QUIZ 8 FOR MATH 304-06, 11/8 YOUR NAME SOLUTIONS

(1) (10 points) Suppose we have two bases, $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3\}$, for $\mathbb{R}^3$ and also we have a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$. And suppose

$$[T(\mathbf{b}_1)]_{\mathcal{C}} = \mathbf{c}_1, \quad [T(\mathbf{b}_2)]_{\mathcal{C}} = \mathbf{c}_2, \quad [T(\mathbf{b}_3)]_{\mathcal{C}} = \mathbf{c}_3.$$

What is the matrix M of T relative to the bases $\mathcal{B}$ and $\mathcal{C}$?

$$M = \begin{bmatrix} [T(\mathbf{b}_1)]_{\mathcal{C}} & [T(\mathbf{b}_2)]_{\mathcal{C}} & [T(\mathbf{b}_3)]_{\mathcal{C}} \end{bmatrix} = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3 \end{bmatrix}.$$

The answer is the last matrix. The point of this question is to see if you know how to use the data given to you.

(2) (10 points) Suppose we have two bases, $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3\}$, for $\mathbb{R}^3$ and also we have a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$. And suppose the matrix M of T relative to the bases $\mathcal{B}$ and $\mathcal{C}$ is

$$M = \begin{bmatrix} 3 & 2 & 1 \\ 6 & 8 & 10 \\ -5 & -4 & -3 \end{bmatrix}.$$

What is the coordinate vector $[T(\mathbf{b}_2)]_{\mathcal{C}}$?

Since $M = \begin{bmatrix} [T(\mathbf{b}_1)]_{\mathcal{C}} & [T(\mathbf{b}_2)]_{\mathcal{C}} & [T(\mathbf{b}_3)]_{\mathcal{C}} \end{bmatrix}$, the answer is the second column,

$$[T(\mathbf{b}_2)]_{\mathcal{C}} = \begin{bmatrix} 2 \\ 8 \\ -14 \end{bmatrix}.$$

The point of this question is also to see if you know how to use the data given to you.

(3) (10 points) An orthogonal basis for $\mathbb{R}^2$ is $\mathcal{B} = \{\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix}\}$. Find $[\mathbf{x}]_{\mathcal{B}}$ where $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ without solving a linear system or using matrices.

Call the basis vectors $\mathbf{b}_1, \mathbf{b}_2$. Then

$$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} \frac{\mathbf{x}\cdot\mathbf{b}_1}{\mathbf{b}_1\cdot\mathbf{b}_1} \\ \frac{\mathbf{x}\cdot\mathbf{b}_2}{\mathbf{b}_2\cdot\mathbf{b}_2} \end{bmatrix} = \begin{bmatrix} 3/5 \\ -1/5 \end{bmatrix}.$$