Here are two bases for $\mathbb{R}^3$:

$$\mathcal{E} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}, \text{ where } \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \text{ are the usual standard basis vectors, and}$$

$$\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}, \text{ where } \mathbf{b}_1 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \ \mathbf{b}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \ \mathbf{b}_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}.$$

(1) Find the basis-change matrix $\underset{\mathcal{E}\leftarrow\mathcal{B}}{P}$ from $\mathcal{B}$ to $\mathcal{E}$.

(2) Find the basis-change matrix $\underset{\mathcal{B}\leftarrow\mathcal{E}}{P}$ from $\mathcal{E}$ to $\mathcal{B}$.

(3) Use the appropriate basis-change matrix to find the coordinates of $\mathbf{x} = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$ in the basis $\mathcal{B}$.