A

Review of Algebra

Arithmetic Operations

The real numbers have the following properties:

$a + b = b + a \qquad ab = ba$		(Commutative Law)
$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$	(Associative Law)
$a(b + c) = ab + ac$		(Distributive Law)

In particular, putting $a = -1$ in the Distributive Law, we get

$$-(b + c) = (-1)(b + c) = (-1)b + (-1)c$$

and so

$$-(b + c) = -b - c$$

Example 1

(a) $(3xy)(-4x) = 3(-4)x^2y = -12x^2y$

(b) $2t(7x + 2tx - 11) = 14tx + 4t^2x - 22t$

(c) $4 - 3(x - 2) = 4 - 3x + 6 = 10 - 3x$ ■

If we use the Distributive Law three times, we get

$$(a + b)(c + d) = (a + b)c + (a + b)d = ac + bc + ad + bd$$

This says that we multiply two factors by multiplying each term in one factor by each term in the other factor and adding the products. Schematically, we have

$$(a + b)(c + d)$$

In the case where $c = a$ and $d = b$, we have

$$(a + b)^2 = a^2 + ba + ab + b^2$$

(1) or

$$\boxed{(a + b)^2 = a^2 + 2ab + b^2}$$

Similarly, we obtain

(2)

$$\boxed{(a - b)^2 = a^2 - 2ab + b^2}$$

Example 2

(a) $(2x + 1)(3x - 5) = 6x^2 + 3x - 10x - 5 = 6x^2 - 7x - 5$

(b) $(x + 6)^2 = x^2 + 12x + 36$

(c) $3(x - 1)(4x + 3) - 2(x + 6) = 3(4x^2 - x - 3) - 2x - 12$
$= 12x^2 - 3x - 9 - 2x - 12$
$= 12x^2 - 5x - 21$

Fractions

To add two fractions with the same denominator, we use the Distributive Law:

$$\frac{a}{b} + \frac{c}{b} = \frac{1}{b} \times a + \frac{1}{b} \times c = \frac{1}{b}(a + c) = \frac{a + c}{b}$$

Thus it is true that

$$\boxed{\frac{a + c}{b} = \frac{a}{b} + \frac{c}{b}}$$

But remember to avoid the following common error:

$$\frac{a}{b + c} \neq \frac{a}{b} + \frac{a}{c}$$

(For instance, take $a = b = c = 1$ to see the error.)

To add two fractions with different denominators, we use a common denominator:

$$\boxed{\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}}$$

We multiply such fractions as follows:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

In particular, it is true that

$$\frac{-a}{b} = -\frac{a}{b} = \frac{a}{-b}$$

To divide two fractions, we invert and multiply:

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Example 3

(a) $\frac{x+3}{x} = \frac{x}{x} + \frac{3}{x} = 1 + \frac{3}{x}$

(b) $\frac{3}{x-1} + \frac{x}{x+2} = \frac{3(x+2) + x(x-1)}{(x-1)(x+2)} = \frac{3x + 6 + x^2 - x}{x^2 + x - 2}$

$$= \frac{x^2 + 2x + 6}{x^2 + x - 2}$$

(c) $\frac{s^2t}{u} \cdot \frac{ut}{-2} = \frac{s^2t^2u}{-2u} = -\frac{s^2t^2}{2}$

(d) $\frac{\frac{x}{y} + 1}{1 - \frac{y}{x}} = \frac{\frac{x+y}{y}}{\frac{x-y}{x}} = \frac{x+y}{y} \times \frac{x}{x-y} = \frac{x(x+y)}{y(x-y)} = \frac{x^2 + xy}{xy - y^2}$

Factoring

We have used the Distributive Law to expand certain algebraic expressions. We sometimes need to reverse this process (again using the Distributive Law) by factoring an expression as a product of simpler ones. The easiest situation occurs when the expression has a common factor as follows:

$$\xrightarrow{\text{Expanding}}$$

$$3x(x-2) = 3x^2 - 6x$$

$$\xleftarrow{\text{Factoring}}$$

To factor a quadratic of the form $x^2 + bx + c$ we note that

$$(x + r)(x + s) = x^2 + (r + s)x + rs$$

so we need to choose numbers r and s so that $r + s = b$ and $rs = c$.

Example 4 Factor $x^2 + 5x - 24$.

Solution The two integers that add to give 5 and multiply to give -24 are -3 and 8. Therefore

$$x^2 + 5x - 24 = (x - 3)(x + 8)$$

Example 5 Factor $2x^2 - 7x - 4$.

Solution Even though the coefficient of x^2 is not 1, we can still look for factors of the form $2x + r$ and $x + s$, where $rs = -4$. Experimentation reveals that

$$2x^2 - 7x - 4 = (2x + 1)(x - 4)$$

Some special quadratics can be factored by using Equations 1 or 2 (from right to left) or by using the formula for a difference of squares:

(3) $$a^2 - b^2 = (a - b)(a + b)$$

The analogous formula for a difference of cubes is

(4) $$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

which you can verify by expanding the right side. For a sum of cubes we have

(5) $$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Example 6
(a) $x^2 - 6x + 9 = (x - 3)^2$ (Equation 2; $a = x$, $b = 3$)
(b) $4x^2 - 25 = (2x - 5)(2x + 5)$ (Equation 3; $a = 2x$, $b = 5$)
(c) $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$ (Equation 5; $a = x$, $b = 2$)

Example 7 Simplify $\dfrac{x^2 - 16}{x^2 - 2x - 8}$.

Solution Factoring numerator and denominator, we have

$$\frac{x^2 - 16}{x^2 - 2x - 8} = \frac{(x - 4)(x + 4)}{(x - 4)(x + 2)} = \frac{x + 4}{x + 2}$$

To factor polynomials of degree 3 or more, we sometimes use the following fact.

The Factor Theorem (6) If P is a polynomial and $P(b) = 0$, then $x - b$ is a factor of $P(x)$.

Example 8 Factor $x^3 - 3x^2 - 10x + 24$.

Solution Let $P(x) = x^3 - 3x^2 - 10x + 24$. If $P(b) = 0$, where b is an integer, then b is a factor of 24. Thus the possibilities for b are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$. We find that $P(1) = 12$, $P(-1) = 30$, $P(2) = 0$. By the Factor Theorem, $x - 2$ is a factor. Instead of substituting further, we use long division as follows:

$$\begin{array}{r} x^2 - x - 12 \\ x - 2 \overline{) x^3 - 3x^2 - 10x + 24} \\ \underline{x^3 - 2x^2} \\ -x^2 - 10x \\ \underline{-x^2 + 2x} \\ -12x + 24 \\ \underline{-12x + 24} \end{array}$$

Therefore

$$\begin{aligned} x^3 - 3x^2 - 10x + 24 &= (x - 2)(x^2 - x - 12) \\ &= (x - 2)(x + 3)(x - 4) \end{aligned}$$

■

Completing the Square

Completing the square is a useful technique for graphing parabolas (as in Example 2 in Section 6 in Review and Preview) or integrating rational functions (as in Example 6 in Section 7.4). Completing the square means rewriting a quadratic $ax^2 + bx + c$ in the form $a(x + p)^2 + q$ and can be accomplished by:

1. Factoring the number a from the terms involving x.
2. Adding and subtracting the square of half the coefficient of x.

In general, we have

$$\begin{aligned} ax^2 + bx + c &= a\left[x^2 + \frac{b}{a}x\right] + c \\ &= a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c \\ &= a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right) \end{aligned}$$

Example 9 Rewrite $x^2 + x + 1$ by completing the square.

Solution The square of half the coefficient of x is $\frac{1}{4}$. Thus

$$x^2 + x + 1 = x^2 + x + \tfrac{1}{4} - \tfrac{1}{4} + 1 = \left(x + \tfrac{1}{2}\right)^2 + \tfrac{3}{4}$$

■

Example 10

$$\begin{aligned} 2x^2 - 12x + 11 &= 2[x^2 - 6x] + 11 = 2[x^2 - 6x + 9 - 9] + 11 \\ &= 2[(x - 3)^2 - 9] + 11 = 2(x - 3)^2 - 7 \end{aligned}$$

■

Quadratic Formula

By completing the square as above we can obtain the following formula for the roots of a quadratic equation.

The Quadratic Formula (7)

The roots of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 11 Solve the equation $5x^2 + 3x - 3 = 0$.

Solution With $a = 5$, $b = 3$, $c = -3$, the quadratic formula gives the solutions

$$x = \frac{-3 \pm \sqrt{3^2 - 4(5)(-3)}}{2(5)} = \frac{-3 \pm \sqrt{69}}{10}$$

The quantity $b^2 - 4ac$ that appears in the quadratic formula is called the **discriminant**. There are three possibilities:

1. If $b^2 - 4ac > 0$, the equation has two real roots.
2. If $b^2 - 4ac = 0$, the roots are equal.
3. If $b^2 - 4ac < 0$, the equation has no real root. (The roots are complex.)

These three cases correspond to the fact that the number of times the parabola $y = ax^2 + bx + c$ crosses the x-axis is 2, 1, or 0 (see Figure 1). In case (3) the quadratic $ax^2 + bx + c$ cannot be factored and is called **irreducible.**

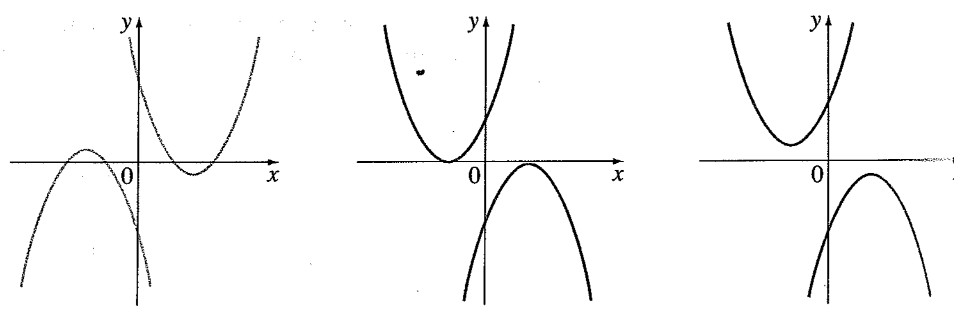

Figure 1
Possible graphs of $y = ax^2 + bx + c$

(a) $b^2 - 4ac > 0$ (b) $b^2 - 4ac = 0$ (c) $b^2 - 4ac < 0$

Example 12 The quadratic $x^2 + x + 2$ is irreducible because its discriminant is negative:

$$b^2 - 4ac = 1^2 - 4(1)(2) = -7 < 0$$

Therefore it is impossible to factor $x^2 + x + 2$.

The Binomial Theorem

Recall the binomial expansion from Equation 1:

$$(a + b)^2 = a^2 + 2ab + b^2$$

If we multiply both sides by $(a + b)$ and simplify, we get the binomial expansion

(8) $$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

Repeating this procedure, we get

$$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

In general, we have the following formula (which is a special case of the Binomial Theorem of Section 10.10):

The Binomial Theorem (9)

If k is a positive integer, then

$$\begin{aligned}(a + b)^k = a^k + ka^{k-1}b &+ \frac{k(k-1)}{1 \cdot 2}a^{k-2}b^2 \\ &+ \frac{k(k-1)(k-2)}{1 \cdot 2 \cdot 3}a^{k-3}b^3 \\ &+ \cdots + \frac{k(k-1)\cdots(k-n+1)}{1 \cdot 2 \cdot 3 \cdot \cdots \cdot n}a^{k-n}b^n \\ &+ \cdots + kab^{k-1} + b^k\end{aligned}$$

Example 13 Expand $(x - 2)^5$.

Solution Using the Binomial Theorem with $a = x$, $b = -2$, $k = 5$, we have

$$\begin{aligned}(x - 2)^5 &= x^5 + 5x^4(-2) + \frac{5 \cdot 4}{1 \cdot 2}x^3(-2)^2 + \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3}x^2(-2)^3 \\ &\quad + 5x(-2)^4 + (-2)^5 \\ &= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32\end{aligned}$$

■

Radicals

The most commonly occurring radicals are square roots. The symbol $\sqrt{}$ means "the positive square root of." Thus

$$x = \sqrt{a} \quad \text{means} \quad x^2 = a \quad \text{and} \quad x \geq 0$$

Since $a = x^2 \geq 0$, the symbol $\sqrt{a}$ makes sense only when $a \geq 0$. Here are two rules for working with square roots:

(10)

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \qquad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

However there is no similar rule for the square root of a sum. In fact, you should remember to avoid the following common error:

$$\sqrt{a + b} \not= \sqrt{a} + \sqrt{b}$$

(For instance, take $a = 9$ and $b = 16$ to see the error.)

Example 14

(a) $\dfrac{\sqrt{18}}{\sqrt{2}} = \sqrt{\dfrac{18}{2}} = \sqrt{9} = 3$

(b) $\sqrt{x^2y} = \sqrt{x^2}\sqrt{y} = |x|\sqrt{y}$

Notice that $\sqrt{x^2} = |x|$ because $\sqrt{}$ indicates the positive square root. (See Section 1 of Review and Preview.)

In general, if n is a positive integer,

$x = \sqrt[n]{a}$ means $x^n = a$
If n is even, then $a \geq 0$ and $x \geq 0$.

Thus $\sqrt[3]{-8} = -2$ because $(-2)^3 = -8$, but $\sqrt[4]{-8}$ and $\sqrt[6]{-8}$ are not defined. The following rules are valid:

$$\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b} \qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Example 15 $\sqrt[3]{x^4} = \sqrt[3]{x^3x} = \sqrt[3]{x^3}\,\sqrt[3]{x} = x\sqrt[3]{x}$

To **rationalize** a numerator or denominator that contains an expression such as $\sqrt{a} - \sqrt{b}$, we multiply both the numerator and the denominator by the conjugate radical $\sqrt{a} + \sqrt{b}$. Then we can take advantage of the formula for a difference of squares:

$$\left(\sqrt{a} - \sqrt{b}\right)\left(\sqrt{a} + \sqrt{b}\right) = \left(\sqrt{a}\right)^2 - \left(\sqrt{b}\right)^2 = a - b$$

Example 16 Rationalize the numerator in the expression $\dfrac{\sqrt{x+4} - 2}{x}$.

Solution We multiply the numerator and the denominator by the conjugate radical $\sqrt{x+4} + 2$:

$$\frac{\sqrt{x+4} - 2}{x} = \left(\frac{\sqrt{x+4} - 2}{x}\right)\left(\frac{\sqrt{x+4} + 2}{\sqrt{x+4} + 2}\right) = \frac{(x+4) - 4}{x\left(\sqrt{x+4} + 2\right)}$$

$$= \frac{x}{x\left(\sqrt{x+4} + 2\right)} = \frac{1}{\sqrt{x+4} + 2}$$

Exponents

Let a be any positive number and let n be a positive integer. Then, by definition,

1. $a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}}$
2. $a^0 = 1$
3. $a^{-n} = \dfrac{1}{a^n}$
4. $a^{1/n} = \sqrt[n]{a}$

   $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$ $\quad m$ is any integer

Laws of Exponents (11)

> Let a and b be positive numbers and let r and s be any rational numbers (that is, ratios of integers). Then
>
> 1. $a^r \times a^s = a^{r+s}$ 2. $\dfrac{a^r}{a^s} = a^{r-s}$ 3. $(a^r)^s = a^{rs}$
>
> 4. $(ab)^r = a^r b^r$ 5. $\left(\dfrac{a}{b}\right)^r = \dfrac{a^r}{b^r} \quad b \neq 0$

In words, these five laws can be stated as follows:

1. To multiply two powers of the same number, we add the exponents.
2. To divide two powers of the same number, we subtract the exponents.
3. To raise a power to a new power, we multiply the exponents.
4. To raise a product to a power, we raise each factor to the power.
5. To raise a quotient to a power, we raise both numerator and denominator to the power.

Example 17

(a) $2^8 \times 8^2 = 2^8 \times (2^3)^2 = 2^8 \times 2^6 = 2^{14}$

(b) $$\frac{x^{-2} - y^{-2}}{x^{-1} + y^{-1}} = \frac{\dfrac{1}{x^2} - \dfrac{1}{y^2}}{\dfrac{1}{x} + \dfrac{1}{y}} = \frac{\dfrac{y^2 - x^2}{x^2y^2}}{\dfrac{y + x}{xy}} = \frac{y^2 - x^2}{x^2y^2} \cdot \frac{xy}{y + x}$$

$$= \frac{(y - x)(y + x)}{xy(y + x)} = \frac{y - x}{xy}$$

(c) $4^{3/2} = \sqrt{4^3} = \sqrt{64} = 8$ Alternative solution: $4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$

(d) $\dfrac{1}{\sqrt[3]{x^4}} = \dfrac{1}{x^{4/3}} = x^{-4/3}$

(e) $\left(\dfrac{x}{y}\right)^3 \left(\dfrac{y^2x}{z}\right)^4 = \dfrac{x^3}{y^3} \cdot \dfrac{y^8x^4}{z^4} = x^7y^5z^{-4}$ ■

Mathematical Induction

The principle of mathematical induction is useful when proving a statement S_n about the positive integer n. For instance, if S_n is the statement

$$(ab)^n = a^nb^n$$

then S_1 says that $ab = ab$

S_2 says that $(ab)^2 = a^2b^2$

and so on.

Principle of Mathematical Induction

> Let S_n be a statement about the positive integer n. Suppose that
> 1. S_1 is true
> 2. S_{k+1} is true whenever S_k is true.
>
> Then S_n is true for all positive integers n.

This is reasonable because, since S_1 is true, it follows from condition 2 (with $k = 1$) that S_2 is true. Then, using condition 2 with $k = 2$, we see that S_3 is true. Again using condition 2, this time with $k = 3$, we have that S_4 is true. This procedure can be followed indefinitely.

In using the principle of mathematical induction, there are three steps.

Step 1: Prove that S_n is true when $n = 1$.

Step 2: Assume that S_n is true when $n = k$ and deduce that S_n is true when $n = k + 1$.

Step 3: Conclude that S_n is true for all n by the principle of mathematical induction.

Example 18 If a and b are real numbers, prove that $(ab)^n = a^n b^n$ for every positive integer n.

Solution Let S_n be the given statement.

1. S_1 is true because $(ab)^1 = ab = a^1 b^1$.
2. Assume that S_k is true, that is, $(ab)^k = a^k b^k$. Then

$$\begin{aligned}(ab)^{k+1} &= (ab)^k(ab) = a^k b^k ab \\ &= (a^k a)(b^k b) = a^{k+1} b^{k+1}\end{aligned}$$

This says that S_{k+1} is true.

3. Therefore, by the principle of mathematical induction, S_n is true for all n; that is, $(ab)^n = a^n b^n$ for every positive integer n. ■

Example 19 Prove that, for every positive integer n,

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

Solution Let S_n be the given statement.

1. S_1 is true because

$$1 = \frac{1(1+1)}{2}$$

2. Assume that S_k is true, that is,

$$1 + 2 + \cdots + k = \frac{k(k+1)}{2}$$

Then

$$\begin{aligned}1 + 2 + \cdots + (k+1) &= (1 + 2 + \cdots + k) + (k+1) \\ &= \frac{k(k+1)}{2} + k + 1 \\ &= \frac{k(k+1) + 2(k+1)}{2} \\ &= \frac{(k+1)(k+2)}{2}\end{aligned}$$

Thus $$1 + 2 + \cdots + (k+1) = \frac{(k+1)[(k+1)+1]}{2}$$

which shows that S_{k+1} is true.

3. Therefore S_n is true for all n by mathematical induction, that is,

$$1 + 2 + \cdots + (n+1) = \frac{n(n+1)}{2}$$

for every positive integer n. ■

APPENDIX A Exercises

In Exercises 1–16 expand and simplify.

1. $(-6ab)(0.5ac)$
2. $-(2x^2y)(-xy^4)$
3. $2x(x-5)$
4. $(4-3x)x$
5. $-2(4-3a)$
6. $8-(4+x)$
7. $4(x^2-x+2)-5(x^2-2x+1)$
8. $5(3t-4)-(t^2+2)-2t(t-3)$
9. $(4x-1)(3x+7)$
10. $x(x-1)(x+2)$
11. $(2x-1)^2$
12. $(2+3x)^2$
13. $y^4(6-y)(5+y)$
14. $(t-5)^2-2(t+3)(8t-1)$
15. $(1+2x)(x^2-3x+1)$
16. $(1+x-x^2)^2$

In Exercises 17–28 perform the indicated operations and simplify.

17. $\dfrac{2+8x}{2}$
18. $\dfrac{9b-6}{3b}$
19. $\dfrac{1}{x+5}+\dfrac{2}{x-3}$
20. $\dfrac{1}{x+1}+\dfrac{1}{x-1}$
21. $u+1+\dfrac{u}{u+1}$
22. $\dfrac{2}{a^2}-\dfrac{3}{ab}+\dfrac{4}{b^2}$
23. $\dfrac{x/y}{z}$
24. $\dfrac{x}{y/z}$
25. $\left(\dfrac{-2r}{s}\right)\left(\dfrac{s^2}{-6t}\right)$
26. $\dfrac{a}{bc}\div\dfrac{b}{ac}$
27. $\dfrac{1+\dfrac{1}{c-1}}{1-\dfrac{1}{c-1}}$
28. $1+\dfrac{1}{1+\dfrac{1}{1+x}}$

In Exercises 29–48 factor the given expression.

29. $2x+12x^3$
30. $5ab-8abc$
31. x^2+7x+6
32. x^2-x-6
33. x^2-2x-8
34. $2x^2+7x-4$
35. $9x^2-36$
36. $8x^2+10x+3$
37. $6x^2-5x-6$
38. $x^2+10x+25$
39. t^3+1
40. $4t^2-9s^2$
41. $4t^2-12t+9$
42. x^3-27
43. x^3+2x^2+x
44. x^3-4x^2+5x-2
45. x^3+3x^2-x-3
46. $x^3-2x^2-23x+60$
47. $x^3+5x^2-2x-24$
48. $x^3-3x^2-4x+12$

In Exercises 49–54 simplify the given expression.

49. $\dfrac{x^2+x-2}{x^2-3x+2}$
50. $\dfrac{2x^2-3x-2}{x^2-4}$
51. $\dfrac{x^2-1}{x^2-9x+8}$
52. $\dfrac{x^3+5x^2+6x}{x^2-x-12}$
53. $\dfrac{1}{x+3}+\dfrac{1}{x^2-9}$
54. $\dfrac{x}{x^2+x-2}-\dfrac{2}{x^2-5x+4}$

In Exercises 55–60 complete the square.

55. x^2+2x+5
56. $x^2-16x+80$
57. $x^2-5x+10$
58. x^2+3x+1
59. $4x^2+4x-2$
60. $3x^2-24x+50$

In Exercises 61–68 solve the given equation.

61. $x^2+9x-10=0$
62. $x^2-2x-8=0$
63. $x^2+9x-1=0$
64. $x^2-2x-7=0$
65. $3x^2+5x+1=0$
66. $2x^2+7x+2=0$
67. $x^3-2x+1=0$
68. $x^3+3x^2+x-1=0$

Which of the quadratics in Exercises 69–72 are irreducible?

69. $2x^2 + 3x + 4$

70. $2x^2 + 9x + 4$

71. $3x^2 + x - 6$

72. $x^2 + 3x + 6$

In Exercises 73–76 use the Binomial Theorem to expand the given expression.

73. $(a + b)^6$

74. $(a + b)^7$

75. $(x^2 - 1)^4$

76. $(3 + x^2)^5$

In Exercises 77–82 simplify the given radicals.

77. $\sqrt{32}\sqrt{2}$

78. $\dfrac{\sqrt[3]{-2}}{\sqrt[3]{54}}$

79. $\dfrac{\sqrt[4]{32x^4}}{\sqrt[4]{2}}$

80. $\sqrt{xy}\sqrt{x^3y}$

81. $\sqrt{16a^4b^3}$

82. $\dfrac{\sqrt[5]{96a^6}}{\sqrt[5]{3a}}$

In Exercises 83–100 use the Laws of Exponents to rewrite and simplify the given expression.

83. $3^{10} \times 9^8$

84. $2^{16} \times 4^{10} \times 16^6$

85. $\dfrac{x^9(2x)^4}{x^3}$

86. $\dfrac{a^n \times a^{2n+1}}{a^{n-2}}$

87. $\dfrac{a^{-3}b^4}{a^{-5}b^5}$

88. $\dfrac{x^{-1} + y^{-1}}{(x + y)^{-1}}$

89. $3^{-1/2}$

90. $96^{1/5}$

91. $125^{2/3}$

92. $64^{-4/3}$

93. $(2x^2y^4)^{3/2}$

94. $(x^{-5}y^3z^{10})^{-3/5}$

95. $\sqrt[5]{y^6}$

96. $\left(\sqrt[4]{a}\right)^3$

97. $\dfrac{1}{\left(\sqrt{t}\right)^5}$

98. $\dfrac{\sqrt[8]{x^5}}{\sqrt[4]{x^3}}$

99. $\sqrt[4]{\dfrac{t^{1/2}\sqrt{st}}{s^{2/3}}}$

100. $\sqrt[4]{r^{2n+1}} \times \sqrt[4]{r^{-1}}$

In Exercises 101–108 rationalize the given expression.

101. $\dfrac{\sqrt{x} - 3}{x - 9}$

102. $\dfrac{\left(1/\sqrt{x}\right) - 1}{x - 1}$

103. $\dfrac{x\sqrt{x} - 8}{x - 4}$

104. $\dfrac{\sqrt{2 + h} + \sqrt{2 - h}}{h}$

105. $\dfrac{2}{3 - \sqrt{5}}$

106. $\dfrac{1}{\sqrt{x} - \sqrt{y}}$

107. $\sqrt{x^2 + 3x + 4} - x$

108. $\sqrt{x^2 + x} - \sqrt{x^2 - x}$

In Exercises 109–116 state whether or not the given equation is true for all values of the variable.

109. $\sqrt{x^2} = x$

110. $\sqrt{x^2 + 4} = |x| + 2$

111. $\dfrac{16 + a}{16} = 1 + \dfrac{a}{16}$

112. $\dfrac{1}{x^{-1} + y^{-1}} = x + y$

113. $\dfrac{x}{x + y} = \dfrac{1}{1 + y}$

114. $\dfrac{2}{4 + x} = \dfrac{1}{2} + \dfrac{2}{x}$

115. $(x^3)^4 = x^7$

116. $6 - 4(x + a) = 6 - 4x - 4a$

In Exercises 117–126 n represents a positive integer. Use mathematical induction to prove the given statement.

117. $2^n > n$

118. $3^n > 2n$

119. $(1 + x)^n \geqslant 1 + nx$ (where $x \geqslant -1$)

120. If $0 \leqslant a < b$, then $a^n < b^n$.

121. $7^n - 1$ is divisible by 6.

122. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

123. $1 + 3 + 5 + \cdots + (2n - 1) = n^2$

124. $2 + 6 + 12 + \cdots + n(n + 1) = \dfrac{n(n + 1)(n + 2)}{3}$

125. $\dfrac{1}{2} + \dfrac{1}{6} + \dfrac{1}{12} + \cdots + \dfrac{1}{n(n + 1)} = \dfrac{n}{n + 1}$

126. $a + ar + ar^2 + \cdots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}$ $(r \neq 1)$