Math 222-06 QUIZ 16 2008/12/1 Name_____

- Total points: 10+10 quiz points.
- Show *complete work*—that is, all the steps needed to completely justify your answer.
- *Simplify* your answers as much as possible.

(1) Write the definition of $\binom{-2}{n}$ and simplify as far as possible.

Definition:

$$\binom{-2}{n} = \frac{(-2)(-3)\cdots(-2-n+1)}{n!}$$

Simplification:

$$= (-1)^n \frac{(2)(3)\cdots(n+1)}{n!} = (-1)^n \frac{(1)(2)(3)\cdots(n)(n+1)}{n!} = (-1)^n(n+1).$$

Answer: $(-1)^n(n+1)$

(2) A curve $y = f(x)$ has arc length function $s(x) = x + x^3$. Find the square of the slope of the curve, i.e., $f'(x)^2$. Where is the curve horizontal?

Solution: Remember that $(ds)^2 = (dx)^2 + (dy)^2$. Thus,

$$\left(\frac{ds}{dx}\right)^2 = \left(\frac{dx}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^2 = 1 + f'(x)^2.$$

Therefore,

$$f'(x)^2 = \left(\frac{ds}{dx}\right)^2 - 1 = (1+3x^2)^2 - 1 = 3x^2(3x^2+2).$$

The curve is horizontal at $3x^2(3x^2+2) = 0$, thus at $x = 0$. (N.B. There's no way of knowing the height of the curve from the information given in the problem.)