MATH 304 - Linear Algebra
Exam 1

Problem 1: A certain company is investing 20 thousand dollars into four areas: research, manufacture, marketing, and support. Together, marketing and support should cost a quarter of the company's total investment. The company wants to invest 7 thousand more in manufacture and support than in marketing. And they know that manufacturing will cost 6 thousand more than marketing. How much will the company spend on research, manufacture, marketing, and support?

Problem 2:

a. Row reduce the matrix A into reduced echelon form.

$$A = \begin{pmatrix} 1 & 2 & 5 & 4 & 1 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 6 & 2 & 1 & 3 \\ 2 & 0 & 1 & 0 & 0 \end{pmatrix}$$

b. What is the rank of A? Why?

c. Is the linear transform $T(v) = Av$ (where v is a vector in $\mathbb{R}^5$) onto? Why?

d. Is it one-to-one? Why?

Problem 3: Calculate the following: A^TB, AB, AB^2, and $(AB)^2$.

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}$$

Problem 4: Calculate A^{-1} and write it as a product of elementary matrices.

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

Problem 5: Write the out the matrix representing the linear transform that rotates the plane by $\pi/3$ counterclockwise (60 degrees).

Problem 6: Let T be a function from $\mathbb{R}^4$ to $\mathbb{R}^4$ defined by $T(a, b, c, d) = (a + b, d + c, b, c + 2b)$. Verify that T is a linear transformation.

5pt Bonus Write down the definition of a vector space.