QUIZ 9 FOR MATH 304-06, 11/13 YOUR NAME SOLUTIONS

(1) (5 points) What is the Asteroid Belt?

The ring of solid bodies that orbit the Sun between the orbits of Mars and Jupiter. (You could add extra information, such as that they are all much smaller than planets, and that the largest is Ceres with a diameter of 588 mi.)

(2) (10 points) An orthogonal basis for $\mathbb{R}^3$ is $\mathcal{B} = \{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix} \}$. Find $[\mathbf{x}]_{\mathcal{B}}$ where $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ without solving a linear system or using matrices.

Call the basis vectors $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$ for short. Compute inner products (in this case, dot products).

$$\mathbf{x} \cdot \mathbf{b}_1 = 8, \quad \mathbf{x} \cdot \mathbf{b}_2 = 0, \quad \mathbf{x} \cdot \mathbf{b}_3 = -6,$$
$$\mathbf{b}_1 \cdot \mathbf{b}_1 = 6, \quad \mathbf{b}_2 \cdot \mathbf{b}_2 = 5, \quad , \mathbf{b}_3 \cdot \mathbf{b}_3 = 11.$$

Then

$$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} \frac{\mathbf{x} \cdot \mathbf{b}_1}{\mathbf{b}_1 \cdot \mathbf{b}_1} \\ \frac{\mathbf{x} \cdot \mathbf{b}_2}{\mathbf{b}_2 \cdot \mathbf{b}_2} \\ \frac{\mathbf{x} \cdot \mathbf{b}_3}{\mathbf{b}_3 \cdot \mathbf{b}_3} \end{bmatrix} = \begin{bmatrix} 4/3 \\ 0 \\ -6/11 \end{bmatrix}.$$

(3) (10 points) Diagonalize $\begin{bmatrix} 2 & 5 \\ 5 & -2 \end{bmatrix}$, if possible. Do not do extra work.

Step 1. Find eigenvalues.

$$\det \begin{bmatrix} 2 - \lambda & 5 \\ 5 & -2 - \lambda \end{bmatrix} = (2 - \lambda)(-2 - \lambda) - 25 = \lambda^2 - 29.$$

So $\lambda = \pm\sqrt{29}$.

Step 2. Find eigenvectors. This example is ugly. I'll skip it. For a better example see Ungraded Assessment 3.