Math 222-06 **QUIZ 10** **Solutions** **2008/10/29**

- Total points: 10+10+5 quiz points.
- Show *complete work*—that is, all the steps needed to completely justify your answer.
- *Simplify* your answers as much as possible.
- If you need extra space, work on the back and make a note on the front.

(1) There is a sequence $a_1, a_2, \ldots, a_n, \ldots$ in which $a_n = \cos(3/n^2)$. Does this sequence converge, and if so, what is its limit?

Solution: It converges; I prove it by producing the limit.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \cos(3/n^2) = \cos \lim_{n\to\infty} 3/n^2 = \cos 0 = 1.$$

(2) Does the series $\sum_{n=1}^{\infty} \ln \dfrac{n^3+1}{2n^3+1}$ converge?

Solution: No. If it converged, the terms $a_n = \ln \dfrac{n^3+1}{2n^3+1}$ would have to approach 0. However:

$$a_n = \frac{n^3+1}{2n^3+1} = \frac{1}{2} + \frac{1}{2}\frac{1}{2n^3+1}$$

so that

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(\frac{1}{2} + \frac{1}{2}\frac{1}{2n^3+1}\right) = \frac{1}{2} + \frac{1}{2}\lim_{n\to\infty}\frac{1}{2n^3+1} = \frac{1}{2} + \frac{1}{2}\cdot 0 = \frac{1}{2}.$$

Therefore,

$$\lim_{n\to\infty} \ln \frac{n^3+1}{2n^3+2} = \ln\left(\lim_{n\to\infty} \frac{n^3+1}{2n^3+2}\right) = \ln \frac{1}{2} \neq 0.$$

Since $\lim_{n\to\infty} a_n \neq 0$ (the terms do not approach 0), the series has no sum, i.e., it diverges.

(3) Does the series $\sum_{n=1}^{\infty} \dfrac{1}{n}$ converge? (You should recognize this series and know its properties. I don't expect a proof.)

No—despite the fact that $\lim_{n\to\infty} a_n = 0$. This is the harmonic series.