QUIZ 7 FOR MATH 304-06, 11/6 YOUR NAME SOLUTIONS

(1) (5 points) For vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$, what does $\mathbf{u} \perp \mathbf{v}$ mean? The answer should be a meaning, not an equation.

It means that the two vectors are orthogonal, or their dot product $= 0$ (either answer is okay).

(2) (5 points) For vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$, what equation corresponds to the statement "$\mathbf{u} \perp \mathbf{v}$"?

$\mathbf{u} \cdot \mathbf{v} = 0$.

(3) (10 points) Let $\mathbf{x} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.

(a) How long is the vector $\mathbf{x}$?
(b) Are $\mathbf{x}$ and $\mathbf{y}$ orthogonal?
(c) Find a vector that is orthogonal to $\mathbf{y}$.

(a) Length $= \|\mathbf{x}\| = \sqrt{50} = 5\sqrt{2}$.
(b) $\mathbf{x} \cdot \mathbf{y} = -1 \neq 0$, so they are not orthogonal.
(c) Cute answer: $\mathbf{0}$. Expected answer: $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. Smart answer: any vector of the form $\begin{bmatrix} a \\ a \\ b \end{bmatrix}$ for any real numbers a, b.

(4) (10 points) Suppose we have two bases, $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3\}$, for $\mathbb{R}^3$ and also we have a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$. And suppose

$$[T(\mathbf{b}_1)]_{\mathcal{C}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad [T(\mathbf{b}_2)]_{\mathcal{C}} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad [T(\mathbf{b}_3)]_{\mathcal{C}} = \begin{bmatrix} 4 \\ 2 \\ 7 \end{bmatrix}.$$

What is the matrix M of T relative to the bases $\mathcal{B}$ and $\mathcal{C}$?

$$M = \begin{bmatrix} 1 & 1 & 4 \\ 2 & 0 & 2 \\ 3 & -1 & 7 \end{bmatrix}.$$