Math 222-06 QUIZ 13 2008/11/17 SOLUTION

- Total points: 20 quiz points: 5 each.
- Show *complete work*—that is, all the steps needed to completely justify your answer.
- If you need extra space, work on the back and make a note on the front.

The power series $\sum_{n=0}^{\infty} c_n(x-4)^n$ converges conditionally for $x = 0$. Does it converge for these values of x? Explain your answers.

(a) $x = 7$? Yes No Can't tell

(b) $x = 8$? Yes No Can't tell

(c) $x = 9$? Yes No Can't tell

(d) $x = 10$? Yes No Can't tell

Analysis.

The center of the interval of convergence is at $x = 4$ because the series is in powers of $x - 4$. For any x inside the radius of convergence the series converges absolutely. For any x outside the radius of convergence the series diverges. Therefore, 0 must be an endpoint of the interval of convergence.

Conclusions: $R = 4$, so the interval has endpoints 0 and 8. Then $x = 7$ is inside the interval and $x = 9, 10$ are outside.

$x = 8$ is tricky. In fact, since it's an endpoint and we aren't given any information about it, we have no way of knowing whether the series converges there. We're only sure the series can't converge absolutely, but it might converge conditionally. We can't assume the series is alternating at $x = 0$ or $x = 4$; the signs of the terms could be very mixed up.