UNGRADED ASSESSMENT 4, MATH 304-06, 11/22 YOUR NAME (OPTIONAL)_______

Point values suggest approximately what a similar question would be worth on a test.

(1) (5 points) What is the Cauchy–Schwartz Inequality?

(2) (0 points) What is the Cauchy–Schwartz Inequality used for?

(3) (5 points) What is the Triangle Inequality (for vectors)?

(4) (5 points) Prove that, if P is an orthogonal matrix and D is diagonal, then PDP^{-1} is symmetric.

(5) (20 points) in $\mathcal{P}_2$, there is an inner product (nicknamed "Betsy") $\langle p(t), q(t)\rangle := p(-1)q(-1)+p(0)q(0)+p(1)q(1)$. The vectors $\mathbf{u}_1 = 1 = t^0$ and $\mathbf{u}_2 = t$ are orthogonal with respect to "Betsy". With respect to the inner product "Betsy",

(a) what is the orthogonal projection of $\mathbf{x} := t^2$ onto the subspace $\operatorname{span}\{t-1, t\}$?
(b) what is the nearest point to $\mathbf{x}$ in the same subspace, $\operatorname{span}\{t-1, t\}$?

(6) (25 points) In $\mathbb{R}^3$, there is an inner product (nicknamed "Arthur") defined by $\langle \mathbf{u}, \mathbf{v}\rangle := 2u_1v_1 + 2u_2v_2 + (u_1 + u_3)(v_1 + v_3)$. Let $\mathcal{E} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be the standard basis for $\mathbb{R}^3$.

(a) Which pairs of vectors in $\mathcal{E}$ are orthogonal with respect to the inner product "Arthur"?
(b) Use Gram–Schmidt orthogonalization to turn $\mathcal{E}$ into an orthogonal basis with respect to "Arthur".