QUIZ 5 Math 304-06 Print your name Solution Wizard

Oct. 6, 2023

(1) (2 points each) Circle your answer to each question. You do not have to give a reason for your answer.

(a) True <u>False</u> $\mathbb{P}_3$ has a basis that has three elements. [Reason: $\dim \mathbb{P}_3 = 4$. A basis has 4 elements.]

(b) True <u>False</u> The dimension of $\mathbb{P}_3$ is 3.

(c) True <u>False</u> A basis for $\mathbb{R}^2$ is $\mathbf{e}_1, \mathbf{e}_2$. [A basis is a set. This is not a set.]

(d) <u>True</u> False A basis for $\mathbb{R}^2$ is $\{\mathbf{e}_1, \mathbf{e}_2\}$. [This is a set of 2 linearly independent vectors, or equally good they span the space, and it's the right number because $\dim \mathbb{R}^2 = 2$, as you all know already.]

(e) True <u>False</u> A basis for $\mathbb{P}_2$ is $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$. [Not a set! Much worse: not one of them belongs to the set $\mathbb{P}_2$.]

(f) True <u>False</u> A basis for $\mathbb{P}_2$ is x^1, x^0. [Not a set! Worse, there's the wrong number, since $\dim \mathbb{P}_2 = 3$.]

(g) True <u>False</u> A basis for $\mathbb{P}_2$ is $\{x^1, x^1 + 1\}$. [It's a set but $\dim \mathbb{P}_2 = 3$.]

(h) True <u>False</u> A basis for $\mathbb{P}_2$ is x^2, x^1, x^0. [Not a set! Is this getting tedious? But you get the message. Remember it on a test.]

(i) <u>True</u> False A basis for $\mathbb{P}_2$ is $\{x^1, x^1 + 1, x^2\}$. [Hooray! A set! It has the right number of elements. But do they span $\mathbb{P}_2$? Yes; that may be obvious.]

(j) True <u>False</u> A basis for $\mathbb{P}_2$ is $\{x^1, x^1 + 1, x^2 + 1, x^2\}$. [Alas, this is too, too much. We should have 3 polynomials, not 4.]

PLEASE TURN OVER FOR QUESTION 2

(2) (10 points) In $\mathbb{R}^3$, find the coordinates of $\begin{bmatrix}1\\2\\3\end{bmatrix}$ with respect to the basis $\mathcal{B} = \{\begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}\}$. Show all necessary work to justify your answer.

Solution 1. We want the numbers a_1, a_2, a_3 such that $\begin{bmatrix}1\\2\\3\end{bmatrix} = a_1 \begin{bmatrix}0\\0\\1\end{bmatrix} + a_2 \begin{bmatrix}0\\1\\1\end{bmatrix} + a_3 \begin{bmatrix}1\\1\\1\end{bmatrix}$. That means we set up a linear system whose augmented matrix is

$$\begin{bmatrix}0 & 0 & 1 & | & 1\\0 & 1 & 1 & | & 2\\1 & 1 & 1 & | & 3\end{bmatrix} \to \begin{bmatrix}0 & 0 & 1 & | & 1\\0 & 1 & 0 & | & 1\\1 & 0 & 0 & | & 1\end{bmatrix},$$

giving the solution $a_1 = a_2 = a_3 = 1$. (Solve this in any way. You can go all the way to RREF, but in this simple example that isn't necessary.) The coordinate vector is

$$[\begin{bmatrix}1\\2\\3\end{bmatrix}]_{\mathcal{B}} = \begin{bmatrix}1\\1\\1\end{bmatrix}.$$

Solution 2. We notice that $\begin{bmatrix}1\\2\\3\end{bmatrix} = 1\begin{bmatrix}0\\0\\1\end{bmatrix} + 1\begin{bmatrix}0\\1\\1\end{bmatrix} + 1\begin{bmatrix}1\\1\\1\end{bmatrix}$. Thus, $[\begin{bmatrix}1\\2\\3\end{bmatrix}]_{\mathcal{B}} = \begin{bmatrix}1\\1\\1\end{bmatrix}$.

We know this is the only answer because a vector has only one coordinate vector, given the basis.

Remember: The question asks for a vector; you have to give a vector. A coordinate vector is always in $\mathbb{R}^n$; in this case $\mathbb{R}^3$.