Math 222-06 QUIZ 15 2008/11/24 SOLUTIONS

- Total points: 10+10 quiz points.
- Show *complete work*—that is, all the steps needed to completely justify your answer.
- *Simplify* your answers as much as possible.

(1) Write in Σ notation the power series (centered at 0) for $\dfrac{1}{8+x^3}$.

Answer: $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{3n}}{8^{n+1}}$

Find the interval of convergence.

Answer: $(-2,2)$

Explanation: I use the geometric series,

$$\frac{1}{1-y}=\sum_{n=0}^{\infty}y^n,$$

which converges for $|y|<1$ (interval of convergence $(-1,1)$). First, transform the fraction into the exact form of a geometric series (which means you'll have a constant factor outside):

$$\frac{1}{8+x^3}=\frac{1}{8(1+x^3/8)}=\frac{1}{8}\,\frac{1}{1-[-x^3/8]}$$

then apply the geometric series formula:

$$=\frac{1}{8}\sum_{n=0}^{\infty}\left(-\frac{x^3}{8}\right)^n=\frac{1}{8}\sum_{n=0}^{\infty}(-1)^n\frac{x^{3n}}{8^n}=\sum_{n=0}^{\infty}(-1)^n\frac{x^{3n}}{8^{n+1}}$$

Either of the last two sums is a good answer.

The interval of convergence comes from the interval of convergence for a geometric series, using $y=-x^3/8$. Thus, we need $|x^3/8|<1$. Solve this: $|x|^3<8$, so $|x|<2$. The interval is $(-2,2)$; and if you want the radius of convergence, it's $R=2$.

(2) Write in Σ notation the power series (centered at 1, *not* at 0) for $\dfrac{1}{1+x}$.

Answer: $\displaystyle\sum_{n=0}^{\infty}\frac{(x-1)^n}{2^{n+1}}$

Find the interval of convergence.

Answer: $(-1,3)$

Explanation: We need a power series in $x-1$, not x, so let $z=x-1$. Thus, $x=z+1$, and the fraction becomes $1/(2+z)$, which you treat like the series in problem (1):

$$\frac{1}{2+z}=\frac{1}{2(1+[z/2])}=\frac{1}{2}\,\frac{1}{1+[z/2]}=\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n=\frac{1}{2}\sum_{n=0}^{\infty}\frac{(x-1)^n}{2^n}=\sum_{n=0}^{\infty}\frac{(x-1)^n}{2^{n+1}}$$

Either of the last two sums is a good answer.

The interval of convergence is also done as in problem (1). We have convergence when $|y|<1$, where $y=z/2=(x-1)/2$. That is, $|x-1|<2$, so $R=2$ and the interval is $(-1,3)$.