QUIZ 6 Math 304-06 Print your name ____________________

Oct. 16, 2023

No consultation!—that includes no electronics.

(1) (3 points each) Read **very** carefully! In this problem, $\mathbb{R}^n$ is a vector space with two bases, $\mathcal{B}$ and $\mathcal{C}$. Circle your answers.

(a) True False $\underset{\mathcal{B}\leftarrow\mathcal{C}}{P}[\mathbf{v}]_{\mathcal{C}} = [\mathbf{v}]_{\mathcal{B}}$.

(b) True False $\underset{\mathcal{B}\leftarrow\mathcal{C}}{P}[\mathbf{v}]_{\mathcal{B}} = [\mathbf{v}]_{\mathcal{C}}$.

(c) True False $\underset{\mathcal{B}\leftarrow\mathcal{C}}{P}\ \underset{\mathcal{C}\leftarrow\mathcal{B}}{P} = I$ (identity matrix).

(d) True False $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}\ \underset{\mathcal{B}\leftarrow\mathcal{C}}{P} = I$ (identity matrix).

(2) (10 points) V is a 4-dimensional vector space and $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4\}$ is a basis for V. What is the coordinate vector $[2\mathbf{b}_1 - 8\mathbf{b}_2 + \mathbf{b}_4]_{\mathcal{B}}$?

(3) (10 points) V is a 3-dimensional vector space and $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ is a basis for V. A vector $\mathbf{v} \in V$ has coordinates $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Write $\mathbf{v}$ as a linear combination of the basis elements.

(4) (8 points) A square matrix B has rank $36 = 6^2$ and nullity $64 = 8^2$. How big is B?

(5) (3 points each) Is the statement about an $n \times n$ square matrix A equivalent to saying A is invertible? Circle your answers.

(a) Yes No The rows of A form a basis for $\mathbb{R}^n$.

(b) Yes No The columns of A form a basis for $\mathbb{R}^n$.

(c) Yes No A has rank n.

(d) Yes No A has rank 0.

(e) Yes No A has nullity n.

(f) Yes No A has nullity 0.