Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $E = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

1. (12 points) On the line provided for each part, write the letters of all matrices, $A - E$, which answer the question. If none of the matrices answer the question, write the letter **N**. No justifications are needed for this question.

(a) Which of the matrices have rank equal to 1?

(a) **B,E**

(b) Which of the matrices have nullity equal to 2?

(b) **B,E**

(c) Which of the matrices have column space equal to $\mathbb{R}^3$?

(c) **C only**

(d) Which of the matrices have row space equal to $\mathbb{R}^2$?

(d) **N**

(e) Which of the matrices can be a change of basis matrix?

(e) **C only**

(f) Which of the matrices has $\mathbf{e}_1 - \mathbf{e}_2$ in its null space?

(f) **A, B**

Let $Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $Y = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, $X = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, $W = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, $V = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

1. (12 points) On the line provided for each part, write the letters of all matrices, $Z - V$, which answer the question. If none of the matrices answer the question, write the letter **N**. No justifications are needed for this question.

(a) Which of the matrices have rank equal to 1?

(a) **Z,X**

(b) Which of the matrices have nullity equal to 2?

(b) **Z,X**

(c) Which of the matrices have column space equal to $\mathbb{R}^3$?

(c) **Y,W**

(d) Which of the matrices have row space equal to $\mathbb{R}^2$?

(d) **N**

(e) Which of the matrices can be a change of basis matrix?

(e) **Y,W**

(f) Which of the matrices has $\mathbf{e}_3 - \mathbf{e}_1$ in its null space?

(f) **Z, X, V**

2. (12 points) Let $S = \{1 + t,\ t + t^2,\ t^2 + t^3,\ 1 + t^3\}$ be a collection of vectors in $\mathbb{P}_3$.

(a) Is S a linearly independent set of vectors in $\mathbb{P}_3$? Justify your answer.

No. A possible dependence relation is $(1 + t^3) = (1 + t) - (t + t^2) + (t^2 + t^3)$.

Alternate solution: Using coordinate vectors, row reduce the matrix $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

There is a dependence relation: The fourth vector is a linear combination of the other three vectors.

(b) Does S span $\mathbb{P}_3$? Justify your answer.

No. The dependence relation above implies that the span of S is at most 3-dimensional.
Since $\mathbb{P}_3$ is 4-dimensional, S cannot span $\mathbb{P}_3$.

2. (12 points) Let $S = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ be a collection of vectors in $\mathbb{R}_{2\times 2}$.

(a) Is S a linearly independent set of vectors in $\mathbb{R}_{2\times 2}$? Justify your answer.

No. A possible dependence relation is $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Alternate solution: Using coordinate vectors, row reduce the matrix $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

There is a dependence relation: The fourth vector is a linear combination of the other three vectors.

(b) Does S span $\mathbb{R}_{2\times 2}$? Justify your answer.

No. The dependence relation above implies that the span of S is at most 3-dimensional.
Since $\mathbb{R}_{2\times 2}$ is 4-dimensional, S cannot span $\mathbb{R}_{2\times 2}$.

3. (16 points) Let $\mathcal{B} = \left\{ \begin{bmatrix} 4 & 3 \\ 1 & 4 \end{bmatrix}, \begin{bmatrix} 4 & 4 \\ 2 & 6 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 3 & 6 \end{bmatrix}, \begin{bmatrix} 4 & 4 \\ 0 & 8 \end{bmatrix} \right\}$ and $\mathcal{D} = \left\{ \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \right\}$ be bases for $\mathbb{R}_{2\times 2}$.

(a) Find $\underset{\mathcal{D}\leftarrow\mathcal{B}}{P}$, the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{D}$.

$$\left[\begin{array}{cccc|cccc} 2 & 1 & 0 & 1 & 4 & 4 & 3 & 4 \\ 1 & 1 & 0 & 1 & 3 & 4 & 3 & 4 \\ 0 & 0 & 1 & 0 & 1 & 2 & 3 & 0 \\ 1 & 1 & 0 & 2 & 4 & 6 & 6 & 8 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 & 3 & 4 \end{array}\right]. \quad \text{So } \underset{\mathcal{D}\leftarrow\mathcal{B}}{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 4 \end{bmatrix}.$$

(b) Which matrix $\mathbf{x}$ in $\mathbb{R}_{2\times 2}$ has $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$?

$$-\begin{bmatrix} 4 & 3 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 2 & 2 \end{bmatrix}$$

(c) Let $\mathbf{b} = \begin{bmatrix} 4 & 4 \\ 0 & 8 \end{bmatrix}$. Find $[\mathbf{b}]_D$ the coordinate vector of $\mathbf{b}$ with respect to $\mathcal{D}$.

$\mathbf{b}$ is the fourth $\mathcal{B}$-basis vector, which is a scalar multiple of the fourth $\mathcal{D}$-basis vector. So $[\mathbf{b}]_D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \end{bmatrix}$.

Alternate solution: $\left[\begin{array}{cccc|c} 2 & 1 & 0 & 1 & 4 \\ 1 & 1 & 0 & 1 & 4 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 2 & 8 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{array}\right]$, which gives $[\mathbf{b}]_D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \end{bmatrix}$.

3. (16 points) Let $\mathcal{B} = \{4 + 3t + t^2 + 4t^3,\ 4 + 4t + 2t^2 + 6t^3,\ 3 + 3t + 3t^2 + 6t^3,\ 4 + 4t + 8t^3\}$ and $\mathcal{D} = \{2 + t + t^3,\ 1 + t + t^3,\ t^2,\ 1 + t + 2t^3\}$ be bases for $\mathbb{P}_3$.

(a) Find $\underset{\mathcal{D}\leftarrow\mathcal{B}}{P}$, the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{D}$.

$$\left[\begin{array}{cccc|cccc} 2 & 1 & 0 & 1 & 4 & 4 & 3 & 4 \\ 1 & 1 & 0 & 1 & 3 & 4 & 3 & 4 \\ 0 & 0 & 1 & 0 & 1 & 2 & 3 & 0 \\ 1 & 1 & 0 & 2 & 4 & 6 & 6 & 8 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 & 3 & 4 \end{array}\right]. \quad \text{So } \underset{\mathcal{D}\leftarrow\mathcal{B}}{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 4 \end{bmatrix}.$$

(b) Which polynomial $\mathbf{x}$ in $\mathbb{P}_3$ has $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$?

$-(4 + 3t + t^2 + 4t^3) + (3 + 3t + 3t^2 + 6t^3) = -1 + 2t^2 + 2t^3$

(c) Let $\mathbf{b} = 4 + 4t + 8t^3$. Find $[\mathbf{b}]_D$ the coordinate vector of $\mathbf{b}$ with respect to $\mathcal{D}$.

$\mathbf{b}$ is the fourth $\mathcal{B}$-basis vector, which is a scalar multiple of the fourth $\mathcal{D}$-basis vector. So $[\mathbf{b}]_D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \end{bmatrix}$.

Alternate solution: $\left[\begin{array}{cccc|c} 2 & 1 & 0 & 1 & 4 \\ 1 & 1 & 0 & 1 & 4 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 2 & 8 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{array}\right]$, which gives $[\mathbf{b}]_D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \end{bmatrix}$.

4. (4 points) Let A be an $n \times n$ **singular** (non-invertible) matrix.

(a) Use the Invertible Matrix Theorem to provide any equivalent statement about the columns of A.

Any of: columns are dependent, do not span $\mathbb{R}^n$, do not form a basis of $\mathbb{R}^n$, at least one does not have a pivot, $\text{Col}(A) \neq \mathbb{R}^n$, $\dim \text{Col}(A) \neq n$.

(b) Use the Invertible Matrix Theorem to provide any equivalent statement about $\det(A)$.

$\det(A) = 0$.

5. (4 points) Let $T : \mathbb{R}^m \to \mathbb{R}^m$ be a linear transformation which **is surjective**.

(a) Use the Invertible Matrix Theorem to provide any equivalent statement about the standard matrix for T.

Any statement equivalent to invertible.

(b) Use the Invertible Matrix Theorem to provide any equivalent statement about the codomain of T.

Is equal to range of T. For every $\mathbf{b} \in \mathbb{R}^m$, $T(\mathbf{x}) = \mathbf{b}$ has a solution. $T(\mathbf{x}) = \mathbf{0}$ has a unique solution.

4. (4 points) Let A be an $n \times n$ **invertible** matrix.

(a) Use the Invertible Matrix Theorem to provide any equivalent statement about the columns of A.

Any of: columns are independent, span $\mathbb{R}^n$, form a basis of $\mathbb{R}^n$, pivot in each, $\text{Col}(A) = \mathbb{R}^n$, $\dim \text{Col}(A) = n$.

(b) Use the Invertible Matrix Theorem to provide any equivalent statement about $\det(A)$.

$\det(A) \neq 0$.

5. (4 points) Let $T : \mathbb{R}^m \to \mathbb{R}^m$ be a linear transformation which **is not injective**.

(a) Use the Invertible Matrix Theorem to provide any equivalent statement about the standard matrix for T.

Any statement equivalent to singular.

(b) Use the Invertible Matrix Theorem to provide any equivalent statement about the codomain of T.

Is not equal to range of T. There is at least one $\mathbf{b}$ where $T(\mathbf{x}) = \mathbf{b}$ does not have a solution. $T(\mathbf{x}) = \mathbf{0}$ does not have a unique solution.

6. (16 points) Consider the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 3 & 4 & 1 \\ 2 & 2 & 2 & k \end{bmatrix}$.

(a) Determine all values of k such that Nul(A) is 2-dimensional.

$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 3 & 4 & 1 \\ 2 & 2 & 2 & k \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & 0 & k-2 \end{bmatrix}$. $k = 2$ is the only solution.

(b) For each value of k from part a, provide a basis for Nul(A).

Row reduce to $\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Parameterize to get basis $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(c) For each value of k from part a, provide a basis for Col(A).

$\left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix} \right\}$ or equivalent.

6. (16 points) Consider the matrix $B = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 3 & 4 & 5 & 2 \\ 3 & 3 & 3 & m \end{bmatrix}$.

(a) Determine all values of m such that Nul(B) is 2-dimensional.

$B = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 3 & 4 & 5 & 2 \\ 3 & 3 & 3 & m \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & 0 & m-3 \end{bmatrix}$. $m = 3$ is the only solution.

(b) For each value of m from part a, provide a basis for Nul(B).

Row reduce to $\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Parameterize to get basis $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(c) For each value of m from part a, provide a basis for Col(B).

$\left\{ \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix} \right\}$ or equivalent.

7. (12 points) Let $A = \begin{bmatrix} 6 & 0 & 2 & 5 \\ 4 & 1 & 7 & 5 \\ 0 & 0 & 3 & 0 \\ 3 & 0 & 2 & 1 \end{bmatrix}$. Parts (b), (c), and (d) do not require any justification.

(a) Compute det(A) using any valid technique(s) from the course.

$\det(A) = -27$

(b) Using your result from part a provide $\det(A^{-1})$ if it exists, otherwise write DNE.

(b) $\frac{-1}{27}$

(c) Using your result from part a provide $\det\left(\frac{1}{2}A\right)$. **Do not attempt to simplify your answer.**

(c) $\frac{-27}{16}$

(d) Using your result from part a provide $\det(AA^T)$. **Do not attempt to simplify your answer.**

(d) $(-27)^2 = 729$

7. (12 points) Let $A = \begin{bmatrix} 2 & 9 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 1 & 7 & 3 & 4 \\ 3 & 5 & 0 & 2 \end{bmatrix}$. Parts (b), (c), and (d) do not require any justification.

(a) Compute det(A) using any valid technique(s) from the course.

$\det(A) = -15$

(b) Using your result from part a provide $\det(A^{-1})$ if it exists, otherwise write DNE.

(b) $\frac{-1}{15}$

(c) Using your result from part a provide $\det\left(\frac{1}{2}A\right)$. **Do not attempt to simplify your answer.**

(c) $\frac{-15}{16}$

(d) Using your result from part a provide $\det(A^T A)$. **Do not attempt to simplify your answer.**

(d) $(-15)^2 = 225$

Version A

8. (12 points) **Select all answer choices which apply**, by filling in the circles directly to the left of your choices. Ambiguous markings will be scored as incorrect. **Questions on this page may have multiple correct answers.**

Suppose T is a linear transformation from $\mathbb{R}^5 \to \mathbb{R}^3$ and A is the standard matrix for T.

(a) What is the maximum possible value for the rank of A?

○ 1 ★ **3** ○ 5 ○ 7 ○ None of these.

(b) What is the minimum possible value for the rank of A?

★ **0** ○ 2 ○ 4 ○ 6 ○ None of these.

(c) What is the maximum possible value for the nullity of A?

○ 1 ○ 3 ★ **5** ○ 7 ○ None of these.

(d) What is the minimum possible value for the nullity of A?

○ 0 ★ **2** ○ 4 ○ 6 ○ None of these.

Define $S : \mathbb{R}^5 \to \mathbb{R}^3$ via $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \mapsto \begin{bmatrix} 2x_1 \\ 0 \\ -4x_1 \end{bmatrix}$.

(e) What is the dimension of the range of S?

○ 5 ○ 4 ○ 3 ○ 2 ★ **None of these.**

(f) What is the dimension of the kernel of S?

○ 5 ★ **4** ○ 3 ○ 2 ○ None of these.

Version B

8. (12 points) **Select all answer choices which apply**, by filling in the circles directly to the left of your choices. Ambiguous markings will be scored as incorrect. **Questions on this page may have multiple correct answers.**

Suppose T is a linear transformation from $\mathbb{R}^5 \to \mathbb{R}^2$ and A is the standard matrix for T.

(a) What is the maximum possible value for the rank of A?

○ 0 ★ **2** ○ 4 ○ 6 ○ None of these.

(b) What is the minimum possible value for the rank of A?

★ **0** ○ 2 ○ 4 ○ 6 ○ None of these.

(c) What is the maximum possible value for the nullity of A?

○ 1 ○ 3 ★ **5** ○ 7 ○ None of these.

(d) What is the minimum possible value for the nullity of A?

○ 1 ★ **3** ○ 5 ○ 7 ○ None of these.

Define $S : \mathbb{R}^5 \to \mathbb{R}^2$ via $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \mapsto \begin{bmatrix} -2x_4 \\ 5x_4 \end{bmatrix}$.

(e) What is the dimension of the range of S?

○ 5 ○ 4 ○ 3 ○ 2 ★ **None of these.**

(f) What is the dimension of the kernel of S?

○ 5 ★ **4** ○ 3 ○ 2 ○ None of these.

Version A

9. (12 points) Let $H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \,\middle|\, x + y = c \right\}$, the subset of vectors in $\mathbb{R}^2$ whose entries add up to a fixed number c.

(a) Prove that when $c = 0$, H is a subspace of $\mathbb{R}^2$.

- Since $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ has $0 + 0 = 0$, H is non empty.
- Let $h_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}, h_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \in H$. Then $x_1 + y_1 = 0 = x_2 + y_2$.
  $h_1 + h_2 = \begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \end{bmatrix}$. Since $x_1 + x_2 + y_1 + y_2 = 0$, H is closed under vector addition.
- Let $k \in \mathbb{R}$. $kh_1 = \begin{bmatrix} kx_1 \\ kx_2 \end{bmatrix}$. Since $k(x_1 + x_2) = 0$, H is closed under scalar multiplication.

(b) When $c \neq 0$, H cannot be a subspace of $\mathbb{R}^2$. Why not?

Any valid justification.

- $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \notin H$.
- If $x_1 + y_1 = c$ and $x_2 + y_2 = c$, then $x_1 + x_2 + y_1 + y_2 = 2c \neq c$, for $c \neq 0$.
- If $x + y = c$ and $k \neq 0$ then $kx + ky = kc \neq c$ for $c \neq 0$.

Version B

9. (12 points) Let $H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \,\middle|\, x + y = c \right\}$, the subset of vectors in $\mathbb{R}^2$ whose entries add up to a fixed number c.

(a) Prove that when $c = 0$, H is a subspace of $\mathbb{R}^2$.

- Since $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ has $0 + 0 = 0$, H is non empty.
- Let $h_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}, h_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \in H$. Then $x_1 + y_1 = 0 = x_2 + y_2$.
  $h_1 + h_2 = \begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \end{bmatrix}$. Since $x_1 + x_2 + y_1 + y_2 = 0$, H is closed under vector addition.
- Let $k \in \mathbb{R}$. $kh_1 = \begin{bmatrix} kx_1 \\ kx_2 \end{bmatrix}$. Since $k(x_1 + x_2) = 0$, H is closed under scalar multiplication.

(b) When $c \neq 0$, H cannot be a subspace of $\mathbb{R}^2$. Why not?

Any valid justification.

- $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \notin H$.
- If $x_1 + y_1 = c$ and $x_2 + y_2 = c$, then $x_1 + x_2 + y_1 + y_2 = 2c \neq c$, for $c \neq 0$.
- If $x + y = c$ and $k \neq 0$ then $kx + ky = kc \neq c$ for $c \neq 0$.