MATH 304 - Linear Algebra
Exam 2

Problem 1: Let $S = \{1+x+2x^2, 2+x+x^2, 1+2x+x^2, 1-x, 1-x^2\}$ be a set of quadratics, and $P = \{a_0 + a_1x + a_2x^2\}$. Prove that $span(S) = P$. Further find a basis for P contained in S.

Problem 2: Let $A = \begin{pmatrix} 1 & 3 & 1 & -1 & 2 \\ 0 & 1 & 2 & 0 & 1 \\ 1 & 2 & 2 & 2 & 4 \\ 2 & 5 & 3 & 1 & 6 \end{pmatrix}$.

a. Find a basis for $Col(A)$.

b. Find a basis for $Null(A)$.

Problem 3: Calculate the determinants of A, B, AB^{-1}, and $A+B$.

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \\ -1 & 0 & 1 \end{pmatrix}$$

Problem 4: Find the eigen values of T, and its corresponding eigen spaces. Is T diagonalizable? ($T(v) = Av$, where v is given in the standard basis.)

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \\ -3 & 1 & 2 \end{pmatrix}$$

Problem 5: Let T be a linear transformation given below, calculate T^k. ($T(v) = Av$, where v is given in the standard basis.)

$$A = \begin{pmatrix} -3 & 0 \\ -2 & -2 \end{pmatrix}$$

Problem 6: Prove the going down lemma: If S is a set of vectors such that $span(S) = V$ then S contains (as a subset) a basis for V.

Problem 7: Define and eigen value and an eigenvector. Use this to show that if λ is an eigenvalue of A, then λ^3 is an eigenvalue of A^3.

Problem 8: Prove that $det(A) = 0$ if and only if A is not invertible.