QUIZ 6 Math 304-06 Print your name <u>Solution Warlock</u>
Oct. 16, 2023

(1) (3 points each) Read **very** carefully! In this problem, $\mathbb{R}^n$ is a vector space with two bases, $\mathcal{B}$ and $\mathcal{C}$. Circle your answers.

(a) <u>True</u> False $\underset{\mathcal{B}\leftarrow\mathcal{C}}{P}[\mathbf{v}]_{\mathcal{C}} = [\mathbf{v}]_{\mathcal{B}}$.

(b) True <u>False</u> $\underset{\mathcal{B}\leftarrow\mathcal{C}}{P}[\mathbf{v}]_{\mathcal{B}} = [\mathbf{v}]_{\mathcal{C}}$.

(c) <u>True</u> False $\underset{\mathcal{B}\leftarrow\mathcal{C}}{P}\underset{\mathcal{C}\leftarrow\mathcal{B}}{P} = I$ (identity matrix).

(d) <u>True</u> False $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}\underset{\mathcal{B}\leftarrow\mathcal{C}}{P} = I$ (identity matrix).

(2) (6 points) V is a 4-dimensional vector space and $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4\}$ is a basis for V. What is the coordinate vector $[2\mathbf{b}_1 - 8\mathbf{b}_2 + \mathbf{b}_4]_{\mathcal{B}}$?

Answer: $\begin{bmatrix} 2 \\ -8 \\ 0 \\ 1 \end{bmatrix}$.

(3) (6 points) V is a 3-dimensional vector space and $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ is a basis for V. A vector $\mathbf{v} \in V$ has coordinates $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Write $\mathbf{v}$ as a linear combination of the basis elements.

Answer: $\mathbf{v} = a\mathbf{b}_1 + b\mathbf{b}_2 + c\mathbf{b}_3$.

(4) (8 points) A square matrix B has rank $36 = 6^2$ and nullity $64 = 8^2$. How big is B?

Answer: B has size 100×100, or if you prefer, $10^2 \times 10^2$. (With thanks to the Pythagorean Theorem).

(5) (3 points each) Which of these statements about an $n \times n$ square matrix A is equivalent to saying A is invertible? Circle your answers.

(a) <u>Yes</u> No The rows of A form a basis for $\mathbb{R}^n$.

(b) <u>Yes</u> No The columns of A form a basis for $\mathbb{R}^n$.

(c) <u>Yes</u> No A has rank n.

(d) Yes <u>No</u> A has rank 0.

(e) Yes <u>No</u> A has nullity n.

(f) <u>Yes</u> No A has nullity 0.