Math 221 2007/12/5 **QUIZ 13** SOLUTION

- Show all your work for each problem; show enough work to fully justify your answer.
- Simplify all answers as far as possible.
- All numerical answers must be in terms of actual numbers and standard constants like π.

(1) [Points: 10] Find $\dfrac{d}{dx}\int_1^{x^2}\sqrt{1+t^4}\,dt$. Quick! (it's short).

Solution. We use the chain rule together with the Fundamental Theorem of Calculus. First, the chain rule: let $u = x^2$. Then

$$\frac{d}{dx}\int_1^{x^2}\sqrt{1+t^4}\,dt \;=\; \frac{du}{dx}\;\cdot\;\frac{d}{du}\int_1^{u}\sqrt{1+t^4}\,dt.$$

We had to do this so we could apply the F.T.C., which only applies if we differentiate with respect to a simple variable in the upper limit of integration. The next step is to apply it to the derivative of the integral:

$$\frac{du}{dx}\;\cdot\;\frac{d}{du}\int_1^{u}\sqrt{1+t^4}\,dt = \frac{du}{dx}\;\cdot\;\sqrt{1+u^4}.$$

Finally, substitute x^2 for u:

$$\frac{dx^2}{dx}\;\cdot\;\sqrt{1+(x^2)^4} = 2x\sqrt{1+x^8}.$$

This is the answer:

$$\frac{d}{dx}\int_1^{x^2}\sqrt{1+t^4}\,dt \;=\; 2x\sqrt{1+x^8}.$$