QUIZ 4 Math 304-06 Print your name Solution Wizard

Sept. 29 (Oct. 2, sadly), 2023

(1) (10 points) $\mathbb{R}^n$ is a type of vector space. Name two other types of vector space (you may use their symbolic names).

Some good answers are $\mathbb{P}_n$, Nul(A), Col(A), Ker(T), Range(T), $\mathcal{M}_{m\times n}$ or $\mathbb{R}_{m\times n}$.

(2) (10 points) In the polynomial vector space $\mathbb{P}_2$, are the polynomials

$$p(x) = x^2 - 3x + 1, \quad q(x) = 2x^2 - 4x - 1, \quad r(x) = x^2 - x - 2$$

linearly independent? Justify your answer.

Answer 1: They are linearly dependent because $q(x) = p(x) + r(x)$.

Answer 2: Set up a linear dependence relation, derive a matrix, and find its rank. The linear dependence relation is

$$a_1 p(x) + a_2 q(x) + a_3 r(x) = 0$$

(the zero polynomial = zero vector), from which you derive the linear system

$$\begin{bmatrix} 1 & 2 & 1 \\ -3 & -4 & -1 \\ 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

The matrix of this homogeneous linear system is $\begin{bmatrix} 1 & 2 & 1 \\ -3 & -4 & -1 \\ 1 & -1 & -2 \end{bmatrix}$ and it has rank 2, with 2 pivot positions, leaving one free variable. Thus, there is a nonzero solution $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$, so there is a linear dependence relation among the three polynomials.

Using the transpose of this matrix is an error and received -2 points.

(3) (10=2+4+3+1 points) Let $S = \{p(x) \in \mathbb{P}_3 : p(1) = 0\}$. Is S a subspace of $\mathbb{P}_3$? Justify your answer.

Yes! But why? We check three properties. Each property must be verified for full credit. 1 point for only remembering the property.

(a) Is the $\mathbf{0}$ vector in the set S? The zero vector in $\mathbb{P}_3$ is the zero polynomial, $z(x) = 0x^3 + 0x^2 + 0x + 0$. Substituting 1 gives $z(1) = 0$, so $z(x) \in S$.

(b) Is the sum of two vectors in S also in S? A vector being a polynomial, I take two polynomials $p(x), q(x) \in S$, which means $p(1) = 0$ and $q(1) = 0$. Their sum is $p(x) + q(x)$. Substituting $x = 1$, I get $p(1) + q(1) = 0 + 0 = 0$. Therefore, $p(x) + q(x) \in S$.

(c) Is any scalar multiple of a vector in S also in S? I take a polynomial, $p(x)$, and a scalar, c, and multiply to get $cp(x)$. Evaluating at $x = 1$, I see that $cp(1) = c0 = 0$. Thus, $cp(x) \in S$.

The three properties are satisfied so the answer is Yes! (One point for the correct answer if justified.)