MATH 304 - Linear Algebra
Exam 3 (Each problem is worth 20 points.)

Problem 1: (Each part 5 points.)Compute $u \bullet v$ in each of the following cases:

a.) $u = (1, 2, 3)$, $v = (3, 2, 1)$

b.) $u = (1, 0, 1, 2)$, $v = (0, 1, 3, 1)$

c.) $u = (2, 3, 3, 1)$, $v = (1, 3, 1, 1)$

d.) $u = (-1, 3, -1, 0)$, $v = (2, 1, 1, 5)$

Problem 2: Let $W = \{(1, 3, 1, 3), (2, -1, -2, 1)\}$ and $v = (1, 2, 3, 4)$.

a.) (5 points.) Verify that W is an orthogonal set.

b.) (5 points.) Find the projection of v on $span(W)$.

c.) (10 points.)Find a basis for the orthogonal complement of W. This basis need not be orthogonal.

Problem 3: Use Gram-Shmidt to turn $\{(1, 1, 1), (2, 0, 1), (0, 1, 3)\}$ into an orthogonal basis. What is the corresponding orthonormal basis?

Problem 4: Find the least squares regression **affine** solution to the following experiment.

x_1	x_2	y
1	1	0
1	2	2
2	1	1
2	2	3

Problem 5: (Each part 10 points)Prove the following:

a.) Prove the Pythagorean theorem. (i.e. For vectors a and b, $||a||^2 + ||b||^2 = ||a + b||^2$ if and only if $a \bullet b = 0$.)

b.) Give the definition of an orthogonal matrix. Prove that an orthogonal matrix preserves the dot product. (i.e. If A is orthogonal, and u, v are vecors then $Au \bullet Av = u \bullet v$.)