

The following set  $\mathcal{B}$  is a basis for the vector space  $\mathbb{P}_2$ :

$$\mathcal{B} = \{t + 1, t^2 + 1, 2t^2 - 2t + 5\}.$$

$p(t) \in \mathbb{P}_2$  is a polynomial whose coordinate vector with respect to  $\mathcal{B}$  is  $[p(t)]_{\mathcal{B}} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ .

(1) What polynomial is  $p(t)$ ?

(2) A linear transformation  $T : \mathbb{P}_2 \rightarrow \mathbb{R}^2$  has the property that

$$T(t + 1) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad T(t^2 + 1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad T(2t^2 - 2t + 5) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Find the value of  $T(p(t))$ .