TEST 2 FOR MATH 304-06, 10/30/19 YOUR NAME ______________________________

Show **all** the work or explanation necessary to justify the answer.

(1) (4×3 points) Which of the following is a vector space? Answer Yes or No in the exam book. Give a reason for each No answer. A reason is not needed for a Yes answer.
  (a) The set $W_3 \subseteq \mathcal{P}_3$ that consists of all polynomials of degree exactly 3.
  (b) The set $Z \subseteq \mathcal{P}_2$ that consists of all polynomials in $\mathcal{P}_3$ whose constant term is 1.
  (c) The set $K \subseteq \mathbb{R}^3$ that consists of all $\mathbf{x} \in \mathbb{R}^3$ such that $x_1x_2x_3 = 0$.
  (d) The null space of a matrix.

(2) (5 points) We have a function between two vector spaces, namely $T : \mathbb{R}^3 \to \mathcal{P}_4$. List the properties of T that make it a linear transformation.

(3) (5 points) Is the following function $T : \mathbb{R}^3 \to \mathbb{R}^2$ a linear transformation? Answer Yes or No in the exam book. Give a reason for a No answer. A reason is not needed for a Yes answer.

$$T(\mathbf{x}) = \begin{bmatrix} x_1x_2 \\ x_1 + x_3 \end{bmatrix}.$$

(4) (20 points) Let V be the vector space $\mathcal{P}_2$ with basis $\mathcal{B} = \{p_1(t), p_2(t), p_3(t)\}$, where

$$p_1(t) = t^2 + t + 1, \quad p_2(t) = 2t^2 + 4, \quad p_3(t) = t^2 - t.$$

Also, let $\mathcal{E} = \{t^2, t, t^0\}$ (the standard basis for $\mathcal{P}_2$).

  (a) There is a vector $\mathbf{u} \in \mathcal{P}_2$ whose coordinate vector with respect to $\mathcal{E}$ is $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. What polynomial is $\mathbf{u}$?

  (b) There is a vector $\mathbf{v} \in \mathcal{P}_2$ whose coordinate vector with respect to $\mathcal{B}$ (not $\mathcal{E}$!) is $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. What polynomial is $\mathbf{v}$?

  (c) Find the change-of-basis matrix $\underset{\mathcal{E} \leftarrow \mathcal{B}}{P}$.

(5) (5 points) In $\mathbb{R}^3$ there are two bases, $\mathcal{B}$ and $\mathcal{C}$. How is the change-of-basis matrix $\underset{\mathcal{C} \leftarrow \mathcal{B}}{P}$ related to the change-of-basis matrix $\underset{\mathcal{B} \leftarrow \mathcal{C}}{P}$?

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(6) (33 points) The matrix for this problem is $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$.

(a) Find $\det A$.
(b) What can you say about the rank of A?
(c) Is A invertible? (Do not find the inverse.)
(d) Find $\det A^3$ without computing A^3.
(e) Find $\det A^T$ without using A^T.
(f) Find the eigenvalues of A.
(g) Find a basis for each eigenspace of A.
(h) Is A diagonalizable? If so, diagonalize it. If not, say why not.

(7) (20 points) The matrix for this problem is $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$.

(a) Find the eigenvalues of A.
(b) Is A invertible? Use the eigenvalues to decide this. (Do not find the inverse.)
(c) Find a basis for each eigenspace of A.
(d) Is A diagonalizable? If so, diagonalize it. If not, say why not.