QUIZ 8 FOR MATH 304-06, 11/8   YOUR NAME ______________________________

(1) (10 points) Suppose we have two bases, $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3\}$, for $\mathbb{R}^3$ and also we have a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$. And suppose

$$[T(\mathbf{b}_1)]_{\mathcal{C}} = \mathbf{c}_1, \quad [T(\mathbf{b}_2)]_{\mathcal{C}} = \mathbf{c}_2, \quad [T(\mathbf{b}_3)]_{\mathcal{C}} = \mathbf{c}_3.$$

What is the matrix M of T relative to the bases $\mathcal{B}$ and $\mathcal{C}$?

(2) (10 points) Suppose we have two bases, $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3\}$, for $\mathbb{R}^3$ and also we have a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$. And suppose the matrix M of T relative to the bases $\mathcal{B}$ and $\mathcal{C}$ is

$$M = \begin{bmatrix} 3 & 2 & 1 \\ 6 & 8 & 10 \\ -5 & -4 & -3 \end{bmatrix}.$$

What is the coordinate vector $[T(\mathbf{b}_2)]_{\mathcal{C}}$?

(3) (10 points) An orthogonal basis for $\mathbb{R}^2$ is $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$. Find $[\mathbf{x}]_{\mathcal{B}}$ where $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ without solving a linear system or using matrices.