TEST 1, MATH 304-06, 9/25/19 MAKEUP   YOUR NAME ________________________

Show all the work or explanation necessary to justify the answer.

(1) (5 points) What are the three properties that define a subspace of $\mathbb{R}^n$?

(2) (15 points) Is this set X linearly independent? If it is not, (a) find a linear dependence relation and (b) find a vector in X that you can remove without changing the span.

$$X = \left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \end{bmatrix} \right\}.$$

(3) ($45 = 9 \times 5$ points) Here are a matrix A and vector $\mathbf{b}$:

$$A = \begin{bmatrix} 3 & 0 & 6 \\ 1 & 2 & 4 \\ 1 & 3 & 5 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}.$$

Let T be the linear transformation defined by $T(\mathbf{x}) := A\mathbf{x}$. Don't forget to explain (except in (a)). You may combine the work for different parts if that saves time.

(a) Find the reduced row echelon form of A.
(b) Solve $A\mathbf{x} = \mathbf{b}$.
(c) Solve the homogeneous equation $A\mathbf{x} = \mathbf{0}$.
(d) Find a basis for the column space $\text{Col}(A)$.
(e) Find a basis for the null space $\text{Nul}(A)$.
(f) What is the domain of T? What is the codomain of T?
(g) What is the range of T?
(h) Is A invertible?
(i) Is T invertible?

(4) (10 points) Find the inverse matrix, or show there is none.

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 6 & 0 & 3 \\ 0 & 2 & 2 \end{bmatrix}$$

(5) ($25 = 5 \times 5$ points) Suppose $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation such that $T(\mathbf{e}_1) = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$ and $T(\mathbf{e}_2) = \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$. As usual, $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, etc.

(a) What is the standard matrix of T?
(b) Find $T(\begin{bmatrix} 2 \\ 4 \end{bmatrix})$.
(c) Is T one-to-one?
(d) Is T onto?
(e) Is T invertible?