Math 222-06 QUIZ 14 2008/11/19 SOLUTIONS

- Total points: 15 quiz points.
- Show *complete work*—that is, all the steps needed to completely justify your answer.
- *Simplify* your answers as much as possible.
- If you need extra space, work on the back and make a note on the front.

Here is a power series: $\displaystyle\sum_{n=1}^{\infty} \frac{(x+3)^n}{n!}$.

What is the center of the power series? Ans.: -3, because $x - a = x + 3 = x - (-3)$.

What is the radius of convergence? Ans.: $R = \infty$, because I know the series (see below).

What is the interval of convergence? Answer: $(-\infty, \infty)$.

Do you recognize this series? If so, write a formula for its sum.

Ans.: This is e^{x+3} since the exponential series is $e^y = \displaystyle\sum_{n=0}^{\infty} \frac{y^n}{n!}$.

Comment.

Several people applied the ratio test but came up with the wrong radius and interval of convergence, namely $R = 1$ and interval $[-4, -2]$ (or similar). The reason was usually a slightly tricky algebra error. When you apply the ratio test, $a_n = (x+3)^n/n!$. Thus,

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(x+3)^{n+1}/(n+1)!}{(x+3)^n/n!}\right| = \left|\frac{(x+3)^{n+1}}{(x+3)^n}\frac{n!}{(n+1)!}\right| = \left|(x+3)\frac{1}{n+1}\right| \to 0 \text{ as } n \to \infty.$$

The algebra error is to cancel $\frac{n!}{(n+1)!}$ carelessly, getting 1 so the ratio appears to be $|x+3|$, which would give $R = 1$, etc. It's necessary to pay close attention when working with factorials.