QUIZ 2 Math 304-06 Print your name ____________________
Sept. 8(11), 2023

Show full work for each question. You must show work for credit.
No consultation!—that includes no electronics.

(1) This matrix $A = \begin{bmatrix} 0 & 2 & 2 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$ is the coefficient matrix of a *homogeneous* linear system. Solve the system by finding the reduced echelon form of A. For full credit, express your answer in vector form with the variables x_1, x_2, x_3, x_4 separated.

(2) Decide whether these four vectors are linearly dependent or independent.

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \quad \mathbf{a}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{a}_4 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}.$$

If they are linearly dependent, show a specific linear dependence relation (with the coefficients).

(3) A transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ is defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 + x_2 \\ x_2 - x_1 \\ x_1 \end{bmatrix}.$$

(a) Evaluate $T\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix} \right)$.

(b) Prove that T is a linear transformation using the definition of linear transformations.