QUIZ 3 Math 304-06 SOLUTIONS
Sept. 18, 2023

(1) Does $A = \begin{bmatrix} 2 & 0 & 2 \\ 3 & 3 & 6 \\ 1 & 1 & 7 \end{bmatrix}$ have an inverse A^{-1}? If it does, find A^{-1}.

10 points. Here is the setup $\begin{bmatrix} A \mid I \end{bmatrix}$, followed by the calculation that we hope ends with $\begin{bmatrix} I \mid A^{-1} \end{bmatrix}$. I'm combining simple reductions to save space.

$$\begin{bmatrix} 2 & 0 & 2 & 1 & 0 & 0 \\ 3 & 3 & 6 & 0 & 1 & 0 \\ 1 & 1 & 7 & 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 & \frac{1}{2} & 0 & 0 \\ 1 & 1 & 2 & 0 & \frac{1}{3} & 0 \\ 1 & 1 & 7 & 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 & \frac{1}{2} & 0 & 0 \\ 1 & 1 & 2 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 5 & 0 & -\frac{1}{3} & 1 \end{bmatrix}$$
$$\to \begin{bmatrix} 1 & 0 & 1 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 1 & -\frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{15} & \frac{1}{5} \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{15} & -\frac{1}{5} \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 1 & 0 & -\frac{1}{15} & \frac{1}{5} \end{bmatrix}.$$

This says that

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{15} & -\frac{1}{5} \\ -\frac{1}{2} & \frac{2}{5} & -\frac{1}{5} \\ 0 & -\frac{1}{15} & \frac{1}{5} \end{bmatrix}.$$

(I believe there is an error in row 2 but it is too late at night for me to fix it. Can you fix it?)

I did not check your work for arithmetical errors. If you had the right setup and got $\begin{bmatrix} I \mid X \end{bmatrix}$ where X is a reasonable matrix, even if wrong, you got the credit.

(2) Use A^{-1} from problem 1 to solve the equation $A\mathbf{x} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$.

10 points. The solution is to multiply both sides by A^{-1}, which gives $A^{-1}(A\mathbf{x}) = A^{-1} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$. Since $A^{-1}(A\mathbf{x}) = (A^{-1}A)\mathbf{x} = I\mathbf{x} = \mathbf{x}$, we get

$$\mathbf{x} = \begin{bmatrix} \frac{1}{2} & \frac{1}{15} & -\frac{1}{5} \\ -\frac{1}{2} & \frac{2}{5} & -\frac{1}{5} \\ 0 & -\frac{1}{15} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{47}{30} \\ -\frac{1}{10} \\ -\frac{1}{15} \end{bmatrix}.$$

(This is using my erroneous A^{-1}. If you got a wrong A^{-1} and used it here, you will get full credit for this problem.)

No credit for solving the equation. The credit is for using A^{-1} to solve the equation.

TURN OVER FOR PROBLEM 3

(3) Which of these properties characterizes invertible matrices A? Circle Yes or No for each question. Read carefully.
10 points: 2 per part.

(a) Yes No A is square.
Reason: Here's a counterexample: O_n, the $n \times n$ matrix of all 0's, has no inverse.

(b) Yes No A is $n \times n$ and has n pivot columns.
Reason: This is one of the important ways to know a matrix is invertible.

(c) Yes No The columns of A are linearly independent.
Reason: A must be square

(d) Yes No A is square and its columns are linearly independent.
Reason: If the columns are linearly independent and A is square, then every column is a pivot column. See (b).

(e) Yes No There is a matrix B such that $AB = I$, an identity matrix.
Reason: A must be square. (I made a mistake in discussing this quiz. The answer is not Yes.)