Math 221 2007/11/7 **QUIZ 10** SOLUTIONS

- Show all your work for each problem; show enough work to fully justify your answer.
- Simplify all answers as far as possible.

(1) [Points: 10] Let $f(x) = \sin 3x \cos^3 3x \tan 3x$.
(a) Simplify $f(x)$ as far as possible.

Solution.
$\sin 3x \cos^3 3x \tan 3x = \sin 3x \cos^3 3x(\sin 3x/\cos 3x) = \sin^2 3x \cos^2 3x$
$= (\sin 3x \cos 3x)^2 = (\frac{1}{2}\sin 6x)^2 = \frac{1}{4}\sin^2 6x = \frac{1}{4}\frac{1-\cos 12x}{2}$
$= \frac{1}{8} - \frac{1}{8}\cos 12x$.

(b) Differentiate $f(x)$.

Solution. There are many possible answers, all of which are equal according to the trig identities. From my simplification above, I would get

$$f'(x) = \frac{d}{dx}(\frac{1}{8} - \frac{1}{8}\cos 12x) = 0 - \frac{d}{dx}\frac{1}{8}\cos 12x = -\frac{1}{8}(-\sin 12x)\frac{d}{dx}(12x) = \frac{3}{2}\sin 12x.$$

(2) [Points: 10] Evaluate:
(a) $\sin^{-1}(\sin \frac{3}{4}\pi)$.

Solution. First you need to find $\sin \frac{3}{4}\pi$. You can use the identity $\sin(\pi - x) = \sin x$ with $x = \frac{3}{4}\pi$ or $x = \frac{1}{4}\pi$. It helps if you know the sine curve very well (study hint!). The answer is $\sqrt{2}/2$.

Then you find $\sin^{-1}\sqrt{2}/2$, which is an angle in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$. This angle is $\frac{1}{4}\pi$. That's the answer.

(b) $\sin(\sin^{-1}\frac{3}{4})$.

Solution. The answer is sin(angle whose sine is $\frac{3}{4}$). That's $\frac{3}{4}$.

(3) [Points: 5] Evaluate $\frac{d}{dx}\tan^{-1} x^2$.
Solution. We know

$$\frac{d}{du}\tan^{-1} u = \frac{1}{1+u^2}.$$

In our problem $u = x^2$, so we need the chain rule. (Always think of the chain rule!) Thus,

$$\frac{d}{dx}\tan^{-1} u = (\frac{d}{du}\tan^{-1} u)\frac{du}{dx} = \frac{1}{1+u^2}2x = \frac{1}{1+(x^2)^2}2x = \frac{2x}{1+x^4}.$$