Class Problem 2 | Math 304-06 | Oct. 13, 2023

Consultation is fine, but no electronics, please.

(1) (20 points) In $\mathbb{R}^2$, one basis is $\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$ and another basis is $\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$. Find the two basis change matrices $\underset{\mathcal{C} \leftarrow \mathcal{B}}{P}$ and $\underset{\mathcal{B} \leftarrow \mathcal{C}}{P}$.

(2) (10 points) Let $\mathbf{x} \in \mathbb{R}^2$. Use one of your basis change matrices to convert the coordinate vector $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$ to the coordinate vector $[\mathbf{x}]_{\mathcal{C}}$, *without finding the vector* $\mathbf{x}$.

TURN OVER FOR MORE! MORE!

(3) (20 points) In $\mathbb{P}_2$, one basis is $\mathcal{E} = \{1, x, x^2\}$ and another is $\mathcal{A} = \{x+x^2, 1+x^2, 1+x\}$. Find the two basis change matrices $\underset{\mathcal{E}\leftarrow\mathcal{A}}{P}$ and $\underset{\mathcal{A}\leftarrow\mathcal{E}}{P}$. Hint: One of them is easy.