Math 304 2004/2/21 **TEST I** Name ____________________

- Show all your work for each problem; show enough work to fully justify your answer. **No credit without complete work!**
- Start each numbered problem on a *fresh page.*
- **Notation:** The vector $\mathbf{e}_i$ in $\mathbb{R}^n$ is all zero except for a 1 in the i-th row.
- **Notation:** If A is a matrix, then f_A is the associated function defined by $f_A(\mathbf{x}) = A\mathbf{x}$.

(1) [Points: 10+5+8+3] Let $A = \begin{bmatrix} 3 & 4 \\ 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 3 & 4 \end{bmatrix}$.

(a) Find A^{-1}.

(b) Use your work from part (a) to express A as a product of elementary matrices.

(c) Solve the matrix equation $AX = B$ by using A^{-1} from part (a). (You must use A^{-1}, not any other method.)

(d) Find B^{T}.

(2) [Points: 10+10] For each of the following linear systems, decide whether it is:

- inconsistent,
- consistent with a unique solution, or
- consistent with infinitely many solutions.

(Don't forget to give justification for your decisions.)

(a)

$$\begin{aligned} x_1 - 2x_2 - 4x_3 &= 3 \\ x_1 + 3x_2 + 6x_3 &= 6 \\ 4x_1 - 3x_2 - 6x_3 &= 15 \end{aligned}$$

(b)

$$\begin{aligned} -x_1 - x_2 - 2x_3 &= 4 \\ x_1 + 3x_2 + 5x_3 &= 3 \\ 4x_1 + 2x_2 + 5x_3 &= 2 \end{aligned}$$

(3) [Points: 5] Express the vector $\mathbf{u} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \in \mathbb{R}^2$ as a linear combination of $\mathbf{e}_1$ and $\mathbf{e}_2$.

(4) [Points: 4] Let M be a 6×3 matrix and f_M its associated function. What are the domain and codomain of f_M?

(5) [Points: 10] Suppose A and B are invertible $n \times n$ matrices. Prove that AB has an inverse matrix and it is $B^{-1}A^{-1}$.

(6) [Points: 5] Consider all matrices of rank 3. Which of them are invertible?

(7) [Points: 10] Suppose A is an invertible 5×5 matrix. Can you say anything about the rank? About whether the associated function f_A is one-to-one? onto?

(8) [Points: 5+5] Define an $n \times n$ matrix B by the rule: $b_{ij} = i + j$.

(a) Prove that B is symmetric.

(b) Find $B\mathbf{e}_2$.

(9) [Points: 10] Define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by the rule: T takes a point $\mathbf{x}$ to its reflection in the vertical axis (that's the x_2-axis). Find the matrix M that represents T.