1069.05021.

The Tutte polynomial of a parallel connection of colored graphs or matroids. (**SGc: Gen: M: Invar**)

2006a On the colored Tutte polynomial of a graph of bounded treewidth. *Discrete Appl. Math.* 154 (2006), no. 6, 1032–1036. MR 2212555 (2006j:05199). Zbl 1091.05027.

Polynomial-time computability for colored graphs of bounded tree width. [Also see Makowsky (2005a).] (**SGc: Gen: Invar: Alg, Knot**)

Marián Trenkler

See S. Jezný.

Nenad Trinajstić

See also A. Graovac.

1983a *Chemical Graph Theory.* 2 vols. CRC Press, Boca Raton, Florida, 1983. MR 86g:92044.

Vol. I: Ch. 3, § VI: " Möbius graphs." Ch. 5, § VI: "Extension of Sachs formula to Möbius systems." § VII: "The characteristic polynomial of a Möbius cycle." Ch. 6, § VIII: "Eigenvalues of Möbius annulenes." (**SG: Chem, Adj: Exp**)

1992a *Chemical Graph Theory, Second Ed.* CRC Press, Boca Raton, Florida, 1992. MR 93g:92034.

Ch. 3, § V.B: "Möbius graphs." Ch. 4, § I: "The adjacency matrix": see pp. 42–43. Ch. 5: "The characteristic polynomial of a graph", § II.B: "The extension of the Sachs formula to Möbius systems"; § III.D: "Möbius cycles". Ch. 6, § VIII: "Eigenvalues of Möbius annulenes" (i.e., unbalanced circles); § IX: "A classification scheme for moncyclic systems" (i.e., characteristic polynomials of circles). (**SG: Adj, Chem**)

Ch. 7: "Topological resonance energy," § V.C: "Möbius annulenes"; § V.G: "Aromaticity in the lowest excited state of annulenes". (**Chem**)

K. Truemper

See also Gerards *et al.* (1990a).

1976a An efficient scaling procedure for gain networks. *Networks* 6 (1976), 151–159. MR 56 #10882. Zbl 331.90027. (**gg: GN, sg: Bal, Sw**)

1977a On max flows with gains and pure min-cost flows. *SIAM J. Appl. Math.* 32 (1977), 450–456. MR 55 #5197. Zbl 352.90069. (**GG, OG, GN, Bal**)

1977b Unimodular matrices of flow problems with additional constraints. *Networks* 7 (1977), 343–358. MR 58 #20352. Zbl 373.90023. (**sg: Incid: Bal**)

1978a Optimal flows in nonlinear gain networks. *Networks* 8 (1978), 17–36. MR 57 #5041. Zbl 381.90039. **(GN)**

††1982a Alpha-balanced graphs and matrices and GF(3)-representability of matroids. *J. Combin. Theory Ser. B* 32 (1982), 112–139. MR 83i:05025. Zbl 478.05026.

A $0, \pm 1$-matrix is called "balanced" if it contains no submatrix that is the incidence matrix of a negative circle. More generally, α-balance of a $0, \pm 1$-matrix corresponds to prescribing the signs of holes in a signed graph. Main theorem characterizes the sets of holes (chordless circles) in a graph that can be the balanced holes in some signing. [A major result. See Conforti and Kapoor (1998a) for a new proof and discussion of applications.] **(sg: Bal, Incid)**

1992a *Matroid Decomposition.* Academic Press, San Diego, 1992. MR 93h:05046. Zbl 760.05001.

§12.1: "Overview." §12.2: "Characterization of alpha-balanced graphs," exposition of (1982a). **(sg: Bal, Sw)**

1992b A decomposition theory for matroids. VII. Analysis of minimal violation matrices. *J. Combin. Theory Ser. B* 55 (1992), 302–335. MR 93e:05021. Zbl 809.05024.

According to Cornuéjols (2001a), this paper contains the following theorem: A bipartite graph is "balanceable" (has a ± 1-weighting (mod 4) in which all polygons have sum 0 (mod 4)) iff it does not contain an induced subgraph that is a subdivided odd wheel or a theta graph with nodes in opposite color classes. [The weights are not gains because they are not oriented. However, this has major applications to signed hypergraphs; *cf.* Rusnak (2009a).] [*Problem.* Generalize to arbitrary graphs.] [Note that in a bipartite graph the sum around a polygon has to be 0 or 2 (mod 4) and therefore belongs to a group $\cong \mathbb{Z}_2$ so can be considered a sign. However, it may not be possible to relabel the edges from $\mathbb{Z}_2$ so as to get the same polygon sums. I.e., the polygon signing may not be derivable from a signed graph.] **(wg: bal)**

Anke Truss

See S. Böcker.

Marcello Truzzi

See F. Harary.

S.V. Tsaranov

See also F.C. Bussemaker, P.J. Cameron, and J.J. Seidel.

1992a On spectra of trees and related two-graphs. In: Jaroslav Nešetřil and Miroslav Fiedler, eds., *Fourth Czechoslovak Symposium on Combinatorics, Graphs and Complexity* (Prachatice, 1990), pp. 337–340. Ann. Discrete Math., Vol. 51. North-Holland, Amsterdam, 1992. MR 93i:05004 (book). Zbl 776.05077.

A two-graph whose points are the edges of a tree T and whose triples are the nonseparating triples of edges of T (from Seidel and Tsaranov (1990a) via Cameron (1994a)). An associated signed complete graph Σ_T on vertex set $E(T)$ is obtained by orienting T arbitrarily, then taking $\sigma_T(ef) = +$ or $-$ depending on whether e and f are similarly or oppositely oriented in the path of T that contains both. Reorienting edges corresponds to switching Σ_T. Thm.: Letting $n = |V(T)|$, the matrices $3I_n + A(\Sigma_T)$ and $2I_{n+1} - A(T)$ have the same numbers of zero and negative eigenvalues. **(TG: Adj, Geom)**

1993a Trees, two-graphs, and related groups. In: D. Jungnickel and S.A. Vanstone, eds., *Coding Theory, Design Theory, Group Theory* (Proc. Marshall Hall Conf., Burlington, Vt., 1990), pp. 275–281. Wiley, New York, 1993. MR 94j:05062.

New proof of theorem on the group (Seidel and Tsaranov 1990a) of the two-graph (Tsaranov 1992a) of a tree. (**TG: Adj, Geom**)

Michael J. Tsatsomeros
See C.R. Johnson and S. Kirkland.

Thomas W. Tucker
See J.L. Gross.

Vanda Tulli
See A. Bellacicco.

Edward C. Turner
See R.Z. Goldstein.

Daniel Turzík
See S. Poljak.

W.T. Tutte

†1967a Antisymmetrical digraphs. *Canad. J. Math.* 19 (1967), 1101–1117. MR 214512 (35 #5362). Zbl 161, 209e (e: 161.20905).

Integral (u, u)-flows on a signed graph with edge capacities, presented in the language of integral $(\tilde{u}, \tilde{u}^*)$-flows on a digraph with edge capacities, with an orientation-reversing, fixed-point free, capacity-preserving involution *. [Such a digraph is the double covering digraph of a bidirected graph, thus the capacities and flows are equivalent to (u, u)-flows on a capacitated signed graph.] Analog of the Min-Flow Max-Cut Theorem (see 3.3). Structure of flows. Application to undirected graph factors. [*Problem*. Convert the entire paper to the language of signed graphs. Express the structure of (u, u)-flows in terms of signed-graphic objects such as unbalanced unicyclic subgraphs. Extract the implicit matroid theory, including the structure of cocircuits (*cf.* Chen and Wang (2009a)).] [Annot. 9 Sept 2010, 12 Jan 2012.] (**sg: ori, cov: Flows**)

†1981a On chain-groups and the factors of graphs. In: L. Lovász and Vera T. Sós, eds., *Algebraic Methods in Graph Theory* (Proc. Colloq., Szeged, 1978), Vol. 2, pp. 793–818. Colloq. Math. Soc. János Bolyai, 25. János Bolyai Math. Soc., Budapest, and North-Holland, Amsterdam, 1981. MR 83b:05104. Zbl 473.05023.

The chain-group approach to the dual even-cycle matroid, $G(-\Gamma)^*$. Developed entirely in terms of the group $\Delta(\Gamma)$ [topologically, $B^1(\Gamma, \mathbb{Z})$] of integral 1-coboundaries. Assuming Γ connected: "Dendroids of $\Delta(\Gamma)$" = bases of $G(-\Gamma)$; Thms. 8.6–7 give their structure in the bipartite and nonbipartite cases. Support of an elementary coboundary = circuit of $G(-\Gamma)^*$; this is a bond of Γ if Γ is bipartite (Thm. 7.5) and a minimal balancing set otherwise (Thm. 7.6). Thm. 7.8: Any coboundary times some power of 2 is a sum of primitive coboundaries. [*Problem*. Explain how this is related to total dyadicity of the incidence matrix.] "Rank of $\Delta(\Gamma)$" = $\operatorname{rk} G(-\Gamma)$; its value is given at the end of §8. §9 develops a relationship between "homomorphisms" of $\Delta(\Gamma)$ (linear functionals) and graph factors. §10: The dual chain group; characterization of circuits of $\operatorname{rk} G(-\Gamma)$. [It is amazing what can be done with nothing but integral 1-coboundaries. *Problem* 1. Extend Tutte's theory of integral chain groups to all signed graphs. Grossman, Kulkarni, and Schochetman (1994a) have a development over a field but this is very different, even aside from their opposite viewpoint that goes from matroids to vector spaces. *Problem* 2. Extend to signed hypergraphs, where each hyperedge has a function $\tau_e : V(e) \to \{+, -\}$, not distinguished from $-\tau_e$—as with bidirected graphs, choosing one of them corresponds to orienting e.]

[Tutte knew and lectured on $G(-\Gamma)^*$ and/or $G(-\Gamma)$ before anyone (Doob 1973a, Simões-Pereira 1973a) published it.—information from Neil Robertson.] (**sg: EC, D, incid**)

Zsolt Tuza
See S. Poljak.

Frank Uhlig
See C.R. Johnson.

J.P. Uhry
See F. Barahona and I. Bieche.

Włodzimierz Ulatowski
See also J. Topp.

1991a On Kirchhoff's voltage law in Z_n. *Discussiones Math.* 11 (1991), 35–50. MR 93g:05121. Zbl 757.05058.

Examines injective, nowhere zero, balanced gains (called "graceful labellings") from Z_{m+1}, $m = |E|$, on arbitrarily oriented circles and variously oriented paths. [*Question.* Does this work generalize to bidirected circles and paths?] (**GD: bal: Circles, Paths**)

N.B. Ul'janov [N.B. Ul'yanov]
See N.B. Ul'yanov.

N.B. Ul'yanov
See D.O. Logofet.

Gurunath Rao Vaidya
See P. Siva Kota Reddy.

J.F. Valdés, J. Cartes, and E.E. Vogel

2000a Polyhedra as $\pm J$ closed Ising lattices. *Rev. Mexicana Fís.* 46 (2000), no. 4, 348–356. MR 1783780 (2001g:82035).

Physics on a signed polyhedral graph. Quantities depend on $x := |E^+|/|E|$. [There is a mistake. The properties are switching invariant but x is not.] [Annot. 17 Jun 2012.] (**Phys: SG**)

James Van Buskirk
See T.J. Lundy.

Edwin R. van Dam
See E.R. van Dam (under "D").

Arnout van de Rijt

2011a The micro-macro link for the theory of structural balance. *J. Math. Sociology* 35 (2011), no. 1-3, 94–113. Zbl 1214.91094. (**SG: Fr**)

Pauline van den Driessche
See J. Bélair, M. Catral, C. Jeffries, C.R. Johnson, and V. Klee.

J.L. van Hemmen
See under "H".

Marc A.A. van Leeuwen

1996a The Robinson-Schensted and Schützenberger algorithms, an elementary approach. *Electronic J. Combin.* 3 (1996), no. 2, #R15, 32 pp. MR 1392500 (97e:05200). Zbl 852.05080.

Elements of the hyperoctahedral group $\mathfrak{O}_d$ (signed permutations) of even degree $d = 2n$ permute $\pm[n]$ and of odd degree $d = 2n+1$ permute $[-n, n]$ (pp. 22f. The natural involution is $\pi \mapsto -\bar{\pi}$, where $\bar{\pi}$ is the reverse of π [reminiscent of signed graph coloring]. [*Cf.* Bloss (2003a) and Parvathi (2004a).] [Annot. 19 Mar 2011.] (**sg: Algeb**)

M.E. Van Valkenburg

See W. Mayeda.

Kevin N. Vandermeulen

See M.S. Cavers and D.A. Gregory.

A. Vannelli

See C.J. Shi.

Jean Vannimenus

See also B. Derrida and G. Toulouse.

J. Vannimenus, S. Kirkpatrick, F.D.M. Haldane, and C. Jayaprakash

1989a Ground-state morphology of random frustrated XY systems. *Phys. Rev. B* 39 (1989), no. 7, 4634–4643. MR 986455 (89m:82087).

XY means signed graphs with complex-unit vertex spins. (**Phys: sg**)

J. Vannimenus, J.M. Maillard, and L. de Sèze

1979a Ground-state correlations in the two-dimensional Ising frustration model. *J. Phys. C: Solid State Phys.* 12 (1979), 4523–4532. (**Phys: SG**)

J. Vannimenus and G. Toulouse

1977a Theory of the frustration effect: II. Ising spins on a square lattice. *J. Phys. C: Solid State Phys.* 10 (1977), L537–L541. (**SG: Phys**)

Ebrahim Vatandoost

See G.R. Omidi.

Vijay V. Vazirani and Mihalis Yannakakis

1988a Pfaffian orientations, 0/1 permanents, and even cycles in directed graphs. In: Timo Lepistö and Arto Salomaa, eds., *Automata, Languages and Programming* (Proc. 15th Int. Colloq., Tampere, Finland, 1988), pp. 667–681. Lect. Notes in Computer Sci., Vol. 317. Springer-Verlag, Berlin, 1988. MR 90k:68078. Zbl 648.68060.

Slightly abridged version of (1989a). (**SD: Adj, Bal: Alg**)

Vijay V. Vazirani and Milhalis [Mihalis] Yannakakis

1989a Pfaffian orientations, 0–1 permanents, and even cycles in directed graphs. *Discrete Appl. Math.* 25 (1989), 179–190. MR 91e:05080. Zbl 696.68076.

"Evenness" of a digraph (i.e., every signing contains a positive cycle) is polynomial-time equivalent to evaluability of a certain 0–1 permanent by a determinant and to parts of the existence and recognition problems for Pfaffian orientations of a graph. Briefly expounded in Brundage (1996a).] (**SD: Adj, Bal: Alg**)

Renata R. Del-Vecchio

See Renata R. Del-Vecchio (under D).

Fernando Vega-Redondo

See G.C.M.A. Ehrhardt.

Venkat Venkatasubramanian
See M.R. Maurya.

Véronique Ventos
See P. Berthomé and D. Forge.

Dirk Vertigan
See J. Geelen and J. Oxley.

Adrian Vetta
See S. Fiorini and J. Geelen.

Fabien Vignes-Tourneret

2009a The multivariate signed Bollobás–Riordan polynomial. *Discrete Math.* 309 (2009), no. 20, 5968–5981. MR 2552629 (2011a:05162). Zbl 1228.05183. arXiv:-0811.1584v1.

Multivariate version of the Chmutov and Pak (2007a) and Chmutov (2009a) signed ribbon-graph polynomials for orientation-embedded signed graphs. [*Cf.* Krushkal (2011a).] [Annot. 12 Jan 2012.] (**SGc: Top, Invar**)

S. Vijay
See V. Lokesha and P. Siva Kota Reddy.

G.K. Vijayakumar
See G.R. Vijayakumar.

G.R. Vijayakumar
See also P.D. Chawathe, D.K. Ray-Chaudhuri, and N.M. Singhi.

1984a (As "G.K. Vijayakumar") A characterization of generalized line graphs and classification of graphs with eigenvalues at least 2 [misprint for -2]. *J. Combin. Inform. Syst. Sci.* 9 (1984), 182–192. MR 89g:05055. Zbl 629.05046. (**sg: Adj, lg**)

1987a Signed graphs represented by D_∞. *European J. Combin.* 8 (1987), 103–112. MR 88b:05111. Zbl 678.05058.

The finite signed simple graphs represented (see 1993a) by a root system D_n have a characterization by forbidden induced subgraphs, the largest of which has order 6. [The complete list is given by Chawathe and Vijayakumar (1990a). The representable signed graphs are the reduced line graphs of simply signed graphs without loops or half edges; see Zaslavsky (2010b, 20xxa, 20xxb).] Also, remarks on signed graphs representable by $\mathbb{R}^\infty$. (**SG: adj, Geom, incid, lg**)

1992a Signed graphs represented by root system E_8. Combinatorial Math. and Appl. (Proc., Calcutta, 1988). *Sankhya Ser. A* 54 (1992), 511–517. MR 94d:05072. Zbl 882.05118.

The finite signed simple graphs represented (see 1993a) by the root system E_8 are characterizable by forbidden induced subgraphs. The largest of these subgraphs has order 10. (**SG: adj, Geom, incid**)

1993a Algebraic equivalence of signed graphs with all eigenvalues ≥ -2. *Ars Combin.* 35 (1993), 173–191. MR 93m:05134. Zbl 786.05059.

Finite signed simple graphs only. $\lambda_1(\Sigma)$:= least eigenvalue of $A(\Sigma)$. Σ is "represented" by $W \subseteq \mathbb{R}^\infty$ if the vertices can be mapped into W so that inner products equal signs of edges, where we interpret $\sigma(u,v) \in \{+1,-1,0\}$. Thm. 1: $\lambda_1(\Sigma) \geq -2 \iff \Sigma$ is represented by $\mathbb{R}^\infty \iff \Sigma$ is represented by D_∞ or E_8 (the root systems).

Σ' is "algebraically equivalent" to Σ if it is obtained from Σ by a sequence of switchings and algebraic transforms. The latter means taking two positively adjacent vertices a, b contained in no negative triangle, switching b, removing edges from b to all common neighbors of a and b, and adding an edge xb, for each $x \in N(a) \setminus N(b)$, with the same sign as xa. Thm. 5: If Σ is connected and $\lambda_1(\Sigma) > -2$, then Σ is algebraically equivalent to the Dynkin diagram of A_n, D_n, or E_k ($k = 6, 7, 8$).

There are other, similar results.

Dictionary: $(\varphi^*(u,v))_{V\times V}$ is the Kirchhoff matrix $2I + A(\Sigma) = \mathrm{H}(-\Sigma) \cdot \mathrm{H}(-\Sigma)^{\mathrm{T}}$ of $-\Sigma$. (**SG: adj, Geom, incid**)

1994a Representation of signed graphs by root system E_8. *Graphs Combin.* 10 (1994), 383–388. MR 96a:05128. Zbl 821.05040.

An algebraic characterization of the forbidden induced subgraphs for simple signed graphs represented by the root system E_8. *Cf.* (1992a) and for definitions (1993a). (**SG: adj, Geom, incid**)

2007a Partitions of the edge set of a graph into internally disjoint paths. *Australas. J. Combin.* 39 (2007), 241–245. MR 2008i:05156. Zbl 1134.05088.

A connected, contrabalanced biased graph $(\Gamma, \varnothing)$ has a covering by $\xi+1$ internally disjoint paths, where ξ = cyclomatic number, iff every $(\Gamma \setminus v, \varnothing)$ has no balanced components. [*Question* 1. Can this generalize to all connected biased graphs? Paths should become balanced subgraphs that are "path-like" (have at most two vertices of attachment). ξ should become a measure of the number of independent unbalanced circles. *Question* 2. Is there a recursive decomposition of a 2-connected biased graph into ξ path-like balanced subgraphs, generalizing the standard ear decomposition of a 2-connected, (contrabalanced biased) graph?] [Annot. 8 Mar 2008.] (**gg: Str**)

2008a A graph labeling related to root lattices. In: B.D. Acharya, S. Arumugam, and Alexander Rosa, eds., *Labelings of Discrete Structures and Applications* (Mananthavady, Kerala, 2006), pp. 175–179. Narosa, New Delhi, 2008. MR 2391786 (2009e:05281) (book). Zbl 1180.05111.

A "2-fold labeling" of a signed simple graph Σ is a nonzero $f \in \mathrm{Nul}[2I + A(\Sigma)]$. Thm. 3: If Σ has a 2-fold labeling, it has one such that, for some $v \in V$, $f(v) = 1$ and every $f(w)$ is an integral multiple of $f(v)$. Applied to prove via signed graphs the classification of root systems with root length $\sqrt{2}$. (**SG: Adj, Geom**)

2009a A method of classifying all simply laced root systems. *J. Algebra Appl.* 8 (2009), no. 4, 533–537. MR 2555519 (2010k:17016). Zbl 1172.05337.

Thm. 6: For a connected signed simple graph with eigenvalues ≤ 2, $2I - A(\Sigma)$ is the Gram matrix of a subset of root system D_n or E_8. (**SG: Adj, Geom**)

2011a A property of weighted graphs without induced cycles of nonpositive weights. *Discrete Math.* 311 (2011), no. 14, 1385–1387. MR 2795549 (2012f:05127).

Let Γ be 2-connected. Thm.: If $\varphi : E \to \mathbb{Z}_{\leq 1}$, extended to $S \subset E$ by $\varphi(S) := \sum_{e\in S} f(e)$, satisfies $\varphi(C) > 0$ for every induced circle, then $\varphi(E) > 0$. Cor.: If $\varphi : E \to \mathbb{R}$ satisfies $\varphi(C) > 0$ for every circle (not necessarily induced), then $\varphi(E) > 0$. Cor.: If Σ has $|E^+(C)| > |E^-(C)|$

for every induced circle, then $|E^+(\Sigma)| > |E^-(\Sigma)|$, a conjecture from B.G. Xu (2009a). [*Cf.* Balakrishnan and Sudharsanam (1982a) where equality is treated.] [Annot. 16 Oct 2011.] (**SGw: Gen**)

2011b Equivalence of four descriptions of generalized line graphs. *J. Graph Theory* 67 (2011), no. 1, 27–33. MR 2809559 (2012e:05318). Zbl 1226.05185. (**sg: LG**)

20xxb Spectral numbers related to signed graphs. Submitted.

Let $\mathrm{Spec}(\Sigma)$:= spectrum of $A(\Sigma)$, $I(\Sigma) := [\min \mathrm{Spec}(\Sigma), \max \mathrm{Spec}(\Sigma)]$. If $\rho \in \mathbb{R}$ is such that $\rho \in I(\Sigma) \implies \rho \in \mathrm{Spec}(\Sigma')$ for some $\Sigma' \subseteq \Sigma$, then $\rho = 0, \pm 1, \pm 2$. If Σ' must be an induced subgraph, $\rho = 0, \pm 1, \pm\sqrt{2}, \pm 2$. (Also, similar results for graphs.) (**SG: Adj**)

G.R. Vijayakumar (as "Vijaya Kumar"), S.B. Rao, and N.M. Singhi

1982a Graphs with eigenvalues at least -2. *Linear Algebra Appl.* 46 (1982), 27–42. MR 83m:05099. Zbl 494.05044.

A minimal forbidden induced graph has order at most 10, which is best possible. [Annot. 2 Aug 2010.] (**sg: adj, Geom, lg**)

G.R. Vijayakumar and N.M. Singhi

1990a Some recent results on signed graphs with least eigenvalues ≥ -2. In: Dijen Ray-Chaudhuri, ed., *Coding Theory and Design Theory Part I: Coding Theory* (Proc. Workshop IMA Program Appl. Combin., Minneapolis, 1987–88), pp. 213–218. IMA Vol. Math. Appl., Vol. 20. Springer-Verlag, New York, 1990. MR 91e:05069. Zbl 711.05033. (**SG: Geom, lg, adj: Exp**)

K.S. Vijayan

See S.B. Rao.

Jacques Villain

1977a Spin glass with non-random interactions. *J. Phys. C: Solid State Phys.* 10 (1977), no. 10, 1717–1734.

Partition function of "fully frustrated" Ising model on signed square lattice graph, i.e., all squares ("plaquettes") are unbalanced ("frustrated"). [Annot. 17 Jun 2012.] (**SG: Phys, Sw**)

1977b Two-level systems in a spin-glass model: I. General formalism and two-dimensional model. *J. Phys. C: Solid State Phys.* 10 (1977), 4793–4803. (**Phys: SG**)

1978a Two-level systems in a spin-glass model: II. Three-dimensional model and effect of a magnetic field. *J. Phys. C: Solid State Phys.* 11 (1978), no. 4, 745–752. (**Phys: SG**)(**GG: Phys, Sw, Bal**)

Daniele Vilone

See F. Radicchi.

Andrew Vince

1983a Combinatorial maps. *J. Combin. Theory Ser. B* 34 (1983), 1–21. MR 84i:05048. Zbl 505.05054.

See Theorem 6.1. (**sg: bal: Top**)

E. Vincent, J. Hammann, and M. Ocio

20xxa Slow dynamics in spin glasses and other complex systems. In: D.H. Ryan, ed., *Recent Progress in Random Magnets*, pp. 207–236. World Scientific, Singapore, 1992.

Surveys experiments with spin glass materials, especially aging behavior. Observations tend to support a landscape of graph signatures with numerous metastable states, subdividing as temperature decreases. [Presumably, the states correspond to clusters of low-frustration states, separated by high-frustration barriers, subdivided into smaller clusters by lower-frustration barriers, and so on. A mathematical examination is needed.] [Annot. 27 Aug 1998, 24 Aug 2012.] (**Phys: sg: Fr**)

Miguel Angel Virasoro
See M. Mézard.

Krishnamurthy Viswanathan
See R.M. Roth.

J. Vlach
See C.J. Shi.

E.E. Vogel
See J.F. Valdés.

Jan Vondrák
See A. Galluccio.

Heinz-Jürgen Voss
See also D. Král'.

1991a *Cycles and Bridges in Graphs.* Math. Appl. (E. Europ. Ser.), Vol. 49. Kluwer, Dordrecht, and Deutscher Verlag der Wissenschaften, Berlin, 1991. MR1131525 (92m:05118). Zbl 731.05031 .

§3.4, "The length of bridges of longest odd and even cycles." §3.9, "The 'odd circumference' in bridges of longest odd cycles." §7.6, "Longest odd and even cycles. ...." §8.4, "Odd and even cycles with a given number of diagonals." §10.1, "Long cycles and long even cycles with many diagonals." §10.3, "Long odd cycles with many diagonals in non-bipartite graphs." [*Problem.* Generalize results on even and odd circles to signed graphs. *Cf.*, e.g., Conlon (2004a).] (**sg: par: Circles**)

Jože Vrabek
See T. Pisanski.

Damir Vukičevic
See T. Došlić.

Kristina Vušković
See M. Conforti and T. Kloks.

Michelle L. Wachs
See E. Gottlieb.

Donald K. Wagner
See also V. Chandru and C.R. Coullard.

††1985a Connectivity in bicircular matroids. *J. Combin. Theory Ser. B* 39 (1985), 308–324. MR 87c:05041. Zbl 584.05019.

Prop. 1 and Thm. 2 show that n-connectivity of the bicircular matroid $BG(\Gamma)$ is equivalent to "n-biconnectivity" of Γ.

When do two 3-biconnected graphs have isomorphic bicircular matroids? §5 proves that 3-biconnected graphs with > 4 vertices have isomorphic bicircular matroids iff one is obtained from the other by a sequence of operations called "edge rolling" and "3-star rotation". This is the bicircular analog of Whitney's circle-matroid isomorphism theorem, but it is complicated. [An important theorem, generalized to all bicircular matroids in Coullard, del Greco, and Wagner (1991a). *Major Research Problems.* Generalize to bias matroids of biased graphs. Find the analog for lift matroids.] (**Bic: Str**)

1988a Equivalent factor matroids of graphs. *Combinatorica* 8 (1988), 373–377. MR 90d:05071. Zbl 717.05022.

"Factor matroid" = even-cycle matroid $G(-\Gamma)$. Decides when $G(-\Gamma) \cong G(B)$ where B is a given bipartite, 4-connected graph. (**EC: Str**)

H. Wagner

See K. Drühl.

Magnus Wahlström

See S. Böcker.

Bronislaw Wajnryb

See R. Aharoni.

M.H. Waldor, W.F. Wolff, and J. Zittartz

1985a Ising models on the pentagon lattice. *Z. Phys. B* 59 (1985), no. 1, 43–51. MR 788876 (86e:82054).

Physics of all-positive and all-negative ("fully frustrated": all girth circles are negative) signs as examples. § II, "Thermodynamics", b) "Homogeneous case": The all-negative signature has multiple ground states that have energy $-J$ (J = bond strength) per vertex because the frustrated edges form a perfect matching in a ground state. [I.e., frustration index $l(-\Gamma) = |V|$ for these pentagonal lattice graphs, assuming no boundary as, e.g., when the lattice is toroidal.] [Annot. 18 Jun 2012.] (**Phys: SG: fr**)

Derek A. Waller

See also F.W. Clarke.

1976a Double covers of graphs. *Bull. Austral. Math. Soc.* 14 (1976), 233–248. MR 53 #10662. Zbl 318.05113. (**SG: Cov**)

Hai-Feng Wang

See M.L. Ye.

Jianfeng Wang and Francesco Belardo

See also F. Belardo.

2011a A note on the signless Laplacian eigenvalues of graphs. *Linear Algebra Appl.* 435 (2011), no. 10, 2585–2590. MR 2811140 (2012d:05242). Zbl 1225.05176. (**Par: Adj**)

JianFeng Wang, Francesco Belardo, QiongXiang Huang, and Bojana Borovićanin

2010a On the two largest Q-eigenvalues of graphs. *Discrete Math.* 310 (2010), no. 21, 2858–2866. MR 2677645 (2011j:05218). Zbl 1208.05079. (**Par: Adj**)

Jianfeng Wang, Francesco Belardo, QiongXiang Huang, and Enzo M. Li Marzi

20xxa On graphs whose Laplacian index does not exceed 4.5. *Linear Algebra Appl.*, in press.

The signless Laplacian $Q(\Gamma) = K(-\Gamma)$ (Wang, Huang, Belardo, and Li Marzi 2009a) is employed to derive results on the Laplacian $K(\Gamma)$. See Wang, Huang, Belardo, and Li Marzi (2009a). [Annot. 20 Dec 2011.] (**Par: Adj**)

Jianfeng Wang and Qiongxiang Huang

2011a Maximizing the signless Laplacian spectral radius of graphs with given diameter or cut vertices. *Linear Multilinear Algebra* 59 (2011), no. 7, 733–744. MR 1222.05182. Zbl 1222.05182.

Thm. 3.1: Fixing diameter d, the largest eigenvalue $q_1(K(-\Gamma))$ is maximized uniquely by a path of length d with a clique joined to its three middle vertices (K_n if $d \leq 2$). Thm. 3.2: Fixing $\nu := n - d$, $q_1 \nearrow 4\nu^2/(2\nu - 1)$. Thm. 4.1: Fixing the number k of cutpoints, q_1 is maximized uniquely by K_{n-k} with paths of nearly equal length attached to every vertex. Fix $\mu := n - k$. Thm. 4.2: If $k \leq n/2$, $q_1 < 2(\mu - 1) + k\mu/[2(\mu-1)^2-n]$. Thm. 4.3: If $n/2 < k \leq n-3$, $q_1 \nearrow 2\mu-1+1/(2\mu-3)$. [*Question.* Can these results generalize to signed graphs?] [Annot. 16 Jan 2012.] (**Par: Adj**)

Jianfeng Wang, Qiongxiang Huang, Xinhui An, and Francesco Belardo

2010a Some results on the signless Laplacians of graphs. *Appl. Math. Lett.* 23 (2010), no. 9, 1045–1049. MR 2659136 (2011e:05161). Zbl 1209.05148. (**Par: Adj**)

JianFeng Wang, QiongXiang Huang, Francesco Belardo, and Enzo M. Li Marzi

2009a On graphs whose signless Laplacian index does not exceed 4.5. *Linear Algebra Appl.* 431 (2009), no. 1-2, 162–178. MR 2522565 (2010g:05238). Zbl 1171.05035.

See Cvetković, Rowlinson, and Simić (2007a). See Wang, Belardo, Huang, and Li Marzi (20xxa). (**Par: Adj**)

2010a On the spectral characterizations of ∞-graphs. *Discrete Math.* 310 (2010), 1845–1855. MR 2629903 (2011m:05188). Zbl 1231.05174.

The spectra of $K(-\Gamma)$ and $K(+\Gamma)$ are compared, where Γ is a tight handcuff (or, figure-eight graph). [*Conjecture.* The results certainly hold for $K(-\Gamma)$ vs. all (Γ, σ).] [Annot. 20 Jan 2012.] (**Par: Adj**)

Jue Wang

See also B. Chen.

2007a Algebraic Structures of Signed Graphs. Doctoral dissertation, Hong Kong University of Science and Technology, 2007.

Cycle and cocycle spaces, interpretations and relationships. New formulation of matrix-tree generalization. (**SG: Incid, Str**)

Ligong Wang

See Y.Q. Chen and K. Li.

Long Wang

See X.B. Ma.

Longqin Wang, Zhengke Miao, and Chao Yan

2009a Local bases of primitive non-powerful signed digraphs. *Discrete Math.* 309 (2009), no. 4, 748–754. MR 2502184 (2010f:05092). Zbl 1168.05029. (**SD**)

Longqin Wang, Lihua You, and Hongping Ma

2011a Primitive non-powerful sign pattern matrices with base 2. *Linear Multilinear Algebra* 59 (2011), no. 6, 693–700. MR 2801362 (2012j:05272). Zbl 1223.05104. (**SD: QM**)

Lusheng Wang

See X.L. Li.

Shiying Wang, Jing Li, Wei Han, and Shangwei Lin

2010a The base sets of quasi-primitive zero-symmetric sign pattern matrices with zero trace. *Linear Algebra Appl.* 433 (2010), 595–605. MR 2653824 (2011g:15056). Zbl 1195.15031. (**QM: SD**)

Shujing Wang

See S.C. Li.

Suijie Wang
See X.G. Liu.

Tianfei Wang
2007a Several sharp upper bounds for the largest Laplacian eigenvalue of a graph. *Sci. in China Ser. A Math.* 50 (2007), no. 12, 1755–1764. MR 2390486 (2009a:05131) (*q.v.*). Zbl 1134.05064.
See MR for the formulas, which apply to $q_1(K(-\Gamma))$ [hence to signed simple graphs]. The proofs use a normalized Laplacian. [Annot. 16 Jan 2012.] (**Par: Adj**)

Tianfei Wang, Jin Yang, and Bin Li
2011a Improved upper bounds for the laplacian spectral radius of a graph. *Electron. J. Combin.* (2011), Article P35, 11 pp. MR 2776811 (2012e:05244). Zbl 1205.05154.
Some proofs involve $K(-\Gamma)$. [Annot. 16 Jan 2012.] (**Par: Adj**)

Wei Wang
See X.-J. Tian.

Xioafeng Wang
See E.L. Wei.

Xing Ke Wang
See S.W. Tan.

Yi Wang
See also Y.Z. Fan and J. Zhou.

Yi Wang and Yi-Zheng Fan
2012a The least eigenvalue of signless Laplacian of graphs under perturbation. *Linear Algebra Appl.* 436 (2012), no. 7, 2084–2092. (**Par: Adj**)

Yue Wang
See Y.Z. Fan.

Egon Wanke
See also F. Höfting.

1993a Paths and cycles in finite periodic graphs. In: Andrzej M. Borzyszkowski and Stefan Sokołowski, eds., *Mathematical Foundations of Computer Science 1993* (Proc., 18th Int. Sympos., MFCS '93, Gdańsk, 1993), pp. 751–760. MR 95c:05077. Zbl 925.05038.
Broadly resembles Höfting and Wanke (1994a) but omits those edges of $\tilde{\Phi}$ that are affected by the modulus α. (**GD(Cov): Alg**)

Ian M. Wanless
2005a Permanents of matrices of signed ones. *Linear Multilinear Algebra* 52 (2005), no. 6, 427–433. MR 2162064 (2006g:15013). Zbl 1085.15006.
$\mathrm{per}(B) := \sum_M \sigma(M)$, summed over all perfect matchings $M \subseteq \Sigma = (K_{n,n}, \sigma)$, where $A(\Sigma) = \begin{pmatrix} B & O \\ O & B \end{pmatrix}$. [Annot. 22 Aug 2012.] (**sg: Adj**)

G.H. Wannier
1950a Antiferromagnetism. The triangular Ising net. *Phys. Rev.* (2) 79 (1950), 357–364. MR 12, 576. Zbl 38, 419 (e: 038.41904). [Errata.] *Phys. Rev. B* 7 (1973), 5017.
Physical consequences of frustration with Ising spins, i.e. $\zeta : V \to \{+1, -1\}$, in the all-negative triangular lattice graph. [See also R.M.F.

Houtappel (1950a,b), G.F. Newell (1950b), I. Syôzi (1950a).] [Annot. 16 Jun 2012.] (**Phys: SG: Par: Fr**)

Dan Warner
See C.R. Johnson.

Stanley Wasserman and Katherine Faust
1994a *Social Network Analysis: Methods and Applications.* Structural Anal. Soc. Sci., 8. Cambridge Univ. Press, Cambridge, 1994. Zbl 980.24676.

§1.2: "Historical and theoretical foundations." A brief summary of various network methods in sociometry, signed graphs and digraphs among them. §4.4: "Signed graphs and signed directed graphs." Mathematical basics. §4.5: "Valued graphs and valued directed graphs." Mentions unweighted and positively weighted signed (di)graphs. Ch. 6: "Structural balance and transitivity." Application of balance of signed (di)graphs and of ensuing notions like clusterability, historically evolving into transitivity of unsigned digraphs. History and evaluation. §6.1: "Structural balance." Balance, indices of imbalance. §6.2: "Clusterability." All graphs, and complete graphs, as in Davis (1967a). §6.3: "Generalizations of clusterability." §6.3.2: "Ranked clusterability." As in Davis and Leinhardt (1972a). [Annot. 28 Apr 2009.]
(**PsS, SG, SD: Bal, Fr, Clu, Gen: Exp, Ref**)

William C. Waterhouse
1977a Some errors in applied mathematics. *Amer. Math. Monthly* 84 (January, 1977), no. 1, 25–27. Zbl 376.9001 (*q.v.*).

Criticizes Roberts and Brown (1975a, 1977a). See rebuttal in the Zbl review.

John J. Watkins
See R.J. Wilson.

William Watkins
See M. Lien.

Kevin D. Wayne
See È. Tardos.

Nikolai Weaver
See E. Flapan.

Jeffrey R. Weeks and Kenneth P. Bogart
1979a Consensus signed digraphs. *SIAM J. Appl. Math.* 36 (1979), 1–14. MR 81i:92026. Zbl 411.05042. (**SD**)

Erling Wei, Wenliang Tang, and Xioafeng Wang
2011a Flows in 3-edge-connected bidirected graphs. *Frontiers Math. China* 6 (2011), no. 2, 339–348. MR 2780896 (2012b:05137). Zbl 1226.05130.

A nowhere-zero 25-flow exists. [Annot. 6 June 2011.] (**SG: Flows**)

Fuyi Wei
See also M.H. Liu.

Fu-yi Wei and Muhuo Liu
2011a [as "Fi-yi Wei" and Muhuo Liu] Ordering of the signless Laplacian spectral radii of unicyclic graphs. *Australasian J. Combin.* 49 (2011), 255–264. MR 2790977 (2011m:05189). Zbl 1228.05208. (**Par: Adj**)

20xxa More results on the ordering of the signless Laplacian spectral radii of unicyclic graphs. Submitted.

See Cvetković, Rowlinson, and Simić (2007a). **(Par: Adj)**

20xxb On the signless Laplacian spectral radii of bicyclic graphs. Submitted.
See Cvetković, Rowlinson, and Simić (2007a). **(Par: Adj)**

Li Juan Wei
See Y.P. Hou.

Martin Weigt
See A.K. Hartmann.

Gerry M. Weiner
See J.S. Maybee.

Volkmar Welker

1997a Colored partitions and a generalization of the braid arrangement. *Electronic J. Combin.* 4 (1997), no. 1, Article R4, 12 pp. MR 98b:57026. Zbl 883.52010.

The arrangement is the affine part (that is, where $x_0 = 1$) of the projective representation of $G(\Phi)$, where Φ is the complex multiplicative gain graph $\Phi = \{1\}K_{n+1} \cup \{re_{0i} : 1 \le i \le n \text{ and } 2 \le r \le s\}$. Here the vertex set is $\{0, 1, \ldots, n\}$, s is any positive integer, and re_{0i} (in the paper, $e_{0i}(r)$) denotes an edge v_0v_i with gain r. The topics of interest are those related to the complex complement. The study is based on the combinatorics of the intersection semilattice [that is, the geometric semilattice $\mathrm{Lat}^{\mathrm{b}}\,\Phi$ of balanced flats], including the Poincaré polynomial of the arrangement [equivalent to the balanced chromatic polynomial of Φ]. **(gg: M, Geom, Invar)**

Albert L. Wells, Jnr.
See also P.J. Cameron and Y. Cheng.

1982a Regular generalized switching classes and related topics. D.Phil. thesis, Oxford Univ., 1982. **(SG: Sw, Adj, Enum, TG, Geom, Cov, Aut)**

1984a Even signings, signed switching classes, and $(-1, 1)$-matrices. *J. Combin. Theory Ser. B* 36 (1984), 194–212. MR 85i:05206. Zbl 527.05007. **(SG: Sw, Enum, Aut)**

D.J.A. Welsh [Dominic Welsh]
See also L. Lovász and W. Schwaärzler.

1976a *Matroid Theory.* L.M.S. Monographs, Vol. 8. Academic Press, London, 1976. MR 55 #148. Zbl 343.05002.

§11.4: "Partition matroids determined by finite groups", sketches the most basic parts of Dowling (1973b). **(gg: M: Exp)**

1992a On the number of knots and links. In: G. Halász, L. Lovász, D. Miklós, and T. Szönyi, eds., *Sets, Graphs and Numbers* (Proc., Budapest, 1991), pp. 713–718. Colloq. Math. Soc. János Bolyai, Vol. 60. János Bolyai Math. Soc., Budapest, and North-Holland, Amsterdam, 1992. MR 94f:57010. Zbl 799.57001.

The signed graph of a link diagram is employed to get an upper bound. **(SGc: Enum)**

1993a *Complexity: Knots, Colourings and Counting.* London Math. Soc. Lect. Note Ser., 186. Cambridge Univ. Press, Cambridge, Eng., 1993. MR 94m:57027. Zbl 799.68008.

Includes very brief treatments of some appearances of signed graphs.

§2.2, "Tait colourings", defines the signed graph of a link diagram, mentioned again in observation (2.3.1) on alternating links and Prop

(5.2.16) on "states models" (from Schwärzler and Welsh (1993a)). §5.6, "Thistlethwaite's nontriviality criterion": the criterion depends on the signed graph.

§2.5, "The braid index and the Seifert index of a link", defines the Seifert graph, a signed graph based on splitting the link diagram. (**SGc, Knot**)

§5.7, "Link invariants and statistical mechanics", defines a relatively simple spin model for signed graphs, with an arbitrary finite number of possible spin values. The partition function is related to link diagrams.

§4.2, "The Ising model", introduces the basic concepts in mathematical terms. §6.4, "The complexity of the Ising model", "Computing ground states of spin systems", pp. 105–107, discusses finding a ground state of the Ising model. This is described as the min-weight cut problem with weights the negatives [this is an error] of the Ising bond interaction values: that is, the weighted frustration index problem in the negative [erroneous] of the Ising graph. It is the max-cut problem when the Ising graph is balanced (ferromagnetic) [should be antibalanced (antiferromagnetic)]. For external magnetic field, follows Barahona (1982a). (**sg: Fr, Phys**)

§3.6, "Ice models", counts "ice configurations" (certain graph orientations) via poise gains modulo 3, although the counting function is not gain-graphic. (**gg, Invar, Phys**)

§4.4: "The Ashkin–Teller–Potts model". This treatment of the Potts model has a different Hamiltonian from that of Fischer and Hertz (1991a). [It does not seem that Welsh intends to admit edge signs. If they are allowed then the Hamiltonian (without edge weights) is $-\sum \sigma(e_{ij})(\delta(s_i, s_j) - 1)$. Up to halving and a constant term, this is Doreian and Mrvar's (1996a) clusterability measure $P(\pi)$, with $\alpha = .5$, of the vertex partition induced by the state.] [Also *cf.* Fischer and Hertz (1991a).] (**clu, Phys**)

1993b The complexity of knots. In: John Gimbel, John W. Kennedy and Louis V. Quintas, eds., *Quo Vadis, Graph Theory?*, pp. 159–171. Ann. Discrete Math., Vol. 55. North-Holland, Amsterdam, 1993. MR 94c:57021. Zbl 801.-68086.

Link diagrams $\leftrightarrow$ dual pairs of sign-colored plane graphs: based on Yajima and Kinoshita (1957a). Unsolved algorithmic problems about knots based on link diagrams; in particular, triviality of diagrams is equivalent to *Problem* 4.2: A polynomial-time algorithm to decide whether the graphical Reidemeister moves can convert a given signed plane graph to one with edges all of one sign. (**SGc: D, Knot: Alg, Exp**)

1993c Knots and braids: some algorithmic questions. In: Neil Robertson and Paul Seymour, eds., *Graph Structure Theory* (Proc., Seattle, 1991), pp. 109–123. Contemp. Math., Vol. 147. Amer. Math. Soc., Providence, R.I., 1993. MR 94g:57014. Zbl 792.05058.

§1 presents the sign-colored graph of a link diagram and §5, "Reidemeister graphs", describes Schwärtzler and Welsh (1993a). §3 defines the sign-colored Seifert graph. (**SGc. Sc(M): Invar, Alg, Knot: Exp**)

1997a Knots. In: Lowell W. Beineke and Robin J. Wilson, eds., *Graph Connections: Relationships between Graph Theory and other Areas of Mathematics*, Ch. 12, pp. 176–193. The Clarendon Press, Oxford, 1997. MR 99a:05001 (book). Zbl

878.57001.

Mostly describes the signed graph of a link diagram and its relation to knot theory, including knot properties deducible directly from the signed graph, the Kauffman bracket and two-variable polynomials, etc. Similar to relevant parts of (1993a). (**SGc: Knot: Invar: Exp**)

Emo Welzl

See H. Edelsbrunner and J. Hage.

D. de Werra

See C. Benzaken.

Arthur T. White

1984a *Graphs, Groups and Surfaces.* Completely revised and enlarged edn. North Holland Math. Stud., Vol. 8. North-Holland, Amsterdam, 1984. MR 86d:05047. Zbl 551.05037.

Ch. 10: "Voltage graphs". (**GG: Top, Cov**)

1994a An introduction to random topological graph theory. *Combinatorics, Probability and Computing* 3 (1994), 545–555. MR 95j:05083. Zbl 815.05027.

Take a graph Γ with cyclomatic number k and randomly sign it so that each edge is negative with probability p. The probability that (Γ, σ) is balanced $= 2^{-k}$ if $p = \frac{1}{2}$ [obvious] and $\leq [\max(p, 1-p)]^k$ in general [not obvious] (this has an interesting asymptotic consequence due to Gimbel, given in this paper). [Related: Frank and Harary (1979a).] (**SG: Rand, Bal**)

2001a *Graphs of Groups on Surfaces: Interactions and Models.* North-Holland Math. Stud., 188. North-Holland (Elsevier), Amsterdam, 2001. MR 1852593 (2002k:-05001). Zbl 1054.05001.

§10-2, "Voltage graphs": Voltage graphs and the covering graph. Thm. 10-8 is similar to Biggs (1974a), Thm. 19.5. Construction of surface embeddings. §11-3, "Nonorientable voltage graph imbeddings": Rotation schemes supplemented by edge signatures as in Ringel (1977a), Stahl (1978a), and Zaslavsky (1992a). (**GG, SG: Top, Cov**)

Neil L. White

See also A. Björner.

1986a A pruning theorem for linear count matroids. *Congressus Numerantium* 54 (1986), 259–264. MR 88c:05047. Zbl 621.05009. (**Bic: Gen**)

Neil White and Walter Whiteley

1983a A class of matroids defined on graphs and hypergraphs by counting properties. Unpublished manuscript, 1983.

See Whiteley (1996a) for an exposition and extension. (**Bic: Gen**)

Walter Whiteley

See also N. White.

1991a The combinatorics of bivariate splines. In: Peter Gritzman and Bernd Sturmfels, eds., *Applied Geometry and Discrete Mathematics: The Victor Klee Festschrift*, pp. 587–608. DIMACS Ser. Discrete Math. Theor. Comput. Sci., Vol. 4. American Math. Soc., Providence, R.I., 1991. MR 1116378 (92m:41038). Zbl 741.41014.

"Balance" used for circles with identity gain (in a gain graph with additive matrix gains), independently of Harary (1953a). §3, "Splines and matrices on graphs": The matrix gains are L_{hi}^{r+1} (p. 592) and the balance equation is $(*)$ (p. 593). [Annot. 12 Jun 2012.] (**gg: bal**)

1996a Some matroids from discrete applied geometry. In: Joseph E. Bonin, James G. Oxley, and Brigitte Servatius, eds., *Matroid Theory* (Proc., Seattle, 1995), pp. 171–311. Contemp. Math., Vol. 197. Amer. Math. Soc., Providence, R.I., 1996. MR 97h:05040. Zbl 860.05018.

Appendix: "Matroids from counts on graphs and hypergraphs", which expounds and extends Loréa (1979a), Schmidt (1979a), and especially White and Whiteley (1983a), describes matroids on the edge sets of graphs (and hypergraphs) that generalize the bicircular matroid. The definition: given $m \geq 0$ and $k \in \mathbb{Z}$, S is independent iff $\varnothing \subset S' \subseteq S$ implies $|S'| \leq m|V(S')| + k$. [Suggested name: "Linearly bounded matroids," since they are defined by a linear bound on the rank.] (**Bic: Gen**)(**Ref**)

Geoff Whittle

See also J. Geelen, J. Oxley, and C. Semple.

1989a Dowling group geometries and the critical problem. *J. Combin. Theory Ser. B* 47 (1989), 80–92. MR 90g:51008. Zbl 628.05018.

A Dowling-lattice version of Crapo and Rota's critical problem. Some minimal matroids whose critical exponent is k (i.e., tangential k-blocks) are given, one being Dowling's rank-n matroid of $\{+,-\}$, $G(\pm K_n^\circ)$. [Annot. 25 May 2009.] (**gg: M: Invar**)

1989b A generalisation of the matroid lift construction. *Trans. Amer. Math. Soc.* 316 (1989), 141–159. MR 90b:05038. Zbl 684.05014.

Examples include bicircular and frame matroids. (**GG: M, Bic**)

2005a Recent work in matroid representation theory. *Discrete Math.* 302 (2005), 285–296. MR 2179649 (2006m:05053). Zbl 1076.05022. annot P. 288: The "free spike Φ_r" is $L(2C_r, \varnothing)$. Pp. 290–291: Biased graphs and the bias [i.e., frame] matroid. *Conjecture* 5.2: With few exceptions, a highly connected matroid that is representable over more than one characteristic is a frame or dual frame matroid. P. 294: The "free swirl Ψ_k" is $G(2C_k, \varnothing)$. $U_{3,6} = L(2C_3, \varnothing) = G(2C_3, \varnothing)$ [the latter because there are no vertex-disjoint unbalanced circles]. [Annot. 25 May 2009.] (**gg: M: Exp**)

Avi Wigderson

See S. Hoory.

Chris Wiggins

See E. Ziv.

J.K. Williams

See also B.G.S. Doman.

1981a Ground state properties of frustrated Ising chains. *J. Phys. C* 14 (1981), 4095–4107.

§2, "The random-bond Ising chain in a uniform field: $(T = 0)$": A path with random edge signs, weighted J, magnetic field B [interpretable as an extra all-positive vertex, as in ??]. Continued in Doman and Williams (1982a), §2. [Annot. 28 Aug 2012.] (**Phys, SG, WG: fr**)

Richard C. Wilson and Ping Zhu

2008a A study of graph spectra for comparing graphs and trees. *Pattern Recognition* 41 (2008), no. 9, 2833–2841. Zbl 1154.68505.

The spectra of $K(+\Gamma)$ and $K(-\Gamma)$ appear to be similarly effective in distinguishing small graphs and better than $A(\Gamma)$. [Annot. 20 Dec

2011.] **(Par: Adj)**

Robin J. Wilson and John J. Watkins

1990a *Graphs: An Introductory Approach. A First Course in Discrete Mathematics.* Wiley, New York, 1990. MR 91b:05001. Zbl 712.05001.

§3.2: "Social Sciences" (pp. 51–53) applies signed graphs. §5.1: "Signed digraphs" (pp. 96–98) discusses positive and negative feedback (i.e., positive and negative cycles) in applications. Based on Open University (1981a). **(SG, PsS, SD: Exp)**

Steve Wilson

1989a Cantankerous maps and rotary embeddings of K_n. *J. Combin. Theory Ser. B* 47 (1989), no. 3, 262–273. MR 90j:05115. Zbl 687.05018.

Cantankerous map: graph in a surface, signed so every edge belongs to a negative digon, and whose map automorphisms act transitively on flags. Rotary map: map with automorphisms that are cyclic permutations around a face and around a vertex on the face. Thm.: A rotary map is either cantankerous or a kind of branched covering. [See Li and Širáň (2007a) for more on cantankerous maps.] **(sg: Top: Aut)**

Shmuel Winograd

See R.M. Karp.

Wayland H. Winstead

See J.R. Burns.

Anthony Wirth

See M. Charikar and T. Coleman.

H.S. Witsenhausen

See Y. Gordon.

C. Witzgall and C.T. Zahn, Jr.

1965a Modification of Edmonds' maximum matching algorithm. *J. Res. Nat. Bur. Standards (U.S.A.) Sect. B* 69B (1965), 91–98. MR 32 #5548. Zbl 141.21901. **(par: ori)**

Jakub Onufry Wojtaszczyk

See M. Cygan.

W.F. Wolff

See also P. Hoever and M.H. Waldor.

W.F. Wolff and J. Zittartz

1982a Correlations in inhomogeneous Ising models. I. General methods, the "fully-frustrated square lattice" and the "chessboard" model. *Z. Phys. B* 47 (1982), no. 4, 341–352. MR 675258 (85d:82104).

§ III, "The fully-frustrated square lattice model (FFS)": Square lattice graph signed so every square ("plaquette") is negative. § IV, "The chessboard model": Square lattice graph with alternate squares negative and positive. [Annot. 28 Aug 2012.] **(Phys, SG: Fr)**

1983a Spin glasses and frustration models: Analytical results. In: J.L. van Hemmen and I. Morgenstern, eds., *Heidelberg Colloquium on Spin Glasses* (Proc., Heidelberg, 1983), pp. 252–271. Lect. Notes in Physics Vol. 192. Springer-Verlag, Berlin, 1983.

Early theoretical physics study of frustrated graphs based on Toulouse (1977a). Signed square lattice with translational sign symmetry and limited variation of signs and edge weights. § II, "Layered Ising models". Dictionary: "plaquette" = square, "frustration index" = sign of a plaquette. [Annot. 24 May 2012.] (**Phys, SG: Fr**)

Paul Wollan
See B. Guenin.

A. Wongseelashote
1976a An algebra for determining all path-values in a network with application to K-shortest-paths problems. *Networks* 6 (1976), 307–334. MR 56 #14628. Zbl 375.90030. (**gg: Paths**)

R. Kevin Wood
See G.G. Brown.

Chai Wah Wu
2005a On Rayleigh–Ritz ratios of a generalized Laplacian matrix of directed graphs. *Linear Algebra Appl.* 402 (2005), 207–227. MR 2141085 (2005m:05108). Zbl 1063.05065.

The graphs are weighted mixed graphs, i.e., bidirected graphs without introverted edges, and the matrices are digraph matrices, i.e., (weighted) outdegree matrices. The "Laplacian" is $D - A$ where A is the adjacency matrix and D is the diagonal outdegree matrix. [Annot. 23 Mar 2009.] (**sg, sd: ori: incid, Adj**)

Leting Wu, Xiaowei Ying, Xintao Wu, Aidong Lu, and Zhi-Hua Zhou
2011a Spectral analysis of k-balanced signed graphs. In: Joshua Zhexue Huang, Longbing Cao and Jaideep Srivastava, eds., *Advances in Knowledge Discovery and Data Mining* (Proc. 15th Pacific-Asia Conf., PAKDD 2011, Shenzen, Part II), pp. 1–12. Lecture Notes in Computer Science, Vol. 6635. Springer, Berlin, 2011.

Spectral analysis of clusterable signed graphs. Dictionary: "k-balance" = k-clusterability. [Annot. 26 Apr 2012.] (**SG: Clu: Adj**)

20xxa Examining spectral space of complex networks with positive and negative links. *Int. J. Social Network Mining*, to appear. (**SG: Adj: Clu, Bal**)

Qiang Wu
See G.Z. Liu.

Shu-Hui Wu
See B.S. Tam.

Xiao Li Wu, Jing Jing Jiang, Ji Ming Guo, and Shang Wang Tan
2011a The minimal signless Laplacian spectral radius of graphs with diameter $n-4$. (In Chinese.) *Acta Math. Sinica (Chin. Ser.)* 54 (2011), no. 4, 601–608. MR 2868198 (2012i:05176). (**Par: Adj**)

Xintao Wu
See L.T. Wu.

Yarong Wu
See G.L. Yu.

Yuhan Wu
See L.H. You.

Zhaoyang Wu
2003a On the number of spikes over finite fields. *Discrete Math.* 265 (2003), 261–296. MR 2004b:05057. Zbl 1014.05015.

A spike is $L_0(\Omega)$ where $\|\Omega\| = 2C_n$. (**gg: M: Enum**)

Donald C. Wunsch
See Harary, Lim, *et al.*

Bai Xiao, Song Yi-Zhe, and Peter Hall
2011a Learning invariant structure for object identification by using graph methods. *Computer Vision Image Understanding* 115 (2011), 1023–1031.
Empirical tests of usefulness of the “feature vector”, consisting of the eigenvalues of $K(-\Gamma)$. [Annot. 24 Jan 2012.] (**Par: Adj: Appl**)

Guang-Hui Xu
See S.C. Gong.

Lei Xu
See Z.H. Chen.

Rui Xu and Cun-Quan Zhang
2005a On flows in bidirected graphs. *Discrete Math.* 299 (2005), 335–343. MR 2168714 (2006e:05081). Zbl 1073.05033.
Σ has a nowhere-zero 6-flow if it is coloop-free and edge 6-connected. [Annot. 5 Feb 2010.] (**SG: Flows**)

Shaoji Xu
See also F.S. Roberts.
1998a *Cycle Space: Cycle Bases, Signed Graphs and Marked Graphs.* Doctoral dissertation, Rutgers Center for Operations Res., Rutgers Univ., 1998.
(**SG, VS: Bal, Alg, PsS**)
1998b The line index and minimum cut of weighted graphs. *European J. Operational Res.* 109 (1998), no. 3, 672–685. Zbl 972.05026.

Takeshi Yajima and Shin'ichi Kinoshita
1957a On the graphs of knots. *Osaka Math. J.* 9 (1957), 155–163. MR 20 #4845. Zbl 80, 170b (e: 080.17002).
Examines the relationship between the two dual sign-colored graphs, Σ and Σ', of a link diagram (Bankwitz 1930a), translating the Reidemeister moves into graph operations and showing that they will convert Σ into Σ'. (**SGc: Knot**)

Takeo Yamada and Harunobu Kinoshita
2002a Finding all the negative cycles in a directed graph. *Discrete Appl. Math.* 118 (2002), 279–291. MR 2002m:05187. Zbl 999.05057.
In a real-weighted digraph, “negative” means the sum of weights is negative. (**WG**)

Takeo Yamamoto
See T. Nakamura.

Chao Yan
See L.Q. Wang.

Jing-Ho Yan, Ko-Wei Lih, David Kuo, and Gerard J. Chang
1997a Signed degree sequences of signed graphs. *J. Graph Theory* 26 (1997), 111–117. MR 98i:05160. Zbl 980.04848.
Net degree sequences of signed simple graphs. Thm. 2 improves the Havel–Hakimi-type theorem from Chartrand, Gavlas, Harary, and Schultz (1992a) by determining the length parameter. Thm. 7 characterizes the net degree sequences of signed trees. [There seems to be room to strengthen the characterization and generalize to weighted degree sequences: see notes on Chartrand *et al.* (1994a).] (**SGw: ori: Invar**)

Bo Yang, William K. Cheung, and Jiming Liu

2007a Community mining from signed social networks. *IEEE Trans. Knowledge Data Engineering* 19 (2007), no. 10, 1333–1348.

Given a (positively weighted) signed (di)graph, the authors provide an algorithm for an approximate clustering. Input: The graph and a length parameter l. Step 1: Construct transition probabilities $pij :=[\sigma ij w_{ij}]^{+}/d(v_i)$. Step 2: Apply the probabilities in a random walk of length $\leq l$ on positive edges; the matrix of l-step probabilities is $(p_{ij})^l$. Combine in a cluster the vertices that have high probabilities from a given starting point. "High" and l are based on the network structure.

Also, a cut algorithm for approximate clustering. A cluster is $X \subset V$ such that the total net degree $d^{\pm}(\Sigma{:}X) \geq d^{\pm}(X, X^c)$ and $d^{\pm}(X^c, X) \leq d^{\pm}(\Sigma{:}X^c)$. [Annot. 11 Feb 2009.] (**SG: WG: Clu: Alg**)

Dan Yang

See Y.Z. Fan.

Jin Yang

See T.F. Wang.

Weiling Yang and Fuji Zhang

2007a The Kauffman bracket polynomial of links and universal signed plane graph. In: Jin Akiyama et al., eds., *Discrete Geometry, Combinatorics and Graph Theory* (7th China-Japan Conf., CJCDGCGT 2005, Tianjin and Xi'an, China, 2005), pp. 228–244. Lect. Notes in Computer Sci., Vol. 4381. Springer, Berlin, 2007. MR 2364767 (2009b:57031). Zbl 1149.05308.

The "chain polynomials" of sign-colored plane graphs with cyclomatic number ≤ 5 are obtained systematically. [*Cf.* Jin and Zhang (2005a, 2007a).] [Annot. 5 July 2009.] (**SGc: Invar**)

Mihalis Yannakakis

See Esther M. Arkin and V.V. Vazirani.

Milhalis Yannakakis [Mihalis Yannakakis]

See Mihalis Yannakakis.

Yan Hong Yao

See L. Feng.

Zahra Yarahmadi

2010a The bipartite edge frustration of extension of splice and link graphs. *Appl. Math. Letters* 23 (2010), no. 9, 1077–1081. MR 2659141 (2011e:05219). Zbl 1210.05115.

Dictionary: "bipartite edge frustration" of Γ = frustration index $l(-\Gamma)$. (**sg: Par: Fr**)

Zahra Yarahmadi and Ali Reza Ashrafi

2011a The bipartite edge frustration of graphs under subdivided edges and their related sums. *Computers Math. Appl.* 62 (2011), no. 1, 319–325. MR 2821848 (no rev). Zbl 1228.05132. (**sg: Par: Fr**)

2011a Extremal properties of the bipartite vertex frustration of graphs. *Appl. Math. Letters* 24 (2011), 1774–1777. MR 2812210 (2012h:05163). Zbl 1234.05134.

Dictionary: "bipartite vertex frustration" of Γ = frustration number $l_0(-\Gamma)$. (**sg: Par: Fr**)

Z. Yarahmadi, T. Došlić, and A.R. Ashrafi

2010a The bipartite edge frustration of composite graphs. *Discrete Appl. Math.* 158

(2010), no. 14, 1551–1558. MR 2659170 (2011g:05301). Zbl 1215.05094. (**sg: Par: Fr**)

Miao-Lin Ye, Yi-Zheng Fan, and Hai-Feng Wang

See also J. Sheng.

2010a Maximizing signless Laplacian or adjacency spectral radius of graphs subject to fixed connectivity. *Linear Algebra Appl.* 433 (2010), no. 6, 1180–1186. MR 2661684 (2011i:05142). Zbl 1207.05125.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Yeong-Nan Yeh

See I. Gutman and S.Y. Lee.

Anders Yeo

See N. Alon and G. Gutin.

Shu Yong Yi and Li Hua You

2011a The local bases and bases of primitive anti-symmetric signed digraphs with no loops. *J. South China Normal Univ. Natur. Sci. Ed.* (2011), no. 1, 39–42. MR 2839257 (no rev). (**SD: Adj**)

Xiaowei Ying

See L.T. Wu.

Xuerong Yong

See X.G. Liu and Y.P. Zhang.

En Sup Yoon

See G. Lee.

Young-Jin Yoon

1997a A characterization of supersolvable signed graphs. *Commun. Korean Math. Soc.* 12 (1997), 1069–1073. MR 99j:05165. Zbl 945.05051.

Attempts to characterize supersolvability of $G(\Sigma)$ in terms of [bias-]simplicial vertices. [Fundamental conceptual and technical errors vitiate the entire paper; see Koban (2004a). For correct results see Zaslavsky (2001a) and Koban (2004a).] (**SG: M: Str**)

Takeo Yoshikawa, Takashi Iino, and Hiroshi Iyetomi

2011a Market structure as a network with positively and negatively weighted links. In: Junzo Watada, Gloria Phillips-Wren, Lakhmi C. Jain, and Robert J. Howlett, eds., *Intelligent Decision Technologies* (Proc. 3rd Int. Conf., IDT'2011), pp. 511–518. Smart Innovation, Systems and Technologies, Vol. 10. Springer-Verlag, Berlin, 2011.

Preliminary report of (2012a). [Annot. 26 Jun 2012.] (**SG, WG: Clu: Appl**)

2012a Observation of frustrated correlation structure in a well-developed financial market. *Progress Theor. Phys.* Suppl. No. 194 (2012), 55–63.

Application of correlation clustering to the Tokyo stock market. The "frustration" of a clustering $\pi = \{B_1, \ldots, B_k\} \in \Pi_V$ in a weighted signed graph (Σ, w) is $F(\pi) := -\sum_i \sum_{e \in E:B_i} w_e$ (*cf.* Traag and Bruggeman (2009a)). [Annot. 26 Jun 2012.] (**SG, WG: Clu: Appl**)

Lihua You

See also L.Q. Wang and S.Y. Yi.

Lihua You, Jiayu Shao, and Haiying Shan

2007a Bounds on the bases of irreducible generalized sign pattern matrices. *Linear Algebra Appl.* 427 (2007), 285–300. MR 2351360 (2008g:15009). Zbl 1179.15034. (**QM: SD**)

Lihua You and Yuhan Wu

2011a Primitive non-powerful symmetric loop-free signed digraphs with given base and minimum number of arcs. *Linear Algebra Appl.* 434 (2011), no. 5, 1215–1227. MR 2763581 (2012c:05201). Zbl 1204.05051. (**SD: QM**)

Zhifu You

See also B.L. Liu.

Zhifu You and Bolian Liu

2011a The signless Laplacian separator of graphs. *Electronic J. Linear Algebra* 22 (2011), 151–160. MR 2781043 (2012a:05209). Zbl 1226.05174. (**Par: Adj**)

A.P. Young

See K. Binder.

J.W.T. Youngs

1968a Remarks on the Heawood conjecture (nonorientable case). *Bull. Amer. Math. Soc.* 74 (1968), 347–353. MR 36 #3675. Zbl 161.43303.

Introducing "cascades": current graphs with bidirected edges. A "cascade" is a bidirected graph, not all positive, that is provided with both a rotation system (hence it is orientation embedded in a surface) and a current (which is a special kind of bidirected flow). Dictionary: "broken" means a negative edge. (**sg: Ori: Appl, Flows**)

1968b The nonorientable genus of K_n. *Bull. Amer. Math. Soc.* 74 (1968), 354–358. MR 36 #3676. Zbl 161.43304.

"Cascades": see Youngs (1968b). (**sg: Ori: Appl**)

Cheng-Ching Yu

See C.C. Chang.

Guihai Yu

See also L.H. Feng.

2008a On the maximal signless Laplacian spectral radius of graphs with given matching number. *Proc. Japan Acad. Ser. A Math. Sci.* 84 (2008), no. 9, 163–166. MR 2483600 (2009m:15012). Zbl 1175.05090.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Guanglong Yu, Yarong Wu, and Jinlong Shu

2011a Signless Laplacian spectral radii of graphs with given chromatic number. *Linear Algebra Appl.* 435 (2011), no. 8, 1813–2096. MR 2810629 (2012e:05247). Zbl 1221.05244. (**Par: Adj**)

2011a Sharp bounds on the signless Laplacian spectral radii of graphs. *Linear Algebra Appl.* 434 (2011), no. 3, 683–687. MR 2746075 (2012e:05246). Zbl 1225.05178. (**Par: Adj**)

Jianming Yu

See G. Jiang.

Xi-Ying Yuan, Yue Liu, and Miaomiao Han

2011a The Laplacian spectral radius of trees and maximum vertex degree. *Discrete Math.* 311 (2011), no. 8-9, 761–768. MR 2774232 (2011m:05191). Zbl 1216.05013.

§3: $Q := K(-\Gamma)$ is used to prove results about trees. [Annot. 21 Jan 2012.] **(Par: Adj)**

Raphael Yuster and Uri Zwick

1994a Finding even cycles even faster. In: Serge Abiteboul and Eli Shamir, eds., *Automata, Languages and Programming* (Proc. 21st Int. Colloq., ICALP 94, Jerusalem, 1994), pp. 532–543. Lect. Notes Computer Sci., Vol. 820. Springer-Verlag, Berlin, 1994. MR 96b:68002 (book). Zbl 844.00024 (book).

Abbreviated version of (1997a). **(par: Cycles: Alg)**

1997a Finding even cycles even faster. *SIAM J. Discrete Math.* 10 (1997), 209–222. MR 98d:05137. Zbl 867.05065.

For fixed *even* k, a very fast algorithm for finding a k-gon. Also, one for finding a shortest even circle. [*Question.* Are these the all-negative cases of similarly fast algorithms to find positive k-gons, or shortest positive circles, in signed graphs?] **(par: Cycles: Alg)**

Sergey Yuzvinsky

2004a Realization of finite abelian groups by nets in $\mathbb{P}^2$. *Compos. Math.* 140 (2004), no. 6, 1614–1624. MR 2005g:52057. Zbl 1066.52027.

Prop. 3.3: A k-net in $\mathbb{CP}^2$ whose classes are pencils is the canonical representation of the jointless Dowling geometry $Q^\dagger(\mathbb{Z}_m) = G(\mathbb{Z}_m K_3)$ of a finite cyclic group. If a k-net in $\mathbb{CP}^2$ represents $G(\mathfrak{A}K_3)$ for a finite abelian group $\mathfrak{A}$, then $\mathfrak{A}$ is a subgroup of a 2-torus (Thm. 4.4) or has small invariant factors (Thm. 5.4); in particular it cannot be $\mathbb{Z}_2^3$ (Thm. 4.2). The author conjectures more definitive characterizations.

(gg: Geom)

C.T. Zahn, Jr.

See also C. Witzgall.

1973a Alternating Euler paths for packings and covers. *Amer. Math. Monthly* 80 (1973), 395–403. MR 51 #10137. Zbl 274.05112. **(par: ori)**

Robert B. Zajonc

1968a Cognitive theories in social psychology. In: Gardner Lindzey and Elliot Aronson, eds., *The Handbook of Social Psychology*, Second Edition, Vol. 1, Ch. 5, pp. 320–411. Addison-Wesley, Reading, Mass., 1968.

"Structural balance," pp. 338–353. "The congruity principle," pp. 353–359. **(PsS: SD, SG, Bal: Exp, Ref)**

Giacomo Zambelli

See A. Del Pia.

Wenan Zang

1998a Coloring graphs with no odd-K_4. *Discrete Math.* 184 (1998), 205–212. MR 99e:05056. Zbl 957.05046.

An algorithm, based in part on Gerards (1994a), that, given an all-negative signed graph, finds a subdivided $-K_4$ subgraph or a 3-coloring of the underlying graph. [*Question.* Is there a generalization to all signed graphs?] [See also Thomassen (2001b).] **(sg: par: Col, Alg, Ref)**

Thomas Zaslavsky

See also M. Beck, P. Berthomé, E.D. Bolker, S. Chaiken, R. Flórez, D. Forge, K.A. Germina, C. Greene, P. Hanlon, N. Reff, K. Rybnikov, D.C. Slilaty, and P. Solé.

1977a Biased graphs. Unpublished manuscript, 1977.

Published, greatly expanded, as (1989a, 1991a, 1995b) and more; as well as (but restricted to signed graphs) (1982a, 1982b). (**GG: M**)

1979a Line graphs of digraphs. Abstract 768-05-3, *Notices Amer. Math. Soc.* 26 (August, 1979), no. 5, A-448.) (**SG: LG: Ori, Incid, Adj(LG). Sw**)

1980a Voltage-graphic geometry and the forest lattice. In: *Report on the XVth Denison-O.S.U. Math. Conf.* (Granville, Ohio, 1980), pp. 85–89. Dept. of Math., The Ohio State Univ., Columbus, Ohio, 1980. (**GG: M, Bic**)

1981a The geometry of root systems and signed graphs. *Amer. Math. Monthly* 88 (1981), 88–105. MR 82g:05012. Zbl 466.05058.

Signed graphs correspond to arrangements of hyperplanes in $\mathbb{R}^n$ of the forms $x_i = x_j$, $x_i = -x_j$, and $x_i = 0$. Consequently, one can compute the number of regions of the arrangement from graph theory, esp. for arrangements corresponding to "sign-symmetric" graphs, i.e., having both or none of each pair $x_i = \pm x_j$. Simplified account of parts of (1982a, 1982b, 1982c), emphasizing geometry. (**SG: M, Geom, Invar**)

1981b Characterizations of signed graphs. *J. Graph Theory* 5 (1981), 401–406. MR 83a:05122. Zbl 471.05035.

Characterizes the sets of circles that are the positive ones in some signing of a graph. (**SG: Bal**)

1981c Is there a theory of signed graph embedding? In: *Report on the XVIth Denison-O.S.U. Math. Conf.* (Granville, Ohio, 1981), pp. 79–82. Dept. of Math., The Ohio State Univ., Columbus, Ohio, 1981.

See (1997a). (**SG: Top, M**)

††1982a Signed graphs. *Discrete Appl. Math.* 4 (1982), 47–74. MR 84e:05095a. Zbl 476.05080. Erratum. *Ibid.* 5 (1983), 248. MR 84e:05095b. Zbl 503.05060.

$G(\Sigma)$ Basic results on: Switching (§3). Minors (§4). The bias matroid $G(\Sigma)$ in many cryptomorphisms (§5) (some erroneous: Thm. 5.1(f,g); partly corrected in the Erratum [and fully in (1991a)]), consistency of matroid with signed-graph minors; separators of $G(\Sigma)$. The signed covering graph $\tilde{\Sigma}$ (§6).

In §8A, the incidence and Kirchhoff matrices and matrix-tree theorem [different from that of Murasugi (1989a)] [generalized by Chaiken (1982a) to a weighted, all-minors version, both directed and undirected]. In §8B, vector representation of the matroid $G(\Sigma)$ by the incidence matrix [as multisubsets of root systems $B_n \cup C_n$].

Conjectures about the interrelation between representability in characteristic 2 and unique representability in characteristic 0 [since answered by Geoff Whittle (A characterisation of the matroids representable over GF(3) and the rationals. *J. Combin. Theory Ser. B* 65 (1995), 222–261. MR 96m:05046. Zbl 835.05015) as developed by Pagano (1998a, 20xxc)].

Examples (§7) include: Sign-symmetric graphs and signed expansions $\pm\Gamma$. The all-negative graph $-\Gamma$, whose matroid (Cor. 7D.3; partly corrected in the Erratum) is the even-circle matroid (see Doob 1973a) and whose incidence matrices include the unoriented incidence matrix of Γ. Signed complete graphs.

Generalizations to gain graphs (called "voltage graphs") mentioned in §9. (**SG, GG: M, Bal, Sw, Cov, Incid, Geom; EC, KG**)

††1982b Signed graph coloring. *Discrete Math.* 39 (1982), 215–228. MR 84h:05050a. Zbl 487.05027.

χ_Σ A "proper k-coloring" of Σ partitions V into a special "zero" part, possibly void, that induces a stable subgraph, and up to k other parts (labelled from a set of k colors), each of which induces an antibalanced subgraph. A "zero-free proper k-coloring" is similar but without the "zero" part. [The suggestion is that a signed analog of a stable vertex set is one that induces an antibalanced subgraph. *Problem.* Use this insight to develop generalizations of stable-set notions, such as cliques and perfection. *Example.* Let $\alpha(\Sigma)$, the "antibalanced vertex set number", be the largest size of an antibalance-inducing vertex set. Then $\alpha(\Gamma) = \alpha(+\Gamma \cup -K_n)$.] §2, "Counting the coloring ways": One gets two related chromatic polynomials. The chromatic polynomial, $\chi_\Sigma(2k+1)$, counts all proper k-colorings; it is essentially the characteristic polynomial of the bias matroid. It can often be most easily computed via the zero-free chromatic polynomial, $\chi^*_\Sigma(2k)$, which counts proper zero-free colorings: see (1982c). Contraction-deletion formulas; subset expansions, where the zero-free polynomial sums only over balanced edge sets. §3, "Pairs of colorings and orientations": Compatible and proper pairs. Contraction and improper pairs. Counting formulas. (Generalizing R.P. Stanley, Acyclic orientations of graphs, *Discrete Math.* 5 (1973), 171–178. MR 47 #6537. Zbl 258.05113.)

Continued in (1982c). (**SG, GG: M, Col, Invar, Cov, Ori, Geom**)

1982c Chromatic invariants of signed graphs. *Discrete Math.* 42 (1982), 287–312. MR 84h:05050b. Zbl 498.05030.

Continuation of (1982b). §1, "Balanced expansion formulas": The fundamental balanced expansion formulas, that express the chromatic polynomial in terms of the zero-free chromatic polynomial. §2, "Counting by color magnitudes and signs". More complicated expansion formulas. §§2–7: Many special cases, treated in great detail: antibalanced graphs, signed graphs that contain $+K_n$ or $-K_n$, signed K_n's (a.k.a. two-graphs), etc. §3, "Sign-symmetric graphs". §4, "Addition and deletion formulas". §5, "All-negative graphs; the even-circle chromatic polynomial". §6, "Partial matching numbers and ordinary chromatic coefficients". §7, "Signed complete graphs". §8, "Orientations": formulas for numbers of acyclic orientations in the examples (*cf.* 1991b). [Annot. Rev 26 Feb 2012.] (**SG, GG: M, Invar, Col, Cov, Ori, Geom; EC, KG**)

1982d Bicircular geometry and the lattice of forests of a graph. *Quart. J. Math. Oxford* (2) 33 (1982), 493–511. MR 84h:05050c. Zbl 519.05020.

The set of all forests in a graph forms a geometric lattice. The set of spanning forests forms a geometric semilattice. The characteristic polynomials count (spanning) forests. (**GG: M, Bic, Geom, Invar**)

1982e Voltage-graphic matroids. In: Adriano Barlotti, ed., *Matroid Theory and Its Applications* (Proc. Session of C.I.M.E., Varenna, Italy, 1980), pp. 417–423. Liguore Editore, Naples, 1982. MR 863015. Zbl 1225.05002. Repr.: C.I.M.E. Summer Schools, Vol. 83, Springer, Heidelberg, and Fondazione C.I.M.E., Florence, 2010. MR 2768789. Zbl 1225.05001.

The frame matroid of a gain graph. (**GG: M, EC, Bic, Invar: Exp**)

1984a How colorful the signed graph? *Discrete Math.* 52 (1984), 279–284. MR 86m:-

05045. Zbl 554.05026.

Studies zero-free chromatic number χ^*, and in particular that of a complete signed graph (which may have parallel edges). The signed graphs whose χ^* is largest or smallest. **(SG: Col)**

1984b Multipartite togs (analogs of two-graphs) and regular bitogs. In: Proc. Fifteenth Southeastern Conf. on Combinatorics, Graph Theory and Computing (Baton Rouge, 1984), Vol. III. *Congressus Numer.* 45 (1984), 281–293. MR 86d:05109. Zbl 625.05044.

A modestly successful attempt to generalize two-graphs along the cohomological lines of Cameron and Wells (1986a). [Annot. 6 July 2011.] **(SG: TG: Gen: Adj, Sw)**

1984c Line graphs of switching classes. In: *Report of the XVIIIth O.S.U. Denison Maths Conference* (Granville, Ohio, 1984), pp. 2–4. Dept. of Math., Ohio State Univ., Columbus, Ohio, 1984.

$\Lambda(\Sigma)$ The line graph of a switching class $[\Sigma]$ of signed graphs is a switching class of signed graphs; call it $[\Lambda'(\Sigma)]$. The reduced line graph Λ is formed from Λ' by deleting parallel pairs of oppositely signed edges. Then $A(\Lambda) = A(\Lambda') = 2I - \mathrm{H}^{\mathrm{T}}\,\mathrm{H}$, where H is an incidence matrix of Σ. Thm. 1: $A(\Lambda)$ has all eigenvalues ≤ 2. Examples: For an ordinary graph Γ, $\Lambda(-\Gamma) = -\Lambda(\Gamma)$. Example: taking $-\Gamma$ and attaching any number of pendant negative digons to each vertex yields (the negative of) Hoffman's generalized line graph. Additional results are claimed but there are no proofs. [See also (20xxa).] [This work is intimately related to that of Vijayakumar *et al.*, which was then unknown to the author, and to Cameron (1980a) and Cameron, Goethals, Seidel, and Shult (1976a).] **(SG: LG: Sw, Adj, Incid)**

1987a The biased graphs whose matroids are binary. *J. Combin. Theory Ser. B* 42 (1987), 337–347. MR 88h:05082. Zbl 667.05015.

For the frame (bias), lift, and extended lift matroids: forbidden-minor and structural characterizations. The latter for signed-graphiic frame matroids is superseded by a result of Pagano (1998a).

[Error in Cor. 4.3: In the last statement, omit "$G(\Omega) = L(\Omega)$." That is true when Ω has no loops, but may not be if Ω has a loop e (because Theorem 3(3) applies with unbalanced block e, but $(E \setminus e, e)$ is not a 2-separation).] **(GG: M: Str)**

1987b Balanced decompositions of a signed graph. *J. Combin. Theory Ser. B* 43 (1987), 1–13. MR 89c:05058. Zbl 624.05056.

Decompose $E(\Sigma)$ into the fewest balanced subsets (generalizing the biparticity of an unsigned graph), or balanced connected subsets. These minimum numbers are δ_0 and δ_1. Thm. 1: $\delta_0 = \lceil \chi^* \rceil + 1$, where χ^* is the zero-free chromatic number of $-\Sigma$. Thm. 2: $\delta_0 = \delta_1$ if Σ is complete. *Conjecture* 1. Σ partitions into δ_0 balanced, connected, and spanning edge sets (whence $\delta_0 = \delta_1$) if it has δ_0 edge-disjoint spanning trees. [Solved and generalized to basepointed matroids by D. Slilaty.] *Conjecture* 2 is a formula for δ_1 in terms of δ_0 of subgraphs. [It has been thoroughly disproved by Slilaty.] **(SG: Fr)**

1987c Vertices of localized imbalance in a biased graph. *Proc. Amer. Math. Soc.* 101 (1987), 199–204. MR 88f:05103. Zbl 622.05054.

Such a vertex (also, a "balancing vertex") is a vertex of an unbalanced graph whose removal leaves a balanced graph. Some elementary results. (**GG: Fr**)

1987d The Möbius function and the characteristic polynomial. In: Neil White, ed., *Combinatorial Geometries*, Ch. 7, pp. 114–138. Encycl. Math. Appl., Vol. 29. Cambridge Univ. Press, Cambridge, 1987. MR 88g:05048 (book). Zbl 632.05017.

Pp. 134–135 expound the geometrical version of Dowling lattices as in Dowling (1973a). (**gg: Geom, m, Invar: Exp**)

1988a Togs (generalizations of two-graphs). In: M.N. Gopalan and G.A. Patwardhan, eds., *Optimization, Design of Experiments and Graph Theory* (Proc. Sympos. in Honour of Prof. M.N. Vartak, Bombay, 1986), pp. 314–334. Indian Inst. of Technology, Bombay, 1988. MR 90h:05112. Zbl 689.05035.

An attempt to generalize two-graphs (here [alas?] called "unitogs") in a way similar to that of Cameron and Wells (1986a) although largely independently. The notable new example is "Johnson togs", based on the Johnson graph of k-subsets of a set. "Hamming togs" are based on a Hamming graph (that is, a Cartesian product of complete graphs) and generalize examples of Cameron and Wells. Other examples are as in (1984b). (**SG: TG: Gen**)

1988b The demigenus of a signed graph. In: *Report on the XXth Ohio State-Denison Mathematics Conference* (Granville, Ohio, 1988). Dept. of Math., Ohio State Univ., Columbus, Ohio, 1988. (**SG: Top, M**)

1989a Biased graphs. I. Bias, balance, and gains. *J. Combin. Theory Ser. B* 47 (1989), 32–52. MR 90k:05138. Zbl 714.05057.

Ω, Φ Fundamental concepts and lemmas of biased graphs. Bias from gains; switching of gains; characterization of balance [for which see also Harary, Lindstrom, and Zetterstrom (1982a)]. (**GG: Bal, Sw**)

1990a Biased graphs whose matroids are special binary matroids. *Graphs Combin.* 6 (1990), 77–93. MR 91f:05097. Zbl 786.05020.

A complete list of the biased graphs Ω such that $G(\Omega)$, $L_0(\Omega)$, or $L(\Omega)$ is one of the traditional special binary matroids, $G(K_5), G(K_{33}), F_7$, their duals, and $G(K_m)$ (for $m \geq 4$) and R_{10}. [Unfortunately omitted are nonbinary matroids like the non-Fano plane and its dual.]

[Error: The graphs $\langle +K_n^\circ \rangle$ were overlooked in the last statement of Lemma 1H—due to an oversight in (1987a) Cor. 4.3—and thus in Props. 2A and 5A. A corrected last statement of Lemma 1H: "If Ω has no two vertex-disjoint negative circles, then $G(\Omega) = M \iff L(\Omega) = M$." In Prop. 2A, add $\Omega = \langle +K_3^\circ \rangle$ to the list for $G(K_4)$. In Prop. 5A, add $\Omega = \langle +K_{m-1}^\circ \rangle$ to the list for $G(K_m)$. Thanks to Stefan van Zwam (25 July 2007).] (**GG: M**)

††1991a Biased graphs. II. The three matroids. *J. Combin. Theory Ser. B* 51 (1991), 46–72. MR 91m:05056. Zbl 763.05096.

G, L, L_0 Basic theory of the bias [or better, "frame"] matroid G (§2) and the lift and complete lift matroids, L and L_0 (§3), of a gain graph or biased graph. Infinite graphs. Matroids that are intermediate between the bias and lift matroids. Several questions and conjectures. (**GG: M**)

1991b Orientation of signed graphs. *European J. Combin.* 12 (1991), 361–375. MR 93a:05065. Zbl 761.05095.

Oriented signed graph = bidirected graph. The oriented matroid of an oriented signed graph. A "cycle" in a bidirected graph is a bias circuit (a balanced circle, or a handcuff with both circles negative) oriented to have no source or sink. Cycles in Σ are compared with those in its signed (i.e., derived) covering graph $\tilde{\Sigma}$. The correspondences among acyclic orientations of Σ and regions of the hyperplane arrangements of Σ and $\tilde{\Sigma}$, and dually the faces of the acyclotope of Σ. Thm. 4.1: the net degree vector $d(\tau)$ of an orientation τ belongs to the face of the acyclotope that is determined by the union of all cycles. Cor. 5.3 (easy): a finite bidirected graph has a source or sink. (**SG: Ori, M, Cov, Geom**)(**SGw: Invar**)

1992a Orientation embedding of signed graphs. *J. Graph Theory* 16 (1992), 399–422. MR 93i:05056. Zbl 778.05033.

Positive circles preserve orientation, negative ones reverse it. The minimal embedding surface of a one-point amalgamation of signed graphs. The formula is almost additive. (**SG: Top**)

1992b Strong Tutte functions of matroids and graphs. *Trans. Amer. Math. Soc.* 334 (1992), 317–347. MR 93a:05047. Zbl 781.05012.

Suppose that a function of matroids with labelled points is defined that is multiplicative on direct sums and satisfies a Tutte–Grothendieck recurrence with coefficients (the "parameters") that depend on the element being deleted and contracted, but not on the particular minor from which it is deleted and contracted: specifically, $F(M) = a_e F(M \backslash e) + b_e F(M/e)$ if e is not a loop or coloop in M. Thm. 2.1 completely characterizes such "strong Tutte functions" for each possible choice of parameters: there is one general type, defined by a rank generating polynomial $R_M(a, b; u, v)$ (the "parametrized rank generating polynomial") involving the parameters $a = (a_e)$, $b = (b_e)$ and the variables u, v, and there are a few special types that exist only for degenerate parameters. All have a Tutte-style basis expansion; indeed, a function has such an expansion iff it is a strong Tutte function (Thms. 7.1, 7.2). The Tutte expansion is a polynomial within each type. If the points are colored and the parameters of a point depend only on the color, one has a multicolored matroid generalization of Kauffman's (1989a) Tutte polynomial of a sign-colored graph. Kauffman's particular choices of parameters are shown to be related to matroid and color duality.

For a graph, "parametrized dichromatic polynomial" $Q_\Gamma = u^{\beta_0(\Gamma)} R_{G(\Gamma)}$, where G = graphic matroid and β_0 = number of connected components. A "portable strong Tutte function" of graphs is multiplicative on disjoint unions, satisfies the parametrized Tutte–Grothendieck recurrence, and has value independent of the vertex set. Thm. 10.1: Such a function either equals Q_Γ or is one of two degenerate exceptions. Prop. 11.1: Kauffman's (1989a) polynomial of a sign-colored graph equals $R_{G(|\Sigma|),\sigma}(a, b; d, d)$ for connected Σ, where $a_+ = b_- = B$ and $a_- = b_+ = A$. [*Cf.* Traldi 1989a).]

[This paper differs from other generalizations of Kauffman's polynomial, by Przytycka and Przytycki (1988a) and Traldi (1989a) (and partially anticipated by Fortuin and Kasteleyn (1972a)), who also develop the

parametrized dichromatic polynomial of a graph, principally in that it characterizes *all* strong Tutte functions; also in generalizing to matroids and in having little to say about knots. Schwärzler and Welsh (1993a) generalize to signed matroids (and characterize their strong Tutte functions) but not to arbitrary colors. Bollobás and Riordan (1999a) initiate the study of the underlying commutative algebra.]

(**Sc(M), SGc: Gen: Invar, D, Knot**)

1993a The projective-planar signed graphs. *Discrete Math.* 113 (1993), 223–247. MR 94d:05047. Zbl 779.05018.

$\mathbb{P}^2$ Characterized by six forbidden minors or eight forbidden topological subgraphs, all small. A close analog of Kuratowski's theorem; the proof even has much of the spirit of the Dirac–Schuster proof of the latter, and all but one of the forbidden graphs are simply derived from the Kuratowski graphs. [Paul Seymour showed me an alternative proof from Kuratowski's theorem that explains this; but it uses sophisticated results, as yet unpublished, of Robertson, Seymour, and Shih.] (**SG: Top**)

[Related: "projective outer-planarity" (POP): embeddable in the projective plane with all vertices on a common face. I have found most of the 40 or so forbidden topological subgraphs for POP of signed graphs (finding the rest will be routine); the proof is long and tedious and will probably not be published. *Problem.* Find a reasonable proof.]

(**SG: Top**)

1994a Frame matroids and biased graphs. *European J. Combin.* 15 (1994), 303–307. MR 95a:05021. Zbl 797.05027.

A simple matroidal characterization of the bias, or "frame", matroids of biased graphs. (**GG: M**)

1995a The signed chromatic number of the projective plane and Klein bottle and antipodal graph coloring. *J. Combin. Theory Ser. B* 63 (1995), 136–145. MR 95j:05099. Zbl 822.05028.

Introducing the signed Heawood problem: what is the largest signed, or zero-free signed, chromatic number of any signed graph that orientation embeds in the sphere with h crosscaps? Solved for $h = 1, 2$.

(**SG: Top, Col**)

††1995b Biased graphs. III. Chromatic and dichromatic invariants. *J. Combin. Theory Ser. B* 64 (1995), 17–88. MR 96g:05139. Zbl 857.05088.

Polynomials of gain and biased graphs: the fundamental object is a four-variable polynomial, the "polychromial" ("polychromatic polynomial"), that specializes to the chromatic, dichromatic, and Whitney-number polynomials. The polynomials come in two flavors: unrestricted and balanced, depending on the edge sets that appear in their defining sums. (They can be defined in the even greater abstraction of "two-ideal graphs", which clarifies the most basic properties.)

§4: "Gain-graph coloring". In $\Phi = (\Gamma, \varphi, \mathfrak{G})$, a "zero-free k-coloring" is a mapping $f : V \to [k] \times \mathfrak{G}$; it is "proper" if, when $e{:}vw$ is a link or loop and $f(v) = (i, g), f(w) = (i, h)$, then $h \neq g\varphi(e; v, w)$. A "k-coloring" is similar but the color set is enlarged by inclusion of a color 0; propriety requires the additional restriction that $f(v)$ and $f(w)$ are not both 0 (and $f(v) \neq 0$ if v supports a half edge). In particular, a "group-coloring" of Φ is a zero-free 1-coloring (ignoring the irrelevant numerical part of the color). A "partial group-coloring" is a group-coloring of an induced subgraph [which can only be proper if the uncolored vertices form a stable set]. The unrestricted and balanced chromatic polynomials count, respectively, unrestricted and zero-free proper k-colorings; the two Whitney-number polynomials count all colorings, proper and improper, by their improper edge sets.

§5: "The matroid connection". The various polynomials are, in essence, bias matroid invariants and closely related to corresponding lift matroid and extended lift matroid invariants.

Almost infinitely many identities, some of them (esp., the balanced expansion formulas in §6) essential. Innumerable examples worked in detail. [The first half, to the middle of §6, is fundamental. The rest is more or less ornamental. Most of the results are, intentionally, generalizations of properties of ordinary graphs.] (**GG: Invar, M, Col**)

1996a The order upper bound on parity embedding of a graph. *J. Combin. Theory Ser. B* 68 (1996), 149–160. MR 98f:05055. Zbl 856.05030.

The smallest surface that holds K_n with loops, if odd circles reverse orientation, even ones preserve it (this is parity embedding). I.e., the demigenus $d(-K_n^\circ)$. (**Par: Top**)

1997a Is there a matroid theory of signed graph embedding? *Ars Combinatoria* 45 (1997), 129–141. MR 97m:05084. Zbl 933.05067. (**SG: M, Top**)

1997b The largest parity demigenus of a simple graph. *J. Combin. Theory Ser. B* 70 (1997), 325–345. MR 99e:05043. Zbl 970.37744.

Like (1996a), but without loops. *Conjecture* 1. The minimal surface for parity embedding K_n is sufficient for orientation embedding of any signed K_n. *Conjectures* 3–4. The minimal surfaces of $\pm K_n^\circ$ and $\pm K_n$ are the smallest permitted by the lower bound obtained from Euler's polyhedral formula. (**Par: KG: Top**)

1997c Avoiding the identity. Problem 10606, *Amer. Math. Monthly* 104 (Aug.–Sept., 1997), no. 7, 664.

Find an upper bound on $f(m)$ = largest r such that any group of order $\geq r$ has m elements such that no product of any subset, possibly with inverted elements, equals the identity. Solution by Stephen M. Gagola (1999a).

[The solution implies that $(*)$ $f_1(m) \leq \lceil 2^{m-1}(m-1)!\sqrt{e}\rceil$, where $f_1(m)$ = smallest r such that every group of order $\geq r$ is a possible gain group for every contrabalanced gain graph of cyclomatic number m. *Problem* 1. Find a good upper bound on f_1. $(*)$ is probably weak. *Problem* 2. Find a good lower bound. *Problem* 3. Estimate f_1 asymptotically.] (**gg**)

1998a Signed analogs of bipartite graphs. *Discrete Math.* 179 (1998), 205–216. MR 2000b:05067. Zbl 980.06737.

Basically, they are the antibalanced and bipartite signed graphs; but the exact description depends on the characterization one chooses for biparticity: whether it is evenness of circles, closed walks, face boundaries in surface embeddings, etc. Characterization by chromatic number leads to a slightly more different list of analogs. (**SG: Str, Top**)

1998b A mathematical bibliography of signed and gain graphs and allied areas. *Elec-*

tronic J. Combin., Dynamic Surveys in Combinatorics (1998), No. DS8. MR 2000m:05001a. Zbl 898.05001.

Complete and annotated—or as nearly so as I can make it. In preparation in perpetuum. Hurry, hurry, write an article!
(**SG, Ori, GG, GN, SD, VS, TG, . . . , Chem, Phys, Biol, PsS, Appl**)

Published edns.: Edn. 6a (Edition 6, Revision a), 20 July 1998 (iv + 124 pp.). Edn. 7, 22–26 Sept. 1999 (vi + 151 pp.). Edn. 8, 8 Sept. 2012 (vi + 341 pp.).

1998c Glossary of signed and gain graphs and allied areas. *Electronic J. Combin.*, Dynamic Surveys in Combinatorics (1998), No. DS9. MR 2000m:05001b. Zbl 898.05002.

A complete (or so it is intended) terminological dictionary of signed, gain, and biased graphs and related topics; including necessary special terminology from ordinary graph theory and mathematical interpretations of the special terminology of applications.
(**SG, Ori, GG, GN, SD, VS, TG, . . . , Chem, Phys, PsS, Appl**)

Published edns.: 21 July 1998 (25 pp.). Second edn. 18 September 1998 (41 pp.).

2001a Supersolvable frame-matroid and graphic-lift lattices. *European J. Combin.* 22 (2001), 119–133. MR 2001k:05051. Zbl 966.05013.

Biased graphs whose bias and lift matroids are supersolvable are characterized by a form of simplicial vertex ordering—with a few exceptions. As preliminary results, modular copoints are characterized [but incompletely in the bias-matroid case, as observed by Koban (2004a)]. §4: "Examples": 4a: "Group expansions and biased expansions"; 4b: "Near-Dowling and Dowling lift lattices"; 4c: "An extension of Edelman and Reiner's theorem" to general gain groups (see Edelman and Reiner (1994a)); 4d: "Bicircular matroids". [Written in 1992 and long delayed. Correction in Koban (2004a). Independently, Yoon (1997a) incorrectly attempted the case of $G(\Sigma)$. Jiang and Yu rediscovered the case of a signed K_n.] (**GG, SG: M, Geom**)

2001b The largest demigenus of a bipartite signed graph. *Discrete Math.* 232 (2001), 189–193. MR 2001m:05100. Zbl 982.05041.

The smallest surface for orientation embedding of $\pm K_{r,s}$. (**SG: Top**)

2002a Perpendicular dissections of space. *Discrete Comput. Geom.* 27 (2002), 303–351. MR 2003i:52026. Zbl 1001.52011.

Given an additive real gain graph Φ on n vertices and n reference points Q_i in $\mathbb{E}^d$, use Φ to specify perpendicular hyperplanes to each of the lines Q_iQ_j by means of the "Pythagorean coordinate" along Q_iQ_j. For generic points, the number of regions is computable based on the fact that the generic hyperplane intersection lattice is $\mathrm{Lat}^{\mathrm{b}}\,\Phi$. Modifications of Pythagorean coordinates give intersection lattice $\mathrm{Lat}^{\mathrm{b}}(\|\Phi\|, \varnothing)$ or a slightly more complex variant, still for generic reference points.
(**GG: Geom, M, Invar**)

2003a Faces of a hyperplane arrangement enumerated by ideal dimension, with application to plane, plaids, and Shi. *Geom. Dedicata* 98 (2003), 63–80. MR 2004f:52025. Zbl 1041.52021.

§6, "Affinographic arrangements": hyperplane arrangements that rep-

resent the extended lift matroid $L_0(\Phi)$ where Φ is an additive real gain graph. Examples: the weakly-composed-partition, extended Shi, and extended Linial arrangements. The faces are counted in terms of dimension and dimension of the infinite part. Ehrenborg (20xxa) has more explicit formulas for Shi. **(GG: m, Geom, Invar)**

††2003b Biased graphs IV: Geometrical realizations. *J. Combin. Theory Ser. B* 89 (2003), no. 2, 231–297. MR 2005b:05057. Zbl 1031.05034.

§§2–4: Various ways in which to represent the bias and lift matroids of a gain or biased graph over a skew field F. Bias matroid: canonical vector and hyperplanar representations (generalizing those of a graph) based on a gain group $\subseteq F^{\times}$, Menelæan and Cevian representations (generalizations of theorems of Menelaus and Ceva), switching vs. change of ideal hyperplane, equational logic. Lift matroid: canonical vector and hyperplanar representations (the latter generalizing the Shi and Linial arrangements among others) based on a gain group $\subseteq F^{+}$, orthographic representation (an affine variation on canonical representation), Pythagorean representation (Zaslavsky 2002a). Both: effect of switching, nonunique gain-group embedding. §5: Effect of Whitney operations, separating vertex. §6: Matroids characterized by restricted general position. §7, "Thick graphs": A partial unique-representation theorem for biased graphs with sufficient edge multiplicity. §8: The 7 biased K_4's. **(GG: M, Geom, Invar)**

2006a Quasigroup associativity and biased expansion graphs. *Electron. Res. Announc. Amer. Math. Soc.* 12 (2006), 13–18. MR 2006i:20081. Zbl 1113.05044.

Summary of (2012a). **(GG: Str)**

2007a Biased graphs. VII. Contrabalance and antivoltages. *J. Combin. Theory Ser. B* 97 (2007), no. 6, 1019–1040. MR 2008h:05025. Zbl 1125.05048.

Contrabalanced graphs, whose gains are called antivoltages. Emphasis on the existence of antivoltages in $\mathbb{Z}_\mu$, $\mathbb{Z}$, and $\mathbb{Z}_p^k$ for application to canonical representation of the contrabalanced bias and lift matroids. The number of such antivoltages is a polynomial function of the group order or (for $\mathbb{Z}$) the bound on circle gains. **(GG: M, bic, Geom, Invar)**

2009a Totally frustrated states in the chromatic theory of gain graphs. *European J. Combinatorics* 30 (2009), 133–156. MR 2460223 (2009k:05100). Zbl 1125.05048.

Given a set Q of "spins", a state is $s : V \to Q$. The gain group $\mathfrak{G}$ acts on the spin set. In a permutation gain graph Φ with gain group $\mathfrak{G}$, edge $e{:}vw$ is "satisfied" if $s(w) = s(v)\varphi(e)$, otherwise "frustrated". A totally frustrated state (every edge is frustrated) generalizes a proper coloring. Enumerative theory, including deletion/contraction, a monodromy formula for the number of totally frustrated states, and a multivariate chromatic polynomial. An abstract partition function in the edge algebra. **(GG: Col: Gen: Invar, M)**

2010a Six signed Petersen graphs. In: *International Conference on Recent Trends in Graph Theory and Combinatorics (ICRTBC-2010)* (Cochin, 2010) [Summaries], pp. 75–76. Dept. of Mathematics, Cochin Univ. of Science and Technology, 2010.

Extended abstract of (2012b). There are six ways to sign the Petersen graph P up to switching isomorphism. Their frustration indices, automorphism and switching automorphism groups, chromatic numbers, and clusterability indices. [This abstract is not entirely reliable.] [Annot. 30 Aug, 26 Dec 2010.] (**SG: Fr, Aut, Col, Clu**)

2010b Matrices in the theory of signed simple graphs. In: B.D. Acharya, G.O.H. Katona, and J. Nesetril, eds., *Advances in Discrete Mathematics and Applications: Mysore, 2008* (Proc. Int. Conf. Discrete Math. 2008, ICDM-2008, Mysore, India, 2008), pp. 207–229. Ramanujan Math. Soc. Lect. Notes Ser., No. 13. Ramanujan Mathematical Soc., Mysore, India, 2010. MR 2766941 (2012d:05017). Zbl 1231.05120.

The adjacency, incidence, and Kirchhoff ("Laplacian") matrices, along with the adjacency matrices of line graphs. Balance, vertex degrees, eigenvalues, line graphs, strong regularity, etc. A survey, emphasizing work of Seidel, Vijayakumar, and Zaslavsky (some of which is unpublished).

Abelson and Rosenberg's (1958a) adjacency matrix is mentioned. (**SG: Adj, Incid, LG: Exp**)

2012a Associativity in multiary quasigroups: The way of biased expansions. *Aequationes Math.* 83 (2012), no. 1, 1–66. Zbl 1235.05059.

An n-ary quasigroup (Q, f) is essentially equivalent, up to isotopy, to a biased expansion $m \cdot C_{n+1}$. Factorizations of f appear as chords in a maximal extension of $m \cdot C_{n+1}$. Thm.: A biased expansion of a 3-connected graph (order ≥ 4) is a group expansion. Cor.: If $n \geq 3$ and the factorization graph of (Q, f) is 3-connected, (Q, f) is isotopic to an iterated group. Thm.: For a biased expansion of a 2-connected graph of order ≥ 4, if all minors of order 4 are group expansions, so is the whole expansion. Cor.: If in (Q, f) $(n \geq 3)$ all ternary residual quasigroups are iterated group isotopes, so is (Q, f). Cor.: (Q, f) is an iterated group isotope if $|Q| = 3$.

Other results: complete structural decomposition of nongroup biased expansions, or partially reducible multiary quasigroups, in terms of groups and either irreducible expansions or multiary quasigroups, respectively. (**GG: Str**)

2012b Six signed Petersen graphs, and their automorphisms. Recent Trends in Graph Theory and Combinatorics (Cochin, 2010). *Discrete Math.* 312 (2012), no. 9, 1558–1583. Zbl 1239.05086.

There are six ways to sign the Petersen graph P up to switching isomorphism. The frustration indices, automorphism and switching automorphism groups (in extensive detail), chromatic numbers, and clusterability indices of them and their negatives. All but automorphisms and clusterability are switching invariant, thus are solved for all signed P's. [Annot. 26 Dec 2010.] (**SG: Fr, Aut, Col, Clu**)

20xxa Line graphs of signed graphs and digraphs. In preparation.

$\Lambda(\Sigma)$ Line graphs of signed graphs are, fundamentally, (bidirected) line graphs of bidirected graphs. Then the line graph of a signed graph is a polar graph, i.e., a switching class of bidirected graphs; the line graph of a polar graph is a signed graph; and the line graph of a sign-biased graph, i.e., of a switching class of signed graphs, is a sign-biased graph. In particular, the line graph of an antibalanced switching class

is an antibalanced switching class. (Partly for this reason, ordinary graphs should usually be regarded as antibalanced, i.e., all negative, in line graph theory.) Since a digraph is an oriented all-positive signed graph, its line graph is a bidirected graph whose positive part is the Harary–Norman line digraph. Among the line graphs of signed graphs, some reduce by cancellation of parallel but oppositely signed edges to all-negative graphs; these are precisely Hoffman's generalized line graphs of ordinary graphs, a fact which explains their line-graph-like behavior. [Attempts at a completely descriptive line graph of a digraph were Muracchini and Ghirlanda (1965a) and Hemminger and Klerlein (1979a). The geometry of line graphs and signed graphs has been developed by Vijayakumar *et al.* (*q.v.*). See also Zaslavsky (1979a, 1984c), Zelinka (1976a) *et al.*] (**SG: LG: Ori, Incid, Adj(LG), Sw**)

20xxb Signed graphs and geometry. Int. Workshop on Set-Valuations, Signed Graphs, Geometry and Their Appl. (IWSSG-2011, Mananthavady, Kerala, 2011). *J. Combin. Inform. Syst. Sci.*, to appear. (**SG: Bal, Fr, Geom, Incid, Adj, M**)

20xxc What is a strongly regular signed graph? In preparation. (**SG: Adj**)

20xxd Geometric lattices of structured partitions: I. Gain-graphic matroids and group-valued partitions. Manuscript, 1985 *et seq.* (**GG: M, Invar, col**)

20xxe Geometric lattices of structured partitions: II. Lattices of group-valued partitions based on graphs and sets. Manuscript, 1985 *et seq.* (**GG: M, Invar, col**)

20xxf The canonical vertex signature and the cosets of the complete binary cycle space. Submitted.

$\partial(E) :=$ {odd-degree vertices} of Γ. Modify Γ by set summation with (a) an even-degree subgraph, (b) a circle in Γ or Γ^c, (c) a circle made up of a path in Γ and another in Γ^c. (a) gives all Γ' with the same ∂. (b) does if $n > 4$. (c) does if $\partial(E) \neq V$ (n even, $\varnothing$ (n odd).

The canonical vertex signature of Σ is $\partial\sigma(v) = (-1)^{\#(\text{negative edges at } v)}$ (Sampathkumar (1972a, 1984a)). For simple $|\Sigma|$, $(\partial\sigma)^{-1}(-1) = \partial(E^-)$. [Annot. 27 May 2010.] (**SG, VS: Str**)

20xxg Universal and topological gains for biased graphs. In preparation. (**GG: Top**)

20xxi Big flats in a box. In preparation.

The naive approach to characteristic polynomials via lattice point counting (in characteristic 0) and Möbius inversion (as in Blass and Sagan 1998a) can only work when one expects it to. (This is a theorem!) (**GG: Geom, M, Invar, col**)

20xxj Biased graphs. V. Group and biased expansions. In preparation. (**GG: M, Geom, Invar**)

20xxk Petersen signed graphs. In preparation.

There are 6 signatures of the Petersen graph P, up to switching isomorphism. For four of them ($+P$, $-P$, $P_I =$ the antipodal quotient of the icosahedral graph, P_1 with one negative edge), many facets are examined closely. (**SG: Sw, Bal, Fr, Clu, Cov, Top, Col, M: Exp**)

20xxm Biased graphs. VIII. A cornucopia of examples. In preparation.

Numerous types of examples of biased graphs, many having particular theory of their own, e.g., Hamiltonian bias. (**GG: M, Geom**)

20xxn The least possible eigenvalue of a super line multigraph. Submitted.
(**SG, LG: Gen**)

20xxo Frustration vs. clusterability in two-mode signed networks (signed bipartite graphs). Submitted.

Compares the frustration index l and the majority biclusterability indices $M(k_1, k_2)$, the latter based on Mrvar and Doreian (2009a), for bipartite signed graphs, especially signed $K_{2,n}$'s. [Annot. 8 Jan 2010.]
(**SG: Fr, Clu**)

Morris Zelditch, Jr.

See J. Berger.

Bohdan Zelinka

See also R.L. Hemminger.

1973a Polare und polarisierte Graphen. In: *XVIII. Int. Wiss. Kolloqu.* (Ilmenau, 1973), Vol. 2, Vortragsreihe "Theorie der Graphen und Netzwerke", pp. 27–28. Technische Hochschule, Ilmenau, 1973. Zbl 272.05102.

See (1976a). [This appears to be a very brief abstract of a lecture.]
(**sg: Ori, sw**)

1973b Quasigroups and factorisation of complete digraphs. *Mat. Časopis* 23 (1973), 333–341. MR 50 #12799. Zbl 271.20039.

Establishes correspondences between quasigroups, algebraic loops, and groups on one hand, and 1-factored complete digraphs on the other, and between automorphisms of the latter and autotopies of the former.
(**GG: Aut**)

1974a Polar graphs and railway traffic. *Aplikace Mat.* 19 (1974), 169–176. MR 49 #12066. Zbl 283.05116.

See (1976a) for definitions. Railway tracks and switches modeled by edges and vertices of a polar graph. Forming its derived graph (see (1976d)), thence a digraph obtained therefrom by splitting vertices into two copies and adjusting arcs, the time for a train to go from one segment to another is found by a shortest path calculation in the digraph. A similar method is used to solve the problem for several trains.
(**sg: Ori, sw: LG: Appl**)

1976a Isomorphisms of polar and polarized graphs. *Czechoslovak Math. J.* 26(101) (1976), 339–351. MR 58 #16429. Zbl 341.05121.

Basic definitions (Zítek 1972a): "Polarized graph" B = bidirected graph (with no negative loops and no parallel edges sharing the same bidirection). "Polar graph" $P \cong$ switching class of bidirected graphs (that is, we forget which direction at a vertex is in and which is out—here called "north" and "south" poles—but we remember that they are different).

Thms. 1–6. Elementary results about automorphisms, including finding the automorphism groups of the "complete polarized" and polar graphs. (The "complete polarized graph" has every possible bidirected link and positive loop, without repetition.) Thm. 7: With small exceptions, any (ordinary) graph can be made polar as, say, P so that Aut P is trivial.

Thms. 8–10. Analogs of Whitney's theorem that the line graph almost always determines the graph. The "pole graph" B* of B or [B]: Split each vertex into an "in" copy and an "out" copy and connect the edges appropriately. [Generalizes splitting a digraph into a bipartite graph. It appears to be a "twisted" signed double covering graph.] Thm. 8. The pole graph is determined, with two exceptions, by the edge relation $e \sim_1 f$ if both enter or both leave a common vertex. (A trivial consequence of Whitney's theorem.) Thm. 9. A polar graph [B] with enough edges going in and out at each vertex is determined by the edge relation $e \sim_2 f$ if one enters and the other exits a common vertex. (Examples show that too few edges going in and out leave [B] undetermined.) Thm. 10. Knowing $\sim_1$, $\sim_2$, and which edges are parallel with the same sign, and if no component of the simplified underlying graph of B is one of twelve forbidden graphs, then [B] is determined. [*Problem* 1. Improve Thm. 10 to a complete characterization of the bidirected graphs that are reconstructible from their line graphs (which are to be taken as bidirected; see Zaslavsky (2010b, 20xxa)). In connection with this, see results on characterizing line graphs of bidirected (or signed) graphs by Vijayakumar (1987a). *Problem* 2. It would be interesting to improve Thm. 9.] (**sg: Ori, sw: Aut, lg**)

1976b Analoga of Menger's theorem for polar and polarized graphs. *Czechoslovak Math. J.* 26(101) (1976), 352–360. MR 58 #16430. Zbl 341.05122.

See (1976a) for basic definitions. Here is the framework of the 8 theorems. Given a bidirected or polar graph, B or P, vertices a and b, and a type X of walk, let s_X $[s'_X]$ = the fewest vertices [edges] whose deletion eliminates all (a, b) walks of type X, and let d_X $[d'_X]$ = maximum number of suitably pairwise internally vertex-disjoint [or, suitably pairwise edge-disjoint] walks of type X from a to b. [My notation.] By "suitably" I mean that a common internal vertex or edge is allowed in P (but not in B) if it is used oppositely by the two walks using it. (See the paper for details.) Thms. 1–4_1 (there are two Theorems 4) concern all-positive and all-introverted walks in a bidirected ("polarized") graph, and are simply the vertex and edge Menger theorems applied to the positive and introverted subgraphs. Thms. 4_2–7 concern polar graphs and have the form $s_X \leq d_X \leq 2s_X$ $[s'_X \leq d'_X \leq 2s'_X]$, which is best possible. Thms. 4_2–5 concern type "heteropolar" (equivalently, directed walks in a bidirected graph). The proofs depend on Menger's theorems in the double covering graph of the polar graph. [Since this has 2 vertices for each 1 in the polar graph, the range of d_X $[d'_X]$ is explained.] Thms. 6–7 concern type "homopolar" (i.e., antidirected walks). The proofs employ the pole graph (see (1976a)). (**sg: Ori, sw: Paths**)

1976c Eulerian polar graphs. *Czechoslovak Math. J.* 26(101) (1976), 361–364. MR 58 #21869. Zbl 341.05123.

See (1976a) for basic definitions. An Eulerian trail in a bidirected graph is a directed trail containing every edge. [Equivalently, a heteropolar trail that contains all the edges in the corresponding polar graph.] It is closed if the endpoints coincide and the trail enters at one end and departs at the other. The fewest directed trails needed to cover a connected bidirected graph is $\frac{1}{2}$ the total of the absolute differences between in-degrees and out-degrees at all vertices, or 1 if in-degree = out-degree everywhere. (**sg: Ori, sw: Paths**)

1976d Self-derived polar graphs. *Czechoslovak Math. J.* 26(101) (1976), 365–370. MR 58 #16431. Zbl 341.05124.

See (1976a) for basic definitions. The "derived graph" of a bidirected graph [this is equivalent to the author's terminology] is essentially the positive part of the bidirected line graph. The theorem can be restated, somewhat simplified: A finite connected bidirected graph B is isomorphic to its derived graph iff B is balanced and contains exactly one circle. (**sg: Ori, sw: LG**)

1976e Groups and polar graphs. *Časopis Pěst. Mat.* 101 (1976), 2–6. MR 58 #21790. Zbl 319.05118.

See (1976a) for basic definitions. A polar graph $PG(\mathfrak{G}, A)$ of a group and a subset A is defined. [It is the Cayley digraph.] In bidirected language: a (bi)directed graph is "homogeneous" if it has automorphisms that are transitive on vertices, both preserving and reversing the orientations of edges, and that induce an arbitrary permutation of the incoming edges at any given vertex, and similarly for outgoing edges. It is shown that the Cayley digraph $PG(\mathfrak{G}, A)$, where $\mathfrak{G}$ is a group and A is a set of generators, is homogeneous if A is both arbitrarily permutable and invertible by $\operatorname{Aut}\mathfrak{G}$. [Bidirection—i.e., the polarity—seems to play no part here.] (**sg: Ori, sw: Aut**)

1982a On double covers of graphs. *Math. Slovaca* 32 (1982), 49–54. MR 83b:05072. Zbl 483.05057.

Is a simple graph Γ a double cover of some signing of a simple graph? An elementary answer in terms of involutions of Γ. Further: if there are two such involutions α_0, α_1 that commute, then Γ/α_i has involution induced by α_{1-i}, so is a double cover of $\Gamma/\langle\alpha_0, \alpha_1\rangle$, which is not necessarily simple. [No properties of particular interest for signed covering are treated.] (**sg: Cov**)

1983a Double covers and logics of graphs. *Czechoslovak Math. J.* 33(108) (1983), 354–360. MR 85k:05098a. Zbl 537.05070.

The double covers here are those of all-negative simple graphs (hence are bipartite). Some properties of these double covers are proved, then connections with a certain lattice (the "logic") of a graph. (**par: Cov: Aut**)

1983b Double covers and logics of graphs II. *Math. Slovaca* 33 (1983), 329–334. MR 85k:05098b. Zbl 524.05058.

The second half of (1983a). (**par: Cov: Aut**)

1988a A remark on signed posets and signed graphs. *Czechoslovak Math. J.* 38(113) (1988), 673–676. MR 90g:05157. Zbl 679.05067 (*q.v.*).

Harary and Sagan (1983a) asked: which signed graphs have the form $S(P)$ for some poset P? Zelinka gives a rather complicated answer for all-negative signed graphs, which has interesting corollaries. For instance, Cor. 3: If $S(P)$ is all negative, and P has $\hat{0}$ or $\hat{1}$, then $S(P)$ is a tree. (**SG, Sgnd**)

Hans-Olov Zetterström

See Harary, Lindstrom, and Zetterström (1982a).

Mingqing Zhai, Ruifang Liu, and Jinlong Shu

2011a An edge-grafting theorem on Laplacian spectra of graphs and its application. *Linear Multilinear Algebra* 59 (2011), no. 3, 303–315. MR 2774085 (2012c:05202).

Zbl 1226.05175. (**Par: Adj**)

Bingyan Zhang

See Y.P. Zhang.

Cun-Quan Zhang

See also R. Xu.

1993a Even-cycle decomposition. Problem 4.2, p. 681, in Nathaniel Dean, Open problems. In: Neil Robertson and Paul Seymour, eds., *Graph Structure Theory* (Proc., Seattle, 1991), pp. 677–688. Contemp. Math., Vol. 147. Amer. Math. Soc., Providence, R.I., 1993.

Conj. 12 is a sufficient condition for $-\Gamma$ to decompose into balanced circles. [*Problem.* Solve the obvious generalization to signed graphs. Is that easier because minors exist?] [Annot. 11 Jun 2012.] (**sg: par: Str**)

De Long Zhang and Shang Wang Tan

2003a On the strongly regular graphs and the Seidel switching. (In Chinese.) *Math. Appl. (Wuhan)* 16 (2003), no. 2, 145–148. MR 1979481 (no rev). Zbl 1030.05076 (no rev). (**TG**)

Fuji Zhang

See X.A. Jin and W. Yang.

Guang-Jun Zhang and Xiao-Dong Zhang

2011a The p-Laplacian spectral radius of weighted trees with a degree sequence and a weight set. *Electronic J. Linear Algebra* 22 (2011), 267–276. MR 2788647 (2012i:05051). Zbl 1227.05190.

Generalizes Biyikoğlu, Marc Hellmuth, and Josef Leydold (2009a) to positively edge-weighted graphs. [*Problem.* Generalize to signed graphs.] [Annot. 21 Jan 2012.] (**Par: Adj: Gen**)

Jianbin Zhang

See X.L. Li.

Jianghua Zhang

See G. Jiang.

Li Zhang

See S.C. Li.

Li Jun Zhang

See X.H. Hao.

Minjie Zhang

See S.C. Li.

Ping Zhang

1997a The characteristic polynomials of subarrangements of Coxeter arrangements. *Discrete Math.* 177 (1997), 245–248. MR 98i:52016. Zbl 980.06614.

Blass and Sagan's (1998a) geometrical form of signed-graph coloring is used to calculate (I) characteristic polynomials of several versions of k-equal subspace arrangements (these are the main results) and (II) [also in Zhang (2000a)] the chromatic polynomials (in geometrical guise) of ordinary graphs extending K_n by one vertex, signed graphs extending $\pm K_n^\circ$ by one vertex, and $\pm K_n$ with any number of negative loops adjoined. (**sg: Invar, Geom, col**)

2000a The characteristic polynomials of interpolations between Coxeter arrangements. *J. Combin. Math. Combin. Comput.* 34 (2000), 109–117. MR 2001b:05220. Zbl

968.32017.

Uses signed-graph coloring (in geometrical guise) to evaluate the chromatic polynomials (in geometrical guise) of all signed graphs interpolating between (1) $+K_n$ and $+K_{n+1}$ [i.e., ordinary graphs extending a complete graph by one vertex]; (2) $\pm K_{n-1}^\circ$ and $\pm K_n^\circ$; (3) $\pm K_n$ and $\pm K_n^\circ$ [known already by several methods, including this one]; (4a) $\pm K_{n-1}$ and $\pm K_{n-1} \cup +K_n$; (4b) $\pm K_{n-1} \cup +K_n$ and $\pm K_n$; and certain signed graphs interpolating (by adding negative edges one vertex at a time, or working down and removing them one vertex at a time) between (5) $+K_n$ and $\pm K_n^\circ$; (6) $+K_n$ and $\pm K_n$. In cases (1)–(3) the chromatic polynomial depends only on how many edges are added [which is obvious from the coloring procedure, if it were not disguised by geometry].

(**sg: Invar, col, Geom**)

Xiankun Zhang

See Z.H. Chen and H.J. Lai.

Xiao-Dong Zhang

See also Y.H. Chen, B.A. He, Y. Hong, and G.J. Zhang.

2004a Two sharp upper bounds for the Laplacian eigenvalues. *Linear Algebra Appl.* 376 (2004), 207–213. MR 2015534 (2004m:05173). Zbl 1037.05032.

§4, Remark 2: The main results extend to signed graphs ("mixed graphs"). [Annot. 23 Mar 2009.] (**sg: Adj**)

2004b Bipartite graphs with small third Laplacian eigenvalue. *Discrete Math.* 278 (2004), no. 1-3, 241–253. MR 2035402 (2004m:05172). Zbl 1033.05073.

See Cvetković, Rowlinson, and Simić (2007a). [*Problem.* Explain in terms of signed graphs, generalizing to the Kirchhoff matrix $K(-\Gamma)$ (the signless Laplacian.] (**Par: Adj**)

2009a The signless Laplacian spectral radius of graphs with given degree sequences. *Discrete Appl. Math.* 157 (2009), no. 13, 2928–2937. MR 2537494 (2011a:05210). Zbl 1213.05153.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Xiao-Dong Zhang and Jiong-Sheng Li

2002a The Laplacian spectrum of a mixed graph. *Linear Algebra Appl.* 353 (2002), 11–20. MR 1918746 (2003d:05138). Zbl 1003.05073.

Spectrum and spectral radius of the Kirchhoff matrix of a signed simple graph. [For this topic, orientation is irrelevant so the results apply to all signed simple graphs, although they are stated for oriented signed graphs in the guise of mixed graphs.] Dictionary: "mixed graph" = bidirected graph where all negative edges are extraverted; "Laplacian matrix" = Kirchhoff matrix; "quasibipartite" = balanced; "line graph" = $-\Lambda(\Sigma)$ (the negative of the line graph of Σ). [Annot. 23 Mar 2009.]

(**sg: Adj, LG**)

Xiao-Dong Zhang and Rong Luo

2003a The Laplacian eigenvalues of mixed graphs. *Linear Algebra Appl.* 362 (2003), 109–119. MR 1955457 (2003m:05128). Zbl 1017.05078.

Σ is a signed simple graph. $\lambda_1(K(\Sigma)) \leq$ max edge degree + 2 (same as Hou, Li, and Pan (2003a), Thm. 3.5(1)). Also, other bounds on λ_1. Thm. 2.5: The second smallest eigenvalue is $\leq \kappa(|\Sigma|)$ if there exists a minimum separating vertex set X such that $\Sigma \setminus X$ is balanced. Dictionary: See X.D. Zhang and Li (2002a). [Annot. 23 Mar 2009.] (**sg: Adj**)

2006a Non-bipartite graphs with third largest Laplacian eigenvalue less than three. *Acta Math. Sinica (Engl. Ser.)* 22 (2006), no. 3, 917–934. MR 2220184 (2007b:05141). Zbl 1102.05040.

[*Problem.* Generalize to connected, unbalanced signed graphs.] (**Par: Adj**)

Xiao-Peng Zhang

See X.-J. Tian.

Y. Zhang

See B. Dasgupta.

Yuanping Zhang

See also X.G. Liu.

Yuanping Zhang, Xiaogang Liu, Bingyan Zhang, and Xuerong Yong

2009a The lollipop graph is determined by its Q-spectrum. *Discrete Math.* 309 (2009), 3364–3369. MR 2526754 (2010g:05239). Zbl 1182.05084.

[Correction by Hamidzade and Kiani (2010a).] (**Par: Adj**)

Guopeng Zhao

See K. Li.

Qing Yu Zheng

See also H.S. Du.

Qing Yu Zheng and Qing Jun Ren

2001a Quasi-Laplacian characteristic polynomials of graphs [or, The quasi-Laplacian characteristic polynomial]. (In Chinese.) *Qufu Shifan Daxue Xuebao Ziran Kexue Ban* [*J. Qufu Normal Univ., Nat. Sci.*] 27 (2001), no. 4, 40–43. MR 1873005 (no rev). Zbl 1013.05050. (**Par: Adj**)

Zhehui Zhong

See G. Adejumo.

Bo Zhou

See also I. Gutman and G.X. Tian.

2008a A connection between ordinary and Laplacian spectra of bipartite graphs. *Linear Multilinear Algebra* 56 (2008), no. 3, 305–310. MR 2384657 (2008k:05139). Zbl 1163.05043.

Γ is bipartite. Subdivide every edge of Γ once. The eigenvalues are the square roots of the Laplacian eigenvalues of Γ, and 0. [*Problem* 1. Generalize to all graphs and the "signless Laplacian". *Problem* 2. Generalize to signed graphs via negative subdivision (every positive edge is subdivided into two negative edges).] [Annot. 28 Aug 2011.] (**Par: Adj**)

2010a Signless Laplacian spectral radius and Hamiltonicity. *Linear Algebra Appl.* 432 (2010), no. 2-3, 566–570. MR 2577702 (2011c:05219). Zbl 1188.05086.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

Bo Zhou and Aleksandar Ilić

2010a On the sum of powers of Laplacian eigenvalues of bipartite graphs. *Czechoslovak Math. J.* 60(135) (2010), no. 4, 1161–1169. MR 2738977 (2011m:05192). Zbl 1224.05333.

See Tian, Huang, and Zhou (2009a). Lower and upper bounds in terms of sums of squared degrees. Thus, bounds on incidence energy *et al.* [Annot. 24 Jan 2012.] (**par: Adj**)

Haijun Zhou

2005a Long-range frustration in a spin-glass model of the vertex-cover problem. *Phys. Rev. Letters* 94 (2005), no. 21, Article 217203.

"Long-range frustration" means correlation between spins (± 1) of vertices at considerable distance, within the same "state" (a configuration domain separated by energy barriers). [This should be generalized to signed graphs.] [Annot. 12 Sept 2010.] (**Phys**)

Hou Chun Zhou

See H.S. Du.

Jiang Zhou

See C.J. Bu.

Jun Zhou, Yi-Zheng Fan and Yi Wang

See also Y.Z. Fan.

2007a On the second largest eigenvalue of a mixed graph. *Discuss. Math. Graph Theory* 27 (2007), no. 2, 373–384. MR 2355728 (2008j:05223). Zbl 1134.05067.

Sufficient condition for $\lambda_2 \geq d_2$, the second largest eigenvalue and degree. Dictionary: See X.D. Zhang and Li (2002a). [Annot. 28 Oct 2011.] (**sg: Adj**)

Xiangqian Zhou

See H. Qin.

Zhi-Hua Zhou

See L.T. Wu.

Bao-Xuan Zhu

2010a On the signless Laplacian spectral radius of graphs with cut vertices. *Linear Algebra Appl.* 433 (2010), no. 5, 928–933. MR 2658643 (2011e:05163). Zbl 1215.05108.

See Cvetković, Rowlinson, and Simić (2007a). (**Par: Adj**)

2011a Bounds on the eigenvalues of graphs with cut vertices or edges. *Linear Algebra Appl.* 434 (2011), 2030–2041. MR 2780398 (2012g:05150). Zbl 1216.05082. (**Par: Adj**)

Li Zhu

See R. Huang.

Ping Zhu

See R.C. Wilson.

Xiao Xin Zhu, Zhi Ren Sun, and Chun Zheng Cao

2008a A bound for the quasi-Laplacian spectral radius of connected graphs [or, A bound on quasi-Laplacian spectral radius of connected graphs]. (In Chinese.) *J. Nanjing Normal Univ. Nat. Sci. Ed.* 31 (2008), no. 2, 27–30. MR 2444764 (2009e:05196). Zbl 1174.05448. (**Par: Adj**)

Xuding Zhu

See A. Raspaud.

Zhongxun Zhu

2011a The signless Laplacian spectral radius of bicyclic graphs with a given girth. *Electronic J. Linear Algebra* 22 (2011), 378–388. MR 2788652 (2012i:05178). Zbl 1227.05191. (**Par: Adj**)

G.M. Ziegler

See A. Björner and L. Lovász.

F. Zítek

1972a Polarisované grafy. [Polarized graphs.] Lecture at the Czechoslovak Conf. on Graph Theory, Štiřín, May, 1972.

For definitions see Zelinka (1976a). For work on these objects see many papers of Zelinka. (**sg: Ori, sw**)

J. Zittartz

See P. Hoever, M.H. Waldor, and W.F. Wolff.

Alejandro Zuñiga

See J. Aracena.

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The arc signed graph $\Lambda_Z(\vec\Gamma)$ of a digraph $\vec\Gamma$ (simple Γ) is the line graph $\Lambda(\Gamma)$ with $\sigma_Z(\vec{uv}\vec{wv}), \sigma_Z(\vec{vu}\vec{vw}) := +$ and $\sigma_Z(\vec{uv}\vec{vw}) := -$. [Thus, it is $-\Lambda(+\Gamma)$ where $+\Gamma$ has orientation $\vec\Gamma$; *cf.* Zaslavsky (20xxb, 20xxa).] Thm. 1: A Krausz-type characterization of Λ_Z. Cor. 1: Λ_Z determines $\vec\Gamma$ up to isolated vertices and reversing the orientation. Thm. 2: Characterization by induced subgraphs: a finite list plus antibalanced circles of length ≥ 4. Cor. 2: $\Lambda_Z(\vec\Gamma)$ graphs can be recognized and $\vec\Gamma$ reconstructed in polynomial time. Dictionary: "(+)-complete" means $+K_n$; "bicomplete" means complete and balanced. [Antibalanced circles are forbidden due to having all-positive base graphs.] (**SG: LG**)

A. Zverovitch

See N. Gülpinar.

Uri Zwick

See R. Yuster.

Krzysztof Zwierzyński

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