SUPPLEMENTARY UNSOLVED PROBLEMS
Math 221-11, 17
Fall, 2007

X10.1. Decide whether the function is one-to-one or not. If not, find an interval on which it is one-to-one, and explain why it is one-to-one on that interval.
(a) $f(x) = 4x^2 + 4$ (b) $f(x) = (x+1)^2 + 2x$ (c) $f(t) = t^6 + 8t^3$

X10.2. Assume y is a function of x, and find the derivative dy/dx. Also, find the equation of the tangent line to the curve where $y = 1$.
(a) $x = y^5 + 5y^2 + 10$ (b) $x = g(y) = \dfrac{y^3 + 4}{y^2 + 4y + 4}$

X11.1. Suppose we have a curve for which $y^3 - 6xy + x^3 = 0$. Find the equation of the tangent line at the point $(3, 3)$.

X11.2. A curve $y = f(x)$ satisfies $y^5 = 4xy + 24$. Find a formula for y' and the slope of the tangent line at the point $(1, 2)$.

X11.3. A curve satisfies $y^4 - 4xy^2 = 16$. Find a formula for y' and the slope of the tangent line at the point $(0, 2)$.

X17.1. Time for fun with trig identities!
(a) Express $\tan 2x$ in terms of functions of x.
(b) Simplify $(\tan 2x)(\sec^2 x - 2)$.
(c) Express $\cot 2x$ in terms of functions of x.
(d) Simplify $\cot x / \cot 2x$.

X17.2. Find the first and second derivatives; simplify if possible:
(a) $f(x) = \tan \frac{x}{2}$.
(b) $g(x) = \sec^2 x - 1/\cos 3x$.

X17.3. In Problem 29(e) in the book, show that the function equals $\cos 3x$.

X21.1. Find the antiderivatives of these functions:
(a) $3x^2 - 25x^2 + 4$
(b) $2\sqrt{x} - 81 + \dfrac{12}{x^3}$
(c) 1
(d) -1
(e) x^n where n is any rational number
(f) $\sqrt{x} \cdot (x^5 - 8x^4)$

X23.1. Consider the integral $\int_0^1 x^2\,dx$. For parts (a, b) divide the interval $[0, 1]$ into n equal subintervals.

(a) Evaluate the Riemann sum (23.1) for $n = 4$ using the "left endpoint rule", where $x_k^* = x_{k-1}$.

(b) Evaluate the Riemann sum (23.1) for $n = 4$ using the "right endpoint rule", where $x_k^* = x_k$.

(c) Compare your numerical answers to the exact value obtained in Solved Problem 4.

(d) If you feel ambitious, it will be interesting to do (a) or (b) with a larger value of n; say $n = 8$. Compare your answer with the one you got from $n = 4$ as well as with the exact value.