Here are three polynomials in \mathbb{P}_3 :

$$p(t) = t^3 - 3t^2 + 4t,$$

$$q(t) = t^3 + 4t^2 + t,$$

$$r(t) = 2t^3 + 15t^2 - t.$$

(1) (10 points) Prove they are linearly dependent. Find a linear dependence relation that they satisfy.

- (2) (5 points) Use your answer to question (1) to express one of the polynomials as a linear combination of the other two.
- (3) (10 points) Find a largest linearly independent subset of $\{p(t), q(t), r(t)\}$; explain why it is independent and largest.