Math 221 2007/11/2 **QUIZ 9** SOLUTIONS

- Show all your work for each problem; show enough work to fully justify your answer.
- Simplify all answers as far as possible.
- All numerical answers must be in terms of actual numbers and standard constants like π and e.

(1) [Points: 10] Evaluate the limits:

(a) $\lim\limits_{h\to 0^+} \dfrac{\sin 5h}{h}$

Solution.
$$\lim_{h\to 0^+} \frac{\sin 5h}{h} = \lim_{h\to 0^+} 5\frac{\sin 5h}{5h} = 5 \lim_{5h\to 0^+} \frac{\sin 5h}{5h} = 5 \lim_{x\to 0} \frac{\sin x}{x} = 5(1) = 5.$$

(b) $\lim\limits_{h\to 0^+} \dfrac{\cos 5h}{h}$

Solution.
$$\lim_{h\to 0^+} \frac{\cos 5h}{h} = \lim_{h\to 0^+} 5\frac{\cos 5h}{5h} = 5 \lim_{x\to 0^+} \frac{\cos x}{x}.$$
Now you need to think. The numerator approaches 1, which is a positive number. The denominator is positive and approaches 0. That means the quotient approaches $+\infty$. The limit is therefore equal to $+\infty$.

(2) [Points: 5] Simplify to a single trig function: $\sin 4x \cos 4x$.
Solution.
Using the identity $\sin 2u = 2 \sin u \cos u$ we get $\frac{1}{2} \sin 8x$.

(3) [Points: 10] Find y' if $y = \sin^2 3x \cos^2 3x$.
Solution.
For the easy answer, first simplify by a trig identity as in part (a). You get $y = (\frac{1}{2} \sin 6x)^2 = \frac{1}{4} \sin^2 6x$. Then you can differentiate, getting $y' = \frac{1}{4} 2 \sin 6x(\cos 6x)\frac{d}{dx}6x = \frac{3}{2} 2 \sin 6x \cos 6x = \frac{3}{2} \sin 12x$.
Or, you could have continued to simplify y using another trig identity, getting $y = \frac{1}{4}\frac{1-\cos 12x}{2} = \frac{1}{8}(1 - \cos 12x)$, whose derivative is $-\frac{1}{8}(-\sin 12x)(12) = \frac{3}{2}\frac{3}{2} \sin 12x$.