Math 304-02 (Zaslavsky) **Problem Set VII** **Hand in Wed., 3/22/2006**

Part I.

(1) In this question, $M = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 7 & 5 \\ 1 & 2 & 6 & 5 \end{bmatrix}$.

(a) Find the rank of M.
(b) Find a basis for the null space, $\mathrm{Nul}(M)$.
(c) Find a basis for the column space, $\mathrm{Col}(M)$.
(d) Find a basis for the row space, $\mathrm{Row}(M)$.
(e) The columns of M form a set of 4 vectors. Are they linearly independent? If not, find a linear dependence among them.
(f) Which subsets of the columns are linearly independent? Do you see any patterns?

(2) In this question, $F : \mathbb{R}^4 \to \mathbb{R}^3$ is the function defined by

$$F(\mathbf{x}) = \begin{bmatrix} x_1 + 2x_2 + x_3 \\ 2x_1 + 4x_2 + 7x_3 + 5x_4 \\ x_1 + 2x_2 + 6x_3 + 5x_4 \end{bmatrix}.$$

(a) Find the matrix of F, i.e., the matrix N such that $F = f_N$.
(b) Find a basis for the kernel, $\mathrm{Ker}(F)$.
(c) Find a basis for the image, $\mathrm{Image}(F)$.

Part II.

(3) Suppose A is an invertible matrix. Prove that $f_A(\mathbf{x}) = \mathbf{0} \iff \mathbf{x} = \mathbf{0}$. (That means: $f_A(\mathbf{0}) = \mathbf{0}$ and, if $\mathbf{x} \neq \mathbf{0}$, then $f_A(\mathbf{x}) \neq \mathbf{0}$.) Don't use the next question to solve this question.

(4) Suppose A is an invertible $n \times n$ matrix. Prove that $f_{A^{-1}}(f_A(\mathbf{x})) = \mathbf{x}$ and $f_A(f_{A^{-1}}(\mathbf{x})) = \mathbf{x}$ for every $\mathbf{x} \in \mathbb{R}^n$.